Peer Assessment for Coursera Statistical Inference Class

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2015-04-26 - Part 1

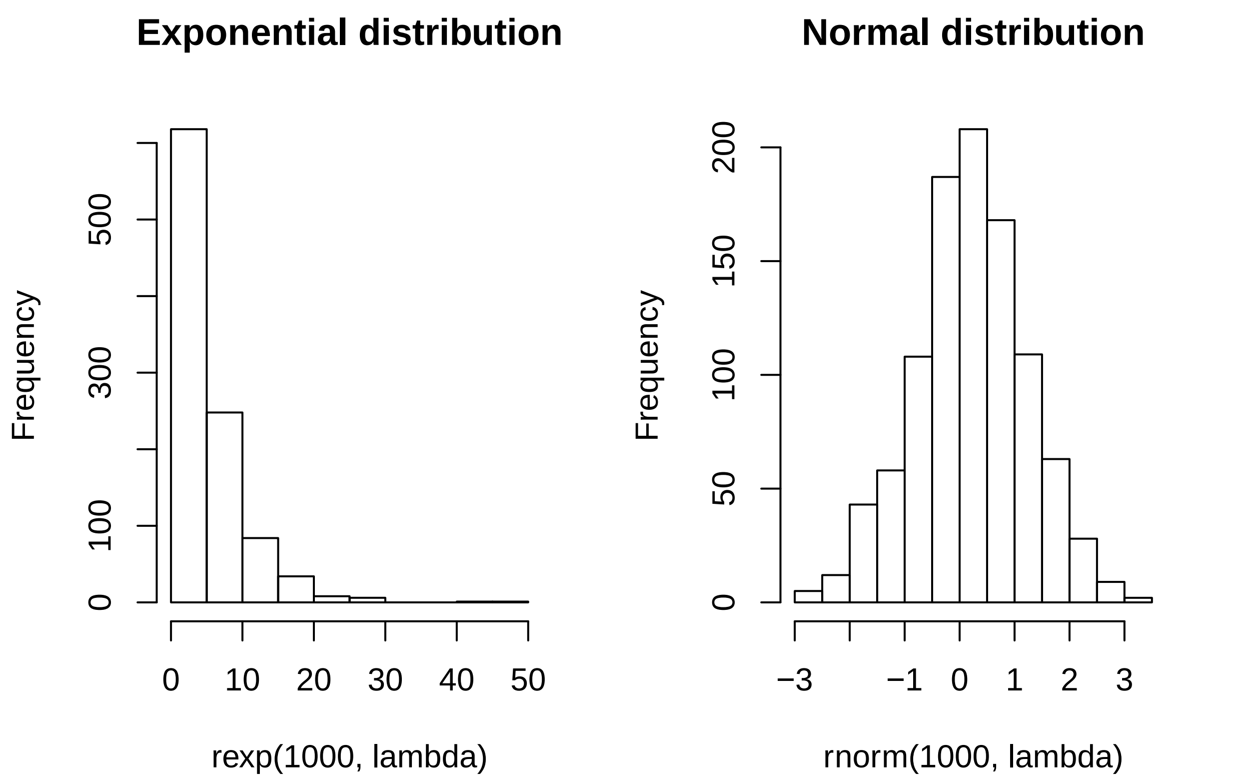
In this project we investigate the exponential distribution in the context of the central limit theorem (CLT) using R. The exponential distribution can be simulated in R with rexp(n, lambda) where lambda is the rate parameter and n is the number of random exponentials. The mean of this distribution is 1/lambda and the standard deviation is also 1/lambda.

We investigate the sampling distribution of the mean of random exponentials by running a large number of simulations. In this project, lambda is set to 0.2 and the sample size to 40 in all of the 1,000 simulations that follow.

```
lambda = 0.2 # rate parameter for the exponential distribution
n = 40 # sample size
b = 1000 # number of simulations
```

First, we look at the distribution of random exponentials (the exponsential distribution) using a histogram. It is most definitively not bell-shaped like the normal distribution.

```
par(mfrow=c(1,2))
hist(rexp(1000, lambda), main="Exponential distribution")
hist(rnorm(1000, lambda), main="Normal distribution")
```



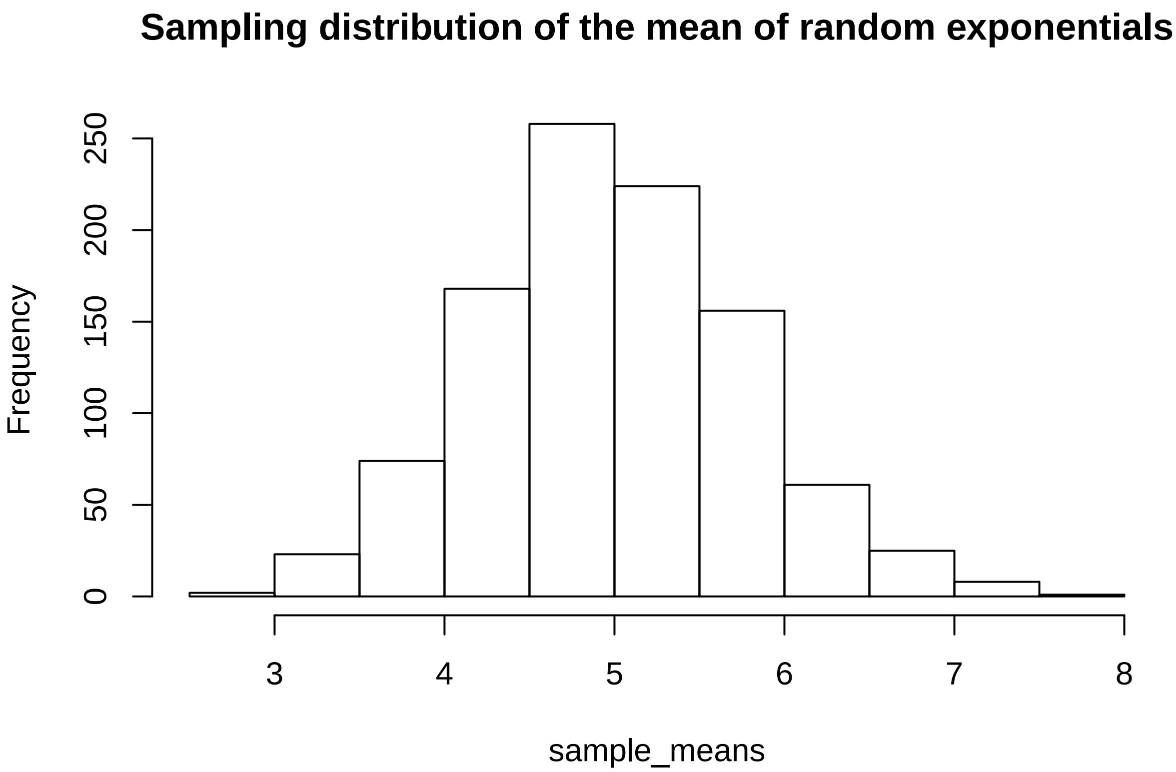
However, according to the CLT, the sampling distribution of the mean of random exponentials should be normal with mean 1/lambda and standard deviation 1/(lambda*sqrt(n)).

In order to verify this, we first collect the means of 1,000 samples of random exponentials, each of size 40.

```
sample_means = NULL
for (i in 1 : b) sample_means = c(sample_means, mean(rexp(n, lambda)))
```

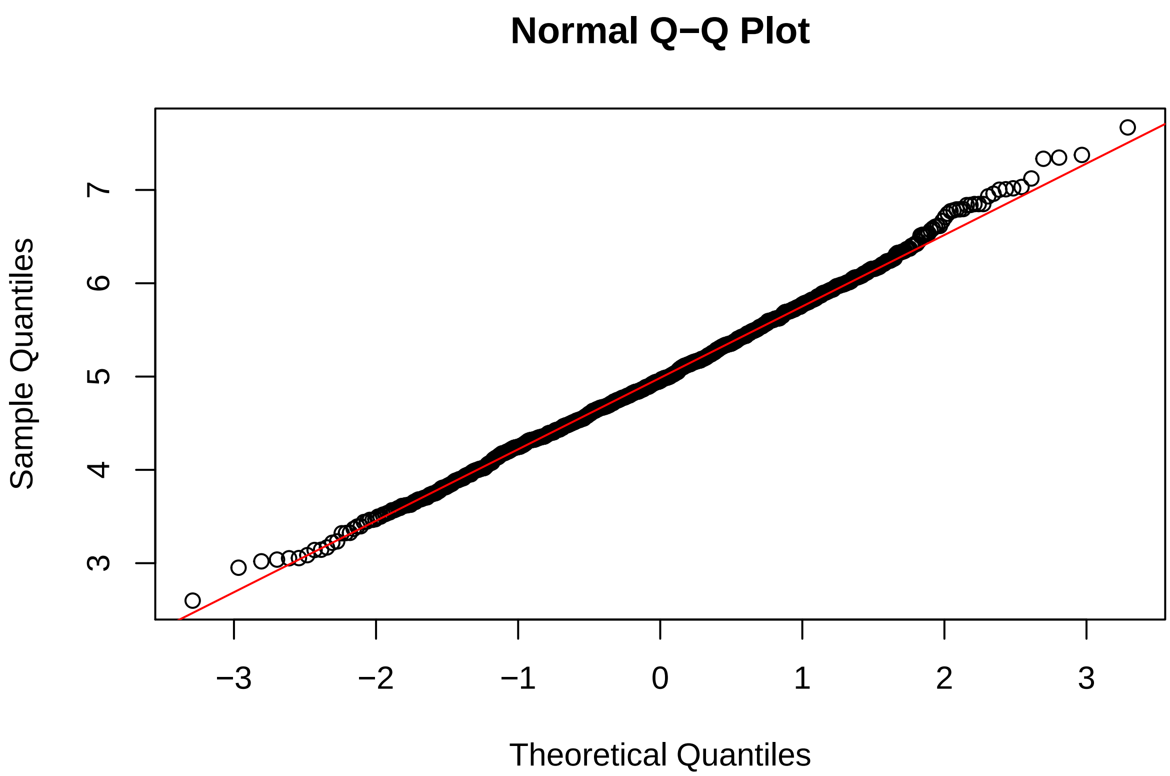
Second, we inspect the distribution of these sample means, which (in accordance to the CLT) is bell-shaped.

```
par(mfrow=c(1,1))
hist(sample_means, main = "Sampling distribution of the mean of random exponentials")
```



We then further verify that this distribution is approximately normal by inspecting the normal Q-Q plot.

```
qqnorm(sample_means)
qqline(sample_means, col=2)
```



As expected, the normal Q-Q plot reveals that the distribution of these sample means is approximately normal with mean:

```
(sample_means_mean = mean(sample_means))
```

```
## [1] 4.991449
```

and standard deviation:

```
(sample_means_sd = sd(sample_means))
```

```
## [1] 0.7791023
```

Therefore, in accordance with the CLT, the empirical mean of this distribution (sample_means_mean) is very similar to the theoretical mean 1/lambda:

```
(theoretical_mean = 1/lambda)
```

```
## [1] 5
```

and the empirical standard deviation of this distribution (sample_means_sd) is very similar to the theoretical standard deviation, i.e., the standard error of the mean 1/(lambda*sqrt(n))

```
(theoretical_sd = 1/(lambda*sqrt(n)))
```

```
## [1] 0.7905694
```