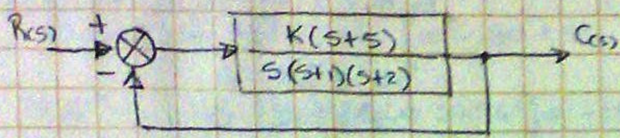
LGR

Ej. #5 - Construir el LGR



$$G(s)H(s) = \frac{K(S+5)}{S(S+1)(S+2)}$$

① POLOS = 3 ; $S=0$, $S=-1$, $S=-2$

② CEROS FINITOS = 1 ; $S=-5$

③ CEROS INFINITOS = 2 (No. AS)

x ⟶ 0, ~~∞~~

x ⟶ ~~0~~, ∞ ✓

x ⟶ ~~0~~, ∞ ✓

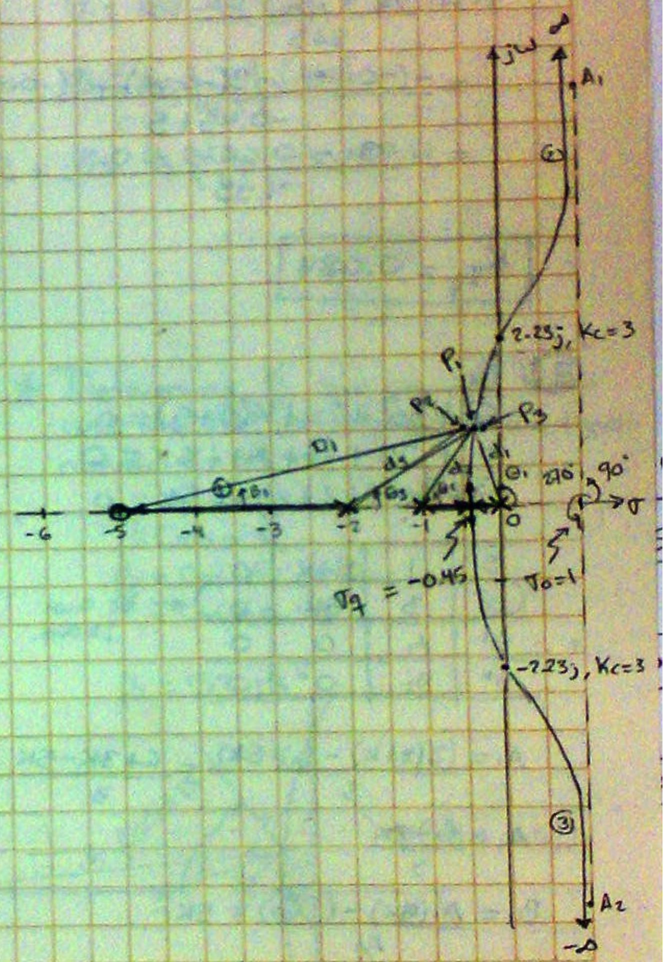
a) No. As = $P - Z = 3 - 1 = 2$

b) $\sigma_0 = \frac{[0-1-2]-[-5]}{2} = \frac{-3+5}{2} = \frac{2}{2}$

$\sigma_0 = 1$

c) $a=0$ $\measuredangle As_1 = \frac{180° \pm (0)360°}{2} = \frac{180}{2} = 90°$

$a=1$ $\measuredangle As_2 = \frac{180° \pm (1)360°}{2} = \frac{540}{2}$ $270°$



④ Existencia: $0 \to -1$

$-2 \to -5$

⑤ σ_q

$1 + G(s)H(s) = 0$

$$1 + \frac{K(S+5)}{S(S+1)(S+2)} = 0$$

$S(S+1)(S+2) + K(S+5) = 0$

$(S^2+S)(S+2) + K(S+5) = 0$

$S^3 + 2S^2 + S^2 + 2S + K(S+5) = 0$

$S^3 + 3S^2 + 2S + K(S+5) = 0$

a) $K = \frac{-S^3 - 3S^2 - 2S}{(S+5)}$

b) $\frac{dK}{dS} \Rightarrow \frac{(S+5)(-3S^2-6S-2)-(-S^3-3S^2-2S)(1)}{(S+5)^2}$

$$\frac{\cancel{-3S^3} - \cancel{6S^2} - \cancel{2S} - 15S^2 - 30S - 10 + \cancel{S^3} + \cancel{3S^2} + \cancel{2S}}{(S+5)^2}$$

$$\frac{-2S^3 - 18S^2 - 30S - 10}{(S+5)^2}$$

c) $\frac{dK}{dS} = 0$ $\quad \frac{-2S^3 - 18S^2 - 30S - 10}{(S+5)^2} = 0$

$-2S^3 - 18S^2 - 30S - 10 = 0 \left(-\frac{1}{2}\right)$

$\underbrace{S^3 + 9S^2 + 15S + 5 = 0}$

$S_1 = -0.45$

$S_2 = -1.6$

$S_3 = -6.94$

$\therefore \ \sigma_q \doteq -0.45$

⑥ K_{Tq}

$$K = \frac{-s^3 - 3s^2 - 2s}{s+5}$$

$$= \frac{-(-0.45)^3 - 3(-0.45)^2 - 2(-0.45)}{-0.45+5}$$

$$= \frac{0.091 - 0.6075 + 0.9}{4.55} = \frac{0.3835}{4.55}$$

$$\boxed{K_{Tq} = 0.084}$$

⑦ K_c

$$s^3 + 3s^2 + 2s + K(s+5) = 0$$
$$s^3 + 3s^2 + 2s + Ks + 5K = 0$$
$$s^3 + 3s^2 + (2+K)s + 5K = 0$$

s^3	1	$2+K$	0
s^2	3	$5K$	0
s^1	A_1	0	0
s^0	B_1	0	0

← Ec. aux 2º ord.

$$A_1 = \frac{(3)(2+K) - (1)(5K)}{3} = \frac{6+3K-5K}{3}$$

$$A_1 = \frac{6-2K}{3}$$

$$B_1 = \frac{\cancel{A_1}(5K) - (3)\cancel{(0)}}{\cancel{A_1}} = 5K$$

Para Sist. Estable

$+1$
$+3$
$\frac{6-2K}{3} > 0$
$5K > 0$

$$\frac{6-2K}{3} > 0$$
$$6 - 2K > 0$$
$$6 > 2K$$
$$\frac{6}{2} > K \Rightarrow 3 > K$$

$5K > 0$

$K > 0$

Rango de Estabilidad

$$\boxed{0 < K < 3}$$

$$\boxed{K_c = 3}$$

⑧ W_c

Ec. aux 2º orden, $K = K_c$, $s = jW_c$

$$3s^2 + 5K = 0$$
$$3s^2 + 5K_c = 0$$
$$3s^2 + 5(3) = 0$$

$$s_{1,2} = \frac{0 \pm \sqrt{(0)^2 - 4(3)(15)}}{2(3)}$$

$$= \frac{\pm\sqrt{-180}}{6} = \frac{\pm 13.41j}{6}$$

$$s = \pm 2.23j$$

$$\boxed{W_c = \pm 2.23j}$$

⑨ Para $P_1(-0.4, j)$

$$\beta_1 - [\Theta_1 + \Theta_2 + \Theta_3] = -180° \pm 5°$$

$$\beta_1 = tg^{-1}\left|\frac{1}{4.6}\right| = 12.26°$$

$$\Theta_1 = 180 - \alpha_1 = 180 - tg^{-1}\left|\frac{1}{0.4}\right|$$
$$= 180 - 68.19$$
$$= 111.8°$$

$$\Theta_2 = tg^{-1}\left|\frac{1}{0.6}\right| = 59.03°$$

$$\Theta_3 = tg^{-1}\left|\frac{1}{1.6}\right| = 32°$$

$$\Rightarrow 12.26° - [111.8° + 59.03° + 32°] = -180° \pm 5°$$
$$12.26° - 202.83° = -180° \pm 5°$$
$$-190.57° = -180° \pm 5°$$

$\therefore P_1$ NO PERTENECE

Para $P_2(-0.5, j)$

$$\beta_1 - [\Theta_1 + \Theta_2 + \Theta_3] = -180° \pm 5°$$
$$12.52° - [116.56° + 63.43° + 33.7°] = -180° \pm 5°$$
$$12.52° - 213.68 = -180° \pm 5°$$
$$-201.16 = -180° \pm 5°$$

$\therefore P_2$ NO PERTENECE

Para $P_3(-0.3, j)$

$\beta_1 - [\theta_1 + \theta_2 + \theta_3] = -180° \pm 5°$

$12.01° - [106.69° + 55° + 30.46°] = -180° \pm 5°$

$12.01° - 192.15° = -180° \pm 5°$

$\underline{-180.14 = -180° \pm 5°}$

$\beta_1 = tg^{-1}\left|\frac{1}{4.7}\right| = 12.01°$

$\theta_1 = 180° - tg^{-1}\left|\frac{1}{0.3}\right| = 106.69°$

$\theta_2 = tg^{-1}\left|\frac{1}{0.7}\right| = 55°$

$\theta_3 = tg^{-1}\left|\frac{1}{1.7}\right| = 30.46°$

$\therefore$ P_3 SI PERTENECE

(10) K_{P_3}

$$K_{P_3} = \frac{d_1 d_2 d_3}{C_K D_1}$$

$d_1 = \sqrt{(0.3)^2 + (1)^2} = 1.04\text{ cm}$

$d_2 = \sqrt{(0.7)^2 + (1)^2} = 1.22\text{ cm}$

$d_3 = \sqrt{(1.7)^2 + (1)^2} = 1.97\text{ cm}$

$D_1 = \sqrt{(4.7)^2 + (1)^2} = 4.8\text{ cm}$

$$K_{P_3} = \frac{(1.04)(1.22)(1.97)}{(1)(4.8)}$$

$\boxed{K_{P_3} = 0.52}$

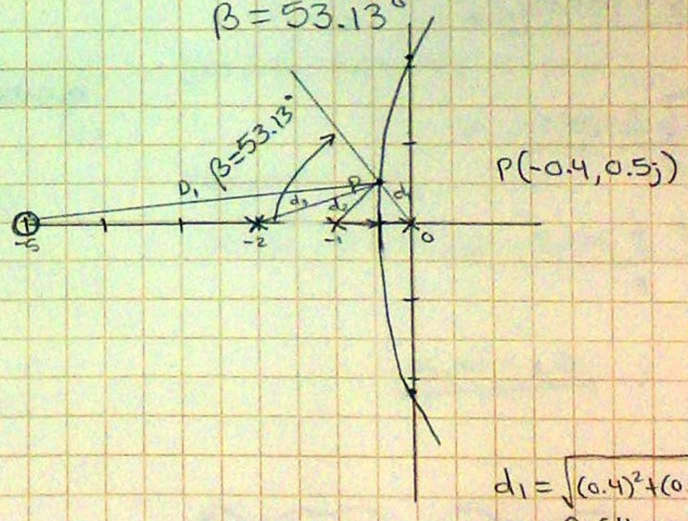
* Determine $\underline{\underline{K}}$ que permite tener una respuesta con relación de amortiguamiento $\delta = 0.6$

$\beta \cos = \delta$

$\beta = \cos^{-1}\delta$

$= \cos^{-1}(0.6)$

$\beta = 53.13°$



$d_1 = \sqrt{(0.4)^2 + (0.5)^2}$
$= 0.64\text{ cm}$

$d_2 = \sqrt{(0.6)^2 + (0.5)^2}$
$= 0.78\text{ cm}$

$d_3 = \sqrt{(1.6)^2 + (0.5)^2}$
$= 1.67\text{ cm}$

$D_1 = \sqrt{(4.6)^2 + (0.5)^2}$
$= 4.62\text{ cm}$

$$K_P = \frac{d_1 d_2 d_3}{C_K D_1} = \frac{(0.64)(0.78)(1.67)}{(1)(4.62)}$$

$\boxed{K_P = 0.180}$

RESP TRANSITORIA ; Rango de K = ?
para:

- Sobreamortiguado $\Rightarrow$ $0 < K < K_{\sigma q}$
  $\underline{0 < K < 0.084}$
- Criticamente amortiguado $\Rightarrow$ $K = K_{\sigma q}$
  $\underline{K = 0.084}$
- Subamortiguado $\Rightarrow$ $K_{\sigma q} < K < K_c$
  $\underline{0.084 < K < 3}$
- Sin amortiguamiento $\Rightarrow$ $K = K_c$
  $\underline{K = 3}$

P pertenece al LGR?

$$\beta_1 - [\Theta_1 + \Theta_2 + \Theta_3] = -180° \pm 5°$$

$$6.2° - [128.66° + 39.8° + 17.35°] = -180° \pm 5°$$

$$6.2° - 185.81° = -180° \pm 5°$$

$$\underbrace{-179.61 = -180° \pm 5°}$$

$\therefore$ P_1 SI PERTENECE

$-1 \quad -0.5 \quad 0$

$$\underline{s^3 + 9s^2 + 15s + 5 = 0}$$

$s + 0.5 = 0$

$s = -0.5$

$$\begin{array}{l|l} \cancel{s^3} + 9s^2 + 15s + 5 & s + 0.5 \longrightarrow \text{RAIZ } s = -0.5 \\ \hline -\cancel{s^3} - 0.5s^2 & s^2 + 8.5s + 10.75 \\ \quad \cancel{8.5s^2} + 15s & \\ \quad -\cancel{8.5s^2} - 4.25s & \\ \qquad \cancel{10.75s} + 5 & \\ \qquad -\cancel{10.75s} - 5.37 & \\ \qquad\quad -0.37 & \end{array}$$

$$\begin{array}{l|l} \cancel{s^3} + 9s^2 + 15s + 5 & s + 0.45 \longrightarrow \boxed{\text{RAIZ } s = -0.45} \\ \hline -\cancel{s^3} - 0.45s^2 & \underline{s^2 + 8.55s + 11.15} \\ \quad \cancel{8.55s^2} + 15s & \\ \quad -\cancel{8.55s^2} - 3.85s & \\ \qquad 11.15s + 5 & \\ \qquad -11.15s - 5.01 & \\ \qquad\quad -0.01 \text{ RESIDUO} & \end{array}$$

$$s^2 + 8.55s + 11.15 \longrightarrow s^2 + 8.55s + 11.15 = 0$$

$$s_{1,2} = \frac{-8.55 \pm \sqrt{(8.55)^2 - 4(1)(11.15)}}{2(1)}$$

$$= \frac{-8.55 \pm \sqrt{73.10 - 44.6}}{2}$$

$$= \frac{-8.55 \pm \sqrt{28.5}}{2}$$

$$= \frac{-8.55 \pm 5.34}{2}$$

$$\underline{s_1 = -1.6} \; ; \; \underline{s_2 = -6.94}$$

$\Rightarrow$

$s_1 = -0.45$

$s_2 = -1.6$

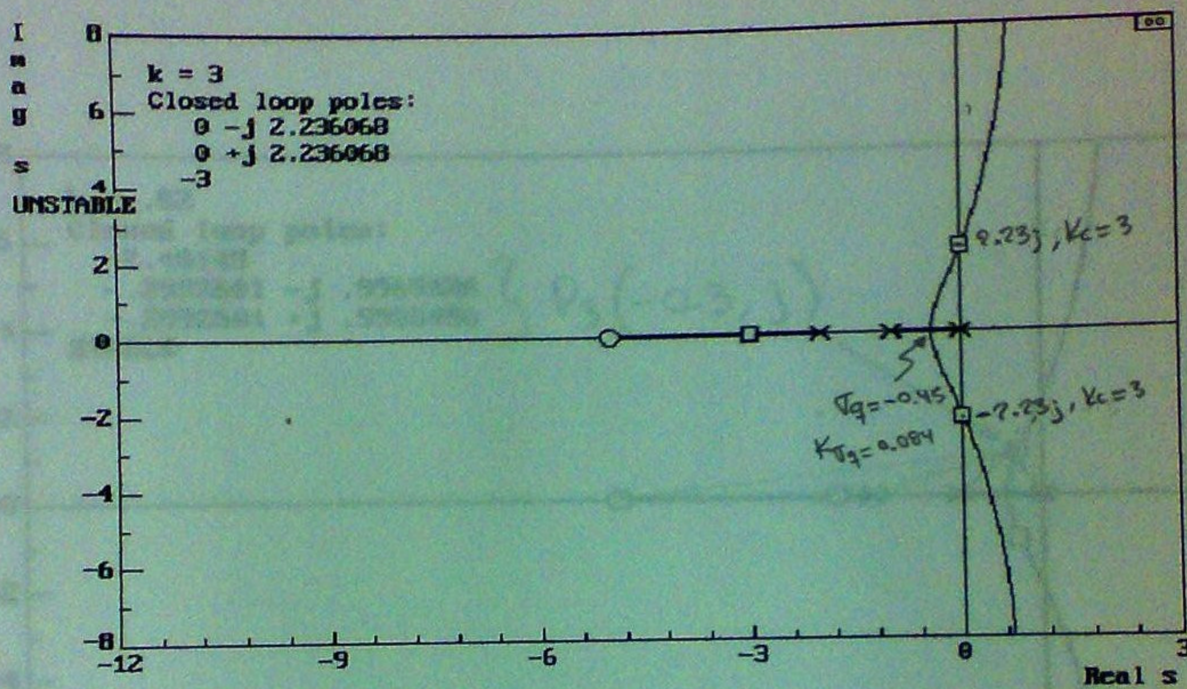
$s_3 = -6.94$

$\therefore \; \sigma_q = -0.45$

En MATLAB:

```
>> x=[1 9 15 5]
x=
   1 9 15 5
>> roots(x)
ans=
   -6.9434
   -1.6091
   -0.4475
```

$$G(s)H(s) = \frac{K(s+5)}{s(s+1)(s+2)}$$



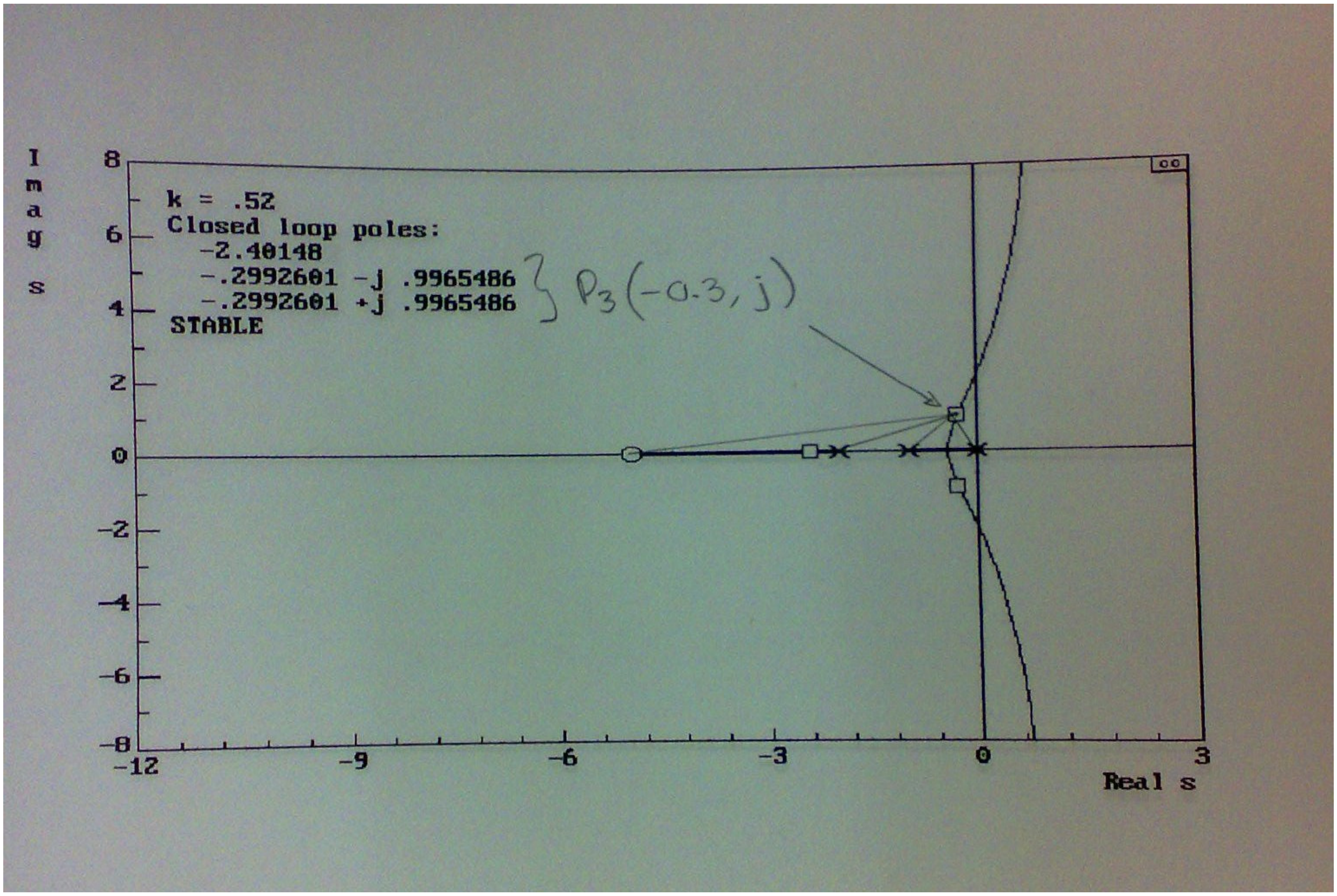
Open loop poles,	Angle of departure
0	-180
-1	0
-2	-180

Open loop zeros,	Angle of arrival
-5	0

Center of gravity = 1
Asymptotic infinite patterns = 2
Angles :-90 90

Breakpoints,	Gain
-6.943381 ,	-104.9713
-1.609094 ,	-.1129854
-.4475254 ,	8.431541E-02

Stable for gain ranges : 0 to 3



k = .52
Closed loop poles:
-2.40148
-.2992601 -j .9965486
-.2992601 +j .9965486
STABLE
P3(-0.3, j)
Imag s
Real s
8
6
4
2
0
-2
-4
-6
-8
-12
-9
-6
-3
0
3