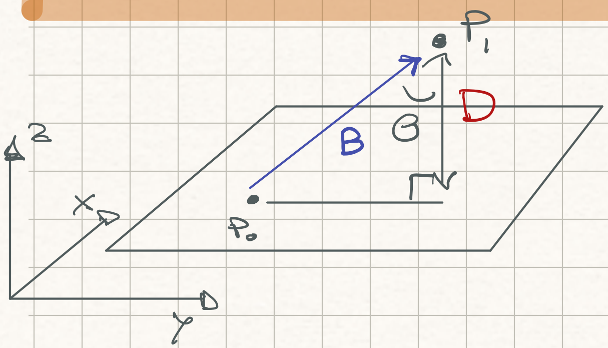
Distance between a point and a plane

Point $P_1(x_1, y_1, z_1)$

Plane $ax_0 + by_0 + cz_0 + d = 0$

the goal is to find the distance between those two



- We want to calculate the shortest distance D
- Let's say we have a point P_0 that lyes on the plane. We can draw the vector B
- θ is the angle between B - D

→ Our Task is to find D ←

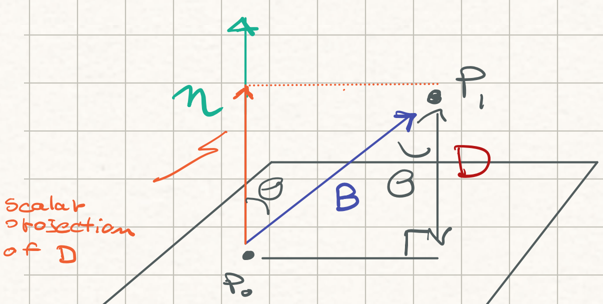
First we can define B

$P_0 = (x_0, y_0, z_0)$ $\quad P_1 = (x_1, y_1, z_1)$

$$B = \overrightarrow{P_0 P_1} = \langle x_1 - x_0, y_1 - y_0, z_1 - z_0 \rangle$$

to define a plane we need

- A point on the plane
- A vector that is perpendicular to the plane (i.e. the normal vector = n)



→ to find D we need the scalar projection of D on n

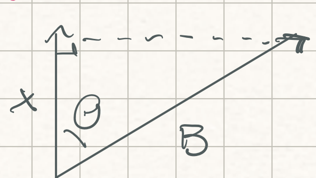
→ we can say that

$$D = \underbrace{\text{comp}_n B}_{\text{scalar projection}} = |B| \cos\theta$$

scalar projection

component of B that is parallel to n

Just a reminder



$\cos\theta = \dfrac{x}{|B|}$ $\qquad x = |B| \cos\theta$

→ But we don't know θ ←

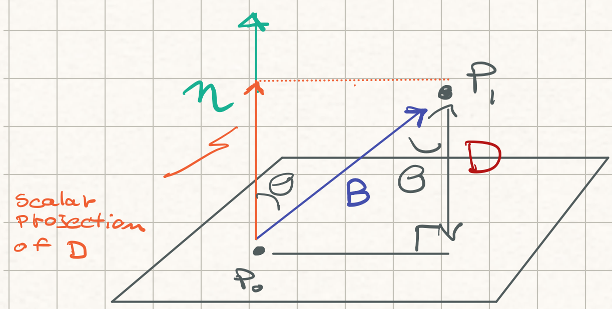
consider two vectors

dot product

$$a \cdot b = |a|\,|b| \cos\theta$$

$$\frac{a \cdot b}{|a|} = |b| \cos\theta$$

For our case



because we want it positive

$$\frac{|n \cdot B|}{|n|} = |B| \cos\theta = D$$

↳ we want to know this

From above we have

$$B = \overrightarrow{P_0 P_1} = \langle x_0 - x_1,\ y_0 - y_1,\ z_0 - z_1 \rangle$$

vector n is:

$n = \langle a, b, c \rangle$ → Plane $ax_0 + by_0 + cz_0 + d = 0$

the coefficients of the plane

$$D = \frac{|a(x_1 - x_0) + b(y_1 - y_0) + c(z_1 - z_0)|}{\sqrt{a^2 + b^2 + c^2}}$$

this is the magnitude of n

→ if we do some algebra

$$D = \frac{|(ax_1 + by_1 + cz_1) - (ax_0 + by_0 + cz_0)|}{\sqrt{a^2 + b^2 + c^2}}$$

→ consider the equation of our plane

$$ax_0 + by_0 + cz_0 + d = 0 \longrightarrow ax_0 + by_0 + cz_0 = -d$$

$$\boxed{D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2+b^2+c^2}}}$$

this is the formula we need to calculate the distance of a point to a plane