Abstract

This article contains a backbone for an article over the computational investigation of the hardness the Multidimensional Knapsack Problem (MKP) as well as the on performance of algorithms for the solution of instances.

1 Introduction

The 0-1 Multidimensional Knapsack Problem (MKP) is a generalization of the Knapsack Problem where an item expends more than a single resource type. A MKP having n itens and m dimensions can be defined as follows.

$$\begin{aligned} \text{maximize} & \sum_{j=1}^{n} p_j x_j \\ \text{subject to} & \sum_{j=1}^{n} c_{ij} x_j \leq b_i \quad i \in \{1, \ldots, m\} \\ & x_j \in \{0, 1\}, \quad j \in \{1, \ldots, n\}. \end{aligned}$$

The problem can be applied on budget planning scenarios, subset project selections, cutting stock problems, task scheduling and allocation of processors and databases in distributed computer programs. The problem is a generalization of the well-known Knapsack Problem (KP) in which $m = 1$.

Several contributions have been made addressing exact, heuristic, approximation and probabilistic approaches for the MKP. The purpose of the work is to evaluate the performance the main approaches presented on literature over different instances of the problem and iddentify those instances which are hard to solve by all the approaches.

Section 2 briefly addresses each one of the main approaches for solving MKP, Section 3 discusses about the instances of MKP used on literature and Section 4 presents some research questions we hope to answer with the article.

2 The main approaches for solving MKP

Due its simple definition and challenging difficulty, the MKP turned a quite addressed problem for experiments with metaheuristics in recent years, although few of them have exhibit competitive performance compared to comercial MIP solvers (at least for the instances dealed by literature). Among the heuristics reporting competitive performance compared to MIP solvers the newest ones are the one proposed by Fleszar and Hindi [3] and one proposed by Hanafi and Wilbaut [5].

According to literature MKP does not allow a FPTAS (unless $P = NP$) but a PTAS is allowed and one was proposed by Frieze and Clarke [4]. Dyer and

Frieze [2] proposed a probabilistic which, given a $\epsilon > 0$ answers computes the optimal solution for the problem with probability at least $1 - \epsilon$ with polynomial time.

At the present moment the most powerful exact method for solving the MKP seems to be the multi-level search strategy proposed by Boussier [1] which, until now, is the only approach which have found the optimal solution for some popular instances with 500 itens and 10 contraints.

3 Instances

The most popular instances for the MKP are those from the OR-LIBRARY and they are used as a reference for state-of-art methods. The generation of thoses instances are generaly guided by two parameters: the tightness δ of the knapsacks and the profit-weight correlation of the items.

The tightness of a knapsack rules the relative capacity of the knapsack. Given a tightness $\delta \in [0, 1]$ the capacity b_i is setted to $b_i = \delta \sum_{i=1}^{n} x_i$.

4 Research Questions and Expectations

References

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