



Quantum Wave ToolBox

Documentation of Algorithms

**QWTB version 0.1**

<https://qwtb.github.io/qwtb/>

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# 1

## Introduction

This document gives overview of the algorithms implemented in Quantum Wave ToolBox (QWTB).

Toolbox was realized within the EMRP-Project SIB59 Q-Wave. The EMRP is jointly funded by the EMRP participating countries within EURAMET and the European Union.



## 2

# ADEV – Allan Deviation

## Description

.id — ADEV

.name — Allan Deviation

.desc — Compute the Allan deviation for a set of time-domain frequency data.

.citation — D.W. Allan, "The Statistics of Atomic Frequency Standards", Proc. IEEE, Vol. 54, No. 2, pp. 221-230, Feb. 1966. Implementation by M. A. Hopcroft, mhopeng@gmail.com, Matlab Central, online: <http://www.mathworks.com/matlabcentral/fileexchange/13246-allan> Test data by W. J. Riley, "The Calculation of Time Domain Frequency Stability", online: <http://www.wiley.com/paper1ht.htm>

.remarks — If tau is empty array, tau values are automatically generated. Tau values must be divisible by 1/fs. Invalid values are ignored. For tau values really used in the calculation see the output. Using sigma as uncertainty is probably not correct.

.license — BSD License

.requires

- y — sampled values
- fs — sampling frequency
- tau — observation time

.returns

- adev — Allan deviation
- tau — observation time of result values

.providesGUF — yes  
.providesMCM — no

## Example

### Allan Deviation

Example for algorithm ADEV.

ADEV is an algorithm to compute the Allan deviation for a set of time-domain frequency data.

See also W. J. Riley, "The Calculation of Time Domain Frequency Stability". Implementation: M. A. Hopcroft, [mhopeng@gmail.com](mailto:mhopeng@gmail.com), Matlab Central.'

### Contents

- Generate sample data
- Call algorithm
- Display results

### Generate sample data

```
DI = [];  
%!demo  
%ysin=2.*sin(2.*pi.*1/300.*[1:1:1e3]);  
%! [x y s errors]=adev(1, ysin, 1, 'best averaging time  
    is 300 s, i.e. cca one sine period');  
% A random numbers with normal probability distribution  
    function will be geneated into data input |DI.y.v|.   
DI.y.v = normrnd(1.5,3,1,1e3);  
% Next a drift is added:  
DI.y.v = DI.y.v + [1:1:1e3]./100;  
% Lets suppose a sampling frequency is 1 Hz:  
DI.fs.v = 1;  
% Let the algorithm generate all possible tau values  
    automatically:  
DI.tau.v = [];
```

## Call algorithm

Use QWTB to apply algorithm ADEV to data DI.

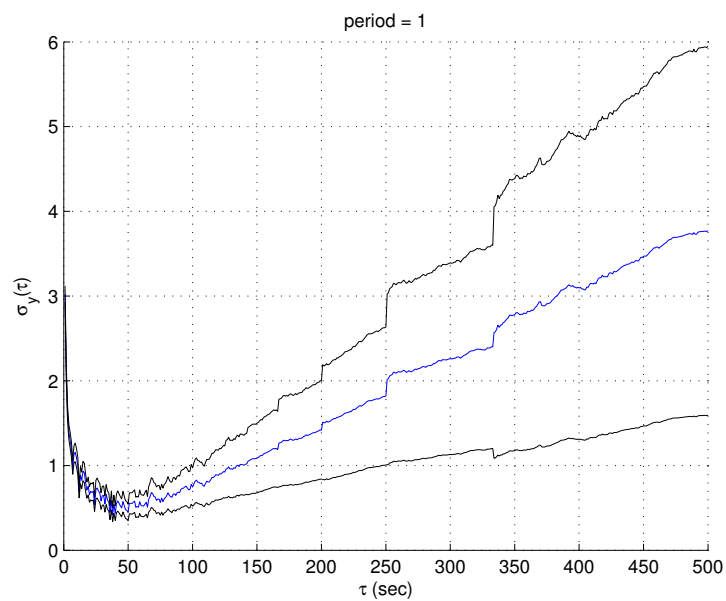
```
DO = qwtb('ADEV', DI);
```

*QWTB: no uncertainty calculation*

## Display results

Log log figure is the best to see allan deviation results:

```
figure; hold on
loglog(DO.tau.v, DO.adev.v, '-b')
loglog(DO.tau.v, DO.adev.v + DO.adev.u, '-k')
loglog(DO.tau.v, DO.adev.v - DO.adev.u, '-k')
xlabel('\tau (sec)');
ylabel('\sigma_y(\tau)');
title(['period = ' num2str(DI.fs.v)]);
grid('on'); hold off
```



# 3

## FPSWF – Four Parameter Sine Wave Fitting

### Description

.id — FPSWF

.name — Four Parameter Sine Wave Fit

.desc — Fits a sine wave to the recorded data by means of least squares fitting using 4 parameter (frequency, amplitude, phase and offset) model. An estimate of signal frequency is required. Due to non-linear characteristic, converge is not always achieved. When run in Matlab, function 'lsqnonlin' in Optimization toolbox is used. When run in GNU Octave, function 'leasqr' in GNU Octave Forge package optim is used.

.citation —

.remarks — Algorithm works essentially only for simple sine wave. Algorithm is very sensitive to distortion. Algorithm requires good estimate of signal frequency.

.license — MIT License

.requires

t — Time series of sampled data

y — Sampled values

fest — Estimate of signal frequency

.returns

f — Frequency of main signal component

A — Amplitude of main signal component

ph — Phase of main signal component

O — Offset of signal

.providesGUF — no

.providesMCM — no

## Example

### Four parameter sine wave fitting

Example for algorithm FPSWF.

FPSWF is an algorithm for estimating the frequency, amplitude, and phase of the sine waveform. The algorithm use least squares method. Algorithm requires good estimate of frequency.

### Contents

- Generate sample data
- Call algorithm
- Display results

### Generate sample data

Two quantities are prepared: t and y, representing 1 second of sinus waveform of nominal frequency 1 kHz, nominal amplitude 1 V, nominal phase 1 rad and offset 1 V sampled at sampling frequency 10 kHz.

```
Dl = [];  
Anom = 2; fnom = 100; phnom = 1; Onom = 0.2;  
Dl.t.v = [0:1/1e4:1-1/1e4];  
Dl.y.v = Anom*sin(2*pi*fnom*Dl.t.v + phnom) + Onom;
```

Lets make an estimate of frequency 0.2 percent higher than nominal value:

```
Dl.fest.v = 100.2;
```



### Call algorithm

Use QWTB to apply algorithm FPSWF to data DI.

```
CS.verbose = 1;  
DO = qwtb( 'FPSWF' , DI , CS );
```

*QWTB: no uncertainty calculation*

*Fitting started*

*Local minimum found.*

*Optimization completed because the **size** of the **gradient** is less than the default value of the **function** tolerance.*

*Fitting finished*

### Display results

Results is the amplitude, frequency and phase of sampled waveform.

```
A = DO.A.v  
f = DO.f.v  
ph = DO.ph.v  
O = DO.O.v
```

*A =*

*2.0000*

*f =*

100.0000

$ph =$

1.0000

$O =$

-0.2000

Errors of estimation in parts per milion:

```
Aerrppm = (DO.A.v - Anom)/Anom .* 1e6  
ferrppm = (DO.f.v - fnom)/fnom .* 1e6  
pherrppm = (DO.ph.v - phnom)/phnom .* 1e6  
Oerrppm = (DO.O.v - Onom)/Onom .* 1e6
```

$Aerrppm =$

$4.8894e-07$

$ferrppm =$

$-1.4211e-10$

$pherrppm =$

$1.7542e-08$

$Oerrppm =$

$-2.0000e+06$

## 4

# INL – Integral Non-Linearity of ADC

## Description

.id — INL

.name — Integral Non-Linearity of ADC

.desc — Calculates Integral Non-Linearity of an ADC. ADC has to sample sinewave, ADC codes are required.

.citation — Virosztek, T., Pálfi V., Renczes B., Kollár I., Balogh L., Sárhegyi A., Márkus J., Bilau Z. T., ADCTest project site: <http://www.mit.bme.hu/projects/adctest> 2000-2014

.remarks — Based on the ADCTest Toolbox v4.3, November 25, 2014.

.license — UNKNOWN

.requires

    t — time series of sampled data

    codes — Sampled values represented as ADC codes (not converted to voltage)

.returns

    INL — INL

.providesGUF — no

.providesMCM — no

## Example

### Integral Non Linearity of ADC

Example for algorithm INL

INL is an algorithm for estimating Integral Non-Linearity of an ADC. ADC has to sample sinewave, ADC codes are required.

See also 'Virosztek, T., Pálfi V., Renczes B., Kollár I., Balogh L., Sárhegyi A., Márkus J., Bilau Z. T., ADCTest project site: <http://www.mit.bme.hu/projects/adctest> 2000-2014';

### Contents

- Generate sample data
- Call algorithm

### Generate sample data

Suppose a sine wave of nominal frequency 10 Hz and nominal amplitude 1 V is sampled by ADC with bit resolution of 4. First quantities `t` with time of samples and quantity `bits` with number of bits are prepared and put into input data structure `DI`.

```
DI = [];  
DI.t.v = [0:1/1e4:1-1/1e4];  
DI.bits.v = 4;
```

Waveform is constructed.

```
Anom = 1; fnom = 2; phnom = 0;  
wvfrm = Anom*sin(2*pi*fnom*DI.t.v + phnom);
```

Next code values are calculated. It is simulated by quantization and scaling of the sampled waveform. In real measurement code values can be obtained directly from the ADC. Suppose ADC range is -1..1.

```

codes = wvfrm;
rmin = -1; rmax = 1;
levels = 2.^DI.bits.v - 1;
codes(codes<rmin) = rmin;
codes(codes>rmax) = rmax;
codes = round((codes-rmin)./2.*levels);

```

Now lets introduce ADC error. Instead of generating code 2 ADC erroneously generates code 3 and instead of 10 it generates 11.

```

codes(codes==2) = 3;
codes(codes==10) = 11;
codes = codes + min(codes);

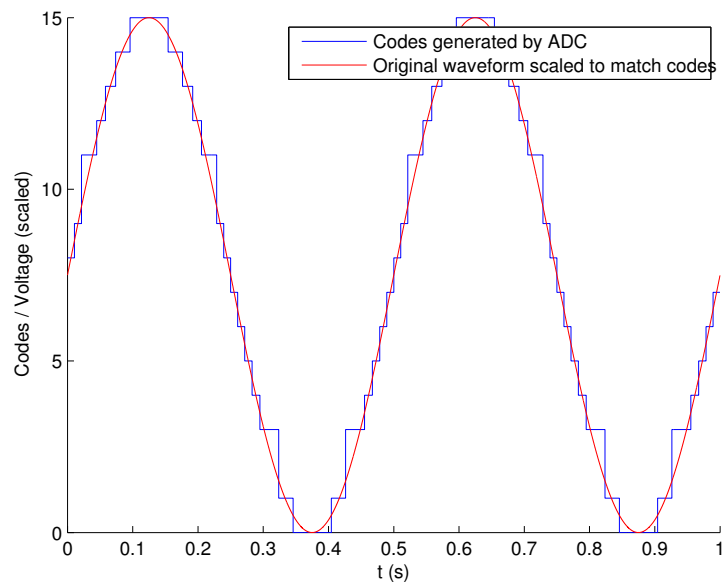
```

Create quantity codes and plot a figure with sampled sine wave and codes.

```

DI.codes.v = codes;
figure
%plot(t, (y+1).*1/2.*levels, t, codes);
hold on
stairs(DI.t.v, codes);
wvfrm = (wvfrm + Anom).*levels./2;
plot(DI.t.v, wvfrm, '-r');
xlabel('t (s)')
ylabel('Codes / Voltage (scaled)');
legend('Codes generated by ADC', 'Original waveform
scaled to match codes');
hold off

```



### Call algorithm

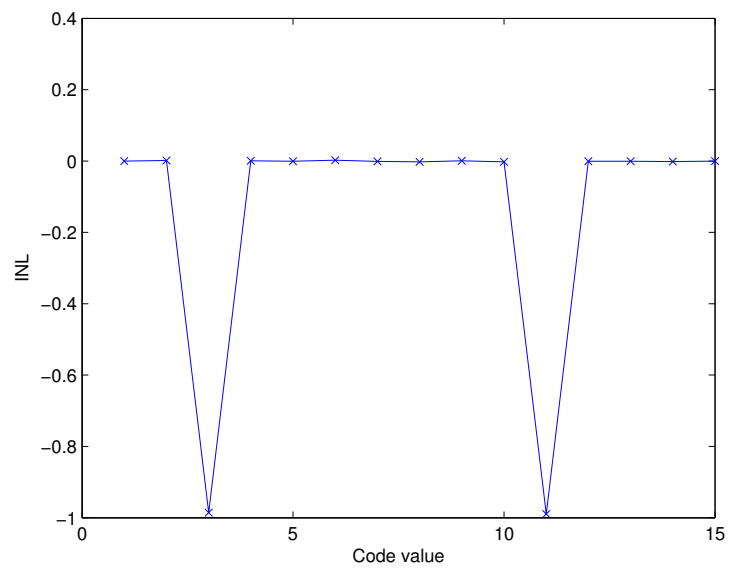
Apply INL algorithm to the input data DI.

```
DO = qwtb('INL', DI);
```

*QWTB: no uncertainty calculation*

Plot results of integral non-linearity. One can clearly observe defects on codes 3 and 11.

```
figure
plot(DO.INL.v, '-x');
xlabel('Code value')
ylabel('INL')
```



## 5

# MADEV – Modified Allan Deviation

## Description

.id — MADEV

.name — Modified Allan Deviation

.desc — Compute the modified Allan deviation for a set of time-domain frequency data.

.citation — D.W. Allan and J.A. Barnes, "A Modified Allan Variance with Increased Oscillator Characterization Ability", Proc. 35th Annu. Symp. on Freq. Contrl., pp. 470-474, May 1981. Implementation: Implementation by M. A. Hopcroft, mhopen@gmail.com, Matlab Central, online: <http://www.mathworks.com/matlabcentral/fileexchange/26637-allan-modified>  
Test data by W. J. Riley, "The Calculation of Time Domain Frequency Stability", online: <http://www.wiley.com/paper1ht.htm>

.remarks — If tau is empty array, tau values are automatically generated. Tau values must be divisible by 1/fs. Invalid values are ignored. For tau values really used in the calculation see the output. Using sigma as uncertainty is probably not correct.

.license — BSD License

.requires

y — sampled values

fs — sampling frequency

tau — observation time

.returns



madev — modified Allan deviation  
tau — observation time of result values

.providesGUF — yes  
.providesMCM — no

## Example

### Modified Allan Deviation

Example for algorithm MADEV.

MADEV is an algorithm to compute the modified Allan deviation for a set of time-domain frequency data.

See also W. J. Riley, "The Calculation of Time Domain Frequency Stability". Implementation: M. A. Hopcroft, [mhopeng@gmail.com](mailto:mhopeng@gmail.com), Matlab Central.'

### Contents

- Generate sample data
- Call algorithm
- Display results

### Generate sample data

```
DI = [];  
%!demo  
%ysin=2.*sin(2.*pi.*1/300.*[1:1:1e3]);  
%! [x y s errors]=madev(1, ysin, 1, 'best averaging  
    time is 300 s, i.e. cca one sine period');  
% A random numbers with normal probability distribution  
    function will be geneated into data input |DI.y.v|.   
DI.y.v = normrnd(1.5,3,1,1e3);  
% Next a drift is added:  
DI.y.v = DI.y.v + [1:1:1e3]./100;  
% Lets suppose a sampling frequency is 1 Hz:  
DI.fs.v = 1;  
% Let the algorithm generate all possible tau values  
    automatically:
```

```
DI.tau.v = [];
```

### Call algorithm

Use QWTB to apply algorithm MADEV to data DI.

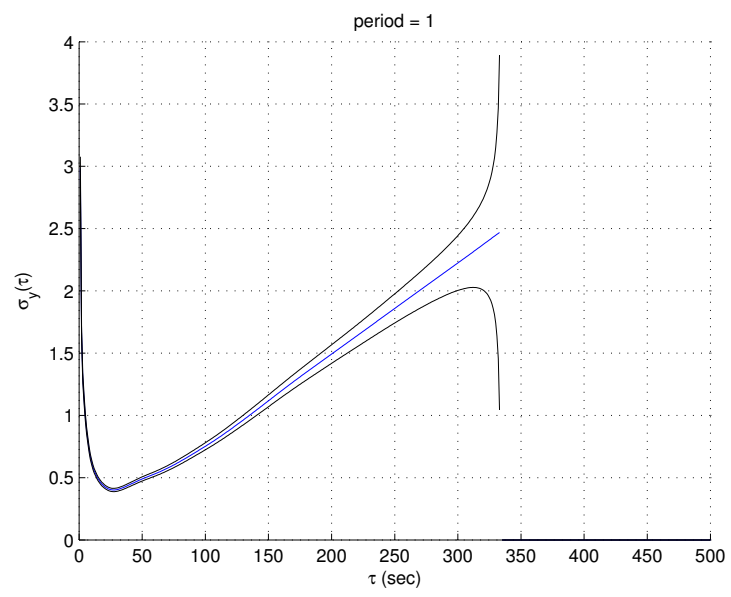
```
DO = qwtb('MADEV', DI);
```

*QWTB: no uncertainty calculation*

### Display results

Log log figure is the best to see modified allan deviation results:

```
figure; hold on
loglog(DO.tau.v, DO.madev.v, '-b')
loglog(DO.tau.v, DO.madev.v + DO.madev.u, '-k')
loglog(DO.tau.v, DO.madev.v - DO.madev.u, '-k')
xlabel('\tau (sec)');
ylabel('\sigma_y(\tau)');
title(['period = ' num2str(DI.fs.v)]);
grid('on'); hold off
```



## 6

# OADEV – Overlapping Allan Deviation

## Description

.id — OADEV

.name — Overlapping Allan Deviation

.desc — Compute the overlapping Allan deviation for a set of time-domain frequency data.

.citation — D.A. Howe, D.W. Allan and J.A. Barnes, "Properties of Signal Sources and Measurement Methods", Proc. 35th Annu. Symp. on Freq. Contrl., pp. 1-47, May 1981. Implementation: Implementation by M. A. Hopcroft, mhopeng@gmail.com, Matlab Central, online: <http://www.mathworks.com/matlabcentral/fileexchange/26441-allan-overlap> Test data by W. J. Riley, "The Calculation of Time Domain Frequency Stability", online: <http://www.wiley.com/paper1ht.htm>

.remarks — If tau is empty array, tau values are automatically generated. Tau values must be divisible by 1/fs. Invalid values are ignored. For tau values really used in the calculation see the output. Using sigma as uncertainty is probably not correct.

.license — BSD License

.requires

y — sampled values

fs — sampling frequency

tau — observation time

.returns

oadev — overlapping Allan deviation

tau — observation time of result values

.providesGUF — yes

.providesMCM — no

## Example

### Allan Overlapping Deviation

Example for algorithm OADEV.

OADEV is an algorithm to compute the overlapping Allan deviation for a set of time-domain frequency data.

See also W. J. Riley, "The Calculation of Time Domain Frequency Stability". Implementation: M. A. Hopcroft, [mhopeng@gmail.com](mailto:mhopeng@gmail.com), Matlab Central.'

### Contents

- Generate sample data
- Call algorithm
- Display results

### Generate sample data

```
DI = [];  
%!demo  
%ysin=2.*sin(2.*pi.*1/300.*[1:1:1e3]);  
%! [x y s errors]=adev(1, ysin, 1, 'best averaging time  
    is 300 s, i.e. cca one sine period');  
% A random numbers with normal probability distribution  
    function will be geneated into data input |DI.y.v|.   
DI.y.v = normrnd(1.5,3,1,1e3);  
% Next a drift is added:  
DI.y.v = DI.y.v + [1:1:1e3]./100;  
% Lets suppose a sampling frequency is 1 Hz:  
DI.fs.v = 1;
```

```
% Let the algorithm generate all possible tau values
    automatically:
DI.tau.v = [];
```

### Call algorithm

Use QWTB to apply algorithm OADEV to data DI.

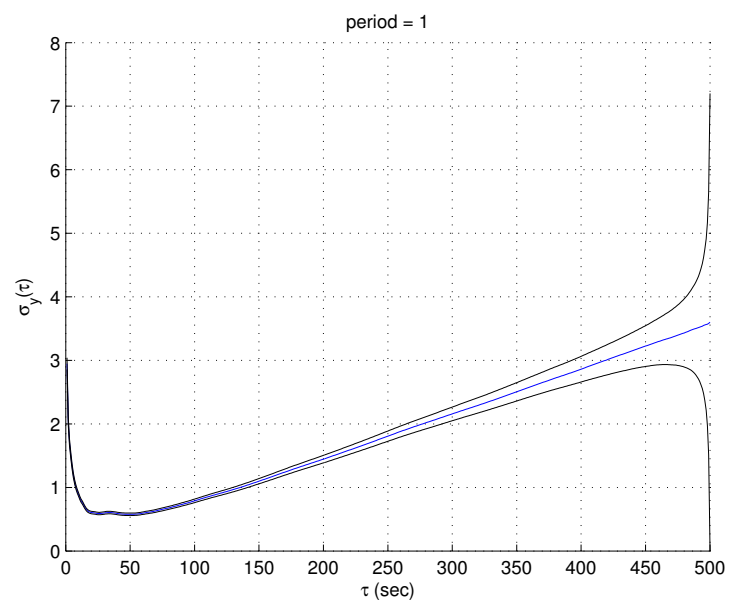
```
DO = qwtb('OADEV', DI);
```

*QWTB: no uncertainty calculation*

### Display results

Log log figure is the best to see allan deviation results:

```
figure; hold on
loglog(DO.tau.v, DO.oadev.v, '-b')
loglog(DO.tau.v, DO.oadev.v + DO.oadev.u, '-k')
loglog(DO.tau.v, DO.oadev.v - DO.oadev.u, '-k')
xlabel('\tau (sec)');
ylabel('\sigma_y(\tau)');
title(['period = ' num2str(DI.fs.v)]);
grid('on'); hold off
```



# 7

## PSFE – Phase Sensitive Frequency Estimator

### Description

.id — PSFE

.name — Phase Sensitive Frequency Estimator

.desc — An algorithm for estimating the frequency, amplitude, and phase of the fundamental component in harmonically distorted waveforms. The algorithm minimizes the phase difference between the sine model and the sampled waveform by effectively minimizing the influence of the harmonic components. It uses a three-parameter sine-fitting algorithm for all phase calculations. The resulting estimates show up to two orders of magnitude smaller sensitivity to harmonic distortions than the results of the four-parameter sine fitting algorithm.

.citation — Lapuh, R., "Estimating the Fundamental Component of Harmonically Distorted Signals From Noncoherently Sampled Data," Instrumentation and Measurement, IEEE Transactions on , vol.64, no.6, pp.1419,1424, June 2015, doi: 10.1109/TIM.2015.2401211, URL: <http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=7061456&isnumber=7104190>

.remarks — Very small errors, effective for harmonically distorted signals.

.license — MIT License

.requires

t — time series of sampled data

y — sampled values



.returns

f — Frequency of main signal component

A — Amplitude of main signal component

ph — Phase of main signal component

.providesGUF — no

.providesMCM — no

## Example

### Phase Sensitive Frequency Estimator

Example for algorithm PSFE.

PSFE is an algorithm for estimating the frequency, amplitude, and phase of the fundamental component in harmonically distorted waveforms. The algorithm minimizes the phase difference between the sine model and the sampled waveform by effectively minimizing the influence of the harmonic components. It uses a three-parameter sine-fitting algorithm for all phase calculations. The resulting estimates show up to two orders of magnitude smaller sensitivity to harmonic distortions than the results of the four-parameter sine fitting algorithm.

See also Lapuh, R., "Estimating the Fundamental Component of Harmonically Distorted Signals From Noncoherently Sampled Data," Instrumentation and Measurement, IEEE Transactions on , vol.64, no.6, pp.1419,1424, June 2015, doi: 10.1109/TIM.2015.2401211, URL: <http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=7061456&isnumber=7104190>

### Contents

- Generate sample data
- Call algorithm
- Display results

### Generate sample data

Two quantities are prepared: t and y, representing 1 second of sinus waveform of nominal frequency 100 Hz, nominal amplitude 1 V and nominal phase 1 rad, sampled at sampling frequency 10 kHz.

```

DI = [];
Anom = 1; fnom = 100; phnom = 1;
DI.t.v = [0:1/1e4:1-1/1e4];
DI.y.v = Anom*sin(2*pi*fnom*DI.t.v + phnom);

```

Add noise:

```

DI.y.v = DI.y.v + normrnd(0, 1e-3, size(DI.y.v));

```

### Call algorithm

Use QWTB to apply algorithm PSFE to data DI.

```

DO = qwtb('PSFE', DI);

```

*QWTB: no uncertainty calculation*

### Display results

Results is the amplitude, frequency and phase of sampled waveform.

```

f = DO.f.v
A = DO.A.v
ph = DO.ph.v

```

```

f =
    100.0000

A =
    1.0000

```

*ph* =

*1.0000*

Errors of estimation in parts per milion:

```
ferrppm = (DO.f.v - fnom)/fnom .* 1e6  
Aerrppm = (DO.A.v - Anom)/Anom .* 1e6  
pherrppm = (DO.ph.v - phnom)/phnom .* 1e6
```

*ferrppm* =

*0.0926*

*Aerrppm* =

*8.2655*

*pherrppm* =

*-10.5950*

## 8

# SP-FFT – Spectrum by means of Fast Fourier Transform

### Description

.id — SP-FFT

.name — Spectrum by means of Fast Fourier Transform

.desc — Calculates frequency and phase spectrum by means of Fast Fourier Transform algorithm. Result is normalized.

.citation —

.remarks —

.license — MIT License

.requires

    y — Sampled values

    fs — Sampling frequency

.returns

    f — Frequency series

    A — Amplitude series

    ph — Phase series

.providesGUF — no

.providesMCM — no

## Example

### Signal Spectrum by means of Fast fourier transform

Example for algorithm SP-FFT.

Calculates frequency and phase spectrum by means of Fast Fourier Transform algorithm. Result is normalized.

#### Contents

- Generate sample data
- Call algorithm
- Display results

#### Generate sample data

Two quantities are prepared:  $y$  and  $fs$ , representing 1 second of signal containing 5 harmonic components and one inter-harmonic component. Main signal component has nominal frequency 1 kHz, nominal amplitude 2 V, nominal phase 1 rad and offset 1 V sampled at sampling frequency 10 kHz.

```
DI = [];  
fsnom = 1e4; Anom = 2; fnom = 100; phnom = 1; Onom =  
    0.2;  
t = [0:1/fsnom:1-1/fsnom];  
DI.y.v = Anom*sin(2*pi*fnom*t + phnom);  
for i = 2:45  
    DI.y.v = DI.y.v + Anom./i*sin(2*pi*fnom*i*t +  
        phnom + i - 1);  
end  
DI.y.v = DI.y.v + 1*sin(2*pi*fnom*1.456*t + phnom);  
DI.fs.v = fsnom;
```

#### Call algorithm

Use QWTB to apply algorithm SP-FFT to data DI.

```
DO = qwtb('SP-FFT', DI);
```

*QWTB: no uncertainty calculation*

## Display results

Results is the amplitude and phase spectrum.

```
figure
plot(DO.f.v, DO.A.v, '-x')
xlabel('f (Hz)'); ylabel('A (V)'); title('Amplitude
    spectrum of the signal');
figure
plot(DO.f.v, DO.ph.v, '-x')
xlabel('f (Hz)'); ylabel('phase (rad)'); title('Phase
    spectrum of the signal');
```

