Centroid of a Triangle

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1 Centroid of a Triangle

1.1 General formula to find the centroid of a triangle

Let **ABC** be any triangle defined by the vertices $\mathbf{A} = (x_1, y_1)$, $\mathbf{B} = (x_2, y_2)$, and $\mathbf{C} = (x_3, y_3)$. The general formula for calculating the centroid **C** of the triangle is as follows:

$$\mathbf{C} = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right). \tag{1}$$

1.2 Example

Let be a triangle **ABC** whose vertices are the points $\mathbf{A} = (-1, 0)$, $\mathbf{B} = (2, 3)$, $\mathbf{C} = (5, -3)$. Find the equation of the line that passes through the centroid of the triangle and is perpendicular to the side **AB** of the triangle.

Solution:

Applying Equation (1), we obtain the centroid of the triangle:

$$\mathbf{C_{ABC}} = \left(\frac{-1 + 2 + 5}{3}, \frac{0 + 3 - 3}{3} \right) = (2, 0)\,. \tag{2}$$

Remember that the equation of a straight line $\mathcal{L}$ is defined by a normal vector **N** and a point $\mathbf{P_0}$ that is on the line:

$$\mathcal{L} = \{\mathbf{P} \mid \mathbf{N} \cdot (\mathbf{P} - \mathbf{P_0}) = 0\}\,. \tag{3}$$

For this specific problem, $\mathbf{N}$ is calculated as follows

$$\mathbf{N} = \mathbf{B} - \mathbf{A} = (2,3) - (-1,0) = (3,3) = 3(1,1) = (1,1). \tag{4}$$

Finally, $\mathbf{P_0}$ will be the centroid because this point must be on the line:

$$\mathbf{P_0} = \mathbf{C_{ABC}} = (2,0)\,. \tag{5}$$

Substituting (4) and (5) in (3), we obtain the desired solution:

$$\mathcal{L} = \{(x,y) \mid (1,1) \cdot ((x,y) - (2,0)) = 0\}\,. \tag{6}$$

$$\mathcal{L} = \{(x,y) \mid (1,1) \cdot (x-2,y) = 0\}\,. \tag{7}$$

$$\mathcal{L} = \{(x,y) \mid x + y - 2 = 0\}\,. \tag{8}$$