# A Hybrid Heuristic for the Multi-objective Knapsack Problem

## Doctoral thesis proposal

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This reason motivates the development of heuristic methods.

## Proposal

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As a performance improvement for the approach, a multi-dimensional indexing strategy will be used for handling the large amount of solutions.

The SCE has been successfully used for solving optimization problems, among then, the multi-dimensional knapsack problem (MKP) (also a contribution of this work).

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$$dom(\boldsymbol{a}, \boldsymbol{b}) = \begin{cases} \forall i \in \{1, 2, \ldots, m\} : f_i(\boldsymbol{a}) \geqslant f_i(\boldsymbol{b}) \text{ and} \\ \exists j \in \{1, 2, \ldots, m\} : f_j(\boldsymbol{a}) > f_j(\boldsymbol{b}) \end{cases}$$

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The set of all efficient solutions.

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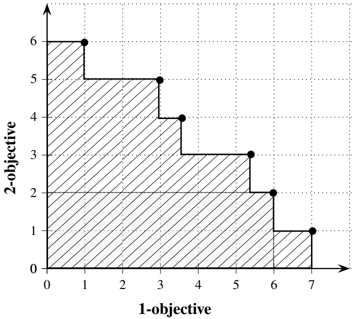


Figure: Example of a Pareto set for a bi-objective problem.

# The multi-objective knapsack problem

## Problem Definition

A problem instance with $n$ items and $m$ objectives:

$$\begin{aligned} \max\ f(\boldsymbol{x}) &= \big(f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \ldots, f_m(\boldsymbol{x})\big) \\ \text{subject to } w(\boldsymbol{x}) &\leqslant W \\ \boldsymbol{x} &\subseteq \{1, \ldots, n\} \end{aligned}$$

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A $\mathcal{NP}$-Hard problem for which the Pareto optimal set tends to grow exponentially with the number of objectives.

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## Knapsack dominance

Consider two solutions $\boldsymbol{x}, \boldsymbol{y}$ for a MOKP.

We say $\boldsymbol{x}$ knapsack-dominates $\boldsymbol{y}$, denoted as $dom_k(\boldsymbol{x}, \boldsymbol{y})$, if $\boldsymbol{x}$ dominates $\boldsymbol{y}$ and does not weight more than $\boldsymbol{y}$.

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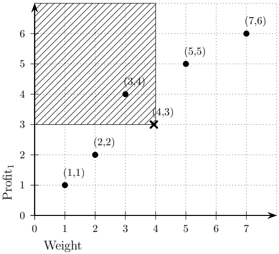


Figure: A knapsack-dominated solution.

# Multi-dimensional Indexing of MOKP solutions

## Dominance check operation

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However, if solutions are mapped into points in a multi-dimensional space, this operation corresponds on checking whether a point exists in a certain region.

# Dominance check operation

Formally:

$$\text{if } \ dom_k(\boldsymbol{y}, \boldsymbol{x}) \ \text{ then } \ pnt(\boldsymbol{y}) \in R(\boldsymbol{x})$$

where

$$pnt(\boldsymbol{x}) = \Big(f_1(\boldsymbol{x}), \ldots, f_m(\boldsymbol{x}), w(\boldsymbol{x})\Big)$$

$$R(\boldsymbol{x}) = \Big\{ a \in \mathbb{R}^{m+1} \ \Big| \ a_{m+1} \leqslant w(\boldsymbol{x}) \ \text{ and } \ a_i \geqslant f_i(\boldsymbol{x}), \ i \in \{1, \ldots, m\} \Big\}$$

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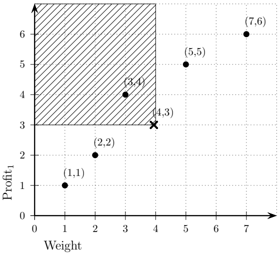

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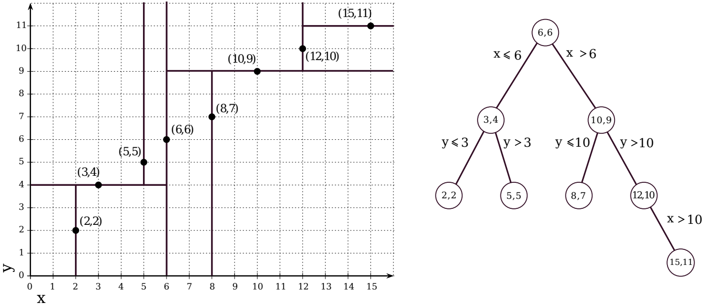
For efficiency reason range search operations is usually executed with the assistance of a *k*-d tree.

The *k*-d tree is a type of binary search tree for indexing multi-dimensional data with simple construction and low space usage.

# Multidimensional solution indexing

Unlike a standard binary tree, that uses only one key for all levels of the tree, the *k*-d tree uses *k* keys and cycles through these keys for successive levels of the tree.



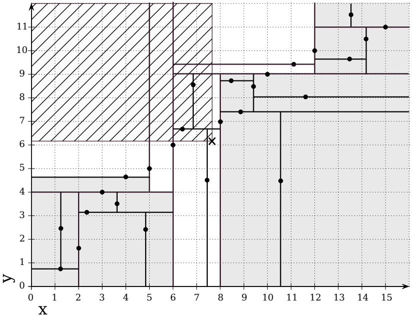

(a) points displayed on a plane

(b) points arranged on a kd-tree

Figure: Example of points indexed in a *k*-d tree.

# Multidimensional solution indexing

Example of dominance check operation using $k$-d tree:



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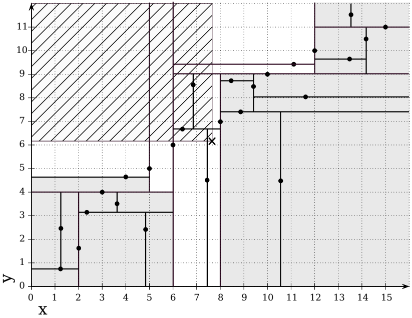


The efficiency of this pruning action grows with the amount of points.

## Multi-dimensional Indexing of MOKP solutions

Use case on an exact method

## Nemhauser and Ullmann's algorithm

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        S^k = {x | ∄a ∈ S^k_* : dom_k(a, x)}            ▷ partial dominance filter
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# Bazgan's dynamic programming algorithm

**function** $\textsc{BazDP}(\boldsymbol{p}, \boldsymbol{w}, W)$
    $S^0 = \left\{\emptyset\right\}$

# Bazgan's dynamic programming algorithm

Item reordering

1: **function** $\textsc{BazDP}(\boldsymbol{p}, \boldsymbol{w}, W)$
2: $S^0 = \left\{\emptyset\right\}$
3: $o_1, \ldots, o_n = \mathcal{O}^{max}$

## Bazgan's dynamic programming algorithm

**function** $\textsc{BazDP}(\boldsymbol{p}, \boldsymbol{w}, W)$

$\quad S^0 = \left\{\emptyset\right\}$

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$\quad$ **for** $k \leftarrow 1, n$ **do**

# Bazgan's dynamic programming algorithm

Feasible extension

1: **function** BAZDP($\boldsymbol{p}, \boldsymbol{w}, W$)
2: $S^0 = \left\{\emptyset\right\}$
3: $o_1, \ldots, o_n = \mathcal{O}^{max}$
4: **for** $k \leftarrow 1, n$ **do**
5: $S_*^k = \left\{\boldsymbol{x} \cup \{o_k\} \;\middle|\; \boldsymbol{x} \in S^{k-1} \wedge w(\boldsymbol{x}) + w_{o_k} \leqslant W\right\}$

# Bazgan's dynamic programming algorithm

Defficiency avoidance

1: **function** $\text{BAZDP}(\boldsymbol{p}, \boldsymbol{w}, W)$
2: $S^0 = \{\emptyset\}$
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4: **for** $k \leftarrow 1, n$ **do**
5: $S_*^k = \left\{ \boldsymbol{x} \cup \{o_k\} \,\middle|\, \boldsymbol{x} \in S^{k-1} \wedge w(\boldsymbol{x}) + w_{o_k} \leqslant W \right\}$
6: $\cup \left\{ \boldsymbol{x} \,\middle|\, \boldsymbol{x} \in S^{k-1} \wedge w(\boldsymbol{x}) + w_{o_k} + \ldots + w_{o_n} > W \right\}$

# Bazgan's dynamic programming algorithm

Unpromising elimination

**function** $\text{BAZDP}(\boldsymbol{p}, \boldsymbol{w}, W)$
$S^0 = \left\{\emptyset\right\}$
$o_1, \ldots, o_n = \mathcal{O}^{max}$
**for** $k \leftarrow 1, n$ **do**
$S^k_* = \left\{\boldsymbol{x} \cup \{o_k\} \;\middle|\; \boldsymbol{x} \in S^{k-1} \wedge w(\boldsymbol{x}) + w_{o_k} \leqslant W\right\}$
$\cup \left\{\boldsymbol{x} \;\middle|\; \boldsymbol{x} \in S^{k-1} \wedge w(\boldsymbol{x}) + w_{o_k} + \ldots + w_{o_n} > W\right\}$
$S^k = \left\{\boldsymbol{x} \in S^k_* \;\middle|\; (\nexists \boldsymbol{a} \in S^k_*)\left[dom_k(\boldsymbol{a}, \boldsymbol{x}) \vee dom\Big(lb(\boldsymbol{a}), ub(\boldsymbol{x})\Big)\right]\right\}$

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    **for** $k \leftarrow 1, n$ **do**
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            $\cup \left\{\boldsymbol{x} \;\middle|\; \boldsymbol{x} \in S^{k-1} \wedge w(\boldsymbol{x}) + w_{o_k} + \ldots + w_{o_n} > W\right\}$
        $S^k = \left\{\boldsymbol{x} \in S_*^k \;\middle|\; (\nexists \boldsymbol{a} \in S_*^k)\left[dom_k(\boldsymbol{a}, \boldsymbol{x}) \vee dom\Big(lb(\boldsymbol{a}), ub(\boldsymbol{x})\Big)\right]\right\}$
    **end for**
    **return** $S^n$
**end function**

# Bazgan's dynamic programming algorithm

Dominance check operation

**function** $\textsc{BazDP}(\boldsymbol{p}, \boldsymbol{w}, W)$
$\quad S^0 = \left\{\emptyset\right\}$
$\quad o_1, \ldots, o_n = \mathcal{O}^{max}$
$\quad$ **for** $k \leftarrow 1, n$ **do**
$$S^k_* = \left\{\boldsymbol{x} \cup \{o_k\} \;\middle|\; \boldsymbol{x} \in S^{k-1} \wedge w(\boldsymbol{x}) + w_{o_k} \leqslant W\right\}$$
$$\cup \left\{\boldsymbol{x} \;\middle|\; \boldsymbol{x} \in S^{k-1} \wedge w(\boldsymbol{x}) + w_{o_k} + \ldots + w_{o_n} > W\right\}$$
$$S^k = \left\{\boldsymbol{x} \in S^k_* \;\middle|\; (\nexists \boldsymbol{a} \in S^k_*)\left[\underline{dom_k}(\boldsymbol{a}, \boldsymbol{x}) \vee \underline{dom}\big(lb(\boldsymbol{a}), ub(\boldsymbol{x})\big)\right]\right\}$$
$\quad$ **end for**
$\quad$ **return** $S^n$
**end function**

## Computational Experiments

Computational experiments on bi-dimensional instances:

A) Random instances:
$p_i^j \in [1, 1000]$,
$w_i \in [1, 1000]$.

B) Unconflicting instances:
$p_i^1 \in [111, 1000]$,
$p_i^2 \in [p_i^1 - 100, p_i^1 + 100]$,
$w_i \in [1, 1000]$.

C) Conflicting instances:
$p_i^1 \in [1, 1000]$,
$p_i^2 \in [max\{900 - p_i^1; 1\}, min\{1100 - p_i^1, 1000\}]$,
$w_i \in [1, 1000]$.

D) Conflicting instances with correlated weight:
$p_i^1 \in [1, 1000]$,
$p_i^2 \in [max\{900 - p_i^1; 1\}, min\{1100 - p_i^1, 1000\}]$,
$w_i \in [p_i^1 + p_i^2 - 200, p_i^1 + p_i^2 + 200]$.

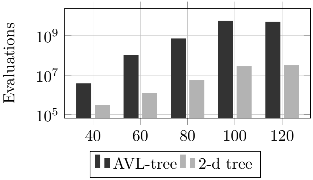
## Computational Experiments

Average CPU-time for bi-objective instances:

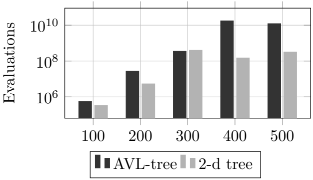
| Instance | | | AVL tree | 2-d tree | |
|---|---|---|---|---|---|
| Type | $n$ | $\|ND\|$ | time (s) | time (s) | speedup |
| A | 40 | 38.1 | **0.06** | **0.06** | 1.0 |
| | 60 | 73.1 | 1.12 | **0.88** | 1.3 |
| | 80 | 125.6 | 19.81 | **11.89** | 1.7 |
| | 100 | 180.4 | 165.24 | **76.50** | 2.2 |
| | 120 | 233.9 | 708.53 | **361.87** | 2.0 |
| B | 100 | 3.1 | **0.02** | 0.08 | 0.3 |
| | 200 | 10.0 | **0.80** | 5.09 | 0.2 |
| | 300 | 24.9 | **9.45** | 88.30 | 0.1 |
| | 400 | 36.2 | **95.39** | 730.04 | 0.1 |
| | 500 | 53.7 | **255.57** | 2824.65 | 0.1 |
| C | 20 | 36.6 | **0.00** | **0.00** | 1.0 |
| | 40 | 102.8 | 0.65 | **0.42** | 1.5 |
| | 60 | 231.9 | 28.98 | **14.09** | 2.1 |
| | 80 | 358.0 | 564.10 | **241.54** | 2.3 |
| | 100 | 513.8 | 3756.57 | **1605.19** | 2.3 |
| D | 20 | 174.9 | 0.15 | **0.12** | 1.3 |
| | 30 | 269.3 | 16.82 | **7.60** | 2.2 |
| | 40 | 478.0 | 395.76 | **186.67** | 2.1 |
| | 50 | 553.4 | 2459.48 | **1417.94** | 1.7 |

# Computational Experiments

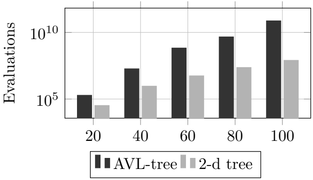
Average number of solution evaluations for bi-objective instances:



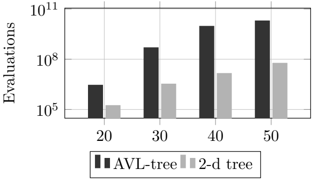

(a) Type A




(b) Type B




(c) Type C




(d) Type D

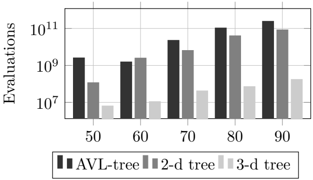
## Computational Experiments

Average CPU-time for 3-objective instances:

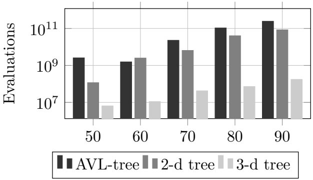
| Instance | | | AVL tree | 2-d tree | | 3-d tree | |
|---|---|---|---|---|---|---|---|
| Type | $n$ | $\|ND\|$ | time (s) | time (s) | speedup | time (s) | speedup |
| A | 50 | 557.5 | 41.2 | 21.3 | 1.9 | **18.5** | 2.2 |
| | 60 | 1240.0 | 485.9 | 247.8 | 1.9 | **79.9** | 6.0 |
| | 70 | 1879.3 | 3179.5 | 1038.0 | 3.0 | **614.5** | 5.1 |
| | 80 | 2540.5 | 6667.9 | 3796.0 | 1.7 | **2943.9** | 2.2 |
| | 90 | 3528.5 | 24476.5 | 12916.7 | 1.8 | **3683.7** | 6.6 |
| B | 100 | 18.0 | **0.1** | 0.3 | 0.3 | 0.3 | 0.3 |
| | 200 | 65.4 | **11.4** | 34.4 | 0.3 | 29.1 | 0.4 |
| | 300 | 214.2 | **307.7** | 631.5 | 0.5 | 583.2 | 0.5 |
| | 400 | 317.0 | **4492.9** | 8464.9 | 0.5 | 5402.2 | 0.8 |
| C | 20 | 254.4 | 0.06 | 0.05 | 1.2 | **0.03** | 2.17 |
| | 30 | 1066.6 | 9.69 | 4.18 | 2.3 | **1.30** | 7.46 |
| | 40 | 2965.5 | 471.68 | 153.21 | 3.1 | **30.50** | 15.5 |
| D | 20 | 4087.7 | 23.6 | 10.9 | 2.2 | **1.9** | 12.5 |
| | 30 | 8834.5 | 8914.2 | 3625.3 | 2.5 | **1019.5** | 8.7 |

# Computational Experiments

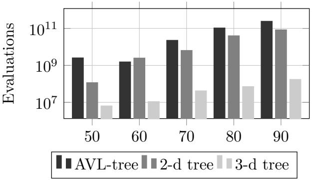
Average number of solution evaluations for 3-objective instances:



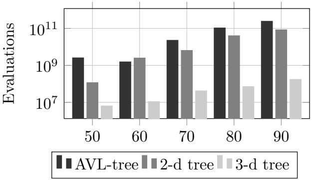

(a) Type A




(b) Type B




(c) Type C




(d) Type D

## Conclusions

The multi-dimensional indexing is applicable to the problem requiring considerably less solution evaluations, especially on hard instances.

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Algorithm speedup 2.3 for bi-dimensional cases and up to 15.5 on 3-dimensional cases.

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The multi-dimensional indexing was not efficient on *easy* instances for which the set of solutions is relatively small.

Several instances are still intractable due the large number of intermediate solutions.

For this reason this work proposes to use the indexing strategy in conjunction with an evolutionary metaheuristic.

# The Shuffled Complex Evolution

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In this shuffling process the first individual goes to the first complex, the second individual goes to the second complex, individual $N$ goes to $N$-th complex, individual $(M + 1)$-th goes back to the first complex, etc;

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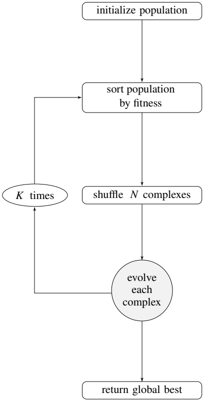

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This new solution is generated by the crossing of the worst individual and an other individual with better fitness;

At first the best individual of the subcomplex is considered for the crossing;

If the new solution is not better than the worst one, the best individual of the complex is considered for a crossing;

If the latter crossing did not result in any improvement, the best individual of whole population is considered;

# The Shuffled Complex Evolution

In each step a subcomplex of $P$ individuals is selected from the complex;

After the selection of the subcomplex, its worst individual is identified to be replaced by a new generated solution;

This new solution is generated by the crossing of the worst individual and an other individual with better fitness;

At first the best individual of the subcomplex is considered for the crossing;

If the new solution is not better than the worst one, the best individual of the complex is considered for a crossing;

If the latter crossing did not result in any improvement, the best individual of whole population is considered;

Finally, if all the crossing steps couldn't generate a better individual, the worst individual of the subcomplex is replaced by a new random solution taken from the feasible solution space.

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In each step a subcomplex of $P$ individuals is selected from the complex;

After the selection of the subcomplex, its worst individual is identified to be replaced by a new generated solution;

This new solution is generated by the crossing of the worst individual and an other individual with better fitness;

At first the best individual of the subcomplex is considered for the crossing;

If the new solution is not better than the worst one, the best individual of the complex is considered for a crossing;

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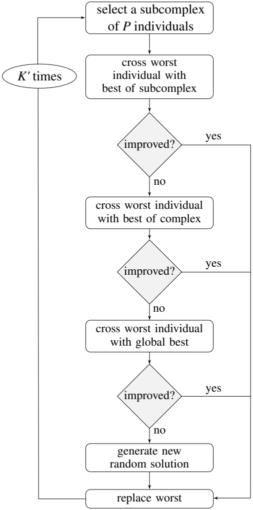

# The Shuffled Complex Evolution

Use case for the MKP

# The SCE for MKP

the SCE is easily applied to any optimization problem. The only steps needed to be specified is **(a)** the creation of a new random solution and **(b)** the crossing procedure of two solutions.

# The SCE for MKP

New random solution generation for the MKP.

1: **function** NEW RANDOM SOLUTION

## The SCE for MKP

New random solution generation for the MKP.

```
function NEW RANDOM SOLUTION
    v ← shuffle(1, 2, . . . , n)
```

# The SCE for MKP

New random solution generation for the MKP.

**function** NEW RANDOM SOLUTION
    $v \leftarrow \text{shuffle}(1, 2, \ldots, n)$
    $s \leftarrow \emptyset$

# The SCE for MKP

New random solution generation for the MKP.

```
function NEW RANDOM SOLUTION
    v ← shuffle(1, 2, ..., n)
    s ← ∅
    for i ← 1 : n do
        s ← s ∪ {v_i}
```

# The SCE for MKP

New random solution generation for the MKP.

**function** NEW RANDOM SOLUTION
    $v \leftarrow \text{shuffle}(1, 2, \ldots, n)$
    $s \leftarrow \emptyset$
    **for** $i \leftarrow 1 : n$ **do**
        $s \leftarrow s \cup \{v_i\}$
        **if** $s$ is not feasible **then**

# The SCE for MKP

New random solution generation for the MKP.

```
function NEW RANDOM SOLUTION
    v ← shuffle(1, 2, ..., n)
    s ← ∅
    for i ← 1 : n do
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        if s is not feasible then
            s ← s − {v_i}
        end if
```

# The SCE for MKP

New random solution generation for the MKP.

```
function NEW RANDOM SOLUTION
    v ← shuffle(1, 2, ..., n)
    s ← ∅
    for i ← 1 : n do
        s ← s ∪ {v_i}
        if s is not feasible then
            s ← s − {v_i}
        end if
    end for
    return s
end function
```

## The SCE for MKP

Crossing procedure for the MKP.

1: **function** CROSSING($x^w$ : worst individual, $x^b$ : better individual, $c$)

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1: **function** CROSSING($x^w$ : worst individual, $x^b$ : better individual, $c$)
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# The SCE for MKP

Crossing procedure for the MKP.

```
function CROSSING(x^w : worst individual, x^b : better individual, c)
    v ← shuffle(1, 2, ..., n)
    for i ← 1 : c  do
        j ← v_i
```

# The SCE for MKP

Crossing procedure for the MKP.

```
function CROSSING(x^w : worst individual, x^b : better individual, c)
    v ← shuffle(1, 2, ..., n)
    for i ← 1 : c do
        j ← v_i
        x_j^w ← x_j^b
    end for
```

# The SCE for MKP

Crossing procedure for the MKP.

**function** CROSSING($x^w$ : worst individual, $x^b$ : better individual, $c$)
    $v \leftarrow \text{shuffle}(1, 2, \ldots, n)$
    **for** $i \leftarrow 1 : c$ **do**
        $j \leftarrow v_i$
        $x^w_j \leftarrow x^b_j$
    **end for**
    **if** $s^w$ is not feasible **then**
        repair $s^w$
    **end if**

# The SCE for MKP

Crossing procedure for the MKP.

**function** CROSSING($x^w$ : worst individual, $x^b$ : better individual, $c$)
    $v \leftarrow$ shuffle$(1, 2, \ldots, n)$
    **for** $i \leftarrow 1 : c$ **do**
        $j \leftarrow v_i$
        $x_j^w \leftarrow x_j^b$
    **end for**
    **if** $s^w$ is not feasible **then**
        repair $s^w$
    **end if**
    return $s^w$
**end function**

## Computational Experiments

Parameters used for SCE:

- $N = 20$: number of complexes;
- $M = 20$: number of individuals in each complex;
- $P = 5$: number of individuals in each subcomplex;
- $K = 300$: number of algorithm iterations;
- $K' = 20$: number of iterations used in the complex evolving process;
- $c = n/5$: number of genes carried from parent in crossing process.

# Computational Experiments

SCE and SCEcr performance on Chu-Beasley problems.

| n | m | $\alpha$ | time (s) SCE | time (s) SCEcr | quality(%) SCE | quality(%) SCEcr |
|---|---|---|---|---|---|---|
| 100 | 5 | 0.25 | 1.22 | **0.17** | 96.51 | **99.73** |
| | | 0.50 | 1.34 | **0.18** | 97.42 | **99.86** |
| | | 0.75 | 1.37 | **0.17** | 98.87 | **99.91** |
| | 10 | 0.25 | 1.32 | **0.25** | 95.68 | **99.53** |
| | | 0.50 | 1.51 | **0.25** | 96.65 | **99.76** |
| | | 0.75 | 1.46 | **0.27** | 98.54 | **99.96** |
| | 30 | 0.25 | 1.74 | **1.20** | 95.38 | **97.96** |
| | | 0.50 | 1.79 | **0.89** | 96.41 | **99.18** |
| | | 0.75 | 1.72 | **0.95** | 98.18 | **99.52** |
| 250 | 5 | 0.25 | 2.87 | **0.69** | 93.22 | **99.86** |
| | | 0.50 | 2.82 | **0.70** | 94.88 | **99.94** |
| | | 0.75 | 2.93 | **0.69** | 97.57 | **99.96** |
| | 10 | 0.25 | 3.08 | **0.87** | 93.14 | **99.58** |
| | | 0.50 | 3.03 | **0.79** | 94.55 | **99.79** |
| | | 0.75 | 3.12 | **0.84** | 97.16 | **99.88** |
| | 30 | 0.25 | 3.74 | **1.52** | 93.10 | **98.42** |
| | | 0.50 | 3.74 | **1.36** | 94.20 | **99.33** |
| | | 0.75 | 3.99 | **1.48** | 96.64 | **99.59** |
| 500 | 5 | 0.25 | 5.62 | **1.25** | 91.37 | **99.77** |
| | | 0.50 | 5.72 | **1.24** | 93.39 | **99.88** |
| | | 0.75 | 5.88 | **1.20** | 96.42 | **99.92** |
| | 10 | 0.25 | 5.97 | **1.41** | 91.62 | **99.51** |
| | | 0.50 | 6.11 | **1.36** | 93.09 | **99.77** |
| | | 0.75 | 5.47 | **1.21** | 96.24 | **99.84** |
| | 30 | 0.25 | 6.20 | **1.96** | 91.37 | **98.76** |
| | | 0.50 | 6.26 | **1.82** | 92.56 | **99.42** |
| | | 0.75 | 6.05 | **1.73** | 95.97 | **99.67** |

# Computational Experiments

SCEcr performance on Glover-Kochenberger problems.

| # | n | m | time(s) SCE | time(s) SCEcr | quality(%) SCE | quality(%) SCEcr |
|---|---|---|---|---|---|---|
| 01 | 100 | 15 | 1.47 | **0.08** | 97.66 | **99.24** |
| 02 | 100 | 25 | 1.61 | **0.09** | 97.94 | **98.94** |
| 03 | 150 | 25 | 2.51 | **0.09** | 97.22 | **99.09** |
| 04 | 150 | 50 | 3.56 | **0.09** | 97.40 | **98.52** |
| 05 | 200 | 25 | 3.55 | **0.09** | 96.88 | **99.28** |
| 06 | 200 | 50 | 4.81 | **0.10** | 97.68 | **98.90** |
| 07 | 500 | 25 | 7.30 | **0.10** | 97.12 | **99.54** |
| 08 | 500 | 50 | 12.20 | **0.11** | 97.27 | **99.33** |
| 09 | 1500 | 25 | 24.61 | **0.12** | 95.40 | **98.22** |
| 10 | 1500 | 50 | 33.79 | **0.13** | 97.50 | **99.64** |
| 11 | 2500 | 100 | 121.28 | **0.15** | 97.95 | **99.70** |

## Conclusions

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At least 99.02% of best known, in less than 2 seconds for every instance.

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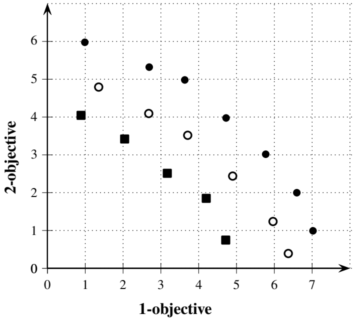
The fact that the MKP shares the same solution representation as the MOKP allow us to use the same procedures used on MKP.

# Non-dominated Sort

Fitness computation for multi-objective solutions.

**function** NDSORT($S$ : solution set)
    $i = 0$
    **while** $S \neq \emptyset$ **do**
        **for** $\boldsymbol{s} \in S$ **do**
            **if** $\nexists \boldsymbol{x} \in S : dom(\boldsymbol{x}, \boldsymbol{s})$ **then**
                $o_s = i$
                $S = S - x$
            **end if**
        **end for**
        $i = i + 1$
    **end while**
**end function**



# Final Doctoral Schedule

| Activities | Weeks | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | November | | | December | | | | January | | | |
| | 2° | 3° | 4° | 1° | 2° | 3° | 4° | 1° | 2° | 3° | 4° |
| Literature review | ● | ● | ● | | | | | | | | |
| Implementation and adjustments | ● | ● | ● | ● | | | | | | | |
| Computational experiments | | | ● | ● | ● | | | | | | |
| CEC Paper writing | | | | | | ● | ● | ● | | | |
| CEC Paper submission | | | | | | | | | ● | | |
| Thesis writing | | | | ● | ● | ● | ● | ● | ● | ● | |