Performance of MIMO systems with channel inversion

Conference Paper · May 2002

DOI: 10.1109/VTC.2002.1002659 · Source: IEEE Xplore

Thomas Haustein
Fraunhofer-Institut für Nachrichtentechnik, Heinrich-Hertz-Institut

Eduard Jorswieck
Technische Universität Braunschweig

Volker Jungnickel
Fraunhofer-Institut für Nachrichtentechnik, Heinrich-Hertz-Institut

Volker Pohl
Technische Universität München

PERFORMANCE OF MIMO SYSTEMS WITH CHANNEL INVERSION

T. Haustein, C.von Helmolt, E. Jorswieck, V. Jungnickel, V. Pohl
Heinrich-Hertz-Institut für Nachrichtentechnik Berlin GmbH, Einsteinufer 37,10587 Berlin, GERMANY
e-mail: haustein@hhi.de

Abstract— **The paper discusses channel inversion which is a spatial equalization technique when channel state information is available at the transmitter. Channel inversion is a straight-forward concept without iterations and it might be useful when the data transmission is critical with time e.g. high data rate applications. We discuss performance degradation caused by channel estimation errors, clipping due to a limited range of the transmitted power and the effect of co-channel interference. These results give insight into technical constrains of this transmission technique and show how these critical issues can be limited or reduced.**

I. Introduction

Wireless communication systems using multiple antennas at the transmitter (Tx) and the receiver (Rx) have the potential to increase the channel capacity dramatically [1]. Various transmission schemes based on channel knowledge at the Rx and eventually at the Tx have been proposed [2] and promising experimental results have been shown [3]. This paper focuses on channel inversion (CI) which has a simple linear algebraic structure and therefore might be useful for high data rate applications. We discuss the bit error rate (BER) performance of CI with respect to channel estimation errors, limited dynamics at the Tx and co-channel interference. We derive an analytical expression for the impact of channels estimation errors on the BER performance for CI in the down-link and we compare the robustness of CI, VBLAST [2] and Zero-Forcing (ZF) against channel estimation errors. Next the effect of clipping at the Tx power amplifier is investigated in detail. Finally we show the effect of co-channel interference which proves to be critical for MIMO systems in general. We show that Individual Power Control (IPC) in each data stream can reduce the corresponding performance degradation.

II. Channel Inversion

Assuming n_T transmit and m_R receive antennas, the flat fading channel can be described with an $n_T \times m_R$-channel matrix H and the transmission equation is given by: $y = Hx + n$ (y : receive vector, x : signal transmit vector, n : noise vector). With the assumption that $Rank(H) = min(n_T, m_R) = m$, data transmission can be performed over m parallel sub-channels.

If channel state information (CSI) is available at the transmitter, CI can be performed at the Tx by inverting the channel before transmission according to the following formulas:

Down-link CI ($n_T > m_R$)
T_x pre-processing: $\tilde{x}^{CI}_{Down} = H^{\dagger}x$.
Reconstructed data at R_x

$$x' = H\tilde{x}^{CI}_{Down} + n = x + n. \quad (1)$$

The signals to transmit (x) are pre-processed by a multiplication with $H^{\dagger}$ denoting the Moore-Penrose pseudo-inverse of H, thus equalizing the signal-to-noise-ratio (SNR) in all transmission channels. In this way each data signal is fully reconstructed at one Rx antenna and forced-to-zero at all other antennas by destructive interference thus creating parallel channels at the Rx without post-processing for signal separation.

Up-link CI ($n_T < m_R$)
T_x pre-processing: $T_x : \tilde{x}^{CI}_{Up} = VD^{-1}V^H x$.
Reconstructed data at R_x

$$x' = UV^H(H\tilde{x}^{CI}_{Up} + n) = x + UV^H n. \quad (2)$$

(The matrices U, D, V are derived from singular value decomposition of H ($H = UDV^H$). $[\cdot]^H$ means Hermitean Conjugate, $[\cdot]^{-1}$ means inverse matrix.)

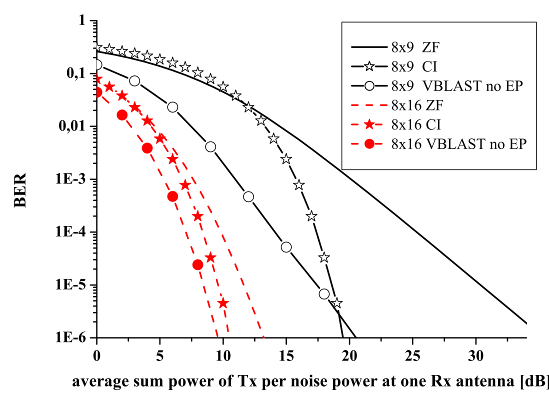

Fig. 1. BER performance for two MIMO systems using ZF (no symbols), VBLAST without error propagation (○) and CI (⋆) with 8 Tx antennas and 9 (–) / 16 (- -) Rx antennas, perfect CSI is assumed.

Qualitatively, CI performs like data transmission over a channel with additive white Gaussian noise (AWGN). In the following, channel reprocity is always assumed which enables the Tx to obtain CSI prior to the transmission from a channel measurement into the opposite direction.

Fig.1 shows the simulated BER of MIMO systems with perfect CSI and BPSK modulation which use Zero-Forcing (ZF), VBLAST or CI. Taking into consideration that for CI the average Tx power depends on the number of Tx and Rx antennas [4] the abscissa of Fig.1 shows the average transmitted sum power per noise power at one receive antenna to give a fair comparison of the transmission techniques. CI performs better than ZF with BERs of less than 10^{-3} but performs worse than VBLAST until a BER of 10^{-5} or below where the curves cross each other. If the antenna diversity is sufficiently high (8 Tx / 16 Rx), all techniques perform quite similar (dashed curves).

III. Performance Degradation

A. Channel Estimation Errors

We assume a channel estimation that is based on a correlation of orthogonal pilot sequences (e.g. Gold- or Hadamard-sequences with minimum length of n_T) transmitted in an extra time slot to identify all Tx antennas simultaneously. If H is the channel matrix for the up-link ($n_T < m_R$) then we expect a channel estimation error which is determined by the following statistics [5]:

$\Delta H = \sigma_{Corr} \cdot N$, ($\sigma_{Corr} = \sqrt{\frac{n_T}{L \cdot SNR}}$)

with N being a matrix of the same size as H which follows statistics of independently and identically distributed complex Gaussian entries with zero mean and unit variance, L: length of the pilot sequence, SNR: average signal-to-noise-ratio per receive antenna. Based on this channel estimation we perform CI in the down-link (the transmission channel is now H^T). The estimate of the transmitted data symbols can then be written as:

$$x' = H^T \tilde{x} + n \quad \text{with:} \quad \tilde{x} = (H^T + \Delta H^T)^{\dagger} x$$

Using linear Taylor expansion for the estimated transmit matrix we obtain the following expression for the Down-link CI

$$x' = x + n + \sigma_{Corr} N^T (H^T)^{\dagger} x.$$

It can be shown that the expectation values of the two following expressions are equal (for a sufficient number of antennas).

$$\langle N^T (H^T)^{\dagger} x \rangle = \langle H^{\dagger} N x \rangle \tag{3}$$

Using (3) we can give an analytical expression for the data estimate at the Rx

$$x' = x + n + \sigma_{Corr} H^{\dagger} N x. \tag{4}$$

The last term of (4) is cross-talk which is similar to additional noise that is enhanced by $H^{\dagger}$

$$x' = x + n + H^{\dagger} \tilde{n} \tag{5}$$

with $\tilde{n}$: being additional noise but independent on noise n caused by the Rx.

This means that for a pilot sequence of the length $L = n_T$ (worst case) and for high SNR (when the last term is the dominating error term) CI performs with an identical slope like ZF. With a sufficient long training sequence ($L > 32$ for the 10x8 example) CI performs better than ZF but for high SNR there is always a bend in the curve for the BER because of the term $H^{\dagger}\tilde{n}$ in (5) which still depends on the antenna diversity gain. At high SNR the second term n can be neglected and the slope of the curve is dominated by the $m_R - n_T + 1$-branch diversity of the MIMO antenna configuration (see Fig.2). The curve for perfect CSI is shifted against the AWGN curve because of the fact that more average Tx power is needed given by the factor $\frac{m_R}{n_T - m_R} = 4$ (see (6)) [4].

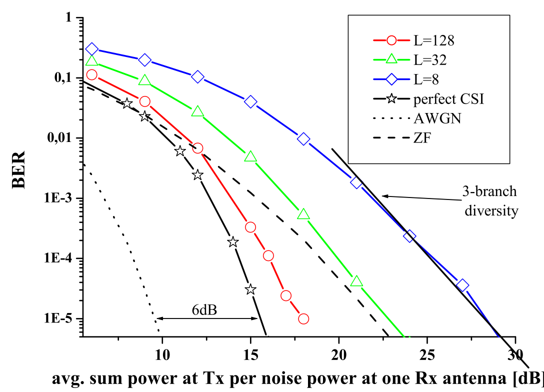

Fig. 2. Down-link-CI, MIMO 10x8, BPSK, length of sequence L variable.

The sensitivity against channel estimation errors for various transmission schemes (ZF, VBLAST, Down-link-CI, Up-link-CI) is compared in Fig.3. To be consistent we plotted the average transmitted sum power per noise at one Rx antenna needed to obtain a BER of 10^{-5} over the length of the training sequence. It is found that for a 12x8 antenna configuration we need a training sequence with $L > 16$ to achieve the same performance like ZF with perfect CSI (see dotted line). Up-link-CI and down-link-CI show the same BER performance in front of

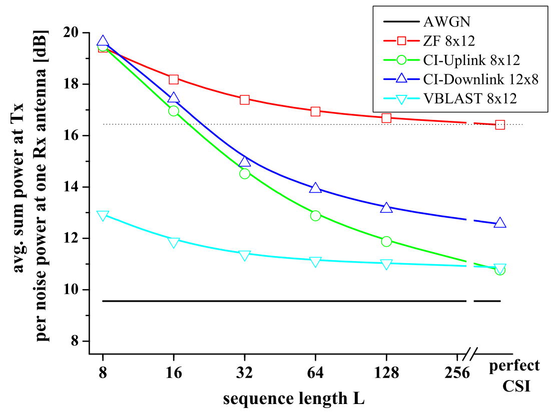

Fig. 3. Sensitivity against channel estimation errors for a BER performance of 10^{-5} (BPSK) using ZF, VBLAST and CI, variable sequence length L.

the decision unit (when the receive antenna gain is already exploited). Since CI is based on CSI at the Tx in general a better channel estimation is required than with ZF or VBLAST. Given this and a reasonable amount of antenna diversity, CI shows a good BER performance with all advantages of its simple linear algebraic structure.

B. *Limited Transmitter Dynamics*

CI requires antenna diversity, otherwise the transmitted power is not limited. This is easily understood when the expectation value of the term $\tilde{x} = H^{\dagger}x$ is discussed. If the expectation value of the data stream symbols $\langle x_i \rangle$ is normalized and $n_T > m_R$ (down-link-CI) then the expectation value of the transmitted power is

$$\langle \tilde{x}^H \tilde{x} \rangle = \langle x^H H^{\dagger H} H^{\dagger} x \rangle = \sum_{i=1}^{m} \frac{1}{\lambda_i} \|x_i\|^2$$

$$\langle \sum_{i=1}^{m} \frac{1}{\lambda_i^2} \rangle = \frac{1}{n_T - m_R} \quad \Rightarrow \quad \langle \tilde{x}^H \tilde{x} \rangle = \frac{m_R}{n_T - m_R} \quad (6)$$

with λ_i^2 being the Eigenvalues of H. We found this result by simulations for various antenna number configurations. This very important relation was also studied by [6] and [7] and its general validity could be shown by [4] for the case of m_R and n_T going to infinity with $\frac{m_R}{n_T} = const.$ If $m_R = n_T$ then the average transmit power in (6) goes to infinity. Practically this means that CI works only with a reasonable amount of antenna diversity. The additional antennas reduce the necessary dynamic range (DR) of the transmitted power, which we define as the peak to average Tx power per antenna. A small DR is favorable with regard to amplifier requirements and to peak power. If the input power at the amplifier exceeds a certain level, amplitude clipping and phase distortions are observed.

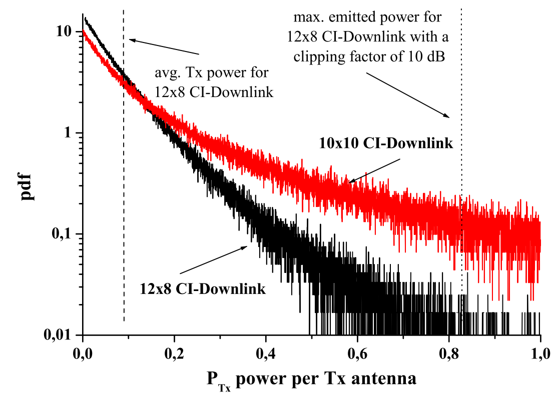

Fig. 4. PDF of the transmitted power per Tx antenna for MIMO systems using Down-link-CI (12 / 10 Tx and 8/10 Rx antennas)

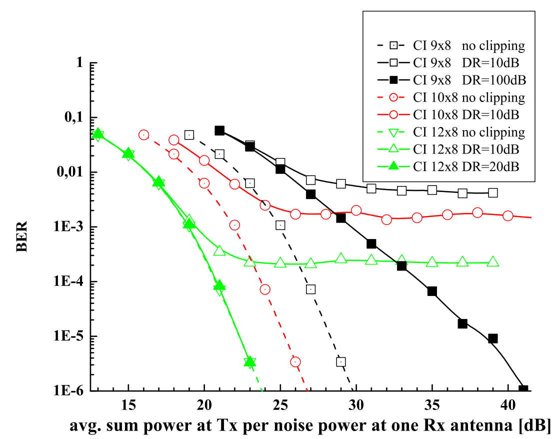

Fig. 5. Power clipping for MIMO using Up-link-CI (16-QAM, L=128), various sets of dynamic range (DR) and antenna diversity

Fig. 4 shows the simulated pdf of the transmitted power from one Tx antenna if CI is used. The black curve represents the emitted power distribution for a MIMO system with 12 Tx and 8 Rx antennas, while the grey curve is a system with 10 antennas on each side. It is obvious that the integral above the power clipping line which is also depicted in Fig.4 is significantly smaller when more Tx antennas are used.

Fig.5 illustrates how clipping at the Tx causes error floors depending on antenna diversity and on the DR of the amplifier. The floor can be estimated from the DR and from the power distribution per antenna like in Fig.4. In general, a DR = $P_{max}/\langle P \rangle$ of 20 dB seems to be sufficient when 12 Tx and 8 Rx antennas are used.

IV. Co-Channel Interference

A. BER Degradation

In reality, interference with devices operating in the same frequency band like the MIMO system have to be considered. The received interference power can be described by Rayleigh or Rician statistics, which means that the interference power may vary significantly at different Rx antennas. The resulting effect on the average BER has to be expected as follows

$$\begin{array}{rll} \text{Up-link ZF} & (n_T < m_R) & x' = x + H^{\dagger}(n+i) \\ \text{Up-link CI} & (n_T < m_R) & x' = x + VU^{H}(n+i) \\ \text{Down-link CI} & (n_T > m_R) & x' = x + (n+i). \end{array}$$

For high SNR (n is negligible) the BER is dominated by the signal-to-interference-ratio (SIR) and an error floor is expected.

B. Individual Power Control for Channel Inversion

Since the interference power per Rx antenna may vary significantly and the channel between the interference sources and the Rx antennas is generally unknown apriori, little can be done by the Rx. But a simple strategy can help out, at least partially. We propose to measure the relative noise-plus-interference power per data stream in front of the decision unit at the Rx unit and to feed back this information to the Tx. With this additional information, the Tx adapts the power in each data stream at the Tx to equalize the SINR of all data streams.

This Individual Power Control (IPC) requires little data load in the feed-back channel but it allows reduction of co-channel interference. The best results are reached with up-link-CI (see Fig.6). Down-link-CI suffers more degradation because no receive diversity is available at the Rx.

C. IPC for ZF and VBLAST

Analyzing the effect of IPC for CI, we found that it can be used far more generally. Systems which use e.g. ZF or VBLAST can also benefit from this technique. Like for CI the SINR of all data streams can be equalized. In the special case

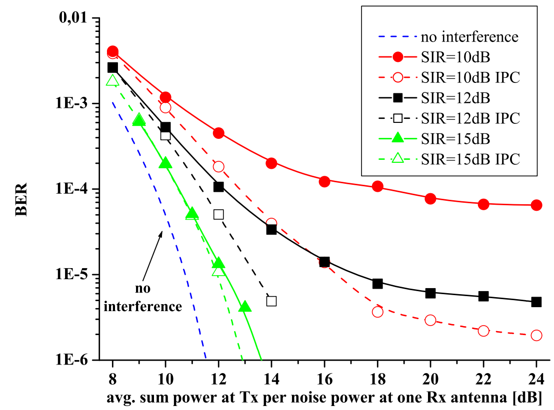

Fig. 6. IPC to combat co-channel interference for a 8x12 MIMO system using up-link CI, BPSK, two BPSK interference sources. Achievable low BER for SIR > 12dB.

of no interference sources IPC can balance the effect of noise enhancement caused by $H^{\dagger}n$. ZF will benefit directly from equalizing noise enhancement (see dotted line in Fig.7) and the transmitted power per data stream, while the BER curve for a system using VBLAST can be adopted to specific needs. For example, if we have a very good SNR, the BER can be reduced by transmitting more power into the data stream that is first to be detected (lowest noise enhancement). This first detected data stream dominates the BER at high SNR while all latter detected data streams are nearly free of errors.

So the technique of IPC is applicable to all MIMO transmis-

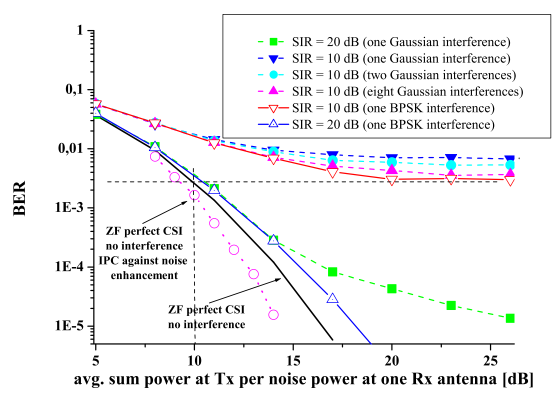

Fig. 7. Bit Error floors for 8x12 MIMO systems with co-channel interference (data BPSK modulated, ZF at Rx, interference: BPSK or Gaussian)

sion schemes at little expense.

The efficiency of IPC depends on the number of antennas, the sort of interference and the number of interference sources. Generally, IPC works the better the more parallel data streams are to be balanced. Another important issue is the distribution of the data streams over several Rx antennas. A high receive diversity improves the efficiency of IPC.

To study how the performance of IPC depends on the interference symbols we did simulations for the BER (Fig.7). We have used interference sources with a Gaussian symbol alphabet or BPSK / M-QAM as it was used for the MIMO data transmission. We found that one interference source using a Gaussian alphabet causes an error floor three times higher than one or many interference sources using BPSK or M-QAM modulation. A sufficient number of Gaussian interferers behave like an interferer using BPSK or M-QAM. This can be explained by the multiplication of only one Gaussian distributed symbol stream with the Gaussian distributed entries of the interference matrix resulting in a 2^{nd}-order Bessel distribution for the received interference power. BPSK or M-QAM symbols transmitted over the same interference channel have a power distribution at the Rx which is χ^2. The superposition of several (8-10) Gaussian interference signals performs similar like BPSK or M-QAM interference. This makes clear that only for Gaussian interference the number of sources is of importance. For interference caused by sources using BPSK or M-QAM alphabets the average SIR per Rx antenna alone determines the error floor. Then in the simple case of ZF the error floor can be predicted from the curve without interference. We just look at the BER of an system free of interference at a SNR of the same value as the SIR (see Fig.7).

V. Conclusions

This paper discussed some practical constrains of channel inversion which might be an interesting transmission technique if very simple and therefore mainly linear approaches for data transmission are requested e.g. high data rate applications. Channel inversion is based on channel knowledge at the Tx and therefore the BER performance suffers more degradation if the CSI is imperfect than with ZF or VBLAST. With a reasonable quality of CSI, channel inversion shows a good performance with BER curves similar to an AWGN channel.

We showed that the effect of power clipping at the Tx can be drastically reduced with antenna diversity.

Finally we investigated the concept of Individual Power Control at the Tx to reduce co-channel interference. Beside its benefits against co-channel interference IPC is also applicable without any interference sources to equalize noise enhancement at the Rx.

Concluding, channel inversion can provide a good BER performance if some constrains are chosen properly while it has a simple and linear structure.

References

[1] E. Telatar. Capacity of multi-antenna gaussian channels. Technical report, AT & T Bell Laboratories Internal Technical Memorandum, June 1995.

[2] G.J. Foschini. Layered space-time architecture for wireless communication in a fading environment when using multiple antennas. *Bell Labs Tech. J.*, pages pp. 41–59, 1996. 1,(2).

[3] Wolniansky et al. An architecture for realising very high data rates over the rich-scattering wireless channel. In *ISSSE-98, Pisa, Italy*, Sept. 1998. invited paper.

[4] E. Jorswieck, G. Wunder, V. Jungnickel, and T. Haustein. Inverse eigenvalue statistics for rayleigh and rician mimo channels. *IEE Professional Network on Antennas and Propagation*, pages 3/1–3/6, Dec. 2001. Seminar.

[5] T. Haustein, E. Jorswieck, V. Jungnickel, U. Krüger, V. Pohl, and C. v. Helmolt. Bit Error Rates for a MIMO System in Rayleigh and Rician Channels. In *IEEE VTC-Fall, Atlantic City, N.J. U.S.A.*, 7.-11.10. 2001.

[6] J. Salz H.R. Karimi, M. Sandell. Comparison between transmitter and receiver array processing ti achieve interference nulling and diversity. In *IEEE PIMRC*, pages 997–1001, 1999.

[7] D.Tse and S.Hanly. Linear multiuser receivers: Effective interference, effective bandwidth and user capacity. *IEEE Transactions on Information Theory*, 45(2):641–657, March 1999.