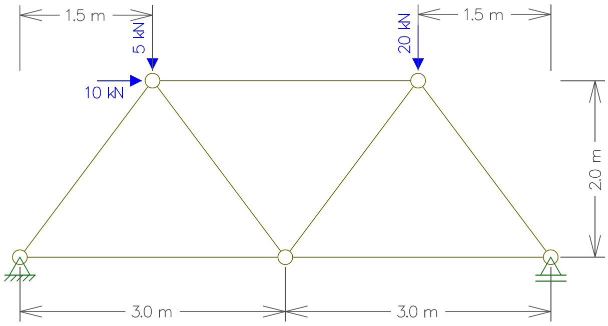
Exemplos MEF2Dframe

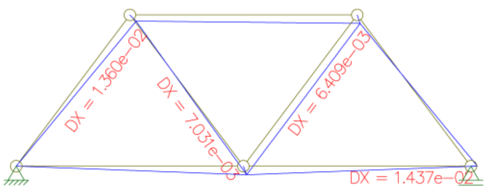
1) Exemplo1: Análise estática de treliça

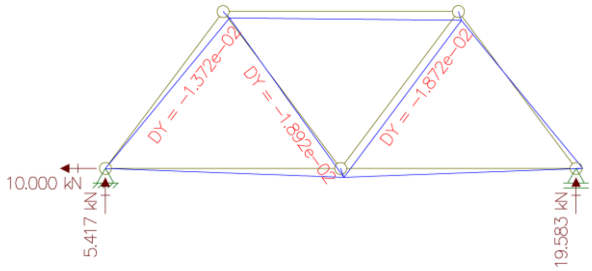


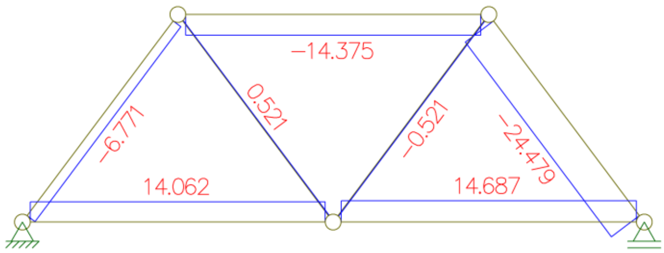
$E = 2 \times 10^{8}$ kN/m^{2}

$A = 3 \times 10^{-5}$ m^{2}

Resultados Ftool:







Arquivo Exemplo1.mef:

Arquivo de entrada de geometria
Nodes
0 0
3 0
6 0
1.5 2
4.5 2
end

Elements
0 0 1
0 1 2
0 0 3
0 3 1

0 1 4
0 4 2
0 3 4
end

Arquivo Exemplo1.loa:

Arquivo de entrada de carregamento

Dados Analise
truss estatica

Material
0 3e-5 2.e8 0
end

C Contorno
0 0 0.
0 1 0.
2 1 0.
end

F nodais
3 10. -5. 0.
4 0. -20. 0.
end

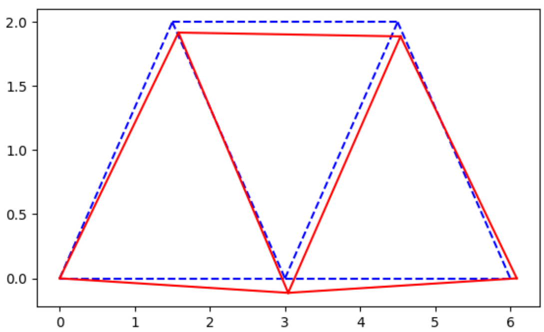
F elementares
end

Apoios elasticos
end

Resultados MEF2Dframe:

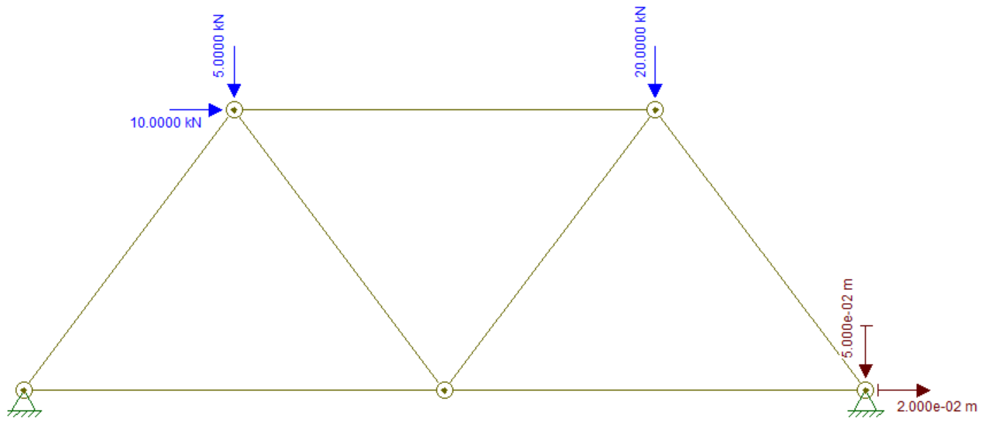
```
Deslocamentos: [ 0.          0.          0.00703125 -0.01891927  0.014375    0.
  0.01359664 -0.01372396  0.00640914 -0.01872396]
Reacoes de apoio: [-10.          5.41666667  19.58333333]
```

```
Esforços internos nodais: [Ni,  Nj]
Elemento 0: [14.0625 14.0625]
Elemento 1: [14.6875 14.6875]
Elemento 2: [-6.77083333 -6.77083333]
Elemento 3: [0.52083333 0.52083333]
Elemento 4: [-0.52083333 -0.52083333]
Elemento 5: [-24.47916667 -24.47916667]
Elemento 6: [-14.375 -14.375]
```

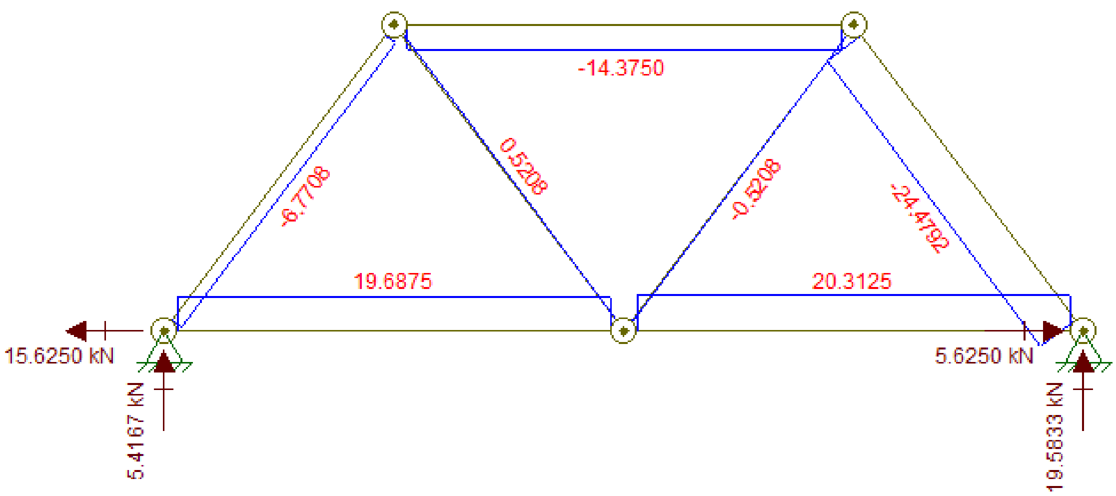


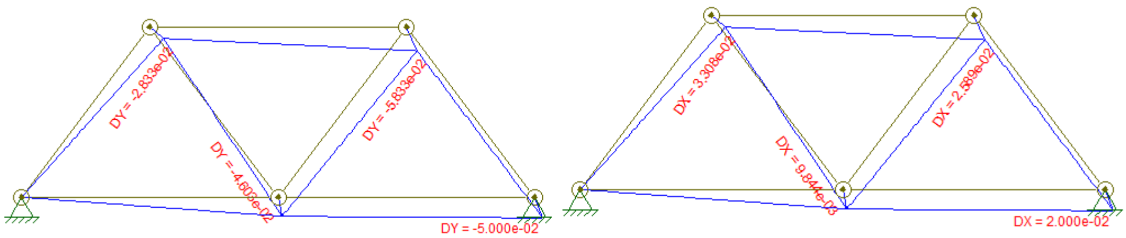
2) Exemplo2: Análise estática de treliça com recalque de apoio

Mesma geometria e carregamento do Exemplo1, mas com o apoio da direita transformado em articulado fixo com 2 recalques.



Resultados Ftool:





Arquivo Exemplo2.mef:

Idêntico ao Exemplo1.mef

Arquivo Exemplo2.loa:

Arquivo de entrada de carregamento

Dados Analise
truss estatica

Material
0 3e-5 2.e8 0
end

C Contorno
0 0 0.
0 1 0.
2 0 2e-2
2 1 -5e-2
end

F nodais
3 10. -5. 0.
4 0. -20. 0.
end

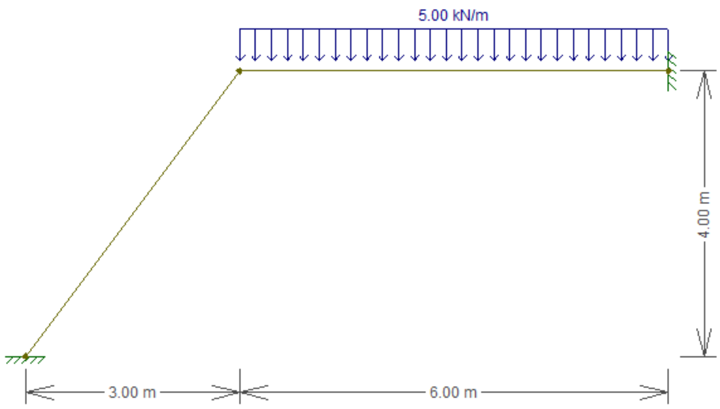
F elementares
end

Apoios elasticos
end

Resultados MEF2Dframe:

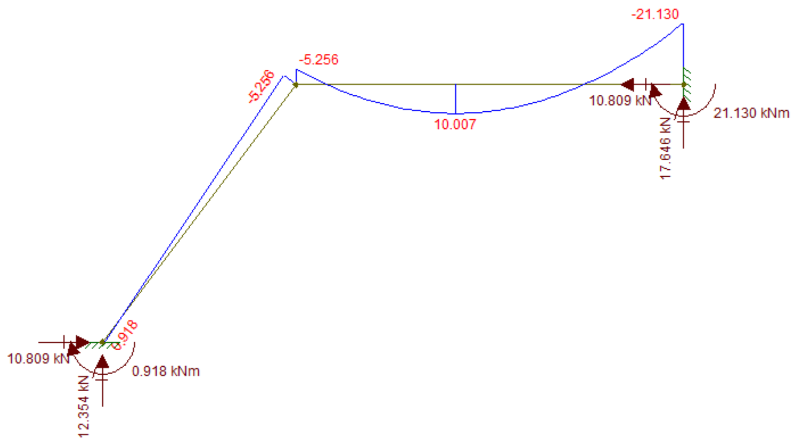
```
Deslocamentos: [ 0.          0.          0.00984375 -0.04602865  0.02       -0.05
  0.03307581 -0.02833333  0.02588831 -0.05833333]
Reacoes de apoio: [-15.625        5.41666667   5.625       19.58333333]
Esforços internos nodais: [Ni,  Nj]
Elemento 0: [19.6875 19.6875]
Elemento 1: [20.3125 20.3125]
Elemento 2: [-6.77083333 -6.77083333]
Elemento 3: [0.52083333 0.52083333]
Elemento 4: [-0.52083333 -0.52083333]
Elemento 5: [-24.47916667 -24.47916667]
Elemento 6: [-14.375 -14.375]
```

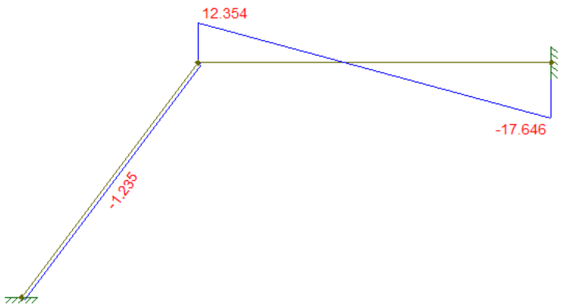
3) <u>Exemplo3: Análise estática de pórtico</u>

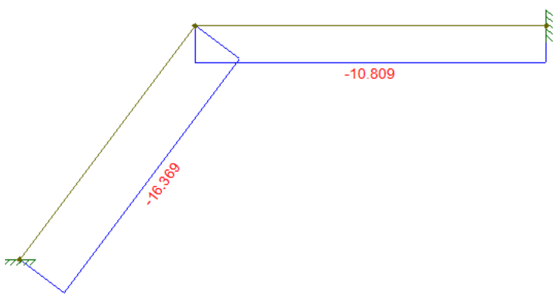


Resultados Ftool:

```
Nodal
Displacements:
 Dx = 4.504e-04 m
 Dy = -1.048e-03 m
 Rz = -7.530e-04 rad
```







Arquivo Exemplo3.mef:

Arquivo de entrada de geometria
Nodes
0 0
3 4
9 4
end

Elements
0 0 1
0 1 2
end

Arquivo Exemplo3.loa:

Arquivo de entrada de carregamento

Dados Analise
frame estatica

Material
1 1.2e-2 1.2e7 1.2e-3
end

C Contorno
0 0 0.
0 1 0.
0 2 0.
2 0 0.
2 1 0.
2 2 0.
end

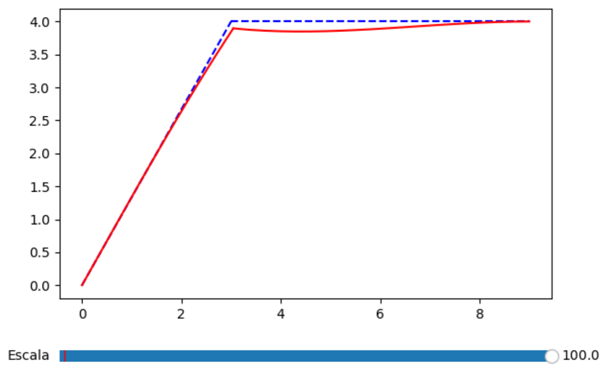
F nodais
end

F elementares
1 0 -5. -5.
end

Apoios elasticos
end

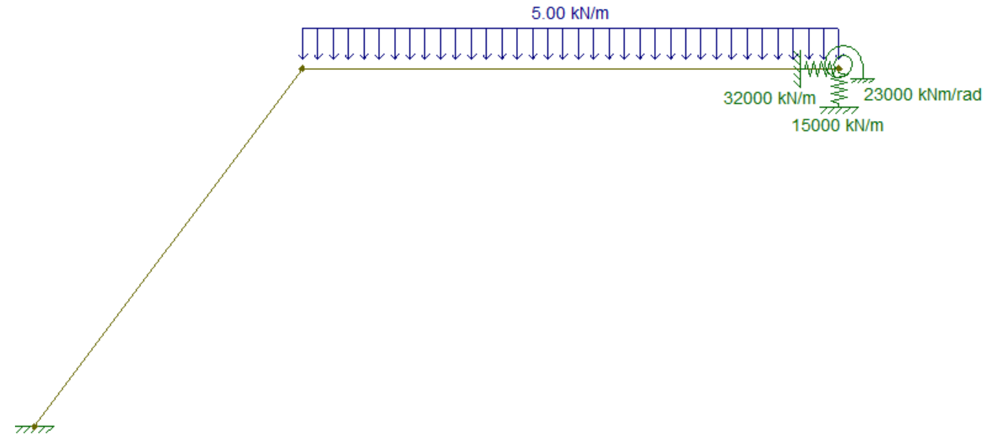
Resultados MEF2Dframe:

```
Deslocamentos: [ 0.          0.          0.          0.00045038 -0.00104824 -0.00075299
  0.          0.          0.        ]
Reacoes de apoio: [ 10.80915655  12.35424007  -0.91835269 -10.80915655  17.64575993
 -21.13011287]
Esforços internos nodais: [Ni, Qi, Mi, Nj, Qj, Mj]
Elemento 0: [-16.36888599  -1.2347812    0.91835269 -16.36888599  -1.2347812
  -5.25555329]
Elemento 1: [-10.80915655  12.35424007  -5.25555329 -10.80915655 -17.64575993
 -21.13011287]
```

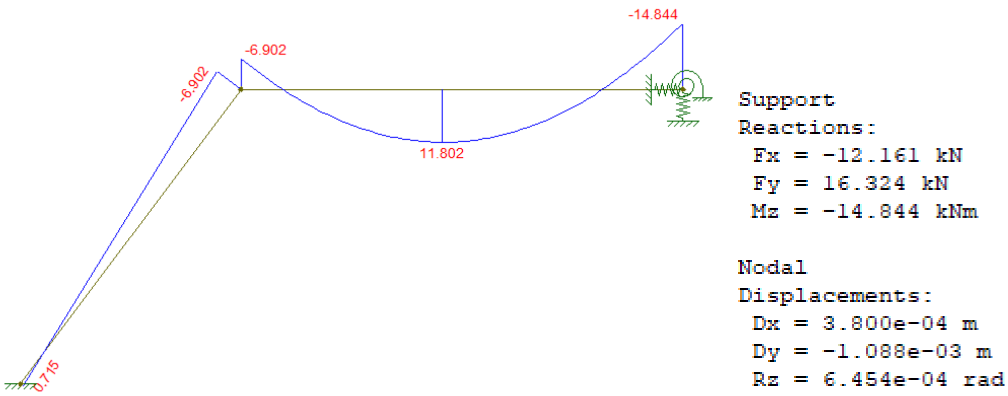


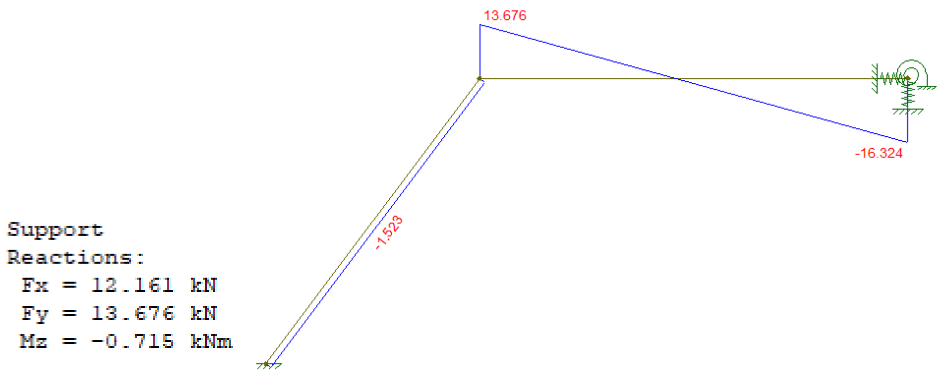
4) <u>Exemplo4: Análise estática de pórtico com apoio elástico</u>

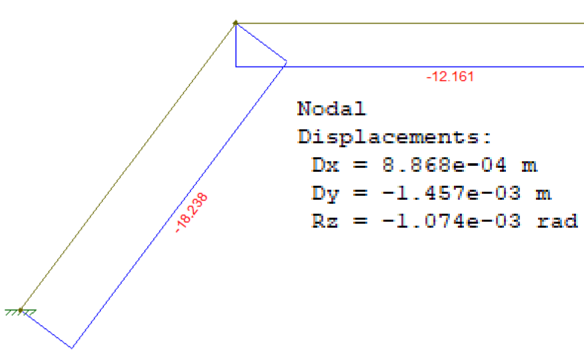
Mesma geometria e carregamento do Exemplo3, mas com o apoio da direita transformado em apoio elástico.



Resultados Ftool:







Arquivo Exemplo4.mef:

Idêntico ao Exemplo3.mef

Arquivo Exemplo4.loa:

Arquivo de entrada de carregamento

Dados Analise
frame estatica

Material
1 1.2e-2 1.2e7 1.2e-3
end

C Contorno
0 0 0.
0 1 0.
0 2 0.
end

F nodais
end

F elementares
1 0 -5. -5.
end

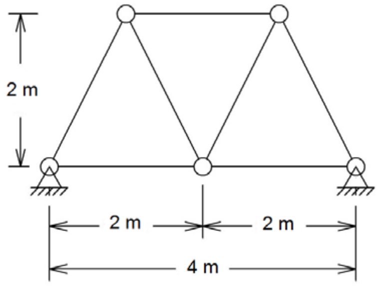
Apoios elasticos
2 32000 15000 23000
end

Resultados MEF2Dframe:

```
Deslocamentos: [ 0.          0.          0.          0.00088677 -0.00145665 -0.00107415
  0.00038004 -0.00108825  0.0006454 ]
Reacoes de apoio: [12.16140748 13.67628448 -0.71484155]
Esforços internos nodais: [Ni, Qi, Mi, Nj, Qj, Mj]
Elemento 0: [-18.23787208  -1.5233553    0.71484155 -18.23787208  -1.5233553
  -6.90193494]
Elemento 1: [-12.16140748  13.67628448  -6.90193494 -12.16140748 -16.32371552
 -14.84422805]
```

5) **Exemplo5: Análise modal de treliça**

Área da seção transversal A = 0,001 m², massa específica ρ = 8000 kg/m³ e módulo de elasticidade E = 2,1 x 10¹¹ N/m²



Resultados Arndt (2009):

i	MEF (7e) ngl[a] = 6	MC[b] (7e 1c) ngl = 13	MC[b] (7e 2c) ngl = 20	MC (7e 5c) ngl = 41	MEFG (7e) n_l = 1, $\beta_1 = \pi$ ngl = 34	MEFG Adap.[c] (7e 3i) 1x 6gl + 2x 34gl
	ω_i (rad/s)	ω_i (rad/s)	ω_i (rad/s)	ω_i (rad/s)	ω_i (rad/s)	ω_i (rad/s)
1	1683,521413	1648,516148	1648,258910	1647,811939	1647,785439	1647,784428
2	1776,278483	1741,661466	1741,319206	1740,868779	1740,840343	1740,839797
3	3341,375203	3119,123132	3113,835167	3111,525066	3111,326191	3111,322715
4	5174,353866	4600,595156	4567,688849	4562,562379	4561,819768	4561,817307
5	5678,184561	4870,575795	4829,702095	4824,125665	4823,253509	4823,248678
6	8315,400602	7380,832845	7379,960217	7379,515018	7379,482416	7379,482322

Arquivo Exemplo5.mef:

Arquivo de entrada de geometria
Nodes
0 0
1 2
2 0
3 2
4 0
end

Elements
0 0 1
0 0 2
0 1 2
0 1 3
0 2 3
0 2 4
0 3 4
end

Arquivo Exemplo5.loa:

Arquivo de entrada de carregamento

Dados Analise
truss modal

Material
8000 0.001 2.1e11 0
end

C Contorno
0 0 0.
0 1 0.
4 0 0.
4 1 0.
end

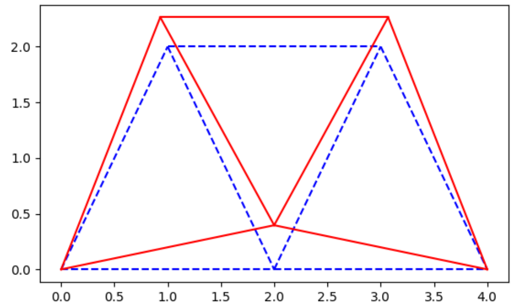
F nodais
end

F elementares
end

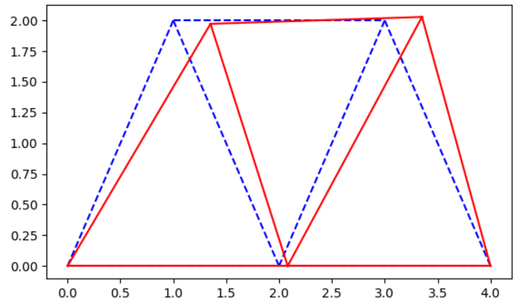
Apoios elasticos
end

Resultados MEF2Dframe:

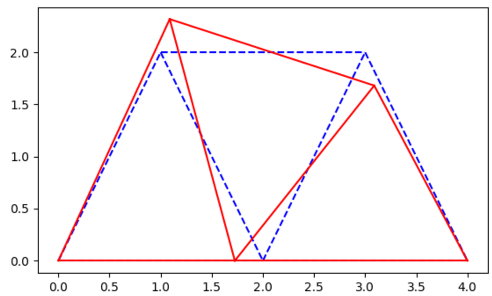
```
Frequências: [1683.52141299 1776.27848685 3341.37520503 5174.35386537 5678.18455758
 8315.40059884]
Modo 0: [-3.47926351e-02  1.31716901e-01 -1.49978055e-16  1.98195515e-01
  3.47926351e-02  1.31716901e-01]
Modo 1: [ 2.12054834e-01 -1.69384176e-02  4.92478553e-02  7.31958426e-16
  2.12054834e-01  1.69384176e-02]
Modo 2: [ 4.79451551e-02  1.71813628e-01 -1.47719353e-01  1.03224906e-16
  4.79451551e-02 -1.71813628e-01]
Modo 3: [-3.37582124e-02  1.54378632e-01  2.33635895e-01  1.12608989e-16
 -3.37582124e-02 -1.54378632e-01]
Modo 4: [-1.62374038e-01 -1.74791659e-01 -2.15774879e-16  1.80893949e-01
  1.62374038e-01 -1.74791659e-01]
Modo 5: [-2.80280410e-01  1.11958504e-01  3.09632113e-17 -2.37590380e-01
  2.80280410e-01  1.11958504e-01]
```



Modo 0

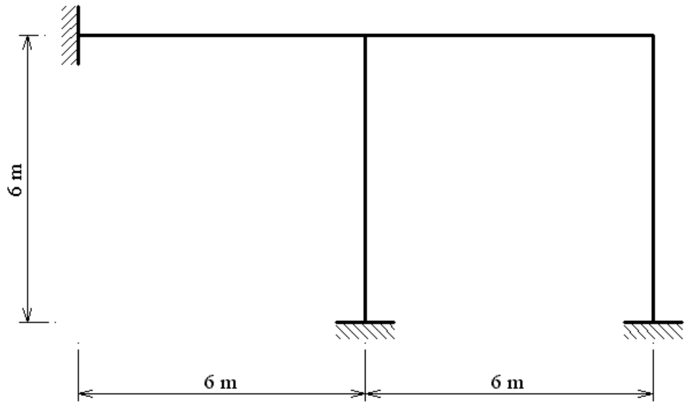


Modo 1



Modo 2

6) <u>Exemplo6: Análise modal de pórtico</u>



Área da seção transversal A = 0,1 m², momento de inércia I = 1 x 10^{-2} m⁴, massa específica ρ = 7800 kg/m³ e módulo de elasticidade E = 1 x 10^{8} N/m².

Resultados Arndt (2009):

	MEF (4e) ngl = 6	MEF (40e) ngl = 114	MC (4e 14c) ngl = 62	MC (4e 30c) ngl = 126	MEFG Adap. (4e 3i) 1x 6gl + 2x 54gl
i	ω_i (rad/s)	ω_i (rad/s)	ω_i (rad/s)	ω_i (rad/s)	ω_i (rad/s)
1	12,412232	11,792155	11,791295	11,791255	11,791251
2	14,304731	12,299299	12,298978	12,298965	12,298964
3	19,197774	15,837412	15,836681	15,836628	15,836624
4	26,070176	20,123625	20,121807	20,121732	20,121724
5	31,037720	21,703787	21,700282	21,700112	21,700096
6	41,495314	25,290751	25,282759	25,282180	25,282132

Arquivo Exemplo6.mef:

Arquivo de entrada de geometria
Nodes
0 0
6 0
12 0
6 -6
12 -6
end

Elements
0 0 1
0 1 2
0 1 3
0 2 4
end

Arquivo Exemplo6.loa:

Arquivo de entrada de carregamento

Dados Analise
frame modal

Material
7800 0.1 1.e8 1.e-2
end

C Contorno
0 0 0.
0 1 0.
0 2 0.
3 0 0.
3 1 0.
3 2 0.
4 0 0.
4 1 0.
4 2 0.
end

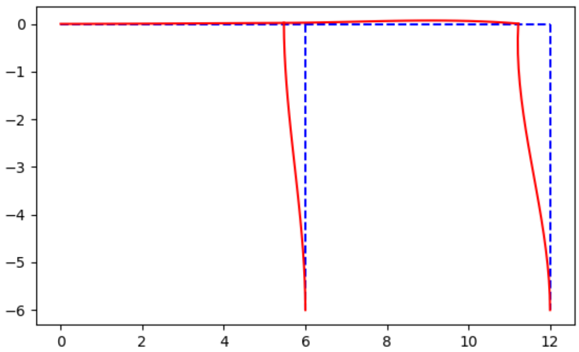
F nodais
end

F elementares
end

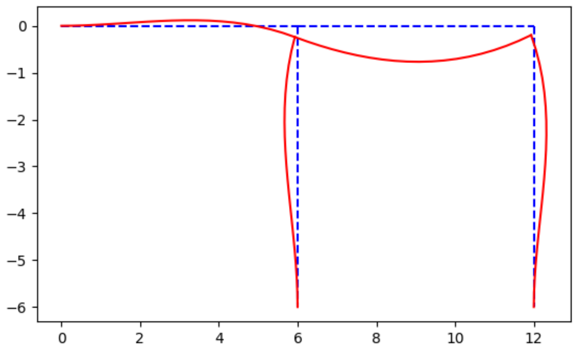
Apoios elasticos
end

Resultados MEF2Dframe:

```
Frequências: [12.41223173 14.30473147 19.19777373 26.07017564 31.03771979 41.49531415]
Modo 0: [-8.17443685e-03  3.21436798e-04  5.59639853e-05 -1.20812269e-02
  1.03481221e-04 -1.02033466e-03]
Modo 1: [-0.00143897 -0.00491385 -0.00645132 -0.00141246 -0.00405732  0.0085526 ]
Modo 2: [ 8.11942092e-05  1.22773771e-02 -6.85434032e-03  1.38324991e-03
 -9.38322751e-04  1.44402715e-03]
Modo 3: [-0.00176596 -0.0034228  -0.00627562  0.00230978  0.0173688   0.00321678]
Modo 4: [ 0.00772969 -0.00460966 -0.01154718  0.0018427  -0.00470845 -0.0170942 ]
Modo 5: [-0.01031815 -0.00224206  0.0004401   0.01671454 -0.00843288 -0.01482881]
```

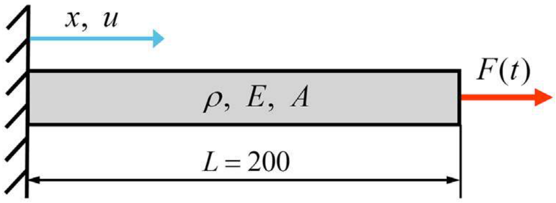


Modo 0



Modo 1

7) <u>Exemplo7: Análise transiente de barra</u>

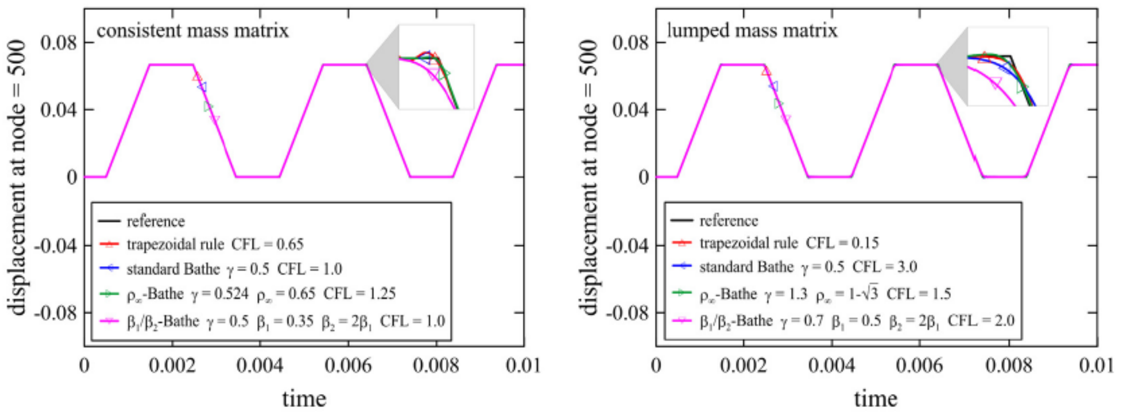


Área da seção transversal A = 1, massa específica ρ = 0,00073 e módulo de elasticidade E = 3 x 10^7. Força constante (patamar) de 10000. Sem amortecimento.

Resultados de Kwon, Bathe, Noh (2020):

Referência: KWON, Sun-Beom; BATHE, Klaus-Jürgen; NOH, Gunwoo. An analysis of implicit time integration schemes for wave propagations. **Computers & Structures**, v. 230, 2020, p. 106188.

Resultados para o nó central (x = 100) utilizando 1000 elementos finitos de 2 nós e matrizes de massa agrupada e consistente.



(a) Consistent mass matrix (b) Lumped mass matrix

Fig. 13. Time history of displacements at the center of the bar using various Bathe time integration schemes and trapezoidal rule.

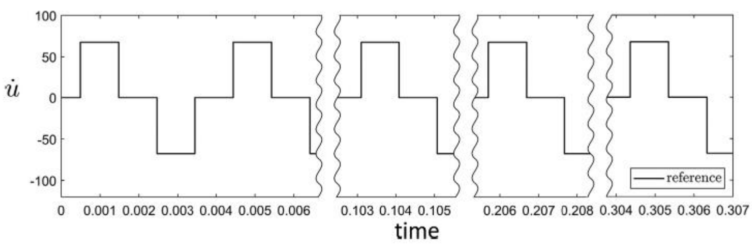


Fig. 12. A clamped bar excited by a step load, and the exact velocity at the center, $x = 100$. Young's modulus $E = 3 \times 10^7$, mass density $\rho = 0.00073$, cross-sectional area $A = 1$, length $L = 200$ [36]. The receiver is located at the center.

Arquivo Exemplo7.mef: (malha com 20 elementos)

Arquivo de entrada de geometria (linha livre para acrescentar comentários)
Nodes
0 0
10 0
20 0

30 0
40 0
50 0
60 0
70 0
80 0
90 0
100 0
110 0
120 0
130 0
140 0
150 0
160 0
170 0
180 0
190 0
200 0
end

Elements
0 0 1
0 1 2
0 2 3
0 3 4
0 4 5
0 5 6
0 6 7
0 7 8
0 8 9
0 9 10
0 10 11
0 11 12
0 12 13
0 13 14
0 14 15
0 15 16
0 16 17
0 17 18
0 18 19
0 19 20
end

Arquivo Exemplo7.loa: (Método de Newton Aceleração Linear com $\Delta t = 9{,}88\ 10^{-7}$ e $T = 0{,}015$)

Arquivo de entrada de carregamento (linha livre para comentários)

Dados Analise
truss transiente NAL 0 hav 0 9.88e-7 0.015

Material
0.00073 1. 30.e6 0.
end

C Contorno
0 0 0.
0 1 0.
1 1 0.
2 1 0.
3 1 0.
4 1 0.
5 1 0.
6 1 0.
7 1 0.
8 1 0.
9 1 0.
10 1 0.
11 1 0.
12 1 0.
13 1 0.
14 1 0.
15 1 0.
16 1 0.
17 1 0.
18 1 0.
19 1 0.
20 1 0.
end

F nodais
20 10000. 0. 0.
end

F elementares
end

Apoios elasticos
end

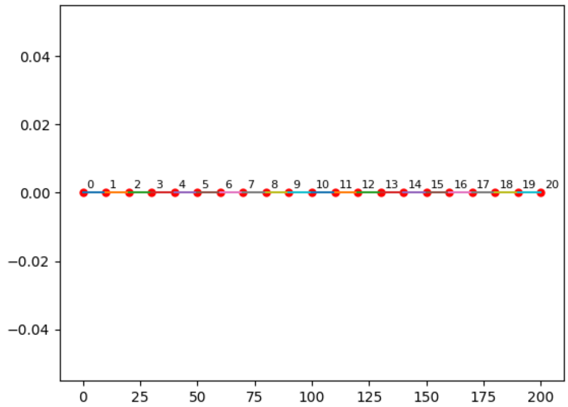
os blocos abaixo só precisam estar presente na análise transiente
C inicial desloc
0 0. 0. 0.
1 0. 0. 0.
2 0. 0. 0.

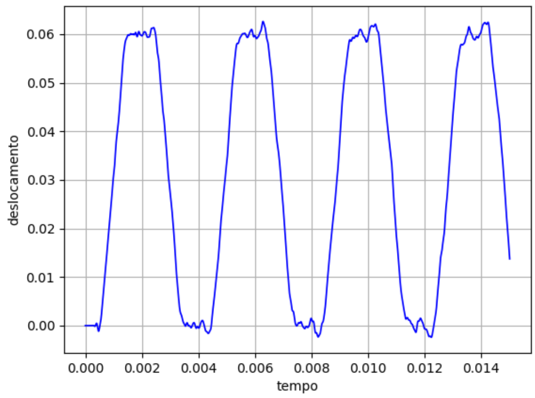
3 0. 0. 0.
4 0. 0. 0.
5 0. 0. 0.
6 0. 0. 0.
7 0. 0. 0.
8 0. 0. 0.
9 0. 0. 0.
10 0. 0. 0.
11 0. 0. 0.
12 0. 0. 0.
13 0. 0. 0.
14 0. 0. 0.
15 0. 0. 0.
16 0. 0. 0.
17 0. 0. 0.
18 0. 0. 0.
19 0. 0. 0.
20 0. 0. 0.
end

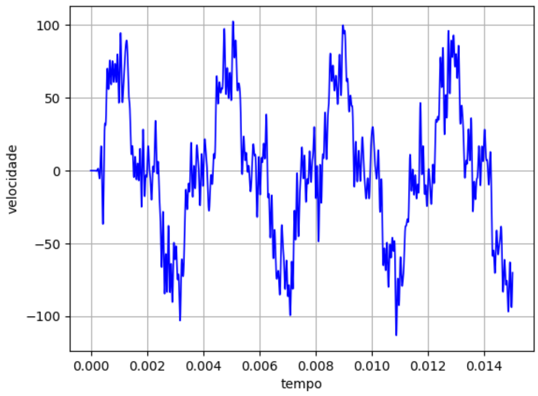
C inicial vel
0 0. 0. 0.
1 0. 0. 0.
2 0. 0. 0.
3 0. 0. 0.
4 0. 0. 0.
5 0. 0. 0.
6 0. 0. 0.
7 0. 0. 0.
8 0. 0. 0.
9 0. 0. 0.
10 0. 0. 0.
11 0. 0. 0.
12 0. 0. 0.
13 0. 0. 0.
14 0. 0. 0.
15 0. 0. 0.
16 0. 0. 0.
17 0. 0. 0.
18 0. 0. 0.
19 0. 0. 0.
20 0. 0. 0.
end

Resultados MEF2Dframe:

Resultados obtidos para o nó central, Método de Newmark Aceleração Linear, com 20 elementos de 2 nós e matriz de massa consistente:







Resultados obtidos para o nó central, Método de Newmark Aceleração Constante, com 1000 elementos de 2 nós e matriz de massa consistente: