

## Interpreting OLS Coefficients

*Different functional forms imply different interpretations of coefficients.  
Understanding these distinctions is fundamental for empirical analysis.*

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# 1. Linear-Linear Model

$$y = \beta_0 + \beta_1 x + u$$

## Interpretation:

- A one-unit increase in  $x$  changes  $y$  by  $\beta_1$  units.

**Example:** If  $\beta_1 = 2.5$ , then increasing  $x$  by 1 increases  $y$  by 2.5 units.

## 2. Log-Linear Model (Semi-elasticity)

$$\ln(y) = \beta_0 + \beta_1 x + u$$

### Interpretation:

- A one-unit increase in  $x$  changes  $y$  by approximately

$$100 \times \beta_1 \%.$$

- This approximation is valid for  $|\beta_1| < 0.1$  (roughly).

- For large  $\beta_1$ , use the exact change:

$$100(e^{\beta_1} - 1)\%.$$

**Example:** If  $\beta_1 = 0.04$ , then a 1-unit increase in  $x$  increases  $y$  by about 4%.

### 3. Linear-Log Model (Semi-elasticity)

$$y = \beta_0 + \beta_1 \ln(x) + u$$

#### Interpretation:

- A 1% increase in  $x$  changes  $y$  by:

$$0.01 \times \beta_1 \text{ units} \quad (\text{i.e., } \frac{\beta_1}{100} \text{ units}).$$

**Example:** If  $\beta_1 = 8$ , then a 1% increase in  $x$  increases  $y$  by  $0.01 \times 8 = 0.08$  units.

## 4. Log–Log Model (Elasticity)

$$\ln(y) = \beta_0 + \beta_1 \ln(x) + u$$

### Interpretation:

- $\beta_1$  is an **elasticity**.
- A 1% increase in  $x$  changes  $y$  by  $\beta_1\%$ .

**Example:** If  $\beta_1 = 0.7$ , then increasing  $x$  by 1% increases  $y$  by 0.7%.

## 5. Dummy Variable in Linear Model

$$y = \beta_0 + \beta_1 D + u$$

where  $D \in \{0, 1\}$ .

### Interpretation:

- $\beta_1$  is the difference in the mean of  $y$  between the two groups:

$$\beta_1 = E[y|D = 1] - E[y|D = 0].$$

**Example:** If  $\beta_1 = 5$ : when  $D = 1$  (vs.  $D = 0$ ),  $y$  is approximately 5 units higher on average.

## 6. Dummy Variable in Log-Linear Model

$$\ln(y) = \beta_0 + \beta_1 D + u$$

### Interpretation:

- The percentage difference between  $D = 1$  and  $D = 0$  is:

$$100(e^{\beta_1} - 1)\%.$$

- For small  $\beta_1$ : approximately  $100\beta_1\%$ .

**Example:** If  $\beta_1 = 0.2$ :  $100(e^{0.2} - 1) \approx 22.14\% \Rightarrow$

That is, when  $D = 1$  (vs  $D = 0$ ),  $y$  is approximately 22% higher.

## 7. Interaction of Continuous Variable and Dummy

$$y = \beta_0 + \beta_1 x + \beta_2 D + \beta_3 (x \cdot D) + u$$

### Interpretation:

- Slope for group  $D = 0$ :  $\beta_1$
- Slope for group  $D = 1$ :  $\beta_1 + \beta_3$
- Difference in slopes:  $\beta_3$

**Used for:** heterogeneous effects, gender differences, treatment interactions.

## Summary Table: Interpretation of $\beta$

Model	Interpretation of $\beta_1$
$y$ vs. $x$	$\Delta y = \beta_1 \Delta x$
$\ln y$ vs. $x$	1 unit $\uparrow$ in $x \rightarrow \approx 100\beta_1\%$ change in $y$
$y$ vs. $\ln x$	1% $\uparrow$ in $x \rightarrow 0.01\beta_1$ units change in $y$
$\ln y$ vs. $\ln x$	elasticity (1% in $x \rightarrow \beta_1\%$ in $y$ )
$y$ vs. dummy $D$	difference in means
$\ln y$ vs. dummy $D$	$100(e^{\beta_1} - 1)\%$ difference

## Common Mistakes to Avoid

- ▶ Using  $100\beta_1\%$  when  $\beta_1$  is large (use exact formula).
- ▶ Treating dummy coefficients in log models as additive (they are multiplicative).
- ▶ Misinterpreting elasticities when logs are missing.
- ▶ Forgetting that log-linear models require  $y > 0$ .
- ▶ Forgetting heterogeneity when interactions are present.

## Final Takeaways

- ✓ Always check the functional form before interpreting coefficients.
- ✓ Logs turn units into percentages or elasticities.
- ✓ Dummy variables shift levels or percentages depending on log/not-log.
- ✓ Interaction terms change slopes.
- ✓ When in doubt: compute marginal effects explicitly.