Sparse Delay-Doppler Channel Estimation in Rapidly Time-Varying Channels for Multiuser OTFS on the Uplink

Rasheed O K, G. D. Surabhi, and A. Chockalingam
Department of ECE, Indian Institute of Science, Bangalore 560012

***Abstract*—Orthogonal time frequency space (OTFS) modulation is a new modulation scheme that operates in the delay-Doppler (DD) domain. In this paper, we consider the problem of *delay-Doppler channel estimation* in OTFS multiple access (OTFS-MA) systems on the uplink. Recognizing the inherent sparse nature of the delay-Doppler (DD) representation of time-varying channels, we model the DD channel estimation problem as a sparse signal recovery problem. To solve this problem, we employ compressed sensing (CS) based estimation techniques. Specifically, we present orthogonal matching pursuit (OMP) and modified subspace pursuit (MSP) based algorithms for DD channel estimation in uplink OTFS-MA. We compare the performance of the proposed CS-based estimation schemes with that of the impulse based channel estimation scheme reported in the OTFS literature. The proposed CS-based algorithms are shown to achieve superior normalized mean squared error and bit error performance compared to those of the impulse based channel estimation scheme.**

keywords: *OTFS modulation, multiuser OTFS, delay-Doppler domain, channel estimation, sparse signal recovery.*

I. INTRODUCTION

Next generation wireless systems (5G and beyond) are envisioned to facilitate seamless and reliable communication in high-mobility environments that include scenarios such as high-speed bullet trains, aircrafts, vehicle-to-vehicle (V2V), and vehicle-to-infrastructure (V2X) communications. Orthogonal time frequency space (OTFS) is a recent modulation technique that has been shown to have robust performance in high-mobility environments [1]-[4]. The fundamental premise of OTFS modulation lies in the representation of the channel and the multiplexing of information symbols in the delay-Doppler (DD) domain, rather than in the time-frequency (TF) domain as done in conventional multicarrier modulation techniques. A key advantage of DD representation of wireless channels is that the rapidly time-varying channels appear almost invariant in time when viewed in the DD domain. The channel in DD domain appears stationary for a longer duration of time, and hence the problem of channel estimation in rapidly time-varying channels is simplified in DD domain compared to channel estimation in TF domain. Also, the channel in the DD domain has a sparse representation, requiring only a few parameters to be estimated at the receiver.

Several works on detection and channel estimation in OTFS modulation have been reported in the literature [5]-[8], [11]-[12]. The detection of OTFS signals using message passing and Markov chain Monte Carlo (MCMC) based algorithms are presented in [5] and [6], respectively. While [5] assumes perfect knowledge of the channel at the receiver for the signal detection, [6] presents a channel estimation scheme based on pseudo-random noise (PN) pilot and uses the estimated channel for signal detection. A channel estimation technique that uses impulses in the DD domain as pilots has also been reported in [1], which is further extended to OTFS in MIMO settings in [7]. In [8]-[10], OTFS based multiple access systems have been investigated. Also, [8] has extended the impulse based channel estimation scheme to multiuser uplink setting. The idea of embedding impulses as pilots along with the data symbols in the DD plane has been proposed in [11]. In [12], an orthogonal matching pursuit based channel estimation for OTFS has been reported in the context of multiuser downlink.

In this paper, we consider OTFS based multiple access (OTFS-MA) system in an uplink setting and address the problem of DD channel estimation. The channel in the DD representation is sparse and hence can be estimated using CS-based estimation techniques. Recognizing the above fact, we model the problem of DD channel estimation as a sparse recovery problem. We propose orthogonal matching pursuit (OMP) and modified subspace pursuit (MSP) based DD channel estimation algorithms for uplink OTFS-MA. Our results show that the CS-based estimation schemes achieve superior normalized mean squared error (NMSE) and bit error rate (BER) performance compared to those with impulse based channel estimation.

II. MULTIUSER OTFS ON THE UPLINK

Consider an OTFS-MA system with K_u uplink users communicating with the base station (BS) receiver as shown in Fig. 1. Each user is equipped with a single transmit antenna and OTFS modulation is employed for uplink transmission. The receiver at the BS is equipped with a single receive antenna. The information bits from different users are multiplexed on an $N \times M$ delay-Doppler grid Γ, given by

$$\Gamma = \{(\tfrac{k}{NT}, \tfrac{l}{M\Delta f}), k = 0, 1, \cdots, N-1, l = 0, 1, \cdots, M-1\}, \quad (1)$$

where $1/NT$ and $1/M\Delta f$ are the resolutions of the Doppler shift and the delay shift, respectively, with N being the number of Doppler bins and M being the number delay bins. Let $\tau_{\max}$ denote the maximum delay spread and $\nu_{\max}$ the maximum Doppler spread of the multiuser channel. Then, the Δf is chosen such that $\nu_{\max} < \Delta f < 1/\tau_{\max}$ is satisfied. As in [8], we call a bin on the DD grid Γ as a DD resource block (DDRB). Let $x_u[k, l]$,

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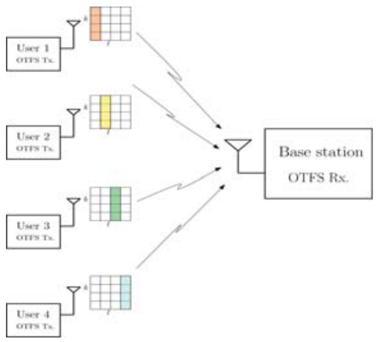


Fig. 1: Multiuser OTFS on the uplink.

$k = 0, 1, \cdots N-1$, $l = 0, 1, \cdots M-1$, and $u = 0, 1, \cdots K_u - 1$ denote the information symbol transmitted by the uth user on (k, l)th DDRB, such that $x_u[k, l] \in \mathbb{A}$, where $\mathbb{A}$ is a modulation alphabet.

The information symbols of uth user $x_u[k, l]$ are mapped from the DD domain to the TF domain using inverse symplectic finite Fourier transform (ISFFT). The modulated TF signal corresponding to the uth user is given by

$$X_u[n, m] = \frac{1}{\sqrt{MN}} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} x_u[k, l] e^{j2\pi\left(\frac{nk}{N} - \frac{ml}{M}\right)}. \quad (2)$$

The obtained TF signal is then transformed to the time domain using Heisenberg transform for transmission over the time-varying channel. The transmitted time domain signal of the uth user is given by

$$x_u(t) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} X_u[n, m] g_{tx}(t - nT) e^{j2\pi m \Delta f (t - nT)}, \quad (3)$$

where $g_{tx}(t)$ denotes the transmit pulse shape.

The channel is represented in the DD domain whose complex baseband channel response for the uth user is denoted by $h_u(\tau, \nu)$, where τ denotes delay and ν denotes the Doppler shift. The transmitted signal $x_u(t)$ experiences the rapidly time-varying channel before it is received at the BS. The received time domain signal $y(t)$ at the BS is given by

$$y(t) = \sum_{u=0}^{K_u-1} \int_{\nu} \int_{\tau} h_u(\tau, \nu) x_u(t - \tau) e^{j2\pi\nu(t-\tau)} \mathrm{d}\tau \mathrm{d}\nu + n(t), \quad (4)$$

where $n(t)$ denotes the additive white Gaussian noise at the receiver. The received signal is transformed back to the DD domain by first taking it to the TF domain using Wigner transform and then to the DD domain by using symplectic finite Fourier transform (SFFT). The TF signal obtained as the output of the Wigner transform is given by

$$Y[n, m] = A_{g_{rx}, y}(t, f)|_{t=nT, f=m\Delta f}, \quad (5)$$

where $g_{rx}(t)$ denotes the receive pulse shape and $A_{g_{rx}, y}(t, f)$ denotes the cross-ambiguity function given by

$$A_{g_{rx}, y}(t, f) = \int g_{rx}^*(t' - t) y(t') e^{-j2\pi f (t' - t)} \mathrm{d}t'. \quad (6)$$

The transmit and receive pulses, $g_{tx}(t)$ and $g_{rx}(t)$, are designed such that the biorthogonality and robustness conditions are satisfied [4]. The received signal in the DD domain after SFFT on the TF signal $Y[n, m]$ is given by

$$y[k, l] = \frac{1}{\sqrt{MN}} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} Y[n, m] e^{-j2\pi\left(\frac{nk}{N} - \frac{ml}{M}\right)}. \quad (7)$$

Let $h_u(\tau, \nu)$ denote the DD channel corresponding to the uth user. If $h_u(\tau, \nu)$ has finite support bounded by $(\tau_{\max}, \nu_{\max})$ and if $A_{g_{rx} g_{tx}}(t, f) = 0$ for $t \in (nT - \tau_{\max}, nT + \tau_{\max})$, $f \in (m\Delta f - \nu_{\max}, m\Delta f + \nu_{\max})$, $\forall (n, m) \neq (0, 0)$, the end-to-end input-output relation in DD domain for the considered uplink OTFS-MA system can be written as

$$\begin{aligned} y[k', l'] = & \frac{1}{MN} \sum_{u=0}^{K_u-1} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} x_u[k, l] \\ & \cdot \tilde{h}_u[(k' - k)_N, (l' - l)_M] + n[k', l'], \end{aligned} \quad (8)$$

where $(.)_N$ denotes modulo-N operation, $n[k', l']$ denotes the additive white Gaussian noise, and $\tilde{h}_u(k, l)$ is as described in [4]. The 2D circular convolution of symbols transmitted by each user with the corresponding channel in (8) can be written in a vectorized form as in the case of single user setting in [5]. Denoting the OTFS symbol vector transmitted by uth user by $\mathbf{x}_u \in \mathbb{C}^{MN \times 1}$ ($\mathbf{x}_{u\,k+Nl} = x_u[k, l]$) and the channel matrix of uth user by $\mathbf{H}_u \in \mathbb{C}^{MN \times MN}$, the input-output relation in uplink OTFS-MA can be written as [8]

$$\mathbf{y} = \sum_{u=0}^{K_u-1} \mathbf{H}_u \mathbf{x}_u + \mathbf{n}, \quad (9)$$

where $\mathbf{y} \in \mathbb{C}^{MN \times 1}$ is the received vector at the BS and $\mathbf{n}$ is the additive white Gaussian noise vector with $\mathbf{n}_{k+Nl} = n[k, l]$.

The DDRBs in the delay-Doppler grid can be allocated to the uplink users in different ways [8]. Here, we consider an allocation scheme in which the users are multiplexed along the delay axis. That is, disjoint and contiguous bins along the delay axis are allocated to each user such that each user gets M/K_u columns of the DD grid for transmission. The DD grid of the uth user will have

$$x_u[k, l] = \begin{cases} a \in \mathbb{A} & \text{if } k \in \{0, 1 \cdots, N-1\} \ \& \\ & l \in \{u\frac{M}{K_u}, \cdots, (u+1)\frac{M}{K_u} - 1\} \\ 0 & \text{otherwise.} \end{cases} \quad (10)$$

Figure 2 shows an illustration of the DDRB allocation on an $N \times M = 16 \times 16$ delay-Doppler grid for $K_u = 4$ users, where the users are allocated DDRBs along the delay axis.

III. Sparse delay-Doppler channel estimation

In this section, we propose CS-based schemes for channel estimation in the DD domain for uplink OTFS-MA.

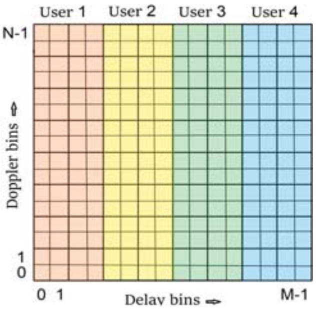


Fig. 2: DDRB allocation in $N \times M$ DD grid along the delay axis.

A. Impulse based DD channel estimation

Impulse based channel estimation technique uses an impulse function $\delta(k, l)$ in the DD domain as the pilot. The pilot corresponding to the uth user is an impulse denoted by $\delta(k_u^p, l_u^p)$, such that (k_u^p, l_u^p) is a DDRB allocated to the uth user for channel estimation [8]. Let x_p denote the pilot and '0' denote the guard symbol. The pilot corresponding to the uth user is allocated as

$$x_u[k,l] = \begin{cases} x_p & k = k_u^p, l = l_u^p, \\ 0 & \text{otherwise.} \end{cases} \tag{11}$$

The interaction of pilot with the channel results in a two-dimensional periodic convolution of the DD impulse response with the pilot. The received pilot in the DD domain corresponding to the uth user is given by

$$y_u^p[k', l'] = \frac{1}{MN} \tilde{h}_u[(k' - k_u^p)_N, (l' - l_u^p)_M] + v[k', l'], \tag{12}$$

which gives the estimate for the uth user's channel, where $k' \in \{0, \cdots, N-1\}$ and $l' \in \{l_u^p, \cdots, l_u^p + \max_i \alpha_{u,i}\}$, and $\alpha_{u,i} = M \Delta f \tau_{u,i}$ and $\tau_{u,i}$ denotes the delay of ith path in the uth user's channel.

B. DD channel estimation as sparse recovery problem

The vectorized input-output relation in (9) can be rewritten in an alternate form as

$$\begin{aligned} \mathbf{y} &= \sum_{u=0}^{K_u-1} \mathbf{X}_u \mathbf{h}_u + \mathbf{n}, \\ &= [\mathbf{X}_0 \mathbf{X}_1 \cdots \mathbf{X}_{K_u-1}] \begin{bmatrix} \mathbf{h}_0 \\ \mathbf{h}_1 \\ \vdots \\ \mathbf{h}_{K_u-1} \end{bmatrix} + \mathbf{n}, \\ &= \mathbf{X}\mathbf{h} + \mathbf{n}, \end{aligned} \tag{13}$$

where $\mathbf{h}_u \in \mathbb{C}^{MN \times 1}$ denotes the equivalent DD channel of uth user in the vectorized form such that $\mathbf{h}_{u\ k+Nl} = h_u[k, l]$ and $\mathbf{X}_u \in \mathbb{C}^{MN \times MN}$ is obtained from (8). It is noted that the vectors $\mathbf{h}_u, u = 0, \ldots, K_u - 1$ are sparse with only P non-zeros out of the NM entries. Therefore, the channel $\mathbf{h}$ in (13) is a sparse vector with $K_u P$ non-zeros out of the $K_u NM$ entries of the vector, leading to a sparsity factor of $\frac{P}{NM}$. For example, if $N = M = 32$, $P = 6$, and $K_u = 4$, then the sparsity factor of the OTFS-MA channel $\mathbf{h}$ is $\frac{P}{NM} = \frac{6}{32 \times 32} = \frac{6}{1024}$. This inherent sparsity can be exploited to achieve efficient channel estimation in the DD domain using CS-based algorithms. The uplink OTFS-MA channel estimation problem can therefore be formulated as a sparse recovery problem as follows:

$$\min \|\mathbf{h}\|_0 \qquad \text{s.t} \qquad \mathbf{y} = \mathbf{X}\mathbf{h} + \mathbf{n}. \tag{14}$$

If the number of paths P of the channel is known apriori, standard greedy sparse recovery algorithms such as orthogonal matching pursuit (OMP) [13] and subspace pursuit (SP) [14] can be used to solve (14) and estimate the channel $\mathbf{h}$. However, we do not assume the knowledge of the number of paths in the channel. Although OMP can be readily modified to work without the knowledge of the sparsity, modifying the SP to work in the absence of the knowledge of sparsity is non-trivial. In the following subsections, we present OMP and modified SP algorithms for OTFS-MA uplink channel estimation.

C. OMP based DD channel estimation

The OMP algorithm for uplink OTFS-MA channel estimation is listed in **Algorithm 1**. The algorithm takes the received signal vector $\mathbf{y}$ and the pilot matrix $\mathbf{X}$ as the input. In the kth iteration, the residue of the previous iteration $\mathbf{r}^{k-1}$ is projected on the columns of $\mathbf{X}$ and the index of the column which results in the highest correlation (denoted by T^k) is added to the current support S^k (lines 5 and 6). Then, the non-zero values corresponding to the obtained support is computed using the least squares (line 7). The residue is updated (line 8) such that the effect of previously updated support is removed, ensuring a new support in the next iteration. The above process is continued until a stopping criterion is satisfied. We use threshold ϵ on the energy of the residue as the stopping criterion. The OMP algorithm, as presented above, does not require the knowledge of the number of paths in the channel. Both the number of paths as well as the channel gains on these paths are estimated from the received vector $\mathbf{y}$ and the pilot matrix $\mathbf{X}$. We use i.i.d complex Gaussian random sequences in the DD domain as pilots for all the users.

Algorithm 1 OMP based uplink OTFS-MA channel estimation

1: Inputs: $\mathbf{y}, \mathbf{X}, \epsilon$
2: Initialize: $k = 0$, $\mathbf{h}^0 = \mathbf{0}$, $S^0 = \phi$, $\mathbf{r}^0 = \mathbf{y}$
3: **while** $\left(\|\mathbf{r}^k\|_2 > \epsilon\right)$
4: $\quad k = k + 1$
5: $\quad T^k = \underset{j}{\text{argmax}}\, |\mathbf{X}^H \mathbf{r}^{k-1}|$ ▷ Support estimation
6: $\quad S^k = S^{k-1} \cup T^k$ ▷ Support update
7: $\quad \mathbf{h}_{S^k} = \mathbf{X}_{S^k}^{\dagger} \mathbf{y}$ ▷ Calculate non-zeros
8: $\quad \mathbf{r}^k = \mathbf{y} - \mathbf{X}_{S^k} \mathbf{h}_{S^k}$ ▷ Residue update
9: **end**
10: Output: Estimated OTFS-MA uplink channel vector $\widehat{\mathbf{h}}_{S^k} = \mathbf{X}_{S^k}^{\dagger} \mathbf{y}$ and $\widehat{\mathbf{h}}_{\bar{S}^k} = \mathbf{0}$

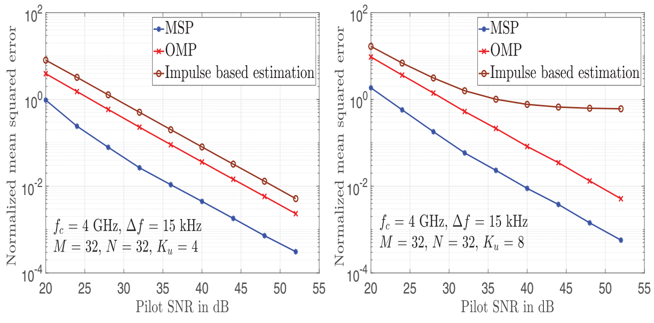
D. Modified SP (MSP) based DD channel estimation

SP has been shown to achieve more robust sparse recovery performance compared to OMP algorithm. However, unlike OMP, the knowledge of underlying sparsity is crucial for achieving good performance with SP. Therefore, we propose a modified SP (MSP) which iteratively estimates the support (and hence the number of paths P) and subsequently computes the values corresponding to the estimated support (channel gains). The listing of MSP algorithm is presented in **Algorithm 2**.

Algorithm 2 MSP based uplink OTFS-MA channel estimation

1: Inputs: $\mathbf{y}, \mathbf{X}, \epsilon_1$
2: Initialize: $i = 1$, $\mathbf{r}_1 = \mathbf{y}$
3: **while** $(\|\mathbf{r}_i\|_2 - \|\mathbf{r}_{i-1}\|_2 > \epsilon_1)$
4: Initialize: $k = 0$, $\mathbf{h}_i{}^0 = \mathbf{0}$, $S_i{}^0 = \{l_1^0, \ldots, l_i^0\}$ are indices of i max. entries of $|\mathbf{X}^H\mathbf{y}|$, $\mathbf{a}_i^0 = \mathbf{X}^{\dagger}_{S_i^0}\mathbf{y}$, $\mathbf{r}_i{}^0 = \mathbf{y} - \mathbf{X}_{S_i^0}\mathbf{a}_i^0$
5: **while** $(k \leq k_{max}\ )$
6: $k = k + 1$
7: $\tilde{S}_i^k = S_i^k \cup \Theta_i^k$, where Θ_i^k is set of i indices corresponding to the i max. entries of $|\mathbf{X}^H\mathbf{r}_i^{k-1}|$
8: $\mathbf{z}_i^k = \mathbf{X}_{\tilde{S}_i^k}\mathbf{y}$
9: $S_i^k = \{l_1^k, \ldots, l_i^k\}$ are i entries from $\tilde{S}_i^k$ which leads to i max. entries of $|\mathbf{z}_i^k|$
10: $\mathbf{a}_i^k = \mathbf{X}^{\dagger}_{S_i^k}\mathbf{y}$
11: $\mathbf{r}_i^k = \mathbf{y} - \mathbf{X}_{S_i^k}\mathbf{a}_i^k$
12: **end**
13: $\mathbf{r}_i = \mathbf{r}_i^{k_{max}}$
14: $S_i = S_i^{k_{max}}$
15: $i = i + 1$
16: **end**
17: Output: Estimated OTFS-MA uplink channel vector $\widehat{\mathbf{h}}_{S_i} = \mathbf{X}^{\dagger}_{S_i}\mathbf{y}$ and $\widehat{\mathbf{h}}_{\bar{S}_i} = \mathbf{0}$

The main idea behind **Algorithm 2** is to obtain the sparse solutions using SP by varying the sparsity value i from $i = 1$ to $i = Q$, such that for $i > Q$ the decrease in residue is negligible. For each sparsity value i, the algorithm starts with an initial support S_i^0, which contains the initial estimates of the non-zero positions of $\mathbf{h}$. To find this initial estimate S_i^0, the algorithm computes the projection of the received vector $\mathbf{y}$ on the columns of the pilot matrix $\mathbf{X}$ and selects the indices of i columns of $\mathbf{X}$ which results in the i maximum entries in the projection. The initial estimate of the non-zero values $\mathbf{a}_i^0$ and the initial residue $\mathbf{r}_i^0$ corresponding to the sparsity value i are computed as shown in step 4 of the algorithm. With this initialization, for each sparsity value i, the algorithm then refines the support in each iteration k (S_i^k in the algorithm) and computes the non-zero values corresponding to the refined support ($\mathbf{a}_i^k$) as done in the conventional SP algorithm (steps 5 to 12 of **Algorithm 2**). The residue obtained for the sparsity value i, denoted as $\mathbf{r}_i$, is then compared with the residue obtained for the sparsity value $i-1$, denoted as $\mathbf{r}_{i-1}$. If the difference $(\|\mathbf{r}_i\|_2 - \|\mathbf{r}_{i-1}\|_2)$ is greater than a threshold ϵ_1 (which is chosen sufficiently small), then the sparsity value i is incremented and the steps 4 to 15 are repeated. Where as, if the difference $(\|\mathbf{r}_i\|_2 - \|\mathbf{r}_{i-1}\|_2)$ is less than the threshold ϵ_1, the algorithm is stopped and the estimate of the uplink OTFS-MA channel is obtained as shown in step 17 of the algorithm. The pilot matrix $\mathbf{X}$ used for the MSP algorithm is same as that used in the OMP algorithm.



(a) $K_u = 4$ (b) $K_u = 8$

Fig. 3: NMSE performance of i) MSP, ii) OMP, and iii) impulse based channel estimation scheme in uplink OTFS-MA.

IV. Results and discussions

In this section, we present the performance of CS-based channel estimation algorithms presented in Sec. III. Figures 3a and 3b show the normalized mean squared error (NMSE) performance of the proposed algorithms as a function of pilot SNR for four and eight uplink users ($K_u = 4, 8$), respectively. A DD grid with $M = 32$ bins along the delay axis and $N = 32$ bins along the Doppler axis is considered. A carrier frequency of 4 GHz, subcarrier spacing of 15 kHz, and BPSK modulation are used. The DD channel corresponding to the uth user is given by $h_u(\tau, \nu) = \sum_{i=1}^{P_u} h_{u,i}\delta(\tau - \tau_{u,i})\delta(\nu - \nu_{u,i})$, where $h_{u,i}$s are assumed to be i.i.d and $\tau_{u,i}$ and $\nu_{u,i}$ denote the delay and Doppler shifts, respectively, corresponding to the ith path of the uth user's channel. We consider a channel with five paths ($P = 5$) with an exponential power-delay profile. The delay-Doppler profile used for simulations is given in Table I.

TABLE I: Delay-Doppler profile

Path index (i)	1	2	3	4	5
Delay $\tau_{u,i}$ (μs)	2.08	4.164	6.246	8.328	10.41
Doppler $\nu_{u,i}$ (Hz)	0	470	940	1410	1880

From Figs. 3a and 3b, we observe that the NMSE performance of the CS-based estimation techniques is superior compared to that of the impulse based channel estimation scheme. Among

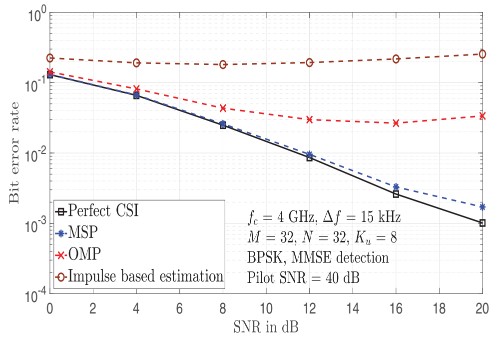


Fig. 4: BER performance of uplink OTFS-MA for $K_u = 8$ with channel estimated using $i)$ MSP, $ii)$ OMP, and $iii)$ impulse based estimation scheme using a pilot SNR of 40 dB.

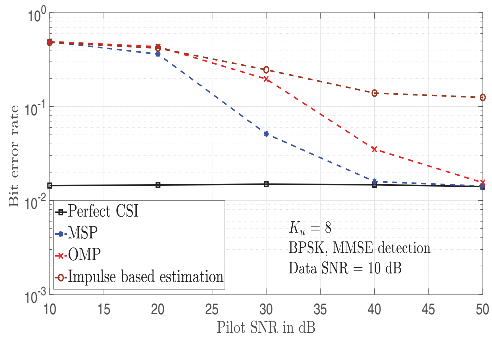


Fig. 5: BER performance as a function of pilot SNR for $K_u = 8$.

the CS-based techniques, the proposed MSP algorithm shows a better NMSE performance compared to that of OMP. Further, it can be seen that the NMSE performance of impulse based channel estimation scheme shows severe degradation with increase in the number of uplink users. This is because the impulse based channel estimation scheme does not involve any steps to combat multiuser interference, and hence it suffers from multiuser interference.

Figure 4 shows the BER performance of uplink OTFS-MA systems for $K_u = 8$, with the channel estimated using $i)$ MSP, $ii)$ OMP, and $iii)$ impulse based estimation scheme. The BER performance with perfect CSI is also shown for comparison. A pilot SNR of 40 dB is used and MMSE detection is performed. All other parameters used for the simulations are the same as those used for Fig. 3. From the figure we observe that the BER performance of uplink OTFS-MA system with impulse based channel estimation is very poor and floors at 0.2. The BER performance with channel estimated using OMP is better than that with the impulse based estimation. However, the BER with OMP based channel estimation also floors at 0.02. The BER performance of uplink OTFS-MA system with channel estimated using MSP is very close to the performance with perfect CSI.

Figure 5 shows the BER as a function of pilot SNR for channel estimated using OMP, MSP, and impulse based channel estimation, for $K_u = 8$ users. The BER is computed for a data SNR of 10 dB for all the systems. The BER with perfect CSI is also shown in the figure for comparison. From the figure, we observe that lower BERs are achieved with increasing pilot SNRs. Also, it can be observed that the BER performance of OTFS-MA system using MSP based channel estimation almost overlaps with the BER using perfect CSI at smaller pilot SNRs compared to OMP and impulse based channel estimation schemes.

V. Conclusions

We considered uplink multiple access using the recently proposed OTFS modulation and addressed the problem of DD channel estimation. Recognizing the inherent sparsity in the DD channel, we proposed the use of CS-based algorithms for DD channel estimation. We presented OMP and modified SP based algorithms and compared their NMSE and BER performances with those of impulse based channel estimation technique reported in the literature. Our results showed that the CS-based channel estimation schemes significantly outperform impulse based channel estimation scheme, highlighting the superiority of CS-based delay-Doppler channel estimation schemes.

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