Growth of Functions

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1 $\mathcal{O}$-Notation

1.1 Definition of $\mathcal{O}$-Notation

$\mathcal{O}(g(n)) = \{f(n)$: there exist positive constants c and n_0 such that $0 \leq f(n) \leq cg(n)$ for all $n \geq n_0\}$.

1.2 $\mathcal{O}$-Notation: Exercises

1. If $T(n) = a_k n^k + \cdots + a_1 n + a_0$. $T(n) = \mathcal{O}(n^k)$?

Solution. In this case, with $n_0 = 1$ and $c = |a_k| + \cdots + |a_1| + |a_0|$. For every $n \geq n_0$ then $T(n) \leq cn^k = |a_k|n^k + \cdots + |a_1|n^k + |a_0|n^k$.

2. Is $n^k = \mathcal{O}(n^{k-1})$?

Solution. By contradiction, suppose $n^k = \mathcal{O}(n^{k-1})$ then, there exist constants n_0 and c such that $n^k \leq cn^{k-1}$ for all $n \geq n_0$. If the mentioned inequality is simplified, we obtain $n \leq c$ for all $n \geq n_0$ wich is clearly false. Thus $n^k \neq \mathcal{O}(n^{k-1})$.

3. If $T(n) = \dfrac{1}{2}n^2 + 3n$, it is true that $T(n) = \mathcal{O}(n^3)$?

Solution. With $n_0 = 1$ and $c = 4$. For every $n \geq n_0$ then $T(n) \leq cn^3$.