A Hybrid Heuristic for the Multi-objective Knapsack Problem

Doctoral thesis proposal

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This reason motivates the development of heuristic methods.

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The SCE has been successfully used for solving optimization problems, among then, the multi-dimensional knapsack problem (MKP) (also a contribution of this work).

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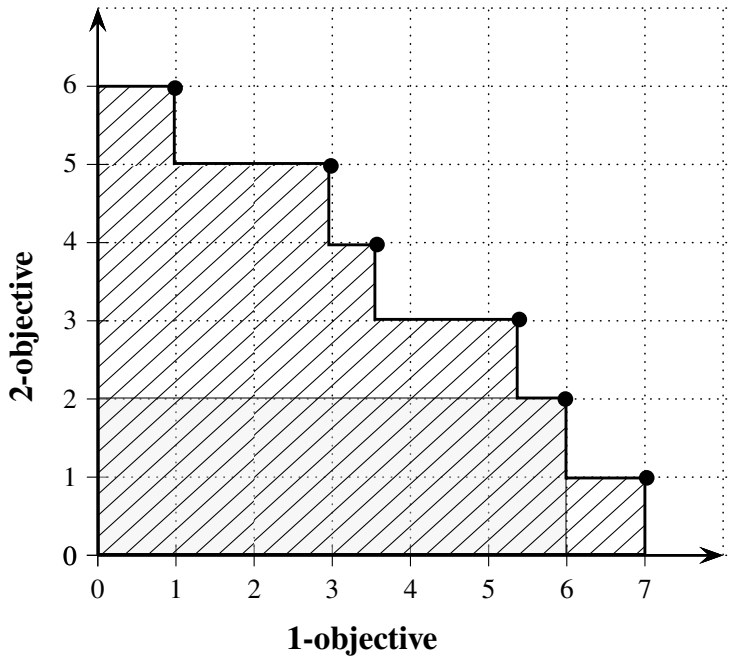

Figure: Example of a Pareto set for a bi-objective problem.

The multi-objective knapsack problem

Problem Definition

A problem instance with n items and m objectives:

$$\begin{aligned} \max\ & f(\boldsymbol{x}) = \Big(f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \ldots, f_m(\boldsymbol{x})\Big) \\ \text{subject to}\ & w(\boldsymbol{x}) \leqslant W \\ & \boldsymbol{x} \subseteq \{1, \ldots, n\} \end{aligned}$$

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A $\mathcal{NP}$-Hard problem for which the Pareto optimal set tends to grow exponentially with the number of objectives.

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Knapsack dominance

Consider two solutions $\boldsymbol{x}, \boldsymbol{y}$ for a MOKP.

We say $\boldsymbol{x}$ knapsack-dominates $\boldsymbol{y}$, denoted as $dom_k(\boldsymbol{x}, \boldsymbol{y})$, if $\boldsymbol{x}$ dominates $\boldsymbol{y}$ and does not weight more than $\boldsymbol{y}$.

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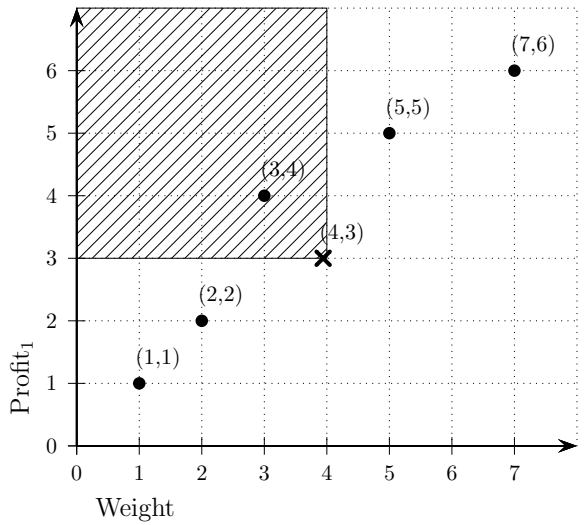

Figure: A knapsack-dominated solution.

Multi-dimensional Indexing of MOKP solutions

Dominance check operation

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However, if solutions are mapped into points in a multi-dimensional space, this operation corresponds on checking whether a point exists in a certain region.

Dominance check operation

Formally:

$$\text{if } \; dom_k(\boldsymbol{y}, \boldsymbol{x}) \; \text{ then } \; pnt(\boldsymbol{y}) \in R(\boldsymbol{x})$$

where

$$pnt(\boldsymbol{x}) = \Big(f_1(\boldsymbol{x}), \ldots, f_m(\boldsymbol{x}), w(\boldsymbol{x})\Big)$$

$$R(\boldsymbol{x}) = \Big\{ a \in \mathbb{R}^{m+1} \;\Big|\; a_{m+1} \leqslant w(\boldsymbol{x}) \text{ and } a_i \geqslant f_i(\boldsymbol{x}),\ i \in \{1, \ldots, m\} \Big\}$$

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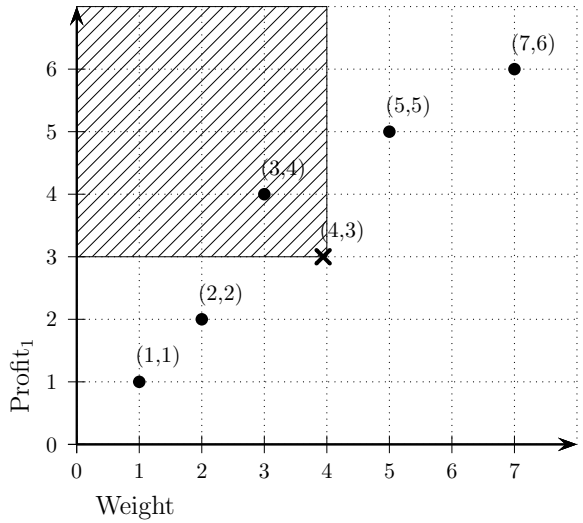

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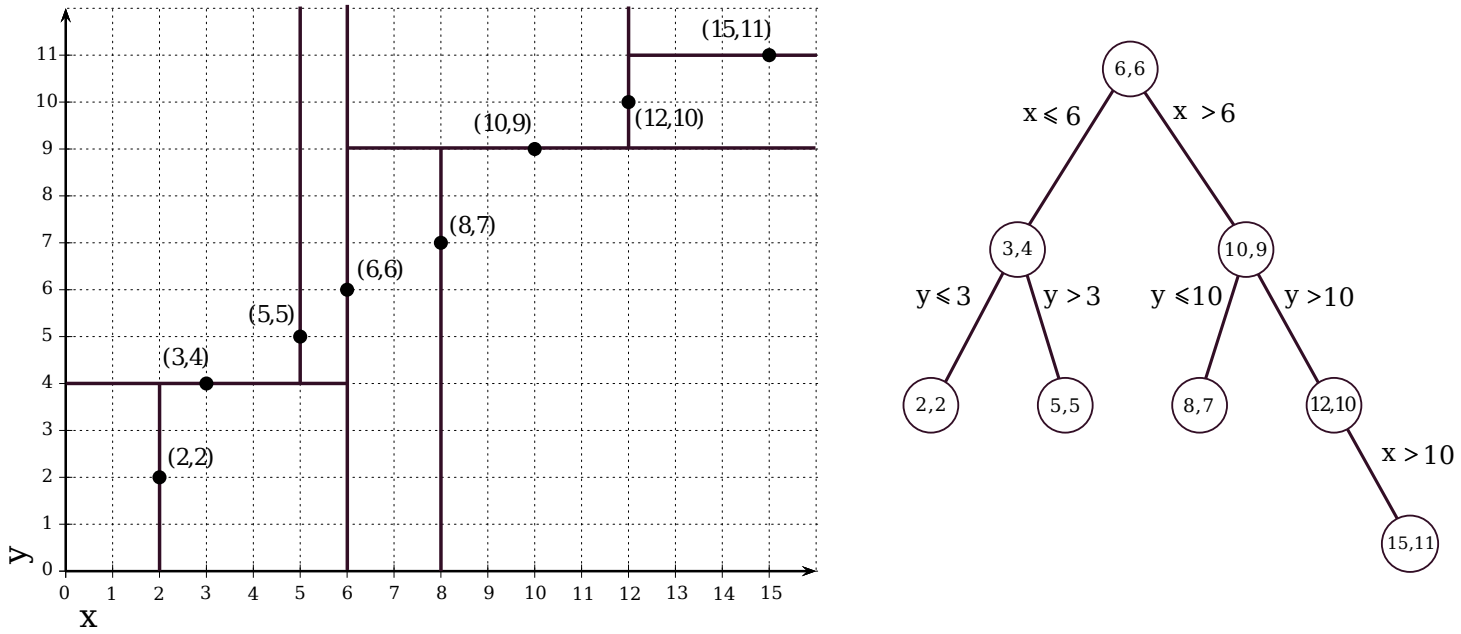
The k-d tree is a type of binary search tree for indexing multi-dimensional data with simple construction and low space usage.

Multidimensional solution indexing

Unlike a standard binary tree, that uses only one key for all levels of the tree, the k-d tree uses k keys and cycles through these keys for successive levels of the tree.



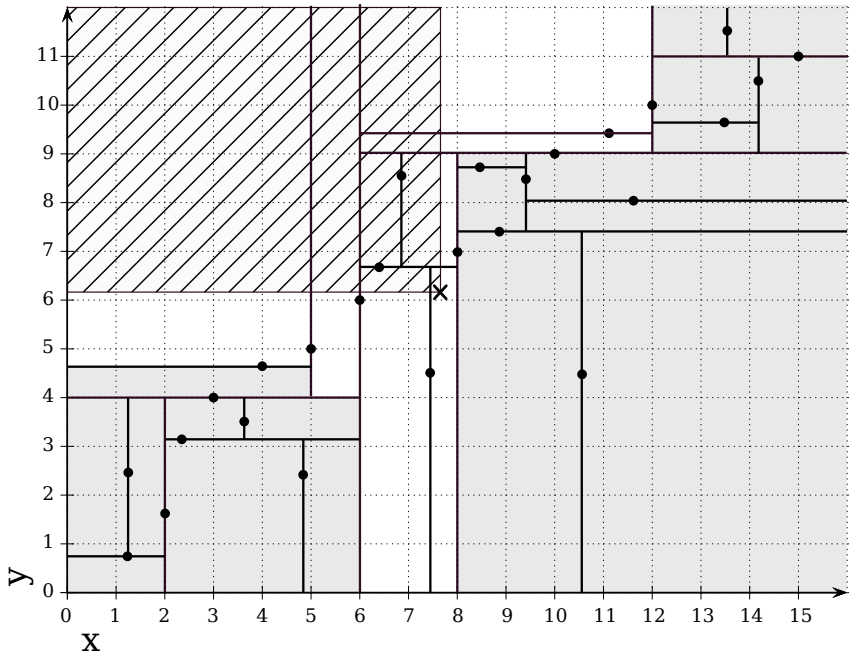
(a) points displayed on a plane

(b) points arranged on a kd-tree

Figure: Example of points indexed in a k-d tree.

Multidimensional solution indexing

Example of dominance check operation using k-d tree:



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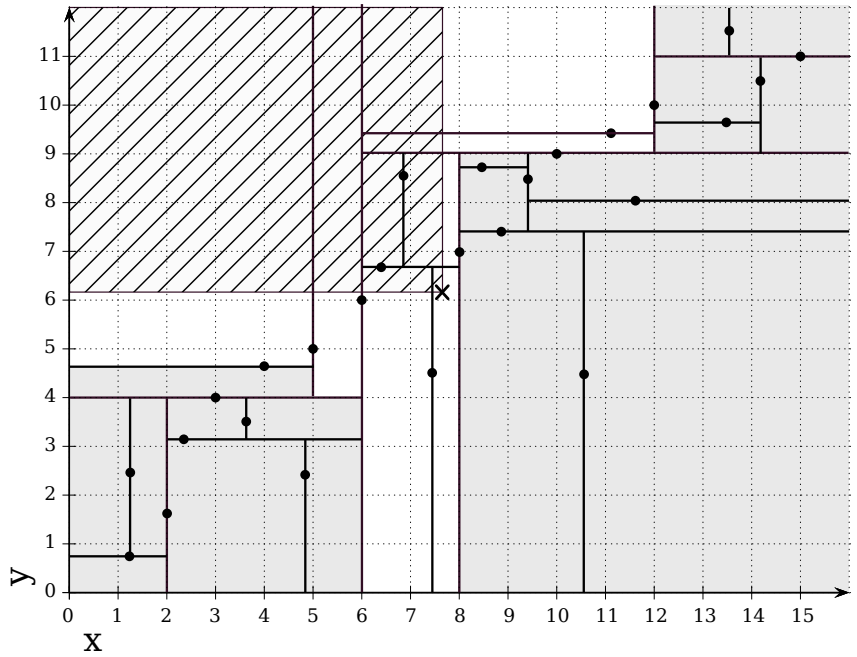

The efficiency of this pruning action grows with the amount of points.

Multi-dimensional Indexing of MOKP solutions

Use case on an exact method

Nemhauser and Ullmann's algorithm

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6: $\quad$ **end for**
7: $\quad P = \{\boldsymbol{x} \mid \nexists \boldsymbol{a} \in S^n : dom(\boldsymbol{a}, \boldsymbol{x}) \mid w(\boldsymbol{x}) \leqslant W\}$ ▷ dominance/feasibility

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5: $\quad\quad S^k = \{\boldsymbol{x} \mid \nexists \boldsymbol{a} \in S^k_* : dom_k(\boldsymbol{a}, \boldsymbol{x})\}$ ▷ partial dominance filter
6: $\quad$ **end for**
7: $\quad P = \{\boldsymbol{x} \mid \nexists \boldsymbol{a} \in S^n : dom(\boldsymbol{a}, \boldsymbol{x}) \mid w(\boldsymbol{x}) \leqslant W\}$ ▷ dominance/feasibility
8: $\quad$ **return** P
9: **end function**

Bazgan's dynamic programming algorithm

1: **function** BAZDP($\boldsymbol{p}, \boldsymbol{w}, W$)
2: $S^0 = \{\emptyset\}$

Bazgan's dynamic programming algorithm

Item reordering

1: **function** BazDP($\boldsymbol{p}, \boldsymbol{w}, W$)
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Bazgan's dynamic programming algorithm

Feasible extension

1: **function** BazDP($\boldsymbol{p}, \boldsymbol{w}, W$)
2: $S^0 = \{\emptyset\}$
3: $o_1, \ldots, o_n = \mathcal{O}^{max}$
4: **for** $k \leftarrow 1, n$ **do**
5: $S^k_* = \left\{ \boldsymbol{x} \cup \{o_k\} \;\middle|\; \boldsymbol{x} \in S^{k-1} \wedge w(\boldsymbol{x}) + w_{o_k} \leqslant W \right\}$

Bazgan's dynamic programming algorithm

Defficiency avoidance

1: **function** BazDP($\boldsymbol{p}, \boldsymbol{w}, W$)
2: $S^0 = \left\{\emptyset\right\}$
3: $o_1, \ldots, o_n = \mathcal{O}^{max}$
4: **for** $k \leftarrow 1, n$ **do**
5: $S^k_* = \left\{\boldsymbol{x} \cup \{o_k\} \;\middle|\; \boldsymbol{x} \in S^{k-1} \wedge w(\boldsymbol{x}) + w_{o_k} \leqslant W\right\}$
6: $\cup \left\{\boldsymbol{x} \;\middle|\; \boldsymbol{x} \in S^{k-1} \wedge w(\boldsymbol{x}) + w_{o_k} + \ldots + w_{o_n} > W\right\}$

Bazgan's dynamic programming algorithm

Unpromising elimination

1: **function** BazDP($\boldsymbol{p}, \boldsymbol{w}, W$)
2: $S^0 = \{\emptyset\}$
3: $o_1, \ldots, o_n = \mathcal{O}^{max}$
4: **for** $k \leftarrow 1, n$ **do**
5: $S^k_* = \left\{ \boldsymbol{x} \cup \{o_k\} \;\middle|\; \boldsymbol{x} \in S^{k-1} \wedge w(\boldsymbol{x}) + w_{o_k} \leqslant W \right\}$
6: $\cup \left\{ \boldsymbol{x} \;\middle|\; \boldsymbol{x} \in S^{k-1} \wedge w(\boldsymbol{x}) + w_{o_k} + \ldots + w_{o_n} > W \right\}$
7: $S^k = \left\{ \boldsymbol{x} \in S^k_* \;\middle|\; (\nexists \boldsymbol{a} \in S^k_*) \left[dom_k(\boldsymbol{a}, \boldsymbol{x}) \vee dom\Big(lb(\boldsymbol{a}), ub(\boldsymbol{x})\Big) \right] \right\}$

Bazgan's dynamic programming algorithm

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8: **end for**
9: **return** S^n
10: **end function**

Bazgan's dynamic programming algorithm

Dominance check operation

1: **function** BazDP($\boldsymbol{p}, \boldsymbol{w}, W$)
2: $S^0 = \{\emptyset\}$
3: $o_1, \ldots, o_n = \mathcal{O}^{max}$
4: **for** $k \leftarrow 1, n$ **do**
5: $S^k_* = \left\{ \boldsymbol{x} \cup \{o_k\} \;\middle|\; \boldsymbol{x} \in S^{k-1} \wedge w(\boldsymbol{x}) + w_{o_k} \leqslant W \right\}$
6: $\cup \left\{ \boldsymbol{x} \;\middle|\; \boldsymbol{x} \in S^{k-1} \wedge w(\boldsymbol{x}) + w_{o_k} + \ldots + w_{o_n} > W \right\}$
7: $S^k = \left\{ \boldsymbol{x} \in S^k_* \;\middle|\; (\nexists \boldsymbol{a} \in S^k_*) \left[\underline{dom_k}(\boldsymbol{a}, \boldsymbol{x}) \vee \underline{dom}\Big(lb(\boldsymbol{a}), ub(\boldsymbol{x})\Big) \right] \right\}$
8: **end for**
9: **return** S^n
10: **end function**

Computational Experiments

Computational experiments on bi-dimensional instances:

A) Random instances:
$p_i^j \in [1, 1000]$,
$w_i \in [1, 1000]$.

B) Unconflicting instances:
$p_i^1 \in [111, 1000]$,
$p_i^2 \in [p_i^1 - 100, p_i^1 + 100]$,
$w_i \in [1, 1000]$.

C) Conflicting instances:
$p_i^1 \in [1, 1000]$,
$p_i^2 \in [max\{900 - p_i^1; 1\}, min\{1100 - p_i^1, 1000\}]$,
$w_i \in [1, 1000]$.

D) Conflicting instances with correlated weight:
$p_i^1 \in [1, 1000]$,
$p_i^2 \in [max\{900 - p_i^1; 1\}, min\{1100 - p_i^1, 1000\}]$,
$w_i \in [p_i^1 + p_i^2 - 200, p_i^1 + p_i^2 + 200]$.

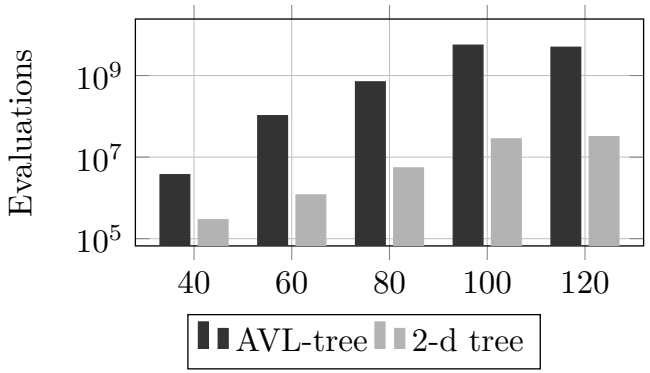
Computational Experiments

Average CPU-time for bi-objective instances:

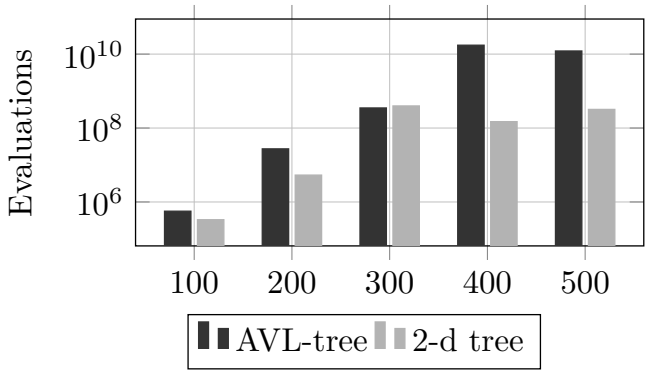
Instance			AVL tree	2-d tree	
Type	n	$\|ND\|$	time (s)	time (s)	speedup
A	40	38.1	**0.06**	**0.06**	1.0
	60	73.1	1.12	**0.88**	1.3
	80	125.6	19.81	**11.89**	1.7
	100	180.4	165.24	**76.50**	2.2
	120	233.9	708.53	**361.87**	2.0
B	100	3.1	**0.02**	0.08	0.3
	200	10.0	**0.80**	5.09	0.2
	300	24.9	**9.45**	88.30	0.1
	400	36.2	**95.39**	730.04	0.1
	500	53.7	**255.57**	2824.65	0.1
C	20	36.6	**0.00**	**0.00**	1.0
	40	102.8	0.65	**0.42**	1.5
	60	231.9	28.98	**14.09**	2.1
	80	358.0	564.10	**241.54**	2.3
	100	513.8	3756.57	**1605.19**	2.3
D	20	174.9	0.15	**0.12**	1.3
	30	269.3	16.82	**7.60**	2.2
	40	478.0	395.76	**186.67**	2.1
	50	553.4	2459.48	**1417.94**	1.7

Computational Experiments

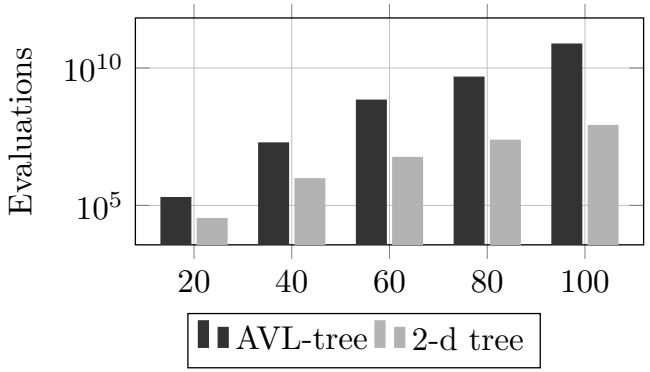
Average number of solution evaluations for bi-objective instances:



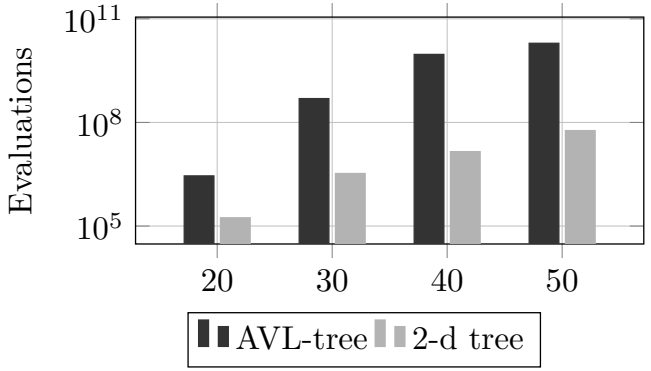
(a) Type A



(b) Type B



(c) Type C



(d) Type D

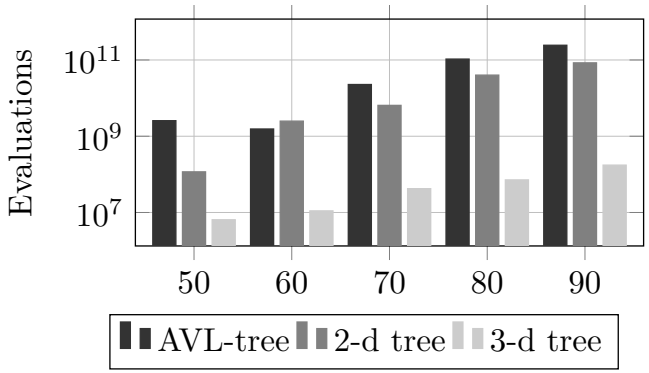
Computational Experiments

Average CPU-time for 3-objective instances:

Instance			AVL tree	2-d tree		3-d tree	
Type	n	$\|ND\|$	time (s)	time (s)	speedup	time (s)	speedup
A	50	557.5	41.2	21.3	1.9	**18.5**	2.2
	60	1240.0	485.9	247.8	1.9	**79.9**	6.0
	70	1879.3	3179.5	1038.0	3.0	**614.5**	5.1
	80	2540.5	6667.9	3796.0	1.7	**2943.9**	2.2
	90	3528.5	24476.5	12916.7	1.8	**3683.7**	6.6
B	100	18.0	**0.1**	0.3	0.3	0.3	0.3
	200	65.4	**11.4**	34.4	0.3	29.1	0.4
	300	214.2	**307.7**	631.5	0.5	583.2	0.5
	400	317.0	**4492.9**	8464.9	0.5	5402.2	0.8
C	20	254.4	0.06	0.05	1.2	**0.03**	2.17
	30	1066.6	9.69	4.18	2.3	**1.30**	7.46
	40	2965.5	471.68	153.21	3.1	**30.50**	15.5
D	20	4087.7	23.6	10.9	2.2	**1.9**	12.5
	30	8834.5	8914.2	3625.3	2.5	**1019.5**	8.7

Computational Experiments

Average number of solution evaluations for 3-objective instances:



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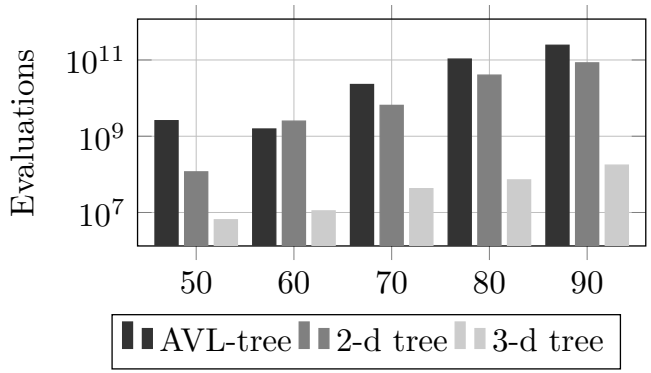
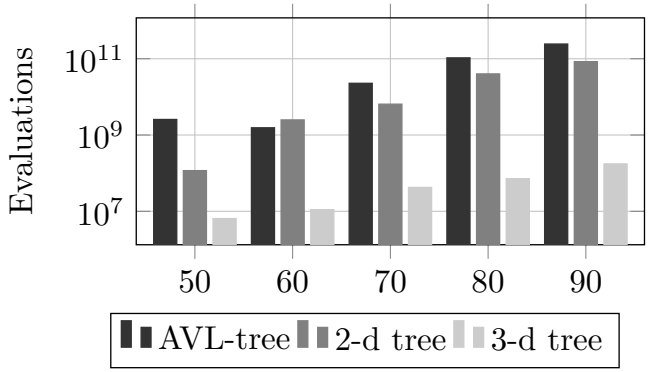
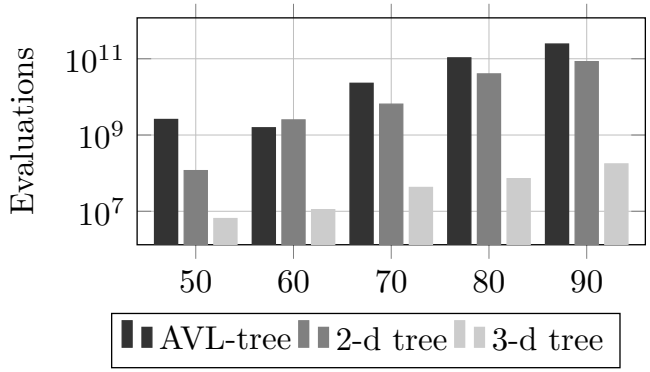


(b) Type B



(c) Type C



(d) Type D

Conclusions

The multi-dimensional indexing is applicable to the problem requiring considerably less solution evaluations, especially on hard instances.

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Several instances are still intractable due the large number of intermediate solutions.

For this reason this work proposes to use the indexing strategy in conjunction with an evolutionary metaheuristic.

The Shuffled Complex Evolution

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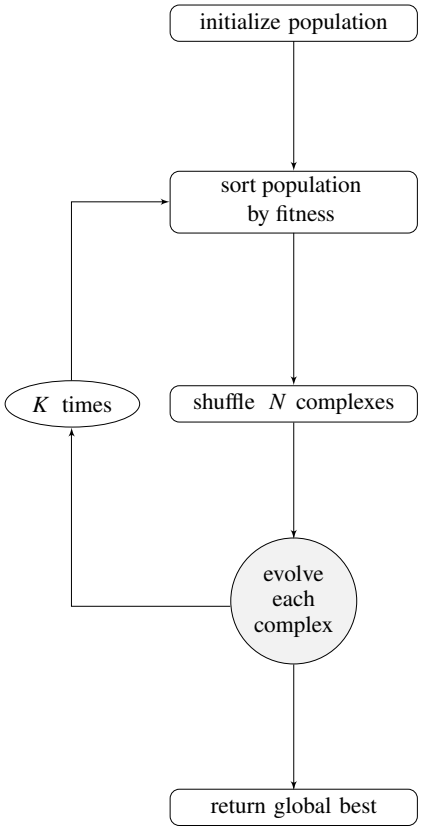

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If the latter crossing did not result in any improvement, the best individual of whole population is considered;

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If the latter crossing did not result in any improvement, the best individual of whole population is considered;

Finally, if all the crossing steps couldn't generate a better individual, the worst individual of the subcomplex is replaced by a new random solution taken from the feasible solution space.

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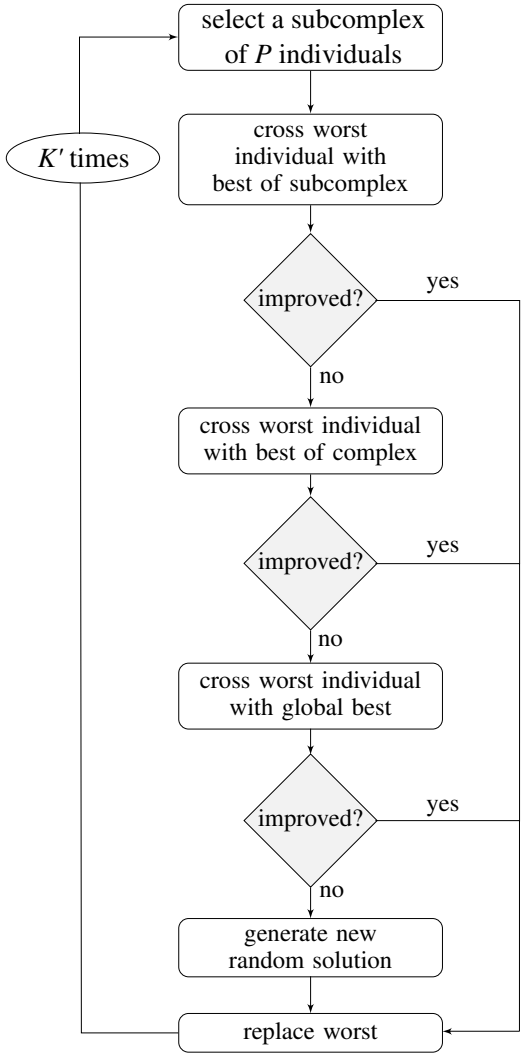

The Shuffled Complex Evolution

Use case for the MKP

The SCE for MKP

the SCE is easily applied to any optimization problem. The only steps needed to be specified is **(a)** the creation of a new random solution and **(b)** the crossing procedure of two solutions.

The SCE for MKP

New random solution generation for the MKP.

1: **function** NEW RANDOM SOLUTION

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2: $v \leftarrow \text{shuffle}(1, 2, \ldots, n)$
3: $s \leftarrow \emptyset$
4: **for** $i \leftarrow 1 : n$ **do**
5: $s \leftarrow s \cup \{v_i\}$

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6: **if** s is not feasible **then**
7: $s \leftarrow s - \{v_i\}$
8: **end if**

The SCE for MKP

New random solution generation for the MKP.

```
1:  function NEW RANDOM SOLUTION
2:      v ← shuffle(1, 2, ..., n)
3:      s ← ∅
4:      for i ← 1 : n do
5:          s ← s ∪ {v_i}
6:          if s is not feasible then
7:              s ← s − {v_i}
8:          end if
9:      end for
10:     return s
11: end function
```

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8: repair s^w
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6: **end for**
7: **if** s^w is not feasible **then**
8: repair s^w
9: **end if**
10: return s^w
11: **end function**

Computational Experiments

Parameters used for SCE:

- $N = 20$: number of complexes;
- $M = 20$: number of individuals in each complex;
- $P = 5$: number of individuals in each subcomplex;
- $K = 300$: number of algorithm iterations;
- $K' = 20$: number of iterations used in the complex evolving process;
- $c = n/5$: number of genes carried from parent in crossing process.

Computational Experiments

SCE and SCEcr performance on Chu-Beasley problems.

n	m	α	time (s)		quality(%)	
			SCE	SCEcr	SCE	SCEcr
100	5	0.25	1.22	**0.17**	96.51	**99.73**
		0.50	1.34	**0.18**	97.42	**99.86**
		0.75	1.37	**0.17**	98.87	**99.91**
	10	0.25	1.32	**0.25**	95.68	**99.53**
		0.50	1.51	**0.25**	96.65	**99.76**
		0.75	1.46	**0.27**	98.54	**99.96**
	30	0.25	1.74	**1.20**	95.38	**97.96**
		0.50	1.79	**0.89**	96.41	**99.18**
		0.75	1.72	**0.95**	98.18	**99.52**
250	5	0.25	2.87	**0.69**	93.22	**99.86**
		0.50	2.82	**0.70**	94.88	**99.94**
		0.75	2.93	**0.69**	97.57	**99.96**
	10	0.25	3.08	**0.87**	93.14	**99.58**
		0.50	3.03	**0.79**	94.55	**99.79**
		0.75	3.12	**0.84**	97.16	**99.88**
	30	0.25	3.74	**1.52**	93.10	**98.42**
		0.50	3.74	**1.36**	94.20	**99.33**
		0.75	3.99	**1.48**	96.64	**99.59**
500	5	0.25	5.62	**1.25**	91.37	**99.77**
		0.50	5.72	**1.24**	93.39	**99.88**
		0.75	5.88	**1.20**	96.42	**99.92**
	10	0.25	5.97	**1.41**	91.62	**99.51**
		0.50	6.11	**1.36**	93.09	**99.77**
		0.75	5.47	**1.21**	96.24	**99.84**
	30	0.25	6.20	**1.96**	91.37	**98.76**
		0.50	6.26	**1.82**	92.56	**99.42**
		0.75	6.05	**1.73**	95.97	**99.67**

Computational Experiments

SCEcr performance on Glover-Kochenberger problems.

#	n	m	time(s)		quality(%)	
			SCE	SCEcr	SCE	SCEcr
01	100	15	1.47	**0.08**	97.66	**99.24**
02	100	25	1.61	**0.09**	97.94	**98.94**
03	150	25	2.51	**0.09**	97.22	**99.09**
04	150	50	3.56	**0.09**	97.40	**98.52**
05	200	25	3.55	**0.09**	96.88	**99.28**
06	200	50	4.81	**0.10**	97.68	**98.90**
07	500	25	7.30	**0.10**	97.12	**99.54**
08	500	50	12.20	**0.11**	97.27	**99.33**
09	1500	25	24.61	**0.12**	95.40	**98.22**
10	1500	50	33.79	**0.13**	97.50	**99.64**
11	2500	100	121.28	**0.15**	97.95	**99.70**

Conclusions

The SCE algorithm for MKP proved to be able to achieve fast convergence ratio, finding good quality near optimal solutions, demanding small amount of computational time.

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At least 99.02% of best known, in less than 2 seconds for every instance.

A SCE for the MOKP

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As seen in previous sections the SCE is easily applied to any optimization problem.

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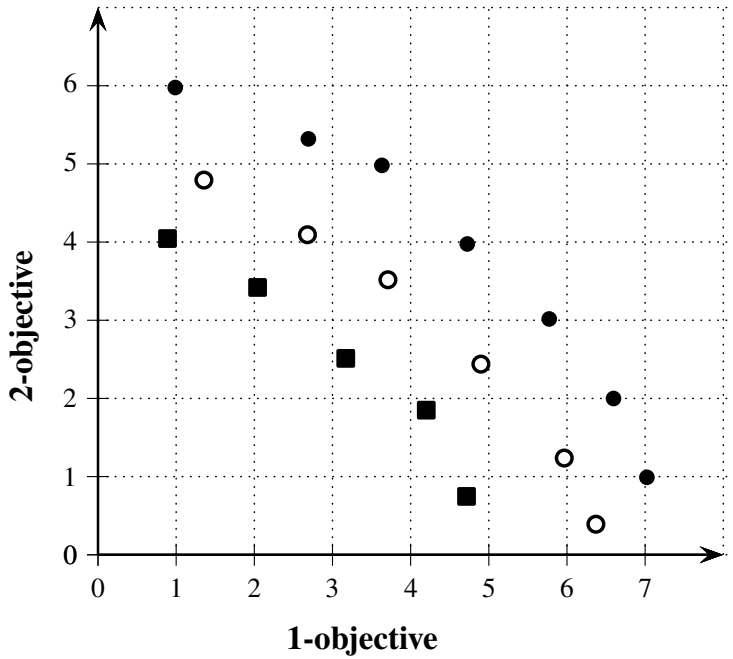
The fact that the MKP shares the same solution representation as the MOKP allow us to use the same procedures used on MKP.

Non-dominated Sort

Fitness computation for multi-objective solutions.

```
1:  function NDSort(S : solution set)
2:      i = 0
3:      while S ≠ ∅ do
4:          for s ∈ S do
5:              if ∄x ∈ S : dom(x, s) then
6:                  o_s = i
7:                  S = S − x
8:              end if
9:          end for
10:         i = i + 1
11:     end while
12: end function
```



Non-dominated Sort

Fitness computation for multi-objective solutions.

Preliminary Computational Experiments

Time and quality for bi-dimensional instances:

Type	n	time (s)	time (s)	Hypervolume
A	120	479.5	46.5	97.0 %
B	500	503.6	98.3	98.6 %
C	80	559.1	16.2	94.4 %
D	40	462.0	92.4	97.2 %

Final Doctoral Schedule

Activities	Weeks											
	November			December				January				
	2°	3°	4°	1°	2°	3°	4°	1°	2°	3°	4°	
Literature review	●	●	●									
Implementation and adjustments	●	●	●	●								
Computational experiments			●	●	●							
CEC Paper writing						●	●	●				
CEC Paper submission									●			
Thesis writing				●	●	●	●	●	●	●		