PROBLEM 1

$$I_h = \int_{x_0}^{x_2} f(x)dx \approx \frac{h}{3}(f_0 + 4f_1 + f_2) - \frac{h^5}{90} f^{(4)}(\xi_1)$$

$$I_{\frac{h}{2}} = \frac{h}{6}(f_0 + 4f_{0.5} + f_1) + \frac{h}{6}(f_1 + 4f_{1.5} + f_2) - \frac{\left(\frac{h}{2}\right)^5}{90} f^{(4)}(\xi)$$
$$= \frac{h}{6}(f_0 + 4f_{0.5} + 2f_1 + 4f_{1.5} + f_2) - \frac{h^5}{2880} f^{(4)}(\xi)$$

$$I_{improved} = \frac{16I_{h/2} - I_h}{15} = \frac{h(7f_0 + 32f_{0.5} + 12f_1 + 32f_{1.5} + 7f_2)}{45} - \frac{h^5}{2700} f^{(4)}(\xi_1)$$
$$\Rightarrow [h \to 2h] \Rightarrow \frac{2h(7f_0 + 32f_1 + 12f_2 + 32f_3 + 7f_4)}{45} - \frac{8h^5}{675} f^{(4)}(\xi_1)$$

The most similar formula is:

$$\int_{x_0}^{x_4} f(x)dx \quad = \quad \frac{2h}{45}(7f_0 + 32f_1 + 12f_2 + 32f_3 + 7f_4) - \frac{8h^7}{945} f^{(6)}(\xi_1)$$