Beyond Expectations: Does Belief Uncertainty Matter for Parental Investment Decisions?

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A growing body of research measures and subsequently accounts for how parents perceive the average returns to investment in the child development process and how this uncertainty impacts their actual investment in children. This paper moves this research beyond the mean and is the first to allow for uncertainty in parental beliefs. I first develop and collect an elicitation procedure of parental subjective belief distributions concerning the technology of skill production that is guided by a model of parental investment. Second, I extend existing measurement error correction methods that are required with belief estimation. I show that parents hold downward-biased mean beliefs about the returns to investment. Moreover, parents who have higher mean beliefs also hold lower levels of uncertainty about their beliefs. Both mean beliefs and uncertainty correlate with actual investment measures. Finally, I estimate a model of parental investment with reference-dependent preferences and show that even though parents hold low mean beliefs, they have a strong incentive to invest if their child is at risk of experiencing a developmental delay.

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1. Introduction

Understanding why some parents invest in their children more than others is a fundamental question to inform policies targeting lifetime inequality. Investment gaps that happen early in the childhood can lead to skill gaps in the future that are difficult to remedy (Cunha, Heckman, and Schennach 2010; Heckman et al. 2010). Recent papers highlight the role of parental information about the process of child development (Cunha, Elo, and Culhane 2022; Boneva and Rauh 2018; Attanasio, Cunha, and Jervis 2019; Dizon-Ross 2019).[1] While this body of work shows that parents from low socio-economic backgrounds often underestimate the returns on investment, and that these beliefs are correlated with lower investment, it does not consider any uncertainty that parents may face about what the returns to investment are.

This paper addresses this gap by first developing and implementing an empirical framework to measure how belief uncertainty about the returns to early investment affects parental investment decisions. The framework combines a theoretical model of parental investment with a tailored belief-elicitation survey that recovers individual subjective distributions over returns to investment. Second, I use a deconvolution procedure to remove measurement error from the survey data, and subsequently estimate a structural model with reference-dependent preferences that incorporates the full distribution of beliefs to assess how both the mean and uncertainty of beliefs shape parental investment decisions.

This paper addresses this gap by developing and implementing an empirical framework to measure how belief uncertainty about the returns to early investment affects parental behavior. The framework combines a theoretical model of parental investment with a tailored belief-elicitation survey that recovers individual subjective distributions over returns to investment. Using this data, I estimate a structural model with reference-dependent preferences that incorporates the full distribution of beliefs to assess how both the mean and uncertainty of beliefs shape investment decisions.

Whether and how belief uncertainty affects investment is important for both theory and policy. First, the direction of this effect is ambiguous. Studies of adolescent schooling suggest that increased perceived earnings or unemployment risk can reduce investment (Attanasio and Kaufmann 2014; Wiswall and Zafar 2015), whereas learning models postulate that higher uncertainty can raise investment by increasing perceived learning

[1] Past research has highlighted the role of family resources (Dahl and Lochner 2012; Caucutt et al. 2020) and parental characteristics such as maternal education and cognitive skills (Currie and Moretti 2003; Aizer and Stroud 2010; Arendt, Christensen, and Hjorth-Trolle 2021).

opportunities (Mira 2007; Badev and Cunha 2012). This paper brings this discussion to early childhood by exploring the joint role of mean beliefs and belief uncertainty in shaping parental behavior.

Second, ignoring belief uncertainty could have large consequences for policy. The incorporation of belief uncertainty first can shed light on the puzzling finding of why interventions that educate parents about child development sometimes fail to increase investment, even when mean beliefs change (Attanasio, Cunha, and Jervis 2019; List, Pernaudet, and Suskind 2021). My findings show that incorporating belief uncertainty changes the interpretation of other structural parameter estimates in a policy relevant manner. The results also imply that interventions which would increase the extent of belief uncertainty in the population alters the potential risks of developmental delays and could be effective at stimulating parental investments.

To study parental beliefs, I design a survey that elicits subjective belief distributions and investment decisions, since such beliefs cannot be identified from observational data alone (Manski 2004). Parents respond to hypothetical scenarios describing a child's health and potential investments, providing age-range predictions for developmental milestones. From these responses, I recover the full subjective belief distribution for each parent, capturing both the expected return to investment and the degree of uncertainty surrounding it. This represents a key contribution of the paper, as prior work typically measures only mean beliefs and overlooks the role of belief uncertainty in shaping parental behavior. In a second set of scenarios, parents indicate the trade-off between leisure and time spent in active engagement with the child, allowing estimation of subjective costs of investment. To ensure accurate recovery of beliefs, I develop an estimation approach that explicitly accounts for measurement error in survey responses and allows for correlated errors between the mean and variance of beliefs.

I collect data through an online panel from a sample of women who i) are aged 18-40, ii) have a child, and iii) the eldest child is aged five years or younger. This sample should have recent direct familiarity with the developmental milestones presented in the survey.[2] Respondents report that children complete more difficult activities at older ages, and they assign higher ages under scenarios in which the child has poor health and low parental investment. These patterns reflect an understanding of developmental trade-offs and milestones, providing evidence that respondents engage meaningfully with the survey.

[2]Individuals too far removed from parenting experience may not have reflected much on child development and could respond randomly or without sufficient engagement.

This paper yields three main findings, which I explore in turn. First, parents systematically underestimate the returns to early investment and report low levels of uncertainty about their beliefs. Second, higher perceived returns are associated with lower uncertainty, indicating that parents who have higher expected returns also tend to be more confident. Third, estimates from a structural model show that increasing either mean beliefs or uncertainty raises parental investment, with effects concentrated among parents who begin with low expected returns.

Mothers in this sample report low mean beliefs about the returns to investment, consistent with prior studies (e.g., Cunha, Elo, and Culhane (2013); Boneva and Rauh (2018)). On average, mothers estimate that a 10% increase in investment leads to a 1.03% increase in their child's human capital, roughly half the return estimated using objective data (see, Attanasio, Cunha, and Jervis 2019). Belief uncertainty is also low, with a coefficient of variation around 0.53. Mothers with higher mean beliefs tend to be more confident: a one standard deviation increase in mean beliefs is associated with a 38% reduction in uncertainty.

To examine the behavioral relevance of these beliefs, I regress actual parental time investment on the estimated beliefs. Higher mean beliefs predict greater investment: a one standard deviation increase corresponds to a 20% standard deviation increase in daily investment hours. By contrast, greater uncertainty is associated with lower investment. A one standard deviation increase in uncertainty predicts a 13% standard deviation decrease in daily investment hours.

The belief elicitation data also allow me to estimate a structural model of parental investment that incorporates reference-dependent preferences. In this framework, investment depends on how parents perceive their child's development relative to specific milestones. Using their own beliefs, parents form comparisons that shape investment behavior, enabling estimation of how much they value avoiding developmental delays. I find that parents place substantial value on their child's skill relative to leisure and household consumption, and exhibit strong incentives to invest if their child is at risk of falling behind on key developmental milestones. Even with low beliefs about average returns, parents maintain strong motivation to prevent delays.

Counterfactual analysis shows that increasing either mean beliefs or belief uncertainty raises investment. This may appear to contradict the reduced-form result, where higher uncertainty is associated with lower investment. The discrepancy arises because mean beliefs and uncertainty are negatively correlated: individuals with higher mean beliefs tend to have lower uncertainty. Reduced-form correlations thus conflate the effects, while counterfactual simulations isolate the separate contributions of mean beliefs and uncertainty within the structural model.

Examining the effect of increasing uncertainty reveals substantial heterogeneity. Parents who respond to higher uncertainty by increasing investment are typically those with very low mean beliefs and correspondingly low initial investment. For these parents, increased uncertainty signals a higher perceived risk that the child will fall below developmental thresholds, motivating additional investment.

This paper contributes to several strands of literature. First, it builds on a large body of work that uses elicited subjective expectations to inform economic models and study decision-making under uncertainty (Manski 1993, 2004; Delavande 2008; Jensen 2010; Zafar 2013; Almås, Attanasio, and Jervis 2023; Boneva et al. 2025). I extend this literature by explicitly eliciting not only parents' expectations but also their subjective uncertainty about returns to early childhood investment, a dimension largely overlooked in prior work.

Second, it relates to research on parental beliefs and information in early childhood development (Cunha, Elo, and Culhane 2013; Boneva and Rauh 2018; Attanasio, Cunha, and Jervis 2019; Dizon-Ross 2019; List, Pernaudet, and Suskind 2021). While existing studies emphasize mean expectations about educational outcomes or evaluate information interventions, I develop a novel elicitation procedure that jointly measures the level and uncertainty of parental beliefs about the skill formation process. Related work has examined adolescent education decisions (Delavande and Zafar 2018; Patnaik et al. 2022; Wiswall and Zafar 2015; Stinebrickner and Stinebrickner 2014), but my focus on early childhood highlights a context where beliefs are less informed and more heterogeneous.

Finally, the paper connects to research on uncertainty, risk preferences, and parental investment (Giustinelli 2016; Attanasio and Kaufmann 2014; Carneiro and Ginja 2016; Tabetando 2019; Sovero 2018; Tanaka and Yamano 2015; Basu and Dimova 2022). Whereas prior studies emphasize risk aversion or credit constraints, I instead examine how uncertainty in subjective beliefs rather than risk preferences shapes investment behavior. This distinction helps explain why parents may underinvest even when perceived returns are high.

The remainder of this paper is structured as follows. Section 2 describes the economic model and specifies the technology of skill formation, as well as the concepts of subjective expectation and uncertainty. Section 3 describes the survey instrument, how the collected data identifies model primitives and the estimation method. Section

4 describes the data. Section 5 discusses the results, while Section 6 concludes.

2. Model

Consider a mother who must decide how much to invest in her child within a one-period model.[3] Let y_i denote her income, x_i denote her time investment in her child, and c_i denote her consumption. She faces a budget constraint given by

$$c_i + p_i x_i = y_i, \tag{1}$$

where p_i is the relative price of parental investment. Moreover, she faces a time constraint where time investment x_i is bounded by total available time:

$$x_i + h_{l_i} + h_{w_i} = 16. \tag{2}$$

Thus, the mother has 16 hours per day, assuming she sleeps 8 hours, to allocate between her child (x_i), leisure (h_{l_i}), and work (h_{w_i}). I assume that work is not a choice variable but given exogenously.

Let $\theta_{i,0}$ and $\theta_{i,1}$ denote the stock of human capital of child i at birth, $t = 0$, and at 24 months, $t = 1$. Let x_i denote the investment in human capital made by the mother between birth and 24 months. Finally, let ξ_i denote a shock to the development process unknown to the parents. I assume that the *objective* technology of skill formation is given by a Cobb-Douglas formulation:

$$\ln \theta_{i,1} = \delta_0 + \delta_1 \ln \theta_{i,0} + \delta_2 \ln x_i + \xi_i. \tag{3}$$

Parental preferences depend on household consumption, leisure, and child development at the end of the period, $u_i(c_i, h_{l_i}, \theta_{i,1}; \boldsymbol{\alpha})$, where $\boldsymbol{\alpha}$ denotes the vector of parental preferences. The parents maximize their expected utility conditional on the agent's information set Ω_i at the time of the decision. Parents know their own preferences $\boldsymbol{\alpha}$, their income y_i, the price of investment goods p_i, their work hours h_{w_i}, and their child initial stock of human capital, $\theta_{i,0}$. However, they do not know the parameters of the objective technology function, $\delta_0, \delta_1, \delta_2$. Parents have beliefs about these parameters, so from their point of view they are random variables. Consequently, the human capital of the child at $t = 1$ will also be a random variable and I denote its distribution by $G_i(\cdot)$.

[3] I will use "parent" and "mother" interchangeably, as this is a single-agent model.

Therefore, the information set of the parents is the set $\Omega_i = \{\boldsymbol{\alpha}, p_i, y_i, h_{w_i}, \theta_{i,0}, G_i(\cdot)\}$.

The distribution $G_i(\cdot)$ has a mean equal to $E[\ln\theta_{i,1}|\Omega_i]$ and a variance equal to $Var(\ln\theta_{i,1}|\Omega_i)$. Assume that: (i) $E[\xi_i|\Omega_i] = 0$; (ii) $E[\delta_k|\Omega_i] = \mu_{i,\delta_k}$ and $Var(\delta_k|\Omega_i) = \sigma^2_{i,\delta_k}$; (iii) $Cov(\delta_1, \delta_2|\Omega_i) = \sigma_{i,\delta_1,\delta_2}$ and $Cov(\delta_k, \delta_l|\Omega_i) = 0$ for all others $k \neq l$; and (iv) $Cov(\xi_i, \delta_k|\Omega_i) = 0$ for all k. In other words, parental beliefs about production shocks have mean zero and are uncorrelated with beliefs about the other parameters of the skill production function, and beliefs about δ_0, which defines the location of the baseline skill, are uncorrelated with beliefs about δ_1 and δ_2.[4]

Under these assumptions, we can write the *subjective* parental expectation and uncertainty about the skill of technology formation as

(4)
$$\mu_{i,\theta_1} \equiv E[\ln\theta_{i,1}|\Omega_i] = \mu_{i,\delta_0} + \mu_{i,\delta_1}\ln\theta_{i,0} + \mu_{i,\delta_2}\ln x_i,$$
$$\sigma^2_{i,\theta_1} \equiv Var(\ln\theta_{i,1}|\Omega_i) = (\sigma^2_{i,\delta_0} + \sigma^2_{i,\xi_i}) + \sigma^2_{i,\delta_1}\ln\theta^2_{i,0} + \sigma^2_{i,\delta_2}\ln x^2_{i,0} + \sigma_{i,\delta_1,\delta_2}\ln\theta_{i,0}\ln x_i.$$

These definitions derive directly from using the expectation and variance operators on (3). The parameter μ_{i,δ_2} is the parental subjective expectation of returns to investment, while σ^2_{i,δ_2} denotes the uncertainty about δ_2.

Parents maximize their expected utility function conditional on their information set:

$$\max_{x_i}\{E[u_i(c_i, h_{l_i}, \theta_{i,1}; \boldsymbol{\alpha})|\Omega_i]\}, \tag{5}$$

subject to the budget constraint (1), the time constraint (2), and the technology of skill formation (3). The optimal investment function depends on parental preferences, their income, the price of investment, their working hours, and on their subjective expectation and uncertainty of the returns to investment, μ_{i,δ_2} and σ^2_{i,δ_2}:

$$x_i^* = f(\boldsymbol{\alpha}, y_i, p_i, h_{w_i}, \mu_{i,\delta_2}, \sigma^2_{i,\delta_2}, \sigma_{i,\delta_1,\delta_2}).$$

Previous papers consider models that only depended on $\boldsymbol{\alpha}$, y_i, p_i and δ_2, which imply that parents have complete knowledge about the technology of skill formation (e.g., Del Boca, Flinn, and Wiswall (2014)). Without additional information about subjective

[4] The assumption that $E[\xi_i|\Omega_i] = 0$ also implies that beliefs about shocks to the process are uncorrelated with covariates from the production function, $\ln\theta_0$ and x. As will be shown later, this assumption is satisfied due to the design of the survey.

beliefs, it is not possible to estimate a model that does not assume complete knowledge. Cunha, Elo, and Culhane (2013) and Attanasio, Cunha, and Jervis (2019) assume a Cobb-Douglas utility function on consumption and child's skill which by construction removes the uncertainty of beliefs from the optimal investment function. This formulation of a parental investment model which incorporates subjective beliefs about the technology of skill production generalizes models in past research.

3. Methodological Framework

The objective is to elicit from parents their subjective expectations and uncertainty about the skill formation process, represented by the parameters of equation 4. Conceptually, the procedure can be viewed as an experiment that varies the inputs on the right-hand side, initial human capital (θ_0) and parental investment (x), and asks parents to state the expected outcome and uncertainty on the left-hand side—the child's human capital. In other words, respondents provide the mean and variance of their subjective belief distribution about child outcomes conditional on given inputs.

A challenge is that the variable representing child human capital, θ, is latent and lacks a natural metric. While various methods exist to address this issue in observational data, belief elicitation requires directly inferring individuals' perceptions of such latent constructs. To do so, I adapt the strategies developed by Cunha, Elo, and Culhane (2022) and Attanasio, Cunha, and Jervis (2019). I briefly summarize the procedure below and provide full details in Appendix XYZ. Building on this framework, I then present an identification strategy and corresponding estimator.

3.1. Survey Instrument

To link parental beliefs to child human capital, I rely on standard developmental milestone assessments that provide cardinal measures of early childhood skills. Following Cunha, Elo, and Culhane (2022), I base my elicitation module on the Motor and Social Development (MSD) scale from the NHANES 1988 survey, a well-established instrument covering motor, language, and numeracy milestones. The MSD consists of asking yes/no questions about whether a child can perform specific developmentally appropriate activities alongside their age in months. Examples of activities are ABC,123,XYZ.

I use Item Response Theory (IRT) to summarize the MSD data into a latent measure of developmental age, expressed in months. In this model, each milestone is treated as an imperfect indicator of an underlying developmental factor, and the probability of achieving a milestone depends on the child's age and latent ability. The estimated model identifies which milestones are most informative, anchors the latent factor on a meaningful scale (log developmental age), and provides a nationally representative distribution of child development.

This IRT framework is critical for the belief elicitation design. It provides a way to translate parental expectations about milestone achievement ages into beliefs about the latent variable θ, which represents child human capital in equation 4. In practice, the IRT model serves as a bridge between observable milestone ages and the underlying developmental scale that enters the subjective skill formation equation.

3.1.1. Subjective Belief Instrument

To elicit parents' subjective expectations about the skill-formation process, respondents are presented with a module built around the same developmental milestones used in the MSD-NHANES assessment. For each item, they are asked to indicate the ages at which they believe a hypothetical child would achieve the activity under given levels of initial human capital (θ_0) and parental investment (x). The instrument thus records how respondents map different inputs (θ_0, x) into expected outcomes—the child's human capital, providing data to recover both the mean and variance of their belief distribution.

Following Cunha, Elo, and Culhane (2022), the survey uses hypothetical rather than self-referential scenarios.[5] Although self-beliefs are often more directly tied to economic behavior, hypothetical families allow exogenous variation of inputs and improve comparability across respondents. Empirical work shows that such beliefs correlate strongly with actual behavior in domains such as education, labor markets, and parental investment (e.g., Wiswall and Zafar 2015; Cunha, Elo, and Culhane 2022; Attanasio, Cunha, and Jervis 2019).[6] In this design, respondents imagine an unspecified family, neither fully abstract nor tied to their own circumstances, reducing cognitive burden while maintaining interpretability.

To keep the exercise manageable, I focus on the four milestones identified by the IRT analysis as very informative about child development:

1. Speak a partial sentence of 3 words;

2. Count 3 objects correctly;

[5] See Appendix A for the full instrument.

[6] See Boneva et al. (2025) for discussion of the conceptual link between self-beliefs and population-level beliefs.

3. Say first and last name together;

4. Know age and sex.

The question that is asked to the respondent is the following:

What do you think are the youngest, most likely, and oldest ages a child will learn how to do [MSD ACTIVITY]?[7]

Each response thus provides three data points, the minimum, mode, and maximum perceived ages of achievement, which together describe the respondent's subjective distribution for that activity.

Scenarios vary two dimensions exogenously: initial health and parental investment intensity. Initial health takes two values, normal ($\bar{\theta}_0$) and poor ($\underline{\theta}_0$), corresponding respectively to term and premature birth profiles taken from the CNLSY data (9 vs. 7 months gestation, 8 vs. 5 pounds, 20 vs. 18 inches).[8] Parental investment also takes two values, high ($\bar{x}$) and low ($\underline{x}$), operationalized through active interaction time. Active interaction, such as playing, soothing, or feeding, is contrasted with passive interaction, such as performing chores or using a phone near the child.[9] High intensity corresponds to roughly six hours of active time per day and low intensity to two hours, based on PSID time-diary percentiles.[10] The cross-combination of these two dimensions produces four scenarios: $(\bar{\theta}_0, \bar{x})$, $(\bar{\theta}_0, \underline{x})$, $(\underline{\theta}_0, \bar{x})$, and $(\underline{\theta}_0, \underline{x})$.

For each scenario-milestone pair (θ_0, x, j), respondents adjust three sliders bounded between 0 and 48 months—representing the youngest $\underline{a}_{i,j,k}$, most likely $\dot{a}_{i,j,k}$, and oldest $\bar{a}_{i,j,k}$ ages. These three points define a subjective distribution over the expected age of achievement, which is later mapped through the IRT-based developmental-age metric to obtain the respondent's belief distribution over latent human capital $G_{\ln \theta_{i,1}}(\cdot; \theta_0, x)$.

[7]An explanation is given to respondents about how to think about youngest, most likely, and oldest. In summary, they were instructed to think about a group of children that all just learned a specific activity. Then, the youngest and oldest would be the child's age that learned the fastest and oldest respectively. For most likely, it is the age at which most children in this set learned this activity.

[8]These numbers are the same used in Cunha, Elo, and Culhane (2022), and were obtained from the Children of the National Longitudinal Survey of the Youth/1979 (CNLSY). They estimate a factor model using gestation, weight at birth, and height at birth as measures. The low scenario describes a premature birth: gestation lasts seven months (percentile 1 in the CNLSY/79), the birth weight is five pounds (percentile 4), and the length at birth is 18 inches (percentile 11). The normal scenario describes children born in a normal term. The gestation lasts nine months (percentile 85), weighs eight pounds (percentile 69), and the length at birth is 20 inches (percentile 85). Given factor score estimates and predicted scores, they compute the implied value of the latent variable θ_0 under a cardinal scale.

[9]Examples follow Folbre et al. (2005); Bono et al. (2016); Guryan, Hurst, and Kearney (2008).

[10]These numbers are the 25th and 90th percentile of the distribution of time spent with their child in the Panel Study of Income Dynamics time diaries. Differently from the initial health data, hours of investment is already a cardinal measure.

3.1.2. From age ranges to belief distributions

From the subjective belief instrument, I obtain the youngest age $\underline{a}_{i,j,k}$, most likely age $\dot{a}_{i,j,k}$, and oldest age $\bar{a}_{i,j,k}$ that respondent i believes a child, under scenario k, will learn how to perform MSD item j. These responses can be viewed as moments from a subjective distribution $H_{a_{i,j,k}}(\cdot)$ with mean $\mu_{a_{i,j,k}}$ and variance $\sigma^2_{a_{i,j,k}}$. The mean $\mu_{a_{i,j,k}}$ represents the expected age at which individual i believes a child will acquire the skill corresponding to item j in scenario k, while the variance $\sigma^2_{a_{i,j,k}}$ captures the degree of uncertainty about that developmental timing.[11]

To construct $H_{a_{i,j,k}}(\cdot)$ from the three reported ages, we must specify how the responses map to points in the distribution. The youngest and oldest ages can be interpreted as either endpoints of a bounded distribution or as approximate lower and upper percentiles of an unbounded one. The most likely age can be interpreted as the mode, median, or mean, depending on assumptions about the respondent's interpretation of "most likely."[12] [13]

Note that $H_{a_{i,j,k}}(\cdot)$ represents a distribution over *ages* at which a skill is acquired, rather than a distribution over *child skills* themselves. To express subjective beliefs in terms of child skill, the latent construct of interest, we require a transformation that maps developmental age to an underlying level of human capital. Intuitively, earlier mastery of a skill implies a higher perceived level of ability, holding other factors constant. I therefore transform $H_{a_{i,j,k}}(\cdot)$ into a corresponding belief distribution $G_{\ln\theta_{i,1}}(\cdot)$ over the latent variable $\ln\theta_{i,1}$, characterized by mean $E[\ln\theta_{i,1}|\Omega_i]$ and variance $Var(\ln\theta_{i,1}|\Omega_i)$.

To illustrate, suppose an individual believes that the median child learns to "walk three steps" at 20 months. If population-level evidence indicates that most children acquire this skill earlier (say, by 15 months), we can interpret this as a belief that the

[11]This variance reflects both the respondent's uncertainty about individual child development and perceived natural variation in milestone attainment across children.

[12]These assumptions are studied and tested in the context of developing countries by Delavande, Giné, and McKenzie (2011b,a). The authors show that minimum and maximum type of questions are consistent with representing either the 5th or 10th (90th or 95th) percentiles of subjective distributions. While they do not study specifically a question asking what is the most likely outcome, they do examine questions of the type "what do you expect." They show that interpreting this answer as a mean value has very poor fit with respect to implied means from subjective probability questions, and poor prediction power with respect to realized future outcomes in follow-up surveys, while median and modes exhibit much better performance.

[13]Cunha, Elo, and Culhane (2022) asks youngest and oldest ages in the same context and assumes that $\underline{a}_{i,j,k}$ and $\bar{a}_{i,j,k}$ are the 10th and 90th percentile of a Normal distribution, while providing robustness checks with alternative distributions and percentile choices. They show that there are no substantial qualitative differences in results. Attanasio and Kaufmann (2014), in a context of future earnings, assume a triangular distribution and treat minimum and maximum as endpoints.

representative child's development is delayed relative to the population norm. Define this perceived developmental delay as $\delta_{i,j,k} = \mu_{a_{i,j,k}} - a_j^*$, where a_j^* is a reference age at which 50% of children in the population can perform item j. Then, the subjective expectation about the child's latent skill level under scenario k can be expressed as a monotonic transformation of this relative timing:

$$\ln \theta_{i,j,k,1} = \ln(A - \delta_{i,j,k}),$$

where A is a scaling constant representing the benchmark developmental age.[14] In this representation, smaller (earlier) $\delta_{i,j,k}$ implies higher perceived child skill.

The variance of the transformed belief distribution, $Var(\ln \theta_1 | \Omega_i)$, reflects both the respondent's perceived uncertainty about the skill formation process and their beliefs about heterogeneity in child development. For example, two parents might provide identical modal ages $\dot{a}_{i,j,k}$ but differ in their reported youngest and oldest ages. The parent giving a wider range may be expressing either greater uncertainty or a belief that the milestone is inherently more variable across children. The model preserves this distinction by maintaining the shape and dispersion implied by $H_{a_{i,j,k}}(\cdot)$ when constructing $G_{\ln \theta_{i,1}}(\cdot)$.

To quantify this dispersion, I use the interquartile range (IQR) of $H_{a_{i,j,k}}(\cdot)$, a common summary of belief dispersion in household surveys (Bruine de Bruin et al. 2023). The IQR is defined as:

$$IQR_H = H^{-1}(0.75) - H^{-1}(0.25).$$

I impose that the transformed distribution $G_{\ln \theta_{i,1}}(\cdot)$ preserves this IQR, ensuring that the spread of beliefs over skill levels corresponds to the spread implied by age responses. Under normality, this provides a straightforward link between IQR and the implied variance, since $IQR = 1.348\sigma$.

The procedures for recovering the subjective mean and variance of $\ln \theta_1$ differ. The mean, $E[\ln \theta_1 | \Omega_i]$, must be adjusted using the IRT-based population mapping because reported ages refer to observable milestones rather than latent skill levels. This step anchors the reported expectations to the reference distribution of development in the population. In contrast, the variance, $Var(\ln \theta_1 | \Omega_i)$, reflects the respondent's own uncertainty about the developmental process and cannot be identified from the IRT. The IRT informs population-level heterogeneity in development but provides no infor-

[14]The scaling constant A is arbitrary, but must be consistent with the possible support of ages that children learn specific milestones. In this paper, A is set to 24 months.

mation about individual subjective uncertainty. Accordingly, the variance is obtained by preserving the dispersion of the reported age distribution $H_{a_i}(\cdot)$ when mapping it to $G_{\ln\theta_{i,1}}(\cdot)$. While, in principle, variation in the IRT-based estimates could be used to study differences between uncertainty about population heterogeneity and uncertainty about the developmental process itself, such a decomposition would require additional identifying assumptions beyond those available here.

From this procedure, I obtain individual-specific means and variances of subjective beliefs about the child's skill level under different investment scenarios, expressed in a common latent metric. These serve as the primary inputs to the identification and estimation strategy discussed next.

3.2. Identification and estimation of the subjective expectation and uncertainty from belief distribution

To achieve identification, I need to define specific distributions for: (i) $H_{a_{i,j,k}}(\cdot)$, the distribution of beliefs about the age a child will learn MSD item j under scenario k, and (ii) $G_{\ln\theta_{i,1}}(\cdot)$, the distribution of beliefs about the future human capital $\ln\theta_1$.

I assume that $H_{a_{i,j,k}}(\cdot)$ follows a triangular distribution with mean $\mu_{a_{i,j,k}}$ and variance $\sigma^2_{a_{i,j,k}}$. This choice allows the mode and median to differ and lie anywhere within the support, unlike the Normal distribution. The triangular distribution is defined by three points: $\underline{a}_{i,j,k}$, $\dot{a}_{i,j,k}$, and $\bar{a}_{i,j,k}$, corresponding to the minimum, most likely, and maximum age a child is expected to achieve MSD item j under scenario k.[15]

For $G_{\ln\theta_{i,1}}(\cdot)$, I assume it is approximately normal with mean $E[\ln\theta_{i,1}|\Omega_i]$ and variance $Var[\ln\theta_{i,1}|\Omega_i]$. Specifically, I only impose that the interquartile range (IQR) of $G_{\ln\theta_{i,1}}(\cdot)$ behaves approximately like that of a Normal distribution, $IQR_G = 1.348\sigma$, ensuring approximate symmetry and limited skewness. Alternative assumptions for $H_{a_{i,j,k}}(\cdot)$ and robustness checks are discussed in Appendix D.

In principle, one could estimate the parameters of interest, μ_{i,δ_2} and σ^2_{i,δ_2}, by directly computing differences between the high and low investment scenarios ($\bar{x}$ and $\underline{x}$), conditional on health levels, for $\mu_{\theta_{i,j,k,1}}$ and $\sigma^2_{\theta_{i,j,k,1}}$. However, this approach ignores potential measurement error in the collected responses.

To address measurement error, define $T = J \times K$ as an index aggregating all activity-

[15] A choice remains about whether to interpret the most likely question as the mode or median. In the survey, the most likely question was described as imagining the age that most children in a sample would learn the activity. In the following sections, I use the median, but results using the mode are in Appendix D.

scenario combinations. Let $\varepsilon^{\mu}_{i,t} = (\varepsilon^{\mu}_{i,1}, \ldots, \varepsilon^{\mu}_{i,T})'$ and $\varepsilon^{\sigma}_{i,t} = (\varepsilon^{\sigma}_{i,1}, \ldots, \varepsilon^{\sigma}_{i,T})'$ denote the measurement errors associated with $\mu_{\theta_{i,t,1}}$ and $\sigma^2_{\theta_{i,t,1}}$, respectively. Define covariates $z^{\mu}_{i,t} = (1.0, \ln\theta_{i,t,0}, \ln x_{i,t})$ and $z^{\sigma}_{i,t} = (1.0, (\ln\theta_{i,t,0})^2, (\ln x_{i,t})^2, \ln x_{i,t}\ln\theta_{i,t,0})$, and aggregate $\mu_{\theta_{i,1}}$, $\sigma^2_{\theta_{i,1}}$, z^{μ}_i, and z^{σ}_i accordingly. Denote the random coefficients as $\beta_{i,\mu} = (\mu_{i,\delta_0}, \mu_{i,\delta_1}, \mu_{i,\delta_2})$ and $\beta_{i,\sigma} = (\sigma^2_{i,0}, \sigma^2_{i,\delta_1}, \sigma^2_{i,\delta_2}, \sigma_{i,\delta_1,\delta_2})$. The measurement error model can then be written as:

$$\mu_{\theta_{i,1}} = z^{\mu}_i \beta_{i,\mu} + \varepsilon^{\mu}_i, \tag{6}$$

$$\sigma^2_{\theta_{i,1}} = z^{\sigma}_i \beta_{i,\sigma} + \varepsilon^{\sigma}_i. \tag{7}$$

This system resembles a random coefficients model. Since $\mu_{\theta_{i,1}}$ and $\sigma^2_{\theta_{i,1}}$ are constructed from the same responses, the measurement errors ε^{μ}_i and ε^{σ}_i are likely correlated, making standard estimators like Swamy (1970) inappropriate. I propose an estimator accommodating seemingly unrelated equations with random coefficients in a panel setting.[16]

Aggregating the two equations, define $y_{i,t} = (\mu_{\theta_{i,t,1}}, \sigma^2_{\theta_{i,t,1}})'$, $Z_{i,t} = \begin{pmatrix} z^{\mu}_{i,t} & 0 \\ 0 & z^{\sigma}_{i,t} \end{pmatrix}$, $\beta_i = (\beta_{i,\mu}, \beta_{i,\sigma})$, and $\varepsilon_{i,t} = (\varepsilon^{\mu}_{i,t}, \varepsilon^{\sigma}_{i,t})$. Let $y_i = (y_{i,1}, \ldots, y_{i,T})$, $Z_i = \begin{pmatrix} z^{\mu}_i & 0 \\ 0 & z^{\sigma}_i \end{pmatrix}$, and $\varepsilon_i = (\varepsilon_{i,1}, \ldots, \varepsilon_{i,T})$. Denote $\beta_i = \beta + \eta_i$. The system becomes:

$$y_i = Z_i(\beta + \eta_i) + \varepsilon_i = Z_i\beta + u_i,$$

where $u_i = \varepsilon_i + Z_i\eta_i$.

The following assumptions are made:

A1. $E[\varepsilon_{i,t}|Z_{i,t}] = 0$ and $E[\eta_i|Z_i] = 0$,

A2. $E[\varepsilon_{i,t}\varepsilon'_{i,t}|Z_{i,t}] = \Omega_i$ and $E[\eta_i\eta'_i|Z_i] = \Delta$,

A3. $E[u_iu'_i|Z_i] = Z_i\Delta Z'_i + (I_T \otimes \Omega_i)$,

where $\otimes$ is the Kronecker product between two matrices, and I_T is a $T \times T$ identity matrix. Assumption *A*1. is respected in this experimental setting since the covariates Z_i are exogenously manipulated and fixed for all individuals. Crucially, this does not

[16] Balestra and Negassi (1992) develop a similar estimator for simultaneous equations with cross-equation dependence.

impose that individuals are correct about their beliefs on average. Assumption $A2$. restricts the measurement error to be correlated across equations in a person specific fashion. Assumption $A2$. also allows for a non-diagonal covariance matrix of the random coefficients. Assumption $A3$. follows directly from the two previous ones.[17]

I provide a detailed description of the estimator in Appendix C, which has the flavor of a Feasible Generalized Least Squares (FGLS) estimator. In the first step, individual-by-individual Seemingly Unrelated Regression estimates of the parameter vector are obtained. Identification requires that the number of informative observations in each equation exceeds the number of parameters to be estimated. There are 16 scenario-activity combinations and 7 parameters in the system. For each scenario-activity pair, I elicit both a mean and a variance of the belief distribution, yielding 16 observations for the mean equation and 16 for the variance equation. While the regressors in the variance equation are functions of those in the mean equation (e.g., squared terms), the exogenous variation across scenarios provides sufficient information to identify all parameters. Using these estimates, feasible estimators of the covariance matrices of the measurement error and random coefficients are computed, allowing us to obtain population-level estimates of β. To produce efficient estimates of the individual-level random coefficients β_i, the estimator exploits the difference between the population-level estimate and the first-stage individual estimates.[18]

3.3. Stated choice instrument

I now describe the second instrument, intended to elicit the individual's opportunity cost of investment. The instrument consists of a series of stated choice experiments. I create a series of hypothetical scenarios of monthly household income, initial baby's health, and how many hours the individual spends at work. Then, I ask the respondent to answer a question: how much would they be willing to pay to spend one hour of leisure instead of taking care of the baby.

I describe a situation in which the parent would like to spend one hour away from the

[17] An alternative would be to allow a more flexible measurement error assumption that allows some form of correlation within individuals. However, there are important trade-offs to consider. The current estimator does not need to impose parametric restrictions on the distribution of the random coefficients and allows for arbitrary correlation both between the random coefficients and between measurement error of equations. A factor model approach that allows for more specific forms of measurement error correlation would substantially increase the computation costs of estimating this problem and would restrict the distribution of the random coefficients and of the measurement error.

[18] Note that the parameters of equation (7) are variances, and therefore need to be strictly positive. I impose a non-negativity constraint on parameters when estimating.

baby every weekday of the month. I highlight that this one hour would be for personal leisure. A friend offers to take care of the baby during this 1 hour, and while they will not be engaged in active interaction, the baby will be safe. I ask the individual to choose the highest hourly rate they would be willing to pay to their friend for the whole month. They choose out of a slider that ranges from $0 to $30. As they move the slider, they can see how much the hourly rate means in monthly expenses, assuming 20 weekdays in a month. If the person would rather not spend one hour away, I ask them to choose $0. The intention is to elicit their price of one hour of daily investment in a month. Also, it is important that the respondent does not see this extra hour of leisure as a perfect complement of child investment, such that there is an explicit tradeoff. Therefore, I emphasize that this is a friend and not a professional caretaker such as a nanny.[19]

For each question, I establish the following scenarios. The monthly household income can be $Y = \{\$2000, \$4000, \$6000\}$, the initial baby's health can be either $H = \{normal, poor\}$, and the working hours of the individual can be $HR = \{0, 4, 8\}$. These variables are chosen according to the model in Section 2.[20] The household income is chosen to reflect different percentiles of income in the population. A $2,000 household income puts the household around the poverty line as established by the United States Federal Government, while a household income of $6,000 puts the household around the median income.

3.4. Model Identification and Estimation

I describe the model identification and estimation strategy. I assume that the respondent's utility is linearly separable in the endogenous variables $c_i, h_{l_i}, \theta_{i,1}$. By following the steps described in the previous section, I can estimate for each individual the parameters of their subjective skill production function. I use them as inputs in the estimation procedure. The remaining parameters left are the preference parameters $\boldsymbol{\alpha}$, and the subjective price of one hour of investment, p_i.

Once again, aggregate the set of possible scenarios to a single index $t \in T = Y \times H \times HR$. Additionally, denote by $p_{i,t}$ the maximum amount the individual i would pay their friend

[19]The slider can be seen in Figure A3. The exact wording of the question is:

> Think about the time you have available outside of work during weekdays. Imagine that for 1 month during weekdays (20 days), you want to spend 1 hour of leisure time away from your baby. A friend offers to take care of your baby during this 1 hour for the month, in exchange for a payment. Your friend will take good care of your baby, but they will not be engaged in active interaction with your baby.

[20]The baby's initial health $\theta_{i,0}$ does not enter the decision function, so it can be used to test model implications.

under scenario t. I asssume that $p_{i,t}$ is a noisy measure of the true subjective price of one hour of investment, p_i. I estimate p_i under a flexible factor model:

$$p_{i,t} = p_i + \gamma_i x_t + \varepsilon_{i,t}, \tag{8}$$

where x_t denote the scenario variables, γ_i is the vector of coefficients associated with the scenarios, and $\varepsilon_{i,t}$ is a measurement error. I estimate the parameters of the model using the Swamy (1970) estimator.[21]

The optimal investment choice implied by the model is $x(\alpha)$, while the econometrician observes x^*. Due to measurement error, these values differ. Let η_x denote the associated measurement error. Assume that:

$$x^* = x(\alpha)e^{\eta_x}, \tag{9}$$

where η_x is normally distributed with mean $\mu_{\eta_x} = -\frac{\sigma^2_{\eta_x}}{2}$ and variance $\sigma^2_{\eta_x}$.

Given these assumptions, the likelihood function is given by, where ϕ is the pdf of the standard normal distribution:

$$l(\alpha) = \sum_i^N \Big(\ln \phi(\ln x_i^* - \ln x_i(\alpha)) \Big). \tag{10}$$

To construct the model implied optimal investment choice $x(\alpha, p_i)$, one must compute the indirect utility function over a fine grid of possible values of x. I define the space of possible values of x as the interval $X = (0.0, 20.0]$[22] For a given value of $\alpha = \bar{\alpha}$, I construct a grid over this interval composed of 100 equidistant points, and compute the indirect utility function for each value. Then, I find the value $x^1(\bar{\alpha})$ which gives the maximum utility. I then construct a smaller interval around $x^1(\bar{\alpha})$ with 100 equidistant points, and proceed with the same algorithm and obtain $x^2(\bar{\alpha})$. I repeat this process until the difference between any two steps is smaller than 10^{-10}. This process is repeated for each new guess of α.

[21] Results of this estimation can be found in Appendix E.

[22] Optimal investment is measured in terms of daily hours. The lower bound of the interval is set to 10^{-6}.

4. Data

I collect the data using Qualtrics, a company that holds online panels of individuals who are willing to take surveys for a small fee. The survey was conducted during April, 2023. My sample consists of 507 women between the ages of 18 and 40 who had at least one child, but no child older than 5 years old.[23] In this section, I present the answers for all survey segments and descriptive statistics. I also present evidence that each survey segment has consistent and economically meaningful participant answers.

4.1. Sample Characteristics and Actual Investment

The survey participants answer the questions from the subjective belief instrument and the stated choice instrument. Additionally, participants answer several demographic questions and report actual investment in their oldest child. Table 1 describes the sample characteristics. The average age of participants is 27.9, with about 1.4 children on average, while the average age of children is 2.2. The sample is also relatively well educated, with 43.4% of the sample having at least a 4 year college degree. Personal and household income are distributed evenly across the categories. The sample is also relatively diverse, with 22.3% of individuals being Hispanic, 27.4% being Non-Hispanic Black, and 39.1% being Non-Hispanic White. Around 79% of women are working on average 5.4 hours a day, while 32% are in school.

In Table 2, I present statistics of actual investment. The participants are asked how many hours they spend with their child on a typical weekday and weekend day on reading, talking, playing inside, and playing outside. Total hours spent is the sum of all 4 activities for each respondent. Table 2 shows that the average participant spends 2.9 hours on a typical weekday and 4.1 hours on a typical weekend day with their child. The average participant spends 0.4 hours reading, 1.1 hours talking, 0.9 hours playing inside, and 0.5 hours playing outside on a typical weekday. On a typical weekend day, the average participant spends 0.5 hours reading, 1.4 hours talking, 1.2 hours playing inside, and 0.9 hour playing outside. The table also shows that the standard deviation of the hours of investment is relatively large, suggesting that there is significant heterogeneity in the sample. These patterns show that parents spend more time with their child during weekends, and that talking is the most common activity.

[23] The full sample consisted originally of 723 women. The number was reduced to 507 after removing individuals that did not pass attention checks, gave inconsistent answers, or chose to not answer some questions.

TABLE 1. Sample Composition

Variable	Mean	St. Dev.
Age of Respondent	27.876	5.678
Number of Children	1.377	0.611
Age of Oldest Child	2.203	1.295
Working	0.787	0.410
Daily Work Hours	5.400	3.625
In School	0.320	0.467
Ethnicity		
Hispanic	0.223	0.417
Non-Hispanic White	0.391	0.488
Non-Hispanic Black	0.274	0.447
Other	0.112	0.316
Marital Status		
Single	0.292	0.455
Married or Cohabitating	0.657	0.475
Separated	0.051	0.221
Education		
Dropout or GED	0.085	0.279
High School	0.162	0.369
Some College	0.320	0.467
College Degree	0.434	0.496
Household Income		
Less than $25,000	0.179	0.384
$25-$50,000	0.205	0.404
$50-$100,000	0.341	0.475
More than $100,000	0.274	0.447

Note: This table reports descriptive statistics for the sample of 507 respondents. All variables are proportions except for Age of Respondent, Number of Children, Age of Oldest Child, and Daily Work Hours. The Daily Work Hours variable is based only on respondents who reported being employed.

TABLE 2. Actual Investment Statistics

Variable	Mean	Median	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
On A **Typical Weekday**, How Many Hours Do You Spend with your Child on							
Reading	0.365	0.250	0.461	0.000	0.033	0.500	5.883
Talking	1.083	0.500	1.469	0.000	0.083	1.500	8.333
Playing Inside	0.952	0.500	1.381	0.000	0.083	1.000	10.000
Playing Outside	0.517	0.333	0.660	0.000	0.033	0.833	6.000
Total	2.918	2.000	3.186	0.000	0.250	4.425	15.500
On A **Typical Weekend**, How Many Hours Do You Spend with your Child on							
Reading	0.544	0.333	0.725	0.000	0.033	0.750	6.000
Talking	1.443	0.667	1.909	0.000	0.083	2.000	10.000
Playing Inside	1.246	0.600	1.601	0.000	0.067	2.000	10.000
Playing Outside	0.874	0.500	1.177	0.000	0.050	1.000	10.667
Total	4.106	2.833	4.407	0.000	0.267	7.000	18.667

Note: This table reports descriptive statistics for actual investment measures based on a sample of 507 individuals. Respondents reported time spent (in minutes) on each activity during a typical weekday and weekend day. Total time reflects the sum of all reported activities. A small number of respondents were excluded due to implausible time reports. See Appendix B for more details.

To understand the determinants of investment, I estimate a linear regression of hours of total actual investment on individual's socio-economic variables. All continuous variables are standardized. The results are presented in Table 3. The table shows who those that currently work or go to school spend less time interacting with their child than those who do not. Additionally, the number of hours spent on a typical weekend is positively correlated with the number of hours worked during the weekday. This is consistent with the idea that parents face a time constraint on investment, but compensate on weekends by investing more.

Table 3 also shows that those with some college education or more spend more time interacting with their child than those with less education, while Non-Hispanic Black and Hispanics spend less time interacting with their child than Non-Hispanic Whites Finally, the table shows that those with a higher household income spend more time interacting with their child than those with a lower household income, although the impact is weak and not for all income brackets. Overall, the collected investment measures show patterns that are consistent with the literature: there is an richer, more educated parents invest more in their child, and there exists ethnicity differences in investment.

4.2. Subjective Belief Instrument Responses

I present survey participants' responses to the age range questions. Table 4 shows the mean and standard deviation of the youngest, most likely, and oldest ages for each activity asked for each scenario of investment and health. The activities are ordered by difficulty according to the IRT model. I highlight two data features. First, we see that the mean age responses tends to increase with the difficulty of the activity for all types of ages. This is consistent with participants paying attention to each activity and understanding that they have different difficulty levels for a child.

Second, we see that as the scenario moves from high investment and normal health to low investment and poor health, the mean age responses tend to increase. This is also seen in Figure 1, where I show histograms for the activity of speaking a partial sentence.[24] The histograms show that the distribution of age responses is shifted to the right as the scenario moves from high investment and normal health to low investment and poor health. This is consistent with the idea that participants are responding to the scenario and not just randomly answering the questions.

[24]The histograms for the other activities are similar and are presented in Appendix E.

TABLE 3. Correlations of Investment with Demographics

(a) Demographics – Part 1

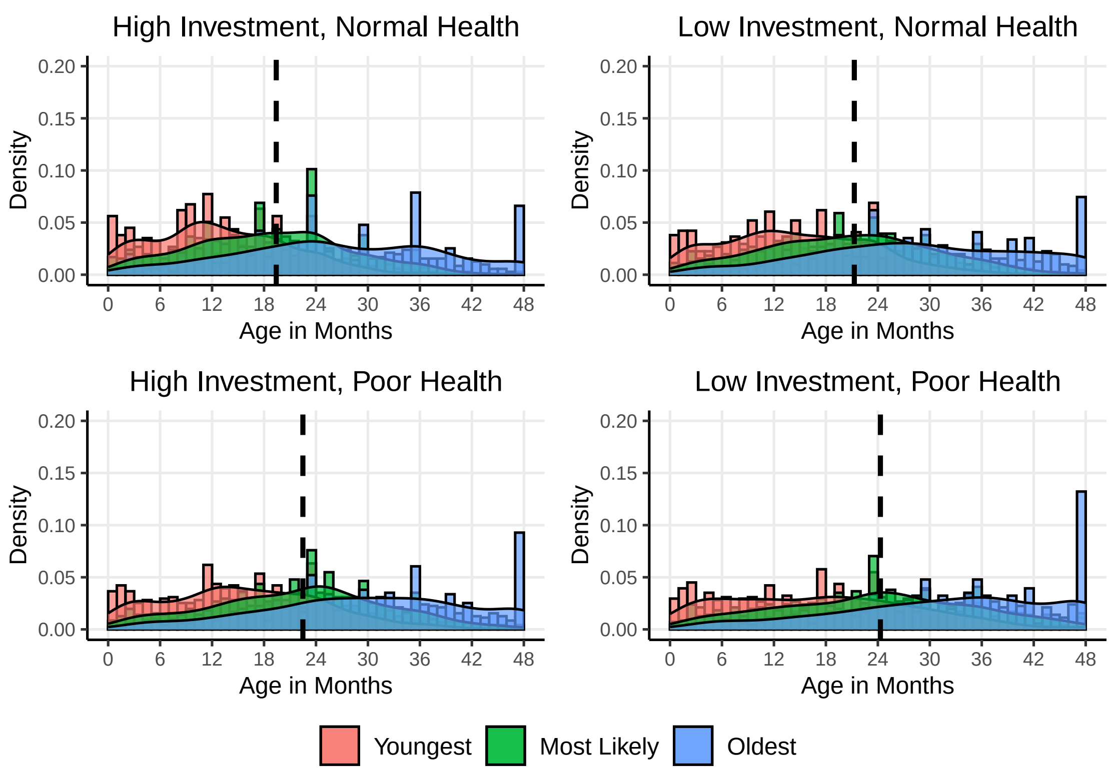
	Weekday Hours	Weekend Hours
Age $\leq$ 30	0.033	–0.120
	(0.103)	(0.109)
# Children	–0.004	–0.060
	(0.070)	(0.072)
Oldest Child $\leq$ 3	0.117	0.087
	(0.099)	(0.099)
Working	-0.599^{***}	-0.486^{***}
	(0.174)	(0.149)
Daily Work Hours	0.017	0.047^{***}
	(0.019)	(0.017)
In School	-0.330^{***}	-0.380^{***}
	(0.099)	(0.100)
Ethnicity (Omitted: Non-Hisp White)		
Non-Hisp Black	-0.337^{***}	-0.376^{***}
	(0.115)	(0.120)
Hispanic	–0.154	-0.309^{***}
	(0.118)	(0.111)
Other	–0.052	-0.242^{*}
	(0.146)	(0.138)
Marital Status (Omitted: Single)		
Married	0.138	0.131
	(0.115)	(0.111)
Separated	0.010	–0.071
	(0.192)	(0.183)

(b) Demographics – Part 2

	Weekday Hours	Weekend Hours
Education (Omitted: Dropout)		
High School	–0.172	–0.065
	(0.181)	(0.158)
Some College	0.366^{**}	0.424^{***}
	(0.161)	(0.132)
College Degree	0.271^{*}	0.320^{**}
	(0.155)	(0.132)
Household Income (Omitted: $0-$25,000)		
$25-$50,000	0.291^{**}	0.285^{**}
	(0.141)	(0.123)
$50-$100,000	0.125	0.222^{*}
	(0.125)	(0.119)
$100,000+	–0.065	0.033
	(0.147)	(0.139)
Constant	0.128	0.047
	(0.277)	(0.235)
Observations	507	507

Note: Robust standard errors in parentheses. *p<0.1; **p<0.05; ***p<0.01. This presents the estimates of regressing a typical weekday or weekend daily investment hours of a parent on their oldest child on socio-economic variables of the parent. The table is split into two for ease of view, but coefficients are all from the same linear regression. All continuous variables are standardized.

FIGURE 1. Distribution of Ages by Scenario for "Speak a Partial Sentence of 3 Words"



Note: This figure displays the histogram and density of the youngest, most likely, and oldest ages for the activity of speaking a partial sentence of 3 words. The histograms are colored by the type of age. The dashed line represents the mean of the most likely age.

TABLE 4. Mean and Standard Deviation of Responses to the Belief Instrument

	High Investment and Normal Health					
	Youngest Age		Most Likely Age		Oldest Age	
	Mean	SD	Mean	SD	Mean	SD
Partial Sentence	13.12	7.96	18.67	9.24	26.44	11.99
Age and Sex	15.50	8.74	21.83	9.59	29.31	11.16
First and Last Name	18.39	10.61	24.99	10.79	32.04	11.87
Counts 3 Objects	16.83	10.11	23.19	10.59	30.84	12.15
	Low Investment and Normal Health					
	Youngest Age		Most Likely Age		Oldest Age	
	Mean	SD	Mean	SD	Mean	SD
Partial Sentence	14.72	8.54	20.68	9.88	28.45	12.08
Age and Sex	17.41	9.56	23.91	10.17	30.86	11.47
First and Last Name	20.16	11.42	26.71	11.31	33.37	11.91
Counts 3 Objects	18.26	10.71	24.91	10.92	32.24	12.11
	High Investment and Poor Health					
	Youngest Age		Most Likely Age		Oldest Age	
	Mean	SD	Mean	SD	Mean	SD
Partial Sentence	15.82	9.23	22.04	10.16	29.61	11.81
Age and Sex	18.04	10.12	24.46	10.37	31.67	11.56
First and Last Name	20.87	11.53	27.41	11.45	34.10	11.87
Counts 3 Objects	18.83	11.02	25.40	11.37	32.76	12.19
	Low Investment and Poor Health					
	Youngest Age		Most Likely Age		Oldest Age	
	Mean	SD	Mean	SD	Mean	SD
Partial Sentence	17.58	10.99	23.84	11.38	31.48	12.36
Age and Sex	20.48	11.83	26.98	11.73	34.07	12.25
First and Last Name	22.45	13.09	28.77	12.43	35.36	12.24
Counts 3 Objects	21.10	12.45	27.72	12.17	34.52	12.54

Note: This table presents the mean and standard deviation of the youngest, most likely, and oldest ages for each activity asked for each scenario of investment and health. The activities are ordered by difficulty according to the IRT model, with the easiest activity being "Partial Sentence" and the hardest being "Counts 3 Objects".

4.3. Price of Investment Instrument

Table A2 displays descriptive statistics for the stated choice instrument. Respondents were asked to choose the maximum amount in dollars they would pay a friend to take care of their child. I present the mean and standard deviation for each scenario of health, household income, and daily working hours.

The table shows evidence of the consistency of the participants answers. The standard deviation in the sample across all scenarios remains constant, indicating that the variability in answers is similar regardless of the scenarios. There is no discernible difference across health scenarios. The means of each combination of work hour and income are similar whether the health of the baby is good or poor. This can be confirmed in Table A3, where the response is regressed on the scenario variables. The coefficients for baby health are all not statistically significant.

On both Tables, there is a strong gradient in work hours and household income. As they work more and earn more, their willingness-to-pay also increases. For example, conditional on good health, in the scenario of 0 working hours and $2,000 income, the mean response is of $8.32. Then, at 8 working hours, the mean response increases ot $12.51. Conversely, at $6,000 income, the mean increases to $10.86. This is again confirmed in Table A3. Increasing the working hours by 1 hour increases the willingness-to-pay by $0.52, while increasing the income by $1,000 increases the willingness-to-pay by $0.61.

These patterns are consistent with a model where parents value leisure. As their available hours of leisure decrease, they value it more. However, income is a significant factor, since as it increases, their budget constraint expands and they are able to allocate more resources to "buy" leisure. These correlations give support to the model presented in Section 2.

5. Results

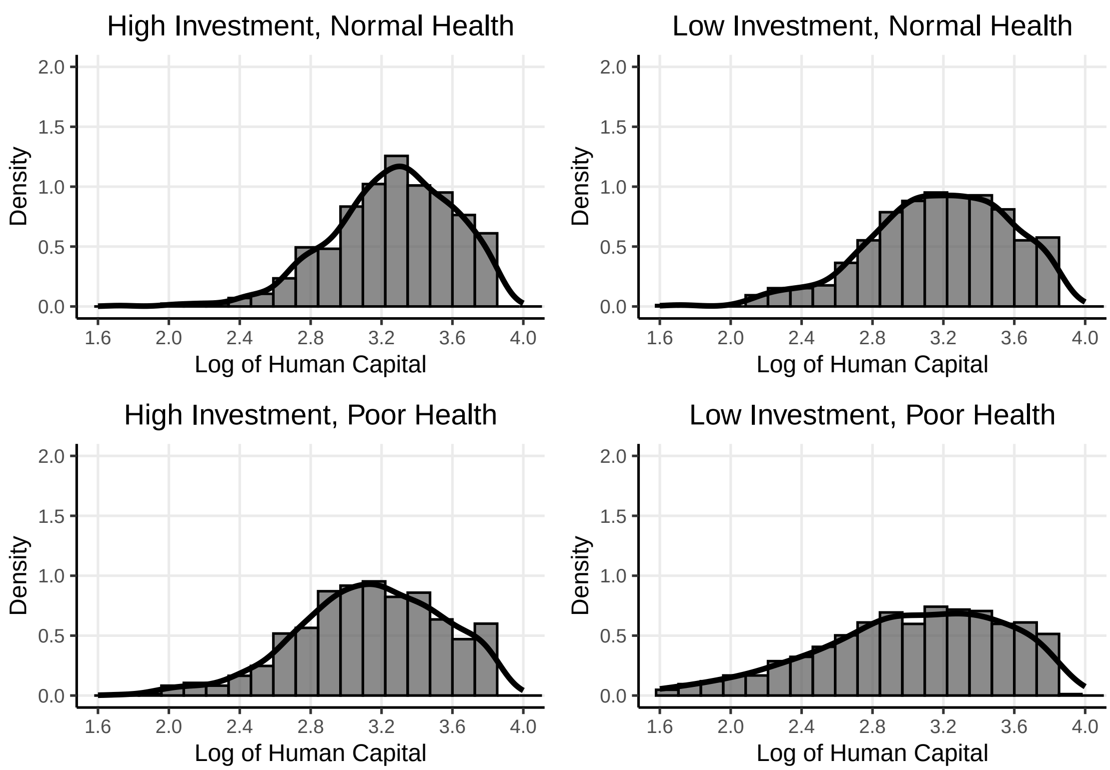
In this section, I first present the estimation results of the subjective production function parameters. Then, I discuss how they are related to each other, to demographic variables, and to actual investment measures. I then illustrate the use of parental subjective distributions by estimating an investment model with reference-dependent preferences, and discuss its interpretation.

5.1. Parental Beliefs

I now present the estimates of the subjective expectation and uncertainty about the future human capital of the child. These estimates assume that the age range distribution $H(\cdot)$ follows a triangular distribution.[25]

In Figure 2, I plot the histogram and distribution of $E[\ln\theta_{i,1}|\Omega_i]$ for each of the four possible hypothetical scenarios. For each scenario, there are four estimates of $E[\ln\theta_{i,1}|\Omega_i]$, one for each activity j. Therefore, I average $E[\ln\theta_{i,1}|\Omega_i]$ across all four activities. I find that the distribution of $E[\ln\theta_{i,1}|\Omega_i]$ is more dispersed and has a lower mean for the low investment and poor health scenario, while it is more concentrated around a higher mean for the high investment and normal health scenario. This indicates that parental beliefs are positively correlated with investment and health, as predicted by the model.

FIGURE 2. Distribution of Error-ridden Expectation of Log of Human Capital by Scenario



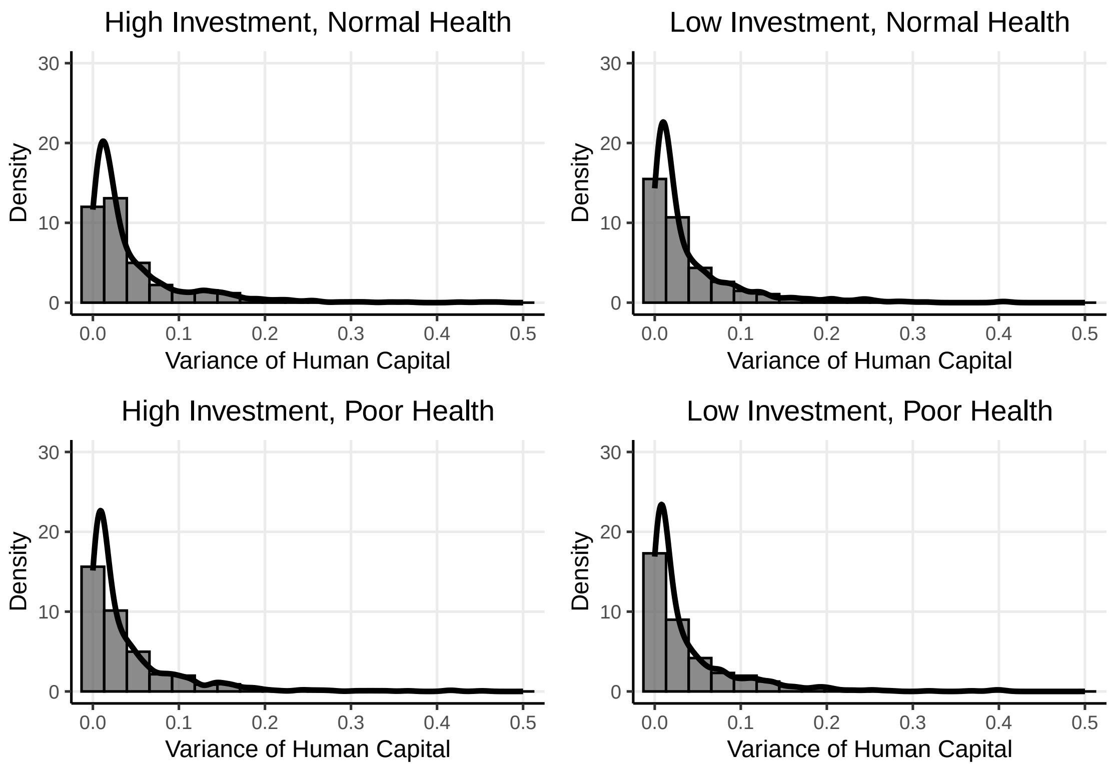
Note: This figure plots the histogram of the error ridden measures of $E[\ln\theta_{i,1}|H_i]$ conditional on the hypothetical scenario that was presented to the respondent.

In Figure 3, I plot the distribution of the error-ridden variance of the log of human

[25] Results for different combinations of distributional assumptions can be found in Appendix D.

capital. I find that the distribution is more concentrated for the low investment and poor health scenario, while it is more dispersed around a lower mean for the high investment and normal health scenario. Therefore, parents are more uncertain about the future human capital of their child under a scenario of higher returns.

FIGURE 3. Distribution of Error-ridden Variance of Log of Human Capital by Scenario



Note: This figure plots the histogram of the error ridden measures of $Var(\ln \theta_{i,1}|H_i)$ conditional on the hypothetical scenario that was presented to the respondent.

Table 5 shows the mean and standard deviation of $E[\ln \theta_{i,1}|\Omega_i]$ and $Var(\ln \theta_{i,1}|\Omega_i)$ and summarizes the above discussion. The mean values of $E[\ln \theta_{i,1}|\Omega_i]$ decrease as the scenarios move from the best to the worst, while its standard deviation increases. For the "best" scenario, where there is high investment and normal health, parents believe that the child's skill will be 3.260, which translates to a developmental age of 26 months. Note that the skill is anchored at the age of 24 months, so parents are slightly optimistic under this scenario. For the "worst" scenario of low investment and poor health, parents believe the child's skill will be 2.990, or 19.8 months, a developmental delay of about 4 months.

In contrast, the mean and standard deviation of $Var(\ln \theta_{i,1}|\Omega_i)$ decrease as the sce-

TABLE 5. Summary Statistics for $E[\ln \theta_{i,1}|H_i]$ and $Var(\ln \theta_{i,1}|H_i)$ by Scenario

	$E[\ln \theta_{i,1}\|H_i]$		$Var(\ln \theta_{i,1}\|H_i)$	
Scenario	Mean	St. Dev.	Mean	St. Dev.
High Investment, Normal Health	3.260	0.341	0.049	0.075
Low Investment, Normal Health	3.175	0.392	0.043	0.065
High Investment, Poor Health	3.131	0.414	0.044	0.069
Low Investment, Poor Health	2.990	0.562	0.041	0.062

Note: This table shows summary statistics for $E[\ln \theta_{i,1}|H_i]$ and $Var(\ln \theta_{i,1}|H_i)$ conditional on the scenario that was presented to the respondent.

narios move from the best to the worst. This is consistent with parents having more pessimistic and less uncertain beliefs about the future human capital of the child when investment is lower and when the child is in poor health.

5.2. Estimates of the Subjective Production Function and Subjective Cost

Table 6 presents the estimates of the subjective production function parameters, μ_{δ_k} and $\sigma^2_{\delta_k}$, from (6) and (7) using the SRC estimator. Additionally, I estimate individual-level coefficients, μ_{i,δ_k} and σ^2_{i,δ_k}, and their standard errors.

Panel A of Table 6 displays the aggregate estimates. All parameters are statistically significant at the 1% level. I focus the discussion on the estimates of the subjective returns to investment parameters, δ_2, as they are the most relevant for the analysis. I find that the subjective mean of the returns to investment parameter, i.e. μ_{i,δ_2}, is 0.103. On average mothers believe that a 10% increase in investments would lead to a 1.03% increase in the child human capital by 24 months. As a comparison, Cunha, Elo, and Culhane (2022) report a subjective elasticity of investment of around 0.17, while Cunha, Elo, and Culhane (2013) report an objective elasticity of investment of around 0.26 using similar methods. Therefore, mothers in this sample are more pessimistic about the returns to investment than the previous literature. The estimated subjective variance, $\sigma^2_{\delta_2}$, is 0.003. Together with the mean estimate, these parameters give a coefficient of variation ($CV = \frac{\sigma}{\mu}$) of 0.53 thereby suggesting a low degree of uncertainty in beliefs of the average parent in the sample. At the individual level, only about 29% of the sample have a coefficient of variation higher than 1.0.

In Panel B, I conduct a significance test for each individual-level parameter. I report the percentage of the estimates whose p-values are lower than 10% confidence region. Focusing on the subjective returns to investment parameters, I find that 43.20% of the estimates of μ_{i,δ_2} are statistically significant at the 10% level, while 27.22% of the estimates of σ^2_{i,δ_2} are statistically significant at the 10% level. The percentage value for the mean estimates are in line with Cunha, Elo, and Culhane (2022).[26]

[26]The percentage of significant coefficients for the variance estimates is low. This is likely due to the fact that the variance estimates are more affected by measurement errors that are not fully corrected by the methodology I propose. On the other hand, it could also be due to model misspecification. For example, the full variance specification, assuming uncorrelation of production shock beliefs to all other parameters, would be:

$$Var(\ln\theta_{i,1}|H_i) = (\sigma^2_{i,\delta_0} + \sigma^2_{i,\varepsilon}) + \sigma^2_{i,\delta_1}\ln\theta^2_{i,0} + \sigma^2_{i,\delta_2}\ln x_i^2 + \sigma_{\delta_0,\delta_1}\ln\theta_{i,0} + \sigma_{\delta_0,\delta_2}\ln x_i + \sigma_{\delta_1,\delta_2}\ln\theta_{i,0}\ln x_i.$$

TABLE 6. Estimates of Mean Subjective Production Function Parameters

Panel A: Aggregate Estimates			
	$E[\ln\theta_{i,1}\|H_i]$		$Var(\ln\theta_{i,1}\|H_i)$
μ_{δ_0}	2.897***	$\sigma^2_{\delta_0}$	0.031***
	(0.0285)		(0.0023)
μ_{δ_1}	0.064***	$\sigma^2_{\delta_1}$	0.001***
	(0.0045)		(0.0002)
μ_{δ_2}	0.103***	$\sigma^2_{\delta_2}$	0.003***
	(0.0084)		(0.0006)
		$\sigma_{1,2}$	−0.002***
			(0.0006)
Panel B: Individual Estimates			
Parameter	% Significant	Parameter	% Significant
μ_{i,δ_0}	100.00%	σ^2_{i,δ_0}	81.46%
μ_{i,δ_1}	45.17%	σ^2_{i,δ_1}	28.80%
μ_{i,δ_2}	43.20%	σ^2_{i,δ_2}	27.22%
		$\sigma_{i,1,2}$	16.17%

Note: Robust standard errors in parenthesis. *p<0.1; **p<0.05; ***p<0.01. This table shows aggregate estimates of equations (6) and (7) and their individual-level predicted estimates. The percentage of significant estimates is calculated using a 10% significance level, and the null hypothesis is that the coefficient is equal to zero.

Figures A7 and Table 7 display the distribution and summary statistics of the estimated individual level parameters of the subjective beliefs of the production function and the subjective costs.[27] The mean values of the production function parameters are by construction equal to the estimates in Table 6. The standard deviation and selected percentiles of the distribution of the individual-level parameters are also reported and give an idea of the heterogeneity in beliefs. Focusing on the parameters related to the returns to investment, i.e. δ_2, I find that in general the mean beliefs exhibit variation across individuals, with the standard deviation of μ_{i,δ_2} being 0.094 and a coefficient of variation close to 1.0. The standard deviation of the variance estimates, σ^2_{i,δ_2}, is 0.004, which produces a similar coefficient of variation.

The mean price estimate shows that on average mothers price one hour of investment compared to leisure at around \$12.41, with the overall distribution of prices not being very dispersed, as the 25th and 75 percentiles are close to the mean. It implies a monthly cost of 1 hour care of children everyday on weekdays at around \$273.

TABLE 7. Individual Level Coefficient Distributions

Variable	Mean	St. Dev.	25th Percentile	Median	75th Percentile
p_i	12.410	5.178	9.018	12.358	15.607
μ_{i,δ_0}	2.916	0.570	2.527	2.924	3.389
μ_{i,δ_1}	0.063	0.055	0.019	0.053	0.098
μ_{i,δ_2}	0.100	0.094	0.022	0.086	0.163
$\sigma^2_{i,0}$	0.033	0.039	0.005	0.016	0.049
σ^2_{i,δ_1}	0.001	0.001	0.0001	0.001	0.002
σ^2_{i,δ_2}	0.004	0.004	0.0004	0.002	0.006
$\sigma_{i,\delta_1,\delta_2}$	−0.002	0.004	−0.004	−0.001	0.0003

Note: This table shows certain moments from the distribution of individual level coefficients. Price p_i was estimated using a linear regression random coefficients model, while the production function subjective parameters were estimated using the seemingly unrelated random coefficients model.

The estimation of this model is not feasible due to the nature of the estimation strategy. While theoretically feasible, the scenarios ($Z = \{\theta_0, x\}$) are simplified to be variables with only two distinct values and thus the within-individual variation of right-hand side variables is very small. Therefore, there would be severe multicollinearity in estimating the full model. Nevertheless, as I show in the next section, these estimates contain information that is relevant to demographic variables and real investment.

[27] I present the estimates of the factor model that produces the price coefficients in Appendix E.

5.3. Relationship between Subjective Variables, Demographics, and Actual Investment

I now investigate how observable characteristics of mothers relate to their subjective beliefs. I focus on the subjective returns to investment parameters, μ_{i,δ_2} and σ^2_{i,δ_2}, and the price of care, p_i. I standardize these variables to have a mean equal to 0 and a standard deviation equal to 1 to keep the relationships comparable. Then I run a linear regression on the demographic variables described in the Data section.[28] I report the results in Table 8.

First, I find that younger mothers tend to have lower mean beliefs and to be more uncertain about the return to investment in children. Second, Non-hispanic Black mothers are more pessimistic and more uncertain about the return to investment than White mothers. Third, I find that married mothers are less uncertain about the return to investment, and price investments much lower than non-married ones. Fourth, I find a strong education gradient in mean beliefs. Finally, while income is not significantly correlated with beliefs, there is a strong income gradient in the cost of care.

While no causal relations can be extracted from these results, they suggest certain patterns that are consistent with the literature. For example, the racial difference in mean beliefs is consistent with the findings of Cunha (2014). The positive education gradient in beliefs is conceptually similar to the use of parental education as a proxy for parental knowledge of the benefits of early investment. Lower and more uncertain beliefs for younger mothers suggests that mothers learn throughout the process of raising children, although an extension incorporating learning in the production function is beyond the scope of this paper. Similarly, it is not surprising that the price of care is more costly for higher income mothers, as they are more likely to be employed and have higher opportunity costs of time. Finally, the negative correlation between marriage and uncertainty and price shows that family structure can be an important determinant in belief formation. This link has been noted in the literature on children's outcomes.

Next, I explore how the estimated beliefs relate to each other. I regress the subjective mean on subjective uncertainty and cost of care, and control for the observable characteristics. I also estimate the same model but using the subjective uncertainty as the dependent variable to obtain the correlation between cost of care as well. Table 9 reports the results. I find that mothers that have lower beliefs about the returns to

[28] I exclude labor supply variables in this regression as they are "transitory" variables that do not reflect fixed characteristics of individuals. However, results do not change qualitatively when included.

TABLE 8. Correlation of Subjective Variables and Socio-Economic Variables

	μ_{i,δ_2}	σ^2_{i,δ_2}	p_i
Age $\leq$ 30	−0.234**	0.200**	0.046
	(0.107)	(0.090)	(0.101)
# Children	−0.096	0.219***	0.018
	(0.067)	(0.083)	(0.077)
Oldest Child $\leq$ 3	−0.052	0.163	−0.144
	(0.114)	(0.102)	(0.123)
Ethnicity (Omitted Group: Non-Hisp White)			
Non-Hisp Black	−0.267**	0.362***	0.062
	(0.123)	(0.131)	(0.123)
Hispanic	−0.288**	0.139	0.110
	(0.118)	(0.108)	(0.128)
Other	−0.060	0.058	−0.075
	(0.145)	(0.125)	(0.135)
Marital State (Omitted Group: Single)			
Married	0.140	−0.255**	−0.391***
	(0.109)	(0.113)	(0.117)
Separated	−0.094	−0.053	−0.060
	(0.203)	(0.235)	(0.213)
Education Level (Omitted Group: Dropout/GED)			
High School	0.153	−0.094	0.003
	(0.161)	(0.182)	(0.212)
Some College	0.461***	−0.148	−0.096
	(0.157)	(0.179)	(0.195)
College Degree	0.267*	−0.052	0.022
	(0.161)	(0.183)	(0.203)
Household Income (Omitted Group: $0-$25,000)			
$25-$50,000	0.193	−0.060	0.260*
	(0.140)	(0.154)	(0.158)
$50-$100,000	0.149	−0.046	0.338**
	(0.140)	(0.144)	(0.156)
$100,000+	−0.008	−0.156	0.360**
	(0.168)	(0.162)	(0.176)
Constant	0.003	−0.371	0.042
	(0.240)	(0.264)	(0.268)
Observations	507	507	507

Note: Robust standard errors in parenthesis. *p<0.1; **p<0.05; ***p<0.01. All continuous variables are standardized. This table presents estimates of a linear regression model where the dependent variable is the mean and variance of the belief distribution about returns to investment. Independent variables consist of socio-economic indicators.

investment tend to be more uncertain as well. This result holds as we add demographic controls and the price of care. Further, mothers that have higher costs of care tend to have lower mean beliefs.

TABLE 9. Correlation between beliefs and cost of care

	μ_{δ_2}			
	(1)	(2)	(3)	(4)
σ^2_{i,δ_2}	−0.431***		−0.411***	−0.381***
	(0.034)		(0.034)	(0.036)
p_i		−0.201***	−0.142***	−0.139***
		(0.042)	(0.040)	(0.041)
	$\sigma^2_{\delta_2}$			
	(1)	(2)	(3)	(4)
μ_{i,δ_2}	−0.431***		−0.419***	−0.379***
	(0.040)		(0.041)	(0.042)
p_i		0.143***	0.058	0.051
		(0.039)	(0.038)	(0.038)
Observations	507	507	507	507
Demographics	No	No	No	Yes

Note: Robust standard errors in parenthesis. *p<0.1; **p<0.05; ***p<0.01. All continuous variables are standardized. This table presents estimates of a linear regression model where the dependent variable is the mean and variance of the belief distribution about returns to investment. Independent variables consist of beliefs and socio-economic indicators.

Finally, I explore how the estimated beliefs relate to actual investment. Two different measures of time investment are collected in the data, the number of hours spent with the child on a typical weekday and weekend. Therefore, I estimate two regressions using these two measures as the dependent variables, and controlling for observable variables. Table 10 reports the correlation between the subjective returns to investment parameters and the actual investment measures. The first column uses only the mean belief as the independent variable, as in the previous literature, and I progressively add the subjective variance, the price of care, and demographic controls.

I find that the subjective mean of the returns to investment parameter, i.e. μ_{δ_2}, is

TABLE 10. Correlation of Real Investment with Beliefs

	(1)	(2)	(3)	(4)
		Weekday Hours		
μ_{i,δ_2}	0.353^{***}	0.288^{***}	0.260^{***}	0.197^{***}
	(0.042)	(0.049)	(0.049)	(0.052)
σ^2_{i,δ_2}		-0.153^{***}	-0.142^{***}	-0.132^{***}
		(0.041)	(0.040)	(0.045)
p_i			-0.160^{***}	-0.133^{***}
			(0.040)	(0.039)
		Weekend Hours		
μ_{i,δ_2}	0.390^{***}	0.291^{***}	0.265^{***}	0.200^{***}
	(0.041)	(0.050)	(0.049)	(0.052)
σ^2_{i,δ_2}		-0.229^{***}	-0.219^{***}	-0.178^{***}
		(0.041)	(0.040)	(0.042)
p_i			-0.153^{***}	-0.146^{***}
			(0.040)	(0.039)
Observations	507	507	507	507
Demographics	No	No	No	Yes

Note: Robust standard errors in parenthesis. *p<0.1; **p<0.05; ***p<0.01. All continuous variables are standardized. This table presents estimates of a linear regression model where the dependent variable consists of actual time investment measures as reported by individuals. Each column adds additional controls such as belief distribution parameters and socio-economic indicators.

positively correlated with the actual investment measures. This finding is consistent with the previous literature (see, Cunha, Elo, and Culhane (2013, 2022); Boneva and Rauh (2018)). A one standard deviation increase in mean beliefs leads to a 23% standard deviation increase in daily hours of investment, whether on weekdays or weekends. However, higher uncertainty predicts lower investments, with a one standard deviation increase associated with a 14% standard deviation decrease in investments. We see a similar pattern in the price of care, but the reduction of investment is larger for weekends than for weekdays.

Overall, these patterns provide reassurance that the elicitation methods provided meaningful data since results for mean beliefs follow the previous literature. Furthermore, the results for the subjective uncertainty suggests that increased uncertainty may play an important role in influencing investment, and future interventions that target belief improvements may find useful to also measure uncertainty.

5.4. An Application to a Model With Reference Dependent Preferences

Previous research either ignore subjective beliefs or assume specific functional forms for the utility function that rules out higher order beliefs. The stated choice data together with the subjective belief estimates for individuals allows me to estimate a flexible model of parental investment in children. In this section, I estimate a model of parental investment in children with reference dependent preferences that incorporates subjective uncertainty on the decision making of individuals. Parental preferences depend on household consumption, leisure, child development at the end of the period, and the relative development of the child compared to a reference level of development, θ_{ref}. I assume that the mother's preferences are given by:

$$\begin{aligned} u_i(c_i, h_{l_i}, \theta_{i,1}) =& \alpha_1 \ln c_i + \alpha_2 \ln h_{l_i} + \alpha_3 \ln \theta_{i,1} + \\ & \alpha_4 (\ln \theta_{ref} - \ln \theta_{i,1}) \mathbb{1}\{(\ln \theta_{i,1} \leq \ln \theta_{ref})\}. \end{aligned}$$

The parameter α_1 captures the preference for consumption and α_2 captures the preference for leisure. The coefficient α_3 captures how parents value the end-of-period human capital of their child, while α_4 denotes how parents value their child's skill being below the reference point. It is reasonable to assume that $\alpha_3 > 0$, since parents would like their child's human capital to increase. However, it is important to look at both α_3 and α_4. Since the reference point introduces a sharp discontinuity in θ_1, the sign and magnitude of α_4 relative to α_3 will determine how much parents value human capital beyond the reference point. If $\alpha_3 > \alpha_4 > 0$, parents will invest more after crossing the reference point. On the other hand, if $\alpha_3 > 0$, but $\alpha_4 < 0$, parents will have a strong incentive to invest only up to the reference point, and have a weak preference for human capital beyond the reference.

The reference point θ_{ref} is the level of development that the parents consider to be "desirable" or "satisfactory". In a similar context of parental child investment, Wang et al. (2022) and Kinsler and Pavan (2021) show that parents use their local peers as a reference point[29] In my context, this definition of a "local" peer is not feasible. Since child development is anchored in developmental age, a natural reference point is a threshold for developmental delay.

The expected utility function which parents maximize is:

$$E[u_i(c_i, h_{l_i}, \theta_{i,1})|\Omega_i] = \alpha_1 \ln c_i + \alpha_2 \ln h_{l_i} + \alpha_3 E[\ln \theta_{i,1}|\Omega_i] + \alpha_4 \int_{\theta_{i,1}} (\ln \theta_{ref} - \ln \theta_{i,1}) \mathbb{1}\{(\ln \theta_{i,1} \leq \ln \theta_{ref})\} dG_i,$$

where $G_i(\cdot)$ is the distribution of subjective beliefs. Given the assumption that $G_i(\cdot)$ is a Normal distribution, it follows that the integral can be simplified to:[30]

$$\int_{\theta_{i,1}} (\ln \theta_{ref} - \ln \theta_{i,1}) \mathbb{1}\{(\ln \theta_{i,1} \leq \ln \theta_{ref})\} dG_i = (\ln \theta_{ref} - \mu_{i,\theta_1}) \Phi\left(\frac{\ln \theta_{ref} - \mu_{i,\theta_1}}{\sigma_{i,\theta_1}}\right) + \sigma_{i,\theta_1} \phi\left(\frac{\ln \theta_{ref} - \mu_{i,\theta_1}}{\sigma_{i,\theta_1}}\right),$$

where Φ and ϕ denote the cdf and pdf of a standard Normal distribution, respectively, and μ_{i,θ_1} and σ_{i,θ_1} are the mean and standard deviation of the subjective beliefs of the production function. Then, the estimation of the model parameters $\boldsymbol{\alpha}$ follows the procedure described in section 3.4.

Table 11 displays the estimation of the preference parameters of the investment

[29] In Wang et al. (2022), the reference is the average development of children in the village, while in Kinsler and Pavan (2021) it is the peers in the same school and classroom. In both papers, parental beliefs are related to what the reference point is, not about the skill production function.

[30] Note that the integral involves the expected value of a Truncated Normal Distribution. Specifically,

$$\int_{\theta_{i,1}} (\ln \theta_{ref} - \ln \theta_{i,1}) \mathbb{1}\{(\ln \theta_{i,1} \leq \ln \theta_{ref})\} dG_i = E[\ln \theta_{ref} - \ln \theta_{i,1}|\Omega_i, \ln \theta_{i,1} \leq \ln \theta_{ref}] Pr(\ln \theta_{i,1} \leq \ln \theta_{ref})$$

.

model.[31] Note that I use the subjective production function parameters and the subjective cost of care as inputs in the estimation procedure. Therefore, the estimates of the preference parameters take into account that different individuals face different costs of care and have different beliefs about the returns to investment.

TABLE 11. Model Estimates

Parameter	Estimates			
	θ_{ref} = 18	θ_{ref} = 20	θ_{ref} = 22	θ_{ref} = 24
α_2	1.132***	1.090***	0.555***	0.674***
	(0.354)	(0.375)	(0.176)	(0.275)
α_3	3.137***	2.071***	1.183***	1.282***
	(0.745)	(0.525)	(0.239)	(0.311)
α_4	-3.077***	-2.031***	-1.159***	-1.262***
	(0.684)	(0.487)	(0.219)	(0.291)

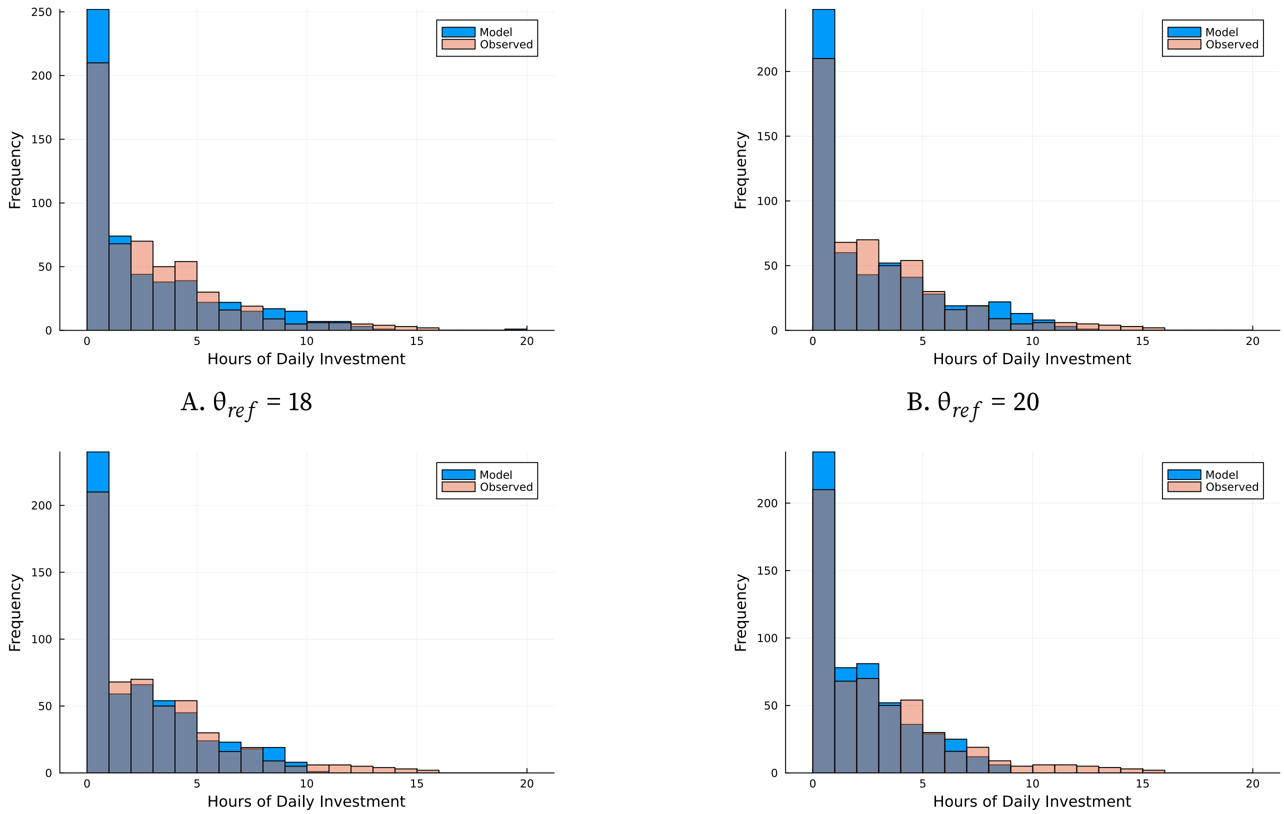
Note: Bootstrapped standard errors in parenthesis. *p<0.1; **p<0.05; ***p<0.01. This table shows the estimates of the preference parameters of the investment model for different reference points. α_2 denotes the preference parameter for leisure, α_3 for the child human capital, and α_4 for the positive distance between the child's human capital and the reference point when it would fall below the reference.

I estimate the model for different reference points θ_{ref}. The elicitation of the subjective beliefs targets the developmental age of 24 months. In other words, the activities that are used to elicit beliefs from parents were typical for children around 2 years old, and the anchoring of the beliefs to a cardinal metric used the 24 months milestone. Then, the reference point θ_{ref} should be a developmental age around 24 months. I estimate the model using $\theta_{ref} = \{18, 20, 22, 24\}$ months. A $\theta_{ref} = 18$ indicates that the reference point is a developmental age of 18 months, which represents a developmental delay of 6 months compared to the typical child of 2 years old. The estimate of α_4 will determine the preference of the mother for investing in their child so that they reach the reference point or go beyond it.

The preference for consumption, α_1 is fixed to 1 for identification. All parameters are statistically significant. The preference for leisure, α_2 is positive for all reference points. The preference for the child's future skill, α_3 is also positive. However, the estimates for α_4 are all negative but smaller than α_3 in absolute value. This means that parents have a strong incentive to invest in their child up to the reference point, but beyond that their

[31]The sample used to estimate this model excludes 54 individuals that violate some model constraints. More details can be found in Appendix B.

FIGURE 4. Model Predicted and Observed Hours of Investment



A. $\theta_{ref} = 18$

B. $\theta_{ref} = 20$

C. $\theta_{ref} = 22$

D. $\theta_{ref} = 24$

Note: This figure displays histograms of actual time investment, as reported by respondents, alongside predicted investment levels generated from the model using estimated preferences at various reference points.

preferences are weaker. This highlights that parents' preferences for being above the reference point are stronger when the reference point is lower, that is, they are more concerned about their child being in a situation of severe developmental delay.[32]

Figure 4 shows the distribution of daily investment hours that is implied by the model under the estimated parameters and compares it to the observed investment. In general, the estimated parameters using any references θ_{ref} have a good fit, but the model overestimates investments close to the bounds of possible investment hours, 0 and 20.

[32]This is consistent with pilot interviews, where mothers were asked whether they valued having a child be developmentally advanced by age 2. While at the exploratory stage, mothers typically responded that they cared mostly about avoiding developmental delays, and did not have strong preferences about having a developmentally advanced child.

I now simulate a policy that increases the mean belief of the returns to investment, μ_{δ_2}. I simulate the policy for the different references points, and report the results in Table 12. This kind of simulation replicates the types of interventions that target parental beliefs. However, these interventions do not measure the uncertainty of the parents.

I first increase all individual beliefs about the returns to investment, μ_{i,δ_2}, by 10% for all individuals. I find that the increase in the mean belief leads to between 4% and 6% increase in the investment in children. The same is done for the uncertainty about returns to investment, σ^2_{i,δ_2}, and it also leads to an increase of about 3%, although with a decreasing trend as the reference points get relaxed.

While the structural model predicts that increasing uncertainty raises parental investment, the reduced-form correlation between reported investment and uncertainty is negative. One natural explanation lies in residual measurement error. Observed investment, x, is a noisy measure of latent optimal investment, x^*, as we only observe a single reported value per parent. Similarly, the estimated belief uncertainty, σ^2_{i,δ_2}, is constructed from the proposed estimator, which reduces but does not eliminate measurement error. If the remaining errors in investment and uncertainty are negatively correlated, for example, if parents who underreport time with their children also tend to overstate uncertainty, the observed correlation between investment and uncertainty can be negative even when the latent effect of uncertainty on x^* is positive.

A second complementary explanation is a case of omitted variable bias through preferences. Given that the optimal investment function depends not only on subjective beliefs but also preferences, if more uncertain parents have lower preferences for human capital, the omitted variable bias could potentially flip the sign of the true relationship between uncertainty and investment.[33]

The structural estimation accounts for the optimal investment decision under reference dependent preferences, explicitly modeling x^* as a function of uncertainty, mean beliefs, and other parameters, while conditioning on the observed σ^2_{i,δ_2} and μ_{i,δ_2}. In contrast, a simple OLS regression conflates the latent effect of uncertainty on optimal investment with the effects of measurement error and unobserved preferences, which can produce a downward-biased or even negative slope.

The result that increasing uncertainty about returns to investment leads to an in-

[33]In a simple model $x^* = \beta\,\sigma^2 + \gamma\,\alpha + \varepsilon$, $\beta > 0$, $\gamma > 0$, omitting α produces an observed slope on uncertainty $\tilde{\beta} = \beta + \gamma \frac{\mathrm{Cov}(\sigma^2,\alpha)}{\mathrm{Var}(\sigma^2)}$. If $\mathrm{Cov}(\sigma^2, \alpha) < 0$, the OLS estimate $\tilde{\beta}$ can be negative even though the true causal effect β is positive.

TABLE 12. Percent Change in Daily Investment Hours Due to Change in Beliefs

	10% μ_{i,δ_2}	10% σ^2_{i,δ_2}
$\theta_{ref} = 18$	3.92	3.39
$\theta_{ref} = 20$	5.63	3.09
$\theta_{ref} = 22$	4.93	3.03
$\theta_{ref} = 24$	5.28	2.71

Note: This table shows the impact of increasing several belief variables on investment as predicted by the structural model and preference estimates for all four reference points. The first column shows the impact of an overall increase of 10% in individuals beliefs about the returns to investment. The second column shows the impact of an overall increase of 10% in individual uncertainty about the returns to investment.

crease in investment may seem counterintuitive, particularly from the perspective of an asset investment problem, where higher uncertainty and risk aversion leads to lower investment overall, or a shift towards safe assets. While we cannot rule out that parents are risk-lovers with respect to child investment, it seems unlikely that this is the case.

The reason that we see this positive effect of uncertainty in this context is twofold. First, the estimates from model indicate that parents have a stronger incentive to invest in children when they believe that their child human capital is below the reference point. Second, most parents tend to have low mean beliefs about their child future human capital. By increasing the uncertainty about the returns to investment, parents now more "at-risk" of their child falling below specific developmental thresholds. This in turn leads them to invest more in their child.

This effect can be seen in Table 13, where I break down the 10% increase in σ^2_{i,δ_2}. The first column shows the percentage of individuals for which the increase in uncertainty leads to an increase or decrease in investment. About 64.5% observe an increase in investment, while 34% reduce their investments, with the remaining 1.5% not changing. The second column shows the average change in investment in hours. The third and fourth column shows the average mean beliefs of the individuals that increase their investment. Finally, the last column shows the average baseline investment of the individuals that increase their investment.

TABLE 13. Breakdown of Change in Investment Due to a 10% Increase in σ^2_{i,δ_2} - $log(\theta_{ref}) = 18$

	% by Sign	Nominal Change (Hours)	μ_2	μ_1	Baseline $\hat{x}$
Increase in Investment	64.46	0.05	0.05	0.04	0.96
Decrease in Investment	34.00	-0.01	0.17	0.10	3.95
No Change in Investment	1.54	0.00	0.26	0.15	12.15

Note: This table breaks down the impact of increasing belief uncertainty about the returns to investment by 10% on investment as predicted by the structural model and preference estimates for $log(\theta_{ref}) = 18$. Results for other reference points can be found in the Appendix E. Each line shows baseline investment, mean beliefs, and the nominal change in hours conditional on whether the increase in uncertainty leads to an increase, decrease, or no change in investment.

The model predicts heterogeneous responses to increased uncertainty. Parents whose expected returns place them near or below the reference point increase investment to reduce the likelihood of loss realizations. By contrast, parents whose expected returns are comfortably above the reference point behave as standard risk-averse agents:

additional uncertainty lowers the expected utility of investment, leading them to reduce effort. Consistent with this mechanism, the 35% of parents who decrease investment tend to hold substantially higher mean beliefs about returns.

6. Conclusion

This paper develops a methodology that elicits both mean beliefs and belief uncertainty about the parameters of the skill production function in early childhood. The methodology and data collection procedure is motivated by a model of parental investment in children in which parents have do not have full knowledge of the skill production function. I elicit the belief distribution and allow for a flexible distribution of beliefs. I ask parents to report the youngest, most likely, and oldest ages a hypothetical child would learn to do some specific activities under different scenarios of initial skill and investment. I develop a simple estimation procedure that allows for correlation of measurement error across equations.

Additionally, I explore a new measure of parental subjective costs of investment in children. Recognizing that time investments represent opportunity costs for parents, I ask parents to report the amount of money they would be willing to pay to trade off investment and leisure. Together with the subjective belief distribution, I use this measure to estimate a model of parental investment in children which takes into account subjective beliefs.

I show that the collected data exhibit patterns that are consistent with the proposed theoretical model. Respondents report lower ages for the youngest, most likely, and oldest age of learning under the scenario of high investment and normal health, consistent with a model where children reach developmental milestones faster under more investment. They also report higher opportunity costs of investment under scenarios where they work more hours a day and have higher household income.

I estimate parental beliefs and find that parents in this sample have low mean beliefs about the returns to investment and uncertainty is relatively small. Nevertheless, individuals that have higher mean beliefs also tend to have lower subjective uncertainty. I also find that both mean beliefs and uncertainty correlate with actual time investment measures.

To illustrate the use of this data, I combine the literature on subjective beliefs and reference points and estimate a model of parental investment that incorporates these two aspects. I find that parents strongly value their child skill even when holding low mean beliefs. Moreover, they have a strong incentive to invest if their child is at risk of being at a developmental delay.

Counterfactual simulations show that an increase in uncertainty leads to an increase in investment. This seemingly counterintuitive result is due to the fact that those that increase their investment are ones that have low mean beliefs and low investment. The increase in uncertainty puts their child more "at-risk" of falling below developmental benchmarks, which induces an increase in investment.

In general, my findings indicate that belief heterogeneity in returns to investment is important in predicting actual time investment in children, and uncertainty about beliefs can be a relevant target for policy interventions. Moreover, the methodology developed in this paper is flexible enough to be transported to other contexts of child investments. Since I find that more pessimist parents tend to be more impacted by increases in uncertainty about their beliefs, it suggests that information interventions may have larger gains for less uncertain parents.

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Appendix A. Survey Instrument

A.1. Belief Elicitation - Instructions

Throughout this section, we will refer to different scenarios of health of the baby and hours of interaction the mother spends with the baby.

1) A "normal health" baby is one whose gestation lasted 9 months, weighed 8 pounds, and measured 20 inches at birth. A "poor health" baby is one whose gestation lasted 7 months, weighed 5 pounds, and measured 18 inches at birth.

2) A "high intensity" interaction is one in which the mothers spends 6 hours a day with the baby in active interaction, while a "low intensity" one the mother spends 2 hours a day with the baby in active interaction. These interactions includes activities such as:

- soothing the baby when he/she is upset;
- moving the baby's arms and legs around playfully;
- playing peek-a-boo with the baby;
- singing songs with the baby;
- speaking to the baby;
- feeding, nursing, bathing, attending to health needs;

We would like you to consider a hypothetical scenario involving a mother and her baby. In this scenario, the baby's health can either be good or poor, and the mother's interaction with the baby can be either high intensity or low intensity. After considering these factors, we would like you to determine the youngest, most likely, and oldest age (in months) at which the baby in this specific situation will learn to perform a certain activity.

To illustrate, let us consider the example of a baby with "normal health" and a "low intensity" interaction between mother and baby. In this case, we would like you to provide your personal belief on the youngest, most likely, and oldest ages at which this baby will learn to walk at least 5 steps by itself. To help you understand the youngest, most likely, and oldest age, we suggest imagining 10 identical babies, with some learning to perform the activity at different ages. In this way, the youngest age would be the earliest at which any of the babies learn the activity, the most likely age would be the age at which most of the babies learn the activity, and the oldest age would be the latest at which any of the babies learn the activity.

In total, there will be 16 questions of this nature, each pertaining to a different scenario and activity. While there are no right or wrong answers, we ask that you carefully consider each situation and activity before giving us your honest personal belief.

A.2. Stated Choice - Instructions

In this section, we will refer to the active interaction time a mother spends with her baby as hours of mother-child interaction. Here are some examples of activities a mom does during active time interactions:

- soothing the baby when he/she is upset;
- moving the baby's arms and legs around playfully;
- playing peek-a-boo with the baby;
- singing songs with the baby;
- speaking to the baby;
- feeding, nursing, bathing, attending to health needs;

What is important to highlight is that active interaction time is one where the main and sole focus of the mother is in the baby.

When the mother is at home but not in active interaction time, we call this leisure or passive interaction time. The mother can still be together with the baby, but the baby is not the main focus of the activity. Here are some examples of activities a mom does during passive time interactions:

- Grocery shopping with baby;
- Browsing social media apps on smartphone with baby at your side;
- Nap time for baby;
- Household chores (cleaning, cooking, etc) while baby is at your side;
- Exercising at home or at the gym;

- Watching TV;

In all these activities, although the mother may check on the baby every 5-10 minutes, she is not exclusively focused on the baby during these activities.

We will also refer to the health of the baby.

A "normal health" baby is one whose gestation lasted 9 months, the baby weighed 8 pounds and measured 20 inches at birth. A "poor health" baby is one whose gestation lasted 7 months, the baby weighed 5 pounds and measured 18 inches at birth.

In this section of the survey, we will ask you to imagine yourself in a new household, composed of you, a partner, and a hypothetical baby (that is, not one of your current children). We will present different situations of household income and health of the baby.

Then, we will ask you to imagine a situation where you want to spend 1 hour away from your baby after work every day for one month (not on weekends). For example, you may want to set a time for your personal rest, or you want to practice a hobby. You ask a friend to look after your baby for that hour. Your friend is a careful person who will ensure that your baby is well taken care of, but they will not engage in active interaction with your baby.

We will ask you to choose the highest hourly rate you would be willing to pay your friend during weekdays for one month under different working hour situations. We assume that there are 20 weekdays in the month. If you would not be willing to stay 1 hour away from the baby, please select $0 dollars.

For example, suppose that when you work 8 hours a day every day, the highest rate you would pay your friend is $10. This is equal to 1 hour times 20 weekdays in a month times $10/hour you are willing to pay, which is equal to $200 per month. The answer would look like this:

We know these questions are not easy to answer. Note that there is no right or wrong, good or bad, answer, we are just interested in what you personally think. Please try to consider each scenario carefully and tell us what you personally believe is the best option. We ask that you make an effort to thoughtfully answer all questions.

Appendix B. Data Details

This appendix provides additional details on the data cleaning procedures used in the analysis.

The survey was completed by 723 women between the ages of 18 and 40, all of whom had at least one child, with the oldest child no older than five years old. Qualtrics, the platform contracted to administer the online survey, aimed to match respondents to U.S. Census benchmarks for education and household income. However, due to limitations in their available panel and challenges in fulfilling the target quotas, the final sample ended up skewing toward individuals with higher educational attainment and household income.

The survey design did not require responses to any questions beyond the consent form and initial screening. Participants were free to skip any subsequent question. Following the methodology in Stantcheva (2023), we included a question on self-reported honesty. Twelve individuals indicated that they had not answered the survey truthfully and were therefore excluded from the sample. We further dropped 91 individuals who failed to provide responses to all required questions, leaving a total of 620 respondents.

Next, we identified cases of non-substantive responses, where individuals gave mechanically repetitive or non-informative answers, which we interpret as non-response behavior. For instance, in the belief elicitation questions asking for youngest, median, and oldest ages for milestone completion, some respondents entered values such as 0, 0, 0 or 20, 20, 20. Similarly, in the price sensitivity instrument, some individuals reported a value of zero across all scenarios. We also drop individuals that did not answer or skipped at least one scenario. Based on these criteria, we excluded 31 individuals due to uniform responses in the belief module and 82 individuals for the same issue in the price instrument, for a total of 113 additional removals. This step reduced the working sample to 507 respondents.

Finally, for model estimation, we applied additional consistency checks. We dropped individuals whose reported time allocations exceeded the feasible upper bound of 16 hours per day for either labor supply or child investment. We also calculated available household income using respondents' answers in the price module and excluded those with implied negative disposable income. These constraints led to the removal of 54 more respondents, resulting in a final sample of 453 individuals used in the structural estimation.

Appendix C. Seemingly Unrelated Random Coefficients Estimator

Let N denote the number of individuals, T the number of observations for each individual, and L the number of parameters to be estimated. We would like to estimate the following system:

$$y_i = Z_i\beta + (\varepsilon_i + Z_i\eta_i) = Z_i\beta + u_i,$$

First, for each $i = 1, \dots, N$, run an OLS regression to prepare for the FGLS Seemingly Unrelated regression (SUR) estimator:

$$\begin{aligned}
\hat{b}_{0,i} &= (Z_i'Z_i)^{-1}(Z_i'y_i),\\
\hat{\Omega}_{0,i} &= \hat{u}_{0,i}'\hat{u}_{0,i}/(T-L),\\
\hat{V}_{0,i} &= (Z_i'(\hat{\Omega}_{0,i}^{-1} \otimes I_T)Z_i)^{-1}.
\end{aligned}$$

Then, compute the feasible SUR estimates for each individual:

$$\begin{aligned}
\hat{b}_{1,i} &= (Z_i'(\hat{\Omega}_{0,i}^{-1} \otimes I_T)Z_i)^{-1}(Z_i'(\hat{\Omega}_{0,i}^{-1} \otimes I_T)\,y_i),\\
\hat{\Omega}_{1,i} &= \hat{u}_{1,i}'\hat{u}_{1,i}/(T-L),\\
\hat{V}_{1,i} &= (Z_i'(\hat{\Omega}_{1,i}^{-1} \otimes I_T)Z_i)^{-1}.
\end{aligned}$$

In the second step, we obtain the feasible estimates for the covariance matrices of the random coefficients and measurement error.[34] Denote $\bar{\hat{b}}_1 = \frac{1}{N}\sum_{i=1}^{N}\hat{b}_{1,i}$:

$$\begin{aligned}
\hat{\Delta} &= \frac{1}{N-1}\sum_{i=1}^{N}(\hat{b}_{1,i}\hat{b}_{1,i}') - N\bar{\hat{b}}_1\bar{\hat{b}}_1',\\
\hat{\Pi}_i &= Z_i\hat{\Delta}Z_i' + (I_T \otimes \hat{\Omega}_{1,i}),\\
\hat{V}_{1,i} &= (Z_i'(\hat{\Omega}_{1,i}^{-1} \otimes I_T)Z_i)^{-1}.
\end{aligned}$$

Then, the efficient SUR estimator is:

$$\begin{aligned}
\hat{\beta} &= (\sum_{i=1}^{N} Z_i'(\hat{\Pi})^{-1}Z_i)^{-1}(\sum_{i=1}^{N} Z_i'(\hat{\Pi})^{-1}\,y_i),\\
Var(\hat{\beta}) &= \frac{1}{N}\sum_{i=1}^{N}(\hat{\Delta} + \hat{V}_{1,i})^{-1}.
\end{aligned}$$

[34] Technically, the estimator for $\hat{\Delta}$ also includes the term $-\frac{1}{N}\sum_{i=1}^{N} V_i$. However, as pointed out by Swamy (1970), this often leads to a computationally nonpositive definite matrix, and this latter term is negligible in large samples.

To obtain an efficient estimator of the individual level paramater β_i, I adapt the estimator for Swamy (1970) found in Judge et al. (1988). Denote $A_i = (\hat{\Delta}^{-1} + \hat{V}_i^{-1})^{-1}\hat{\Delta}^{-1}$. Then:

$$\hat{\beta}_i = (\hat{\Delta}^{-1} + \hat{V}_i^{-1})^{-1}(\hat{\Delta}^{-1}\hat{\beta} + \hat{V}_i^{-1}\hat{b}_{1,i}),$$
$$Var(\hat{\beta}_i) = Var(\hat{\beta}) + (I_L - A_i)(\hat{V}_i^{-1} - Var(\hat{\beta}))(I_L - A_i)'.$$

Appendix D. Robustness of Distributional Assumptions

In this section I present estimates for the parameters of the subjective production function using different distributional assumptions for respondent's answers. The main results of the paper assumes that respondent's answers follow a Triangular distribution, where the oldest and youngest ages reported are the extremes of the triangular distribution, and the most likely answer corresponds to the mode, which is the peak of the triangle.

This kind of elicitation procedure is also known as "three-point estimation." One commonly used distribution is the PERT distribution, which is a transformation of the more general Beta distribution. In practice, it resembles a smoothed triangular distribution. From the minimum a, most likely, b, and maximum values c, the mean of this distribution is given by $\mu = \frac{a+4b+c}{6}$, its median by $Med = \frac{a+6b+c}{8}$, and variance by $\frac{(\mu-a)(c-\mu)}{7}$.

Delavande, Giné, and McKenzie (2011b) and Delavande, Giné, and McKenzie (2011a) show that in the context of some developing countries (namely, India and Tonga), respondents do not interpret minimum and maximum questions as representing 0 and 100 chance probabilities of the cumulative distribution function. They show that most respondents interpret these bounds as representing anywhere between the 90th and 95th percentiles for the maximum. As such, I also estimate the parameters but instead assuming that the youngest and oldest answers represent either the 1st and 99th, 5th and 95th, and 10th and 90th percentiles. This involves solving a simple system of equations I solve $a = F(x_{n\text{–th}}; a, b, c)$; $c = F(x_{(100-n)\text{–th}}; a, b, c)$, where $x_{(n\text{–th}}$ and $x_{(100-n)\text{–th}}$ are the percentile values I want to find and $F(; a, b, c)$ is the cdf of the triangular distribution with parameters a, b, c.

Table A1 presents the results for these different assumption, including the baseline assumption of a triangular distribution. The parameters of the mean subjective production function are stable for all distributions. However, the variance estimates are somewhat sensitive to distributional assumptions, particularly the ones that extend the distribution beyond the youngest and oldest values. This is not surprising, since the outcome variable $Var(\ln \theta_{i,1}|\Omega_i)$ is based off the IQR of the distribution of answers. Therefore, the distributions that extend participant answers also inflate IQR estimates. The estimates for $\sigma^2_{\delta_2}$ using the 10th and 90th percentiles are 3 times larger than the baseline assumption.

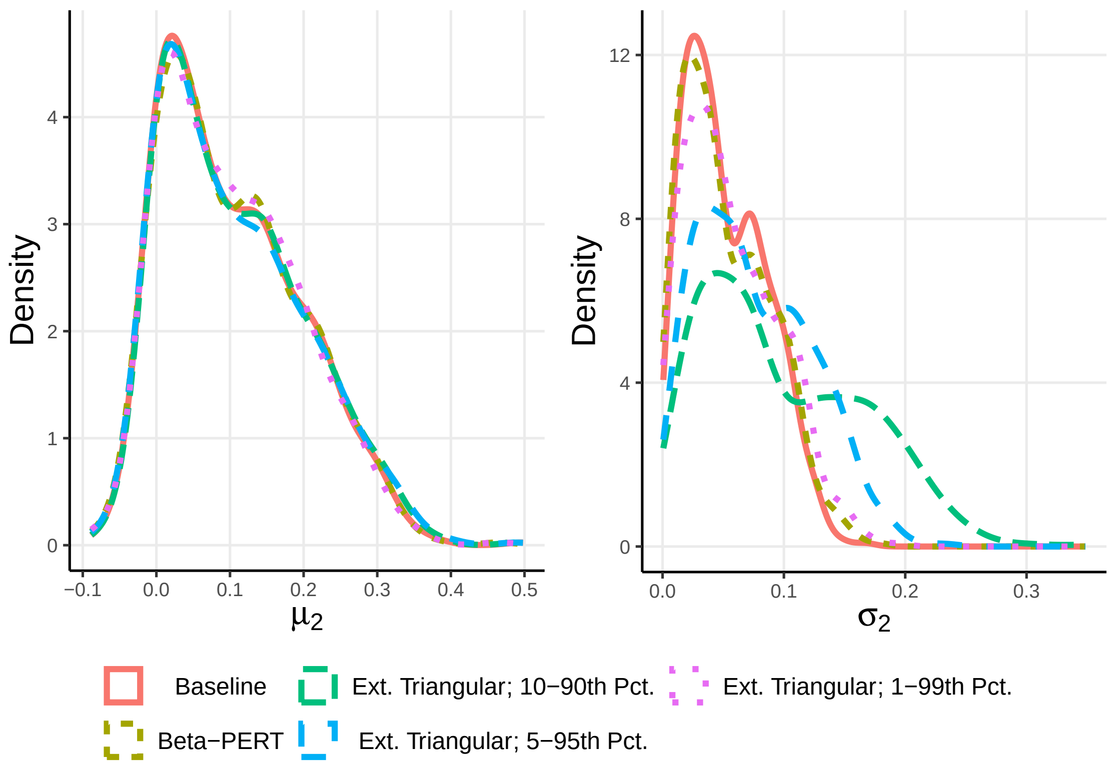
This discrepancy can be seen in Figure A1. The distribution of μ_{i,δ_2} does not vary much by assumption, while the distribution of σ_{i,δ_2} becomes more spread out the larger the extension of the extreme values.

TABLE A1. Estimates of Mean Subjective Production Function Parameters - Robustness

	Baseline	PERT	Triag. 1–99th	Triag. 5–95th	Triag. 10–90th
μ_{δ_0}	2.897***	2.892***	2.889***	2.893***	2.891***
	(0.0285)	(0.0286)	(0.0282)	(0.0291)	(0.0286)
μ_{δ_1}	0.064***	0.063***	0.064***	0.064***	0.064***
	(0.0045)	(0.0046)	(0.0045)	(0.0046)	(0.0045)
μ_{δ_2}	0.103***	0.103***	0.102***	0.105***	0.106***
	(0.0084)	(0.0088)	(0.0086)	(0.0087)	(0.0085)
σ^2_0	0.031***	0.031***	0.030***	0.063***	0.090***
	(0.0023)	(0.0023)	(0.0027)	(0.0045)	(0.0063)
$\sigma^2_{\delta_1}$	0.001**	0.001**	0.001**	0.002***	0.004***
	(0.0002)	(0.0002)	(0.0002)	(0.0003)	(0.0006)
$\sigma^2_{\delta_2}$	0.004***	0.004***	0.004***	0.007***	0.013***
	(0.0006)	(0.0007)	(0.0008)	(0.0011)	(0.0020)
$\sigma_{\delta_1,\delta_2}$	−0.002**	−0.002**	−0.003***	−0.004***	−0.007***
	(0.0006)	(0.0007)	(0.0008)	(0.0011)	(0.0019)

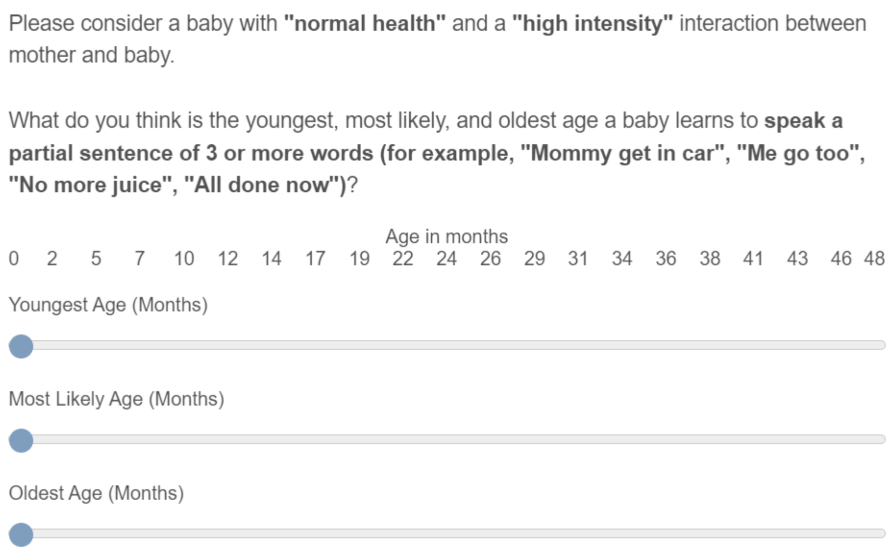
Note: Robust standard errors in parenthesis. *p<0.1; **p<0.05; ***p<0.01. This table shows aggregate estimates of equations (6) and (7) with different distributional assumptions about answers from youngest, most likely, and oldest ages. All distributions use the most likely answer as the mode. Baseline and PERT uses the youngest and oldest as the extremes of a triangular distribution. The last three columns assumes that the youngest and oldest matches specific percentiles of a triangular distribution.

FIGURE A1. Distribution of Estimates of μ_{i,δ_2} and σ_{i,δ_2}



Note: This figures shows the distribution of estimates of μ_{i,δ_2} and σ_{i,δ_2} with different distributional assumptions about answers from youngest, most likely, and oldest ages. All distributions use the most likely answer as the mode. Baseline and PERT uses the youngest and oldest as the extremes of a triangular distribution. The last three columns assumes that the youngest and oldest matches specific percentiles of a triangular distribution.

FIGURE A2. Subjective Belief Instrument



Please consider a baby with **"normal health"** and a **"high intensity"** interaction between mother and baby.

What do you think is the youngest, most likely, and oldest age a baby learns to **speak a partial sentence of 3 or more words (for example, "Mommy get in car", "Me go too", "No more juice", "All done now")**?

Age in months

0 2 5 7 10 12 14 17 19 22 24 26 29 31 34 36 38 41 43 46 48

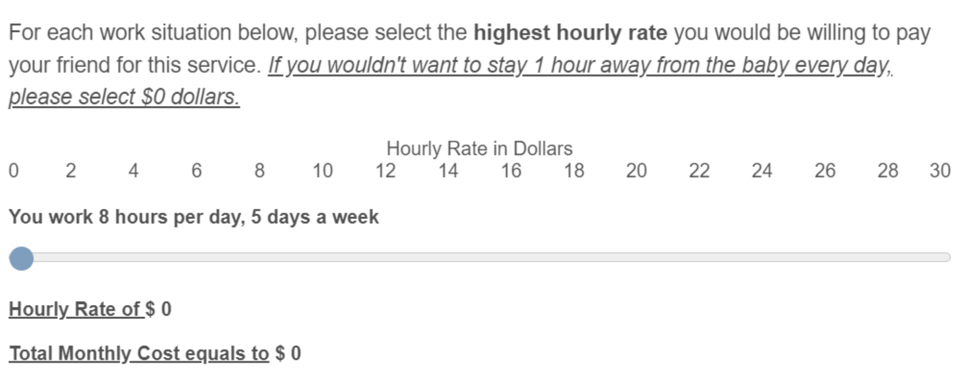
Youngest Age (Months)

Most Likely Age (Months)

Oldest Age (Months)

Note: This figure shows the main survey instrument presented to respondents. It starts by outlining the hypothetical scenario to be considered, followed by the belief question. Highlighted in bold are the hypothetical scenario values and MSD activities. Respondents are restricted to give ascending order answers from youngest, most likely, and oldest ages.

FIGURE A3. Stated Choice Instrument - Willingness to Pay



For each work situation below, please select the **highest hourly rate** you would be willing to pay your friend for this service. *If you wouldn't want to stay 1 hour away from the baby every day, please select $0 dollars.*

Hourly Rate in Dollars

0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30

You work 8 hours per day, 5 days a week

Hourly Rate of $ 0

Total Monthly Cost equals to $ 0

Appendix E. Additional Tables and Figures

FIGURE A4. Distribution of Ages by Scenario for "Count 3 Objects Correctly"

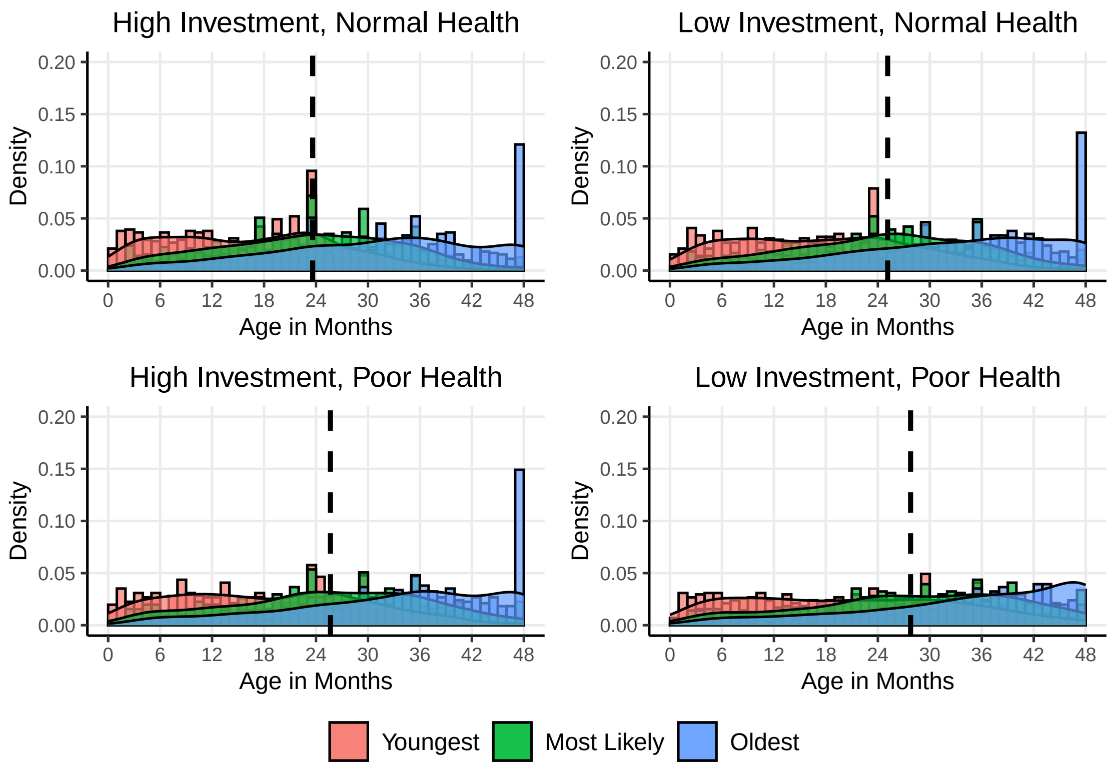


Figure A5. Distribution of Ages by Scenario for "Say First and Last Name Together"

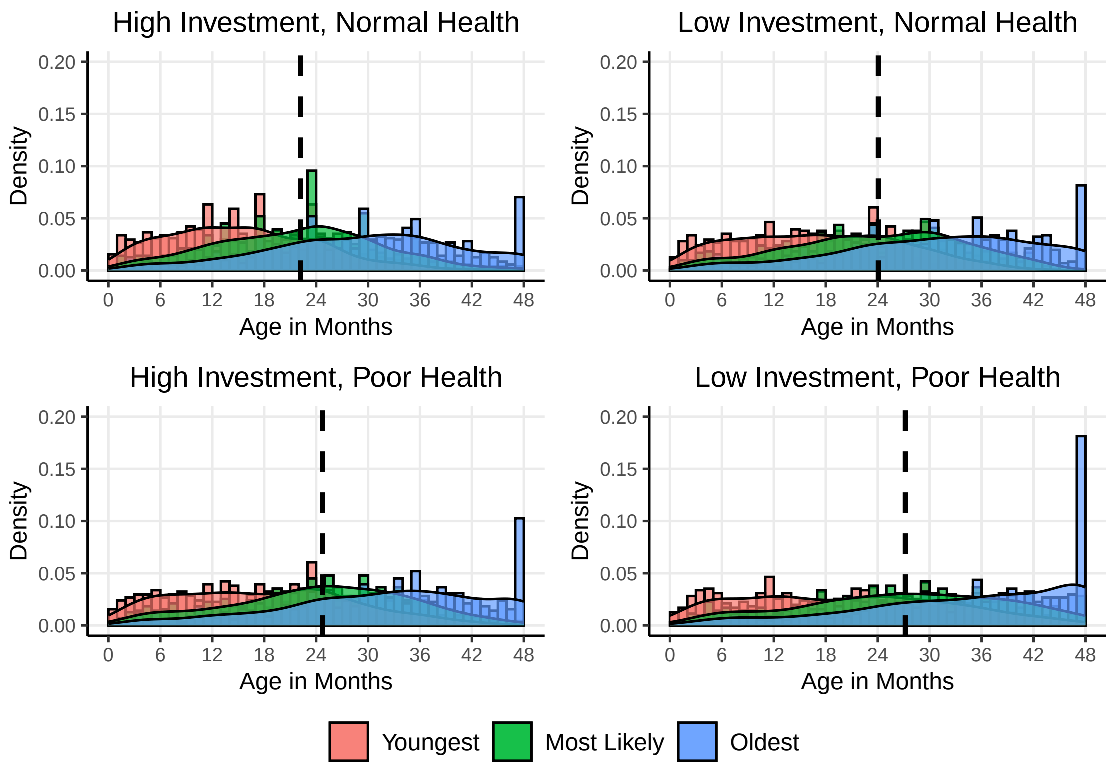


FIGURE A6. Distribution of Ages by Scenario for "Know Own Age and Sex"

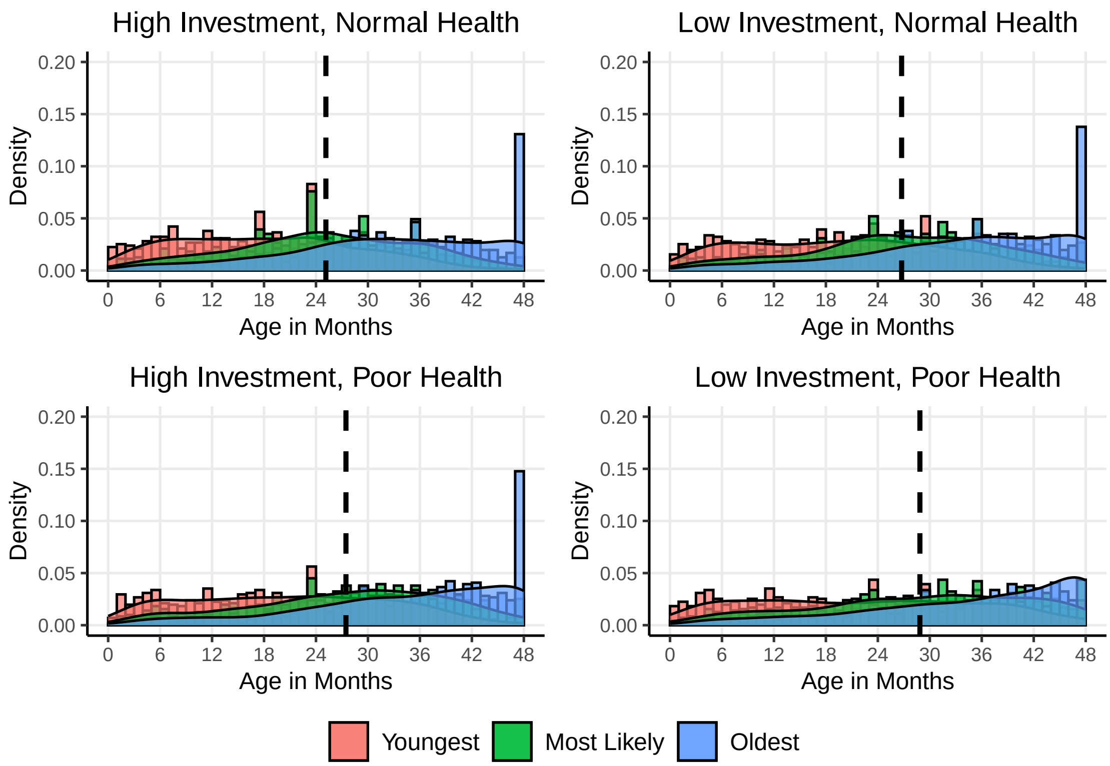


TABLE A2. Descriptive Statistics - Willingness-to-Pay

	Conditional on Good Health					
	Household Income					
	$2,000		$4,000		$8,000	
	Mean	SD	Mean	SD	Mean	SD
Work 0 Hours	8.32	7.58	9.50	7.91	10.86	8.26
Work 4 Hours	11.29	6.90	12.41	6.51	13.47	6.90
Work 8 Hours	12.51	6.73	14.27	6.51	15.36	7.25
	Conditional on Poor Health					
	Household Income					
	$2,000		$4,000		$8,000	
	Mean	SD	Mean	SD	Mean	SD
Work 0 Hours	9.12	8.44	9.74	8.30	11.22	8.95
Work 4 Hours	11.42	7.74	12.34	7.62	13.55	8.05
Work 8 Hours	12.58	8.13	13.56	7.95	15.05	8.69

Note: This table presents the mean and standard deviation of the maximum willingness-to-pay for the each scenario of health, household income, and daily working hours.

TABLE A3. Correlations of Willingness to Pay and Scenarios

	(1)	(2)	(3)	(4)
Hours of Work	0.52***			0.52***
	(0.03)			(0.03)
Household Income (in $ thousands)		0.61***		0.61***
		(0.04)		(0.04)
Baby Health			-0.06	-0.06
			(0.19)	(0.19)
Constant	10.18***	9.85***	12.30***	7.79***
	(0.27)	(0.28)	(0.27)	(0.35)
Observations	10,944	10,944	10,944	10,944

Note: Clustered standard errors at the individual level in parenthesis. *p<0.1; **p<0.05; ***p<0.01. The four columns show the correlation between willingness to pay and scenario variables. Hours of Work can be 0, 4, or 8. Household Income can be $2,000, $4,000, or $6,000. Baby Health can be good or poor.

TABLE A4. Descriptive Statistics Investment

	Work 8 Hours		Work 4 Hours		Work 0 Hours	
	Good Health, Household Income of $2,000					
	Mean	SD	Mean	SD	Mean	SD
Willingness to Pay ($)	12.40	6.72	11.10	6.89	8.14	7.51
	Good Health, Household Income of $4,000					
	Mean	SD	Mean	SD	Mean	SD
Willingness to Pay ($)	14.15	6.51	12.29	6.56	9.32	7.91
	Good Health, Household Income of $6,000					
	Mean	SD	Mean	SD	Mean	SD
Willingness to Pay ($)	15.28	7.24	13.30	6.90	10.55	8.16
	Poor Health, Household Income of $2,000					
	Mean	SD	Mean	SD	Mean	SD
Willingness to Pay ($)	12.42	8.14	11.24	7.73	8.87	8.39
	Poor Health, Household Income of $4,000					
	Mean	SD	Mean	SD	Mean	SD
Willingness to Pay ($)	13.40	8.00	12.18	7.63	9.51	8.31
	Poor Health, Household Income of $6,000					
	Mean	SD	Mean	SD	Mean	SD
Willingness to Pay ($)	14.92	8.74	13.47	8.12	11.03	8.98

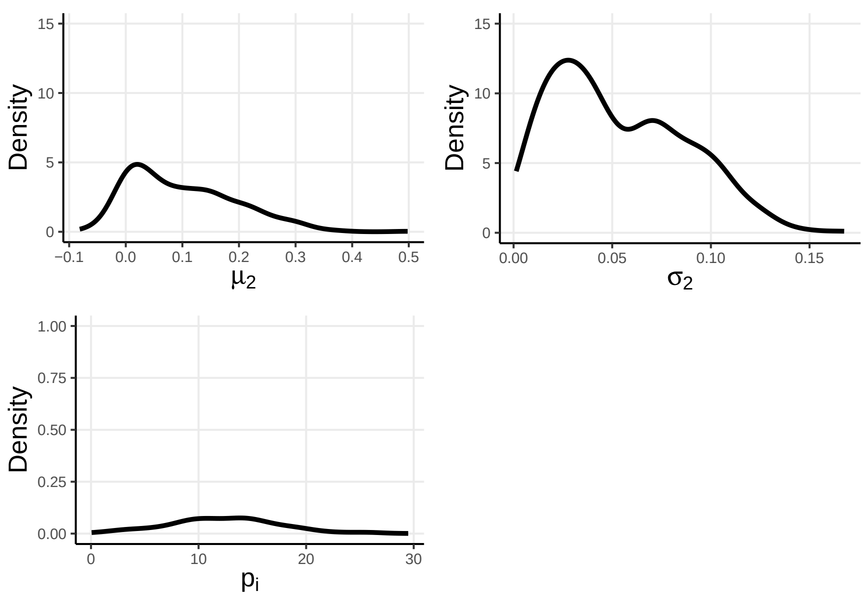
This table shows

TABLE A5. Estimates of Willingness to Pay System

Panel B: Aggregate Estimates	
p_i	12.238***
	(0.225)
Work Hours	1.663***
	(0.108)
Income	1.007***
	(0.079)
θ_0	−0.039
	(0.101)
Panel B: Individual Estimates	
Parameter	% Significant
p_i	99.34%

Note: Robust standard errors in parenthesis. *p<0.1; **p<0.05; ***p<0.01. This table shows aggregate estimates of the willingness to pay equations 8 and the individual-level predicted estimates of p_i. The percentage of significant estimates is calculated using a 10% significance level, and the null hypothesis is that the coefficient is equal to zero.

FIGURE A7. Distribution of Individual Level Coefficients



Note: This figure displays the distribution of the efficient individual level estimates of the subjective expected returns to investment μ_{i,δ_2} and subjective uncertainty of returns to investment σ_{i,δ_2}.

TABLE A6. Breakdown of Change in Investment Due to a 10% Increase in σ^2_{i,δ_2} - $log(\theta_{ref}) = 20$

	% by Sign	Nominal Change (Hours)	μ_2	μ_1	Baseline $\hat{x}$
Increase in Investment	62.25	0.05	0.05	0.03	0.96
Decrease in Investment	35.32	0.00	0.17	0.10	3.34
No Change in Investment	2.43	0.00	0.27	0.15	11.61

Note: This table breaks down the impact of increasing belief uncertainty about the returns to investment by 10% on investment as predicted by the structural model and preference estimates for $log(\theta_{ref}) = 20$. Each line shows baseline investment, mean beliefs, and the nominal change in hours conditional on whether the increase in uncertainty leads to an increase, decrease, or no change in investment.

TABLE A7. Breakdown of Change in Investment Due to a 10% Increase in σ^2_{i,δ_2} - $log(\theta_{ref}) = 22$

	% by Sign	Nominal Change (Hours)	μ_2	μ_1	Baseline $\hat{x}$
Increase in Investment	62.25	0.05	0.05	0.04	0.83
Decrease in Investment	34.66	0.00	0.16	0.10	3.31
No Change in Investment	3.09	0.00	0.25	0.15	10.78

Note: This table breaks down the impact of increasing belief uncertainty about the returns to investment by 10% on investment as predicted by the structural model and preference estimates for $log(\theta_{ref}) = 22$. Each line shows baseline investment, mean beliefs, and the nominal change in hours conditional on whether the increase in uncertainty leads to an increase, decrease, or no change in investment.

TABLE A8. Breakdown of Change in Investment Due to a 10% Increase in σ^2_{i,δ_2} - $log(\theta_{ref}) = 24$

	% by Sign	Nominal Change (Hours)	μ_2	μ_1	Baseline $\hat{x}$
Increase in Investment	60.49	0.05	0.05	0.04	0.88
Decrease in Investment	34.88	0.00	0.16	0.09	3.27
No Change in Investment	4.64	0.00	0.22	0.13	10.55

Note: This table breaks down the impact of increasing belief uncertainty about the returns to investment by 10% on investment as predicted by the structural model and preference estimates for $log(\theta_{ref}) = 24$. Each line shows baseline investment, mean beliefs, and the nominal change in hours conditional on whether the increase in uncertainty leads to an increase, decrease, or no change in investment.