Grayscale Image Segmentation with K-Means

Marcos Daniel Calderón-Calderón

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1 Grayscale Image Segmentation with K-Means

Let $I_{m \times n}$ a grayscale image with m rows and n columns.

Choose, randomly, an initial vector of K-means:

$$V = \{m_1,\ m_2,\ m_3,\ \ldots\ ,\ m_K\}\,. \tag{1}$$

For each pixel $x_{i,j} \in I$ $(1 \leq i \leq m,\ 1 \leq j \leq n)$, the euclidean distance, in absolute value, is calculated with each of the elements of the vector S:

$$d_{i,j}^{p} = \mid x_{i,j} - m_p \mid,\ \forall\ m_p \in V,\ 1 \leq p \leq K. \tag{2}$$

In an new image $I'_{m \times n}$, each pixel $x'_{i,j} \in I'$ is initialized with the index p of the value m_p that minimizes the euclidean distance calculated in equation (2):

$$x'_{i,j} = p_{min}. \tag{3}$$

The next step is to calculate a new vector of means V. Each new $m_p \in V$ is calculated by adding all the pixels $x_{i,j} \in I$ assigned to the same mean index stored in I' and dividing the sum with the number of pixels with the same mean index assigned:

$$m_p = \frac{\sum x_{i,j}}{N_p} \quad if \quad x'_{i,j} = p \quad \forall\ m_p \in V. \tag{4}$$

Finally, the pixels $x_{i,j}$ of the original image I will be replaced by the value of the mean indicated in the index stored in $x'_{i,j}$, (see Figure 1).

With the new image I calculated in the previous step, and the new vector V calculated in equation (4),the above procedure is iterated until a convergence condition is reached. For example, if the norm of two consecutive vectors: V_i, V_{i-1} is less than a very small given value ϵ =0.002.

1	2	5	4	80
4	3	4	5	86
200	250	251	244	90
217	213	234	245	90
214	254	255	243	99

(a) Pixels of initial grayscale image I.

	3-Means vector		
Value:	10	100	200
Index:	1	2	3

(b) Initial vector V of 3 means.

1	1	1	1	2
1	1	1	1	2
3	3	3	3	2
3	3	3	3	2
3	3	3	3	2

(a) Image I' with indexes to the nearest mean of V for every pixel in I.

10	10	10	10	100
10	10	10	10	100
200	200	200	200	100
200	200	200	200	100
200	200	200	200	100

(b) New image I replacing the pixel value with the nearest mean in V indicated in I'.

Figure 1: Simple example of the K-Means algorithm.

2 Results

2.1 Image "Student"

(a) Original image "Student".

(b) Segmented image with $k = 2$.

(c) Segmented image with $k = 4$.

(a) Segmented image with $k = 6$.

(b) Segmented image with $k = 8$.

(c) Segmented image with $k = 10$.

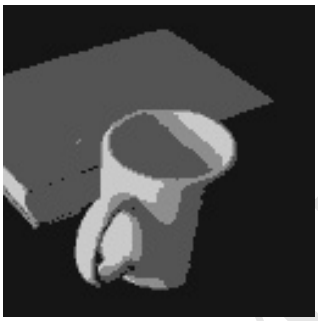
Figure 2: K-Means clustering algorithm applied to image "Student".

2.2 Image "Cup"

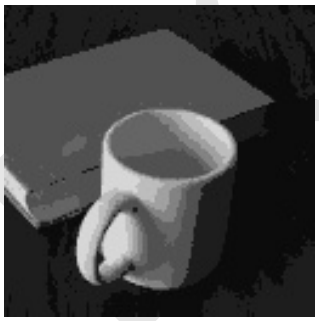
(a) Original image "Cup".

(b) Segmented image with $k = 2$.

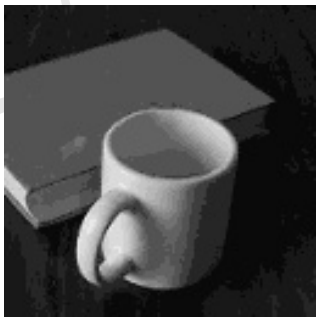
(c) Segmented image with $k = 3$.



(a) Segmented image with $k = 4$.



(b) Segmented image with $k = 8$.



(c) Segmented image with $k = 16$.

Figure 3: K-Means clustering algorithm applied to image "Cup".