



UNIVERSITEIT•STELLENBOSCH•UNIVERSITY
jou kennisvennoot • your knowledge partner

Designing the audio generation core of a hardware synthesiser

Marco Rademan
21561273

Report submitted in partial fulfilment of the requirements of the module
Project (E) 448 for the degree Baccalaureus in Engineering in the Department of Electrical and Electronic
Engineering at Stellenbosch University.

Supervisor: Prof J. Versfeld

November 2021

Acknowledgements

I would like to thank my dog, Muffin. I also would like to thank the inventor of the incubator; without him/her, I would not be here. Finally, I would like to thank Dr Herman Kamper for this amazing report template.



UNIVERSITEIT•STELLENBOSCH•UNIVERSITY
jou kennisvennoot • your knowledge partner

Plagiaatverklaring / Plagiarism Declaration

1. Plagiaat is die oorneem en gebruik van die idees, materiaal en ander intellektuele eiendom van ander persone asof dit jou eie werk is.

Plagiarism is the use of ideas, material and other intellectual property of another's work and to present it as my own.

2. Ek erken dat die pleeg van plagiaat 'n strafbare oortreding is aangesien dit 'n vorm van diefstal is.

I agree that plagiarism is a punishable offence because it constitutes theft.

3. Ek verstaan ook dat direkte vertalings plagiaat is.

I also understand that direct translations are plagiarism.

4. Dienooreenkomstig is alle aanhalings en bydraes vanuit enige bron (ingesluit die internet) volledig verwys (erken). Ek erken dat die woordelike aanhaal van teks sonder aanhalingstekens (selfs al word die bron volledig erken) plagiaat is.

Accordingly all quotations and contributions from any source whatsoever (including the internet) have been cited fully. I understand that the reproduction of text without quotation marks (even when the source is cited) is plagiarism

5. Ek verklaar dat die werk in hierdie skryfstuk vervat, behalwe waar anders aangedui, my eie oorspronklike werk is en dat ek dit nie vantevore in die geheel of gedeeltelik ingehandig het vir bepunting in hierdie module/werkstuk of 'n ander module/werkstuk nie.

I declare that the work contained in this assignment, except where otherwise stated, is my original work and that I have not previously (in its entirety or in part) submitted it for grading in this module/assignment or another module/assignment.

21561273 Studentenommer / Student number	 Handtekening / Signature
MW Rademan Voorletters en van / Initials and surname	November 2021 Datum / Date

Abstract

English

The English abstract.

Afrikaans

Die Afrikaanse uittreksel.

Contents

Declaration	ii
Abstract	iii
List of Figures	vii
List of Tables	viii
Nomenclature	x
1. Introduction	1
1.1. Background and motivation	1
1.2. Problem statement	1
1.3. System specification	2
1.3.1. Scope	2
1.3.2. Functional specification	2
1.3.3. Technical requirements	2
1.4. Summary of work	3
1.5. Report overview	4
2. Literature overview	5
2.1. A brief history of musical synthesisers	5
2.2. An overview of synthesis techniques	5
2.2.1. Additive synthesis (Fourier synthesis)	6
2.2.2. Subtractive synthesis	6
2.2.3. FM synthesis	6
2.2.4. Physical modelling	7
2.2.5. Sampling	7
2.2.6. Wavetable synthesis	7
2.3. Basic modular synthesiser building blocks	7
2.3.1. The VCO	8
2.3.2. The VCF	8
2.3.3. The VCA	8
2.3.4. The ADSR envelope	8
2.4. Prerequisite knowledge	9
2.4.1. Equal temperament tuning	9
2.4.2. Audio	10
2.4.3. Prototype IIR filters	11

3. Design	12
3.1. DSP pipeline	12
3.1.1. Component definition	12
3.1.2. Global user parameters	13
3.2. Top-level system in MCU implementation	14
3.3. LUT	15
3.3.1. Linear interpolation	15
3.3.2. Constructing the basic waveform LUTs	17
3.4. Wavetable	17
3.4.1. Modelling wavetable frequency conversion	18
3.4.2. Implementing frequency scaling	20
3.4.3. Implementing inter-wavetable interpolation	21
3.4.4. Applying FM	21
3.5. IIR filtering	22
3.6. Waveshaper	23
3.6.1. Constructing the hyperbolic tangent LUT	24
3.6.2. Analysing waveshaping frequency content	24
3.6.3. Anti-aliasing filters	25
3.6.4. LUT indexing	26
3.7. ADSR envelope generator	27
3.7.1. Creating the exponential LUT	27
3.7.2. Implementing the state-machine	28
3.8. Generator	29
3.8.1. Note-on/off triggers	29
3.8.2. Vibrato	29
3.8.3. Filter cutoff modulation	30
3.8.4. Volume modulation	30
3.8.5. Stereo width	30
3.9. Generator manager	31
3.9.1. Implementing basic data structures with arrays	32
3.9.2. Note-on/off triggers	33
3.9.3. Sample buffer request	33
4. System testing	35
4.1. Wavetable	35
4.2. ADSR	36
4.3. IIR filters	36
4.4. Waveshaping	37
4.5. Generator	38
4.6. Generator manager	38
5. Summary and Conclusion	40
5.1. Results achieved	40
5.2. Further improvements and work	41

Bibliography	42
A. Additional figures	44
B. Additional sections and information	45
B.1. MIDI	45
B.2. A basic monophonic modular setup	45
B.3. Regarding integer arithmetic using Q-numbers	46
B.4. The implications of linear interpolation in the frequency domain and design considerations .	46
B.5. Optimising harmonic indexing using a LUT	47
B.6. Alternative uses and signal chains for the designed components	47
B.7. Musical test	47
C. Tables	48
D. Code listings	53
E. Test code	60
F. Project Planning Schedule	69
G. Outcomes Compliance	70

List of Figures

2.1. ADSR envelope operation [1].	9
3.1. High-level system description.	13
3.2. System block diagram legend.	15
3.3. The top-level system block diagram.	15
3.4. LUT system block diagram.	15
3.5. Wavetable system block diagram.	18
3.6. Frequency scaling using upsampling and downsampling	19
3.7. The effects of linear interpolation in the frequency domain.	19
3.8. Waveshaper system block diagram	23
3.9. Waveshaping harmonic analyses.	25
3.10. ADSR envelope system block diagram.	27
3.11. System block diagram of the generator.	29
3.12. Generator manager system block diagram	32
3.13. Generator queue freeing algorithm example	34
4.1. Testing inter-wavetable interpolation and harmonic indexing.	35
4.2. Sampled ADSR envelope with retrigger.	36
4.3. HP24-filtered Guassian noise.	36
4.4. Comparing the effect of the anti-aliasing LP12 filter for hyperbolic tangent waveshaping.	37
4.5. Comparing the effect of the anti-aliasing LP12 filter for sinusoidal waveshaping.	37
4.6. Testing vibrato and stereo width.	39
4.7. Testing the generator manager.	39
A.1. Examples of fundamental modules.	44
A.2. Ableton's Wavetable VST	44
B.1. Block diagram of a very basic monophonic modular setup.	45
B.2. The effects of linear interpolation in the frequency domain.	46

List of Tables

3.1. All global parameters	14
3.2. Digital biquad filter denominator coefficients	22
3.3. Digital biquad filter numerator coefficients	22
3.4. Axis scaling values and gradient errors for constructing the hyperbolic tangent LUT.	24
C.1. Bilinear transform substitution [2]	48
C.2. MIDI note IDs and their frequencies [3]	48
C.3. ARM Cortex M4 and M7 FPU instruction set [4]	52

Listings

3.1. Fast modulus for a power of 2	16
3.2. LUT lookup with linear interpolation	16
3.3. Sampling from a wavetable	21
D.1. Efficient frequency scaling using the equal temperament tuning system	53
D.2. Configuring wavetable frequency	54
D.3. Calculating the coefficients for a LP24 filter	54
D.4. Filtering a signal	54
D.5. Trigonometric lookup functions	55
D.6. Sampling from an ADSR envelope state-machine	55
D.7. Applying vibrato FM	56
D.8. Sampling from a generator	56
D.9. Generator manager functions	56
E.1. Code used to generate test data for chapter 4	60

Nomenclature

Variables and functions

$y[n]$	A discrete-time signal with samples indexed by variable n .
$x_1[n] * x_2[n]$	The convolution of discrete-time functions.
$h[n]$	The impulse response of a discrete-time system.
$X(f)$	The discrete-time Fourier transform (DTFT) of a function.
$\lfloor x \rfloor$	The floor of a variable x , corresponding to the integer component of x .
$\{x\}$	The fractional part of x .
$\{n_1, n_2, \dots, n_K\}$	A set of numbers, not to be confused with the fractional function.
η	The frequency scaling factor in wavetable sampling.
$y[n]_{\uparrow L}$	Upsampling a discrete-time signal by a factor L .
$y[n]_{\downarrow M}$	Downsampling a discrete-time signal by a factor M .
$\text{tri}[\frac{n}{L}]$	A discrete triangular pulse beginning at $-L$ and ending at L , with an amplitude of 1.
$\text{rect}[\frac{n}{L}]$	A discrete rectangular pulse beginning at $-\frac{L}{2}$ and ending at $\frac{L}{2}$, with an amplitude of 1.
\mathbb{N}	The set of all natural numbers ($\{1, 2, 3, 4, \dots\}$).
\mathbb{N}_0	The set of all natural numbers including 0.
\mathbb{R}	The set of all real numbers.
\mathbb{Z}	The set of all integers ($\{\dots, -2, -1, 0, 1, 2, \dots\}$).
$\text{lerp}(x_1, x_2, \delta)$	Linear interpolation between points x_1 and x_2 , by a distance factor δ .

Acronyms and abbreviations

VCA	Voltage-controlled amplifier
ADSR	Attack, Decay, Sustain, Release
VCF	Voltage-controller filter
VST	Virtual studio technology
DAW	Digital audio workstation
FM	Frequency modulation
LUT	Lookup table
LFO	Low frequency oscillator
IR	Impulse response
VCO	Voltage-controlled oscillator
CV	Control voltage
LPF	Low-pass filter
HPF	High-pass filter
BPF	Band-pass filter
FIR	Finite impulse response
IIR	Infinite impulse response
MIDI	Musical instrument digital interface
SMF	Standard MIDI file
SQNR	Signal to quantisation noise ratio
PCM	Pulse-code modulation
DPCM	Differential pulse-code modulation
UART	Universal asynchronous receiver/transmitter
SAI	Serial audio interface
DSP	Digital signal processing
LP12	12 dB/octave LPF
HP12	12 dB/octave HPF
BP12	12 dB/octave BPF
LP24	24 dB/octave LPF
HP24	24 dB/octave HPF
MCU	Microcontroller unit
MPU	Microprocessing unit
IC	Integrated circuit
FPU	Floating-point unit
STFT	Short-time Fourier transform

Chapter 1

Introduction

1.1. Background and motivation

Hardware synthesisers play an important role in all of music today, and has seen a cast expanse in a variety of product lines across the globe, since the inception of analogue sound synthesisers in 1928. A hardware synthesiser refers to a physical system that generates audio signals, using external control inputs that can either be pre-programmed, such as a drum-machine, or manually sent by the user, such as a piano keyboard via the MIDI protocol. These audio systems can be digital, where sound is generated by a MCU, or analogue, generated by the manipulation of VCOs, using a variety of control signals. A few user-specified parameters can often be enough to generate a wide array of interesting and complicated sounds, some of which are iconic and recognisable in old and modern music alike.

Due to the highly competitive nature of the market and complexity of hardware audio generation systems, much of the design and techniques used in products are often proprietary. Low-level design for MCUs, such as the STM32 Cortex M4 and M7 series, are often not well-documented or analysed. Open-source code [5] for these processors are often available for DIY projects, but are often minimal and leave many of the frequency domain effects and MCU optimisations unexplored. This thesis aims in designing such a system, in a well-documented and analysed fashion, that can be used as a building block for a wide array of more complicated products.

1.2. Problem statement

Wavetable-based audio generation software must be designed. The system must be able to play up to a fixed number of notes, each with an arbitrary frequency, by using on/off note triggers. This makes the system compatible with must forms of user note inputs, whether it be a button or MIDI messages. This system must be designed conceptually, and implemented in C, such that it considers MCU implementation. The system is especially targeted for ARM-based MCUs, such as the STM32 M4 and M7 series.

The system must produce high-quality stereo audio, while taking speed and memory into account. The system must include all the basic synthesis features: volume modulation; filtering and cut-off modulation; ADSR envelope control signals; FM; waveshaping. When all the basic synthesis techniques are designed, modelled and tested, this thesis must provide the foundation for implementation on MCUs, which can also be altered to produce a more complex product. Thus, the system must be designed in a way that allows for easy implementation on hardware, while being input-type and hardware agnostic, thereby considering the processes required to produce quality stereo audio from hardware.

1.3. System specification

1.3.1. Scope

The scope of this thesis is restricted to synthesis software only. This excludes the design and/or implementation of any hardware-related code, such as SAI drivers or MIDI decoders. Anything that cannot be tested and analysed in a entirely software-based workflow is out of scope. Thus, the audio data that is generated by the system must be stored in a file and analysed externally, using a platform such as MATLAB. The following items are within the scope of this thesis:

1. The audio-synthesis core software, that is triggered by note-on and note-off information.
2. The emulation of MIDI input by interpreting SMFs, for testing.
3. User-parameters that can be changed under test conditions to test system functionality.

Even though no hardware is designed, the hardware must be considered when designing the software, to ensure good support for a variety of topologies. Computational efficiency is emphasised in the design, but timing will not be tested, since it is hardware dependent (x86 vs ARM assembly, clock frequency, hardware peripheral speed, etc.).

1.3.2. Functional specification

The following items specify the audio generation capabilities, which has been formulated with reference to figure B.1, along with including some of the functionality of features provided by other eurorack modules:

1. The user can select the basic waveforms (sine, triangle, sawtooth and square) and interpolate between them, as is common in many wavetable synthesisers (see figure A.2).
2. The user can change the stereo width of the audio by detuning 2 additional oscillators per note and setting stereo width by panning and changing the volume of the additional oscillators.
3. An ADSR envelope, with user set parameters, must control the volume envelope of a single note over time.
4. The user can choose from a selection of filters: HP12, HP24, LP12, LP24 and BP12. The filters can have a user-defined Q, if applicable.
5. The user can set the filter cut-off frequency relative to the fundamental frequency of a note. This is to ensure that all notes have the same timbre at different frequencies. An ADSR envelope, with user set parameters, must control the cut-off frequency off the note over time.
6. The user can perform stereo wave-shaping on the audio, where they can specify a gain value into a waveshaper function. The waveshaper functions available must be the hyperbolic tangent and a sinusoid.
7. The user must be able to apply sinusoidal vibrato at a specified rate and amount.
8. The instrument must be polyphonic, i.e. can play multiple notes at once.

1.3.3. Technical requirements

The functional requirements stipulated in subsection 1.3.2 can be translated into technical requirements for the software audio core, which can be measured and designed. Since the hardware aspect is not explicitly

considered in design, the system must be able to scale according to hardware requirements and performance, so that a multitude of processors and hardware topologies can benefit from the design. The system is targeted for ARM-based MCUs, especially for STM Cortex M4 and M7, which both have FPUs available. However, the system should also efficiently function on other MCUs with included FPUs.

Care must be taken when writing mathematically-intensive code, to ensure optimal use of the hardware, such as leveraging FPU instructions, avoiding data duplication, utilising cache memory etc.

The technical requirements for the system are as follows:

1. An audio system that can generate buffers of a specified size of stereo audio data efficiently.
2. An arbitrary sampling rate may be specified. Common audio sampling rates [6] such as 44.1, 48 and 88.2 kHz can be used.
3. Care must be taken to avoid or minimise aliasing.
4. An arbitrary polyphony number (maximum number of notes) may be specified. A "generator" refers to the object that is generating the audio for a single note.
5. When the number of active notes exceeds the polyphony, the generator containing the oldest playing note must be re-triggered.
6. The system managing the generators must do so efficiently in $O(N)$ time, where N refers to the polyphony number.
7. FPU/CPU intensive operations such as divide or modulus operation must be avoided at all costs, if possible.
8. The DSP chain must consider efficiency, and simplify processing as far as reasonably practical.
9. The core software must be written in C, to allow compilation for any MCU, and to have full control over memory usage and predictability in the assembly code. Therefore, no object-orientation will be used, and functional data manipulation will be the prime consideration.
10. Processing speed takes priority over memory consumption, as far as reasonably practical.
11. An arbitrary wavetable buffer size can be specified, to provide control over the memory consumption and maximum harmonic content of a waveform (see subsection 3.4.2).
12. All DSP must be performed with single-precision floating-point operations, to allow for full 24-bit audio resolution for DAC conversion, which many SAI codex ICs support.
13. Filtering must be computationally efficient. This implies that IIR filtering must be used, since human hearing is insensitive to phase shifts of higher harmonics within the filtered signal.

1.4. Summary of work

The following items summarise the work that has been performed in this thesis:

1. A computationally efficient method of doing table-lookups with linear interpolation for periodic signals is created.
2. The effect of frequency scaling and linear interpolation is modelled in the frequency domain, forming the bases of predicting the effect of higher-order interpolation filters for wavetable frequency scaling.

3. A wavetable schema is designed to avoid aliasing for higher frequency notes.
4. FM and inter-wavetable interpolation was explored.
5. The design and efficient coefficient computation for 5 IIR filters is done.
6. LUTs for waveshaping and a technique for anti-aliasing is done through numerical analysis.
7. An ADSR state-machine that outputs a piecewise-exponential is created.
8. A way to create a note with stereo width is detailed.
9. An efficient way for managing generator objects is devised for managing note on/off triggers.

1.5. Report overview

Literature overview

The history and operation of synthesis is explored in this chapter. A detailed explanation of synthesis concepts and components are detailed by using existing synthesisers and/or eurorack modules as examples. The required musical, audio and mathematical knowledge for this thesis is detailed here. This chapter only covers existing techniques and knowledge, with some elaboration when necessary.

Design

This chapter details the design of the audio system. A top-down system specification with bottom-up component synthesis methodology is applied here. All the mathematics and self-developed techniques, and the application of existing techniques are documented in this chapter.

System testing

Each component is quantitatively tested in this chapter. Due to the complexity of this system, and the amount of user parameters, the whole system cannot be quantitatively tested, as the quality of sound is subjective. However, the nature of this system ensures correct operation if all the components are properly tested, since chaining the components together will not cause system failure. It is up to the user to set parameters, which can lead to **purposeful** system instability. This is similar to how a modular synthesiser system operates: correct system operation depends on the correct operation of the modules. Instead, a musical test is done, with some qualitative comments on the audio.

Conclusion

The design and its performance are discussed, with reference to the system specification. Possible improvements and further applications is detailed.

Chapter 2

Literature overview

2.1. A brief history of musical synthesisers

An instrument usually described as a “synthesiser” or “synth” is any electronic device or software that can generate audio signals. This can be done through a variety of techniques that have been developed and have evolved during the 20th and 21st centuries.

A non-keyboard style of synthesis - modular synthesis - was popularised by the companies Moog and Buchla in the 1960s [7]. Analogue electronics were used for synthesis where multiple modules generate control voltages in tandem to modulate parameters. Common building blocks are VCOs, envelope generators (ADSR), VCFs, VCAs, sequencers, wave-shapers, and noise generators. These are still the fundamental aspects of most synthesisers to date. Emulation or cloning of original Moog or Buchla hardware such as the 4-pole ladder VCF is still sought after in the current commercial market [8].

In contrast, modern Eurorack modules (a standardised modular hardware and electronics specification) offer a wide variety of complex options, which are often analogue or digital-analogue hybrid. An example is the Make Noise MATHS [9] module which is very popular in the modular synth community [10]. It provides features such as amplification, integration, summation, function generation, and is considered an essential module by many influencers.

Due to the complexity of analogue electronics, most synths were still monophonic in the 1960s and 1970s, or had very limited polyphony. Each extra note required duplication of electronics, which, in turn, requires more fine-tuning. The introduction of digital technology in the 1980s allowed for more flexible polyphony at affordable prices. The Yamaha DX7 is an incredibly well-known early digital synthesiser released in 1983 [11], which used FM-synthesis (see section 2.2). The DX7 was used on records by U2, Toto, Queen, Elton John, and jazz virtuoso Chick Corea. Sampling synthesis, which is very similar to wavetable synthesis, was utilised in the late 1980s by other digital keyboard products such as the Roland D-50, the Fairlight CMI, and drum machines used in the conception of the hip-hop genre.

The introduction of more powerful computation led to the development of software synths and VSTs for DAWs which use a variety of synthesis techniques such as FM, additive synthesis, subtractive synthesis, physical modelling, and wavetable synthesis. Wavetable synthesis is very popular in all music genres and sound design for film. Some of the most popular instrument VSTs used are Serum by Xfer Records [12], Massive by Native Instruments [13], and PIGMENTS by Arturia [14]. The aforementioned VSTs focus on wavetable synthesis with sampling, filtering, parameter modulation and FM capabilities.

2.2. An overview of synthesis techniques

Most techniques [15] are often combined in commercial products and operate in a similar way. The similarities, differences and operation principles will be explored in this section. This will also explore why wavetable synthesis can be considered the most flexible and computationally robust technique.

2.2.1. Additive synthesis (Fourier synthesis)

The principle of operation is based on the harmonic series of a time signal:

$$y(t) = \sum_{n=0}^{\infty} a_n \sin(\omega n t + \phi_n)$$

Various sinusoids are added together with different amplitudes and phases to produce a signal. The amplitudes and phases and frequencies may be time varying as well, which ties into physical modelling techniques that accounts for the time dependent timbre of most instruments. Modulating frequency directly ties in with FM synthesis.

Computing and adding many sinusoids in real-time can be computationally expensive – which can be reduced with a sinusoid LUT, as is directly done in wavetable synthesis. If time-varying amplitudes and phases are not present, additive synthesis can be completely replaced by wavetables (LUTs).

2.2.2. Subtractive synthesis

This technique is very simple and is possible in most synthesisers. It requires a harmonically rich source signal (generated by any means, such as direct computation, LUTs or analogue electronics) like a square wave:

$$y(t) = A \cdot \text{sgn}(\sin(\omega n t + \phi_n))$$

It consists of odd harmonics with amplitudes $a_n \propto \frac{1}{n}$. The source signal is then passed through a filter to further shape the harmonics. Any filter can be used. Time varying filters with modulated parameters are usually prevalent. This option is almost always present and/or possible to achieve in most synthesisers that offer filtering capabilities. Many products usually offer a selection of base waveforms which often includes most of or all the basic waveforms (sinusoid, triangle, sawtooth and square).

2.2.3. FM synthesis

This technique uses the same principles as FM for data communication, except in the audible frequency range. The resulting waveform is of the form:

$$y(t) = g(\omega(t))$$

where $g(\omega(t))$ is a periodic function with time-dependent frequency ω . This technique can produce unique and interesting results depending on the functions chosen for g and ω . Emulation of drum-like sounds such as toms and growling sounds often occurring in EDM genres are easily possible with this technique.

The choices offered for chosen for g and ω are product dependent but can often include the basic waveforms for g and ADSR envelopes and LFOs for ω . Multiple oscillators modulating each other's frequency, often in a coupled or recursive manner, is common, as in the stock Ableton VST plugin Operator [16], which is a FM-centric VST. Many non-FM-centric synths also offer a vibrato feature, which requires the use of dedicated vibrato LFO that slightly modulates the source signal's frequency. This is present in VSTs such as Omnisphere 2 by Spectrasonics [17], which is wavetable and sample-based.

FM synthesis is often combined with wavetable synths such as the Serum and Massive VSTs. It is also easily achievable in modular synth setups since most oscillator modules allow for controlling their frequency with a voltage signal.

2.2.4. Physical modelling

This method involves simulating the sound source of interest. It is usually separated in continuous models for bowed or blown instrument or impulsive models such as struck or picked instruments.

A variety of methods can be used, such as IR modelling, analytical simulation (differential equations), frequency domain modelling as mentioned under additive synthesis, and waveguide synthesis such as the Karplus-Strong plucked string algorithm [18] [19].

This type of synthesis is not relevant to the topic of this thesis.

2.2.5. Sampling

Sampling synthesis is the technique of using pre-recorded audio samples to reproduce sounds. An example would be to record every key of a piano at different volumes and then assigning a sample to trigger when conditions are met [19]. The Kontakt player by Native Instruments [20] is a popular sample player plugin into which third-party sample libraries can be loaded into to reproduce high-quality and realistic audio. High quality samples often take enormous amounts of effort to make, which results in a high commercial price point as can be seen in the Omnisphere 2 VST [17] and the Spitfire Audio Kontakt libraries [21].

Recorded samples can also be manipulated to increase or decrease their pitch, allowing for a wide variety of options to the performer. It is very similar to wavetable synthesis, where a predefined buffer (LUT) is used to generate sound. However, sampling often uses large buffers that are not necessarily intended to reproduce a periodic waveform (but sometimes do for continuous sound produced by instruments such as flutes), but instead a one-shot or partially looped triggered signal, ideal for percussive instruments. Samples and wavetables can be manipulated and modulated in the same way. This technique is computationally efficient but may require a large amount of memory to store the samples – often in the order of gigabytes, as for Kontakt libraries.

2.2.6. Wavetable synthesis

Wavetable synthesis is a very powerful, efficient and popular technique used in many modern synthesisers, which includes VST instruments, keyboards and modules.

A periodic waveform is stored in a table [15], which is sampled at a specific rate (see sections 3.4.1 and 3.4.2). A variety of these tables can be stored (even created by the user as in Serum) and manipulated by interpolating between wavetables or manipulating the wavetables themselves, such as with folding or adjustable duty cycle. This technique allows for complete freedom in parameter modulation. This includes techniques such as FM. Figure A.2 in the appendix shows an example of a wavetable VST synthesiser [16].

2.3. Basic modular synthesiser building blocks

This overview focuses on modular synthesiser building blocks directly but is relevant to most forms of synthesis (especially in this thesis) since most standard modern synthesis products are based on the building blocks popularised by Moog and Buchla [22]. Example modules will be shown, discussed, and compared to features present in commercial wavetable synthesisers and features to be considered for design in this thesis. Refer to appendix B.2 for an explanation of a basic modular synthesiser setup. Figure ?? in the appendix shows the modules that are used as examples [23] in this section.

2.3.1. The VCO

VCO modules commonly include 1V/octave CV inputs for frequency and provide the basic waveforms as outputs either separately via a switch or simultaneously. They can be analogue or digital in nature and can use a variety of synthesis techniques to generate their waveforms. They often come with the ability set the offset tuning voltage and can be used to create FM signals through control voltages. Extra features such as wave folding are sometimes also present.

The Doepfer A-111-3 Micro Precision VCO/LFO [24] is an analogue VCO that can also operate in LFO configuration, either with a linear or exponential voltage control. Sync (for phase/frequency syncing) and PWM CV inputs are also available. All the basic waveforms are present, except for the sinusoid which is notoriously difficult to generate with analogue electronics, which is commonly implemented by a high-Q unstable filter.

2.3.2. The VCF

The VCF is an incredibly important module that forms the basis of subtractive synthesis techniques. Most synths also offer filtering capabilities, such as the widely used Nord Stage 3 [25].

Filters can come in many types, often designed with unique characteristics. This can include special control voltage behaviour, feedback path saturation to limit resonance while adding additional harmonics, or the ability to achieve exceptionally high Q values that cause purposeful instability that allow filters to also function as a sinusoidal oscillator (which many VCOs do not generate).

Thus, filters for musical applications are usually not designed to be as “clean” and stable as possible. Instead, they focus on usability and uniqueness. Filter types can include a switchable LPF, BPF or HPF mode, a ladder filter, 12dB/octave or 24dB/octave varieties and a state variable filter configuration.

The IntelliJel UVCF [26] is popular state variable filter that simultaneously outputs a 2-pole low-passed, 2-pole high-passed and 1-pole band-passed signal which has a cut-off that can be modulated by 2 separate 1V/octave control voltages. It can also be set to have a high Q-value so that it can act as a sinusoidal VCO due to filter instability.

2.3.3. The VCA

The VCA has the primary purpose of performing the multiplication of signals for uses in AM and otherwise. It acts as an amplifier with a voltage controllable gain. It is often used in conjunction with an LFO to create a tremolo effect or with an ADSR envelope to shape the transient of signal to emulate bowing or plucking and removing clicks and pops that can occur with the immediate triggering of signal. Many VCOs only output a continuous signal. Hence, a VCA is required to mute any oscillators that are not triggered. The ring modulation effect can also be achieved by multiplying 2 signals in the audible frequency range together.

The MFB VCA [27] is module that has 3 different inputs and 2 CV inputs that modulate the gain. The operation of the various inputs is specific to this module and out of the scope of this thesis.

2.3.4. The ADSR envelope

The ADSR envelope is a critical component in synthesis used to achieve realistic sounds. It is often used to modulate filter cut-off to allow for dynamic subtractive synthesis. It is also used for AM to emulate the natural attack and decay characteristics of real instruments. It can emulate plucking, strumming, bowing, and blowing techniques found in real instruments. It can also be used in FM to recreate the typical pitch modulation found

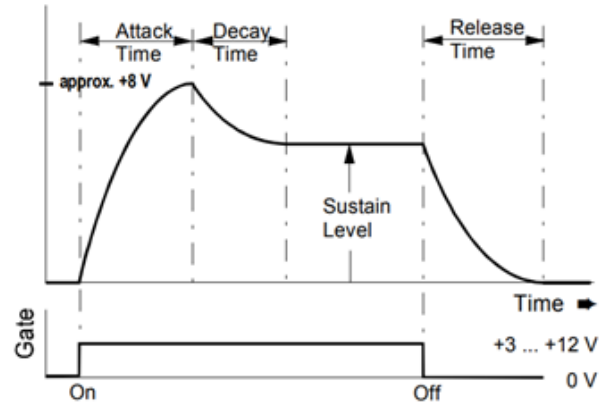
when striking percussive instruments. Due to the logarithmic nature of human hearing [28], an exponential shape is required of the envelope so that a linear change in volume is perceived.

ADSR envelopes are available in most wavetable synthesisers for parameter modulation, such as Serum, Massive and Ableton’s stock Wavetable VST instruments. Many keyboards also include this feature, such as the Nord Stage 3 [25]. The envelope consists of 4 phases. The curve is initiated with a gate “on” trigger after which a rising function is started. Once a threshold is reached, determined by the attack time, the decay state is activated. The decay is specified by a decay time parameter. The function decreases until a sustain level is reached, which is a parameter set by the performer. The sustain phase remains constant until the gate signal changes state to indicate an “off” trigger, initiating the release phase. The release phase is a decreasing function that decreases until zero is reached (or close to zero in the case of an RC circuit), determined by a release time parameter.

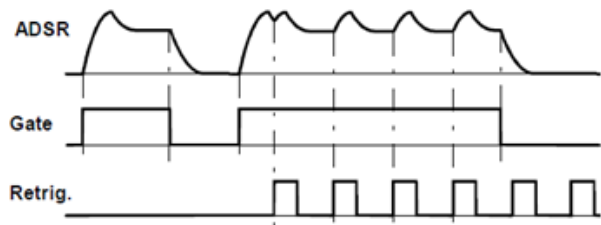
There are thus 4 parameters that can be set by the performer: attack time, decay time, sustain level and release time. The A, D and R phases are usually exponential functions implemented by an RC circuit.

This is well suited for AM and FM, since octaves are exponential in nature (doubling in frequency) and human hearing is logarithmic in nature [28] – an exponential volume change is perceived as linear.

The Doepfer A-140 ADSR Envelope Generator [1] is a classic envelope generator with a gate CV input, an envelope output, and a negated envelope output. It also has a retrigger input that allows the “on” trigger to occur again, reinitiating the attack phase, independent of the current phase of the envelope.



(a) The typical ADSR curve with gate signal.



(b) Typical re-triggering operation.

Figure 2.1: ADSR envelope operation [1].

2.4. Prerequisite knowledge

2.4.1. Equal temperament tuning

Various tuning systems have existed since the inception of standardised instruments. A common problem with dividing the octave into 12 notes (used in most Western music) is the inconsistent ratios between notes when a different root key is chosen. This problem is the result of using the natural harmonic series to define the ratios between pitches. I.e., a perfect fifth is the ratio 3:2, which was used to define the Pythagorean tuning.

There is no way to tune 12 notes in a scale that will result in equal integer ratios for all intervals across all notes and octaves. Thus, equal temperament tuning was introduced to solve this problem [29].

This tuning system uses $\sqrt[12]{2}$ as the relationship between semitones, resulting in equal ratios for all interval across all octaves. The standard for tuning is defined by the frequency of concert A (A4) to be 440 Hz. Consequently, the n 'th semitone after A4 is $440(\sqrt[12]{2})^n$, and the k 'th semitone before A4 is $440(\sqrt[12]{2})^{-k}$.

Furthermore, each semitone is divided into 100 equal steps, which is known as cents, with each cent differing from the next by a ratio of $\sqrt[1200]{2}$. This is a measure of the intonation of note.

A frequency ratio of $c \in \mathbb{R}$ cents, can be split into a combination of semitones ($x \in \mathbb{Z}$) and cents ($y \in [0, 100)$) as shown in equation 2.1.

$$2^{\frac{c}{1200}} = 2^{\frac{x}{12} + \frac{y}{1200}} = 2^{\frac{x}{12}} \cdot 2^{\frac{y}{1200}} \quad (2.1)$$

Where $x = \lfloor \frac{c}{12} \rfloor$ and $y = c - 12x$. Using this type of decomposition for a frequency ratio, allows for easy table lookups into semitone and cent LUTs to allow accuracy in frequency scaling on the cent level, i.e. when $y \in \{0, 1, \dots, 99\}$. Linear interpolation can then approximate these ratios between integer cent values. See code listing D.1 in the appendix for the C implementation.

2.4.2. Audio

Stereo

Most consumer audio is in a stereo format, i.e. having a left and right audio channel. It can then further be converted to mid (M) and side (S) channels using equations 2.2 and 2.3. The mid channel can be considered as the “mono’d” version of stereo audio, which is used by devices such as phone speakers, which cannot play stereo audio. The side channel can be considered as the signal containing all the stereo information, i.e. a measure of audio width. Audio processing is sometimes done on the mid and side channels instead of the left and channels. After processing, the left and right channels are reconstructed.

$$M = \frac{L + R}{2} \quad (2.2)$$

$$S = \frac{L - R}{2} \quad (2.3)$$

Note that if $L = R = y[n]$ (a mono signal played through both stereo channels), then $S = 0$, $M = y[n]$, implying that no stereo information is present. Using equations 2.2 and 2.3, we can then reconstruct the L and R signals using:

$$L = M + S \quad (2.4)$$

$$R = M - S \quad (2.5)$$

Quality

A variety of digital properties can determine the quality of the audio that is streamed. Prime considerations are sampling rate and bit depth. There are other considerations, such as dithering and encoding (PCM, DPCM, etc.) [30].

The sampling rate determines the bandwidth of the audio, as per the Nyquist criterion. Since human hearing is restricted to 20 Hz to 20 kHz [28], sampling rates for high-fidelity audio often exceed 40 kHz. Common rates are 44.1, 48 and 88.2 kHz [6]. The sampling rate is often higher than required, to avoid aliasing when processing the signals. The signal is sometimes up-sampled (often through linear interpolation) to double or quadruple the sampling rate before processing to combat aliasing.

The bit depth refers to the amount of bits used to store the audio data. The way it is stored depends on the encoding, which will yield different quantisation error probability distributions. The bit depth determines

the noise-floor of the signal, expressed via SQNR. Typical bit-depths are 16, 24 and 32 bits, with SQNRs of 96.33, 144.49 and 192.66 dB respectively [31].

2.4.3. Prototype IIR filters

A popular technique for designing an IIR filter is by using a continuous prototype filter $H(s)$ with critical frequencies $\omega_c = 1$ rad/sec, by utilising the bilinear transform and frequency scaling [2].

A prototype filters are converted to the Z-domain to be in the form

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{a_0 + a_1 z^{-1} + a_2 z^{-2}} = \frac{N_0 + N_1 z^{-1} + N_2 z^{-2}}{1 - D_1 z^{-1} - D_2 z^{-2}} \quad (2.6)$$

where N_0, N_1, N_2, D_1, D_2 are the coefficients normalised by a_0 . D_1 and D_2 are negated.

Converting back to the discrete time domain, the output of the filter ($y[n]$) can be calculated as

$$y[n] = N_0 x[n] + N_1 x[n-1] + N_2 x[n-2] + D_1 y[n-1] + D_2 y[n-2] \quad (2.7)$$

The negation of D_1 and D_2 proves useful in this circumstance, since now we can take advantage of the typical multiply-and-accumulate FPU instruction that many processors offer.

A list of substitutions to convert from the S to Z domain using the bilinear transform is shown in table C.1 in the appendix. The table also takes frequency mapping into account. The cut-off frequencies are mapped from 1 rad/s in the prototype filter to $\omega_0 = 2\pi \frac{f_0}{f_s}$ rad/sample (normalised digital frequency).

Chapter 3

Design

In this chapter, we apply bottom-up synthesis for designing our system, by designing, modeling and deriving behaviour of the lowest-level components first. When implementing code for this section, special consideration is made for compilation and FPU instructions. This will not always be discussed. See table C.3 in the appendix for a list of ARM FPU assembly instructions.

Only the most important code implementations are shown in this section. If implementation is trivial or large, the C code is listed in appendix D. Enough information will be by provided by the system block diagrams and accompanying equations to make other implementations possible.

This system was first partially implemented for integer arithmetic using Q-numbers, but was rewritten for floating-point operations after encountered problems, and extra design and implementation considerations. See appendix B.3 for details.

3.1. DSP pipeline

The system consists of a number of functional components which have been detailed in section 1.3.2. Figure 3.1 shows a high-level description of all of the components and the processes that they must perform. The components are numbered according to a top-down design approach, where number 1 refers to the highest level of operation between all components. Components are only dependent on other lower-level components.

3.1.1. Component definition

Each component can be defined in terms of its lower-level component dependencies, the required functionality and its inputs/outputs.

1. **generator manager:** this component manages the generators, by efficiently choosing an available generator, or the oldest playing generator if none are available. It receives note on/off requests, associates it with a generator, and does all triggering and configuration (such as frequency) that is required. The active generators can also be sampled, upon a audio buffer request. In here, generator samples are summed, scaled and clamped to a range $[-1, 1]$. This component uses a MIDI-note hashtable, and a generator queue and stack, to manage note associations and active/inactive generators.
2. **generator:** this component represents the collection of data that is required to produce stereo audio samples for a single triggered note. This component is responsible for retrieving and blending wavetable samples, applying waveshaping (and the anti-aliasing required), sampling the ADSR envelopes for volume and filter modulation, triggering the recalculation of filter coefficients and applying the filtering.
3. **Wavetable:** this component is responsible for all LUT lookups and inter-LUT interpolation, while also managing the periodicity (as per subsection 3.3), frequency and harmonic content of a periodic waveform. It can receive FM input for vibrato. It produces mono samples that is to be used in conjunction with other wavetables in stereo blending.

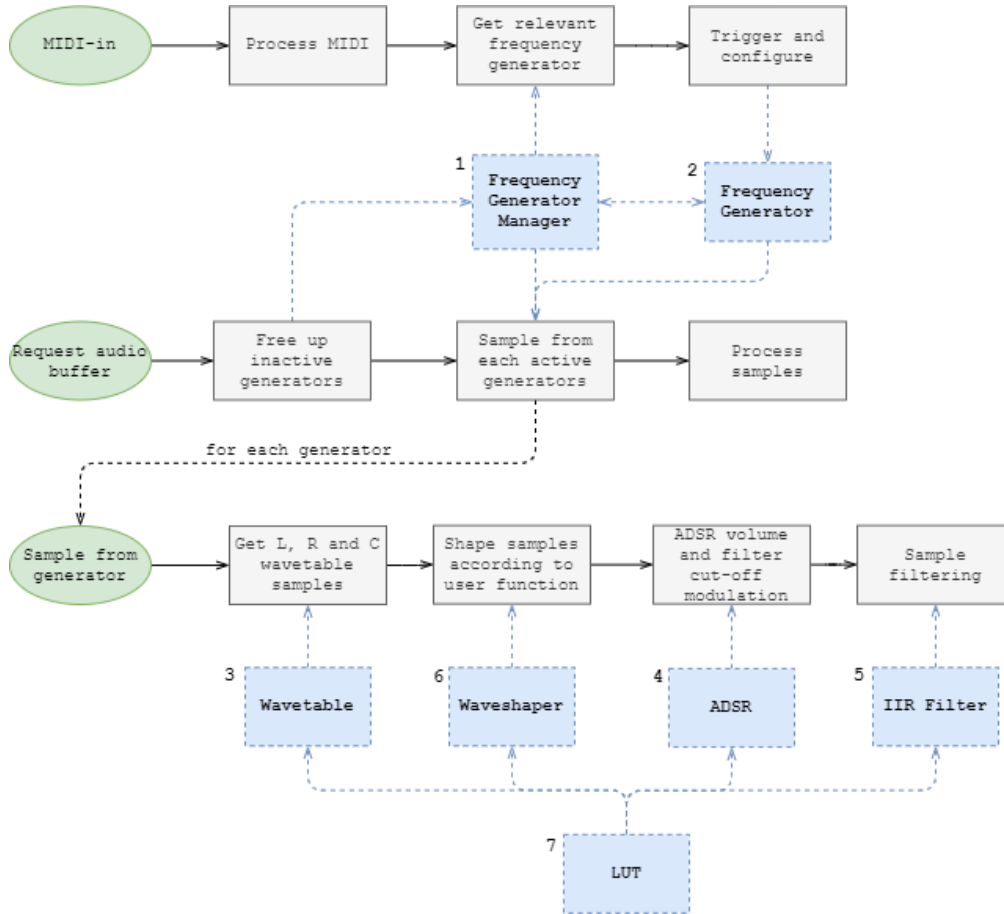


Figure 3.1: High-level system description.

4. **ADSR**: this component produces mono samples from an ADSR curve, using table lookups into an exponential function LUT. It is a state-machine that manages the piece-wise exponential ADSR curve, given ADSR parameters. It can be queried to determine whether the release phase has finished. It can receive on and off triggers, and can be re-triggered to the attack phase, at any moment, if necessary.
5. **IIR filter**: this component contains all the functions from different filter types that can calculate the IIR coefficients. It stores all necessary sample delays, for each filter instance, that are required to perform filtering. It can apply filtering to an input signal, and has the ability to be reset (sample delays set to 0).
6. **Waveshaper**: this component can apply a waveshaping function (either sin, tanh, or none) to an input. It does not handle anti-aliasing (the generator does). It is responsible for doing lookups into the function LUTs and managing periodicity with the sin lookup, and out-of-bounds tanh lookups.
7. **LUT**: this component does a singular table lookup, with linear interpolation, into any array, without checking for an out-of-bounds index. Ensuring a within-bounds lookup index the responsibility of the parent components. This component is the core of the audio engine, and must be fast and efficient.

3.1.2. Global user parameters

From the functional specification in section 1.3.2, we can determine all the global parameters that is applicable to every playing note. These parameters must be separated from the data contained within the components, so that it can easily be changed during an audio stream, to provide the performer with continuous feedback.

Table 3.1 summarises all the global parameters, indicating to which category of processing they are

relevant, and the units they must be stored in. Note that the attack, decay and release parameters are stored in terms of digital frequency, and not seconds. This is so that LUT lookup pointers can easily be updated. All frequencies in this system are stored in digital frequency, as to remain sampling-frequency-agnostic.

Table 3.1: All global parameters

Category	Parameter	Unit(s)
<i>Volume</i>	Envelope: attack, decay, release	cycles/sample
	Envelope: sustain	-
<i>Filtering</i>	Envelope: attack, decay, release	cycles/sample
	Envelope: sustain	-
	Relative frequency start	-
	Relative frequency end	-
	Q	-
	Type	-
<i>Detune</i>	Amount	cents
	Stereo width	-
	Volume	-
<i>Vibrato</i>	Frequency	cycles/sample
	Intensity	cents
<i>Waveshaping</i>	Input gain	-
	Type	-

3.2. Top-level system in MCU implementation

The top-level system is the entry-point of MIDI messages and is illustrated in figure 3.3. In this part of the system, MIDI commands are parsed and added to a queue. An audio buffer request is made, where a fixed amount of stereo samples are retrieved from the generators and stored in a buffer. This is the system that is to be implemented on a MCU, and is thus not within the scope. Instead, a SMF will be used to provide input, used in section ?? to demonstrate functionality using musical test, which will output sound to a wav file.

Figure 3.2 shows the legend that should be used to interpret the block diagrams shown in this chapter.

The note queue is only processed at the beginning of a buffer request, thus adding latency into the system, that the performer will experience. The latency (l) that is added to the system is a function of the size of the audio buffer (N), specified in number of stereo samples and the sampling rate (f_s).

$$l = \frac{N}{f_s} \quad (3.1)$$

The added latency must be minimal to ensure no effect on live performance. Typically, latencies of 10 ms to 15 ms are acceptable and unnoticeable to performers. Thus, at a sampling rate of 44.1 kHz, a maximum size of 661 stereo samples are acceptable for the buffer.

Also note that a note on/off trigger is restricted to a minimum time equal to the latency, otherwise a note on/off trigger pair will cancel out, since they are both processed at the same time. Furthermore, extra equipment in the signal processing chain, such as external effect and amplifiers, may add more latency. Thus, it is better to choose an audio buffer size of around 2 ms, so that the instrument can still be real-time-usable in conjunction with an external signal chain.

The choice of buffer size is not relevant in this thesis, except for note triggering time, since system testing is not done for a real-time application. The system is designed to be compatible for any choice of buffer size

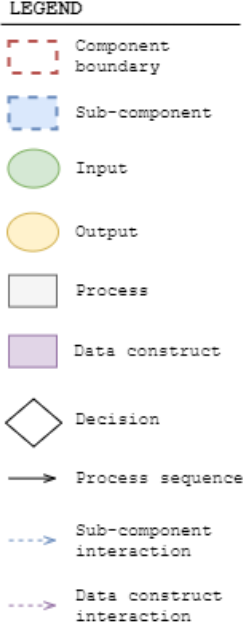


Figure 3.2: System block diagram legend.

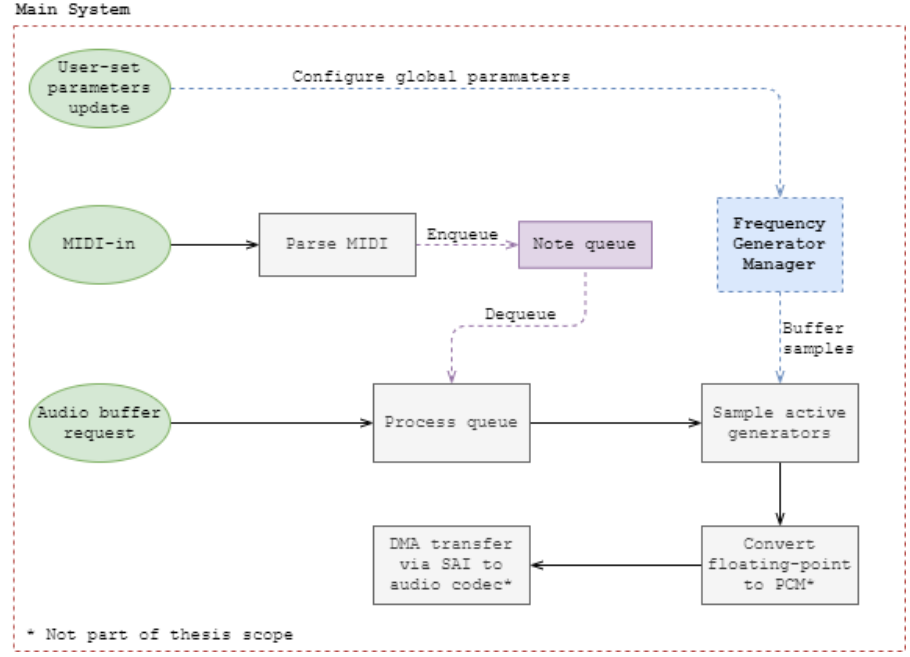


Figure 3.3: The top-level system block diagram.

and sampling rate. Choosing these must be done with the hardware platform in mind. Generally, a higher sampling rate will reduce aliasing, which must be leveraged if the hardware has enough processing power.

3.3. LUT

3.3.1. Linear interpolation

Linear interpolation is process of defining a function in terms of points, and then connecting the points in a piecewise-linear fashion. This is especially relevant for LUT lookups, when a value between points is required for better accuracy. This is the core of the system. Figure 3.4 shows the block diagram for this system component.

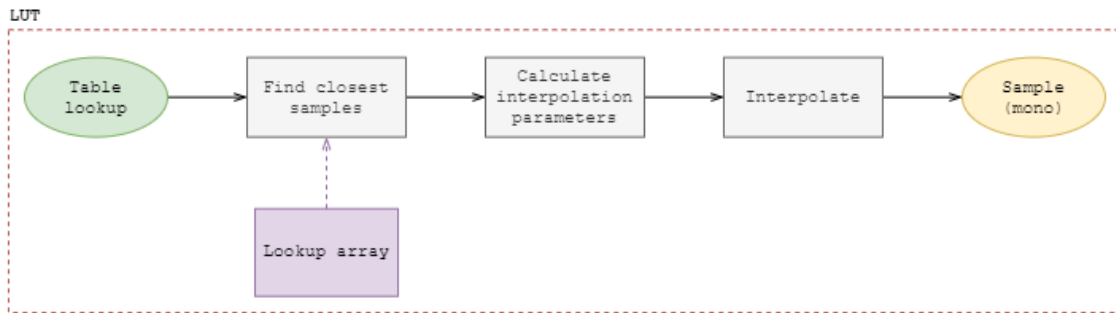


Figure 3.4: LUT system block diagram.

The linear interpolation (lerp) function is defined as follows:

$$\text{lerp}(x_1, x_2, \delta) = x_1 + \delta(x_2 - x_1) \quad (3.2)$$

Where x_1 is the starting point, x_2 is the endpoint, and $\delta \in [0, 1]$ is the interpolation distance.

Assuming that we have a LUT ($L[n]$) storing N samples indexed with $n < N$, $n \in \mathbb{N}_0$, we can find a linearly interpolated point $p(i)$ at position $0 \leq i < N - 1$, $i \in \mathbb{R}$ using equation 3.2.

$$p(i) = \text{lerp}(L[\lfloor i \rfloor], L[\lfloor i \rfloor + 1], \{i\}) \quad (3.3)$$

If periodic behaviour is required from the LUT, we can wrap i around when it exceeds the limits, either through using the fractional function or the modulus function for real numbers.

The modulus operation can be defined through floored division as shown in equation 3.4, which can also provide an extension of the function into the real numbers for $a, r, n \in \mathbb{R}$.

$$a \equiv r \pmod{n} \Leftrightarrow r = a - n \lfloor \frac{a}{n} \rfloor \quad (3.4)$$

Equation 3.4 can be implemented efficiently in code if n is a power of 2, and $a \in \mathbb{N}_0$. Listing 3.1 shows the C implementation for this function. Note that the use of the “inline” keyword is a C++ compiler directive, but is only used to increase the speed of the program by avoiding branch penalties when calling the function. We thus restrict all periodic LUTs to have size that is a power of 2.

```
1 inline uint16_t fast_mod(uint16_t x, uint16_t mod) {
2     return x & (mod - 1);
3 }
```

Listing 3.1: Fast modulus for a power of 2

Listing 3.2 shows the implementation of equation 3.3 and figure 3.4 in C, where special care has been taken to ensure that if $i \in [N - 1, N)$, the interpolation will be performed between the samples $L[N - 1]$ and $L[0]$. This is required for interpolating periodic LUTs.

```
1 inline float lut_lookup(float lut[], uint32_t lut_size, float i) {
2     uint32_t floor_i = (uint32_t) i;
3     float delta = i - (float) floor_i;
4     float x1 = lut[floor_i];
5     float x2 = lut[fast_mod(floor_i + 1, lut_size)]; //wraps i to 0 if i = lut_size
6     return lerp(x1, x2, delta);
7 }
```

Listing 3.2: LUT lookup with linear interpolation

The fractional function can also be used to ensure periodicity. Equation 3.5 shows the definition of the fractional function and its range.

$$\{x\} = x - \lfloor x \rfloor, \{x\} \in [0, 1) \forall x \in \mathbb{R} \quad (3.5)$$

The fractional function is periodic with a frequency of 1 Hz, and contains a straight line starting at (0,0) and approaching (1,1) over $x \in [0, 1)$. Thus, we can create a function ($W(t)$) that wraps the index so it repeats with a period of p , with a shifting factor (k), and an amplitude (A), shown in equation 3.6. This can be used to easily wrap the index into a LUT of size A , with p defining the input range of t , with k setting the starting lookup index ($W(t = 0)$).

$$W(t) = A \left\{ \frac{1}{p} (t - k) \right\} \quad (3.6)$$

Note that when $k = \frac{p}{2}$, then a period of $W(t)$ is contained in $t \in (\frac{-p}{2}, \frac{p}{2})$, having a range $W(t) \in [0, A)$, and a starting lookup index $W(t = 0) = \frac{A}{2}$.

3.3.2. Constructing the basic waveform LUTs

In this section we will investigate how to generate a Fourier series of the basic waveforms (square, triangle and sawtooth) for LUT look-ups and explore memory optimisation techniques. The waveform of interest will be represented as the following series:

$$y[n] = \sum_{k=1}^K a_k \sin(2\pi k \frac{n}{N}) \quad (3.7)$$

Where k is the wavenumber, a_k is the k 'th amplitude coefficient, K is the number of harmonics, and $N = 2^b$ is the size of the LUT that will store a complete period of the waveform, i.e. $0 \leq n < N$, $n \in \mathbb{N}_0$.

With reference to equation 3.7, the coefficients a_k can be derived as follows:

1. Square:

$$a_k = \begin{cases} \frac{1}{k}, & k \equiv 1 \pmod{2} \\ 0, & k \equiv 0 \pmod{2} \end{cases} \quad (3.8)$$

2. Triangle:

$$a_k = \begin{cases} \frac{(-1)^{\frac{k-1}{2}}}{k^2}, & k \equiv 1 \pmod{2} \\ 0, & k \equiv 0 \pmod{2} \end{cases} \quad (3.9)$$

3. Sawtooth:

$$a_k = \frac{(-1)^{k+1}}{k} \quad (3.10)$$

Equations 3.8 and 3.9 imply only odd harmonics, hence for harmonic index (see equation 3.14) $i = 0$, it will represent a simple sinusoid.

Furthermore, since equation 3.7 is a linear combination of harmonic sinusoids with no phase-shift, and $a_k > 0$ for all odd k , $a_k \leq 0$ for all even k in equations 3.8 to 3.10, we can conclude that all basic waveforms and their harmonics are in phase. This makes it possible to interpolate between wavetables (as seen in Ableton's Wavetable VST in figure A.2) without phase cancellation. Furthermore, to allow for range predictability, the samples are normalised so that the range falls within $[-1, 1]$.

The sinusoid is trivial to construct, since it does not contain any other harmonics. Thus, we can store the 4 basic waveforms in a $4 \times K \times N$ array. "4" corresponds to the basic waveforms, ordered by increasing harmonic content (sine, triangle, sawtooth, square); K refers to the amount of copies with different harmonics we want to store of each waveform; $N = 2^b$, $b \in \mathbb{N}$ is the amount of samples per waveform period. The 3D array allows for code simplicity when each note requires an harmonic index and inter-wavetable interpolation (see section 3.4).

In this case, we are duplicating the sinusoid LUT K times. However, we prioritise speed and code simplicity over memory consumption. Adding code to deal with the sinusoid specifically could incur branch penalties, and will require a special array for the sinusoid, which will increase code complexity.

3.4. Wavetable

This component is responsible for generating a specific frequency of an interpolated waveshape using the basic waveform LUTs. Figure 3.5 shows the block diagram.

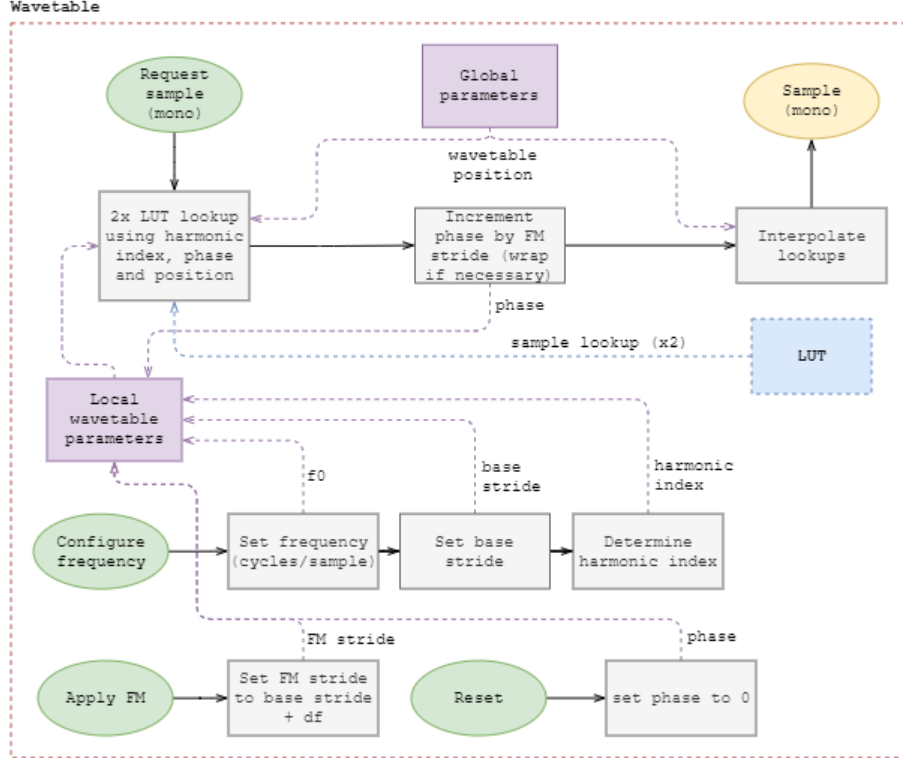


Figure 3.5: Wavetable system block diagram.

Each wavetable requires its own data structure to store relevant values, which includes stride and base stride ($\text{stride} \propto \eta$, see equation 3.13), phase (ϕ), and harmonic index (i). Base stride refers to the center frequency of the wavetable, whereas stride includes FM.

Each sample request results in the appropriate wavetable lookup and inter-wavetable interpolation. The phase is incremented after each request. The frequency must also be configured before use, required to calculate the harmonic index. A reset trigger also resets the phase. The rest of this section provides further explanation, derivation and insight into these parameters.

3.4.1. Modelling wavetable frequency conversion

This section focuses on modelling wavetable sampling and determining the effects of linear interpolation and frequency scaling in the frequency domain.

Suppose we want to store a periodic signal $y[n]$ in a buffer consisting of 2^b samples, where $b \in \mathbb{N}$. The fundamental frequency of the buffer as normalised digital frequency (cycles per sample) is thus 2^{-b} .

If we require the buffer to store H harmonics including the fundamental, we use the Nyquist frequency to determine the maximum number of harmonics the buffer can store. This can be specified by the user of the code, according to the memory restrictions of the hardware platform.

$$H_{\max} 2^{-b} = 0.5 \Rightarrow H_{\max} = 2^{b-1} \quad (3.11)$$

Therefore, a bigger buffer size results in the ability to store more harmonics.

Now we consider the follow process shown in figure 3.6 to scale a sampled signal's frequency ($y[n]$) by a factor of $\eta = \frac{M}{L}$. The signals in the frequency conversion process is modelled as $\hat{y}[n] = (y[n]_{\uparrow L} * h[n])_{\downarrow M}$.

As per the scaling theorem [32], the frequency axis of $DTFT\{y_1[n]\}$ contracts by a factor L . A LPF is used to remove unwanted copies of the spectrum, which would result in aliasing if the frequency axis is



Figure 3.6: Frequency scaling using upsampling and downsampling

expanded by a factor M through downsampling.

There are many choices for the LPF, but for arbitrary frequency scaling, M and L will become large to achieve close approximation for any real number. The simplest filter choice would be the linear interpolator, which is a FIR filter characterised by its impulse response [32]:

$$h[n] = \text{tri}\left[\frac{n}{L}\right] = \frac{1}{L} \text{rect}\left[\frac{n}{L}\right] * \text{rect}\left[\frac{n}{L}\right]$$

Taking the DTFT,

$$H(f) = \frac{1}{L} \text{DTFT}\left\{\text{rect}\left[\frac{n}{L}\right]\right\}^2 = \frac{\sin^2(L\pi f)}{L \sin^2(\pi f)}$$

Note that $H(f) = 1$ for $L = 1$. For $L \neq 1$, it has zeroes at $f = \frac{p}{L}, p \in \mathbb{N} \setminus \{0\}$. Figure 3.7 shows the effect of the linear interpolation process in the frequency domain. The bandwidth of $y[n]$ is represented as a triangular pulse.

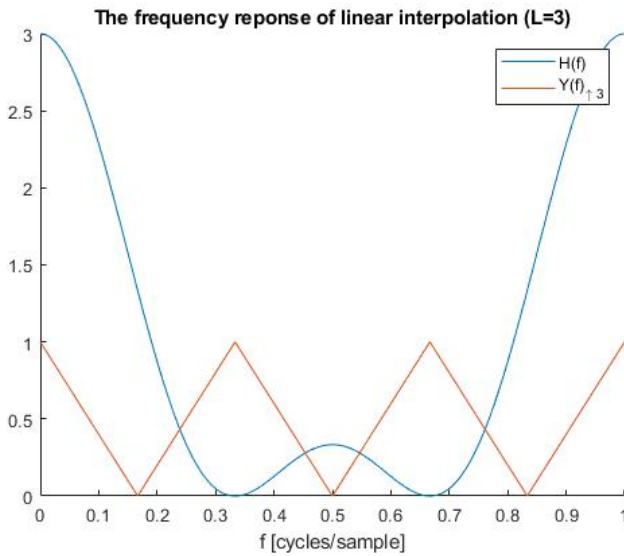


Figure 3.7: The effects of linear interpolation in the frequency domain.

With reference to figure 3.7, notice that $H(f)$ has nulls at the DC component of the upsampled signal spectrum. All the frequencies that are close to DC, i.e. lower frequencies, have high attenuation. Thus, when the sampling rate is increased, the spectral distortion is not affected, since the wavetable fundamental is fixed at 2^{-b} cycles/sample. Increasing b , however, will lower the fundamental frequency and result in less harmonic distortion when interpolating lower frequencies.

The linear interpolation filter has a fixed cutoff which is a function of L . When the frequency is scaled up, i.e. $M > L$, the original spectrum with a full bandwidth will alias since the linear interpolation filter does not account for this. To combat this issue, we can use a variety of buffers for a single waveform, each containing a different number of harmonics, thus band-limiting the signal to combat aliasing for when $M > L$. This problem is not present for $M < L$, i.e. playing lower frequencies than 2^{-b} cycles/sample. Below this threshold, the maximum harmonics played back is limited by b .

Assuming that the waveform is bandlimited to B cycles/sample, we need to ensure that

$$\frac{M}{L}B \leq 0.5$$

as per the Nyquist criterion. Furthermore, B is determined by the number of harmonics h , i.e. $B = h2^{-b}$.

$$h2^{-b} \leq \frac{L}{M}0.5 \Leftrightarrow h \leq 2^{b-1} \frac{L}{M} = 2^{b-1} \frac{1}{\eta} \quad (3.12)$$

We can find the maximum number of harmonics (including the fundamental) h_{max} for a given digital frequency f_0 . First, we find the required frequency scaling factor:

$$2^{-b}\eta = f_0 \Rightarrow \eta = f_02^b \Rightarrow \eta = Nf_0 \quad (3.13)$$

Substituting equation 3.13 into equation 3.12, we find

$$h \leq \frac{0.5}{f} \Rightarrow h_{max} = \frac{0.5}{f_0}$$

If we use K buffers to store a varying number of harmonics, with the buffers indexed with $i \in \{0, 1, \dots, K\}$, and store 2^{i+1} harmonics in each buffer, we are restricted to $K_{max} = b - 2$ as per equation 3.11. Note that this is arbitrary, and only to avoid large memory consumption for storing a linear increase in harmonics. If the MCU has external memory available, it should be considered to store as many waveforms with differing harmonics as possible, using a linear increase instead.

We can find the closes index of a given digital frequency f_0 which will contain the maximum number of harmonics (h_{max}), without aliasing, stored in the buffer i as follows:

$$i = \min(\lfloor \log_2(h_{max}) \rfloor, K) = \min\left(\left\lfloor \log_2\left(\frac{0.5}{f_0}\right) \right\rfloor, K\right) \quad (3.14)$$

Equation 3.14 can be optimised by using a LUT and further memory considerations, to avoid an expensive call to a logarithmic function. See appendix B.5 for details. The C implementation for configuring a wavetable is shown in listing D.2 in the appendix.

Note that the minimum number of stored harmonics is 2 ($i = 0$). Thus we can only play frequencies up to a max of 0.25 cycles/sample. The highest note on an 88-key piano (C8) is 4186.01 Hz in equal temperament tuning. At a standard audio sampling rate of 44.1 kHz, this corresponds to $f_0 = 0.095 < 0.25$ cycles/sample. Almost all of the MIDI notes (see table C.2 in the appendix) are covered in this range, with the highest non-aliased note being E9 (10.54808 KHz), which corresponds to $f_0 = 0.239 < 0.25$ cycles/sample. Arguably, this is well outside the commonly played range. At 44.1 KHz, the only aliased MIDI notes are F9, F#9 and G9. This problem is avoided at a sampling rate of 48 kHz.

3.4.2. Implementing frequency scaling

Frequency scaling can be trivially implemented without any upsampling, downsampling or explicit filtering. The frequency scaling factor η (from equation 3.13) can be used to update a pointer into a LUT ($W[i]$) storing a periodic waveform.

For a table consisting of 2^b samples, the pointer will be considered as the phase $\phi[n]$ of the waveform. The phase must be updated using:

$$\phi[n] = \text{mod}(\phi[n-1] + \eta, 2^b) \quad (3.15)$$

Here, the frequency scaling factor η represents the amount that the table pointer must increase by, which is known as **stride**.

The modulus operation ensures that the phase pointer never falls outside of the range of indices of the table, effectively creating the periodicity required. However, we do not expect $f_0 > 0.5$, thus we can simplify 3.15 to:

$$\phi[n] = \begin{cases} \phi[n-1] + \eta, & \phi[n-1] + \eta < 2^b \\ \phi[n-1] + \eta - 2^b, & \phi[n-1] + \eta > 2^b \end{cases} \quad (3.16)$$

Which eliminates using an expensive floating-point modulus operation.

With $\phi[n] \in [0, 2^b)$ due to the modulus operation, the samples of $W[i]$ can be linearly interpolated to $L[n]$ using equation 3.3:

$$L[n] = \text{lerp}(W[\lfloor \phi[n] \rfloor], W[\text{mod}(\lfloor \phi[n] \rfloor + 1, 2^b)], \{\phi[n]\}) \quad (3.17)$$

Equation 3.3 is easy to implement in code, since the floor of a positive floating-point number can be determined through integer conversion. The fractional part of a floating-point can be determined using 3.5.

Using an if-statement to detect wrapping to the beginning of the table is also an alternative, but not necessarily as efficient, depending on branch penalties in the processor.

3.4.3. Implementing inter-wavetable interpolation

From subsection 3.3.2, the wavetables are stored in a 3D array, notated as $W[t, k, n]$, where $t \in \{0, 1, 2, 3\}$ is the wave type, k is the harmonic index and n the sample index.

Interpolation between 2 wavetables can be specified by $\hat{t} \in [0, 4)$, which is the continuous extension of t . Interpolation can then be done similar to equation 3.17. Equation 3.18 shows this, also allowing wrapping to occur from the square to the sinusoid LUTs. Listing 3.3 shows the C implementation for wavetable sampling.

$$W(\hat{t}, k, n) = \text{lerp}(W[\lfloor \hat{t} \rfloor, k, n], W[\text{mod}(\lfloor \hat{t} \rfloor + 1, 4), k, n], \{\hat{t}\}) \quad (3.18)$$

```

1 inline float wt_sample(wavetable* wt, gen_config* gc) {
2     uint8_t t = (uint8_t)gc->wt_pos;
3     uint8_t tp1 = t + 1;
4     np1 = fast_mod(np1, 4); //wrap LUT (square to sine)
5     float x1 = lut_lookup(basic_luts[t][wt->harmonic_index], LUT_SIZE, wt->phase);
6     float x2 = lut_lookup(basic_luts[tp1][wt->harmonic_index], LUT_SIZE, wt->phase);
7     wt->phase += wt->stride;
8     if (wt->phase > (float)LUT_SIZE) wt->phase -= (float)LUT_SIZE; //wrap phase
9     return lerp(x1, x2, gc->wt_pos - (float)t);
10 }
```

Listing 3.3: Sampling from a wavetable

3.4.4. Applying FM

FM can be applied by modifying the the base stride for each sample. Similar to conventional FM techniques, we have a center/base frequency f_0 , which is modified by by a modulation frequency $f_{FM}[n]$, with $\eta_0 = Nf_0$ and $\eta_{FM}[n] = Nf_{FM}[n]$ as per equation 3.13.

Thus, we store our base stride (η_0) as part of the internal wavetable state, which is set during a note-on trigger. We then have our final stride expressed as $\eta_f[n] = \eta_0 + \eta_{FM}[n]$.

3.5. IIR filtering

As detailed in section 2.2, filters are an essential component in many synthesisers. Filter cut-off can be controlled by external modulation sources. Thus, fast filter coefficient calculation is paramount, since it is calculated on a per-sample basis.

To achieve maximal speed, division operations must be avoided as far as possible, since most processors do not have single-cycle division operations. Furthermore, a biquad IIR filter can be used for speed. This also reflects the typical 2-pole filters often used in analogue synthesisers. The 1-pole filter will also be analysed, since it is often used for a “softer” roll-off and is required for anti-aliasing techniques in waveshaping, explored in section 3.6.

In this section, the 3 most common 2-pole filter types (HP24, BP12, LP24) along with 1-pole filters (HP12, LP12) for synthesis, will be detailed.

This section uses the techniques mentioned in section 2.4.3. Equations 3.19 to 3.23 are the prototype HP24, LP24, BP12, HP12 and LP12 filters respectively.

$$H_{hp24}(s) = \frac{s^2}{s^2 + \frac{1}{Q}s + 1} \quad (3.19)$$

$$H_{lp24}(s) = \frac{1}{s^2 + \frac{1}{Q}s + 1} \quad (3.20)$$

$$H_{bp12}(s) = \frac{s}{s^2 + \frac{1}{Q}s + 1} \quad (3.21)$$

$$H_{hp12}(s) = \frac{s}{s + 1} \quad (3.22)$$

$$H_{lp12}(s) = \frac{1}{s + 1} \quad (3.23)$$

The coefficients of the digital biquad filter can be calculated for the prototype filters. The denominator coefficients are the same for the HP24, LP24 and BP12 filters, whereas the denominator coefficients of the HP12 and LP12 are the same. It is summarised in table 3.2 below.

Table 3.2: Digital biquad filter denominator coefficients

Filter	a_0	a_1	a_2
HP24, LP24, BP12	$2Q + \sin(\omega_0)$	$-2 \cdot 2Q \cos(\omega_0)$	$2Q - \sin(\omega_0)$
HP12, LP12	$(1 - \cos(\omega_0)) + \sin(\omega_0)$	$2(1 - \cos(\omega_0))$	$(1 - \cos(\omega_0)) - \sin(\omega_0)$

The numerator coefficients are shown in table 3.3 below.

Table 3.3: Digital biquad filter numerator coefficients

Filter	b_0	b_1	b_2
HP24	$-\frac{1}{2}b_1$	$-2Q(1 + \cos(\omega_0))$	$-\frac{1}{2}b_1$
LP24	$\frac{1}{2}b_1$	$2Q(1 - \cos(\omega_0))$	$\frac{1}{2}b_1$
BP12	$Q \sin(\omega_0)$	0	$-b_0$
HP12	$-\sin(\omega_0)$	0	$-b_0$
LP12	$\frac{1}{2}b_1$	$2(1 - \cos(\omega_0))$	$\frac{1}{2}b_1$

Calculating the coefficients can be optimised by using a LUT for cosine and sine approximation, and storing $2Q$, $\cos(\omega_0)$, $\sin(\omega_0)$, and $1 - \cos(\omega_0)$ as intermediate values. Unfortunately a single division

operation is unavoidable. All coefficients must be normalised by a_0 to obtain the filter form of equation 2.6. This scaling factor can also be stored as an intermediate value and then further used via multiplication.

Listing D.3 in the appendix shows the C implementation for calculating the LP24 filter coefficients using tables 3.2 and 3.3. Implementation for other filters are similar, utilising pre-negation of a_1 and a_2 to save on MCU clock cycles, storing intermediate values and utilising trigonometric LUTs, which are gained without extra memory consumption since a sinusoid is already stored as a basic waveform. The cosine LUT is also gained without extra memory consumption by using the identity $\cos(x) = \sin(x + \frac{\pi}{2})$. The trigonometric lookup functions' C implementations are shown in appendix listing D.5.

Filtering is then done using equation 2.7. The C implementation is shown in code listing D.4 in the appendix.

3.6. Waveshaper

As per the functional requirements, waveshaping will be done by one of 2 user-chosen functions: the hyperbolic tangent or the sinusoid. LUTs will be used to perform the waveshaping.

The sinusoidal LUT is already present in the basic waveforms, so no extra memory consumption is required. The hyperbolic tangent function will require its own LUT.

Shaping a discrete input function $x[n]$ to $y[n]$ with a continuous waveshaping function $w(x)$ can be described with $y[n] = w(g \cdot x[n])$, where g is the gain into the waveshaping function. If w is an odd function, it will only add odd harmonics, whereas an even function only adds even harmonics. Both of the considered waveshaping functions are odd, so only odd harmonics will be added. The waveshaping function can be considered as a means to add signal distortion to our system.

Adding extra harmonics introduces aliasing into the system, since the waveshaping functions will add harmonics with amplitudes proportional to the input gain g . Aliasing can be reduced in a variety of ways, such as bandlimiting our signal into the waveshaping function, or by oversampling the input signal, or both. Even though oversampling is viable, it will require more processing. Thus, for simplicity and speed, only the bandlimiting option was considered. The system block diagram for this component is shown in figure 3.5.

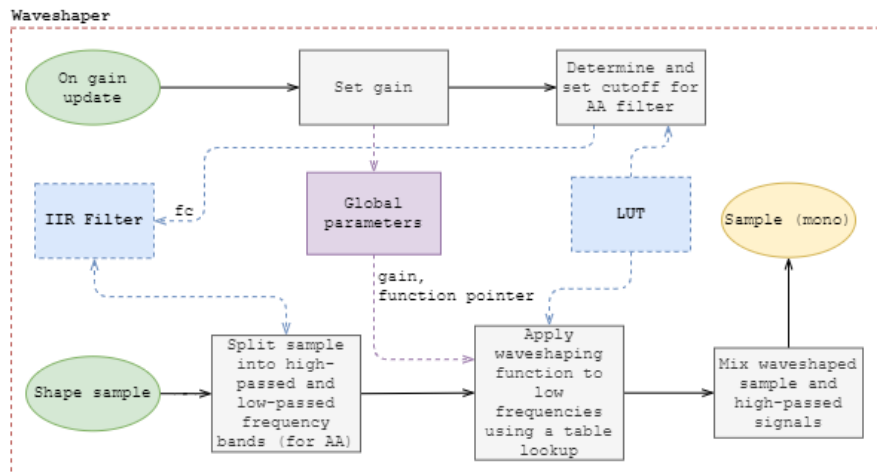


Figure 3.8: Waveshaper system block diagram

3.6.1. Constructing the hyperbolic tangent LUT

To construct a buffer with $N + 1$ samples indexed by $n \in \{0, 1, \dots, N\}$ storing the hyperbolic tangent, we need to consider some of the properties of $\tanh(x)$:

$$\lim_{x \rightarrow +\infty} \tanh(x) = 1 \quad \text{and} \quad \lim_{x \rightarrow -\infty} \tanh(x) = -1 \quad (3.24)$$

$$\frac{d}{dx} \tanh(x) = \text{sech}^2(x) \coth(x) \quad (3.25)$$

$$\lim_{x \rightarrow \pm\infty} \frac{d}{dx} \tanh(x) = 0 \quad (3.26)$$

We can approximate the behaviour of this function with $y[n]$ by storing a section of $\tanh(x)$ mapped to $x = \frac{n}{N} \in [0, 1)$, and then approximating the asymptotes in equation 3.24 by straight lines for $x > 1$ and $x < 0$. Shifting and scaling the function slightly is required, achieved by A (amplitude scaling) and k (input axis scaling). The function is also shifted right by $\frac{1}{2}$ to fit most of the function behaviour in $[0, 1)$.

$$y[n] = A \tanh(k(\frac{n}{N} - \frac{1}{2})) \quad (3.27)$$

Note that the gradient $y'[\frac{N}{2}] = k$. Thus, k is also a measure of the gradient steepness at $y[\frac{N}{2}] = 0$. We want $y[N] = 1$ and $y'[N] = \varepsilon$, where ε is the acceptable gradient error close to 0, from equations 3.25 and 3.26. Solving for these boundary conditions yields $A = \coth(\frac{1}{2}k)$, and values for k as a function of ε is shown in table 3.4.

Table 3.4: Axis scaling values and gradient errors for constructing the hyperbolic tangent LUT.

ε	k
0.1	5.3697
0.01	8.0810
0.001	10.6606
0.0001	13.1750

It should be noted that a higher k corresponds to better accuracy at the edges, but less accuracy within the curve around the x -intercept, since fewer sample points are available there for linear interpolation, due to axis scaling. To balance these factors, a value of $k = 9$ is chosen. Figure ?? in the appendix shows the result of these choices.

3.6.2. Analysing waveshaping frequency content

Analytical analysis of the Fourier series of $\tanh(g \cdot \sin(x))$ is possible by using the Taylor series expansion of $\tanh(x)$. However, the Taylor series only converges for $x \in (-\frac{\pi}{2}, +\frac{\pi}{2})$, which proves to be problematic for any input $|g \cdot x[n]| \geq \frac{\pi}{2}$. Furthermore, the Fourier series of $\sin(g \cdot \sin(x))$ can be evaluated in terms of the Bessel J functions, which would require a LUT to compute efficiently.

For any unknown input $g \cdot f(x)$ into the waveshaping function, finding an analytical solution of the Fourier series is incredibly cumbersome, or impossible. Instead, a numerical approach is taken to determine the effects of waveshaping in the frequency domain, where the effect of g will be investigated on a sinusoid. We will devise our own technique.

For any waveshaping function $w(x)$ and a set of M increasing gain values $\{g_1, g_2, \dots, g_M\}$, we can investigate the effect of g in the discrete time domain, by sampling a 1 Hz sinusoid with N points within a

period, while recording P periods.

$$Y[k, g] = \text{DTFT}\{w(g \cdot \sin[2\pi \frac{n}{N}])\} \quad (3.28)$$

For each $g \in \{g_1, g_2, \dots, g_M\}$, we can record the number of the harmonic that is last in exceeding a predefined amplitude ratio with the fundamental. In this case, our ratio is chosen as $\frac{1}{100}$ (-40 dB) to the fundamental.

We must ensure that we choose N large enough, so that we can avoid significant aliasing in our analysis that could contaminate the results, and we choose P large enough for good frequency domain resolution. Choosing $N = 1000$ (500 times more than the Nyquist limit) and $P = 100$ proved to be adequate for the set of gains that were analysed. From this choice, the fundamental occurs at $k = 100$, with harmonics occurring at $k = 100 \cdot h$, $h \in \mathbb{N}/\{1\}$. Up to 500 harmonics may be analysed. The hyperbolic tangent had $g \in \{1, 2, \dots, 64\}$ analysed. The sinusoid had gains analysed for $g \in \{\frac{\pi}{16}, \frac{2\pi}{16}, \frac{3\pi}{16}, \dots, 4\pi\}$. This limits the gains that the user can apply to 64 and 4π respectively.

Knowing the last harmonic h_f above the -40 dB to the fundamental threshold, we can determine the bandwidth B_w in cycles per sample using the Nyquist criterion, shown in equation 3.29.

$$B_w = \frac{0.5}{h_f} \quad (3.29)$$

Figure 3.9 plots h_f^{-1} as a function of g for both waveshaping functions. These values can be used in a LUT to determine the cutoff frequency for a bandwidth-limiting LPF. For $g = 0$ and $h_f^{-1} = \infty$ (seen in figure 3.9a for $g \in \{\frac{\pi}{16}, \frac{2\pi}{16}\}$), we can set $h_f^{-1} = 2 \frac{20000}{f_s}$, which limits the signal to the audible range as per equation 3.29.

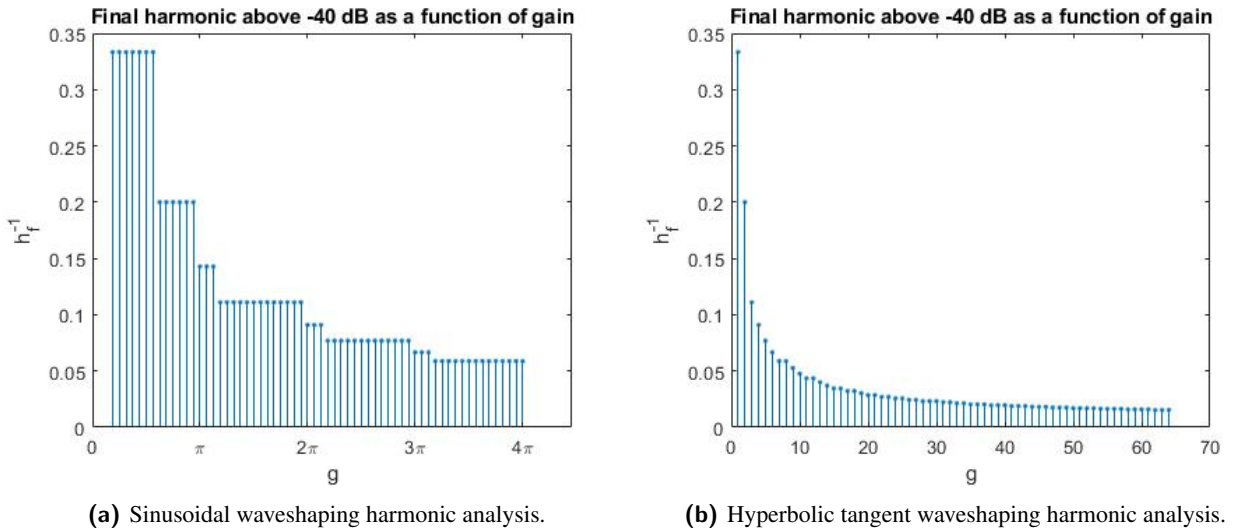


Figure 3.9: Waveshaping harmonic analyses.

3.6.3. Anti-aliasing filters

From the previous section, we can now determine the maximum bandwidth of an input as a function of gain. However, by limiting the bandwidth, we discard the frequency content above the bandwidth. This is undesirable. Hence, we must both high-pass and low-pass our incoming signal, and combine them again after waveshaping has occurred to the low-passed signal.

We have already designed IIR filters in section 3.5. We can choose between LP12 and LP24 filters.

Although a LP24 filter would suppress aliasing better, the LP12 filter is chosen. This is so that we can gain more harmonic content, with aliasing as a trade-off, which is often preferable in musical circumstances. This choice is made with the end-product audio in mind.

The LP12 filter also has a nice property, that could remove the need for a corresponding HP12 filter entirely. From equation 3.23 and 3.22, we can run them in parallel.

$$H_{||}(s) = H_{lp12}(s) + H_{hp12}(s) = \frac{1}{s+1} + \frac{s}{s+1} = 1 \quad (3.30)$$

From , we can then construct the HP12 filter from the LP12-filtered signal as shown in equation 3.31. This eliminates the need for a another filter, and replaces it with a subtraction operation.

$$H_{hp12}(s) = 1 - H_{lp12}(s) \quad (3.31)$$

We can express this in the discrete-time domain as follows:

$$y_{hp12}[n] = x[n] - h_{lp12}[n] * x[n] \quad (3.32)$$

Given a waveshaping-function $w(x)$, we can write the entire waveshaping system with input $x[n]$ and output $y_w[n]$ as shown in equation 3.33, using equations 3.31 and 3.32. We can store the low-passed signal $x_{lp12}[n] = h_{lp12}[n] * x[n]$ as an intermediary value to save clock cycles. We therefore only require a single IIR filter for this component.

$$y_w[n] = x[n] - h_{lp12}[n] * x[n] + w(h_{lp12}[n] * x[n]) \quad (3.33)$$

Equation 3.33 is the concrete form of what is depicted in the system block diagram from figure 3.8. The convolution operator represents the discrete-time difference equation (2.7) from section 2.4.3.

3.6.4. LUT indexing

Hyperbolic tangent

To index into the hyperbolic tangent LUT, we must reverse the transformations applied in equation 3.27. Thus our lookup index n for an input value t is shown by equation 3.34. We do not scale the final output by $\frac{1}{A}$, since $A \approx 1$ for small ϵ . For $x < 0$, we output -1 and for $x > 1$ we output +1.

$$n = \frac{N}{k} \left(x + \frac{1}{2} \right), \quad x \in (0, 1) \quad (3.34)$$

Sinusoid

We already have a LUT storing $\sin(2\pi \frac{n}{N})$ in the basic waveforms. Since we are interested in finding $\sin(x)$, we can find n utilising equation 3.6 (section 3.3), with $p = 2\pi$ and $A = N$ to ensure in-bounds indices for n and periodic behaviour.

$$n = N \left\{ \frac{t}{2\pi} \right\} \quad (3.35)$$

3.7. ADSR envelope generator

The ADSR envelope generator can be considered a state machine that produces a piecewise-defined function of 3 exponentials and a constant, based on a trigger signal, as shown in figure 2.1a. Figure 3.10 shows the block diagram of this component. This is a state-machine with 5 states: attack; decay; sustain; release; not playing. The attack and release states can only be externally triggered, therefore, the sampling mechanism only needs to be concerned with transitions to the decay, release and not playing states. A “playing” state is any state except the “not playing” state.

For code simplicity, states are encoded using integers: $\{0, 1, 2, 3, 4\}$, referring to attack, decay, sustain, release and not playing respectively. Furthermore, the attack, decay and release times must have a minimum value to avoid clicks and pops in the audio. This value was chosen as 1 ms.

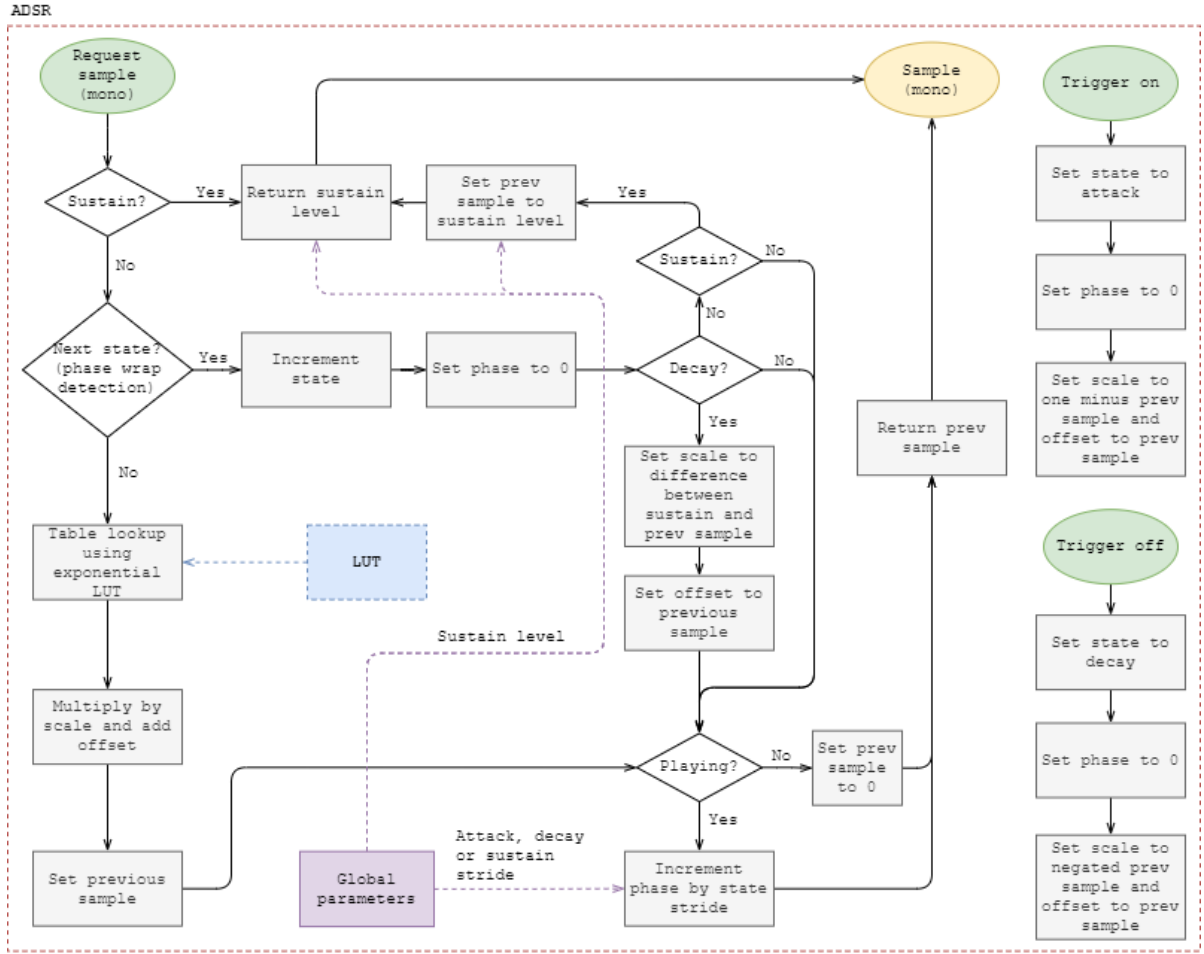


Figure 3.10: ADSR envelope system block diagram.

3.7.1. Creating the exponential LUT

To construct the LUT, we consider an upwards-decaying exponential $E[n]$, with $E[0] = 0$ and $E[N - 1] = 1$, where N is our required number of samples. Equation 3.36 shows the form we are interested in.

$$E[n] = K(1 - R^n), \quad R \in [0, 1) \quad (3.36)$$

Since $\lim_{n \rightarrow \infty} E[n] = K$, we choose a threshold value $p \in (0, 1)$ for our final sample, such that $E[N - 1] = pK$. An analogue circuit would usually use a comparator with such a threshold voltage to charge and discharge a capacitor, which also yields an exponential function. Such a threshold is arbitrary, and usually at the designer's discretion. It will determine the shape of the function stored in the LUT. A higher p will result in a flatter slope as $n \rightarrow N - 1$. A typical value to choose is $p = \frac{2}{3}$. Substituting our boundary conditions into 3.36, we find:

$$K = \frac{1}{p}$$

$$R = \sqrt[N-1]{1-p}$$

3.7.2. Implementing the state-machine

The nature of this component requires many state checks to function properly. Since this can cause branch penalties, care must be taken when considering the amount of branches and their locations, by minimising branching if possible.

Proper re-trigger behaviour is required, as per the specifications. Thus, the ADSR requires storage of its most recent sample, to allow for scaling and offset calculation based off previous output. It cannot be known in advance when triggering will occur, which implies that a release state trigger can occur during any state. This is similar for the attack state. A data structure is required to store all the internal state of a single ADSR envelope. The internal state includes LUT phase, exponential scaling and offset values, the previous sample, and the current state.

As per subsection 3.7.1, we know that the last value of the LUT is 1, and has a range of $[0, 1]$. The previous value ($y[n - 1]$) is also recorded as part of internal state. We need to calculate the offset (b) and the scaling factor (a) such that we can get the required behaviour from $E[n]$. We can deduce the following:

1. A trigger on event must initiate the attack phase, which must rise to 1. Thus, $a = 1 - y[n - 1]$ and $c = y[n - 1]$.
2. A trigger off event must initiate the release phase, which must decay to 0. We then have $a = -y[n - 1]$ and $c = y[n - 1]$.
3. The decay phase must decay to the sustain level (s). We cannot be certain that it transitions from a sample that is exactly 1 (due to linear interpolation and phase wrapping), so thus $a = s - y[n - 1]$ and $c = y[n - 1]$.

To achieve the correct attack, decay and release timings, we can treat the exponential LUT like wavetable, and using stride values that correspond with the required timing. However, we do not want it to exhibit periodic behaviour, so we must ensure that the phase is always less than $N - 1$. A transition and phase reset must occur after the phase exceeds $N - 1$. To calculate the required stride η to sweep over a single exponential ADSR state, given a time T_0 in seconds and a fixed LUT size N , we derive an expression from equation 3.13:

$$\eta = \frac{N}{T_0 f_s} \quad (3.37)$$

The C implementation of the state machine is shown in listing D.6 in the appendix.

3.8. Generator

The generator is a top-level component responsible for managing all sub-components and creating the stereo audio samples for a single note. It receives note on/off triggers that correctly configures all sub-components with the required parameters and subsequently triggers the ADSR envelopes. Figure 3.11 shows the system block diagram of this component. This component also requires internal state, which is not shown in the block diagram to prevent clutter.

The internal state of the generator, which is a function of the configured frequency, must be configured with a note-on trigger. The details of this configuration is discussed further in this section.

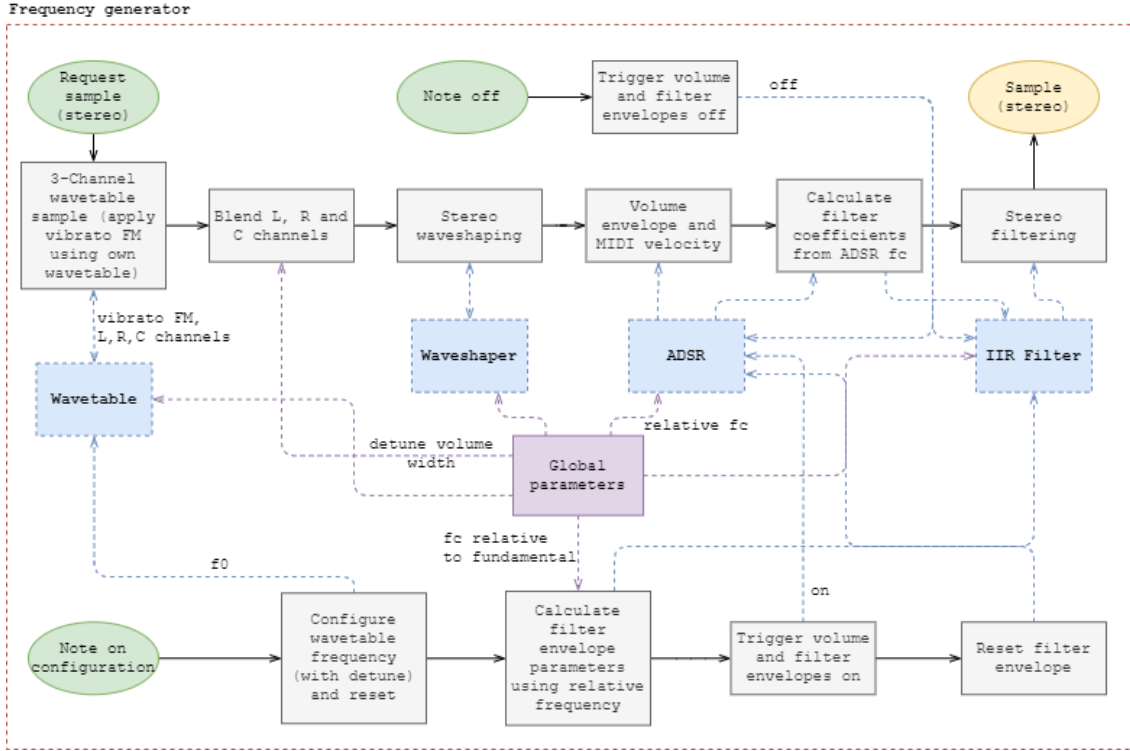


Figure 3.11: System block diagram of the generator.

3.8.1. Note-on/off triggers

With reference to the system block diagram (figure 3.11), the triggering process will be discussed. It can never be known when a note trigger will take place, or at which frequency it takes place. Thus, wavetable frequencies for the center, left and right channels must be configured using the MIDI note ID (see table C.2) which can be used as a lookup index for the required digital frequency. The note frequency, referred to as **base frequency**, is stored as part of the internal state. Although not required, the phase of the wavetables are reset.

A note on/off trigger must also trigger the volume and filter ADSR envelope components on/off.

3.8.2. Vibrato

Vibrato can be applied using the FM capabilities of the wavetable component from section 3.4.4. From the global parameters, the vibrato intensity v is specified as a value in cents, which can be used to calculate the required frequency deviation amplitude A as a function of the base frequency f_0 .

$$A = 2^{\frac{v}{1200}} f_0 - f_0 = (2^{\frac{v}{1200}} - 1) f_0 \quad (3.38)$$

A sinusoidal wavetable with the required vibrato frequency f_v can be used to create the frequency deviation samples $df[n]$. The FM must be applied to the center, left and right channels.

From equation 3.38, the frequency scaling vibrato factor $2^{\frac{v}{1200}}$ can be stored as part of the internal state of the generator, which can be efficiently calculated using the techniques from subsection ???. Equation 3.39 shows the frequency deviation as a result of vibrato. Note that the frequency deviation is expected to be small (< 50 cents), so no recalculation of the harmonic index of the wavetables will be done. On edge cases, this could cause aliasing, but would be unnoticeable.

$$df[n] = A \sin(2\pi f_v n) \quad (3.39)$$

The C implementation is shown in code listing D.7 in the appendix.

3.8.3. Filter cutoff modulation

The filter cutoff frequency $f_c[n]$ in samples/cycle is relative to the digital frequency of the note that is playing (f_0), as the functional requirements. A relative starting and ending cutoff frequency specified by scaling factors r_s and r_e are used to determine amount of cutoff modulation ($m_{filter}[n]$) applied by the filter ADSR envelope. Care must be taken to ensure that the filter cut-off does not exceed the Nyquist limit. To ensure that the cut-off always remains within reasonable limits, it is restricted to the audible range (20 Hz to 20 kHz).

$$f_c[n] = \max(\min(r_s f_0 + f_0(r_e - r_s) m_{filter}[n], \frac{20000}{f_s}), \frac{20}{f_s}) \quad (3.40)$$

From equation 3.40, we can store $r_s f_0$ and $f_0(r_e - r_s)$ in the internal generator state as filter modulation offset and amplitude respectively. This will save some clock cycles when sampling.

3.8.4. Volume modulation

The note volume $v[n] \in [0, 1]$ is determined by the ADSR envelope modulation $m_{vol}[n]$ and the MIDI note velocity $k_{vel} \in \{0, 1, \dots, 127\}$, which is a 7-bit unsigned integer that is specified with a note-on MIDI message [33]. Since $m_{vol}[n] \in [0, 1]$ by design, we need to normalise k_{vel} .

$$v[n] = \frac{k_{vel} m_{vol}[n]}{127} \Rightarrow V[n] \in [0, 1] \quad (3.41)$$

From equation 3.41, we can save on clock cycles by storing $\frac{k_{vel}}{127}$ in the internal generator state with a note-on configuration.

3.8.5. Stereo width

There are a variety of ways and functions that can be used to introduce stereo width, given center ($y_C[n]$), left ($y_L[n]$) and right ($y_R[n]$) channels that need to be mixed into stereo audio.

From subsection 2.4.2, we can use the mid-side form of audio to specify the mono and stereo content of the audio. As per the requirements, the detune width ($\delta_w \in [0, 1]$) and detune volume ($\delta_v \in [0, 1]$) parameters must be used to blend the detuned left and right channels. Volume is a scaling factor, where a width of $\delta_w = 0$ corresponds to no stereo width, and $\delta_w = 1$ to full stereo width. Using these definitions we can define the

mid-side content as follows:

$$M = y_C[n] + \frac{1}{2}\delta_v(2 - \delta_w)(y_R[n] + y_L[n]) \quad (3.42)$$

$$S = \frac{1}{2}\delta_v\delta_w(y_L - y_R) \quad (3.43)$$

Note that when $\delta_w = 1$, the mid channel contains the absolute minimum of $0.5\delta_v$ of the detuned content. This is necessary to ensure that the “mono’d” signal will not result in a complete loss of detune content. The center channel is only contained in the mid information, with the difference between left and right channels (see equation 2.3) is scaled by the detune width and the volume.

Substituting equations 3.42 and 3.43 into equations 2.4 and 2.5, we determine the content of the stereo audio to be as follows:

$$L = y_C + \delta_v(y_L + (1 - \delta_w)y_R)$$

$$R = y_C + \delta_v(y_R + (1 - \delta_w)y_L)$$

The C implementation for sampling from a generator is shown in code listing D.8 in the appendix.

3.9. Generator manager

The generator manager is responsible for triggering inactive generator components. This system stores an array of a finite number of generators, which need to be assigned to notes as appropriate. As per the functional specification, if all generators are active, then the oldest active generator must be re-triggered and configured to play any new incoming notes. The expected amount that re-triggering will be performed depends on user parameters. A generator is only considered as “inactive” or “available” if the release state of the volume envelope has finished. Otherwise it is “active”.

The amount of samples generated will be far greater than the amount of note-on/off requests. However, it is still necessary to make generators available when they exited their volume envelope release phase, which must be done once for every buffer request. The smaller the buffer, the less the latency (see equation 3.1), but the higher the generator polling rate. Thus, speed is of importance here. It cannot be known when a generator will be available until the release phase is over. The release time could also change, if the user changes this parameter while an envelope is in that state. The active generators must be polled at the start of every buffer request to check whether it can be made available.

The system block diagram for this component is shown in figure 3.12.

MCUs often only have a finite amount of memory available to them. We therefore need to be certain about the memory consumption of the generators. For speed, storing them in an array (and not any other structures such as a C++ vector or queue) is best.

Creating and destroying generator data for every note is an unnecessary process that requires memory allocation, whereas allocating a fixed amount of memory for generators, that are stored at successive memory locations is preferable.

Pointers to the generator data can be used to add generators to other data constructs such as queues and stacks for management. Without calling any memory allocation operations (like “malloc”), we can effectively store the generator data at fixed locations within the heap. This may also help with caching.

See the appendix for the full C implementation in code listing D.9.

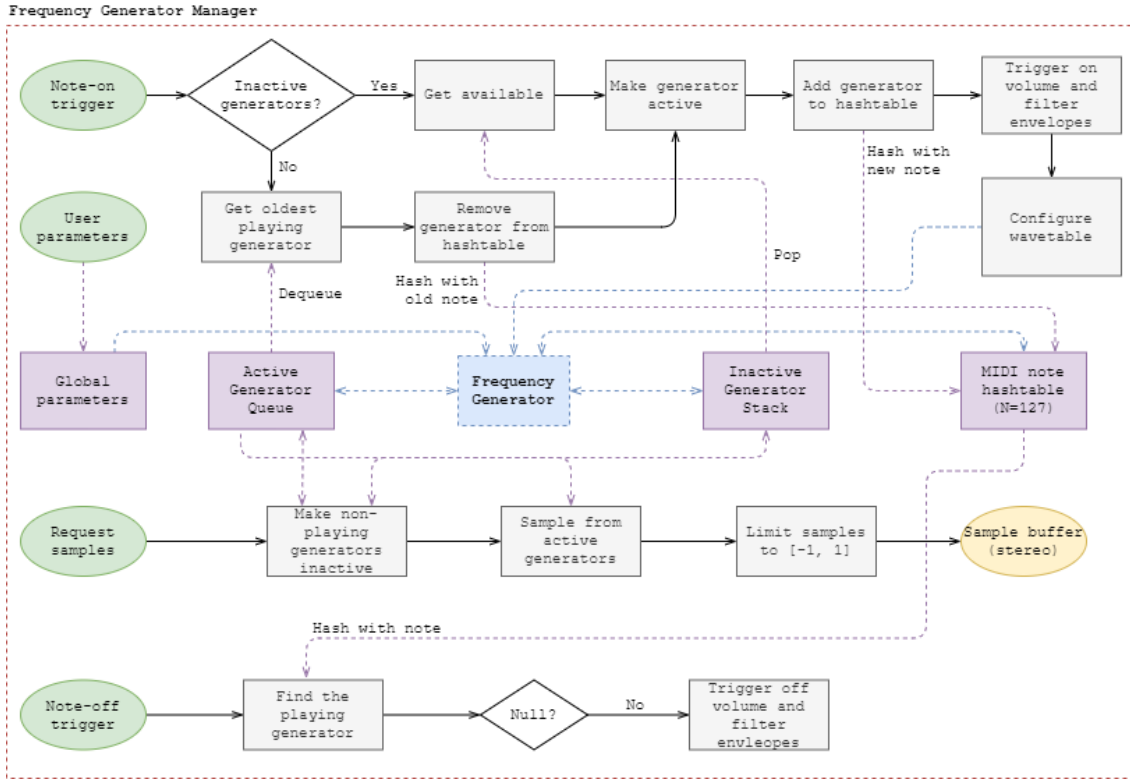


Figure 3.12: Generator manager system block diagram

3.9.1. Implementing basic data structures with arrays

Efficient management of the generators requires the use of 3 data structures:

1. **Queue** - the FIFO nature of this structure is required to keep track of the order in which generators we triggered. The first item in this queue will be the oldest. It also keeps tracks of all the active generators, so that inactive generators are not sampled as well.
2. **Stack** - the stack has an efficient array implementation which allows for quick access to inactive/available generators.
3. **Hash table** - Note on and note off triggers come in pairs. It is necessary to keep track of which generator is playing which note, so that it can easily be triggered off. Since the MIDI protocol only supports 128 notes, a hash table provides a quick way to access that generator in $O(1)$ time complexity, with little memory consumption (512 bytes, for a 32-bit system).

With this management scheme, a generator can either be in the active queue, or the available stack, but not in both.

Suppose that our system contains N generators. The queue and the stack can easily be implemented with an array of size N , storing generator pointers. Another variable is required that points to the location of first available slot in the array - the “head pointer”. Inserting into array, for both the queue and the stack, is done by inserting the item into the head pointer location, and then incrementing the pointer by 1.

Removing an item from the stack, requires the decrementing of the head pointer by 1, and then retrieving the data at that location. This operation has $O(1)$ time complexity. Removing an item from the queue is more expensive, since the data at location 0 must be retrieved. The head pointer is decremented, and all items are shifted one to the left. This operation has $O(N)$ time complexity. In both cases, we never loop past the item before the head pointer, thus deleting items is not required.

Note that a more efficient queue implementation is possible ($O(1)$ time complexity), by having a starting and ending index, and treating the array as a circular buffer. But since N will never realistically be very large (≤ 32), and dequeuing items will not happen often, it adds unnecessary complexity by adding in extra indexing operations.

The hash table is simple to implement, using an array of size 128, which stores generator pointers. The MIDI note value can be used to index into the array. Adding and retrieving items have $O(1)$ time complexity.

3.9.2. Note-on/off triggers

With reference to figure 3.12, there are two cases to consider for a note-on trigger: triggering if inactive generators are present; re-triggering the oldest generator if all generators are active. To check whether any active generators are available is simple, if the head pointer of the inactive generator stack is at index 0, we can be sure that all generators are inactive.

If there are available generators, a generator is popped off the stack. It is associated with the note in the hashtable, and appropriately configured with the correct frequency. This requires two $O(1)$ operations. If no generators are available, then the oldest generator is dequeued, and removed from its associated note in the hashtable which is stored in its internal state. This is required so that the note-off trigger for the previous playing note will be ignored. Triggering, configuring and hashing proceeds as per the previous case. This process requires an $O(1)$ and an $O(N)$ operation. If a note-on trigger occurs for the same note before a note-off (which should not happen unless it is artificially forced or through faulty MIDI), then retrieving the generator from the hashtable is done. If nothing is present, only then may a new generator be assigned to a note, otherwise the old generator is re-triggered. This is required, since it could lock a generator out from ever being triggered off. This is shown in the C implementation, although not explicitly shown in the block diagram.

For note-off triggers, the MIDI note is used for a hashtable lookup. If there is a generator present, it is still associated with that note and is triggered off. Otherwise, the generator has been re-triggered, and nothing is done.

3.9.3. Sample buffer request

Once a sample buffer (of a fixed size M) is requested, all the active generators must be sampled. To maximise the use of caching, a single generator generates the required M samples before moving on to sampling from the next generator.

This is added to the stereo buffer, and scaled according to the number of voices. If we assume that V voices are managed by this component, we can approximate the output of each voice to be between -1 and +1 (high Q filtering and the detuned wavetables might make this range larger). We therefore scale the output of each voice by N^{-1} , to ensure that the output is roughly within the -1 and +1 range. Once all the generators are sampled, the buffer is hard-clipped to $[-1, +1]$, so that we can be certain that no bit overflow will occur when converting the buffer data into an appropriate format for the codex IC. This conversion is hardware and application dependent (such as 24-bit PCM audio samples), and is not within scope.

Within this buffer request, we must free all inactive generators within the active queue, so that they may be used again. If we want to achieve this in $O(N)$ time, we can use a simple algorithm on the active generator queue, demonstrated in figure 3.13. In the figure, 2 buffer requests are shown, with the algorithm being executed twice.

The generators are labeled from A-H, with available generators shown in red and active generators shown in green. The example manages 8 generators, with the dotted box at the end indicating a non-element location,

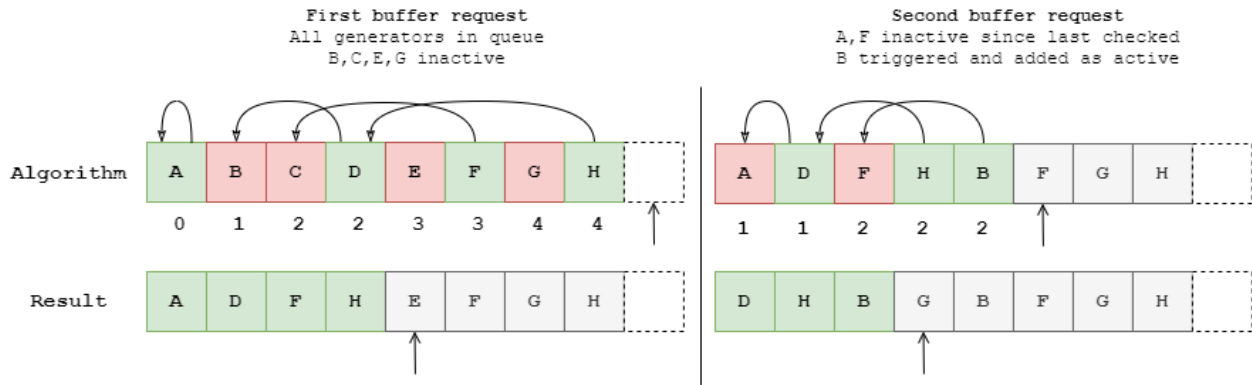


Figure 3.13: Generator queue freeing algorithm example

which the head pointer can be pointed at if the queue is full. The grey elements are items that will never be accessed, since they are past, or at, the head pointer. The algorithm keeps a running count of the number of available generators. If the generator is available, it is pushed onto the available stack and removed from the hash table. Otherwise, if the generator is still active, it is shifted left by the number of counted available generators. The running count is shown below the generators in the figure.

After the algorithm has executed, the head pointer is decremented by the running count. The C implementation of this algorithm (“gm_make_not_playing_available”) is seen in listing D.9 in the appendix.

Chapter 4

System testing

Each component of the system is individually tested, along with a final complete system test, and a musical test with some qualitative comments, which is described in appendix B.7. It is difficult to quantify the

Unless otherwise mentioned, all spectrograms were generated with a window size of 512, a sample overlap of 510 for good time-axis resolution, using a Blackman windowing function for a balance between main-lobe width and side-lobe amplitude. All tests were done at a sampling rate of 44.1 kHz, with 1 second's worth of samples (44100 samples). The data was generated using C code and the implemented components, and analysed using MATLAB.

4.1. Wavetable

A single wavetable set at a frequency of 220 Hz (A3) is sampled with different wavetable positions. This test is done to verify the inter-wavetable interpolation function. Figure 4.1a shows the results. To test the harmonic indexing detailed in section 3.4, a linear square-wave chirp was sampled from a single wavetable component, with frequency being updated once per sample. The spectrogram was then generated. Any aliasing occurring as a result of the harmonic content of the square-wave will be visible. This is shown in figure 4.1b.

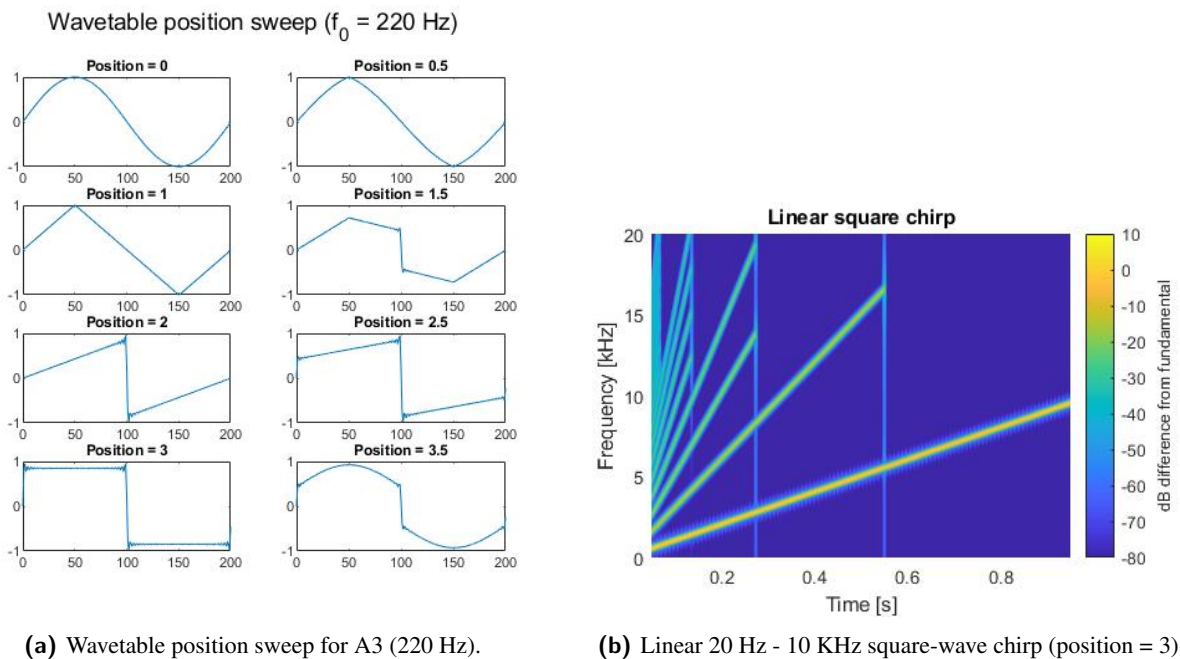


Figure 4.1: Testing inter-wavetable interpolation and harmonic indexing.

The inter-wavetable interpolation works as expected. Harmonic indexing also proves effective against aliasing, with no frequency producing aliased harmonics. The vertical lines in the spectrogram that is observed when harmonic loss occurs is result of the previous harmonics instantaneously disappearing, and can be neglected. Also note that the square wave only has uneven harmonics, but has a reduction in harmonics if the

even harmonics will alias, which is why the harmonics do not sweep all the way up to Nyquist.

4.2. ADSR

An ADSR object was sampled and triggered at different points in time. The envelope had time-parameters set for $t_a = 0.125\text{s}$, $t_d = 0.125\text{s}$, $t_r = 0.125\text{s}$ with a sustain of 0.5. The exponential LUT is constructed with $p = \frac{2}{3}$. Since 44100 samples were obtained, the triggering times were defined in terms of $k = \lfloor \frac{44100}{8} \rfloor$. A trigger-on is done at sample index $n_0 = \lfloor \frac{k}{3} \rfloor$ ($t \approx 0.042\text{s}$). A retrigger is done at $n_1 = \lfloor n_0 + k + \frac{k}{2} \rfloor$ ($t \approx 0.229\text{s}$). A trigger off is done at $n_2 = \lfloor n_1 + 3k \rfloor$ ($t \approx 0.604\text{s}$). Figure 4.2 shows the results.

The envelope operates as expected, with the correct timing, sustain level and range. The envelope was successfully retriggered.

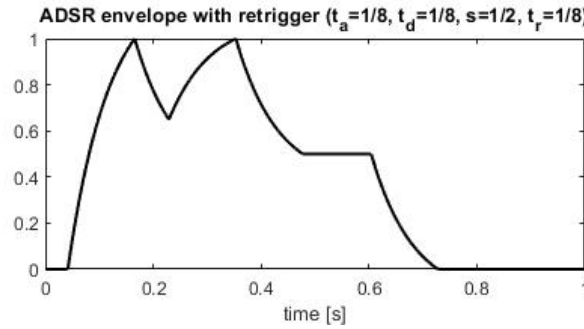


Figure 4.2: Sampled ADSR envelope with retrigger.

4.3. IIR filters

The time-dependent cutoff frequency of these filters were tested, which would simultaneously show correct filter operation. It should be noted that pin-point accurate cutoff frequencies are not tested, since deviation of a few Hertz due to LUT interpolation accuracy is inconsequential in a musical context. It is only the correct variation over time that is critical.

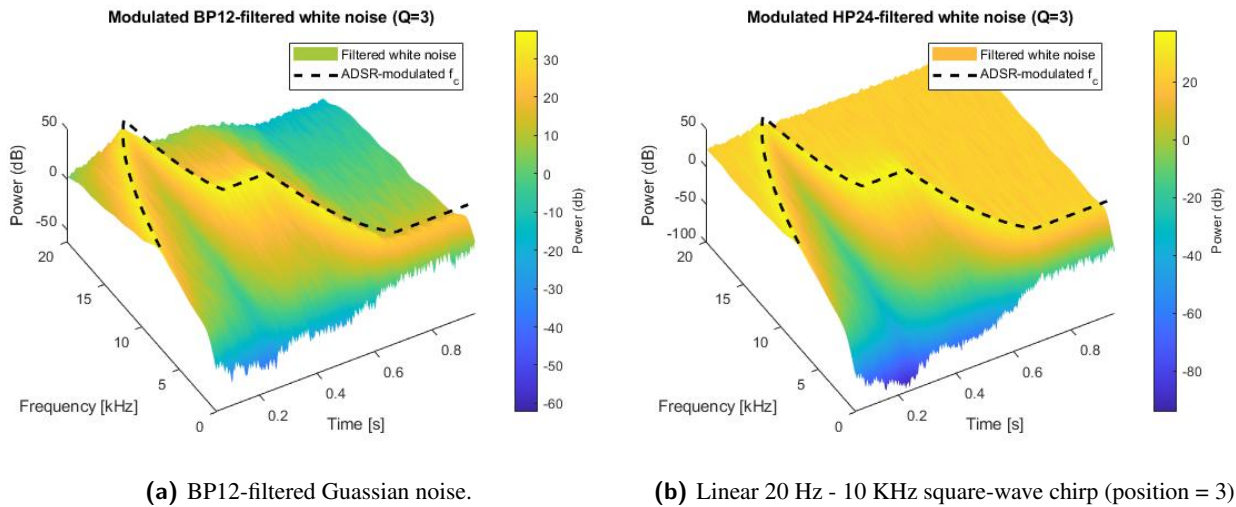


Figure 4.3: HP24-filtered Gaussian noise.

Gaussian white noise ($\mu = 0$, $\sigma^2 = 1$) was generated to test filter operation. The white noise was filtered and a spectrogram was produced. The white noise contains frequency content at all frequencies, but unequal

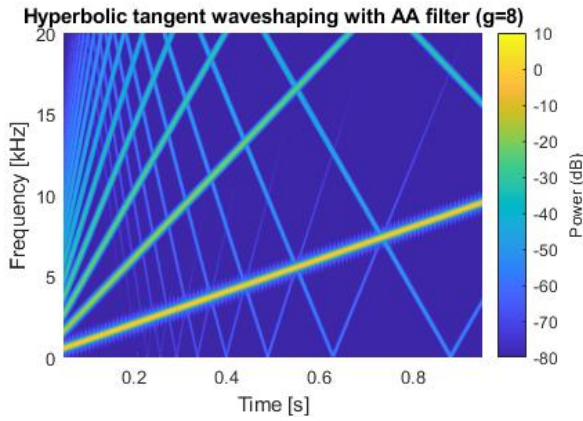
amounts at any instantaneous point within a STFT. An ADSR envelope was used to modulate the signal, with $t_a = 0.25s$, $t_d = 0.1s$, $t_r = 0.2s$ and a sustain of 0.5. The signal was modulated to have a minimum frequency of 1 kHz and a maximum of 20 kHz. The BP12, HP24 and LP24 filters had a Q set to 3, to make the cutoff frequency more apparent. To make the results more interpretable, a 2D Guassion blur was applied to the time-frequency STFT data, using a Guassion smoothing kernel with $\sigma = 15$. This eliminates the jagged edges of the white noise's STFT, but does not change the filter's shape significantly

The envelope's output was also separately recorded, and plotted as a dotted line for reference. The spectrogram had window size of 1024 with a sample overlap of 1020, for high time and frequency resolution.

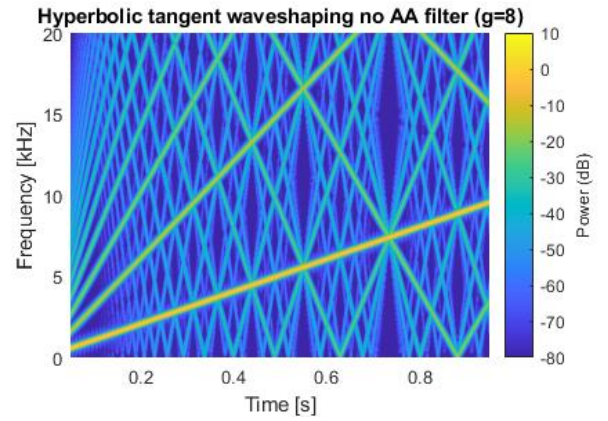
The figures clearly show a correct cutoff frequency modulation, with the required range between 1 kHz and 20 kHz. It should be noted that it does not explicitly start at 1 kHz, since the 1024 samples required to perform the first FFT consumes the start of the envelope.

4.4. Waveshaping

Linear sinuoidal chirps (20 Hz - 10 kHz) generated by a generator component with detune volume set to 0, position set to 0, no filtering applied other than for anti-aliasing, and ADSR envelope times set to the minimum of 1 ms with a sustain of 1. The envelope only affects the signal in the attack phase, which occurs at the start of the chirp. This effect is insignificant over the time-span of the recorded data.

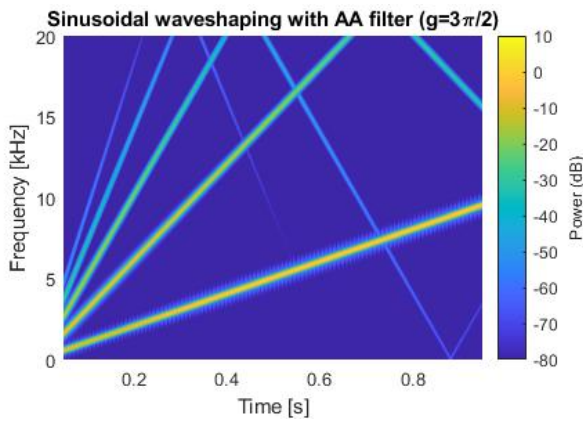


(a) Waveshaped chirp for $g = 8$, with anti-aliasing filter.

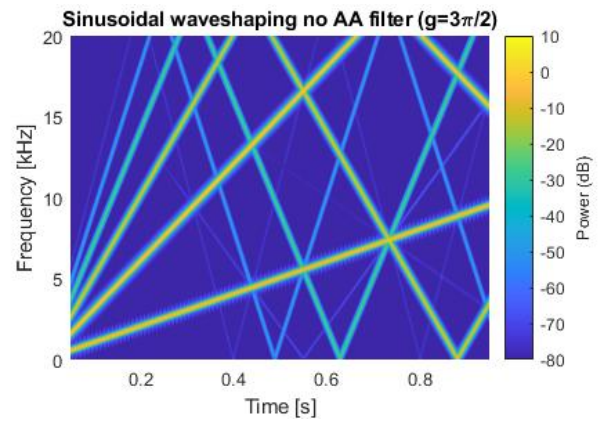


(b) Waveshaped chirp for $g = 8$, without the anti-aliasing filter.

Figure 4.4: Comparing the effect of the anti-aliasing LP12 filter for hyperbolic tangent waveshaping.



(a) Waveshaped chirp for $g = \frac{3}{2}\pi$, with anti-aliasing filter.



(b) Waveshaped chirp for $g = \frac{3}{2}\pi$, without the anti-aliasing filter.

Figure 4.5: Comparing the effect of the anti-aliasing LP12 filter for sinusoidal waveshaping.

The gain into both the sinusoidal and the hyperbolic tangent waveshaping functions with changed, with the anti-aliasing LP12 filter's cut-off set to the appropriate value from the look-up table as described in section 3.6. A spectrogram was generated, and compared to the results from having the anti-aliasing filter set to a cutoff of 20 kHz. The comparison is shown in figures 4.4 and 4.6. The output of the STFT was normalised by the maximum value in the time-frequency data, so that the effect of the harmonics relative to the fundamental can more easily be seen.

The reduction in aliasing for the hyperbolic tangent is clear. It is also apparent that the magnitude of the aliased frequencies in figure 4.4a is smaller than that of 4.4b. Similar observations are made for figure 4.5a and 4.5b, although the extra harmonic content added by sinusoidal waveshaping is less for these examples.

4.5. Generator

The vibrato capabilities of the generator component were tested in this section. All relevant generator parameters were set so that their effect will non-existing or minimal (detune volume set to 0; ADSR envelope times set to 1 ms, with a 1.0 sustain value; no filtering or waveshaping). A sinusoid with a base frequency of 10 kHz, frequency deviation (vibrato intensity) of 500 cents ($10 \cdot 2^{500/1200} - 10$) = 3.35 kHz, and FM frequency of 2 Hz was sampled. A spectrogram was created with the sampled data normalised by the maximum value in the time-frequency data, along with a straight line that shows the expected center frequency of 10 kHz. This is shown in figure 4.6a.

To test the stereo width, the detune volume is changed to 1, and various stereo widths sampled. The additional left and right oscillators were detuned up and down by 1200 cents (1 octave) respectively. A sinusoid was used, with no filtering applied. The base frequency was arbitrarily set to 100 Hz, which does not matter for the following test.

The left and right audio channels were converted to mid and side format as per equations 2.2 and 2.3. An XY-plot (common known as a Lissajous plot in the music industry) was created from these channels, where the Y-component represents the mid channel and the X-component represents the side channel. For a mono signal ($L = R$), a straight vertical line must be observed. For a phase inverted signal ($L = -R$), a straight horizontal line must be observed. The results are shown in figure 4.6b.

Figure 4.6a shows that the sinusoidal vibrato acts as expected. Figure 4.6b clearly indicates an increase in side-channel content as the stereo width increases, which can be seen by the axis of symmetry tending towards the side-channel axis.

4.6. Generator manager

Normal operation of the generator manager was tested. This includes note on/off triggers, below capacity. A set of 8 predefined frequencies $\{0.5, 3, \dots, 8\}$ kHz was associated with the MIDI notes $\{0, 1, \dots, 7\}$. Each sinusoidal tone is triggered on at $t = \{0, 1/16, \dots, 7/16\}$ and triggered off in reverse order at $t = \{8/16, 9/16, \dots, 15/16\}$. Detune volume is set to 0, ADSR time parameters to 1 ms with a sustain of 1, and no filtering or waveshaping is applied. The results are shown in figure 4.7a.

Generator overload is tested, with frequencies $\{1, 3, \dots, 15, 2, 4, \dots, 16\}$ kHz associated with MIDI notes $\{0, 1, \dots, 15\}$. Each sinusoidal tone is sequentially triggered at $t = \{0, 1/16, \dots, 15/16\}$, with no off-triggers occurring. This is shown in figure 4.7b.

For both results, the STFT is normalised by the maximum within the time-frequency data. Also note that the spectral lines at the edges of note on and off triggers are a result of the sudden addition of the tones, and

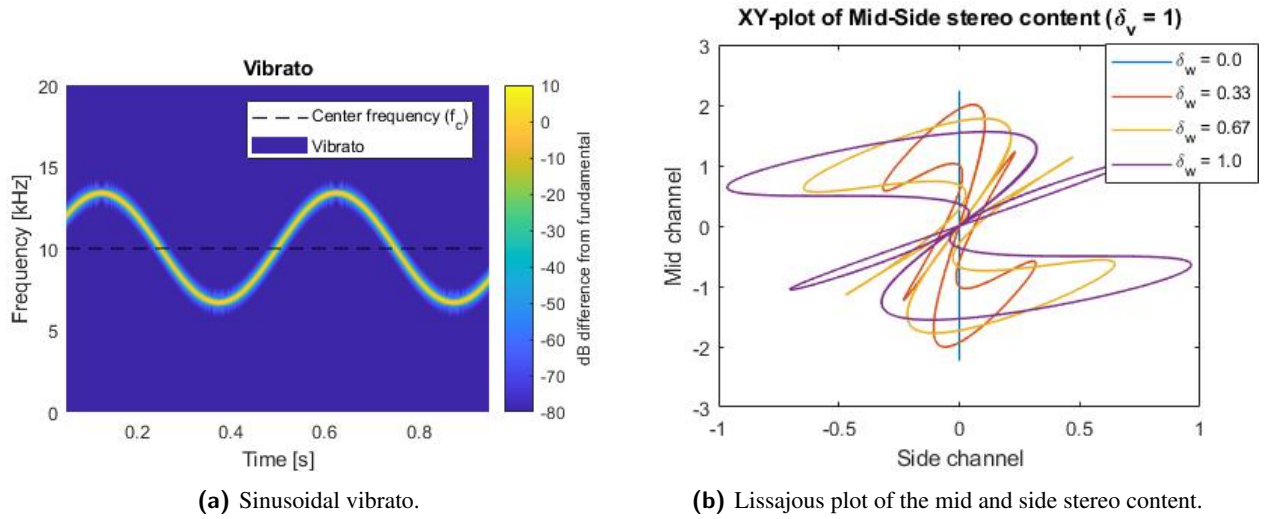


Figure 4.6: Testing vibrato and stereo width.

are not of concern.

The generator manager operates as required, but under normal and overload conditions.

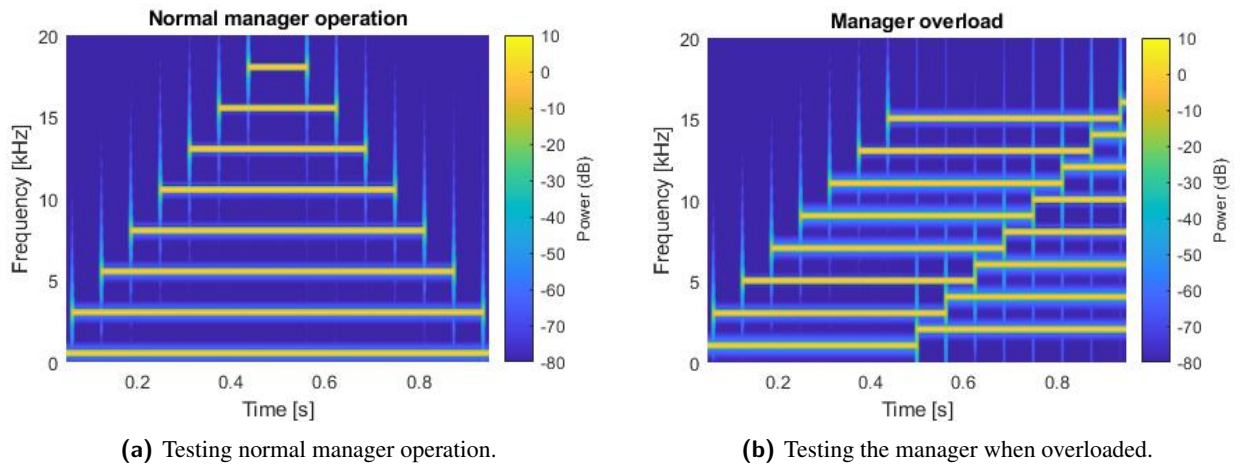


Figure 4.7: Testing the generator manager.

Chapter 5

Summary and Conclusion

5.1. Results achieved

The audio generation core for a hardware synthesiser was successfully designed and implemented in C. The following, with reference to subsection 1.3.2, was successfully achieved:

1. The basic waveforms (sine, triangle, sawtooth and square) can be selected and interpolated according to user parameters, by using the wavetable component. The LUTs required for this task has been effectively designed to combat aliasing, while allowing for an arbitrary amount of copies with increasing harmonics. This system supports any sampling frequency, and can yield a bit depth of up to 24 bits if required.
2. Two additional oscillators can be detuned from the base frequency oscillator, and panned so as to achieve a desired stereo width, using the aforementioned wavetable techniques.
3. An ADSR envelope state machine has been designed that can produce the required piecewise-exponential behaviour, with retriggering capabilities.
4. HP12, HP24, LP12, LP24 and BP24 filters derived from analogue prototypes have been implemented, requiring only 2 table lookups and a single division operation to calculate the FIR coefficients.
5. The cutoff frequency of the available filters can be modulated by an ADSR envelope on a per-sample basis, and set relative to the fundamental frequency.
6. Stereo waveshaping using the hyperbolic tangent and sinusoid functions can be performed, with an effective anti-aliasing method being implemented, using a single LP12 filter, with a cutoff frequency set as a function of the gain into the waveshaper. Waveshaping is done efficiently using LUTs.
7. Sinusoidal vibrato at a specified frequency and intensity can be applied to a stereo waveform using the FM capabilities of the wavetable component.
8. Polyphony is achieved using a manager component that can play up to a fixed number of notes, which can do so efficiently in $O(N)$ time. If a new note is triggered when the generator is at capacity, the oldest playing note is retriggered. The manager can produce stereo samples from active generators, and store it into a buffer of arbitrary size, while provide adequate headroom (samples are within $[-1,1]$ for normal operation, and clipped if necessary).

All the components designed in this thesis are building blocks for creating a more complex synthesis core. The building blocks can easily be rearranged in the signal chain to produce different effects, and can be used on their own for other applications. See appendix B.6 for detail on these alternatives and applications.

5.2. Further improvements and work

Designing a wavetable synthesiser can be a highly creative and product-dependent process, with unique features often being the selling point. This thesis only explored and designed the fundamental aspects of what can become a fully-fledged product. The first step in furthering design would be the implementation of the designed software on hardware. An ARM-based implementation is recommended, such as the STM32 Cortex M7 series. A dual-core processor could be desirable, by using the lower-speed core to process user input and handle display, while using the high-speed core exclusively for DSP. Implementation on a Cortex A7-series MPUs can also be desirable if greater performance is required. Simulating and inspecting the ARM assembly could also be done, where various optimisation techniques can be applied to improve performance. A performance analysis can also be done for specific hardware architectures, and optimised accordingly.

Furthermore, the designed software can also be implemented for VST design, where modifications can be made for concurrency, by sampling generator components on different threads. This could also be beneficial for some Cortex A7 MPUs. The techniques detailed in this thesis could also be implemented on an FPGA-based design.

It should be noted that fully-fledged MIDI support is not integrated into this system. Allowing pitch-bend and hold-pedal functionality is considered critical for products. Pitch-bend and hold-pedal functionality would be a possible augmentation to this system, without requiring significant effort. A MIDI-control based system can also be added, that would allow the MIDI protocol to also change user parameters, so that the instrument can be externally controlled.

Aside from hardware implementation, many additions can be made to the designed components. Among these additions are the implementation of more filters, such as the comb, or peak filter. The filters can also have more “character” added, by utilising saturation in the feedback path. This can be achieved through including hyperbolic tangent waveshaping in the filtering function, which then saturates the output $y[n]$. This would incur further numerical simulation to avoid aliasing for feedback saturation. Furthermore, the addition of oversampling in combination with the described anti-aliasing waveshaping scheme can be introduced for more robust performance. The optimal thresholds to choose for harmonics in the proposed scheme (section 3.6) can also be revisited by doing an in-depth study of acceptable levels of aliasing for musical contexts. The design of the ADSR state-machine can also be revisited for optimisation.

The proposed harmonic indexing scheme can also be revisited, where a more memory-efficient way of storing LUTs with the required number of harmonics can be devised. This could perhaps be achieved by storing a single waveform with many harmonics, and then filtering and temporarily storing it to reduce harmonic content when required. This requires many design considerations (see appendix B.4). This would increase processing requirements, and would be best achieved using a second core, or a multi-threaded system. Such a technique would be hardware dependent, which is why it was not considered in this thesis, which aimed to provide a general foundation for these systems. The current technique only requires an external memory for LUT storage, which can be set up to contain many megabytes of waveforms, loaded by the MCU as required.

Finally, a system can be designed that would allow for arbitrary parameter modulation, by having a selection of modulator functions that the user can assign to modulate arbitrary parameters. This would increase computational and implementation complexity significantly.

Finally, integrating other effects, such as a general EQ, delay, phaser and reverb effects, could be beneficial for the release of a finished product. Such effects would require a powerful processor if it is combined with this system, or it can be offloaded to an additional effects processor.

Bibliography

- [1] *ADSR A-140*, Doepfer. [Online]. Available: https://doepfer.de/a100_man/A140_man.pdf
- [2] *Configure the Coefficients for Digital Biquad Filters in TLV320AIC3xxx Family*, Texas Instruments, 2010.
- [3] MTU Physics, “Tuning: Frequencies for equal-tempered scale, A4 = 440 Hz,” last accessed 20 July 2021. [Online]. Available: <https://pages.mtu.edu/~suits/notefreqs.html>
- [4] ARM Developer, “FPU instruction set,” last accessed 9 October 2021. [Online]. Available: <https://developer.arm.com/documentation/ddi0439/b/BEHJADED>
- [5] Seveen, “STM32 Synth,” 2019, last accessed 16 October 2021. [Online]. Available: <https://github.com/Seveen/stm32-synth>
- [6] audiojs, “List of common audio sample rates,” 2017, last accessed 14 October 2021. [Online]. Available: <https://github.com/audiojs/sample-rate>
- [7] S. Lee, “This is the early history of the synthesizer,” 2018, last accessed 2 October 2021. [Online]. Available: <https://www.redbull.com/gb-en/electronic-music-early-history-of-the-synth>
- [8] AJH Synth, “Transistor ladder filter,” last accessed 2 October 2021. [Online]. Available: <https://ajhsynth.com/VCF.html>
- [9] Make Noise, “Maths,” last accessed 2 October 2021. [Online]. Available: <https://www.makenoisemusic.com/modules/maths>
- [10] MusicRadar, “The 16 best eurorack modules 2021: the right modules for any build, or expansion of your modular synthesizer system,” 2021, last accessed 2 October 2021. [Online]. Available: <https://www.musicradar.com/news/the-best-eurorack-modules-in-the-world>
- [11] Vintage Synth Explorer, “Yamaha DX7,” last accessed 2 October 2021. [Online]. Available: <https://www.vintagesynth.com/yamaha/dx7.php>
- [12] Xfer, “Serum: Advanced wavetable synthesizer,” last accessed 3 October 2021. [Online]. Available: <https://xferrecords.com/products/serum>
- [13] Native Instruments, “Massive,” last accessed 3 October 2021. [Online]. Available: <https://www.native-instruments.com/en/products/komplete/synths/massive/>
- [14] Arturia, “Pigments: Polychrome software synthesizer,” last accessed 4 October 2021. [Online]. Available: <https://www.arturia.com/products/analog-classics/pigments/overview#en>
- [15] D. Karras, “Sound synthesis theory,” 2018, last accessed 3 October 2021. [Online]. Available: https://en.wikibooks.org/wiki/Sound_Synthesis_Theory
- [16] Ableton, “Live,” last accessed 1 October 2021. [Online]. Available: <https://www.ableton.com/en/shop/live/>

- [17] Spectrasonics, “Omnisphere 2.8 - endless possibilities,” last accessed 5 October 2021. [Online]. Available: <https://www.spectrasonics.net/products/omnisphere/>
- [18] P. Mantione, “The fundamentals of physical modeling synthesis,” 2019, last accessed 5 October 2021. [Online]. Available: <https://theproaudiofiles.com/physical-modeling-synthesis/>
- [19] J. Smith, “Virtual acoustic musical instruments: Review and update,” *Journal of New Music Research*, vol. 33, pp. 283–304, 09 2004.
- [20] Native Instruments, “Kontakt 6 player,” last accessed 5 October 2021. [Online]. Available: <https://www.native-instruments.com/en/products/komplete/samplers/kontakt-6-player/>
- [21] Spitfire Audio, “Spitfire audio,” last accessed 5 October 2021. [Online]. Available: <https://www.spitfireaudio.com/shop/>
- [22] M. Apler, “Sound industry history: East vs. west coast synthesis: Moog, buchla, and finding a balance between familiarity and uncertainty in electronic instrument design,” last accessed 7 October 2021. [Online]. Available: <https://hii-mag.com/allposts/industry-history-eastwest>
- [23] Detroit Modular, “Eurorack: Modules,” last accessed 7 October 2021. [Online]. Available: <https://www.detroitmodular.com/eurorack.html>
- [24] Doepfer, “A-111-3 Micro Precision VCO / VCLFO,” last accessed 7 October 2021. [Online]. Available: <https://doepfer.de/A1113.htm>
- [25] Nord, “Nord Stage 3,” last accessed 7 October 2021. [Online]. Available: <https://www.nordkeyboards.com/products/nord-stage-3>
- [26] IntelliJel, “The little filter that could,” last accessed 7 October 2021. [Online]. Available: <https://intellijel.com/shop/eurorack/uvcf/>
- [27] VCA Module, MFB. [Online]. Available: http://mfberlin.de/wp-content/uploads/VCA_english.pdf
- [28] S. Pigeon, “The non-linearities of the human ear,” last accessed 7 October 2021. [Online]. Available: https://www.audiocheck.net/soundtests_nonlinear.php
- [29] HyperPhysics, “Equal temperament,” last accessed 7 October 2021. [Online]. Available: <http://hyperphysics.phy-astr.gsu.edu/hbase/Music/et.html>
- [30] W. Abu-Al-Saud, “Differential pulse code modulation (dpcm),” last accessed 8 October 2021. [Online]. Available: <https://faculty.kfupm.edu.sa/EE/wajih/files/EE%20370/EE%20370,%20Lecture%2025.pdf>
- [31] Wikimedia Foundation, Inc., “Audio bit depth,” 2021, last accessed 8 October 2021. [Online]. Available: https://en.wikipedia.org/wiki/Audio_bit_depth
- [32] J. O. Smith, “Mathematics of the Discrete Fourier Transform (DFT) with Audio Applications, Second Edition,” online book, Last accessed 30 July 2021. [Online]. Available: <http://ccrma.stanford.edu/~jos/mdft/>
- [33] MIDI Association, “Summary of midi 1.0 messages,” 2021, last accessed 9 October 2021. [Online]. Available: <https://www.midi.org/specifications-old/item/table-1-summary-of-midi-message>

Additional figures



(a) Doepfer A-111-3



(b) IntelliJel UVCF



(c) MFB VCA



(d) Doepfer A-140

Figure A.1: Examples of fundamental modules.

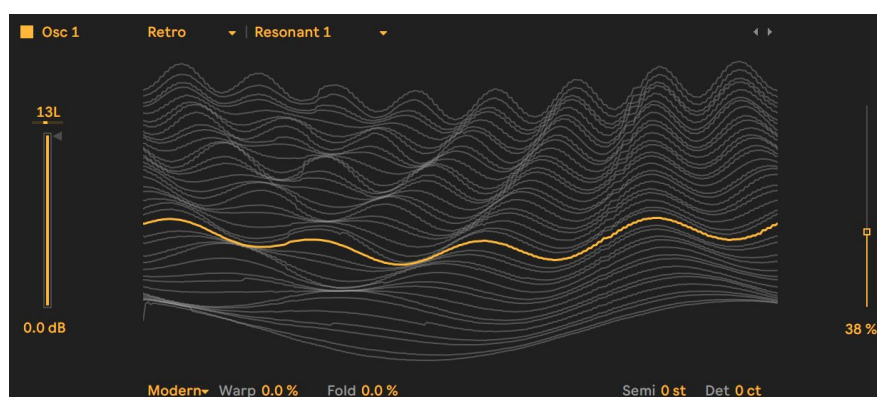


Figure A.2: Ableton's Wavetable VST

Appendix B

Additional sections and information

B.1. MIDI

The MIDI protocol is traditionally a communications standard specified for electronic inter-instrument communication. The protocol has been specified for a variety of mediums, which include USB 1.0 and 2.0, data storage in ".mid" files (SMF), and the original UART-based protocol that interfaces with a 5-pin MIDI (DIN) connector (of which only 3 pins are used).

A MIDI message consists of 3 bytes [33]. The first byte contains status information, which indicates the message type. Messages can be sent on one of 16 channels, which are identified in the status byte if applicable. There are 5 message types:

1. Channel Voice: contains note information, such as note on/off, aftertouch, pressure and control change. This is the most important message for the application at hand.
2. Channel Mode: sound off; reset; all notes off.
3. System Common: manufacturer information; song select and position; tune request.
4. System Real-Time: intended for timing and sequencing, which includes the timing clock (24 times per quarter note) and start and stop sequencing.
5. System Exclusive (Sysex): specific to the device.

INSERT INFO ON SMF AND THE LIBRARY USED HERE

B.2. A basic monophonic modular setup

Figure B.1 shows a block diagram of a typical modular setup. Many keyboard synthesisers follow this type topology.

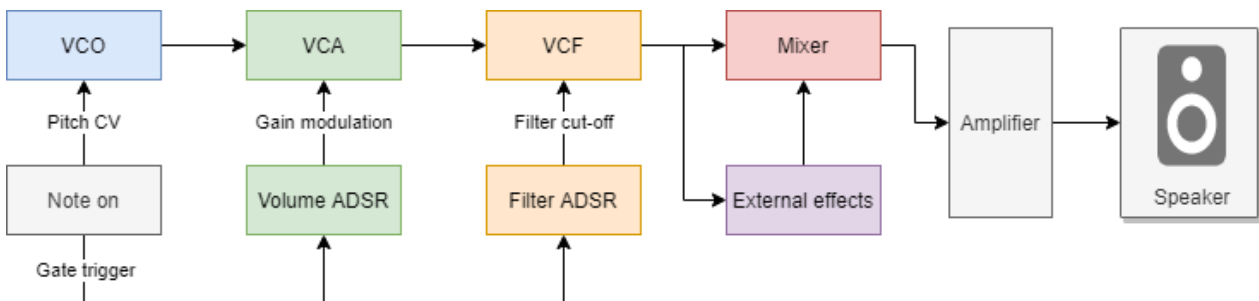


Figure B.1: Block diagram of a very basic monophonic modular setup.

The system consists of a note controller, which sends modulating signals to all of the modules. The modules have user-defined parameters, that is also modulated by ADSR sources.

The VCO generates the audio signal, which is then attenuated accordingly by the VCA and the volume ADSR envelope. The gain-modulated signal then enters the filter, which has a cut-off parameter that is modulated by a filter ADSR envelope. Note that the VCF and VCA can be swapped in order. The ADSR has very low-frequency content, which makes the order of these modules mostly inconsequential. The transient response of the VCF will be affected by the order.

The final modified signal is sent to a mixer, which can apply additional effects such as distortion, reverb or delay (either in series or parallel). These effects could be internally included in the synthesiser, or part of an FX loop.

The amplification and speaker is not necessarily included, depending on the output of the synthesiser (could be a line-out, a digital signal within a DAW or internal speakers).

It should be noted that many in-between steps is often included at the discretion of the designer. An example would be distortion/saturation between the VCO and VCA.

B.3. Regarding integer arithmetic using Q-numbers

QNUM stuff.

B.4. The implications of linear interpolation in the frequency domain and design considerations

For reference, figure B.2 is shown.

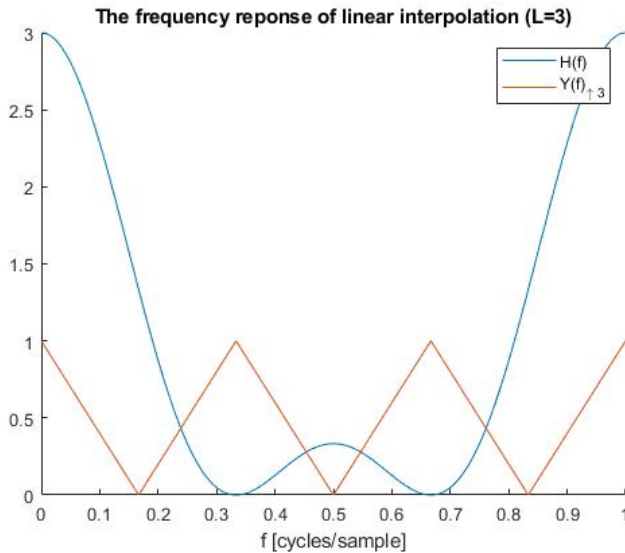


Figure B.2: The effects of linear interpolation in the frequency domain.

With an increase in b , more harmonics can be stored. But this comes with diminishing returns, since even an harmonically rich waveform, like the square wave, has significant attenuation for higher harmonics. For example, if $b = 9 \Rightarrow H_{max} = 256$. The level of the 255th harmonic (only odd harmonics) is $-20\log_{10}(255) \approx -48$ dB lower than the fundamental. The addition of more harmonics will become less and less notable to the human hearing.

The sampling rate does, however, affect the number of harmonics for a certain note that is played without aliasing in the digital domain. Assuming that human hearing is capped at 20 kHz, the lowest note on an 88-key piano is A0 (27.5 Hz), which has 727 harmonics in the audible range. It can be argued that this note is very rarely played, and the amplitude of the higher harmonics is not very audible compared to the fundamental. The human ear is also less sensitive to higher frequencies (the exact amount depends on the person).

There is a trade-off between memory (2^b LUT size) and spectral distortion as a result of interpolation, and number of harmonics. Quantifying the optimal number of harmonics based on the most frequently played

octaves, human ear sensitivity and harmonic audibility compared to the fundamental, is complicated and out of the scope of this thesis. Therefore, memory consumption will be the prime consideration.

B.5. Optimising harmonic indexing using a LUT

Harmonic indexing.

B.6. Alternative uses and signal chains for the designed components

Other uses.

B.7. Musical test

TESTING STUFF

Appendix C

Tables

Table C.1: Bilinear transform substitution [2]

S-domain	Z-domain
1	$(1 + 2z^{-1} + z^{-2})(1 - \cos(\omega_0))$
s	$(1 - z^{-2}) \sin(\omega_0)$
s^2	$(1 - 2z^{-1} + z^{-2})(1 + \cos(\omega_0))$
$1 + s^2$	$2(1 - 2\cos(\omega_0)z^{-1} + z^{-2})$

Table C.2: MIDI note IDs and their frequencies [3]

MIDI (hexadecimal)	Note Name	Frequency (Hz)
0x0		8.18
0x1		8.66
0x2		9.18
0x3		9.72
0x4		10.3
0x5		10.91
0x6		11.56
0x7		12.25
0x8		12.98
0x9		13.75
0xA		14.57
0xB		15.43
0xC		16.35
0xD		17.32
0xE		18.35
0xF		19.45
0x10		20.6
0x11		21.83
0x12		23.12
0x13		24.5
0x14		25.96
0x15	A0	27.5
0x16	A#0/Bb0	29.14
0x17	B0	30.87
0x18	C1	32.7
0x19	C#1/Db1	34.65
0x1A	D1	36.71

0x1B	D#1/Eb1	38.89
0x1C	E1	41.2
0x1D	F1	43.65
0x1E	F#1/Gb1	46.25
0x1F	G1	49
0x20	G#1/Ab1	51.91
0x21	A1	55
0x22	A#1/Bb1	58.27
0x23	B1	61.74
0x24	C2	65.41
0x25	C#2/Db2	69.3
0x26	D2	73.42
0x27	D#2/Eb2	77.78
0x28	E2	82.41
0x29	F2	87.31
0x2A	F#2/Gb2	92.5
0x2B	G2	98
0x2C	G#2/Ab2	103.83
0x2D	A2	110
0x2E	A#2/Bb2	116.54
0x2F	B2	123.47
0x30	C3	130.81
0x31	C#3/Db3	138.59
0x32	D3	146.83
0x33	D#3/Eb3	155.56
0x34	E3	164.81
0x35	F3	174.61
0x36	F#3/Gb3	185
0x37	G3	196
0x38	G#3/Ab3	207.65
0x39	A3	220
0x3A	A#3/Bb3	233.08
0x3B	B3	246.94
0x3C	C4 (middle C)	261.63
0x3D	C#4/Db4	277.18
0x3E	D4	293.66
0x3F	D#4/Eb4	311.13
0x40	E4	329.63
0x41	F4	349.23
0x42	F#4/Gb4	369.99
0x43	G4	392
0x44	G#4/Ab4	415.3
0x45	A4 concert pitch	440

0x46	A#4/Bb4	466.16
0x47	B4	493.88
0x48	C5	523.25
0x49	C#5/Db5	554.37
0x4A	D5	587.33
0x4B	D#5/Eb5	622.25
0x4C	E5	659.26
0x4D	F5	698.46
0x4E	F#5/Gb5	739.99
0x4F	G5	783.99
0x50	G#5/Ab5	830.61
0x51	A5	880
0x52	A#5/Bb5	932.33
0x53	B5	987.77
0x54	C6	1046.5
0x55	C#6/Db6	1108.73
0x56	D6	1174.66
0x57	D#6/Eb6	1244.51
0x58	E6	1318.51
0x59	F6	1396.91
0x5A	F#6/Gb6	1479.98
0x5B	G6	1567.98
0x5C	G#6/Ab6	1661.22
0x5D	A6	1760
0x5E	A#6/Bb6	1864.66
0x5F	B6	1975.53
0x60	C7	2093
0x61	C#7/Db7	2217.46
0x62	D7	2349.32
0x63	D#7/Eb7	2489.02
0x64	E7	2637.02
0x65	F7	2793.83
0x66	F#7/Gb7	2959.96
0x67	G7	3135.96
0x68	G#7/Ab7	3322.44
0x69	A7	3520
0x6A	A#7/Bb7	3729.31
0x6B	B7	3951.07
0x6C	C8	4186.01
0x6D	C#8/Db8	4434.92
0x6E	D8	4698.64
0x6F	D#8/Eb8	4978.03
0x70	E8	5274.04

0x71	F8	5587.65
0x72	F#8/Gb8	5919.91
0x73	G8	6271.93
0x74	G#8/Ab8	6644.88
0x75	A8	7040
0x76	A#8/Bb8	7458.62
0x77	B8	7902.13
0x78	C9	8372.02
0x79	C#9/Db9	8869.84
0x7A	D9	9397.27
0x7B	D#9/Eb9	9956.06
0x7C	E9	10548.08
0x7D	F9	11175.3
0x7E	F#9/Gb9	11839.82
0x7F	G9	12543.85

Table C.3: ARM Cortex M4 and M7 FPU instruction set [4]

Operation	Assembler	Cycles
Absolute value	VABS.F32	1
Addition	VADD.F32	1
Compare	VCMP.F32	1
	VCMPE.F32	1
Convert	VCVT.F32	1
Divide	VDIV.F32	14
Load	VLDM.64	1+2*N
	VLDM.32	1+N
	VLDR.64	3
	VLDR.32	2
Move	VMOV	1
	VMOV	1
	VMOV	2
	VMRS	1
	VMSR	1
Multiply	VMUL.F32	1
	VMLA.F32	3
	VMLS.F32	3
	VNMLA.F32	3
	VNMLS.F32	3
Multiply (fused)	VFMA.F32	3
	VFMS.F32	3
	VFNMA.F32	3
	VFNMS.F32	3
Negate	VNEG.F32	1
	VNMUL.F32	1
Pop	VPOP.64	1+2*N
	VPOP.32	1+N
Push	VPUSH.64	1+2*N
	VPUSH.32	1+N
Square-root	VSQRT.F32	14
Store	VSTM.64	1+2*N
	VSTM.32	1+N
	VSTR.64	3
	VSTR.32	2
Subtract	VSUB.F32	1

Appendix D

Code listings

```
1 float cents_scaling_factor[] = {
2 1.000000000000000f, 1.00057778950655f, 1.00115591285382f, 1.00173437023470,
3 1.00231316184217f, 1.00289228786937f, 1.00347174850950f, 1.00405154395592,
4 1.00463167440205f, 1.00521214004148f, 1.00579294106785f, 1.00637407767497,
5 1.00695555005672f, 1.00753735840711f, 1.00811950292026f, 1.00870198379040,
6 1.00928480121187f, 1.00986795537914f, 1.01045144648676f, 1.01103527472943,
7 1.01161944030192f, 1.01220394339916f, 1.01278878421615f, 1.01337396294802,
8 1.01395947979003f, 1.01454533493752f, 1.01513152858597f, 1.01571806093096,
9 1.01630493216819f, 1.01689214249346f, 1.01747969210269f, 1.01806758119192,
10 1.01865580995729f, 1.01924437859508f, 1.01983328730164f, 1.02042253627348,
11 1.02101212570719f, 1.02160205579949f, 1.02219232674721f, 1.02278293874728,
12 1.02337389199677f, 1.02396518669285f, 1.02455682303280f, 1.02514880121402,
13 1.02574112143402f, 1.02633378389042f, 1.02692678878098f, 1.02752013630354,
14 1.02811382665607f, 1.02870786003665f, 1.02930223664349f, 1.02989695667490,
15 1.03049202032930f, 1.03108742780523f, 1.03168317930136f, 1.03227927501645,
16 1.03287571514939f, 1.03347249989918f, 1.03406962946493f, 1.03466710404588,
17 1.03526492384138f, 1.03586308905088f, 1.03646159987396f, 1.03706045651031,
18 1.03765965915975f, 1.03825920802219f, 1.03885910329766f, 1.03945934518634,
19 1.04005993388848f, 1.04066086960447f, 1.04126215253481f, 1.04186378288011,
20 1.04246576084112f, 1.04306808661868f, 1.04367076041375f, 1.04427378242741,
21 1.04487715286087f, 1.04548087191543f, 1.04608493979253f, 1.04668935669371,
22 1.04729412282063f, 1.04789923837507f, 1.04850470355893f, 1.04911051857422,
23 1.04971668362307f, 1.05032319890772f, 1.05093006463054f, 1.05153728099401,
24 1.05214484820072f, 1.05275276645338f, 1.05336103595484f, 1.05396965690802,
25 1.05457862951601f, 1.05518795398198f, 1.05579763050924f, 1.05640765930119,
26 1.05701804056138f, 1.05762877449346f, 1.05823986130119f, 1.05885130118847,
27 1.05946309435930
28 };
29
30 float semitones_scaling_factor[] = {
31 1.000000000000000f, 1.05946309435930f, 1.12246204830937f, 1.18920711500272,
32 1.25992104989487f, 1.33483985417003f, 1.41421356237310f, 1.49830707687668,
33 1.58740105196820f, 1.68179283050743f, 1.78179743628068f, 1.88774862536339
34 };
35
36 float get_detune_factor_semitones_lut(int st) {
37     int i = abs(st);
38     int oct = i / 12;
39     i = i - oct * 12;
40     return ((float)(1<<oct)) * semitones_scaling_factor[i];
41 }
42
43 float get_detune_factor_cents_lut(float cents) {
44     float xc = fabs(cents);
```

```

45 float xs = floorf(xc * 0.01f); //amount of integer semitones
46 xc = ( xc - 100.0f * xs); //cents in the range [0,100)
47 float stf = get_detune_factor_semitones_lut((int)xs); //scaling from octaves
48 float cf = lut_lookup_no_wrap(cents_scaling_factor, xc); //scaling from cents,
    interpolated
49 float f = stf * cf; //cumulative scaling factor
50 return cents < 0.0f ? 1.0f / f : f; //scaling up or down
51 }
52
53 float get_detune_factor(float cents) {
54     return get_detune_factor_cents_lut(cents);
55 }
56
57 float detune_cents(float orig_freq, float cents) {
58     return orig_freq * get_detune_factor(cents);
59 }

```

Listing D.1: Efficient frequency scaling using the equal temperament tuning system

```

1 inline void wt_config_digital_freq(wavetable* wt, float freq) {
2     wt->stride = freq * (float)LUT_SIZE; //set strides, with no FM applied
3     wt->base_stride = wt->stride;
4     uint16_t harmonics = (uint16_t)(0.5f / freq);
5     harmonics = harmonics > (LUT_SIZE>>1) ? (LUT_SIZE>>1) : harmonics;
6     uint8_t harmonic_index = harmonics < 4 ? 0 : harmonic_indices[(harmonics>>2)-1];
7     wt->harmonic_index = harmonic_index;
8 }

```

Listing D.2: Configuring wavetable frequency

```

1 void iir_calc_lp24_coeff(IIR_coeff* filter, float f0, float q) {
2     float cosw = cos_lookup(f0);
3     float sinw = sin_lookup(f0);
4     float twoq = 2.0f * q;
5     //normalising by a0_recip = 1 / a0
6     float a0_recip = 1.0f / (twoq + sinw); //division is unavoidable here
7     float a1 = 2.0 * twoq * cosw; //pre-negated: d1 = -a1/a0
8     float a2 = sinw - twoq; //pre-negated: d2 = -a2/a0
9     float b0, b1, b2;
10    b1 = twoq * (1.0f - cosw);
11    b0 = b1 * 0.5f;
12    b2 = b0;
13    filter->n0 = b0 * a0_recip;
14    filter->n1 = b1 * a0_recip;
15    filter->n2 = b2 * a0_recip;
16    filter->d1 = a1 * a0_recip;
17    filter->d2 = a2 * a0_recip;
18 }

```

Listing D.3: Calculating the coefficients for a LP24 filter

```

1 inline float iir_filter_sample(IIR_coeff* coeff, IIR_prev_values* prev, float x) {
2     float y = coeff->n0 * x;
3     y += coeff->n1 * prev->xm1; //FPU VFMA.F32 operations
4     y += coeff->n2 * prev->xm2;

```

```

5  y += coeff->d1 * prev->ym1;
6  y += coeff->d2 * prev->ym2;
7  prev->ym2 = prev->ym1; //unit delays
8  prev->ym1 = y;
9  prev->xm2 = prev->xm1;
10 prev->xm1 = x;
11 return prev->ym1;
12 }

```

Listing D.4: Filtering a signal

```

1 //approximates cos(2*pi*f) for f in [0,0.75)
2 inline float cos_lookup(float f) {
3     return lut_lookup(basic_luts[SINE][0] + LUT_SIZE / 4, LUT_SIZE, f * (float)
4     LUT_SIZE);
5 }
6 //approximates sin(2*pi*f) for f in [0,1)
7 inline float sin_lookup(float f) {
8     return lut_lookup(basic_luts[SINE][0], LUT_SIZE, f * (float)LUT_SIZE);
9 }

```

Listing D.5: Trigonometric lookup functions

```

1 inline float adsr_sample(ADSR* adsr, float params[]) {
2     if (adsr->state == SUSTAIN) {
3         return params[SUSTAIN];
4     }
5     if ((adsr->phase < (float)EXP_LUT_SIZE - 1.0f))
6     { //no transition
7         float lookup = lut_lookup(lut_exp, EXP_LUT_SIZE, adsr->phase);
8         adsr->prev_sample = adsr->scale * lookup + adsr->offset;
9     }
10    else { //transition
11        adsr->state += 1;
12        adsr->phase = 0.0f;
13        //calculate scaling and offset
14        if (adsr->state == DECAY) {
15            adsr->scale = params[SUSTAIN] - adsr->prev_sample;
16            adsr->offset = adsr->prev_sample;
17        }
18        else if (adsr->state == SUSTAIN) {
19            adsr->prev_sample = params[SUSTAIN];
20            return params[SUSTAIN];
21        }
22    }
23    if (adsr->state != NOT_PLAYING) {
24        //increment phase
25        adsr->phase += params[adsr->state];
26        return adsr->prev_sample;
27    }
28    else {
29        //return 0 if not playing
30        adsr->prev_sample = 0.0f;
31    }
32    return adsr->prev_sample;

```

33 }

Listing D.6: Sampling from an ADSR envelope state-machine

```

1 inline void gen_vibrato(generator* g, gen_config* gc) {
2     float osc = (wt_sample_no_interpolation(&g->wt_vibrato, 0));
3     float ampl = (gc->vibrato_factor - 1.0f) * g->base_freq;
4     float vib = osc * ampl;
5     wt_apply_fm(&g->wt_left, vib);
6     wt_apply_fm(&g->wt_right, vib);
7     wt_apply_fm(&g->wt_center, vib);
8 }

```

Listing D.7: Applying vibrato FM

```

1 inline void gen_sample(generator* g, gen_config* gc, float* buf_L, float* buf_R) {
2     //apply vibrato using FM
3     gen_vibrato(g, gc);
4     //sample L, R, C channels
5     float sc = wt_sample(&g->wt_center, gc);
6     float sl = wt_sample(&g->wt_left, gc);
7     float sr = wt_sample(&g->wt_right, gc);
8     //blend detuned samples
9     float width = 1.0f - gc->detune_width; //invert to get volume in other channel
10    //detune_width of 1 separates sr and sl into L and R channels
11    //detune_width of 0 is mono
12    float L = sc + gc->detune_volume * (sl + width * sr);
13    float R = sc + gc->detune_volume * (sr + width * sl);
14    //find f0 using envelope
15    float f0 = g->filter_freq_start + adsr_sample(&g->envelope_filter_cutoff, gc->
        filt_adsr_params) * g->filter_envelope_amplitude;
16    f0 = clamp(f0, DIGITAL_FREQ_20HZ, DIGITAL_FREQ_20KHZ); //limit to audible range
17    //calculate filter coefficients
18    (*(gc->filter_coeff_func))(&g->filter_coeff, f0, gc->filter_Q);
19    //waveshape L and R
20    L = gen_waveshape_sample(&g->filter_sat_pv_L, gc, L);
21    R = gen_waveshape_sample(&g->filter_sat_pv_R, gc, R);
22    //apply volume
23    float volume = adsr_sample(&g->envelope_volume, gc->vol_adsr_params) * g->
        velocity;
24    L = L * volume;
25    R = R * volume;
26    //filter channels
27    L = iir_filter_sample(&g->filter_coeff, &g->filter_left_pv, L);
28    R = iir_filter_sample(&g->filter_coeff, &g->filter_right_pv, R);
29    //output
30    *buf_L = L;
31    *buf_R = R;
32 }

```

Listing D.8: Sampling from a generator

```

1 generator* note_played_hash_table[128];
2
3 struct gen_manager {

```

```

4  generator generators[NUM_GENERATORS];
5  generator* available[NUM_GENERATORS];
6  int8_t available_head = NUM_GENERATORS;
7  generator* in_use[NUM_GENERATORS];
8  uint8_t in_use_head = 0;
9  };
10
11 inline void gm_init(gen_manager* gm) {
12     //make all available
13     for (int i = 0; i < NUM_GENERATORS; i++)
14     {
15         gm->available[i] = &gm->generators[i];
16     }
17     gm->available_head = NUM_GENERATORS;
18     gm->in_use_head = 0;
19     //none are in use
20     for (int i = 0; i < NUM_GENERATORS; i++)
21     {
22         gm->in_use[i] = nullptr;
23     }
24     //none are playing notes
25     for (int i = 0; i < 128; i++) {
26         note_played_hash_table[i] = nullptr;
27     }
28 }
29
30 inline void gm_add_to_in_use(gen_manager* gm, generator* g) {
31     gm->in_use[gm->in_use_head] = g;
32     gm->in_use_head++;
33 }
34
35 inline generator* gm_pop_off_back_from_in_use(gen_manager* gm) {
36     generator* g = gm->in_use[0];
37     //shift queue
38     for (int i = 0; i < gm->in_use_head; i++)
39     {
40         gm->in_use[i] = gm->in_use[i + 1];
41     }
42     gm->in_use_head--;
43     return g;
44 }
45
46 inline void gm_add_to_available(gen_manager* gm, generator* g) {
47     gm->available[gm->available_head] = g;
48     gm->available_head++;
49 }
50
51 inline generator* gm_pop_off_front_from_available(gen_manager* gm) {
52     gm->available_head--;
53     return gm->available[gm->available_head];
54 }
55
56 inline void gm_set_gen_playing_note(generator* g, uint8_t midi_note) {
57     note_played_hash_table[midi_note] = g;

```

```

58 }
59
60 //get an available gen, or the oldest playing gen if none are available
61 inline generator* gm_get_gen(gen_manager* gm) {
62     generator* g;
63     if (!gm->available_head == 0) {
64         //pop off front of available
65         g = gm_pop_off_front_from_available(gm);
66     }
67     else { //none are available (this should be a rare case of key-mashing) or long
68         release times
69         g = gm_pop_off_back_from_in_use(gm);
70         gm_set_gen_playing_note(nullptr, g->midi_note); //make sure that the retrigger
71         of a note-off will not affect the new note
72     }
73     gm_add_to_in_use(gm, g);
74     return g;
75 }
76
77 //add all non-playing gens to the available list in O(N) time.
78 inline void gm_make_not_playing_available(gen_manager* gm) {
79     int count = 0;
80     for (int i = 0; i < gm->in_use_head; i++)
81     {
82         generator* g = gm->in_use[i];
83         if (!gen_is_playing(g)) {
84             count++;
85             gm_add_to_available(gm, g);
86             gm_set_gen_playing_note(nullptr, g->midi_note); //remove from hashtable
87         }
88         else {
89             gm->in_use[i - count] = g;
90         }
91     }
92     gm->in_use_head -= count;
93 }
94
95 inline generator* gm_get_gen_playing_note(uint8_t midi_note) {
96     return note_played_hash_table[midi_note];
97 }
98
99 float L_temp[PLAYBACK_BUFFER_SIZE];
100 float R_temp[PLAYBACK_BUFFER_SIZE];
101
102 inline void gm_write_n_samples(gen_manager* gm, gen_config* gc, float bufL[], float
103     bufR[], uint32_t n) {
104     for (int i = 0; i < PLAYBACK_BUFFER_SIZE; i++) //init to zero for addition later
105     {
106         bufL[i] = 0.0f;
107         bufR[i] = 0.0f;
108     }
109     for (int i = 0; i < gm->in_use_head; i++) //for all active voices
110     {
111         gen_write_n_samples(gm->in_use[i], gc, L_temp, R_temp, n); //write to temp

```

```

109     for (int i = 0; i < PLAYBACK_BUFFER_SIZE; i++) //for all samples in each voice
110     {
111         bufL[i] += 0.125f * L_temp[i]; //1/8th for 8 voices to ensure headroom
112         bufR[i] += 0.125f * R_temp[i];
113     }
114 }
115 for (int i = 0; i < PLAYBACK_BUFFER_SIZE; i++) //clamp to +- 1
116 {
117     float L = bufL[i], R = bufR[i];
118     bufL[i] = clamp(L, -1.0f, 1.0f);
119     bufR[i] = clamp(R, -1.0f, 1.0f);
120 }
121 //make generators that finished decay phase available
122 gm_make_not_playing_available(gm);
123 }
124
125 inline void gm_trigger_note_on(gen_manager* gm, gen_config* gc, uint8_t note,
126     uint8_t vel){
127     generator* g = gm_get_gen_playing_note(note);
128     if (g == nullptr) g = gm_get_gen(gm); //prevents any weirdness in retriggers of
129     notes before a note off
130     gen_freq(g, gc, notes_digital_freq[note], vel);
131     g->midi_note = note;
132     gm_set_gen_playing_note(g, note);
133     gen_note_on(g);
134 }
135
136 inline void gm_trigger_note_off(gen_manager* gm, uint8_t note) {
137     generator* g = gm_get_gen_playing_note(note);
138     gm_set_gen_playing_note(nullptr, note);
139     if (g != nullptr) gen_note_off(g); //prevents any weirdness in note off triggers
140     if it's not actually on
141 }

```

Listing D.9: Generator manager functions

Appendix E

Test code

```
1 #include <iostream>
2 #include <string>
3 #include <fstream>
4 #include <vector>
5 #include <random>
6
7 #include "LUT.h"
8 #include "init_luts.h"
9 #include "ADSR.h"
10 #include "IIR.h"
11 #include "wavetable.h"
12 #include "generator.h"
13 #include "play_notes.h"
14 #include "wav.h"
15
16
17 using namespace std;
18
19 //TODO: write code that tests all aspects for report
20
21 //white noise generator
22 void generate_wn(float* buf, int N) {
23     unsigned seed = 42; //LOL
24     std::default_random_engine generator(seed);
25     auto dist = std::normal_distribution<float>(0.0f, 1.0f);
26
27     for (int i = 0; i < N; i++)
28     {
29         buf[i] = dist(generator);
30     }
31 }
32
33 //chirp generator (in digital frequency)
34 void generate_linear_chirp_frequencies(float* buf, int N, float start, float end) {
35     for (int i = 0; i < N; i++)
36     {
37         float x = (float)i / (float)N;
38         float f = start + (end-start) * x;
39         buf[i] = f;
40     }
41 }
42
43 void add_wn(float* wn, int N, float u, float s) {
44     srand(time(NULL));
```



```

45 unsigned seed = rand();
46 std::default_random_engine generator(seed);
47 auto dist = std::normal_distribution<float>(u, s);
48
49 for (int i = 0; i < N; i++)
50 {
51     wn[i] += dist(generator);
52 }
53 }
54
55 void test_adsr(float* buf, int N) {
56     ADSR adsr;
57     int off = 0;
58     int k = N / 8;
59     float t = k / FS;
60     float params[4];
61     adsr_config(params, t, t, 0.5f, t);
62
63     for (int i = 0; i < k / 3; i++)
64     {
65         buf[i] = 0.0f;
66     }
67     off += k / 3;
68     adsr_trigger_on(&adsr);
69
70     for (int i = off; i < off + k + k/2; i++)
71     {
72         buf[i] = adsr_sample(&adsr, params);
73     }
74     adsr_trigger_on(&adsr);
75     off += k + k / 2;
76     for (int i = off; i < off + 3 * k; i++)
77     {
78         buf[i] = adsr_sample(&adsr, params);
79     }
80     off += 3* k;
81     adsr_trigger_off(&adsr);
82     for (int i = off; i < off + N - off; i++)
83     {
84         buf[i] = adsr_sample(&adsr, params);
85     }
86 }
87
88 void test_wavetable(float* buf, int N, float pos) {
89     //wavetables at different pos and freq
90     float f0 = notes_digital_freq[A0];
91     int off = 0;
92     int n = N / 8;
93     gen_config gc;
94     gc.wt_pos = pos;
95
96     wavetable wt;
97
98     for (float f = f0; f <= notes_digital_freq[A8]; f *= 2.0f) //sweep octaves

```

```

99  {
100     wt.phase = 0;
101     wt_config_digital_freq(&wt, f);
102     for (int i = off; i < off + n; i++)
103     {
104         buf[i] = wt_sample(&wt, &gc);
105     }
106     off += n;
107 }
108
109 }
110
111 void test_vibrato(float* buf, int N) {
112     float f0 = 10000.0f/FS;
113     float cents = 500.0f;
114     generator g;
115     gen_config gc;
116     gen_config_default(&gc);
117
118     gen_config_vibrato(&gc, cents, 2.0f / FS);
119
120     gen_apply_vibrato_config(&g, &gc);
121     gen_freq(&g, &gc, f0, 127);
122     gen_note_on(&g);
123
124
125
126     for (int i = 0; i < N; i++)
127     {
128         float temp;
129         gen_sample(&g, &gc, buf + i, &temp);
130     }
131 }
132
133 void test_filter(float* wn, float* out, float* adsr_out, int N, float a, float d,
134     float s, float r, float freqstart, float freqend, float q, void (*
135     filter_coeff_func)(IIR_coeff*, float, float)) {
136     ADSR adsr;
137     IIR_prev_values pv = {0,0,0,0};
138     IIR_coeff coeff;
139     float params[4];
140     adsr_config(params, a, d, s, r);
141     adsr_trigger_on(&adsr);
142
143     for (int i = 0; i < N/2; i++)
144     {
145         float f0 = freqstart + adsr_sample(&adsr, params) * (freqend - freqstart);
146         (*filter_coeff_func>(&coeff, f0, q);
147         out[i] = iir_filter_sample(&coeff, &pv, wn[i]);
148         adsr_out[i] = f0;
149     }
150
151     adsr_trigger_off(&adsr);

```

```

151     for (int i = N/2; i < N; i++)
152     {
153         float f0 = freqstart + adsr_sample(&adsr, params) * (freqend - freqstart);
154         (*filter_coeff_func)(&coeff, f0, q);
155         out[i] = iir_filter_sample(&coeff, &pv, wn[i]);
156         adsr_out[i] = f0;
157     }
158 }
159
160 void test_waveshape(float* buf_tanh, float* buf_sin, int N, float* freq, float
    g_tanh, float g_sin, bool use_aa = true) {
161     generator g;
162     gen_config gc;
163     gen_config_default(&gc);
164     gen_apply_vibrato_config(&g, &gc);
165     gen_note_on(&g);
166     gen_config_tanh_saturator(&gc, g_tanh);
167     gen_config_filter_envelope(&gc, 0.1f, 0.1f, 1.0f, 0.1f, 512.0f, 512.0f, 0.7071f);
168     gen_config_wavetables(&gc, 0.0f, 0.0f, 0.0f, 0.0f);
169     if(!use_aa)
170         iir_calc_lp12_coeff(&gc.filter_sat_AA, DIGITAL_FREQ_20KHZ, 1.0f);
171     for (int i = 0; i < N; i++)
172     {
173         float temp;
174         gen_freq(&g, &gc, freq[i], 127);
175         gen_sample(&g, &gc, buf_tanh + i, &temp);
176     }
177
178     //TODO IMPLEMENT SIN
179     gen_config_sine_saturator(&gc, g_sin);
180     gen_reset_AA_pv(&g); //reset pv from previous step
181     if (!use_aa)
182         iir_calc_lp12_coeff(&gc.filter_sat_AA, DIGITAL_FREQ_20KHZ, 1.0f);
183     for (int i = 0; i < N; i++)
184     {
185         //WHERE IS THE NOISE COMING FROM?!?!?!?!
186         float temp;
187         gen_freq(&g, &gc, freq[i], 127);
188         gen_sample(&g, &gc, buf_sin + i, &temp);
189     }
190 }
191
192 void test_wavetable_sweep(float* buf, int N, float* freq, float pos) {
193     wavetable wt;
194     gen_config gc;
195
196     gc.wt_pos = pos;
197
198     for (int i = 0; i < N; i++)
199     {
200         wt_config_digital_freq(&wt, freq[i]);
201         buf[i] = wt_sample(&wt, &gc);
202     }
203 }

```

```

204
205 void test_stereo_width(float* bufL, float* bufR, int N, float width) {
206     float f0 = 100.0f / FS;
207     float cents = 1200.0f;
208     generator g;
209     gen_config gc;
210     gen_config_default(&gc);
211     gen_apply_vibrato_config(&g, &gc);
212     gen_config_wavetables(&gc, 0.0f, cents, 1.0f, width);
213     gen_freq(&g, &gc, f0, 127);
214     gen_note_on(&g);
215     gen_write_n_samples(&g, &gc, bufL, bufR, N);
216 }
217
218 void test_gm_overload(float buf[], int N) {
219
220     gen_manager gm;
221     gen_config gc;
222     gen_config_default(&gc);
223     gm_init(&gm);
224     gc.filter_coeff_func = &iir_calc_no_coeff;
225     gen_config_wavetables(&gc, 0.0f, 0.0f, 0.0f, 0.0f);
226
227     float* temp = new float[N];
228
229     float freq[16];
230     for (int i = 0; i < 8; i++)
231     {
232         freq[i] = (1000.0f + i * 2000.0f) / FS; //1k, 3k, ..., 15k
233         freq[i+8] = (2000.0f + i * 2000.0f) / FS; //2k, 4k, ..., 16k
234     }
235
236     int L = N / 16;
237     int end = L * 16;
238     int note = 0;
239     for (int off = 0; off < end; off += L)
240     {
241         gm_trigger_note_on_freq(&gm, &gc, note, 127, freq[note]);
242         gm_write_n_samples(&gm, &gc, buf + off, temp + off, L);
243         note++;
244     }
245
246     //fill remaining with 0
247     for (int i = 0; i < N-end; i++)
248     {
249         buf[end + i] = 0.0f;
250     }
251 }
252
253
254
255 void test_gm_operation(float buf[], int N) {
256
257     gen_manager gm;

```

```

258 gen_config gc;
259 gen_config_default(&gc);
260 gm_init(&gm);
261 gc.filter_coeff_func = &iir_calc_no_coeff;
262 gen_config_wavetables(&gc, 0.0f, 0.0f, 0.0f, 0.0f);
263
264 float* temp = new float[N];
265
266 float freq[8];
267 for (int i = 0; i < 8; i++)
268 {
269     freq[i] = (500.0f + i * 2500.0f) / FS; //0k5, 3k, 5k5, ..., 18k
270 }
271
272 int L = N / 16;
273 int end = L * 16;
274 int note = 0;
275 int count = 0;
276 //trigger all on, then trigger all off by LIFO
277 for (int off = 0; off < end; off += L)
278 {
279     if (count < 8)
280     {
281         gm_trigger_note_on_freq(&gm, &gc, note, 127, freq[note]);
282         note++;
283     }
284     else {
285         gm_trigger_note_off(&gm, note);
286         note--;
287     }
288
289     gm_write_n_samples(&gm, &gc, buf + off, temp + off, L);
290     count++;
291 }
292
293 //fill remaining with 0
294 for (int i = 0; i < N - end; i++)
295 {
296     buf[end + i] = 0.0f;
297 }
298 }
299
300
301 void test_freq_scaling() {
302     cout << "Detune factor 125.57: " << get_detune_factor_cents_lut(125.57f) << endl;
303 }
304
305 void print_to_file(float arr[], int size, const string& name) {
306     ofstream f;
307     f.open(name);
308     f.precision(10);
309     for (int i = 0; i < size; i++) {
310         f << arr[i] << endl;
311     }

```

```

312     f.close();
313 }
314
315
316 void print_iir_coeff(IIR_coeff& f) {
317     cout << "[" << f.n0 << ", " << f.n1 << ", " << f.n2 << "], [ 1, -" << f.d1 << ",
318         "-" << f.d2 << "]" << endl;
319 }
320
321 void print_basic_waveforms(){
322     string names[] = { "sine", "triangle", "sawtooth", "square" };
323     for (int i = 0; i < 4; i++) {
324         for (int j = 0; j < 8; j++) {
325             print_to_file(basic_luts[i][j], LUT_SIZE, "..\\basic waveforms\\" + names[i]
326                 + "\\\" + to_string(j) + ".txt");
327         }
328     }
329 }
330
331 void run_tests() {
332     int N = 44100;
333     float* wn = new float[N];
334     generate_wn(wn, N);
335
336     float* chirp_freq = new float[N];
337     generate_linear_chirp_frequencies(chirp_freq, N, DIGITAL_FREQ_20HZ,
338         DIGITAL_FREQ_20KHZ*0.5f); //20 to 10k
339
340     float* out = new float[N];
341     float* out2 = new float[N];
342     float* adsr = new float[N];
343
344     //FILTERS
345     cout << "-----" << endl;
346     cout << "STARTING FILTER TESTS" << endl;
347     cout << "-----" << endl;
348
349     cout << "HP24" << endl;
350     print_to_file(wn, N, "..//testfiles//filter//white.txt");
351     test_filter(wn, out, adsr, N, 0.25f, 0.1f, 0.5f, 0.2f, 1000.0f / FS, 20000.0f /
352         FS, 5.0f, &iir_calc_hp24_coeff);
353     print_to_file(out, N, "..//testfiles//filter//hp24.txt");
354     cout << "LP24" << endl;
355     test_filter(wn, out, adsr, N, 0.25f, 0.1f, 0.5f, 0.2f, 1000.0f / FS, 20000.0f /
356         FS, 5.0f, &iir_calc_lp24_coeff);
357     print_to_file(out, N, "..//testfiles//filter//lp24.txt");
358     cout << "BP12" << endl;
359     test_filter(wn, out, adsr, N, 0.25f, 0.1f, 0.5f, 0.2f, 1000.0f / FS, 20000.0f /

```

```

    / FS, 5.0f, &iir_calc_hp12_coeff);
360 print_to_file(out, N, "../testfiles//filter//hp12.txt");
361 cout << "LP12" << endl;
362 test_filter(wn, out, adsr, 44100, 0.25f, 0.1f, 0.5f, 0.2f, 1000.0f / FS, 20000.0f
    / FS, 5.0f, &iir_calc_lp12_coeff);
363 print_to_file(out, N, "../testfiles//filter//lp12.txt");
364 cout << "WRITING ENVELOPE" << endl;
365 print_to_file(adsr, N, "../testfiles//filter//adsr.txt");
366 int i = 0;
367
368 cout << endl;
369 cout << "-----" << endl;
370 cout << "STARTING WAVETABLE TESTS" << endl;
371 cout << "-----" << endl;
372 //WAVETABLE
373 for (float p = 0.0; p < 4.0f; p += 0.5f)
374 {
375     cout << "POS = " << p << endl;
376     test_wavetable(out, N, p);
377     print_to_file(out, N, "../testfiles//wavetable//" + to_string(i) + ".txt");
378     i++;
379 }
380
381 cout << "SQUARE CHIRP" << endl;
382 test_wavetable_sweep(out, N, chirp_freq, 3.0f);
383 print_to_file(out, N, "../testfiles//wavetable//square_chirp.txt");
384
385 cout << endl;
386 cout << "-----" << endl;
387 cout << "STARTING GENERATOR TESTS" << endl;
388 cout << "-----" << endl;
389 //GENERATOR
390 cout << "VIBRATO" << endl;
391 test_vibrato(out, N);
392 print_to_file(out, N, "../testfiles//generator//vibrato.txt");
393
394 float w[] = { 0.0f, 0.33f, 0.67f, 1.0f };
395
396 for (int i = 0; i < 4; i++)
397 {
398     cout << "WIDTH = " << w[i] << endl;
399     test_stereo_width(out, out2, N, w[i]);
400     print_to_file(out, N, "../testfiles//generator//width L " + to_string(i) + ".
        txt");
401     print_to_file(out2, N, "../testfiles//generator//width R " + to_string(i) + ".
        txt");
402 }
403
404 cout << endl;
405 cout << "-----" << endl;
406 cout << "STARTING WAVESHAPER TESTS" << endl;
407 cout << "-----" << endl;
408
409

```

```

410 //WAVESHAPE
411 for (int i = 0; i < 5; i++)
412 {
413     float ip1 = (float)i + 1.0f;
414     float g_sin = PI / 2.0f * ((float)i + 1.0f);
415     float g_tanh = ip1 * ip1 * 2.0f;
416
417     cout << "G_TANH = " << g_tanh << " G_SIN = " << g_sin << endl;
418
419     test_waveshape(out, out2, N, chirp_freq, g_tanh, g_sin, true);
420     print_to_file(out, N, "../testfiles/waveshape/tanh " + to_string(i) + ".txt"
421 );
422     print_to_file(out2, N, "../testfiles/waveshape/sin " + to_string(i) + ".txt"
423 );
424
425     test_waveshape(out, out2, N, chirp_freq, g_tanh, g_sin, false);
426     print_to_file(out, N, "../testfiles/waveshape/tanh " + to_string(i) + " no
427 aa.txt");
428     print_to_file(out2, N, "../testfiles/waveshape/sin " + to_string(i) + " no
429 aa.txt");
430 }
431
432 cout << endl;
433 cout << "-----" << endl;
434 cout << "STARTING ADSR TESTS" << endl;
435 cout << "-----" << endl;
436 cout << "ADSR WITH RETRIGGER" << endl;
437 test_adsr(out, N);
438 print_to_file(out, N, "../testfiles/adsr/adsr retrigger.txt");
439
440 cout << endl;
441 cout << "-----" << endl;
442 cout << "STARTING MANAGER TESTS" << endl;
443 cout << "-----" << endl;
444 cout << "TESTING MANAGER OPERATION" << endl;
445 test_gm_operation(out, N);
446 print_to_file(out, N, "../testfiles/manager/operation.txt");
447
448 cout << "TESTING MANAGER OVERLOAD" << endl;
449 test_gm_overload(out, N);
450 print_to_file(out, N, "../testfiles/manager/overload.txt");
451
452 }

```

Listing E.1: Code used to generate test data for chapter 4

Appendix F

Project Planning Schedule

This is an appendix.

Appendix G

Outcomes Compliance

This is another appendix.