

Non-thermal Particle Production in the Early Universe

Marcos A. G. García

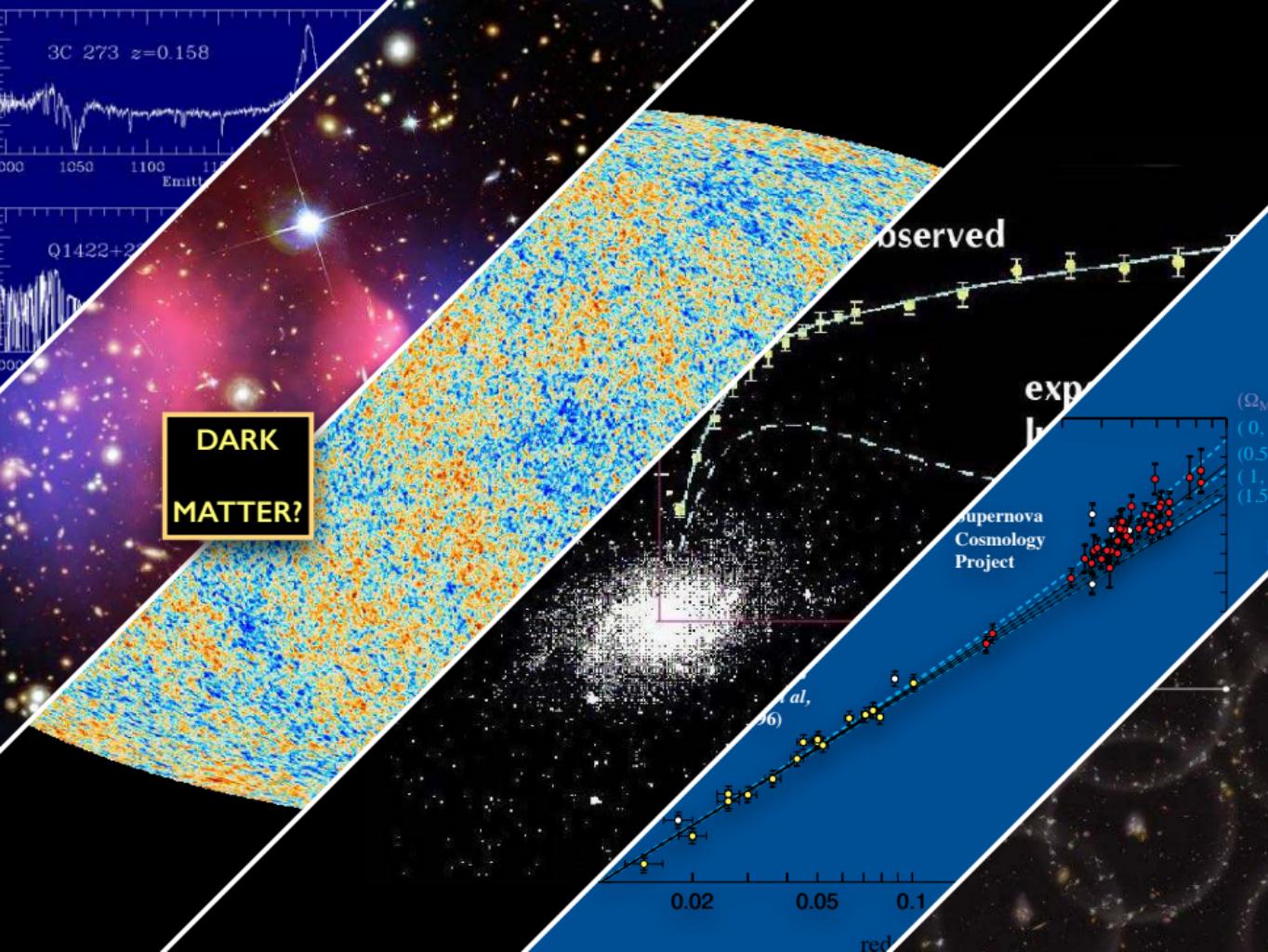
Rice University

1709.01549, MG, Y. Mambrini, K. Olive, M. Peloso

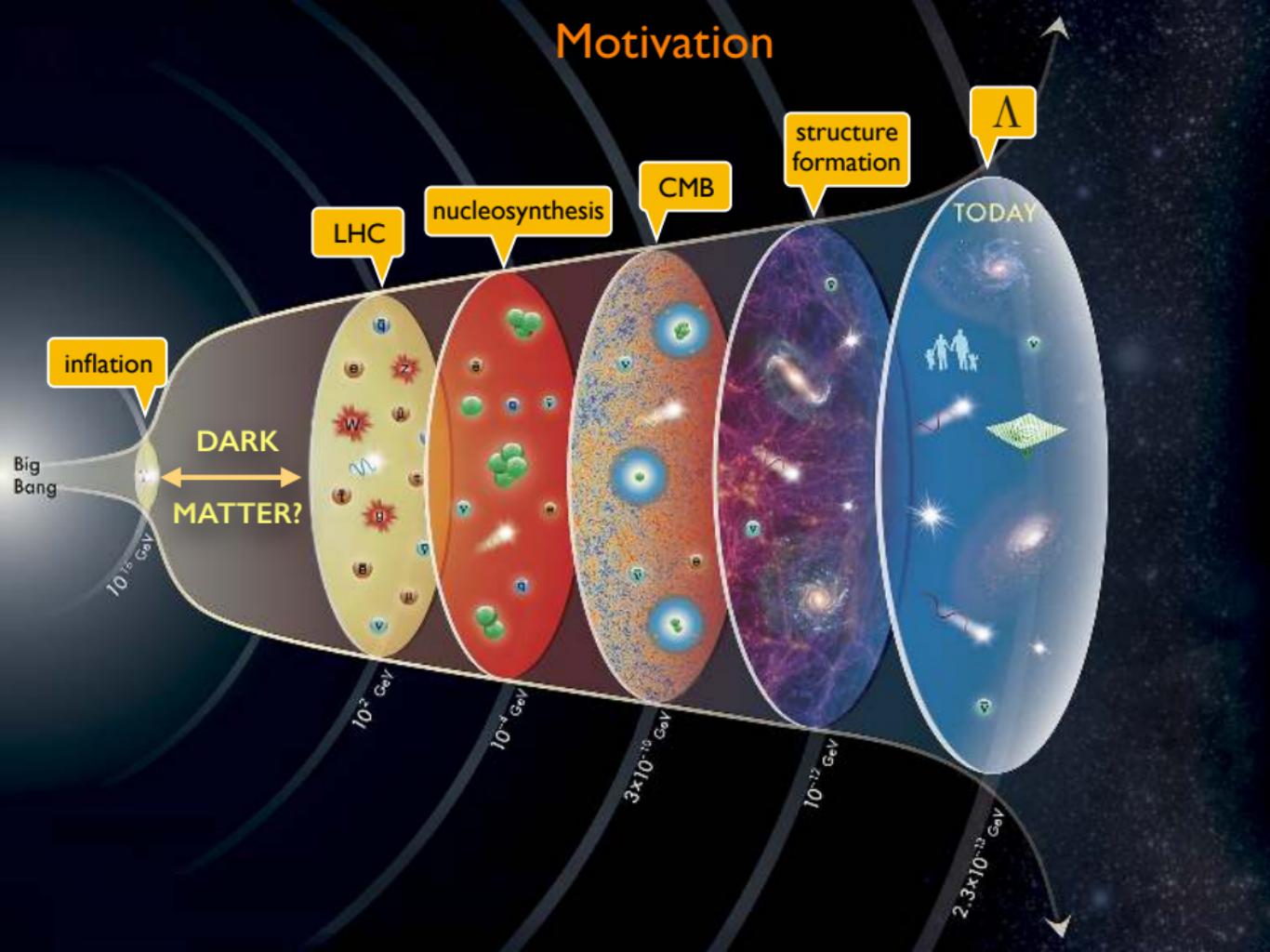
1806.01865, MG, M. Amin

1706.02319, M. Amin, MG, H. Xie, O. Wen

18xx.xxxxx, MG, M. Amin, D. Green

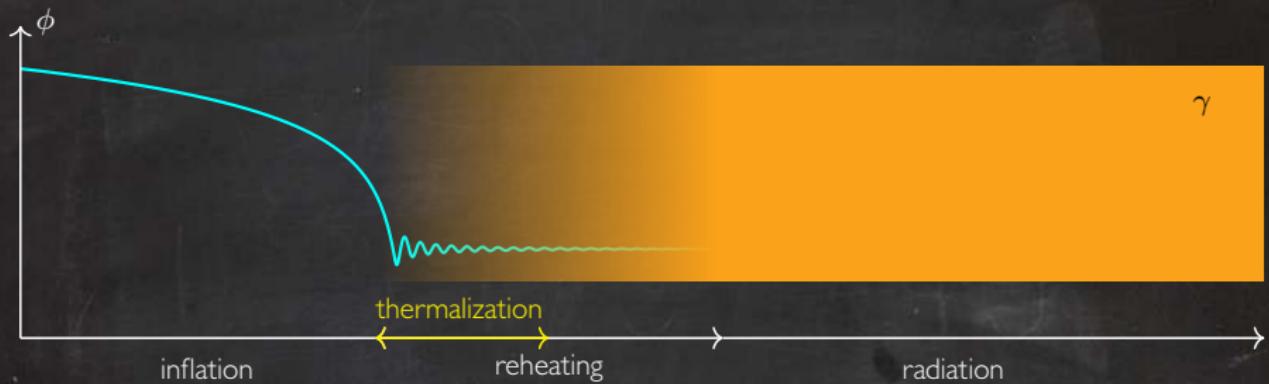


Motivation



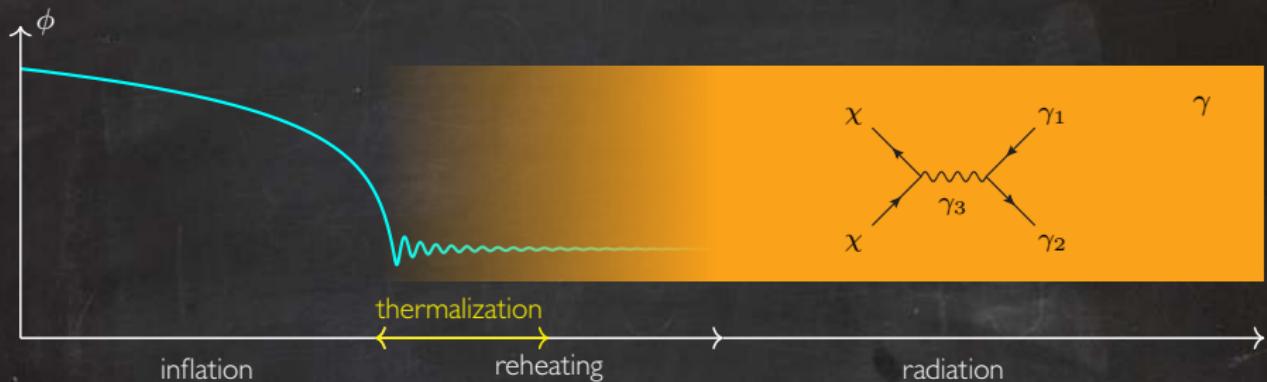
Freeze-in vs. Freeze-out

The reheating and thermalization processes after inflation have a finite duration



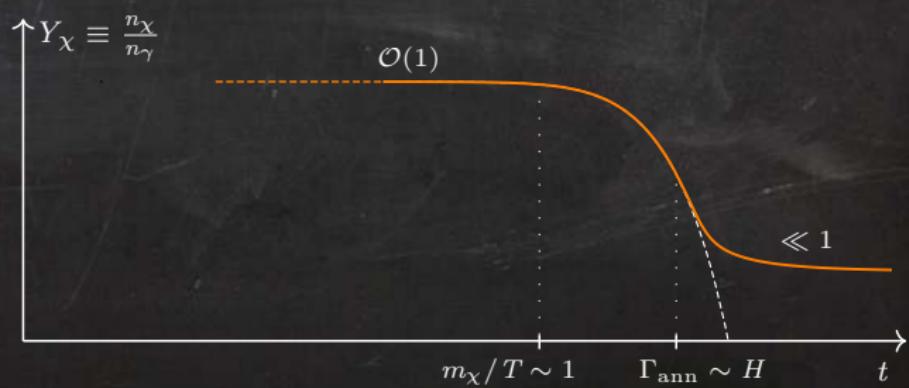
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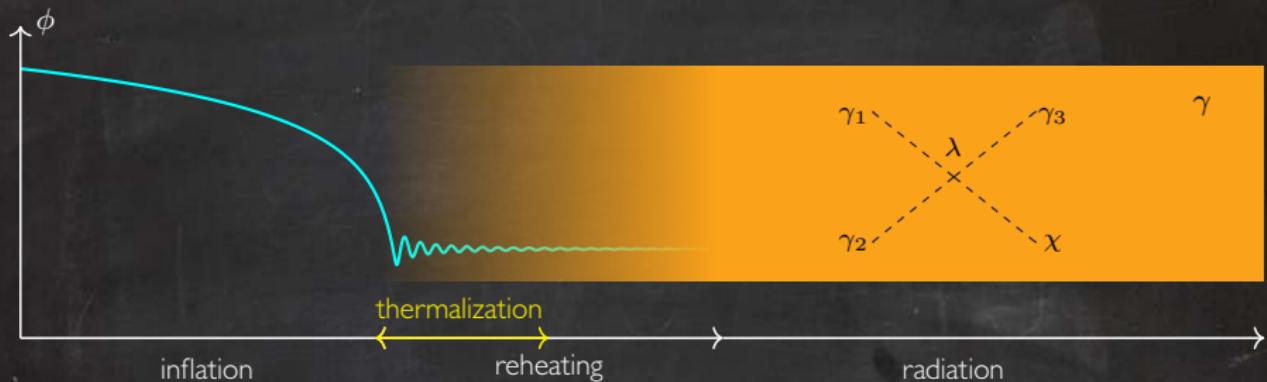
Freeze-out:

- Thermal equilibrium
- Thermal production
- $Y_\chi \longleftrightarrow \langle \sigma v \rangle_{\text{ann}}$
- IR dominated



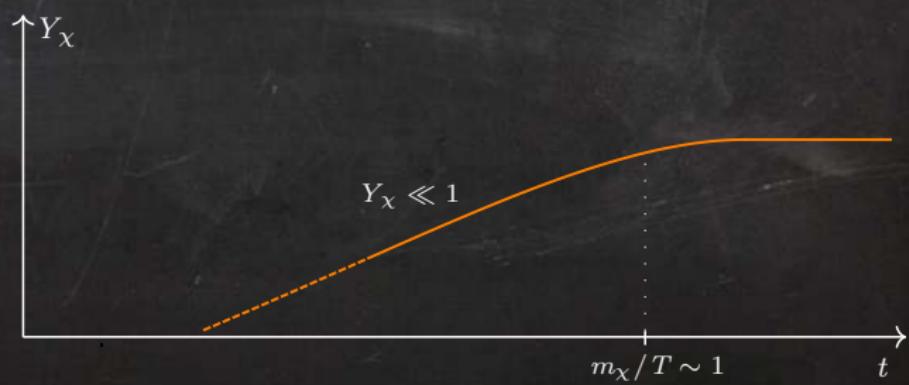
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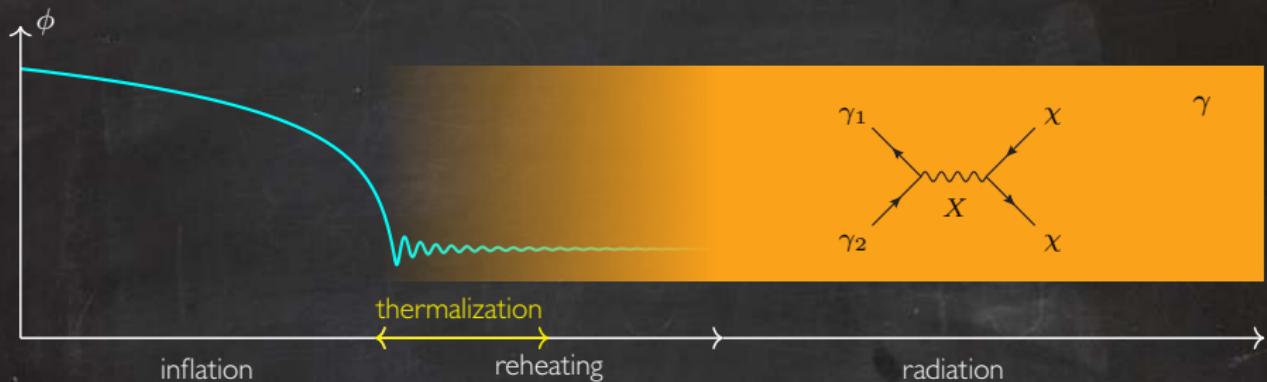
Freeze-in:

- Not in equilibrium
- Thermal production
- $Y_\chi \longleftrightarrow \langle \sigma v \rangle_{\text{prod}}$
- IR dominated



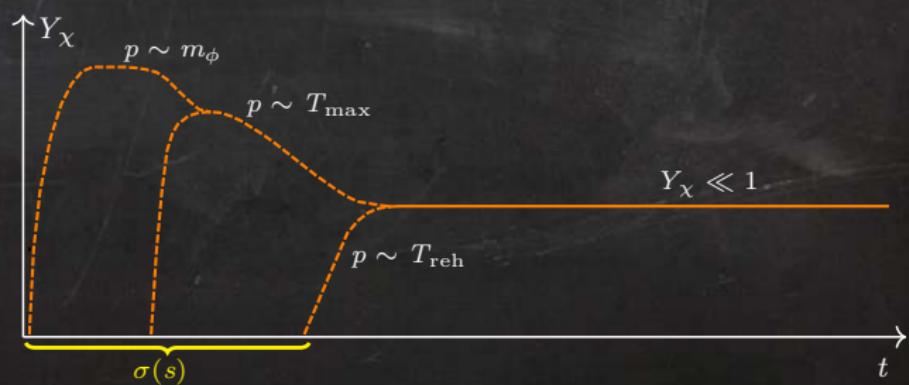
Freeze-in vs. Freeze-out

The reheating and thermalization processes after inflation have a finite duration



Freeze-in:

- Not in equilibrium
- Non-thermal and thermal production
- $Y_\chi \longleftrightarrow \langle \sigma v \rangle_{\text{prod}}$
- UV dominated



Reheating and Thermalization

(i) Instantaneous inflaton decay *and* thermalization:

$$T_{\text{reh}} = \left(\frac{40}{\pi^2 g_{\text{reh}}} \right)^{1/4} (\Gamma_\phi M_P)^{1/2}$$

Reheating and Thermalization

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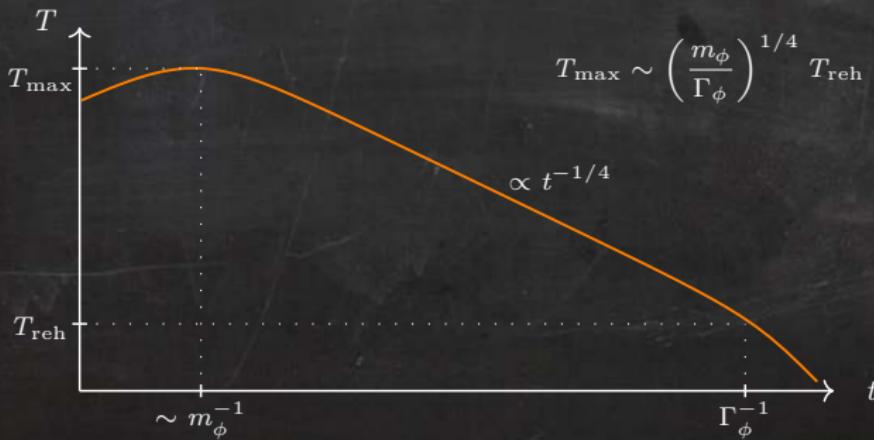
$$T_{\text{reh}} = \left(\frac{40}{\pi^2 g_{\text{reh}}} \right)^{1/4} (\Gamma_\phi M_P)^{1/2}$$

(ii) Instantaneous thermalization:

$$\dot{\rho}_\phi + 3H\rho_\phi + \Gamma_\phi \rho_\phi = 0$$

$$\dot{\rho}_\gamma + 4H\rho_\gamma - \Gamma_\phi \rho_\phi = 0$$

$$\rho_\phi + \rho_\gamma = 3M_P^2 H^2$$



(iii) The path to thermal equilibrium:

Immediately after reheating starts, n_γ is *not* determined by a temperature, but by counting

How many inflatons are left over?

$$n_\phi = \frac{\rho_\phi}{m_\phi} = \frac{\rho_{\text{end}}}{m_\phi} \left(\frac{a}{a_{\text{end}}} \right)^{-3} e^{-\Gamma_\phi t}$$

How many inflatons have decayed?

$$n_\gamma \simeq \frac{\rho_{\text{end}}}{m_\phi} \left(\frac{a}{a_{\text{end}}} \right)^{-3} \left(1 - e^{-\Gamma_\phi t} \right)$$

In thermal equilibrium we would have

$$\tilde{g}n_\gamma^T \equiv \left(\sum_B g_B + \frac{3}{4} \sum_F g_F \right) \frac{\zeta(3)}{\pi^2} T^3$$

At T_{max} , $\tilde{g}n_\gamma^T/n_\gamma > 1$ for $\Gamma_\phi/m_\phi \lesssim 10^{-10}$ (Planck suppressed)

The spectrum of γ is determined by the decay of the ϕ -condensate, $f_\phi = (2\pi)^3 n_\phi \delta^3(\mathbf{p})$

$$\frac{\partial f_\gamma}{\partial t} - Hk \frac{\partial f_\gamma}{\partial k} = \frac{2\pi^2}{k^2} n_\phi \Gamma_\phi \delta(k - m_\phi/2)$$

The solution predicts very energetic particles, $\langle k \rangle \sim m_\phi$,

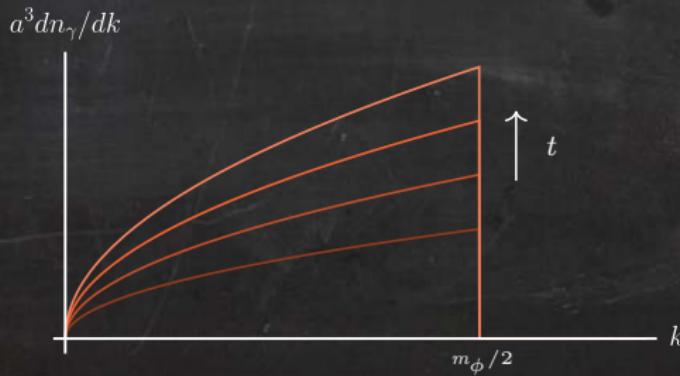
$$f_\gamma(k) \simeq 24\pi^2 \frac{n_\gamma}{m_\phi^3} \left(\frac{m_\phi}{2k} \right)^{3/2} \theta(m_\phi/2 - k)$$

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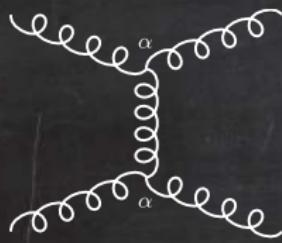
These particles need to slow-down and multiply

Thermalization through gauge interactions is *tricky*

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If a Boltzmann eq. can be used to track the evolution of $f_\gamma(p)$, the elastic collision term would be

$$\begin{aligned} \mathcal{C}^{2\leftrightarrow 2}[f_\gamma(p)] &= \frac{1}{2} \int \frac{d^3\mathbf{k} d^3\mathbf{p}' d^3\mathbf{k}'}{(2\pi)^9} \frac{|\mathcal{M}(p, k; p', k')|^2}{(2p_0)(2k_0)(2p'_0)(2k'_0)} (2\pi)^4 \delta^{(4)}(p + k - p' - k') \\ &\times \left\{ f_p f_k [1 \pm f_{p'}] [1 \pm f_{k'}] - f_{p'} f_{k'} [1 \pm f_p] [1 \pm f_k] \right\}, \end{aligned}$$



$$\frac{|\mathcal{M}(p, k; p', k')|^2}{(2p_0)(2k_0)(2p'_0)(2k'_0)} \sim \frac{\alpha^2}{|\mathbf{k} - \mathbf{k}'|_\perp^4}$$

Screening scale determined by effect of successive scatterings

$$m_s^2 \sim \alpha \int d^3\mathbf{k} f_\gamma(k)/k \quad \Rightarrow \quad \frac{\Gamma_{\text{elas}}}{H} \sim \alpha m_\phi t$$

(A. Kurkela, G. Moore, 1107.5050; J. Ellis, MG et. al., 1512.05701)

Thermalization through gauge interactions is *tricky*

Inelastic processes can have formation times which exceed the mean free time between scatterings

$$\left(\text{---} \left(\begin{array}{c} \text{---} \\ \text{---} \end{array} \right) \right)^* \left(\text{---} \left(\begin{array}{c} \text{---} \\ \text{---} \end{array} \right) \right) \neq 0$$

The diagram shows two horizontal lines representing particle paths. The top line has a wavy line above it, representing a gauge boson exchange. The bottom line has several vertical wavy lines (Z bosons) attached to it at different points. Ellipses indicate intermediate vertices. The entire left-hand diagram is enclosed in parentheses with a superscript asterisk (*), indicating it is the complex conjugate of the right-hand diagram.

(L. Landau, I. Pomeranchuk, Dokl. Akad. Nauk Ser. Fiz. 92 (1953) 535; A. Migdal, Phys. Rev. 103 (1956) 1811)

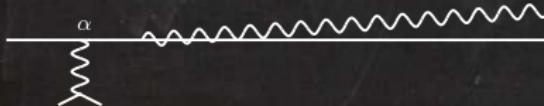
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Hard branching including LPM effect



$$t_\gamma \sim \sqrt{\frac{\tau E}{q_\perp^2}}$$

(P. Arnold, G. Moore, L. Yaffe, hep-ph/0209353)

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Hard branching including LPM effect



$$\frac{\Gamma_{\text{inelas}}}{H} \sim \alpha \left(\frac{M_P}{m} \right) (\Gamma_\phi t)^{1/2}$$

(J. Ellis, MG et. al., 1512.05701)

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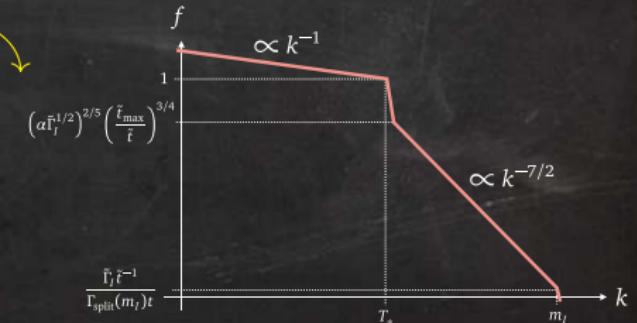
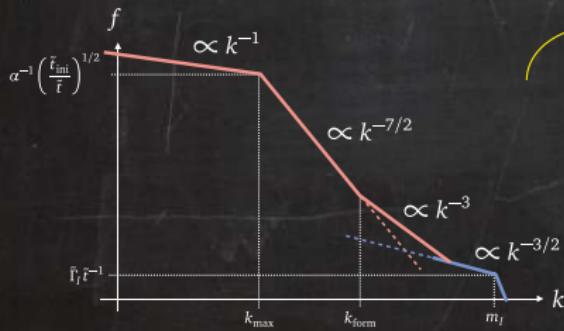
$$\frac{\partial f_\gamma}{\partial t} - Hk \frac{\partial f_\gamma}{\partial k} = \left| \overbrace{\text{---}}^{\text{String}} \right|^2 + \left| \overbrace{\text{---}}^{\text{Wavy}} \right|^2 + \dots$$
$$\equiv -\mathcal{C}^{2 \leftrightarrow 2}[f_\gamma] - \mathcal{C}^{“1 \leftrightarrow 2”}[f_\gamma] + \dots$$

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(K. Harigaya, K. Mukaida, 1312.3097; K. Mukaida, M. Yamada, 1506.07661)



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$$\frac{\partial f_\gamma}{\partial t} - Hk \frac{\partial f_\gamma}{\partial k} = \left| \overbrace{\text{---}}^{\text{oscillations}} \right|^2 + \left| \overbrace{\text{---}}^{\text{oscillations}} \right|^2 + \dots$$

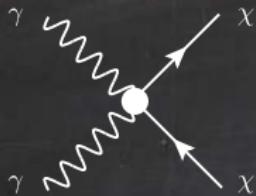
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$$\Gamma_\phi t_{\text{th}} \simeq \alpha^{-16/5} \left(\frac{\Gamma_\phi m_\phi^2}{M_P^3} \right)^{2/5} \ll 1$$

$$T_{\text{th}} \simeq \alpha^{4/5} m_\phi \left(\frac{24}{\pi^2 g_{\text{reh}}} \right)^{1/4} \left(\frac{\Gamma_\phi M_P^2}{m_\phi^3} \right)^{2/5}$$

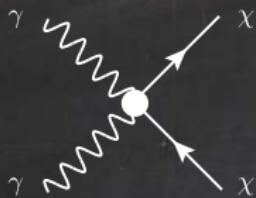
Non-equilibrium DM production



The interaction between DM and the relativistic background is very weak

$$\begin{aligned} \frac{\partial f_\chi}{\partial t} - H p_1 \frac{\partial f_\chi}{\partial p_1} = & -\frac{1}{2p_1} \int \frac{g_\chi d^3 \mathbf{p}_2}{(2\pi)^3 2p_2} \frac{g_\gamma d^3 \mathbf{k}_1}{(2\pi)^3 2k_1} \frac{g_\gamma d^3 \mathbf{k}_2}{(2\pi)^3 2k_2} (2\pi)^4 \delta^{(4)}(k_1 + k_2 - p_1 - p_2) \\ & \times \left[\begin{array}{l} |\mathcal{M}|_{\chi\chi \rightarrow \gamma\gamma}^2 f_\chi(p_1) f_\chi(p_2) [1 + f_\gamma(k_1)] [1 + f_\gamma(k_2)] \\ - |\mathcal{M}|_{\gamma\gamma \rightarrow \chi\chi}^2 f_\gamma(k_1) f_\gamma(k_2) [1 - f_\chi(p_1)] [1 - f_\chi(p_2)] \end{array} \right] \end{aligned}$$

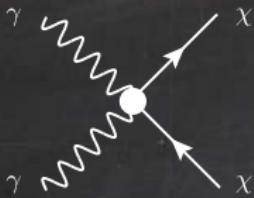
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Integrating,

$$\frac{dn_\chi}{dt} + 3Hn_\chi = 4g_\chi^2 g_\gamma^2 \int \frac{d^3 \mathbf{k}_1}{(2\pi)^3 2k_1} \frac{d^3 \mathbf{k}_2}{(2\pi)^3 2k_2} (k_1 \cdot k_2) \sigma(s)_{\gamma\gamma \rightarrow \chi\chi} f_\gamma(k_1) f_\gamma(k_2)$$

Non-thermal vs. thermal production

Before thermalization:

$$\dot{n}_\chi + 3Hn_\chi = 18g_\chi^2 g_\gamma^2 \frac{n_\gamma^2}{m_\phi^3} \int_0^{m_\phi^2} ds \sqrt{s} \sigma(s) \left[\ln \left(\frac{m_\phi + \sqrt{m_\phi^2 - s}}{\sqrt{s}} \right) - \frac{\sqrt{m_\phi^2 - s}}{m_\phi} \right]$$


 $\langle \sigma v \rangle_{\text{NT}} n_\gamma^2$

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 $\langle \sigma v \rangle_{\text{NT}} n_\gamma^2$

After thermalization:

$$\begin{aligned} \dot{n}_\chi + 3Hn_\chi &= \frac{g_\chi^2 g_\gamma^2}{8\pi^4} \int dk_1 dk_2 d\cos\theta_{12} \frac{(k_1 k_2)^2 (1 - \cos\theta_{12})}{(e^{k_1/T} \pm 1)(e^{k_2/T} \pm 1)} \sigma(s) \\ &\quad \downarrow \text{M.B.} \\ &\simeq \frac{g_\chi^2 g_\gamma^2 T}{2(2\pi)^4} \int_0^\infty ds s^{3/2} \sigma(s) K_1(\sqrt{s}/T) \end{aligned}$$


 $\langle \sigma v \rangle_{\text{T}} (n_\gamma^{\text{T}})^2$

Let $\sigma(s) \sim \frac{s^{n/2}}{M^{n+2}}$ so that $\frac{\langle \sigma v \rangle}{g_\chi^2 g_\gamma^2} \propto \frac{1}{M^{n+2}} \begin{cases} T^n & (\text{T}) \\ m_\phi^n & (\text{NT}) \end{cases}$

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Thermal:

$$Y_\chi^T(T_{\text{reh}}) \propto \frac{M_P T_{\text{reh}}^7}{g_{\text{reh}}^{1/2} M^{n+2}} \times \begin{cases} \frac{1}{n-6} (T_{\text{max}}^{n-6} - T_{\text{reh}}^{n-6}), & n > -1, n \neq 6 \\ \ln \left(\frac{T_{\text{max}}}{T_{\text{reh}}} \right), & n = 6 \end{cases}$$

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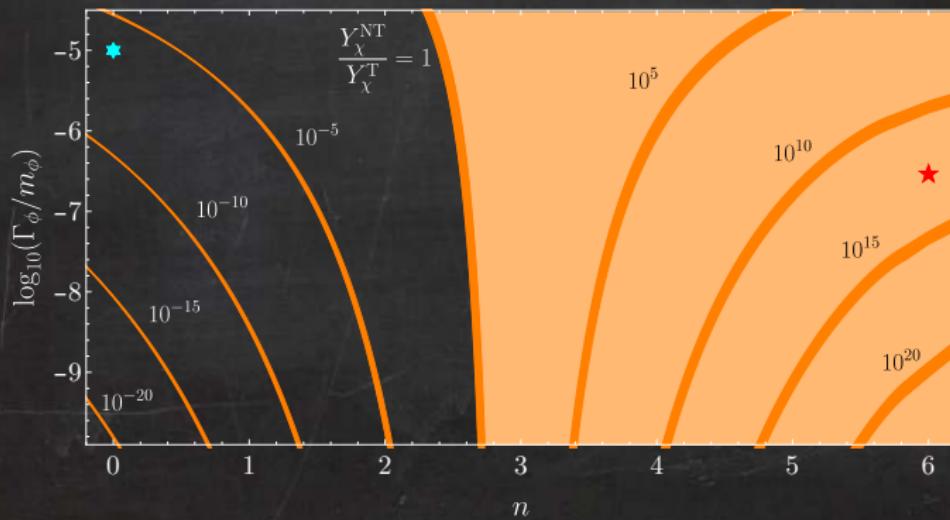
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Non-thermal:

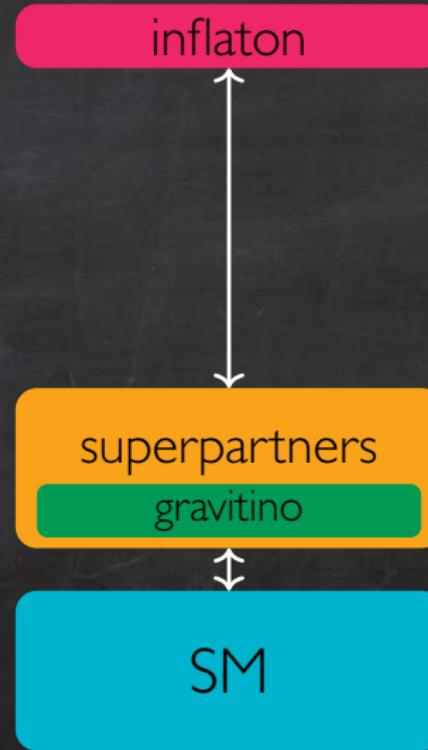
$$Y_\chi^{\text{NT}}(T_{\text{reh}}) \propto g_{\text{reh}}^{3/2} \frac{T_{\text{reh}}^3 M_P m_\phi^{n-2}}{M^{n+2}} (\Gamma_\phi t_{\text{th}})$$

Let $\sigma(s) \sim \frac{s^{n/2}}{M^{n+2}}$ so that $\frac{\langle \sigma v \rangle}{g_\chi^2 g_\gamma^2} \propto \frac{1}{M^{n+2}} \begin{cases} T^n & (\text{T}) \\ m_\phi^n & (\text{NT}) \end{cases}$



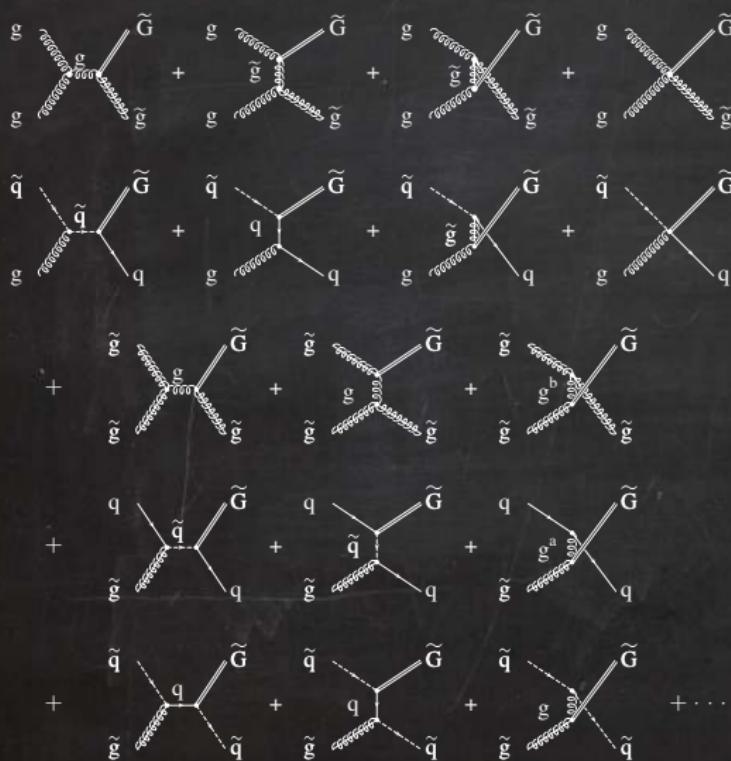
Light Gravitino

$\phi \rightarrow g + g$ and weak scale supersymmetry



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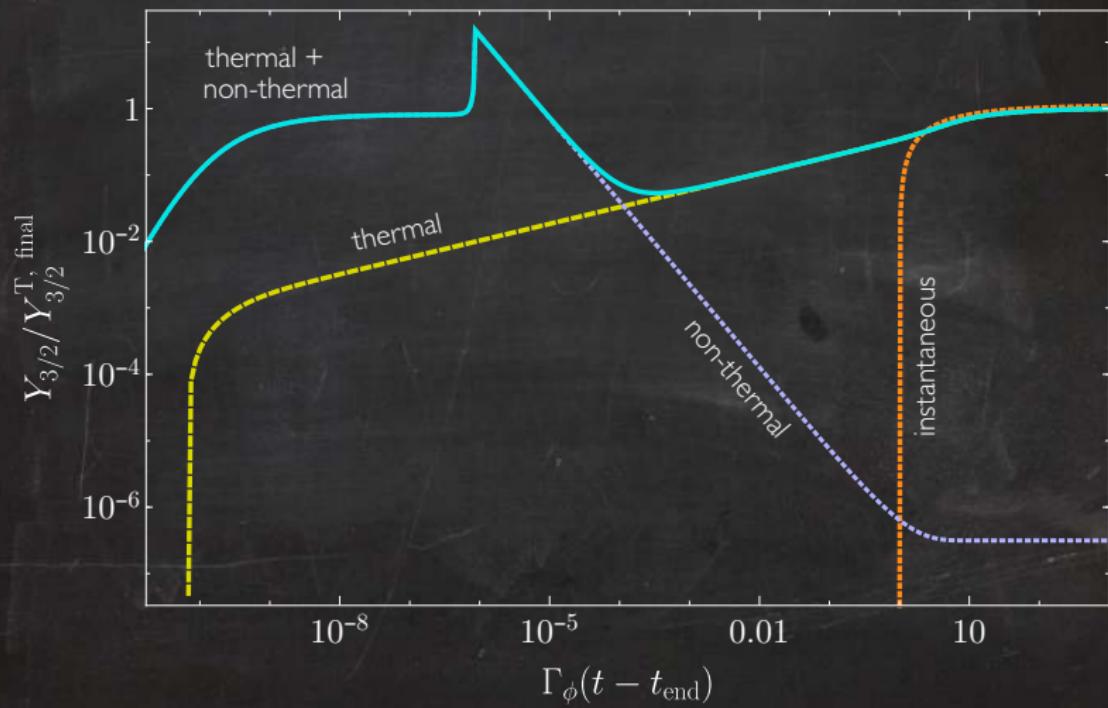
$$\left. \begin{array}{l} \langle \sigma v \rangle_{\text{NT}} = \\ \sum_{i=1}^3 \frac{16\pi\alpha_i}{M_P^2} |f^{abc}|^2 \left(1 + \frac{m_{\tilde{g}_i}^2}{3m_{3/2}^2} \right) \\ \\ \langle \sigma v \rangle_{\text{T}} = \\ \sum_{i=1}^3 \frac{3\pi^2 c_i \alpha_i}{4\zeta(3) M_P^2} \left(1 + \frac{m_{\tilde{g}_i}^2}{3m_{3/2}^2} \right) \ln \left(\frac{k_i}{g_i} \right) \end{array} \right\}$$

(M. Bolz et. al., hep-ph/0012052)

(V. Rychkov, A. Strumia, hep-ph/0701104)

Light Gravitino

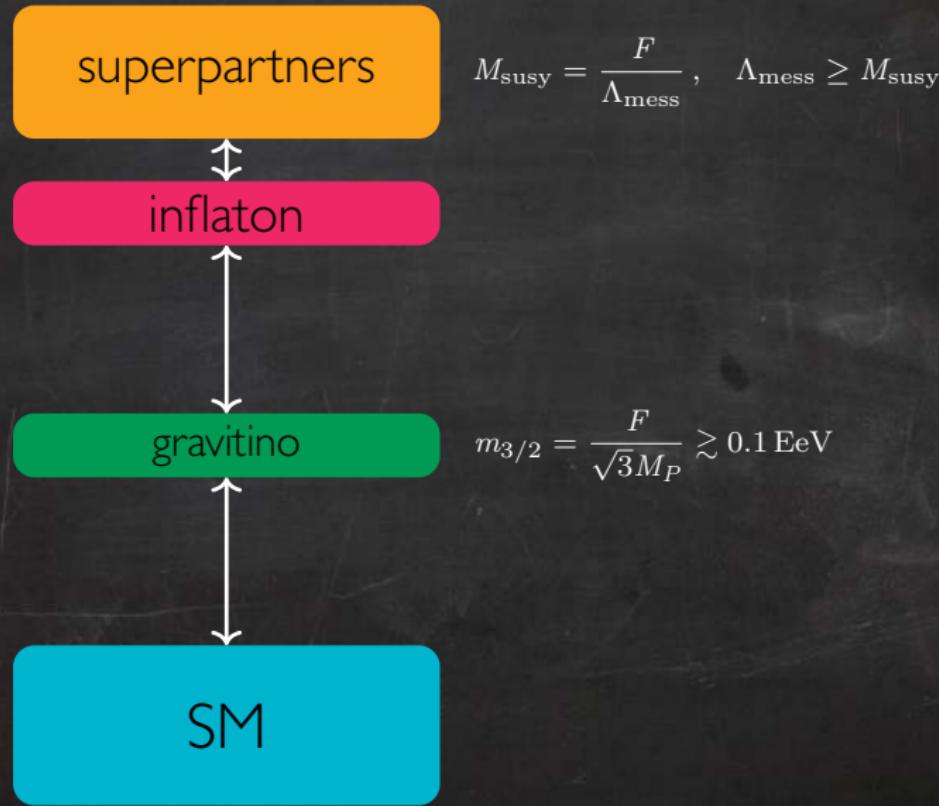
$\phi \rightarrow g + g$ and weak scale supersymmetry



Heavy Gravitino

$\phi \rightarrow g + g$ and high scale supersymmetry

(E. Dudas, Y. Mambrini, K. Olive, 1704.03008)

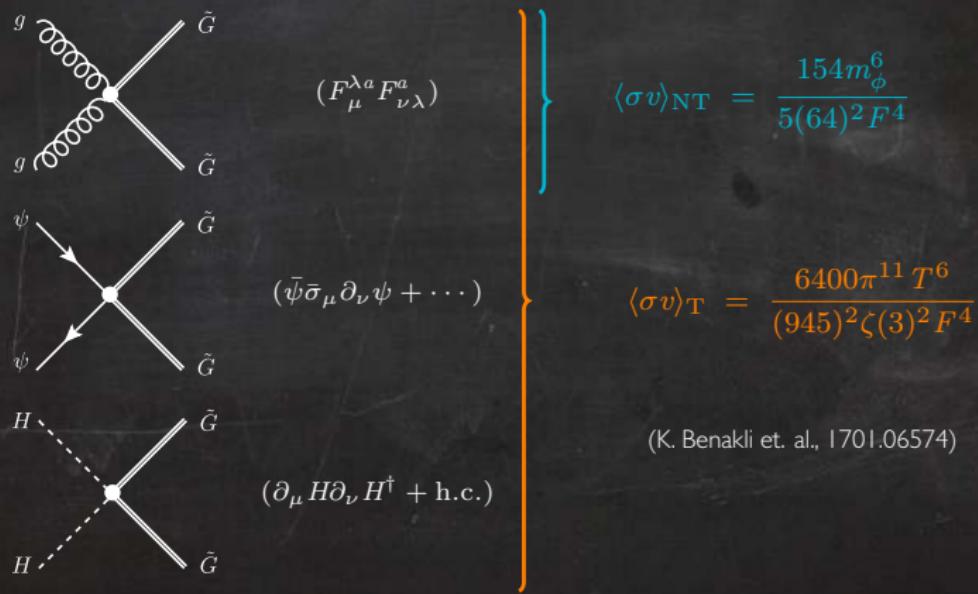


Heavy Gravitino

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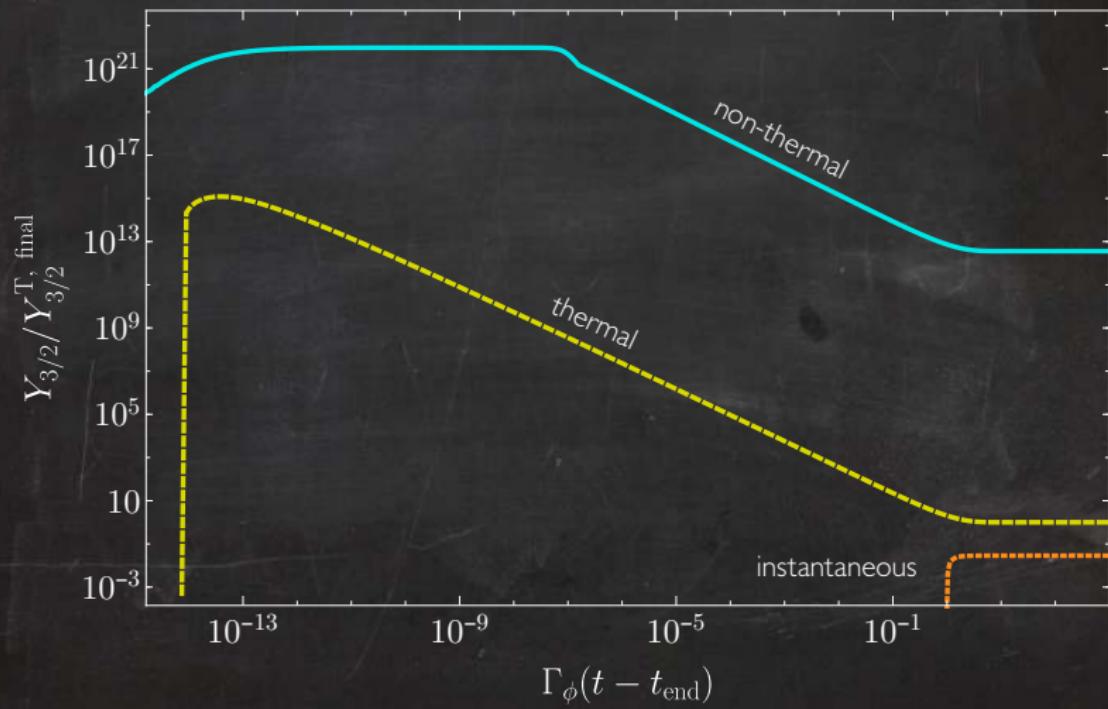
Leading-order universal Goldstino-matter interactions ($F = \sqrt{3}m_{3/2}M_P$):

$$\mathcal{L}_{2G} = \frac{i}{2F^2} (G\sigma^\mu \partial^\nu \bar{G} - \partial^\nu G\sigma^\mu \bar{G}) T_{\mu\nu}$$



Heavy Gravitino

$\phi \rightarrow g + g$ and high scale supersymmetry



Heavy Gravitino

$\phi \rightarrow g + g$ and high scale supersymmetry

Assuming instantaneous reheating and thermalization...

$$\Omega_{3/2}^{\text{inst}} h^2 \simeq 0.11 \left(\frac{0.1 \text{ EeV}}{m_{3/2}} \right)^3 \left(\frac{T_{\text{reh}}}{2.2 \times 10^{10}} \right)^7$$

vs. accounting for their finite duration...

$$\Omega_{3/2} h^2 \simeq 0.11 \left(\frac{0.1 \text{ EeV}}{m_{3/2}} \right)^3 \left(\frac{T_{\text{reh}}}{2.2 \times 10^8} \right)^{19/5} \left(\frac{m_\phi}{3 \times 10^{13} \text{ GeV}} \right)^{24/5} \left(\frac{0.030}{\alpha_3} \right)^{16/5}$$

(similar analysis applies to DM production through heavy spin-2 mediators, N. Bernal et. al. 1803.01866)

Freezing-in dark matter through a heavy invisible Z'

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Univ. Paris-Sud, Université Paris-Saclay, 91405 Orsay, France

(1806.00016 [hep-ph])

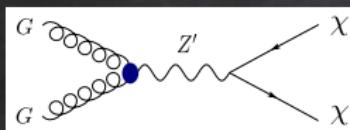


FIG. 1: Production of dark matter through gluon fusion in the early Universe

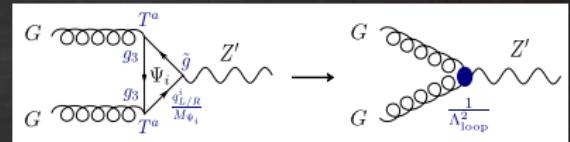


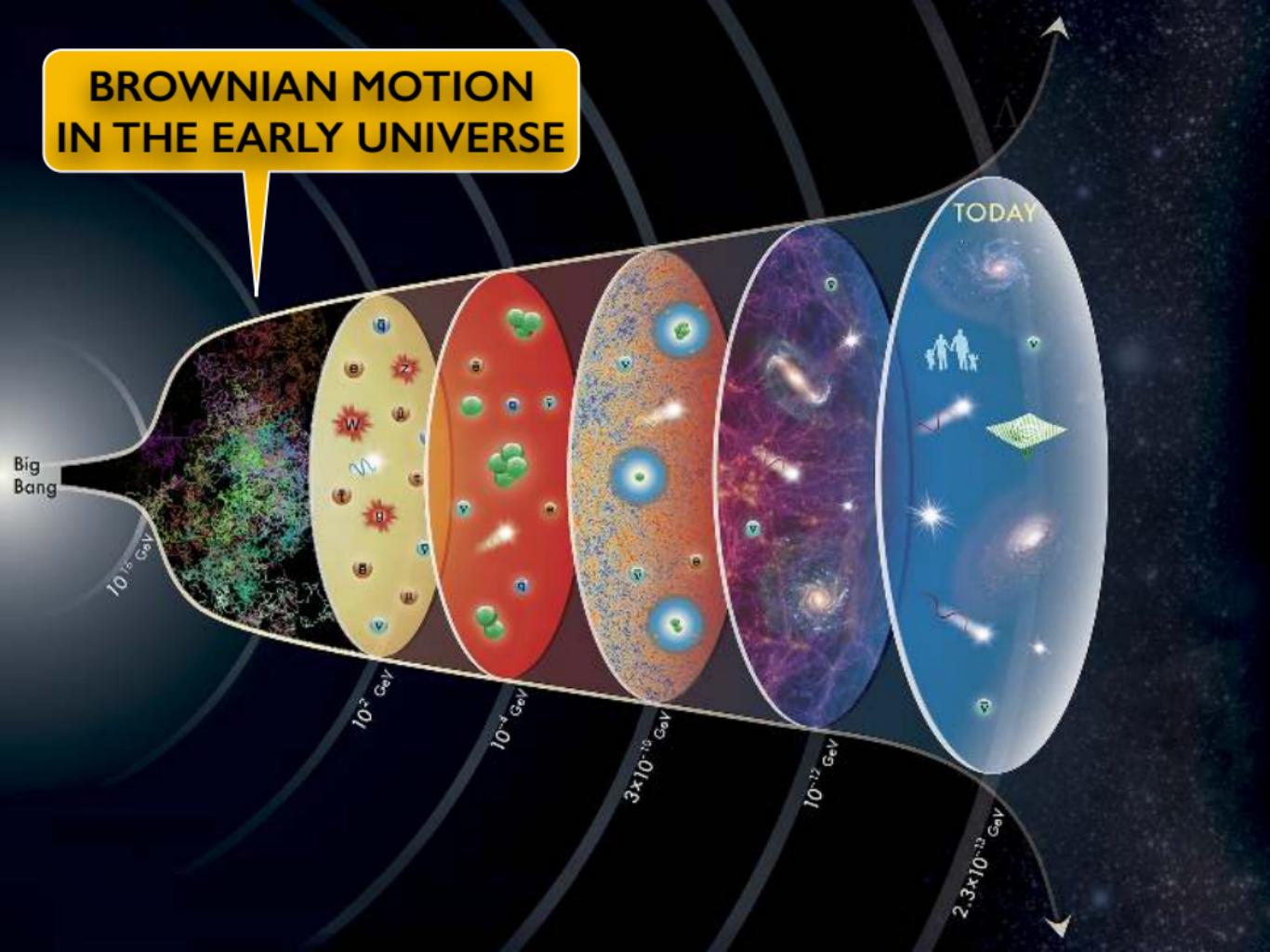
FIG. 4: Triangle diagram generated containing heavy chiral fermions Ψ_i (left panel), and the resulting effective vertex at low energy (right panel).

$$\langle \sigma v \rangle n_\gamma^2 = R(T) \approx \begin{cases} 2 \times 10^2 \frac{\alpha^2}{\Lambda^4} \frac{m_\chi^2}{M_{Z'}^4} T^{10} & (\text{fermionic DM}) \rightarrow n = 4 \\ 10^4 \frac{\beta^2}{\Lambda^4 M_{Z'}^4} T^{12} & (\text{abelian DM}) \rightarrow n = 6 \\ 2 \times 10^9 \frac{\gamma^2}{\Lambda^4 M_{Z'}^4} T^{16} & (\text{non-abelian DM}) \rightarrow n = 10 \end{cases} \quad (14)$$

Conclusion

- UV-dominated freeze-in during reheating is realized for $\sigma(s) \sim s^{n/2}$, $n > 2$
- Thermalization time-scale determines the DM abundance at late times
- Effect important for DM production in very high scale susy models, or for heavy spin-2 mediators
- Preheating? N_{eff} ?

BROWNIAN MOTION IN THE EARLY UNIVERSE



Complexity in the Early Universe

What if the early universe is not simple?

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$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2} G_{ij} \partial_\mu \phi^i \partial^\mu \phi^j - V(\phi) \right]$$

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$$[(\partial_\tau^2 + k^2) + \mathbf{P}(\tau) \partial_\tau + \mathbf{M}(\tau)] \cdot \chi = 0$$

where $d\tau = dt/a$, χ^i canonically normalized, gauge invariant perturbations, and

$$(\mathbf{P})_j^i = 2\Gamma_{jk}^i \phi^{k'}$$

$$(\mathbf{M})_j^i = -\frac{a''}{a} \delta_j^i - 2H\Gamma_{jk}^i \phi^{k'} + a^2 \left[\frac{\dot{\phi}^i}{H} V_{,k} + (G^{ij} V_{,j}),_k + \dots \right]$$

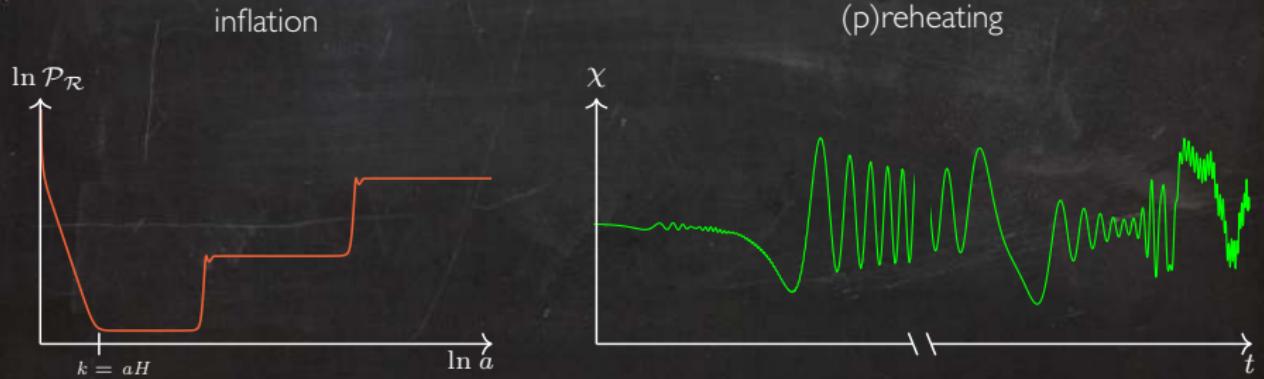
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Stochastic Particle Production

Spectator field in non-expanding background

(M. Amin, D. Baumann 1512.02637)

$$\left(\frac{d^2}{dt^2} + k^2 + M^2 + m^2(t) \right) \chi(t) = 0$$



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After the j -th event,

$$\chi_j(t) \equiv \frac{1}{\sqrt{2\omega}} \left[\beta_j e^{i\omega t} + \alpha_j e^{-i\omega t} \right],$$

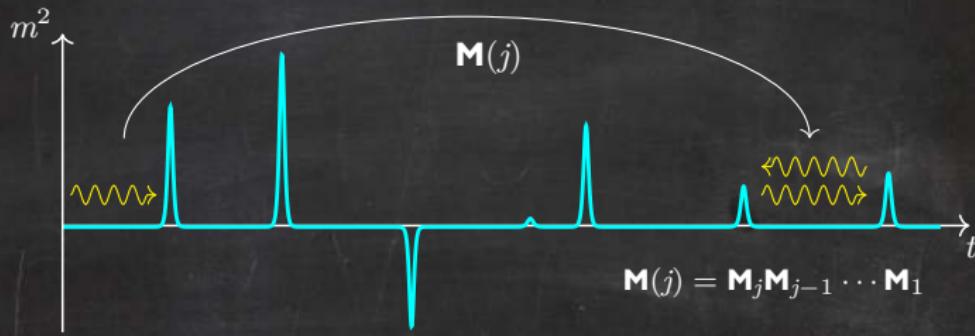
$$\begin{pmatrix} \boldsymbol{\beta}_j \\ \boldsymbol{\alpha}_j \end{pmatrix} = \mathbf{M}_j \begin{pmatrix} \boldsymbol{\beta}_{j-1} \\ \boldsymbol{\alpha}_{j-1} \end{pmatrix}$$

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$$\left(\frac{d^2}{dt^2} + k^2 + M^2 + m^2(t) \right) \chi(t) = 0$$



A random walk (with drift) for the occupation number

$$n(j) = \frac{1}{2\omega} \left(|\dot{\chi}_j|^2 + \omega^2 |\chi_j|^2 \right) - \frac{1}{2} = |\beta_j|^2$$

Stochastic Particle Production

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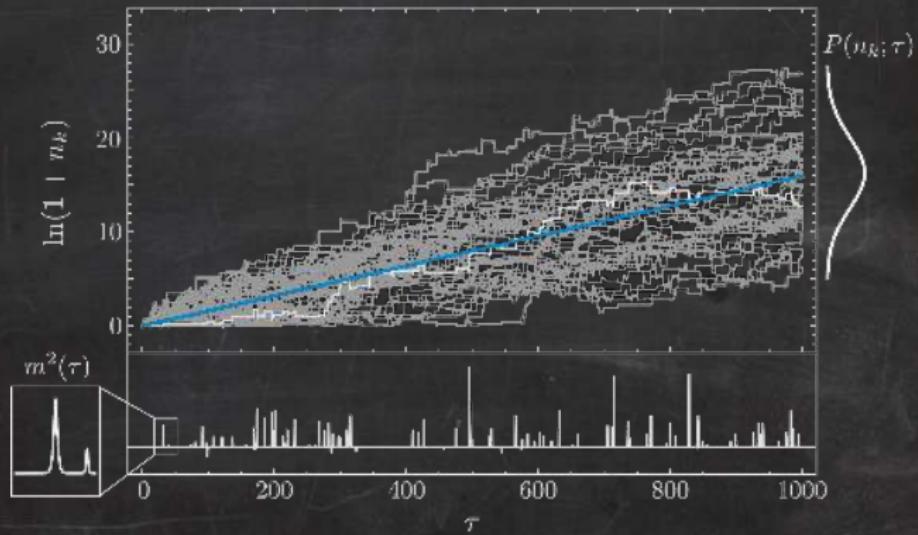
$$\left(\frac{d^2}{dt^2} + k^2 + M^2 + m^2(t) \right) \chi(t) = 0$$



$$\partial_t P(\mathbf{M}; t) = -\partial_{\mathbf{M}} \left[\frac{\langle \delta \mathbf{M} \rangle_{\mathbf{M}_2}}{\delta t} P(\mathbf{M}; t) \right] + \frac{1}{2!} \partial_{\mathbf{M}}^2 \left[\frac{\langle \delta \mathbf{M}^2 \rangle_{\mathbf{M}_2}}{\delta t} P(\mathbf{M}; t) \right] + \dots$$

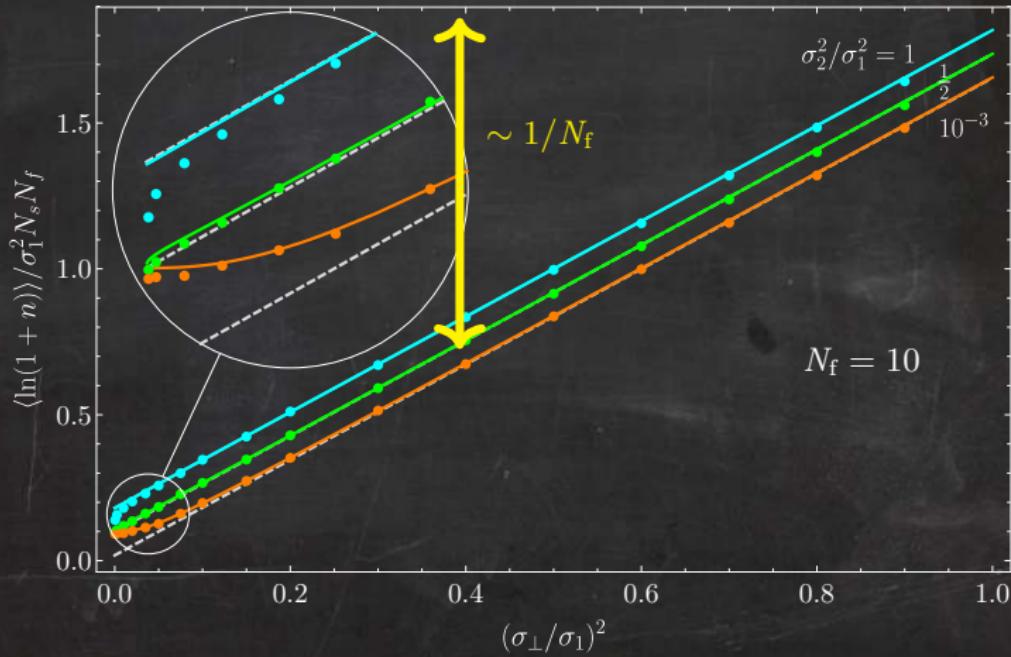
Single field:

$$\frac{1}{\sigma^2} \frac{\partial}{\partial t} P(n; t) = \frac{\partial}{\partial n} \left[n^2 \frac{\partial}{\partial n} P(n; t) \right]$$



- log-normal
- late-time equilibrium \longleftrightarrow maximal entropy

Multiple fields:



Similar

Conformal Spectator in dS

Include now the effect of expansion

$$\left(\frac{d^2}{dt^2} + 3H\frac{d}{dt} + \frac{k^2}{a^2} + M^2 + m^2(t) \right) \chi_{\mathbf{k}}(t) = 0$$

$$\begin{array}{ccc} & \downarrow & \\ a = a_0 e^{H(t-t_0)} & M^2 = 2H^2 & m^2(t) = \sum_j m_j \delta(t - t_j) \\ (\text{de Sitter}) & (\text{conformal mass}) & (\text{non-adiabatic}) \end{array}$$

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Or, with $X_{\mathbf{k}} = a\chi$, $\tau = a_0/a$

$$\frac{d^2 X_{\mathbf{k}}}{d\tau^2} + \left(\frac{k^2}{(a_0 H)^2} + \sum_j \frac{m_j}{\tau H} \delta(\tau - \tau_j) \right) X_{\mathbf{k}} = 0$$

In between scatterings

$$X_{\mathbf{k}}(\tau) = \alpha_{\mathbf{k},j} f_{\mathbf{k}}(\tau) + \beta_{\mathbf{k},j} f_{\mathbf{k}}^*(\tau), \quad f_{\mathbf{k}}(\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}}$$

Junction conditions

$$\begin{pmatrix} \beta_j \\ \alpha_j \end{pmatrix} = \mathbf{M}_j \begin{pmatrix} \beta_{j-1} \\ \alpha_{j-1} \end{pmatrix} = \mathbf{M}(j) \begin{pmatrix} \beta_0 \\ \alpha_0 \end{pmatrix}$$

with

$$\mathbf{M}_j = \mathbb{1} + im_j a_j \begin{pmatrix} |f(\tau_j)|^2 & f(\tau_j)^2 \\ -f(\tau_j)^{*2} & -|f(\tau_j)|^2 \end{pmatrix}$$

and

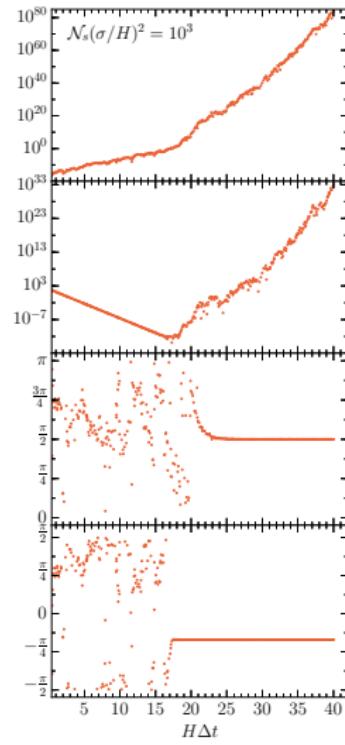
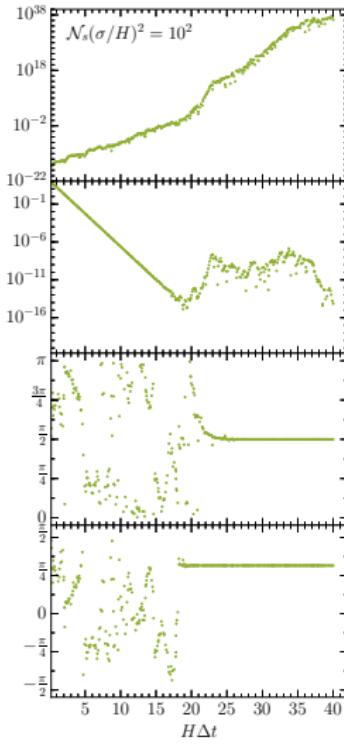
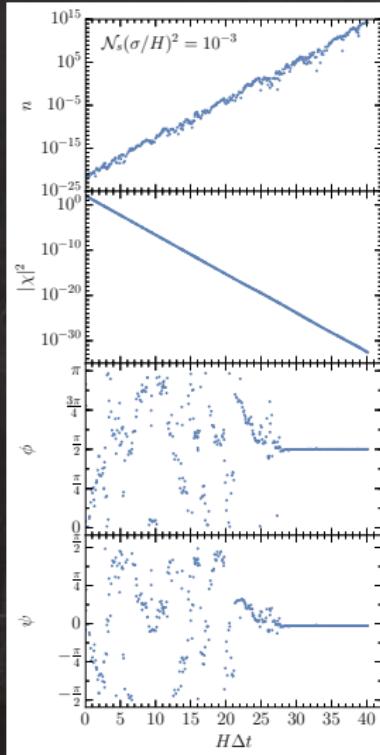
$$\mathbf{M}(j) = \mathbf{M}_j \mathbf{M}_{j-1} \cdots \mathbf{M}_1 = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix} \begin{pmatrix} \sqrt{1+n} & \sqrt{n} \\ \sqrt{n} & \sqrt{1+n} \end{pmatrix} \begin{pmatrix} e^{i\psi} & 0 \\ 0 & e^{-i\psi} \end{pmatrix}$$

$$\phi(j) = \frac{1}{2} \arg (\beta_j \alpha_j^*) , \quad \psi(j) = -\frac{1}{2} \arg (\beta_j \alpha_j) , \quad n(j) = |\beta_j|^2$$

Strength of interaction (perturbativity) is quantified by

$$\frac{N_s}{H\Delta t} \frac{\text{Var}[m_j]}{H^2} \equiv \mathcal{N}_s \frac{\sigma^2}{H^2}$$

Perturbative (Fokker-Planck)



Non-perturbative (Backreaction)

$$\text{With } \lambda = 2n+1, \quad \tilde{\lambda} = \sqrt{\lambda^2 - 1}, \quad \xi = \frac{N_s}{2} \left(\frac{\sigma}{2k_{\text{phys}}} \right)^2$$

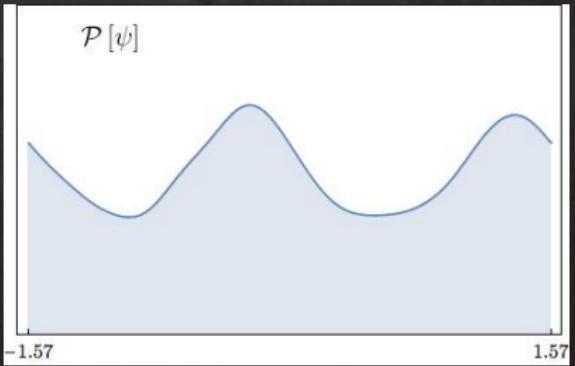
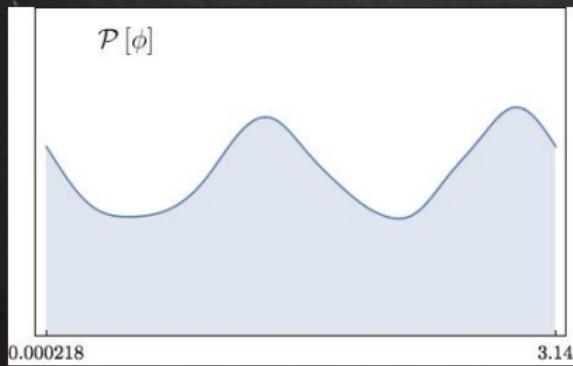
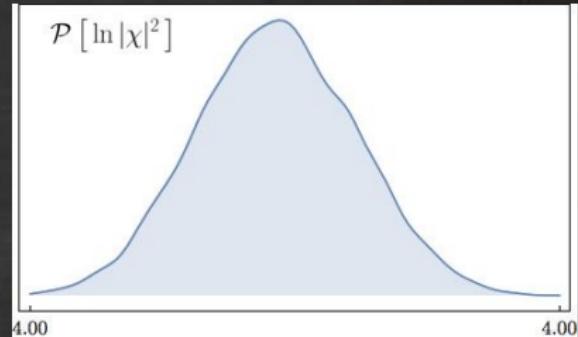
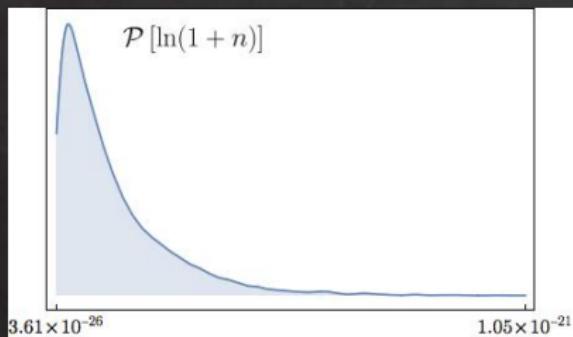
Sub-horizon:

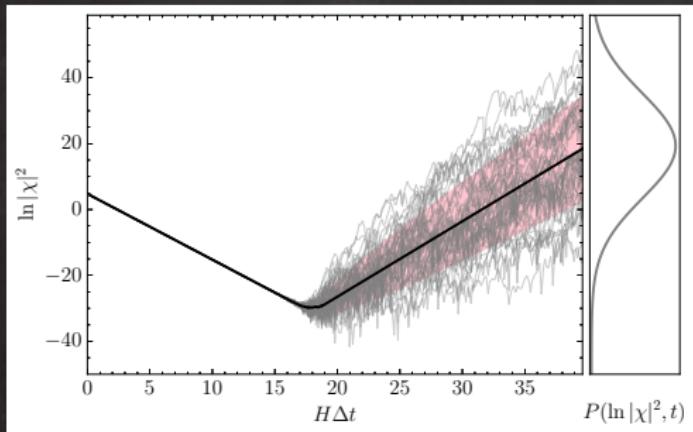
$$\frac{\partial P}{\partial \xi} = -2 \frac{\partial}{\partial \lambda} (\lambda P) + \frac{\partial^2}{\partial \lambda^2} (\tilde{\lambda}^2 P)$$

Super-horizon:

$$\begin{aligned} \frac{1}{4} \frac{\partial P}{\partial \xi} = & -\frac{\partial}{\partial \lambda} [\lambda \cos^2 \phi P] + \frac{\partial}{\partial \phi} [\cos^3 \phi \sin \phi P] + \frac{1}{2} \frac{\partial^2}{\partial \lambda^2} [\lambda^2 \sin^2 2\phi P] \\ & + 2 \frac{\partial^2}{\partial \lambda \partial \phi} [\lambda \cos^3 \phi \sin \phi P] + \frac{1}{2} \frac{\partial^2}{\partial \phi^2} [\cos^4 \phi P] \end{aligned}$$

$$N_s = 1000, \quad N_{\text{real}} = 1000, \quad \mathcal{N}_s(\sigma/H)^2 = 10^{-4}$$



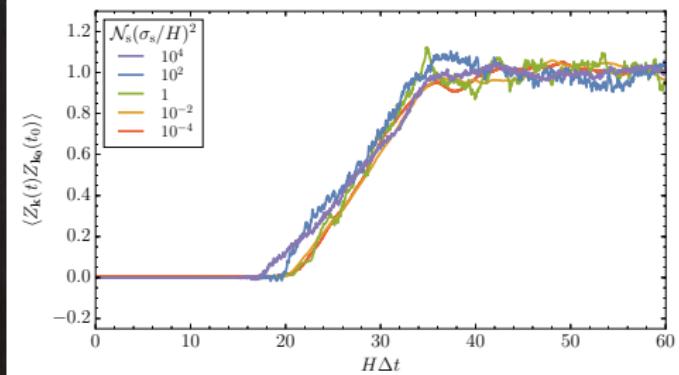


Mean

$$\langle \ln |\chi|^2 \rangle \propto H(t - t_*)$$

Variance

$$\text{Var} [\ln |\chi|^2] \propto H(t - t_*)$$



$$\langle Z_{\mathbf{k}}(t)Z_{\mathbf{k}_0}(t_0) \rangle \propto \min(t - t_*, t_0 - t_*) \times \theta(t - t_*)$$

where

$$Z_{\mathbf{k}}(t) = \ln |\chi|^2 - \langle \ln |\chi|^2 \rangle$$

GEOMETRIC RANDOM WALK

Power Spectrum

$$\text{background dynamics} \longrightarrow \text{particle production} \longleftrightarrow \text{curvature fluctuations}$$

$$\langle \chi_{\mathbf{k}_1} \chi_{\mathbf{k}_2} \dots \rangle \qquad \qquad \qquad \langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \dots \rangle$$

(D. Green, B. Horn, L. Senatore, E. Silverstein 0902.1006
D. L. Nacir, R. Porto, L. Senatore, M. Zaldarriaga 1109.4192
M. Dias, J. Frazer, D. Marsh 1604.05970
R. Flauger, M. Mirbabayi, L. Senatore, E. Silverstein 1606.00513)

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$$\Delta_{\zeta}^2 \sim \left. \Delta_{\zeta}^2 \right|_{\chi=0} + C \times \left(\left. \Delta_{\zeta}^2 \right|_{\chi=0} \right)^2 \times \mathcal{N}_s \frac{\sigma^2}{H^2}$$

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$$\langle \zeta^n \rangle \sim \langle \zeta^n \rangle_{\chi=0} + \mathcal{C}' \times \langle \zeta^2 \rangle_{\chi=0}^n \times \exp \left[\frac{n^2}{2} F \left(\mathcal{N}_s \frac{\sigma^2}{H^2} \right) \right]$$

from log-normality

(work in progress!)

The Future

- Non-conformal (massless) spectator
- Tensor fluctuations?
- (P)reheating
- Strong scattering regime: Random Matrix Theory?
- Non-linearity / Backreaction