PROBLEM I

4. Setting a very small tolerance (below machine precision) may lead to excessive iterations without meaningful improvement in accuracy due to floating-point limitations. The results might also not converge to an approximation.

PROBLEM III

A1. The solver would create problems if J(x,y) is near-singular (determinant close to zero) which would make the inversion step in Newton's method become unstable. This happens if y=0, which makes the first row of the Jacobian matrix [0, x], causing divergence. Also, poor initial guesses far from the solutions could lead the method to diverge or get stuck in oscillations. Finally, if initial guesses are close to each other, the method may converge to the same solution multiple times.

A2. The requirements for convergence in this methods are: Contraction mapping (The functions g_1 and g_2 should be contraction mappings in the neighborhood of the solution, i.e. $|g'(x)|<1$), good initial guesses and correct sign chosen for the $g_2(x,y)$ function.

For the various roots, the Newton method generally has faster convergence when the initial guess is close to the solution, as it uses the Jacobian to more directly navigate towards the root. The fixed-point method is generally slower and more prone to divergence if the initial guess is not close to the solution or if the contraction condition isn't met.

When starting from (x,y)=(2,1), the function does not converge. This point is far from the solutions, and the initial value for y leads g_1 and g_2 to give values that make the system diverge or result in complex numbers.