Low-Complexity Linear MIMO-OTFS Receivers

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Abstract—**Orthogonal time-frequency space (OTFS) scheme, which transforms a time and frequency selective channel into an almost non-selective delay-Doppler domain channel, establishes reliable wireless communication links for high-speed moving devices. This work designs and analyzes low-complexity zero-forcing and minimum mean square error receivers for multiple-input multiple-output (MIMO)-OTFS systems for both perfect and imperfect channel state informations. The proposed receivers provide exactly the same solution as that of their conventional counterparts, and reduce the complexity by exploiting the doubly-circulant nature of the MIMO-OTFS channel matrix, the block-wise inverse, and *Schur* complement. We numerically show the same bit error rate and lower complexity of the proposed designs over the conventional ones.**

I. Introduction

A practical wireless channel, due to multiple propagation paths and Doppler shift, is both time- and frequency-selective. Cyclic prefix (CP)-aided orthogonal frequency division multiplexing (OFDM) is commonly used to combat the frequency-selectivity. High Doppler shift, due to high-speed relative movement between transmitter and receiver, however, disturbs inter-subcarrier orthogonality in an OFDM system, which degrades its performance [1]. The orthogonal time frequency space (OTFS) scheme tackles this impairment by multiplexing transmit symbols in the delay-Doppler domain [1].

Designing computationally-efficient receivers for OTFS systems has gained significant research attention [2]–[8]. Reference [2] derived the diversity of single-input single-output (SISO)/multiple-input multiple-output (MIMO)-OTFS systems with maximum-likelihood (ML) decoding. Raviteja *et al.* in [3], by assuming perfect receive channel state information (CSI), proposed a reduced complexity iterative message passing (MP)-aided data detection algorithm for SISO-OTFS systems. This algorithm exploits the inherent OTFS channel sparsity by using factor graphs. Ramachandran *et al.* in [4] investigated an MP receiver for MIMO-OTFS systems. Surabhi *et al.* in [5], by exploiting the doubly-circulant OTFS channel structure, proposed reduced complexity zero-forcing (ZF) and minimum mean square error (MMSE) receivers for SISO-OTFS systems. Tiwari *et al.* in [6], by using sparsity and quasi-banded structure of matrices involved in the demodulation process, investigated a low-complexity linear MMSE (LMMSE) receiver for SISO-OTFS systems. The authors in [7] developed low-complexity ZF/MMSE receivers for 2×2 MIMO-OTFS systems. Reference [8] proposed an aproximate maximum a posteriori detector for OTFS systems using the variational Bayes approach.

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We see that the existing OTFS literature in [2]–[7] has not yet investigated computationally-efficient receivers for MIMO-OTFS systems, exception being the MP algorithm in [4] and ZF/MMSE receivers for 2×2 MIMO-OTFS systems in [7]. Conventional ZF and MMSE receivers, due to inherent matrix inversion, has $\mathcal{O}(N_t^3 M^3 N^3)$ complexity for MIMO-OTFS systems [5], where N_t, M and N denote the number of transmit antennas, delay bins and Doppler bins, respectively. For practical systems, the parameters M, N, N_t take large values, which radically increases the complexity of conventional ZF and MMSE receivers for MIMO-OTFS systems [3], [5]. The MP receiver, which reduces complexity by using the Gaussian approximation of interference, and by exploiting sparsity in the OTFS channel, is widely used for MIMO-OTFS systems [4]. The MP receiver, however, as shown later in Section-IV, has significantly higher complexity than the proposed designs. In the light of above observations, this work focuses on designing low-complexity receivers for generic MIMO-OTFS systems with its following **main contributions**.

- We propose a novel computationally-efficient algorithm to invert the matrices $\mathbf{H}^H\mathbf{H}$ and $\mathbf{H}^H\mathbf{H} + \rho\mathbf{I}$ by exploiting the inherent doubly-circulant structure of MIMO-OTFS channel $\mathbf{H} \in \mathbb{C}^{N_r MN \times N_t MN}$, block-wise inverse property of block matrices and Schur complement. We use the proposed algorithm to construct low-complexity ZF (LZ) and MMSE (LM) receivers for MIMO-OTFS systems, which yield exactly the same solution, and consequently the same bit error rate (BER), as that of the conventional ZF and MMSE receivers. The BER of proposed receivers is further reduced by integrating them with likelihood ascent search (LAS) technique [9].
- We analytically show that the proposed LZ and LM receivers have significantly lower computational complexity than conventional ZF and MMSE receivers [7], and the MP-based receivers [3].
- We numerically show that the proposed designs, for both perfect and imperfect receive CSI, have significantly lower BER and complexity than the existing designs [2]–[4].

II. MIMO-OTFS System Model

For a better understanding of MIMO-OTFS system model, we first briefly explain the SISO-OTFS model, and then extend it to MIMO. We begin by considering an OTFS frame in the delay-Doppler domain with N Doppler bins and M delay bins

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[3]. Let $x[k,l]$ be a QAM symbol, which is to be transmitted on the kth Doppler and the lth delay bin in the delay-Doppler frame, where $k = 0, 1, \ldots, N-1$ and $l = 0, 1, \ldots, M-1$. The delay-Doppler domain wireless channel $h(\tau, \nu)$, with delay parameter τ and Doppler parameter ν, is represented as [3]

$$h(\tau, \nu) = \sum_{i=1}^{L_h} h_i \delta(\tau - \tau_i) \delta(\nu - \nu_i). \tag{1}$$

Here L_h is the number of channel paths due to L_h clusters of reflectors, where each cluster introduces a delay and a Doppler shift [3]. The three-tuple (h_i, τ_i, ν_i) denotes complex channel gain $h_i \sim \mathcal{CN}(0, \sigma^2_{L_{h_i}})$ [2], delay τ_i and the Doppler ν_i for the ith cluster. The delay and Doppler taps for the ith path are

$$\tau_i = \frac{l_i}{M\Delta f},\ \nu_i = \frac{k_i}{NT}, \tag{2}$$

where l_i and k_i are integer indices corresponding to the delay τ_i and Doppler ν_i, respectively. Both delay and Doppler values need not be integer multiple of taps l_i and k_i [2]; their discretization, however, allows us to model the channel with fewer delay and Doppler taps [4].

We assume, similar to [2], [10], a pulse at transmitter and receiver, which satisfies the bi-orthogonal property. Thus, the input-output relation in the delay-Doppler domain for the channel model in (1), similar to [3], can be expressed for a SISO-OTFS system as

$$y[k,l] = \sum_{i=1}^{L_h} h'_i x\left[(k-k_i)_N, (l-l_i)_M\right] + v[k,l]. \tag{3}$$

Here $v[k,l]$ is zero mean circularly symmetric complex Gaussian noise with variance σ_v^2, and $h'_i = h_i \exp(-j2\pi\nu_i\tau_i)$. Note that since $h_i \sim \mathcal{CN}(0, \sigma^2_{h_i})$, so is h'_i. The SISO-OTFS receive signal in (3) is expressed in the vector form as [5]

$$\mathbf{y}_{\text{s}} = \mathbf{H}_{\text{s}}\mathbf{x}_{\text{s}} + \mathbf{v}_{\text{s}}, \tag{4}$$

where, for $k = 0, \ldots, N-1$ and $l = 0, \ldots, M-1$, the $(k+Nl)$th element of $\mathbf{x}_{\text{s}} \in \mathbb{C}^{MN\times 1}$ is $x_{k+Nl} = x[k,l]$. The received vector $\mathbf{y}_{\text{s}} \in \mathbb{C}^{MN\times 1}$ and noise vector $\mathbf{v}_{\text{s}} \in \mathbb{C}^{MN\times 1}$ also have the same structure.

Observation 1: Here $\mathbf{H}_{\text{s}} \in \mathbb{C}^{MN\times MN}$ is a doubly-block circulant matrix, i.e., it is a block circulant matrix with M circulant blocks, each of size $N \times N$ [5].

The OTFS system model in (4) can now be extended to the MIMO scenario. We consider a spatially-multiplexed MIMO-OTFS system with N_t transmit and $N_r \geq N_t$ receive antennas [11]. The signal at the rth receive antenna, using (4), is

$$\mathbf{y}_r = \sum_{t=1}^{N_t} \mathbf{H}_{r,t}\mathbf{x}_t + \tilde{\mathbf{v}}_r. \tag{5}$$

Here $\mathbf{x}_t \in \mathbb{C}^{NM\times 1}$ and $\mathbf{y}_r \in \mathbb{C}^{NM\times 1}$ are transmit and receive vectors for the tth transmit and rth receive antenna, respectively. Each element of $\mathbf{x}_t$ is independent and identically distributed (i.i.d.) with mean zero and variance P_x. Each element of the noise vector $\tilde{\mathbf{v}}_r \in \mathbb{C}^{NM\times 1}$ at the rth receive antenna is $\mathcal{CN}(0, \sigma_v^2)$. The matrix $\mathbf{H}_{r,t} \in \mathbb{C}^{NM\times NM}$ represents the OTFS channel in the delay-Doppler domain between the tth transmit and the rth receive antenna.

Observation 2: The matrix $\mathbf{H}_{r,t}$, similar to $\mathbf{H}_{\text{s}}$, is a doubly-block circulant matrix, i.e., it is a block circulant matrix with M circulant blocks, each of size $N \times N$ [3].

We concatenate receive vectors in (5) as $\mathbf{y} = [\mathbf{y}_1^T, \mathbf{y}_2^T, \ldots, \mathbf{y}_{N_r}^T]^T \in \mathbb{C}^{N_r MN\times 1}$ to obtain

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \tilde{\mathbf{v}}. \tag{6}$$

Here $\mathbf{x} = [\mathbf{x}_1^T, \mathbf{x}_2^T, \ldots, \mathbf{x}_{N_t}^T]^T \in \mathbb{C}^{N_t MN\times 1}$ is transmit symbol vector and $\tilde{\mathbf{v}} = [\tilde{\mathbf{v}}_1^T, \tilde{\mathbf{v}}_2^T, \ldots, \tilde{\mathbf{v}}_{N_r}^T]^T \in \mathbb{C}^{N_r MN\times 1}$, with probability density function (pdf) $\mathcal{CN}(\mathbf{0}, \sigma_v^2\mathbf{I}_{N_r MN})$, denotes noise vector. The block channel matrix $\mathbf{H} \in \mathbb{C}^{N_r MN\times N_t MN}$ for MIMO-OTFS system in (6) is given as follows

$$\mathbf{H} \triangleq \begin{bmatrix} \mathbf{H}_{1,1} & \mathbf{H}_{1,2} & \cdots & \mathbf{H}_{1,N_t} \\ \mathbf{H}_{2,1} & \mathbf{H}_{2,2} & \cdots & \mathbf{H}_{2,N_t} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_{N_r,1} & \mathbf{H}_{N_r,2} & \cdots & \mathbf{H}_{N_r,N_t} \end{bmatrix}. \tag{7}$$

Let $\mathbf{G}_{\text{A}} \in \mathbb{C}^{N_r MN\times N_t MN}$, with A $\in \{\text{ZF}, \text{MMSE}\}$, be the receiver matrix for detecting the transmit vector $\mathbf{x}$, whose estimate is $\hat{\mathbf{x}} = \mathbf{G}_{\text{A}}^H\mathbf{y}$. For conventional ZF and MMSE receivers, the matrix $\mathbf{G}_{\text{A}}$ can be expressed as [12]

$$\mathbf{G}_{\text{A}} = \begin{cases} \mathbf{H}\left(\mathbf{H}^H\mathbf{H}\right)^{-1} & \text{for ZF} \\ \mathbf{H}\left(\mathbf{H}^H\mathbf{H} + \rho\mathbf{I}_{N_t MN}\right)^{-1} & \text{for MMSE,} \end{cases} \tag{8}$$

where $\rho = \sigma_v^2/P_x$ = 1/SNR. These receivers invert a matrix of size $N_t MN \times N_t MN$ with $\mathcal{O}(N_t^3 M^3 N^3)$ complexity. The number of delay bins M, Doppler bins N and transmit antennas N_t for practical systems take large values [3]. The conventional ZF and MMSE receivers for MIMO-OTFS systems are thus computationally inefficient. We propose low-complexity ZF (LZ) and MMSE (LM) receivers, which exploit the inherent structure of the MIMO-OTFS channel matrix $\mathbf{H}$, and have only $\mathcal{O}(MN) + \mathcal{O}(MN\log_2 MN)$ complexity.

III. Proposed low-complexity receivers

The proposed LZ and LM receivers use the following key ideas: a) split MIMO-OTFS channel $\mathbf{H}$ in terms of DFT and diagonal block matrices by exploiting its inherent circulant structure; and b) invoke block-wise inverse property for inverting the matrix in the MMSE and ZF receivers.

Preliminaries: Before discussing the proposed receivers, we define in Table I, various sets of matrices which will be used frequently in the sequel. We see from Table-I that the

TABLE I: Definition of different sets used in the work.

Set	Definition
$\mathcal{B}_{M,N}$	Set of circulant matrices with M circulant blocks, each of size $N \times N$.
$\mathcal{C}_{N_r N_t, MN}$	Set of block matrices with $N_r N_t$ blocks of $MN \times MN$ diagonal matrices
$\mathcal{C}_{t^2, MN}$	Set of block square matrices with t^2 blocks of $MN \times MN$ diagonal matrices

set $\mathcal{B}_{M,N}$ denotes a set of circulant matrices with M circulant blocks, each of size $N \times N$. If a matrix $\mathbf{B} \in \mathcal{B}_{M,N}$, it can be represented as $\mathbf{B} = \texttt{CIRC}(\mathbf{B}_1, \mathbf{B}_2, \ldots, \mathbf{B}_M)$, where $\texttt{CIRC}(\cdot)$ is circulant operation and $\mathbf{B}_i \in \mathbb{C}^{N \times N}$ is the ith circulant block of $\mathbf{B}$. We now state a lemma from [13].

Lemma 1: If a matrix $\mathbf{B} \in \mathcal{B}_{M,N}$, it can be diagonalized using the DFT matrices $\mathbf{F}_M \in \mathbb{C}^{M \times M}$ and $\mathbf{F}_N \in \mathbb{C}^{N \times N}$ as $\mathbf{B} = (\mathbf{F}_M \otimes \mathbf{F}_N)^H \mathbf{\Lambda} (\mathbf{F}_M \otimes \mathbf{F}_N)$. The diagonal matrix $\mathbf{\Lambda} \in \mathbb{C}^{MN \times MN}$, which consists of the eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_{MN}$ of the matrix $\mathbf{B}$, can be expressed as

$$\mathbf{\Lambda} = \sum_{i=0}^{M-1} \mathbf{\Omega}_M^i \otimes \mathbf{\Lambda}^i. \tag{9}$$

Here $\otimes$ denotes the matrix Kronecher product. The diagonal matrix $\mathbf{\Lambda}^i \in \mathbb{C}^{N \times N}$ consists of eigenvalues of the ith circulant block $\mathbf{B}_i$ and the diagonal matrix $\mathbf{\Omega}_M = \text{diag}[1, e^{j2\pi/M}, e^{j4\pi/M}, \ldots, e^{j2\pi(M-1)/M}]$.

We recall from *Observation 2* that each sub-matrix of the MIMO-OTFS channel matrix $\mathbf{H}$ in (7) belongs to the set $\mathcal{B}_{M,N}$. Using *Lemma* 1, the (r,t)th sub-matrix $\mathbf{H}_{r,t} \in \mathbb{C}^{MN \times MN}$ of $\mathbf{H}$ can, therefore, be decomposed as $\mathbf{H}_{r,t} = (\mathbf{F}_M \otimes \mathbf{F}_N)^H \mathbf{D}_{r,t} (\mathbf{F}_M \otimes \mathbf{F}_N)$, where the diagonal matrix $\mathbf{D}_{r,t} = \text{diag}[\lambda_1^{r,t}, \lambda_2^{r,t}, \ldots, \lambda_{MN}^{r,t}] \in \mathbb{C}^{MN \times MN}$ consists of eigenvalues of $\mathbf{H}_{r,t}$. From (9), we have $\mathbf{D}_{r,t} = \sum_{i=1}^{M} \mathbf{\Omega}_M^i \otimes \mathbf{D}_{r,t}^i$, where diagonal matrix $\mathbf{D}_{r,t}^i$ consists of eigenvalues of the ith circulant block of $\mathbf{H}_{r,t}$. The MIMO-OTFS channel in (7), using this decomposition, can be partitioned as

$$\mathbf{H} = \mathbf{\Psi}_\text{R}^H \mathbf{D} \mathbf{\Psi}_\text{T}. \tag{10}$$

Here $\mathbf{\Psi}_\text{R} \in \mathbb{C}^{N_r MN \times N_r MN} = \mathbf{I}_{N_r} \otimes \mathbf{F}_M \otimes \mathbf{F}_N$ and $\mathbf{\Psi}_\text{T} \in \mathbb{C}^{N_t MN \times N_t MN} = \mathbf{I}_{N_t} \otimes \mathbf{F}_M \otimes \mathbf{F}_N$. Since $\mathbf{F}_M$ and $\mathbf{F}_N$ are DFT matrices, $\mathbf{\Psi}_\text{R} \mathbf{\Psi}_\text{R}^H = \mathbf{\Psi}_\text{R}^H \mathbf{\Psi}_\text{R} = \mathbf{I}_{N_r MN}$ and $\mathbf{\Psi}_\text{T} \mathbf{\Psi}_\text{T}^H = \mathbf{\Psi}_\text{T}^H \mathbf{\Psi}_\text{T} = \mathbf{I}_{N_t MN}$. The block matrix $\mathbf{D} \in \mathbb{C}^{N_r MN \times N_t MN}$, which consists of eigenvalues of $\mathbf{H}$, is given as

$$\mathbf{D} = \begin{bmatrix} \mathbf{D}_{1,1} & \mathbf{D}_{1,2} & \cdots & \mathbf{D}_{1,N_t} \\ \mathbf{D}_{2,1} & \mathbf{D}_{2,2} & \cdots & \mathbf{D}_{2,N_t} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{D}_{N_r,1} & \mathbf{D}_{N_r,2} & \cdots & \mathbf{D}_{N_r,N_t} \end{bmatrix}. \tag{11}$$

We see that the eigenvalue matrix $\mathbf{D}$*, unlike its counterpart in SISO-OTFS channel [5], is not diagonal, which is a key difference between the SISO- and MIMO-OTFS system models.* It follows from Table I that the matrix $\mathbf{D}$ belongs to the set $\mathcal{C}_{N_r N_t, MN}$. Using the decomposition in (10), and the properties $\mathbf{\Psi}_\text{R} \mathbf{\Psi}_\text{R}^H = \mathbf{\Psi}_\text{R}^H \mathbf{\Psi}_\text{R} = \mathbf{I}_{N_r MN}$ and $\mathbf{\Psi}_\text{T} \mathbf{\Psi}_\text{T}^H = \mathbf{\Psi}_\text{T}^H \mathbf{\Psi}_\text{T} = \mathbf{I}_{N_t MN}$, the equivalent combiner matrices for the proposed LZ and LM receivers are derived from (8) as follows

$$\mathbf{G}_\text{A} = \begin{cases} \mathbf{\Psi}_\text{R}^H \mathbf{D} \left(\mathbf{D}^H \mathbf{D}\right)^{-1} \mathbf{\Psi}_\text{T} & \text{for LZ} \\ \mathbf{\Psi}_\text{R}^H \mathbf{D} \left(\mathbf{D}^H \mathbf{D} + \rho \mathbf{I}_{N_t MN}\right)^{-1} \mathbf{\Psi}_\text{T} & \text{for LM.} \end{cases} \tag{12}$$

Observation 3: The proposed LZ and LM receivers in (12) yield exactly the same solutions as that of the conventional ZF and MMSE receivers in (8). This is because, to derive (12) from (8), we replace the MIMO-OTFS channel matrix $\mathbf{H}$ by its decomposition in (10).

For the sets $\mathcal{C}_{N_r N_t, MN}$ and $\mathcal{C}_{t^2, MN}$ in Table-I, we now state the following lemma from [14].

Lemma 2: If $\mathbf{X}$, $\mathbf{Y} \in \mathcal{C}_{N_r N_t, MN}$, the matrices obtained using operations $\mathbf{X}^T$, $\mathbf{X}^H$, $\mathbf{XY}(=\mathbf{YX})$, $a_1\mathbf{X} + a_2\mathbf{Y}$ and $\sum_{n=1}^{N} a_n \mathbf{X}_n$ also belong to the set $\mathcal{C}_{N_r N_t, MN}$, where $a_1, a_2, \ldots, a_n$ are scalars. Additionally, if $\mathbf{X}$, $\mathbf{Y} \in \mathbb{C}^{N_r MN \times N_t MN}$, the matrices $\mathbf{XY}^H$ and $\mathbf{X}^H\mathbf{Y}$ belong to the set $\mathcal{C}_{N_r^2, MN}$ and $\mathcal{C}_{N_t^2, MN}$, respectively.

We see from (8) that the conventional ZF and MMSE receivers invert matrices $\mathbf{H}^H\mathbf{H}$ and $\mathbf{H}^H\mathbf{H} + \rho \mathbf{I}_{N_t MN}$ respectively with $\mathcal{O}(N_t^3 M^3 N^3)$ complexity [7]. The proposed LZ and LM receivers use the equivalent formulation in (12), and therefore, invert $\mathbf{D}_\text{LZ} = \mathbf{D}^H\mathbf{D}$ and $\mathbf{D}_\text{LM} = \mathbf{D}^H\mathbf{D} + \rho \mathbf{I}_{N_t MN}$ respectively. By exploiting the properties of sets $\mathcal{C}_{N_r N_t, MN}$ and $\mathcal{C}_{t^2, MN}$, and *Lemma* 2, we now propose a low-complexity algorithm for computing $\mathbf{D}_\text{LZ}^{-1}$ and $\mathbf{D}_\text{LM}^{-1}$, which as shown later in this section, has $\mathcal{O}(MN)$ complexity.

A. Low-complexity ZF (LZ) and MMSE (LM) receivers

It follows from *Lemma* 2 that the matrix $\mathbf{D}_\text{A} \in \mathbb{C}^{N_t MN \times N_t MN}$, for A $\in$ {LZ, LM}, belongs to the set $\mathcal{C}_{t^2, MN}$, where $t = N_t$. The matrix $\mathbf{D}_\text{A}$, thus consists of blocks of $MN \times MN$ diagonal matrices, and therefore, it can always be partitioned as follows

$$\mathbf{D}_\text{A} = \left[\begin{array}{cccc|c} \mathbf{D}_{\text{A}_{1,1}} & \mathbf{D}_{\text{A}_{1,2}} & \cdots & \mathbf{D}_{\text{A}_{1,N_t-1}} & \mathbf{D}_{\text{A}_{1,N_t}} \\ \mathbf{D}_{\text{A}_{2,1}} & \mathbf{D}_{\text{A}_{2,2}} & \cdots & \mathbf{D}_{\text{A}_{2,N_t-1}} & \mathbf{D}_{\text{A}_{2,N_t}} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{D}_{\text{A}_{N_t-1,1}} & \mathbf{D}_{\text{A}_{N_t-1,2}} & \cdots & \mathbf{D}_{\text{A}_{N_t-1,N_t-1}} & \mathbf{D}_{\text{A}_{N_t-1,N_t}} \\ \hline \mathbf{D}_{\text{A}_{N_t,1}} & \mathbf{D}_{\text{A}_{N_t,2}} & \cdots & \mathbf{D}_{\text{A}_{N_t,N_t-1}} & \mathbf{D}_{\text{A}_{N_t,N_t}} \end{array}\right]. \tag{13}$$

Here each $\mathbf{D}_{\text{A}_{i,j}}$, for $1 \leq i, j \leq N_t$, is an $MN \times MN$ diagonal matrix. If a matrix $\mathbf{X}$ can be partitioned in to four sub-matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$, it can be inverted block-wise as [15]

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{S}^{-1} & -\mathbf{S}^{-1}\mathbf{B}\mathbf{D}^{-1} \\ -\mathbf{D}^{-1}\mathbf{C}\mathbf{S}^{-1} & \mathbf{D}^{-1} + \mathbf{D}^{-1}\mathbf{C}\mathbf{S}^{-1}\mathbf{B}\mathbf{D}^{-1} \end{bmatrix}. \tag{14}$$

This holds if the matrix $\mathbf{D}$ and its *Schur* complement $\mathbf{S} = \mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C}$ are invertible.

Observation 4: We observe from (13) and (14) that the inverse of the matrix $\mathbf{D}_\text{A}$ can be performed block-wise, provided sub-matrix $\mathbf{D}_{\text{A}_{N_t,N_t}}$ in (13) and its *Schur* complement are always invertible. *This is a key design aspect of the proposed low-complexity receivers for MIMO-OTFS systems.* To perform block-wise inverse of the matrix $\mathbf{D}_\text{A}$ using (14) for reducing complexity, we next prove that the sub-matrix $\mathbf{D}_{\text{A}_{N_t,N_t}}$ in (13) and its *Schur* complement are always invertible. To this end, we state the next lemma whose proof is relegated to Appendix A.

Lemma 3: If a matrix $\mathbf{X} \in \mathcal{C}_{t^2, MN}$, then its inverse $\mathbf{X}^{-1} \in \mathcal{C}_{t^2, MN}$, and it always exists.

To prove *Lemma* 3, we exploit i) the property from (13) that the matrix $\mathbf{X}$ can be partitioned in terms of the diagonal blocks $\mathbf{X}_{i,j}$ for $1 \leq i, j \leq t$, since $\mathbf{X}$ belongs to the set $\mathcal{C}_{t^2, MN}$; and ii) *Lemma* 2 that the *Schur* complement of the diagonal matrix

$\mathbf{X}_{t,t}$ belongs to the set $\mathcal{C}_{(t-1)^2,MN}$, and therefore, inverse of the *Schur* complement can also be calculated using the block-wise inverse property from (14).

We now use *Lemma* 2, *Lemma* 3 and the results given in (13) and (14) to design a low-complexity Algorithm-1 for computing the inverse $\mathbf{D}_{\text{A}}^{-1}$, i.e., $\mathbf{D}_{\text{LZ}}^{-1}$ and $\mathbf{D}_{\text{LM}}^{-1}$, for the proposed LM and LZ receivers. The proposed Algorithm-1 operates in two steps: a) matrix partitioning; and b) backtracking. We next explain the detailed operation of the proposed Algorithm-1.

Algorithm 1: Pseudo-code for computing $\mathbf{D}_{\text{A}}^{-1}$.

Input: Matrix $\mathbf{D} \in \mathcal{C}_{N_r N_t, MN}$
Output: Matrix $\mathbf{D}_{\text{A}}^{-1} \in \mathcal{C}_{N_t^2, MN}$

1 Compute $\mathbf{D}_{\text{A}} = \mathbf{D}^H\mathbf{D}$, if A $\in$ LZ; else $\mathbf{D}_{\text{A}} = \mathbf{D}_{\text{LM}} = \mathbf{D}^H\mathbf{D} + \rho\mathbf{I}_{N_t MN}$
2 **Initialization:** $\mathbf{P}_{N_t-1} = [\ ], \mathbf{P}_{N_t-2} = [\ ], \ldots, \mathbf{P}_1 = [\ ]$ and $\mathbf{P}_0 = \mathbf{D}_{\text{A}} \in \mathcal{C}_{t^2, MN}$
3 */ Matrix Partitioning */
4 **for** $i = 1 : 1 : N_t - 1$ **do**
5 Partition $\mathbf{P}_0$ using (13) and store in $\mathbf{P}_{N_t-i}$ as
$$\mathbf{P}_{N_t-i} = \begin{bmatrix} \mathbf{A}_{N_t-i} & \mathbf{B}_{N_t-i} \\ \mathbf{C}_{N_t-i} & \mathbf{D}_{N_t-i} \end{bmatrix}.$$
6 Replace $\mathbf{P}_0$ by the *Schur* complement of $\mathbf{D}_{N_t-i}$ as
$$\mathbf{P}_0 = \mathbf{A}_{N_t-i} - \mathbf{B}_{N_t-i}\mathbf{D}_{N_t-i}^{-1}\mathbf{C}_{N_t-i}.$$
7 **end**
8 */ Backtracking */
9 Compute $\mathbf{S}_1^{-1} = \mathbf{P}_0^{-1} = (\mathbf{A}_1 - \mathbf{B}_1\mathbf{D}_1^{-1}\mathbf{C}_1)^{-1}$
10 **Initialize** the matrix $\mathbf{F}$ using (14) as
$$\mathbf{F} = \begin{bmatrix} \mathbf{S}_1^{-1} & -\mathbf{S}_1^{-1}\mathbf{B}_1\mathbf{D}_1^{-1} \\ -\mathbf{D}_1^{-1}\mathbf{C}_1\mathbf{S}_1^{-1} & \mathbf{D}_1^{-1} + \mathbf{D}_1^{-1}\mathbf{C}_1\mathbf{S}_1^{-1}\mathbf{B}_1\mathbf{D}_1^{-1} \end{bmatrix}.$$
11 **for** $i = 2 : 1 : N_t - 1$ **do**
12 Compute block-wise inverse using (14) as
$$\mathbf{S}_{i+1}^{-1} = \begin{bmatrix} \mathbf{F} & -\mathbf{F}\mathbf{B}_i\mathbf{D}_i^{-1} \\ -\mathbf{D}_i^{-1}\mathbf{C}_i\mathbf{F} & \mathbf{D}_i^{-1} + \mathbf{D}_i^{-1}\mathbf{C}_i\mathbf{F}\mathbf{B}_i\mathbf{D}_i^{-1} \end{bmatrix}.$$
13 Replace $\mathbf{F} = \mathbf{S}_{i+1}^{-1}$
14 **end**
15 **return**: $\mathbf{D}_{\text{A}}^{-1} = \mathbf{F}$

a) Matrix Partitioning (lines $4-7$): It is an iterative process. Initially (for $i = 1$), by using (13), we partition $\mathbf{D}_{\text{A}} \in \mathcal{C}_{N_t^2, MN}$ in terms of the sub-matrices $\mathbf{A}_{N_t-i} \in \mathbb{C}^{(N_t-i)MN \times (N_t-i)MN}$, $\mathbf{B}_{N_t-i} \in \mathbb{C}^{(N_t-i)MN \times MN}$, $\mathbf{C}_{N_t-i} \in \mathbb{C}^{MN \times (N_t-i)MN}$ and $\mathbf{D}_{N_t-i} \in \mathbb{C}^{MN \times MN}$, and store these partitions in the matrix $\mathbf{P}_{N_t-1}$. For the block-wise inversion of $\mathbf{D}_{\text{A}}$ according to (14), we now need to invert the *Schur* complement $\mathbf{S}_{N_t-1} = (\mathbf{A}_{N_t-1} - \mathbf{B}_{N_t-1}\mathbf{D}_{N_t-1}^{-1}\mathbf{C}_{N_t-1})$. We know from *Lemma* 2 that $\mathbf{S}_{N_t-1}$ belongs to the set $\mathcal{C}_{(N_t-1)^2, MN}$. It can therefore be inverted block-wise using (13) and (14). We thus partition $\mathbf{S}_{N_t-1}$ in terms of $\mathbf{A}_{N_t-2}$, $\mathbf{B}_{N_t-2}$, $\mathbf{C}_{N_t-2}$ and $\mathbf{D}_{N_t-2}$ using (13), and subsequently evaluate $\mathbf{S}_{N_t-2} = (\mathbf{A}_{N_t-2} - \mathbf{B}_{N_t-2}\mathbf{D}_{N_t-2}^{-1}\mathbf{C}_{N_t-2})$ which, from *Lemma* 2, belongs to the set $\mathcal{C}_{(N_t-2)^2, MN}$. We, therefore, next split $\mathbf{S}_{N_t-2}$ in terms of the sub-matrices $\mathbf{A}_{N_t-3}$, $\mathbf{B}_{N_t-3}$, $\mathbf{C}_{N_t-3}$ and $\mathbf{D}_{N_t-3}$ for computing $\mathbf{S}_{N_t-3}$ and so on. To summarize, for $i = 2, \ldots, N_t - 1$, we partition the *Schur* complement $\mathbf{S}_{N_t-i}$ using (13), and store these partitions in the corresponding matrices $\mathbf{P}_{N_t-i}$. With $i = N_t - 1$, we reach $\mathbf{S}_1$ which is an $MN \times MN$ diagonal matrix, and therefore, $\mathbf{S}_1^{-1}$ does not require further partitioning.

b) Backtracking (lines $9-13$): The partitioned matrix obtained in the first phase is now used to calculate $\mathbf{D}_{\text{A}}^{-1}$ by exploiting *Lemma* 3 and the result from (14). As shown in line-10, we initialize backtracking process by matrix $\mathbf{F}$, which is obtained using $\mathbf{S}_1^{-1}$ and sub-matrices $\{\mathbf{A}_1, \mathbf{B}_1, \mathbf{C}_1, \mathbf{D}_1\}$ stored in the matrix $\mathbf{P}_1$. Note that $\mathbf{F}$ is the inverse of the *Schur* complement $\mathbf{S}_2$. We know from *Lemma* 3 that if $\mathbf{S}_i$ belongs to the set $\mathcal{C}_{i^2, MN}$, its inverse $\mathbf{S}_i^{-1}$ also belongs to the same set $\mathcal{C}_{i^2, MN}$. Exploiting this property and the block-wise inverse from (14), in the lines $11-13$, we recursively compute $\mathbf{S}_{i+1}^{-1}$ using $\mathbf{S}_i^{-1}$ and the sub-matrices $\{\mathbf{A}_i, \mathbf{B}_i, \mathbf{C}_i, \mathbf{D}_i\}$ stored in $\mathbf{P}_i$. We see that, when $i = N_t - 1$, the inverse $\mathbf{S}_{N_t}^{-1}$ in the line 12 is computed with the help of *Schur* complement $\mathbf{F} = \mathbf{S}_{N_t-1}^{-1}$ and the sub-matrices $\{\mathbf{A}_{N_t-1}, \mathbf{B}_{N_t-1}, \mathbf{C}_{N_t-1}, \mathbf{D}_{N_t-1}\}$ as shown in (15) (on the top of the next page).

By comparing (15) with (14), we see that $\mathbf{S}_{\mathbf{N_t}}^{-1} = \mathbf{D}_{\text{A}}^{-1}$. In other words, in the final step of the backtracking phase, the matrix $\mathbf{F}$ returns $\mathbf{D}_{\text{A}}^{-1}$. *The LZ and LM receivers thus give the same solution as the conventional ZF and MMSE receivers. Their complexity, as shown next, is significantly lower than their conventional counterparts.*

IV. Complexity of the Proposed Receivers

We compute the complexity by considering multiplication/division and addition/subtraction as operations [5].

1) Computation of $\mathbf{D}_A^{-1}$: We state the following lemma, which is proved in Appendix B.

Lemma 4: Total number of operations required for computing $\mathbf{D}_{\text{A}}^{-1}$, for A $\in$ {LZ, LM}, is

$$\begin{aligned} \mu_{\text{D}_{\text{LZ}}} &= [2N_t^3 - 3N_t^2 + N_t + 2N_t^2 N_r]MN \\ \mu_{\text{D}_{\text{LM}}} &= [2N_t^3 - 3N_t^2 + 3N_t + 2N_t^2 N_r]MN. \end{aligned} \tag{16}$$

We see that the computational complexity for calculating $\mathbf{D}_{\text{A}}^{-1}$ with Algorithm-1 is $\mathcal{O}(MN)$, since in practice $N_t, N_r \ll MN$. This is unlike conventional ZF and MMSE receivers which calculate their respective inverse operations with $\mathcal{O}(N_t^3 M^3 N^3)$ complexity. We now evaluate the complexity of the proposed LZ and LM receivers in (12) for processing the received vector $\mathbf{y}$, i.e., the operation $\mathbf{G}_{\text{A}}^H\mathbf{y}$ for A $\in$ {LZ, LM}.

2) Computation of $\mathbf{G}_A^H\mathbf{y}$: We state the following lemma which is proved in Appendix C.

Lemma 5: Total number of operations required for $\mathbf{G}_{\text{A}}^H\mathbf{y}$ operation is given as follows $\mu_{\text{G}_{\text{A}}} = \left[2N_t^2 N_r + N_t N_r - N_t\right]MN + \left[N_t + N_r\right]\mathcal{O}(MN\log_2 MN)$.
Also i) calculation of the matrix $\mathbf{D}_{r,t}^i$ in the expression $\mathbf{D}_{r,t} = \sum_{i=1}^{M} \mathbf{\Omega}_M^i \otimes \mathbf{D}_{r,t}^i$ requires $\mathcal{O}(MN\log_2 N)$ operations [2]; and ii) $\mathcal{O}(MN\log_2 M)$ operations are needed to compute the eigenvalue matrix $\mathbf{D}_{r,t}$ of the matrix $\mathbf{H}_{r,t}$ [2]. The complexity of computing the eigenvalue matrix $\mathbf{D}$ for the MIMO-OTFS channel matrix $\mathbf{H}$ is thus $N_r N_t \mathcal{O}(MN\log_2 MN)$. By exploiting this result, *Lemma* 4 and *Lemma* 5, we calculate the overall receiver complexity as

$$\begin{aligned} \mu_{\text{LZ}} = \mu_{\text{D}_{\text{LZ}}} + \mu_{\text{G}_{\text{A}}} &= \left[2N_t^3 - 3N_t^2 + 4N_t^2 N_r + N_t N_r\right]MN \\ &\quad + \left[N_t + N_r + N_t N_r\right]\mathcal{O}(MN\log_2 MN) \\ \mu_{\text{LM}} &= \mu_{\text{LZ}} + 2N_t MN. \end{aligned} \tag{17}$$

We observe from (17) that the complexity of LZ and LM receivers is significantly lower than the conventional ZF and MMSE receivers, which have $\mathcal{O}(N_t^3 M^3 N^3)$ complexity.

$$\mathbf{S}_{\mathbf{N_t}}^{-1} = \begin{bmatrix} \mathbf{S}_{N_t-1}^{-1} & -\mathbf{S}_{N_t-1}^{-1}\mathbf{B}_{N_t-1}\mathbf{D}_{N_t-1}^{-1} \\ -\mathbf{D}_{N_t-1}^{-1}\mathbf{C}_{N_t-1}\mathbf{S}_{N_t-1}^{-1} & \mathbf{D}_{N_t-1}^{-1} + \mathbf{D}_{N_t-1}^{-1}\mathbf{C}_{N_t-1}\mathbf{S}_{N_t-1}^{-1}\mathbf{B}_{N_t-1}\mathbf{D}_{N_t-1}^{-1} \end{bmatrix}. \tag{15}$$

Further, the complexity of the MP-based receiver in SISO-OTFS systems with/without bi-orthogonal pulse varies as $\mathcal{O}(N_I MNSQ)$ [3], where N_I is the number iterations required for the MP algorithm to converge, Q is the constellation size, and S is the number of non-zero elements in each row or column of SISO-OTFS channel matrix $\mathbf{H}_s \in \mathbb{C}^{MN\times MN}$. Since there are $N_r N_t$ number of links between the MIMO-OTFS transmitter and receiver, the complexity of the MP algorithm for data detection in MIMO-OTFS systems can be obtained as $\mathcal{O}(N_I N_r N_t MNSQ)$. This complexity, due to the presence of the parameters N_I, S, and Q, is significantly higher than the proposed LZ and LM designs.

V. Simulation results

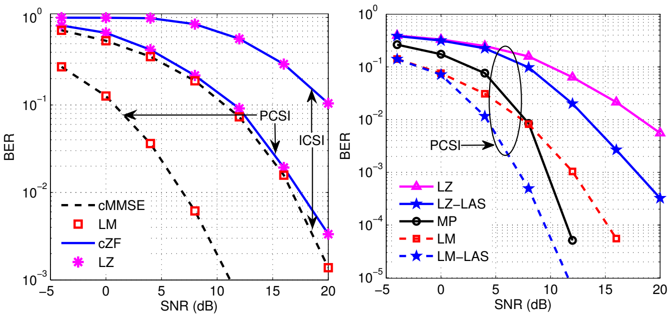
We now investigate the BER of proposed designs for a spatially-multiplexed MIMO-OTFS system with a pulse which satisfies the bi-orthogonality property [2], [5]. We consider a $N_r \times N_t = 4\times 4$ MIMO set-up, with either BPSK or QPSK constellation. We assume the number of delay bins M and the Doppler bins N as $M = N = 32$, carrier frequency of 4 GHz, and a subcarrier spacing of 15 KHz. We use a 5-tap delay-Doppler channel with its parameters given in Table- II, and define SNR as P_x/σ_v^2 . We abbreviate the i) conventional ZF/MMSE as cZF/cMMSE; ii) perfect and imperfect receive CSI scenarios as PCSI and ICSI, respectively.

TABLE II: Delay-Doppler channel parameters

Channel tap no.	1	2	3	4	5
Delay (μs)	2.08	5.20	8.328	11.46	14.80
Doppler (Hz)	0	470	940	1410	1851
Channel tap power (dB)	1	-1.804	-3.565	-5.376	-8.860

Figure 1(a) shows the BER of conventional and proposed ZF/MMSE receivers for a MIMO-OTFS system with QPSK constellation, perfect and imperfect receive CSI. The estimate of MIMO-OTFS channel $\mathbf{H}$ is modeled as $\widehat{\mathbf{H}} = \mathbf{H} + \Delta\mathbf{H}$. The matrix $\Delta\mathbf{H} \in \mathbb{C}^{N_r MN\times N_t MN}$ is the CSI error, which is independent of $\mathbf{H}$. Since $\mathbf{H}$ belongs to the set $\mathcal{C}_{N_r N_t, MN}$, $\Delta\mathbf{H}$ also belongs to the set $\mathcal{C}_{N_r N_t, MN}$. We assume that the non-zero entries in a row or column of the block $\Delta\mathbf{H}_{r,t}$ of the error matrix $\Delta\mathbf{H}$ are i.i.d. as $\mathcal{CN}(0,\sigma_e^2)$, with error variance $\sigma_e^2 = \rho/N_t$ [11]. We see that the BER of LZ and LM receivers exactly match their conventional counterparts with both perfect and imperfect CSI. This is because the proposed designs do not make any approximation and exploit the following inherent properties i) doubly-circulant structure of the MIMO-OTFS channel matrix $\mathbf{H}$ by decomposing it as $\mathbf{H} = \mathbf{\Psi}_{\mathrm{R}}^H \mathbf{D}\mathbf{\Psi}_{\mathrm{T}}$; ii) fact that $\mathbf{D} \in \mathcal{C}_{N_r N_t, MN}$; and iii) block-wise inverse of matrices and *Schur* Complement.

Figure 1(b) compares the BER of proposed receivers and the widely-used non-linear MP-based receiver for MIMO-OTFS systems with BPSK modulation and perfect receive CSI. For a fair comparison with non-linear MP receiver, we also plot the



(a) (b)

Fig. 1: a) BER comparison of the proposed LZ and LM receivers: a) with their conventional counterparts by considering QPSK, and perfect/imperfect CSI; (b) with and without LAS technique, and MP receiver with BPSK.

BER of LZ-LAS and LM-LAS receivers wherein the proposed LZ and LM receivers are followed by a low-complexity local-search-based non-linear likelihood ascent search (LAS) operation [9]. The LAS receiver begins with an initial solution provided by the LZ or LM receiver, and searches for good solutions in the neighborhood until a local optimum is reached [9]. We see that the i) proposed LM receiver comprehensively outperforms the MP receiver at the low SNR; ii) LM-LAS receiver has significantly lower BER than the MP receiver for all SNR values. As the LAS has $\mathcal{O}(N_t MN)$ complexity, the LM-LAS receiver has $\mu_{\text{LM}} + \mathcal{O}(N_t MN)$ complexity [5], which is almost similar to that of the proposed LM design.

VI. Conclusions

We proposed novel low-complexity ZF (LZ) and MMSE (LM) receivers, which exploit MIMO-OTFS channel characteristics to achieve lower complexity than the conventional ZF, MMSE and message passing (MP) receivers. We showed that the BER of i) both LZ and LM receivers is same as that of their conventional counterparts; and ii) LM receiver at low SNR values is lower than the non-linear MP receiver. The LM receiver, when combined with non-linear likelihood ascent search technique, outperforms MP receiver at all SNR values.

Appendix A

To begin with, let $t = N_t$. The matrix $\mathbf{X}$, similar to (13), can always be partitioned into four sub-matrices, namely $\mathbf{A}_{N_t-1} \in \mathbb{C}^{(N_t-1)MN\times(N_t-1)MN}$, $\mathbf{B}_{N_t-1} \in \mathbb{C}^{(N_t-1)MN\times MN}$, $\mathbf{C}_{N_t-1} \in \mathbb{C}^{MN\times(N_t-1)MN}$ and $\mathbf{D}_{N_t-1} \in \mathbb{C}^{MN\times MN}$. With the above partitioning of $\mathbf{X}$, it follows from (14), that $\mathbf{X}^{-1}$ can be computed block-wise, provided the matrix $\mathbf{D}_{N_t-1}$ and its *Schur* complement $\mathbf{S}_{N_t-1} = \mathbf{A}_{N_t-1} - \mathbf{B}_{N_t-1}\mathbf{D}_{N_t-1}^{-1}\mathbf{C}_{N_t-1}$ are invertible. Since $\mathbf{X} \in \mathcal{C}_{N_t^2, MN}$, the sub-matrix $\mathbf{D}_{N_t-1}$ is always an $MN \times MN$ diagonal matrix with all its elements are > 0 [5]. Thus, $\mathbf{D}_{N_t-1}^{-1}$ always exists. For $\mathbf{X}^{-1}$ to exist, we have to next prove that $\mathbf{S}_{N_t-1}^{-1}$ exists. We see that from *Lemma* 2, the *Schur* complement $\mathbf{S}_{N_t-1} \in \mathcal{C}_{(N_t-1)^2, MN}$. Thus, similar to (13), $\mathbf{S}_{N_t-1}^{-1}$ can be calculated block-wise by partitioning $\mathbf{S}_{N_t-1}$ in terms of the sub-matrices $\mathbf{A}_{N_t-2}$, $\mathbf{B}_{N_t-2}$, $\mathbf{C}_{N_t-2}$ and $\mathbf{D}_{N_t-2}$, and by imposing the conditions that $\mathbf{D}_{N_t-2}$ and its *Schur* complement $\mathbf{S}_{N_t-2} = \mathbf{A}_{N_t-2} - \mathbf{B}_{N_t-2}\mathbf{D}_{N_t-2}^{-1}\mathbf{C}_{N_t-2}$ are invertible. Proceeding in this way, let $\mathbf{S}_{N_t-i} = \mathbf{A}_{N_t-i} - \mathbf{B}_{N_t-i}\mathbf{D}_{N_t-i}^{-1}\mathbf{C}_{N_t-i}$ be the *Schur* complement of the matrix $\mathbf{D}_{N_t-i}$ at the ith step with the corresponding sub-matrices $\mathbf{A}_{N_t-i} \in \mathbb{C}^{(N_t-i)MN\times(N_t-i)MN}$, $\mathbf{B}_{N_t-i} \in \mathbb{C}^{(N_t-i)MN\times MN}$, $\mathbf{C}_{N_t-i} \in \mathbb{C}^{MN\times(N_t-i)MN}$ and $\mathbf{D}_{N_t-i} \in \mathbb{C}^{MN\times MN}$, where the index $i = 1,\ldots,N_t-1$. Since all the matrices $\mathbf{A}_{N_t-i}$, $\mathbf{B}_{N_t-i}$, $\mathbf{C}_{N_t-i}$ and $\mathbf{D}_{N_t-i}$ consist of blocks of $MN \times MN$ diagonal matrices, it follows from *Lemma* 2 that at the ith step, $\mathbf{S}_{N_t-i} \in \mathcal{C}_{(N_t-i)^2,MN}$. Therefore, $\mathbf{D}_{N_t-i}^{-1}$ always exists and the inverse of $\mathbf{S}_{N_t-i}$ can always be computed by employing the results in (14). At the final step when $i = N_t - 1$, $\mathbf{S}_1 = (\mathbf{A}_1 - \mathbf{B}_1\mathbf{D}_1^{-1}\mathbf{C}_1) \in \mathcal{C}_{1,MN}$, and computation of $\mathbf{S}_1^{-1}$ does not need any further partitioning, because $\mathbf{S}_1$ reduces to a diagonal matrix. This completes the proof that inverse of matrix $\mathbf{X} \in \mathcal{C}_{t^2,MN}$ always exists. The property that the *Schur* complement $\mathbf{S}_{N_t-i} \in \mathcal{C}_{(N_t-i)^2,MN}$ ensures that the inverse $\mathbf{X}^{-1} \in \mathcal{C}_{t^2,MN}$.

Appendix B

Since the matrix $\mathbf{D} \in \mathcal{C}_{N_rN_t,MN}$, computing $\mathbf{D}_{\mathrm{A}} = \mathbf{D}^H\mathbf{D}$ for A $\in$ {LZ}, and $\mathbf{D}_{\mathrm{A}} = \mathbf{D}^H\mathbf{D} + \rho\mathbf{I}_{N_tMN}$ for A $\in$ {LM} requires $[2N_t^2N_r - N_t^2]MN$ and $[2N_t^2N_r - N_t^2 + 2N_t]MN$ operations, respectively. For calculating $\mathbf{D}_{\mathrm{A}}^{-1}$, as shown in Algorithm-1, for $1 \le i \le N_t - 1$, we need to compute $\mathbf{D}_{N_t-i}^{-1}$ and the corresponding *Schur* complement $\mathbf{S}_{N_t-i}$. This is followed by the computation of $\mathbf{S}_{N_t-i}^{-1}$. Independent of the index i, $\mathbf{D}_{N_t-i}$ is always an $MN \times MN$ diagonal matrix. Computing of $\mathbf{D}_{N_t-i}^{-1}$ requires MN multiplications. We see that the computation of $\mathbf{S}_{N_t-i}$ costs $(N_t-i)MN + (N_t-i)^2MN + MN$ multiplications and $(N_t-i)^2MN$ additions. Following the result in (14), we now need to compute four sub-matrices in terms of the matrices $\mathbf{S}_{N_t-i}^{-1}$, $\mathbf{A}_{N_t-i}$, $\mathbf{B}_{N_t-i}$, $\mathbf{C}_{N_t-i}$ and $\mathbf{D}_{N_t-i}$. Let α_{N_t-i} be the number of operations required for computing $\mathbf{S}_{N_t-i}^{-1}$. By using the fact from *Lemma* 3 that $\mathbf{S}_{N_t-i}^{-1} \in \mathcal{C}_{(N_t-i)^2,MN}$, we see that computing $\mathbf{S}_{N_t-i}^{-1}\mathbf{B}_{N_t-i}\mathbf{D}_{N_t-i}^{-1}$ and $\mathbf{D}_{N_t-i}^{-1}\mathbf{C}_{N_t-i}\mathbf{S}_{N_t-i}^{-1}$ require $(N_t-i)^2MN + (N_t-i-1)(N_t-i)MN$ operations each. Computing $\mathbf{D}_{N_t-i}^{-1} + \mathbf{D}_{N_t-i}^{-1}\mathbf{C}_{N_t-i}\mathbf{S}_{N_t-i}^{-1}\mathbf{B}_{N_t-i}\mathbf{D}_{N_t-i}^{-1}$ requires $(N_t-1)MN + (N_t-i-1)MN + MN$ operations. For each i, total operations μ_{N_t-i} are $\mu_{N_t-i} = (1 + 2N_t + 6N_t^2)MN + (6i^2 + 12N_ti - 2i)MN + \alpha_{N_t-i}$. It follows from Algorithm-1 that α_{N_t-i} can also be calculated by using the above procedure. Consequently, the number of operations required for computing $\mathbf{D}_{\mathrm{A}}^{-1}$ are $\sum_{i=1}^{N_t-1}\mu_{N_t-i} = [2N_t^3 - 2N_t^2 + N_t]MN$. Finally, the total number of operations $\mu_{\mathrm{D_{LZ}}}$ required for computing $\mathbf{D}_{\mathrm{A}}^{-1}$, for A $\in$ {LZ}, can now be evaluated by adding $[N_t^2N_r + N_t^2(N_r-1)]MN$ and $\sum_{i=1}^{N_t-1}\mu_{N_t-i}$, which yields the desired result for $\mu_{\mathrm{D_{LZ}}}$ in *Lemma* 4. Next, the addition of $[N_t^2N_r + N_t^2(N_r-1) + 2N_t]MN$ and $\sum_{i=1}^{N_t-1}\mu_{N_t-i}$ gives the desired result for $\mu_{\mathrm{D_{LM}}}$ in *Lemma* 4.

Appendix C

Let the matrix $\tilde{\mathbf{D}}_{\mathrm{A}}$ be defined as $\tilde{\mathbf{D}}_{\mathrm{A}} = \mathbf{D}\mathbf{D}_{\mathrm{A}}^{-1}$. Since $\mathbf{D} \in \mathcal{C}_{N_rN_t,MN}$ and, we know from *Lemma* 3 that $\mathbf{D}_{\mathrm{A}}^{-1} \in \mathcal{C}_{N_t^2,MN}$, calculation of $\tilde{\mathbf{D}}_{\mathrm{A}}$ requires $[N_t^2N_r + (N_t-1)N_tN_r]MN$ operations. It follows from *Lemma* 2 that the matrix $\tilde{\mathbf{D}}_{\mathrm{A}} \in \mathcal{C}_{N_rN_t,MN}$. The receiver matrix $\mathbf{G}_{\mathrm{A}}$ in (12) can now be decomposed as $\mathbf{G}_{\mathrm{A}} = \mathbf{\Psi}_{\mathrm{R}}^H\tilde{\mathbf{D}}_{\mathrm{A}}\mathbf{\Psi}_{\mathrm{T}}$. For performing $\mathbf{G}_{\mathrm{A}}^H\mathbf{y}$, we first compute $\tilde{\mathbf{y}} = \mathbf{\Psi}_{\mathrm{R}}\mathbf{y}$. Since $\mathbf{\Psi}_{\mathrm{R}}$ is a block diagonal matrix whose each block is $\mathbf{F}_M \otimes \mathbf{F}_N$, vector $\tilde{\mathbf{y}}$ is expressed as

$$\tilde{\mathbf{y}} = \left[((\mathbf{F}_M \otimes \mathbf{F}_N)\mathbf{y}_1)^T, \ldots, ((\mathbf{F}_M \otimes \mathbf{F}_N)\mathbf{y}_{N_r})^T\right]^T. \quad (18)$$

Let $\tilde{\mathbf{Y}}_r$, for $1 \le r \le N_r$, be the matrices such that $\mathrm{vec}(\tilde{\mathbf{Y}}_r) = \tilde{\mathbf{y}}_r$. The vector $(\mathbf{F}_M \otimes \mathbf{F}_N)\mathbf{y}_r$ can then be rewritten as $\mathrm{vec}(\mathbf{F}_N\tilde{\mathbf{Y}}_r\mathbf{F}_M^H)$, and can be evaluated by computing M-point IDFT along the rows of $\tilde{\mathbf{Y}}_r$ and N-point IDFT along the columns of $\tilde{\mathbf{Y}}_r$. Computing $\tilde{\mathbf{y}}$ in (18) thus requires $N_r\mathcal{O}(MN\log_2 MN)$ operations [5]. Computing vector $\mathbf{z} = \tilde{\mathbf{D}}_{\mathrm{A}}^H\tilde{\mathbf{y}}$ requires $N_tN_rMN + N_t(N_r-1)MN$ operations. After this, $\mathbf{\Psi}_T^H\mathbf{z}$ can be computed using $N_t\mathcal{O}(MN\log_2 MN)$ operations. The number of operations required to process $\mathbf{y}$ are

$$\mu_{\mathrm{G_A}} = \left[N_t^2N_r + N_tN_r(N_t-1) + N_tN_r + N_t(N_r-1)\right]MN + \left[N_t + N_r\right]\mathcal{O}(MN\log_2 MN). \quad (19)$$

By solving (19), we get the desired result stated in *Lemma* 5.

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