PROBLEM 1

$$y(x) = y_0 + y_0'(x - x_0) + \frac{1}{2}y_0''(x - x_0)^2 + \frac{1}{6}y_0^{(3)}(x - x_0)^3 + \frac{1}{24}y_0^{(4)}(x - x_0)^4 + O(x^5)$$

$$y_1 = y(x_0 + h) = y_0 + y_0'h + \frac{1}{2}y_0''h^2 + \frac{1}{6}y_0^{(3)}h^3 + \frac{1}{24}y_0^{(4)}h^4 + O(h^5)$$

$$y_{-1} = y(x_0 - h) = y_0 - y_0'h + \frac{1}{2}y_0''h^2 - \frac{1}{6}y_0^{(3)}h^3 + \frac{1}{24}y_0^{(4)}h^4 + O(h^5)$$

$$\left.\begin{aligned} y_1 &= y_0 + y_0'h + \frac{1}{2}y_0''h^2 + \frac{1}{6}y_0^{(3)}h^3 + \frac{1}{24}y_0^{(4)}h^4 + O(h^5) \\ y_{-1} &= y_0 - y_0'h + \frac{1}{2}y_0''h^2 - \frac{1}{6}y_0^{(3)}h^3 + \frac{1}{24}y_0^{(4)}h^4 + O(h^5) \end{aligned}\right\}$$

$$y_1 + y_{-1} = 2y_0 + y_0''h^2 + \frac{1}{12}y_0^{(4)}h^4 + O(h^6)$$

$$y_0'' = \frac{y_1 - 2y_0 + y_{-1}}{h^2} - \frac{1}{12}y_0^{(4)}h^2 + O(h^4)$$

PROBLEM 2

$$y(x) = y_0 + y_0'(x - x_0) + \frac{1}{2}y_0''(x - x_0)^2 + \frac{1}{6}y_0^{(3)}(x - x_0)^3 + \frac{1}{24}y_0^{(4)}(x - x_0)^4 + O(x^5)$$

$$\begin{aligned} y_2 = y(x_0 + 2h) &= y_0 + y_0'\,2h + \frac{y_0''(2h)^2}{2} + \frac{y_0^{(3)}(2h)^3}{6} + \frac{y_0^{(4)}(2h)^4}{24} + \frac{y_0^{(5)}(2h)^5}{120} + \frac{y_0^{(6)}(2h)^6}{720} + O(h^7) \\ &= y_0 + 2y_0'h + 2y_0''h^2 + \frac{4}{3}y_0^{(3)}h^3 + \frac{2}{3}y_0^{(4)}h^4 + \frac{4}{15}y_0^{(5)}h^5 + \frac{4}{45}y_0^{(6)}h^6 + O(h^7) \end{aligned}$$

$$\begin{aligned} y_{-2} = y(x_0 - 2h) &= y_0 - y_0'\,2h + \frac{y_0''(2h)^2}{2} - \frac{y_0^{(3)}(2h)^3}{6} + \frac{y_0^{(4)}(2h)^4}{24} - \frac{y_0^{(5)}(2h)^5}{120} + \frac{y_0^{(6)}(2h)^6}{720} + O(h^7) \\ &= y_0 - 2y_0'h + 2y_0''h^2 - \frac{4}{3}y_0^{(3)}h^3 + \frac{2}{3}y_0^{(4)}h^4 - \frac{4}{15}y_0^{(5)}h^5 + \frac{4}{45}y_0^{(6)}h^6 + O(h^7) \end{aligned}$$

$$\left.\begin{aligned} y_2 + y_{-2} &= 2y_0 + 4y_0''h^2 + \frac{4}{3}y_0^{(4)}h^4 + \frac{8}{45}y_0^{(6)}h^6 + O(h^8) \\ y_1 + y_{-1} &= 2y_0 + y_0''h^2 + \frac{1}{12}y_0^{(4)}h^4 + \frac{1}{360}y_0^{(6)}h^6 + O(h^8) \end{aligned}\right\}$$

$$16(y_1 + y_{-1}) - (y_2 + y_{-2}) = 30y_0 + 12y_0''h^2 - \frac{2}{15}y_0^{(6)}h^6 + O(h^8)$$

$$y_0'' = \frac{-y_2 + 16y_1 - 30y_0 + 16y_{-1} - y_{-2}}{12h^2} - \frac{1}{90}y_0^{(6)}h^4 + O(h^6)$$