

Exercise 12.1. Show that the torus \mathbb{T}^n is orientable.

Exercise 12.2. Show that a product $M_0 \times \cdots \times M_{k-1}$ of several oriented manifolds is naturally oriented.

Exercise 12.3. Let M be a differentiable n -manifold. Show that the sign $\text{sgn } \omega$ of a nonvanishing n -form on M , defined by

$$(\text{sgn } \omega)|_p(X^0, \dots, X^{n-1}) := \text{sgn}(\omega|_p(X^0, \dots, X^{n-1}))$$

for $p \in M$ and $X^0, \dots, X^{n-1} \in T_p M$, is an orientation on M , and every orientation on M is the sign of some nonvanishing n -form.

Exercise 12.4 (Projective plane). Show that the projective plane \mathbb{P}^2 is not orientable.

Exercise 12.5. Show that the Mobius band is not orientable.

1. MANIFOLDS WITH BOUNDARY

Exercise 12.6. Let M be an n -dimensional \mathcal{C}^r manifold with boundary.

- (a) Show that $\text{Int } M$ and ∂M are disjoint sets.
- (b) Show that ∂M , endowed with the subspace topology, can be given the structure of an $n - 1$ -dimensional \mathcal{C}^r manifold (without boundary) such that the inclusion map into M is \mathcal{C}^r .