# Quantitative Social Science Methods, I, Lecture Notes: Missing Data

Gary King[1]
Institute for Quantitative Social Science
Harvard University

August 17, 2020

[1] GaryKing.org

Overview

Missingness Assumptions

Application Specific Methods

Multiple Imputation

Computational Algorithms

What Can Go Wrong

Time Series, Cross-Sectional Imputations

# Summary

- Biased or inefficient missing data practices
  - Make up numbers: e.g., change Party ID "don't knows" to "independent"
  - Listwise deletion: used by 94% pre-2000 in AJPS/APSR/BJPS
  - Many other ad hoc approaches: some intuitive, most biased
- Application-specific methods: efficient, but model-dependent, hard to develop and use
- Multiple imputation: An easy-to-use, statistically appropriate alternative
  - fill in $\approx 5$ data sets with different imputations for missing values
  - Convenience: analyze each one as you would without missingness
  - Use a special method to combine the results
  - Robust: separate missingness and analysis models

# Readings

- Gary King; James Honaker; Anne ~~Joseph~~ O'Connell; Kenneth Scheve. "Analyzing Incomplete Political Science Data: An Alternative Algorithm for Multiple Imputation" APSR, 2001.
- James Honaker and Gary King. "What to do about Missing Values in Time Series Cross-Section Data" AJPS, 2010.
- Blackwell, Matthew, James Honaker, and Gary King. "A Unified Approach to Measurement Error and Missing Data: {Overview, Details and Extensions}" SMR, 2017.
- Amelia II: A Program for Missing Data

j.mp/MisMeas

# Notation

$$D = \begin{pmatrix} 1 & 2.5 & 432 & 0 \\ 5 & 3.2 & 543 & 1 \\ 2 & 7.4 & 219 & 1 \\ 6 & 1.9 & 234 & 1 \\ 3 & 1.2 & 108 & 0 \\ 0 & 7.7 & 95 & 1 \end{pmatrix}, \qquad M = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix}$$

$D_{mis}$ = missing elements in $D$
$D_{obs}$ = observed elements in $D$

- If you treat some elements as missing: make sure they exist (what's your view on the National Helium Reserve?)
- Quiz: If some values don't exist, what analysis would you do?

# Missingness Assumptions

Please excuse the literature's crazy nomenclature

| Assumption | Acronym | Can predict $M$ with: |
|---|---|---|
| Missing Completely At Random | MCAR | — |
| Missing At Random | MAR | $D_{obs}$ |
| Nonignorable | NI | $D_{obs}$ & $D_{mis}$ |

# Missingness Assumptions, again

1. MCAR (naive): Coin flips determine answering survey questions

$$P(M|D) = P(M)$$

2. MAR (empirical): missingness is a function of measured variables

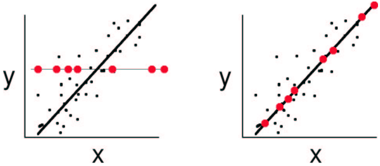
$$P(M|D) \equiv P(M|D_{obs}, D_{mis}) = P(M|D_{obs})$$

   - e.g., Independents are less likely to answer vote choice question (with PID measured)
   - e.g., Some occupations are less likely to answer the income question (with occupation measured)

3. NI (fatalistic): missingness depends on unobservables
   - $P(M|D)$ doesn't simplify
   - e.g., censoring income if income is $> \$100K$ <u>and</u> you can't predict high income with other measured variables
   - Adding variables to predict income can change NI to MAR

# Existing General Purpose Missing Data Methods

- Listwise deletion. MCAR: RMSE is 1 SE off; MAR: biased
- Best guess imputation: Depends on guesser!
- Impute 0, add indicator to control: biased
- Pairwise deletion: biased unless MCAR
- Hot deck imputation: Inefficient, SEs wrong
- Mean substitution: attenuation bias




- $y$-hat Regression Imputation: Optimistic (scatter when observed, perfectly linear when unobserved?); SEs too small
- $y$-hat + $\epsilon$ Regression Imputation: Ignores estimation uncertainty; impossible with scattered missingness; getting there

# Application-specific Methods: Overview

- Examples: Models of censoring, truncation, etc.
- Likelihood Inference: Unknowns given knowns, $P(\theta|Y_{obs})$.
- Optimal, if model is correct
- Can be model dependent: sensitive to distributional assumptions
- Often difficult practically: Unless code already exists
- Rare with scattered missingness in $X$ and $Y$
- Important to understand: even if you use another approach

# Creating Application-Specific Methods

- Suppose: $D = \{D_{obs}, D_{mis}\}$ is observed; ($M$ always is)
- Likelihood: $\mathrm{P}(D, M|\theta, \gamma) = \mathrm{P}(D|\theta)\mathrm{P}(M|D, \gamma)$
- Quiz: Can we drop $P(M|D, \gamma)$? (MAR: Stoch.& Param. Indep.)
- Suppose now, as usual: $D$ is observed only when $M$ is 1
- Likelihood: integrate out the missing observations

$$
\begin{aligned}
P(D_{obs}, M|\theta, \gamma) &= \int \mathrm{P}(D|\theta)\mathrm{P}(M|D, \gamma) dD_{mis} \\
&= P(D_{obs}|\theta)P(M|D_{obs}, \gamma), \\
&\propto P(D_{obs}|\theta)
\end{aligned}
$$

- With MAR: $P(D_{obs}|\theta)$ specialized; no help with $X$ missingness
- Example: Censored Exponential Model (you've seen)
- Without MAR: missingness model is NI, can't be dropped
- Little known about $M$: Specifying $P(M|D_{obs}, \gamma)$ hard
- NI models: (Heckman, many others) often don't do well when can be evaluated

# Multiple Imputation

## Point estimates: consistent, efficient; SEs are right (or conservative)

- Impute $m$ values for each missing element
  - $m$ is bigger with more uncertainty and more missingness
  - Imputation method (we'll describe later) assumes MAR
  - Model must include all (estimation, fundamental) uncertainty
  - Produces independent imputations
- Create $m$ completed data sets
  - Observed data are the same across the data sets
  - Imputations of missing data differ
    - Cells we predict well don't differ much
    - Cells we can't predict well differ a lot
- Run whatever statistical method you would have with no missing data for each completed data set

# Combining Analyses from Separate Imputed Datasets

- By averaging (simple, easy to understand)
  - Point estimate: average individual point estimates, $q_j$ ($j = 1, \dots, m$)

$$\bar{q} = \frac{1}{m} \sum_{j=1}^{m} q_j$$

  - Standard error:

$$\begin{aligned} \mathrm{SE}(q)^2 &= \mathrm{mean}(\mathrm{SE}_j^2) + \mathrm{variance}(q_j)\,(1 + 1/m) \\ &= \text{within} + \text{between} \end{aligned}$$

$(1 + 1/m)$ vanishes as $m$ increases

- By simulation (simpler, fits with our procecures)
  - draw $1/m$ of needed sims of the QOI from each data set
  - combine (i.e., concatenate into a larger set of QOI simulations)
  - make inferences from combined sims as usual

# An Imputation Model

- Assume $D$ is complete and: $\underset{N\times 1}{D_i} \sim N(D_i|\mu, \Sigma)$ (SURM w/o $X$)
- Seems crazy, but will imply regression of each var on all others
- Likelihood: $L(\mu, \Sigma|D) \propto \prod_{i=1}^{n} N(D_i|\mu, \Sigma)$
- With $D$ partially missing

$$\begin{aligned} L(\mu, \Sigma|D_{obs}) &\propto \prod_{i=1}^{n} \int N(D_i|\mu, \Sigma) dD_{mis} \\ &= \prod_{i=1}^{n} N(D_{i,obs}|\mu_{obs}, \Sigma_{obs}) \end{aligned}$$

  since marginals of MVN's are normal.
- Simple theoretically: merely a likelihood model for data $(D_{obs}, M)$ and same parameters as when fully observed $(\mu, \Sigma)$
- Complicated computationally: $D_{i,obs}$ has different elements observed for each $i$; each observation is informative about different pieces of $(\mu, \Sigma)$

# An Imputation Model (continued)

- Hard Statistically: params increase fast in number of vars ($p$)

$$\begin{aligned}\text{parameters} &= \text{parameters}(\mu) + \text{parameters}(\Sigma)\\ &= p + p(p+1)/2 = p(p+3)/2.\end{aligned}$$

  E.g., for $p = 5$, parameters= 20; for $p = 40$ parameters= 860 (with what sample size?)
- Specialized Models, such as for categorical or mixed variables, are harder to apply and do not usually perform better
- For social science survey data (mostly ordinal scales): this is a reasonable choice for imputation, even if not for analysis

# Creating Imputations From This Model

- Suppose $D = \{Y, X\}$ with 2 vars; missingness only in $Y$
- $\rightsquigarrow$ $D$ is bivariate normal

$$D \sim N(D|\mu, \Sigma) = N\left[\begin{pmatrix} Y \\ X \end{pmatrix} \middle| \begin{pmatrix} \mu_y \\ \mu_x \end{pmatrix}, \begin{pmatrix} \sigma_y & \sigma_{xy} \\ \sigma_{xy} & \sigma_x \end{pmatrix}\right]$$

- Conditionals of bivariate normals are normal

$$Y|X \sim N\left[y|E(Y|X), V(Y|X)\right]$$

- CEF is regression of $Y$ on all $X$'s (recall: SURM has no $X$)

$$E(Y|X) = \mu_y + \beta(X - \mu_x)$$

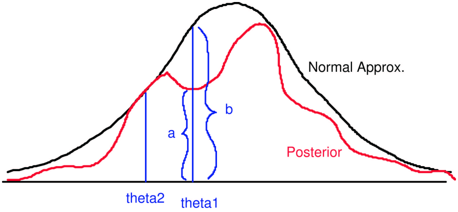
- Other details: $\beta = \sigma_{xy}/\sigma_x, \quad V(Y|X) = \sigma_y - \sigma_{xy}^2/\sigma_x$
- To simulate:
  - Maximize likelihood; draw sims of $\tilde{\mu}$ and $\tilde{\Sigma}$
  - Calculate $\tilde{\beta}$, $\tilde{\sigma}$s; then sims of $E(Y|X)$ and $V(Y|X)$
  - Draw a simulation of the missing $Y$ from the conditional normal
  - Equivalent to drawing from $\tilde{y}_i = X\tilde{\beta} + \tilde{\epsilon}_i$ (estimation, fundamental uncertainty)

# Computational Algorithms: Optim, EM, EMs (p.1 of 3)

- Optim with hundreds of parameters would work very slowly
- EM (expectation maximization): fast algorithm for maximum
  - Much faster than optim
  - Intuition:
    - Without missingness, estimating $\beta$ is easy: run LS
    - If $\beta$ is known, imputation is easy: draw $\tilde{\epsilon}$, use $\tilde{y} = x\beta + \tilde{\epsilon}$
  - EM works by iterating between
    - Impute $\hat{Y}$ with $x\hat{\beta}$, given current estimates, $\hat{\beta}$
    - Estimate $\beta$ (by LS) with current imputations of $Y$
  - Can easily do imputation via $x\hat{\beta} + \tilde{\epsilon}$
  - Problem: SEs too small due to no estimation uncertainty ($\hat{\beta} \neq \beta$); i.e., we need to draw $\beta$ from its posterior first
- EMs: EM for param maximization and then simulation
  - Add estimation uncertainty by drawing $\tilde{\beta} \sim N(\hat{\beta}, \hat{V}(\hat{\beta}))$
  - The central limit theorem guarantees that this works as $n \longrightarrow \infty$, but for real sample sizes it may be inadequate

# Computational Algorithms: EMis, EMB (p.2 of 3)

- EMis: EM with simulation via importance resampling (probabilistic rejection sampling to draw from the posterior)



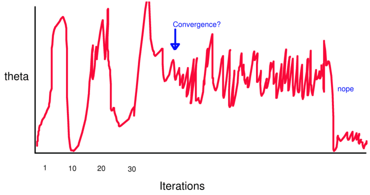

Keep $\tilde{\theta}_1$ with probability $\propto a/b$ (the importance ratio).
Keep $\tilde{\theta}_2$ with probability 1.

- EMB: EM With Bootstrap
  - Draw $m$ sets of $n$ obs (with replacement) from the data
  - Use EM to estimate $\beta$ and $\Sigma$ in each (for estimation uncertainty)
  - Impute $D_{mis}$ from the model (for fundamental uncertainty)
  - Lightning fast; works with very large data sets
  - Basis for Amelia II

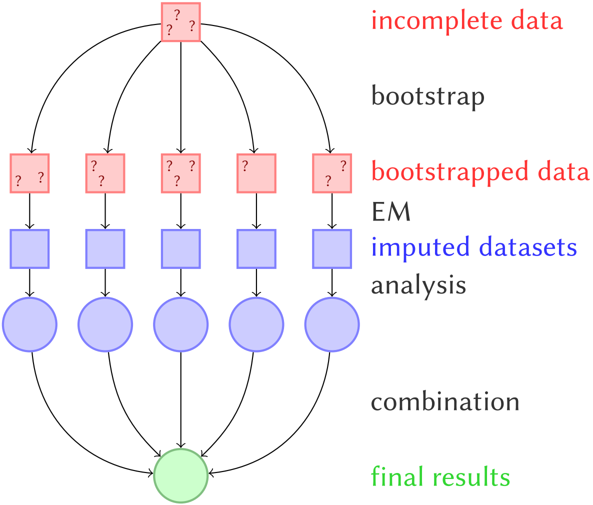
# Computational Algorithms: MCMC, IP (p.3 of 3)

- The IP algorithm: Repeat until convergence ($\approx$ stochastic EM)
  - I-Step: draw $D_{mis}$ from $P(D_{mis}|D_{obs}, \tilde{\theta})$ (i.e., $\tilde{y} = x\tilde{\beta} + \tilde{\epsilon}$)
  - P-Step: draw $\theta$ from $P(\theta|D_{obs}, \tilde{D}_{mis})$
- IP is an example of MCMC (Markov Chain Monte Carlo)
  - 1 of 1990+'s most important developments in stats
  - enabled statisticians to do things they never dreamed possible
  - requires considerable expertise; doesn't help others as much
  - Few MCMC routines are in canned packages
- Hard to know when MCMC algorithms converge

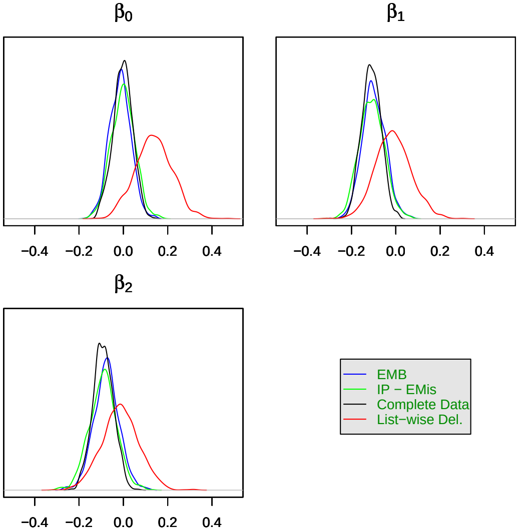
- Convergence is asymptotic
- Plot traces hard to interpret
- Worst-converging parameter controls the system



# Multiple Imputation: Amelia Style



# Comparisons of Posterior Density Approximations




EMB: similar results, much faster, far easier to program

# What Can Go Wrong and What to Do

- Risk: Inference is learning about facts we don't have with facts we have; we assume the 2 are related!
- Robustness: Imputation and analysis separated $\rightsquigarrow$ imputation affects only missing observations; High missingness reduces the property
- Vars to include: At least as much information in the imputation model as in the analysis model: all vars in analysis model; others that would help predict (e.g., All measures of a variable, post-treatment variables)
- Transform: Fit imputation model distributional assumptions by transformation to unbounded scales: $\sqrt{\text{counts}}$, $\ln(p/(1-p))$, ln(money), etc.
- Code ordinal variables: as close to interval as possible

# What Can Go Wrong and What to Do (continued)

- Strongly nonlinear relationships: use transformations or added quadratic terms
- "Uncongenial" imputations: If imputation model has as much info as analysis model, but the specification (such as the functional form) differs, CIs are conservative (e.g., ≥ 95% CIs)
- Super-efficiency: When imputation model includes more information than analysis model, it can be more efficient than the "optimal" application-specific model
- Quiz: If $X$ is randomly imputed is there attenuation (the usual consequence of random measurement error in an explanatory variable)?
- Quiz: If $X$ is imputed with information from $Y$, is there endogeneity?
- Answer to both: draws from the joint posterior are put back into the data; not changing the joint distribution

# The Best Case for Listwise Deletion

Listwise deletion is better than MI when all 4 hold:

- The analysis model is conditional on $X$ (like regression) and functional form is correct (so listwise deletion is consistent and the characteristic robustness of regression is not lost when applied to data with slight measurement error, endogeneity, nonlinearity, etc.)
- NI missingness in $X$ and no external variables are available that could be used in an imputation stage to fix the problem
- Missingness in $X$ is not a function of $Y$
- $n$ left after listwise deletion is large so that the efficiency loss does not counter balance biases induced by the other conditions

I.e., you don't trust data to impute $D_{mis}$ but trust it to analyze $D_{obs}$

# Example: Support for Perot, Data Inputs

- Research question: were voters who didn't share in economic recovery more likely to support Perot in the 1996 election?
- Data: 1996 National Election Survey (n=1714)
- Dependent variable: Perot Feeling Thermometer
- Key explanatory variables: retrospective and propsective evaluations of national economic performance and personal financial circumstances
- Control variables: age, education, family income, race, gender, union membership, ideology
- Extra variables for imputation model to help prediction: attention to the campaign; feeling thermometers for Clinton, Dole, Democrats, Republicans; PID; Partisan moderation; vote intention; martial status; Hispanic; party contact, number of organizations R is a paying member of, and active member of.
- Nonlinear terms: $\text{age}^2$
- Transform: to more closely approximate distributional assumptions: logged number of organizations participating in

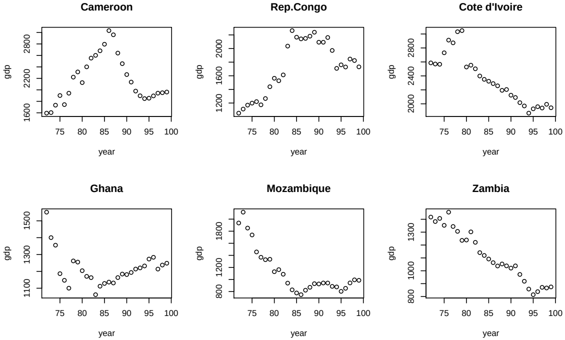
# Example: Support for Perot, Results

- Analysis model: linear regression
- Amelia for Imputation: generate 5 imputed data sets
- Coefficient on retrospective economic evaluations (range: 1–5):

| | |
|---|---|
| Listwise deletion | .43 |
| | (.90) |
| Multiple imputation | 1.65 |
| | (.72) |

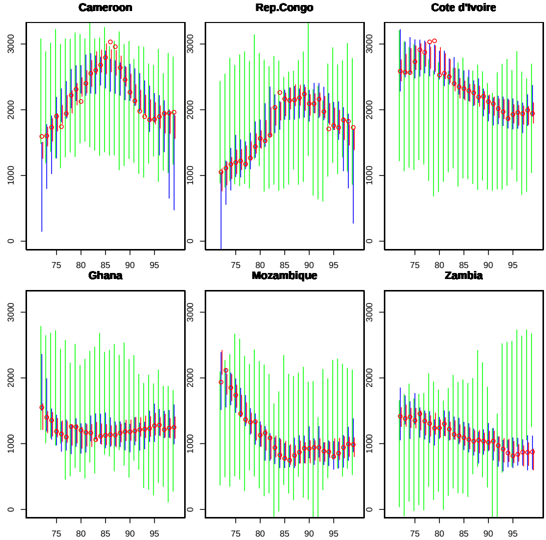
- Effect: $(5-1) \times 1.65 = 6.6$, also % of the range of $Y$
- MI estimator is more efficient, with a smaller SE
- MI estimator is 4 times larger
- Based on listwise deletion: no evidence that perception of poor economic performance is related to support for Perot
- Based on MI estimator: R's with negative retrospective economic evaluations more likely to have favorable views of Perot

# MI in Time Series Cross-Section Data



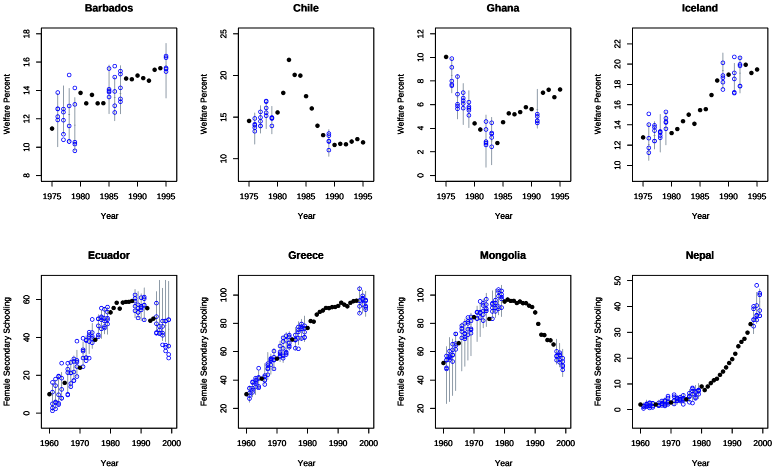

Include: (1) fixed effects, (2) time trends, and (3) priors for cells

# Imputation one Observation at a time




Circles=true GDP; green=no time trends; blue=polynomials; red=LOESS

# Replication of Baum and Lake: Imputation Model Fit




Black = observed. Blue circles = five imputations; Bars = 95% CIs