

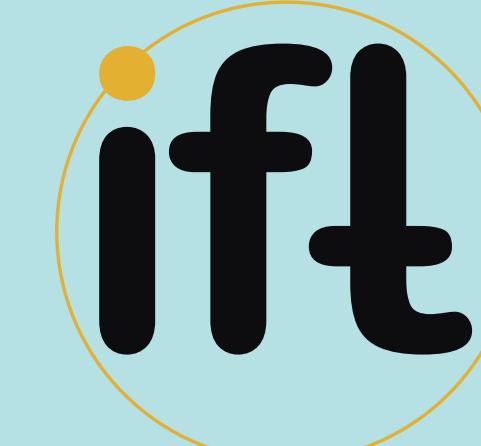
30/11/2020



How warm are non-thermal relics?

Marcos A. G. García
IFT-UAM

- | | |
|------------|--|
| 2011.13458 | with G. Ballesteros and M. Pierre |
| 2006.03325 | with Y. Mambrini, K. Olive and S. Verner |
| 2004.08404 | with Y. Mambrini, K. Olive and K. Kaneta |
| 1806.01865 | with M. Amin |
| 1709.01549 | with Y. Mambrini, K. Olive and M. Peloso |



Instituto de
Física
Teórica
UAM-CSIC

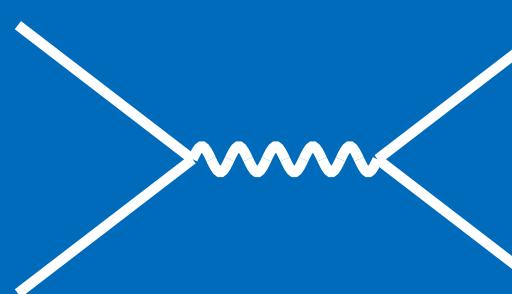
UAM
Universidad Autónoma
de Madrid

 **CSIC**
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

1. Reheating



2. Freeze-in



3. Lyman- α



4. The end?

The Goals



Understand the (perturbative) repopulation of the Universe after the end of inflation (rates, energy densities, temperatures, distributions,..)



Characterize the out-of-equilibrium production of dark matter (from particle/condensate decays, scatterings, misalignment...)



In the absence of a direct signal, can we say more beyond Ω_{DM} ?
Use the Ly- α measurement of the power spectrum to impose constraints



A short summary of what we (don't) understand

1. Reheating



Accelerated expansion can be driven by a slowly rolling scalar field

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - V(\Phi) \right]$$

$$ds^2 = dt^2 - a(t)^2 dx^2$$

$$G_{\mu\nu} = M_P^{-2} T_{\mu\nu}$$

$$\rho_\Phi = \frac{1}{2} \dot{\Phi} + V(\Phi)$$

$$P_\Phi = \frac{1}{2} \dot{\Phi} - V(\Phi)$$

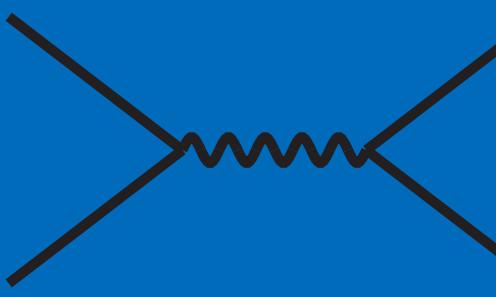
$$H \equiv \frac{\dot{a}}{a} = \left(\frac{\rho_\Phi}{3M_P^2} \right)^{1/2}$$

$$\dot{\rho}_\Phi + 3H(\rho_\Phi + P_\Phi) = 0$$

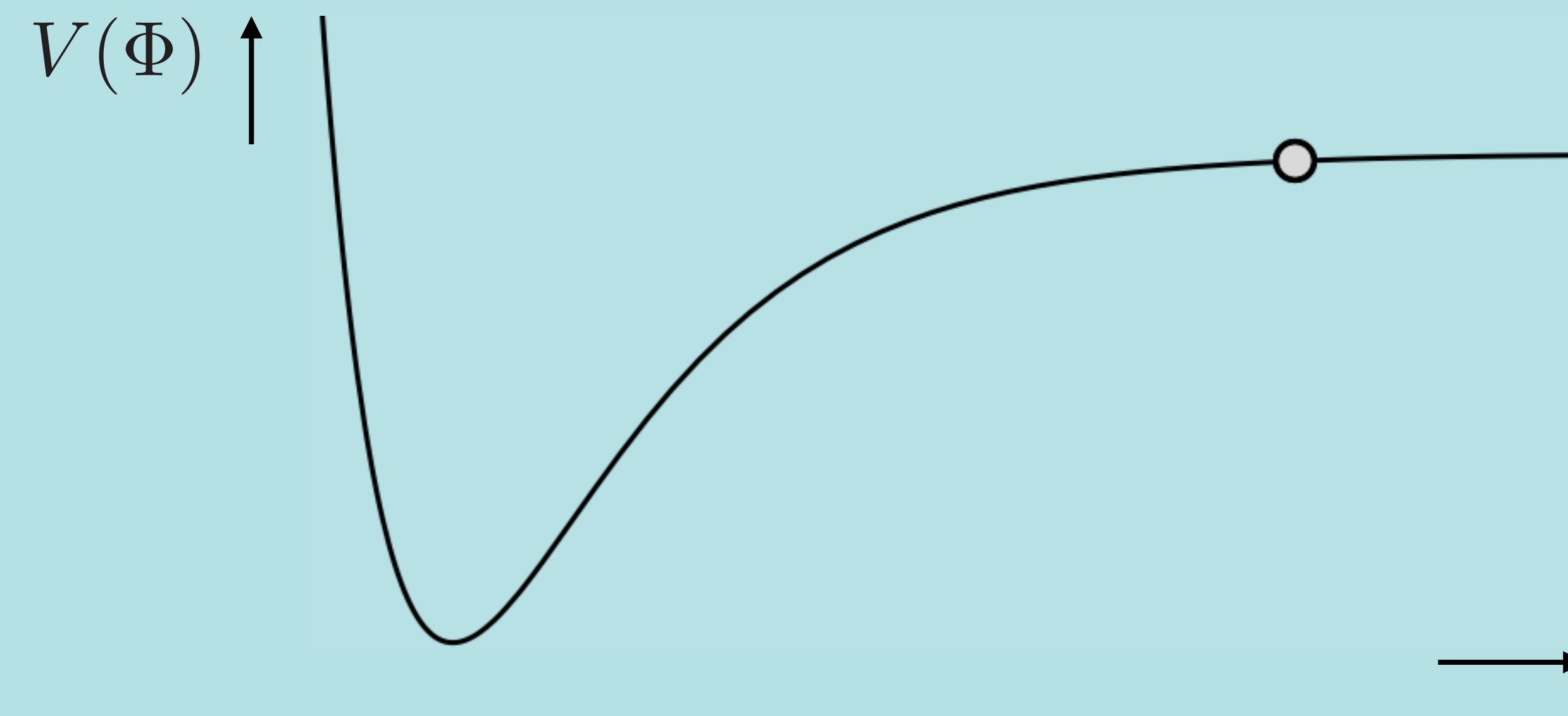
or

$$\ddot{\Phi} + 3H\dot{\Phi} + V'(\Phi) = 0$$

2. Freeze-in



3. Lyman- α



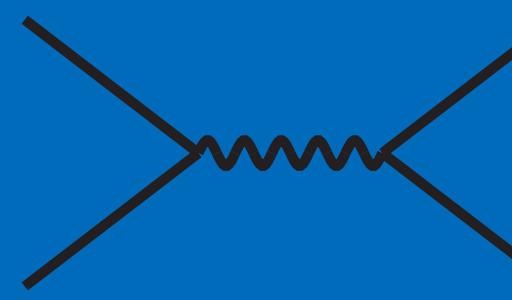
4. The end?

1. Reheating



Accelerated expansion can be driven by a slowly rolling scalar field

2. Freeze-in

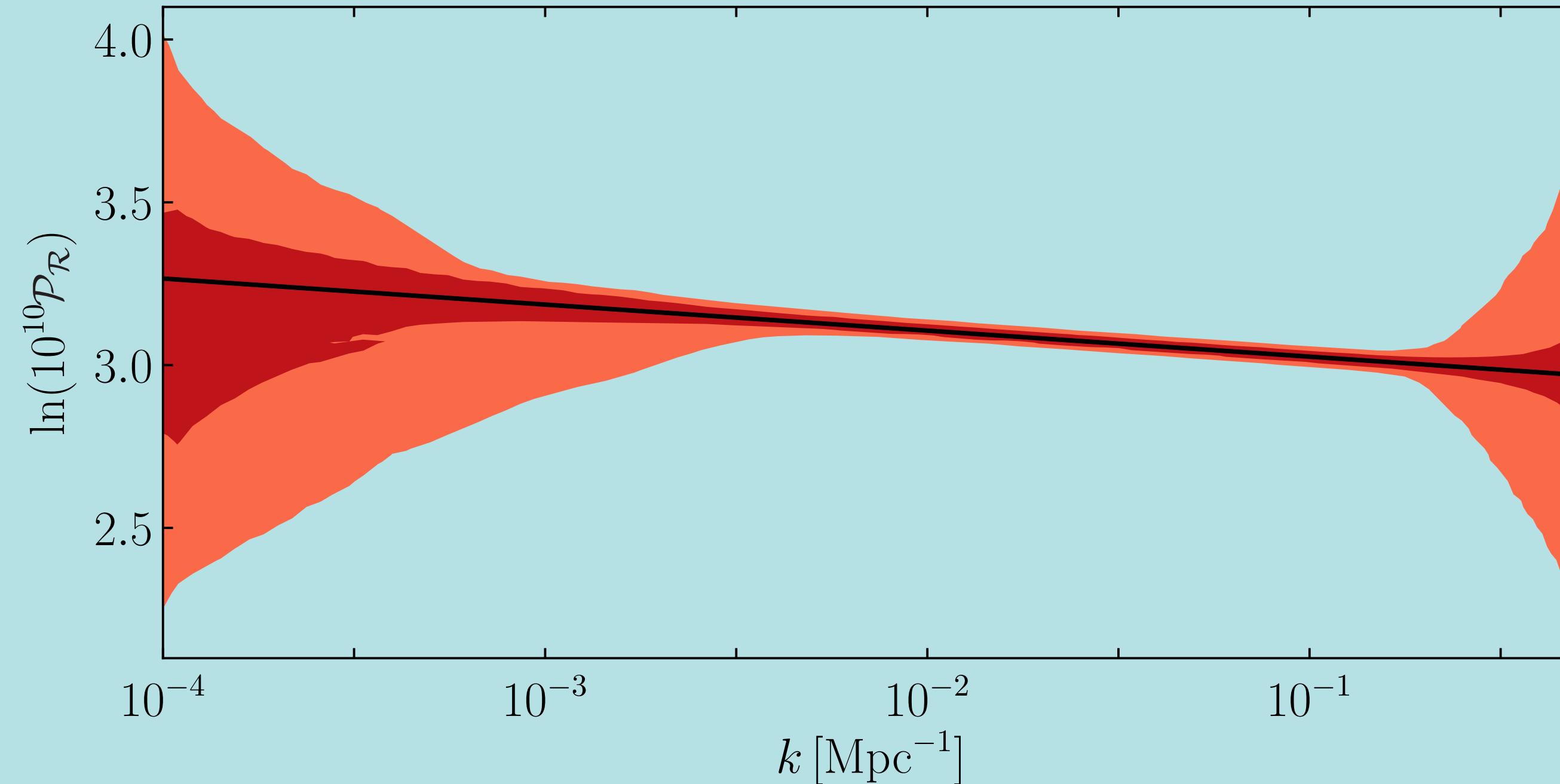


$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - V(\Phi) \right]$$

$$ds^2 = dt^2 - a(t)^2 dx^2$$

$$\begin{aligned}\Phi &\rightarrow \Phi + \delta\Phi \\ g &\rightarrow g + \delta g\end{aligned}$$

Planck TT, TE, EE + lowE + lensing + BK15 (■ 1 σ , ■ 2 σ)



$$\mathcal{P}_R = \frac{H_*^4}{4\pi^2 \dot{\phi}_*^2} \left(\frac{k}{aH} \right)^{n_s - 1}$$

$$\mathcal{P}_T = \frac{2}{\pi^2} H_*^2 \left(\frac{k}{aH} \right)^{n_T}$$

4. The end?



1. Reheating



After inflation ends, reheating begins

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - V(\Phi) \right]$$

$$ds^2 = dt^2 - a(t)^2 dx^2$$

$$G_{\mu\nu} = M_P^{-2} T_{\mu\nu}$$

$$\rho_\Phi = \frac{1}{2} \dot{\Phi} + V(\Phi)$$

$$P_\Phi = \frac{1}{2} \dot{\Phi} - V(\Phi)$$

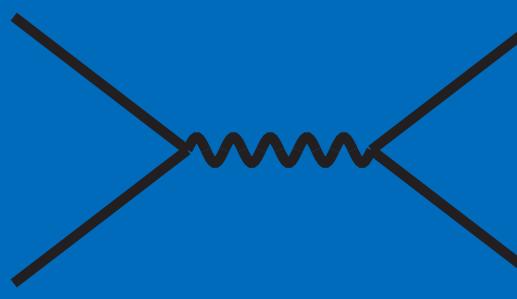
$$H \equiv \frac{\dot{a}}{a} = \left(\frac{\rho_\Phi}{3M_P^2} \right)^{1/2}$$

$$\dot{\rho}_\Phi + 3H(\rho_\Phi + P_\Phi) = 0$$

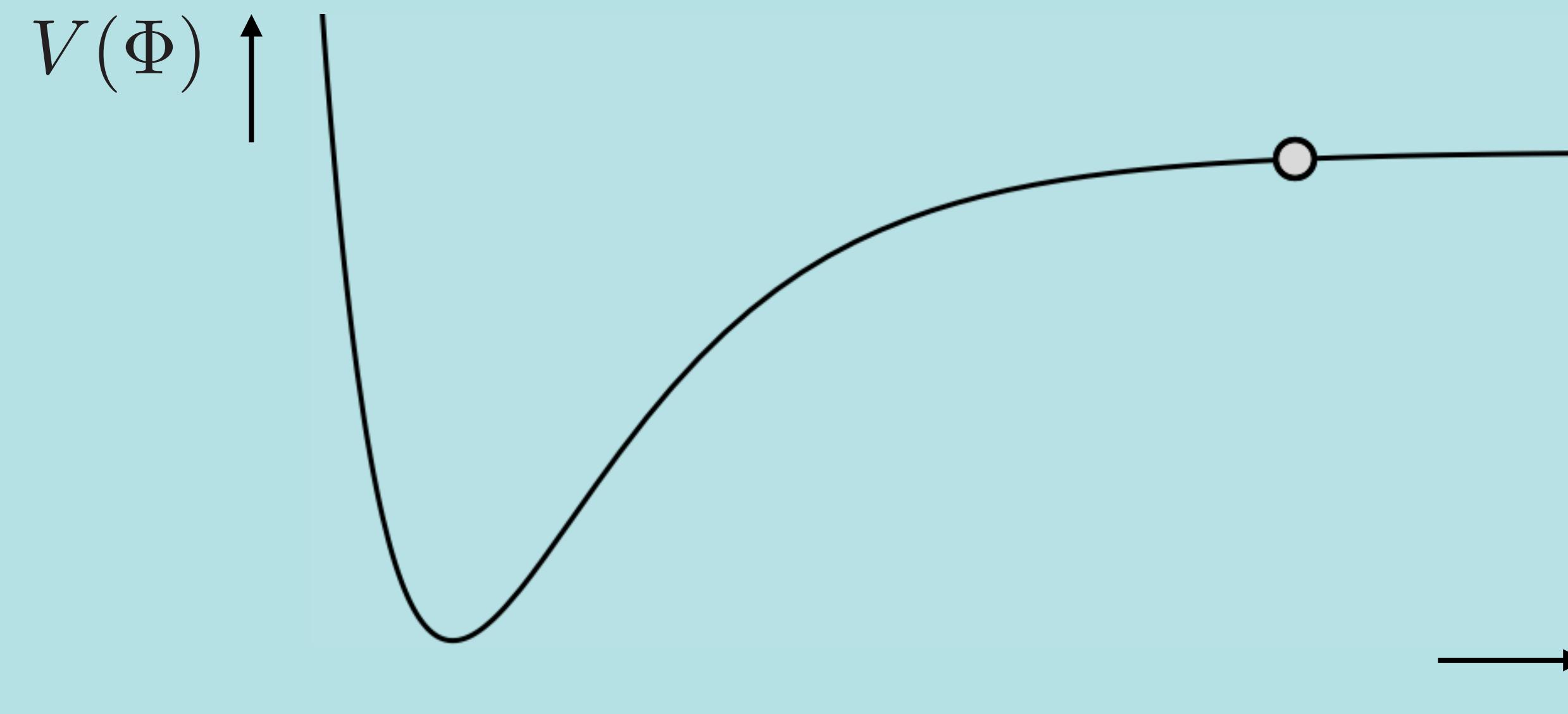
or

$$\ddot{\Phi} + 3H\dot{\Phi} + V'(\Phi) = 0$$

2. Freeze-in

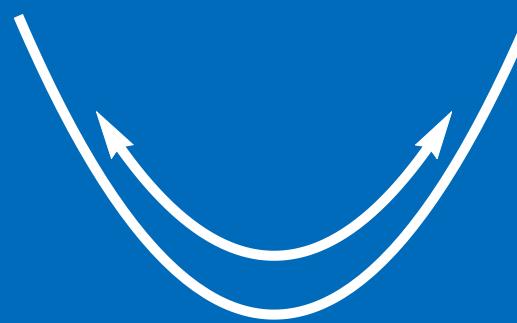


3. Lyman- α

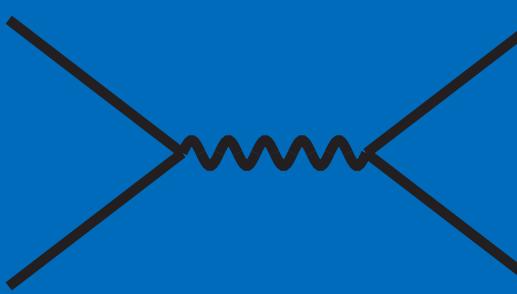


4. The end?

1. Reheating



2. Freeze-in

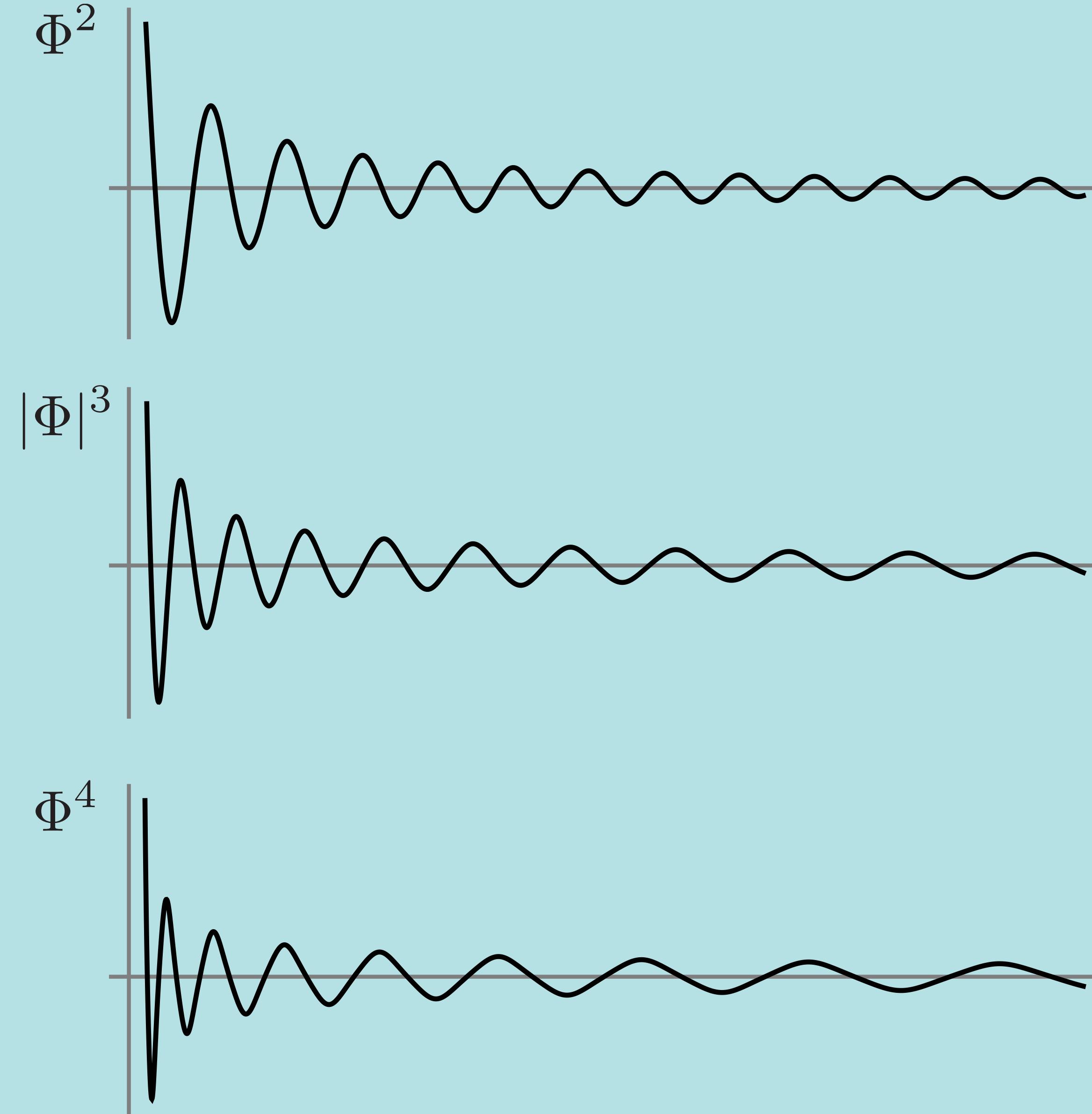


3. Lyman- α



4. The end?

Reheating depends on the shape of the potential



$$V(\Phi) \propto |\Phi|^k$$



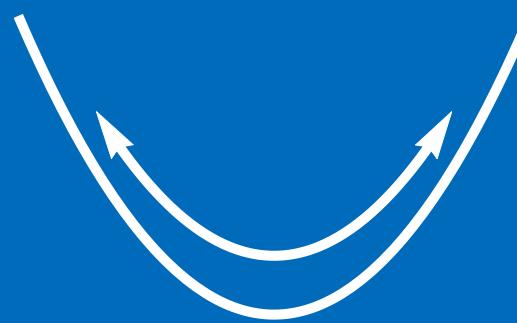
over one oscillation,

$$\langle \dot{\Phi}^2 \rangle \simeq \langle \Phi V'(\Phi) \rangle$$



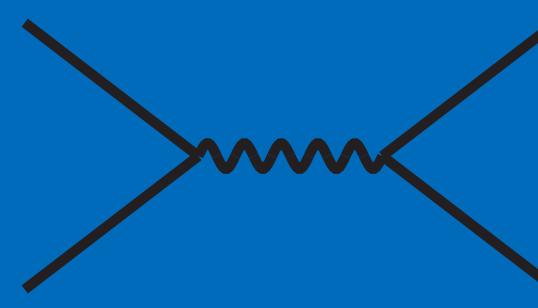
$$\langle P_\Phi \rangle = \frac{k-2}{k+2} \langle \rho_\Phi \rangle$$

1. Reheating



Reheating depends on the shape of the potential

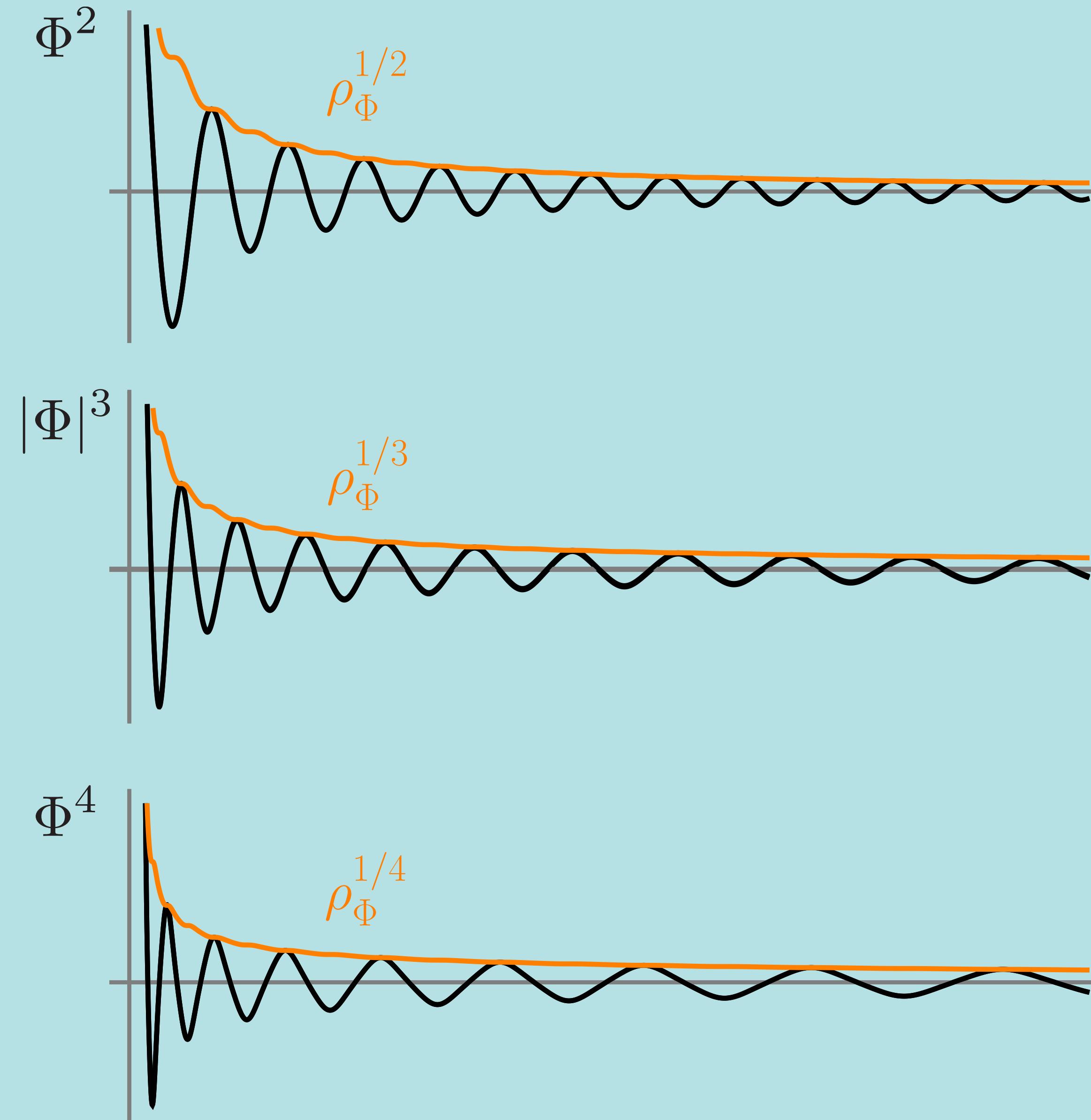
2. Freeze-in



3. Lyman- α



4. The end?



~ matter

$$\rho_\Phi = \rho_{\text{end}} \left(\frac{a}{a_{\text{end}}} \right)^{-\frac{6k}{k+2}}$$

$$a \propto t^{\frac{k+2}{3k}}$$

~ radiation

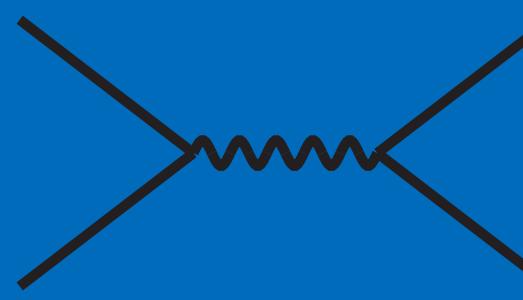
1. Reheating



Particle production in the expanding Universe

$$\chi + a + b + \cdots \longleftrightarrow i + j + \cdots$$

2. Freeze-in



3. Lyman- α



$$\frac{\partial f_\chi}{\partial t} - H|\mathbf{p}| \frac{\partial f_\chi}{\partial |\mathbf{p}|} = \mathcal{C}[f_\chi(|\mathbf{p}|, t)]$$

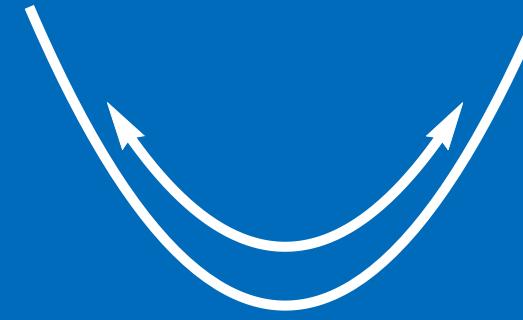
$$\begin{aligned} \mathcal{C}[f_\chi] = & -\frac{1}{2p_0} \int \frac{g_a d^3 \mathbf{p}_a}{(2\pi)^3 2p_{a0}} \frac{g_b d^3 \mathbf{p}_b}{(2\pi)^3 2p_{b0}} \cdots \frac{g_i d^3 \mathbf{p}_i}{(2\pi)^3 2p_{i0}} \frac{g_j d^3 \mathbf{p}_j}{(2\pi)^3 2p_{j0}} \cdots \\ & \times (2\pi)^4 \delta^{(4)}(p_\chi + p_a + p_b + \cdots - p_i - p_j - \cdots) \\ & \times \left[|\mathcal{M}|_{\chi+a+b+\cdots \rightarrow i+j+\cdots}^2 f_a f_b \cdots f_\chi (1 \pm f_i) (1 \pm f_j) \cdots \right. \\ & \left. - |\mathcal{M}|_{i+j+\cdots \rightarrow \chi+a+b+\cdots}^2 f_i f_j \cdots (1 \pm f_a) (1 \pm f_b) \cdots (1 \pm f_\chi) \right] \end{aligned}$$

$$n_\chi(t) = \frac{g_\chi}{(2\pi)^3} \int d^3 \mathbf{p} f_\chi(p_0, t)$$

$$\rho_\chi(t) = \frac{g_\chi}{(2\pi)^3} \int d^3 \mathbf{p} p_0 f_\chi(p_0, t)$$

4. The end?

1. Reheating

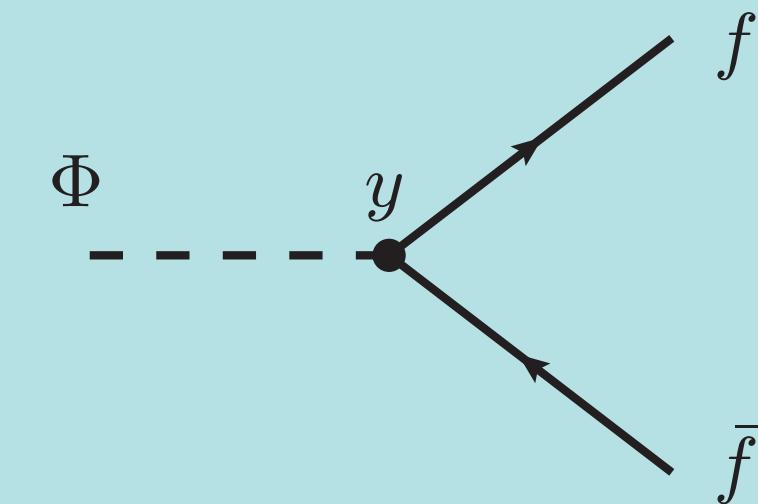


Integrated Boltzmann equations

$$\dot{\rho}_\Phi + 3 \left(\frac{2k}{k+2} \right) H \rho_\Phi = -\Gamma_\Phi(t) \rho_\Phi$$

$$\dot{\rho}_R + 4H\rho_R = \Gamma_\Phi(t)\rho_\Phi$$

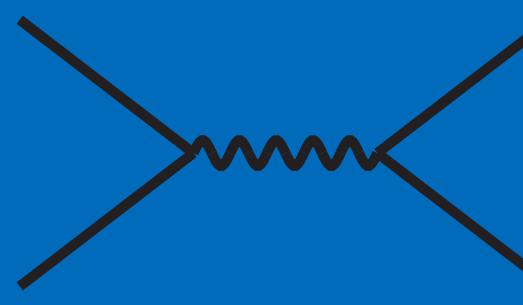
$$3M_P^2 H^2 = \rho_\Phi + \rho_R$$



$$\Gamma_\Phi = \frac{y^2}{8\pi} m_\Phi(t),$$

$$m_\Phi^2 \equiv \partial_\phi^2 V(\Phi) \propto \rho_\Phi^{\frac{k-2}{k}}$$

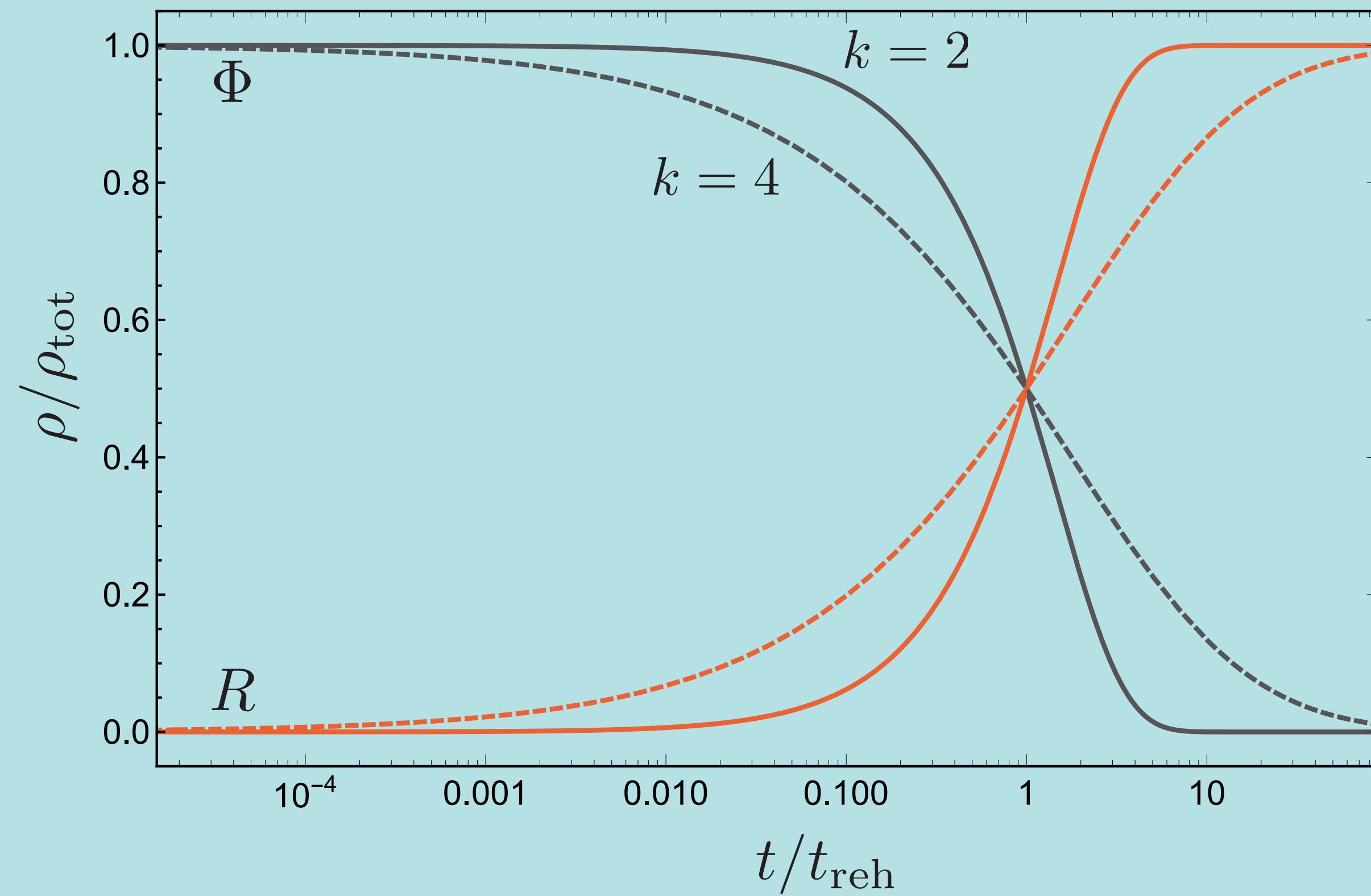
2. Freeze-in



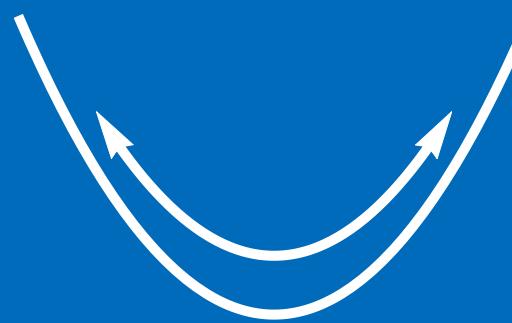
3. Lyman-α



4. The end?



1. Reheating



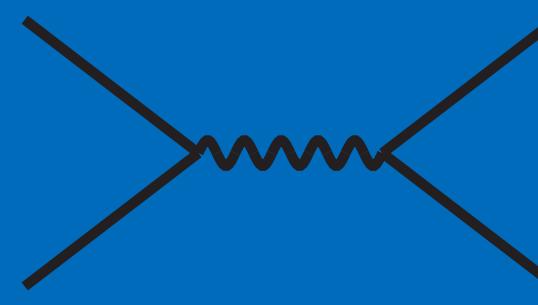
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$$\dot{\rho}_\Phi + 3 \left(\frac{2k}{k+2} \right) H \rho_\Phi = -\Gamma_\Phi(t) \rho_\Phi$$

$$\dot{\rho}_R + 4H\rho_R = \Gamma_\Phi(t)\rho_\Phi$$

$$3M_P^2 H^2 = \rho_\Phi + \rho_R$$

2. Freeze-in

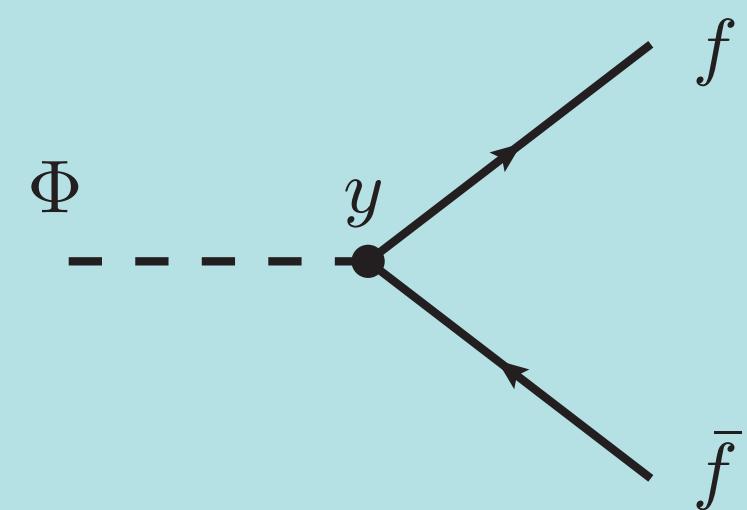


3. Lyman- α



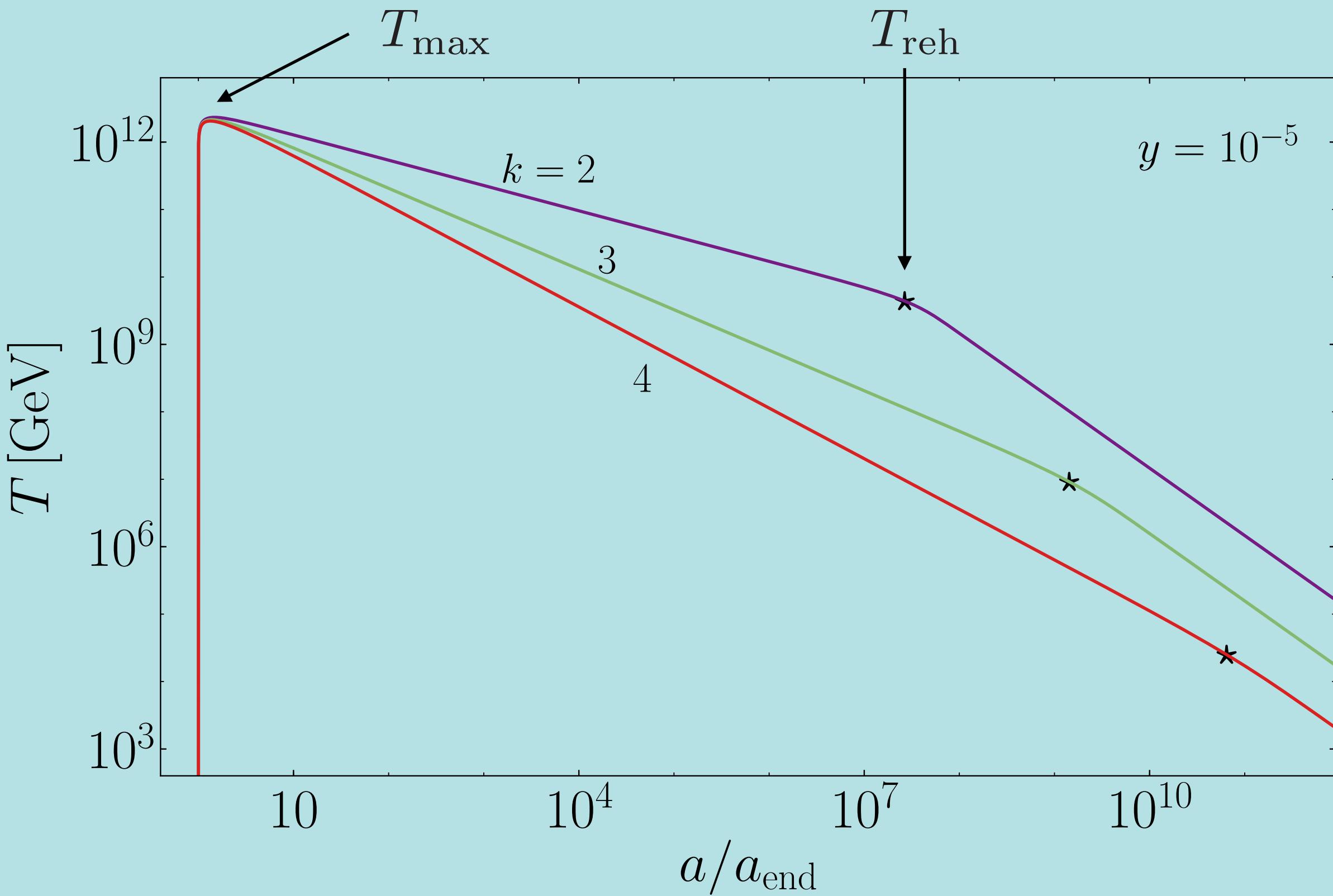
$$T = \left(\frac{30\rho_R}{\pi^2 g_*} \right)^{1/4}$$
$$\propto a^{-\frac{3k-3}{2k+4}}$$

4. The end?



$$\Gamma_\Phi = \frac{y^2}{8\pi} m_\Phi(t),$$

$$m_\Phi^2 \equiv \partial_\phi^2 V(\Phi) \propto \rho_\Phi^{\frac{k-2}{k}}$$



1. Reheating

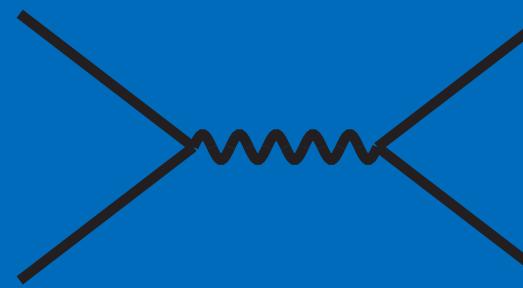


A closer look: thermalization is not instantaneous! (quadratic only)

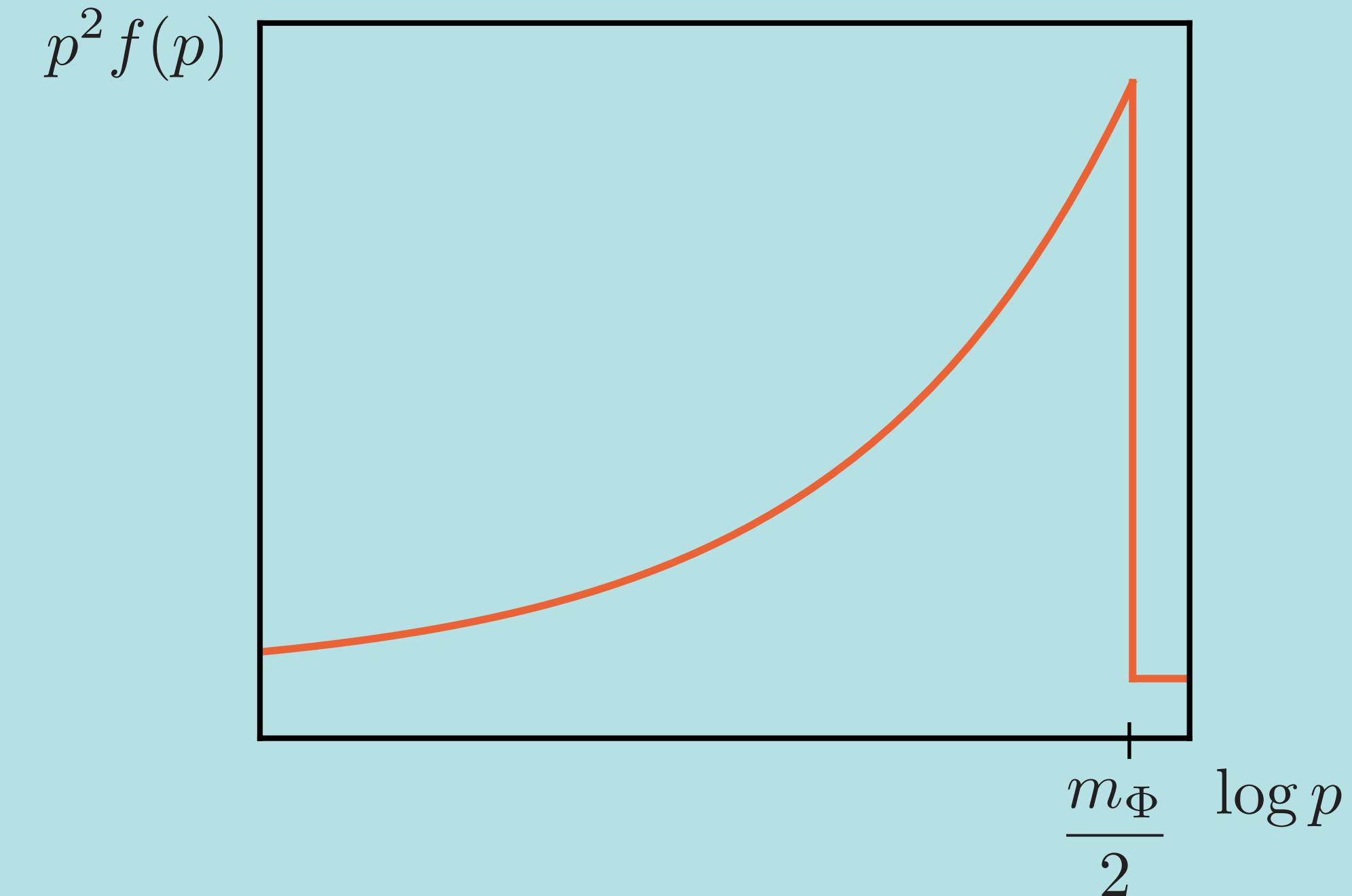
$$\mathcal{C}[f_\chi(p, t)] = \frac{8\pi^2}{g_\chi m_\Phi^2} n_\Phi \Gamma_{\Phi \rightarrow \chi\psi} \delta(p - m_\Phi/2)$$

$$f_\chi(p, t) = \frac{16\pi^2 \Gamma_{\Phi \rightarrow \chi\psi} n_\Phi(\hat{t})}{g_\chi m_\Phi^3 H(\hat{t})} \theta(t - \hat{t}), \quad \frac{a(t)}{a(\hat{t})} = \frac{m_\Phi}{2p}$$

2. Freeze-in



3. Lyman- α



$$f_\chi(p, t) \simeq \frac{24\pi^2 n_\chi(t)}{g_\chi m_\Phi^3} \left(\frac{m_\Phi}{2p} \right)^{3/2} \theta(m_\Phi/2 - p) \quad (t \ll t_{\text{reh}})$$

4. The end?

1. Reheating

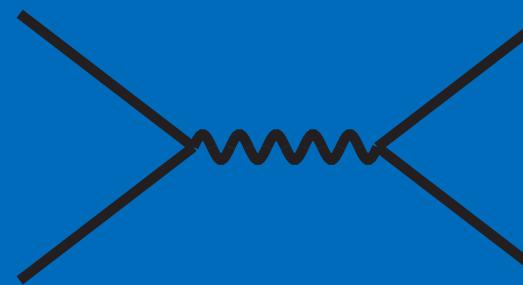


A closer look: thermalization is not instantaneous! (quadratic only)

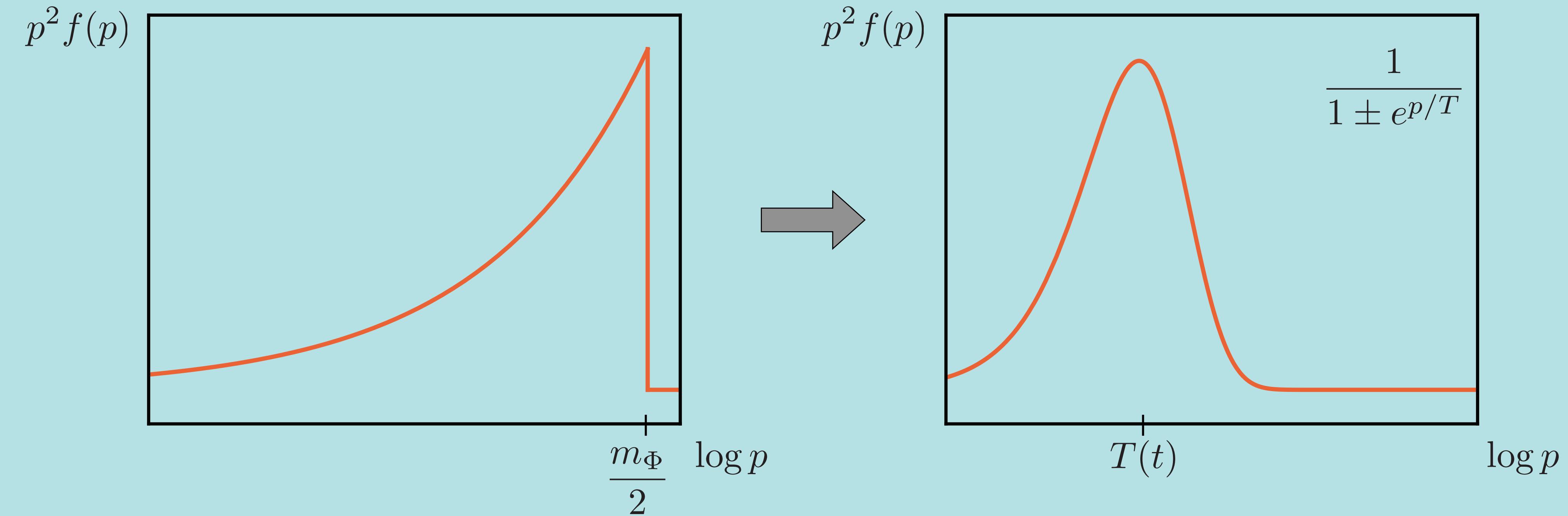
$$\mathcal{C}[f_\chi(p, t)] = \frac{8\pi^2}{g_\chi m_\Phi^2} n_\Phi \Gamma_{\Phi \rightarrow \chi\psi} \delta(p - m_\Phi/2)$$

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2. Freeze-in

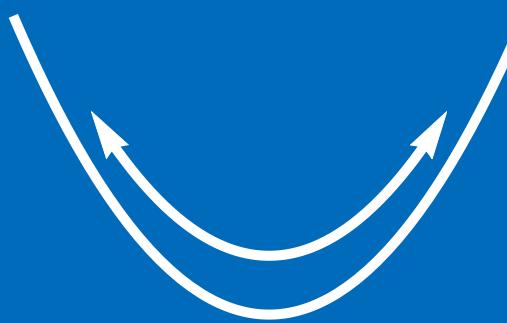


3. Lyman- α



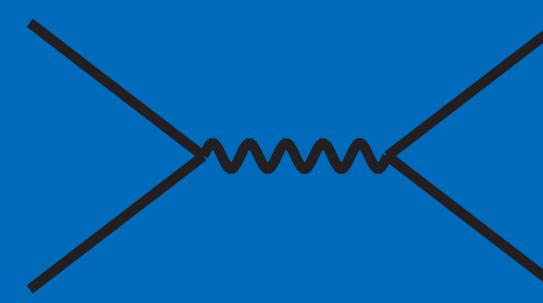
4. The end?

1. Reheating



$$\frac{\partial f_\chi}{\partial t} - H p \frac{\partial f_\chi}{\partial p} = \left| \begin{array}{c} \text{---} \\ | \end{array} \right|^2 + \left| \begin{array}{c} \text{---} \\ | \end{array} \right|^2 + \dots$$
$$\equiv -\mathcal{C}^{2 \leftrightarrow 2}[f_\chi] - \mathcal{C}^{\text{"}1 \leftrightarrow 2\text{"}}[f_\chi] + \dots,$$

2. Freeze-in

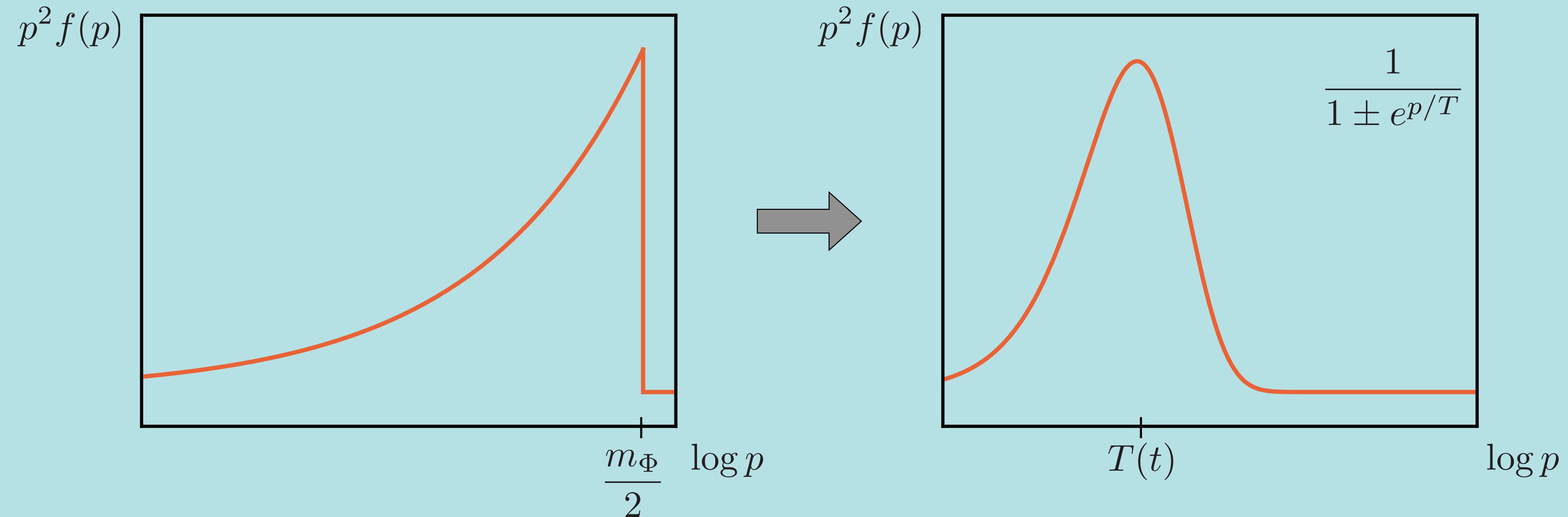


K. Harigaya, K. Mukaida, JHEP 05 (2014) 006

K. Mukaida, M. Yamada, JCAP 02 (2016) 003

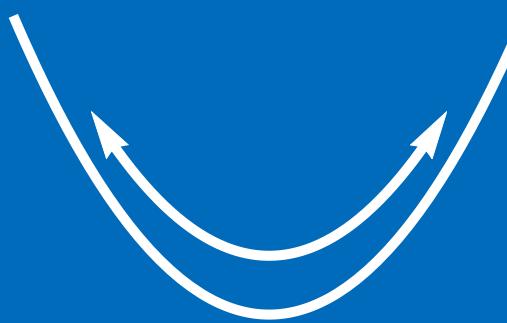
$$\Gamma_\Phi t_{\text{th}} \simeq \alpha_{\text{SM}}^{-16/5} \left(\frac{\Gamma_\Phi m_\Phi^2}{M_P^3} \right)^{2/5}$$

3. Lyman- α

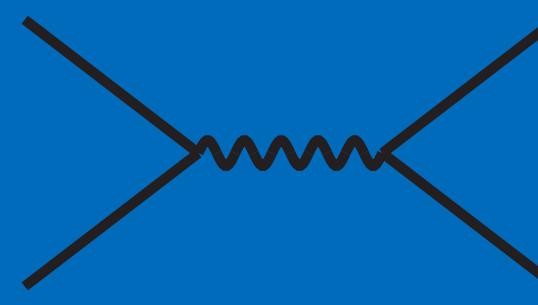


4. The end?

1. Reheating



2. Freeze-in

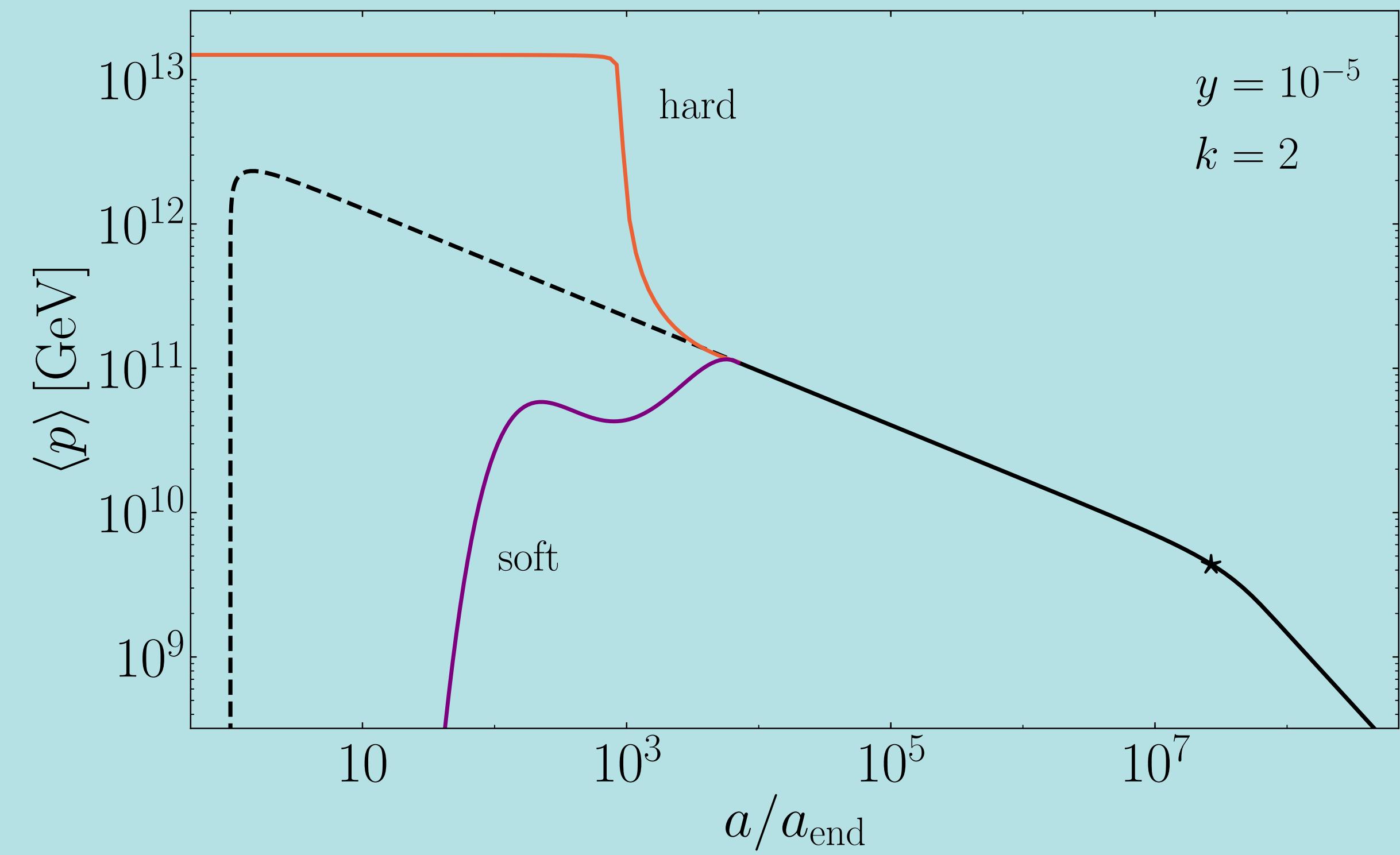


3. Lyman- α

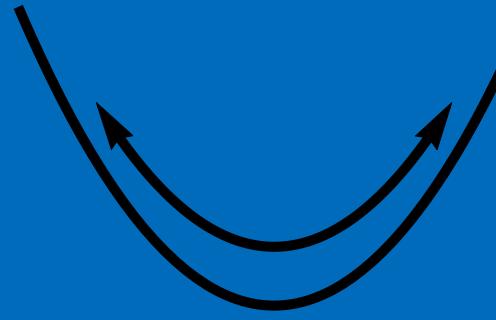


4. The end?

$$\frac{\partial f_\chi}{\partial t} - H p \frac{\partial f_\chi}{\partial p} = \left| \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \right|^2 + \left| \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \right|^2 + \dots$$
$$\equiv -\mathcal{C}^{2 \leftrightarrow 2}[f_\chi] - \mathcal{C}^{\text{"}1 \leftrightarrow 2\text{"}}[f_\chi] + \dots,$$

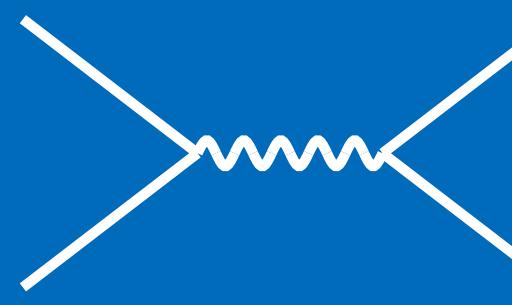


1. Reheating



Freeze-in: producing dark matter out-of-equilibrium

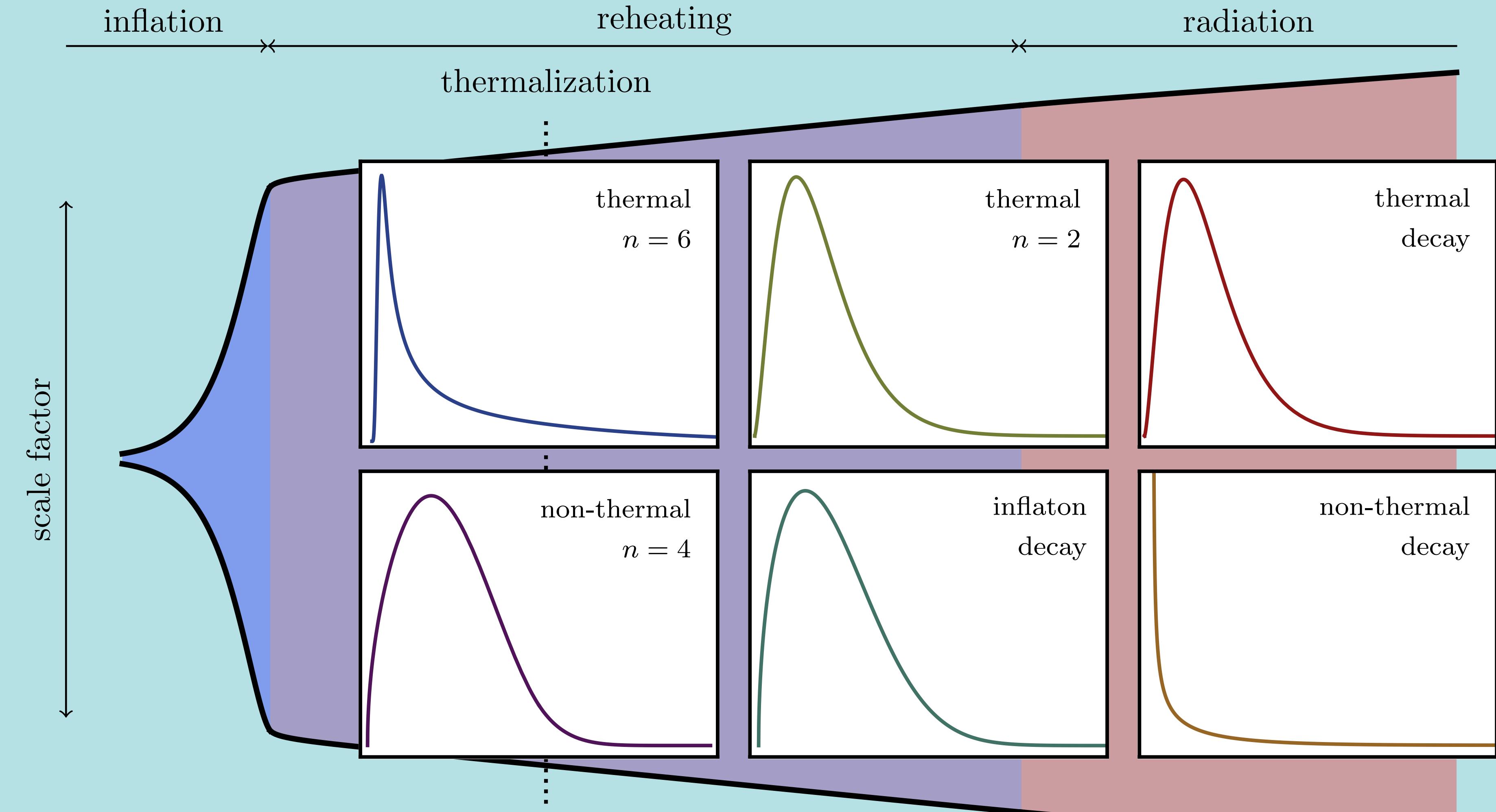
2. Freeze-in



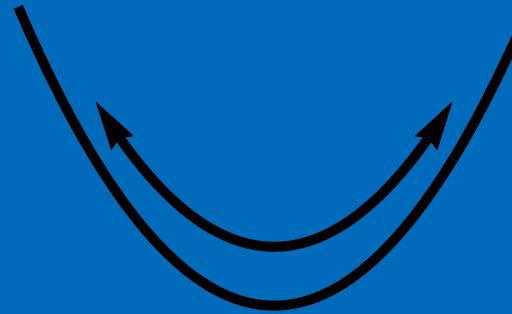
3. Lyman- α



4. The end?

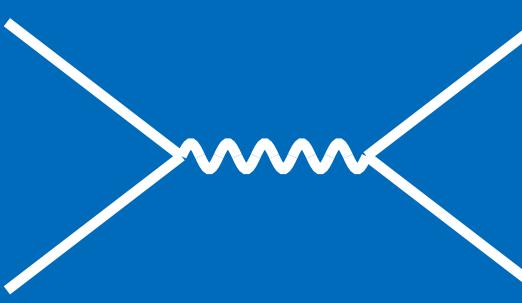


1. Reheating



Decay of an oscillating condensate (inflaton, non-stabilized moduli, ...)

2. Freeze-in



$$f_\chi \left(\frac{p}{p_0} \frac{a(t)}{a_*} \right) = f_\chi \left(\frac{p a(t)/a_0}{p_0 a_*/a_0} \right) = f_\chi \underbrace{\left(\frac{p_{\text{co}}}{p_0 a_*/a_0} \right)}_{T_* \quad (\text{or } T_{\text{NCDM}})} \equiv f_\chi(q)$$

3. Lyman- α



Standard in literature and cosmology codes (CLASS)

C. Ma, E. Bertschinger, *Astrophys. J.* 455 (1995) 7

D. Blas, J. Lesgourgues, T. Tram, *JCAP* 07 (2011) 034

4. The end?

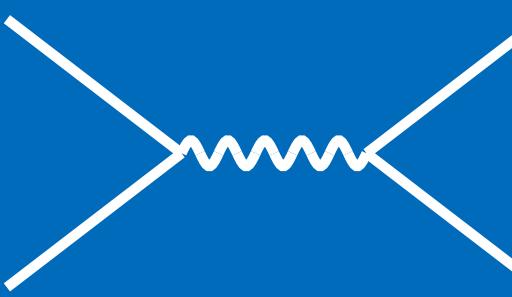
1. Reheating



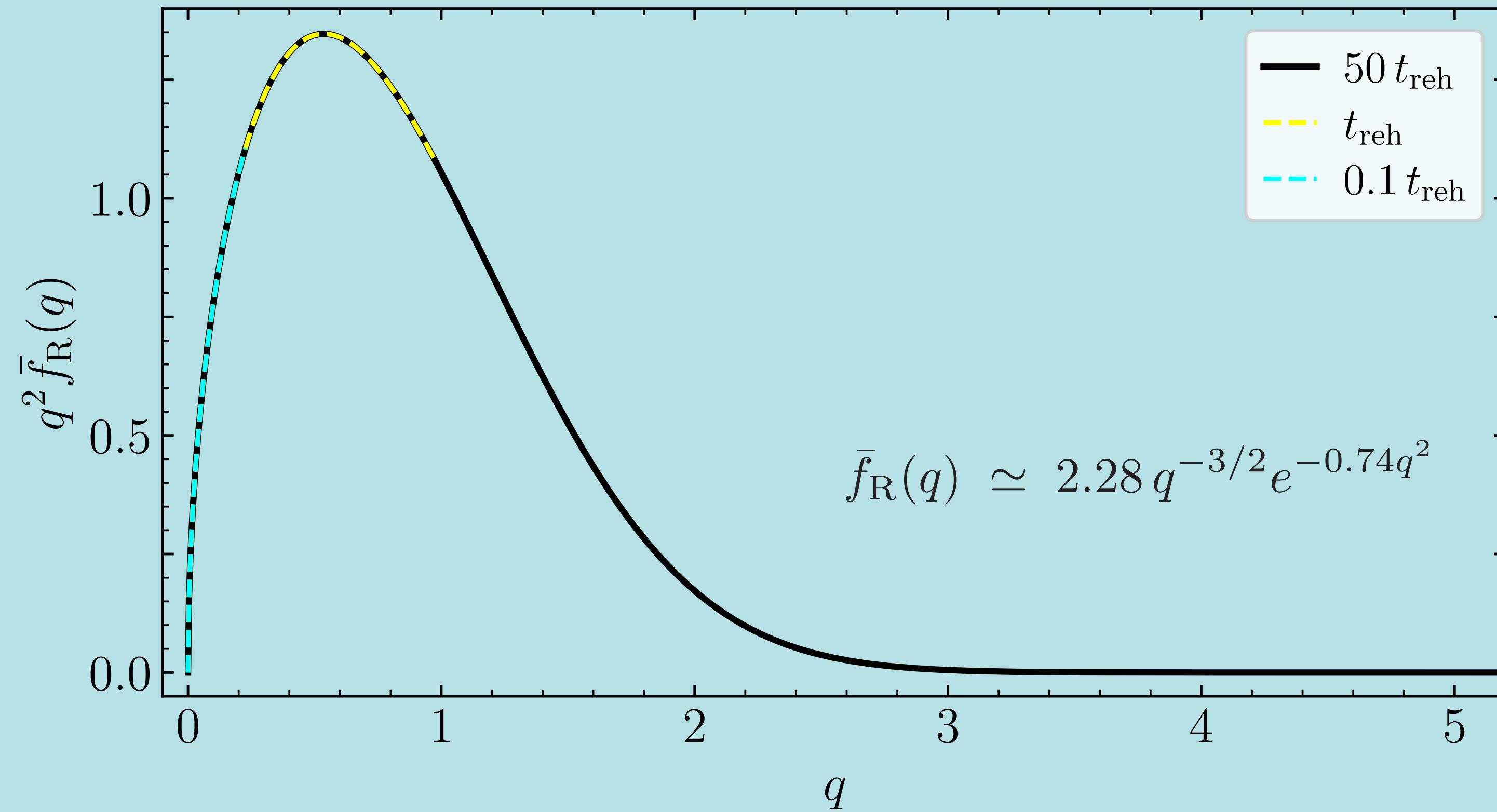
Decay of an oscillating condensate (inflaton, non-stabilized moduli, ...)

$$f_\chi(p, t) d^3 p = \frac{4\pi^4 \text{Br}_\chi g_{*s}^{\text{reh}}}{5g_\chi} \left(\frac{T_{\text{reh}}}{m_\Phi} \right)^4 \left(\frac{a_0}{a(t)} \right)^3 T_*^3 \bar{f}_R(q) d^3 q, \quad T_* = \left(\frac{g_{*s}^0}{g_{*s}^{\text{reh}}} \right)^{1/3} \frac{m_\Phi}{2T_{\text{reh}}} T_0$$

2. Freeze-in



3. Lyman- α



4. The end?

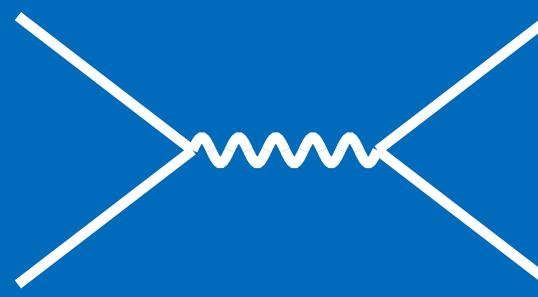
1. Reheating



Decay of an oscillating condensate (inflaton, non-stabilized moduli, ...)

$$f_\chi(p, t) d^3 p = \frac{4\pi^4 \text{Br}_\chi g_{*s}^{\text{reh}}}{5g_\chi} \left(\frac{T_{\text{reh}}}{m_\Phi} \right)^4 \left(\frac{a_0}{a(t)} \right)^3 T_*^3 \bar{f}_{\text{R}}(q) d^3 q, \quad T_* = \left(\frac{g_{*s}^0}{g_{*s}^{\text{reh}}} \right)^{1/3} \frac{m_\Phi}{2T_{\text{reh}}} T_0$$

2. Freeze-in



$$n_\chi(t) \simeq 0.70\pi^2 \text{Br}_\chi g_{*s}^{\text{reh}} \left(\frac{T_{\text{reh}}}{m_\Phi} \right)^4 \left(\frac{a_0}{a(t)} \right)^3 T_*^3$$

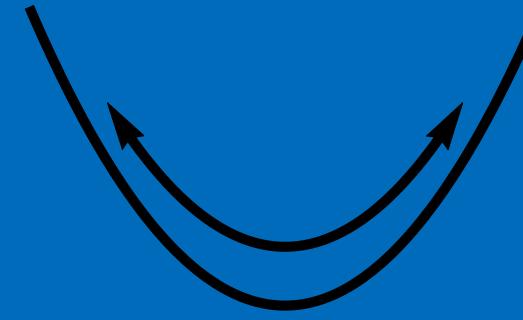
3. Lyman- α



$$\Omega_\chi h^2 \simeq 0.1 \left(\frac{\text{Br}_\chi}{5.5 \times 10^{-4}} \right) \left(\frac{m_{\text{DM}}}{1 \text{ MeV}} \right) \left(\frac{T_{\text{reh}}}{10^{10} \text{ GeV}} \right) \left(\frac{3 \times 10^{13} \text{ GeV}}{m_\Phi} \right)$$

4. The end?

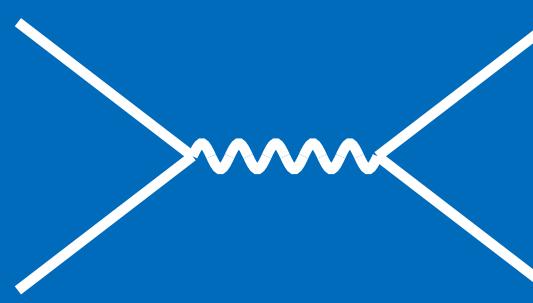
1. Reheating



Out-of-equilibrium decay chain $\Phi \rightarrow A \rightarrow \chi$

$$\Omega_\chi h^2 \simeq 0.1 \left(\frac{\text{Br}_\chi}{5.5 \times 10^{-4}} \right) \left(\frac{m_{\text{DM}}}{1 \text{ MeV}} \right) \left(\frac{T_{\text{reh}}}{10^{10} \text{ GeV}} \right) \left(\frac{3 \times 10^{13} \text{ GeV}}{m_\Phi} \right)$$

2. Freeze-in



$$f_\chi(p, t) d^3 p = \frac{24\pi^3 \sqrt{10g_{*s}^{\text{reh}}} \text{Br}_\chi \text{Br}_A \Gamma_A M_P}{5g_A m_A^2} \left(\frac{T_{\text{reh}}}{m_\phi} \right)^2 \mathcal{F}(q, \mathcal{R}) \left(\frac{a_0}{a(t)} \right)^3 T_*^3 d^3 q$$

3. Lyman- α



$$\mathcal{F}(q, \mathcal{R}) = q^{-2} \int_0^\mathcal{R} dy y^2 \int_{\left| q - \frac{y^2}{q} \right|}^\infty \frac{z dz}{\sqrt{q^2 + 4y^2}} \bar{f}_R(z) \simeq \begin{cases} \bar{f}_{\text{D}, \text{NR}}(q), & \mathcal{R} \gg 1, \\ \frac{\mathcal{R}^3}{3} \bar{f}_{\text{D}, \text{R}}(q), & \mathcal{R} \ll 1. \end{cases}$$

$$\mathcal{R} \equiv \left(\frac{g_{*s}^{\text{reh}}}{g_{*s}^{\text{dec}}} \right)^{1/3} \frac{m_A T_{\text{reh}}}{m_\phi T_{\text{dec}}}$$

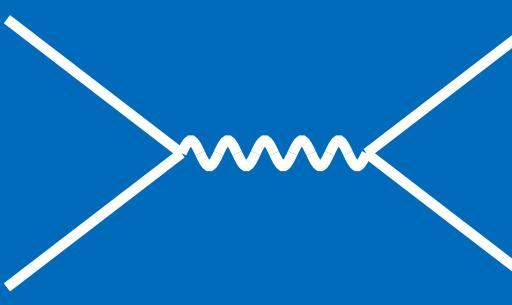
4. The end?

(for thermal decays, see also K. Petraki and A. Kusenko, Phys. Rev. D 77 (2008) 065014;
K. J. Bae, A. Kamada, S. P. Liew and K. Yanagi, JCAP 01 (2018) 054)

1. Reheating



2. Freeze-in

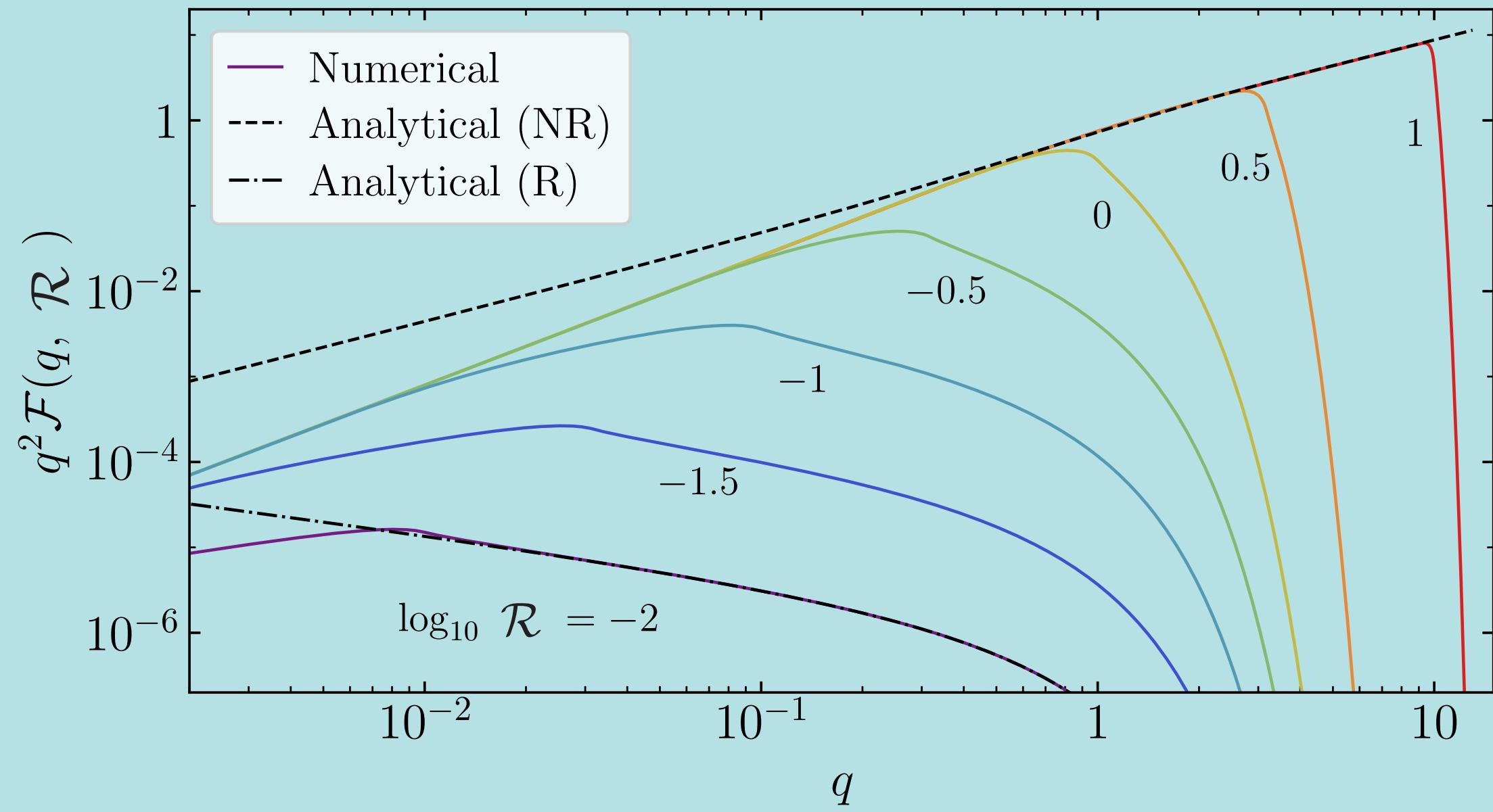


3. Lyman- α

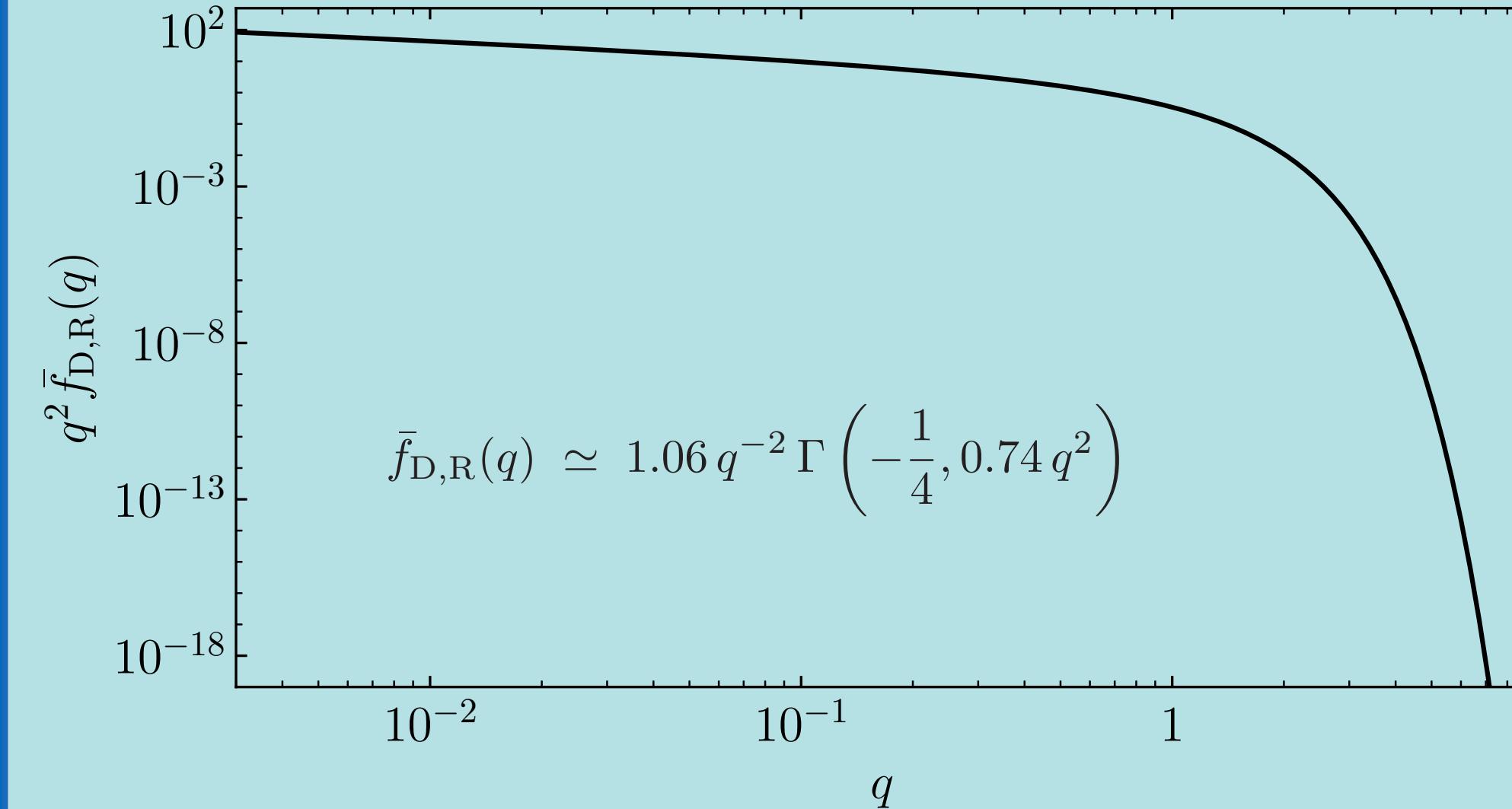


4. The end?

Out-of-equilibrium decay chain $\Phi \rightarrow A \rightarrow \chi$

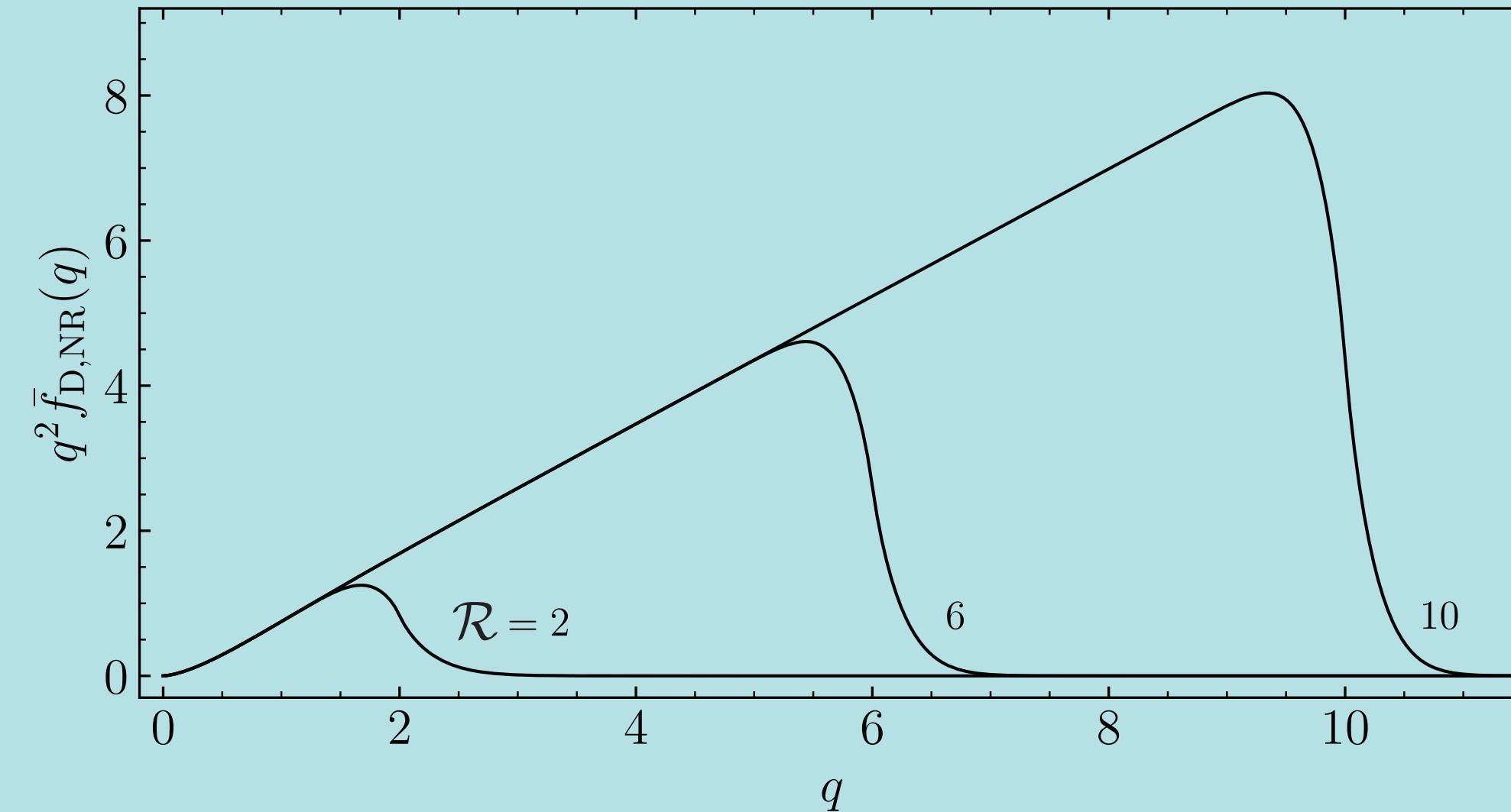


R



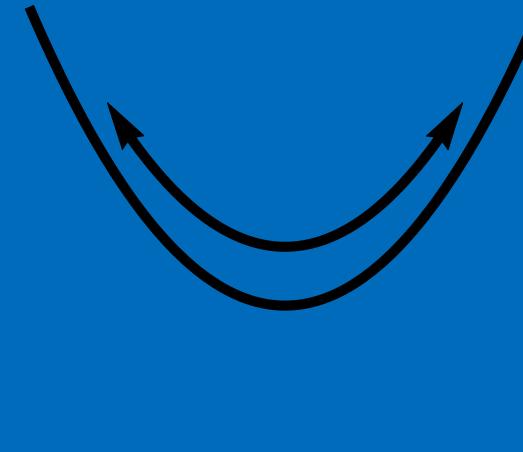
q

NR

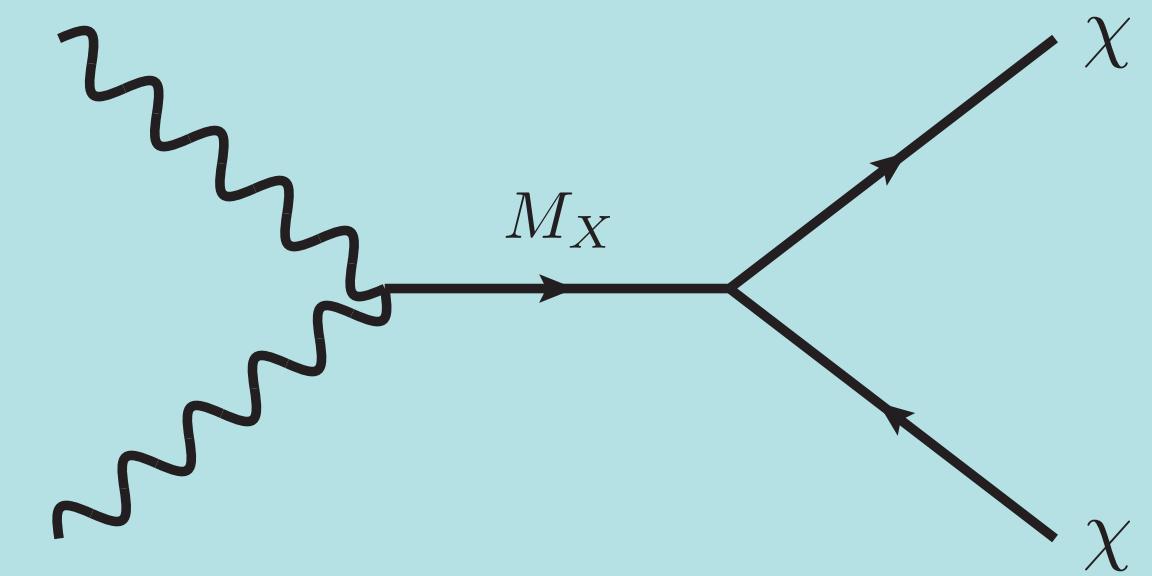


q

1. Reheating

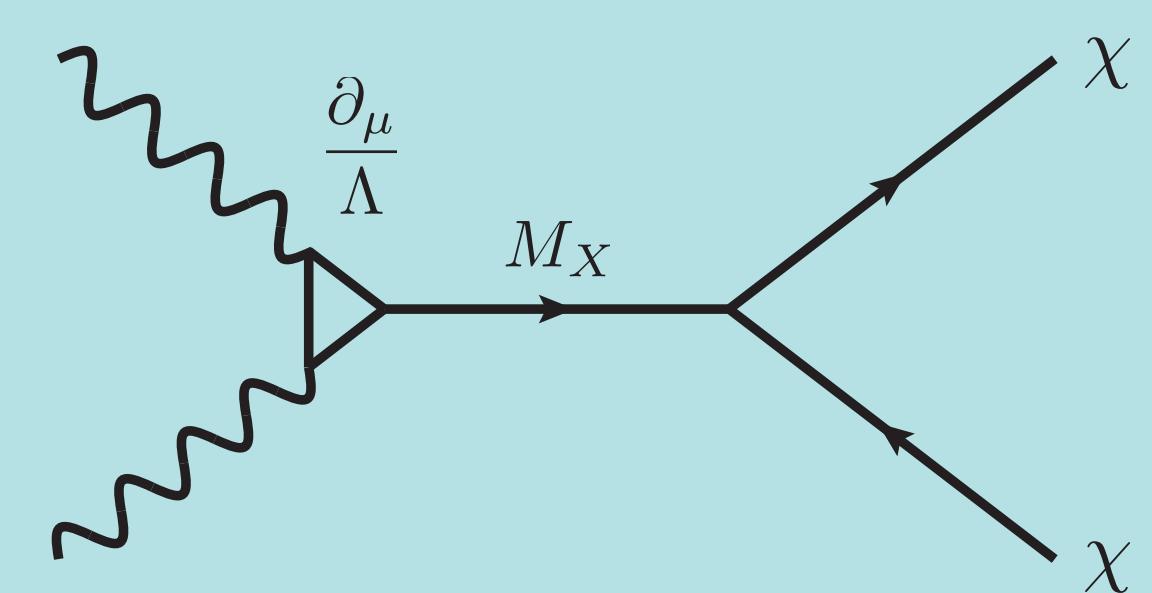
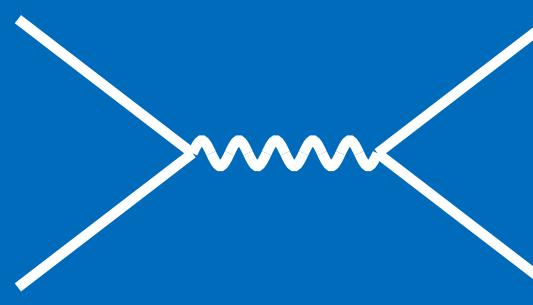


Freezing-in through scatterings



$$\sigma(s) = \frac{s}{M_X^4}$$

2. Freeze-in



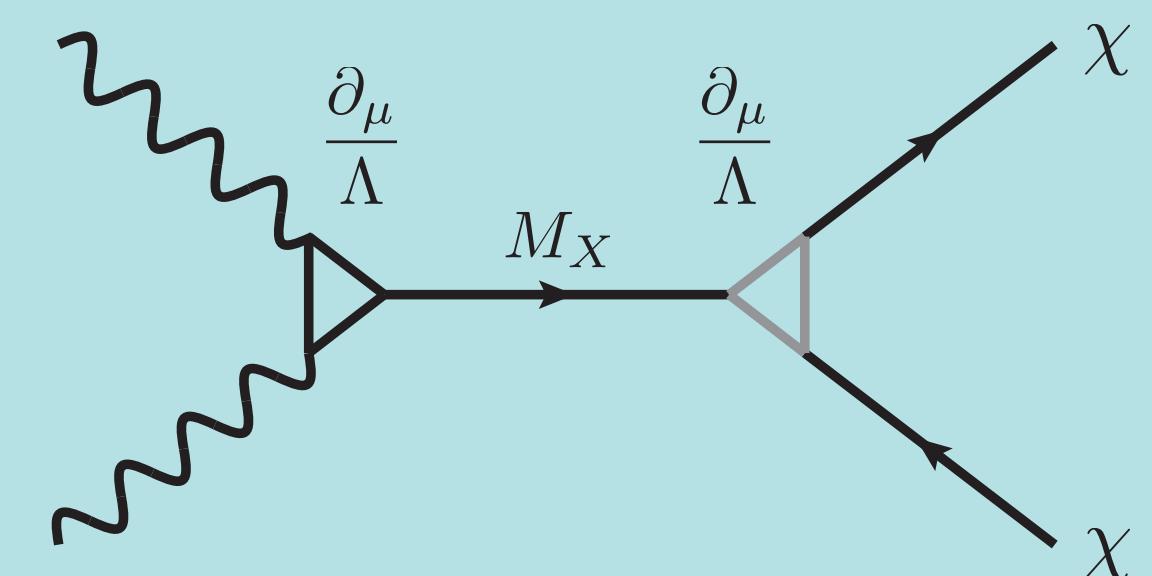
$$\sigma(s) = \frac{s^2}{\Lambda^2 M_X^4}$$

Production at $t \leq t_{\text{reh}}$ if

$$\sigma(s) = \frac{s^{n/2}}{\Lambda^{n+2}}$$

with $n > -1$

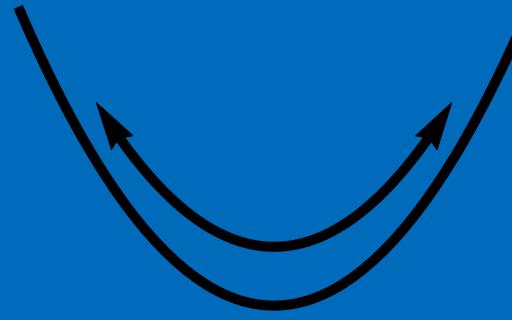
3. Lyman-alpha



$$\sigma(s) = \frac{s^3}{\Lambda^4 M_X^4}$$

4. The end?

1. Reheating



Freezing-in through scatterings (examples)

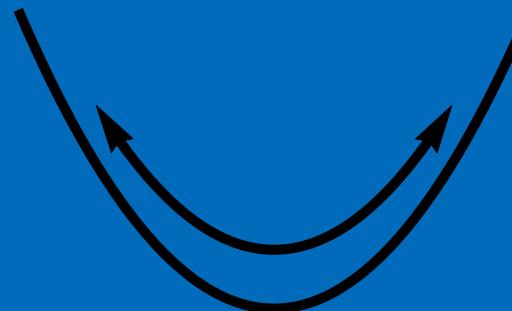
- $n = 0$: Low scale susy gravitino, $\sigma \propto M_P^{-2}$. Axino, $\sigma \propto f_a^{-2}$
V. Rychkov, A. Strumia, Phys. Rev. D 75 (2007) 075011 ; A. Strumia, JHEP 06 (2010) 036
- $n = 2$: $SU(10) \rightarrow SU(4) \times SU(2)_L \times U(1)_R \rightarrow$ SM (heavy DM), $\sigma \propto s/M_{\text{int}}^4$
Light spin-2 mediator (light DM), $\sigma \propto s/M_P^4$
Y. Mambrini et al., Phys. Rev. Lett. 110 (2013) 241306; N. Bernal et al., Phys. Rev. D 97 (2018) 115020
- $n = 4$: Non-susy spin-3/2 DM + sterile neutrino, $\sigma(s) \propto (s/m_{3/2} m_R M_P)^2$
- $n = 6$: High scale susy gravitino (heavy DM), $\sigma \propto s^3/(m_{3/2} M_P)^4$
Heavy spin-2 mediator (light DM), $\sigma \propto s^3/(m_{\tilde{h}} M_P)^4$
K. Benakli et al., Phys. Rev. D 95 (2017) 095002; N. Bernal et al., Phys. Rev. D 97 (2018) 115020
- $n > 6$: Vector non-Abelian DM with heavy Z'
G. Bhattacharyya, M. Dutra, Y. Mambrini, M. Pierre, Phys. Rev. D 98 (2018) 035038

3. Lyman- α



4. The end?

1. Reheating

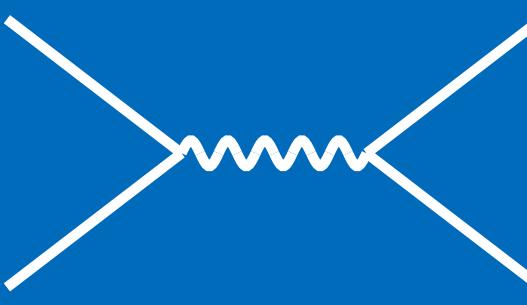


Thermal freeze-in

$n < 6$:

$$\Omega_{\chi}^{(n)} h^2 \simeq \frac{g_A g_B g_\psi g_\chi \sqrt{c} 2^{n+3} \Gamma(\frac{n}{2} + 3)^2 \zeta(\frac{n}{2} + 3)^2 \mathcal{S}(n)}{(6 - n)(n + 4)} \left(\frac{106.75}{g_{*s}^{\text{reh}}} \right)^{3/2} \left(\frac{T_{\text{reh}}}{\Lambda} \right)^{n+1} \left(\frac{10^{16} \text{ GeV}}{\Lambda} \right) \left(\frac{m_{\text{DM}}}{1 \text{ keV}} \right)$$

2. Freeze-in



$n = 6$:

$$\Omega_{\chi}^{(6)} h^2 = g_A g_B g_\psi g_\chi \sqrt{c} \mathcal{S}(6) \left(\frac{106.75}{g_{*s}^{\text{reh}}} \right)^{3/2} \left(\frac{m_{\text{DM}}}{1.2 \text{ keV}} \right) \left(\frac{T_{\text{reh}}}{10^6 \text{ GeV}} \right)^7 \left(\frac{10^8 \text{ GeV}}{\Lambda} \right)^8 \ln \left(\frac{T_{\text{max}}}{T_{\text{reh}}} \right)$$

3. Lyman- α

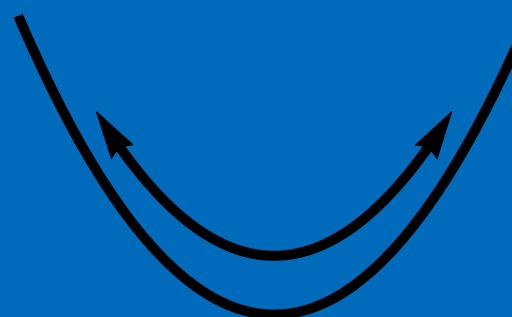


$n > 6$:

$$\begin{aligned} \Omega_{\chi}^{(n)} h^2 \simeq & \frac{g_A g_B g_\psi g_\chi \sqrt{c} 2^{n+3} \Gamma(\frac{n+4}{2}) \Gamma(\frac{n+6}{2})}{n - 6} \left(\frac{106.75}{g_{*s}^{\text{reh}}} \right)^{3/2} \\ & \times \left(\frac{T_{\text{max}}}{\Lambda} \right)^{n+1} \left(\frac{T_{\text{reh}}}{T_{\text{max}}} \right)^7 \left(\frac{10^{16} \text{ GeV}}{\Lambda} \right) \left(\frac{m_{\text{DM}}}{1.8 \text{ keV}} \right) \end{aligned}$$

4. The end?

1. Reheating

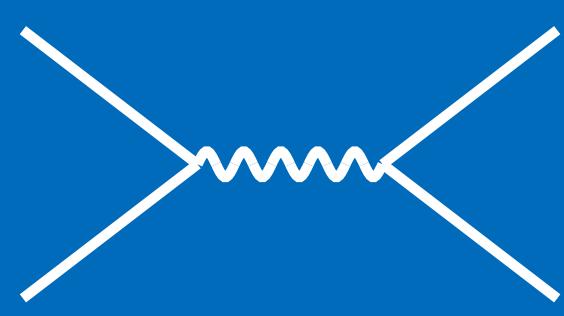


Thermal freeze-in

$n < 6$:

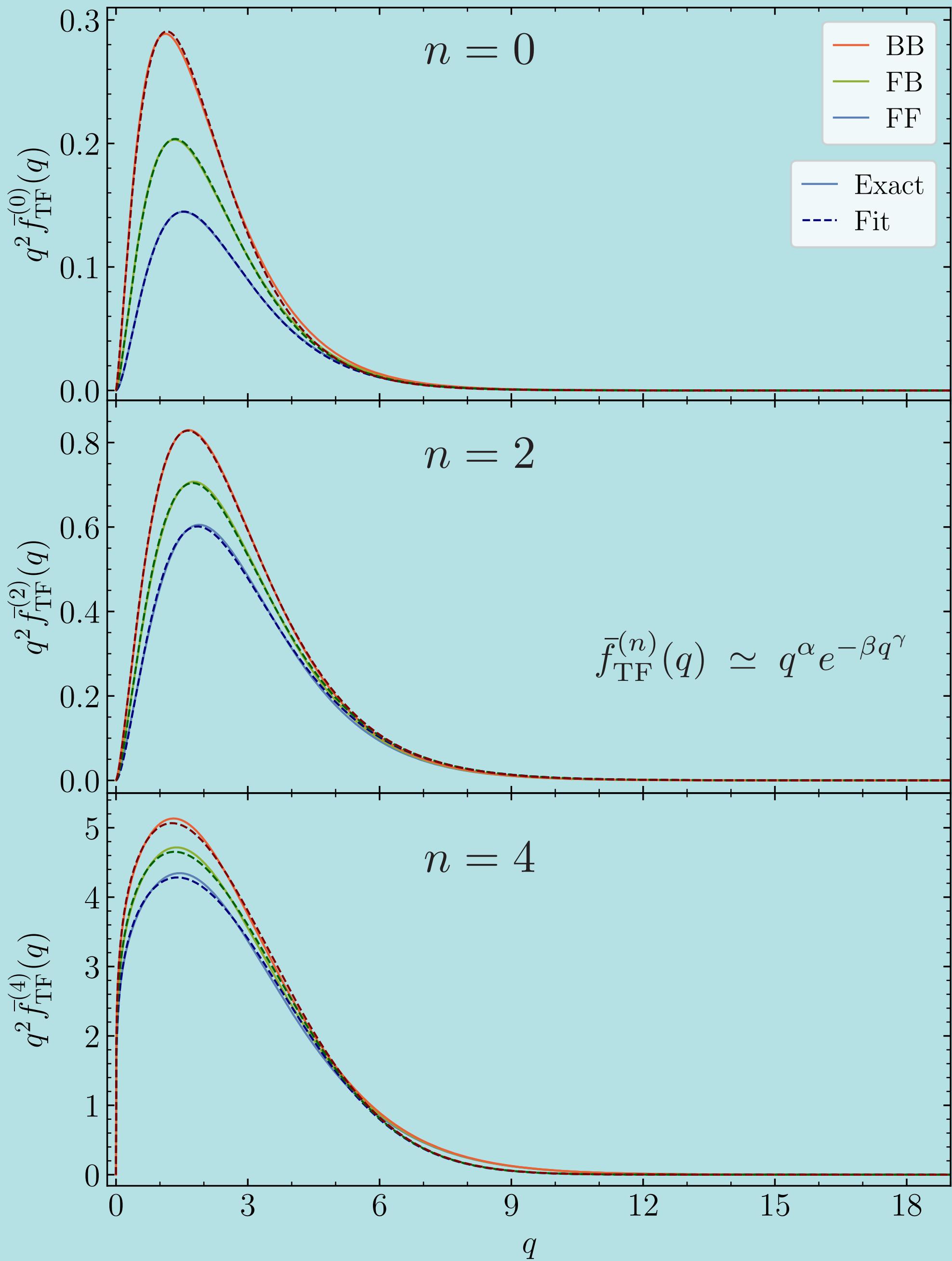
$$f_\chi(p, t) d^3 p \simeq \frac{3 \cdot 2^{n+6} \Gamma(\frac{n+4}{2}) g_A g_B g_\psi M_P T_{\text{reh}}^{n+1}}{5(2\pi)^3 \Lambda^{n+2}} \\ \times \left(\frac{6c}{g_{*s}^{\text{reh}}} \right)^{1/2} \left(\frac{a_0}{a(t)} \right)^3 T_*^3 \bar{f}_{\text{TF}}^{(n)}(q) d^3 q$$

2. Freeze-in



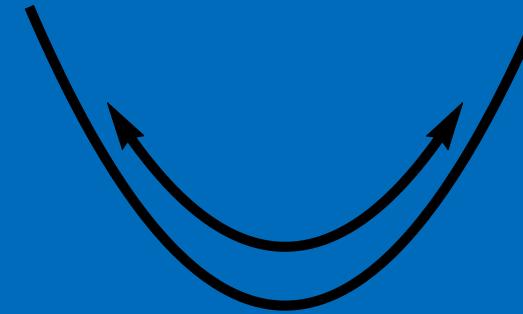
$$T_* = \left(\frac{g_{*s}^0}{g_{*s}^{\text{reh}}} \right)^{1/3} T_0$$

3. Lyman- α

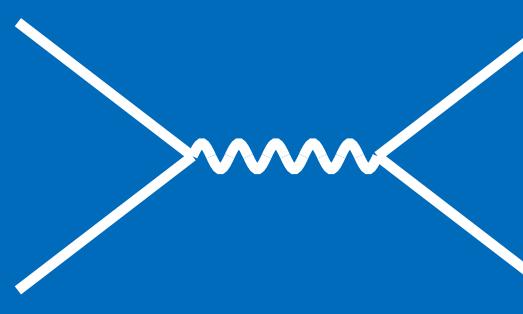


4. The end?

1. Reheating



2. Freeze-in



3. Lyman- α

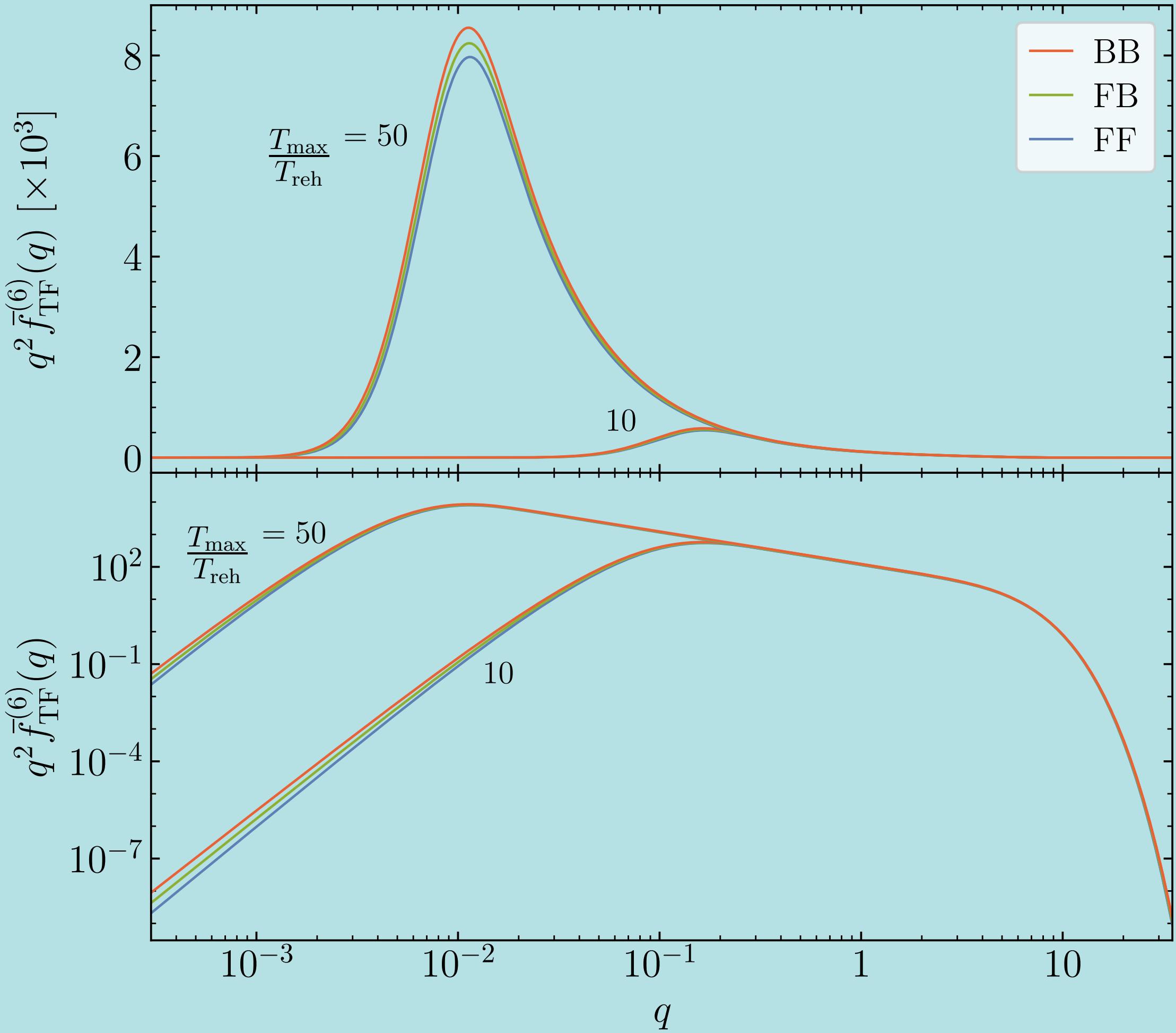


4. The end?

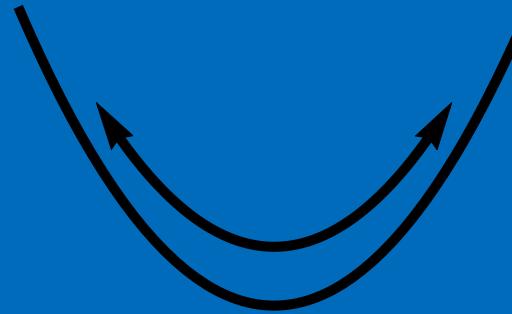
Thermal freeze-in

$n \geq 6$:

$$\bar{f}_{\text{TF}}^{(n)}(q) \simeq q^{\frac{3}{5}(1-n)} \left[\Gamma \left(\frac{11}{10}n - \frac{3}{5}, q \right) - \Gamma \left(\frac{11}{10}n - \frac{3}{5}, q \left(\frac{T_{\max}}{T_{\text{reh}}} \right)^{5/3} \right) \right]$$

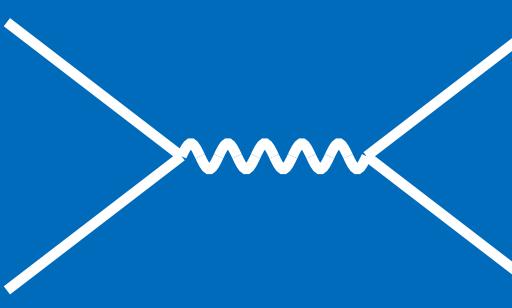


1. Reheating



Non-thermal freeze-in

2. Freeze-in

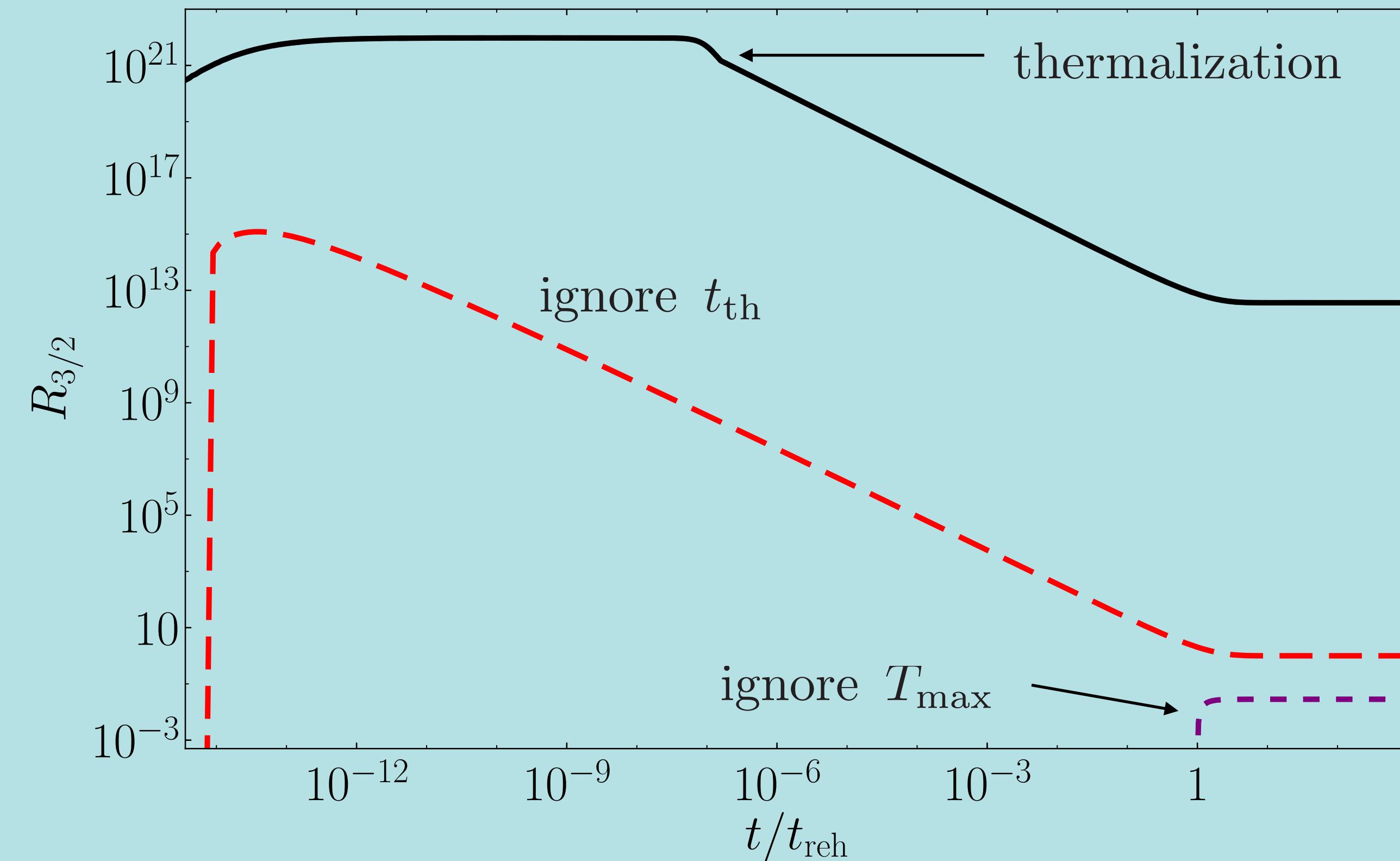


3. Lyman- α



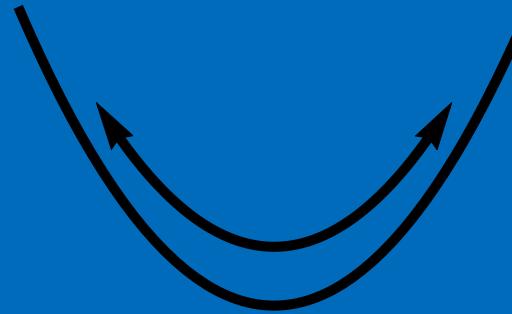
4. The end?

High scale supersymmetry breaking: $m_{\text{susy}} \gg m_\Phi \gg m_{3/2}$ ($n = 6$)

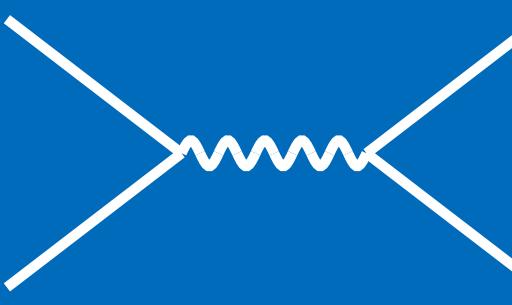


$$R_{3/2} \equiv \frac{n_{3/2}^{\text{NT}}}{n_{3/2}^{\text{T}}}$$

1. Reheating



2. Freeze-in

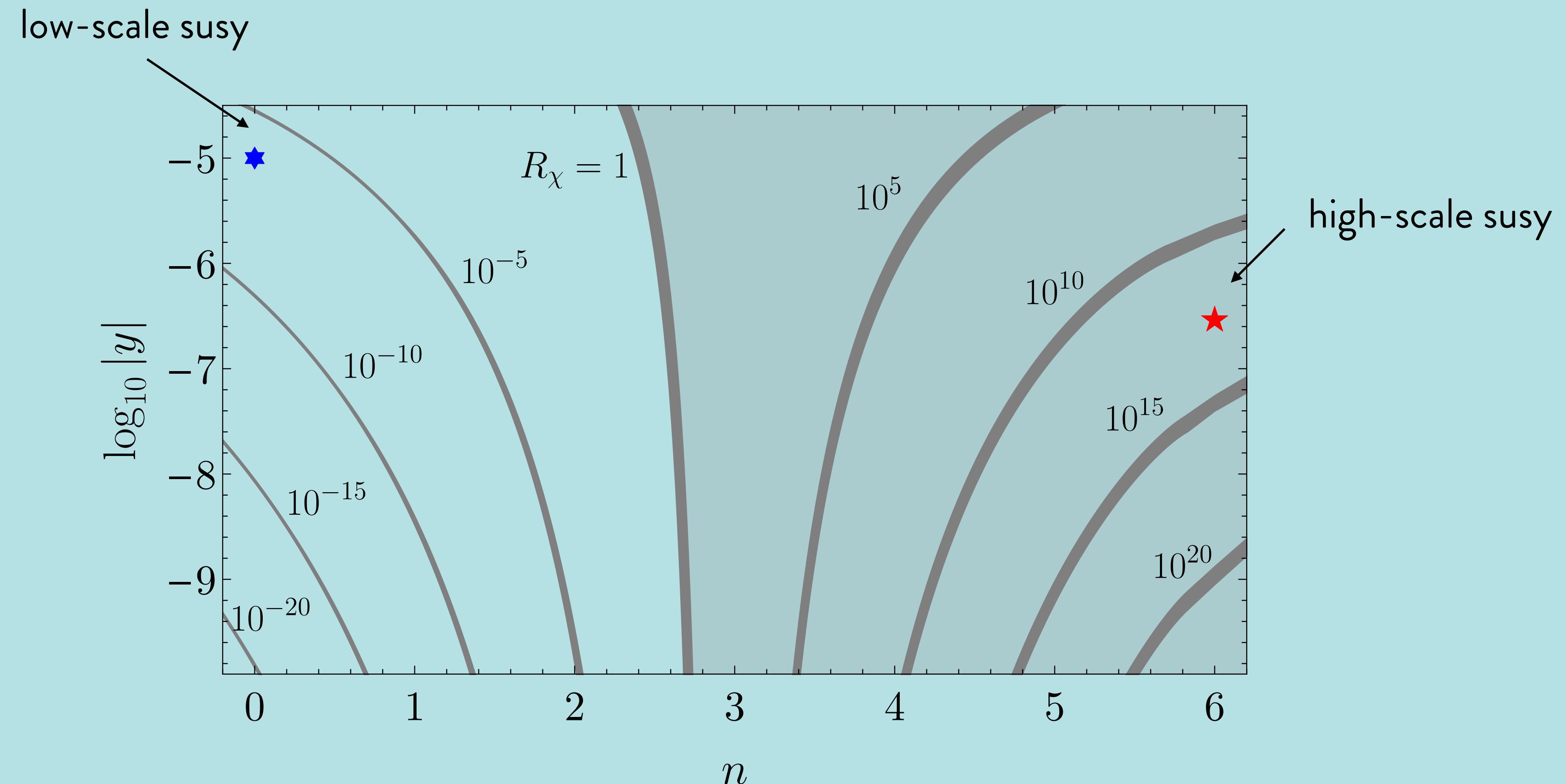


3. Lyman- α



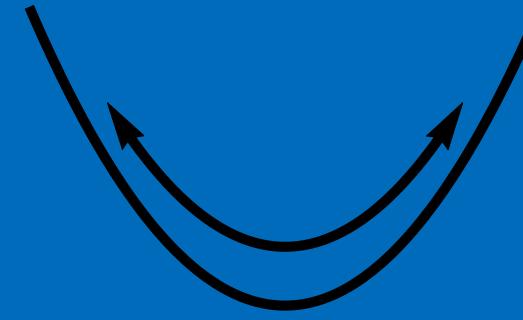
4. The end?

Non-thermal freeze-in



$$\Omega_\chi \propto \frac{m_\Phi^{n-2} M_P T_{\text{reh}}^3}{M^{n+2}} (\Gamma_\Phi t_{\text{th}})$$

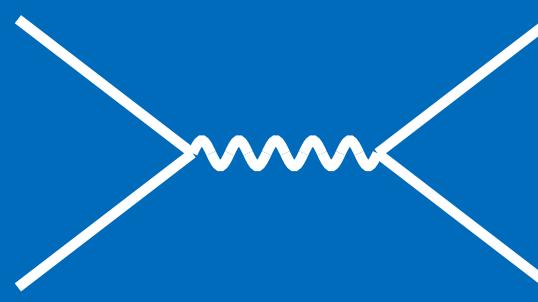
1. Reheating



Non-thermal freeze-in ($n = 4$)

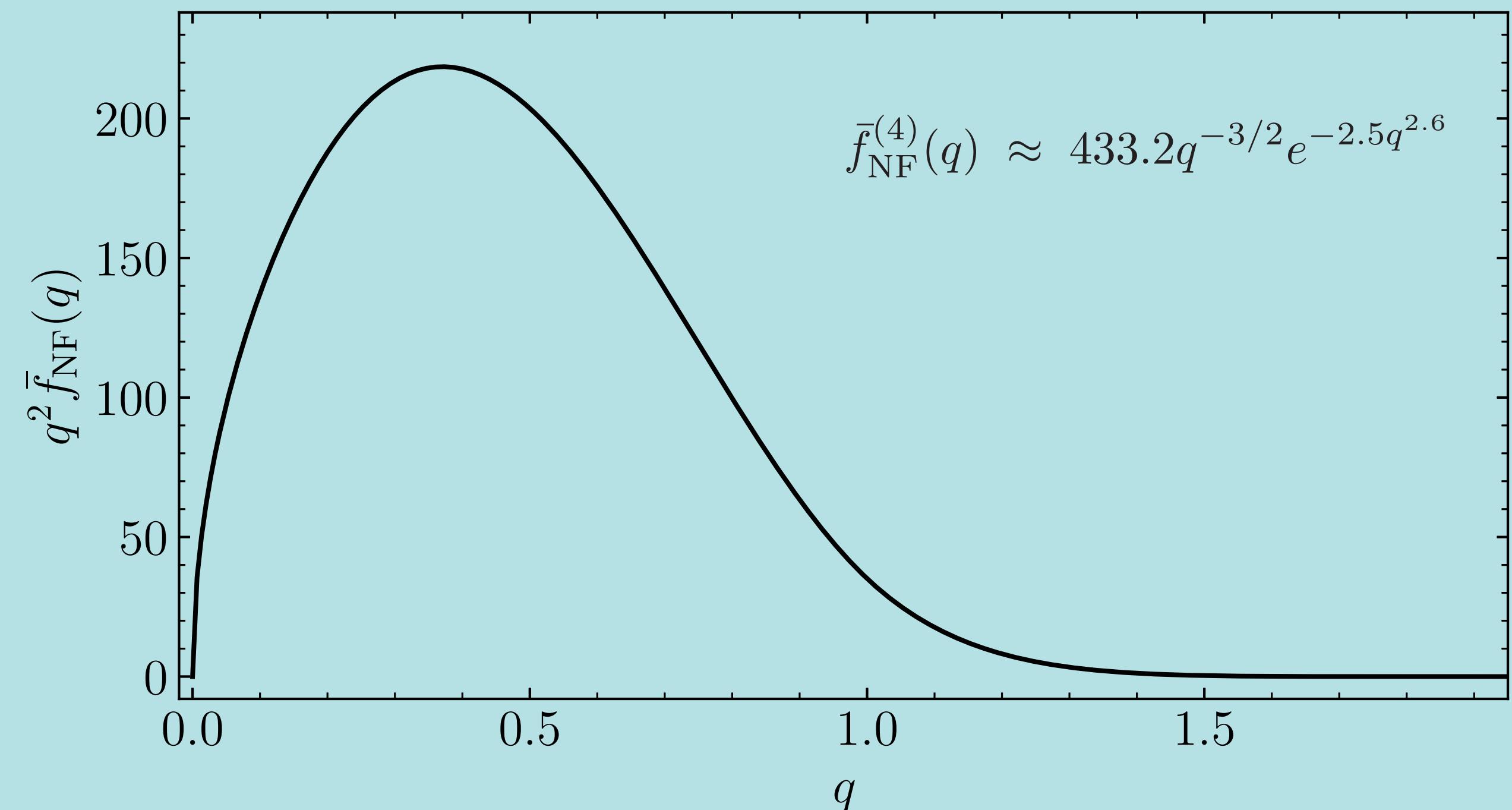
$$f_\chi(p, t) d^3 p \simeq \frac{256\pi^2 g_\psi}{15015 \Lambda^6} \left(\frac{\pi^2 c g_{*s}^{\text{reh}}}{24} \right)^{13/10} \left(\frac{\alpha_{\text{SM}}^{16} T_{\text{reh}}^{26} M_P^{13}}{m_\Phi^9} \right)^{1/5} \left(\frac{a_0}{a(t)} \right)^3 T_*^3 \bar{f}_{\text{NF}}^{(4)}(q) d^3 q$$

2. Freeze-in



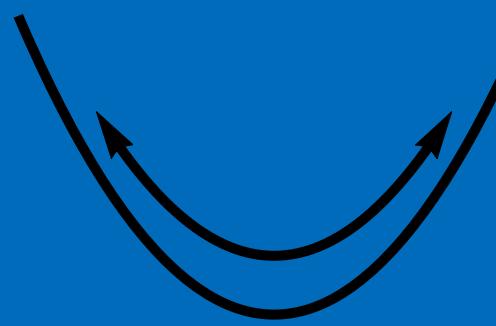
$$T_* = \frac{\alpha_{\text{SM}}^{-32/15}}{2} \left(\frac{g_{*s}^0}{g_{*s}^{\text{reh}}} \right)^{1/3} \left(\frac{\pi^2 c g_{*s}^{\text{reh}}}{24} \right)^{2/15} \left(\frac{m_\Phi}{T_{\text{reh}}} \right)^{7/15} \left(\frac{m_\Phi}{M_P} \right)^{16/15} T_0$$

3. Lyman- α

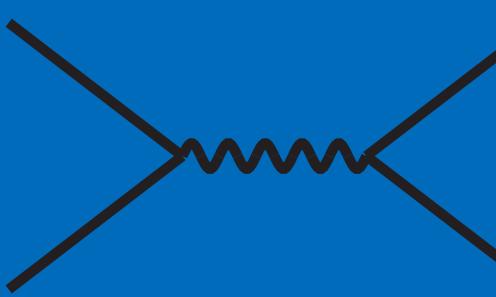


4. The end?

1. Reheating



2. Freeze-in

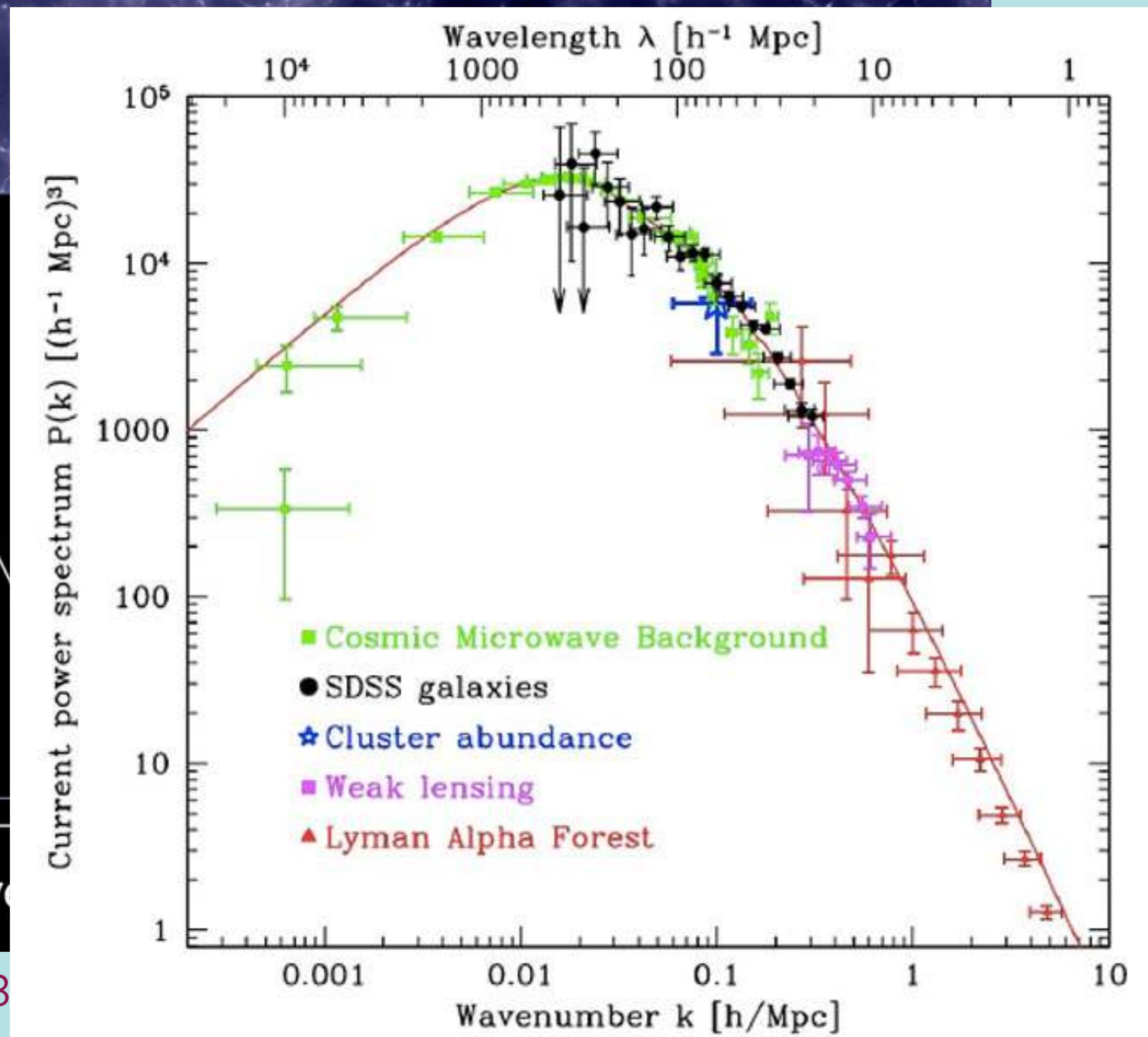
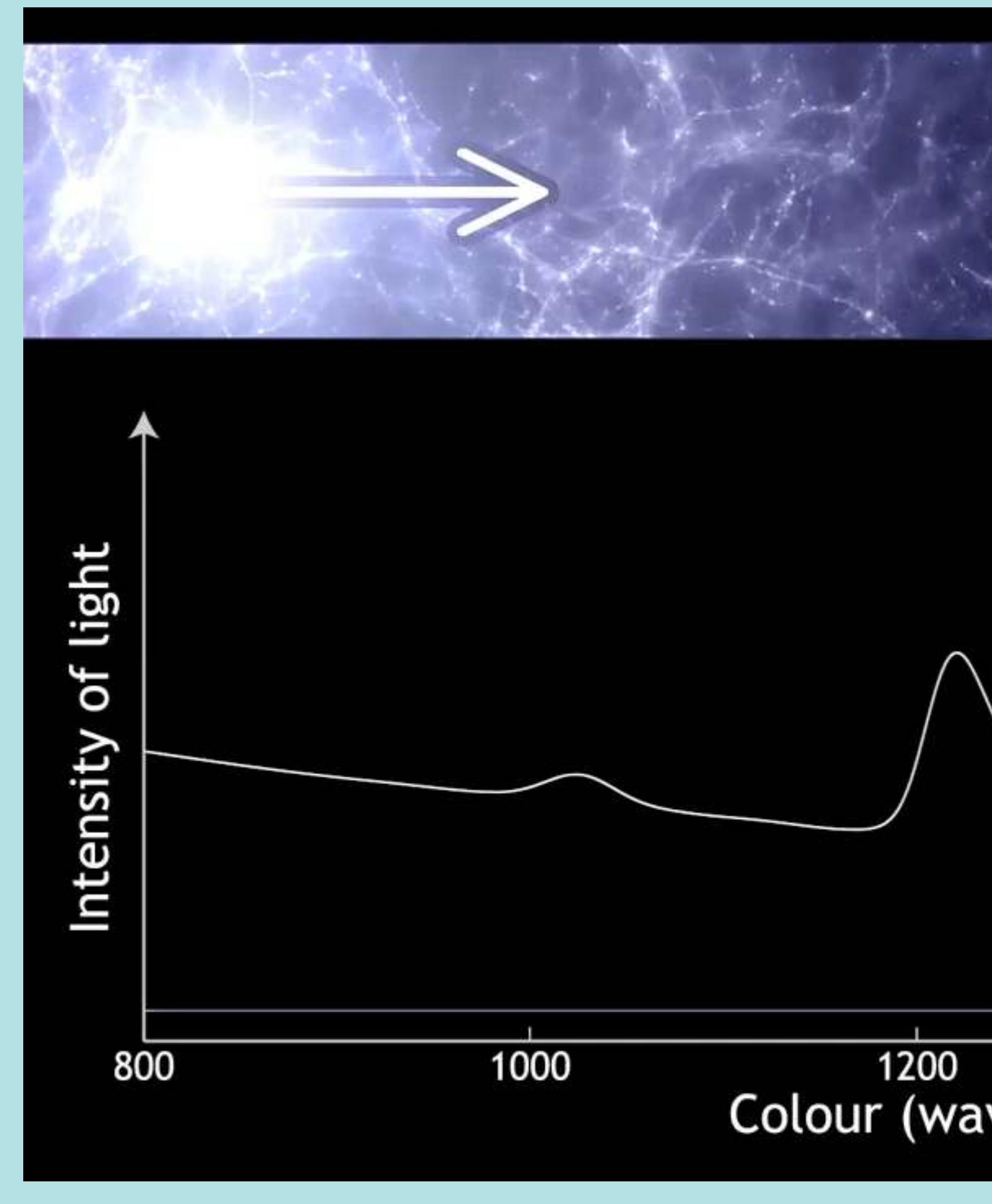


3. Lyman- α



4. The end?

The Lyman- α constraint on Warm Dark Matter

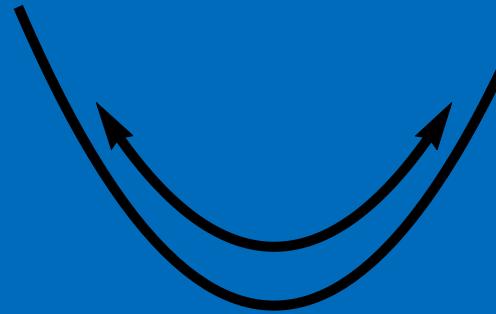


V. Iršič, et al., Mon.Not.Roy.Astron.Soc. 466 (2017) 4, 433

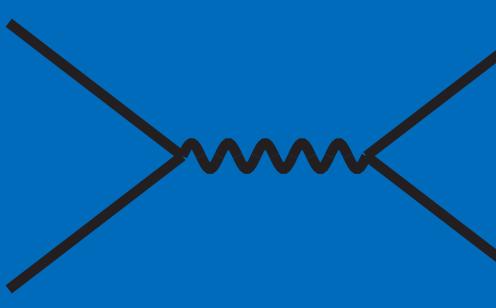
N. Palanque-Delabrouille et al., Astron. Astrophys. 559 (2013) A85;

S. Chabanier et al., JCAP 07 (2019) 017

1. Reheating



2. Freeze-in



3. Lyman- α

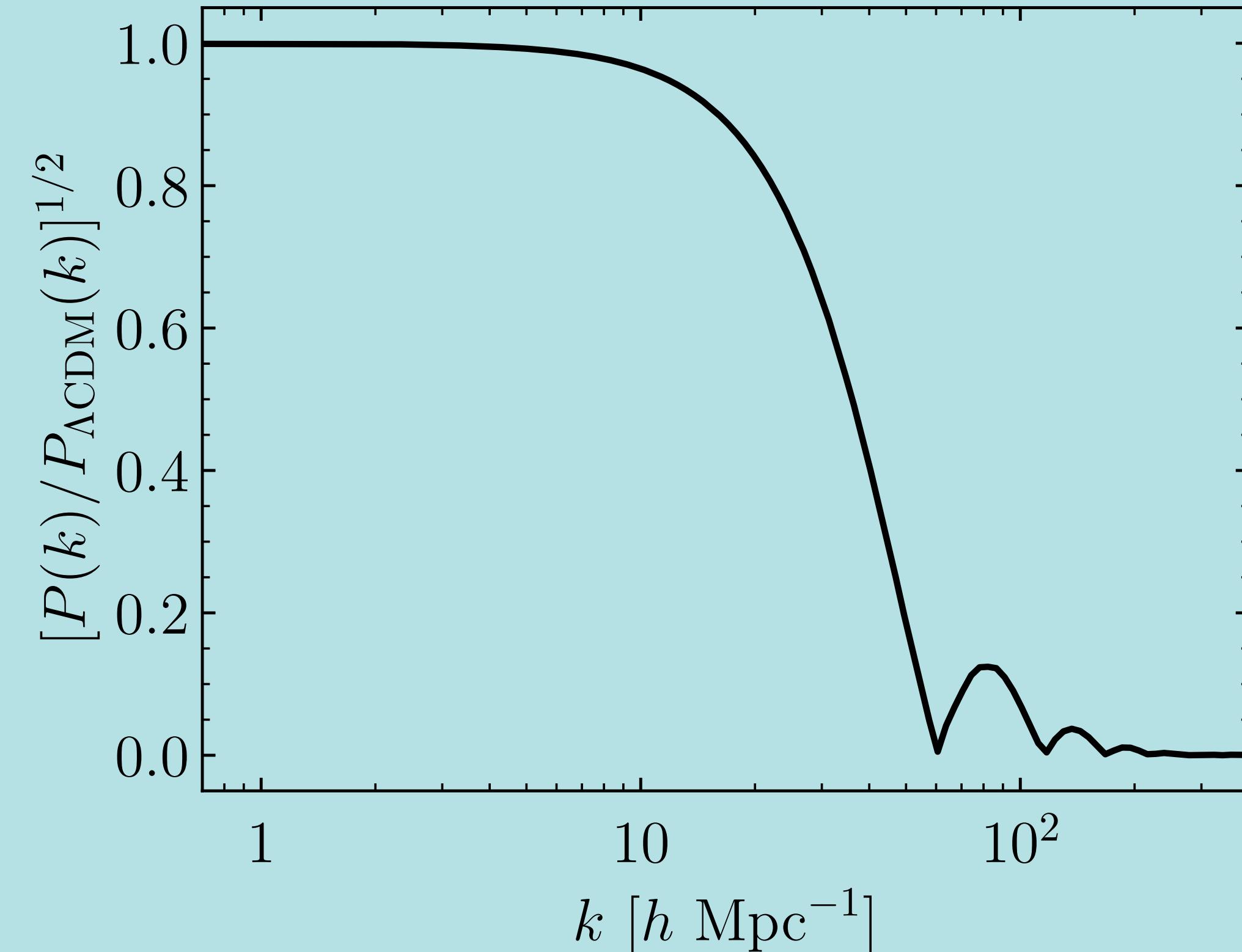


4. The end?

The perturbed Boltzmann equation

$$f(x, \mathbf{p}, \tau) = f_0(|\mathbf{p}|, \tau)[1 + \Psi(x, \mathbf{p}, \tau)] \longrightarrow$$

$$\ddot{\delta} + \mathcal{H}\dot{\delta} - \frac{3}{2}\mathcal{H}^2 \left(1 - w \frac{10}{9} \frac{k^2}{\mathcal{H}^2}\right) \delta = 0$$



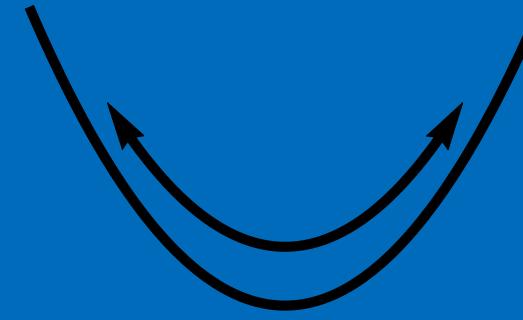
$$k_{\text{FS}}(a) = \frac{9}{10} \frac{\mathcal{H}^2}{w}$$

$$k_H(a) \equiv \left[\int_0^a \frac{1}{k_{\text{FS}}(\tilde{a})} \frac{d\tilde{a}}{\tilde{a}} \right]^{-1}$$

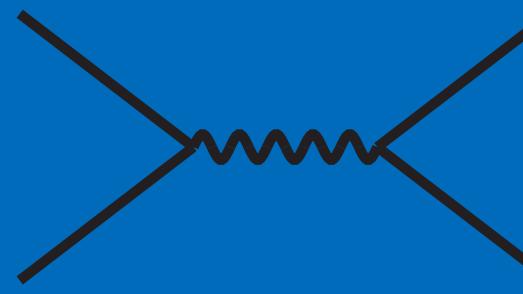
$$w \simeq \frac{\delta P}{\delta \rho} = \frac{T_*^2}{3m_{\text{DM}}^2} \frac{\langle q^2 \rangle}{a^2}$$

J. Lesgourgues, T. Tram, JCAP 09 (2011) 032

1. Reheating



2. Freeze-in

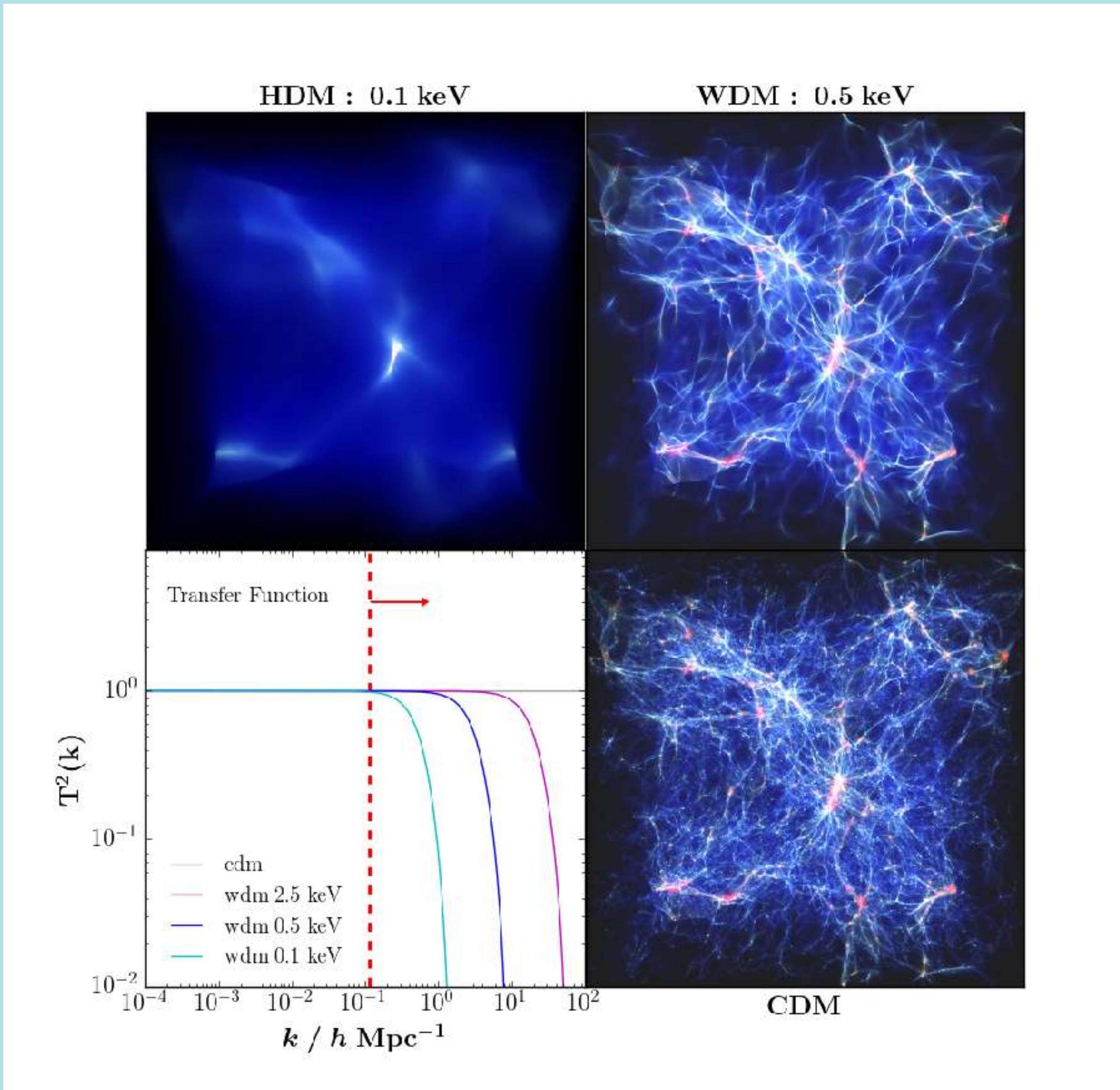


3. Lyman- α



4. The end?

The perturbed Boltzmann equation



$$\ddot{\delta} + \mathcal{H}\dot{\delta} - \frac{3}{2}\mathcal{H}^2 \left(1 - w\frac{10}{9}\frac{k^2}{\mathcal{H}^2}\right)\delta = 0$$

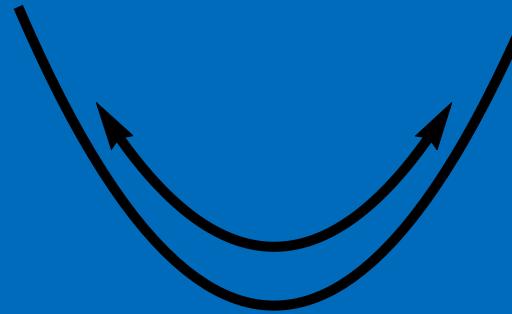
$$k_{\text{FS}}(a) = \frac{9}{10} \frac{\mathcal{H}^2}{w}$$

$$k_H(a) \equiv \left[\int_0^a \frac{1}{k_{\text{FS}}(\tilde{a})} \frac{d\tilde{a}}{\tilde{a}} \right]^{-1}$$

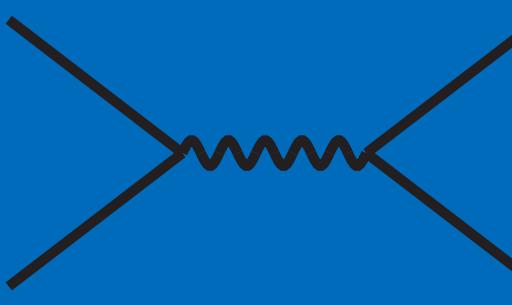
J. Baur et al., JCAP 08 (2016) 012

R. Murguia et al., JCAP 11 (2017) 046

1. Reheating



2. Freeze-in



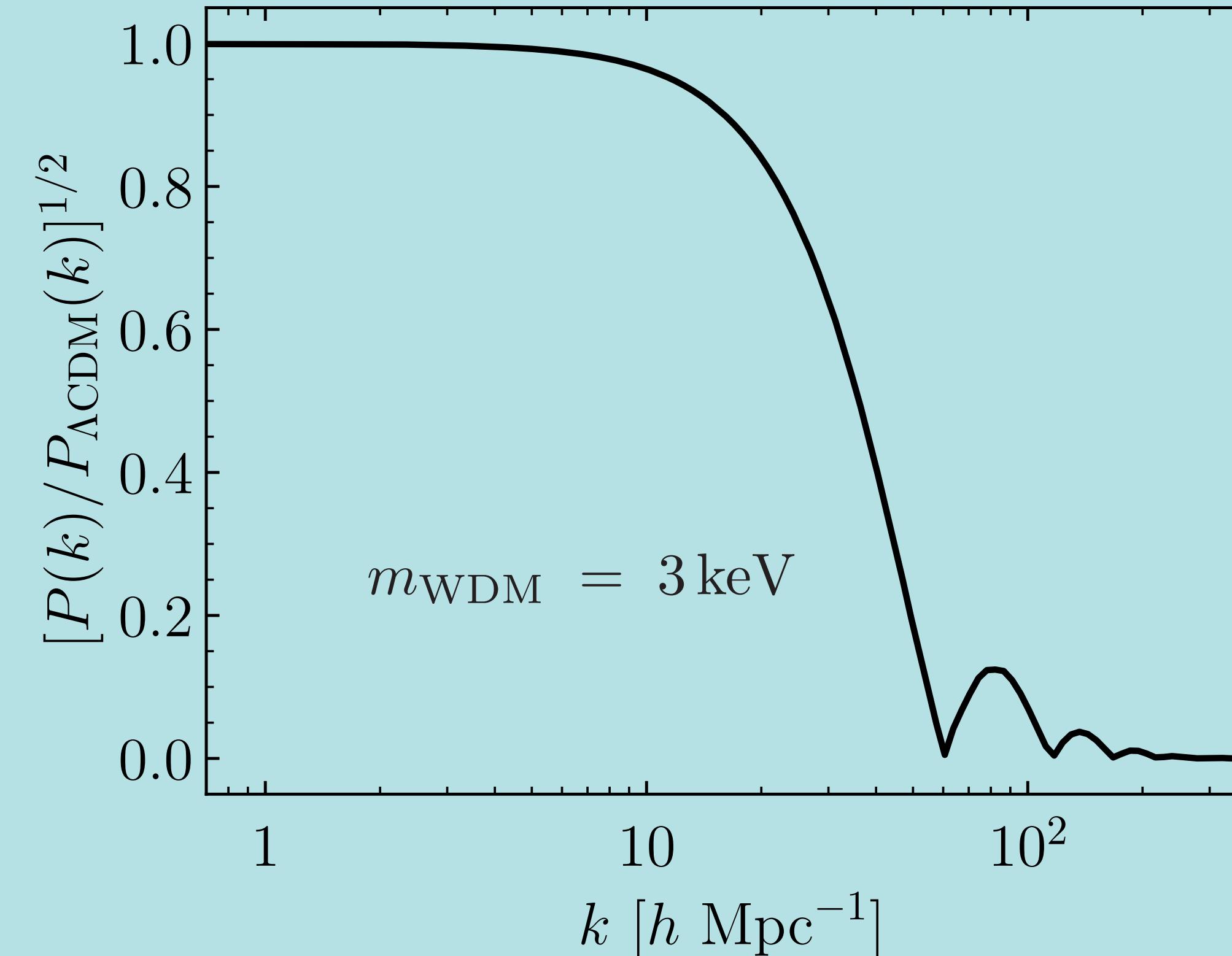
3. Lyman- α



4. The end?

The perturbed Boltzmann equation

$$f(x, \mathbf{p}, \tau) = f_0(|\mathbf{p}|, \tau)[1 + \Psi(x, \mathbf{p}, \tau)] \longrightarrow \ddot{\delta} + \mathcal{H}\dot{\delta} - \frac{3}{2}\mathcal{H}^2 \left(1 - w \frac{10}{9} \frac{k^2}{\mathcal{H}^2}\right) \delta = 0$$



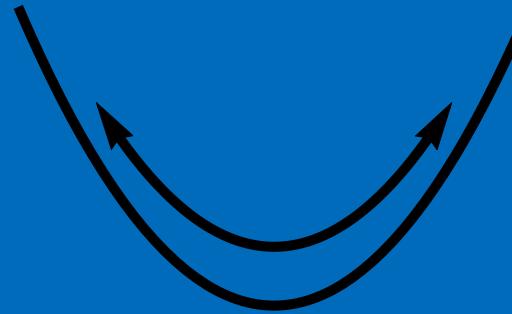
$$k_H(a=1) \simeq 3.5 h \text{ Mpc}^{-1}$$

$$k_{\text{FS}}(a) = \frac{9}{10} \frac{\mathcal{H}^2}{w}$$
$$k_H(a) \equiv \left[\int_0^a \frac{1}{k_{\text{FS}}(\tilde{a})} \frac{d\tilde{a}}{\tilde{a}} \right]^{-1}$$

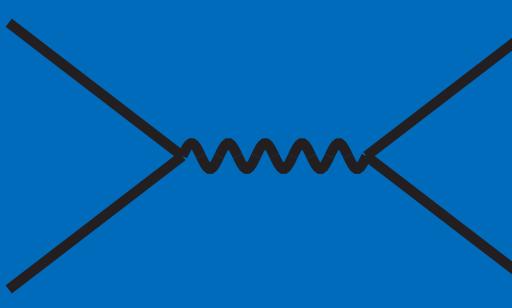
$$w \simeq \frac{\delta P}{\delta \rho} = \frac{T_*^2}{3m_{\text{DM}}^2} \frac{\langle q^2 \rangle}{a^2}$$

$$w_{\text{WDM}}(a) \simeq 6 \times 10^{-15} a^{-2} \left(\frac{\text{keV}}{m_{\text{WDM}}} \right)^{8/3}$$

1. Reheating



2. Freeze-in



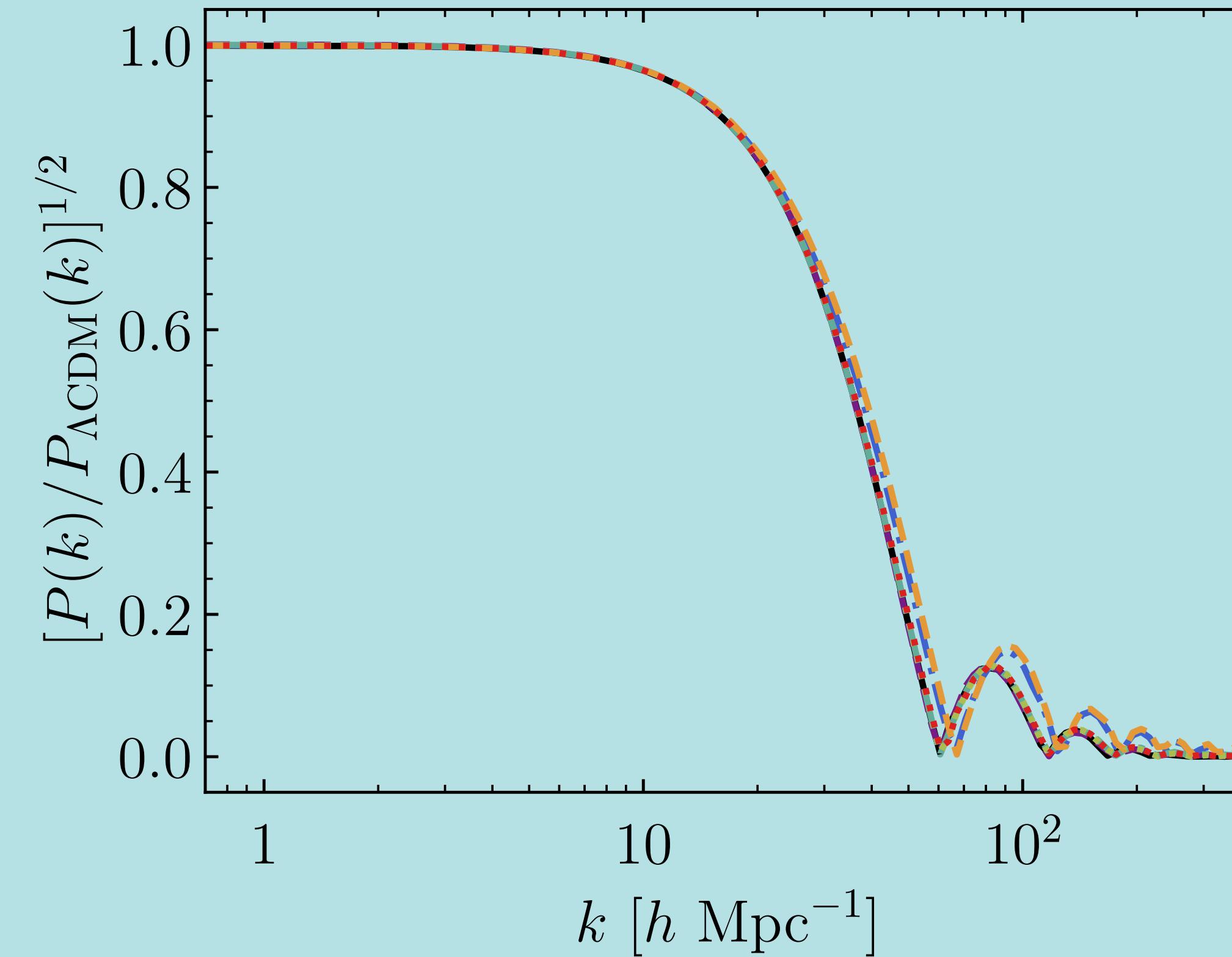
3. Lyman- α



4. The end?

The perturbed Boltzmann equation

$$f(x, \mathbf{p}, \tau) = f_0(|\mathbf{p}|, \tau)[1 + \Psi(x, \mathbf{p}, \tau)] \longrightarrow \ddot{\delta} + \mathcal{H}\dot{\delta} - \frac{3}{2}\mathcal{H}^2 \left(1 - w\frac{10}{9}\frac{k^2}{\mathcal{H}^2}\right)\delta = 0$$



$$w(m_{\text{DM}}) = w_{\text{WDM}}(m_{\text{WDM}})$$

$$k_{\text{FS}}(a) = \frac{9}{10} \frac{\mathcal{H}^2}{w}$$

$$k_H(a) \equiv \left[\int_0^a \frac{1}{k_{\text{FS}}(\tilde{a})} \frac{d\tilde{a}}{\tilde{a}} \right]^{-1}$$

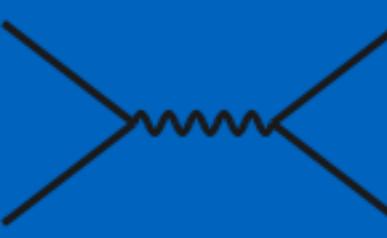
$$w \simeq \frac{\delta P}{\delta \rho} = \frac{T_*^2}{3m_{\text{DM}}^2} \frac{\langle q^2 \rangle}{a^2}$$

$$m_{\text{DM}} = m_{\text{WDM}} \left(\frac{T_*}{T_{\text{WDM}}} \right) \sqrt{\frac{\langle q^2 \rangle}{\langle q^2 \rangle_{\text{WDM}}}}$$

1. Reheating



2. Freeze-in

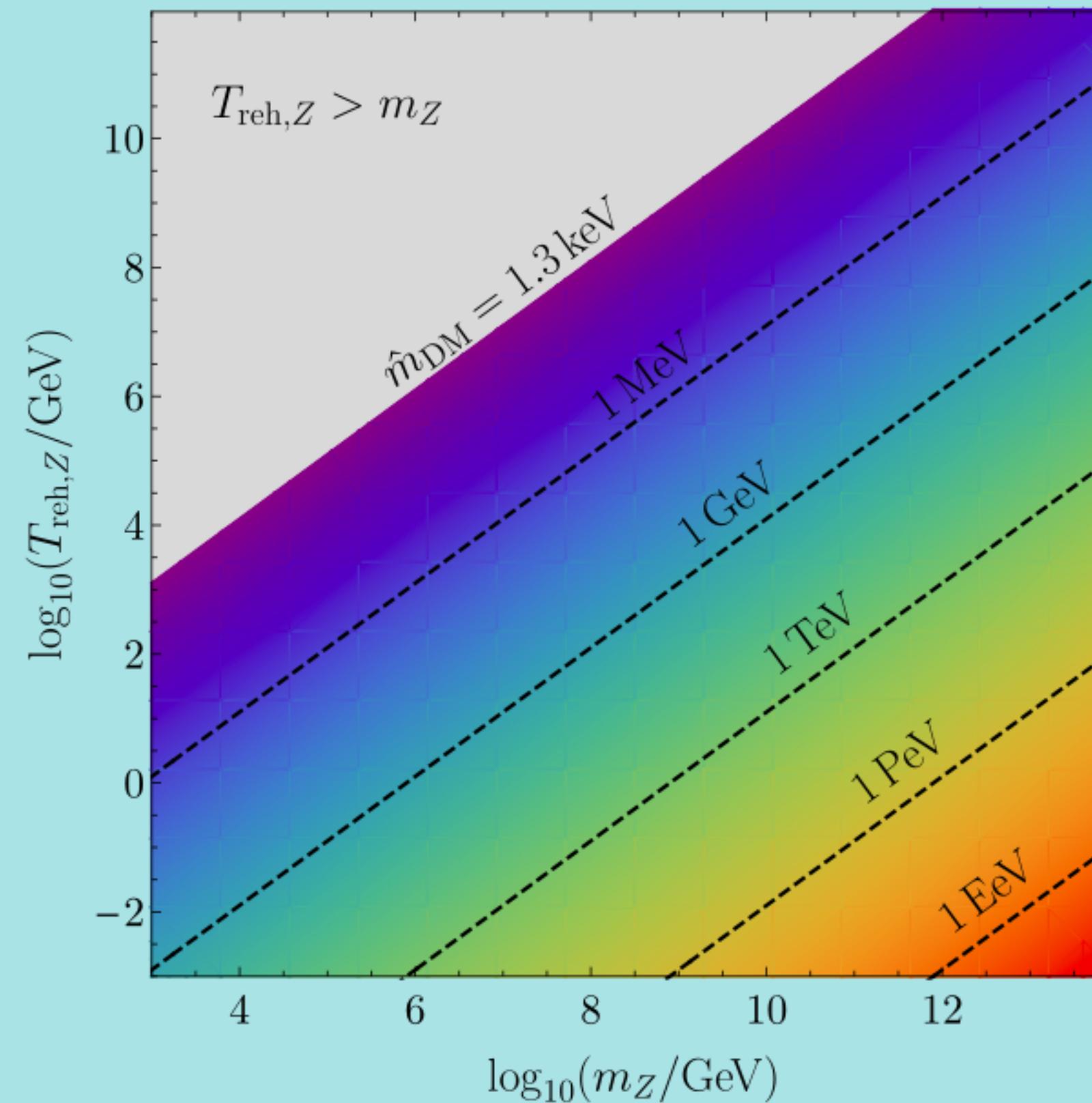


3. Lyman- α



4. The end?

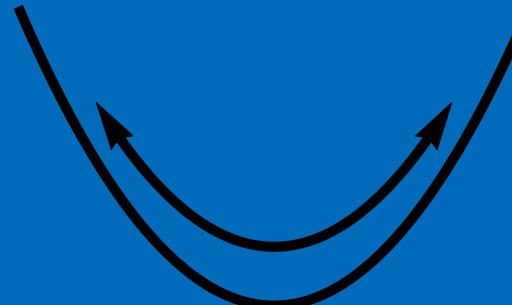
Constraints on dark matter from condensate decay (inflaton, moduli)



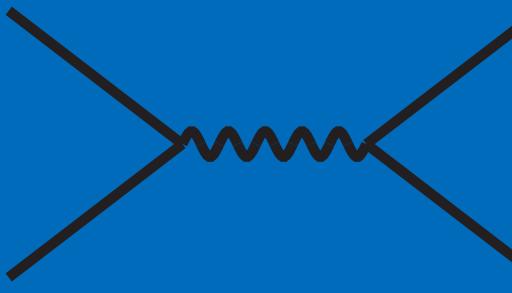
$$m_{\text{DM}} > 3.78 \text{ MeV} \left(\frac{m_{\text{WDM}}}{3 \text{ keV}} \right)^{4/3} \left(\frac{106.75}{g_{*s}^{\text{reh}}} \right)^{1/3} \times \left(\frac{m_\Phi}{3 \times 10^{13} \text{ GeV}} \right) \left(\frac{10^{10} \text{ GeV}}{T_{\text{reh}}} \right)$$

$$\text{Br}_\chi \lesssim 1.5 \times 10^{-4} \left(\frac{g_{*s}^{\text{reh}}}{106.5} \right)^{1/3} \left(\frac{3 \text{ keV}}{m_{\text{WDM}}} \right)^{4/3} \quad (\text{absolute})$$

1. Reheating



2. Freeze-in



3. Lyman- α



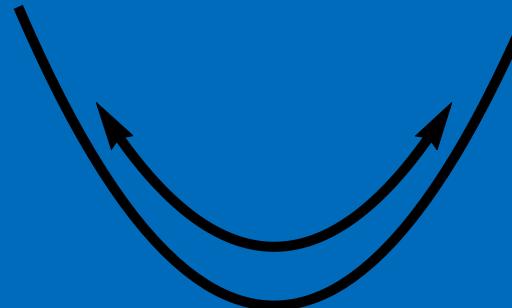
4. The end?

Constraints on dark matter from chain decay

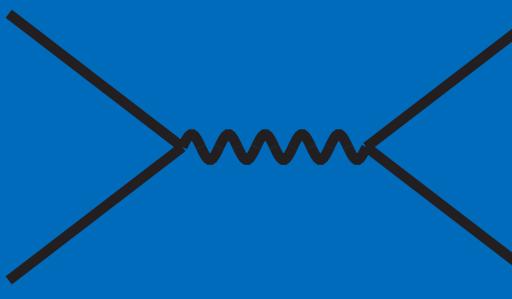
$$\Phi \rightarrow A \rightarrow \chi$$

$$m_{\text{DM}} > \left(\frac{m_{\text{WDM}}}{3 \text{ keV}} \right)^{4/3} \times \begin{cases} 1.23 \text{ MeV} \left(\frac{106.75}{g_{*s}^{\text{reh}}} \right)^{1/3} \left(\frac{m_\Phi}{3 \times 10^{13} \text{ GeV}} \right) \left(\frac{10^{10} \text{ GeV}}{T_{\text{reh}}} \right), & \text{R} \\ 2.7 \text{ MeV} \left(\frac{106.75}{g_{*s}^{\text{dec}}} \right)^{1/3} \left(\frac{m_A}{3 \times 10^{13} \text{ GeV}} \right) \left(\frac{10^{10} \text{ GeV}}{T_{\text{dec}}} \right), & \text{NR} \end{cases}$$

1. Reheating



2. Freeze-in



3. Lyman- α

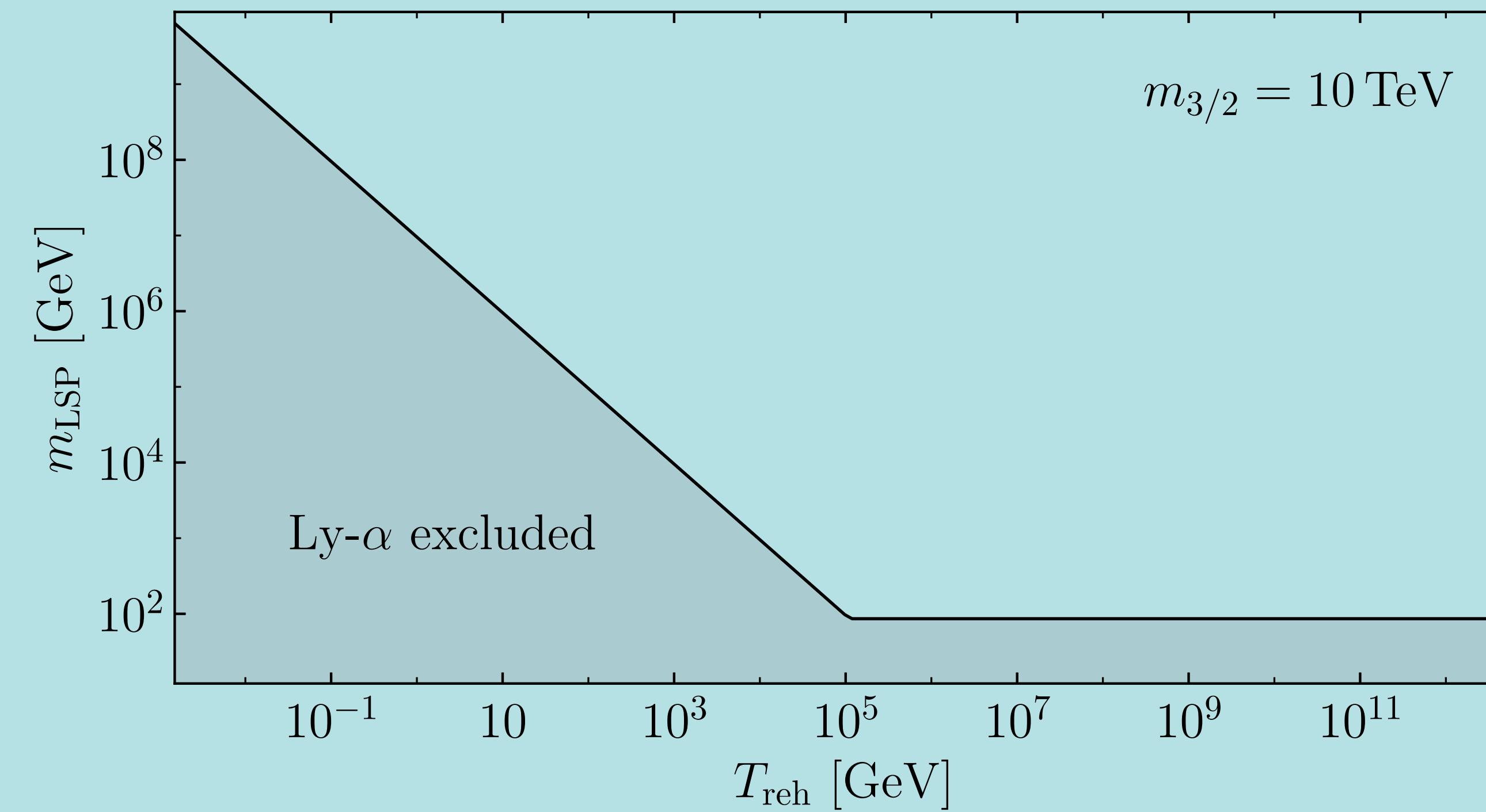


4. The end?

Constraints on dark matter from chain decay

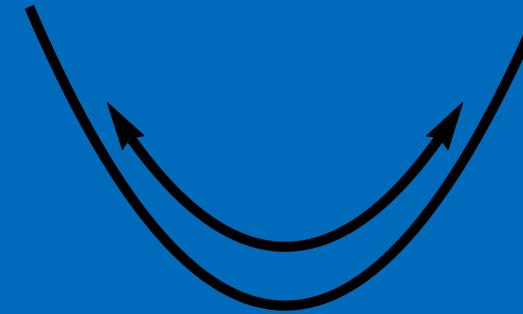
$$\Phi \rightarrow \Psi_{3/2} \rightarrow \text{LSP}$$

$$m_{\text{LSP}} > \begin{cases} 86 \text{ GeV} \left(\frac{m_{\text{WDM}}}{3 \text{ keV}} \right)^{4/3} \left(\frac{10 \text{ TeV}}{m_{3/2}} \right)^{1/2}, & T_{\text{reh}} \gg 10^5 \text{ GeV} \left(\frac{m_{3/2}}{10 \text{ TeV}} \right)^{1/2} \\ 95 \text{ GeV} \left(\frac{m_{\text{WDM}}}{3 \text{ keV}} \right)^{4/3} \left(\frac{10^5 \text{ GeV}}{T_{\text{reh}}} \right), & T_{\text{reh}} \ll 10^5 \text{ GeV} \left(\frac{m_{3/2}}{10 \text{ TeV}} \right)^{1/2} \end{cases}$$

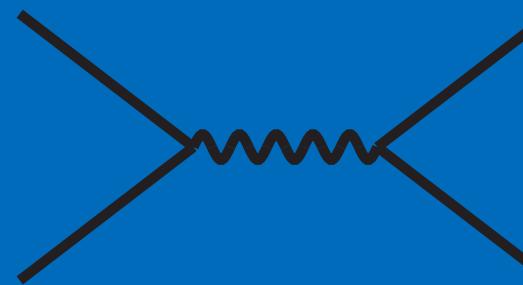


$$\Gamma_{3/2} = \frac{193}{384\pi} \frac{m_{3/2}^3}{M_P^2}$$

1. Reheating



2. Freeze-in



3. Lyman- α



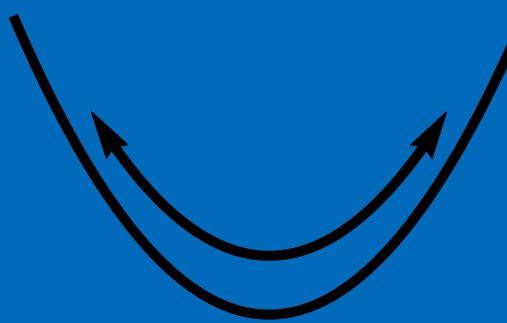
4. The end?

Constraints on dark matter from thermal freeze-in

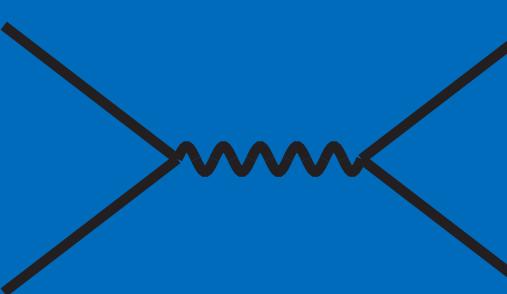
$$m_{\text{DM}} > \left(\frac{m_{\text{WDM}}}{3 \text{ keV}} \right)^{4/3} \left(\frac{106.75}{g_{*s}^{\text{reh}}} \right)^{1/3} \begin{cases} 7.3 \text{ keV}, & (n = 0, \text{ FF}) \\ 8.5 \text{ keV}, & (n = 2, \text{ FF}) \\ 8.5 \text{ keV}, & (n = 4, \text{ FF}) \end{cases}$$

$$m_{\text{DM}} > \left(\frac{m_{\text{WDM}}}{3 \text{ keV}} \right)^{4/3} \left(\frac{106.75}{g_{*s}^{\text{reh}}} \right)^{1/3} \begin{cases} 9 \text{ keV} \ln^{-1/2} \left(\frac{T_{\text{max}}}{T_{\text{reh}}} \right), & n = 6, \\ 23 \text{ keV} \left(\frac{T_{\text{reh}}}{T_{\text{max}}} \right), & n = 8, \\ 52 \text{ keV} \left(\frac{T_{\text{reh}}}{T_{\text{max}}} \right)^{5/3}, & n \geq 10. \end{cases}$$

1. Reheating



2. Freeze-in

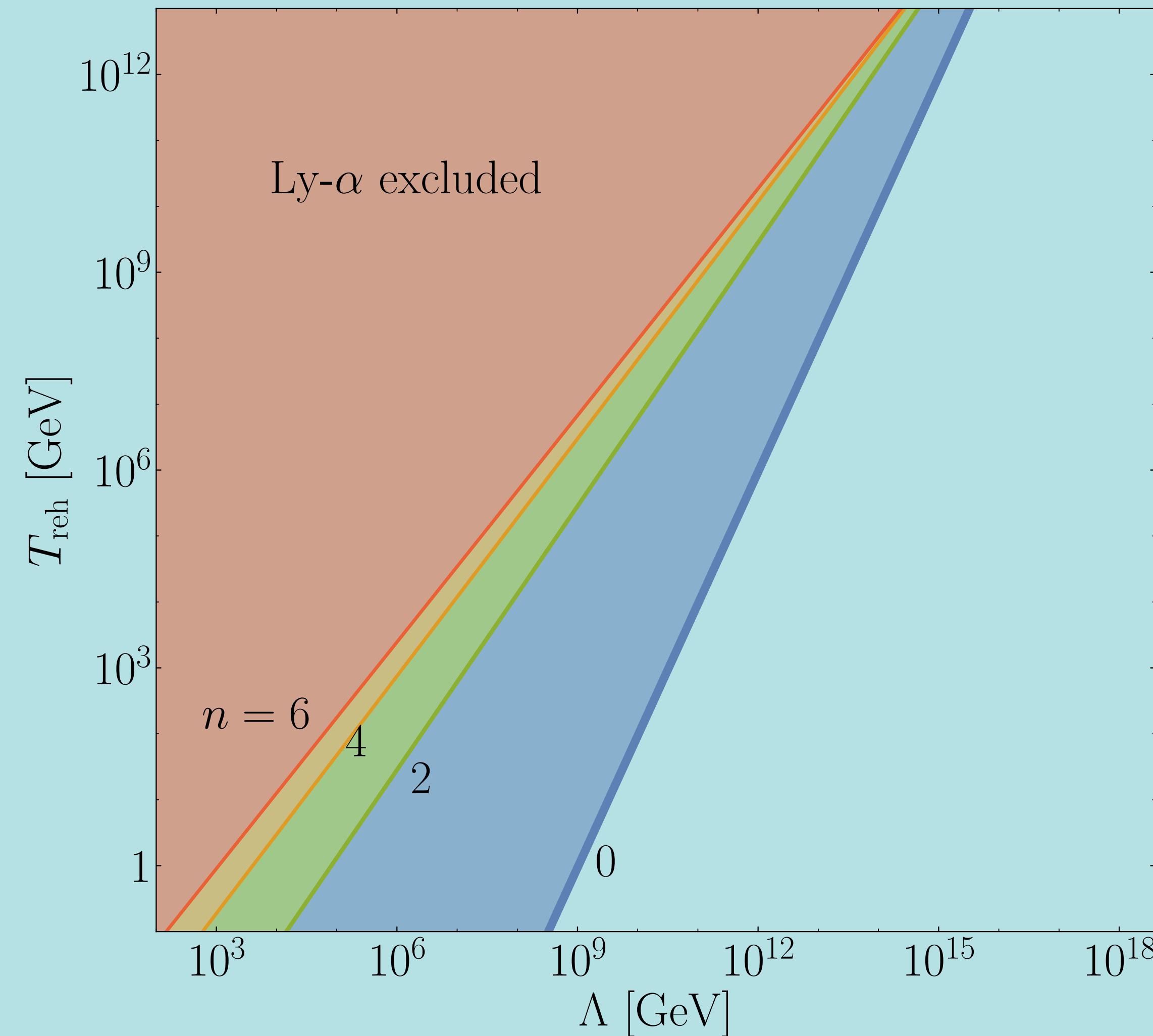


3. Lyman- α



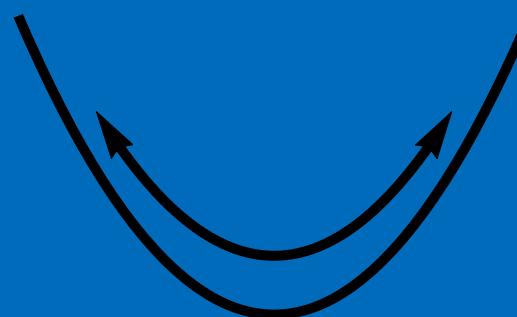
4. The end?

Constraints on dark matter from thermal freeze-in



$$\sigma(s) = \frac{s^{n/2}}{\Lambda^{n+2}}$$

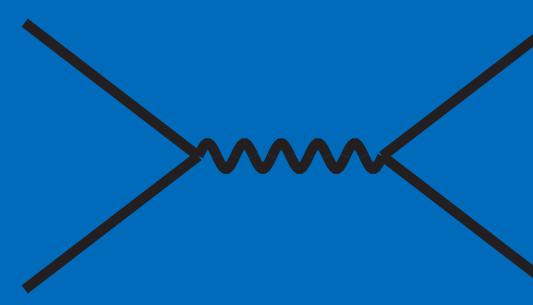
1. Reheating



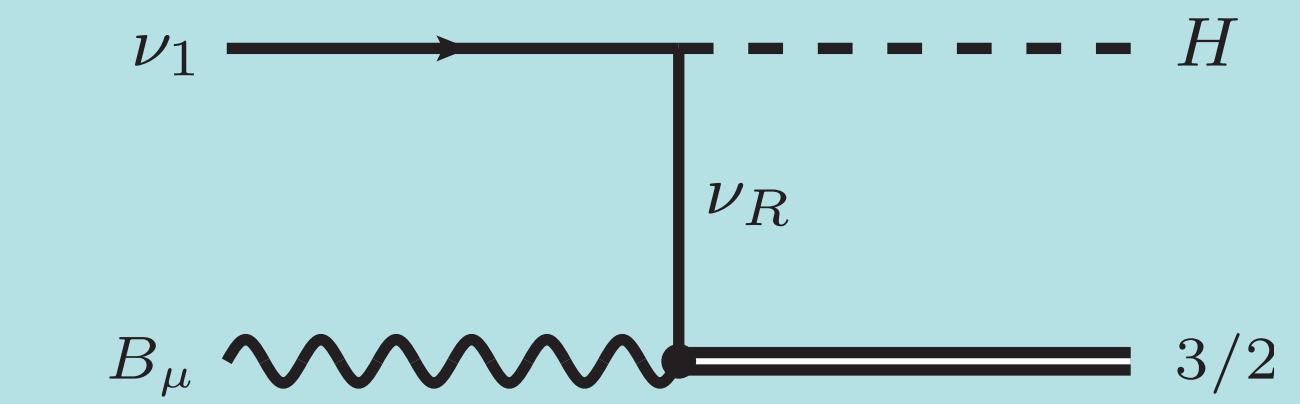
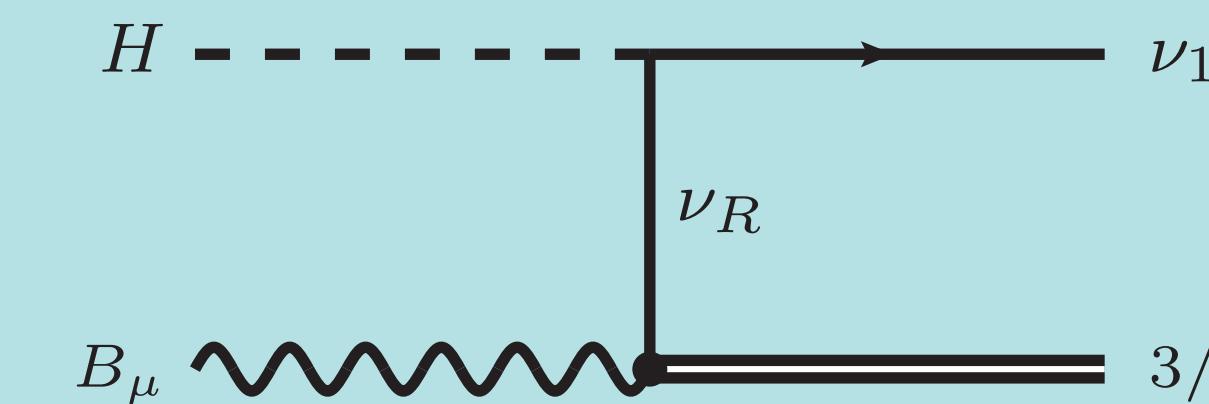
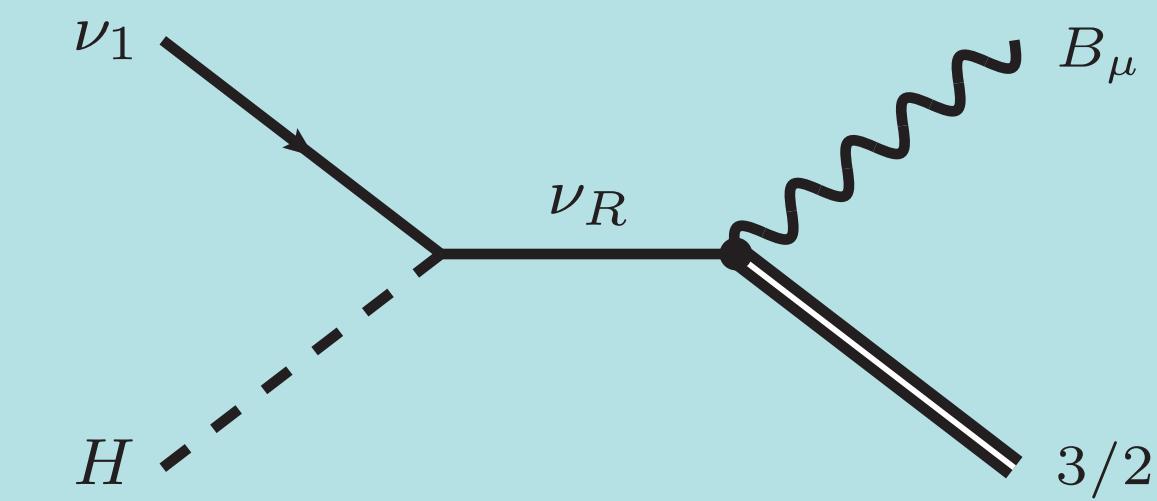
Non-thermal freeze-in: a non-supersymmetric decaying spin-3/2 particle

$$\mathcal{L} = \frac{i\alpha}{2M_P} \bar{\nu}_R \gamma^\mu [\gamma^\rho, \gamma^\sigma] \Psi_\mu F_{\rho\sigma} + \text{h.c.}$$

2. Freeze-in



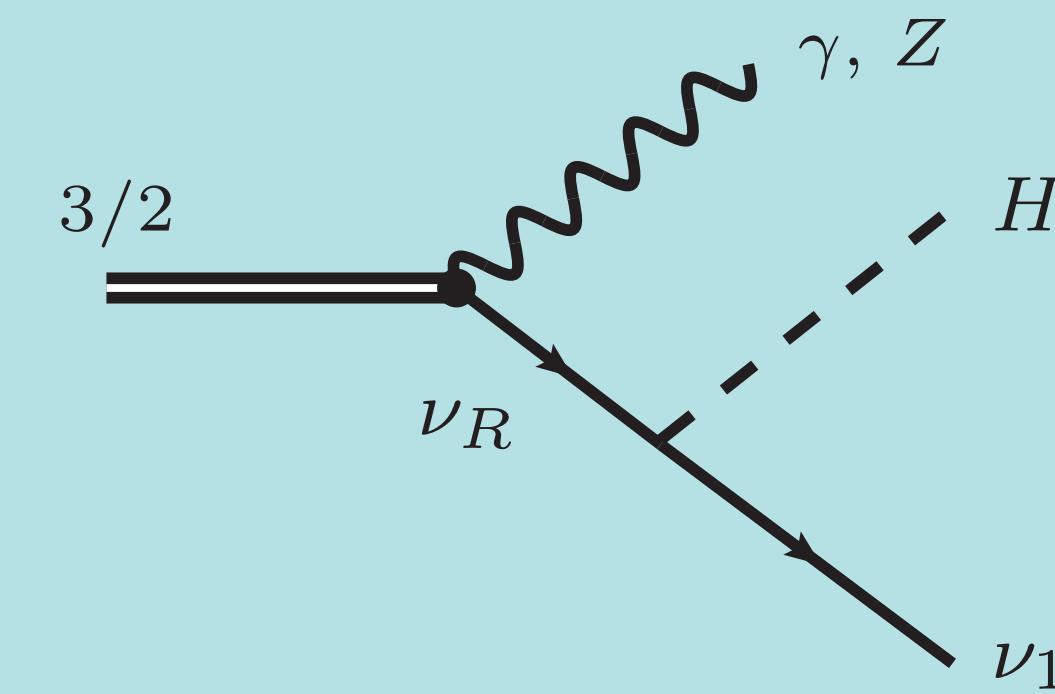
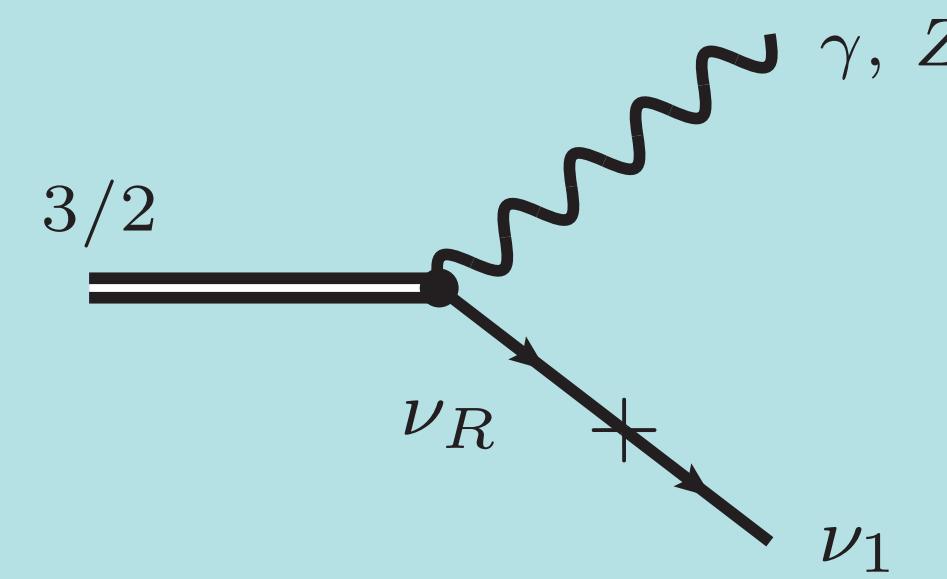
$n = 4$ freeze-in



3. Lyman-alpha

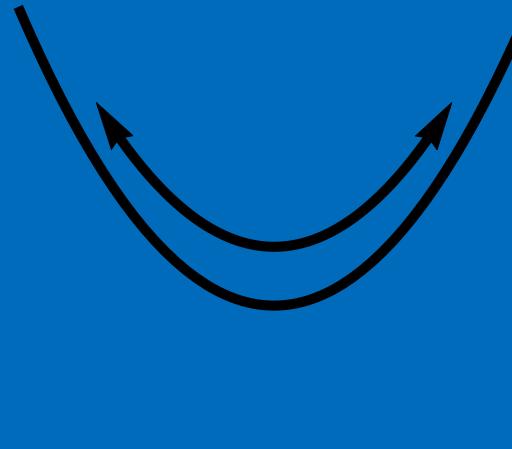


2 & 3-body decays



4. The end?

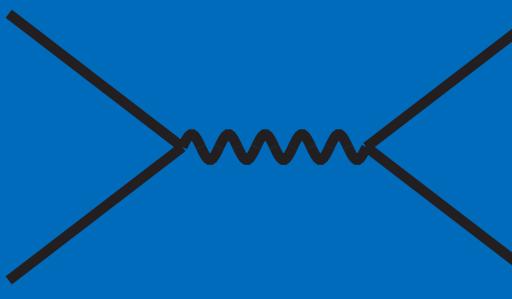
1. Reheating



Non-thermal freeze-in: a non-supersymmetric decaying spin-3/2 particle

$$\Omega_{3/2}^{\text{thermal}} h^2 \simeq 0.1 \left(\frac{\alpha_1}{1.1 \times 10^{-3}} \right)^2 \left(\frac{106.9}{g_{*s}^{\text{reh}}} \right)^{3/2} \left(\frac{T_{\text{reh}}}{10^{10} \text{ GeV}} \right)^5 \left(\frac{m_\nu}{0.15 \text{ eV}} \right) \left(\frac{10^{14} \text{ GeV}}{m_R} \right) \left(\frac{10^4 \text{ GeV}}{m_{3/2}} \right)$$

2. Freeze-in



$$\begin{aligned} \Omega_{3/2}^{\text{non-thermal}} h^2 \simeq & 0.1 \left(\frac{\alpha_1}{1.1 \times 10^{-3}} \right)^2 \left(\frac{g_{*s}^{\text{reh}}}{106.5} \right)^{7/10} \left(\frac{0.030}{\alpha_{\text{SM}}} \right)^{16/5} \left(\frac{m_\Phi}{3 \times 10^{13} \text{ GeV}} \right)^{14/5} \\ & \times \left(\frac{m_\nu}{0.15 \text{ eV}} \right) \left(\frac{10^4 \text{ GeV}}{m_{3/2}} \right) \left(\frac{10^{14} \text{ GeV}}{m_R} \right) \left(\frac{T_{\text{reh}}}{10^{10} \text{ GeV}} \right)^{19/5} \left(\frac{\text{Br}_\nu}{7 \times 10^{-4}} \right) \end{aligned}$$

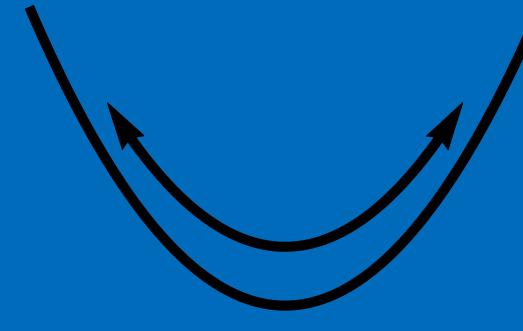
3. Lyman- α



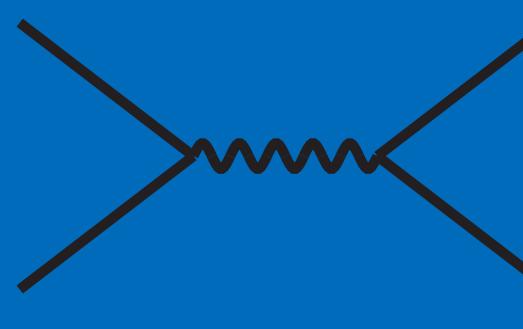
$$\begin{aligned} m_{3/2} > & 0.44 \text{ keV} \left(\frac{m_{\text{WDM}}}{3 \text{ keV}} \right)^{4/3} \left(\frac{\alpha_{\text{SM}}}{0.03} \right)^{-32/15} \left(\frac{c}{3/5} \right)^{2/15} \\ & \times \left(\frac{106.75}{g_{*s}^{\text{reh}}} \right)^{1/5} \left(\frac{10^{10} \text{ GeV}}{T_{\text{reh}}} \right)^{7/15} \left(\frac{m_\Phi}{3 \times 10^{13} \text{ GeV}} \right)^{23/15} \end{aligned}$$

4. The end?

1. Reheating



2. Freeze-in

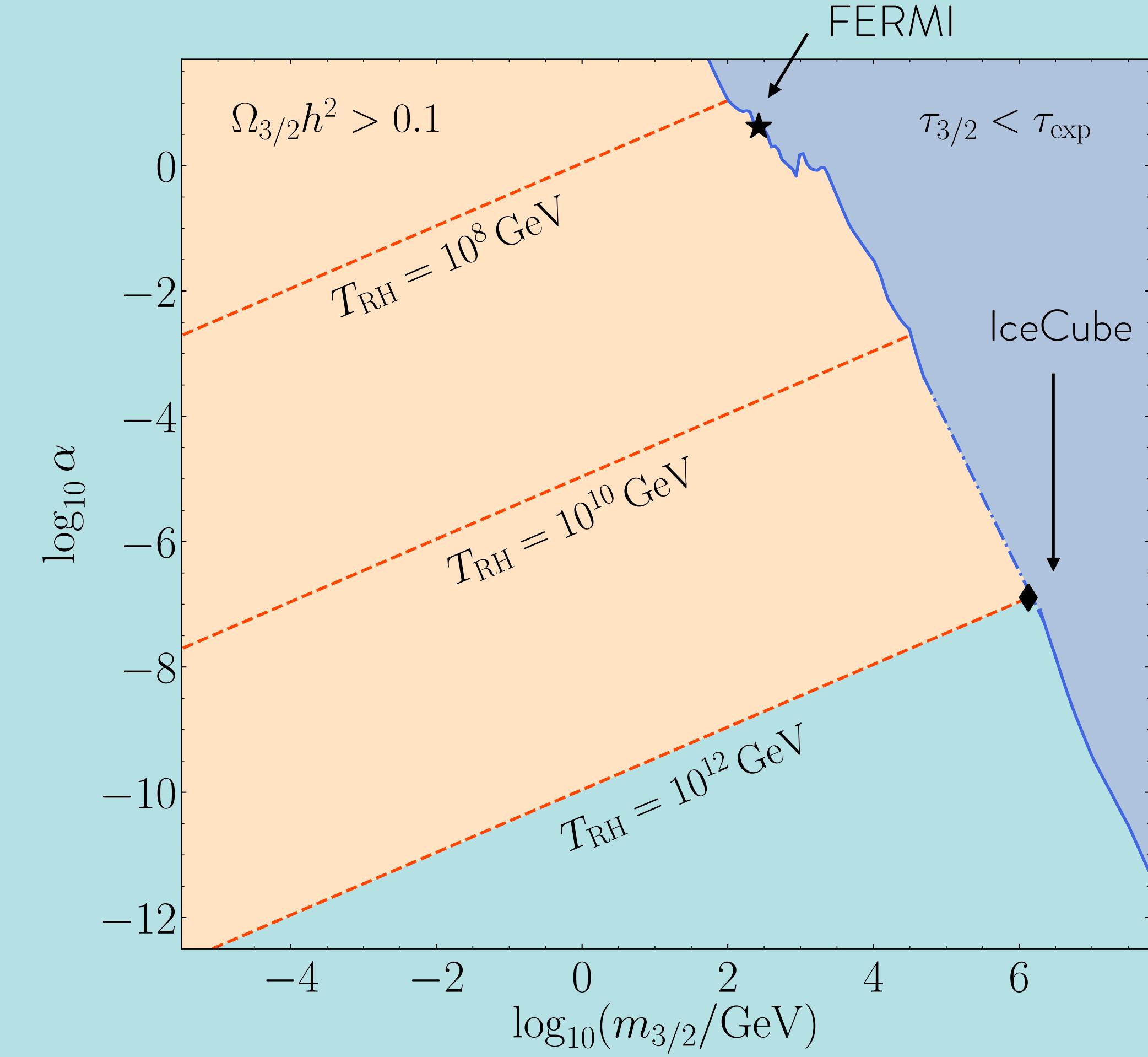
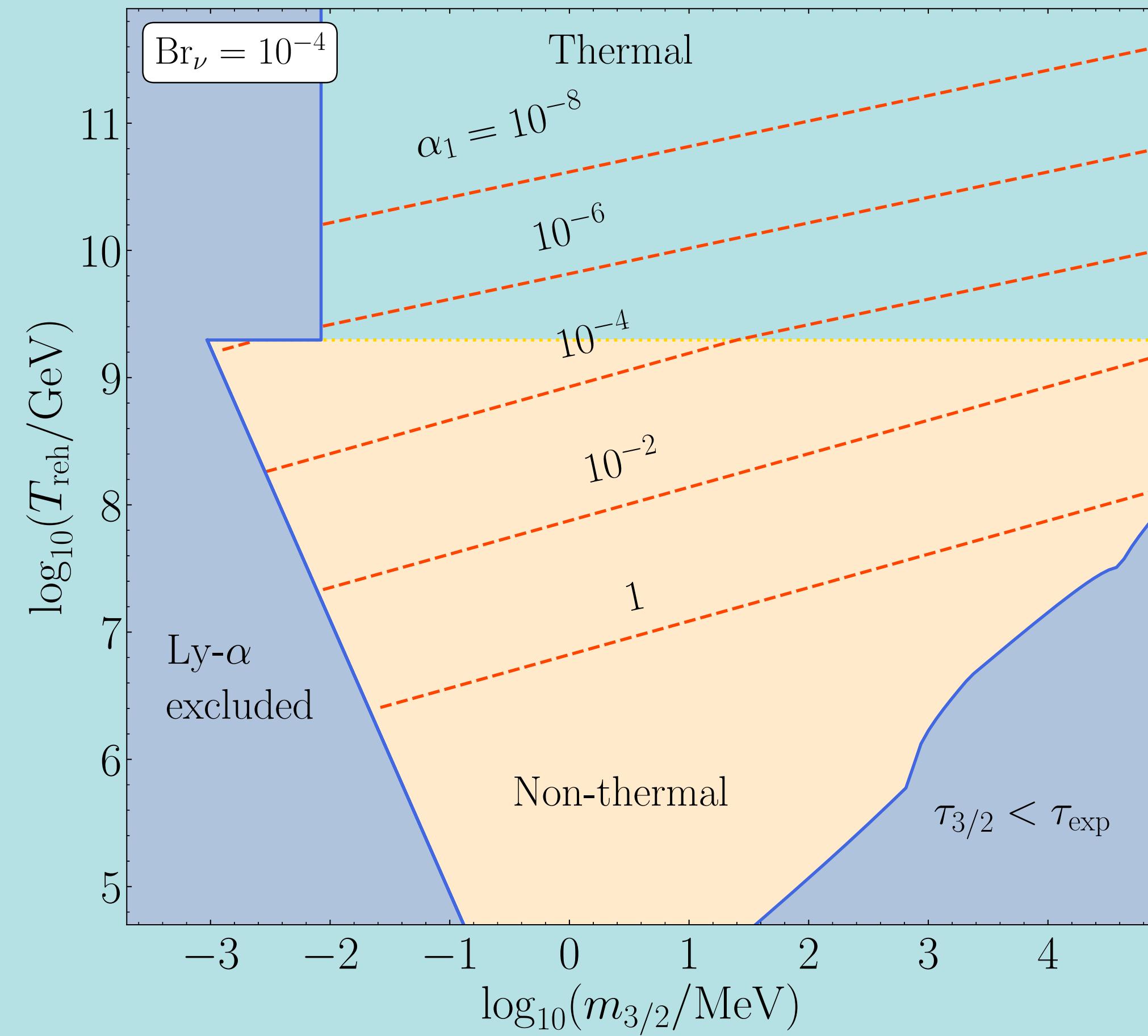


3. Lyman- α

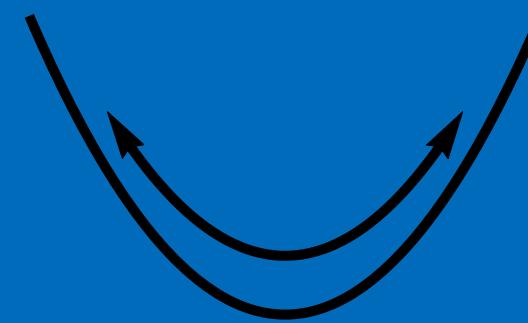


4. The end?

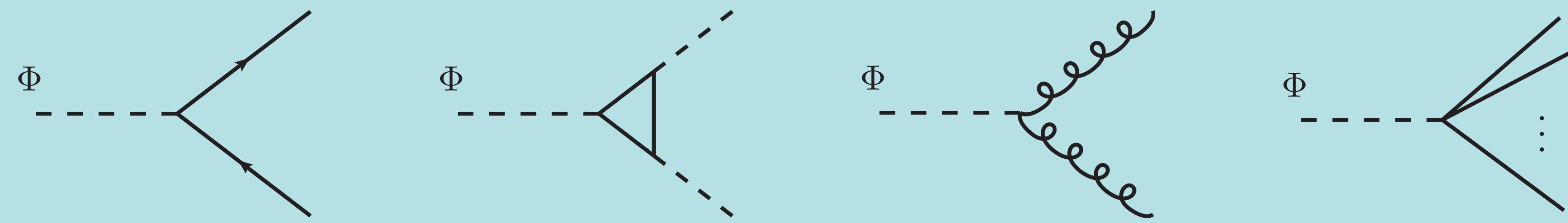
Non-thermal freeze-in: a non-supersymmetric decaying spin-3/2 particle



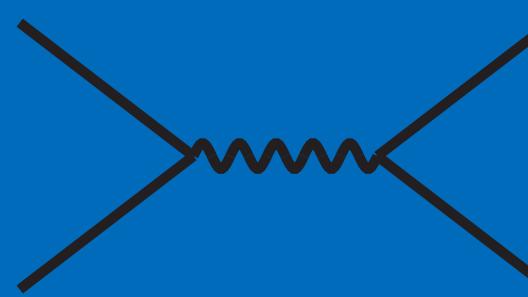
1. Reheating



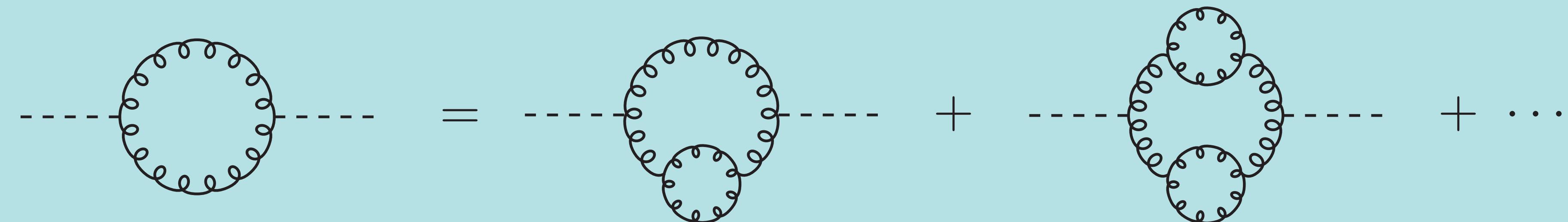
For $k \neq 2$ the nature of the final state matters



2. Freeze-in



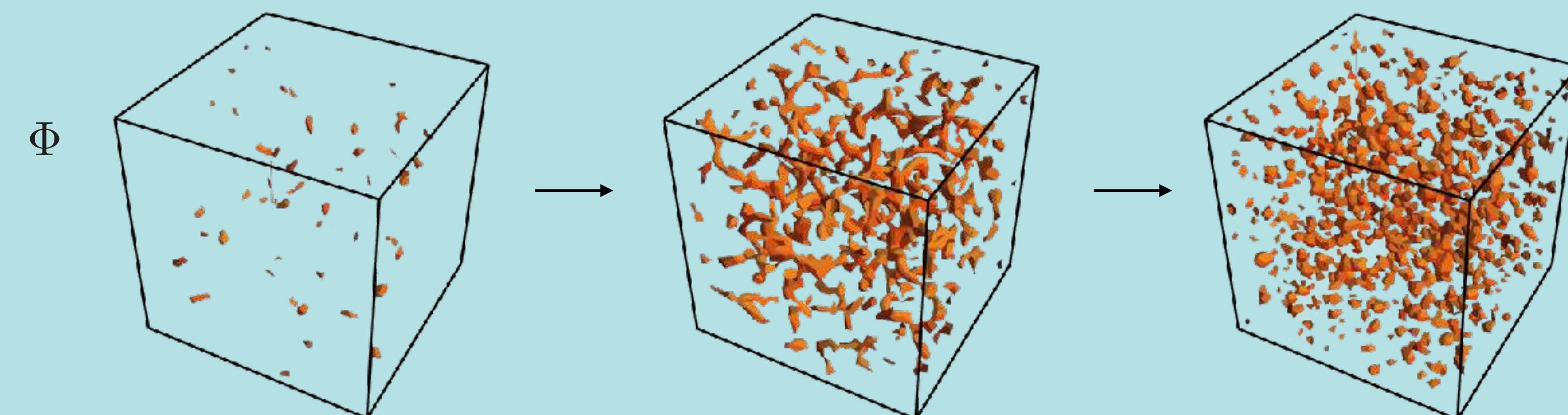
In-medium effects cannot be neglected



3. Lyman-α

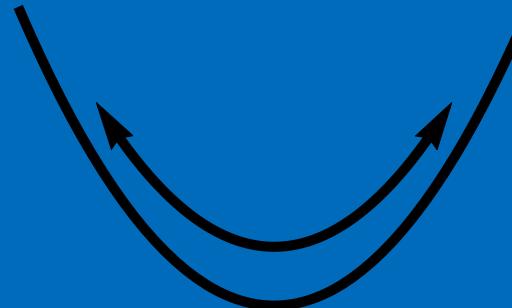


Interactions fragment homogeneous fields



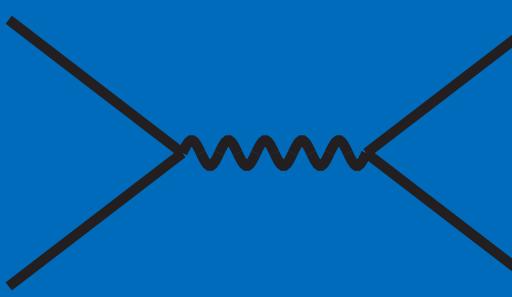
4. The end?

1. Reheating



So, how warm are non-thermal relics?

2. Freeze-in



3. Lyman- α



4. The end?

