A fast dynamic programming for the multidimensional knapsack problem

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Abstract

This work addresses... The Multidimensional knapsack programming. The dynamic programming method... The data structure...

1 Introduction

- Falar do MKP.
- Falar do algoritmo de programao dinamica.
- (Dizer que para o algoritmo de programao dinmica tem comportamento polinomial se fixada as dimenes)
- Falar da estrutura de dados e da proposta de acelerar o algoritmo.

The multidimensional knapsack problem (MKP) is a strongly NP-hard combinatorial optimization problem which can be viewed as a resource allocation problem and defined as follows:

$$\text{maximize} \sum_{j=1}^{n} p_j x_j \tag{1}$$

subject to
$$\sum_{j=1}^{n} w_{ij} x_j \leqslant c_i \quad i \in \{1, \dots, m\}$$
 (2)

$$x_j \in \{0, 1\}, \quad j \in \{1, \dots, n\}.$$
 (3)

The problem can be interpreted as a set of n items with profits p_j and a set of m resources with capacities c_i . Each item j consumes an amount w_{ij} from each resource i, if selected. The objective is to select a subset of items with maximum total profit, not exceeding the defined resource capacities. The decision variable x_j indicates if j-th item is selected. It is considered an integer programming problem (IP) since its variables x_i are restricted to be integers.

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The multidimensional knapsack problem can be applied on budget planning scenarios and project selections [?], cutting stock problems [?], loading problems [?], allocation of processors and databases in distributed computer programs [?].

The problem is a generalization of the well-known knapsack problem (KP) in which m=1. However it is a NP-hard problem significantly harder to solve in practice than the KP. Due its simple definition but challenging difficulty of solving, the MKP is often used to to verify the efficiency of novel metaheuristics.

2 The dynamic programming algorithm

- Breve resumo sobre mtodos de programao dynamica;
- Explicao overview do mtodo para o MKP;
- Definies, Propriedados e teoremas para o MKP: dominancia
- Representao de lista

3 The use of data structure

- Introduo KDTree: estrutura, insero, busca
- Aplicao da KDTree ao algoritmo: validade das propriedades de dominancia

4 Computational experiments

- Base de dados utilizaca
- Parametros dos algoritmos
- Anlise dos resultados (comparao)

5 Conclusions and future remarks

- Concluses dos resultados
- Trabalhos futuros