

# Quantitative Social Science Methods, I, Lecture Notes: Detecting and Reducing Model Dependence in Causal Inference

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<sup>1</sup>[GaryKing.org](http://GaryKing.org)

## Detecting Model Dependence

Matching to Reduce Model Dependence

Three Matching Methods

Problems with Propensity Score Matching

The Matching Frontier

# Readings in Model Dependence

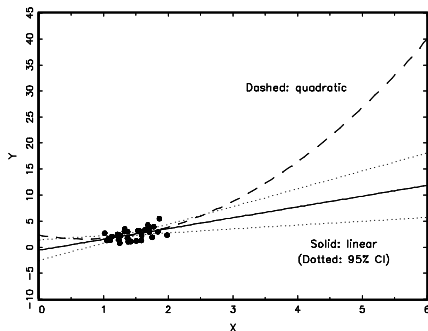
- King, Gary and Langche Zeng. “[The Dangers of Extreme Counterfactuals](#),” *Political Analysis*, 14, 2, (2007): 131-159.
- King, Gary and Langche Zeng. “[When Can History be Our Guide? The Pitfalls of Counterfactual Inference](#),” *International Studies Quarterly*, 2006, 51 (March, 2007): 183–210.
- Related Software: [WhatIf](#), [MatchIt](#), [Zelig](#), [CEM](#)

[j.mp/causalinference](http://j.mp/causalinference)

# Counterfactuals

- Three types:
  1. **Forecasts** What will the mortality rate be in 2025?
  2. **Whatif Questions** What would have happened if the U.S. had not invaded Iraq?
  3. **Causal Effects** What is the causal effect of the Iraq war on World GDP? (a factual minus a counterfactual)
- Counterfactuals are part of most social science research

# Which model would you choose? (Both fit the data well.)



- Compare prediction at  $x = 1.5$  to prediction at  $x = 5$
- How do you choose a model?  $R^2$ ? Some “test”? “Theory”?
- The bottom line: answers to some questions don’t exist in the data. We show how to determine which ones.
- Same for what if questions, predictions, and causal inferences

# Model Dependence Proof

## Model Free Inference

To estimate  $E(Y|X = x)$  at  $x$ , average many observed  $Y$  with value  $x$

## Assumptions (Model-Based Inference)

1. Definition: model dependence at  $x$  is the difference between predicted outcomes for any two models that fit about equally well.
2. The functional form follows strong continuity (think smoothness, although it is less restrictive)

## Result

The maximum degree of model dependence: a function of the distance from the counterfactual to the data

# A Simple Measure of Distance from The Data

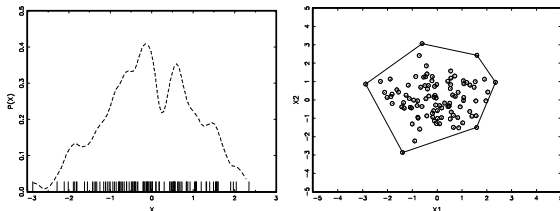


Figure: The Convex Hull

- **Interpolation:** Inside the convex hull
- **Extrapolation:** Outside the convex hull
- Works mathematically for any number of  $X$  variables
- Software to determine whether a point is in the hull (which is all we need) without calculating the hull (which would take forever), so its fast; see [GaryKing.org/whatif](http://GaryKing.org/whatif)

# Model Dependence Example

Replication of Doyle and Sambanis, APSR 2000

(From: King and Zeng, 2007)

- **Data:** 124 Post-World War II civil wars
- **Dependent var:** peacebuilding success
- **Treatment:** multilateral UN peacekeeping intervention (0/1)
- **Control vars:** war type, severity, duration; development status,...
- **Counterfactual question:** Switch UN intervention for each war
- **Data analysis:** Logit model
- **The question:** How *model dependent* are the results?
- **Percent of counterfactuals in the convex hull:** 0%
- $\leadsto$  without estimating any models, we know: inferences will be model dependent
- **For illustration:** let's find an example....

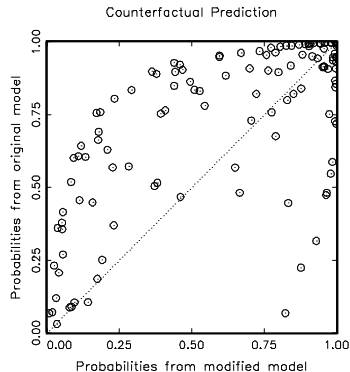
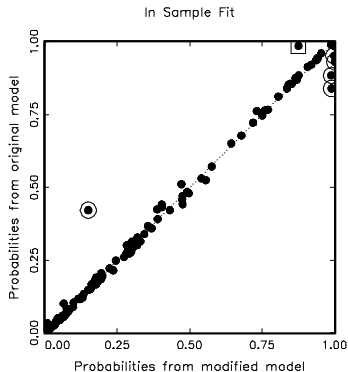


# Two Logit Models, Apparently Similar Results

## Effect of Multilateral UN Intervention on Peacebuilding Success

Variables	Original “Interactive” Model			Modified Model		
	Coeff	SE	P-val	Coeff	SE	P-val
Wartype	-1.742	.609	.004	-1.666	.606	.006
Logdead	-.445	.126	.000	-.437	.125	.000
Wardur	.006	.006	.258	.006	.006	.342
Factnum	-1.259	.703	.073	-1.045	.899	.245
Factnum2	.062	.065	.346	.032	.104	.756
Trnsfcap	.004	.002	.010	.004	.002	.017
Develop	.001	.000	.065	.001	.000	.068
Exp	-6.016	3.071	.050	-6.215	3.065	.043
Decade	-.299	.169	.077	-0.284	.169	.093
Treaty	2.124	.821	.010	2.126	.802	.008
UNOP4	3.135	1.091	.004	.262	1.392	.851
Wardur*UNOP4	—	—	—	.037	.011	.001
Constant	8.609	2.157	0.000	7.978	2.350	.000
N	122			122		
Log-likelihood	-45.649			-44.902		
Pseudo $R^2$	.423			.433		

# Model Dependence: Same Fit, Different Predictions



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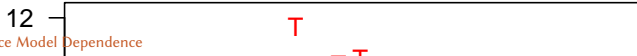
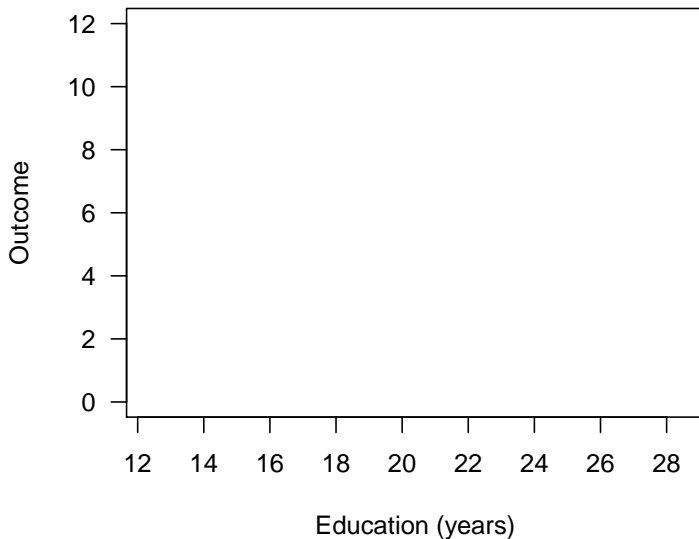
The Matching Frontier

# Readings, Matching

- Do powerful methods have to be complicated?
  - “Causal Inference Without Balance Checking: Coarsened Exact Matching” (PA, 2011. Stefano Iacus, Gary King, and Giuseppe Porro)
- The most popular method (propensity score matching, used in 140,000 articles!) sounds magical:
  - “Why Propensity Scores Should Not Be Used for Matching” (Gary King, Richard Nielsen) (PA, 2019; Gary King and Richard Nielsen)
- Matching methods optimize either imbalance ( $\approx$  bias) or # units pruned ( $\approx$  variance); users need both simultaneously:
  - “The Balance-Sample Size Frontier in Matching Methods for Causal Inference” (AJPS, 2017; Gary King, Christopher Lucas and Richard Nielsen)
- Current practice, matching as preprocessing: violates current statistical theory. So let's change the theory:
  - “A Theory of Statistical Inference for Matching Methods in Causal Research” (Stefano Iacus, Gary King, Giuseppe Porro)

# Matching to Reduce Model Dependence

(Ho, Imai, King, Stuart, 2007: fig.1, *Political Analysis*)



# The Problems Matching Solves

Without Matching: ~~Without Matching:~~

Imbalance ~~Imbalance~~  $\leadsto$  Model Dependence  $\leadsto$  ~~Model Dependence~~  
 $\leadsto$  Researcher discretion  $\leadsto$  ~~Researcher discretion~~  $\leadsto$  Bias  $\leadsto$  ~~Bias~~

A central project of statistics: Automating away human discretion

- Qualitative choice from unbiased estimates = biased estimator
  - e.g., Choosing from *results* of 50 randomized experiments
  - Choosing based on “plausibility” is probably worse<sub>[eff]</sub>
- conscientious effort doesn’t avoid biases (Banaji 2013)<sub>[acc]</sub>
- People do not have easy access to their own mental processes or feedback to avoid the problem (Wilson and Brekke 1994)<sub>[expt]</sub>
- Experts overestimate their ability to control personal biases more than nonexperts, and more prominent experts are the

# What's Matching?

- **Notation:**  $Y_i$  dep var,  $T_i$  (1=treated, 0=control),  $X_i$  confounders
- Treatment Effect for treated observation  $i$ :

$$\begin{aligned} TE_i &= Y_i(1) - Y_i(0) \\ &= \text{observed} - \text{unobserved} \end{aligned}$$

- Estimate  $Y_i(0)$  with  $Y_j$  with a matched ( $X_i \approx X_j$ ) control
- **Quantities of Interest**
  1. SATT: Sample Average Treatment effect on the Treated:

$$SATT = \text{Mean}_{i \in \{T_i=1\}} (TE_i)$$

2. FSATT: Feasible SATT (prune badly matched treateds too)
- **Big convenience:** Follow preprocessing with whatever statistical method you'd have used without matching
  - **Pruning nonmatches makes control vars matter less:** reduces imbalance, model dependence, researcher discretion, & bias

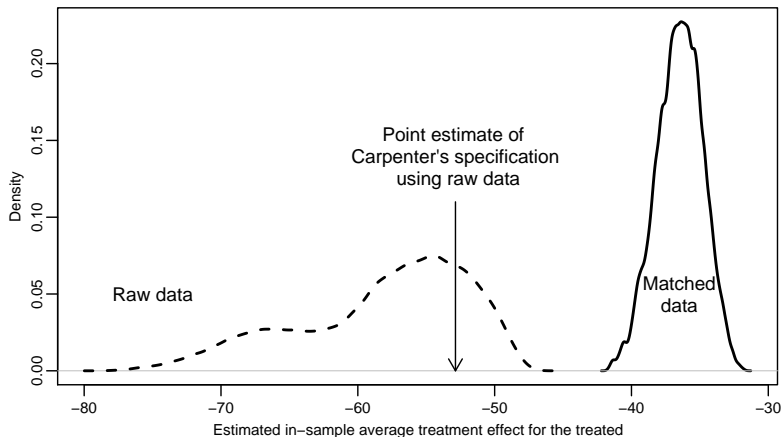
# Evaluating Reduction in Model Dependence

Empirical Illustration: Carpenter, AJPS, 2002

- **Hypothesis:** Democratic senate majorities slow FDA drug approval time
- **Data:**  $n = 408$  new drugs (262 approved, 146 pending)
- **Measured confounders:** 18 (clinical factors, firm characteristics, media variables, etc.)
- **Model:** lognormal survival
- **QOI:** Causal effect of Democratic Senate majority (identified by Carpenter as not robust)
- **Match:** prune 49 units (2 treated, 17 control units)
- **Run:** 262,143 possible specifications; calculate SATT for each
- **Evaluate:** *Variability* in SATT across specifications
- (Normally we'd only use one or a few specifications)

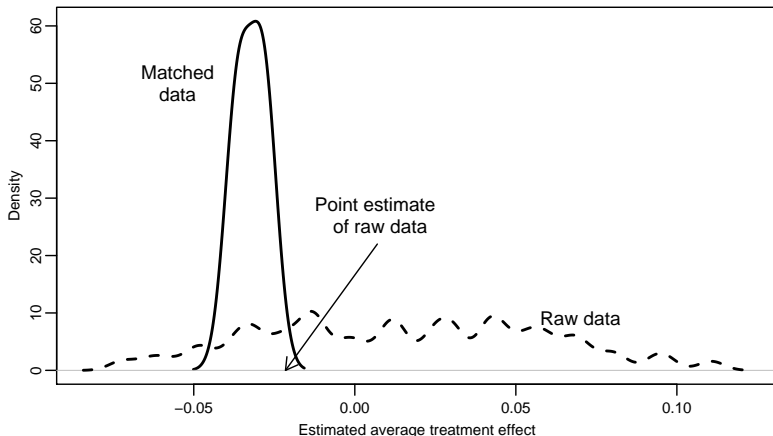


# Reducing Model Dependence



SATT Histogram: Effect of Democratic Senate majority on FDA drug approval time, across 262, 143 specifications

## Another Example: Jeffrey Koch, AJPS, 2002



SATT Histogram: Effect of being a highly visible female  
Republican candidate across 63 possible specifications with the  
Koch data

# Assumptions to Justify Current Practice

## Existing Theory of Inference: Stop What You're Doing!

- Framework: **simple random sampling** from a population
- Exact matching: Rarely possible; but would make estimation easy
- Assumptions:
  - *Unconfoundedness*:  $T \perp Y(0) \mid X$  (Healthy & unhealthy get meds)
  - *Common support*:  $\Pr(T = 1 \mid X) < 1$  ( $T = 0, 1$  are both possible)
- Approximate matching (bias correction, new variance estimation): common, but all current practices would have to change

## Alternative Theory of Inference: It's Gonna be OK!

- Framework: **stratified random sampling** from a population
- Define  $A$ : a stratum in a partition of the product space of  $X$  ("continuous" variables have natural breakpoints)
- We already know and use these procedures: Group strong and weak partisans; Don't match college dropout with 1st year grad student
- Assumptions:
  - *Set-wide Unconfoundedness*:  $T \perp Y(0) \mid A$
  - *Set-wide Common support*:  $\Pr(T = 1 \mid A) < 1$
- Fits all common matching methods & practices; no asymptotics
- Easy extensions for: multi-level, continuous, & mismeasured treatments;  $A$  too wide,  $n$  too small

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# Matching: Finding Hidden Randomized Experiments

## Types of Experiments

Balance	<i>Complete</i>	<i>Fully</i>
Covariates:	<i>Randomization</i>	<i>Blocked</i>
<i>Observed</i>	On average	Exact
<i>Unobserved</i>	On average	On average

→ *Fully blocked* dominates *complete randomization* for: imbalance, model dependence, power, efficiency, bias, research costs, robustness. E.g., Imai, King, Nall 2009: SEs 600% smaller!

## Goal of Each Matching Method (in Observational Data)

- PSM: *complete randomization*
- Other methods: *fully blocked*
- Other matching methods dominate PSM (wait, it gets worse)

# Method 1: Mahalanobis Distance Matching

(Approximates Fully Blocked Experiment)

## Procedure

### 1. Preprocess (Matching)

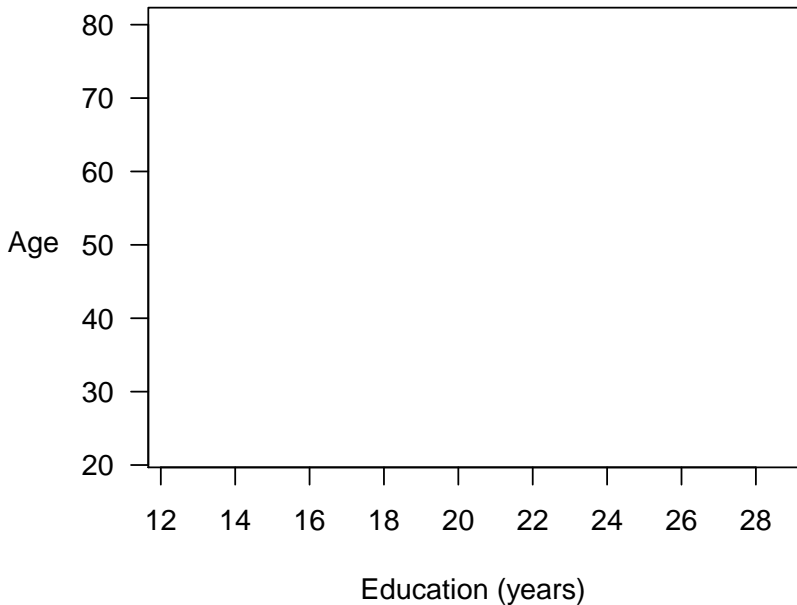
- $\text{Distance}(X_c, X_t) = \sqrt{(X_c - X_t)' S^{-1} (X_c - X_t)}$
- Match each treated unit to the nearest control unit
- Control units: not reused; pruned if unused
- Prune matches if  $\text{Distance} > \text{caliper}$
- (Many adjustments available to this basic method)

### 2. Estimation Difference in means or a model

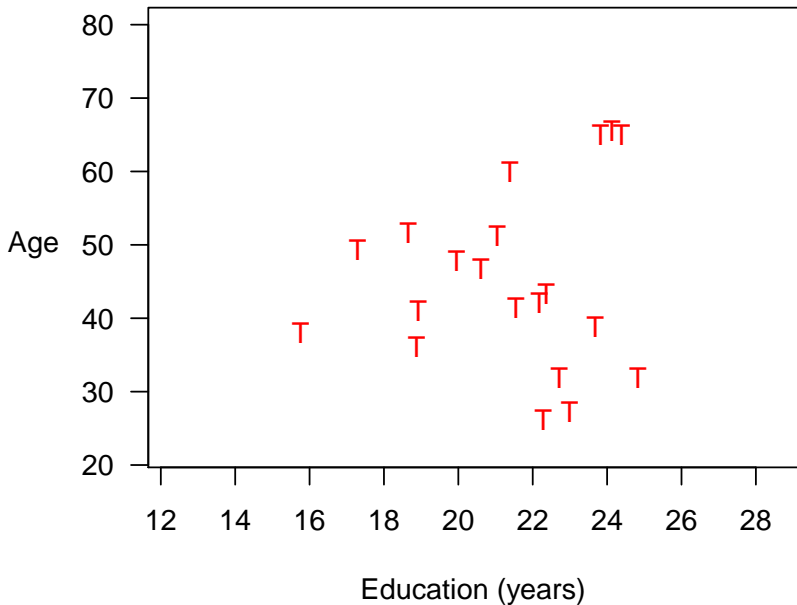
## Interpretation

- Quiz: Do you understand the distance trade offs?
- Quiz: Does **standardization** help?
- ~> Mahalanobis is for methodologists; in applications, use Euclidean!

## Mahalanobis Distance Matching

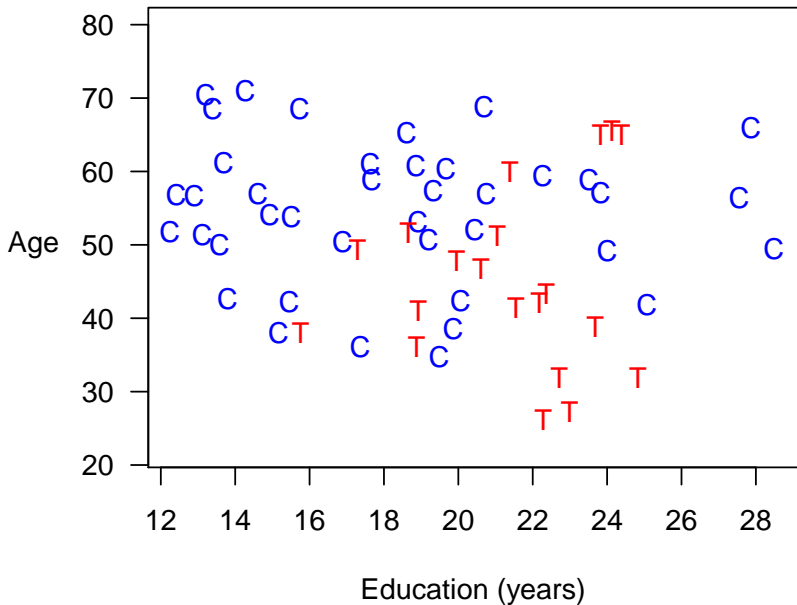


# Mahalanobis Distance Matching

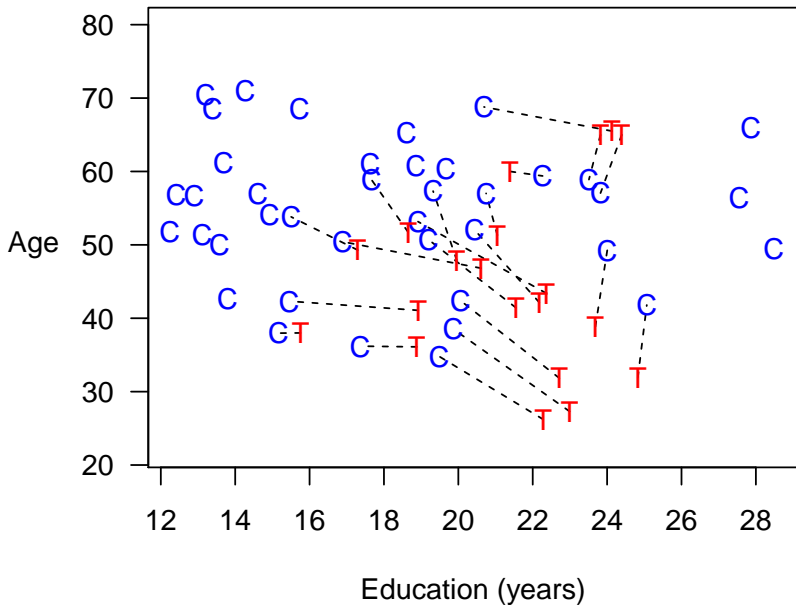




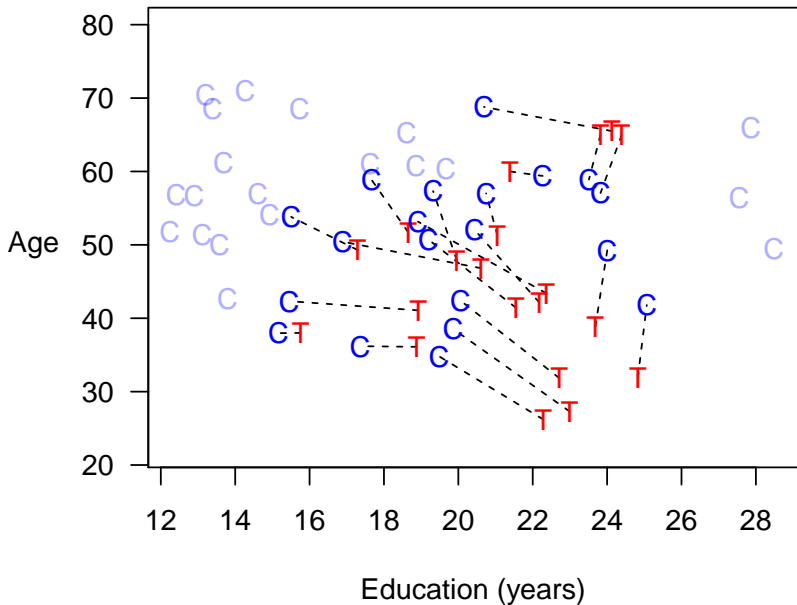
# Mahalanobis Distance Matching



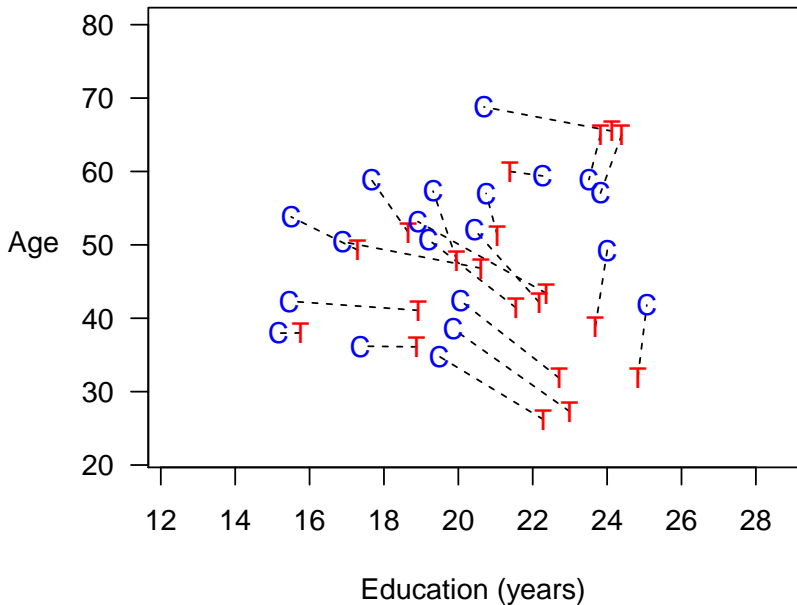
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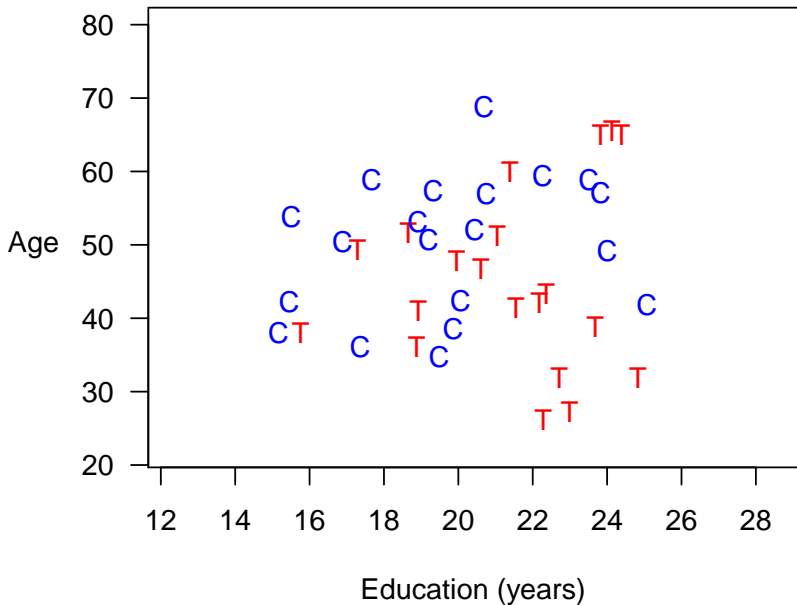
# Mahalanobis Distance Matching



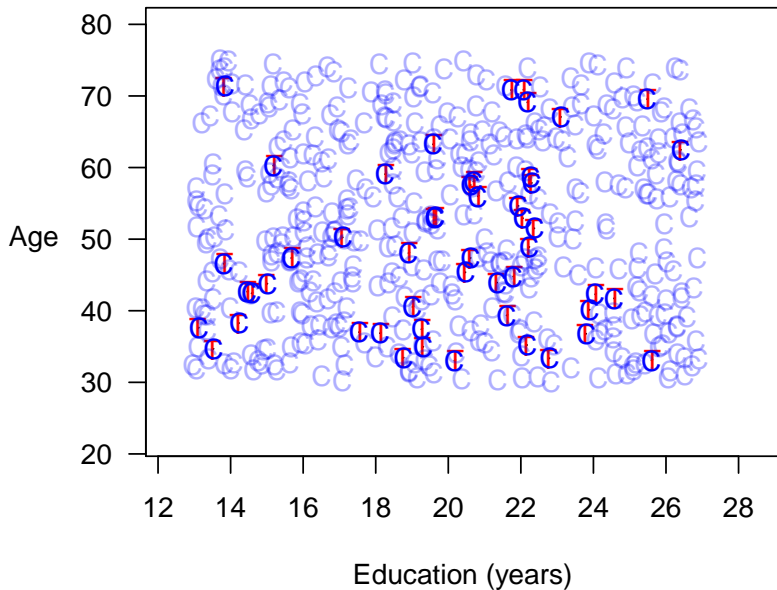
# Mahalanobis Distance Matching



# Mahalanobis Distance Matching



## Best Case: Mahalanobis Distance Matching



# Method 2: Coarsened Exact Matching

(Approximates Fully Blocked Experiment)

## Procedure

### 1. Preprocess (Matching)

- Temporarily coarsen  $X$  as much as you're willing
  - e.g., Education (grade school, high school, college, graduate)
- Apply exact matching to the coarsened  $X$ ,  $C(X)$ 
  - Sort observations into strata, each with unique values of  $C(X)$
  - Prune any stratum with 0 treated or 0 control units
- Pass on original (uncoarsened) units except those pruned

### 2. Estimation Difference in means or a model

- Weight controls in each stratum to equal treated

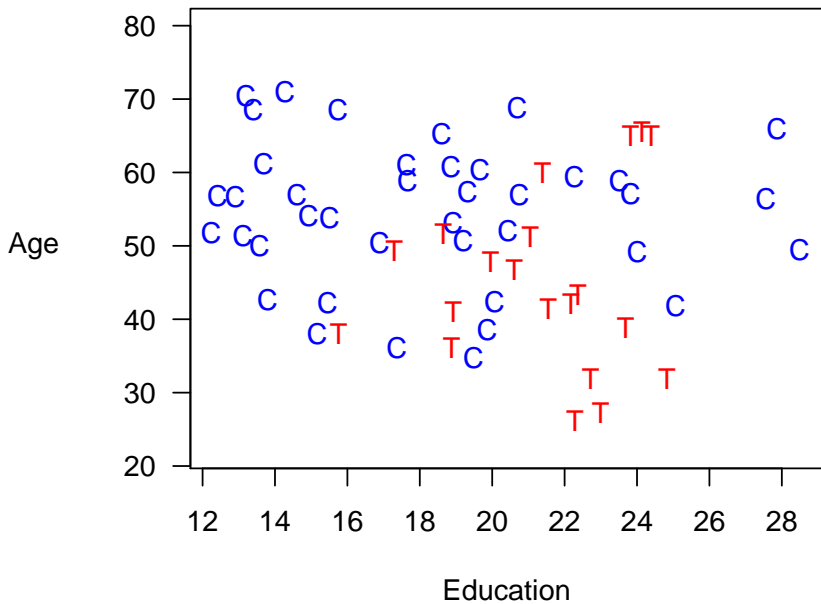
## Interpretation

- Quiz: Do you understand distance trade offs?
- Quiz: What do you do if you have too few observations?

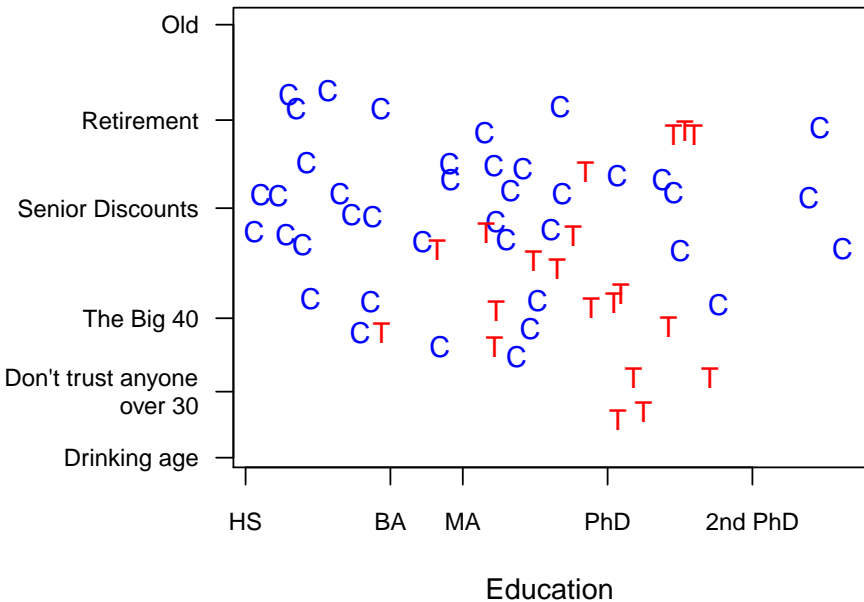
# Coarsened Exact Matching



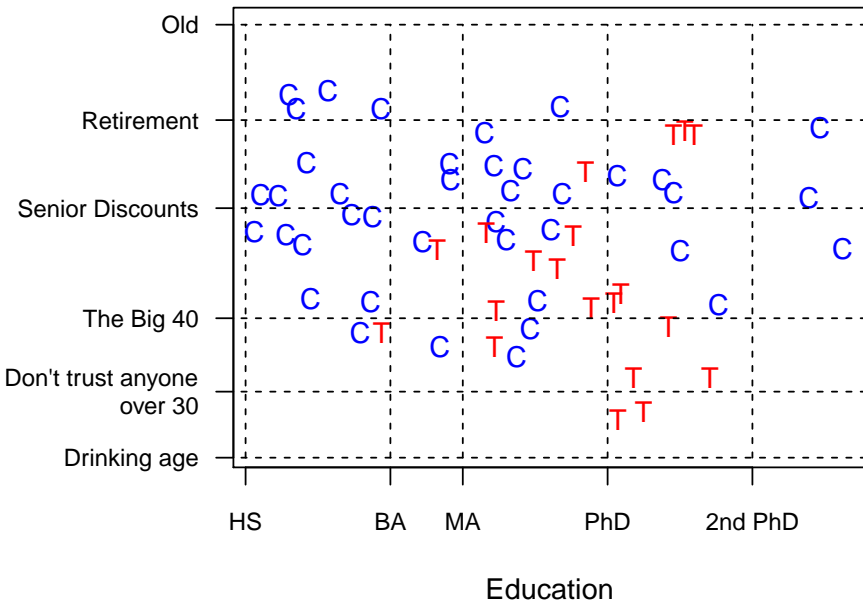
## Coarsened Exact Matching



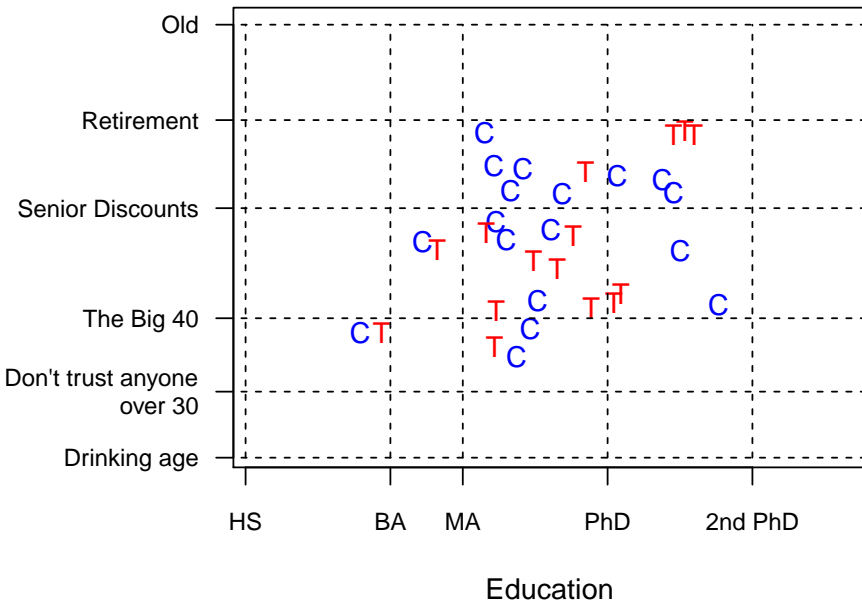
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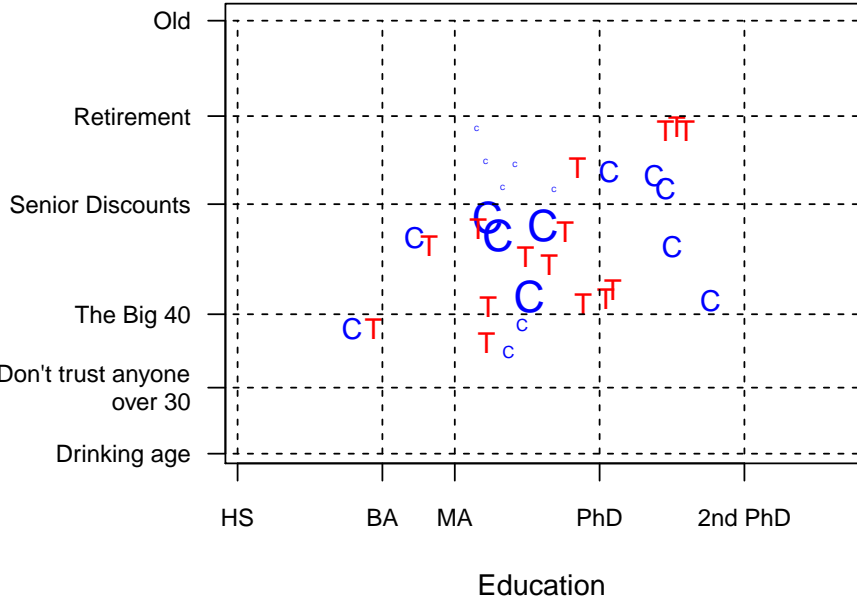
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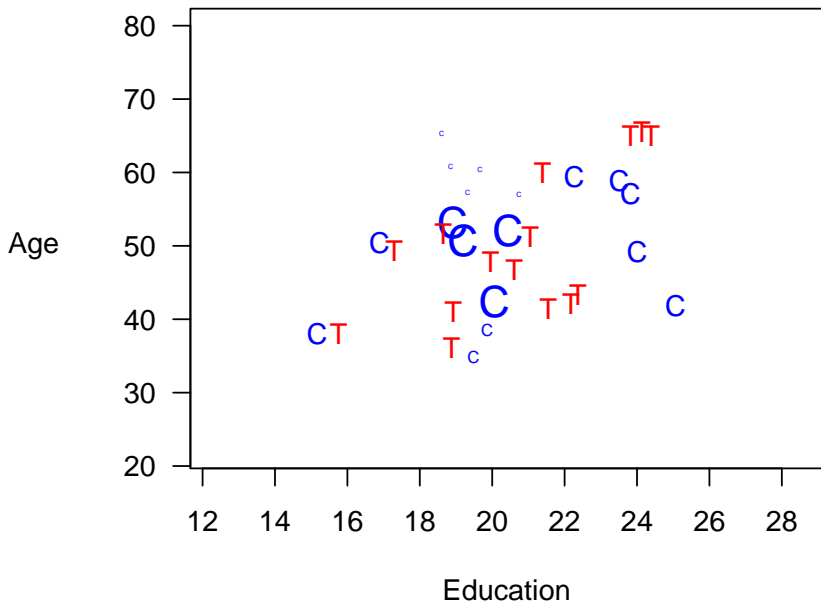
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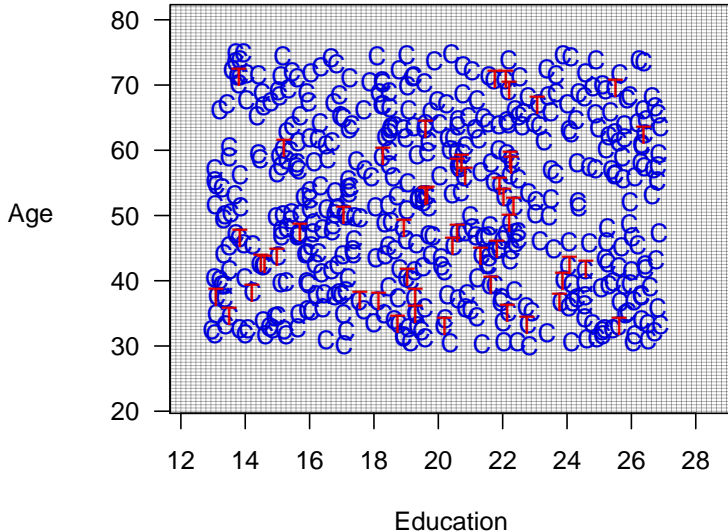
# Coarsened Exact Matching



## Coarsened Exact Matching



## Best Case: Coarsened Exact Matching



# Method 3: Propensity Score Matching

(Approximates Completely Randomized Experiment)

## Procedure

### 1. Preprocess (Matching)

- Reduce  $k$  elements of  $X$  to scalar  $\pi_i \equiv \Pr(T_i = 1|X) = \frac{1}{1+e^{-X_i\beta}}$
- $\text{Distance}(X_c, X_t) = |\pi_c - \pi_t|$
- Match each treated unit to the nearest control unit
- Control units: not reused; pruned if unused
- Prune matches if  $\text{Distance} > \text{caliper}$
- (Many adjustments available to this basic method)

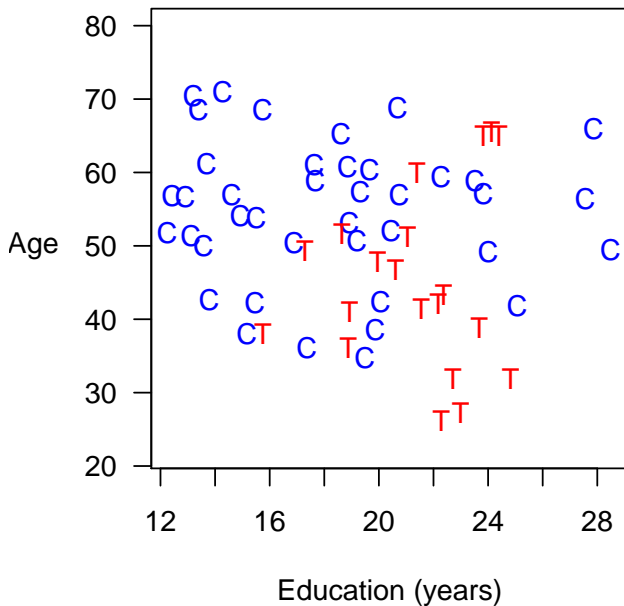
### 2. Estimation Difference in means or a model

## Interpretation

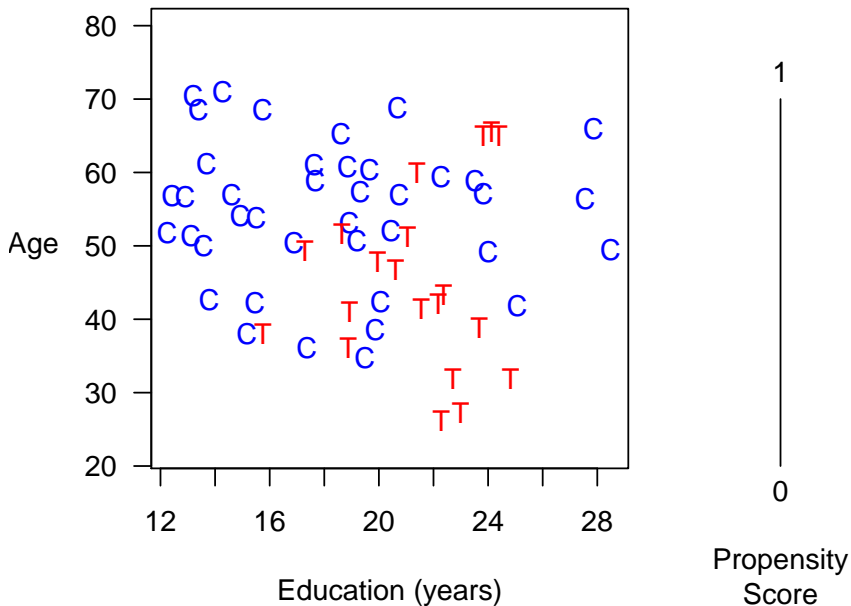
- Quiz: Do you understand distance trade offs?
- Quiz: What do you do when one variable is very important?



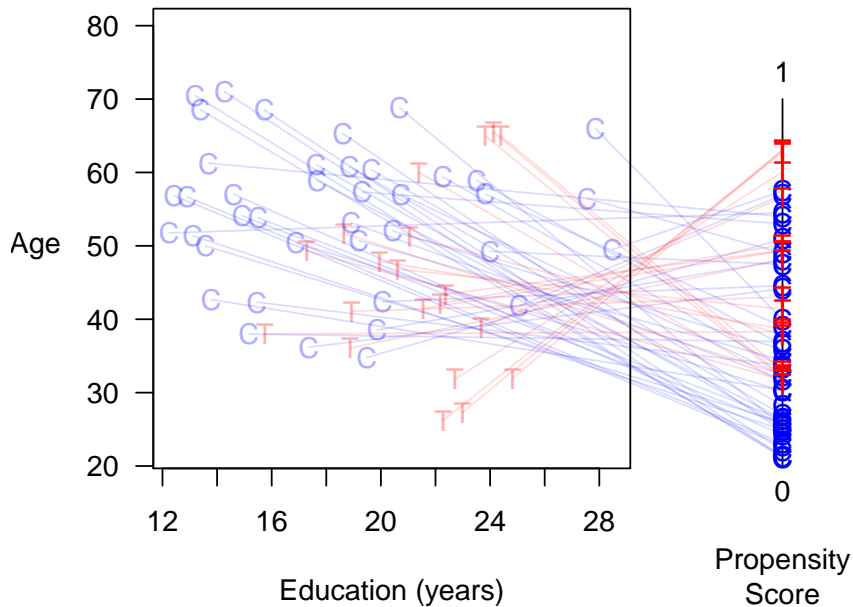
## Propensity Score Matching



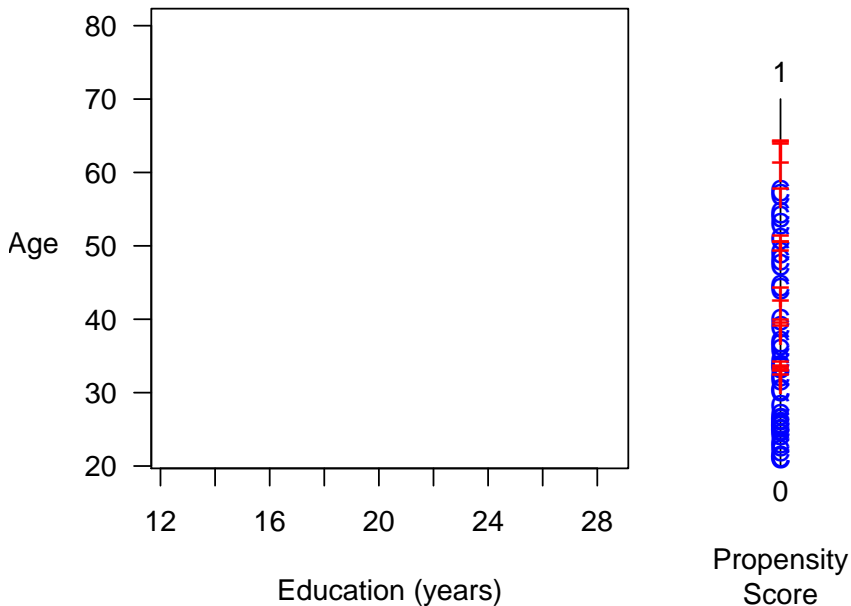
## Propensity Score Matching



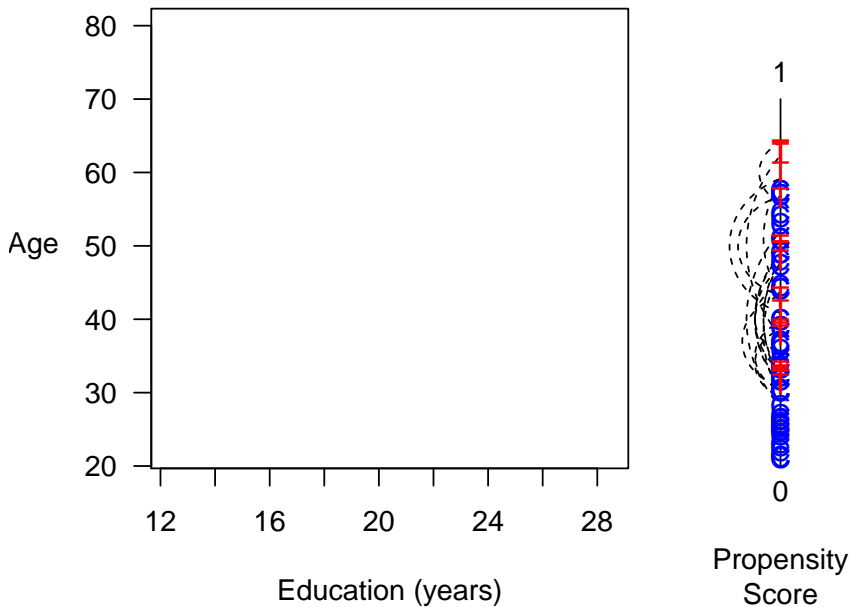
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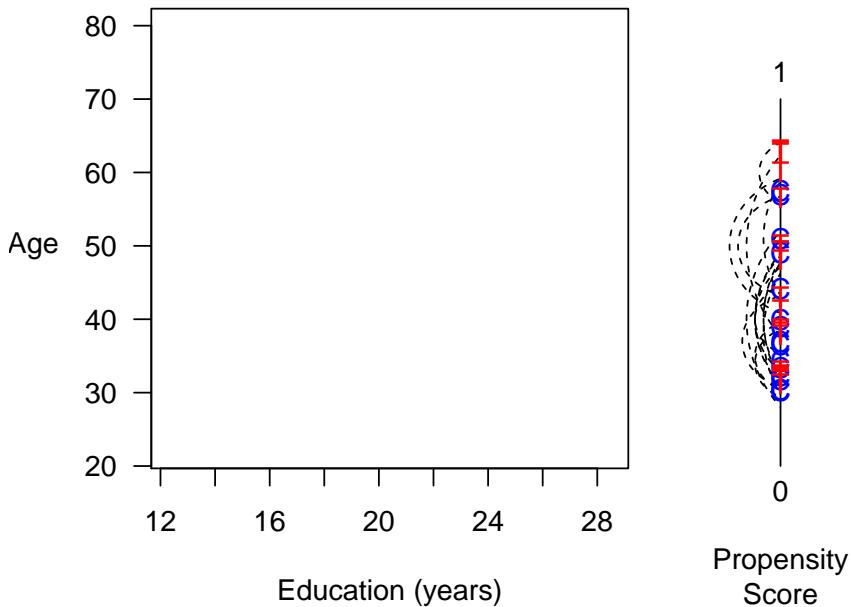
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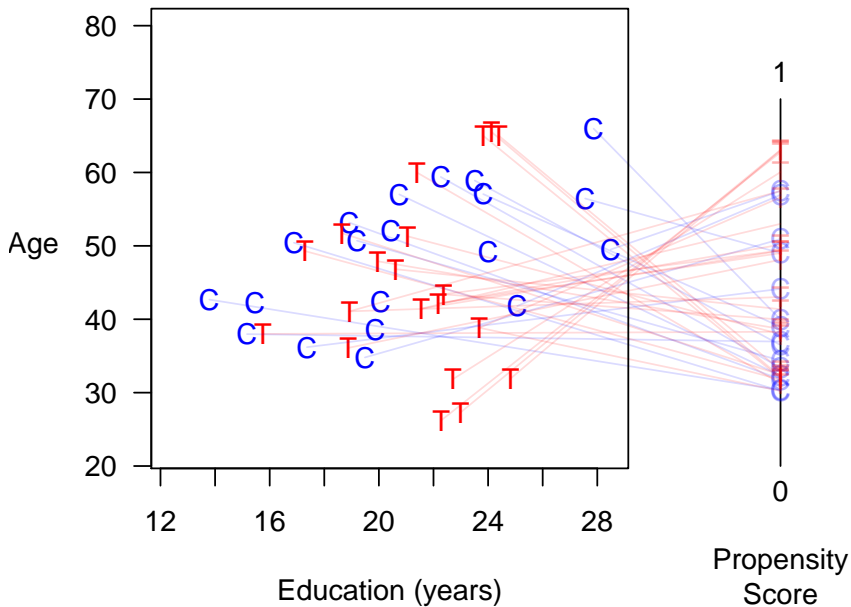
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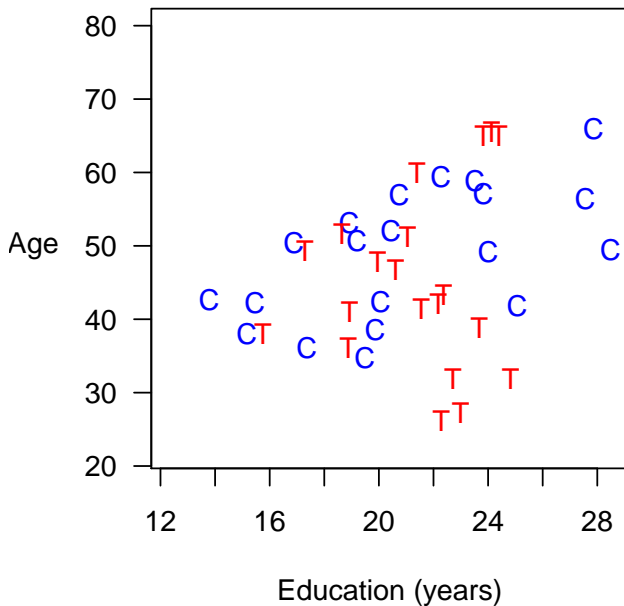
## Propensity Score Matching



## Propensity Score Matching

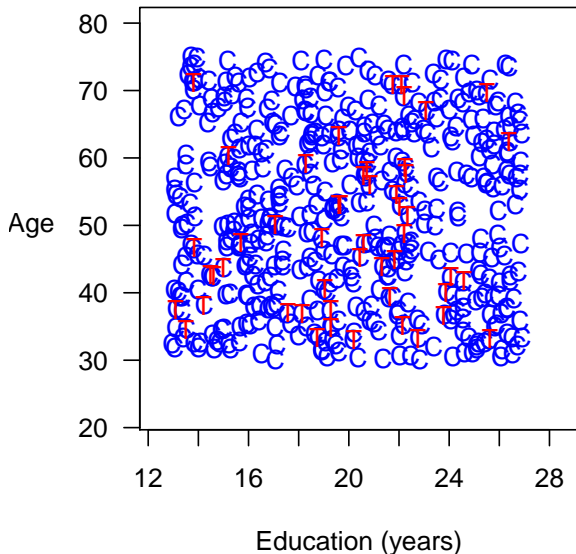


## Propensity Score Matching





# Best Case: Propensity Score Matching is Suboptimal



1  
0

Propensity  
Score



1

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**Problems with Propensity Score Matching**

The Matching Frontier

# Random Pruning Increases Imbalance

Deleting data only helps if you're careful!

- “Random pruning”: pruning process is independent of  $X$
- Discrete example
  - Sex-balanced dataset: treateds  $M_t, F_t$ , controls  $M_c, F_c$
  - Randomly prune 1 treated & 1 control  $\leadsto$  4 possible datasets: 2 balanced  $\{M_t, M_c\}, \{F_t, F_c\}$   
2 imbalanced  $\{M_t, F_c\}, \{F_t, M_c\}$
  - $\implies$  random pruning increases imbalance
- Continuous example
  - Dataset:  $T \in \{0, 1\}$  randomly assigned;  $X$  any fixed variable; with  $n$  units
  - Measure of imbalance: squared difference in means  $d^2$ , where  $d = \bar{X}_t - \bar{X}_c$
  - $E(d^2) = V(d) \propto 1/n$  (note:  $E(d) = 0$ )
  - Random pruning  $\leadsto n$  declines  $\leadsto E(d^2)$  increases
  - $\implies$  random pruning increases imbalance
- Result is completely general

# PSM's Statistical Properties

## 1. Low Standards: Sometimes helps, never optimizes

- *Efficient* relative to complete randomization, but
- *Inefficient* relative to (the more powerful) full blocking
- Other methods dominate:

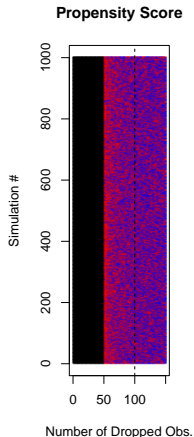
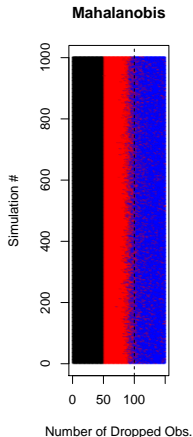
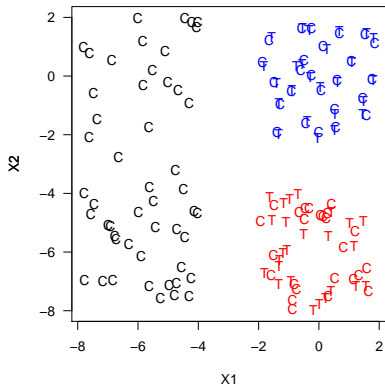
$$X_c = X_t \Rightarrow \pi_c = \pi_t \text{ but}$$

$$\pi_c = \pi_t \not\Rightarrow X_c = X_t$$

## 2. The PSM Paradox: When you do “better,” you do worse

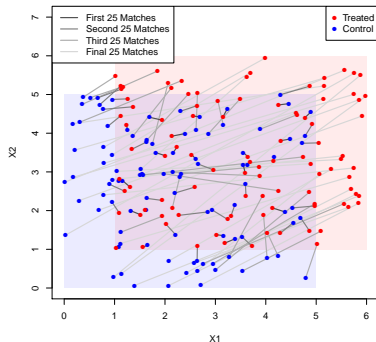
- Background: Random matching increases imbalance
- When PSM approximates complete randomization (to begin with or, after some pruning)  $\rightsquigarrow$  all  $\hat{\pi} \approx 0.5$  (or constant within strata)  $\rightsquigarrow$  pruning at random  $\rightsquigarrow$  Imbalance  $\rightsquigarrow$  Inefficiency  $\rightsquigarrow$  Model dependence  $\rightsquigarrow$  Bias
- If the data have no good matches, the paradox won't be a problem but you're cooked anyway.
- Doesn't PSM solve the curse of dimensionality problem? Nope. The PSM Paradox gets worse with more covariates

# PSM is Blind Where Other Methods Can See

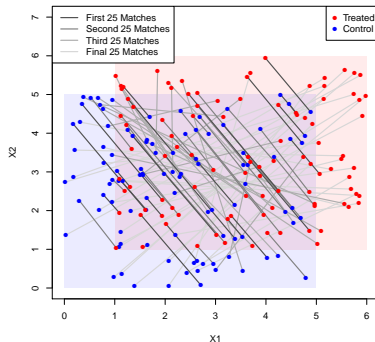


# What Does PSM Match?

## MDM Matches



## PSM Matches

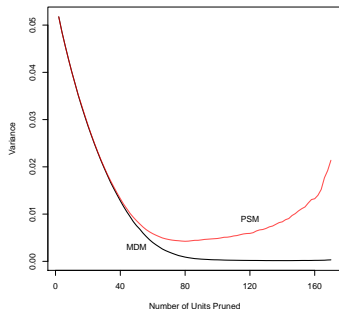


Controls:  $X_1, X_2 \sim \text{Uniform}(0,5)$

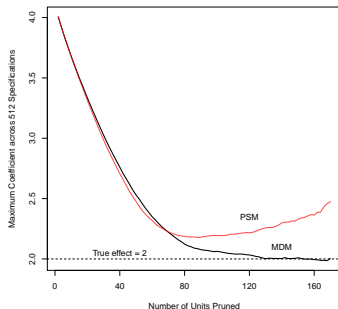
Treateds:  $X_1, X_2 \sim \text{Uniform}(1,6)$

# PSM Increases Model Dependence & Bias

Model Dependence



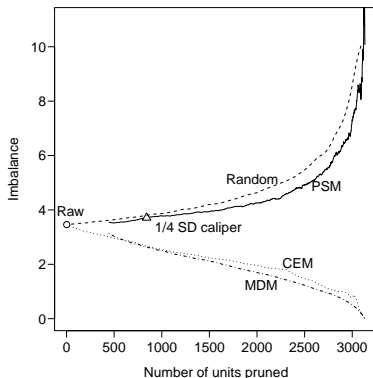
Bias



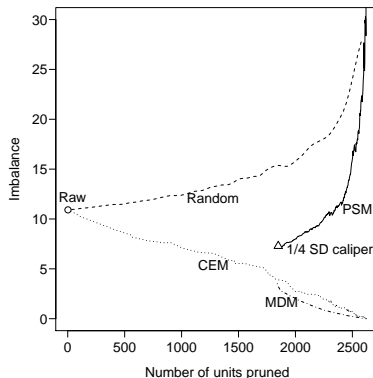
$$Y_i = 2T_i + X_{1i} + X_{2i} + \epsilon_i$$
$$\epsilon_i \sim N(0, 1)$$

# The Propensity Score Paradox in Real Data

Finkel et al. (JOP, 2012)



Nielsen et al. (AJPS, 2011)



Similar pattern for > 20 other real data sets we checked



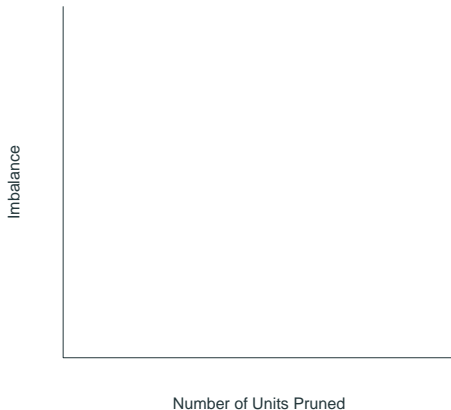


# Tensions in Existing Matching Methods

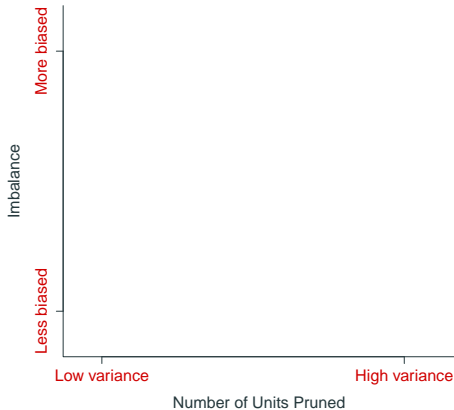
- Maximize one metric; judge against another: Propensity score matching, compared with var-by-var diff in means
- Choose  $n$ ; check imbalance after: Propensity score matching, Mahalanobis
- Choose imbalance; check  $n$  after: exact matching, CEM



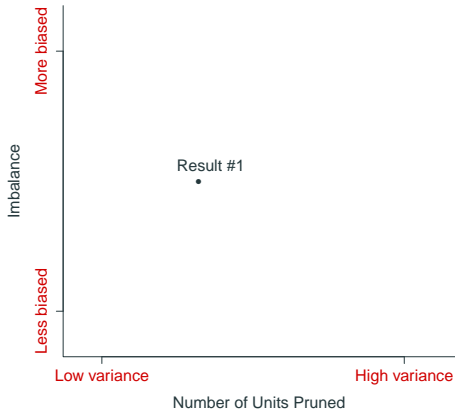
# A Solution: The Matching Frontier



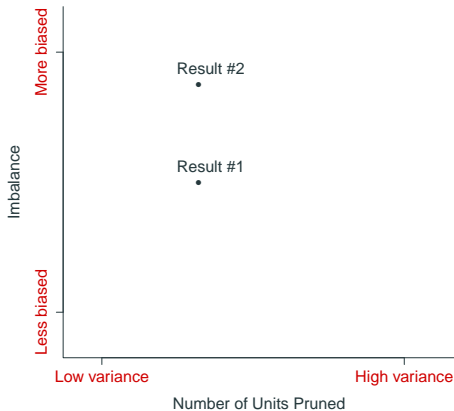
# A Solution: The Matching Frontier



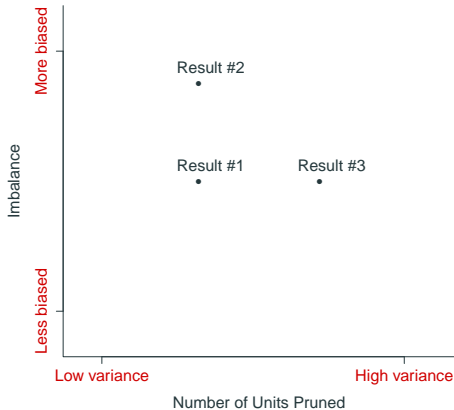
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# A Solution: The Matching Frontier

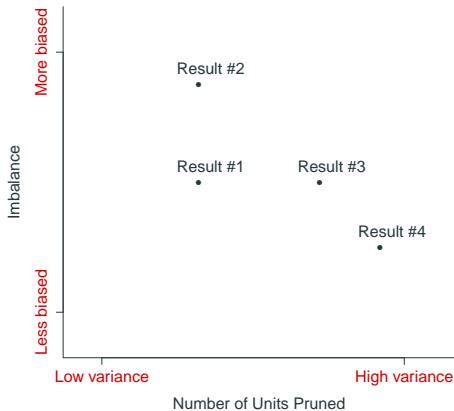


# A Solution: The Matching Frontier

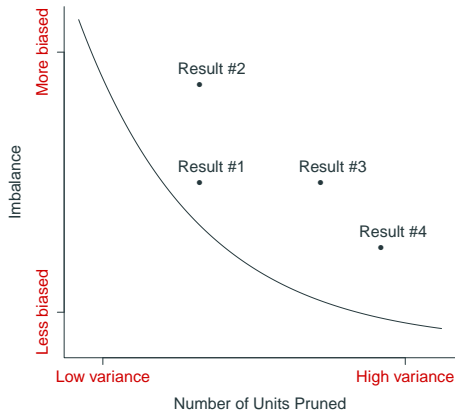




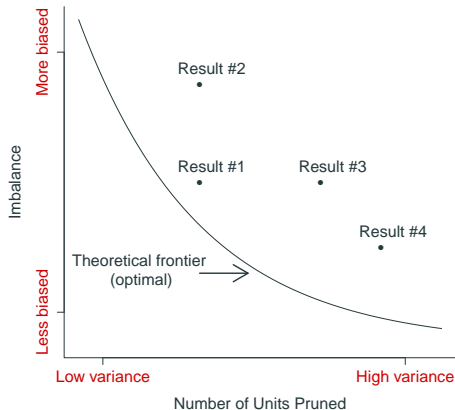
# A Solution: The Matching Frontier



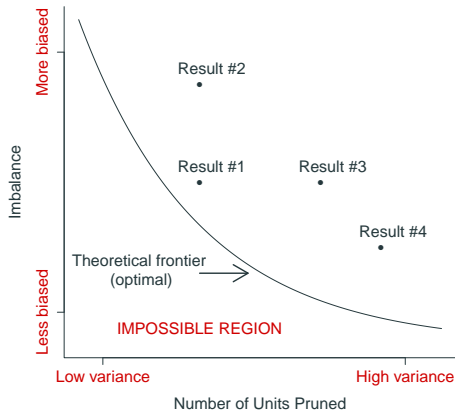
# A Solution: The Matching Frontier



# A Solution: The Matching Frontier



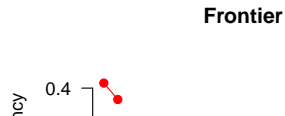
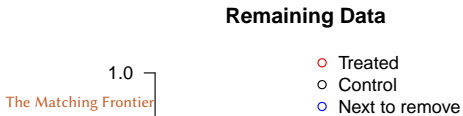
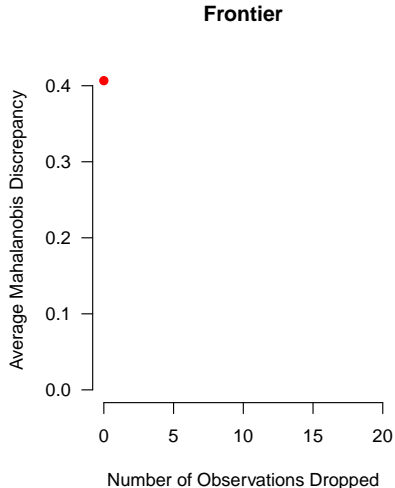
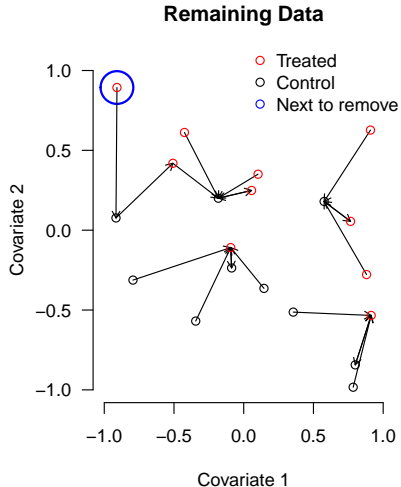
# A Solution: The Matching Frontier



# How hard is the frontier to calculate?

- Consider 1 point on the SATT frontier:
  - Start with matrix of  $N$  control units  $X_0$
  - Calculate imbalance for all  $\binom{N}{n}$  subsets of rows of  $X_0$
  - Choose subset with lowest imbalance
- Evaluations needed to compute the entire frontier:
  - $\binom{N}{n}$  evaluations for each sample size  $n = N, N - 1, \dots, 1$
  - The combination is the (gargantuan) “power set”
  - e.g.,  $N > 300$  requires more imbalance evaluations than elementary particles in the universe
  - $\leadsto$  It's **hard** to calculate!
- We develop algorithms for the (optimal) frontier which:
  - runs very fast
  - operate as “greedy” but we prove are optimal
  - do not require evaluating every subset
  - work with very large data sets
  - is the exact frontier (no approximation or estimation)
  - $\leadsto$  It's **easy** to calculate!

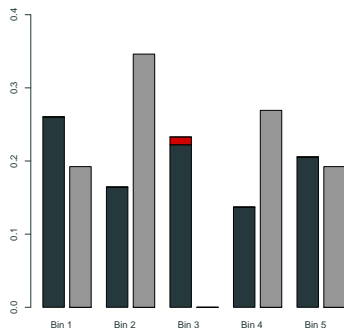
# Constructing the FSATT Mahalanobis Frontier



# Discrete algorithm

Short version:

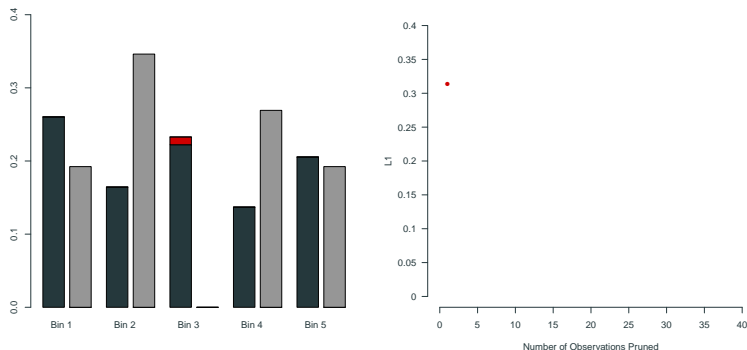
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Short version:

- Calculate bins
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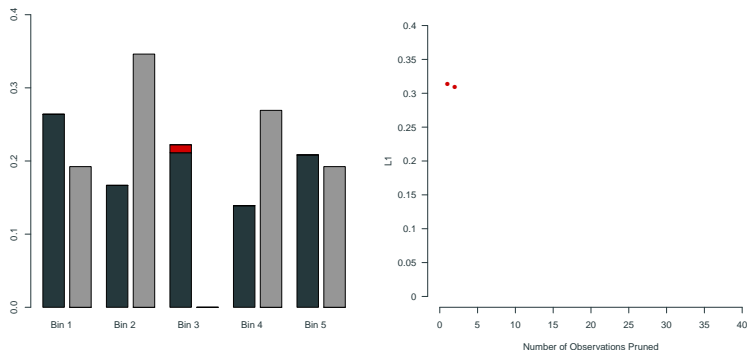




# Discrete algorithm

Short version:

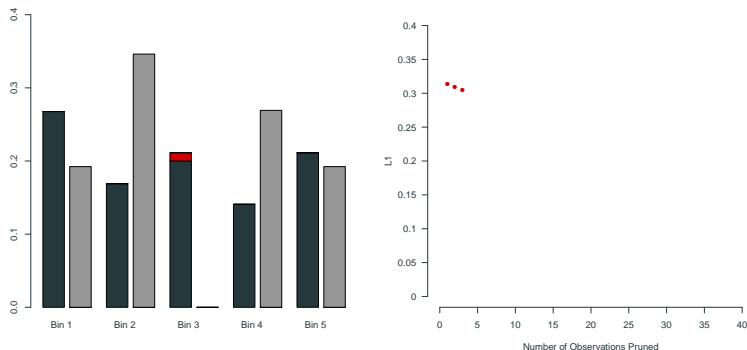
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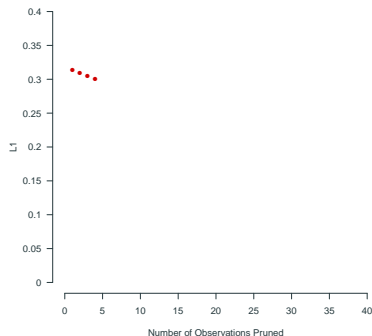
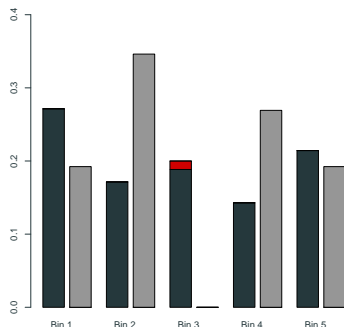
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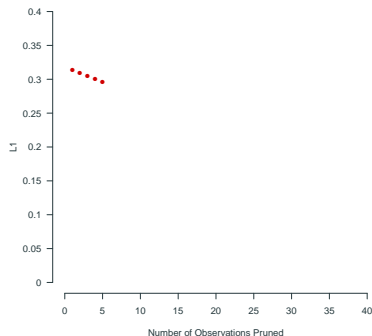
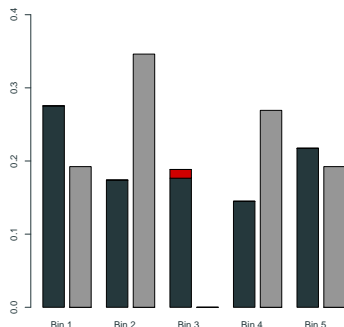
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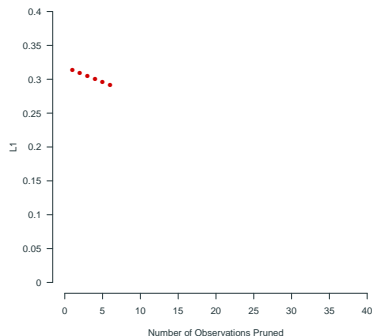
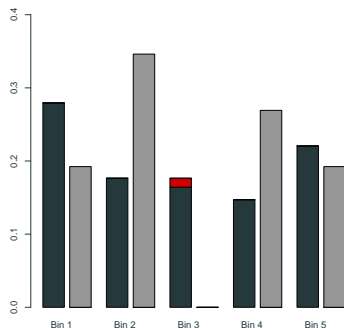
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Short version:

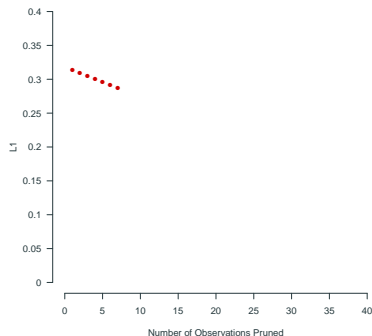
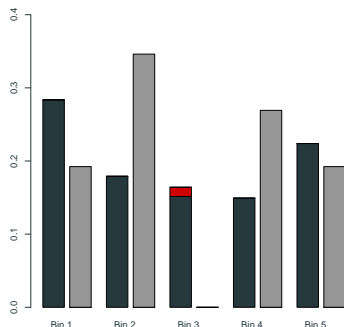
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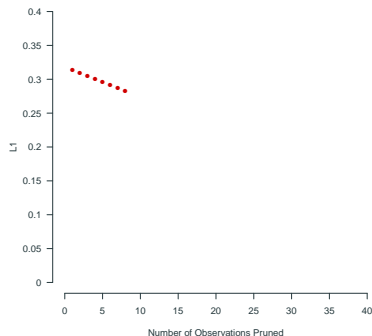
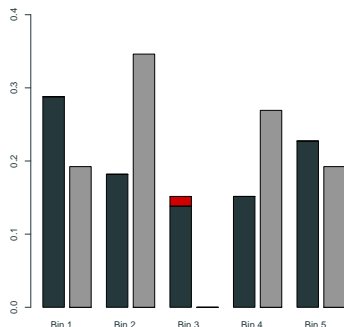
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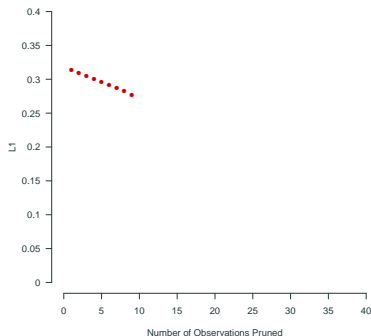
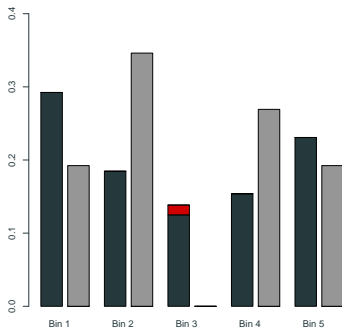
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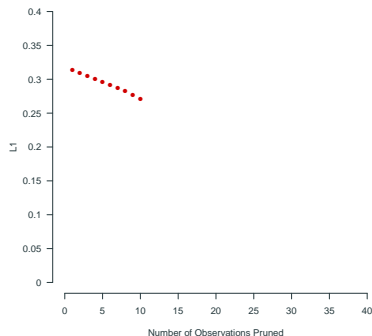
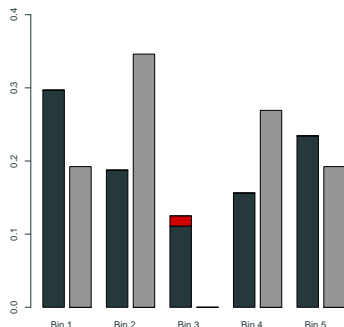




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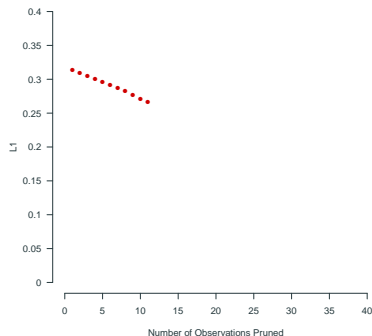
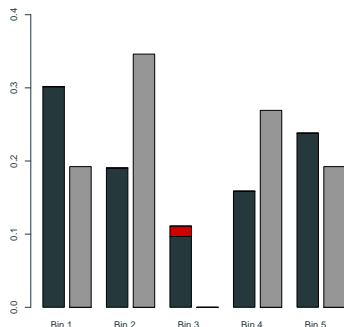
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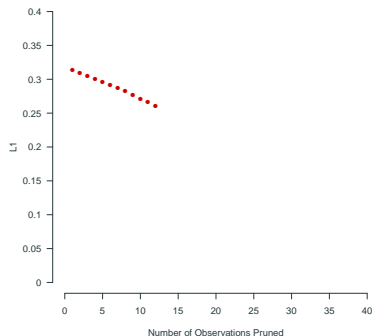
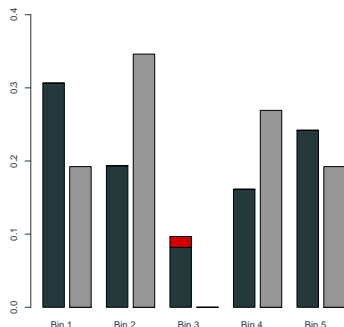
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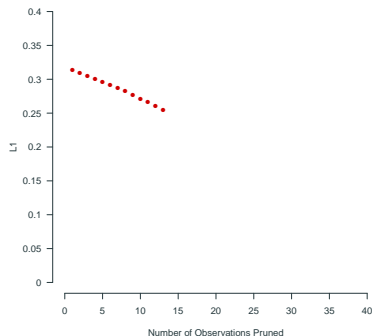
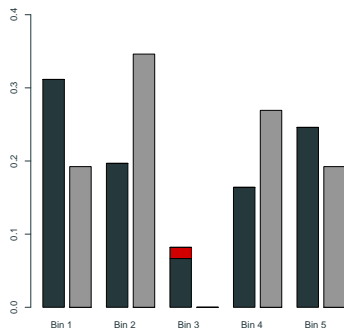
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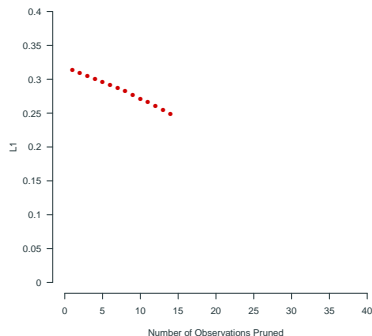
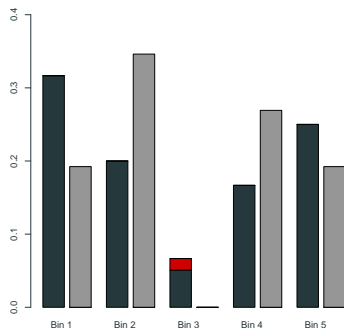
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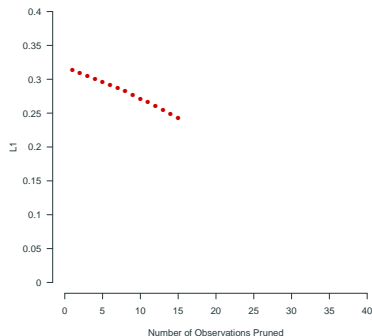
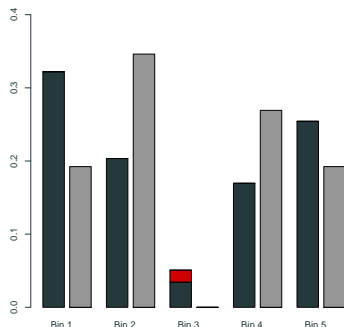
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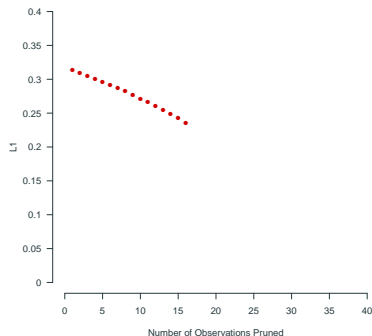
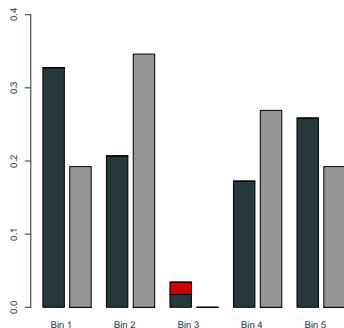
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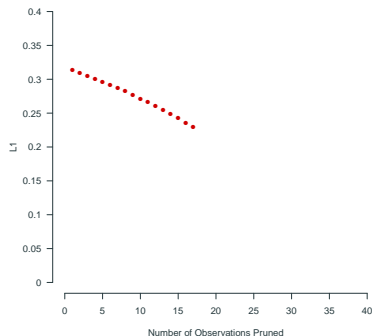
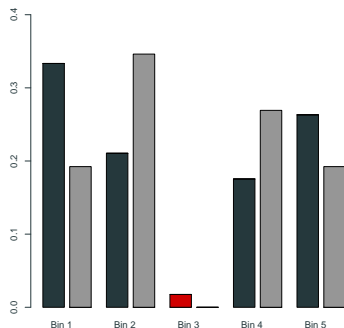
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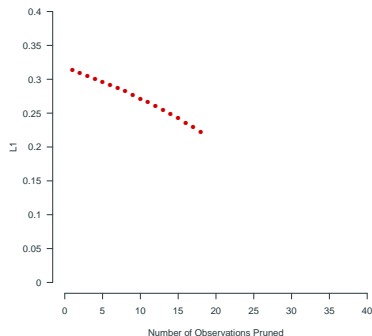
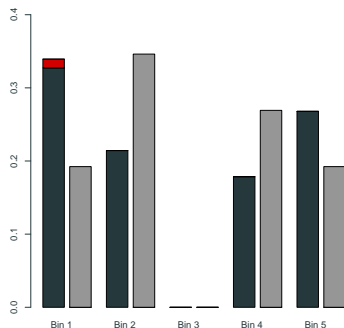




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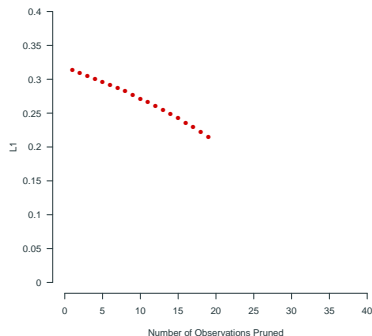
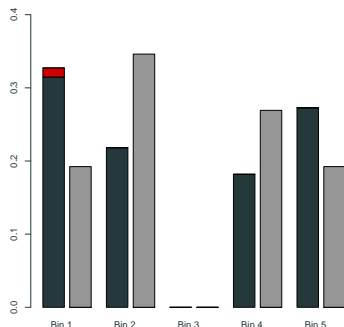
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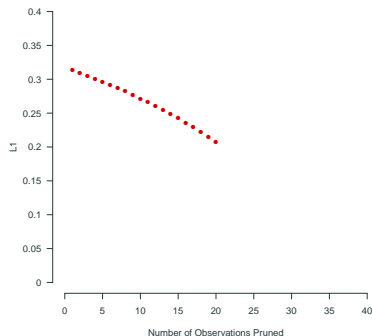
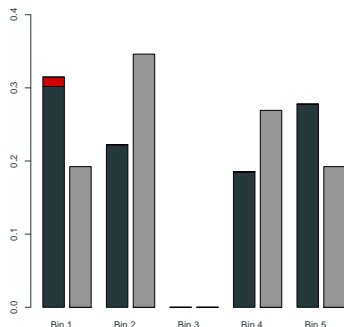
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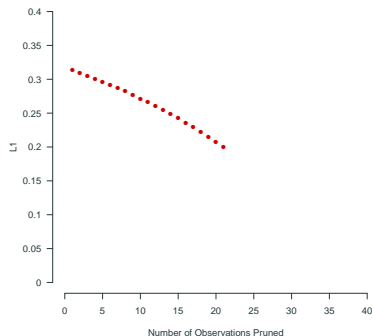
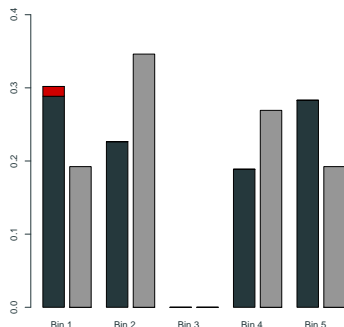
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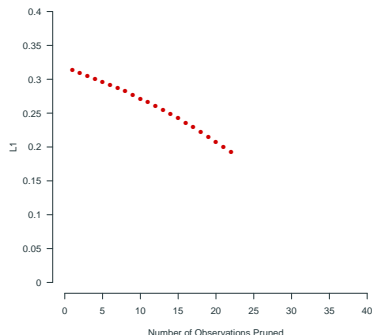
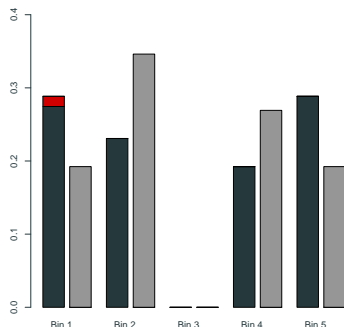
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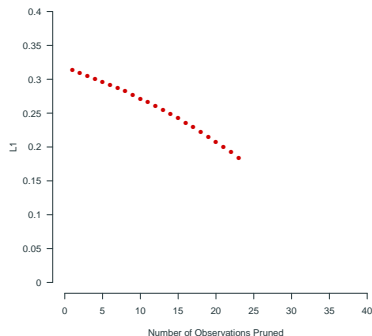
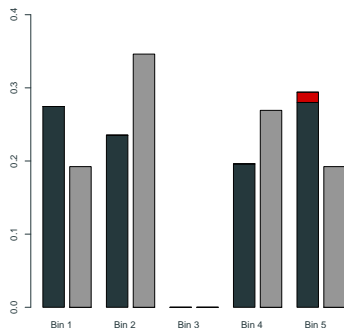
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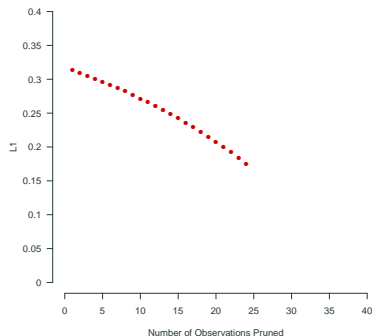
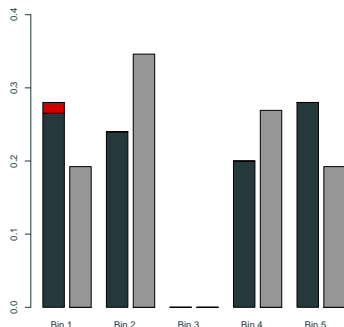
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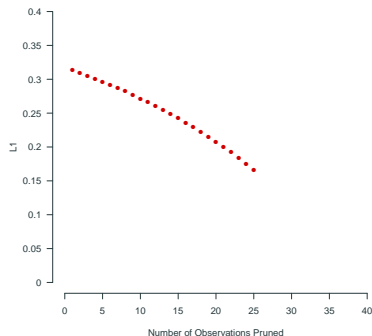
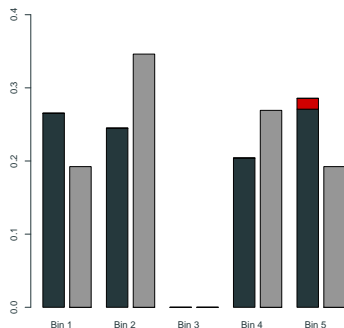
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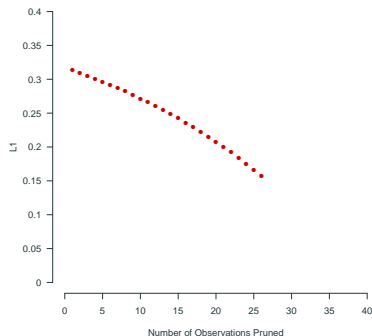
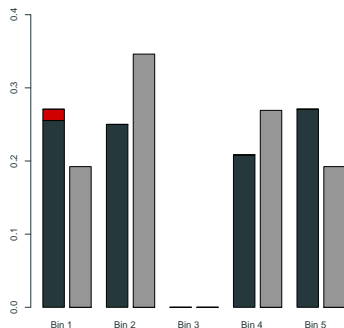




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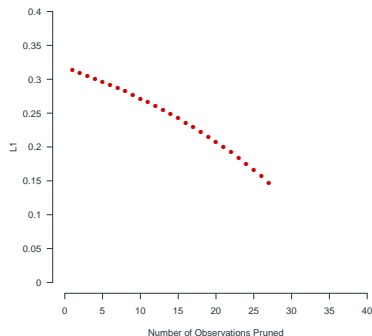
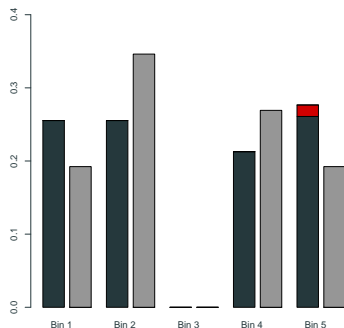
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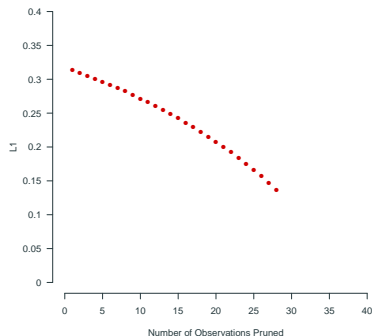
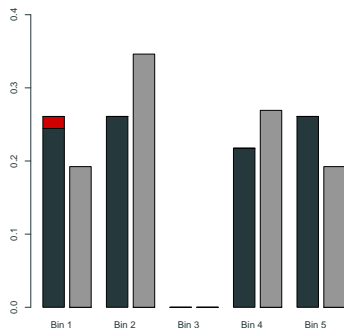
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Short version:

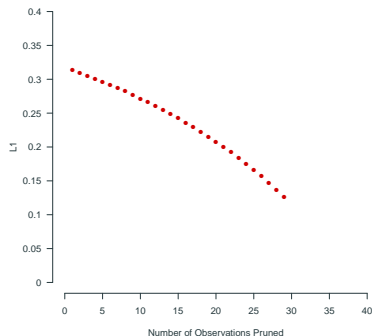
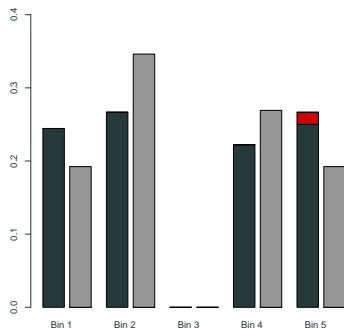
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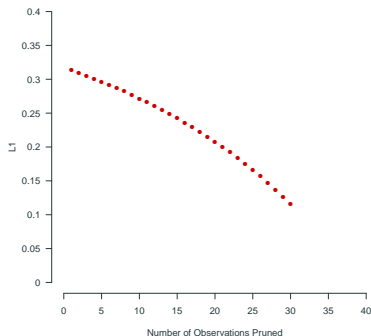
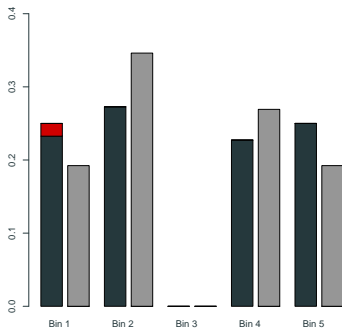
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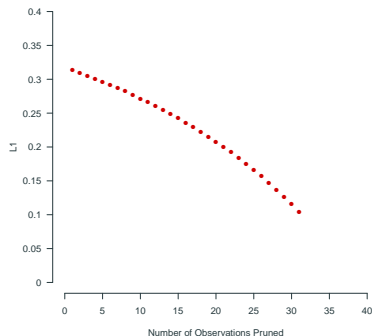
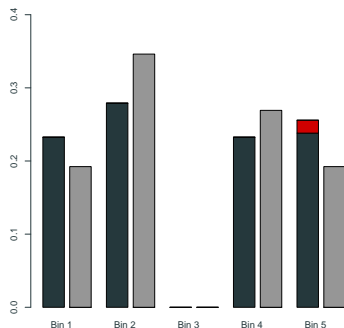
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# Discrete algorithm

Short version:

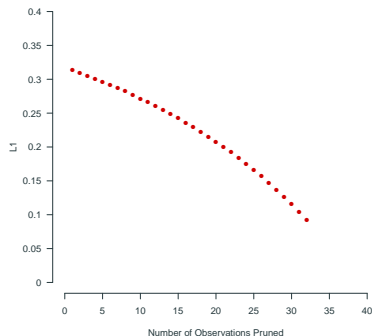
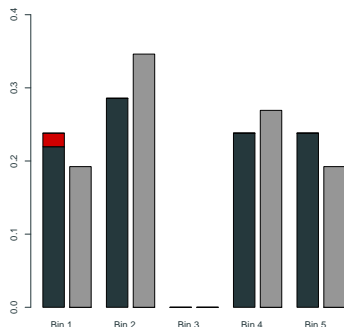
- Calculate bins
- Until balance stops improving, greedily prune a control unit from the bin with the largest proportional difference between control and treated units



# Discrete algorithm

Short version:

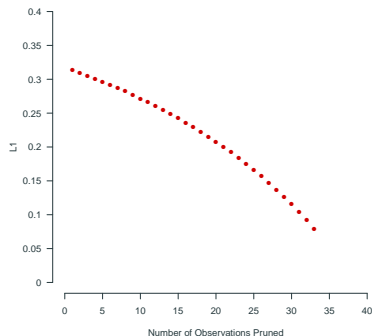
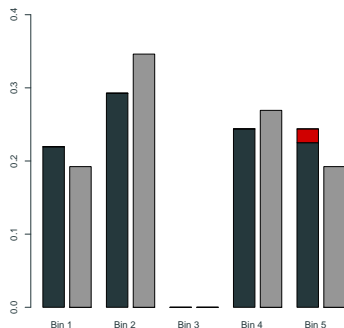
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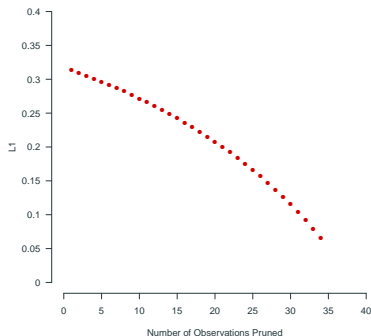
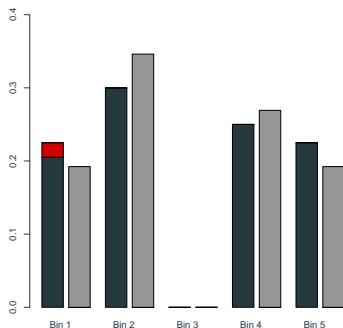




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Short version:

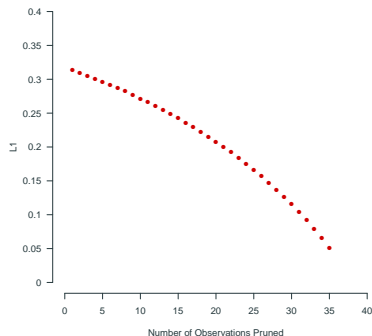
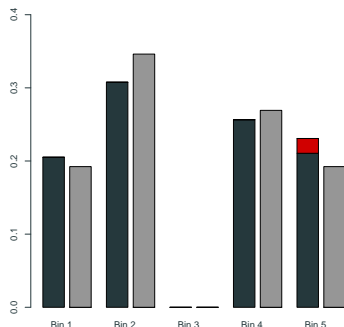
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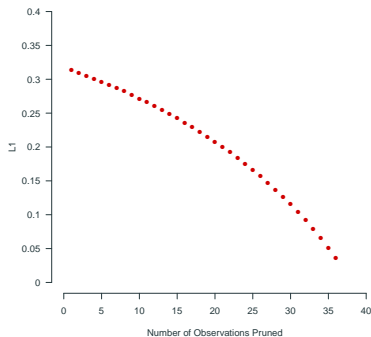
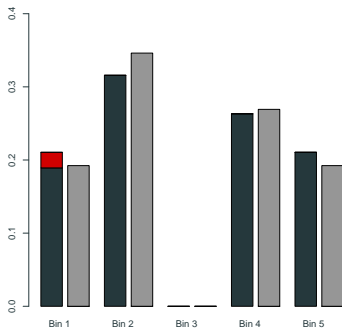
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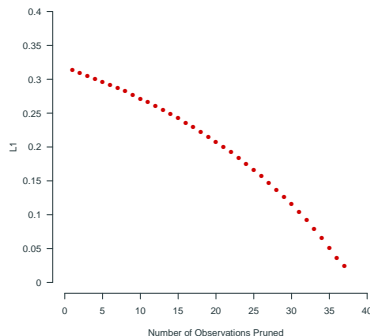
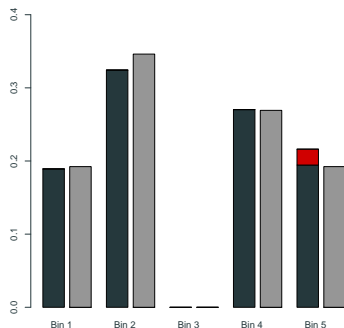
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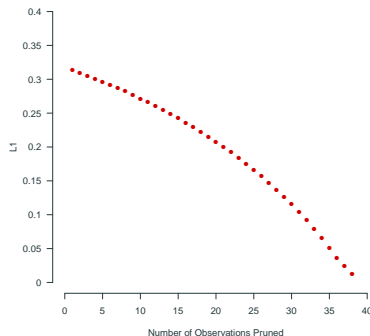
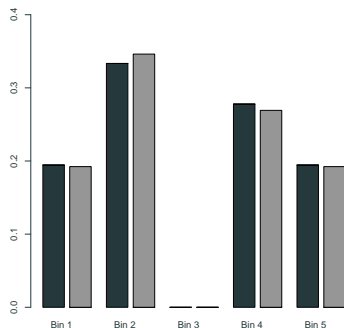
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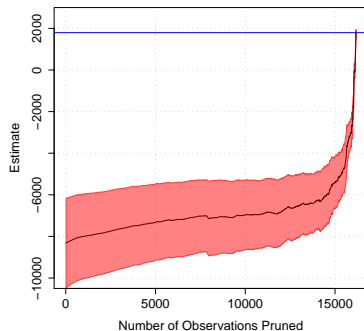
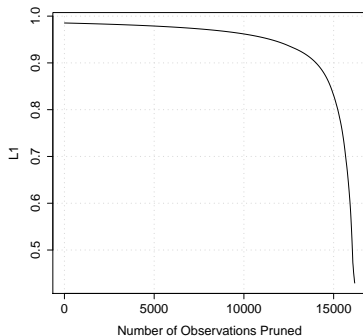
# Discrete algorithm

Short version:

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# Job Training Data: Frontier and Causal Estimates



- 185 Ts; pruning most 16,252 Cs won't increase variance much
- Huge bias-variance trade-off after pruning most Cs
- Estimates converge to experiment after removing bias
- No mysteries: basis of inference clearly revealed