PROBLEM 4

The numerical result is exact (up to rounding errors) because the Newton-Cotes 2nd-order integration method (Simpson's Rule) is exact for polynomials of degree 3 or lower.

This is because Simpson's Rule is derived from integrating a 2nd-degree (quadratic) Lagrange interpolating polynomial. However, it can integrate polynomials of degree up to 3 exactly because higher-order terms in the error cancel out. The error term for Simpson's Rule depends on the fourth derivative of the function.

For any polynomial of degree 3 or less, the fourth derivative is zero everywhere, meaning the theoretical error is exactly zero and any deviation in the result is due to numerical precision (rounding errors).

PROBLEM 5

The error given by both methods for f3 is clearly larger than the ones for f1 and f2. Numerical integration methods perform best on smooth, continuous functions like f1 and f2. For f3, the discontinuity at x=0 leads to reduced convergence rates. The error is dominated by the lack of smoothness, especially for Gauss-Legendre methods.

Simpson's 3/8 Rule is a fixed-order method and relies on evenly spaced points, which may not adapt well to functions with varying complexity, while Gauss-Legendre quadrature adapts better to smooth functions but struggles near discontinuities.

For very small h (large n), round-off errors due to finite machine precision can dominate, causing the convergence rates to flatten out or deviate from the expected rates.

For large h (coarser resolution), the error is larger, but as h decreases, the error reduces steadily. This behavior is consistent with the expected convergence rate for Simpson's 3/8 Rule, which is 4th order for smooth functions.

The orange line is nearly flat, showing a constant error regardless of the step size h. This suggests that the method reaches a point where the error does not reduce further with more precision.

For discontinuous functions adaptive methods dynamically adjust the step size h to place more points where the integrand changes rapidly.

For highly oscillatory functions standard methods struggle due to cancellations and require many points to achieve accuracy. In this case Filon’s method would be more efficient.