Chapter 1

Permutations and Combinations

Show that:

$$
\begin{aligned}
\textbf{(a)}\ n\binom{n}{r} &= (r+1)\binom{n}{r+1} + r\binom{n}{r} \\
&= (r+1)\frac{n!}{(r+1)!(n-(r+1))!} + r.\frac{n!}{(n-r)!r!} \\
&= \frac{(r+1)}{(r+1)!}\frac{n!}{(n-r-1)!} + \frac{r}{r!}\frac{n!}{(n-r)!} \\
&= \frac{1}{r!}\frac{n!}{(n-r-1)!} + \frac{1}{(r-1)!}\frac{n!}{(n-r)!} \\
&= \frac{n!}{r!(n-r-1)!}.\frac{(n-r)}{(n-r)} + \frac{n!}{(r-1)!(n-r)!}.\frac{r}{r} \\
&= \frac{n!(n-r)}{r!(n-r)!} + \frac{n!r}{r!(n-r)!} \\
&= \frac{n!(n-r)+n!r}{r!(n-r)!} \\
&= \frac{n!(n-r+r)}{r!(n-r)!} \\
&= \frac{n!n}{r!(n-r)!} \\
&\boxed{= n\binom{n}{r}} \qquad \blacksquare
\end{aligned}
$$

(b) $\binom{n}{2}\binom{n}{r} = \underbrace{\binom{r+2}{2}\binom{n}{n+2}}_{\star} + 2\underbrace{\binom{r+1}{2}\binom{n}{r+1}}_{\star} + \underbrace{\binom{r}{2}\binom{n}{r}}_{\star}$ $\boxed{\star \binom{n}{m}\binom{m}{k} = \binom{n}{k}\binom{n-k}{m-k}}$

$$= \binom{n}{2}\binom{n-2}{(r+2)-2} + 2\binom{n}{2}\binom{n-2}{(r+1)-2} + \binom{n}{2}\binom{n-2}{r-2}$$

$$= \binom{n}{2}.\Bigg[\underbrace{\binom{n-2}{r} + \binom{n-2}{r-1}}_{*} + \underbrace{\binom{n-2}{r-1} + \binom{n-2}{r-2}}_{*}\Bigg]$$ $\boxed{* \binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}}$

$$= \binom{n}{2}.\Bigg[\underbrace{\binom{n-1}{r} + \binom{n-1}{r-1}}_{*}\Bigg]$$

$$\boxed{= \binom{n}{2}.\binom{n}{r}} \quad \blacksquare$$

Chapter 2

Generations Functions

Chapter 3

The Principle of Inclusion and Exclusion

Chapter 4

The Cycles of Permutations

Chapter 5

Distributions: Occupancy

Chapter 6

Partitions, Compositions, Trees , and Networks

Chapter 7

Permutations with Restricted Position I

Chapter 8

Permutations with Restricted Position II