

## 1 Partition

$$\begin{aligned} & \text{maximize } \sum_{j=1}^n w_j x_j \\ & \text{subject to } \sum_{j=1}^n w_j x_j \leq \frac{1}{2} \sum_{j=1}^n x_j \\ & \quad x_j \in \{0, 1\}, \quad j \in \{1, \dots, n\}. \end{aligned}$$

## 2 Subset-sum

$$\begin{aligned} & \text{maximize } \sum_{j=1}^n w_j x_j \\ & \text{subject to } \sum_{j=1}^n w_j x_j \leq b \\ & \quad x_j \in \{0, 1\}, \quad j \in \{1, \dots, n\}. \end{aligned}$$

$P_{nb}$ : Probability of  $n$  numbers has a subset which sums exactly  $b$ , considering  $M$  as maximum integer (over uniform distribution).

$$\begin{aligned} T_{1b} &= \begin{cases} 1/M & , \text{ if } b \leq M \\ 0 & , \text{ otherwise} \end{cases} \\ T_{2b} &= \begin{cases} 2M - 2 + b & , \text{ if } 0 < b \leq M \\ 2M + 1 - b & , \text{ if } M < b \leq 2M \\ 0 & , \text{ otherwise} \end{cases} \\ T_{nb} &= \begin{cases} 1 & , \text{ if } b = 0 \\ \frac{1}{M} \sum_{i=1}^M P_{(n-1)(b-i)} & , \text{ if } 0 < b \leq nM \\ 0 & , \text{ otherwise} \end{cases} \end{aligned}$$

## 3 Knapsack

$$\begin{aligned} & \text{maximize } \sum_{j=1}^n p_j x_j \\ & \text{subject to } \sum_{j=1}^n w_j x_j \leq b \\ & \quad x_j \in \{0, 1\}, \quad j \in \{1, \dots, n\}. \end{aligned}$$

## 4 Multidimensional Knapsack

$$\begin{aligned} & \text{maximize } \sum_{j=1}^n p_j x_j \\ & \text{subject to } \sum_{j=1}^n w_{ij} x_j \leq b_i \quad i \in \{1, \dots, m\} \\ & \quad x_j \in \{0, 1\}, \quad j \in \{1, \dots, n\}. \end{aligned}$$