PROBLEM 1a

$$P[3,4](x) = \frac{a_0 + a_1x + a_2x^2 + a_3x^3}{1 + b_1x + b_2x^2 + b_3x^3 + b_4x^4}$$

$$(a_0 + a_1x + a_2x^2 + a_3x^3)(1 + b_1x + b_2x^2 + b_3x^3 + b_4x^4) = e^x$$

$$\begin{aligned} a_0 + (a_0b_1 + a_1)x + (a_0b_2 + a_1b_1 + a_2)x^2 + (a_0b_3 + a_1b_2 + a_2b_1 + a_3)x^3 \\ + (a_0b_4 + a_1b_3 + a_2b_2 + a_3b_1)x^4 + (a_1b_4 + a_2b_3 + a_3b_2)x^5 \\ + (a_2b_4 + a_3b_3)x^6 + a_3b_4x^7 \\ = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 + \frac{1}{5040}x^7 \end{aligned}$$

$$\left.\begin{array}{c} a_0 = 1 \\ a_0b_1 + a_1 = 1 \\ a_0b_2 + a_1b_1 + a_2 = \frac{1}{2} \\ a_0b_3 + a_1b_2 + a_2b_1 + a_3 = \frac{1}{6} \\ a_0b_4 + a_1b_3 + a_2b_2 + a_3b_1 = \frac{1}{24} \\ a_1b_4 + a_2b_3 + a_3b_2 = \frac{1}{120} \\ a_2b_4 + a_3b_3 = \frac{1}{720} \\ a_3b_4 = \frac{1}{5040} \end{array}\right\} \Rightarrow \left\{\begin{array}{c} a_0 = 1 \\ a_1 = \frac{3}{7} \\ a_2 = \frac{1}{14} \\ a_3 = \frac{1}{210} \\ b_1 = \frac{-4}{7} \\ b_2 = \frac{1}{7} \\ b_3 = \frac{-2}{105} \\ b_4 = \frac{1}{840} \end{array}\right.$$

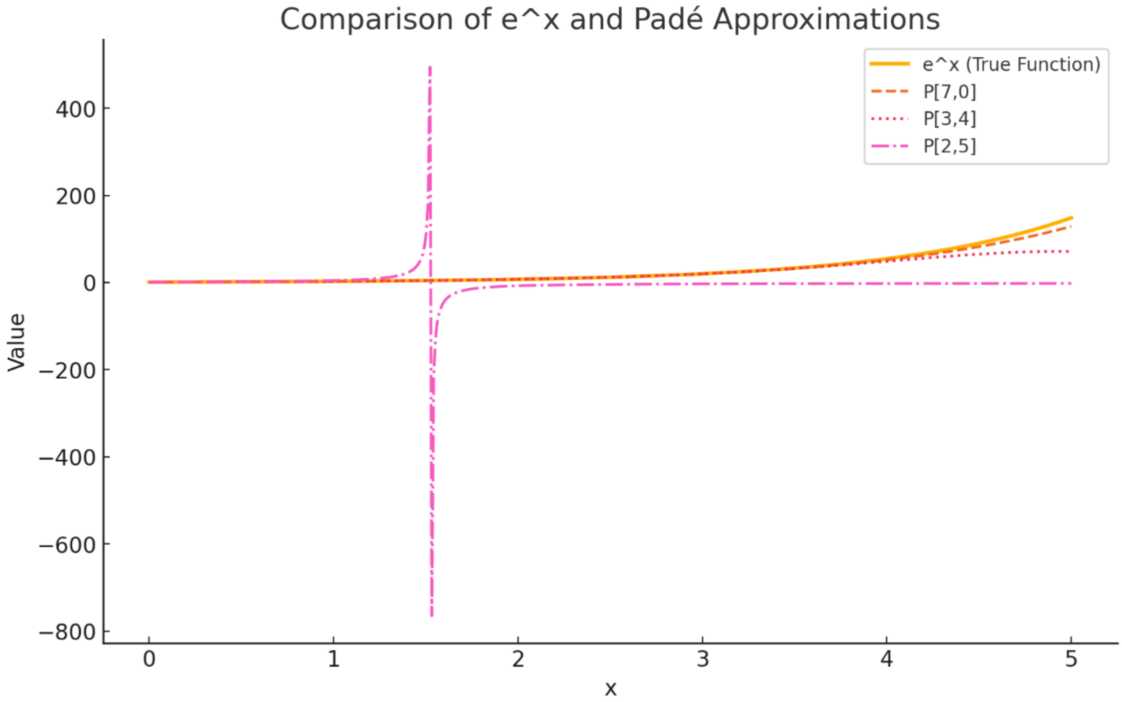
$$P[2,5](x) = \frac{a_0 + a_1x + a_2x^2}{1 + b_1x + b_2x^2 + b_3x^3 + b_4x^4 + b_5x^5}$$

$$(a_0 + a_1x + a_2x^2)(1 + b_1x + b_2x^2 + b_3x^3 + b_4x^4 + b_5x^5) = e^x$$

$$\begin{aligned} a_0 + (a_0b_1 + a_1)x + (a_0b_2 + a_1b_1 + a_2)x^2 + (a_0b_3 + a_1b_2 + a_2b_1)x^3 \\ + (a_0b_4 + a_1b_3 + a_2b_2)x^4 + (a_0b_5 + a_1b_4 + a_2b_3)x^5 + (a_1b_5 + a_2b_4)x^6 \\ + a_2b_5x^7 = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 + \frac{1}{5040}x^7 \end{aligned}$$

$$\left.\begin{array}{c} a_0 = 1 \\ a_0b_1 + a_1 = 1 \\ a_0b_2 + a_1b_1 + a_2 = \frac{1}{2} \\ a_0b_3 + a_1b_2 + a_2b_1 = \frac{1}{6} \\ a_0b_4 + a_1b_3 + a_2b_2 = \frac{1}{24} \\ a_0b_5 + a_1b_4 + a_2b_3 = \frac{1}{120} \\ a_1b_5 + a_2b_4 = \frac{1}{720} \\ a_2b_5 = \frac{1}{5040} \end{array}\right\} \Rightarrow \left\{\begin{array}{c} a_0 = 1 \\ a_1 = \frac{2}{7} \\ a_2 = \frac{1}{42} \\ b_1 = \frac{-5}{7} \\ b_2 = \frac{5}{21} \\ b_3 = \frac{-1}{21} \\ b_4 = \frac{1}{168} \\ b_5 = \frac{-1}{2520} \end{array}\right.$$

x	Error P[0,7]	Error P[3,4]	Error P[2,5]
0.5	1.0254536642e-07	4.9025365989e-09	0.27156995830143
1	2.7860205077e-05	2.25856657154e-06	1.93062858316323
2	0.00810371797827	0.00195932473710	14.8890560989306
5	19.79411148352898	77.0285437179610	150.594977284395



PROBLEM 2

k	x_k	$f(x_k)$	$f[x_k, x_{k+1}]$	$f[x_k, x_{k+1}, x_{k+2}]$	$f[x_k, x_{k+1}, x_{k+2}, x_{k+3}]$
0	4	1	1	3/8	1/12
1	6	3	5/2	7/8	
2	8	8	6		
3	10	20			

$$
\begin{aligned}
P(x) = f(x_0) + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1) \\
+ f[x_0, x_1, x_2, x_3](x - x_0)(x - x_1)(x - x_2) \\
= 1 + (x - 4) + \frac{3}{8}(x - 4)(x - 6) + \frac{1}{12}(x - 4)(x - 6)(x - 8) \\
= \frac{1}{24}(2x^3 - 27x^2 + 142x + 240)
\end{aligned}
$$