Arithmetic Progression and its Sum

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1 Arithmetic Progression and its Sum

1.1 Definition of Arithmetic Progression

An arithmetic progression is a sequence of numbers such that the difference d between the consecutive terms is constant, the first element of the progression will be a_1:

$$a_1, \quad a_1 + d, \quad a_1 + 2d, \quad a_1 + 3d, \quad \dots$$

Therefore, the nth term of the sequence can be calculated as follows:

$$a_n = a_1 + (n-1)d.$$

Below is a very simple example of arithmetic progression:

$$3, \quad 6, \quad 9, \quad 12, \quad \dots$$

1.2 Sum of a Finite Arithmetic Progression

The sum of the n terms of an arithmetic progression is defined as follows:

$$S_n = a_1 \quad + \quad (a_1 + d) \quad + \quad (a_1 + 2d) \quad + \quad \dots \quad + \quad (a_1 + (n-1)d)\,. \quad (1)$$

Expression (1) can be simplified as follows:

$$S_n = na_1 \quad + \quad (d \quad + \quad 2d \quad + \quad \dots \quad + \quad (n-1)d)\,. \quad (2)$$

With $l = (a_1 + (n-1)d)$, the sum represented in equation (1) can also be expressed as follows:

$$S_n = l \quad + \quad (l-d) \quad + \quad (l-2d) \quad + \quad \ldots \quad + \quad (l-(n-1)d)\,. \tag{3}$$

Equation (3) also can be simplified:

$$S_n = nl \quad - \quad (d \quad + \quad 2d \quad + \quad \ldots \quad + \quad (n-1)d)\,. \tag{4}$$

If equation (2) is added with equation (4), the terms containing d are canceled:

$$2S_n = n\,(a_1 + l)\,. \tag{5}$$

And therefore, calculating the sum of an arithmetic progression is very easy:

$$S_n = \frac{n\,(a_1 + l)}{2}. \tag{6}$$

1.2.1 Example

Find the sum of the first 40 terms of the arithmetic sequence: 2, 5, 8, 11, ...

First, the 40th term is calculated. Applying $a_n = a_1 + (n-1)d$:

$$a_{40} = 2 + (39)3 = 119.$$

Finally, the sum of the arithmetic progression is calculated ($S_n = \frac{n\,(a_1 + l)}{2}$):

Then find the sum:

$$S_n = \frac{40\,(2 + 119)}{2} = 2420.$$