A shuffled frog leaping algorithm for the multidimensional knapsack problem

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***Abstract*—The abstract goes here.**

I. Introduction

The Multidimensional Knapsack Problem (MKP) is a strongly NP-hard combinatorial optimization problem which can be viewed as a resource allocation problem and defined as follows:

$$
\begin{aligned}
&\text{maximize} \sum_{j=1}^{n} p_j x_j \\
&\text{subject to} \sum_{j=1}^{n} w_{ij} x_j \leqslant c_i \quad i \in \{1, \ldots, m\} \\
&\qquad\qquad x_j \in \{0, 1\}, \quad j \in \{1, \ldots, n\}.
\end{aligned}
$$

The problem can be interpreted as a set of n itens with profits p_j and a set of m resources with capacities c_i. Each item j consumes an amount w_{ij} from each resource i, if selected. The objective is to select a subset of items with maximum total profit, not exceeding the defined resource capacities. The decision variable x_j indicates if j-th item is selected.

The multidimensional knapsack problem can be applied on budget planning scenarios, subset project selections, cutting stock problems, task scheduling, allocation of processors and databases in distributed computer programs. The problem is a generalization of the well-known knapsack problem (KP) in which $m = 1$.

The MKP is a NP-Hard problem significantly harder to solve in practice than the KP. Despite the existence of a fully polynomial approximation scheme (FPAS) for the KP, finding a FPAS for the MKP is NP-hard for $m \geqslant 2$ [1]. Due its simple definition but challenging difficulty the MKP is often used to to verify the efficiency of novel metaheuristics.

In this paper we address the application of a metaheuristic called shuffled frog leaping algorithm (SFLA) to the multidimensional knapsack problem. The SFLA is a metaheuristic proposed by Eusuff and Lansey [2], [3] which combines concepts from two other widely used metaheuristics: The shuffled complex evolution algorithm (SCE) and the Particle Swarm Optimization (PSO), providing a robust heuristic which has been successfully applied to several optimization problems [4], [5], [6], [7], [8].

The reminder of the paper is organized as follows: Section II presents the shuffled frog leaping algorithm. Section III proposes the application of SFLA for the multidimensional knapsack problem. Section IV comprises several computational experiments. In section V we make our concluding remarks about the experimental results.

II. The shuffled frog leaping

The SFLA is a metaheuristic to solve discrete and combinatorial problems based on the memetics of living beings and recalls the behavior of a group of frogs searching for the location that has the maximum amount of available food. In the following subsections we present the concepts of SCE and PSO to finally present the shuffled frog leaping algorithm.

A. *The shuffled complex evolution*

The shuffled complex evolution ([9]) is a population based evolutionary optimization algorithm that regards a natural evolution happenning simultaneously in independent communities. In the SCE the population is partitioned into communities (complexes), each of which will evolve independently through a number of evolving steps. In each evolving step a subset of the complex (subcomplex) is selected as potencial group of parents. The new offspring replaces the worst individual of the subcomplex. To avoid been trapped in local optimum a new offspring can be occasionally taken from a random location of the feasible space and introduced to the complex. After a defined number of evolutions, the complexes are forced to mix into a single population. This new evolved population are once again partitioned into complexes through a shuffling process and the evolving steps are repeated until a stop condition is satisfied.

III. A SFLA for the MKP

A. *Construction of a new random solution*

To ensure diversification of the population every initial individual (solution) is created through a random constructive procedure that sorts the items at random and attempts to selected each item one by one. If the item fits the knapsack, i.e., its selection does not exceed any resource, the item is included in the solution.

After the initializing the population their individuals are sorted by descending order according to their fitness (profit)

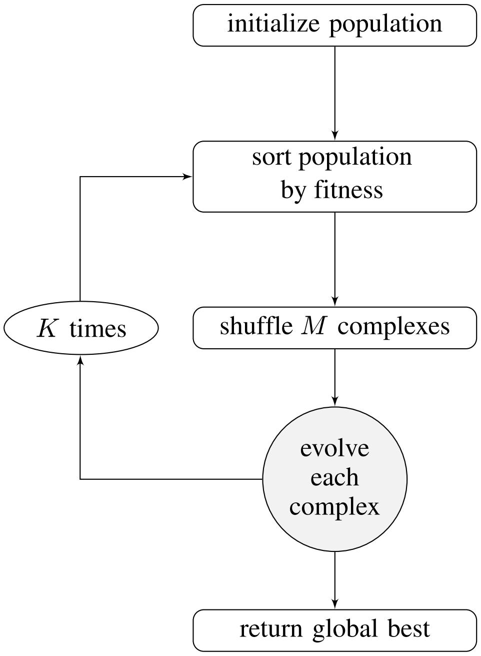

Fig. 1: The SCE algorithm.

and the best global solution is identified as s_{gb}. The entire population is then partitioned into N complexes, each containing M individuals. In this partition process the first individual goes to the first complex, the second individual goes to the second complex, individual N goes to N-th complex, individual $M+1$ goes back to the first complex, etc.

With the complexes constructed the next step is to evolve each complex through a given fixed amount of steps. In each step a subcomplex is formed by selecting P individuals from the complex using a triangular probability distribution, where the i-th individual has a probability $p_i = \frac{2(n+1-i)}{n(n+1)}$ of being selected. After the selection of the subcomplex, its best and worst individual, respectively identified by s_w and s_b are crossed, generating a new solution.

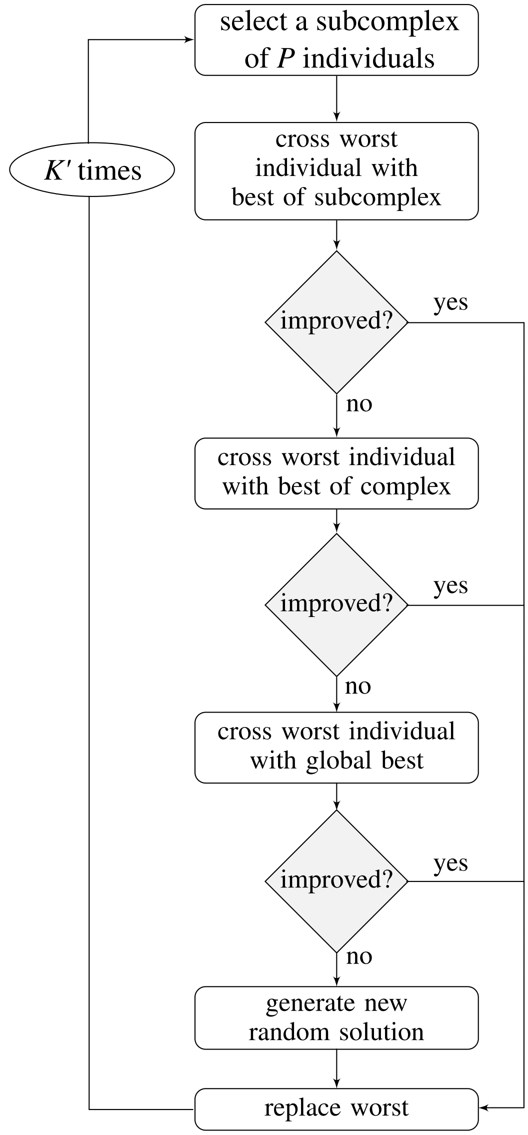

Fig. 2: The evolving procedure for a complex.

IV. Computational experiments

V. Conclusions and future remarks

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