

Chapter 1

Permutations and Combinations

$$\begin{aligned}n\binom{n}{r} &= (r+1)\binom{n}{r+1} + r\binom{n}{r} \\&= (r+1)\frac{n!}{(r+1)!(n-(r+1))!} + r\frac{n!}{(n-r)!r!} \\&= \frac{(r+1)}{(r+1)!}\frac{n!}{(n-r-1)!} + \frac{r}{r!}\frac{n!}{(n-r)!} \\&= \frac{1}{r!}\frac{n!}{(n-r-1)!} + \frac{1}{(r-1)!}\frac{n!}{(n-r)!} \\&= \frac{n!}{r!(n-r-1)!}\cdot\frac{(n-r)}{(n-r)} + \frac{n!}{(r-1)!(n-r)!}\cdot\frac{r}{r} \\&= \frac{n!(n-r)}{r!(n-r)!} + \frac{n!r}{r!(n-r)!} \\&= \frac{n!(n-r) + n!r}{r!(n-r)!} \\&= \frac{n!(n-r+r)}{r!(n-r)!} \\&= \frac{n!n}{r!(n-r)!}\end{aligned}$$

$$\boxed{n\binom{n}{r} = n\binom{n}{r}} \quad \blacksquare$$

$$\binom{n}{2} \binom{n}{r} = \underbrace{\binom{r+2}{2} \binom{n}{n+2}}_{*} + 2 \underbrace{\binom{r+1}{2} \binom{n}{r+1}}_{*} + \underbrace{\binom{r}{2} \binom{n}{r}}_{*} \quad \boxed{* \binom{n}{m} \binom{m}{k} = \binom{n}{k} \binom{n-k}{m-k}}$$

$$= \binom{n}{2} \binom{n-2}{(r+2)-2} + 2 \binom{n}{2} \binom{n-2}{(r+1)-2} + \binom{n}{2} \binom{n-2}{r-2}$$

$$= \binom{n}{2} \cdot \left[\underbrace{\binom{n-2}{r} + \binom{n-2}{r-1}}_{*} + \underbrace{\binom{n-2}{r-1} + \binom{n-2}{r-2}}_{*} \right] \quad \boxed{* \binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}}$$

$$= \binom{n}{2} \cdot \left[\underbrace{\binom{n-1}{r} + \binom{n-1}{r-1}}_{*} \right]$$

$$\boxed{= \binom{n}{2} \cdot \binom{n}{r}} \quad \blacksquare$$

Chapter 2

Generations Functions

Chapter 3

The Principle of Inclusion and Exclusion

Chapter 4

The Cycles of Permutations

Chapter 5

Distributions: Occupancy

Chapter 6

Partitions, Compositions, Trees , and Networks

Chapter 7

Permutations with Restricted Position I

Chapter 8

Permutations with Restricted Position II