Comprehensive Design and Modeling of a Quadcopter

Marco Soto[1] and Emily Lory[2]
University of Illinois at Urbana-Champaign, Champaign, IL, 61820, United States

Ram Velamuri[3]
University of Illinois at Urbana-Champaign, Champaign, IL, 61820, United States

Ella Young[4]
University of Illinois at Urbana-Champaign, Champaign, IL, 61820, United States

This paper details how the required equations of motion were determined to create a controller for a quadrotor to accomplish the goals written in the performance requirements. This included hovering above ground for an allotted amount of time, flying in a circle of a designated radius and height, and following a set flight path. This was done with all engineering requirements maintained and by properly addressing realistic constraints of the quadcopter. Understanding what is required to precisely describe and control a vehicle like a quadcopter is essential for smoothly integrating aerospace vehicles into society for purposes other than traditional transportation.

Nomenclature

A = Jacobian matrix in terms of x
B = Jacobian matrix in terms of u
C_D = coefficient of drag
C_L = coefficient of lift
F = forces on the system
g = force of gravity on Earth
I_{ii} = moment of inertia in position ii
l_z = distance in the z direction
m = mass
p = momentum
$\dot{q}$ = velocity
$\ddot{q}$ = acceleration
R_i = rotation matrix in i
t = time
T_i = thrust force
u_0 = equilibrium point
x = displacement in the x axis
x_0 = equilibrium point
$\dot{x}$ = velocity in the x direction
$\ddot{X}$ = acceleration in the x direction
y = displacement in the Y axis
$\ddot{Y}$ = acceleration in the Y direction
z = displacement in the Z axis
$\ddot{Z}$ = acceleration in the Z direction
θ = pitch
ϕ = roll
ψ = yaw
ω = angular velocity
$\dot{\omega}$ = angular acceleration
τ_i = torques of i

[1] Undergraduate Student and Aerospace Engineering for first author.
[2] Undergraduate Student and Aerospace Engineering for second author.
[3] Undergraduate Student and Aerospace Engineering for third author.
[4] Undergraduate Student and Aerospace Engineering for fourth author.

Introduction

Quadcopters, also known as quadrotors or drones, are unmanned aerial vehicles (UAVs) utilizing four rotors for lift and maneuverability. Their distinct design with four propellers arranged in a cross formation allows them to achieve vertical take-off and landing (VTOL) with exceptional hovering capabilities. This makes them highly versatile for various applications, from photography and videography over search and rescue missions to defense. They are popular due to their ease of control and come in a range of sizes and functionalities.

Beyond established uses, the field of autonomous systems has created new uses for drones. Research continues to focus on improving their flight dynamics, power efficiency, and payload capacities, which can enable applications in areas such as agriculture, package delivery, and even environmental surveying. For all these innovations, the challenge of understanding and modeling the dynamics of drones is key. With this understanding, the drones can have a successful design and deployment.

This project will focus on creating a dynamical model (digital twin) of a four-axis symmetric drone, incorporating both its center of mass motion and rigid body dynamics that describe its attitude control. The state of the drone, as well as its attitude, will only be altered by the change of speed of each rotor blade, which results from a change in motor torque. The motor torque can be adjusted for each blade/motor individually. Overall, our digital model will allow for the simulation and analysis of the drone's performance under realistic constraints and can provide insights into optimizing strategies to control the drone.

The modeling process requires us to address several engineering challenges, including accurately representing the forces and moments generated by the rotors, accounting for the effects of gravity and lift on the drone, and ensuring that the drone is stable during various maneuvers. In addition, the model must operate within design constraints, such as weight, power consumption, and component selection, to ensure the drone’s feasibility and applicability.

The following engineering requirements (ER) must be included in this project:

- ER1: The drone model shall have 6 degrees of freedom (center of mass motion plus attitude) and account for gravity and lift generated from 4 rotors.

- ER2: The drone model shall be able to modify its state and attitude solely through increasing or decreasing the number of revolutions per second of its 4 rotor blades, as determined by the motor torques.

- ER3: The modeled drone shall be able to achieve the performance goals as outlined in section 4, without exhausting its power supply.

- ER4: The model as determined through its parameters such as mass, power consumption, engine torques and rotor blade size shall be realistic and implementable, preferably through the use of off-the-shelf parts. In particular, the mass of the drone shall be between 0.1kg and 10kg.

By following the above engineering requirements, parameters are developed around which to design the drone.

We will also have the following performance requirements for the drone:

1. Hover 1m above ground for 2 minutes.
2. Fly in a circle of radius 2m, at an altitude of 1m above ground at a speed of 0.5m/s for at least 1 minute.
3. Launch from ground and ascend vertically until 1m above ground. Move in a straight line 1m above ground at an average speed of 1m/s for 5m, stop (hover), yaw 90 deg to the left, move in another straight line for 5m, stop (hover), land vertically with a speed of no more than 1cm/s.

These performance requirements are put in place to test the validity of the equations of motion which will be later derived in the project, to demonstrate the drone's capability.

Through this project, we aim to derive the equations of motion that govern the drone by using the engineering requirements and validate the model by demonstrating the drone's ability to achieve specific performance goals.

Mathematical Description

To describe the quadcopter utilizing mathematical models, the governing equations of dynamics must be identified. This includes applying the coordinate transformations utilizing rotation matrices, Newton's Laws, Euler equations, and Tait-Bryan angles. This will result in two sets of equations of motion to describe the system. One will describe the translational dynamics, and the other will describe the rotational dynamics.

Translational Dynamics

The quadcopter requires six degrees of freedom, the position states x, y, z and the rotation states ϕ, θ, ψ. ϕ is the roll angle, θ defines the pitch angle, and ψ is the yaw angle. To start with the translational dynamics, we will begin by focusing on the position states and their transformations. To find the equations of motion for translational dynamics we can utilize Newton's Laws and the proper frames.

The first step is defining the rotation matrices required to transform from the inertial to the body frame. The inertial frame is defined by the drone's position and orientation measured relative to the fixed reference point on the Earth's surface. The body frame is defined as the coordinate system fixed to the drone's physical form with the origin at the drone's center of gravity and the axes aligned with the primary directions of motion of the drone. The image below depicts these two frames.

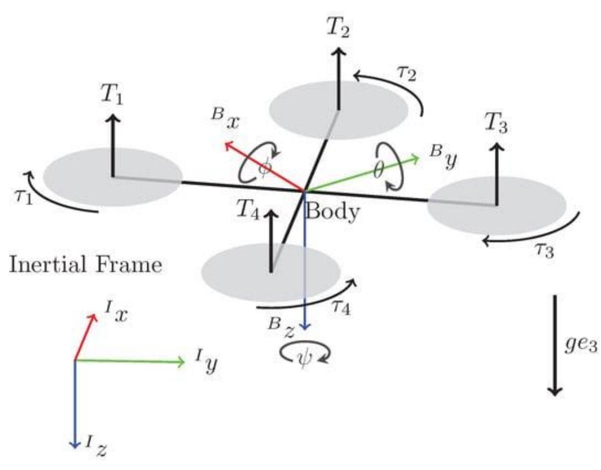

Fig 1. Forces, inertial frame, and body frame in relation to the quadcopter [1].

The rotation can be defined as a series of rotations in each axis matrix multiplied to create a single rotation matrix that can convert the vector from the inertial to the body frame. The three rotation matrices required are defined below.

$$R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{bmatrix} \tag{1}$$

$$R_y(\theta) = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \tag{2}$$

$$R_z(\psi) = \begin{bmatrix} \cos(\psi) & \sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

The equations for the rotation from inertial to body frame and vis versa are shown below.

$$R_{BinI}(\phi, \theta, \psi) = R_z R_y R_x = \tag{4}$$

$$\begin{bmatrix} \cos(\psi)\cos(\theta) & \sin(\phi)\sin(\theta)\cos(\psi) - \sin(\psi)\cos(\phi) & \sin(\phi)\sin(\psi) + \sin(\theta)\cos(\phi)\cos(\psi) \\ \sin(\psi)\cos(\theta) & \sin(\phi)\sin(\psi)\sin(\theta) + \cos(\phi)\cos(\psi) & -\sin(\phi)\cos(\psi) + \sin(\psi)\sin(\theta)\cos(\phi) \\ -\sin(\theta) & \sin(\phi)\cos(\theta) & \cos(\phi)\cos(\theta) \end{bmatrix}$$

$$R_{IinB}(\phi, \theta, \psi) = R_{BinI}(\phi, \theta, \psi)^{-1} = R_{BinI}(\phi, \theta, \psi)^T = \tag{5}$$

$$\begin{bmatrix} \cos(\psi)\cos(\theta) & \sin(\psi)\cos(\theta) & -\sin(\theta) \\ \sin(\phi)\sin(\theta)\cos(\psi) - \sin(\psi)\cos(\phi) & \sin(\phi)\sin(\psi)\sin(\theta) + \cos(\phi)\cos(\psi) & \sin(\phi)\cos(\theta) \\ \sin(\phi)\sin(\psi) + \sin(\theta)\cos(\phi)\cos(\psi) & -\sin(\phi)\cos(\psi) + \sin(\psi)\sin(\theta)\cos(\phi) & \cos(\phi)\cos(\theta) \end{bmatrix}$$

With these rotation matrices, the equations of motion for translational dynamics can be found with Newton's Laws. In this application the only forces necessary to analyze the body frame are the thrust forces for each rotor and how they impact the acceleration of the whole system. In the inertial frame, the earth's gravity must also be factored in. To use the sum of the thrust forces in the inertial frame they must be converted using the rotation matrix calculated above. The application of Newton's Second Law for the quadcopter in the inertial frame is written below based on the free body diagram in Fig 1.

$$\frac{d\boldsymbol{p}}{dt} = \frac{dm}{dt}\dot{\boldsymbol{q}} + m\frac{d\dot{\boldsymbol{q}}}{dt} = \boldsymbol{F} \tag{6}$$

$$m\ddot{\boldsymbol{q}} = R_{IinB}(\boldsymbol{T_1} + \boldsymbol{T_2} + \boldsymbol{T_3} + \boldsymbol{T_4}) - m\boldsymbol{g} = \boldsymbol{F_{total}}$$

Although the thrust of each rotor and the force of gravity act only along the *z*-axis in the **body frame**, the quadcopter's acceleration in the inertial frame must account for all three spatial components (*x*, *y*, and *z*). This is because the quadcopter's roll, pitch, and yaw change the orientation of the thrust vector relative to the inertial frame. By transforming from the body frame to the inertial frame, the *x*, *y*, and *z* components of the thrust vector must be resolved to correctly describe the UAV's dynamics. The resulting equations of motion are as follows:

$$T_{total} = T_1 + T_2 + T_3 + T_4 \tag{7}$$

$$\begin{bmatrix} \ddot{X} \\ \ddot{Y} \\ \ddot{Z} \end{bmatrix} = \frac{1}{m} \begin{bmatrix} T_{total}(\sin(\phi)\sin(\psi) + \sin(\theta)\cos(\phi)\cos(\psi)) \\ T_{total}(-\sin(\phi)\cos(\psi) + \sin(\psi)\sin(\theta)\cos(\phi)) \\ T_{total}(\cos(\phi)\cos(\theta)) \end{bmatrix}$$

The total thrust can be calculated as a function of C_L (coefficient of lift), C_D(coefficient of drag), and τ. Starting with the equation for torques, because of the instantaneous control over the motors we can neglect the angular acceleration term. We can then isolate the squared angular velocity term to relate it to the total thrust equation.

$$\tau = C_D\omega^2 + I\dot{\omega} \tag{8}$$

$$T_{total} = C_L\omega^2 \tag{9}$$

$$T_{total} = \frac{C_L}{C_D}\sum_{i=1}^{4}\tau_i \tag{10}$$

Inserting the total thrust into equations of motion we have already derived gives us the following:

$$\begin{bmatrix} \ddot{X} \\ \ddot{Y} \\ \ddot{Z} \end{bmatrix} = \frac{1}{m}\frac{C_L}{C_D}(\tau_1 + \tau_2 + \tau_3 + \tau_4) \begin{bmatrix} \sin(\phi)\sin(\psi) + \sin(\theta)\cos(\phi)\cos(\psi) \\ -\sin(\phi)\cos(\psi) + \sin(\psi)\sin(\theta)\cos(\phi) \\ \cos(\phi)\cos(\theta) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} \tag{11}$$

With this we can now relate our translational equations of motion to the torques for each motor.

Rotational Dynamics

To find the second set of equations of motion in terms of the rotational dynamics, Euler's equations and the corresponding relevant matrices will be utilized as shown below.

The moment of inertia for our quadcopter can be represented as two rods of equal mass joined at the center making 90^0 angles with its adjacent neighbor rod. In addition to this, point masses will be considered at either end of each rod. Our moment of inertia tensor, I, will be a symmetrical and diagonal 3x3 due to the quadcopter's configuration; two symmetrically arranged rods have its principal axes of rotation align with its body frame axes x, y, and z.

$$I = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \tag{12}$$

Assuming the rods are uniform, their contribution to the moment of inertia is calculated using the perpendicular and parallel components relative to each axis. Due to symmetry, the moment of inertia about the x and y axis are the same, while the moment of inertia about the z-axis will consider four-point masses rather than two:

$$I_{xx} = I_{yy} = \left(\frac{1}{12}\right)(m_{body})l^2 + 2(m_{motor})r_{rotor}^2$$

$$I_{zz} = \left(\frac{1}{12}\right)(m_{body})l^2 + 4(m_{motor})r_{rotor}^2$$

The torque, τ, consists of the torques τ_ϕ, τ_θ, and τ_ψ in the direction of those corresponding angles along the x, y and z axis respectively. We see that roll is induced on the quadcopter when there is a torque imbalance between rotors 1 and 3. Similarly, pitch is induced when there is a torque imbalance between rotors 2 and 4. Lastly, yaw is induced when the torques of two opposite rotors are larger than the others remaining two, and vis versa:

$$\tau = \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} l\frac{C_L}{C_D}(\tau_1 - \tau_3) \\ l\frac{C_L}{C_D}(\tau_2 - \tau_4) \\ \tau_1 - \tau_2 + \tau_3 - \tau_4 \end{bmatrix} \tag{13}$$

The angular equations of rotational motion can be derived using Newton-Euler equations of rigid body motion. The equation in the body frame for the angular acceleration of a quadcopter is given as:

$$\dot{\boldsymbol{\omega}} = \boldsymbol{I}^{-1}(\boldsymbol{\tau} - \boldsymbol{\omega} \times (\boldsymbol{I\omega})) \tag{14}$$

With all this information combined we will have a solution for the body frame angular acceleration of the quadcopter as follows.

$$\dot{\boldsymbol{\omega}} = \begin{bmatrix} \frac{1}{I_{xx}} & 0 & 0 \\ 0 & \frac{1}{I_{yy}} & 0 \\ 0 & 0 & \frac{1}{I_{zz}} \end{bmatrix} \left(\begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} - \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \times \left(\begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \right) \right) \tag{15}$$

$$\dot{\boldsymbol{\omega}} = \begin{bmatrix} \frac{l\frac{C_L}{C_D}(\tau_1 - \tau_3) - (\omega_y I_{zz} \omega_z - \omega_z I_{yy} \omega_y)}{I_{xx}} \\ \frac{l\frac{C_L}{C_D}(\tau_2 - \tau_4) - (\omega_z I_{xx} \omega_x - \omega_x I_{zz} \omega_z)}{I_{yy}} \\ \frac{\tau_1 - \tau_2 + \tau_3 - \tau_4 - (\omega_x I_{yy} \omega_y - \omega_y I_{xx} \omega_x)}{I_{zz}} \end{bmatrix}$$

When considering roll, pitch and yaw rates as a function of orientation and body fixed angular rates, we are also able to create the below relations.

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \omega_x + \omega_y \sin(\phi)\tan(\theta) + \omega_z \cos(\phi)\tan(\theta) \\ \omega_y \cos(\phi) - \omega_z \sin(\phi) \\ \frac{\omega_y \sin(\phi) + \omega_z \cos(\phi)}{\cos(\theta)} \end{bmatrix} \tag{16}$$

To compute the **second derivatives** from the time derivatives of the roll, pitch and yaw angles, we use the relationship between the **body-fixed angular rates** and the **Euler angle rates**. Since we've already derived the first derivatives, we derive the equations again, while accounting for time-dependent terms in the orientation angles and angular rates.

$$\begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} \dot{\omega}_x + \dot{\omega}_y \sin(\phi)\tan(\theta) + \dot{\omega}_z \cos(\phi)\tan(\theta) + \left(\omega_y cos(\phi) tan(\theta) - \omega_z sin(\phi) tan(\theta)\right)\dot{\phi} + \left(\omega_y sin(\phi) + \omega_z cos(\phi)\right) sec^2(\theta)\dot{\theta} \\ \dot{\omega}_y \cos(\phi) - \dot{\omega}_z \sin(\phi) - \left(\omega_y sin(\phi) + \omega_z cos(\phi)\right)\dot{\phi} \\ \frac{\dot{\omega}_y \sin(\phi) + \dot{\omega}_z \cos(\phi) + \left(\omega_y cos(\phi) - \omega_z sin(\phi)\right)\dot{\phi} + \left(\omega_y sin(\phi) + \omega_z cos(\phi)\right) sin(\theta)\dot{\theta}}{cos^2(\theta)} \end{bmatrix} \tag{17}$$

After solving these two equations of motions describing the quadcopter, we understand the path and altitude the quadcopter follows over time. Several factors were excluded in the calculations to simplify the dynamics including but not limited to air resistance, external forces and moments, and thermal effects. Using this information, we can create a controller that helps us determine the necessary torques to accomplish the tasks we are taking on.

Model Performance

To simulate the results of the drone a basic controller was designed to track the trajectory, velocity, and acceleration of the drone as it performed the three tasks required of it. Python was also used as a computational tool to validate the stated equations of motion in all three tasks.

Controller

To start setting up the controller the EOM needed to be linearized about a set of equilibrium points. In state-space form our EOM is represented below as a function of **x** and **u** in addition to the A and B matrices which are found from the Jacobians of our equations of motion with respect to **x** and **u**.

$$\boldsymbol{x} = \begin{bmatrix} x \\ y \\ z \\ \phi \\ \theta \\ \psi \\ \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \boldsymbol{u} = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \end{bmatrix} \tag{18}$$

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + B\boldsymbol{u} \tag{19}$$

Our chosen equilibrium points are shown below.

$$\boldsymbol{x_0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \boldsymbol{u_0} = \frac{9.81 \times C_L}{C_D} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} m^2 \tag{20}$$

Trial and error were used to adjust the Q and R matrices to create an optimal controller utilizing LQR. With this controller and the listed parameters, the quadcopter was able to accomplish the following flight paths.

$$\begin{aligned} m &= 1.552\ kg \\ I_{xx} &= 0.00005\ kg\ m^2 \\ I_{yy} &= 0.00005 kg\ m^2 \\ I_{zz} &= 0.0001 \\ & kg\ m^2 \\ l_z &= 0.04\ m \end{aligned} \tag{21}$$

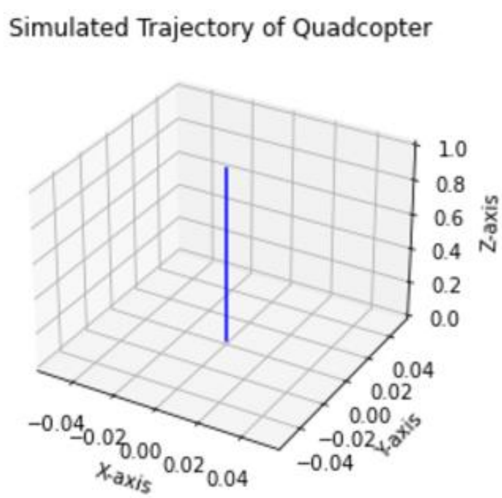

Fig 2. Simulated trajectory of quadcopter moving upwards 1m.

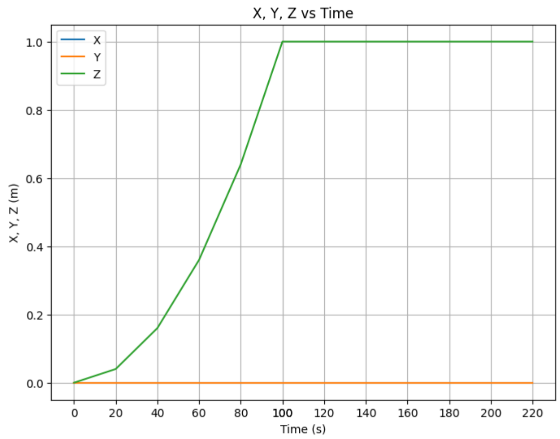

Fig 3. X, Y, and Z over time.

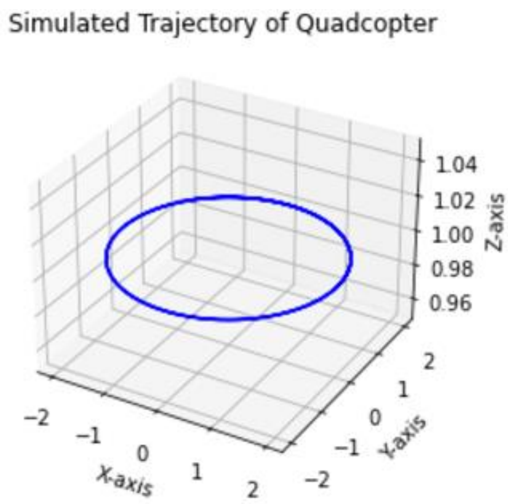

Fig 4. Simulated trajectory of quadcopter flying in a circle of radius 2 m.

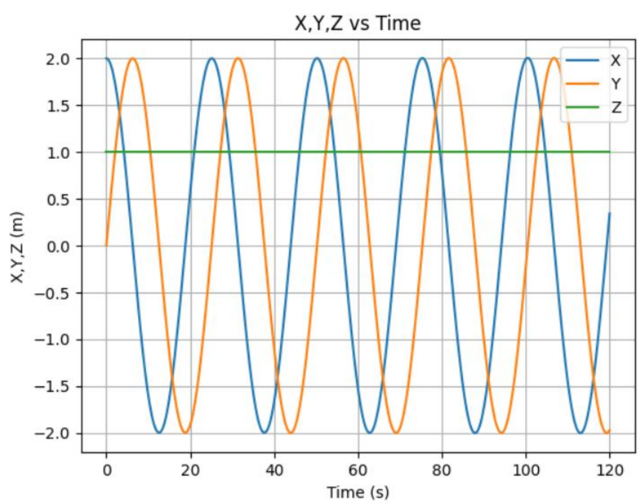

Fig 5. X, Y, and Z over time.

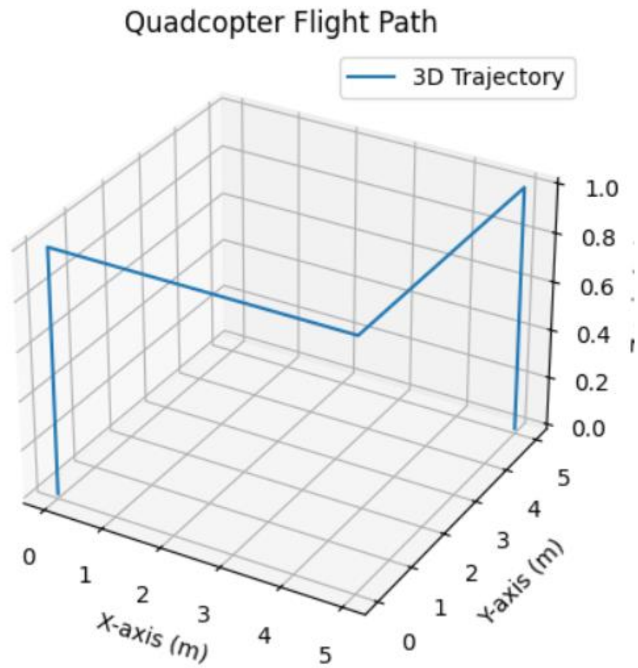

Fig 6. Simulated trajectory of quadcopter flying in a designated path.

Compliance With Engineering Requirements

ER 1 states that the drone model shall have 6 degrees of freedom (center of mass motion plus attitude) and account for gravity and lift generated from 4 rotors. Our model meets this requirement, as we have accounted for both the center of mass motion and attitude. By doing so, we can ensure that the drone is precisely modeled for both its translational and rotational motion which describes the roll, pitch, and yaw. This ensures that the model can represent the drone flying in 3-dimensional space by taking into consideration moments of inertia and torques generated by the rotors, as well as forces such as lift, weight and thrust.

ER 2 says the drone shall modify its state and attitude solely by increasing or decreasing the revolutions per second of the four rotors, these changes are generated by torque. State control refers to the movement of the quadcopter in the cartesian plane (vertical and lateral motion). While torque is not directly responsible for translational motion, it is stabilized to allow movement centralized on one axis. For example, in Fig. 2 the drone flies vertically at a constant velocity and can remain there for 2 minutes. The quadcopter can hover using the knowledge of its mass and the force of gravity. It accounts for the weight of the drone and the four rotors generate identical lift forces to overcome gravity and then provide lift to balance gravity and hover in place. Torque directly comes into play concerning the rotational motion of the drone and attitude control. By adjusting the rotations per second of opposing pairs or rotors, torque can be generated to adjust pitch and roll. Pitch is generated by creating thrust in the rear motors to create a torque to tip the drone forward or backward. Roll is generated by increasing the thrust in the left or right pairs of motors to tilt the drone to either side. Lastly, changing the thrust generated from opposite motors increases the torque in one direction and uses the conservation of angular momentum to have the drone spin and therefore yaw in one direction. In Fig. 3, the drone utilizes all three degrees of freedom as well as centripetal motion to complete its route. The drone can adjust its pitch or roll to induce the centripetal force, tilting the drone towards the center of its rotation. The yaw torque allows the drone to continuously face the center of rotation. Additional forces that play a role in this motion are the balancing lift, the force needed to be produced by the rotors to allow a constant altitude; the centripetal force, needed for additional calculations, depending on the mass, velocity, and radius of its path; and the tilt angle, depends on the mass and moment of inertia of the drone and is calculated using the centripetal force, mass, and force of gravity. For this motion, the degrees of freedom for attitude control are working all at once to create the motion; in Fig. 4 the drone utilizes the degrees of freedom almost in succession. The drone recreates the motion from Fig. 2, creates torque to yaw to the left 90 degrees, creates lateral movement with roll or pitch, and then repeats that process in reverse order to finally land back on the ground. This final trajectory utilizes movements completed by the drone in previous instances but puts them together sequentially.

ER 3 states the drone shall complete the performance goals without exhausting the power supply.

Flight time supported by the battery is found using the Amp-hours provided by the battery divided by the current drawn by the motors.

$$Flight\ Time = \left(\frac{Ah}{4(A)}\right) = \left(\frac{6\ Ah}{4(21\ A)}\right) = \ 0.07\ hours\ =\ 4.29\ \text{min}$$

The drone is required to fly for 3 minutes to complete the first two performance goals, but this does not include the time it would take to take off and land for either requirement or the time to complete the third performance goal. To account for this extra time, the drone can be run at 50% output to save battery power. 50% output power will double the flight time allowing for almost nine minutes of flight, which is sufficient for all three maneuvers without recharging.

ER 4 states the drone shall be realistic in terms of weight, size, and power consumption. Based on the reality check and the COTS materials list, the projected weight of the quadcopter will be 1.5 kg with a rotor radius of 225 mm and a propeller radius of 127 mm. The mass of the body will be the total mass of the drone minus the masses of the motors and the radius of the rotor will be half the length of the rod.

$$I_{xx} = \ I_{yy} = \left(\frac{1}{12}\right)\left(\frac{m_{body}}{2}\right) l^2 \ +\ 2(m_{motor})r_{rotor}^2 = \left(\frac{1}{12}\right)\left(\frac{1.316\ kg}{2}\right)0.45^2 m\ +\ 2(0.046\ kg)0.225^2 m$$

$$I_{xx} = I_{yy} = 0.01576\ kg \cdot m^2$$

$$I_{zz} = \left(\frac{1}{12}\right)\left(\frac{m_{body}}{2}\right) l^2 \ +\ 4(m_{motor})r_{rotor}^2 = \left(\frac{1}{12}\right)\left(\frac{1.316\ kg}{2}\right)0.45^2 m\ +\ 4(0.046\ kg)0.225^2 m$$

$$I_{zz} = 0.02041\ kg \cdot m^2$$

Compliance With Performance Goals

The first performance goal says the drone should hover 1 meter above the ground for 2 minutes. By observing Figure 2, we see that the drone does reach the specified height of 1 meter. Through simulations, we were able to see this height lasting for 2 minutes. In addition, when the drone hovers to 1 meter of height, we see that it goes straight up, without any deviations in its path. The control system was able to keep the drone steady in the vertical direction.

The second performance goal states that the drone should fly in a circle of radius 2 m, at an altitude of 1m above ground at a speed of 0.5m/s for at least 1 minute. From Figure 3, we can see that this is the case, and the drone exactly meets this requirement. Simulations were run to ensure that the drone was able to last at this position for at least 1 minute. The drone had a smooth and steady circular path and did not deviate from its path at all. This shows us that both the translational and rotational dynamics of the drone work together in tandem to keep the drone steady.

The third performance goal states that the drone should launch from the ground and ascend vertically until 1m above ground. Then, it should move in a straight line 1m above ground at an average speed of 1m/s for 5m, stop (hover), yaw 90 deg to the left, move in another straight line for 5m, stop (hover), and land vertically with a speed of no more than 1cm/s. Observing figure 4, we see that the drone is capable of this, and performed the maneuvers smoothly without deviation from its path.

Overall, the drone met all 3 performance requirements, showing that the drone we constructed executes its maneuvers precisely and accurately, as seen in the graphs above.

Constructing the Quadcopter

Building a successful quadcopter first requires the goal of the drone to be fully realized; the size and material used for the frame directly contribute to the maneuverability and stability of its flight. In addition to this, each material has its price point, so the ultimate budget must be noted. In the case of the project's performance goals, requiring precise movements at low altitude and low speed, stability is more important than agility. While the budget is more flexible, assume an undergraduate or graduate student group is working on the project to set a price point [2].

Table 1 COTS Materials

Material	*Quantity*	*Cost (per unit)*	*Description*
F450 Drone Frame and Landing Gear	1	$21.00	The frame is made of glass fiber for durability and has a diameter of 450 mm. Has a net weight of 272 g [3].
QM2812 2212 980KV Clockwise Motor	2	$20.00	Each has a weight of 46 g. The maximum power is 250 W and the maximum current is 21 A [4].
QM2812 2212 980KV Counterclockwise Motor	2	$20.00	Each has a weight of 46 g. The maximum power is 250 W and the maximum current is 21 A [4].
QX-MOTOR 30A ESC	4	$15.00	Protects the system from overheating or producing an abnormal voltage. Each unit is 37 g [5].
1045 Self-locking Propellers Clockwise/ Counterclockwise	2 of each	$15.00	Comes in a pack of 16 blades since they are fragile. The package is 7.8 oz, so each blade is approximately 13.5 g. The radius of each propeller is 5 in, so 127 mm [6].
PIX 2.4.8 Flight Controller	1	$110.00	Enables autonomous navigation, weighing 60 g [7].
M8N GPS and Bracket	1	$20.00	Location system weights 85 g [8].
Distribution Board with XT60-Male Plug	1	$13.00	Connection board to engage the motors, square shape with 50 mm sides [9].

HT-8A Remote Control with F-08A Receiver	1	$12.00	Used to manually control the drone, the controller weights 304 g [10].
XT60 Plug Galvanometer	1	$7.00	Measures the current flowing through the circuit to monitor spikes, and weights 17 g [11].
6000 mAh 3s 35c Lipo Battery (XT60)	1	$30.00	Battery voltage of 11.1V with a weight of 455 g [12].

The compiled list produces a final cost of approximately $368, but all the items can be purchased in a kit for $250 but the kit comes without a battery; in addition, the compiled parts have a mass of approximately 1.5 kg [13]. The drone kit must be assembled by the buyer and requires prior knowledge of the instruments used. During the fabrication process, there must be testing done, to make sure the parts are properly constructed. Throughout testing, it is important to practice proper safety procedures and stay away from other people and property.

This kit is ideal for project goals due to its size, weight, rotor diameter, and its autopilot technology. Its weight is within specifications and the rotor diameter supports stable flight; the autopilot would explain the need for the equations of motion with regards to the performance goals.

Contributions of Team Members

Table 2 Team Member's Contributions

Team Member	*Contribution*
Marco Soto	Abstract, Nomenclature, Mathematical Description, Model
Emily Lory	Mathematical Description, Model Performance
Ram Velamuri	Introduction, portion of verifying that the model meets the engineering requirements, verifying model meets performance requirements, references.
Ella Young	Portion of compliance with engineering requirements, construction of the quadcopter, and conclusion, references.

GitHub

https://github.com/marcoroni-s/quadrotor_project

References

[1] Madeiras, J., and Cardeira, C., "Saturated Trajectory Tracking Controller in the Body-Frame for Quadrotors" *MDPI - Drone Design and Development* [online journal], Vol. 8, No. 4, Paper 1, URL: https://doi.org/10.3390/drones8040163 [retrieved 25 November 2024].

[2]"The Ultimate Guide to Building a Quadcopter From Scratch," *Autodesk Instructables*, URL: https://www.instructables.com/The-Ultimate-Guide-to-Building-a-Quadcopter-From-S/ [retrieved 26 November 2024].

[3] "F450 Drone Frame Kit," *Amazon,* URL: https://www.amazon.com/YoungRC-4-Axis-Airframe-Quadcopter-Landing/dp/B0776WLHX7 [retrieved 26 November 2024].

[4] "QX- Motor QM2812 2212 980KV," *QX- Motor*, URL: https://www.qx-motor.net/products/qx-motor-qm2812-2212-980kv-brushless-motor-cw-ccw-for-f330-f450-f550-multicopter-rc-drone-motor-

parts?srsltid=AfmBOoollz0bTJF0dENzFVb3VxrgUz8wqkc9yEQAnk2r0mPtvWZQxENP [retrieved 27 November 2024]

[5] "QX- Motor 30A ESC 2-3s," *QX-Motor*, URL: https://qx-motor.co/product/esc-30a/ [retrieved 27 November 2024].

[6] "SoloGood 1045 (10x4.5) Propellers," *Amazon*, URL: https://www.amazon.com/SoloGood-10x4-5-Propellers-Quadcopter-Multirotor/dp/B091KGR9Z6/ref=sr_1_4?dib=eyJ2IjoiMSJ9.PStIyJByzgogWKzzhLS-wNptngSaLIsxTp0eW-xfm8EElFYRCQTjh3VVJqTS3A8_1b14tOFkqaRgaoUxA0hl36Vzv3L6FEihsTO-QIoqRn1Jf8NWFpUNXGrLxG2beM9THswRI-KkJy60LvCJHBcCZKpp2s-bT9k2TbO4CjPC_cNWuR6FQODWjwNnG9ar-rVgBnsBl7F8H0HYot6bFcN3a3XYwcg9TsW4pRveGIAOsyw72hzv9iBOojO5zkMxKmN9G4b0ZclhTWOrokuSLVqQkUIz6X1-_WHMXLC7osJ9NY.UVpUrtRRnkv4-hLWWZjICh29Eu4umFctKisgvJ2tJDg&dib_tag=se&hvadid=604584156476&hvdev=c&hvlocphy=9021523&hvnetw=g&hvqmt=e&hvrand=7597589631128097l8&hvtargid=kwd-362866892797&hydadcr=4092_13167887&keywords=1045+propeller&qid=1732742671&s=toys-and-games&sr=1-4 [retrieved 26 November 2024].

[7] "PIXHAWK PX4 PIX 2.4.8 32 BIT FLIGHT CONTROLLER," *The Drone Flight*, URL: https://thedroneflight.com/products/pixhawk-px4-pix-2-4-8-32-bit-flight-controller-autopilot-with-4g-sd-safety-switch-buzzer-ppm-i2c-for-rc-quadcopter-ardupilot?variant=43442823069836. [retrieved 27 November 2024].

[8] "NEO-M8N GPS and Compass for Pixhawk / APM 2.7 / APM 2.8, *SpeedyFPV*, URL: https://speedyfpv.com/products/ublox-neo-m8n-gps-and-compass-for-apm2-7-apm2-8-new?variant=45359802417366 [retrieved 27 November 2024].

[9] "Distribution Board Waterproof Connection Board Soldered T / XT60https://www.qx-motor.net/products/distribution-board-waterproof-connection-board-soldered-t-xt60-plug-and-banana-bullet-connectors-for-remote-control-toy-parts?variant=47457513898269 (aiaa)
," *QX-Motor*, URL: https://www.qx-motor.net/products/distribution-board-waterproof-connection-board-soldered-t-xt60-plug-and-banana-bullet-connectors-for-remote-control-toy-parts?variant=47457513898269 [retrieved 27 November 2024].

[10] "HotRC HT-8A 2.4GHz 8CH 8A RC Transmitter PWM FHSS, *AliExpress*, https://www.aliexpress.us/item/3256806503919644.html?src=google&pdp_npi=4%40dis%21USD%2126.00%2113.78%21%21%21%21%21%40%2112000038020573591%21ppc%21%21%21&src=google&albch=shopping&acnt=708-803-3821&isdl=y&slnk=&plac=&mtctp=&albbt=Google_7_shopping&aff_platform=google&aff_short_key=UneMJZVf&gclsrc=aw.ds&albagn=888888&ds_e_adid=&ds_e_matchtype=&ds_e_device=c&ds_e_network=x&ds_e_product_group_id=&ds_e_product_id=en3256806503919644&ds_e_product_merchant_id=109214453&ds_e_product_country=US&ds_e_product_language=en&ds_e_product_channel=online&ds_e_product_store_id=&ds_url_v=2&albcp=19678427463&albag=&isSmbAutoCall=false&needSmbHouyi=false&gad_source=1&gclid=CjwKCAiAxqC6BhBcEiwAlXp458nFn9JMZwHGih38RahJgBjDrYCE5HHl45vvZZ_akXQIn1essdRumBoCLoIQAvD_BwE&gatewayAdapt=glo2usa. [retrieved 26 November 2024].

[11] "Pixhawk Flight Control Current Sensor," *AliExpress*, URL: https://www.aliexpress.us/item/3256803425331797.html?gatewayAdapt=glo2usa4itemAdapt#nav-description [retrieved 27 November 2024].

[12] “TATTU 6000 mAh 3s 35c Lipo Battery (XT60),” *getfpv*, URL: https://www.getfpv.com/tattu-6000mah-3s-35c-lipo-battery-xt60.html?srsltid=AfmBOopww8GKP-0JMSzTqFemPXh_pUfA4nzd1TbMYshxanAz8FNqmk_K25Q&gQT=1 [retieved 9 December 2024].

[13] “F450 4-axis Quadcopter DIY RC FPV Drone Kit,” *QX-Motors*, URL: https://www.qx-motor.net/products/f450-4-axis-quadcopter-diy-rc-fpv-drone-kit-with-f450-frame-pxi-apm-2-8-flight-controller-980kv-motor-gps-ht-8a-transmitter-f-08a-receiver?srsltid=AfmBOoqdPpdV4FKc_rfElb5aoZSA9z7wZUl5-VDJgWbBAPBSa1UL14rz [retrieved 27 November 2024].