Part 2 Explanation

Marcos Pasa

1 The problem

Hereafter item and item worry level will be used interchangeably. When a monkey process an item it does the fallowing things:

1. Updates the item by applying an operation on it.

2. Check if the item is divisible by some number and pass this item to another monkey.

The problem with this is that the item will increase a lot. To solve it, we need and way of using an equivalent thing to the item that doesn't increase to much.

2 Solution

Consider there is n monkeys, each monkey has a number to use in the divisibility test, let d_i be this number for the i-th monkey.

An item can be written in n different ways as some multiple of d_i plus a remainder. Because items are updated by some operation, after an item was updated $j-1$ times, it's worry level written relative to d_i is

$$k_{ij} \cdot d_i + r_{ij}$$

The operation used to update an item can be of two types: multiply by some number or add by some number. Denoting this operation by T

$$T(x) = \begin{cases} x \cdot a & , \ a \in R \\ x + b & , \ b \in R \end{cases}$$

Therefore, when an item, with initial worry level x, is being processed for the first time by the i-th monkey, we need to calculate the fallowing

$$r_{i2} = T(x) \bmod d_i = T(k_{i1} \cdot d_i + r_{i1}) \bmod d_i$$

Where it was used the form of x relative to d_i. So we have two cases

1. First type of T

   $$[(k_{i1} \cdot d_i + r_{i1}) \cdot a] \bmod d_i = (ak_{i1}d_i + ar_{i1}) \bmod d_i = ar_{i1} \bmod d_i = T(r_{i1}) \bmod d_i$$

2. Second type of T

   $$(k_{i1} \cdot d_i + r_{i1} + b) \bmod d_i = (r_{i1} + b) \bmod d_i = T(r_{i1}) \bmod d_i$$

Therefore, the expression to be calculated is independent of the type of T. Furthermore, we don't need x, just it's remainder by d_i (r_{i1}).

Now let's consider when an item, with first worry level x, is processed by the second time by the j-th monkey. In this case we need to calculate

$$r_{j3} = T_2(T_1(x)) \bmod d_j$$

If we write $T_1(x)$ as some multiple of d_j plus the remainder, we get the same problem as before, so if $T_1(x) = k_{j2} \cdot d_j + r_{j2}$

$$r_{j3} = T_2(r_{j2}) \bmod d_j$$

So we only need to know the remainder of $T_1(x)$ by d_j (r_{j2}), but again, this is the same problem as before, because if $x = k_{j1} \cdot d_j + r_{j1}$

$$r_{j2} = T_1(k_{j1} \cdot d_j + r_{j1}) \bmod d_j = T_1(r_{j1}) \bmod d_j$$

To sumarize

$$\begin{aligned} r_{j1} &= x \bmod d_j \\ r_{j2} &= T_1(r_{j1}) \bmod d_j \\ r_{j3} &= T_2(r_{j2}) \bmod d_j \end{aligned}$$

Therefore, if we represent every item, with initial worry level x, by a vector r which i-th element is the remainder of x by d_i and then update this vector every time this item is processed by the formula

$$r_{new} = T(r_{old}) \bmod d$$

where d is a vector whose i-th element is d_i, to now if this item is divisible by d_i it's enough to check if $r_i = 0$.

To exemplify, before the item is processed, r is

$$r = \begin{bmatrix} r_{11} \\ r_{21} \\ r_{31} \\ \vdots \\ r_{n1} \end{bmatrix}$$

After the item was processed one time, r is

$$r = \begin{bmatrix} r_{12} \\ r_{22} \\ r_{32} \\ \vdots \\ r_{n2} \end{bmatrix}$$

After the item was processed n times, r is

$$r = \begin{bmatrix} r_{1n} \\ r_{2n} \\ r_{3n} \\ \vdots \\ r_{nn} \end{bmatrix}$$

3 Conclusion

The solution to the problem is: instead of calculating item worry level it is better to calculate the vector r, because its i-th element is never greater than d_i