~~TP 3~~ (de la guía).

1) $y' + 2y = 0 \quad ; \quad y(0) = 3$

$sY(s) - Y(0) + 2Y(s) = 0$

$Y(s)[s+2] - 3 = 0 \Rightarrow Y(s) = \frac{+3}{s+2}$

$\mathcal{L}^{-1}[y(s)] = 3e^{-2t} = y(t)$

Comprobamos condinicial p/ $t=0 \Rightarrow y(0) = 3e^{-2\cdot 0} = 3$ ✓

$y'(t) = -6e^{-2t} \Rightarrow -6e^{-2t} + 6e^{-2t} = 0$ ✓

Comprobado

2) y 3)

$y'' + 3y' + 2y = x \qquad CI = 0 \; ; \; x = cte = 8$

$s^2Y(s) - sY(0) - Y'(0) + 3sY(s) - 3Y(0) + 2Y(s) = \frac{8}{s}$

$CI = 0$

$s^2Y(s) + 3sY(s) + 2Y(s) = \frac{8}{s}$

$Y(s)(s^2 + 3s + 2) = \frac{8}{s} \Rightarrow Y(s) = \frac{8}{(s^2+3s+2)s}$

$Y(s) = \frac{8}{(s+1)(s+2)s}$

$A = \lim_{s \to -1} \cancel{(s+1)} \frac{8}{\cancel{(s+1)}(s+2)s} = \frac{8}{(-1+2)\cdot -1} = -8$

$B = \lim_{s \to -2} \cancel{(s+2)} \frac{8}{(s+1)\cancel{(s+2)}s} = \frac{8}{(-2+1)\cdot -2} = 4$

$C = \lim_{s \to 0} \cancel{s} \frac{8}{(s+1)(s+2)\cancel{s}} = \frac{8}{2} = 4$

$Y(s) = \frac{4}{s} + \frac{4}{(s+2)} - \frac{8}{(s+1)}$

$y(t) = 4 + 4e^{-2t} - 8e^{-t}$

VF $\Rightarrow \lim_{s \to 0} sY(s) \Rightarrow 4$

VI $\Rightarrow \lim_{s \to \infty} sY(s) \Rightarrow 0$

4)

$y(t=2) = ?$, $CI \Rightarrow y(0) = 3$

$y'(0) = -7$

$y'' + 5y' + 6y = 0$

$s^2 Y(s) - sY(0) - Y'(0) + 5sY(s) - 5Y(0) + 6Y(s) = 0$

$s^2 Y(s) - 3s + 7 + 5sY(s) - 15 + 6Y(s) = 0$

$Y(s)(s^2 + 5s + 6) - 3s - 8 = 0$

$Y(s) = \frac{3s+8}{s^2+5s+6} = \frac{3s+8}{(s+2)(s+3)} = \frac{A_1}{(s+2)} + \frac{A_2}{(s+3)}$

$A_1 = \lim_{s \to -2} Y(s) \cdot (s+2) = \frac{3s+8}{\cancel{(s+2)}(s+3)} \cancel{(s+2)} = \frac{3 \cdot (-2) + 8}{(-2+3)} = 2$

$A_2 = \lim_{s \to -3} Y(s)(s+3) = \frac{3s+8}{(s+2)\cancel{(s+3)}} \cancel{(s+3)} = \frac{3(-3)+8}{(-3+2)} = 1$

$Y(s) = \frac{2}{s+2} + \frac{1}{s+3} \Rightarrow \mathcal{L}^{-1} \Rightarrow 2e^{-2t} + e^{-3t} = y(t)$

$y(t)|_{t=2} = 39{,}11 \times 10^{-3}$

5)

$CI \Rightarrow x(0) = 8$

$y(0) = 3$

$\left.\begin{aligned} x' &= 2x - 3y \\ y' &= -2x + y \end{aligned}\right\}$

$\left.\begin{aligned} sX(s) - x(0) &= 2X(s) - 3Y(s) \\ sY(s) - y(0) &= -2X(s) + Y(s) \end{aligned}\right\}$

$\left.\begin{aligned} sX(s) - 8 &= 2X(s) - 3Y(s) \\ sY(s) - 3 &= -2X(s) + Y(s) \end{aligned}\right\} \Rightarrow \begin{aligned} -8 &= X(s)[2-s] + Y(s)[-3 \\ -3 &= X(s)[-2] + Y(s)[1- \end{aligned}$

$\begin{bmatrix} -8 \\ -3 \end{bmatrix} = \begin{bmatrix} X(s) \\ Y(s) \end{bmatrix} \times \begin{bmatrix} (2-s) & (-3) \\ (-2) & (1-s) \end{bmatrix}$

$DET = [(2-s)(1-s) - (-3)(-$

$= s^2 - 3s - 4$

$\text{Det}\,x \Rightarrow \begin{bmatrix} -8 & \\ -3 & 1-s \end{bmatrix} \Rightarrow \text{Det}\,x = 8s - 17$

$\text{Det}\,y \Rightarrow \begin{bmatrix} 2-s & -8 \\ -2 & -3 \end{bmatrix} \Rightarrow \text{Det}\,y = 3s - 22$

$\therefore y(s) = \frac{\text{Det}\,y}{\text{Det}} = \frac{3s-22}{s^2-3s-4} = \frac{-2}{s-4} + \frac{5}{s+1}$

$x(s) = \frac{\text{Det}\,x}{\text{Det}} = \frac{8s-17}{s^2-3s-4} = \frac{3}{s-4} + \frac{5}{s+1}$

$$\left.\begin{array}{l} \underline{y(t) = -2e^{4t} + 5e^{-t}} \\ \underline{x(t) = 3e^{4t} + 5e^{-t}} \end{array}\right\} \quad \begin{array}{l} y(0) = 3 \\ x(0) = 8 \end{array}$$

$x'(t) = 12e^{-4t} - 5e^{-t} = 2x(t) - 3y(t) ? *$

$* = 6e^{-4t} + 10e^{-t} - (-6e^{4t} + 15e^{-t})$

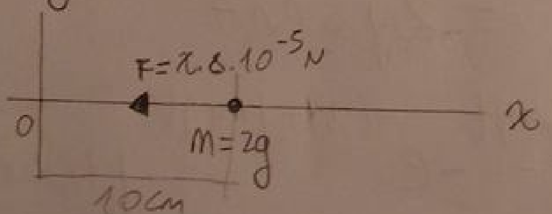
$= 6e^{-4t} + 10e^{-t} + 6e^{4t} - 15e^{-t}$

$= 12e^{-4t} - 5e^{-t}$

b) $m = 2g$

$p/ x = 10_{cm} \Rightarrow \dot{x} = 0$

$= 100 \times 10^{-3} m$



$F = -x \cdot 8 \cdot 10^{-5} N$; $m\ddot{x} = -x\, 8\, 10^{-5} N$

$m[s^2 x(s) - s\,x(0) - x'(0)] = -x(s)\, 8\, 10^{-5} N$

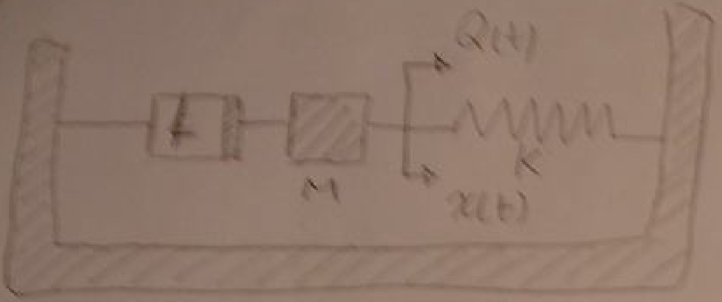
$[s^2 x(s) - s \cdot 100 \times 10^{-3} m - 0] = -x(s)\, 8 \cdot 10^{-5} N$

$s^2 x(s) = -x(s) \cdot 40 \times 10^{-6} + s\, 100 \times 10^{-3}$

$x(s)[s^2 + 40 \times 10^{-6}] = s \cdot 100 \times 10^{-3}$

$x(s) = \frac{s \cdot 100 \times 10^{-3}}{[s^2 + 40 \times 10^{-6}]} \Rightarrow \mathcal{L}^{-1} \Rightarrow \underline{x(t) = 100 \times 10^{-3} m \cdot \cos(\sqrt{40\, 10^{-6}}\, t)}$

7)



$$Kx + f\dot{x} + M\ddot{x} = Q$$ → fricción, masa, fuerza

$Q(t) = cte = 4N$

$t = 0 \Rightarrow$ Reposo. $\Rightarrow \dot{x}(0) = \ddot{x}(0) = 0$

$M = 1kg$; $K = 2 N/m$; $f = 0,2 N s/m$

$$2x + 0,2\dot{x} + \ddot{x} = 4 \Rightarrow 2x(s) + 0,2 s x(s) + s^2 x(s) = \frac{4}{s}$$

$$x(s)\left(2 + 0,2s + s^2\right) = \frac{4}{s} \Rightarrow x(s) = \frac{4}{s(s^2 + 0,2s + 2)}$$

$$x(s) = \frac{-1 + j7,08\times10^{-2}}{s + (0,1 - j1,41)} + \frac{2}{s} + \frac{-1 - j7,08\times10^{-2}}{s + (0,1 + j1,41)}$$

$$\Rightarrow x(t) = (-1 + j7,08\times10^{-2})e^{-(0,1-j1,41)t} + (-1 - j7,08\times10^{-2})e^{-(0,1+j1,41)t} + 2$$

$$e^{j\theta} = \cos\theta + j\,\mathrm{sen}\,\theta \quad ; \quad e^{-j\theta} = \cos\theta - j\,\mathrm{sen}\,\theta$$

$$e^{j\theta} + e^{-j\theta} = 2\cos\theta \quad ; \quad e^{j\theta} - e^{-j\theta} = 2j\,\mathrm{sen}\,\theta$$

$$\Rightarrow -e^{-(0,1-j1,41)t} + j7,08\times10^{-2}e^{-(0,1-j1,41)t} - e^{-(0,1+j1,41)t} - j7,08\times10^{-2}e^{-(0,1+j1,41)t}$$

$$(e^{-0,1t} \cdot e^{j1,41t}) + j7,08\times10^{-2}(e^{-0,1t}e^{j1,41t}) - e^{-0,1t}e^{j1,41t} - j7,08\times10^{-2}(e^{-0,1t}e^{-j\ldots}$$

$$(-1 + j7,08\times10^{-2})e^{-0,1t}e^{j1,41t} + (-1 - j7,08\times10^{-2})e^{-0,1t}e^{-j1,41t} + 2$$

$$[-e^{j1,41t} + j7,08\times10^{-2}e^{j1,41t} - e^{-j1,41t} - j7,08\times10^{-2}e^{-j1,41t}]e^{-0,1t} + 2$$

$$[-(e^{j1,41t} + e^{-j1,41t}) + j7,08\times10^{-2}(e^{j1,41t} - e^{-j1,41t})]e^{-0,1t} + 2$$

$$[-2\cos(1,41t) + j7,08\times10^{-2} \cdot 2j\,\mathrm{sen}(1,41t)]e^{-0,1t} + 2 \Rightarrow$$

$$\left[-2\cos(1{,}41t) - 0{,}141\,\text{sen}(1{,}41t)\right] e^{-0{,}1t} + 2$$ 3

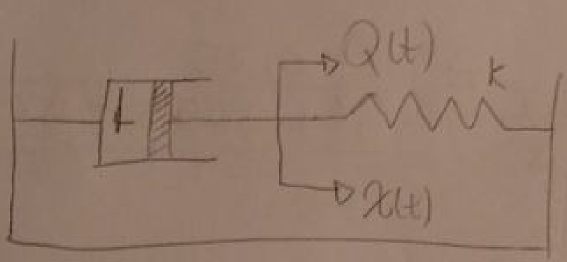
TP 3

1) $y' + 2y = x \Rightarrow sY(s) - Y(0) + 2Y(s) = X(s)$

$Y(s)(s+2) - Y(0) = X(s)$

$$\frac{Y(s)}{X(s)} = \frac{Y(0)}{(s+2)} \qquad \text{P/ CI} = 0 \Rightarrow \frac{Y(s)}{X(s)} = \frac{1}{s+2}$$

2)

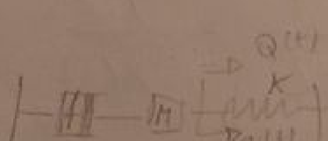


$$f\dot{x} + kx = Q$$

$f\,[s\,x(s) - x(0)] + k\,x(s) = Q(s)$; tomando $\text{CI} = 0$

$\Rightarrow x(s)[fs + k] = Q(s)$

$$\therefore \frac{x(s)}{Q(s)} = \frac{1}{fs + k}$$

P/ el sistema con masa (

$m\ddot{x} + f\dot{x} + kx = Q(t) \Rightarrow m s^2 x(s) + fs\,x(s) + k\,x(s) = Q(s)$

$x(s)[ms^2 + fs + k] = Q(s)$

$$\frac{x(s)}{Q(s)} = \frac{1}{ms^2 + fs + k} = \frac{1}{m}\,\frac{1}{s^2 + \frac{f}{m}s + \frac{k}{m}}$$

P/ 

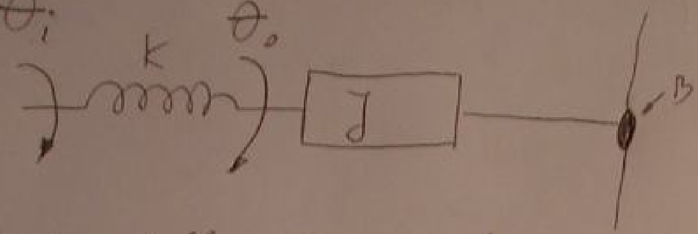
$$\frac{1/M}{s^2 + K/M} = \frac{\sqrt{K/M}}{s^2 + K/M}\;\frac{\sqrt{K/M}}{K}$$

sen $\omega t \Rightarrow \mathcal{L} \Rightarrow \frac{\omega}{s^2 + \omega^2}$

$\Rightarrow \frac{\sqrt{K/M}}{K} \cdot \text{sen}\left(\sqrt{K/M}\,t\right)$

$$\frac{x(s)}{Q(s)} = \frac{1/M}{s^2 + K/M}$$; P/ $Q(t) = \delta(t) \Rightarrow \mathcal{L} \Rightarrow 1 \Rightarrow x(s) = \frac{1/M}{s^2 + K/M}$

3)



$$\sum T(t) = J\ddot{\theta}_o(t)$$

$$-k(\theta_o - \theta_i) - \dot{\theta}_o B = J\ddot{\theta}_o$$

$$-k\theta_o + k\theta_i - \dot{\theta}_o B = J\ddot{\theta}_o$$

$$k\theta_i = J\ddot{\theta}_o + k\theta_o + \dot{\theta}_o B$$

$$k\theta_i(s) = Js^2\theta_o(s) + s\theta_o(s)B + k\theta_o(s)$$

$$k\theta_i(s) = \theta_o(Js^2 + Bs + k)$$

$$\frac{\theta_o(s)}{\theta_i(s)} = \frac{k}{Js^2 + Bs + k} \Rightarrow \frac{\theta_o(s)}{\theta_i(s)} = \frac{k/J}{s^2 + \frac{B}{J}s + \frac{k}{J}}$$

$$\frac{\theta_o(s)}{\theta_i(s)} = \frac{\frac{5N.\cancel{m}/rad}{20kg\,m^{\cancel{2}}}}{s^2 + \frac{1N.\cancel{m}.s/rad}{20kg\,m^{\cancel{2}}}s + \frac{5N.\cancel{m}/rad}{20kg.m^{\cancel{2}}}} = \frac{0,25}{s^2 + 0,05s + 0,25} = \frac{\theta_o(s)}{\theta_i(s)}$$

4)

$$y(t) + 2\int y\,dt = x + \int x\,dt$$

$$Y(s) + 2\frac{Y(s)}{s} = X(s) + \frac{X(s)}{s}$$

$$Y(s)\left[1 + \frac{2}{s}\right] = X(s)\left[1 + \frac{1}{s}\right]$$

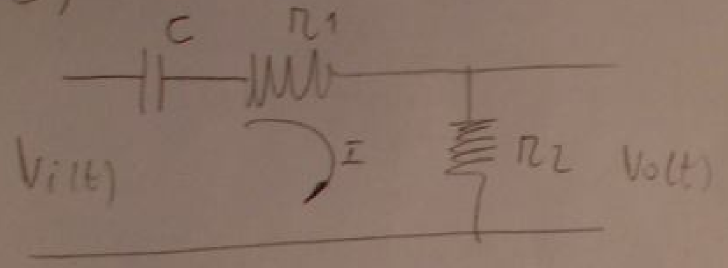
$$\frac{Y(s)}{X(s)} = \frac{\frac{s+1}{\cancel{s}}}{\frac{s+2}{\cancel{s}}} = \frac{s+1}{s+2} = \frac{Y(s)}{X(s)}$$

5)

$$\frac{Y(s)}{X(s)} = \frac{2s+1}{s^2+s+1} \;;\quad s^2Y(s) + sY(s) + Y(s) = 2sX(s) + X(s)$$

$$\ddot{y} + \dot{y} + y = 2\dot{x} + x \quad \text{con } CI = 0$$

6)



$C = 1\,\mu F$

$R_1 = 1\times10^6\,\Omega$

$R_2 = 100k\,\Omega$

$$V_i(t) = \frac{1}{C}\int I\,dt + I(R_1+R_2) \Rightarrow \mathcal{L} \Rightarrow \frac{I(s)}{sC} + I(s)(R_1+R_2)$$

$$V_o(t) = I\,R_2 \Rightarrow \mathcal{L} \Rightarrow I(s)\,R_2$$

$$\frac{V_o}{V_i} = \frac{R_2}{R_1+R_2+\frac{1}{sC}} = \frac{sR_2C}{sC(R_1+R_2)+1}$$

; siempre luego q' dejar la expresión cond término "s" solo en el denominado

$$= \frac{R_2 \cancel{C}}{\cancel{C}(R_1+R_2)} \frac{s}{s+\frac{1}{C(R_1+R_2)}} \Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_1+R_2}\,\frac{s}{s+\frac{1}{C(R_1+R_2)}}$$

$$\frac{V_o}{V_i} = \frac{1}{11}\cdot\frac{s}{s+\frac{10}{11}}$$

7)

$$X(s) \longrightarrow \boxed{\frac{s+1}{s+2}} \longrightarrow \boxed{\frac{5}{s}} \longrightarrow Y(s)$$

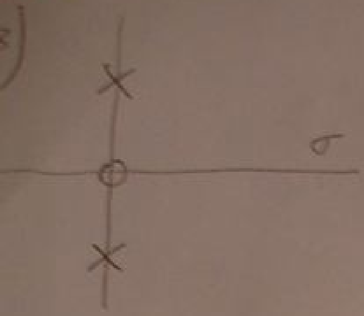
$y(t)\big|_{x(t) = 6\,U(t)}$?

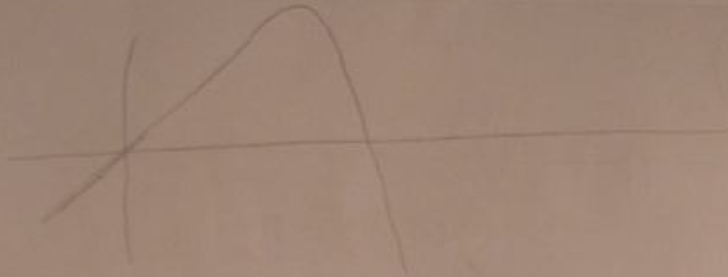
$$y(s) = x(s)\cdot\frac{s+1}{s+2}\cdot\frac{5}{s}$$

$$y(s) = \frac{6}{s}\cdot\frac{s+1}{s+2}\,\frac{5}{s} = 30\cdot\frac{s+1}{s^2(s+2)} = \frac{A_0}{s^2}+\frac{A_1}{s}+\frac{B_0}{s+2}$$

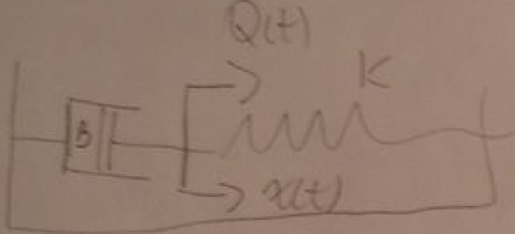
$$A_0 = \lim_{s\to0} s^2\,y(s) = 15$$

$$A_1 = \lim_{s\to0} \frac{d}{ds}\left[30\cdot\frac{s+1}{s+2}\right] = 30\cdot\frac{(s+2)-(s+1)}{(s+2)^2} =$$

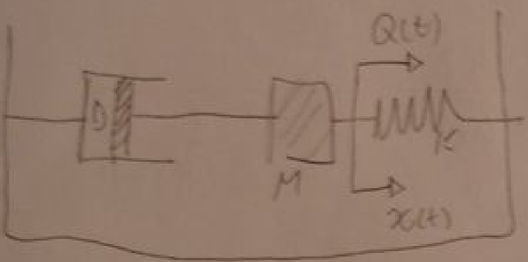




TP N° 1



$$B\dot{x} + kx = Q(t)$$



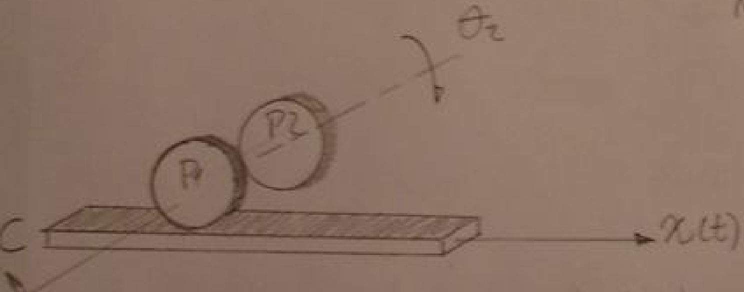
$$\Sigma F = m\ddot{x}$$

$$m\ddot{x} + B\dot{x} + kx = Q(t)$$

$C = 10$ dientes/mm ; $P = \frac{1}{C}$ (paso)

$N_1 = 40$ dientes

$N_2 = 10$ dientes



- $\overbrace{\theta_1 N_1 = \theta_2 N_2}$ → relación por dos engranajes

$$\theta_2 = \theta_1 \cdot \frac{N_1}{N_2}$$

- $\underbrace{C\,x(t)}$ $= N$ → $\frac{\text{Dientes}}{mm} \cdot mm = N$

$$\theta_1(t) = \underbrace{\frac{2\pi}{N_1}}_{\text{cuantos grados implica cada diente}} \cdot \overbrace{C\,x(t)}^{\text{cuantos dientes se movió}}$$

$$\theta_2(t) = \theta_1(t) \cdot \frac{N_1}{N_2} = \frac{2\pi}{\cancel{N_1}} C\,x(t) \cdot \frac{\cancel{N_1}}{N_2}$$

$$\theta_2(t) = \frac{2\pi}{N_2} C\,x(t) \quad ; \quad x(t) = \theta_2(t) \cdot \frac{N_2}{2\pi C}$$

$$x(t) = \theta_2(t) \cdot \frac{N_2}{2\pi} \cdot P$$

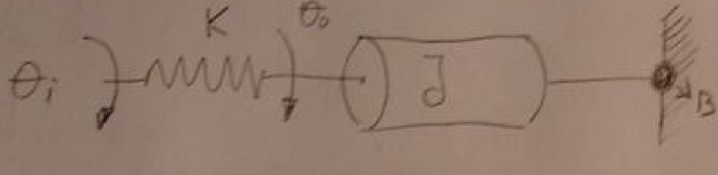
4)

$\sum T(t) = J\ddot{\theta}(t)$

$T(t) - K\theta(t) = J\ddot{\theta}(t)$

$T(t) = J\ddot{\theta}(t) + K\theta(t)$ /

5)



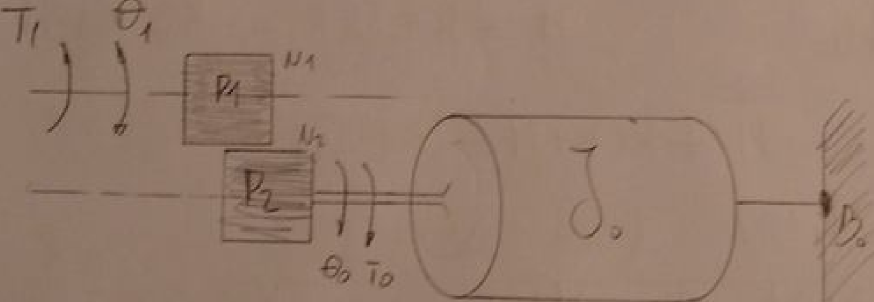
$\sum T(t) = J\ddot{\theta}(t)$

No hay torque aplicado

$\Rightarrow -K(\theta_o - \theta_i) - B\dot{\theta}_o = J\ddot{\theta}_o$

$K\theta_i = J\ddot{\theta}_o + B\dot{\theta}_o + K\theta_o$ /

6)



$N_1\theta_1 = N_2\theta_0$

$\theta_0 = \theta_1 \frac{N_1}{N_2}$

$T_0 = J_0\ddot{\theta}_0 + B\dot{\theta}_0$

$N_1\theta_1 = N_2\theta_0$

$N_1\dot{\theta}_1 = N_2\dot{\theta}_0$

$N_1\ddot{\theta}_1 = N_2\ddot{\theta}_0$

Trabajo

$W = \int F(t)\,dx$

potencia

$dW = F(t)\,dx \Rightarrow p(t) = \frac{dW}{dt} = F(t)\cdot\dot{x}(t)$

Análogamente $\Rightarrow P(t) = T(t)\cdot\dot{\theta}(t)$, $P_1(t)$ y $P_0(t)$ son iguales

Trabajo rotacional

$T_1(t)\cdot\dot{\theta}_1 = T_0(t)\cdot\dot{\theta}_0 = T_0(t)\cdot\frac{N_1}{N_0}\dot{\theta}_1$

$T_1(t) = T_0(t)\frac{N_1}{N_0} \Rightarrow N_0 T_1(t) = N_1 T_0(t)$ /

$\dot{\theta}_0 = \frac{N_1}{N_0}\dot{\theta}_1$; $\ddot{\theta}_0 = \frac{N_1}{N_0}\ddot{\theta}_1$

$T_0 = \frac{N_0}{N_1}T_1(t)$

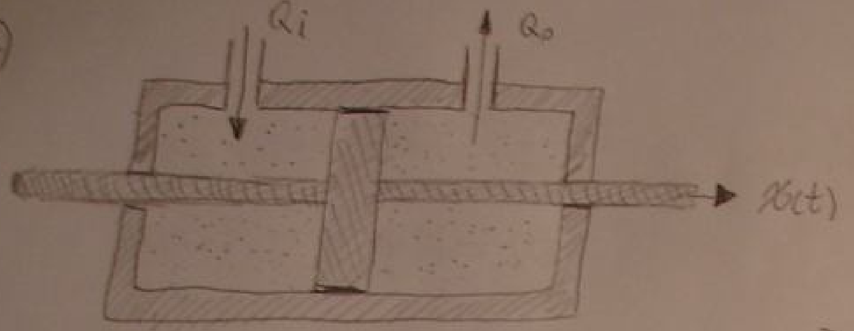
$\frac{N_0}{N_1}T_1(t) = J_0\frac{N_1}{N_0}\ddot{\theta}_1 + B_0\frac{N_1}{N_0}\dot{\theta}_1$

$T_1(t) = \left(\frac{N_1}{N_0}\right)^2\left[J_0\ddot{\theta}_1 + B\dot{\theta}_1\right]$ /

$J_1\text{reflejado} = J_0\left(\frac{N_1}{N_0}\right)^2$

$B_1\text{reflejado} = B_0\left(\frac{N_1}{N_0}\right)^2$

7)



Definición de caudal:

$\cdot Q = A\,v$; $A \Rightarrow$ Area $[L^2][m^2]$

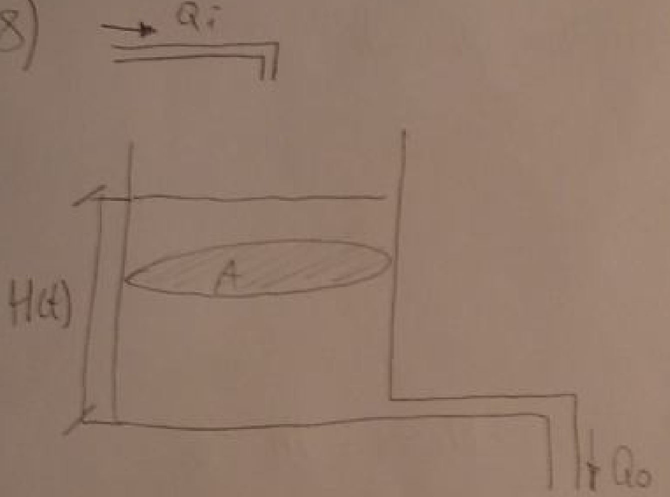
$v \Rightarrow$ Velocidad

$\Rightarrow Q = A \dfrac{dx(t)}{dt}$

$\Rightarrow dx(t) = \dfrac{1}{A} Q(t)\,dt$

$$x(t) = \frac{1}{A}\int Q(t)\,dt$$

8)



$\dfrac{m^3}{seg} = \dfrac{m^2}{seg}\,m \rightarrow RH^{-1}$ ($RH\left[\dfrac{seg}{m^2}\right]$ + Resistencia hidr...

$Q_o = k_1\,H(t)$;

$\dfrac{dH}{dt} = k_2(Q_i - Q_o)$

$\dot{H} = k_2 Q_i - k_2 . k_1 H$

$$\dot{H} + k_2 k_1 H = k_2 Q_i$$

$SH(s) + k_2 k_1 H(s) = k_2 Q_i(s)$

$H(s)\left[s + k_2 k_1\right] = k_2 Q_i(s) \Rightarrow \dfrac{H(s)}{Q_i(s)} = \dfrac{k_2}{s + k_2 k_1}$

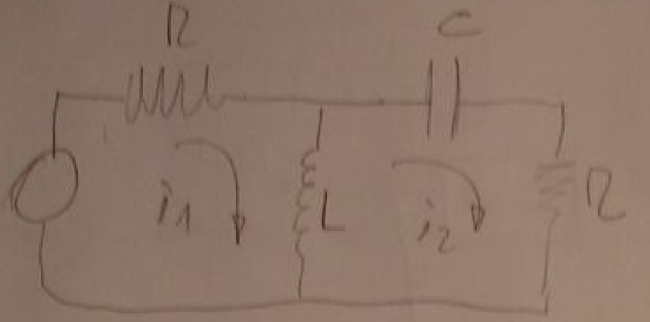
$\Rightarrow k_2 \Rightarrow \dfrac{1}{A}$, $\Rightarrow k_1 = \overbrace{R_H^{-1}}^{\text{resistencia hidráulica}}$ $\quad RH\left[seg/m^2\right]$; $Q\left[m^3/seg\right]$

$\Rightarrow Q_i(t) = A\dfrac{dh(t)}{dt} + \dfrac{1}{RH}.h(t) \Rightarrow \dfrac{H(s)}{Q_i(s)} = \dfrac{1/A}{s + \dfrac{1}{A\,RH}}$

$Q_i(s) = A\,S\,H(s) + \dfrac{1}{RH}H(s)$

$= H(s)\left[AS + \dfrac{1}{RH}\right] \Rightarrow \dfrac{H(s)}{Q_i(s)} = \dfrac{1}{AS + \dfrac{1}{RH}} = \dfrac{1/A}{S + \dfrac{1}{RHA}}$

9)



(6)

$$\begin{cases} E(t) = I_1 R + L\dot{I}_1(t) - L\dot{I}_2(t) \\ 0 = -L\dot{I}_1(t) + L\dot{I}_2(t) + \frac{1}{C}\int i_2\,dt + i_2 R \end{cases}$$

$$\frac{E(s)}{s} = I_1 (R + sL) - I_2 (sL)$$

$$0 = -I_1 (sL) + I_2 \left(sL + \frac{1}{sC} + R\right)$$

$$\begin{bmatrix} E(s)/s \\ 0 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \begin{bmatrix} (R+sL) & -sL \\ -sL & (sL + \frac{1}{sC} + R) \end{bmatrix}$$

$$\Delta = (R+sL)\left(sL + \frac{1}{Cs} + R\right) - (sL)^2$$

$$\Delta_{I_1} = \frac{E(s)}{s} \cdot \left(sL + \frac{1}{Cs} + R\right)$$

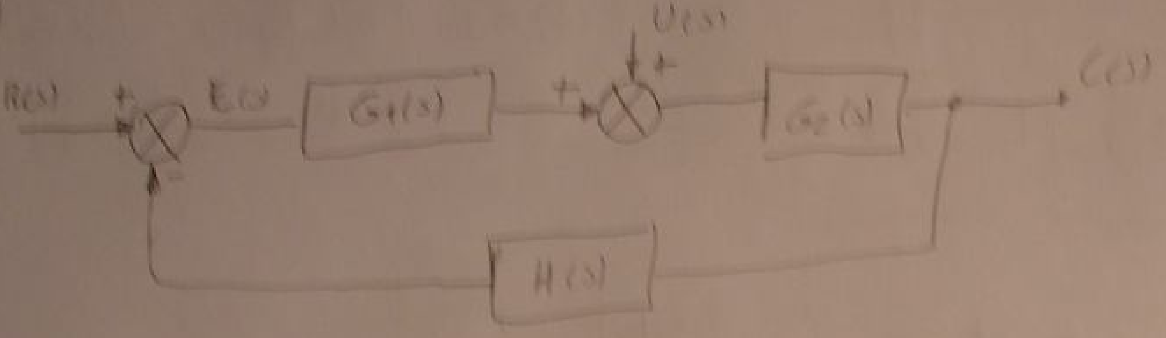
$$\Delta_{I_2} = \frac{E(s)}{s} \cdot sL$$

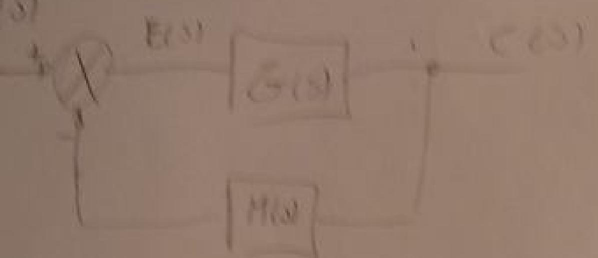
$$I_1 = \frac{E(s)}{s} \cdot \frac{(sL + 1/Cs + R)}{(R+sL)(sL + 1/Cs + R) - (sL)^2}$$

$$I_2 = \frac{E(s)\ sL}{s\,(R+sL)(sL + 1/Cs + R) - (sL)^2}$$

TP4/

1)





$$\begin{cases} C(s) = E(s) \cdot G(s) \\ E(s) = R(s) - C(s)\,H(s) \end{cases}$$

$$C(s) = [R(s) - C(s)\,H(s)]\,G(s)$$

$$C(s) = R(s)\,G(s) - C(s)\,H(s)\,G(s)$$

$$C(s)\,[1 + H(s)\,G(s)] = R(s)\,G(s)$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + \underbrace{H(s)\,G(s)}}$$

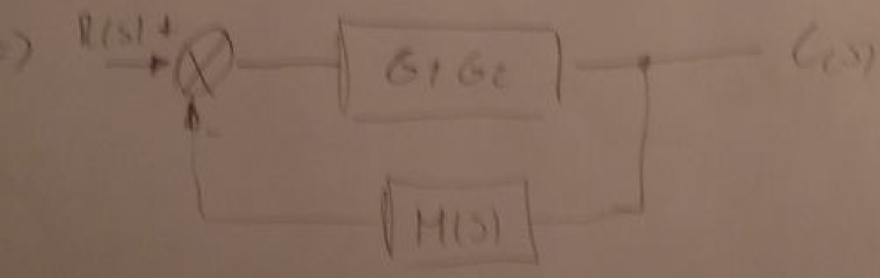
$[1 \pm H(s)\,G(s)]$

+ ⇒ realimentación negativa
− ⇒ realimentación positiva

· $G(s)\,H(s)$ ⇒ fnc de transferencia en lazo abierto

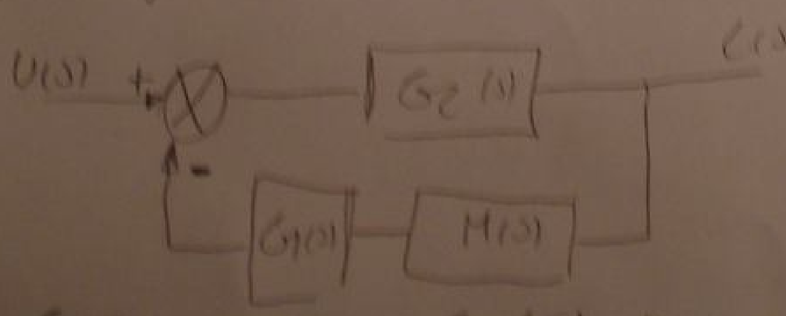
sin U(s)

⇒



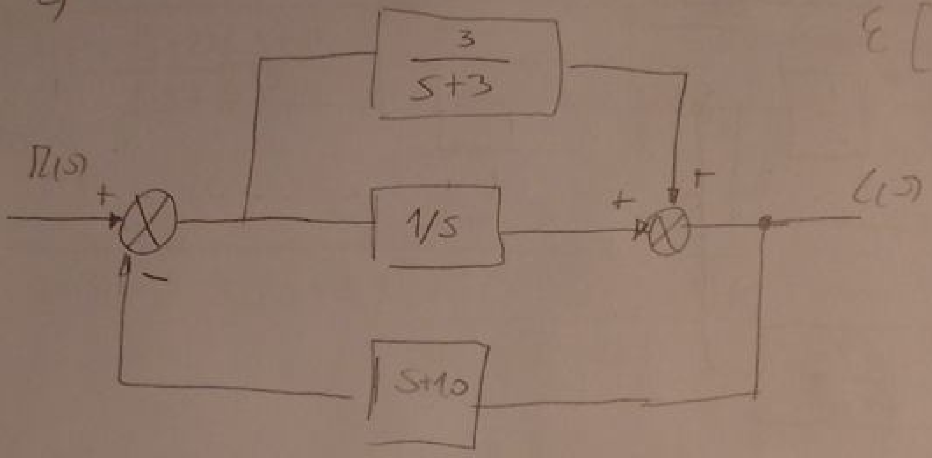
$$\Rightarrow \left.\frac{C(s)}{R(s)}\right|_{U=0} = \frac{G_1(s)\,G_2(s)}{1 + H(s)\,G_1(s)\,G_2(s)}$$

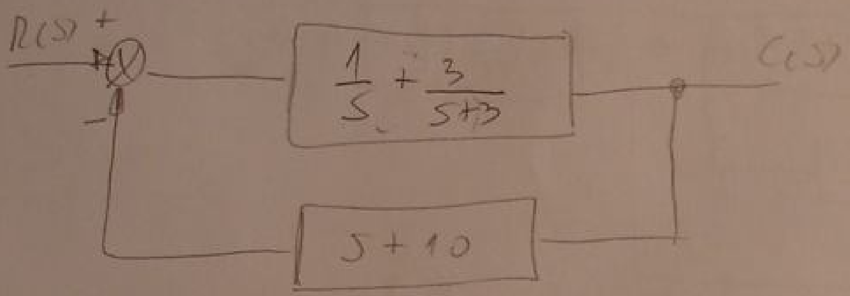
con U(s) y R(s) = 0



$$\Rightarrow \frac{C(s)}{U(s)} = \frac{G_2(s)}{1 + H(s)\,G_1(s)\,G_2(s)}$$

2) ε[7





$$\frac{s+3+3s}{s(s+3)} = \frac{4s+3}{s(s+3)}$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+H(s)G(s)} = \frac{\frac{1}{s}+\frac{3}{s+3}}{1+(s+10)\left[\frac{4s+3}{s(s+3)}\right]}$$

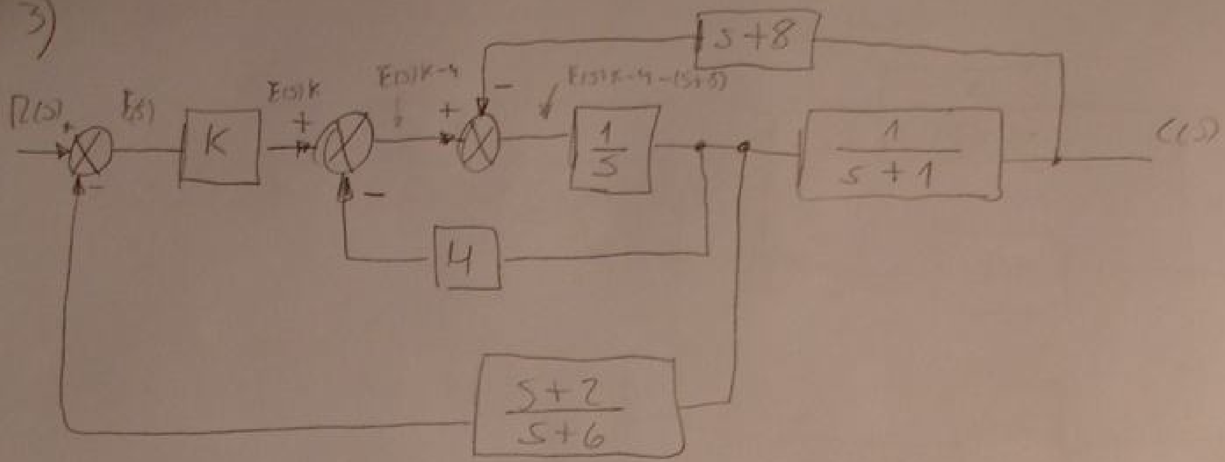
$$= \frac{4s+3}{\left[1+\frac{(s+10)(4s+3)}{s(s+3)}\right]s(s+3)}$$

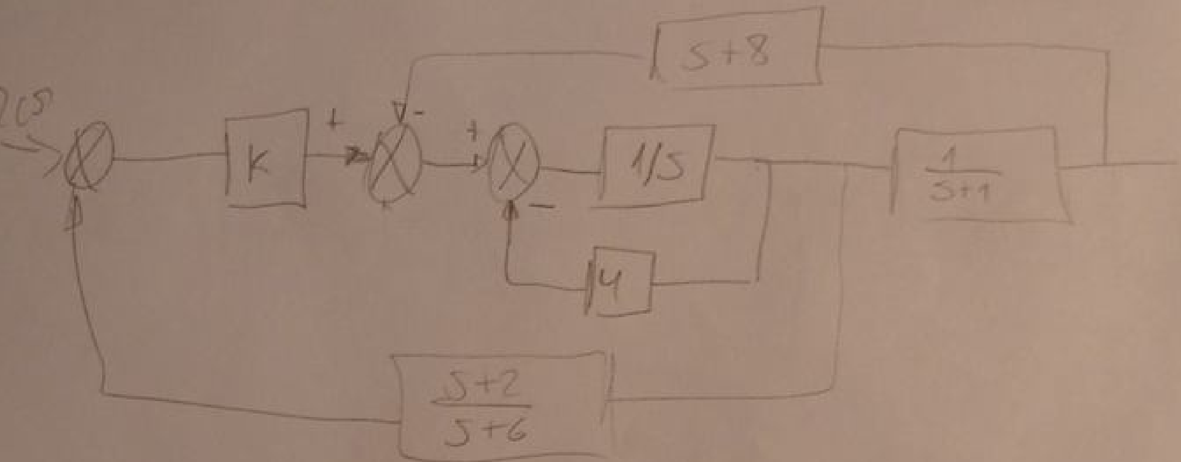
$$= \frac{4s+3}{s(s+3)+4s^2+3s+40s+30} = \frac{4s+3}{s^2+3s+4s^2+3s+40s+30}$$

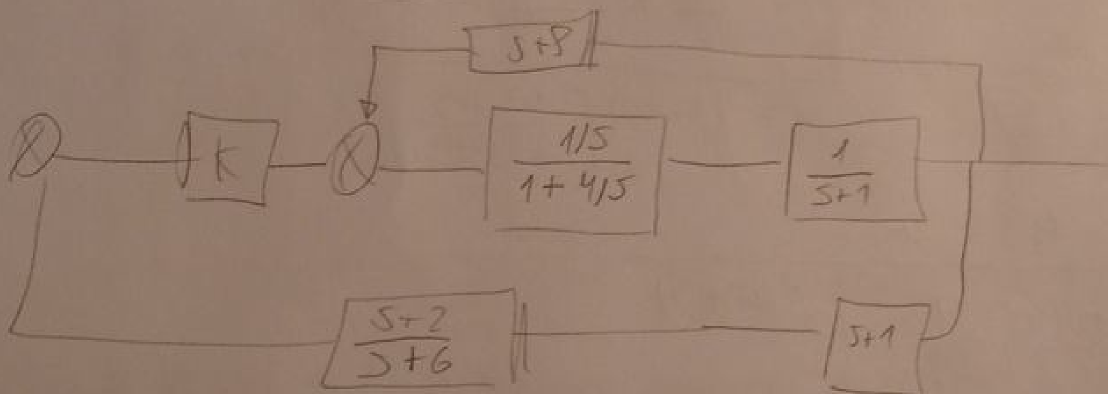
$$= \frac{4s+3}{5s^2+46s+30} = \frac{4}{5}\,\frac{s+3/4}{s^2+\frac{46}{5}s+\frac{30}{5}}$$

$$\frac{C(s)}{R(s)} = 0{,}8\,\frac{s+0{,}75}{s^2+9{,}2s+6}$$

3)

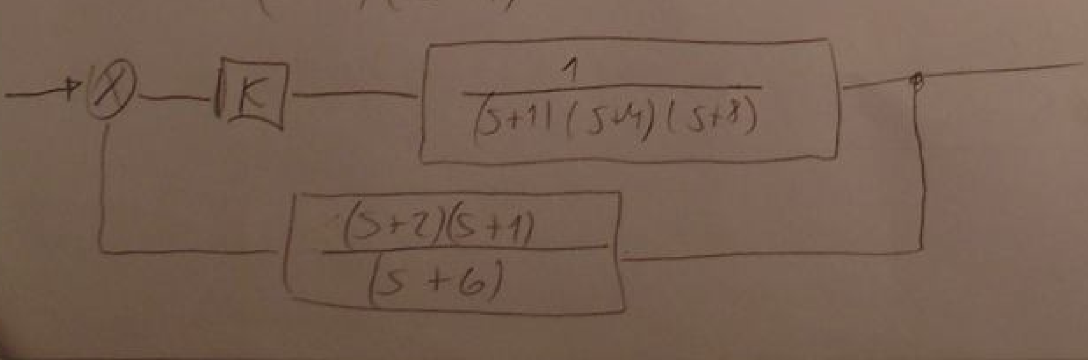






$$\frac{1}{s(1+\frac{4}{s})}\cdot\frac{1}{(s+1)} = \frac{\cancel{s}}{\cancel{s}(s+1)(s+4)} = \frac{1}{(s+1)(s+4)}$$

$$\frac{\frac{1}{(s+1)(s+4)}}{1+(s+8)\frac{1}{(s+1)(s+4)}} = \frac{1}{(s+1)(s+4)+(s+8)}$$



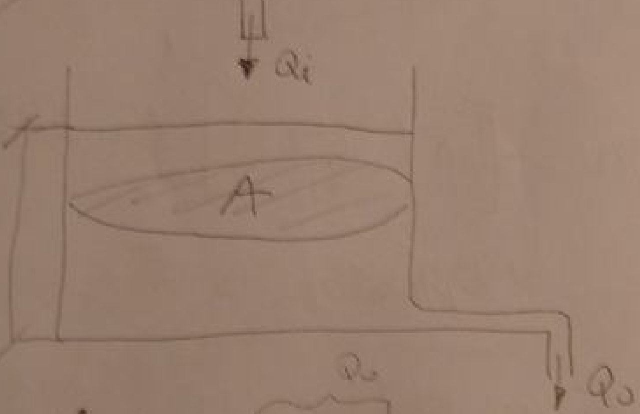
$$\frac{K}{\left[1+\frac{(s+2)(s+1)}{(s+6)}\cdot\frac{K}{(s+1)(s+4)(s+8)}\right](s+1)(s+4)(s+8)}$$

$$=\frac{K}{(s+1)(s+4)(s+8)+\frac{(s+2)(s+1)K}{(s+6)}}$$

$$=\frac{K\quad(s+6)}{(s+1)(s+4)(s+8)(s+6)+(s+2)(s+1)K}$$

TP N° 5

1)



- $h(0)=4m$
- $h(30sg)=2{,}96\,m$
- $\frac{dh(t)}{dt}=K_1 h(t)$
- $Q_0=K_2 h(t)=\frac{1}{R_H}h(t)$

2)

$$Q_i=A\dot{h}(t)+\underbrace{\frac{1}{R_H}h(t)}_{Q_0}\Rightarrow Q_i(s)=AsH(s)+\frac{1}{R_H}H(s)$$

$$=H(s)\left[As+\frac{1}{R_H}\right]$$

$$\frac{H(s)}{Q_i(s)}=\frac{1/A}{s+\frac{1}{AR_H}}$$

b) $Q_i=A\dot{h}(t)+\frac{1}{R_H}h(t)=A\cdot R_H\dot{Q_0}(t)+Q_0(t)$

$$Q_i(s)=AR_H s\,Q_0(s)+Q_0(s)=Q_0(s)\left[AR_H s+1\right]$$

$$\frac{Q_0(s)}{Q_i(s)}=\frac{1/AR_H}{s+\frac{1}{AR_H}}\rightarrow 1/\tau_0$$

c) $\tau = A.R_H$.

R_H?

$$Q_i(s) = A[sH(s) - H(0)] + \frac{1}{R_H} H(s)$$

Tomando en cuenta todos los datos provistos:

$$0 = 2[sH(s) - 4] + \frac{1}{R_H} H(s) = 2sH(s) - 8 + \frac{1}{R_H} H(s)$$

$$0 = H(s)\left[2s + \frac{1}{R_H}\right] - 8 \Rightarrow H(s) = \frac{4}{s + \frac{1}{2R_H}}$$

$$h(t) = 4e^{-t/2R_H} \quad ; \quad h(t)\big|_{t=30\,seg} = 2{,}96m$$

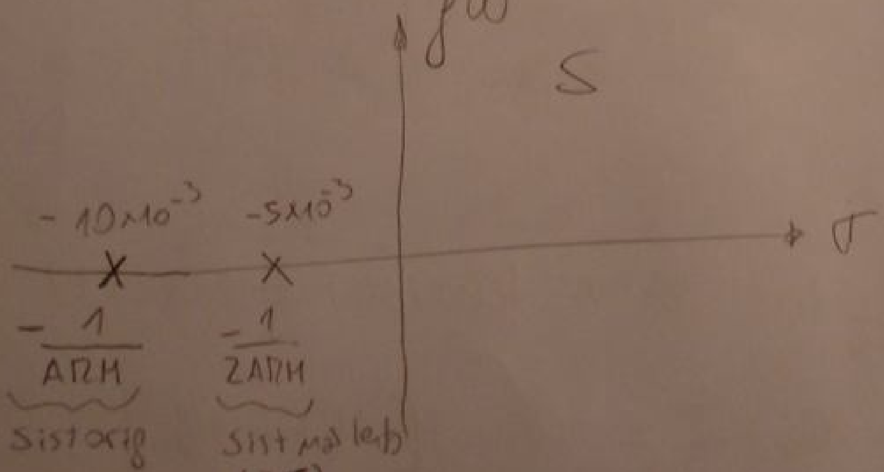
$$\therefore 2{,}96m = 4e^{-30/2R_H}$$

$$\ln\left(\frac{2{,}96m}{4}\right) = -\frac{30}{2R_H} \Rightarrow R_H = -\left[\ln\left(\frac{2{,}96}{4}\right)\right]^{-1} \cdot 15 = 49{,}81\left[\frac{seg}{m^2}\right]$$

$$\therefore \tau = 2 \cdot 49{,}81\,seg \Rightarrow 99{,}633\,seg = \tau$$

d) Depende del A y RH ⇒ forma del tanque y parámetros constructivos.

e) f)



g)

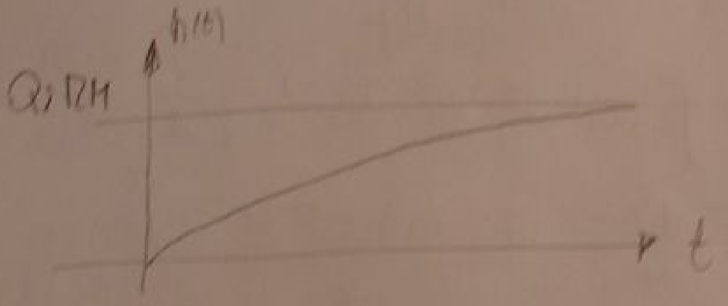
$$\frac{Q_i(s)}{s} = H(s)\left[A s + \frac{1}{RH}\right]$$; suponiendo para $Q_i(s)$ una función escalón.

$h(0) = 0$

$$\frac{H(s)}{Q_i(s)} = \frac{1}{s\left[As + \frac{1}{RH}\right]} = \frac{1/A}{s\left(s + \frac{1}{ARH}\right)}$$

$$\frac{H(s)}{Q_i(s)} = \frac{-50}{x + 10 \cdot 10^{-3}} + \frac{50}{s} \Rightarrow \mathcal{L} \to -50\, e^{-0{,}01t} + 50$$

$$\Rightarrow H(t) = Q_i \cdot RH\left[1 - e^{-0{,}01t}\right]$$



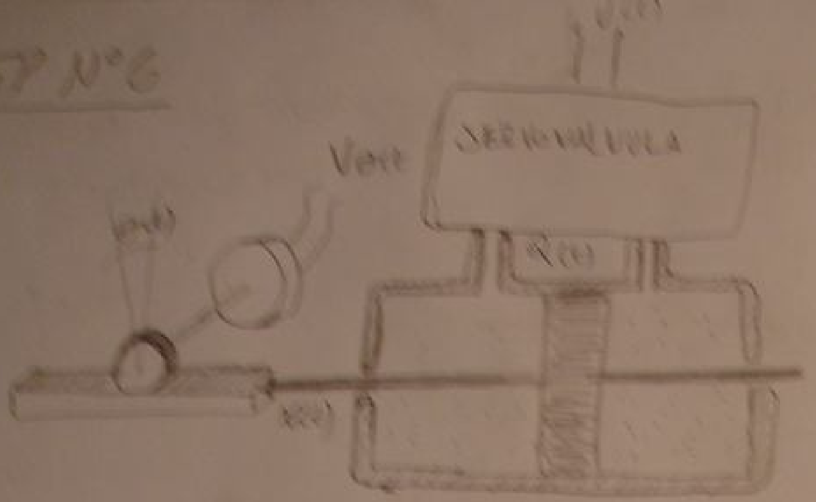
h) $$h(t)\Big|_{Q_i = 50\ell}\Big|_{t\to\infty} = Q_i RH$$; $1\,L = 1\,dm^3$

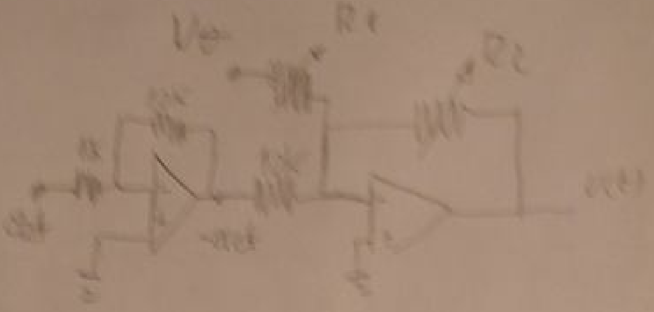
$Q_i\,[L/seg] \Rightarrow [dm^3/seg]$

$$1\,dm^3 \Leftrightarrow 0{,}001\,m^3 \Leftrightarrow 1\,L$$

$$h(t) = 50\,L \cdot \frac{0{,}001\,m^3}{L} \cdot 50\,\frac{seg}{m^2} = 2{,}5\,mts = h(\infty)$$

TP N°6





$e_{ref} \rightarrow U(t) \Rightarrow Q(t) \Rightarrow x(t) \Rightarrow \theta(t) \Rightarrow V_\theta(t)$

$$\frac{Q(s)}{U(s)} = \frac{a\,\omega t}{(s+10)^2}\ , \qquad Q(t) = \frac{dV(t)}{dt} = A\frac{dx(t)}{dt} = A\,\dot{x}(t)$$

$$\dot{x}(t) = \frac{1}{A}\,Q(t) \Rightarrow x(t) = \frac{1}{A}\int Q(t)\,dt \Rightarrow X(s) = \frac{1}{A}\,\frac{Q(s)}{s}$$

$$A = \pi\frac{d^2}{4} = \frac{\pi\cdot(35{,}052\times10^{-3}\,m)^2}{4} =$$

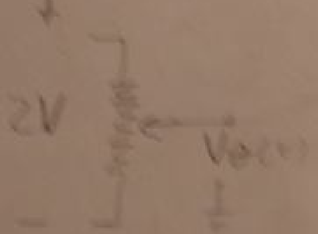
$$\frac{X(s)}{Q(s)} = \frac{1}{AS}$$

$2V$ — 2π

$\theta(t)$

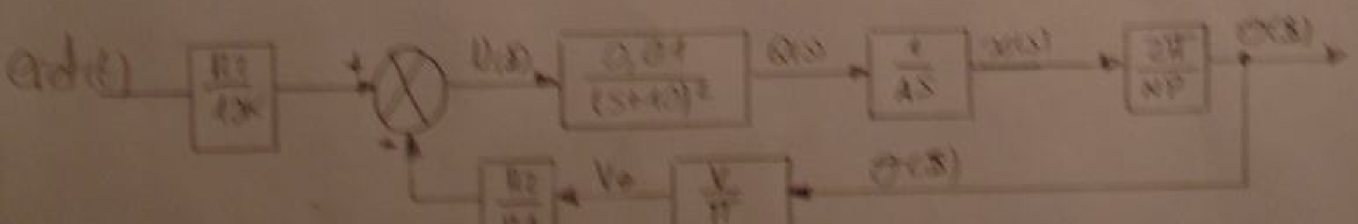
$$\frac{\theta(t)}{x(t)} = \frac{2\pi}{NP}$$

$$\frac{\cancel{2}V\cdot\theta(t)}{\cancel{2}\pi}$$



$$\frac{V_\theta(t)}{\theta(t)} = \frac{V}{\pi}$$

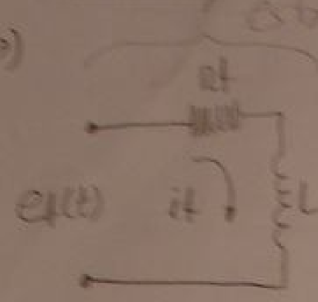
$$U(t) = \frac{R_2}{1k}\cdot e_{ref}(t) - V_\theta(t)\,\frac{R_2}{R_1}$$

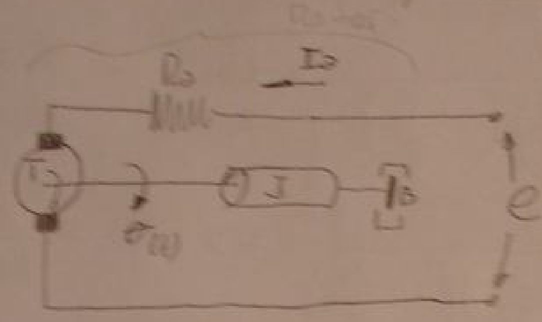


TP N°7

Motor cc controlado por campo. (10)

a)





$\frac{\theta(t)}{Ef(t)}$?

; $e_a = cte$

1) $T = K\,\Phi(t)\, i_a$

2) $\Phi(t) = K'\, i_f$

3) $i_a = K''$

$T = K K' K''\, i_f \Rightarrow \underline{T = K_T\, i_f}$; $K_T = K K' K''$

$\sum T = J\ddot{\theta}(t)$

$T - B\dot{\theta}(t) = J\ddot{\theta}(t)$

$K_T i_f - B\dot{\theta}(t) = J\ddot{\theta}(t)$

$K_T I_f(s) - sB\theta(s) = Js^2\theta(s)$

$K_T \frac{e_f(s)}{L_f\left(s+\frac{R_f}{L_f}\right)} = Js^2\theta(s) + sB\theta(s)$

$= \theta(s)\left[Js^2 + sB\right]$

Buscamos una expresión para $I_f(s)$

$e_f(t) = i_f(t)R_f + L_f\dot{i}_f(t)$

$e_f(s) = I_f(s)R_f + sL_f I_f(s)$

$e_f(s) = I_f(s)\left[R_f + sL_f\right]$

$I_f(s) = \frac{e_f(s)}{R_f + sL_f} = \frac{e_f(s)/L_f}{\left(s+\frac{R_f}{L_f}\right)}$

; $\frac{I_f(s)}{E_f(s)} = \frac{1/L_f}{s + R_f/L_f}$

$$\Rightarrow \frac{\theta(s)}{e_f(s)} = \frac{K_T}{L_f\left(s+\frac{R_f}{L_f}\right)\left(Js^2+sB\right)} = \frac{K_T/(L_f J)}{s\left(s+\frac{R_f}{L_f}\right)\left(s+\frac{B}{J}\right)}$$

$$\frac{\theta(s)}{e_f(s)} = \frac{K_T}{L_f J}\cdot\frac{1}{s\left(s+\frac{R_f}{L_f}\right)\left(s+\frac{B}{J}\right)}$$

b)
- $L_f = 20\,Hy$
- $R_f = 120\,\Omega$
- $J = 1{,}35\,[kg.m^2]$
- $B = 0{,}467\,[N.m.s/rad]$
- $i_a = 15\,[A]$
- $K_T =$

$$\frac{\dot{\theta}(s)}{e_f(s)} = \frac{K_T}{L_f J}\;\frac{1}{\left(s+\frac{R_f}{L_f}\right)\left(s+\frac{B}{J}\right)} ;$$

Podemos calcular K_T usando TVF

$$\frac{\dot{\theta}(s)}{e_f(s)} = \frac{K_T}{L_f . J} \frac{1}{\left(s+\frac{R_f}{L_f}\right)\left(s+\frac{B}{J}\right)} \Rightarrow \lim_{s\to 0} s\cdot F(s) \quad \Big| \ \frac{e_f(s)}{s}$$

$$\lim_{s\to 0} s\dot{\theta}(s) = \lim_{s\to 0} \cancel{s} \frac{K_T}{L_f . J} \frac{e_f(s)}{\cancel{s}\left(s+\frac{R_f}{L_f}\right)\left(s+\frac{B}{J}\right)} \Rightarrow \dot{\theta}(s) = \frac{K_T}{L_f . J} \frac{e_f(s)}{\frac{R_f}{L_f}\ \frac{B}{J}}$$

$$K_T = \frac{\dot{\theta}(s)\ L_f . \cancel{J} . R_f . B}{e_f(s)\ L_f . \cancel{J}} = \frac{\dot{\theta}(s)}{e_f(s)} . R_f . B$$

$1\,rpm \rightarrow 2\pi \frac{rad}{seg}$; $\dot{\theta}(t)\ [rad/seg]$

$1200\,rpm \rightarrow 7{,}539 \times 10^{3} \frac{rad}{seg}$

$$K_T = 7{,}539\times 10^{3} \frac{\cancel{rad}}{seg} \cdot \frac{1}{110V} \cdot 120\Omega \cdot 0{,}667 \frac{Nm . s}{\cancel{rad}} = 5{,}4856\times 10^{3}$$

$K_T = 5{,}4856\times 10^{3}$

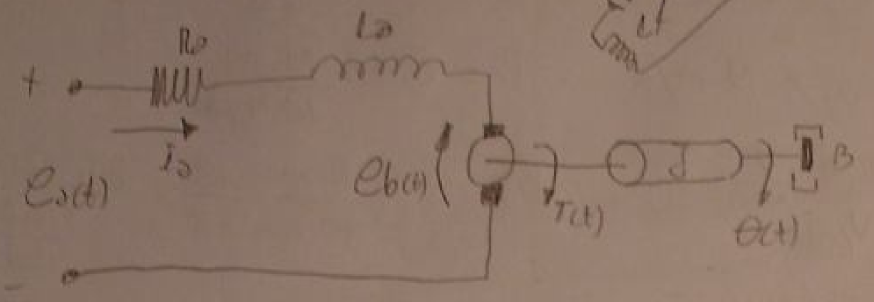
$$\Rightarrow \frac{\dot{\theta}(s)}{E_f(s)} = 74{,}963\times 10^{-3} \frac{1}{(s+6)(s+0{,}5)}$$

TP N° 8

$e_f = c_{te}$

Motor cc controlado por ARMADURA

11



$$\frac{\dot{\theta}(t)}{e_a(t)}\ ?$$

Para este caso.

$T(t) = K_T\, I_a(t)$

$$\left.\begin{aligned} e_a(t) &= i_a(t)\,R_a + L_a\,\dot{i}(t) + e_b(t) \\ T(t) - B\,\dot{\theta}(t) &= J\,\ddot{\theta}(t) \end{aligned}\right\}$$

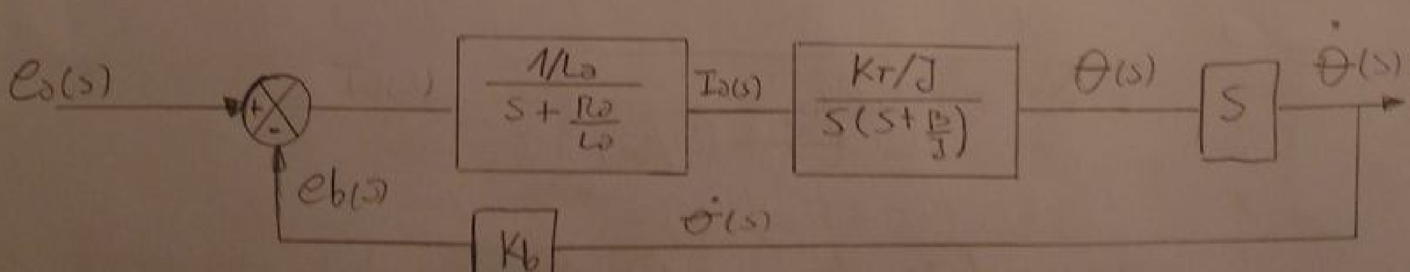
$$e_a(s) = I_a(s)\left[R_a + s L_a\right] + e_b(s) =$$

$$T(s) - \theta(s)\left[sB + s^2 J\right] = 0$$

$$\frac{I_a(s)}{e_a(s) - e_b(s)} = \frac{1/L_a}{s + \frac{R_a}{L_a}}$$

$$\frac{\theta(s)}{T(s)} = \frac{1}{s\left[B + sJ\right]} = \frac{1/J}{s\left(s + \frac{B}{J}\right)} = \frac{\theta(s)}{T(s)}$$

$$\frac{\theta(s)}{i_a(s)} = \frac{K_T/J}{s\left(s + \frac{B}{J}\right)}$$



$$\frac{\dot{\theta}(s)}{e_a(s)} = \frac{\frac{1/L_a}{s + \frac{R_a}{L_a}} \cdot \frac{K_T/J}{\cancel{s}(s + B/J)} \cdot \cancel{s}}{1 + \frac{1/L_a}{s + \frac{R_a}{L_a}}\,\frac{K_T/J}{\cancel{s}(s + B/J)}\,\cancel{s} \cdot K_b} = \frac{\frac{1/L_a}{\left(s + \frac{R_a}{L_a}\right)} \cdot \frac{K_T/J}{(s + B/J)}}{1 + \frac{1/L_a}{\left(s + \frac{R_a}{L_a}\right)}\,\frac{K_T/J}{(s + B/J)}\,K_b} = \frac{\dot{\theta}(s)}{e_a(s)}$$

P/

$e = 150V$ · $R_a = 0,57\,\Omega$ · $J = 0,016\ kg\cdot m^2$

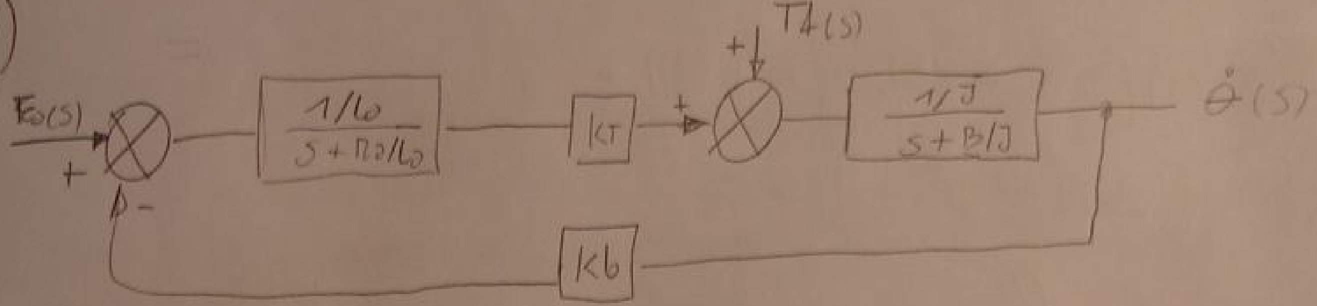
· $L_a = 4\,mH$ · $k_T = 0,8\ N\cdot m/A$ · $B = 0,15\ \frac{N\cdot m}{10^3 rpm}$

· $k_b = 18,9\ \frac{V}{10^3 rpm} = 3\ V/rads$ · $B = 0,15\ \frac{N\cdot M}{10^3 rpm}\ \frac{10^3}{2\pi} = 23,87\times10^{-3}\ \frac{N\cdot M}{rad/s}$

$$\Rightarrow \dot{\theta}(s) = \frac{\frac{1}{L_a \frac{R_a}{L_a}} \cdot k_T \cdot \frac{1}{J\frac{B}{J}}\ e(s)}{1 + k_T k_b \frac{1}{R_a B}} = \frac{k_T\, e}{R_a B\left(1 + \frac{k_T k_b}{R_a B}\right)}$$

$\dot{\theta}(s) = 7,89$ rpm parece...

)



P/ $T_A(s) = 0$

$$\Rightarrow \frac{\dot{\theta}(s)}{E_a(s)} = \frac{k_T}{J L_a}\ \frac{1}{s^2 + \left(\frac{R_a}{L_a} + \frac{B}{J}\right)s + \frac{B R_a + k_T k_b}{J L_a}}$$

/ $E_a(s) \to 0$

$$\frac{\dot{\theta}(s)}{T_A(s)} = \frac{\frac{1/J}{s+B/J}}{1 + \frac{1/J}{s+B/J}\,K_b K_T\,\frac{1/L_a}{s+R_a/L_a}} = \frac{1/J}{\left(s+\frac{B}{J}\right) + \frac{K_b K_T}{J L_a}\ \frac{1}{\cancel{(s+B/J)}(s+R_a/L_a)}}$$

$$= \frac{1}{J}\cdot\frac{1}{s+\frac{B}{J} + \frac{k_b k_T}{J L_a\,(s+R_a/L_a)}}$$

$$= \frac{1}{J}\cdot\frac{J L_a\,(s+R_a/L_a)}{(s+\frac{B}{J})\left[J L_a\,(s+R_a/L_a)\right] + k_b k_T}$$

$$= \frac{\cancel{J} L_a}{J^2 L_a}\ \frac{(s+R_a/L_a)}{(s+B/J)(s+R_a/L_a) + \frac{k_b k_T}{J L_a}}$$

$$= \frac{1}{J} \frac{s + R_a/L_a}{s^2 + s\left(\frac{R_a}{L_a} + \frac{B}{J}\right) + \frac{B}{J}\frac{R_a}{L_a} + \frac{K_b K_T}{J L_a}}$$

12

$$\frac{\dot{\theta}(s)}{T_f(s)} = \frac{1}{J} \frac{s + R_a/L_a}{s^2 + s\left(\frac{R_a}{L_a} + \frac{B}{J}\right) + \frac{B R_a + K_b K_T}{J L_a}}$$

$\therefore$

$$\dot{\theta}(s) = \frac{K_T}{J L_a} \frac{E_a(s)}{s^2 + \left(\frac{R_a}{L_a} + \frac{B}{J}\right)s + \frac{B R_a + K_T K_b}{J L_a}} - \frac{1}{J} \frac{(s + R_a/L_a)\, T_f(s)}{s^2 + s\left(\frac{R_a}{L_a} + \frac{B}{J}\right) + \frac{B R_a + K_b K_T}{J L_a}}$$

$$\dot{\theta}(s) = \left[\frac{1}{J} \cdot \frac{1}{s^2 + \left(\frac{R_a}{L_a} + \frac{B}{J}\right)s + \frac{B R_a + K_T K_b}{J L_a}}\right]\left[\frac{K_T}{L_a} E_a(s) - (s + R_a/L_a)\, T_f(s)\right]$$

5) Disminuir la tensión de campo en 2 y tomando en cuenta la dependencia lineal propuesta nos lleva a:

$K_T' = \frac{K_T}{2}$; $K_B' = \frac{K_B}{2}$;

$e_a(t) = E_a\, \mu(t) \Rightarrow e_a(s) = \frac{E_a}{s}$

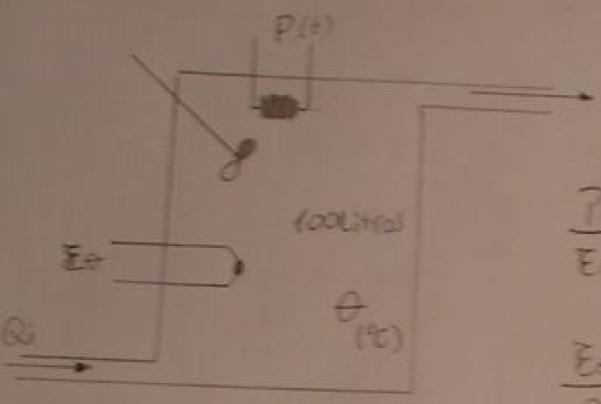
$$\dot{\theta}(s) = \frac{K_T}{J L_a} \frac{E_a}{s^2 + \left(\frac{R_a}{L_a} + \frac{B}{J}\right)s + \frac{B R_a + K_T K_B}{J L_a}} \cdot \frac{1}{s}$$

$$\dot{\theta}(0) \cdot s = \frac{K_T}{\cancel{J L_a}} \frac{E_a}{\frac{B R_a + K_T K_B}{\cancel{J L_a}}} = 49{,}71 \text{ rad/seg}$$

$$\dot{\theta}(0)' s = \frac{K_T}{2\cancel{J L_a}} \frac{E_a}{\frac{4 B R_a + K_T K_B}{4\cancel{J L_a}}} = 97{,}78 \text{ rad/seg}$$

Sube la velocidad x2 $K_T K_B \gg 4 B R_A$ ...

TPNº9 / Modelización de un sistema térmico

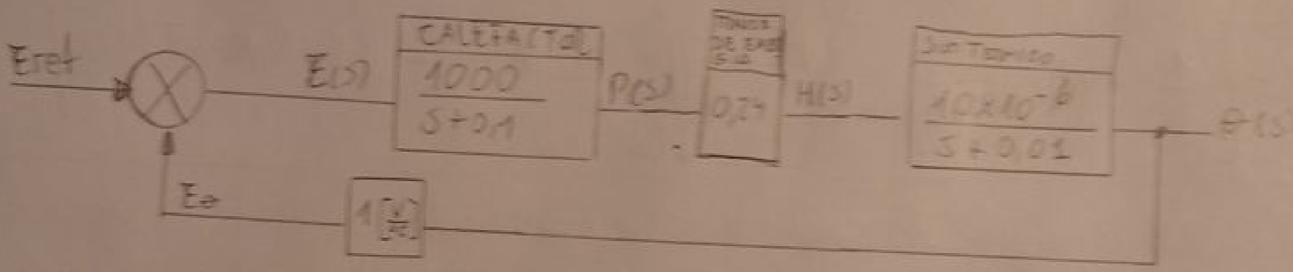


$$Q_i = Q_o = 1\frac{L}{s} = 1\frac{dm^3}{s}$$

$$\frac{P(s)}{E(s)} = \frac{1000}{s+0{,}1} \quad \frac{[W]}{[V]}$$

$$\frac{E_\theta(s)}{\theta(s)} = 1 \ [V/ºC]$$

1) Tensión ref − tensión medida → calefactor → P(t) → Transferencia de calor del calefactor al líquido → → comportamiento del sistema térmico al cual se le ha transferido la energía → Tensión medida



- Transferencia de energía:

$$Q(t) = \frac{1\,Cal}{4{,}186\,Joule}\,w(t) \Rightarrow \text{Equivalente mecánico de calor}$$

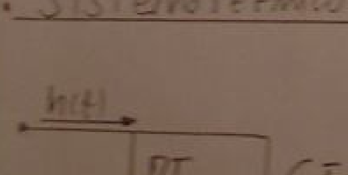
cada caloría ⟷ 4,184 Joule (w(t))

$\frac{dQ(t)}{dt} = h(t) \Rightarrow$ Energía transferida al líquido; Calor que llega en un tiempo dt

$$h(t) = \frac{1\,Cal}{4{,}186\,Joule}\cdot \overbrace{p(t)}^{\text{Potencia}} \Rightarrow h(t) = 0{,}24\frac{Cal}{Joule}\,p(t)$$

$$\frac{H(s)}{P(s)} = 0{,}24\frac{Cal}{Joule}$$

- Sistema térmico: A la transferencia térmica h(t), se le opone una R_T y también influye la presencia de una capacidad térmica C_T



$$R_T = \frac{1}{g\,Ce\,Q} \ ; \quad C_T = m\,Ce \qquad C_T = g\,V\,Ce$$

- g = Peso específico
- Ce = Calor específico
- V = Volumen
- Q = Caudal

$$h(t) = \frac{\theta(t)}{R_T} + C_T\frac{d\theta(t)}{dt}$$

$$H(s) = \theta(s)\left[\frac{1}{R_T} + s\,C_T\right] \Rightarrow \frac{\theta(s)}{H(s)} = \frac{1/C_T}{s + 1/(R_T C_T)}$$

$$C_T = \underbrace{\frac{1\,Kg}{dm^3}}_{\text{Peso específico del agua}}\cdot\underbrace{100\,dm^3}_{\text{Volumen}}\cdot\frac{1\,Cal}{g\,ºC} = 100\times10^3\frac{Cal}{ºC} = C_T \ \Big|\ R_T = \frac{1}{\frac{1\,Kg}{dm^3}\cdot\frac{1\,Cal}{g\,ºC}\cdot\frac{1\,dm^3}{s}} = 1\times10^{-3}\frac{seg\,ºC}{Cal}$$

Ce ⇒ Energía a administrar en forma de calor para elevar la temperatura de un cuerpo en 1 grado centígrado (Definición de calor específico) (13)

$$\frac{\theta(s)}{H(s)} = \frac{10\times10^{-6}}{s+0{,}01}$$

2) $$\frac{\theta}{Ed} = \frac{\frac{1000}{(s+0{,}1)}\cdot 0{,}24\cdot\frac{10\times10^{-6}}{(s+0{,}01)}}{1+\frac{1000}{(s+0{,}1)}\cdot 0{,}24\,\frac{10\times10^{-6}}{(s+0{,}01)}} = \frac{2{,}4\times10^{-3}}{(s+0{,}1)(s+0{,}01)+2{,}4\times10^{-3}}$$

$$\frac{\theta(s)}{Ed(s)} = \frac{2{,}4\times10^{-3}}{s^2+0{,}11s+3{,}4\times10^{-3}}\left[\frac{°C}{V}\right]$$

3)

$$\theta(\infty) = \lim_{s\to0} s\,\theta(s) = \frac{10\cdot2{,}4\times10^{-3}}{3{,}4\times10^{-3}} = 7{,}058\ °C$$ ⇒ para $Ed = 10V$

$\theta(\infty) = 7{,}058\ °C$

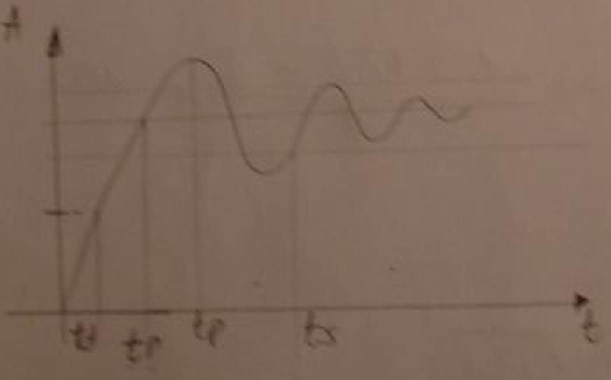
$t_{s5\%} = \frac{3}{\sigma}$; $\xi = \sigma\,\omega_n$

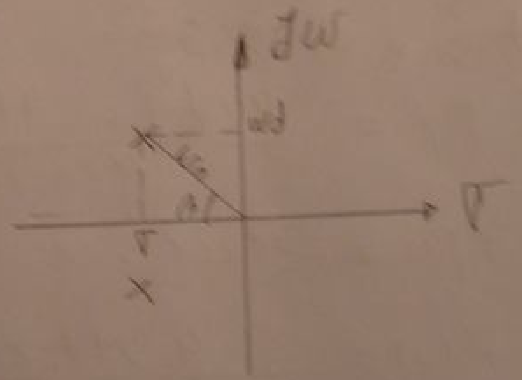
$$\Rightarrow s^2+0{,}11s+3{,}4\times10^{-3} = s^2+2\xi\omega_n s+\omega_n^2$$

$\omega_n = \sqrt{3{,}4\times10^{-3}}$; $2\xi\omega_n = 2\sigma = 0{,}11 \Rightarrow \sigma = 55\times10^{-3}$

$t_{s5\%} = \frac{3}{\sigma} = 54{,}\widehat{54}\ sg$ $\quad(t_{s2\%} = 72{,}\widehat{72}\ sg)$

TP 10: Respuesta Transitoria.





• $t_{s5\%} = \frac{3}{\sigma}$; $t_{s2\%} = \frac{4}{\sigma} \Rightarrow \sigma = \frac{3}{t_{s5\%}}$ ó $\frac{4}{t_{s2\%}}$

• $\omega_n = \sqrt{\sigma^2+\omega d^2}$; $\omega d = \frac{\pi}{tp}$; $\sigma = \xi\,\omega_n \Rightarrow$ • $\xi = \cos\beta$

• $\xi = \frac{\sigma}{\omega_n}$

$$Mp\% = \frac{y(tp) - y(\infty)}{y(\infty)} 100\% = e^{-(\sigma \pi / \omega d)} \cdot 100 = e^{-(\sigma\, tp)} \cdot 100$$

$$= e^{-\left(\frac{\xi \pi}{\sqrt{1-\xi^2}}\right)} \cdot 100$$

$$\frac{\omega n^2}{s^2 + 2\xi \omega n s + \omega n^2}$$

$0{,}95 \sim 1{,}05$

1- $\sigma = \frac{3}{ts_{5\%}} = \frac{3}{0{,}156} = 19{,}23\ sg^{-1}$

2- $\omega n = \sqrt{\sigma^2 + \omega d^2}$

$\omega d = \frac{\pi}{tp} = \frac{\pi}{5\times10^{-2}} = 62{,}83\ rad/sg$

$\Rightarrow \omega n = \sqrt{19{,}23^2 + 62{,}83^2} = \underline{65{,}707 = \omega n}$

3- $\omega d = 62{,}83\ rad/sg$

4- $\xi = \frac{\sigma}{\omega n} = \frac{19{,}23\ sg^{-1}}{65{,}707\ rad/sg} = 0{,}292$

5- $Td = 1{,}8\times10^{-2}\ sg$

6- $Tr = 3\times10^{-2}\ sg$

7- $Tp = 5\times10^{-2}\ sg$

8- $ts_{5\%} = 0{,}156\ sg$

9- $Mp = \frac{y(tp) - y(\infty)}{y(\infty)} 100 = \frac{1{,}3722 - 1}{1} \cdot 100$

$Mp\% = 37{,}22\%$

$$\Rightarrow F(s) = \frac{4{,}317\times10^3}{s^2 + 38{,}37 s + 4{,}317\times10^3}$$

B)

$$\frac{y(s)}{x(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$ => No xq la figura muestra la respuesta de un sistema de primer orden.

$y(t) = 100\left(1 - e^{-t/\tau}\right)$; $x(s) = \frac{50}{s}$; $x(t) = 50\,u(t)$

$86{,}47 = 100\left(1 - e^{-1/\tau}\right)$; $864{,}7 \times 10^{-3} = 1 - e^{-1/\tau}$

$e^{-1/\tau} = 0{,}1353 \Rightarrow -\frac{1}{\tau} = \ln(0{,}1353)$

$\tau = \frac{-1}{\ln(0{,}1353)} = 0{,}5$

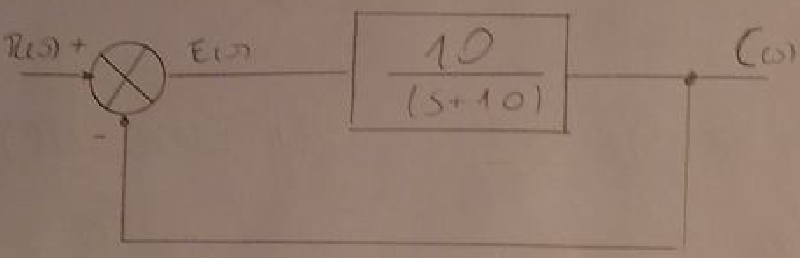
$\Rightarrow y(s) = 100\left(\frac{1}{s} - \frac{1}{s+2}\right)$

$\Rightarrow y(s) = \frac{100}{s} - \frac{100}{s+2} = \frac{100(\cancel{s}+2-\cancel{s})}{s\,(s+2)} = \frac{200}{s}\,\frac{1}{s+2}$

$\frac{y(s)}{x(s)} = \frac{y(s)}{\frac{50}{s}} = \frac{200}{\cancel{s}} \cdot \frac{\cancel{s}}{50}\,\frac{1}{s+2} \Rightarrow \frac{y(s)}{x(s)} = \frac{4}{s+2}$

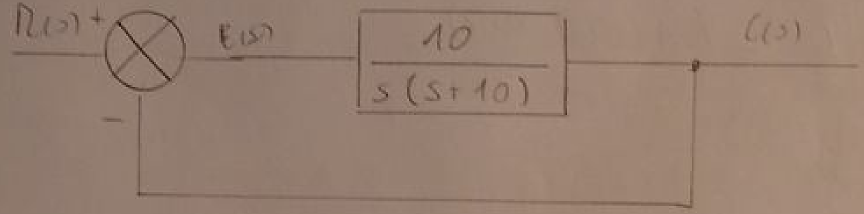
TP N° 12

"A"



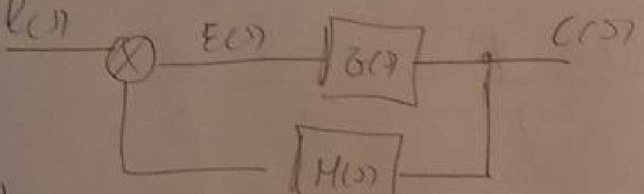
Tipo cero
(orden 1)

"B"



Tipo uno.
(orden dos)

Para todo sistema:



$$\Rightarrow \frac{E(s)}{R(s)} = \frac{1}{1+G(s)H(s)}$$

1)

	A	B	
$K_p = \lim_{s\to 0} G(s)\,H(s)$	$\frac{10}{10} = 1$	∞	K_p
$K_v = \lim_{s\to 0} s\,G(s)\,H(s)$	$0 \cdot \frac{10}{10} = 0$	$\frac{\cancel{s}\,10}{\cancel{s}(s+10)} = 1$	K_v
$K_a = \lim_{s\to 0} s^2\,G(s)\,H(s)$	$\frac{s^2\,10}{(s+10)} = 0$	$\frac{s^{\cancel{2}}\,10}{\cancel{s}(s+10)} = 0$	K_a

2)

R(s)	A	B	ess
Escalón $(1/s)$	$\cancel{s}\cdot\frac{1}{1+K_p}\cdot\frac{1}{\cancel{s}} = 0,5$	$\cancel{s}\,\frac{1}{1+K_p}\cdot\frac{1}{\cancel{s}} = 0$	$\frac{1}{1+K_p}$
Rampa $(1/s^2)$	$\cancel{s}\cdot\frac{1}{1+K_v}\,\frac{1}{s^{\cancel{2}}} = \infty$	$\cancel{s}\,\frac{1}{1+\frac{10}{(s+10)s}}\,\frac{1}{s^{\cancel{2}}} = \frac{1}{s+\frac{10}{s+10}} = 1$	$\frac{1}{K_v}$
Parábola $(1/s^3)$	$\cancel{s}\cdot\frac{1}{1+K_a}\,\frac{1}{s^{\cancel{3}2}} = \infty$	$\cancel{s}\,\frac{1}{1+K_a}\,\frac{1}{s^{\cancel{3}2}} = \infty$	$\frac{1}{K_a}$

TP 13

rampa para "A" es sistema de tipo uno.

$$E(s) = \frac{1}{1+\frac{10}{s+10}} \cdot \frac{1}{s^2} = \frac{s+10}{s^2(s+10+10)} = \frac{s+10}{s^2(s+20)}$$

$$= \frac{A_0}{s^2} + \frac{A_1}{s} + \frac{B_0}{s+20} = \frac{0,5}{s^2} + \frac{0,025}{s} - \frac{0,025}{s+20}$$

$$\mathcal{L}^{-1} \Rightarrow \underline{e(t) = 0,5t + 0,025 - 0,025e^{-20t}}$$

$$e(t) = R(t) - c(t) \Rightarrow R(t) = t$$

P/B $\Rightarrow$ sistema de orden dos.

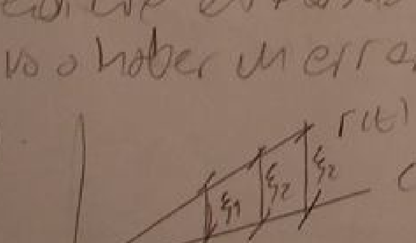
$$E(s) = \frac{1}{1+\frac{10}{(s+10)s}} \frac{1}{s^2} = \frac{(s+10)\cancel{s}}{[(s+10)s+10]s^{\cancel{2}}} = \frac{s+10}{(s^2+10s+10)s}$$

$$= \frac{-1}{s+1,12} + \frac{1}{s} + \frac{0,0164}{s+8,87}$$

$$\mathcal{L}^{-1} \Rightarrow \underline{e(t) = 1 - e^{-1,12t} + 0,0164e^{-8,87t}}$$

$$c(t) = t - e(t) = t - 1 - e^{-1,12t} + 0,0164e^{-8,87t}$$

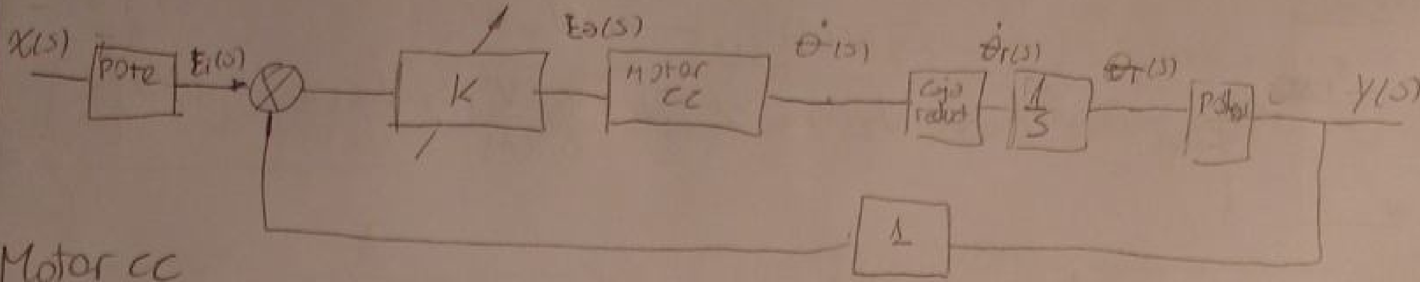
En el sistema "A" en la expresión de error vemos que hay un término dependiente en forma lineal de "t". Esto quiere decir que para "A" va a haber un error creciente con el tiempo para una entrada rampa.



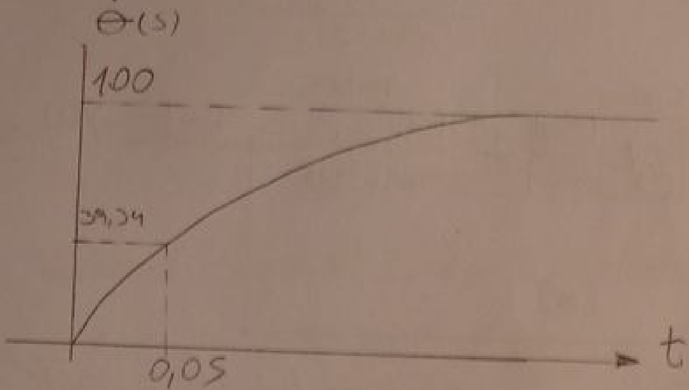
$\xi_1 < \xi_2 < \xi_3 \ldots$

Esto no sucede así, para el sistema "B". El error máximo se da para $t=0$ (creo...) y para $t=\infty$ habrá un error estacionario fijo.

TP 13



Motor cc



$$\dot{\theta}(t) = 100\left(1 - e^{-(t/\tau)}\right)$$

$$39{,}34 = 100\left(1 - e^{-0{,}05/\tau}\right)$$

$$\frac{39{,}34}{100} = 1 - e^{-0{,}05/\tau}$$

$$1 - \frac{39{,}34}{100} = e^{-0{,}05/\tau}$$

$$\ln\left(1 - \frac{39{,}34}{100}\right) = \frac{-0{,}05}{\tau} \Rightarrow \tau = \frac{-0{,}05}{\ln\left(1 - \frac{39{,}34}{100}\right)} = 0{,}1$$

$\Rightarrow \dot{\theta}(t) = 100\left[1 - e^{-(t/0{,}1)}\right]$; esta velocidad, fue producto de haber entrado con $\frac{10}{s} = E_o(s)$

$$\Rightarrow \dot{\theta}(s) = 100\left[\frac{1}{s} - \frac{1}{s+10}\right] = 100\left[\frac{s+10-s}{s(s+10)}\right]$$

$$\dot{\theta}(s) = 100\,\frac{10}{s(s+10)} \;;\; \Rightarrow \frac{\dot{\theta}(s)}{E_o(s)} = 10\cancel{0}\cdot\frac{10}{\cancel{s}(s+10)}\cdot\frac{\cancel{s}}{\cancel{10}}$$

$$\frac{\dot{\theta}(s)}{E_o(s)} = \frac{100}{s+10} \Rightarrow \text{Motor CC}$$

Caja Reductora:

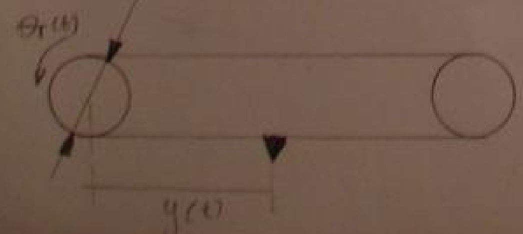
Relación 1/10 $\Rightarrow \dfrac{\dot{\theta}_r(s)}{\dot{\theta}(s)} = 0{,}1\ [\text{Adim}]$

2π — 2 cm

θ_r

$\Rightarrow d = \theta \cdot r$

Polea: 2cm

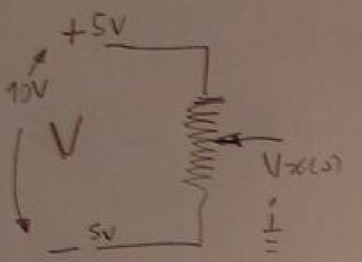


$$y(s) = \theta_r(s)\cdot r$$

$$\Rightarrow \frac{y(s)}{\theta_r(s)} = 0{,}01\left[\frac{m}{rad}\right]$$

2) P/ K = 2,5

Potenciometros de 10 cm de carrera.

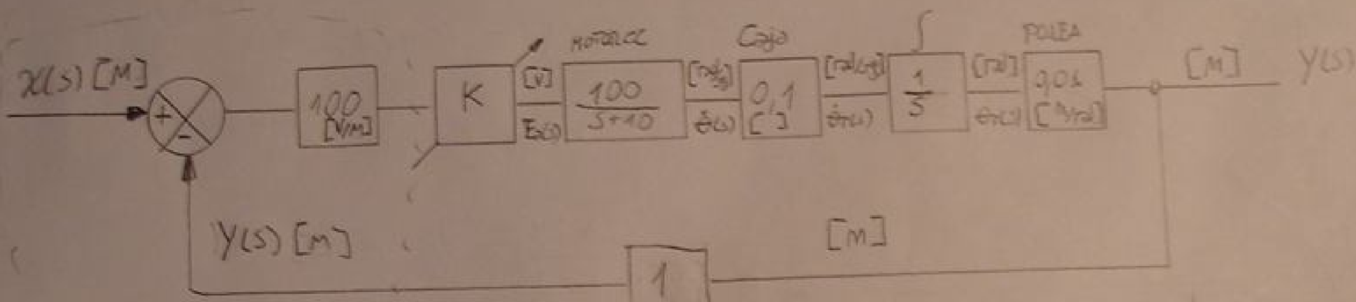


$\Rightarrow$ 10 cm — 10V

$x(s)$ — $V_{x(s)}$

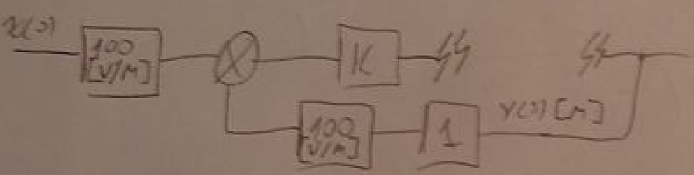
$$\Rightarrow V_{x(s)} = x(s) \cdot \frac{10V}{0,1M} \Rightarrow \frac{V_{x(s)}}{x(s)} = 100 \left[\frac{V}{M}\right]$$

El sistema nos queda de la siguiente forma:

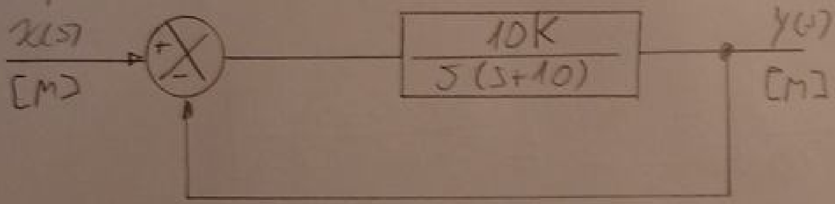


$\Rightarrow$ A fines de análisis se puso así, en realidad esta parte debe...

Queda:



Simplificando:



$$\frac{Y(s)}{x(s)} = \frac{10K}{\left[1 + \frac{10k}{s(s+10)}\right] s(s+10)} = \frac{10K}{s(s+10) + 10k} = \frac{10K}{s^2 + 10s + 10K}$$

$$\frac{Y(s)}{x(s)} = \frac{10.K}{s^2 + 10s + 10.K}$$

Para una entrada de $0,01\, u(t) \Rightarrow \frac{0,01}{s}$

$$\Rightarrow Y(s) = \frac{10 \cdot K}{s^2 + 10s + 10k} \cdot \frac{0,01}{s} \; ; \quad P/ \lim_{s \to 0} Y(s)\, s \Rightarrow \frac{\cancel{10K}\; 0,01}{\cancel{10k}}$$

$Y(\infty) = 0,01$ mts

2) P/ K = 2,5

- Sobrepico
- t_s
- w_d

$\Rightarrow 25 = w_n^2$; $10 = 2\xi w_n \Rightarrow \xi = \frac{10}{2w_n}$

$\xi = 1$ $(w_n = \sqrt{25} = 5)$; $2w_n\xi = 2\sigma \Rightarrow \sigma = 5$

Si $\xi = 1 \Rightarrow$ críticamente amortiguado, no presenta sobrepico

3)

$e(s) = \frac{1}{1 + \frac{10K}{s(s+10)}}$; $e_{ss} = \lim_{s \to 0} s\,e(s) = \frac{s}{1 + \frac{10K}{s(s+10)}}$

Rampa $\Rightarrow = \frac{\cancel{s}(s+10)}{s(s+10) + 10K} \cdot \frac{1}{\cancel{s^2}} = \frac{10}{10K} = \frac{1}{K} = \underline{0,4 = e_{ss}}$

4) Variamos la ganancia de K aumentándola 10 veces:

$\Rightarrow K' = 10K = 25 \Rightarrow e_{ss} = \frac{1}{25} = \underline{0,04 = e_{ss}}$

5)

$Mp\% = e^{-(\sigma \pi / w_d)} \cdot 100 = e^{-\left(\frac{\xi \pi}{\sqrt{1-\xi^2}}\right)} \cdot 100$

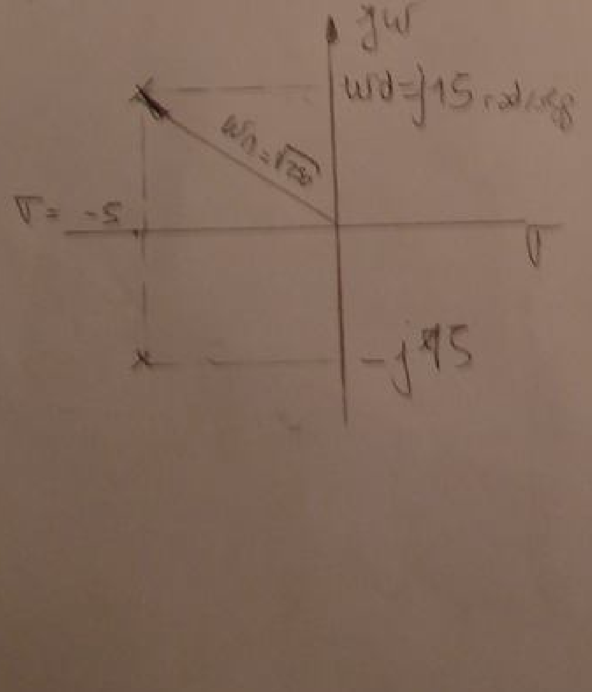
$\Rightarrow 250 = w_n^2 \Rightarrow w_n = \sqrt{250}$; $2 w_n \xi = 10 \Rightarrow \xi = \frac{10}{2w_n}$

$\xi = 0,316$

$\Rightarrow \underline{Mp\% = 35,12\ \%}$

$\cancel{2}\xi w_n = \cancel{2}\sigma = 5$; $w_n^2 = \sigma^2 + w_d^2 \Rightarrow w_d = \sqrt{w_n^2 - \sigma^2}$

$w_d = 15$ rad/seg



__No__ es aceptable pero podría mejorarse con un compensador de atraso

TP N° 15

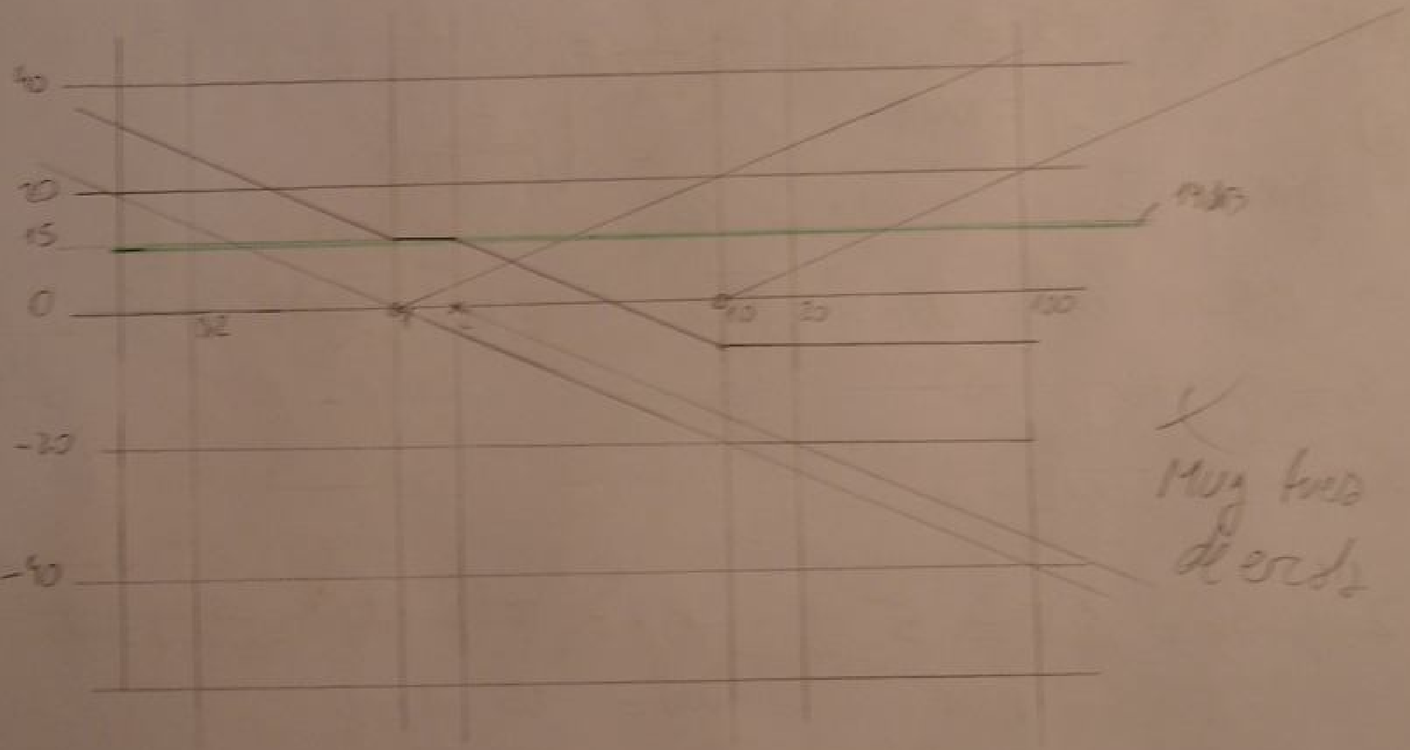
1) $y'' + 2y' = x'' + 11x' + 10x$

$s^2 y(s) + 2s\,y(s) = s^2 x(s) + 11s\,x(s) + 10\,x(s)$

$y(s)\left[s^2 + 2s\right] = x(s)\left[s^2 + 11s + 10\right]$

$$G(s)H(s) = \frac{y(s)}{x(s)} = \frac{s^2 + 11s + 10}{s^2 + 2s} = \frac{(s+1)(s+10)}{s\,(s+2)}$$

$$G(s)H(s) = \frac{10}{2}\;\frac{(s+1)(s/10 + 1)}{s\,(s/2 + 1)} \qquad K_T = 14\,dB$$



Muy bien dicho

2)

$$H(s) = \frac{(s+5)}{s^2+4s+400} \quad ; \quad G(s) = \frac{K}{s^2}(s+1)$$

$$4 = 2\xi\omega_n \; , \; \omega_n^2 = 400 \Rightarrow \omega_n = \sqrt{400} \; ; \; \xi = \frac{4}{2\omega_n} = 0,1$$
$$\omega_n = 20$$

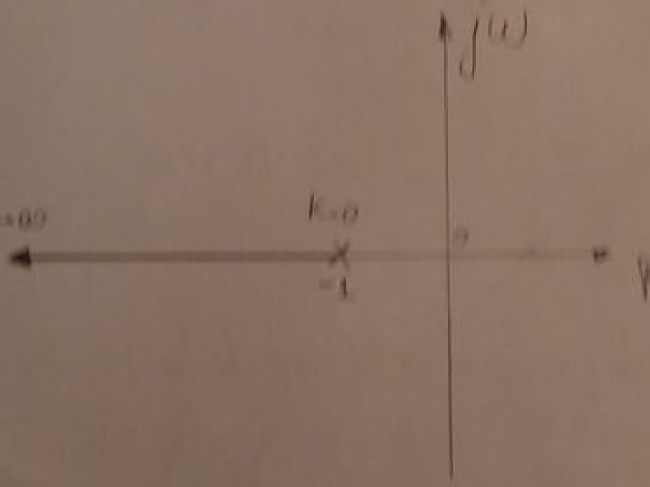
$$\Rightarrow M_p = \frac{1}{2\xi\sqrt{1-\xi^2}} = 5,025 \Rightarrow 20\log|M_p| = \underline{14,02 = |M_p|_{dB}}$$

$$H(s)\,G(s) = \frac{80\,(s+1)\,(s+5)}{(s^2+4s+400)\,s^2} = \frac{80\,(s+1)\,(0,2\,s+1)}{(2,5\times10^{-3}\,s^2+0,01\,s+1)\,s^2}$$

74,12 es el valor correcto para KT (para ξ dado = -13 db)

TP 16

1) $G(s) = \dfrac{K}{s+1} = G(s)\,H(s)$



$$\frac{C(s)}{R(s)} = \frac{K}{s+1+K} = \frac{K}{s+(1+K)}$$

$$R(s) = \frac{1}{s}$$

$$\Rightarrow C(s) = \frac{K}{s\,[s+(1+K)]}$$

$$C(s) = \frac{A}{s} + \frac{B}{s+(1+K)}$$

$$A = \frac{K}{1+K} \quad ; \quad B = \frac{K}{-1-K} \Rightarrow C(s) = \frac{K}{1+K}\,\frac{1}{s} - \frac{K}{1+K}\,\frac{1}{s+(1+K)}$$

$$\mathcal{L}^{-1} \Rightarrow C(t) = \frac{K}{1+K}\left[1 - e^{-t(1+K)}\right] \; ; \quad \tau = \frac{1}{1+K}$$

$$C(t) = \frac{K}{\tau}\left[1 - e^{-t/\tau}\right] \Rightarrow$$ A mayor ganancia, mayor rapidez de respuesta

2)

$$G(s)\,H(s) = \frac{K}{s(s+10)} =$$

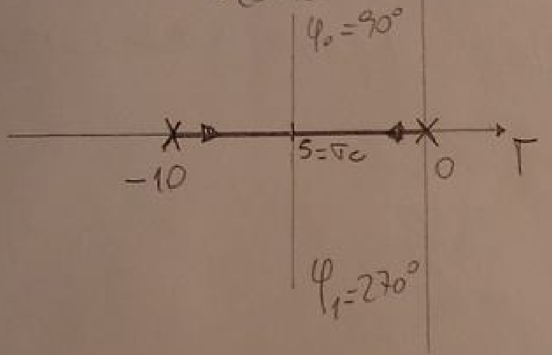
$$\frac{C(s)}{R(s)} = \frac{K}{s(s+10)+K} = \frac{K}{s^2+10s+K}$$

$$s^2+10s+K = s^2 + 2\xi\omega_n s + \omega_n^2$$

$$\Rightarrow \omega_n = \sqrt{K}\ ;\quad 10 = 2\xi\omega_n \Rightarrow \xi = \frac{10}{2\omega_n} = \frac{10}{2\sqrt{K}}$$

$$\xi = \frac{5}{\sqrt{K}}\ ;$$

$$G(s)\,H(s) = \frac{K}{s(s+10)}$$



• N° asint $= p - z$

• $\varphi_k = 180° \dfrac{(2K+1)}{p-z}$; $K = 0, 1, \dots p-z-1$

$\Rightarrow p - z = 2$; $K = 0, 1$

$$\Rightarrow \varphi_0 = 180 \cdot \frac{(2 \cdot 0 + 1)}{2} = 90°$$

$$\Rightarrow \varphi_1 = 180 \cdot \frac{3}{2} = 270°$$

• $\sigma_c = \dfrac{\sum Re[p] - \sum Re[z]}{p-z} = \dfrac{(0+10) - 0}{2} \Rightarrow \sigma_c = 5$

• $\sigma_c \Rightarrow$ Varicentro de asíntotas.

• Punto de bifurcación: Punto en el cual el lugar de raíces deja el eje de los reales

Usamos la ecuación característica igualada a cero.

$1 + G(s)H(s) = 0 \quad \Big| \quad K = -s^2 - 10s$

$s^2 + 10s + K = 0 \quad \Big| \quad \dfrac{\partial K}{\partial s} = -(2s+10)$

Igualamos la 1er derivada de K respecto de s a cero y despejamos el valor de s:

$2s + 10 = 0 \Rightarrow \underline{s = -5} \Rightarrow$ Pto de bifurcación

Este pto es valido solo si cae dentro del lugar de raíces!

- Análisis de estabilidad:

Aplicamos Routh para $\Rightarrow s^2 + 10s + K = 1 + G(s)M(s) = 0$

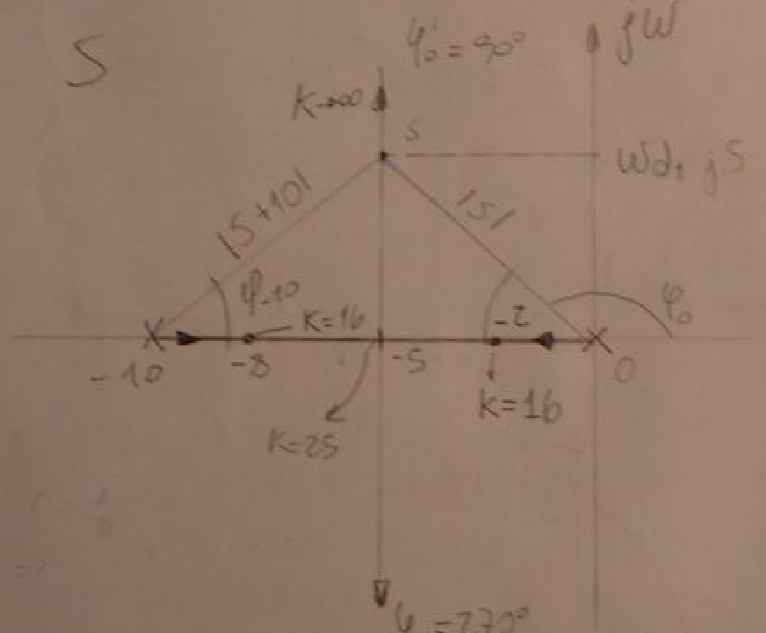
s^2	1	K
s^1	10	0
s^0	K	

$\frac{10K - 1 \cdot 0}{10}$

$\Rightarrow$ El sistema será estable para todo valor de $K \geq 0$ (No hay cambio de signo en la 1ª columna)

- Gráfico de la ecuación característica (lugar de raíces)

$$G(s)M(s) = -1 = -1 + j0 = \underbrace{|1| \angle 180}_{\text{condición para que un punto sea lugar de raíces.}}$$



siempre se forman triángulos equiláteros en este caso.

$w_n^2 = w_d^2 + \sigma^2$

$w_d = \sqrt{w_n^2 - \sigma^2}$

$w_n = \sqrt{K}$ $\quad w_n = \sqrt{w_d^2 + \sigma^2}$

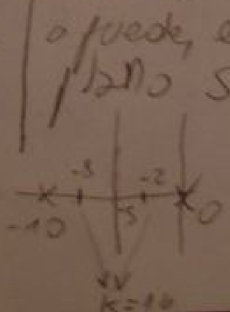
$w_d = \sqrt{K - \sigma^2}$

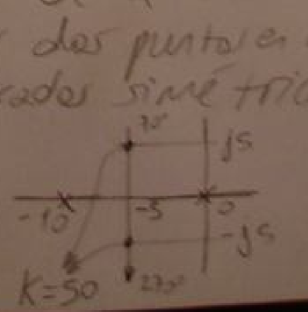
$w_d = \sqrt{K - 25}$

Para $K < 25$ w_d no tiene sentido físico, sería un sistema sobreamortiguado .. y estos sistemas no presentan sobrepico! Así sí estaríamos moviéndonos en el eje de las σ para $K < 25$

Para $s = -2 + j0 \Rightarrow$
$\quad = -8 + j0 \Rightarrow$ $\quad K = 16$

Para un valor de K existen o pueden existir dos puntos en el plano S ubicados simétricamente





P/ $s = 5 + j5$

$|K| = \frac{\Pi |s + p_j|}{\Pi |s + z_j|} = 7{,}07 \cdot 7{,}07 = 50$

3)

$$G(s)H(s) = \frac{K}{s(s+10)(s+20)} = \frac{K}{s^3+30s^2+200s}$$

$$\frac{C(s)}{R(s)} = \frac{K}{s(s+10)(s+20)+K}$$

N° de Asíntotas $\Rightarrow$ $p-z = 3$

$$\varphi_k = 180° \frac{(2k-1)}{p-z} \quad K = 0,1,2 \quad \begin{cases} \varphi_0 = -60° \\ \varphi_1 = 60° \\ \varphi_2 = 180° \end{cases}$$

$$\sigma_c = \frac{\sum Re|P| - \sum Re|Z|}{p-z} = \frac{-10-20}{3} = -10$$

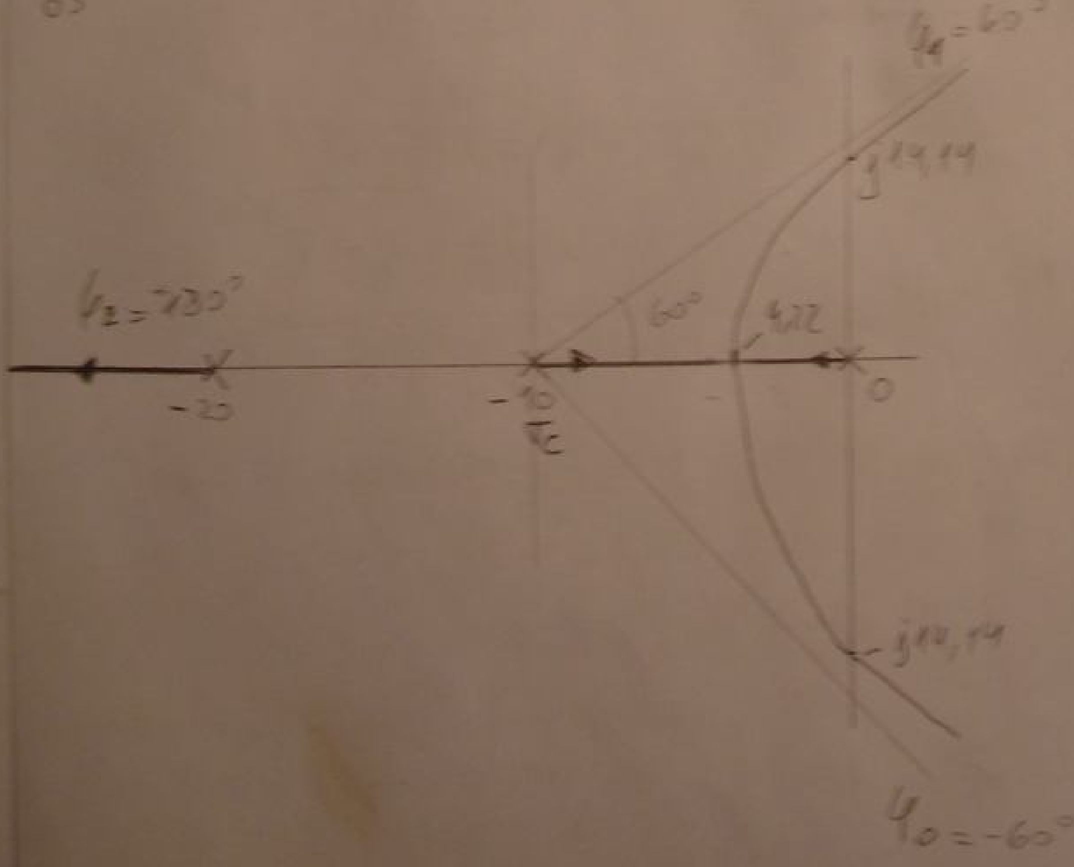
Punto de bifurcación:

$$s^3+30s^2+200s+K = 0$$

$$K = -(s^3+30s^2+200s)$$

$$\frac{\partial K}{\partial s} = -(3s^2+60s+200);$$

$$0 = 3s^2+60s+200 \begin{cases} -15{,}77 = \ldots \\ -4{,}22 = \ldots \end{cases}$$



dando esta 22

21

$\rightarrow 12,$

$$s^3 + 30s^2 + 200s + K = 0$$ 20

s^3	1	200
s^2	30	K
s^1	$200-\frac{K}{30}$	
s^0	K	

$$\frac{6\times10^3 - K}{30} = 200 - \frac{K}{30}$$

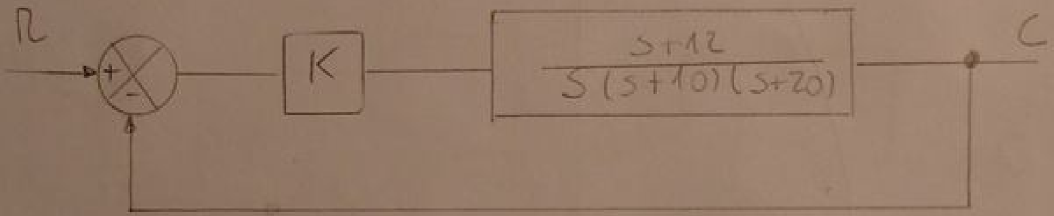
como cond de estabilidad

$$\underline{K \geqslant 6000}$$

de la parte cuadrática de Routh averiguamos por donde estan los cruces al eje $j\omega$

$$\Rightarrow 30s^2 + 6000 = 0 \begin{cases} -j14{,}14 \\ +j14{,}14 \end{cases}$$

4)



$$\frac{C(s)}{R(s)} = \frac{K(s+12)}{s(s+10)(s+20) + K(s+12)} = \frac{K(s+12)}{s^3+30s^2+200s+Ks+12K}$$

$$s^3+30s^2+200s+K(s+12)$$

$$\frac{C(s)}{R(s)} = \frac{K(s+12)}{s^3+30s^2+(200+K)s+12K}$$

$$G(s)H(s) = \frac{K(s+12)}{s^3+30s^2+200s} = \frac{K(s+12)}{s(s+10)(s+20)}$$

N° Asintotontas $P - Z = \underline{2 \text{ asintontas}}$

$$\varphi_k = \frac{180(2K-1)}{P-Z} \; ; \; K = 0, 1 \Rightarrow \begin{matrix} \varphi_0 = -90 \\ \varphi_1 = 90 \end{matrix}$$

$$\sigma_c = \frac{\Sigma Re[P] - \Sigma Re[Z]}{P-Z} = \frac{-30-12}{2} = -9 = \sigma_c \Rightarrow \text{baricentro}$$

Punto de bifurcación: $1 + G(s)H(s) = 0$

$s^3+30s^2+200s+K(s+12)=0 \Rightarrow$ Ecuación característica, probablemente

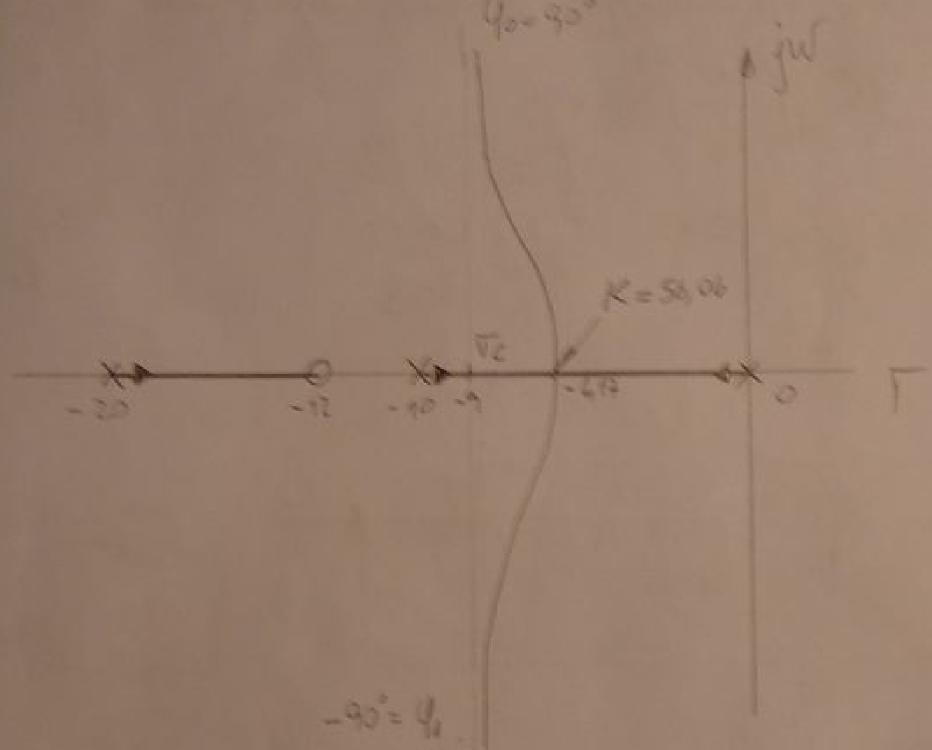
$$K = -\frac{(s^3+30s^2+200s)}{s+12}$$

$$\frac{dK}{ds} = -(2s^3+66s^2+720s+2{,}4\times10^3)=0 \quad \begin{cases} s_0 = -6{,}17 \\ s_1 = -13{,}41 + j\,2{,}8 \\ s_2 = -13{,}41 - j\,2{,}8 \end{cases}$$

$\Downarrow$

$K_{1s_0} = 56{,}06$

2



Estabilidad: $s^3+30s^2+(200+K)s+12K$

s^3	1	$200+K$
s^2	30	$12K$
s^1	$0{,}6K+200$	
s^0	$12K$	

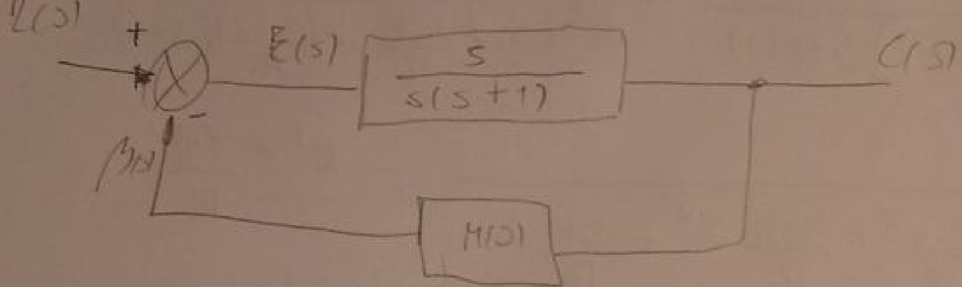
$$\frac{30(200+K)-12K}{30} = 0{,}6K+200$$

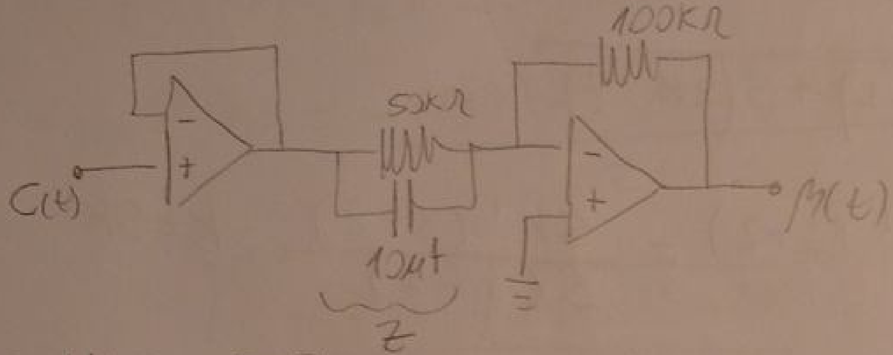
Estable para todo $K \geq 0$

De la línea de s^2 y tomando el valor crítico para K buscamos el corte con el eje $j\omega$ $\Rightarrow$ En este caso, no hay corte en el eje imaginario

5) El 4) tiene igual lugar de raíces.

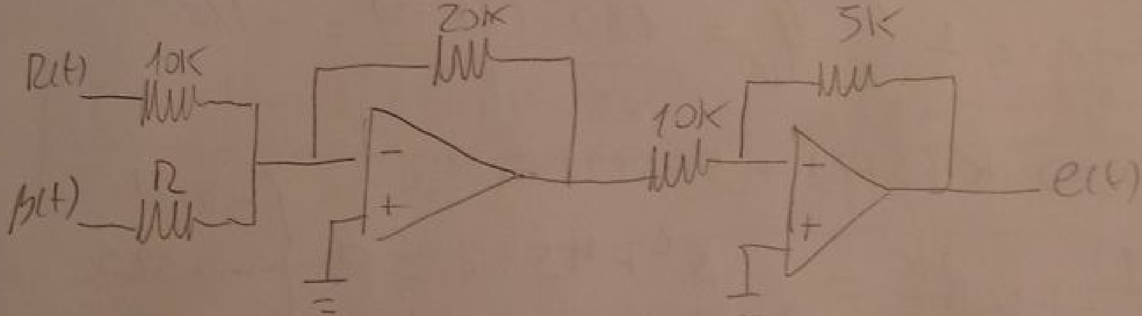
TP 17





$$\frac{\beta(t)}{c(t)} = \frac{100k}{50k \parallel \frac{1}{cs}} = s+2$$

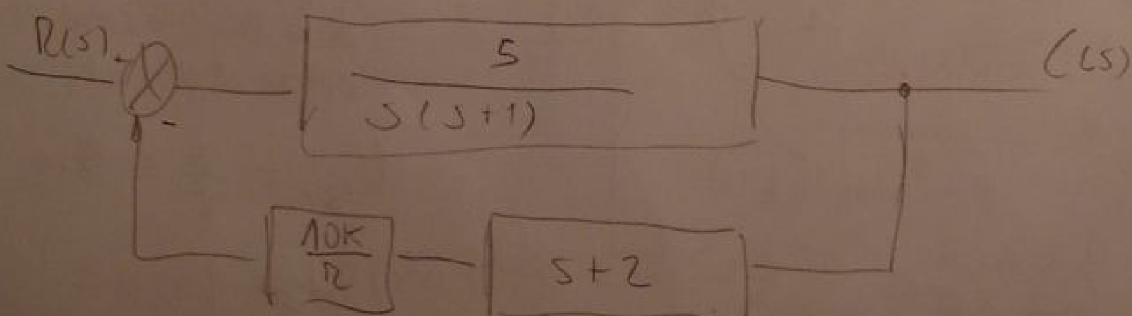
$$\Rightarrow M(s) = s+2$$



$$e(t) = \left[\frac{20k}{10k} R(t) + \frac{20k}{R} \beta(t)\right] \cdot \frac{5k}{10k}$$

$$e(t) = \left[2R(t) + \frac{20k}{R} \beta(t)\right] \cdot 0,5$$

$$e(t) = R(t) + \frac{10k}{R} \beta(t)$$



Buscamos R para q' ⇒ $\xi = 0{,}866$; $t_{s5\%} < 5\,seg$

$$\frac{C(s)}{R(s)} = \frac{5}{\left[1 + \frac{10k}{R}(s+2)\right] s(s+1)} = \frac{5}{s^2 + s + \frac{10k}{R} s(s+2)(s+1)}$$

$$= \frac{5}{s^2 + s + \frac{10k}{R} s^3 + \frac{30k}{R} s^2 + \frac{20k}{R} s} \quad ; \quad k' = \frac{10k}{R}$$

$$= \frac{5}{s^3 k' + s^2(3k'+1) + s(2k'+1)}$$

$$G(s)H(s) = \frac{5}{s(s+1)} k'(s+2) = \frac{5k'(s+2)}{s(s+1)} \; ; \; k = 5k'$$

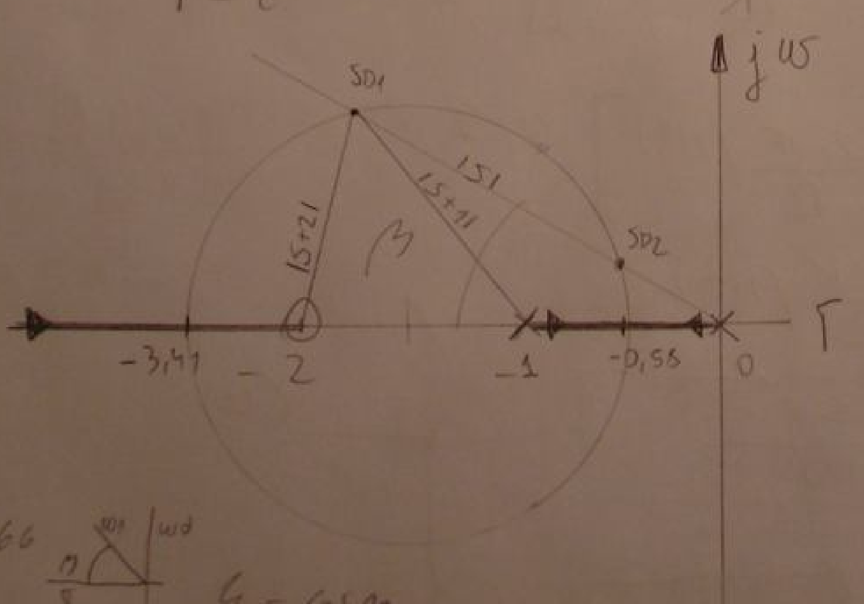
$$G(s)H(s) = \frac{k(s+2)}{s(s+1)}$$

N° de asínt ⇒ $P - Z = 1$; $\varphi_0 = 180°\frac{(2k-1)}{P-Z} = -180°$

$G(s)H(s) + 1 = 0 \Rightarrow k(s+2) + s(s+1) = 0$

$$k = -\frac{s(s+1)}{s+2} \; ; \; \frac{\partial k}{\partial s} = s^2 + 4s + 2 = 0 \begin{cases} -3{,}41 \\ -0{,}585 \end{cases}$$

$$\sigma_c = \frac{\sum Re[P] - \sum Re[Z]}{P-Z} = \frac{1-2}{} = -1$$



Valuando la ecuación del lugar de raíces, $G(s)H(s)+1=0$ encontramos q' tiene la forma de un círculo. Vemos q' en este caso en particular las dos raíces de la ecuación característica entran en el root place.

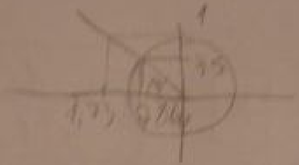
$\xi = 0{,}866$

$\xi = \cos\beta$,
$\beta = \cos^{-1}\xi = 30°$

$$t_{s5\%} = \frac{3}{\sigma} \leq 3\,seg$$

$\Rightarrow |\sigma| \geq 1$

SD_2 no cumple con $|\sigma| \geq 1$, por lo q buscamos donde está ubicado SD_1 23



$$\frac{1 - 3,5}{1,73 - 0,166}$$

$$s^2 + s(1+K) + 2K = 0$$

$$-0,577\, s = \frac{-(1+K) \pm \sqrt{(1+K)^2 - 8K}}{2}$$

$$\left.\begin{aligned} & s^2 + s(1+K) + 2K = 0 \\ & -0,577\, s = \frac{-(1+K) + \sqrt{(1+K)^2 - 8K}}{2} \end{aligned}\right\}$$

$$s = \frac{(1+K) - \sqrt{(1+K)^2 - 8K}}{2 \cdot 0,577}$$

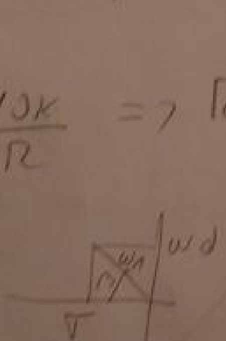
$$\left[\frac{(1+K) - \sqrt{(1+K)^2 - 8K}}{2 \cdot 0,577}\right]^2 + \left[\frac{(1+K) - \sqrt{(1+K)^2 - 8K}}{2 \cdot 0,577}\right](1+K) + 2K = 0$$

Para encontrar el punto, parece ser que hay q usar regla o unvieras dhechos

esto nos dejo en $\Rightarrow SD = -2,35 + j1,35$ (hecho a escala)

$$\Rightarrow K = \frac{|s|\,|s+1|}{|s+2|} = 3,85$$

$$\Rightarrow K = 5K' = 5 \cdot \frac{10k}{R} \Rightarrow R = \frac{5 \cdot 10k}{K} = \underline{13\,k\Omega = R}$$



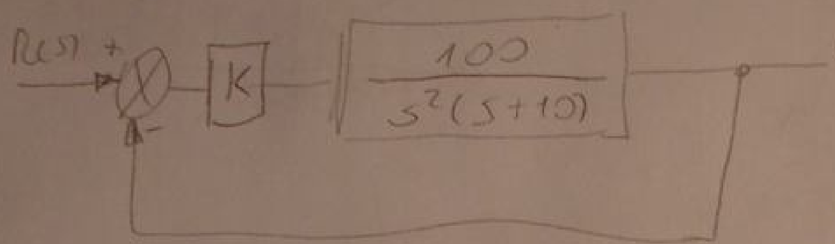
$$\xi = \cos\beta \; ; \; \sigma = \omega_n \xi$$

$$\Rightarrow \omega_n = \frac{|\sigma|}{\xi} = \frac{2,35}{0,866} = 2,7 \frac{rad}{seg}$$

$$\omega_d = \sqrt{\omega_n^2 - \sigma^2} = 1,33 \frac{rad}{seg}$$

$$ts_{5\%} = \frac{3}{\sigma} = 1,27\,seg \ll 5\,seg$$

TP 19



- Estable
- $\xi = 0,5$
- $\omega_d = 5\,rad/seg$
- $e_{ss}\big|_{\frac{1}{s^2}} = 0$

$$G(s)H(s) = \frac{\overbrace{100K}^{Ke}}{s^2(s+10)} \quad ; \quad 100K = Ke$$

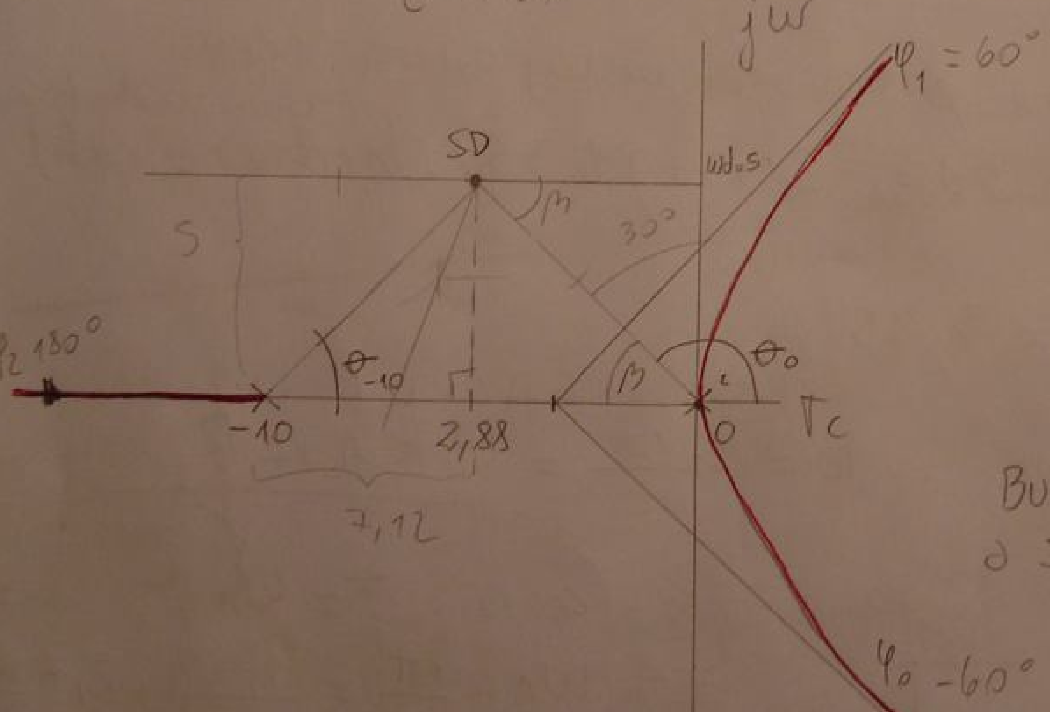
$$G(s)H(s)+1 = 0 \Rightarrow s^2(s+10) + Ke = 0$$

$$N^\circ \text{Asint} = p - z = 3 \quad ; \quad \varphi_k = 180\,\frac{(2K-1)}{p-z} \quad \begin{cases} \varphi_0 = -60 \\ \varphi_1 = 60 \\ \varphi_2 = 180 \end{cases}$$

$$\sigma_c = \frac{\sum Re(P) + \sum Re(Z)}{p-z} = \frac{-10}{3} = -3{,}3\overline{3} = \sigma_c$$

$$Ke = -(s^3 + 10s^2) \quad ; \quad \frac{dKe}{ds} = -(3s^2 + 20s)$$

$$3s^2 + 20s = 0 \begin{cases} P_1 = -6{,}66 \\ P_2 = 0 \end{cases}$$



$$\beta = \arccos(\xi) = 60^\circ$$

$$\omega_n = \frac{\omega_d}{\cos 30^\circ} = 5{,}77\,\frac{rad}{seg}$$

$$\sigma = \xi\,\omega_n = 2{,}88$$

$$\theta_0 = -2\cdot(180-60) = -240^\circ$$

$$\theta_{-10} = -35^\circ$$

Buscamos cuanto le falta a SD para cumplir con la condición de fase para ser lugar de raiz

$$\theta_0 + \theta_{10} = -275^\circ$$

$$\varphi_c - 275 = \pm 180$$

$$\varphi_c = 95^\circ$$

$s^3 + 10s^2 + 0s + K_c = 0$ (2)

$$\begin{array}{lll} s^3 & 1 & 0 \\ s^2 & 10 & K_c \\ s^1 & -\frac{K_c}{10} & \\ s^0 & K_c & \end{array}$$

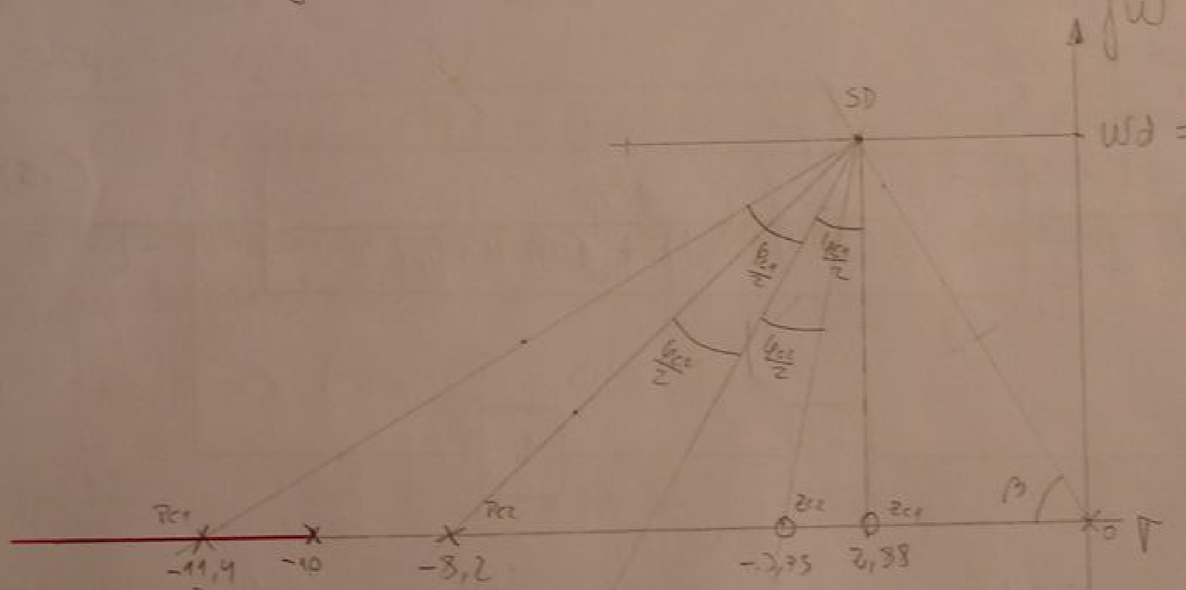
$\frac{10 \cdot 0 - K_c}{10}$

2 cambios de signo. el sistema presenta dos raíces en el lado positivo

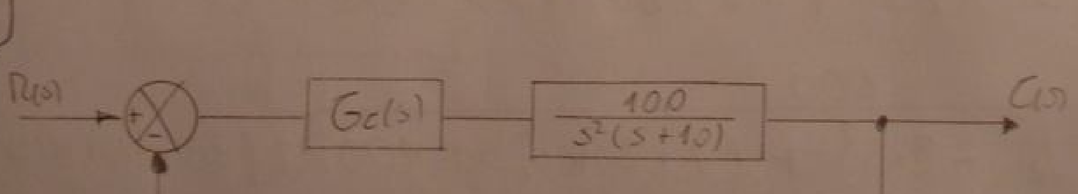
$\underline{K_c = 0}$

$10s^2 + K_c = 0 \Rightarrow 10s^2 = 0 \Rightarrow$ NO corta el eje $j\omega$

Usamos compensador por adelanto siempre menor a 60° para si nos sale un derivador puro. Entonces nos hace falta dos comp. uno de 60° y otro de 35°



$$\left.\begin{array}{l} P_{c1} = -11,4 \\ Z_{c1} = -2,88 \\ P_{c2} = -8,2 \\ Z_{c2} = -3,95 \end{array}\right\} \Rightarrow G_c(s) = K_c \frac{(s+2,88)(s+3,95)}{(s+11,4)(s+8,2)}$$



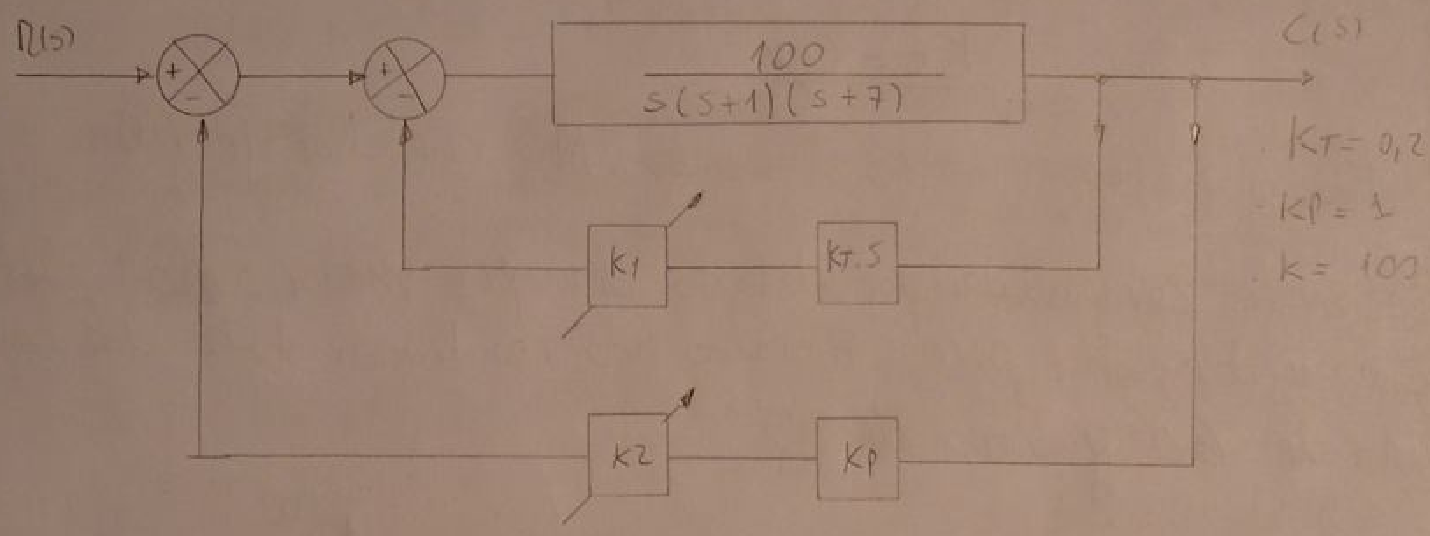
La nueva Ec característica para el lugar de raíces será:

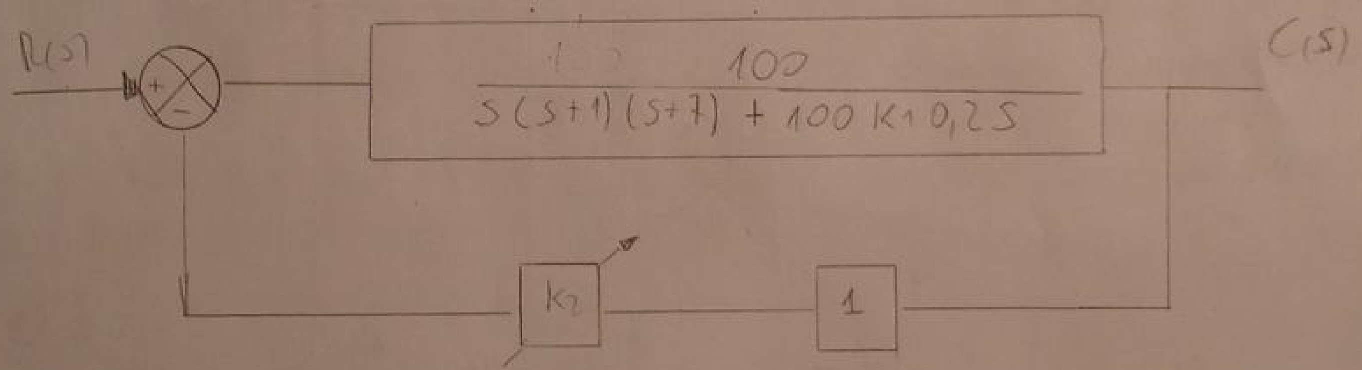
$$1 + \frac{K_c(s+2,88)(s+3,95)}{(s+11,4)(s+8,2)} \frac{100}{s^2(s+10)} = 0$$

$$(s+11,4)(s+8,2)(s+10)s^2+100K_C(s+2,88)(s+3,95)=0$$

$$K_C=\frac{-(s+11,4)(s+8,2)(s+10)s^2}{100(s+2,88)(s+3,95)}=\frac{9,9\cdot 7,3\cdot 3,7\cdot 5,8^2}{100\cdot 5\cdot 5,1}=8,3=K_C$$

B)





$$\frac{C(s)}{R(s)}=\frac{100}{s(s+1)(s+7)+100K_1\,0,2s+100K_2}$$

$$=\frac{100}{s^3+8s^2+7s+20K_1s+100K_2}$$

$$\frac{C(s)}{R(s)}=\frac{100}{s^3+8s^2+(7+20K_1)s+100K_2\}\Rightarrow G(s)H(s)+1}$$

Se pide $M_p=4,32\%$; $t_p=1,57$; $K_v>20$

Valiendonos de la posibilidad de variar K_1 y K_2, debemos generar un lugar de raíces de la ecuación característica que pase por los puntos dados en el enunciado (dado al especificar M_p, t_p y K_v)

26

$$M_p = e^{-\sigma\pi/w_d} \cdot 100 \; ; \; t_p = \frac{\pi}{w_d} \Rightarrow w_d = \frac{\pi}{t_p} = 2\,rad/seg$$ 24

$$\ln\left(\frac{4,32}{100}\right) = -\frac{\sigma\pi}{w_d} \Rightarrow \sigma = -\ln\left(\frac{4,32}{100}\right)\frac{w_d}{\pi} = 2$$

$$\Rightarrow s^2 + 2\xi w_n s + w_n^2 = s^2 + 2\sigma s + (w_d^2 + \sigma^2) = \underline{s^2 + 4s + 8}$$

Esta ecuación cuadrática, contiene como raiz los puntos necesarios de trabajo que contemplan las condiciones pedidas. Estas raices deberán formar parte de la ecuación característica igualada a cero para si el pto de trabajo pertenezca al lugar de raíces

$$
\begin{array}{l|l}
s^3 + 8s^2 + (7+20k_1)s + 100k_2 & s^2 + 4s + 8 \\
\hline
-s^3 - 4s^2 - 8s & s + 4 \\
0 \quad 4s^2 + (20k_1 - 1)s + 100k_2 & \\
\quad -4s^2 - 16s - 32 & \\
\hline
\quad 0 \quad (20k_1 - 17)s + (100k_2 - 32) &
\end{array}
$$

"Si de la división entre dos polinomios se obtiene un resto nulo (y suponiendo que el grado del dividendo es mayor que el del divisor) entonces el divisor contiene raices del dividendo."

$\Rightarrow$ Elegimos valores para k_1 y k_2 de modo que nos haga el resto $= \emptyset$

$$\left.\begin{array}{l} 20k_1 - 17 = 0 \Rightarrow k_1 = \frac{17}{20} = 0,85 \\ 100k_2 - 32 = 0 \Rightarrow k_2 = \frac{32}{100} = 0,32 \end{array}\right\}$$

$$\Rightarrow G(s)H(s) + 1 = s^3 + 8s^2 + 24s + 32$$

$$\frac{C(s)}{R(s)} = \frac{100}{s^3 + 8s^2 + 24s + 32}$$

$$K_V = \lim_{s\to 0} s \cdot G(s)H(s) = \cancel{s} \cdot \frac{32}{\cancel{s}(s^2 + 8s + 24)} \Rightarrow \frac{32}{24} = 1,33$$

$K_V = 1,33 < 20 \Rightarrow$ No cumple en la parte estacionaria

Será necesario compensar por atraso.

Unidad 20

$\xi = 0,5 \ , \ KV = 4 \ seg^{-1}$

$$G(s) = \frac{2\ Kc}{s(s^2+2s+2)} \quad ; \quad H(s) = 2$$

$$G(s)H(s) = \frac{4\ kc}{s^3+2s^2+2s} = \frac{4\ kc}{s(s+1-j)(s+1+j)}$$

$N^\circ asint = P - Z = 3$

$\varphi_K = 180 \frac{(2K-1)}{P-Z}$ $\begin{cases} \varphi_0 = -60 \\ \varphi_1 = +60 \\ \varphi_2 = +180 \end{cases}$

$\sigma_c = \frac{\sum Re|P| - \sum Re|Z|}{P-Z}$

$\sigma_c = \frac{2}{3} = 0,\overline{66}$

$PB \Rightarrow G(s)H(s) + 1 = 0$

$s^3 + 2s^2 + 2s + 4kc = 0$

$kc = -\frac{1}{4}(s^3+2s^2+2s)$

$\frac{dkc}{ds} = -\frac{1}{4}(3s^2+4s+2) = 0$

$PB \begin{cases} -0,\overline{66} - j0,471 \\ -0,\overline{66} + j0,471 \end{cases}$

$s^3 + 2s^2 + 2s + \overbrace{4k}^{k_e}$

$s^3 \quad 1 \quad 2$

$s^2 \quad 2 \quad ke$

$s^1 \quad 2 - 0,5ke$

$s^0 \quad ke$

$\frac{4 - ke}{2} = 2 - 0,5kc$

$K\ critico = 4 \Rightarrow K \leq 4$

$2s^2 + 4 = 0 \begin{Bmatrix} -j1,41 \\ +j1,41 \end{Bmatrix}$ Ptos de corte

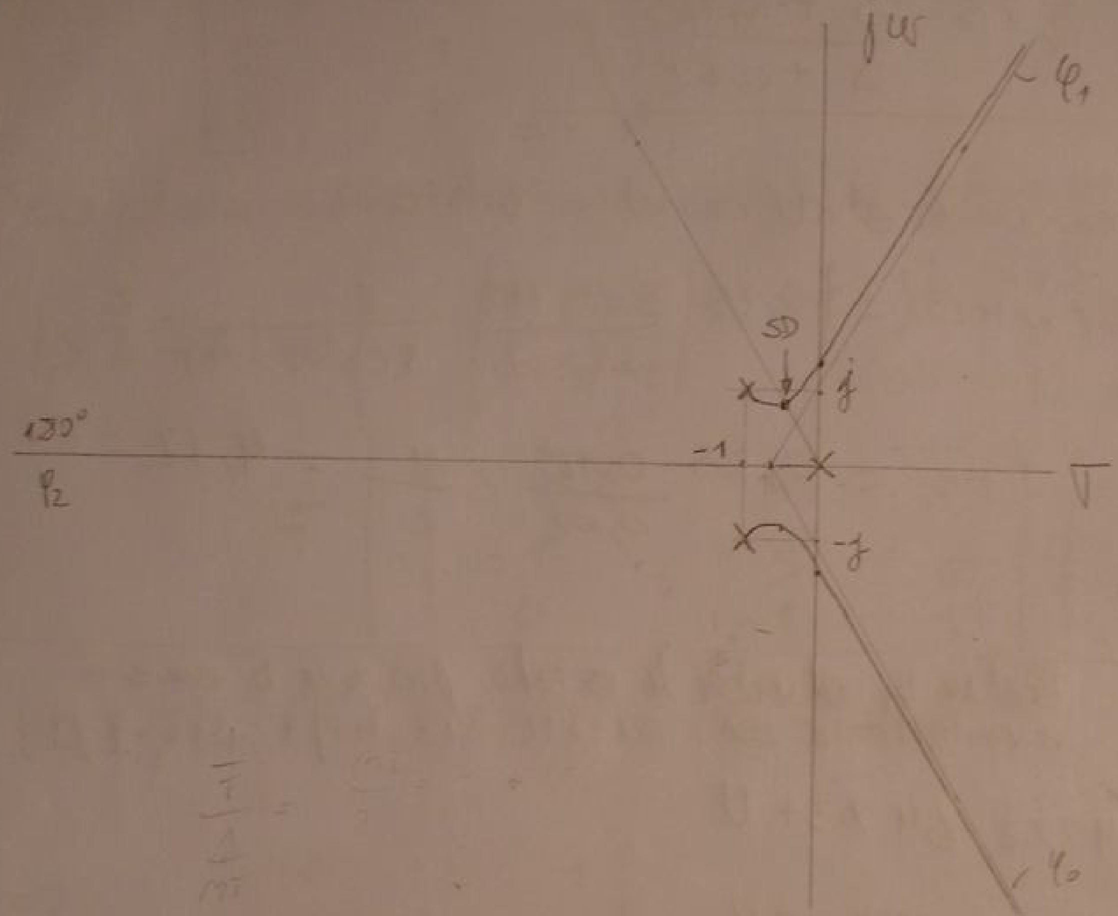
$\xi = 0,5 \Rightarrow A\cos 0,5 = 60^\circ$

Vemos que el punto de trabajo estaría en $-0,5 + j0,866 = S_D$

para un $K_e = 1 \Rightarrow K_v = 0,25$

$KV = \lim_{s \to 0} s\, G(s)H(s) \Rightarrow \frac{\cancel{s}\ 2 \cdot 0,25}{\cancel{s}(s+1-j)(s+1+j)} \Rightarrow \frac{0,5}{(1-j)(1+j)} = 0,25$

$Kv < 4\ s^{-1} \Rightarrow$ hay que corregir.



0,55.1,9.1 =

Para corregir el funcionamiento en regimen estacionario, agregamos el compensador por atrazo

$\Rightarrow \quad K_c \dfrac{s+Z_C}{s+P_C}$; Tomamos el K_C como 0,25 (el q ya habíamos calculado).

$$\Rightarrow \underbrace{\boxed{0{,}25\,\frac{s+Z_C}{s+P_C}}}^{G_C(s)} \cdot \underbrace{\boxed{\frac{2}{s(s^2+2s+2)}}}^{G(s)} ; \quad K_V = \lim_{s\to 0} s\,G(s)H(s)$$

$$K_V = \lim_{s\to 0} \cancel{s}\; 0{,}25\, \frac{s+Z_C}{s+P_C} \cdot \frac{2}{\cancel{s}(s^2+2s+2)} \cdot 2$$

$$= \frac{1}{2} \cdot \frac{Z_C}{P_C} = 0{,}5\,\frac{Z_C}{P_C}$$ esto debe ser igual al K_V pedido

$\dfrac{Z_C}{P_C} = 8$; Para elegir Z_C usamos el siguiente criterio

$Z_C = 1/T$ $\quad \dfrac{\frac{1}{T}}{\frac{1}{\beta T}} = \beta$

$P_C = 1/\beta T$

$0{,}1\ \cancel{\zeta\omega}\ < Z_C < 0{,}5\ \cancel{\zeta\omega}$ → de la parte real de los polos dominantes a lazo cerrado

Tomamos el promedio

$0{,}055 < Z_C < 0{,}275 \Rightarrow Z_C = 0{,}165$

$$\Rightarrow P_C = \frac{0{,}165}{8} = 0{,}02$$

$$\Rightarrow G_c(s) = 0,25 \; \frac{s + 0,165}{s + 0,02}$$

Si calculamos nuevamente el KV con estos valores:

$$KV = \lim_{s \to 0} s\, G(s) H(s) = \cancel{s}\, 0,25 \; \frac{s+0,165}{s+0,02} \cdot \frac{2}{\cancel{s}(s^2+2s+2)} \cdot 2$$

$$= 0,25 \; \frac{0,165}{0,02} \cdot \frac{4}{2} = 4,12 = KV$$

Ahora sí cumple

Se tiene: Realimentar el vector de estado para que la ecuación característica sea: $s_1 = -12$; $s_2 = -4+j4$; $s_3 = -4-j4$

$$\dot{x}_1 = -12x_1 - 40x_2 - 64x_3 + U$$
$$\dot{x}_2 = x_1 + 0x_2 + 0x_3 + 0U$$
$$\dot{x}_3 = 0x_1 + 1x_2 + 0x_3 + 0U$$
$$y = 0x_1 + 4x_2 + 64x_3 + 0U$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -12 & -40 & -64 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} U(t)$$

$$y(t) = \begin{bmatrix} 0 & 4 & 64 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Para pasar a la forma FCC usamos la transformación $x = Pz$ donde

$$P = SM$$

$$\Rightarrow S = [B \quad AB \quad A^2B] \Rightarrow \text{Matriz de controlabilidad}$$

$$M = \begin{bmatrix} a_1 & a_2 & \cdots & a_{n-1} & 1 \\ a_2 & \vdots & & 1 & \\ a_{n-1} & 1 & & 0 & \\ 1 & 0 & & 0 & 0 \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & -12 & 104 \\ 0 & 1 & -12 \\ 0 & 0 & 1 \end{bmatrix}$$

Para calcular la matriz M buscamos los coeficientes a_i

$$|sI - A| = \left| s \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} - \begin{vmatrix} -12 & -40 & -64 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} \right|$$

$$= \left| \begin{vmatrix} s+12 & 40 & 64 \\ -1 & s & 0 \\ 0 & -1 & s \end{vmatrix} \right|$$

$$|sI - A| = \overset{a_3}{s^3} + \overset{a_2}{12 s^2} + \overset{a_1}{40 s} + \overset{a_0}{64}$$

$$\Rightarrow M = \begin{vmatrix} 40 & 12 & 1 \\ 12 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix}$$

$$P = SM = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\left.\begin{aligned} \Rightarrow \bar{A} &= P^{-1} A P \\ \bar{B} &= P^{-1} B \\ \bar{C} &= C P \end{aligned}\right\} \quad A_{FCC} = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -64 & -40 & -12 \end{vmatrix}$$

$$B_{FCC} = \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} \qquad C_{FCC} = |64 \quad 4 \quad 0|$$

$$\left.\begin{aligned} \begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{vmatrix} &= \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -64 & -40 & -12 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} U(t) \\ y(t) &= |64 \quad 4 \quad 0| \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} \end{aligned}\right\}$$

Determinación de la matriz K

$K = [k_1 \quad k_2 \quad k_3]$

$\dot{x} = Ax + BU \quad ; \quad U = -Kx$

$y = Cx$

$\Rightarrow \dot{x} = Ax + B(-Kx)$

$\dot{x} = (A - BK)x$

$|sI - A + BK| = (s+12)(s+4-j4)(s+4+j4)$

$$\begin{vmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{vmatrix} - \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -64 & -40 & -12 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} [k_1 \quad k_2 \quad k_3]$$

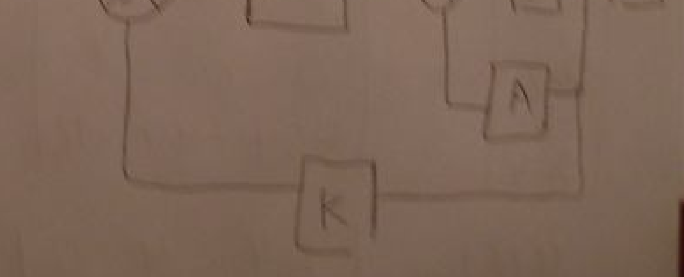
$$\begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ +64 & +40 & (s+12) \end{vmatrix} + \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ k_1 & k_2 & k_3 \end{vmatrix} = \begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ k_1+64 & k_2+40 & s+12+k_3 \end{vmatrix}$$

$s^2(s+12+k_3) + k_1+64 + s(k_2+40) = (s+12)(s+4-j4)(s+4+j4)$

$s^3 + 12s^2 + k_3 s^2 + k_1 + 64 + s k_2 + 40 s =$

$s^3 + s^2(k_3+12) + s(+40+k_2) + k_1 + 64 = s^3 + 20s^2 + 128s + 384$

$$\left.\begin{aligned} k_3 + 12 &= -20 \\ +40 + k_2 &= 128 \\ k_1 + 64 &= 384 \end{aligned}\right\} \begin{aligned} k_3 &= 8 \\ k_2 &= 88 \\ k_1 &= 320 \end{aligned}$$



Se tienen las siguientes ecuaciones dinámicas: (21)

$$\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{vmatrix} = \begin{vmatrix} -5 & 5 & 0 \\ 0 & -3 & 3 \\ 0 & 0 & -1 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} U(t)$$

$$y(t) = \begin{vmatrix} 1 & 0 & 0 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix}$$

a)- Realizar el diagrama de flujo de la señal indicando la configuración representada.

b)- Determinar controlabilidad y observabilidad.

c)- Encontrar $\Phi(t)$

d)- Ubicar los polos en $S_1 = -12$, $S_{2,3} = 4 \pm j$ ⇒ No es el

e)- Graficar el lugar de raíces. (...) no ... el del ...

a) Primero transformamos todo a FCC

$P = SM$; $S = [B \ \ AB \ \ A^2B]$, $M = \begin{vmatrix} a_1 & a_2 & 1 \\ a_2 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix}$

$$S = \begin{vmatrix} 0 & 0 & 15 \\ 0 & 3 & -12 \\ 1 & -1 & 1 \end{vmatrix} , \quad |SI - A| = \underset{a_3}{S^3} + \underset{a_2}{9S^2} + \underset{a_1}{23S} + \underset{a_0}{15}$$

$$M = \begin{vmatrix} 23 & 9 & 1 \\ 9 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} , \quad P = SM = \begin{vmatrix} 15 & 0 & 0 \\ 15 & 3 & 0 \\ 15 & 8 & 1 \end{vmatrix}$$

$$\bar{A} = P^{-1} A P = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -15 & -23 & -9 \end{vmatrix} , \quad \bar{B} = P^{-1}B = \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix}$$

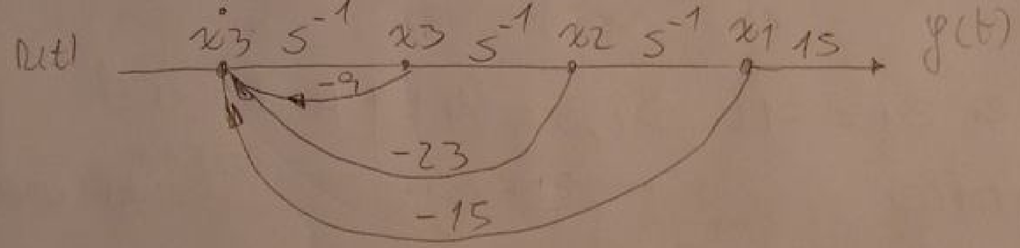
$$\bar{C} = CP = [15 \ \ 0 \ \ 0]$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{pmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -15 & -23 & -9\end{pmatrix}\begin{vmatrix}x_1\\ x_2\\ x_3\end{vmatrix} + \begin{vmatrix}0\\ 0\\ 1\end{vmatrix} U(t)$$

$$y(t) = \begin{bmatrix}15 & 0 & 0\end{bmatrix}\begin{vmatrix}x_1\\ x_2\\ x_3\end{vmatrix} \qquad F(t) = \frac{15}{s^3+9s^2+23s+15}$$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = -15x_1 - 23x_2 - 9x_3 + U(t)$$

$$y(t) = 15x_1$$



b)

$$M = \begin{bmatrix}B & AB & A^2B\end{bmatrix} = \begin{bmatrix}0 & 0 & 1\\ 0 & 1 & -9\\ 1 & -9 & 58\end{bmatrix} \quad \left.\begin{matrix}\text{Rango} = 3\\ \text{Det} = -1 \neq 0\end{matrix}\right\} \text{CONTROLABLE}$$

$$O = \begin{bmatrix}C^T & A^TC^T & A^{2T}C^T\end{bmatrix} = \begin{vmatrix}15 & 0 & 0\\ 0 & 15 & 0\\ 0 & 0 & 15\end{vmatrix} \quad \left.\begin{matrix}\text{Det} \neq 0\\ \text{Rango} = 3\end{matrix}\right\} \text{OBSERVABLE}$$

d) $s_1 = -12$, $s_{2,3} = 4 \pm j4$

$$\dot{x} = Ax + BU$$
$$y(t) = Cx$$
$$U = -K \cdot x$$

$$\dot{x} = Ax - BKx = (A - BK)x$$
$$sx = Ax - BU$$
$$sx - Ax + BKx = [sI - A + BK]x = 0$$

$$\begin{bmatrix}s & 0 & 0\\ 0 & s & 0\\ 0 & 0 & s\end{bmatrix} - \begin{vmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -15 & -23 & -9\end{vmatrix} + \begin{vmatrix}0\\ 0\\ 1\end{vmatrix}\begin{vmatrix}k_1 & k_2 & k_3\end{vmatrix}$$

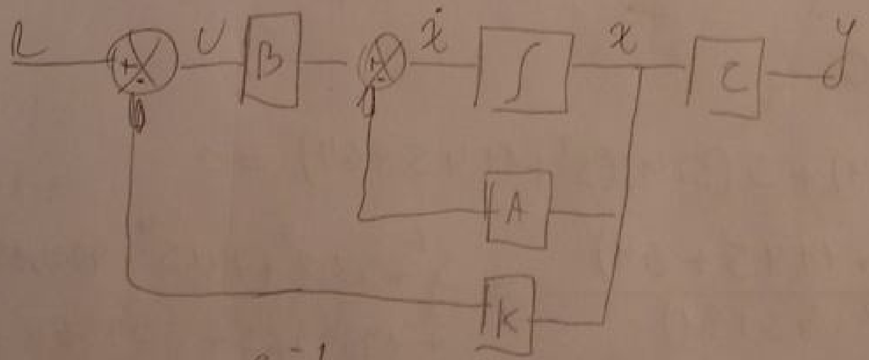
(30)

$$\begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 15 & 23 & (s+9) \end{vmatrix} + \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ k_1 & k_2 & k_3 \end{vmatrix} = \begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ (15+k_1) & (23+k_2) & (s+9+k_3) \end{vmatrix}$$ 28

$$\begin{matrix} s & -1 & 0 \\ 0 & s & -1 \end{matrix}$$

$$\begin{aligned} \text{Det} &= s^2(s+9+k_3) + 15+k_1 + s(23+k_2) \\ &= s^3 + s^2(9+k_3) + s(23+k_2) + 15+k_1 \\ &= (s+12)(s+4-j)(s+4+j) = (s+12)(s^2+8s+17) \\ &= s^3+20s^2+113s+204 \end{aligned}$$

$$\left.\begin{aligned} 9+k_3 &= 20 \\ 15+k_1 &= 204 \\ 23+k_2 &= 113 \end{aligned}\right\} \left.\begin{aligned} k_3 &= 11 \\ k_1 &= 189 \\ k_2 &= 90 \end{aligned}\right\} \quad K = |189 \quad 90 \quad 11|$$



c) $\Phi(t) = \mathcal{L}^{-1}([sI - A]^{-1})$

$$\Rightarrow \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -15 & -23 & -9 \end{bmatrix} = \begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 15 & 23 & (s+9) \end{vmatrix}$$

$$[sI-A]^{-1} = \frac{\text{Adj}\,|sI-A|}{\Delta\,|sI-A|} = \frac{1}{s^3\,9s^2+23s+15} \begin{vmatrix} s^2+9s+23 & -15 & -15s \\ s+9 & s^2+9s & -(23s+15) \\ -1 & -s & +s^2 \end{vmatrix}$$

se pare un procedeu despues asi q' lo salteo

• Realizar el diagrama de LR P/ $-\infty < K < \infty$ de un sistema que tiene ec. función como la siguiente:

$$G(s)H(s) = \frac{K\,(s+10)\,(s^2+9s+81)}{s\,(s+4)\,(s^2+14{,}4s+64)}$$

Indicar el rango de variación de K para que sea estable.

$$G(s)H(s) = \frac{K\,(s+10)(s+4{,}5+j7{,}8)(s+4{,}5-j7{,}8)}{s\,(s+4)\,(s+7{,}2+j3{,}5)\,(s+7{,}2-j3{,}5)}$$

$$N^\circ\ \text{Asint} = P - Z = 4 - 3 = 1$$

$$\varphi_K = 180^\circ \cdot \frac{(2K-1)}{P-Z} = -180^\circ$$

$$\sigma_C = \frac{\Sigma Re(P) - \Sigma Re(Z)}{P-Z} = \frac{(4+7{,}2+7{,}2)-(10+4{,}5+4{,}5)}{1} = -6 = \sigma_c$$

PD $\Rightarrow G(s)H(s) + 1 = 0$

$$\Rightarrow K(s+10)(s^2+9s+81) + s(s+4)(s^2+14{,}4s+64) = 0$$

$$\frac{\partial K}{\partial s} = -\frac{s\,(s+4)\,(s^2+14{,}4s+64)}{(s+10)\,(s^2+9s+81)} = s^6 + 38s^5 + 741s^4 + 9020{,}8s^3 + 60644{,}6s^2 + 196992s + 207360 = 0$$

$P_1 = -1{,}87$

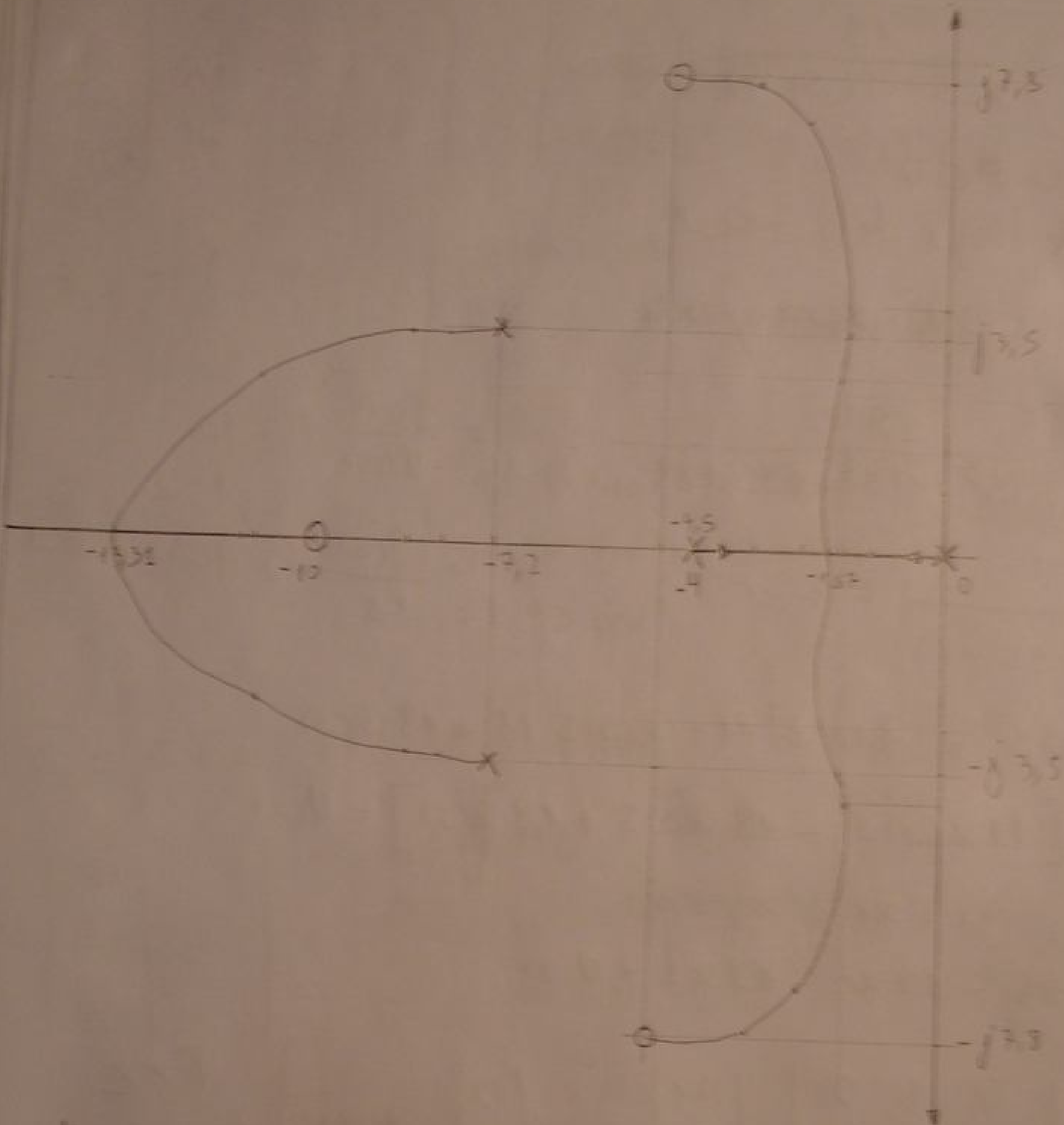
$\left.\begin{array}{l} P_2 = -6{,}031 \pm j2{,}36 \\ P_3 = -5{,}37 \pm j13 \end{array}\right\}$ No entran

$P_4 = -13{,}31$

$G(s)H(s) + 1 = 0$

$$s^4 + s^3(K+18{,}4) + s^2(19K+121{,}6) + s(171K+256) + 810K$$

29



s^4 1 $(19K+121{,}6)$ $810K$

s^3 $(K+13{,}4)$ $(171K+256)$

s^2 $\dfrac{19K^2+300{,}2K+1931{,}44}{K+13{,}4}$ $810K$

s^1 $\dfrac{2439K^4+71267{,}3K^3+627073{,}52K^2+3104315{,}3K+933337}{19K^3+650K^2+750K+36453{,}5}$

s^0 $810K$

$K_c \Rightarrow -13{,}4$, $\cancel{-7{,}9 \pm j6{,}47}$, $\cancel{-2{,}01 \pm j5{,}15}$, $-6{,}79$

$K_c \Rightarrow -6{,}79$

corte en el eje de $j\omega \Rightarrow \dfrac{19K_c^2+300{,}2K_c+1981{,}74}{K_c+13{,}4}s^2+810K_c$ $\begin{cases} [illegible] \\ [illegible] \end{cases}$

(39)

Para $G(s) = \dfrac{18}{s(s+2)(s+9)}$

- Encontrar la FCD
- Diagrama de flujo de la señal
- $\phi(t)$ p/ sistema desacoplado

$$\frac{G(s)}{1+G(s)H(s)} = \frac{18}{s^3+11s^2+18s+18} = \frac{Y(s)}{U(s)}$$

$$\frac{U(s)}{s^3+11s^2+18s+18} = Q(s) \quad ; \quad Q(s) = \frac{Y(s)}{18} \Rightarrow Y(s) = 18\overset{x_1}{Q(s)}$$

$$U(s) = Q(s)s^3 + Q(s)s^2\,11 + Q(s)s\,18 + 18\,Q(s)$$

$$\overset{x_3}{Q(s)}s^3 = -\left[11\overset{x_3}{Q(s)}s^2 + 18\overset{x_2}{Q(s)}s + 18\overset{x_1}{Q(s)}\right] + U(s)$$

$x_1 = Q(s)$; $x_2 = \dot{x}_1$; $x_3 = \dot{x}_2$

$$\dot{x}_3 = -11x_3 - 18x_2 - 18x_1 + U(s)$$

$$\left.\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} &= \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -18 & -18 & -11 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} u(t) \\ y(t) &= \begin{vmatrix} 18 & 0 & 0 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} \end{aligned}\right\} \text{FCC}$$

$$|\lambda I - A| = \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} - \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -18 & -18 & -11 \end{vmatrix} = \begin{vmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ 18 & 18 & \lambda+11 \end{vmatrix}$$

$$\begin{matrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \end{matrix}$$

$$\text{Det} = \lambda^2(\lambda+11) + 18 + 18\lambda = 0$$

$$\lambda \begin{cases} -0{,}866 \pm j1{,}092 \ ; \ -9{,}267 \end{cases}$$

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$$T = \begin{vmatrix} 1 & 1 & 1 \\ \lambda_1 & \lambda_2 & \lambda_3 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ -0{,}866+j1{,}092 & -0{,}866-j1{,}092 & -9{,}267 \\ -0{,}442-j1{,}891 & -0{,}442+j1{,}891 & 85{,}877 \end{vmatrix}$$

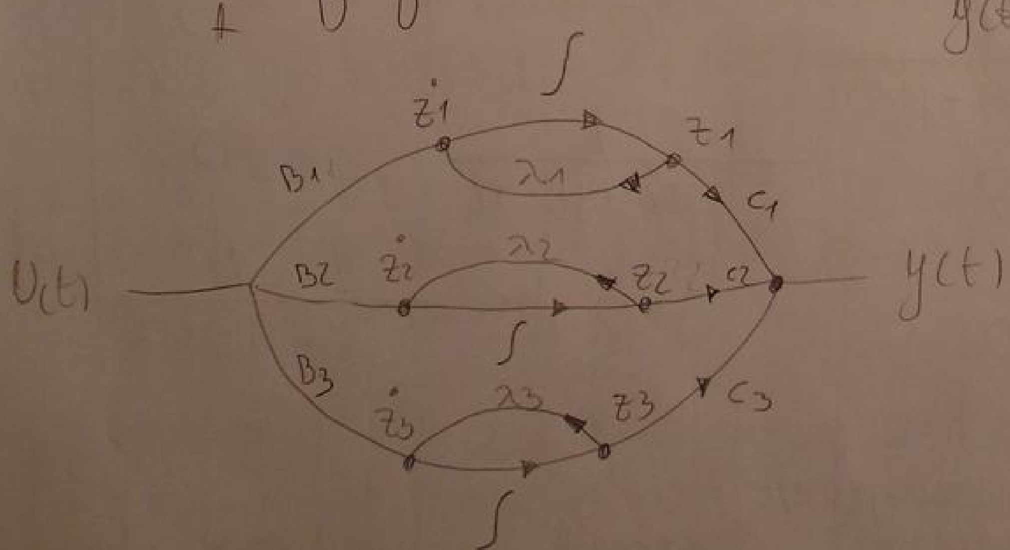
$$\bar{A} = T^{-1} A T = \begin{vmatrix} -0{,}866+1{,}092 & 0 & 0 \\ 0 & -0{,}866-j1{,}092 & 0 \\ 0 & 0 & -9{,}267 \end{vmatrix}$$

$$\bar{B} = T^{-1} B = \begin{vmatrix} -6{,}967\times10^{-3} - j53{,}6\times10^{-3} \\ -6{,}967\times10^{-3} + j53{,}6\times10^{-3} \\ 13{,}93\times10^{-3} \end{vmatrix}$$

$$\bar{C} = CT = \begin{vmatrix} 18 & 18 & 18 \end{vmatrix}$$

$$\begin{vmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{vmatrix} = \begin{vmatrix} -0{,}866+j1{,}092 & 0 & 0 \\ 0 & -0{,}866-j1{,}092 & 0 \\ 0 & 0 & -9{,}267 \end{vmatrix} \begin{vmatrix} z_1 \\ z_2 \\ z_3 \end{vmatrix} + \bar{B}\, U$$

$$y(t) = \begin{vmatrix} 18 & 18 & 18 \end{vmatrix} \begin{vmatrix} z_1 \\ z_2 \\ z_3 \end{vmatrix}$$



$$\phi(t) = \mathcal{L}^{-1}\left[(sI - A)^{-1}\right]$$

$$(sI - A)^{-1} = \frac{\text{Adj cof}}{|sI - A|} \quad ; \quad |sI - A| = s^3 + 11s^2 + 18s + 18 = (s + 0{,}866 + j1{,}092)(s + 0{,}866 - j1{,}092)(s + 9{,}267)$$

$$sI - A = \begin{vmatrix} (s + 0{,}866 - j1{,}092) & 0 & 0 \\ 0 & (s + 0{,}866 + j1{,}092) & 0 \\ 0 & 0 & (s + 9{,}267) \end{vmatrix}$$

$$\text{cof} = \text{Adj cof} = \begin{vmatrix} \frac{(s + 0{,}866 + j1{,}092)(s + 9{,}267)}{\text{Det}} & 0 & 0 \\ 0 & \frac{(s + 0{,}866 - j1{,}092)(s + 9{,}267)}{\text{Det}} & 0 \\ 0 & 0 & \frac{(s + 0{,}866 - j1{,}092)(s + 0{,}866 - j1{,}09}{\text{Det}} \end{vmatrix}$$

$$(sI - A)^{-1} = \begin{vmatrix} \frac{1}{s + 0{,}866 - j1{,}092} & 0 & 0 \\ 0 & \frac{1}{s + 0{,}866 + j1{,}092} & 0 \\ 0 & 0 & \frac{1}{s + 9{,}267} \end{vmatrix}$$

$$\phi(t) = \begin{vmatrix} e^{-(0{,}866 - j1{,}092)t} & 0 & 0 \\ 0 & e^{-(0{,}866 + j1{,}092)t} & 0 \\ 0 & 0 & e^{-9{,}267t} \end{vmatrix}$$

$\dddot{y}(t) + 11\ddot{y}(t) + 36\dot{y}(t) + 36y(t) = 36R(s)$

$$\begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -36 & -36 & -11 \end{vmatrix}$$

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$s^3 y(s) + 11 s^2 y(s) + 36 s y(s) + 36 y(s) = 36 R(s)$

$\frac{y(s)}{R(s)} = \frac{36}{s^3 + 11s^2 + 36s + 36}$; $\frac{y(s)}{36} = Q(s)$; $\frac{R(s)}{s^3 + 11s^2 + 36s + 36} = Q(s)$

$R(s) = Q(s)s^3 + Q(s)11s^2 + 36Q(s)s + 36Q(s)$

$\overbrace{Q(s)s^3}^{\dot{x}_3} = R(s) - 11\overbrace{s^2Q(s)}^{x_3} - 36\overbrace{sQ(s)}^{x_2} - 36\overbrace{Q(s)}^{x_1}$

$\dot{x}_3 = R(s) - 11x_3 - 36x_2 - 36x_1$

$x_1 = Q(s)$
$x_2 = sQ(s) = \dot{x}_1$
$x_3 = s^2Q(s) = \dot{x}_2$

$y(t) = 36x_1$

$$\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -36 & -36 & -11 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} R(s)$$

$$y(t) = \begin{vmatrix} 36 & 0 & 0 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix}$$

$$\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -c & -b & -a \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} - \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ k_1 & k_2 & k_3 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix}$$

Ubicar polos en $\Rightarrow -10$ y $-3 \pm 3j$

$\dot{x}(t) = Ax + BU$
$y(t) = cx$
$U = -kx$

$sx(s) = Ax(s) - BKx(s)$
$sx(s) - Ax(s) + BKx(s) = 0$
$[sI - A + BK]x(s) = 0$

$\dot{x}_3 = -cx_1 - bx_2 - ax_3 - k_1x_1 - k_2x_2 - k_3x_3$

$\dot{x}_3 = -[x_1(c+k_1) + x_2(b+k_2) + x_3(a+k_3)]$

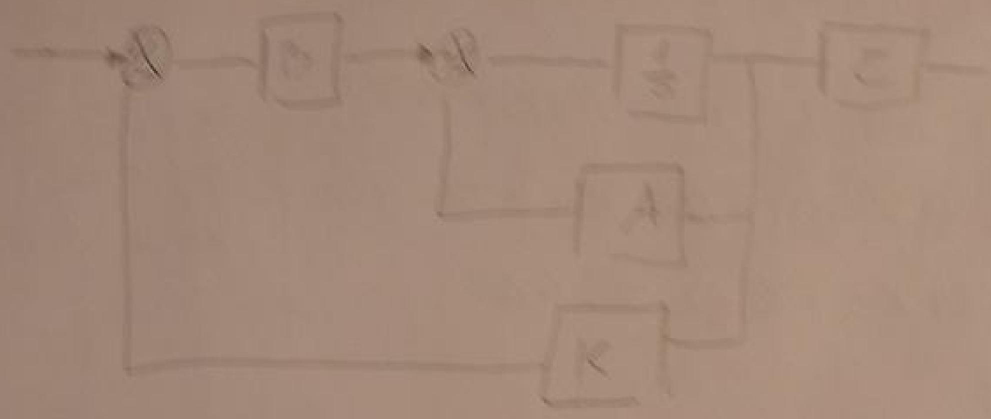
$$\begin{vmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{vmatrix} - \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -36 & -36 & -11 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} \begin{vmatrix} k_1 & k_2 & k_3 \end{vmatrix}$$

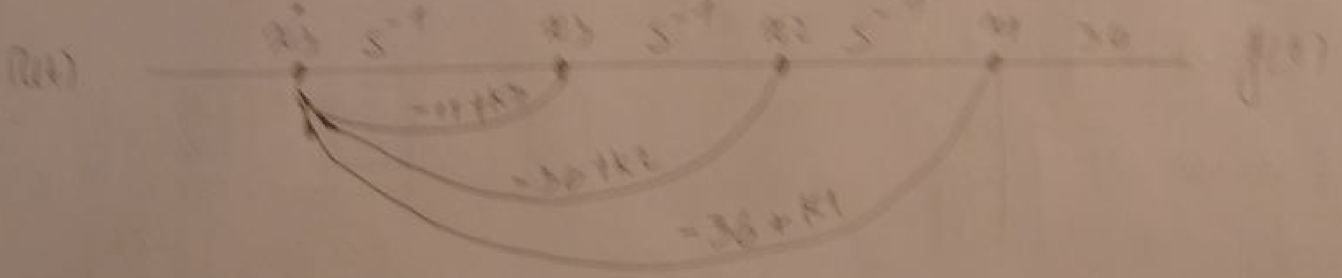
$$\begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 36 & 36 & s+11 \end{vmatrix} + \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ k_1 & k_2 & k_3 \end{vmatrix} = \begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ (36+k_1) & (36+k_2) & (s+11+k_3) \end{vmatrix}$$

$\mathrm{Det} = s^2(s+11+k_3) + 36 + k_1 + s(36+k_2) = s^3 + s^2(11+k_3) + s(36+k_2) + 36 + k_1 \Rightarrow$

$$s^3 + s^2(11+k_3) + s(36+k_2) + 36+k_1 = (s+10)(s+3-j3)(s+3+j3)$$
$$= s^3+16s^2+78s+180$$

$11+k_3 = 16$	$k_3 = 5$
$36+k_2 = 78$	$k_2 = 42$
$36+k_1 = 180$	$k_1 = 144$





setkae

$$\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{vmatrix} = \begin{vmatrix} -5 & 5 & 0 \\ 0 & -3 & 3 \\ 0 & 0 & -1 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} u(t)$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix}$$

$$|\lambda I - A| = \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} - \begin{vmatrix} -5 & 5 & 0 \\ 0 & -3 & 3 \\ 0 & 0 & -1 \end{vmatrix} = \begin{vmatrix} \lambda+5 & -5 & 0 \\ 0 & \lambda+3 & -3 \\ 0 & 0 & \lambda+1 \end{vmatrix}$$

$$(\lambda+5)(\lambda+3)(\lambda+1) = 0 \quad \begin{cases} \lambda_1 = -5 \\ \lambda_2 = -3 \\ \lambda_3 = -1 \end{cases}$$

$$T = \begin{vmatrix} 1 & 1 & 1 \\ \lambda_1 & \lambda_2 & \lambda_3 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ -5 & -3 & -1 \\ 25 & 9 & 1 \end{vmatrix} = T$$ (32)

Valida si A está expresado en la FCC

Si no está en FCC buscamos los autovectores q' conforman T:

$$|\lambda_1 I - A| P_1 = \begin{vmatrix} -5 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -5 \end{vmatrix} - \begin{vmatrix} -5 & 5 & 0 \\ 0 & -3 & 3 \\ 0 & 0 & -1 \end{vmatrix} = \begin{vmatrix} 0 & -5 & 0 \\ 0 & -2 & -3 \\ 0 & 0 & -4 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} = 0$$

$$\left.\begin{array}{l} -5y = 0 \\ -2y - 3z = 0 \\ -4z = 0 \end{array}\right\} \qquad T \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix}$$

$$\begin{vmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{vmatrix} - \begin{vmatrix} -5 & 5 & 0 \\ 0 & -3 & 3 \\ 0 & 0 & -1 \end{vmatrix} = \begin{vmatrix} 2 & -5 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & -2 \end{vmatrix} \begin{vmatrix} P_{21} \\ P_{22} \\ P_{23} \end{vmatrix}$$

$2P_{21} - 5P_{22} = 0 \quad ; \quad 2P_{21} = 5P_{22} \qquad P_{21} = 1, \; P_{22} = 0{,}4$

$$\left.\begin{array}{l} 3P_{23} = 0 \\ -2P_{23} = 0 \end{array}\right\} P_{23} = 0 \qquad \begin{vmatrix} 1 \\ 0{,}4 \\ 0 \end{vmatrix}$$

$$\begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix} - \begin{vmatrix} -5 & 5 & 0 \\ 0 & -3 & 3 \\ 0 & 0 & -1 \end{vmatrix} = \begin{vmatrix} 4 & -5 & 0 \\ 0 & -2 & -3 \\ 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} P_{31} \\ P_{32} \\ P_{33} \end{vmatrix}$$

$4P_{31} - 5P_{32} = 0$

$-2P_{32} - 3P_{33} = 0$

$P_{33} = 1$

$P_{31} = 1{,}875$

$P_{32} = 3/2$

$P_{33} = 1$

$$\begin{vmatrix} 1{,}875 \\ 3/2 \\ 1 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 5 & 1{,}875 \\ 0 & 2 & 1{,}5 \\ 0 & 0 & 1 \end{vmatrix}$$

$0x - 5x + 2x$

$0y - 2y - 3z$

$$P_{32} = -\frac{3P_{33}}{2} = -\frac{3}{2}$$

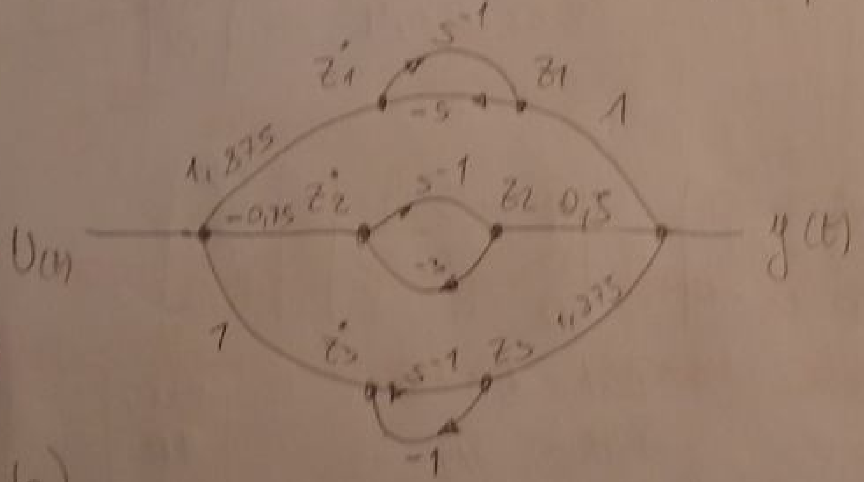
$$P = \begin{vmatrix} 1 & 5 & 1{,}875 \\ 0 & 2 & 1{,}5 \\ 0 & 0 & 1 \end{vmatrix} \Rightarrow P^{-1} = \begin{vmatrix} 1 & -2{,}5 & 1{,}875 \\ 0 & 0{,}5 & -0{,}75 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\bar{A} = P^{-1} A P = \begin{vmatrix} -5 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -1 \end{vmatrix}$$

$$\bar{B} = P^{-1} B = \begin{vmatrix} 1{,}875 \\ -0{,}75 \\ 1 \end{vmatrix}$$

$$\bar{C} = C P = \begin{vmatrix} 1 & 0{,}5 & 1{,}875 \end{vmatrix}$$

$$\left.\begin{aligned} \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} &= \begin{vmatrix} -5 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -1 \end{vmatrix} \begin{vmatrix} z_1 \\ z_2 \\ z_3 \end{vmatrix} + \begin{vmatrix} 1{,}875 \\ -0{,}75 \\ 1 \end{vmatrix} U \\ y(t) &= \begin{vmatrix} 1 & 0{,}5 & 1{,}875 \end{vmatrix} \begin{vmatrix} z_1 \\ z_2 \\ z_3 \end{vmatrix} \end{aligned}\right\}$$



b)

$$M = \begin{vmatrix} B & AB & A^2B \end{vmatrix} = \begin{vmatrix} 1{,}875 & -9{,}375 & 46{,}88 \\ -0{,}75 & 2{,}25 & -6{,}75 \\ 1 & -1 & 1 \end{vmatrix}$$ Rango = 3 controlable

$$O = \begin{vmatrix} C^T & A^T C^T & A^{2^T} C^T \end{vmatrix} = \begin{vmatrix} 1 & -5 & 2{,}5 \\ 0{,}5 & -1{,}5 & 4{,}5 \\ 1{,}875 & -1{,}875 & 1{,}875 \end{vmatrix}$$ Rango = 3 Observable

$$\phi(t) = \mathcal{L}^{-1}\left[(SI - A)^{-1}\right]$$

$$(SI - A) = \begin{vmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{vmatrix} - \begin{vmatrix} -5 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -1 \end{vmatrix} = \begin{vmatrix} s+5 & 0 & 0 \\ 0 & s+3 & 0 \\ 0 & 0 & s+1 \end{vmatrix}$$

$$(SI - A)^{-1} = \frac{\text{Adj cot}}{|SI - A|} = \frac{\begin{vmatrix} [(s+3)(s+1)] & 0 & 0 \\ 0 & [(s+5)(s+1)] & 0 \\ 0 & 0 & [(s+5)(s+3)] \end{vmatrix}}{(s+5)(s+3)(s+1)}$$

$$(SI - A)^{-1} = \begin{vmatrix} \frac{1}{(s+5)} & 0 & 0 \\ 0 & \frac{1}{(s+3)} & 0 \\ 0 & 0 & \frac{1}{(s+1)} \end{vmatrix}$$

$$\cancel{\mathcal{L}^{-1}\left[(SI - A)^{-1}\right]} = \begin{vmatrix} e^{-5t} & 0 & 0 \\ 0 & e^{-3t} & 0 \\ 0 & 0 & e^{-t} \end{vmatrix} = \phi(t) = e^{At}$$

$e^{Dt} \Rightarrow$

(pg 232)
Método 2

Para hallar la $\phi(t)$ siempre partimos de la matriz FCC

Podemos llegar a lo mismo haciendo $\phi(t) = e^{At} = P\, e^{Dt}\, P^{-1}$ Método 1 (pg 231)

$$\phi(t) = \begin{vmatrix} 1 & 5 & -1{,}375 \\ 0 & 2 & 1{,}5 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} e^{-5t} & 0 & 0 \\ 0 & e^{-3t} & 0 \\ 0 & 0 & e^{-t} \end{vmatrix} \begin{vmatrix} 1 & -2{,}5 & 1{,}375 \\ 0 & 0{,}5 & -0{,}75 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} e^{-5t} & 5e^{-3t} & -1{,}375e^{-t} \\ 0 & 2e^{-3t} & 1{,}5e^{-t} \\ 0 & 0 & e^{-t} \end{vmatrix} \cdot \begin{vmatrix} 1 & -2{,}5 & 1{,}375 \\ 0 & 0{,}5 & -0{,}75 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\phi(t) = \begin{vmatrix} e^{-5t} & 2{,}5(e^{-3t}-e^{-5t}) & 1{,}875(e^{-5t}-e^{-t})-3{,}75e^{-3t} \\ 0 & e^{-3t} & 1{,}5(e^{-t}-e^{-3t}) \\ 0 & 0 & e^{-t} \end{vmatrix}$$

Para matlab:

$t = sym('t')$; creamos variable simbólico.

$[V, d] = eig(A)$; guarda en "V" la matriz P y en "d" la matriz $\begin{bmatrix} x_1 & 0 & 0 \\ 0 & x_2 & 0 \\ 0 & 0 & x_3 \end{bmatrix}$

; Método 1:

$eDt = expm(d*t)$

$\phi_1 = V*eDt*V^{-1}$

; Método 2:

$s = sym('s')$

$I = [1\ 0\ 0; 0\ 1\ 0; 0\ 0\ 1]$

$l = (sI - A)^{\wedge}(-1)$

$\phi_2 = ILAPLACE(l)$

---

d)

$\dot{x}(t) = Ax(t) + Bu(t)$

$y(t) = x(t)C$

$U = -Kx$

$sx(s) = Ax(s) - BKx(s)$

$sx(s) - Ax(s) + BKx(s) = 0$

$(sI - A + BK) = 0$

$$\begin{vmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{vmatrix} - \begin{vmatrix} -5 & 5 & 0 \\ 0 & -3 & 3 \\ 0 & 0 & -1 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} \begin{vmatrix} k_1 & k_2 & k_3 \end{vmatrix} = 0$$

$$\begin{vmatrix} s+5 & -5 & 0 \\ 0 & s+3 & -3 \\ 0 & 0 & s+1 \end{vmatrix} + \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ k_1 & k_2 & k_3 \end{vmatrix} = \begin{vmatrix} s+5 & -5 & 0 \\ 0 & s+3 & -3 \\ k_1 & k_2 & (s+1+k_3) \end{vmatrix}$$

$(s+5)(s+3)(s+1+k_3) + 3k_2(s+5) = (s+12)(s+4-4j)(s+4+4j)$

$+3k_1 5$

(37)

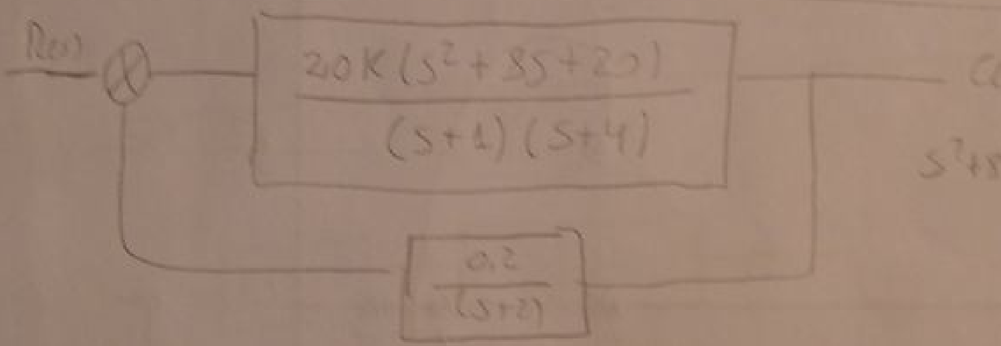
$(s^2+8s+15)(s+1+k_3)+3k_2(s+5)+15k_1$

$s^3+s^2+s^2k_3+8s^2+8s+8sk_3+15s+15+15k_3+3k_2s+15k_2+3k_1 5$

$s^3+s^2(1+k_3+8)+s(8+8k_3+15+3k_2)+15+15k_3+15k_2$

$s^3+s^2(9+k_3)+s(23+8k_3+3k_2)+15+15k_3+15k_2=$
$=s^3+20s^2+128s+384$

$$\left.\begin{array}{l} 9+k_3=20 \\ 23+8k_3+3k_2=128 \\ 15+15k_3+15k_2+15k_1=384 \end{array}\right\} \begin{array}{l} k_3=11 \\ k_2=5{,}66 \\ k_1=7{,}94 \end{array}$$



$s^2+8s+20 \Rightarrow (s+4-j2)(s+4+j2)$

$$\frac{C(s)}{R(s)}=\frac{20K(s+4-j2)(s+4+j2)}{\left(1+\frac{20K(s+4-j2)(s+4+j2)}{(s+1)(s+4)}\frac{0{,}2}{(s+2)}\right)(s+1)(s+4)}$$

$$=\frac{20K(s+4-j2)(s+4+j2)(s+2)}{(s+2)(s+1)(s+4)+4K(s+4-j2)(s+4+j2)}$$

$$=\frac{K(20s^3+200s^2+720s+800)}{s^3+7s^2+14s+8+4Ks^2+32Ks+80K}$$

$$=\frac{K(20s^3+200s^2+720s+800)}{s^3+s^2(7+4K)+s(14+32K)+8+80K}$$ } Sistema tipo 0
No parecen integradores

$$G(s)H(s)=\frac{4K(s+4+j2)(s+4-2j)}{(s+1)(s+4)(s+2)}$$, $P-Z=1$, $\varphi_k=\frac{180(2K-1)}{1}$
$\varphi_0=180°$

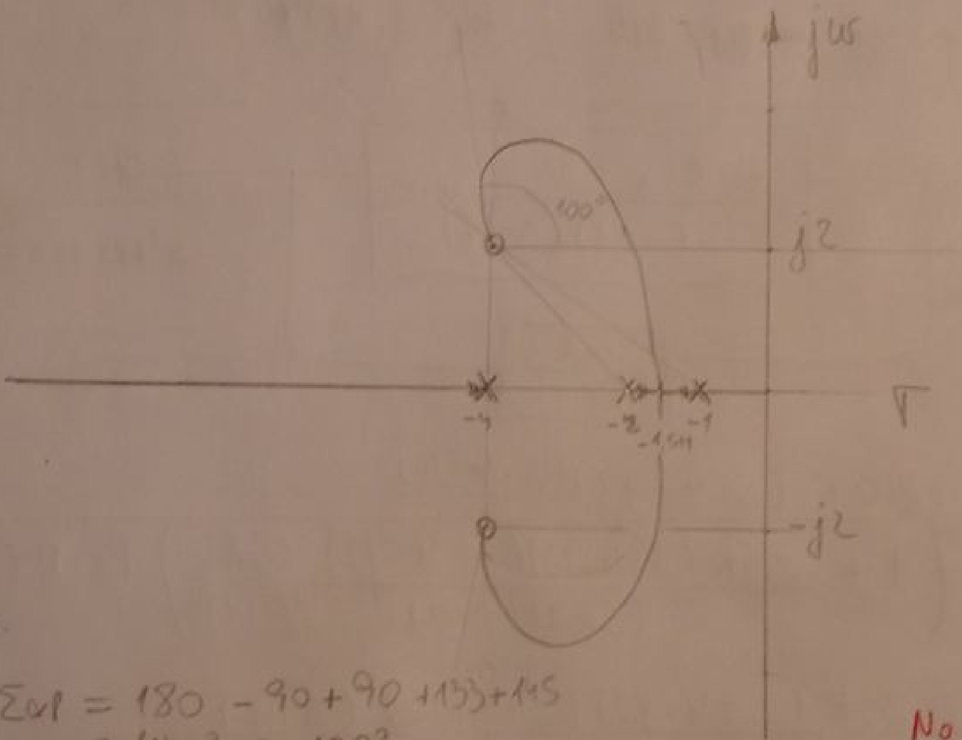
$$\sigma_c=\frac{\sum Re|P|-\sum Re|Z|}{P-Z}=\frac{-(1+4+2)-8}{1}=-1$$

Pb $\Rightarrow G(s)H(s)+1=0$

$4k(s+4+2j)(s+4-2j)+(s+1)(s+4)(s+2)=0$

$\frac{\partial K}{\partial s} \Rightarrow -\frac{(s+1)(s+4)(s+2)}{4(s^2+8s+20)} \rightarrow 0{,}25s^4+4s^3+25{,}5s^2+66s+54=0$

$$\text{Pb}\begin{cases}-1{,}511 \checkmark\\-3{,}327 \times\\ \left.\begin{array}{l}-5{,}581+j3{,}44\\-5{,}581-j3{,}44\end{array}\right\}\times\end{cases}$$



Angulo de llegada

$\alpha = 180-\Sigma\alpha z+\Sigma\alpha p = 180-90+90+135+145$
$= 460° \Rightarrow 100°$

$G(s)H(s)+1 = s^3+7s^2+14s+8+4ks^2+32ks+80K$
$= s^3+s^2(7+4K)+s(14+32K)+8+80K$

s^3	1	$(14+32K)$
s^2	$(7+4K)$	$(8+80K)$
s^1	$\frac{32K^2+50K+22{,}5}{K+1{,}75}$	
s^0	$(8+80K)$	

Sistema estable para

$(7+4K) \longrightarrow -1{,}75 \times$
$(32K^2+50K+22{,}5) \rightarrow \times$
$(8+80K) \longrightarrow -0{,}1 \times (0{,}01)$

No entran en juego valores negativos de K.

ess p/ escalón

$$ess_{escalón} = \frac{1}{1+K_p}$$

$$K_v = \lim_{s \to 0} G(s)H(s) = \frac{4(4+j2)(4-j2)}{1 \cdot 4 \cdot 2} = 10$$

$$ess = 90{,}91 \times 10^{-3}$$

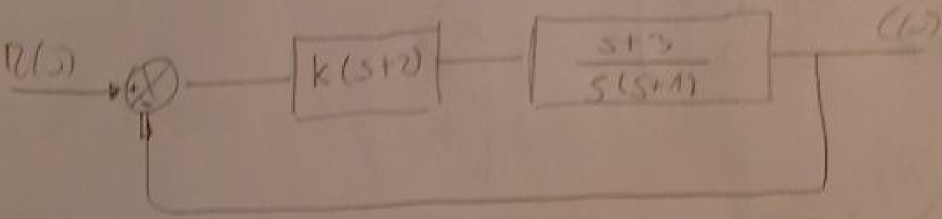
- Lugar de raíces:

A-6-1 ✓
A-6-3 ✓
A-6-4 ✓
A-6-5 ✓

A-7-6

y demás ejercicios de compensación

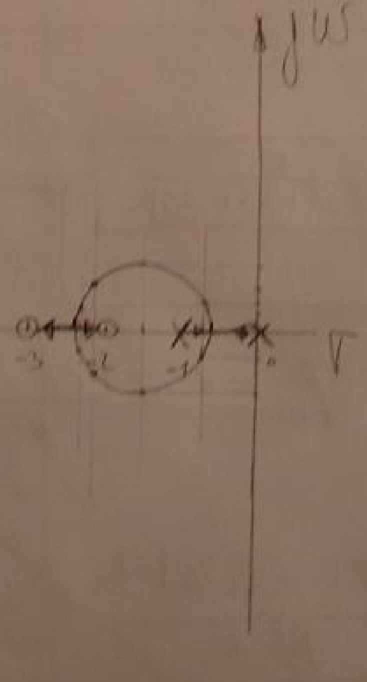
A-6-1) pág 368



$$G(s)H(s) = \frac{K(s+2)(s+3)}{s(s+1)} \quad ; \quad p - z = 0$$

$$\sigma_c = \frac{\sum Re|P| - \sum Re|Z|}{p-z} = \infty$$

s



$$G(s)H(s)+1=0$$

$$K(s+2)(s+3)+s(s+1)=0$$

$$K(s^2+5s+6)+s(s+1)=0$$

$$K=-\frac{s(s+1)}{s^2+5s+6} \quad ; \quad \frac{dK}{ds}=(s+0{,}634)(s+2{,}366)=0$$

$$Pb\begin{Bmatrix}-0{,}634\\-2{,}366\end{Bmatrix}$$ Las dos son pb

$$Ks^2+5Ks+6K+s^2+s=0$$

$$s^2(1+K)+s(5K+1)+6K=0$$

s^2	$(1+K)$	$6K$	$1+K \Rightarrow -1$	Estable para $K \geq 0$
s^1	$(5K+1)$		$5K+1 \Rightarrow -1/5$	
s^0	$6K$		$6K \Rightarrow 0$	

No corta $j\omega$

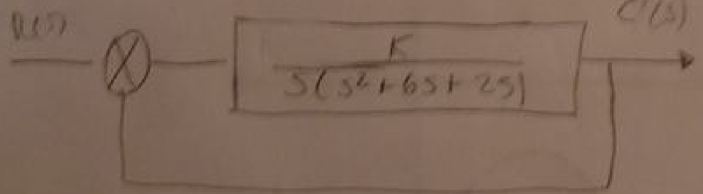
Algunos puntos:

$K=1 \Rightarrow -1{,}5 \pm j0{,}866$

$K=5 \Rightarrow -2{,}167 \pm j0{,}55$

$K=10 \Rightarrow -2{,}318 \pm j0{,}28$

A-6-3 pg 371



$$G(s)H(s)=\frac{K}{s(s^2+6s+25)}=\frac{K}{s(s+3-j4)(s+3+j4)}$$

$$p-z=3 \quad ; \quad \varphi_K=\frac{180(2K-1)}{p-z}\begin{cases}\varphi_0=-60^\circ\\ \varphi_1=60^\circ\\ \varphi_2=180^\circ\end{cases}$$

$$\sigma_c=\frac{\sum Re|P|-\sum Re|z|}{p-z}=-\frac{6}{3}=-2=\sigma_c$$

(3A)
(36)

$G(s)H(s) + 1 = 0$

$K + s(s^2+6s+25) = 0$

$K = -s(s^2+6s+25)$; $\frac{\partial K}{\partial s} = -\frac{\partial}{\partial s}\left[s(s^2+6s+25)\right]$

$\frac{\partial K}{\partial s} = 3s^2+12s+25$

$pb \Rightarrow \begin{cases} -2-j2{,}1 \\ -2+j2{,}1 \end{cases}$

$K + s^3 + 6s^2 + 25s \Rightarrow s^3 + 6s^2 + 25s + K = 0$

$$\begin{array}{lll} s^3 & 1 & 25 \\ s^2 & 6 & K \\ s^1 & \frac{150-K}{150} & \\ s^0 & K & \end{array}$$

$\left.\begin{array}{l} \frac{150-K}{150} \Rightarrow 150 \\ K \Rightarrow 0 \end{array}\right\}$ Estable para $0 \geqslant K \geqslant 150$

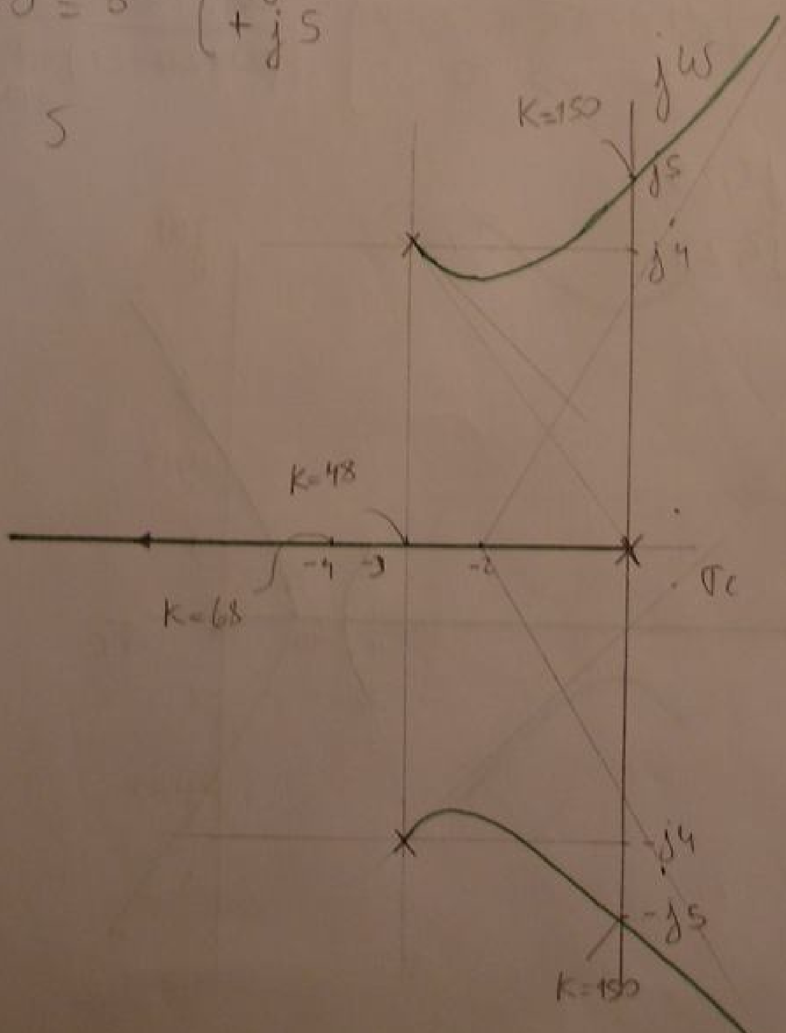
Ang salida

$-(180 + \Sigma \alpha p - \Sigma \alpha) = -(180+90+134) = -404 \Rightarrow -44°$

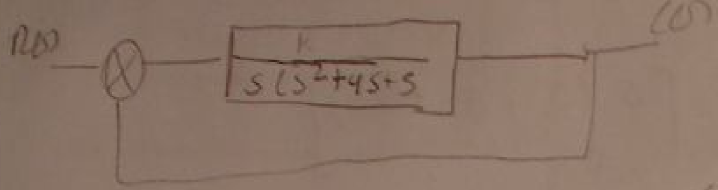
corte:

$6s^2 + 150 = 0$ $\begin{cases} -j5 \\ +j5 \end{cases}$

S



A-6-4) pg 374



$$G(s)H(s) = \frac{K}{s(s^2+4s+5)} = \frac{K}{s(s+2-j)(s+2+j)}$$

$p-z=3$; $\quad \varphi_k = \dfrac{180(2k-1)}{p-z}$ $\begin{cases} \varphi_0 = -60 \\ \varphi_1 = 60 \\ \varphi_2 = 180 \end{cases}$

$$\sigma_c = \frac{\sum Re|P| - \sum Re|Z|}{p-z} = \frac{4}{3} = \sigma_c = 1{,}33$$

$G(s)H(s)+1=0$

$K + s(s+2-j)(s+2+j) = 0$ $\Big|$ $K = -s(s+2-j)(s+2+j)$

$\dfrac{\partial K}{\partial s} = 3s^2+8s+5 = 0$

pb $\begin{cases} -1{,}667 \\ -1 \end{cases}$

$G(s)H(s)+1 = K + s^3 + 4s^2 + 5s = s^3 + s^2 4 + 5s + K = 0$

s^3	1	5
s^2	4	K
s^1	$\frac{20-K}{4}$	
s^0	K	

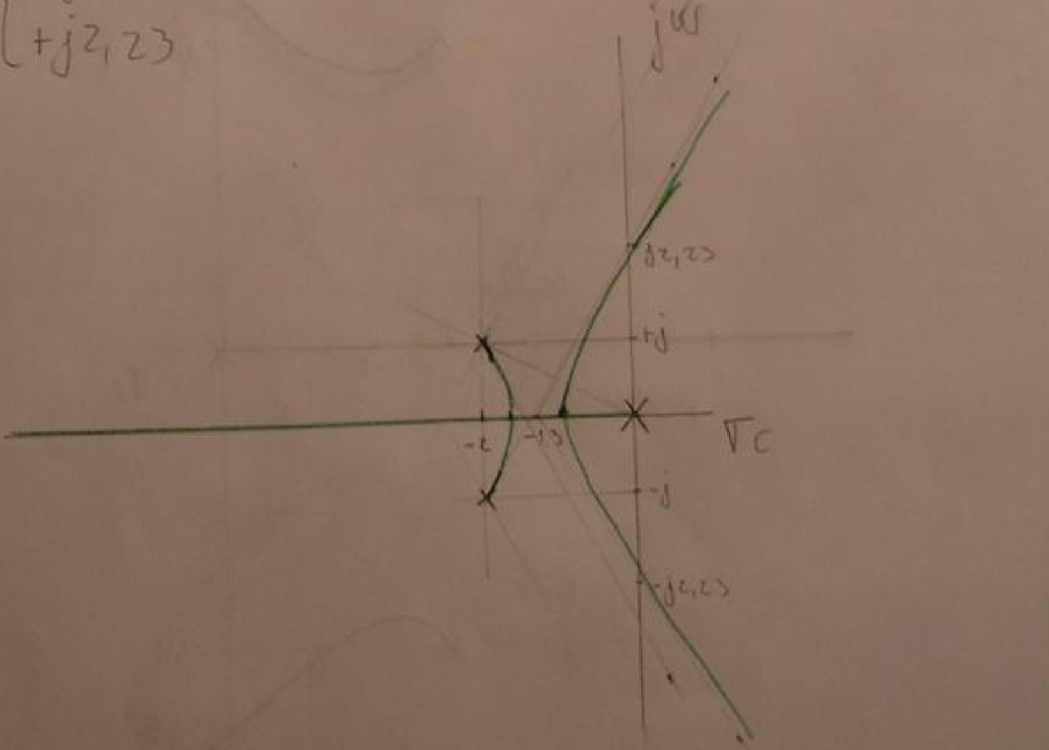
$\dfrac{20-K}{4} \Rightarrow 20$

$K \Rightarrow 0$

Estable para: $0 \leq K \leq 20$

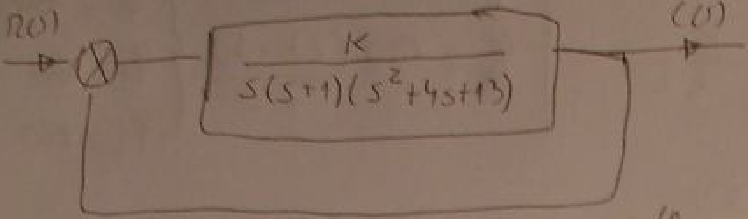
$-(180 + \sum\varphi_p - \sum\varphi_z) = -(180+90+152)$
$= -(422) \Rightarrow -62°$

corte en: $4s^2+20$ $\begin{cases} -j2{,}23 \\ +j2{,}23 \end{cases}$



A-6-5) Pg 375

(37)



$$G(s)H(s) = \frac{K}{s(s+1)(s^2+4s+13)} = \frac{K}{s(s+1)(s+2-j3)(s+2+j3)}$$

$p-z=4$, $\varphi_k = 180\frac{(2k-1)}{p-z}$ $\begin{cases} \varphi_0 = -45 \\ \varphi_1 = 45 \\ \varphi_2 = 135 \\ \varphi_3 = 225 \end{cases}$ $\sigma_c = \frac{\sum Re(p) - \sum Re(z)}{p-z} = -\frac{5}{4} = 1,25$

$G(s)H(s)+1=0$

$K + s(s+1)(s^2+4s+13)$

$K = -s(s+1)(s^2+4s+13)$

$\frac{\partial K}{\partial s} = 4s^3 + 15s^2 + 34s + 13 = 0$

$pb = \begin{cases} -0,466 \\ -1,64 \pm j2,07 \times \end{cases}$

$G(s)H(s)+1 = s^4 + 5s^3 + 17s^2 + 13s + K$

s^4	1	17	K
s^3	5	13	
s^2	14,4	K	
s^1	$\frac{187,2-5K}{14,4}$		
s^0	K		

$\frac{187,2-5K}{14,4} \Rightarrow K = 37,44$

$K \Rightarrow 0$

Estable para

$0 \leq K \leq 37,44$

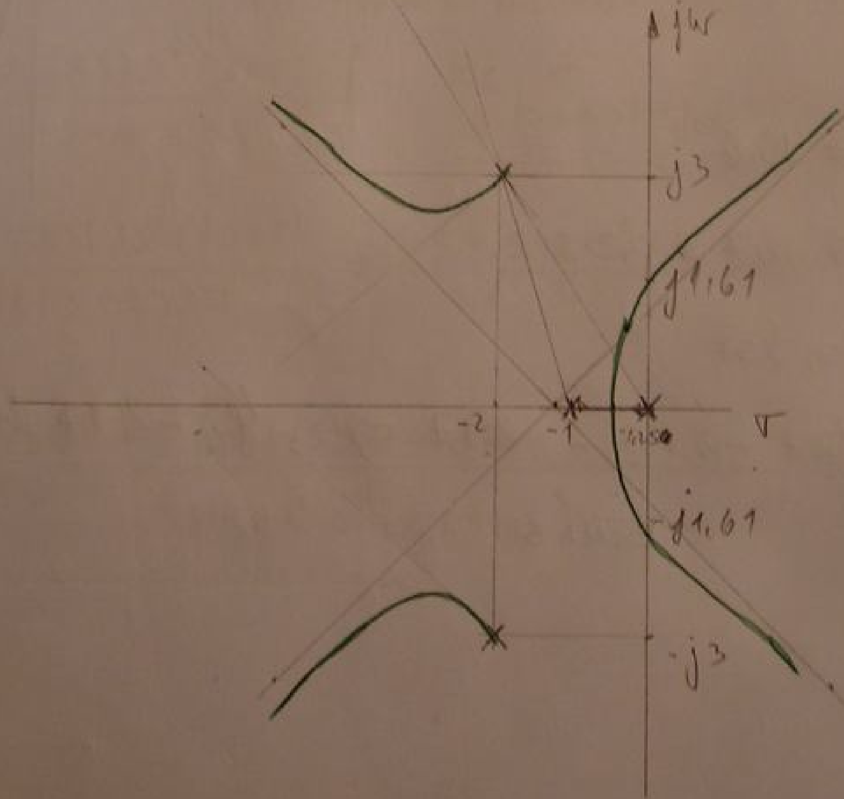
Ang sal:

$-(180 + \sum \varphi p - \sum \varphi z)$

$= -(180 + 123 + 106,190)$

$=$

Corte $\Rightarrow 14,4s^2 + 37,44 \Rightarrow \begin{cases} -j1,61 \\ +j1,61 \end{cases}$



K $K_V - c$ $M_F \geq 50°$

A-7-6) pg 442 $\xi = 0,7$, $\omega_n = 0,5$

$$H(s)\,G(s) = \frac{K}{s^2 \cdot (s+1)} = \frac{K}{s^3 + s^2}$$

$p - z = 3$

$$\varphi_k = 180 \frac{(2k-1)}{p-z} \begin{cases} \varphi_0 = -60 \\ \varphi_1 = +60 \\ \varphi_2 = 180 \end{cases}$$

$$\sigma_c = \frac{\sum Re|P| - \sum Re|z|}{p-z} = \frac{1}{3} = 0,333$$

$$G(s)H(s) + 1 = 0 \Rightarrow K + s^3 + s^2 = 0 \Rightarrow K = -(s^3 + s^2);\ \frac{\partial K}{\partial s} = -(3s^2 + 2s) = 0$$

$$Pb \begin{cases} -0,666 \times \\ 0 \end{cases}$$

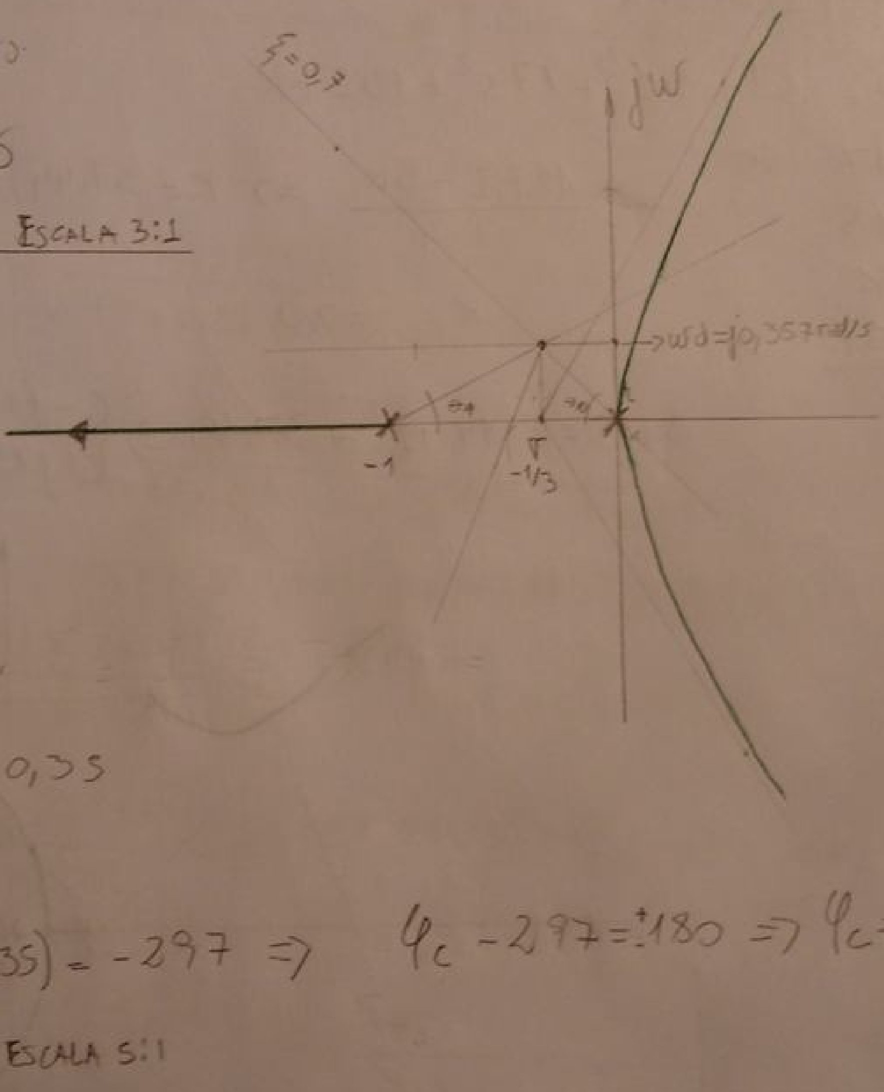
s^3	1	K
s^2	1	
s^1	K	
s^0	K	

$K \geq 0$

No corta.

S

ESCALA 3:1



$$\cos^{-1}\xi = \beta = 45,6°\ ;$$

$$\sigma = \omega_n \xi = 0,5 \cdot 0,7 = 0,35$$

$$\omega_n^2 = \sigma^2 + \omega_d^2$$

$$\omega_d = \sqrt{\omega_n^2 - \sigma^2} = 0,357$$

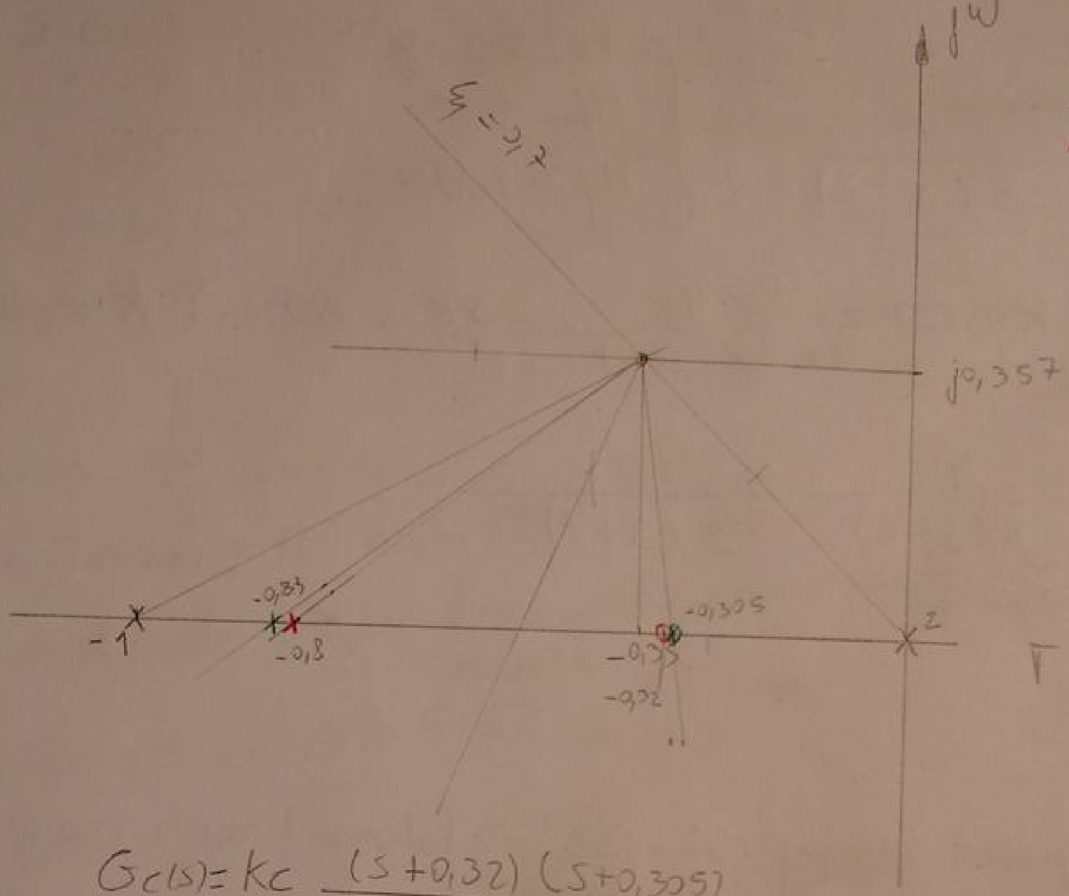
$$2\theta_0 + \theta_1 = -(27° + 2 \cdot 135) = -297 \Rightarrow \quad \varphi_c - 297 = \pm 180 \Rightarrow \varphi_c = 117$$

$\varphi_{c1} = 60°$

ESCALA 5:1

$\varphi_{c2} = 57°$

• 60°
• 57°



$P_{c1} = -0,8$
$z_{c1} = -0,32$
$P_{c2} = -0,83$
$z_{c2} = -0,305$

$$G_c(s) = K_c \frac{(s+0,32)(s+0,305)}{(s+0,8)(s+0,83)}$$

$$\Rightarrow G_c(s)G(s)H(s) = \frac{K_c \cdot (s+0,32)(s+0,305)}{(s+0,8)(s+0,83)\, s^2 (s+1)}$$

$$G_c(s)G(s)H(s) + 1 = 0 \Rightarrow K_c \cdot (s+0,32)(s+0,305) + (s+0,8)(s+0,83)s^2(s+1) = 0$$

$$K_c = -\left| \frac{(s+0,8)(s+0,83)s^2(s+1)}{(s+0,32)(s+0,305)} \right| \qquad K_c \Big|_{s=0,35+j0,357} =$$

$$= -\frac{|s+0,8|\,|s+0,83|\,|s^2|\,|s+1|}{|s+0,32|\,|s+0,305|} = \frac{5,7 \cdot 5,95 \cdot 5^2 \cdot 7,4}{3,6 \cdot 3,7} = 471 = K_c$$

$$G_c(s)G(s)H(s) = K_c \frac{s^2 + 0,625s + 0,0976}{s^5 + 2,63s^4 + 2,294s^3 + 0,664s^2}$$

K ______ $K_v = 5$; $MF \geq 50°$

Compensación con Bode.

Ogata pág 615

$$G(s) = \frac{4}{s(s+2)} \quad \text{especif.} \begin{cases} K_v = 20\ seg^{-1} \\ M_f = 50° \\ M_g \geq 10\,dB \end{cases}$$

$$K_v = \lim_{s \to 0} K s G(s) = \lim_{s \to 0} \frac{\cancel{s}\, 4K}{\cancel{s}(s+2)} = 2K = 20 \therefore K = 10$$

$$\Rightarrow KG(s) = \frac{4K}{s(s+2)} = \frac{40}{s(s+2)} \; ; \; K_{te} = \frac{40}{2} = 20 \Rightarrow 20\log(20) = 2$$

Del bode encontramos que:

$M_f = 22° \Rightarrow$

$M_g = \infty$

$\varphi_c = M\phi$ pedido $- M\phi$ sistema $+ 5° = 33° \Rightarrow$ Podemos hacerlo con un compensador

$$G(s) = K_c \frac{\left(s + \frac{1}{T}\right)}{\left(s + \frac{1}{T\alpha}\right)} = \overbrace{K_c\alpha}^{K} \frac{(Ts+1)}{(\alpha Ts+1)} \; ; \; \operatorname{sen} \varphi_c = \frac{1-\alpha}{1+\alpha}$$

$$0{,}544 + 0{,}544\alpha - 1 + \alpha = 0 \Rightarrow \alpha(0{,}544+1) = 1 - 0{,}544$$

$$\alpha = \frac{1-0{,}544}{0{,}544+1} = 0{,}295$$

del Bode

$$|G(\omega)| = -20\log\frac{1}{\sqrt{\alpha}} = -20\log\frac{1}{\sqrt{0{,}295}} = -5{,}30\,dB \Rightarrow \omega_n = 9\,rad/s$$

$$\omega_n = \frac{1}{\sqrt{\alpha}\,T} \Rightarrow \omega_n\sqrt{\alpha} = \frac{1}{T} = 4{,}88\,rad \Rightarrow \text{zero} \; ; \; \underbrace{\frac{1}{T\alpha} = 16{,}54\,rad/seg}_{\text{polo}}$$

$$K = K_c\alpha \; ; \; 10 = K_c \cdot 0{,}295 \Rightarrow K_c = 34$$

$$\Rightarrow G_c(s)\,G(s) = 34\frac{(s+4{,}88)}{(s+16{,}54)} \cdot \frac{4}{s(s+2)}$$

$$G(s) = \frac{K}{s(s+2)(s+10)} \quad ; \quad K_V = 5 \quad ; \quad MF \geq 50^\circ$$

$$K_V = \lim_{s \to 0} s\,G(s) = \frac{K}{20} = 5 \Rightarrow K = 100$$

$$G(s) = \frac{100}{s(s+2)(s+10)} \quad ; \quad \text{Graficamos el bode}$$

$20 \log k = 20 \log 5 = 14.$

$M_F = 13{,}5^\circ$

$M_G = 6\,dB$

$$\varphi_c = -180^\circ + M\phi_{pedido} + 12^\circ = -118^\circ$$

Buscamos en el Bode a que frecuencia corresponde $-118^\circ \Rightarrow 0{,}8\,rad/s$

Modulo " " $\Rightarrow 16\,db$

$$20 \log \beta = 15 \Rightarrow \beta = 5{,}62 = 10^{15/20}$$

El cero irá:

$$\frac{w_c}{10} < z_c < \frac{w_c}{2} \Rightarrow \frac{0{,}8}{10} < z_c < \frac{0{,}8}{2} \Rightarrow 0{,}08 < z_c < 0{,}4$$

$z_c \Rightarrow 0{,}24$

$$\Rightarrow \text{Polo} = \frac{1}{T\beta} = \frac{z_c}{\beta} = \frac{0{,}24}{5{,}62} = 0{,}04$$

$$G_c(s) = 17{,}8\,\frac{(s+0{,}24)}{(s+0{,}04)} \quad ; \quad k_c = \frac{K}{\beta} = \frac{100}{5{,}62} = 17{,}8.$$

$$G_c(s)\,G(s) = 17{,}8\,\frac{(s+0{,}24)}{(s+0{,}04)} \cdot \frac{1}{s(s+2)(s+10)}$$

$$\frac{17{,}8s + 4{,}272}{s^4 + 12{,}04s^3 + 20{,}48s^2 + 0{,}8s}$$

$\varphi_c = M_{F necesario} - M_{F sist} + 12° = 48{,}5°$

$\operatorname{sen}\varphi_c = \dfrac{1-\alpha}{1+\alpha}$, $\operatorname{sen}\varphi_c + \alpha \operatorname{sen}\varphi_c - 1 + \alpha = 0$

$\alpha(\operatorname{sen}\varphi_c + 1)$; $\dfrac{1-\operatorname{sen}\varphi_c}{1+\operatorname{sen}\varphi_c} = \alpha = 0{,}143$

$|G(\omega)| = -20\log\dfrac{1}{\sqrt{\alpha}} = -8{,}44$

$\hookrightarrow \omega_c = 5 \text{ rad/seg}$

$G_c = k_c \dfrac{(s+\frac{1}{T})}{(s+\frac{1}{\alpha T})} = \alpha k_c \dfrac{(Ts+1)}{(\alpha Ts+1)}$; $\omega_x = \dfrac{1}{\sqrt{\alpha}T}$; $\dfrac{1}{T} = \omega_x\sqrt{\alpha}$

$\Rightarrow$ Zero $\Rightarrow \omega_c\sqrt{\alpha} = 1{,}9 \approx 2$ rad/seg p/ Zero.

$\dfrac{}{T} \Rightarrow$ Pole $\Rightarrow 13{,}3$ rad/seg

$k = k_c\alpha \Rightarrow k_c = \dfrac{k}{\alpha} = \dfrac{100}{0{,}143} = 699{,}3$

$\Rightarrow G_c(s)\,G(s) = 699{,}3 \cdot \dfrac{(s+2)}{(s+13{,}3)} \dfrac{1}{s(s+2)(s+10)}$

$= \dfrac{699{,}3s + 1398{,}6}{s^4 + 25{,}3s^3 + 179{,}6s^2 + 266s}$

$GM(s) = \dfrac{K(s+10)(s^2+9s+81)}{s(s+4)(s^2+14{,}4s+64)}$

$= \dfrac{K(s+10)(s+4{,}5-j7{,}8)(s+4{,}5+j7{,}8)}{s(s+4)(s+7{,}2+j3{,}5)(s+7{,}2-j3{,}5)}$

$N_{asin} = p - z = 4 - 3 = 1$; $\varphi_0 = -180°$, $\sigma_c = \dfrac{\sum Re|P| - \sum Re|z|}{p-z} = -15$

$Pb \Rightarrow GM(s) + 1 = 0$

$K(s+10)(s^2+9s+81) + s(s+4)(s^2+14{,}4s+64) = 0$

$K = -\dfrac{s(s+4)(s^2+14{,}4s+64)}{(s+10)(s^2+9s+81)} \Rightarrow \dfrac{dk}{ds} = s^6 + 38s^5 + 741s^4 + 9020{,}8s^3$
$+ 60641{,}6s^2 + 196992s$
$+ 207360 = 0$

$Pb = \begin{cases} -1{,}87 \\ -13{,}31 \end{cases}$

$s^3K + 19Ks^2 + 171Ks + 810K + s^4 + 18{,}4s^3 + 121{,}6s^2 + 256s = 0$ (40)

$s^4 + s^3(K+18{,}4) + s^2(19K+121{,}6) + s(171K+256) + 810K = 0$

$s^4 \quad 1 \quad (19K+121{,}6) \quad 810K$

$s^3 \quad (K+18{,}4) \quad (171K+256)$

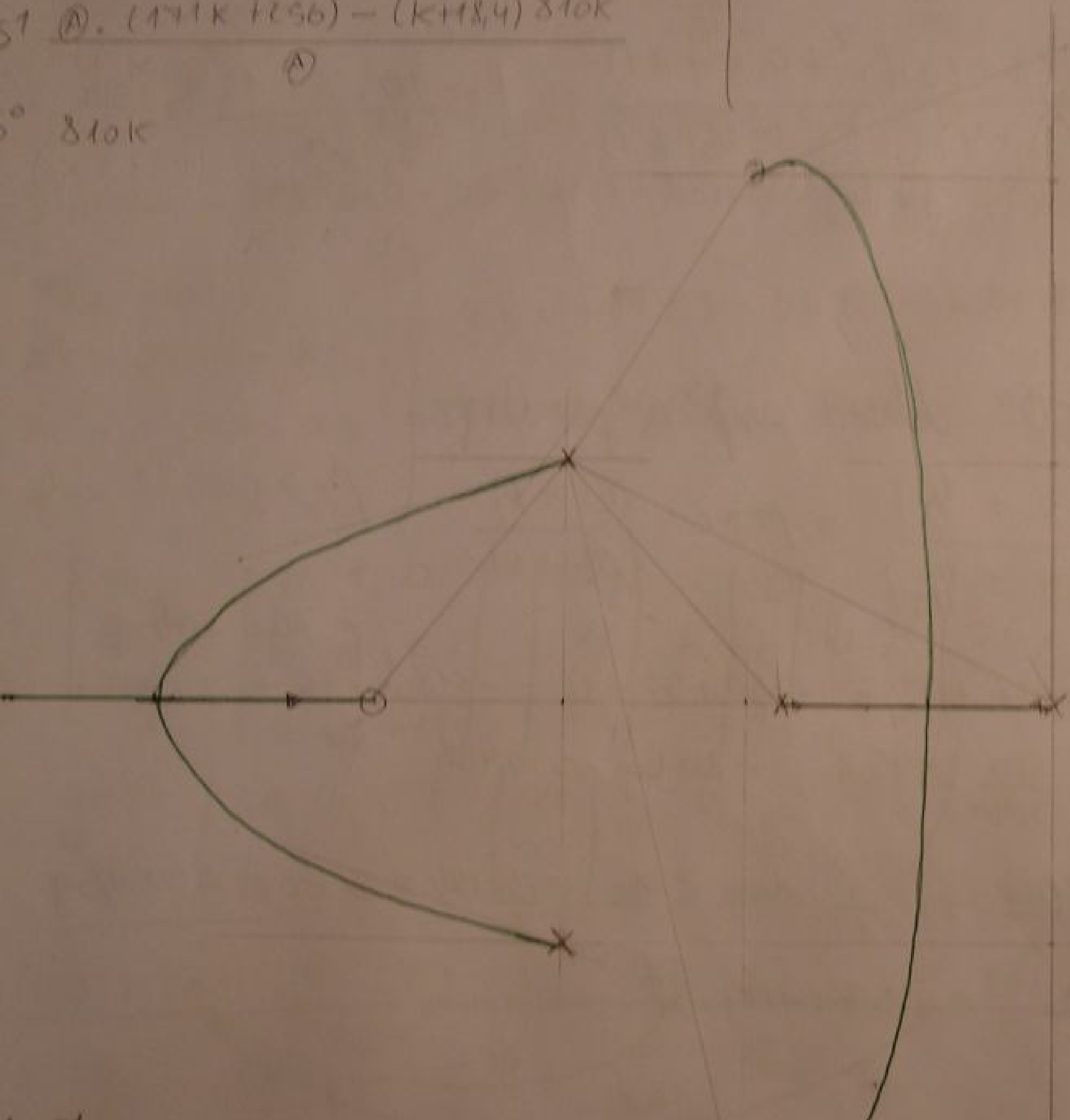
$s^2 \quad \dfrac{\overset{A}{(K+18{,}4)(19K+121{,}6)-(171K+256)}}{(K+18{,}4)} \quad 810K$

$s^1 \quad \dfrac{A\cdot(171K+256)-(K+18{,}4)\,810K}{A}$

$s^0 \quad 810K$

$K = -18{,}4\ ;\ -6{,}4$

Estable para $K \geq 0$



Ang sal

$-(180 - \Sigma\alpha z + \Sigma\alpha p)$

$-(180-(52+23+104)+(131+154+90))$

$= 162$

Ang lleg

$+180 - \Sigma\alpha z + \Sigma\alpha p$

$180-(90°+55)+(95+120+76+57) = 383$

$= 23°$

Tema 3)

$$GM(s) = \frac{k(s+10)(s^2+9s+81)}{s(s+4)(s^2+14{,}4s+64)}$$

$$Kv = \lim_{s\to 0} s\,k\,GM(s) = \frac{k \cdot 10 \cdot 81}{4 \cdot 64} = k \cdot 3{,}164 = 50$$

$\Rightarrow k = 15{,}8$

$$GM(s) = \frac{15{,}8\,(s+10)(s^2+9s+81)}{s(s+4)(s^2+14{,}4s+64)} \; ; \; k = \frac{10 \cdot 81 \cdot 15{,}8}{4 \cdot 64} = 50$$

$|k|_{dB} = 20 \log k = 34\,dB$

$M_G = \infty$

$M\varphi = 67{,}5°$

como se piden 50° deberé compensar por atraso

$$Gc(s) = kc\,\frac{(s+1/T)}{(s+1/\beta T)} = \beta kc\,\frac{(Ts+1)}{(\beta Ts+1)} \; ; \; \beta > 1$$

$\varphi c = -180° + M\varphi_{pedido} + 12° = -180 + 50 + 12 = -118.$ $\left\{ \begin{array}{l} \omega_1 = 3\,rad/seg \\ |G(\omega)| = 34\,dB \end{array} \right.$

$$\beta = \frac{Kv_{pedido}}{Kv_{sist}} = \frac{50}{3{,}164} = 15{,}8 \; ; \; 20 \log \beta = 34\,dB$$

$$\frac{\omega_1}{10} < zc < \frac{\omega_1}{2} \Rightarrow \frac{3}{10}\,rad/seg < zc < \frac{3}{2}\,rad/seg \Rightarrow zc = 0{,}9\,rad/seg$$

$$P = \frac{1}{\beta T} = \frac{zc}{\beta} = 0{,}057\,rad/seg \Rightarrow P$$

0,6

0,033

$$kc = \frac{K}{\beta} =$$

$$\Rightarrow Gc(s)G(s) = \frac{(s+0{,}9)}{(s+0{,}057)} \cdot \frac{(s+10)(s^2+9s+81)}{s(s+4)(s^2+14{,}45s+64)}$$

Un compensador x atraso prácticamente no modifica el LR del sistema original. (solo lo modifica a la cercanía del origen)

Tema 4)

$$\dddot{y}(t)+11\ddot{y}(t)+36\dot{y}(t)+36y(t)=36\,r(t) \qquad \frac{y(t)}{R(t)} \qquad \begin{matrix}\lambda_1=-2\\ \lambda_2=-3\\ \lambda_3=-6\end{matrix}$$

$$s^3y(s)+s^2\,11\,y(s)+s\,36\,y(s)+36y(s)=36\,R(s)$$

$$y(s)\left[s^3+11s^2+36s+36\right]=36\,R(s)$$

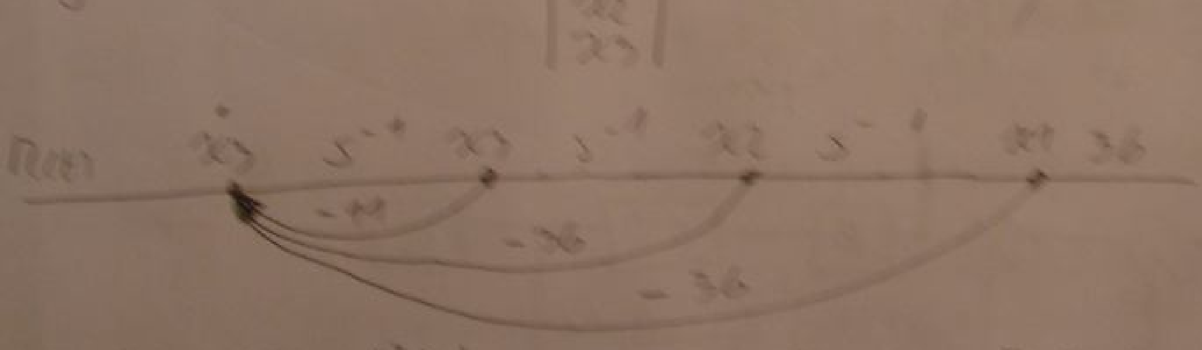
$$\frac{y(s)}{R(s)}=\frac{36}{s^3+11s^2+36s+36}\cdot\frac{Q(s)}{Q(s)} \Rightarrow \frac{y(s)}{36}=\frac{R(s)}{s^3+11s^2+36s+36}=Q(s)$$

$$R(s)=s^3Q(s)+11s^2Q(s)+36sQ(s)+36Q(s)$$

$$s^3Q(s)=-11s^2Q(s)-36sQ(s)-36Q(s)+R(s)$$

$$\left.\begin{aligned} x_1&=Q(s)\\ \dot{x}_1&=sQ(s)=x_2\\ \dot{x}_2&=s^2Q(s)=x_3\\ \dot{x}_3&=s^3Q(s)\end{aligned}\right\} \quad \begin{aligned}\dot{x}_3&=-11x_3-36x_2-36x_1+R(s)\\ y(s)&=36x_1\end{aligned}$$

$$\left.\begin{aligned}\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}&=\begin{vmatrix}0&1&0\\0&0&1\\-36&-36&-11\end{vmatrix}\begin{vmatrix}x_1\\x_2\\x_3\end{vmatrix}+\begin{vmatrix}0\\0\\1\end{vmatrix}R(t)\\ y(t)&=\begin{vmatrix}36&0&0\end{vmatrix}\begin{vmatrix}x_1\\x_2\\x_3\end{vmatrix}\end{aligned}\right\}\ \text{Ec dinámica}$$



$$O=\begin{vmatrix}B & AB & A^2B\end{vmatrix}=\begin{vmatrix}0&0&1\\0&1&-11\\1&-11&85\end{vmatrix} \Rightarrow \text{Rango}=3\ \text{observable}$$

$$C=\begin{vmatrix}C^T & A^TC^T & A^{2T}C^T\end{vmatrix}$$

$$C=\begin{vmatrix}36&0&0\\0&36&0\\0&0&36\end{vmatrix}\ \text{Rango}=3\ \text{controlable}$$

$\dot{x} = Ax(t) + BU(t)$

$y(t) = Cx(t)$

$U(t) = -Kx(t)$

$sx(s) = Ax(s) - BKx(s)$

$sx(s) - Ax(s) + BKx(s) = 0$

$(sI - A + BK)\,x(s) = 0$

$$s\begin{vmatrix}1&0&0\\0&1&0\\0&0&1\end{vmatrix} - \begin{vmatrix}0&1&0\\0&0&1\\-36&-36&-11\end{vmatrix} + \begin{vmatrix}0\\0\\1\end{vmatrix}|k_1\ k_2\ k_3|$$

$$\begin{vmatrix}s&0&0\\0&s&0\\0&0&s\end{vmatrix} - \begin{vmatrix}0&1&0\\0&0&1\\-36&-36&-11\end{vmatrix} + \begin{vmatrix}0&0&0\\0&0&0\\k_1&k_2&k_3\end{vmatrix}$$

$$\begin{vmatrix}s&-1&0\\0&s&-1\\36&36&s+11\end{vmatrix} + \begin{vmatrix}0&0&0\\0&0&0\\k_1&k_2&k_3\end{vmatrix} = \begin{vmatrix}s&-1&0\\0&s&-1\\36+k_1&36+k_2&s+11+k_3\end{vmatrix}\begin{matrix}s&-1&0\\0&s&-1\end{matrix}$$

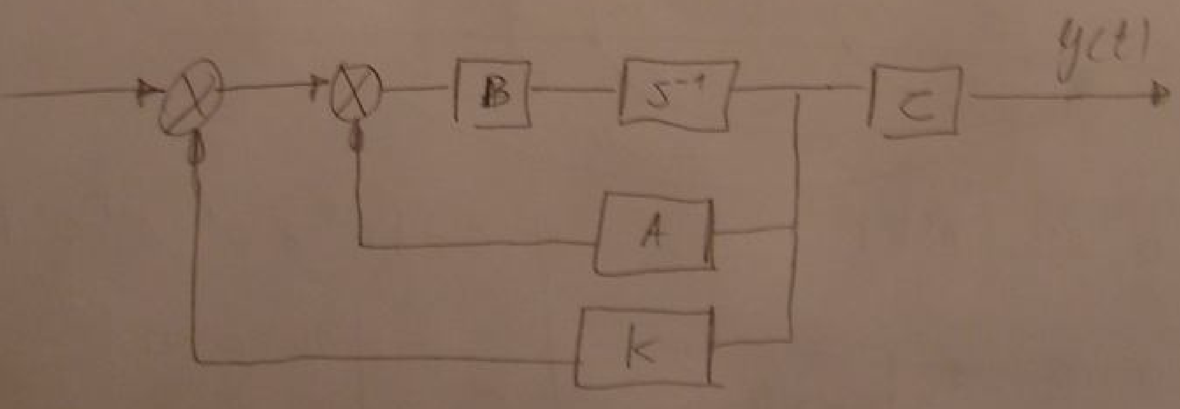
$|sI - A + Bk| = s^2(s+11+k_3) + 36 + k_1 + s(36+k_2)$

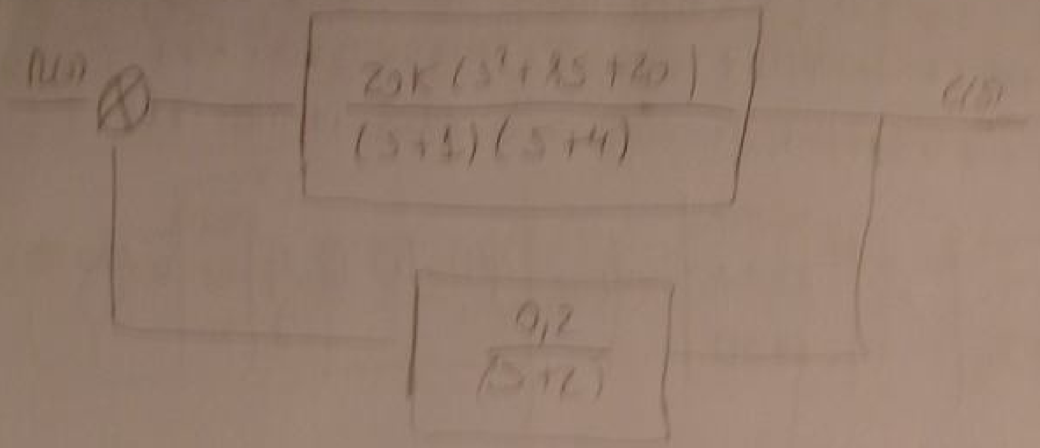
$= s^3 + 11s^2 + k_3 s^2 + 36 + k_1 + 36s + k_2 s$

$= s^3 + s^2(11+k_3) + s(36+k_2) + 36 + k_1 = (s+10)(s+3+3j)(s+3-3j)$

$= s^3 + 16s^2 + 78s + 180$

$$\left.\begin{aligned}11+k_3 &= 16\\36+k_2 &= 78\\36+k_1 &= 180\end{aligned}\right\}\quad\begin{aligned}k_3 &= 5\\k_2 &= 42\\k_1 &= 144\end{aligned}$$





$P/K = 5$

. Respuesta en FCO

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G\,H(s)} = \frac{100(s^2+8s+20)}{\left[1+\frac{100(s^2+8s+20)}{(s+1)(s+4)}\frac{0,2}{(s+2)}\right](s+1)(s+4)}$$

$$= \frac{100(s^2+8s+20)(s+2)}{(s+2)(s+1)(s+4)+100(s^2+8s+20)\,0,2}$$

$$\frac{C(s)}{R(s)} = \frac{100s^3+1000s^2+3600s+4000}{s^3+27s^2+174s+408}$$

$$\frac{C(s)}{R(s)} = \frac{100+1000s^{-1}+3600s^{-2}+4000s^{-3}}{1+27s^{-1}+174s^{-2}+408s^{-3}}$$

$$C(s)\left[1+27s^{-1}+174s^{-2}+408s^{-3}\right] = R(s)\left[100+1000s^{-1}+3600s^{-2}+4000s^{-3}\right]$$

$$C(s) = \left[-27s^{-1}-174s^{-2}-408s^{-3}\right]C(s)+R(s)100+1000s^{-1}R(s)+3600s^{-2}R(s)+$$
$$+4000s^{-3}R(s)$$

$$C(s) = -27s^{-1}C(s)-174s^{-2}C(s)-408s^{-3}C(s)+100R(s)+1000s^{-1}R(s)+3600s^{-2}R(s)+$$
$$+4000s^{-3}R(s)$$

$$= \underbrace{s^{-1}\Big\{-27C(s)+1000R(s)+\underbrace{s^{-1}\Big[-174C(s)+3600R(s)+\underbrace{s^{-1}\{\overbrace{-408C(s)+4000R(s)}^{\dot{x}_1}\}}_{x_1}\Big]}_{x_2}\Big\}}_{x_3} + 100R(s)$$

$\dot{x}_1 = -408\,y(t)+4000R(s)$

$\dot{x}_2 = -174\,y(t)+3600R(s)+x_1$

$\dot{x}_3 = -27\,y(t)+1000R(s)+x_2$,

$y(t) = x_3+100R(s)$

V_i $\dot{x}_1 = -408\,y(t) + 4000R(s) = -408(x_3 + 100R(s)) + 4000R(s) = -408x_3 - 36800R(s)$

$\dot{x}_2 = -174\,y(t) + 3600R(s) = -174(x_3 + 100R(s)) + 3600R(s) = -174x_3 - 13800R(s) + x_1$

$\dot{x}_3 = -27\,y(t) + 1000R(s) + x_2 = -27(x_3 + 100R(s)) + 1000R(s) + x_2 = -27x_3 - 1700R(s) + x_2$

V_i
$$\begin{vmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{vmatrix} = \begin{vmatrix}0 & 0 & -408\\ 1 & 0 & -174\\ 0 & 1 & -27\end{vmatrix}\begin{vmatrix}x_1\\ x_2\\ x_3\end{vmatrix} + \begin{vmatrix}-36800\\ -13800\\ -1700\end{vmatrix} R(s) \qquad y(t) = \begin{vmatrix}0 & 0 & 1\end{vmatrix}\begin{vmatrix}x_1\\ x_2\\ x_3\end{vmatrix} + 100R(s)$$

V_o $C(s) = 100s^3x(s) + 1000s^2x(s) + 3600x(s)s + 4000x(s)$

$100s^3x(s) = C(s) - 1000s^2x(s) - 3600sx(s) - 4000x(s)$

V_o
$$\left.\begin{aligned} s^3x(s) &= \dot{x}_3\\ s^2x(s) &= x_3 = \dot{x}_2\\ sx(s) &= x_2 = \dot{x}_1\\ x(s) &= x_1\end{aligned}\right\} \quad \begin{aligned} C(s) &= 100\dot{x}_3 + 1000x_3 + 3600x_2 + 4000x_1\\ &= -2700x_3 - 17400x_2 - 40800x_1 + 100R(s) + 1000x_3\\ &\quad + 3600x_2 + 4000x_1\\ &= -1700x_3 - 13800x_2 - 36800x_1 + 100R(s)\end{aligned}$$

$\frac{V_o}{V_i}$ $R(s) = (s^3 + 27s^2 + 174s + 408)\,x(s)$

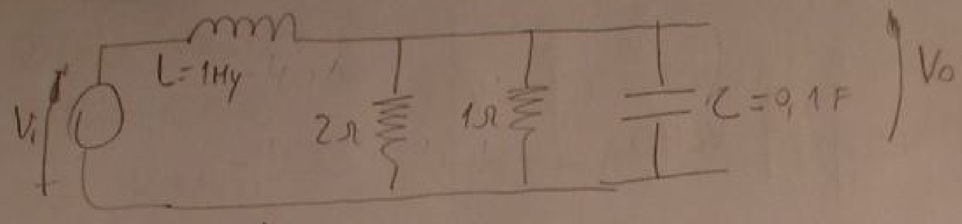
$s^3x(s) = -27s^2x(s) - 174sx(s) - 408x(s) + R(s)$

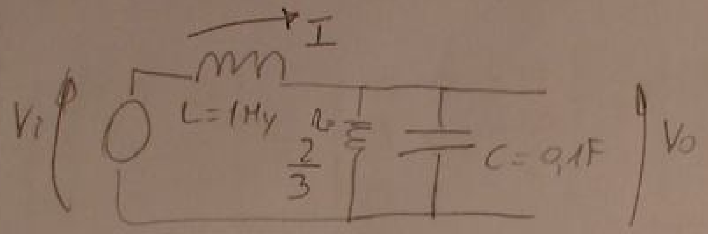
$\dot{x}_3 = -27x_3 - 174x_2 - 408x_1 + R(s)$

$\frac{V_o}{V_i}$
$$\begin{vmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{vmatrix} = \begin{vmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -408 & -174 & -27\end{vmatrix}\begin{vmatrix}x_1\\ x_2\\ x_3\end{vmatrix} + \begin{vmatrix}0\\ 0\\ 1\end{vmatrix} U(s)$$

$\frac{V_o}{V_i}$ $y(t) = \begin{vmatrix}-36800 & -13800 & -1700\end{vmatrix}\begin{vmatrix}x_1\\ x_2\\ x_3\end{vmatrix} + 100\,U(s)$

Vi entrada
Vo salida





$$V_i = LS + \frac{R\ 1/CS}{R + 1/CS}$$

$$= LS + \frac{R}{CS\left(R + \frac{1}{CS}\right)}$$

$$= LS + \frac{R}{SCR + 1}$$

$$= LS + \frac{\cancel{R}}{\left(S + \frac{1}{CR}\right)C\cancel{R}}$$

$$V_i = I(s)\left[LS + \frac{1}{C}\frac{1}{s + 1/CR}\right]$$

$$= I(s)\left[\frac{LCS(S + 1/CR) + 1}{C(S + 1/CR)}\right]$$

$$V_o = I(s)\frac{1}{C\left(S + \frac{1}{CR}\right)}$$

$$\frac{V_o}{V_i} = \frac{\cancel{C(S + 1/CR)}}{\cancel{C\left(S + \frac{1}{CR}\right)}\left[LCS(S + 1/CR) + 1\right]} = \frac{1}{LCS(S + 1/CR) + 1}$$

$$= \frac{1}{LCS^2 + \frac{LC}{CR}S + 1} = \frac{1/LC}{S^2 + \frac{1}{RC}S + \frac{1}{LC}}$$

$$\frac{V_o}{V_i} = \frac{10}{S^2 + 15S + 10}\ \frac{x(s)}{x(s)}$$

$$V_o = 10\,x(s)$$

$$V_i = (S^2 + 15S + 10)\,x(s)$$

$$x(s)S^2 = -15S\,x(s) - 10\,x(s) + V_i$$

$$\dot{x}_2 = x(s)S^2$$

$$x_2 = S\,x(s) = \dot{x}_1$$

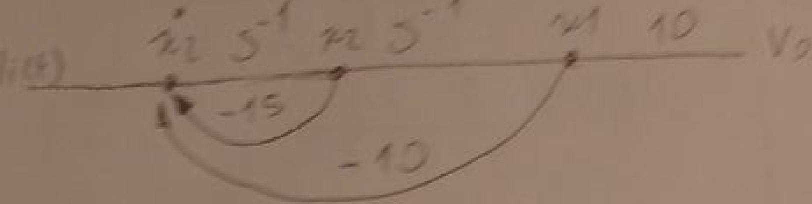
$$x_1 = x(s)$$

$$\dot{x}_2 = -15x_2 - 10x_1 + V_i$$

$$V_o = 10x_1$$

$$\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ -10 & -15 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} + \begin{vmatrix} 0 \\ 1 \end{vmatrix} v_i(t)$$

$$V_0(t) = \begin{vmatrix} 10 & 0 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \end{vmatrix}$$



$$\frac{C(s)}{R(s)} = \frac{12}{(s+1)(s+2)(s+6)}$$

* y(t) si R(t) = 0 y x(0) = $\begin{vmatrix} 1 \\ 1 \\ 0 \end{vmatrix}$

- Represent VE
- Diagrama de estado
- Representar p/ $s_1 = -12$
- $\phi(t)$ *

$s_{2,3} = -3 \pm j3$

$$\frac{C(s)}{R(s)} = \frac{12}{s^3 + 9s^2 + 20s + 12} = \frac{12s^{-3}}{1 + 9s^{-1} + 20s^{-2} + 12s^{-3}}$$

$$C(s) = [-9s^{-1} - 20s^{-2} - 12s^{-3}]\,C(s) + 12R(s)s^{-3}$$

$$= -9s^{-1}C(s) - 20s^{-2}C(s) - 12s^{-3}C(s) + 12R(s)s^{-3}$$

$$= \underbrace{s^{-1}\{-9C(s) + \underbrace{s^{-1}\{-20C(s) + \underbrace{s^{-1}\{-12C(s) + 12R(s)\}}_{x_1}\}}_{x_2}\}}_{x_3}$$

$\dot{x}_1 = -12C(s) + 12R(s)$

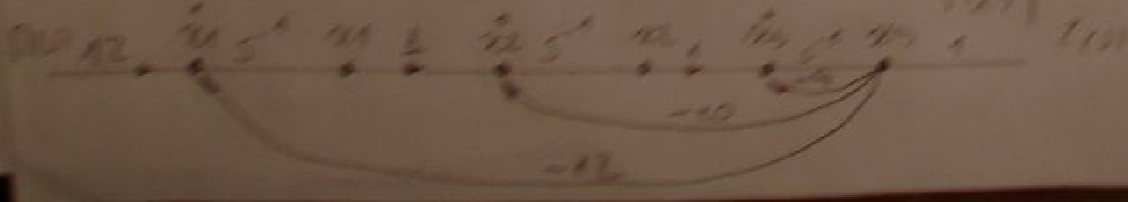
$\dot{x}_2 = -20C(s) + x_1$

$\dot{x}_3 = -9C(s) + x_2$

$x_3 = C(s)$

$$\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{vmatrix} = \begin{vmatrix} 0 & 0 & -12 \\ 1 & 0 & -20 \\ 0 & 1 & -9 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} + \begin{vmatrix} 12 \\ 0 \\ 0 \end{vmatrix} R(t)$$

$$C(t) = \begin{vmatrix} 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix}$$



(44)

$\dot{x}(t) = Ax(t) + Bu(t)$
$y(t) = Cx(t)$
$u = -Kx(t)$

$sx(s) = Ax(s) - BKx(s)$
$sx(s) - Ax(s) + Kx(s)$
$(sI - A + BK)x(s) = 0$

$$\begin{vmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{vmatrix} - \begin{vmatrix} 0 & 0 & -12 \\ 1 & 0 & -20 \\ 0 & 1 & -9 \end{vmatrix} + \underbrace{\begin{vmatrix} 12 \\ 0 \\ 0 \end{vmatrix} |k_1\ k_2\ k_3|}$$

Ojo con la multiplicación!! Había tomado por costumbre $BK = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ k_1 & k_2 & k_3 \end{vmatrix}$

En este caso $BK = \begin{vmatrix} 12k_1 & 12k_2 & 12k_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$

$$\begin{vmatrix} s & 0 & 12 \\ -1 & s & 20 \\ 0 & -1 & 9+s \end{vmatrix} + BK \Rightarrow \begin{vmatrix} s+12k_1 & 12k_2 & 12+12k_3 \\ -1 & s & 20 \\ 0 & -1 & s+9 \end{vmatrix}$$

$$s^3 + 9s^2 + 20s + 12k_1s^2 + 108k_1s + 240k_1 + 12k_2s + 108k_2 + 12 + 12k_3$$

$$s^3 + s^2(9 + 12k_1) + s(20 + 108k_1 + 12k_2) + 240k_1 + 108k_2 + 12 + 12k_3$$
$$= s^3 + 18s^2 + 90s + 216$$

$$\left.\begin{aligned} (9 + 12k_1) &= 18 \\ (20 + 108k_1 + 12k_2) &= 90 \\ (240k_1 + 108k_2 + 12 + 12k_3) &= 216 \end{aligned}\right\} \begin{aligned} k_1 &= 0{,}75 \\ k_2 &= -0{,}916 \\ k_3 &= 10{,}25 \end{aligned}$$

$$\phi(t) = \mathcal{L}^{-1}\left[(sI - A)^{-1}\right]$$

$$sI - A = \begin{vmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{vmatrix} - \begin{vmatrix} 0 & 0 & -12 \\ 1 & 0 & -20 \\ 0 & 1 & -9 \end{vmatrix} = \begin{vmatrix} s & 0 & 12 \\ -1 & s & 20 \\ 0 & -1 & 9+s \end{vmatrix}$$

$$(sI - A)^{-1} = \frac{\mathrm{Adj}^T(\mathrm{cof})}{|sI - A|}$$

$$\mathrm{cof}(sI - A) = \begin{vmatrix} s^2 + 9s + 20 & s+9 & 1 \\ -12 & s^2 + 9s & s \\ -12s & -20s - 12 & s^2 \end{vmatrix}$$

$$\mathrm{Adj}(sI-A) = \left| \begin{matrix} s^4+9s^3+20s^2+12s \\ 0 \\ 12s^3+105s^2+240s+14 \end{matrix} \right.$$

$$(s) = \left| \begin{matrix} \dfrac{s^2+9s+20}{s^3+9s^2+20s+12} & \dfrac{-12}{} & \dfrac{-12s}{} \\ \dfrac{s+9}{} & \dfrac{s^2+9s}{} & \dfrac{-(20s+12)}{} \\ \dfrac{1}{} & \dfrac{s}{} & \dfrac{s^2}{} \end{matrix} \right|$$

$$(s) = \left| \begin{matrix} \dfrac{-1,5}{s+2}+\dfrac{2,4}{s+1}+\dfrac{0,1}{s+6}; & \dfrac{3}{s+2}-\dfrac{2,4}{s+1}-\dfrac{0,6}{s+6}; & \dfrac{-6}{s+2}+\dfrac{2,4}{s+1}+\dfrac{3,6}{s+6} \\ \dfrac{-1,75}{s+2}+\dfrac{1,6}{s+1}+\dfrac{0,15}{s+6}; & \dfrac{3,5}{s+2}-\dfrac{1,6}{s+1}-\dfrac{0,9}{s+6}; & \dfrac{-7}{s+2}+\dfrac{1,6}{s+1}+\dfrac{5,4}{s+6} \\ \dfrac{-0,25}{s+2}+\dfrac{0,2}{s+1}+\dfrac{0,05}{s+6}; & \dfrac{0,5}{s+2}-\dfrac{0,2}{s+1}-\dfrac{0,3}{s+6}; & \dfrac{-1}{s+2}+\dfrac{0,2}{s+1}+\dfrac{1,8}{s+6} \end{matrix} \right|$$

$s^3+9s^2+20s+12 \Rightarrow (s+1)(s+2)(s+6)$

$s^2+9s+20 \Rightarrow (s+5)(s+4)$

$$\frac{(s+5)(s+4)}{(s+1)(s+2)(s+6)} = \frac{A}{s+1}+\frac{B}{s+2}+\frac{C}{s+6} \qquad \begin{cases} A = 2,4 \\ B = -1,5 \\ C = 0,1 \end{cases}$$

$$= \lim_{s\to -1} \cancel{(s+1)} \frac{(s+5)(s+4)}{\cancel{(s+1)}(s+2)(s+6)} = \frac{(-1+5)(-1+4)}{(-1+2)(-1+6)} = 2,4$$

$$\phi(t) = \begin{vmatrix} -1{,}5e^{-2t}+2{,}4e^{-t}+0{,}1e^{-6t}; & 3e^{-2t}-2{,}4e^{-t}-0{,}6e^{-6t}; & -6e^{-2t}+2{,}4e^{-t}+3{,}6e^{-6t} \\ -1{,}75e^{-2t}+1{,}6e^{-t}+0{,}15e^{-6t}; & 3{,}5e^{-2t}-1{,}6e^{-t}-0{,}9e^{-6t}; & -7e^{-2t}+1{,}6e^{-t}+5{,}4e^{-6t} \\ -0{,}25e^{-2t}+0{,}2e^{-t}+0{,}05e^{-6t}; & 0{,}5e^{-2t}-0{,}2e^{-t}-0{,}3e^{-6t}; & -1e^{-2t}+0{,}2e^{-t}+1{,}8e^{-6t} \end{vmatrix}$$

Si $U(t) = \phi$ y $x_0 = \begin{vmatrix} 1 \\ 0 \end{vmatrix}$

$$\Rightarrow \frac{dx(t)}{dt} = Ax(t) + BU(t) \Rightarrow Sx(s) - x_0 = Ax(s) + BU(s)$$

$$Sx(s) - Ax(s) - BU(s) - x_0 = 0$$

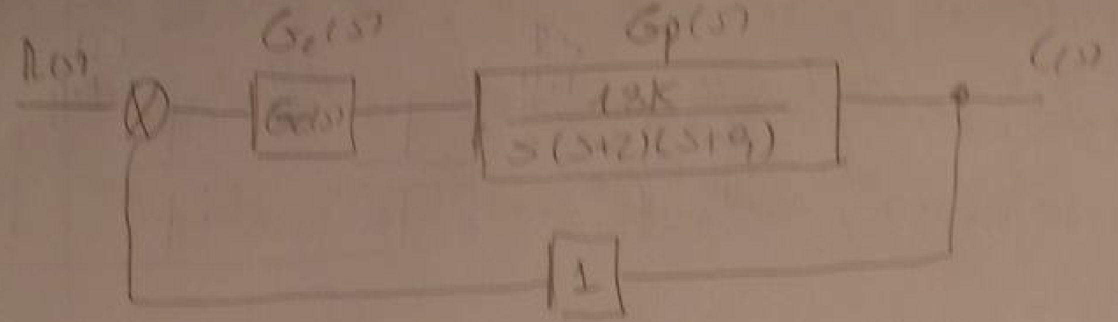
$$(SI - A)x(s) - BU(s) - x_0 = 0$$

$$x(s) = \frac{x_0 + BU(s)}{(SI-A)} = \underbrace{(SI-A)^{-1}}_{\phi(s)} x_0 + (SI-A)^{-1} BU(s)$$

$$x(s) = \phi(s)x_0 + \phi(s)BU(s)$$

$$\mathcal{L}^{-1}[x(s)] \Rightarrow \phi(t)x_0 + \int_0^t \phi(t-\tau)BU(\tau)d\tau = x(t)$$

$$\phi(t)x_0 = \begin{vmatrix} 1{,}5e^{-2t} - 0{,}5e^{-6t} \\ -1{,}75e^{-2t} - 0{,}75e^{-6t} \\ 0{,}25e^{-2t} - 0{,}25e^{-6t} \end{vmatrix}$$



$W_n = 2,828$ rad/seg ; $T_s = 2$ seg a l 2%

A- Compensar mediante P+D

B- Compensar en adelanto en cascada.

$$G_p(s) = \frac{18k}{s(s+2)(s+9)} = \frac{K'}{s(s+2)(s+9)}$$

$N_{asin} = p - z = 3$; $\varphi_k = 180 \frac{(2k-1)}{p-z} = \begin{cases} \varphi_0 = -60 \\ \varphi_1 = 60 \\ \varphi_2 = 180 \end{cases}$

$\sigma_c = \frac{\Sigma Re|p| - \Sigma Re|z|}{p-z} = -\frac{11}{3} = -3,66$

$G(s)H(s)+1 \Rightarrow K' + s(s+2)(s+9) = 0$; $K' = -s(s+2)(s+9)$

$\frac{\partial k'}{\partial s} = 3s^2 + 22s + 18 = 0 \Rightarrow pb = -6,4 ; -0,938$

$K' + s^3 + 11s^2 + 18s \Rightarrow s^3 + 11s^2 + 18s + k' = 0$

s^3 1 18

s^2 11 k'

s^1 $\frac{k' - 198}{11}$

s^0 k'

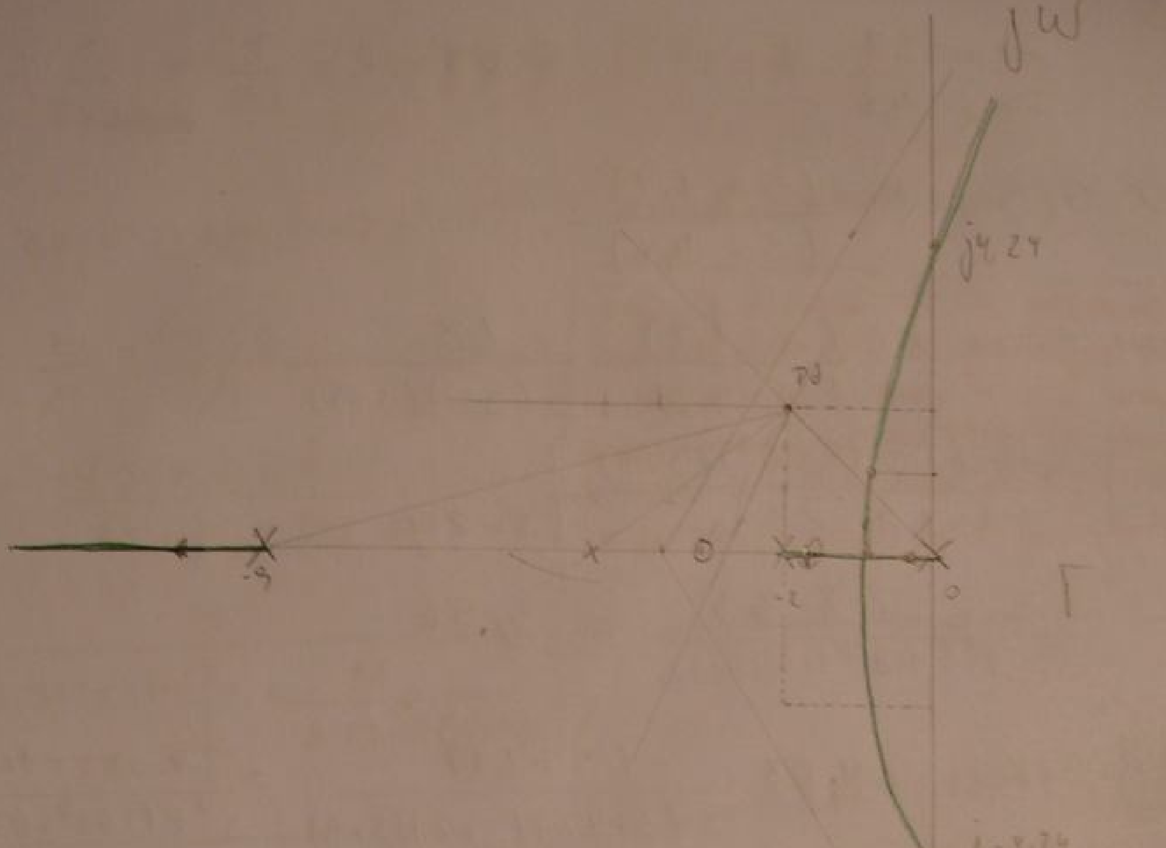
Estable para $k \geq 198$

$11s^2 + 198 = 0 \; \{ \pm j4,24$

$t_{s2\%} = \frac{4}{\sigma} \Rightarrow \sigma = \frac{4}{t_{s2\%}} = 2 = \sigma$

$W_n^2 = \sigma^2 + W_d^2 \Rightarrow W_d = \sqrt{W_n^2 - \sigma^2} = \sqrt{2,828^2 - 2^2} = 2 = W_d$

$P_d = 2 + j2$



$\varphi_{PD} = -16^\circ - 90^\circ - 134 = -240$

$\varphi_c - 240^\circ = \pm 180(2k+1) \Rightarrow \underline{\varphi_c = 60^\circ}$

Ubicamos un cero que nos pueda proveer este aporte de fase.

$$\operatorname{tg}\varphi_c = \frac{\operatorname{Im} p_{pd}}{-[z_c - \operatorname{Re} p_{pd}]} = \frac{2}{-[z_c + 2]} = \operatorname{tg}\varphi_c \Rightarrow 2 = -\operatorname{tg}\varphi_c\, z_c - 2\operatorname{tg}\varphi_c$$

$$\frac{2 + 2\operatorname{tg}\varphi_c}{-\operatorname{tg}\varphi_c} = \frac{-2}{\operatorname{tg}\varphi_c} - 2 = -3,15 = z_c$$

$$G_{PD}(s) = k_p(1 + k_p T_D s) = k_p T_D \left(s + \frac{1}{T_D}\right) \Rightarrow z_c = \frac{1}{T_D} = 3,15$$

$T_D = 0,317$

$$G_{cPD}(s)G(s) = k_p T_D (s + 3,15) \cdot \frac{18}{(s+2)(s+9)s}$$

$$|k_p T_D| = \left| \frac{s(s+2)(s+9)}{18(s+3,15)} \right|_{s=2+j2} = \frac{7,3 \cdot 2 \cdot 2,8}{18 \cdot 2,3} = 1$$

$\Rightarrow k_p = 3,15$

$$\Rightarrow G_{cPD}\,G(s) = \frac{18\,(s+3,15)}{(s+2)(s+9)s} = \frac{18s + 56,7}{s^3 + 11s^2 + 18s}$$

B -

$Zc = 1,7$

$Pc = 4,6$

$$\Rightarrow G_c(s) = k_c \frac{(s+1,7)}{(s+4,6)}$$

$$G_c(s)\,G(s) = k_c \frac{(s+1,7)}{(s+4,6)} \cdot \frac{18}{s(s+2)(s+9)}$$

$$|k_c| = \left| \frac{(s+4,6)\, s\,(s+2)(s+9)}{(s+1,7)\; 18} \right|_{s=2+j2}$$

$$= \frac{7,3 \cdot 2 \cdot 2,8 \cdot 3,3}{2,1 \cdot 18} = 3,56$$

$$\Rightarrow G_c(s)\,G(s) = 64,08 \cdot \frac{(s+1,7)}{s(s+4,6)(s+2)(s+9)} = \frac{64,08s + 108,936}{s^4 + 15,6s^3 + 63,6s^2 + 82,8s}$$

$$G_p = \frac{18}{s^3 + 11s^2 + 18s}$$

Para el mismo $G_p(s)$,

a- Determinar error para $G_c(s) = 1$. k_v? $(k = 1)$

b- $e_{ss} = 0,2 \Rightarrow$ compensar en atraso para tal fin

c- Realizar un compensador PI q' cumpla con $e_{ss} = 0,2$

d- Realizar la representación en VE y obtener la Matriz de transición de estado para FCD. Determinar el valor de salida para $x_1(t)$. Valuar la salida para impulso unitario y escalón unitario.

$$a - k_v = \lim_{s \to 0} s\,G_p(s) = \lim_{s \to 0} \cancel{s} \frac{18}{\cancel{s}(s+2)(s+9)} = \frac{18}{18} = 1$$

$$\Rightarrow e_{ss} = \frac{1}{k_v} = 1$$

$$b - G_c(s) = K_c \frac{(s + 1/T)}{(s + 1/\beta T)} \;;\; \beta > 1 \;=\; \frac{K_c \beta \cancel{T}}{\cancel{T}} \frac{(Ts+1)}{(\beta Ts + 1)}$$

$K_v' = k_v K_c \beta = \frac{1}{e_{ss\,deseado}} = \frac{1}{0{,}2} = 5 = K_c\beta \quad P/\, K_c = 1 \Rightarrow \beta = 5$

original → 18/18

*

$\frac{0{,}866}{10} < z_c < \frac{0{,}866}{2} \Rightarrow 0{,}086 < z_c < 0{,}43 \Rightarrow z_c = 0{,}26$

$P_c = \frac{1}{\beta T} = \frac{z_c}{\beta} = \frac{0{,}26}{5} = 0{,}052$

$\Rightarrow G_c(s) = \frac{(s + 0{,}26)}{(s + 0{,}052)}$

Para la elección de z_c debemos buscar un punto que no modifique la respuesta dinámica del sistema. Para ello, tomamos como referencia que nos ubicaremos hacia la derecha de la parte real de los polos dominantes en lazo cerrado (NO del punto de trabajo).

* $\frac{C(s)}{R(s)} = \frac{18}{s(s+2)(s+9)\left[1 + \frac{18}{s(s+2)(s+9)}\right]} = \frac{18}{s(s+2)(s+9) + 18}$

$= \frac{18}{s^3 + 11s^2 + 18s + 18}$

$S_1 = -0{,}866 - j1{,}091$
$S_2 = -0{,}866 + j1{,}091$ } Polos dominantes
$S_3 = -9{,}267$

- Compensador PI

$G_{c_{PI}}(s) = k_p + \frac{k_p}{T_i s} = k_p\left(1 + \frac{1}{T_i s}\right) = \frac{k_p}{s}\left(s + \frac{1}{T_i}\right)$

$G_{c_{PI}}(s) = \frac{k_p\,(s + 1/T_i)}{s} \Rightarrow \frac{1}{T_i} = \frac{Re\{PD\}}{10} = \frac{0{,}866}{10} = 0{,}0866$

En compensador PI, el polo se ubica a 1/10 de frecuencia del polo dominante a lazo cerrado.

$\Rightarrow G_{c_{PI}}(s)\, G_p(s) = \frac{k_p\,(s + 0{,}0866)}{s} \cdot \frac{18}{s(s+2)(s+9)}$

$= \frac{k_p 18\,(s + 0{,}0866)}{s^2(s+2)(s+9)}$ x ⇒ No está cumpliendo con kv (Ver pag 80 al dorso)

$|k_p| = \left|\frac{s^2(s+2)(s+9)}{18\,(s + 0{,}0866)}\right| = \frac{1{,}4 \cdot 1{,}6 \cdot 8{,}3}{18 \cdot} = 1{,}03$

→ prácticamente se están anulando entre sí d-

$$\frac{C(s)}{R(s)} = \frac{18}{s(s+2)(s+9)+18} \qquad \frac{X(s)}{X(s)} = \frac{18}{s^3+11s^2+18s+18}$$

$$R(s) = s^3 X(s) + 11s^2 X(s) + 18s X(s) + 18 X(s)$$

$$s^3 X(s) = -11s^2 X(s) - 18s X(s) - 18 X(s) + R(s)$$

A $s^3 X(s) = \dot{x}_3$

$s^2 X(s) = x_3 = \dot{x}_2$

$s X(s) = x_2 = \dot{x}_1$

B $X(s) = x_1$

$$\dot{x}_3 = -11x_3 - 18x_2 - 18x_1 + R(s)$$

$$\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -18 & -18 & -11 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} U(t)$$

C $$y(t) = \begin{vmatrix} 18 & 0 & 0 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix}$$

C $|\lambda S - A| = 0 \Rightarrow$ $$\begin{vmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{vmatrix} - \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -18 & -18 & -11 \end{vmatrix} = \begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 18 & 18 & 11+s \end{vmatrix}$$

$$\text{Det} = s^2(11+s) + 18 + 18s = 0 \begin{cases} -0{,}866 + j1{,}09 \\ -0{,}866 - j1{,}09 \\ -9{,}26 \end{cases}$$

A $|\lambda I - A| P_1$ $$= \begin{vmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ 18 & 18 & 11+\lambda \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix}$$

D $\lambda x - y = 0$

$\lambda y - z = 0$

$18x + 18y + (11+\lambda) z = 0$

$\lambda x = y$; $P\ x = 1 \Rightarrow y = -0{,}866 + j1{,}09$

$$z = \frac{-18x - 18y}{11+\lambda} = -0{,}4411 - j1{,}888$$

A $$\begin{vmatrix} 1 \\ -0{,}866 + j1{,}09 \\ -0{,}441 - j1{,}88 \end{vmatrix}$$

$P\ x = 1 \Rightarrow y = -9{,}26$

$z = 85{,}44$

$$\begin{vmatrix} 1 \\ -9{,}26 \\ 85{,}44 \end{vmatrix}$$

$$P = \begin{vmatrix} 1 & 1 & 1 \\ -0,866+j1,09 & -0,866+j1,09 & -9,26 \\ -0,441-j1,88 & -0,441+j1,88 & 85,44 \end{vmatrix}$$

$$A_{FCD} = \begin{vmatrix} -0,866+j1,09 & 0 & 0 \\ 0 & -0,866-j1,09 & 0 \\ 0 & 0 & -9,26 \end{vmatrix} = P^{-1} A P$$

$$B_{FCD} = P^{-1} B = \begin{vmatrix} -0,007-j0,053 \\ -0,007+j0,053 \\ 0,0140 \end{vmatrix} ; \quad C_{FCD} = CP = |18 \quad 18 \quad 18|$$

$$\phi(t) = (sI - A)^{-1} = \frac{Adj(sI-A)}{|sI-A|}$$

$$(sI-A) = \begin{vmatrix} s+0,866-j1,09 & 0 & 0 \\ 0 & s+0,866+j1,09 & 0 \\ 0 & 0 & s+9,26 \end{vmatrix}$$

$$Adj(sI-A) = \begin{vmatrix} (s+0,866+j1,09)(s+9,26) & 0 & 0 \\ 0 & (s+0,866-j1,09)(s+9,26) & 0 \\ 0 & 0 & (s+0,866-j1,09)(s+0,866+j1,09) \end{vmatrix}$$

$$Det = s^3 + 10,992 s^2 + 18s + 18 = (s+0,866-j1,09)(s+0,866+j1,09)(s+9,26)$$

$$\frac{Adj(sI-A)}{|sI-A|} = \begin{vmatrix} \frac{\cancel{(s+0,866+j1,09)}\cancel{(s+9,26)}}{\cancel{(s+0,866+j1,09)}(s+0,866-j1,09)\cancel{(s+9,26)}} & 0 & 0 \\ 0 & \frac{1}{s+0,866+j1,09} & 0 \\ 0 & 0 & \frac{1}{s+9,26} \end{vmatrix}$$

$$\phi(s) = \begin{vmatrix} \frac{1}{s+0,866-j1,09} & 0 & 0 \\ 0 & \frac{1}{s+0,866+j1,09} & 0 \\ 0 & 0 & \frac{1}{s+9,26} \end{vmatrix}$$

$$\phi(t) = \begin{vmatrix} e^{-(0,866-j1,09)t} & 0 & 0 \\ & e^{-(0,866+j1,09)t} & 0 \\ & & e^{-9,26t} \end{vmatrix}$$

$$s\,x(s) - x(0) = A x(s) + B U(s)$$

$$(sI - A)\, x(s) = x(0) + B U(s)$$

$$x(s) = (sI-A)^{-1} x(0) + (sI-A)^{-1} B U(s)$$

$$\mathcal{L}^{-1} \Rightarrow x(t) = \phi(t)\, x(0) + \int_0^t \phi(t-\tau) B\, U(\tau)\, d\tau$$

P/ Impulso unitario $\Rightarrow$ $1 \overset{\mathcal{L}}{\Leftrightarrow} \delta(t)$

Suponiendo $x(0) = 0$

3x3 3x1

$$x(s) = \phi(s)\, B\, U(s) =$$

$$\begin{vmatrix} \frac{1}{s+0,866-j1,09} & 0 & 0 \\ 0 & \frac{1}{s+0,866+j1,09} & \frac{1}{s+9,26} \\ 0 & 0 & \end{vmatrix} \begin{vmatrix} -0,007-j0,053 \\ -0,007+j0,053 \\ 0,0140 \end{vmatrix} |1|$$

$$\begin{vmatrix} \frac{-0,007-j0,053}{s+0,866-j1,09} & 0 & 0 \\ 0 & \frac{-0,007+j0,053}{s+0,866+j1,09} & 0 \\ 0 & 0 & \frac{0,0140}{s+9,26} \end{vmatrix}$$

$$y(s) = C_{FCD}\, x(s) = \frac{-0{,}126 - j0{,}954}{s+0{,}866-j1{,}09} + \frac{-0{,}126 + j0{,}954}{s+0{,}866+j1{,}09} + \frac{0{,}252}{s+9{,}26}$$

$$y(t) = (-0{,}126 - j0{,}954)\, e^{-(0{,}866-j1{,}09)t} + (-0{,}126 + j0{,}954)\, e^{-(0{,}866+j1{,}09)t} + 0{,}252\, e^{-9{,}26t}$$

?/ escalón:

$$\int_0^t \begin{vmatrix} e^{-(0{,}866-1{,}09)(t-\tau)} & & \\ & e^{-(0{,}866+j1{,}09)(t-\tau)} & \\ & & e^{-9{,}26(t-\tau)} \end{vmatrix} B \cdot U\, d\tau$$

$$\int_0^t e^{-a(t-\tau)} = \int_0^t e^{(-at+a\tau)}\, d\tau = \int_0^t e^{-at} \cdot e^{a\tau} = e^{-at} \int_0^t e^{a\tau}\, d\tau$$

$$= e^{-at}\, \frac{1}{a}\left[e^{at} - e^{0}\right] = \frac{1}{a}\left[e^{0} - e^{-at}\right] = \frac{1}{a}\left(1 - e^{-at}\right)$$

$$\Rightarrow \begin{vmatrix} \frac{1}{0{,}866-j1{,}09}\left(1-e^{-(0{,}866-j1{,}09)t}\right) & 0 & 0 \\ 0 & \frac{1}{0{,}866+j1{,}09}\left(1-e^{-(0{,}866+j1{,}09)t}\right) & 0 \\ 0 & 0 & \frac{1}{9{,}26}\left(1-e^{-9{,}26t}\right) \end{vmatrix} B \cdot U$$

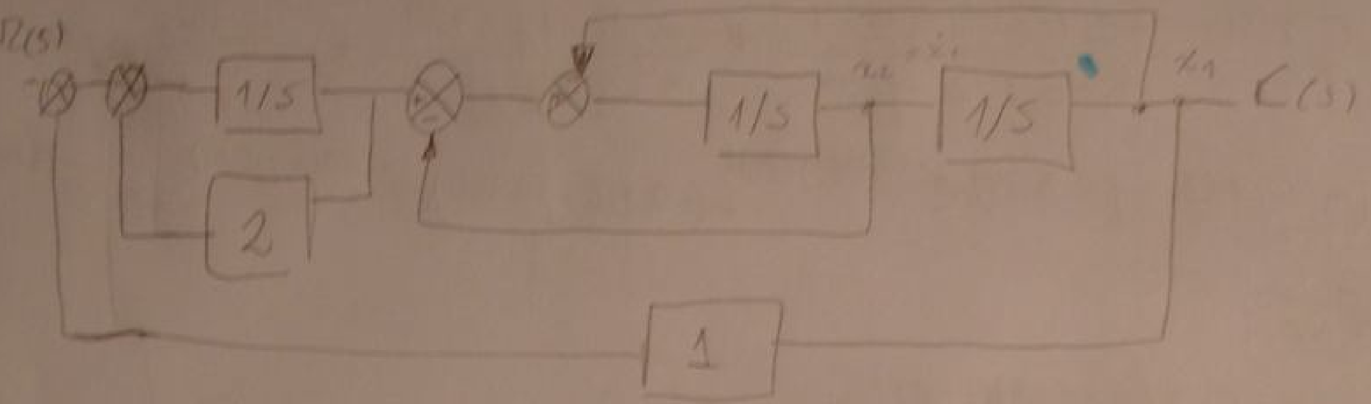
— Hasta acá está bien (se usó un valor de B q/ no es después)

$\Rightarrow y(t) =$

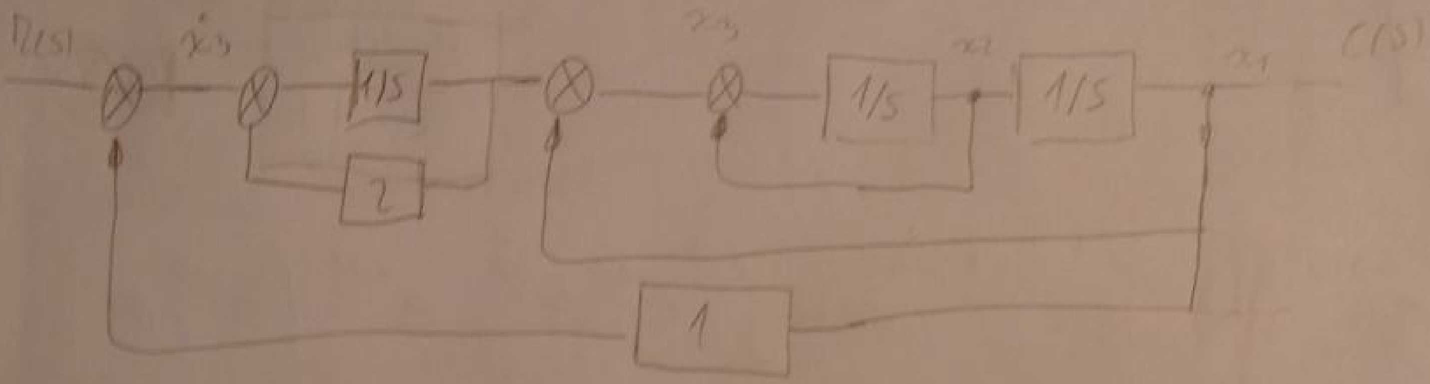
$$\frac{18}{0{,}866-j1{,}09}\left(1-e^{-(0{,}866-j1{,}09)t}\right) + \frac{18}{0{,}866+j1{,}09}\left(1-e^{-(0{,}866+j1{,}09)t}\right) +$$

$$+ \frac{1}{9{,}26}\left(1-e^{-9{,}26t}\right) \quad x \Rightarrow B \text{ nueva } \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \ldots$$ (esto mal el B q/ se usó)

Se tiene:



⇓



$$\frac{C(s)}{R(s)} = \frac{\dfrac{\dfrac{\frac{1/s}{1+1/s}\cdot 1/s}{1+\frac{1/s}{1+1/s}\,1/s}\cdot\dfrac{1/s}{1+2/s}}{1}}{1+\dfrac{\frac{1/s}{1+1/s}\cdot 1/s}{1+\frac{1/s}{1+1/s}\cdot 1/s}\cdot\dfrac{1/s}{1+2/s}}$$

HP

$$\frac{C(s)}{R(s)} = \frac{1}{s^3+3s^2+3s+3}\quad\frac{X(s)}{X(s)}$$

$$R(s) = s^3X(s) + 3s^2X(s) + 3sX(s) + 3X(s)$$

$$s^3X(s) = -3\overbrace{s^2X(s)}^{x_3} - 3\overbrace{sX(s)}^{x_2} - 3\overbrace{X(s)}^{x_1} + R(s)$$

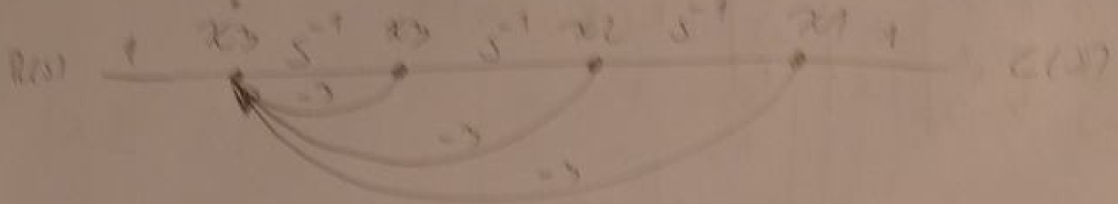
$$\dot{x}_3 = -3x_3 - 3x_2 - 3x_1 + R(s)$$

$$\begin{vmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{vmatrix} = \begin{vmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -3 & -3 & -3\end{vmatrix} \begin{vmatrix}x_1\\ x_2\\ x_3\end{vmatrix} + \begin{vmatrix}0\\ 0\\ 1\end{vmatrix} u(t)$$

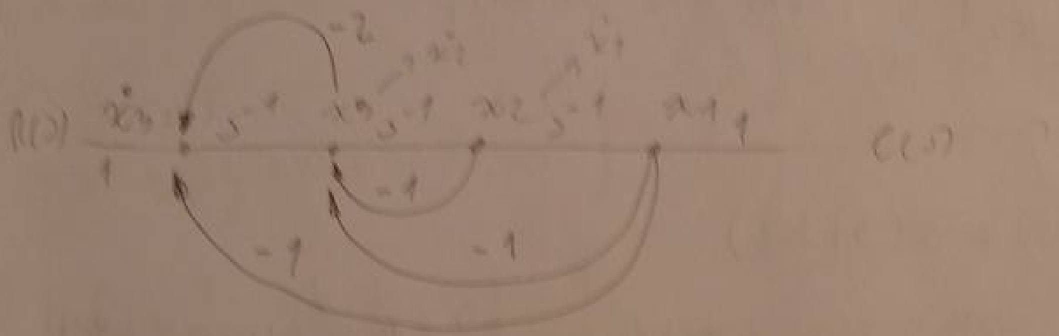
$$y(t) = \begin{vmatrix}1 & 0 & 0\end{vmatrix} \begin{vmatrix}x_1\\ x_2\\ x_3\end{vmatrix}$$

$$\begin{vmatrix}0 & 0 & 1\\ 0 & 1 & -3\\ 1 & -3 & 6\end{vmatrix}$$

es controlable



Si no, del sistema original:



$$\begin{vmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{vmatrix} = \begin{vmatrix}0 & 1 & 0\\ -1 & -1 & 0\\ -1 & 0 & -2\end{vmatrix} \begin{vmatrix}x_1\\ x_2\\ x_3\end{vmatrix} + \begin{vmatrix}0\\ 0\\ 1\end{vmatrix} R(s)$$

$$y(t) = \begin{vmatrix}1 & 0 & 0\end{vmatrix} \begin{vmatrix}x_1\\ x_2\\ x_3\end{vmatrix}$$

$$M = \begin{vmatrix}B & AB & A^2B\end{vmatrix} \Rightarrow \begin{vmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 1 & -2 & 4\end{vmatrix} = Det = 0$$

$\Rightarrow$ No controlable

$$O = \begin{vmatrix}C^T & A^TC^T & A^{T^2}C^T\end{vmatrix} \Rightarrow \begin{vmatrix}1 & 0 & -1\\ 0 & 1 & -1\\ 0 & 0 & 0\end{vmatrix} = Rango = 2$$

$\Rightarrow$ No observable

como la última configuración no es ni observable, ni controlable, usamos la FCC q' sí lo es.

$$sx(s) = Ax(s) + Bu(s) \quad \Big\} \Rightarrow sx(s) = Ax(s) - BKx(s)$$
$$u(s) = -Kx(s) \qquad\qquad sx(s) - Ax(s) + BKx(s) = 0$$

$$(sI - A + BK)\,x(s) = 0$$

$$\begin{vmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{vmatrix} - \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -3 & -3 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} \begin{vmatrix} k_1 & k_2 & k_3 \end{vmatrix}$$

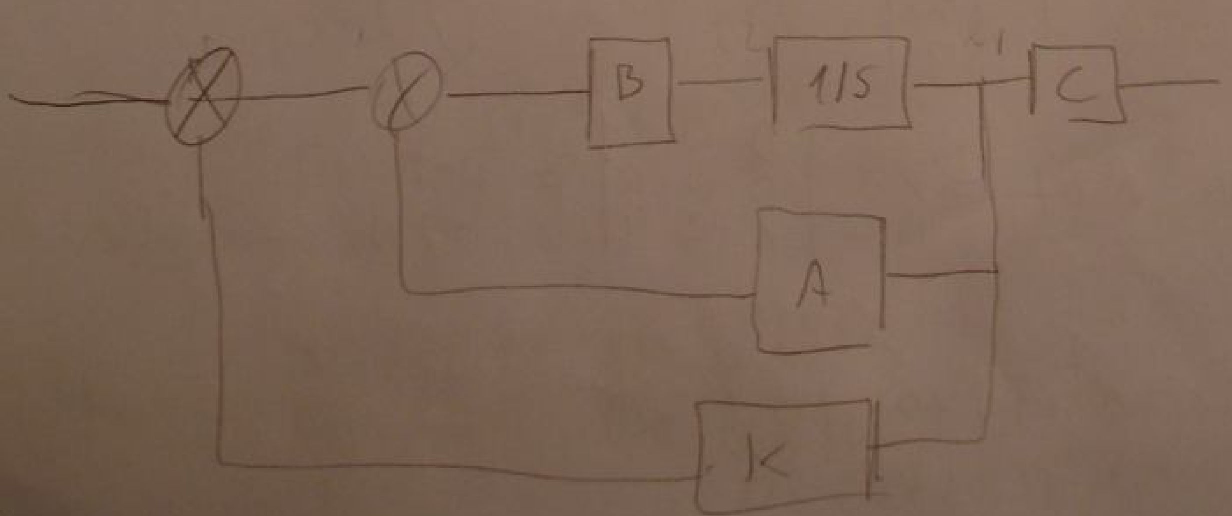
$$\begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 3 & 3 & 3+s \end{vmatrix} + \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ k_1 & k_2 & k_3 \end{vmatrix} = \begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 3+k_1 & 3+k_2 & 3+s+k_3 \end{vmatrix}$$

$$\begin{matrix} s & -1 & 0 \\ 0 & s & -1 \end{matrix}$$

$$s^2(s+3+k_3) + 3 + k_1 + s(3+k_2)$$

$$s^3 + s^2(3+k_3) + s(3+k_2) + 3 + k_1 = s^3 + 15s^2 + 40{,}5s + 54$$

$$\left.\begin{matrix} 3+k_3 = 15 \\ 3+k_2 = 40{,}5 \\ 3+k_1 = 54 \end{matrix}\right\} \begin{matrix} k_3 = 12 \\ k_2 = 37{,}5 \\ k_1 = 51 \end{matrix} \qquad K = \begin{vmatrix} 51 & 37{,}5 & 12 \end{vmatrix}$$



Ahora el sistema queda

51

$$\frac{C(s)}{R(s)} = \frac{1}{s^3+15s^2+40,5s+54} \quad \begin{cases} \lambda_1 = -0,37 + j1,0911 \\ \lambda_2 = -0,37 + j1,0911 \\ \lambda_3 = -2,2599 \end{cases}$$

Como pide la rta al 5 y solo $\phi(t)$ ... tanco diagonalizamos para hacer mas expeditivo el calculo.

$$|\lambda I - A| P \Rightarrow \begin{vmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ 3 & 3 & \lambda+3 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} = 0$$

$$\left.\begin{aligned} \lambda x - y &= 0 \\ \lambda y - z &= 0 \\ 3x + 3y + (\lambda+3) z &= 0 \end{aligned}\right\} \quad \begin{aligned} \lambda x &= y \quad ?\, x = \underline{1} \Rightarrow y = \underline{\lambda} \\ \lambda y &= z \Rightarrow \;\hookrightarrow z = \underline{\lambda^2} \end{aligned}$$

$$P = \begin{vmatrix} 1 & 1 & 1 \\ \lambda_1 & \lambda_2 & \lambda_3 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ -0,37+j1,0911 & -0,37-j1,0911 & -2,26 \\ -1,053-j0,807 & -1,053+j0,807 & 5,107 \end{vmatrix}$$

$$B_{FCD} = \begin{vmatrix} 1 \\ -2,26 \\ 5,107 \end{vmatrix} \; ; \quad A_{FCD} = \begin{vmatrix} -0,37+j1,0911 & 0 & 0 \\ 0 & -0,37-j1,0911 & 0 \\ 0 & 0 & -2,26 \end{vmatrix}$$

$$(sI - A) = \begin{vmatrix} s+(0,37-j1,0911) & 0 & 0 \\ 0 & s+(0,37+j1,0911) & 0 \\ 0 & 0 & s+2,26 \end{vmatrix}$$

$$\text{cof} = \text{Adj} = \begin{vmatrix} (s+(0,37+j1,0911))(s+2,26) & 0 & 0 \\ 0 & (s+(0,37-j1,0911))(s+2,26) & 0 \\ 0 & 0 & (s+(0,37-j1,0911))(s+(0,37+j1,0911)) \end{vmatrix}$$

$$\det = (s + 0{,}37 + j1{,}0944)(s + 0{,}37 - j1{,}0944)(s + 2{,}26)$$

$$(sI - A)^{-1} = \begin{vmatrix} \dfrac{1}{s + 0{,}37 - j1{,}0944} & 0 & 0 \\ 0 & \dfrac{1}{s + 0{,}37 + j1{,}0944} & 0 \\ 0 & 0 & \dfrac{1}{s + 2{,}26} \end{vmatrix}$$

$$\phi(t) = \begin{vmatrix} e^{-\overbrace{(0{,}37 - j1{,}0944)}^{a}t} & 0 & 0 \\ 0 & e^{-\overbrace{(0{,}37 + j1{,}0944)}^{b}t} & 0 \\ 0 & 0 & e^{-\overbrace{2{,}26}^{c}t} \end{vmatrix}$$

$$sX(s) - x_0 = AX(s) + BU(s)$$

$$(sI - A)X(s) = x_0 + BU(s)$$

$$\Rightarrow X(s) = \underbrace{(sI - A)^{-1}}_{\phi(s)} x_0 + (sI - A)^{-1} BU(s)$$

$$x(t) = \phi(t)x(0) + \int_0^t \phi(t - \tau) B U(\tau)\, d\tau$$

$$\int_0^t e^{-a(t-\tau)}\, d\tau = e^{-at} \int_0^t e^{a\tau}\, d\tau = \frac{1}{a} e^{-at} \left[e^{at} - e^{0} \right]$$

$$= \frac{1}{a}\left(e^{-at} e^{at} - e^{-at} \right) = \frac{1}{a}\left(1 - e^{-at} \right)$$

$$x(t) = \begin{vmatrix} \frac{1}{a}(1 - e^{-at}) & 0 & 0 \\ 0 & \frac{1}{b}(1 - e^{-bt}) & 0 \\ 0 & 0 & \frac{1}{c}(1 - e^{-ct}) \end{vmatrix} B\,(1)$$

$$x(t) = \begin{vmatrix} \frac{1}{a}(1-e^{-at}) & 0 & 0 \\ 0 & \frac{1}{b}(1-e^{-bt}) & 0 \\ 0 & 0 & \frac{1}{c}(1-e^{-ct}) \end{vmatrix} \begin{vmatrix} 1 \\ -c \\ c^2 \end{vmatrix} [1]$$

$$x(t) = \frac{1}{a}(1-e^{-at}) - \frac{c}{b}(1-e^{-bt}) + \frac{c^2}{\cancel{c}}(1-e^{-ct})$$

$$x(t) = \frac{1}{0,37-j1,0911} - \frac{e^{-(0,37-j1,0911)t}}{0,37-j1,0911} - \frac{2,26}{0,37+j1,0911} - \frac{2,26\, e^{-(0,37+j1,0911)t}}{0,37+j1,0911} +$$
$$+ 2,26 - 2,26\, e^{-2,26t}$$

$$x(t) = 1,9 + j2,68 - e^{-0,37t}\left(\frac{e^{j1,09t}}{0,37-j1,0911} - \frac{2,26\, e^{-j1,0911t}}{0,37+j1,0911}\right) - $$
$$-2,26\, e^{-2,26t}$$

$$e^{j\theta} - e^{-j\theta} \Rightarrow 2j\,\text{sen}\,\theta$$

TVF

$$\lim s \cdot \frac{1}{s^3 + 15s^2 + 40,5 + 54} \cdot \frac{1}{s} \Rightarrow 1,85 \times 10^{-2} = VF.$$

Compensar en Atrazo por Bode:

$KV = 5$
$M\phi > 40°$
$MG > 10dB$

$$G(s)H(s) = \frac{1}{s(s+1)(0,5s+1)}$$
$$= \frac{2}{s(s+1)(s+2)}$$

$$\lim_{s \to 0} s\,G(s)H(s) = \frac{2}{2} \Rightarrow KV = K1 = 5 \Rightarrow \underline{K=5}$$

$$\Rightarrow \frac{10}{s(s+1)(s+2)} \; ; \; K_T = 20 \log 5 = 14\,dB$$

$$\left.\begin{matrix} M_F = -15,75^\circ \\ M_G = -8dB \end{matrix}\right\} \text{Inestable}$$

10 45
3,5 120

$\varphi_c = -180 + M\phi_{pedido} + 12^\circ = -128 \begin{cases} w_1 = 0,4\ rad/seg \\ G = 22dB \end{cases}$

$\Rightarrow 20\log\frac{1}{\beta} = -22dB \Rightarrow \beta = 12,58$

$$G_c = k_c\frac{(s+1/T)}{(s+1/\beta T)} = k_c\beta\frac{(Ts+1)}{(T\beta s+1)}$$

$$\frac{w_1}{10} < z_c < \frac{w_1}{2} \Rightarrow z_c = \frac{w_1}{10} = 0,04 = \frac{1}{T}$$

$$P_c = \frac{1}{\beta T} = \frac{z_c}{\beta} = 0,00318$$

$$K = k_c\beta \;;\; k_c = \frac{K}{\beta} = \frac{5}{12,58} = 0,397 = k_c$$

$$G_cG(s) = 0,397\frac{(s+0,04)}{(s+0,00318)}\frac{2}{s(s+1)(s+2)}$$

$$G(s)H(s) = \frac{4}{s(s+2)}$$

$K_v = 20$
$M\phi \geq 50$
$M_G > 10$

$$K_v = \lim_{s\to 0} GHs = s\frac{4}{s(s+2)} = 2K = 20 \Rightarrow K = 10$$

$$\Rightarrow \frac{40}{s(s+2)} \;;\; K_{To} = 26.$$

En atraso:

$\varphi_c = -180 + M\phi_{pedido} + 12 = -118 \Rightarrow \begin{cases} w_1 = 1\ rad/seg \\ G = 26dB \end{cases}$

Del Bode $\Rightarrow M\phi = 22,5^\circ \;;\; M_G = \infty$

$20\log\frac{1}{\beta} = -26dB \Rightarrow \beta = 19,95$

$K = k_c\beta$
$k_c = \frac{K}{\beta} = \frac{10}{19,95}$
$k_c = 0,5$

$$z_c = \frac{w_1}{10} = 0,1 \;;\; P_c = \frac{z_c}{\beta} = 0,005$$

45 — 10 (53)
1,22

$$G_cG(s) = 0{,}5 \cdot \frac{(s+0{,}1)}{(s+0{,}005)} \frac{4}{s(s+2)}$$

En adelanto:

$$\varphi_c = M\phi_{pedido} - M\phi_{actual} + 5° = 50° - 22{,}5° + 5° = 32{,}5$$

$$\operatorname{sen} \varphi_c = \frac{1-\alpha}{1+\alpha} \quad ; \quad \alpha = 0{,}3$$

$$-20 \log \frac{1}{\sqrt{\alpha}} = -5{,}22\,db \Rightarrow w_n = \frac{1}{\sqrt{\alpha}\,T} \Rightarrow \frac{1}{T} = w_n \sqrt{\alpha} = 8{,}75\sqrt{0{,}3} = 4{,}8$$

$\hookrightarrow w_n = 8{,}75$

$$\Rightarrow z_c = \frac{1}{T} = 4{,}8\ rad/seg \Rightarrow P_c = \frac{z_c}{\alpha} = 16$$

$$K = K_c\,\alpha \Rightarrow K_c = \frac{K}{\alpha} = \frac{10}{0{,}3} = 33{,}33$$

$$G_cG(s) = 33{,}33 \frac{(s+4{,}8)}{(s+16)} \frac{4}{s(s+2)}$$

---

a) Defino la FdT de un SLIT en términos de la rta al impulso.

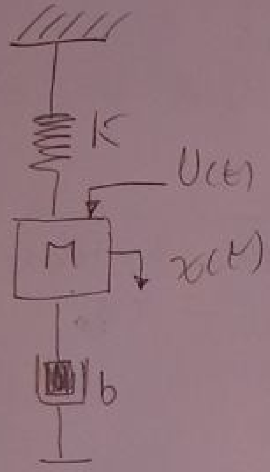
Sea un sistema SLIT. Si consideramos la salida del sistema ante un impulso unitario en la entrada, encontramos que

$$Y(s) = G(s) \cdot C(s) \quad ; \; p/\ C(s) = 1 \Leftrightarrow \delta(t)$$

$\Rightarrow Y(s) = G(s) \Rightarrow$ Encontramos que para un SLIT la respuesta ante un impulso unitario, es la FdT. La FdT y la respuesta al impulso contienen la misma info.

b) La FdT también puede ser vista como la relación entrada-salida del sistema. Dicho concepto está limitado a SLIT.

c) Aquel que presento todos sus polos y ceros del lado negativo.



$$\sum F = m \cdot \ddot{x}$$

$$ky + b\dot{y} + m\ddot{y} = U(t)$$

$$Kx_1 + bx_2 + M\dot{x}_2 = U$$

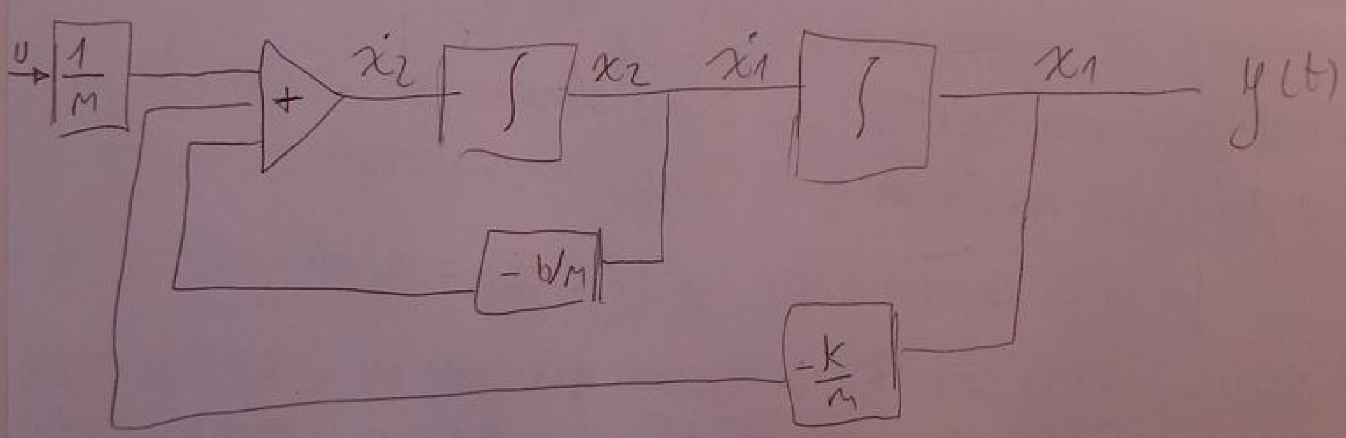
$$x_2 = \dot{x}_1$$

$$\dot{x}_2 = \frac{U}{M} - \frac{K}{M}x_1 - \frac{b}{M}x_2$$

$$x_1 = y(t)$$

$$\begin{vmatrix}\dot{x}_1\\ \dot{x}_2\end{vmatrix} = \begin{vmatrix}0 & 1\\ -\frac{K}{M} & -\frac{b}{M}\end{vmatrix}\begin{vmatrix}x_1\\ x_2\end{vmatrix} + \begin{vmatrix}0\\ 1/M\end{vmatrix}U$$

$$y(t) = \begin{vmatrix}1 & 0\end{vmatrix}\begin{vmatrix}x_1\\ x_2\end{vmatrix}$$



$$sX(s) = AX(s) + BU(s) \quad \Big| \quad Y(s) = CX(s)$$

$$X(s) = (sI - A)^{-1}BU(s) \quad \Big| \quad Y(s) = C(sI-A)^{-1}BU(s)$$

$$G(s) = \frac{Y(s)}{U(s)} \Rightarrow \underline{G(s) = C(sI-A)^{-1}B}$$

$$|sI - A| = \begin{vmatrix} s & 0 \\ 0 & s \end{vmatrix} - \begin{vmatrix} 0 & 1 \\ -\frac{k}{M} & -\frac{b}{M} \end{vmatrix} = \begin{vmatrix} s & -1 \\ \frac{k}{M} & s+\frac{b}{M} \end{vmatrix}$$

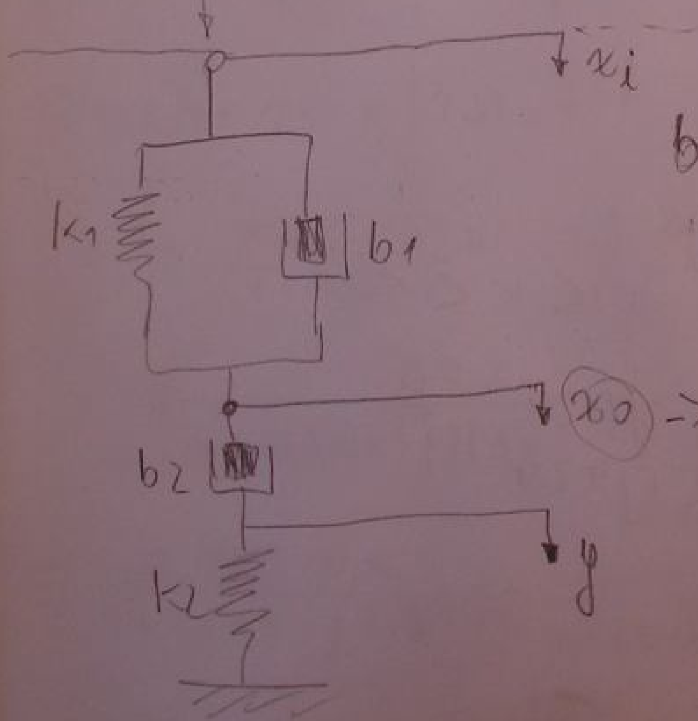
$$(sI-A)^{-1} = \frac{Adj}{|sI-A|} = \frac{1}{s^2 + s\frac{b}{M} + \frac{k}{M}} \begin{vmatrix} s+\frac{b}{M} & 1 \\ -\frac{k}{M} & s \end{vmatrix}$$

$$C(sI-A)^{-1}B = \frac{\begin{vmatrix} s+\frac{b}{M} & 1 \end{vmatrix} \begin{vmatrix} 0 \\ \frac{1}{M} \end{vmatrix}}{s^2 + s\frac{b}{M} + \frac{k}{M}} = \frac{1}{Ms^2 + sb + k}$$

$$y(s)k + sy(s)b + Ms^2y(s) = U(s)$$

$$y(s)(Ms^2 + bs + k) = U(s)$$

$$\frac{y(s)}{U(s)} = \frac{1}{Ms^2 + bs + k}$$

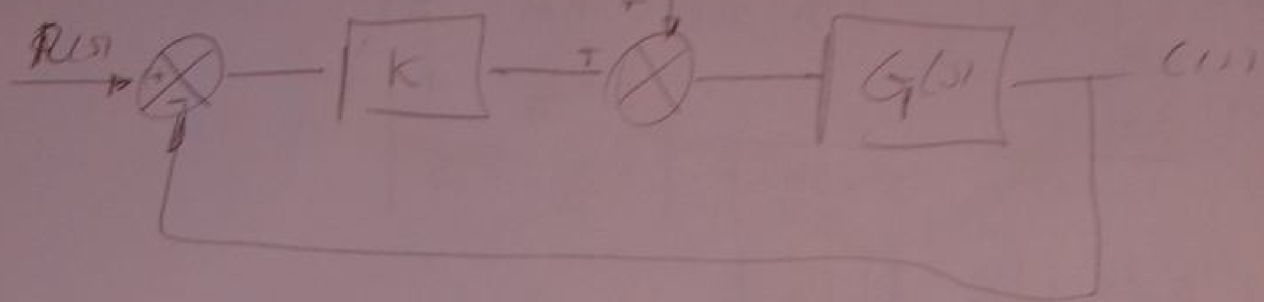


$$b(\dot{x}_i - \dot{x}_o) + k_1(x_i - x_o) + b_2(\dot{x}_o - \dot{y}) + k_2 y = 0$$

x_o -> Ptoret

(Pro aplicacion - pro llegada)

Se dispone del siguiente sistema:



$$G_p(s) = \frac{K}{s(s+3+3j)(s+3-3j)} = \frac{K}{s^3+6s^2+18s}$$

$$N^\circ \text{ Asint} = P - Z = 3 \; ; \; \varphi_k = 180 \frac{(2k-1)}{p-z} \begin{cases} \varphi_0 = -60 \\ \varphi_1 = 60 \\ \varphi_2 = 180 \end{cases}$$

$$\sigma_c = \frac{\sum Re|P| - \sum Re|z|}{p-z} = \frac{6}{3} = 2$$

$$G(s)H(s) + 1 = K + s^3 + 6s^2 + 18s = 0$$

$$K = -(s^3+6s^2+18s) \Rightarrow \frac{dK}{ds} = 3s^2 + 12s + 18 = 0$$

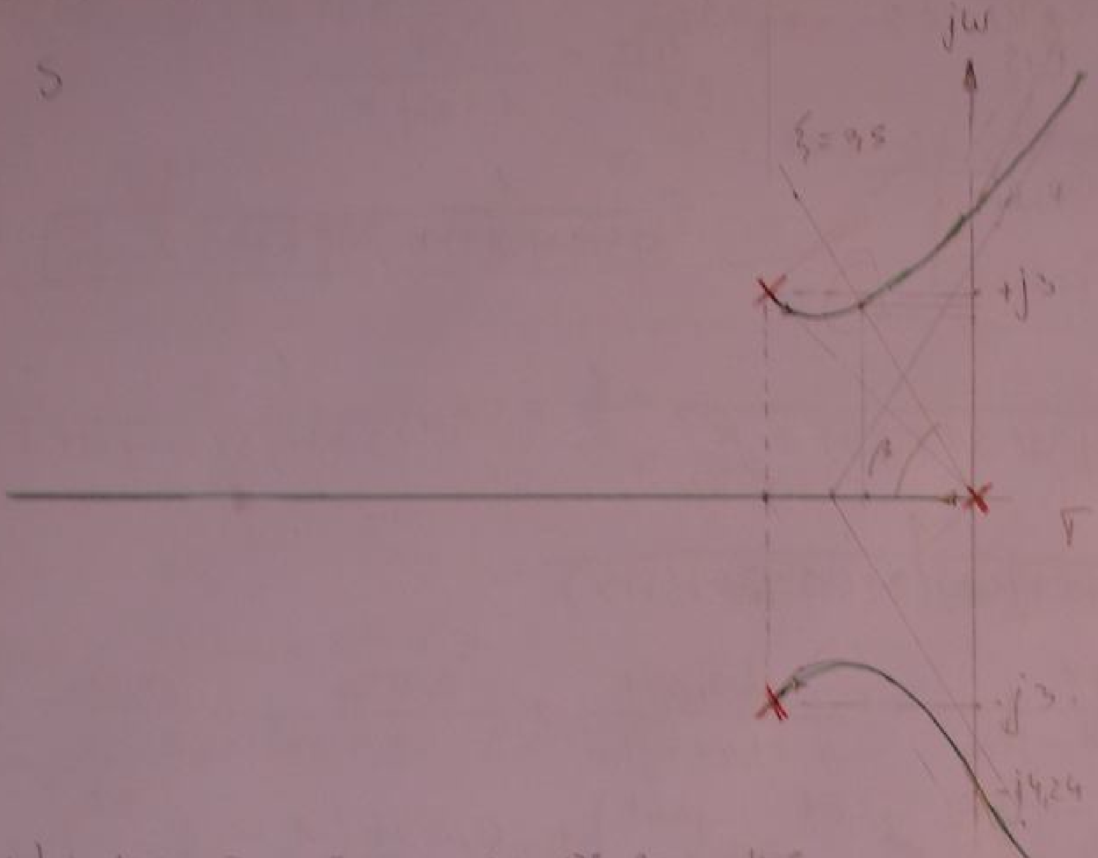
$$pb \begin{cases} -2 \pm j1{,}41 \end{cases}$$

$$s^3 + 6s^2 + 18s + K = 0$$

s^3	1	18
s^2	6	K
s^1	$\frac{108-K}{6}$	
s^0	K	

Estable para $0 < K < 108$

$$6s^2 + 108 = \begin{cases} -j4{,}24 \\ +j4{,}24 \end{cases}$$



Ang salida: $-180-\Sigma\alpha p+\Sigma\alpha z=-180-135-90=-405$
$=-45$

P/ $\xi = 0,5 \Rightarrow K$?

$\beta = A\cos\xi = 60°$

$|K| = |s||s+3+3j||s+3-3j| =$
$= 3,3 \cdot 1,4 \cdot 6 = 27,2$

P/ $R(s) = 1/s$ y $P(s) = 0$ $\qquad \frac{C(s)}{R(s)} = \frac{KGp}{1+KGp}$

$$C(s) = \frac{K\,G(s)}{1+G(s)}R(s) = \frac{1}{s}\,27,2\cdot\frac{1}{s(s+3+3j)(s+3-3j)+K}$$

$$= \frac{27,2}{s^2(s+3+3j)(s+3-3j)+27,2s} = \frac{27,2}{(s+1,48+j2,6)(s+1,48-j2,6)(s+3)s}$$

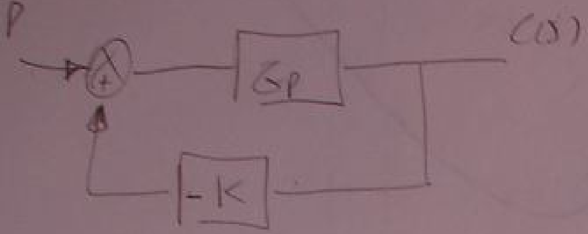
$$= \frac{-0,007+j0,575}{s+(1,48-j2,6)} + \frac{-0,007+0,575}{s+(1,48+j2,6)} + \frac{1}{s} + \frac{-0,985}{s+3}$$

$$= (-0,007+j0,575)e^{-1,48t}\left(e^{-j2,6t}+e^{j2,6t}\right) - 0,985e^{-3t}+1$$

$$= (-0,007+j0,575)e^{-1,48t}\cdot 2j\,\mathrm{sen}\,2,6t - 0,985e^{-3t}+1$$

$$= (-1,15-j0,014)\,e^{-1,48t}\,\mathrm{sen}(2,6t) - 0,985e^{-3t}+1 = C(s)\Big|_{R(s)=1/s}$$

$P/\ P(s) = \frac{1}{s}$ y $R(s) = 0$



$$\frac{P(s)}{C(s)} = \frac{G_p}{1 + G_p K}$$

$$= \frac{1}{s(s+3+j3)(s+3-j3)\left[1 + \frac{K}{(\quad)}\right]}$$

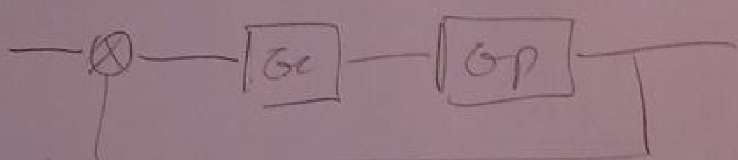
$$= \frac{1}{s(s+3+j3)(s+3-j3) + K} \cdot \frac{1}{s} = \frac{1}{s^2(s+3+j3)(s+3-j3) + Ks}$$

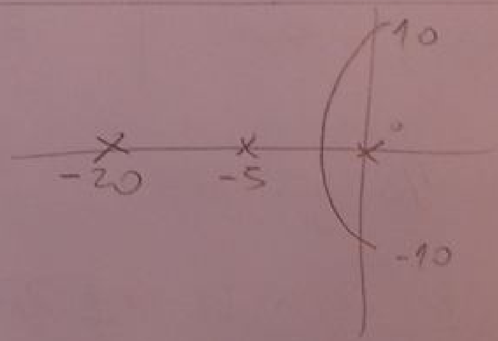
$$= \frac{1}{s(s+1{,}48+j2{,}6)(s+1{,}48-j2{,}6)(s+3)}$$

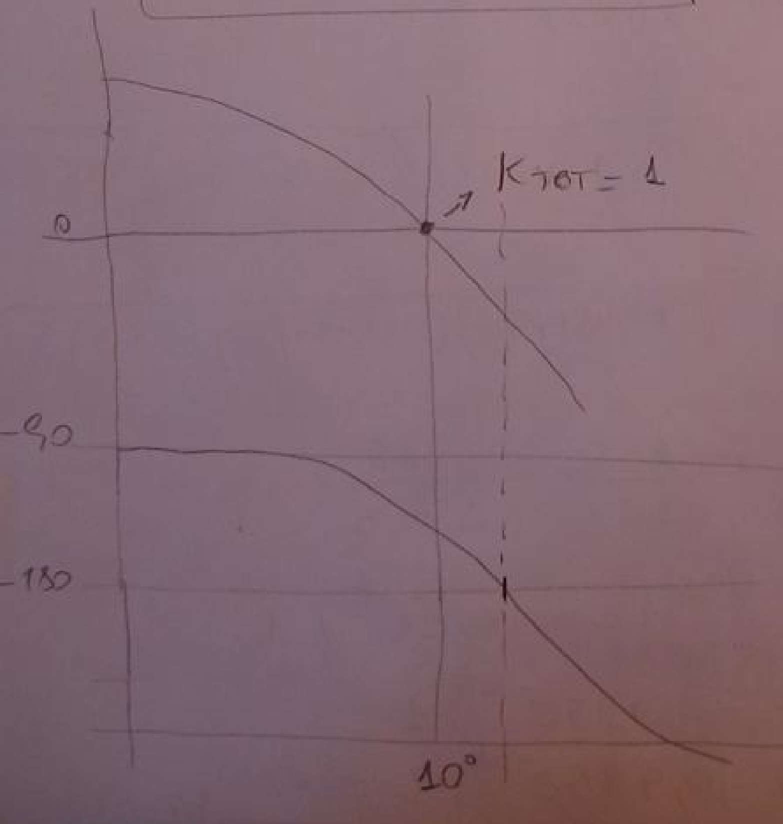
$$= \frac{j0{,}021}{s+1{,}48-j2{,}6} + \frac{-j0{,}021}{s+1{,}48+j2{,}6} + \frac{0{,}036}{s} - \frac{0{,}036}{s+3}$$

$$= j0{,}021\, e^{-1{,}48t}\left(e^{j2{,}6t} - e^{-j2{,}6t}\right) + 0{,}036\left(1 - e^{-3t}\right)$$

$$c(t) = 0{,}042\, e^{-1{,}48t} \operatorname{sen}(2{,}6t) + 0{,}036\left(1 - e^{-3t}\right) = C(s)\Big|_{\substack{P(s)=1/s \\ R(s)=0}}$$







Fr LA, MF, MG, KV

$$G_p = \frac{100}{s(s+20)(s+5)}$$

$$100K + s^3 + 25s^2 + 100s$$

$s^3 + 25s^2 + 100s + \underbrace{100k}_{k'}$

s^3	1	100
s^2	25	k'
s^1	$\frac{2500 - k'}{25}$	
s^0	k'	

$k' < 2500 \Rightarrow k < 25$

$25s^2 + 2500 \begin{cases} \pm j10 \end{cases}$

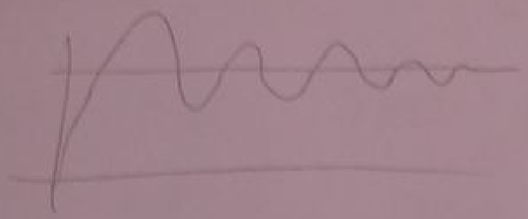
$\frac{45 - 10}{2}$

$M_\phi = 76{,}5$; $M_G = 25dB$; $K_V = 1$

② $LC \Rightarrow \frac{KGp}{1 + KGp} = \frac{100}{s(s+20)(s+5) + 100}$

$$\frac{C(s)}{R(s)} = \frac{100}{s^3 + 25s^2 + 100s + 100}$$

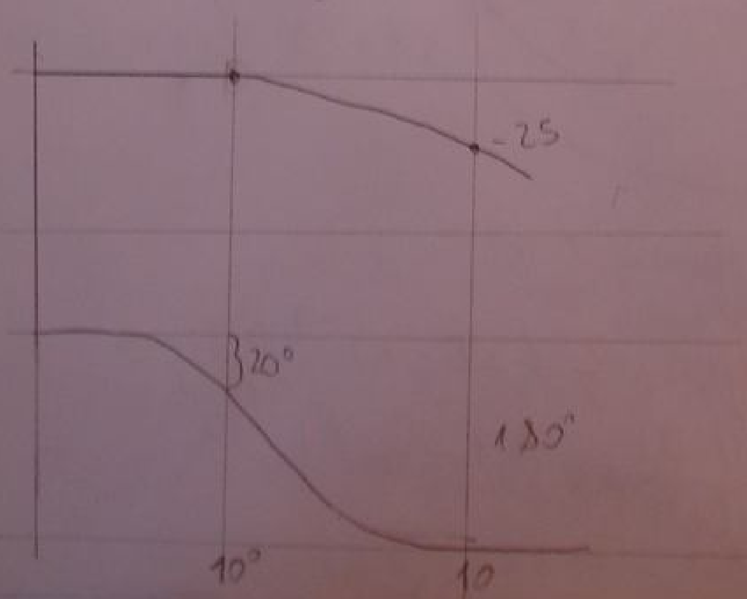
$\Rightarrow (s+5)s + 100 = s^2 + 5s + 100$; $\begin{cases} \omega_n = \sqrt{100} = 10 \\ 2 \cdot \xi \omega_n = 5 \\ \xi = \frac{5}{2\omega_n} = \frac{5}{20} = 0{,}250 \end{cases}$



$\Rightarrow \xi = 0{,}250 \Rightarrow$

$\frac{\pi - 180}{20}$

$\xi \begin{cases} > 1 \Rightarrow \text{Subamortiguado} \\ 1 \Rightarrow \text{Críticamente amortiguado} \\ < 1 \Rightarrow \text{Amortiguado} \\ 0 \Rightarrow \text{oscilatorio} \end{cases}$



P/ $R(t) = 2 \operatorname{sen} t$ $C(\infty)$?

$\operatorname{sen} \omega t = \frac{\omega}{s^2 + \omega^2}$

$R(t) = \frac{2}{s^2 + 1}$

$2 \operatorname{sen}\left(t - \frac{20\pi}{180}\right)$

• $C(\infty) = 2 \operatorname{sen}(t - 9\pi)$

$(sI-A) = \ldots \quad |12+s \;\; 40 \;\; 64|$ (57)

$r_2(t) = 2\,\text{sen}\,10t \Rightarrow \qquad 20\log x = -25$

$$10^{\frac{-25}{20}} = x = 56{,}23\times10^{-3}$$

$$2\cdot10^{\left(\frac{-25}{20}\right)}\text{sen}(10t-\pi) = c(\infty)$$

$\dot{x}_1 = -12x_1 - 40x_2 - 64x_3 + u$

$\dot{x}_2 = x_1$

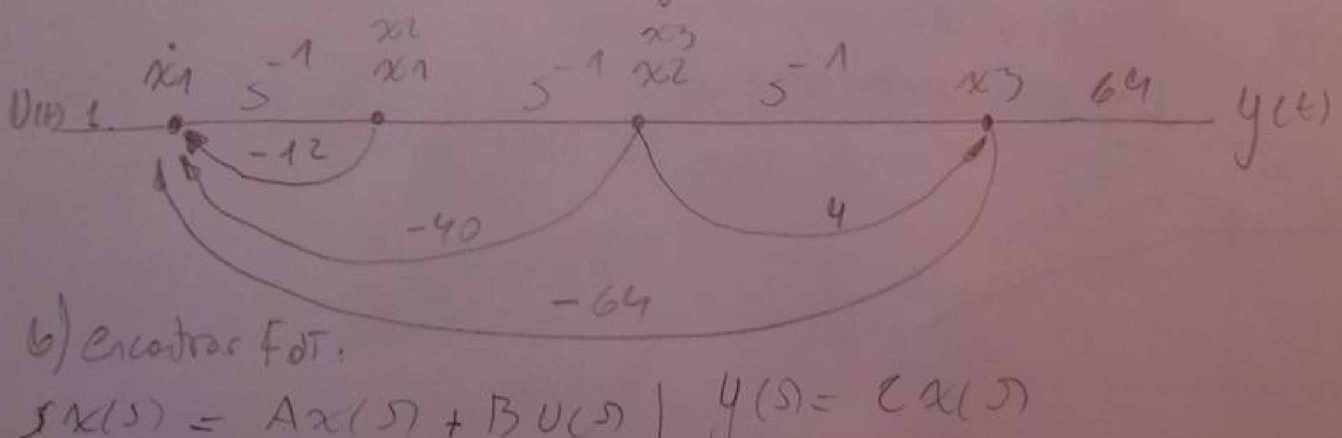
$\dot{x}_3 = x_2$

$y(t) = 4x_2 + 64x_3$

a) Diagrama de flujo ec. matricial

$$\begin{vmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{vmatrix} = \begin{vmatrix}-12 & -40 & -64\\ 1 & 0 & 0\\ 0 & 1 & 0\end{vmatrix}\begin{vmatrix}x_1\\ x_2\\ x_3\end{vmatrix} + \begin{vmatrix}1\\ 0\\ 0\end{vmatrix} u$$

$$y(t) = |0 \;\; 4 \;\; 64| \begin{vmatrix}x_1\\ x_2\\ x_3\end{vmatrix}$$



b) encontrar FdT.

$sX(s) = AX(s) + BU(s)$

$(sI-A)X(s) = BU(s)$

$X(s) = (sI-A)^{-1}BU(s)$

$Y(s) = CX(s)$

$Y(s) = C(sI-A)^{-1}BU(s)$

$$\frac{Y(s)}{U(s)} = C(sI-A)^{-1}B$$

$$(sI-A) = \begin{vmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{vmatrix} - \begin{vmatrix} -12 & -40 & -64 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 12+s & 40 & 64 \\ -1 & s & 0 \\ 0 & -1 & s \end{vmatrix}$$ (57)

$$(sI-A)^{-1} = \frac{Adj\,(sI-A)}{|sI-A|}$$

$$\frac{Adj\,(sI-A)}{|sI-A|} = \begin{vmatrix} s^2 & -40s-64 & -64s \\ s & s^2+12s & -64 \\ 1 & s+12 & s^2+12s+40 \end{vmatrix} \cdot \frac{1}{s^3+12s^2+40s+64}$$

$$C(sI-A)^{-1}B = \frac{4s+64}{s^3+12s^2+40s+64} = \frac{y(s)}{U(s)}$$

c)

Realimentar para ej $\begin{cases} -12 \\ -4 \pm j4 \end{cases}$ $\begin{vmatrix} 1 & -12 & 104 \\ 0 & 1 & -12 \\ 0 & 0 & 1 \end{vmatrix} \Rightarrow Rang = 3$

$$\left.\begin{aligned} s x(s) &= A x(s) + B U(s) \\ U(s) &= -K x(s) \end{aligned}\right\} \Rightarrow \begin{aligned} s x(s) &= A x(s) - B K x(s) \\ (sI - A + BK)\, x(s) &= 0 \end{aligned}$$

$$\begin{vmatrix} 12+s & 40 & 64 \\ -1 & s & 0 \\ 0 & -1 & s \end{vmatrix} + \begin{vmatrix} k_1 & k_2 & k_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

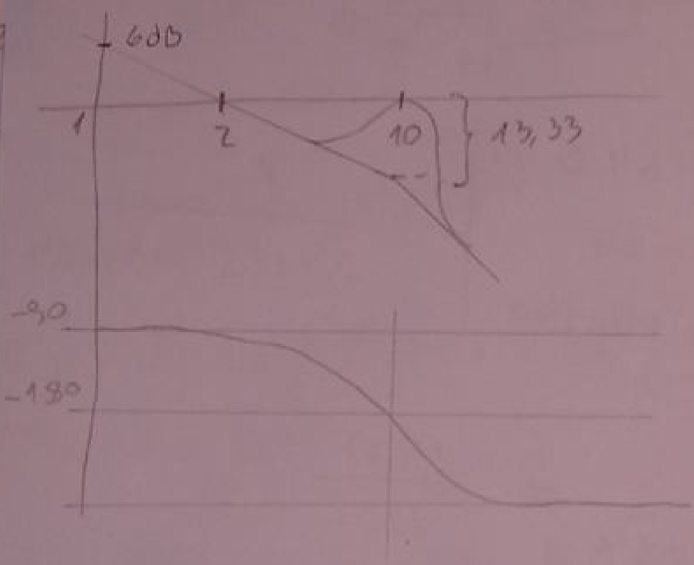
$$\begin{vmatrix} 12+s+k_1 & 40+k_2 & 64+k_3 \\ -1 & s & 0 \\ 0 & -1 & s \end{vmatrix}$$

$$s^2 k_1 + s k_2 + k_3 + s^3 + 12s^2 + 40s + 64$$

$$s^3 + s^2(k_1+12) + s(k_2+40) + 64 + k_3 = s^3 + 20s^2 + 128s + 384$$

$(k_1+12) = 20 \Rightarrow k_1 = 8$; $(k_2+40) = 128 \Rightarrow k_2 = 88$; $(64+k_3) = 384 \Rightarrow k_3 = 320$

$k = 18 \quad 88 \quad 3201$



Tipo 1

$20 \log x = 6dB$

$10^{\frac{6}{20}} = x = 2$

$$\frac{1}{s(s^2 + 2\xi\omega_n + \omega_n^2)}$$

$Mr = 10^{13,33/20} = 4,64$

$$Mr = \frac{1}{2\xi\sqrt{1-\xi^2}} \Rightarrow Mr^2 = \frac{1}{4\xi^2(1-\xi^2)} = \frac{1}{4\xi^2 - 4\xi^4}$$

$$\xi^2 - \xi^4 = \frac{1}{4Mr^2}$$

$$\begin{cases} \xi_1 = -0,108 \; \times \\ \xi_2 = 0,108 \; \checkmark \Rightarrow \xi = 0,108 \\ \xi_3 = -0,99 \; \times \\ \xi_4 = 0,99 \; \times \end{cases}$$

$$\omega_n = \frac{\omega_r}{\sqrt{1-2\xi^2}} = \frac{10}{\sqrt{1-0,0233}} = 10,12$$

$$\Rightarrow \frac{1}{s(s^2 + 2 \cdot 0,108 \cdot 10,12 + 10,12^2)}$$

$$\frac{1}{s(s^2 + 2,186s + 102,4)} \quad ; \quad \frac{1 \cdot k}{102,4} = 2$$

$$\Rightarrow G(s) = \frac{204,8}{s(s^2 + 2,186s + 102,4)}$$

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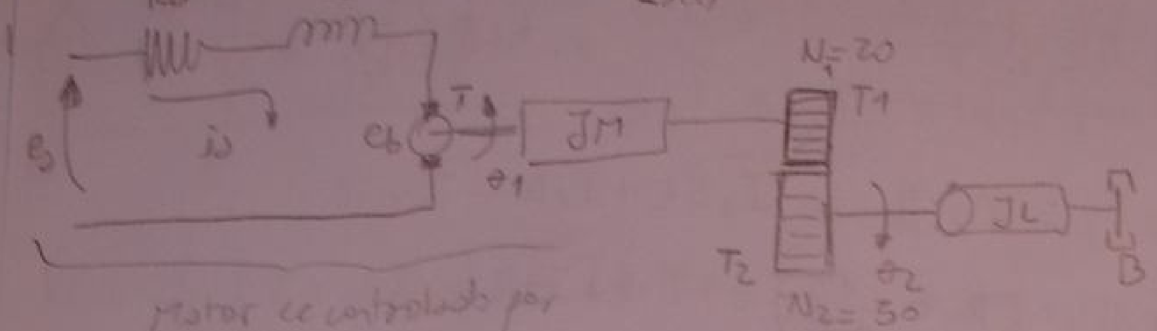
$\frac{\theta_1(s)}{E_a(s)}$?



Motor cc controlado por armadura.

$$e_a = I_a R_a + s I_a L_a + e_b \quad \Big\} \quad e_a = I_a R_a + s I_a L_a + k_b s \theta_1$$

$$T = k_T I_a$$

$$T = \ddot{\theta}_1 J_M + T_1 \Rightarrow k_T I_a = s^2 \theta_1 J_M + T_1$$

$$T_1 = \frac{N_1}{N_2} T_2 \Rightarrow k_T I_a = s^2 \theta_1 J_M + \frac{N_1}{N_2} T_2$$

$$T_2 = J_L \ddot{\theta}_2 + B \dot{\theta}_2 \Rightarrow T_2(s) = s^2 J_L \theta_2 + s B \theta_2$$

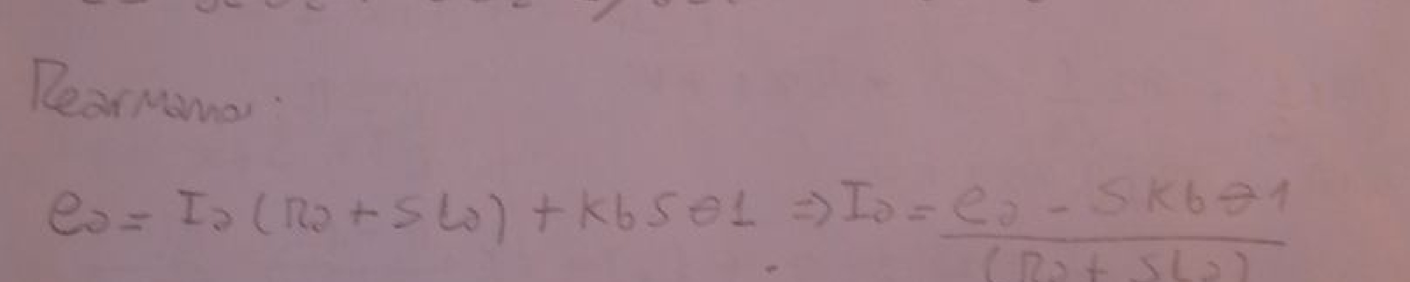
Rearmamos:

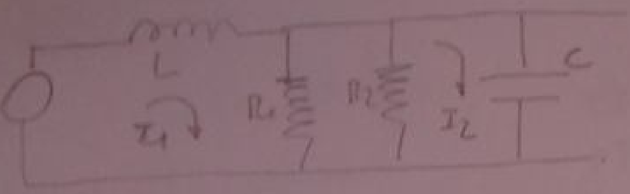
$$e_a = I_a (R_a + s L_a) + k_b s \theta_1 \Rightarrow I_a = \frac{e_a - s k_b \theta_1}{(R_a + s L_a)}$$

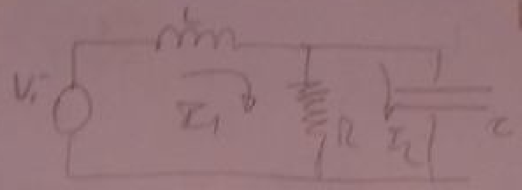
$$I_a = \frac{e_a - e_b}{(R_a + s L_a)} \Rightarrow \frac{I_a}{e_a - e_b} = \frac{1}{R_a + s L_a}$$

$$T = s^2 \theta_1 J_M + \frac{N_1}{N_2} \theta_2 (s^2 J_L + s B), \quad \theta_2 = \theta_1 \frac{N_1}{N_2}$$

$$\theta_1 \left(s^2 J_M + \left(\frac{N_1}{N_2}\right)^2 (s^2 J_L + s B) \right)$$







$V_i \Rightarrow I_1$
$\Rightarrow V_C$

$$V_i = I_1 (sL + R) - I_2 R = I_1 sL + I_1 R - I_2 R$$

$$0 = -I_1 R + I_2 \left(R + \frac{1}{sC}\right) = -I_1 R + I_2 R + \frac{I_2}{sC}$$

$$I_1 = x_1 \Rightarrow \dot{x}_1 = I_1 s$$

$$\frac{I_2}{sC} = x_2 \Rightarrow I_2 = x_2 s C = \dot{x}_2 C$$

$$\left.\begin{aligned} \dot{x}_1 L + R x_1 - \dot{x}_2 C R &= V_i \\ -x_1 R + \dot{x}_2 C R + x_2 &= 0 \end{aligned}\right\}$$

$$\dot{x}_2 = x_1 \frac{1}{C} - x_2 \frac{1}{CR}$$

$$\dot{x}_1 L = \left(x_1 \frac{1}{C} - x_2 \frac{1}{CR}\right) CR - R x_1 + V_i$$

$$= x_1 R - x_2 - R x_1 + V_i$$

$$\frac{R}{L} - R$$

$$\frac{R - LR}{L} = \frac{R(1-L)}{L}$$

$$\dot{x}_1 = -x_2 \frac{1}{L} + V_i/L$$

$$\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \end{vmatrix} = \begin{vmatrix} 0 & -1/L \\ \frac{1}{C} & -\frac{1}{RC} \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} + \begin{vmatrix} 1/L \\ 0 \end{vmatrix} V_i$$

$$C = \begin{vmatrix} 0 & 1 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \end{vmatrix}$$

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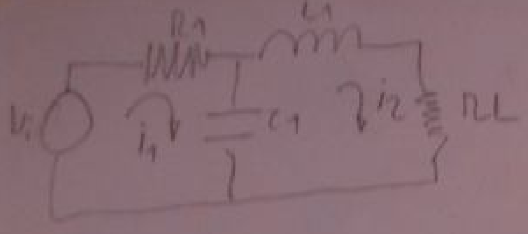


$$V_i = I_1\left(R_1 + \frac{1}{sC}\right) - I_2 \frac{1}{sC}$$

$$0 = -I_1 \frac{1}{sC} + I_2\left(sL_1 + R_L + \frac{1}{sC}\right)$$

$$V_i = \frac{1}{sC}(I_1 - I_2) + I_1 R_1$$

$$0 = -\frac{1}{sC}(I_1 - I_2) + I_2 s L_1 + I_2 R_L$$

$$\left.\begin{array}{l} x_1 = \frac{1}{sC}(I_1 - I_2) \;;\; \dot{x}_1 C = (I_1 - I_2) \\ x_2 = I_2 \Rightarrow \dot{x}_2 = sI_2 \end{array}\right\}$$

$$\left.\begin{array}{l} V_i = x_1 + I_1 R_1 \\ 0 = -x_1 + \dot{x}_2 L + x_2 R_L \end{array}\right| \quad \dot{x}_2 = x_1 \frac{1}{L} - x_2 \frac{R_L}{L}$$

$$I_1 = \dot{x}_1 C + I_2$$

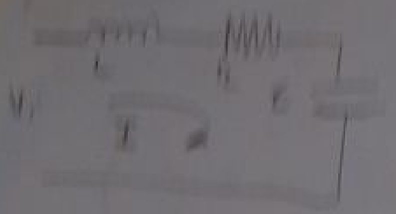
$$V_i = \frac{1}{sC}(\dot{x}_1 C + \cancel{I_2} - \cancel{I_2}) + \dot{x}_1 C R_1 + I_2 R_1$$

$$= x_1 + \dot{x}_1 C R_1 + x_2 R_1$$

$$\dot{x}_1 = V_i \frac{1}{CR_1} - x_1 \frac{1}{CR_1} - x_2 \frac{1}{C}$$

$$\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \end{vmatrix} = \begin{vmatrix} -1/CR_1 & -1/C \\ 1/L & -R_L/L \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} + \begin{vmatrix} 1/R_1C \\ 0 \end{vmatrix} V_i$$

$$y(t) = \begin{vmatrix} 0 & R_L \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \end{vmatrix}$$



$V_i = sL I + I R + I \frac{1}{sC}$

$V_o = I \frac{1}{sC}$

$$\frac{V_o}{V_i} = \frac{1}{sC\left(sL + R + \frac{1}{sC}\right)} = \frac{1}{s^2 LC + sRC + 1}$$

$$\frac{V_o}{V_i} = \frac{1}{s^2 + s\frac{RC}{LC} + \frac{1}{LC}} = \frac{1}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$$

$V_i = x s^2 + x s \frac{R}{L} + x \frac{1}{LC}$ | $x s^2 = \dot{x}_2$

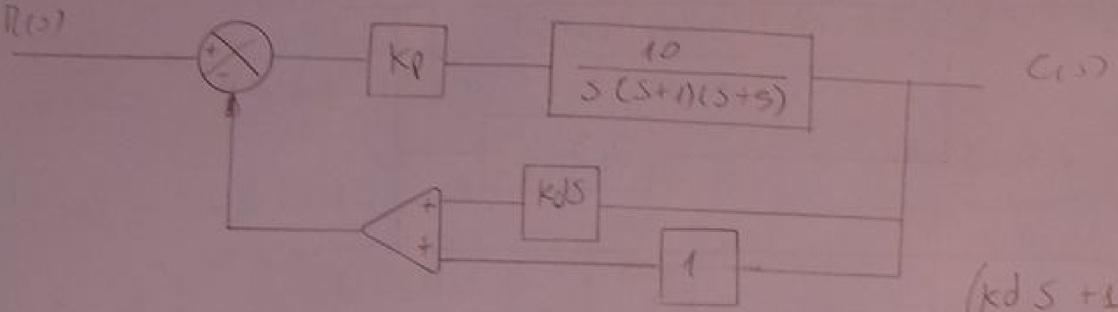
$x s^2 = V_i - x s \frac{R}{L} - x \frac{1}{LC}$ | $x s = x_2 = \dot{x}_1$

$x = x_1$

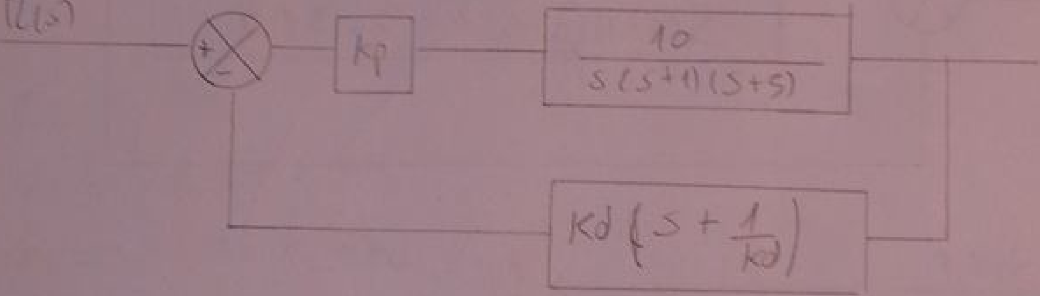
$\dot{x}_2 = -x_1 \frac{1}{LC} - x_2 \frac{R}{L} + V_i$; $V_o = x = x_1$

$$\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{R}{L} \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} + \begin{vmatrix} 0 \\ 1 \end{vmatrix} V_i$$

$$y(t) = \begin{vmatrix} 1 & 0 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \end{vmatrix}$$

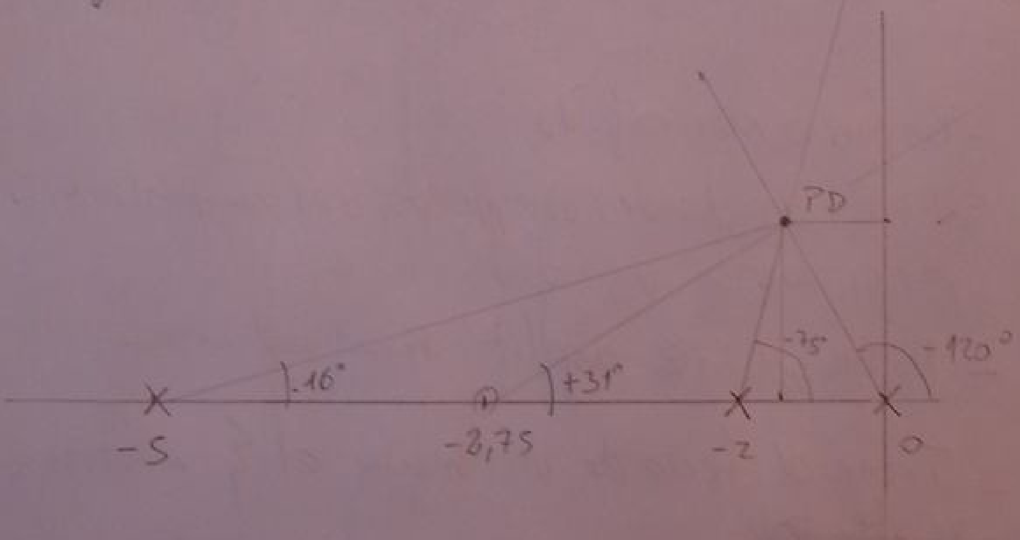


$(kd\,s + 1) = kd\left(s + \frac{1}{kd}\right)$



$\omega_n = \sqrt{2}$; $\sigma = \omega_n \xi = \frac{\sqrt{2}}{2}$; $\omega_d = \sqrt{\omega_n^2 - \sigma^2} = 1{,}22 = \sqrt{1{,}5}$

$\Rightarrow P_d = \frac{\sqrt{2}}{2} + j\sqrt{1.5}$; $A\cos\xi = 60°$



$PD^{\circ} \Rightarrow -16 - 75 - 120° = -211°$

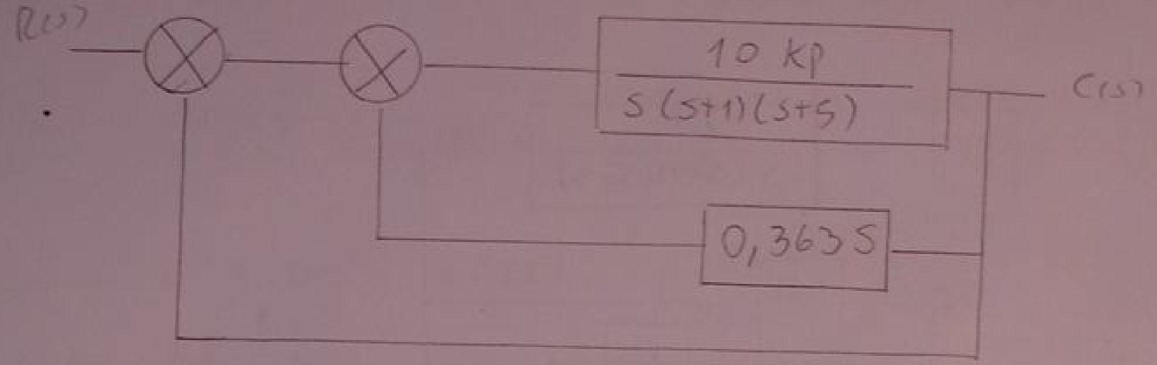
$-211 + \varphi_c = -180 \Rightarrow \varphi_c = 31°$

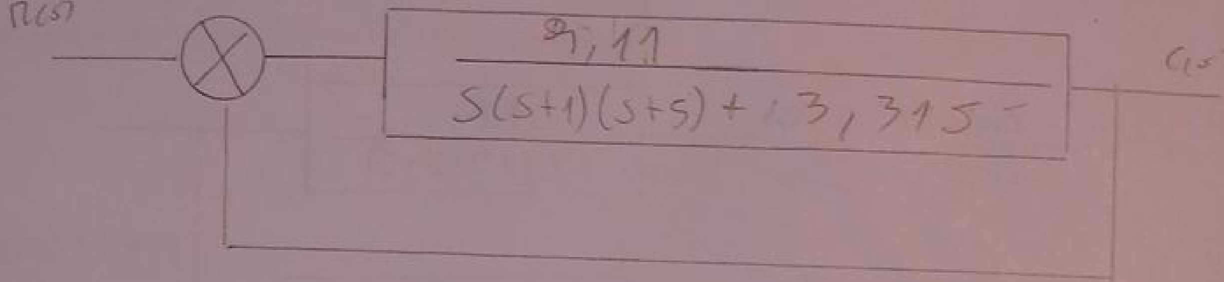
$\frac{Im\{Pd\}}{Re\{Pd\} - Zc} = \tan(180° - 31°) \Rightarrow Zc = -2{,}75$

$Zc = \frac{1}{kd} \Rightarrow kd = 0{,}363$

$K = \frac{4{,}5 \cdot 1{,}25 \cdot \sqrt{2}}{10 \cdot 2{,}4} = 0{,}331$

$K = k_p \cdot kd \Rightarrow k_p = 0{,}911$





$$G(s) = \frac{9,11}{s(s^2+6s+8,31)} \quad ; \; k_p = \infty$$

$\Rightarrow e_{ss} = 0$.

$$\frac{C(s)}{R(s)} = \frac{9,11}{s^3+6s^2+8,31s+9,11}$$

$$\begin{cases} \underbrace{-0,684}_{} \pm j1,224 \\ -0,702 \\ -4,6 \end{cases}$$

[illegible]

Ventajas:
- Respuesta más rápida
- Utilizado donde no se pueden usar compensadores en cascada por problemas de ruido
- No requiere amplificación adicional.
- Tiene el efecto de incrementar el ξ mientras que ω_n no se ve afectado.
- Realimentación "velocidad y posición" ⇒ Taquimétrica

$$\frac{C(s)}{R(s)} = \frac{s+15}{s(s^2+6s+8)} = \frac{s+15}{s^3+6s^2+8s} = \frac{s^{-3}(s+15)}{1+6s^{-1}+8s^{-2}} \qquad (b1)$$

$$\frac{C(s)}{R(s)} = \frac{s^{-2}+15s^{-3}}{1+6s^{-1}+8s^{-2}}; \Rightarrow C(s)(1+6s^{-1}+8s^{-2}) = R(s)(s^{-2}+15s^{-3})$$

$$C(s) = -6C(s)s^{-1} - 8C(s)s^{-2} + R(s)s^{-2} + 15R(s)s^{-3}$$

$$= \underbrace{s^{-1}\Big\{-6C(s) + \underbrace{s^{-1}\big\{R(s) - 8C(s) + \underbrace{s^{-1}\{15R(s)\}}_{x_3}\big\}}_{x_2}\Big\}}_{x_1}$$

$C(s) = x_1$

$\dot{x}_1 = -6x_1 + x_2$

$\dot{x}_2 = -8x_1 + x_3 + u$

$\dot{x}_3 = 15u$

$$\left\{\begin{aligned} \begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{vmatrix} &= \begin{vmatrix} -6 & 1 & 0 \\ -8 & 0 & 1 \\ 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} + \begin{vmatrix} 0 \\ 1 \\ 15 \end{vmatrix} u \\ y &= \begin{vmatrix} 1 & 0 & 0 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} \end{aligned}\right.$$

$$\begin{vmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{vmatrix} - \begin{vmatrix} -6 & 1 & 0 \\ -8 & 0 & 1 \\ 0 & 0 & 0 \end{vmatrix} = \begin{vmatrix} s+6 & -1 & 0 \\ 8 & s & -1 \\ 0 & 0 & s \end{vmatrix}$$

$\begin{matrix} s+6 & -1 & 0 \\ 8 & s & -1 \end{matrix}$

$$(s+6)s^2 + 8s = 0 \quad \begin{cases} 0 \\ -2 \\ -4 \end{cases}$$

$\dot{x} = Ax + BU \quad ; \quad s\,x(s) = A\,x(s) - BK\,x(s) \;\Big|\; (U = -Kx)$

$(sI - A + BK)\,x(s) = 0$

$$\begin{vmatrix} s+6 & -1 & 0 \\ 8 & s & -1 \\ 0 & 0 & s \end{vmatrix} + \begin{vmatrix} 0 & 0 & 0 \\ k_1 & k_2 & k_3 \\ 15k_1 & 15k_2 & 15k_3 \end{vmatrix} = (s+15)(s+3+3j)(s+3-3j)$$
$$= s^3 + 21s^2 + 108s + 270$$

$$\begin{vmatrix} s+6 & -1 & 0 \\ 8+k_1 & s+k_2 & -1+k_3 \\ 15k_1 & 15k_2 & 15k_3+s \end{vmatrix} =$$

$$s^3 + 15s^2k_3 + k_2s^2 + 21k_2s + 6s^2 + 90sk_3 + 90k_2 + 8s + 120k_3 + k_1s + 15k_1$$

$$s^3 + s^2(15k_3 + 6 + k_2) + s(21k_2 + 90k_3 + 8 + k_1) + 90k_2 + 120k_3 + 15k_1$$

$$\left.\begin{aligned} 15k_3 + k_2 + 6 &= 21 \\ 90k_3 + 21k_2 + k_1 + 8 &= 108 \\ 120k_3 + 90k_2 + 15k_1 &= 270 \end{aligned}\right\}$$

$$\begin{vmatrix} 15 & 1 & 0 \\ 90 & 21 & 1 \\ 120 & 90 & 15 \end{vmatrix} \begin{vmatrix} k_3 \\ k_2 \\ k_1 \end{vmatrix} + \begin{vmatrix} 6 \\ 8 \\ 0 \end{vmatrix} = \begin{vmatrix} 21 \\ 108 \\ 270 \end{vmatrix}$$

$$\left.\begin{aligned} k_1 &= 10 \\ k_2 &= 0 \\ k_3 &= 1 \end{aligned}\right\}$$

$$\dot{x} = Ax + B(-Kx + k_1 r)$$
$$= (A - BK)x + Bk_1 r$$

$$(A - BK) = \begin{vmatrix} -6 & 1 & 0 \\ -18 & 0 & 1 \\ -150 & 0 & -15 \end{vmatrix}$$

$$Bk_1 r = \begin{vmatrix} 0 \\ 10 \\ 150 \end{vmatrix} r$$

$$\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{vmatrix} = \begin{vmatrix} -6 & 1 & 0 \\ -18 & 0 & 0 \\ -150 & 0 & -15 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} + \begin{vmatrix} 0 \\ 10 \\ 150 \end{vmatrix} r$$

$$y = \begin{vmatrix} 1 & 0 & 0 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix}$$

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$$\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{vmatrix} = \begin{vmatrix} -5 & 5 & 0 \\ 0 & -3 & 3 \\ 0 & 0 & -1 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} u(t)$$

$$y(t) = |1\ 0\ 0| \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix}$$

$$\dot{x}_1 = -5x_1 + 5x_2$$
$$\dot{x}_2 = -3x_2 + 3x_3$$
$$\dot{x}_3 = -x_3 + U(t)$$
$$y(t) = x_1$$

$$\frac{x_3}{\dot{x}_3} = \frac{1}{s+1} \quad ; \quad \frac{x_2}{\dot{x}_2} = \frac{1}{s+3} \quad ; \quad \frac{x_1}{\dot{x}_1} = \frac{1}{s+5}$$

$$\frac{y(t)}{U(t)} = \frac{1}{s+1} \cdot \frac{3}{s+3} \cdot \frac{5}{s+5} = \frac{15}{s^3 + 9s^2 + 23s + 15}$$

$$s x(s) = A x(s) + B U(s)$$
$$s x(s) - A x(s) = B U(s)$$
$$(sI - A) x(s) = B U(s)$$

$$x(s) = (sI - A)^{-1} B U(s)$$
$$\frac{y(s)}{U(s)} = C (sI - A)^{-1} B$$

b)

$$M = |B \quad AB \quad A^2 B| = \begin{vmatrix} 0 & 0 & 15 \\ 0 & 3 & -12 \\ 1 & -1 & 1 \end{vmatrix} \quad \begin{matrix} \text{rango} = 3 \\ \det \neq 0 \end{matrix}$$

$$O = |C^T \quad A^T C^T \quad A^{2T} C^T| = \begin{vmatrix} 1 & -5 & 25 \\ 0 & 5 & -40 \\ 0 & 0 & 15 \end{vmatrix} \quad \begin{matrix} \text{rango} = 3 \\ \det \neq 0 \end{matrix}$$

Completamente controlable y observable

$$|\lambda I - A| = \left| \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} - \begin{vmatrix} -5 & 5 & 0 \\ 0 & -3 & 3 \\ 0 & 0 & -1 \end{vmatrix} \right|$$

$$= \left| \begin{vmatrix} \lambda+5 & -5 & 0 \\ 0 & \lambda+3 & -3 \\ 0 & 0 & \lambda+1 \end{vmatrix} \right| = (\lambda+5)(\lambda+3)(\lambda+1)$$

$$\begin{matrix} \lambda+5 & -5 & 0 \\ 0 & \lambda+3 & -3 \end{matrix}$$

$\lambda_1 = -5$

$\lambda_2 = -3$

$\lambda_3 = -1$

c)

$$\varphi(t) = \mathcal{L}^{-1}\left[(sI - A)^{-1}\right]$$

$$sI - A = \begin{vmatrix} s+5 & -5 & 0 \\ 0 & s+3 & -3 \\ 0 & 0 & s+1 \end{vmatrix}$$

$$\text{Adj} = \begin{vmatrix} s^2+4s+3 & 5s+5 & 15 \\ 0 & s^2+6s+5 & 3s+15 \\ 0 & 0 & s^2+8s+15 \end{vmatrix}$$

$$\det = s^3 + 9s^2 + 23s + 15 = (s+1)(s+3)(s+5)$$

$$(sI-A)^{-1} = \frac{\text{Adj}(sI-A)}{\det(sI-A)}$$

$$\frac{\text{Adj}}{\det} = \frac{1}{(s+1)(s+3)(s+5)} \begin{vmatrix} (s+3)(s+1) & 5(s+1) & 15 \\ 0 & (s+5)(s+1) & 3(s+5) \\ 0 & 0 & (s+5)(s+3) \end{vmatrix}$$

$$\varphi(s) = \begin{vmatrix} \frac{1}{s+5} & \frac{5}{(s+3)(s+5)} & \frac{15}{(s+1)(s+3)(s+5)} \\ 0 & \frac{1}{(s+3)} & \frac{3}{(s+1)(s+3)} \\ 0 & 0 & \frac{1}{(s+1)} \end{vmatrix}$$

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$$\varphi(s) = \begin{vmatrix} \frac{1}{s+5} & 2{,}5\left(\frac{1}{(s+3)} - \frac{1}{(s+5)}\right) & \frac{-3{,}75}{(s+3)} + \frac{1{,}875}{(s+1)} + \frac{1{,}875}{(s+5)} \\ 0 & \frac{1}{(s+3)} & 1{,}5\left(\frac{1}{(s+1)} - \frac{1}{(s+3)}\right) \\ 0 & 0 & \frac{1}{(s+1)} \end{vmatrix}$$

$$\varphi(t) = \begin{vmatrix} e^{-5t} & 2{,}5\left(e^{-3t} - e^{-5t}\right) & -3{,}75e^{-3t} + 1{,}875\left(e^{-t} + e^{-5t}\right) \\ 0 & e^{-3t} & 1{,}5\left(e^{-t} - e^{-3t}\right) \\ 0 & 0 & e^{-t} \end{vmatrix}$$

Reresolución $\Rightarrow \frac{C(s)}{R(s)} = \frac{\overset{b_1}{}\overset{b_2}{s} + \overset{b_3}{15}}{s^3 + \underset{a_1}{6}s^2 + \underset{a_2}{8}s}$

$a_1 = 6$
$a_2 = 8$
$b_0 = 0$
$b_1 = 0$
$b_2 = 1$
$b_3 = 15$

$x_1 = y - \beta_0 u$

$x_2 = \dot{y} - \beta_0 \dot{u} - \beta_1 u = \dot{x}_1 - \beta_1 u$

$x_3 = \ddot{y} - \beta_0 \ddot{u} - \beta_1 \dot{u} - \beta_2 u = \dot{x}_2 - \beta_2 u$

$\beta_0 = b_0$

$\beta_1 = b_1 - a_1\beta_0$

$\beta_2 = b_2 - a_1\beta_1 - a_2\beta_0$

$\beta_3 = b_3 - a_1\beta_2 - a_2\beta_1 - a_3\beta_0$

$\beta_0 = 0$
$\beta_1 = 0$
$\beta_2 = 1$
$\beta_3 = 9$

=

$x_1 = y$

$x_2 = \dot{x}_1$

$x_3 = \dot{x}_2 + u$

$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3 + u$

$\dot{x}_3 = -8x_2 - 6x_3 + 9u$

$$\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -8 & -6 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} + \begin{vmatrix} 0 \\ 1 \\ 9 \end{vmatrix} u \qquad y = \begin{vmatrix} 1 & 0 & 0 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix}$$

$15k_1 + 0k_2 + 0k_3 + 0$
$9k_1 + 15k_2 - 8k_3 + 8$
$0k_1 + 1k_2 + 9k_3 + 6$

$(sI - A + BK)$

$$\begin{vmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{vmatrix} - \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -8 & -6 \end{vmatrix} + \begin{vmatrix} 0 \\ 1 \\ 9 \end{vmatrix} \begin{vmatrix} k_1 & k_2 & k_3 \end{vmatrix}$$

$$\begin{vmatrix} s & -1 & 0 \\ k_1 & s+k_2 & -1+k_3 \\ 9k_1 & 8+9k_2 & s+6+9k_3 \end{vmatrix} = s^3 + 21s^2 + 108s + 270$$

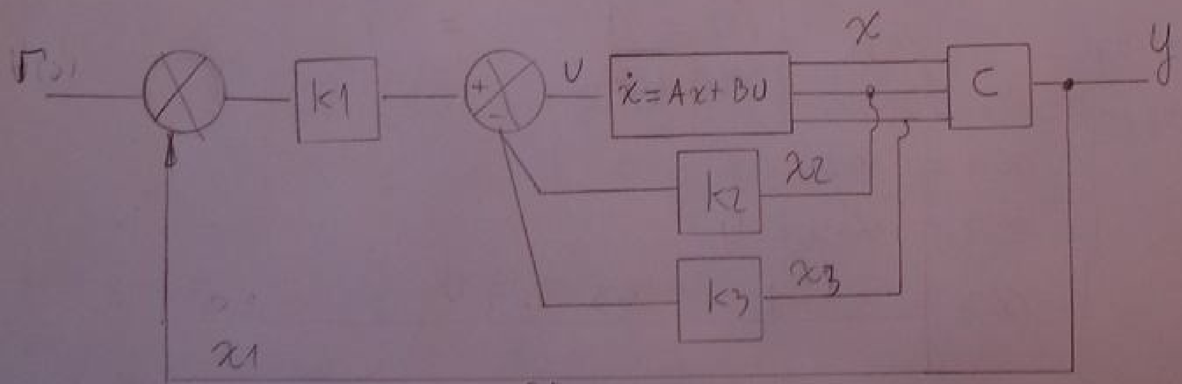
$$s^3 + 6s^2 + 9s^2k_3 + k_2s^2 + 15k_2s + 8s - 8sk_3 + k_1s + 15k_1$$

$$= s^3 + s^2(6 + 9k_3 + k_2) + s(15k_2 + 8 - 8k_3 + k_1) + 15k_1$$

$15k_1 = 270$
$15k_2 + 8 - 8k_3 + k_1 = 108$
$6 + 9k_3 + k_2 = 21$

$k_1 = 18$
$k_2 = 6$
$k_3 = 1$

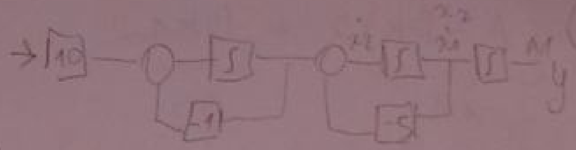
$$\begin{vmatrix} 15 & 0 & 0 \\ 1 & 15 & -8 \\ 0 & 1 & 9 \end{vmatrix} \begin{vmatrix} k_1 \\ k_2 \\ k_3 \end{vmatrix} + \begin{vmatrix} 0 \\ 8 \\ 6 \end{vmatrix} = \begin{vmatrix} 270 \\ 108 \\ 21 \end{vmatrix} \Rightarrow K = \begin{vmatrix} 18 \\ 6 \\ 1 \end{vmatrix}$$



$$\dot{x} = \overbrace{(A - Bk)}^{AA}x + \overbrace{Bk_1}^{BB}r \Rightarrow AA = \begin{vmatrix} 0 & 1 & 0 \\ -18 & -6 & 0 \\ -162 & -62 & -15 \end{vmatrix} ; BB = \begin{vmatrix} 0 \\ 18 \\ 62 \end{vmatrix}$$

$$\Rightarrow FT = \frac{18s + 270}{s^3 + 21s^2 + 108s + 270} ; e_{ss}\Big|_{\frac{1}{s}} = 0$$

$$F_T = \frac{10}{(s+1)(s+5)s} = \frac{Y(s)}{U(s)}$$



Metodo 1: En FCO: (No andó!!)

$$\frac{C(s)}{U(s)} = \frac{10}{s^3+6s^2+5s} = \frac{s^{-3}\,10}{1+6s^{-1}+5s^{-2}}$$

$$C(s) = -6s^{-1}C(s) - 5s^{-2}C(s) + s^{-3}\,10\,U(s)$$

$$C(s) = s^{-1}\underbrace{\left\{-6C(s) + s^{-1}\underbrace{\left\{-5C(s) + s^{-1}\left\{\underbrace{10U(s)}_{\dot{x}_3}\right\}\right\}}_{\dot{x}_2}\right\}}_{\dot{x}_1}$$

$$x_1 = C(s)$$
$$\dot{x}_1 = -6x_1 + x_2$$
$$\dot{x}_2 = -5x_1 + x_3$$
$$\dot{x}_3 = 10U(s)$$

$$\left\{\begin{array}{l} \begin{vmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{vmatrix} = \begin{vmatrix}-6 & 1 & 0\\ -5 & 0 & 1\\ 0 & 0 & 0\end{vmatrix}\begin{vmatrix}x_1\\ x_2\\ x_3\end{vmatrix} + \begin{vmatrix}0\\ 0\\ 10\end{vmatrix}U \\ y = \begin{vmatrix}1 & 0 & 0\end{vmatrix}\begin{vmatrix}x_1\\ x_2\\ x_3\end{vmatrix}\end{array}\right\}$$

$$(sI - A + BK) = \begin{vmatrix}s+6 & -1 & 0\\ 5 & s & -1\\ 10k_1 & 10k_2 & s+10k_3\end{vmatrix}$$

$$\det(sI - A + Bk) = s^3 + 10s^2k_3 + 10k_2s + 6s^2 + 60sk_3 + 60k_2 + 5s + 50k_3 + 10k$$
$$= s^3 + s^2(10k_3+6) + s(10k_2+60k_3+5) + 60k_2+50k_3+10$$
$$= s^3 + s^2\,6{,}014 + s\,8{,}5 + 9{,}2$$

$k_1 = -1{,}1366$	$10k_3 + 6 = 6{,}014$
$k_2 = 0{,}3416$	$10k_2 + 60k_3 + 5 = 8{,}5$
$k_3 = 0{,}0014$	$60k_2 + 50k_3 + 10k_1 = 9{,}2$

$$\dot{x} = \underbrace{(A - BK)}_{AA} x + \underbrace{BK1}_{BB} r \qquad CC = C;\ DD = D$$

$$\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{vmatrix} = \begin{vmatrix} -6 & 1 & 0 \\ -5 & 0 & 1 \\ 11,366 & -3,416 & -0,014 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ -11,366 \end{vmatrix}$$

$$y = \begin{vmatrix} 1 & 0 & 0 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix}$$

$$FT = \frac{-11,366}{s^3 + 6,014 s^2 + 8,5 s + 9,2}$$ ✗ ⇒ No logramos $e_{ss} = 0$ para escalón

Método 2: (Ando!)

$$F(s) = \frac{10}{s^3 + 6s^2 + 5s}$$

$a_1 = 6$	$b_3 = 10$
$a_2 = 5$	$b_2 = 0$
$a_3 = 0$	$b_1 = 0$
	$b_0 = 0$

$x_1 = y - \beta_0 U$

$x_2 = \dot{y} - \beta_0 \dot{U} - \beta_1 U = \dot{x}_1 - \beta_1 U$

$x_3 = \ddot{y} - \beta_0 \ddot{U} - \beta_1 \dot{U} - \beta_2 U = \dot{x}_2 - \beta_2 U$

$$A = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a_n & a_{n-1} & a_{n-2} \end{vmatrix} \qquad B = \begin{vmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{vmatrix}$$

$\beta_0 = b_0$

$\beta_1 = b_1 - a_1 \beta_0$

$\beta_2 = b_2 - a_1 \beta_1 - a_2 \beta_0$

$\beta_3 = b_3 - a_1 \beta_2 - a_2 \beta_1 - a_3 \beta_0$

$\beta_0 = 0$
$\beta_1 = 0$
$\beta_2 = 0$
$\beta_3 = 10$

$\dot{x}_1 = x_2 - \beta_1 U = x_2$

$\dot{x}_2 = x_3 - \beta_2 U = x_3$

$\dot{x}_3 = -5 x_2 - 6 x_3 + 10 U$

$x_1 = y$

$$A = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -5 & -6 \end{vmatrix} \qquad B = \begin{vmatrix} 0 \\ 0 \\ 10 \end{vmatrix}$$

$$C = \begin{vmatrix} 1 & 0 & 0 \end{vmatrix}$$

$sI - A + BK =$

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$$\begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 10k_1 & 5+10k_2 & s+6+10k_3 \end{vmatrix} = s^3 + 6s^2 + 10s^2k_3 + 5s + 10k_2 s + 10k_1$$

$$= s^3 + s^2(6+10k_3) + s(5+10k_2) + 10k_1$$

$$= s^3 + s^2\,6{,}014 + s\,8{,}5 + 9{,}2$$

$$0k_1 + 0k_2 + 10k_3 + 6 = 6{,}014$$
$$0k_1 + 10k_2 + 0k_3 + 5 = 8{,}5$$
$$10k_1 + 0k_2 + 0k_3 + 0 = 9{,}2$$

$$\begin{vmatrix} 0 & 0 & 10 \\ 0 & 10 & 0 \\ 10 & 0 & 0 \end{vmatrix} \begin{vmatrix} k_1 \\ k_2 \\ k_3 \end{vmatrix} + \begin{vmatrix} 6 \\ 5 \\ 0 \end{vmatrix} = \begin{vmatrix} 6{,}014 \\ 8{,}5 \\ 9{,}2 \end{vmatrix} \Rightarrow \begin{vmatrix} k_1 \\ k_2 \\ k_3 \end{vmatrix} = \begin{vmatrix} 0{,}0014 \\ 0{,}350 \\ 0{,}92 \end{vmatrix}$$

$$\dot{x} = \overbrace{(A - Bk)}^{AA} x + \overbrace{B}^{BB} k_1 r$$

$$AA = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0{,}014 & -8{,}5 & -15{,}2 \end{vmatrix} ; \quad BB = \begin{vmatrix} 0 \\ 0 \\ 0{,}014 \end{vmatrix}$$

$$TF = \frac{0{,}014}{s^3 + 15s^2 + 8{,}5s + 0{,}014}$$

Final 24/03/08

$$\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ -6 & -5 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} + \begin{vmatrix} 0 \\ 1 \end{vmatrix} u$$

$$y = |6 \;\; 0| \begin{vmatrix} x_1 \\ x_2 \end{vmatrix}$$

$$\begin{vmatrix} \lambda & 0 \\ 0 & \lambda \end{vmatrix} - \begin{vmatrix} 0 & 1 \\ -6 & -5 \end{vmatrix} = 0$$

$$\begin{vmatrix} \lambda & -1 \\ 6 & \lambda+5 \end{vmatrix} = \lambda(\lambda+5) + 6 = 0$$

$\lambda_1 = -3$

$\lambda_2 = -2.$

$$|\lambda_1 I - A|P = 0 = \left(\begin{vmatrix} -3 & 0 \\ 0 & -3 \end{vmatrix} - \begin{vmatrix} 0 & 1 \\ -6 & -5 \end{vmatrix}\right)\begin{vmatrix} x_1 \\ y_1 \end{vmatrix} = 0$$

$$= \begin{vmatrix} -3 & -1 \\ 6 & 2 \end{vmatrix}\begin{vmatrix} x_1 \\ y_1 \end{vmatrix} = 0$$

$$\begin{array}{l|l} -3x - y = 0 & -3x = y \\ 6x + 2y = 0 & 6x + 2(-3x) = 0 \end{array}$$

$$6x - 6x = 0 \Rightarrow x = 1 \quad y = -3$$

$$|\lambda_2 I - A|P = 0 \left(\begin{vmatrix} -2 & 0 \\ 0 & -2 \end{vmatrix} - \begin{vmatrix} 0 & 1 \\ -6 & -5 \end{vmatrix}\right)\begin{vmatrix} x_2 \\ y_2 \end{vmatrix}$$

$$= \begin{vmatrix} -2 & -1 \\ 6 & 3 \end{vmatrix}\begin{vmatrix} x_2 \\ y_2 \end{vmatrix} \Rightarrow \begin{array}{l|l} -2x_2 - y_2 = 0 & -2x_2 = y_2 \\ 6x_2 + 3y_2 = 0 & 6x_2 - 6x_2 = 0 \end{array}$$

$$\Rightarrow x_2 = 1 \; ; \; y_2 = -2$$

$$P = \begin{vmatrix} 1 & 1 \\ -3 & -2 \end{vmatrix} \; ; \; P^{-1} = \begin{vmatrix} -2 & -1 \\ 3 & 1 \end{vmatrix}$$

$$\Rightarrow A_{FCD} = P^{-1}AP = \begin{vmatrix} -3 & 0 \\ 0 & -2 \end{vmatrix}$$

$$B_{FCD} = P^{-1}B = \begin{vmatrix} -1 \\ 1 \end{vmatrix}$$

$$C_{FCD} = CP = \begin{vmatrix} 6 & 6 \end{vmatrix}$$

$$\left.\begin{array}{l} \begin{vmatrix} \dot{z}_1 \\ \dot{z}_2 \end{vmatrix} = \begin{vmatrix} -3 & 0 \\ 0 & -2 \end{vmatrix}\begin{vmatrix} z_1 \\ z_2 \end{vmatrix} + \begin{vmatrix} -1 \\ 1 \end{vmatrix} U \\ y = \begin{vmatrix} 6 & 6 \end{vmatrix}\begin{vmatrix} z_1 \\ z_2 \end{vmatrix} \end{array}\right\} \quad FCD$$

2) $M = |B \;\; AB| = \begin{vmatrix} 0 & 1 \\ 1 & -5 \end{vmatrix}$ $\begin{array}{l} Det \neq 0 \\ Rank = 2 \end{array}$ $\Rightarrow$ Totalmente controlable

$O = |C^T \;\; A^T C^T| = \begin{vmatrix} 6 & 0 \\ 0 & 6 \end{vmatrix}$ $\begin{array}{l} Rank = 2 \\ Det \neq 0 \end{array}$ $\Rightarrow$ Totalmente observable

3) La transformación no afecta los autovalores $(P^{-1}P = I)$ 66

4) $sX(s) = AX(s) + BU(s)$
$(sI - A)X(s) = BU(s)$

$X(s) = (sI-A)^{-1}BU(s)$
$Y(s) = CX(s)$
$FT = C(sI-A)^{-1}B$

$$(sI - A)^{-1} = \begin{vmatrix} s & 0 \\ 0 & s \end{vmatrix} - \begin{vmatrix} 0 & 1 \\ -6 & -5 \end{vmatrix} = \left(\begin{vmatrix} s & -1 \\ 6 & s+5 \end{vmatrix} \right)^{-1}$$

$$= \frac{Adj(sI-A)}{det(sI-A)} = \frac{1}{(s+3)(s+2)} \begin{vmatrix} s+5 & 1 \\ -6 & s \end{vmatrix}$$

$$= \frac{1}{(s+3)(s+2)} \; |6 \;\; 0| \begin{vmatrix} s+5 & 1 \\ -6 & s \end{vmatrix} \begin{vmatrix} 0 \\ 1 \end{vmatrix}$$

$$FT = \frac{6}{(s+3)(s+2)}$$

5) $\phi(t) = \mathcal{L}^{-1}\left[(sI - A)^{-1}\right]$

$$\phi(s) = \begin{vmatrix} \frac{s+5}{(s+3)(s+2)} & \frac{1}{(s+3)(s+2)} \\ \frac{-6}{(s+3)(s+2)} & \frac{s}{(s+3)(s+2)} \end{vmatrix}$$

$$= \begin{vmatrix} \frac{3}{(s+2)} + \frac{-2}{(s+3)} & \frac{1}{(s+2)} - \frac{1}{(s+3)} \\ \frac{-6}{(s+2)} + \frac{6}{(s+3)} & \frac{-2}{(s+2)} + \frac{3}{(s+3)} \end{vmatrix}$$

$$\phi(t) = \begin{vmatrix} 3e^{-2t} - 2e^{-3t} & -1e^{-3t} + e^{-2t} \\ -6e^{-2t} + 6e^{-3t} & -2e^{-2t} + 3e^{-3t} \end{vmatrix}$$

b)

$$\xi = 0{,}5 \ ; \quad \omega_n = 4 \Rightarrow \sigma = \omega_n \xi = 2$$

$$\omega_n^2 = \omega_d^2 + \sigma^2 \Rightarrow \omega_d = \sqrt{\omega_n^2 - \sigma^2} = 2\sqrt{3}$$

$$= \sqrt{16 - 4} = \sqrt{12} = 2\sqrt{3}$$

$$p_d = -2 \pm j2\sqrt{3}$$

$$\dot{x} = Ax + BU \ , \quad U = -Kx \Rightarrow sx(s) = Ax(s) - BKx(s)$$

$$(sI - A + BK) = (s + 2 - j2\sqrt{3})(s + 2 + j2\sqrt{3})$$

$$= s^2 + 4s + 16$$

$$\begin{vmatrix} s & 0 \\ 0 & s \end{vmatrix} - \begin{vmatrix} 0 & 1 \\ -6 & -5 \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ k_1 & k_2 \end{vmatrix} = \begin{vmatrix} s & -1 \\ 6 & s+5 \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ k_1 & k_2 \end{vmatrix}$$

$$\begin{vmatrix} s & -1 \\ 6k_1 & s+5+k_2 \end{vmatrix} \Rightarrow s(s+5+k_2) + 6k_1$$

$$s^2 + 5s + k_2 s + 6k_1$$

$$s^2 + s(5 + k_2) + 6k_1 = s^2 + 4s + 16.$$

$$\left.\begin{matrix} 5 + k_2 = 4 \\ 6k_1 = 16 \end{matrix}\right\} \Rightarrow \begin{matrix} k_1 = 10 \\ k_2 = -1 \end{matrix}$$

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1) El controlador es del tipo PD

$$\Rightarrow G_{PD} = kd\left(s + \frac{1}{TD}\right) = kd\,TD\,(TD\,s + 1)$$

2)

$$G(s)H(s) = \frac{K(s+2)}{s^2(s+12)} = K\frac{s+2}{s^3+12s^2}$$

$$N = P - Z = 3 - 1 = 2$$

$$\varphi_k = 180\frac{(2k-1)}{P-Z} \quad \begin{cases} \varphi_0 = -90 \\ \varphi_1 = 90 \end{cases}$$

k hasta [(P-Z)-1]

$$\sigma_c = \frac{-\sum Re\{P\} + \sum Re\{Z\}}{P-Z} = \frac{-(12-2)}{2} = -5$$

$$PB \Rightarrow G(s)H(s) + 1 = 0$$
$$K(s+2) + s^3 + 12s^2 = 0$$

$$K = -\frac{(s^3+12s^2)}{(s+2)}$$

$$\frac{\partial k}{\partial s} = 2s^3 + 18s^2 + 48s = 0$$

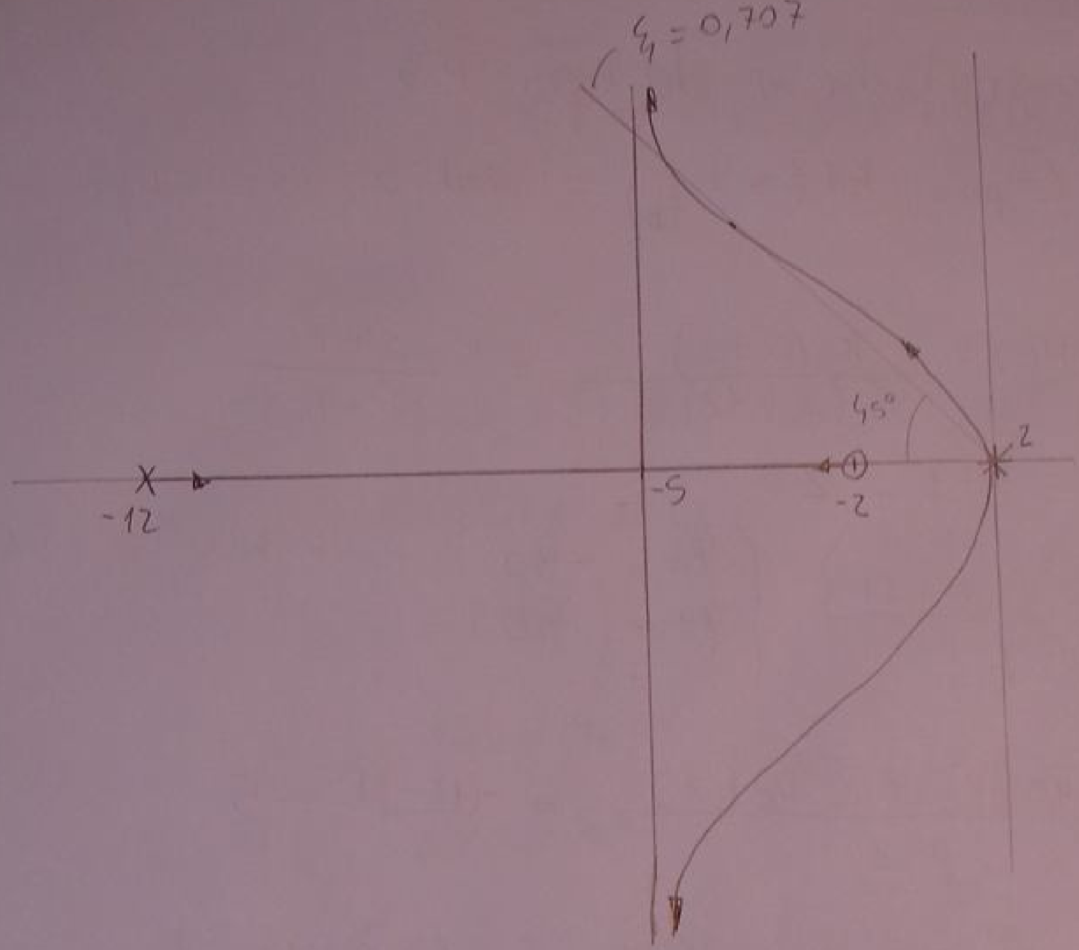
$$\begin{matrix} P_1 = 0 \\ P_2 = -4{,}5 + j1{,}9 \\ P_3 = -4{,}5 - j1{,}9 \end{matrix}$$

$$ks + 2k + s^3 + 12s^2 \Rightarrow s^3 + 12s^2 + ks + 2k = 0$$

s^3	1	k
s^2	12	2K
s^1	$\frac{10k}{12}$	
s^0	2K	

Estable p/ k > 0.

No corta el eje j



$\xi = 0,707 \Rightarrow \beta = A\cos \xi = 45°$; $W_n =$
$= \frac{\sqrt{2}}{2}$.

$$\frac{\frac{K(s+2)}{s^2(s+12)}}{1+\frac{K(s+2)}{s^2(s+12)}} = \frac{K(s+2)}{s^2(s+12)+K(s+2)} = \frac{K(s+2)}{s^3+12s^2+Ks+2K}$$

$$s^2 + 2\xi W_n s + W_n^2 = s^2 + \sqrt{2}\, W_n s + W_n^2 = 0$$

$$s^3 + 12s^2 + Ks + 2K \quad \big|\, s^2 + \sqrt{2}W_n s + W_n^2$$

$$s + (12 - \sqrt{2}W_n)$$

$$-s^3 - \sqrt{2}W_n s^2 - W_n^2 s$$

$$s^2(12-\sqrt{2}W_n) + s(K - W_n^2) + 2K$$

$$-s^2(12-\sqrt{2}W_n) - s\sqrt{2}W_n(12-\sqrt{2}W_n) - W_n^2(12-\sqrt{2}W_n)$$

$$s(W_n^2 + K - 12\sqrt{2}W_n) + 2K - 12W_n^2 + \sqrt{2}W_n^3$$

(71) (68)

$$\omega_n^2 + k - 12\sqrt{2}\,\omega_n = 0 \Rightarrow k = -\omega_n^2 + 12\sqrt{2}\,\omega_n$$

$$2k - 12\omega_n^2 + \sqrt{2}\,\omega_n^3 = 0$$

$$k = -\omega_n^2 + 12\sqrt{2}\,\omega_n$$

$$-2\omega_n^2 + 24\sqrt{2}\,\omega_n - 12\omega_n^2 + \sqrt{2}\,\omega_n^3 = 0$$

$$\sqrt{2}\,\omega_n^3 - 14\omega_n^2 + 24\sqrt{2}\,\omega_n = 0$$

$$\omega_n\left(\sqrt{2}\,\omega_n^2 - 14\omega_n + 24\sqrt{2}\right) = 0 \quad \begin{cases} 4{,}24 = \omega_{n_1} \\ 5{,}65 = \omega_{n_2} \end{cases}$$

$$\Rightarrow \underline{k_1 = 53{,}97} \; ; \; \omega_1 = 4{,}24$$

$$\Rightarrow \underline{k_2 = 63{,}96} \; ; \; \omega_2 = 5{,}65$$

$$\omega_{n_1} = 4{,}24 \Rightarrow \sigma = \omega_n \xi = -3 \; ; \; \omega_d = \sqrt{\omega_n^2 - \sigma^2} = 3$$

$$P_1 = -3 \pm j3$$

$$\omega_{n_2} = 5{,}65 \Rightarrow \sigma = 4 \; ; \; \omega_d = 4$$

$$P_2 = -4 \pm j4$$

Si esos dos puntos son polos, entonces son punto de diseño a lazo cerrado

P_1 es pto de diseño

P_2 es pto de diseño

Para encontrar el k conociendo solo el ξ, debemos partir desde la FTLC y encontrar la ecuación de segundo grado que contenga el ξ, ω_n necesario. Las incógnitas van a ser ω_n y K. Lo que se hace es buscar una expresión q' comparta las raíces entre el denominador FTLC y la función de segundo orden deseada, esto lo hacemos con la división de polinomios.

ess escalón

$$K_p = \lim_{s \to 0} G(s)H(s) = \frac{s+2}{s^3+12s^2} = \infty$$

$$e_{ss} = \frac{1}{1+K_p} = 0$$

Por otro método (TVF)

$$\cancel{s}\,\frac{1}{\cancel{s}}\,\frac{(s+2)}{s^3+12s^2+s+2} = 1$$

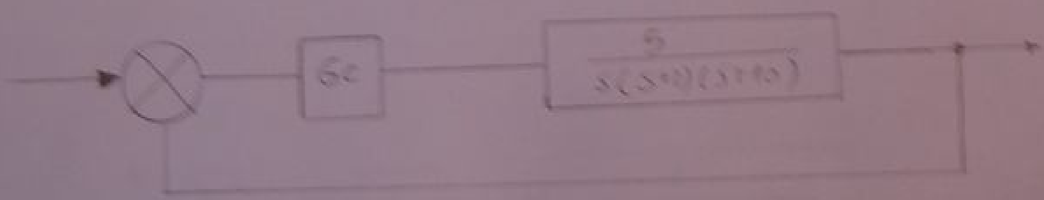
ess rampa:

$$K_v = \lim_{s \to 0} s\,G(s)H(s) = \frac{s+2}{s^2+12s} = \infty$$

$$e_{ss} = \frac{1}{K_v} = 0$$

$$\cancel{s}\,\frac{1}{s^{\cancel{2}}}\,\frac{(s+2)}{s^3+12s^2+s+2} = \frac{2}{0} = \infty$$ (No sirve el TVF en este caso para determinar el ess)

3) Compensación por Bode:



$MF = 60°$

$K_v = 2$

$$G(s)H(s) = \frac{K\,5}{s(s+2)(s+10)} \qquad K_v = \lim_{s \to 0} s\,G(s)H(s) = \frac{K\,5}{20} = 2$$

$$K = 8 \Rightarrow K\,G(s)H(s) = \frac{40}{s(s+2)(s+10)} \quad ; \quad K_T = \frac{40}{20} = 2$$

$|K_T|_{dB} = 20 \log 2 = 6$

$M_g = +4\,dB$

$MF = +34{,}5°$

$\varphi_c = -180 + 60° + 12° = -108°$ $\Rightarrow$ $\omega = 0,5$ rad/seg
$G = 12$ dB

$$20 \log \frac{1}{\beta} = G(\varphi_c) = -12 \text{ dB}$$

$$10^{-12/20} = \frac{1}{\beta} \Rightarrow \beta = \frac{1}{10^{-12/20}} = 4$$

$$G_c(s) = K_c \frac{(s + 1/T)}{(s + 1/\beta T)} = \beta K_c \frac{(Ts + 1)}{(\beta T s + 1)}$$

$$\beta K_c = K = 8 \Rightarrow K_c = \frac{K}{\beta} = 2 = \underline{2 = K_c}$$

$$\frac{\omega_1}{10} < z_c < \frac{\omega_1}{2}$$

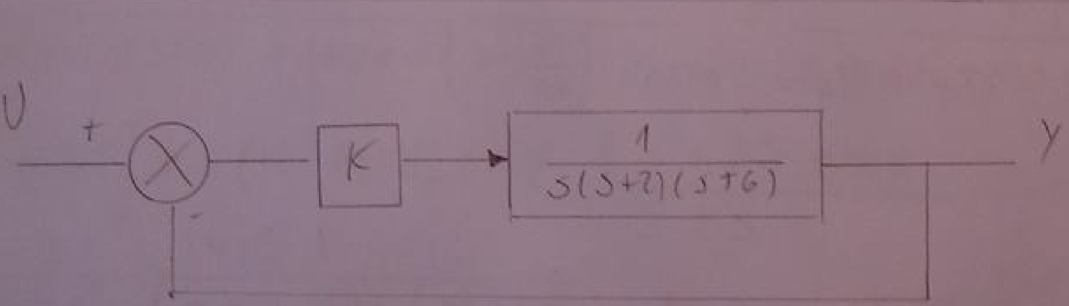
$$z_c = 0,15 \text{ rad/seg} = \frac{1}{T} \Rightarrow p_c = \frac{z_c}{\beta} = 0,0375 \text{ rad/seg}$$

$$\Rightarrow G_c(s) = 2 \frac{(s + 0,15)}{(s + 0,0375)}$$

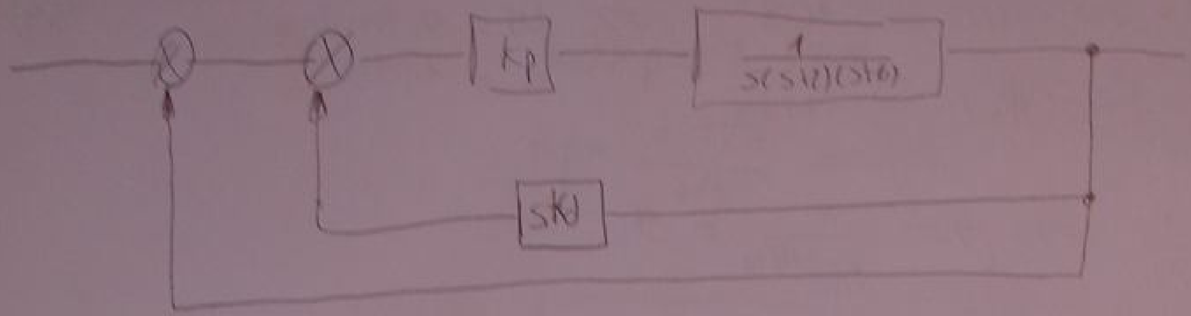
$$G_c G H(s) = \frac{2(s+0,15)}{(s+0,0375)} \cdot \frac{5}{s(s+2)(s+10)}$$

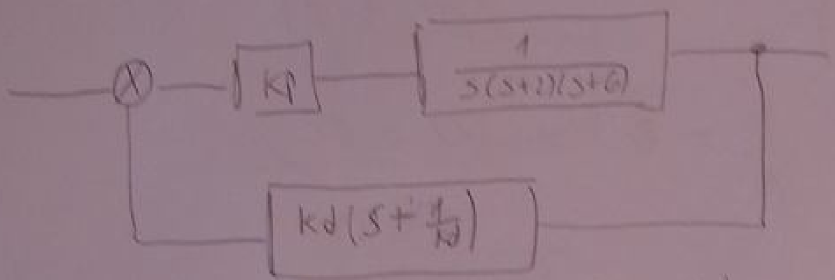
$$= \frac{10s + 1,5}{s^4 + 12,0375 s^3 + 20,45 s^2 + 0,75 s}$$

1)

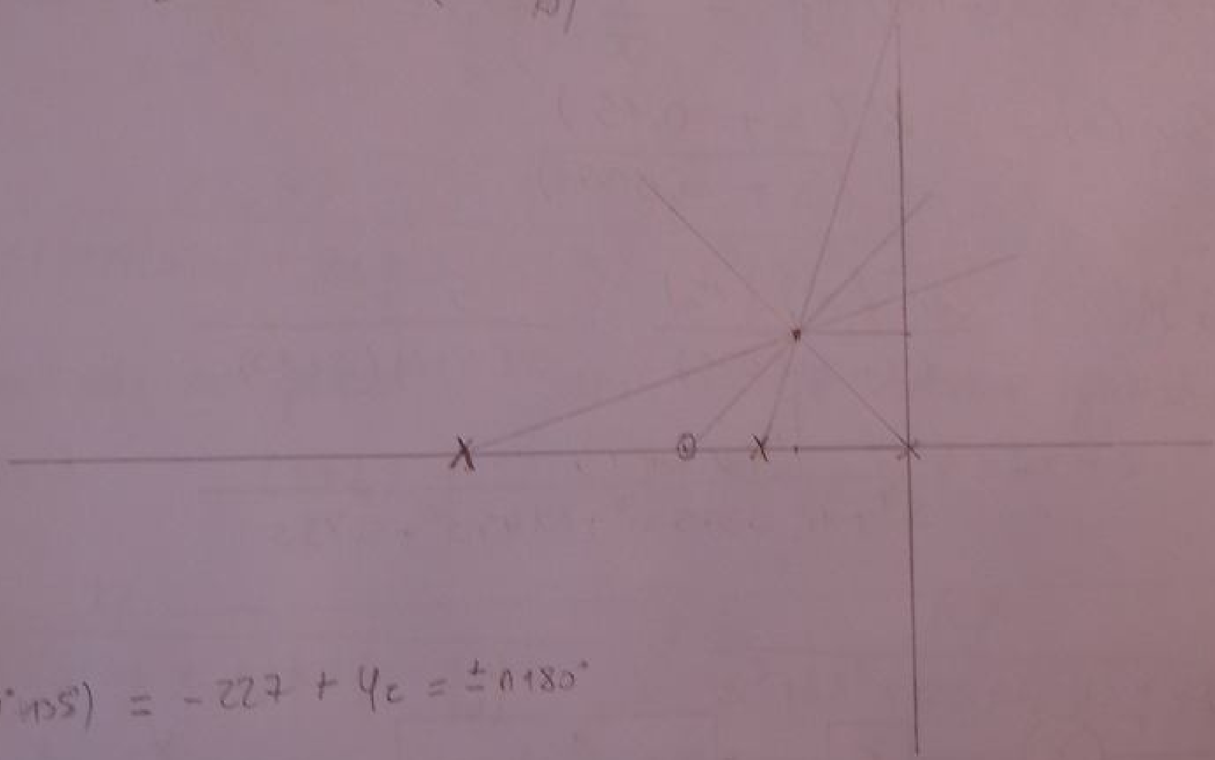


Mediante compensación en la realimentación lograr que el sistema de lazo cerrado presente dos polos complejos conjugados dominantes en $s = -1,5 \pm j1,5$



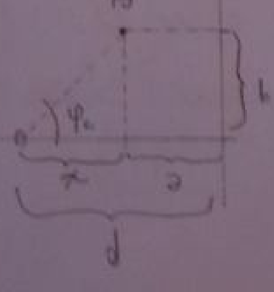


$s k_d + 1 = k_d\left(s + \frac{1}{k_d}\right)$.



$-(20^\circ + 70^\circ + 135^\circ) = -227 + \varphi_c = \pm n 180^\circ$

$\varphi_c = 45$.



$tg\,\varphi_c = \frac{b}{x} \Rightarrow x = -\left(\frac{b}{tg\,\varphi_c} + a\right)$

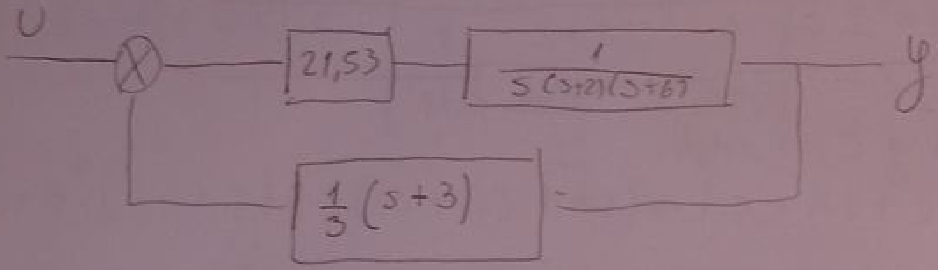
$z_c = -3 = \frac{1}{k_d}$

$k_d = 0,33$

$k = \frac{4,7 \cdot 1,6 \cdot 2,1}{2,2} = 7,178$

$$\frac{kd\,kp\,s}{s(s+2)(s+6)} \;;\quad kd\,kp = 7{,}178 \Rightarrow kp = \frac{7{,}178}{kd}$$

$$kp = \frac{7{,}178}{1/3} = 21{,}534$$



$$FTLC = \frac{\dfrac{21{,}53}{s(s+2)(s+6)}}{1 + \dfrac{21{,}53}{s(s+2)(s+6)} \cdot \dfrac{1}{3}(s+3)}$$

$$= \frac{21{,}53}{s(s+2)(s+6) + \dfrac{21{,}53}{3}(s+3)} = \frac{21{,}53}{s^3+8s^2+19{,}17s+21{,}53}$$

otro metodo mas preciso, seria el de calcular por division de polinomio.

$$(s+1{,}5+j1{,}5)(s+1{,}5-j1{,}5) = s^2+3s+4{,}5$$

$$\frac{kp}{s(s+2)(s+6) + kp\,kd\left(s+\frac{1}{kd}\right)} = \frac{kp}{s^3+8s^2+12s+kp\,kd\,s+kp}$$

$$\begin{array}{l|l} s^3+8s^2+s(12+kp\,kd)+kp & s^2+3s+4{,}5 \\ \cline{2-2} -s^3-3s^2-4{,}5s & s+5 \\ \hline \quad 5s^2+s(7{,}5+kp\,kd)+kp & \\ \quad -5s^2-15s-22{,}5 & \\ \hline \quad s\underbrace{(kp\,kd-7{,}5)}_{=0} \underbrace{-22{,}5+kp}_{=0} & \end{array}$$

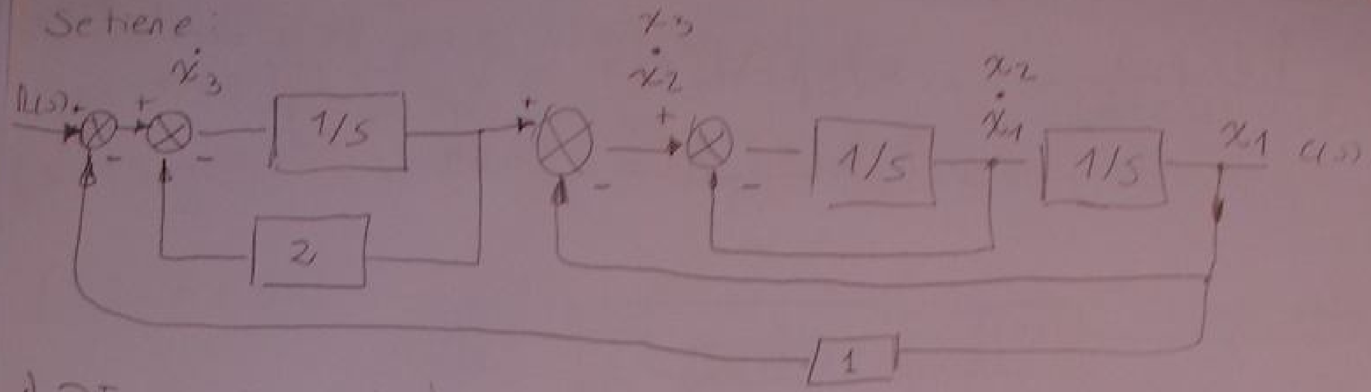
$$\underline{kp = 22{,}5}$$

$$kp\,kd = 7{,}5$$

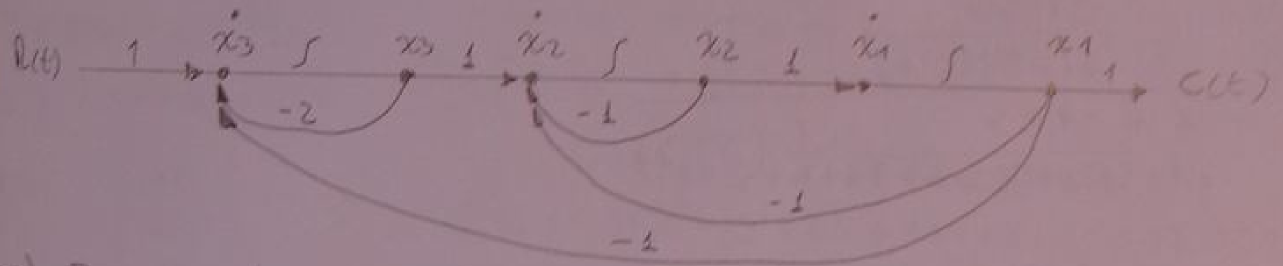
$$kd = \frac{7{,}5}{kp} = \frac{1}{3} = kd$$

$$\Rightarrow FTLC = \frac{22{,}5}{(s+5)(s+1{,}5+j1{,}5)(s+1{,}5-j1{,}5)} = \frac{22{,}5}{s^3+8s^2+19{,}5s+22{,}5}$$

Se tiene:



1) DF con R(t) entrada y C(t) salida.



2) Representar en VE y ecuaciones dinámicas:

$$\left.\begin{aligned}
\dot{x}_3 &= -2x_3 - x_1 + R(t) \\
\dot{x}_2 &= -x_2 - x_1 + x_3 \\
\dot{x}_1 &= x_2 \\
x_1 &= C(t)
\end{aligned}\right\}$$

$$\left.\begin{aligned}
\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{vmatrix} &= \begin{vmatrix} 0 & 1 & 0 \\ -1 & -1 & 1 \\ -1 & 0 & -2 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} R(t) \\
y(t) &= \begin{vmatrix} 1 & 0 & 0 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix}
\end{aligned}\right\}$$

3)

$$M = |B \; AB \; A^2B| = \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & -3 \\ 1 & -2 & 4 \end{vmatrix} \quad \left.\begin{matrix} r = 3 \\ \det \neq 0 \end{matrix}\right\} \text{ Totalmente controlable.}$$

$$O = |C^T \; A^T C^T \; A^{2T} C^T| = \begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{vmatrix} \quad \left.\begin{matrix} r = 3 \\ \det \neq 0 \end{matrix}\right\} \text{ Totalmente observable}$$

4) Realimentar el VE para q' queden los polos en:

$s_{1,2} = -1,5 \pm j1,5 \ ; s_3 = -12$

$$(s+1,5+j1,5)(s+1,5-j1,5)(s+12) = s^3 + 15s^2 + 40,5s + 54$$

$$(sI - A + BK) = \begin{vmatrix} s & -1 & 0 \\ 1 & s+1 & -1 \\ 1+k_1 & k_2 & s+2+k_3 \end{vmatrix} = s^3 + 3s^2 + k_3 s^2 + 3s + k_3 s + k_2 s + 3 + k_3 + k_1$$

$$s^3 + s^2(3+k_3) + s(3+k_3+k_2) + k_3 + k_1 + 3$$

$3 + k_3 = 15$

$3 + k_3 + k_2 = 40,5$

$3 + k_3 + k_1 = 54$

$$\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} \begin{vmatrix} k_1 \\ k_2 \\ k_3 \end{vmatrix} + \begin{vmatrix} 3 \\ 3 \\ 3 \end{vmatrix} = \begin{vmatrix} 15 \\ 40,5 \\ 54 \end{vmatrix}$$

$$\begin{vmatrix} k_1 \\ k_2 \\ k_3 \end{vmatrix} = \begin{vmatrix} 39 \\ 25,5 \\ 12 \end{vmatrix}$$

$s(s+1)(s+2+k_3) + 1 + k_1 + k_2 s + s + 2 + k_3$

$s^2 k_3 + k_3 s + s^3 + 3s^2 + 2s + 1k_1 + k_2 s + s + 2 + k_3$

$$\dot{x} = Ax - BKx = \overbrace{(A - BK)}^{AA} x$$

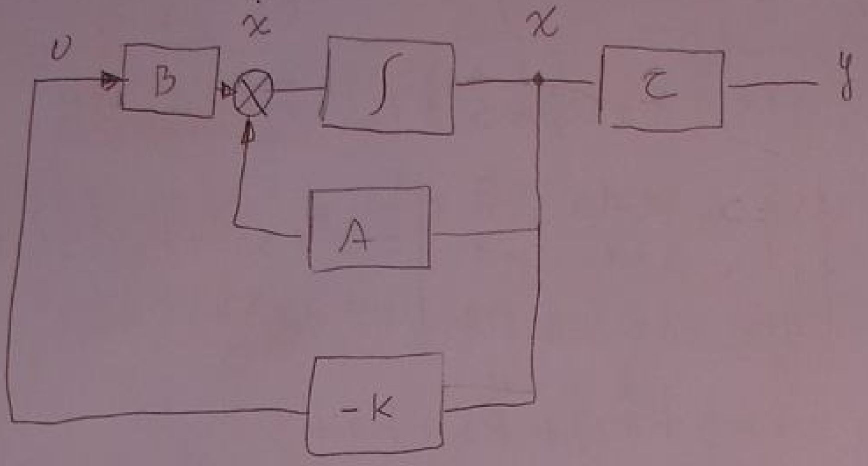
$$AA = \begin{vmatrix} 0 & 1 & 0 \\ -1 & -1 & 1 \\ -48 & -33,5 & -6 \end{vmatrix}$$

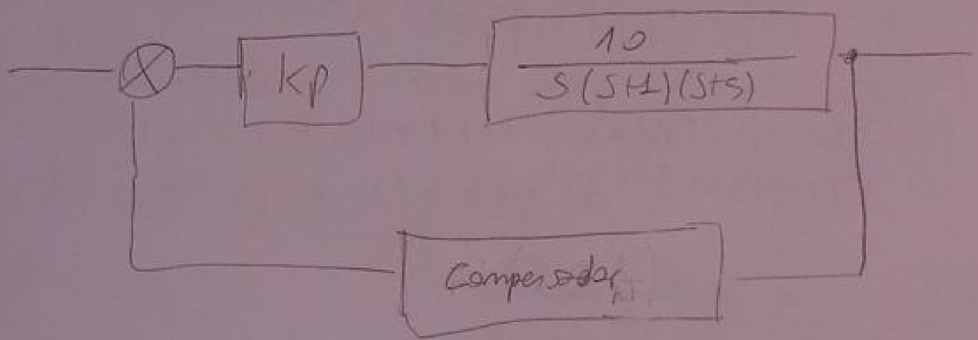
$$\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ -1 & -1 & 1 \\ -40 & -25,5 & -14 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} U(t)$$

$$y(t) = \begin{vmatrix} 1 & 0 & 0 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix}$$

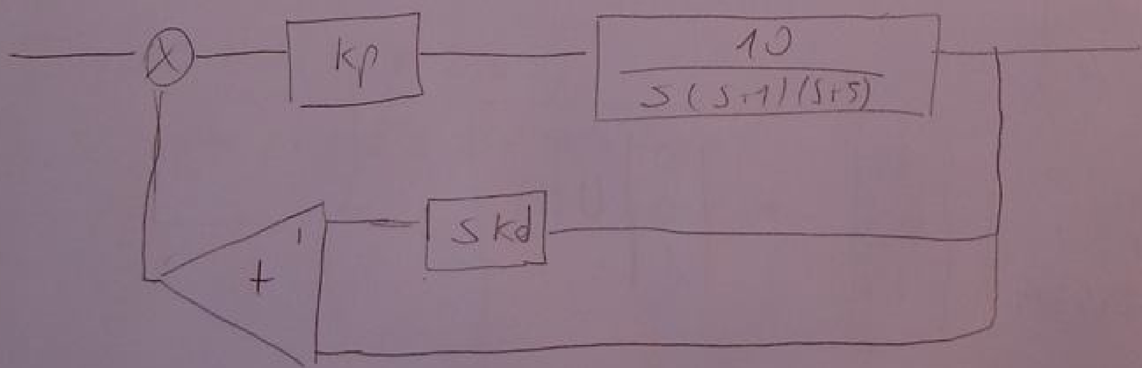
$$F = \frac{1}{s^3 + 15s^2 + 40,5 + 54}$$

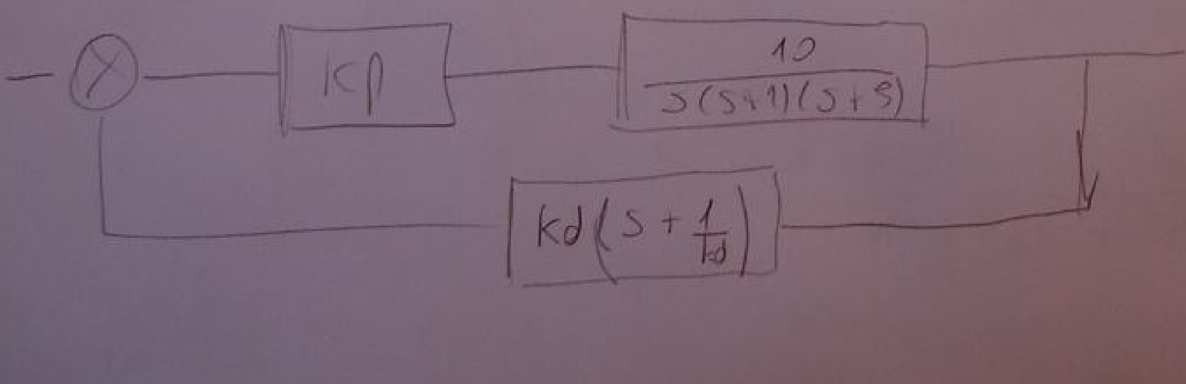
Como se reconstruye el sistema realimentado?





Dejarlo en $\xi = 0,5$ y $\omega_n = 1,4142 = \sqrt{2}$





$$FT = \frac{k_p\, 10}{s(s+1)(s+5) + \underline{10}\, k_p k_d \left(s + \frac{1}{k_d}\right)} = \frac{K_p\, 10}{s^3 + 6s^2 + (10 k_p k_d + 5)s + 10 k_p}$$

OJO! cuando sacarmos FT

$$s^2 + 2\xi w_n + w_n^2 = s^2 + 2\sqrt{2}\, 0{,}5 s + 2 = s^2 + \sqrt{2} s + 2$$

$$\begin{array}{l|l}
s^3 + 6s^2 + (10k_pk_d + 5)s + 10k_p & s^2 + \sqrt{2}s + 2 \\
\hline
-s^3 - \sqrt{2}s^2 - 2s & s + 6 - \sqrt{2} \\
\hline
0 \quad s^2(6-\sqrt{2}) + s(10k_pk_d + 3) + 10k_p & \\
-s^2(6-\sqrt{2}) - s\sqrt{2}(6-\sqrt{2}) - 2(6-\sqrt{2}) & \\
\hline
0 \quad s(10k_pk_d + 3 - \sqrt{2}6 + 2) + 10k_p - 12 + 2\sqrt{2} & \\
\quad s\underbrace{(10k_pk_d + 5 - \sqrt{2}6)}_{=0} \; \underbrace{+10k_p - 12 + 2\sqrt{2}}_{=0} &
\end{array}$$

$$10K_p = 12 - 2\sqrt{2} \Rightarrow \underline{k_p = 0{,}917}\,/$$

$$10k_pk_d = \sqrt{2}6 - 5 \Rightarrow k_d = \frac{\sqrt{2}6 - 5}{10\,(0{,}917)} = \underline{0{,}38 = k_d}\,/$$

$$F_T = \frac{9{,}17}{s^3 + 6s^2 + 8{,}48s + 9{,}17}$$ cumple con $e_{ss} = 0$ para escalón

$$G_c = 0{,}38\,(s + 2{,}63)$$

$\sqrt{2 - 0{,}5} = \sqrt{\frac{3}{2}}$

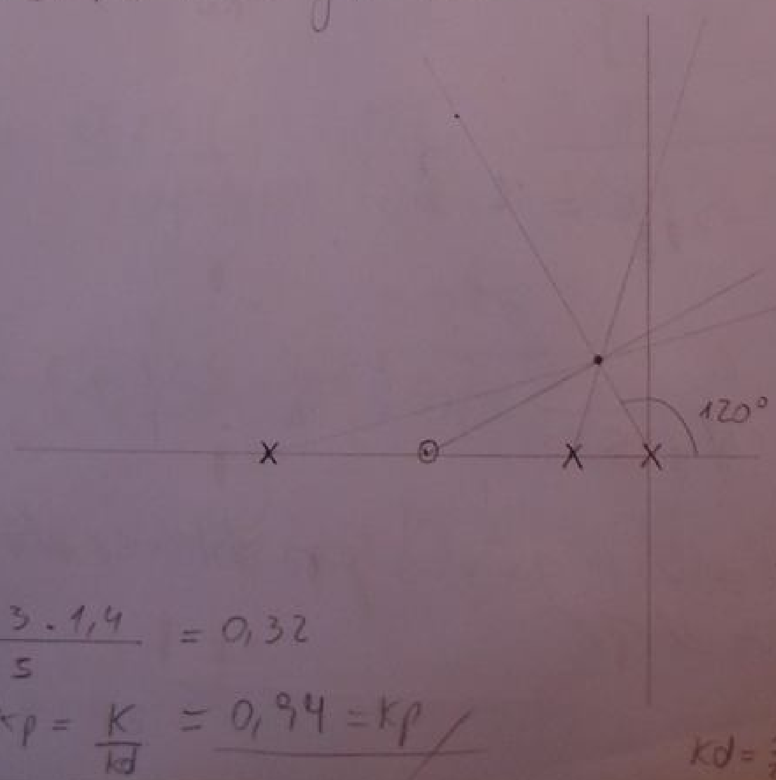
$w_d = \sqrt{w_n^2 - \sigma^2}$

$\sigma = w_n \cdot \xi = \frac{\sqrt{2}}{2}$

$w_n = \sqrt{2}\,;\ \xi = 0{,}5$

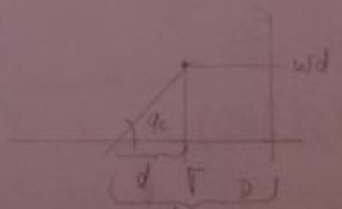
$\beta = \arccos \xi = 60°$

Si no, por el método gráfico:



$-(120 + 73 + 16) = -209$

$\varphi_c = 29°$



$tg\,\varphi_c = \frac{w_d}{d}\ ;\ d = \frac{w_d}{tg\,\varphi_c}$

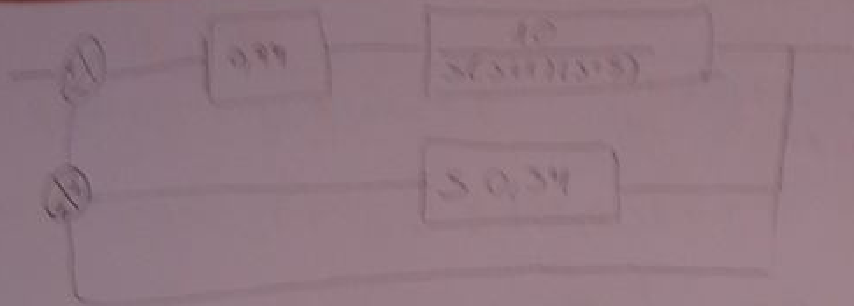
$D = \frac{w_d}{tg\,\varphi_c} + \sigma$

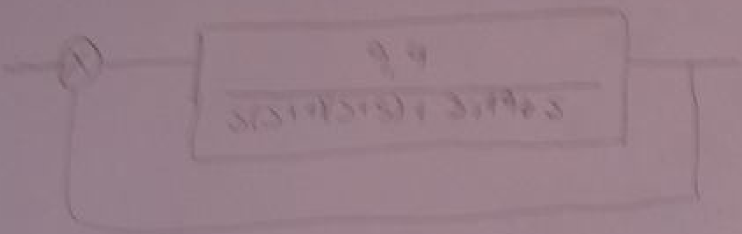
$= \frac{\sqrt{3/2}}{tg\,29°} + \frac{\sqrt{2}}{2} = 2{,}91$

$$K = \frac{4{,}5 \cdot 1{,}3 \cdot 1{,}4}{10 \cdot 2{,}5} = 0{,}32$$

$$k = k_p k_d \Rightarrow k_p = \frac{K}{k_d} = \underline{0{,}94 = k_p}\,/$$

$k_d = \frac{1}{2{,}91} = 0{,}34$



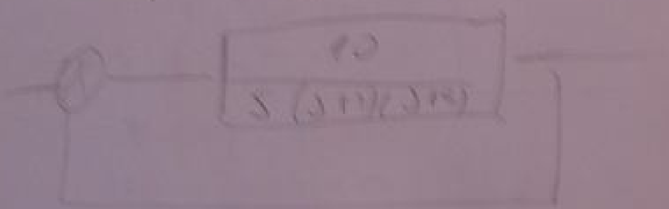


$$F_T = \frac{9,4}{s(s+1)(s+5)+3,196s + 9,4}$$

$$F_T = \frac{9,4}{s^3+6s^2+8,196s+9,4}$$

compensar para $Mf = 50°$ y $e_{ss} = 0,3$ pdo rampa

G



$e_{ss} = \frac{1}{K_V} = 3,\overline{33}$

Ge

$\frac{10 \cdot K}{5} = 3,\overline{33} \Rightarrow K = 1,66$

$K\,G(s)\,H(s) = \frac{16,6}{s(s+1)(s+5)}$; $K_{dB} = 20 \log 3,33 = 10,45$

Si U comprobamos si aumentando la ganancia para obtener el ess pedido cumplimos también con el Mf

$Mf = 9°$
$Mg = 6\,dB$

Compensamos por atraso:

$$G_c = \frac{\beta K_c (Ts + 1)}{(\beta T s + 1)} = \frac{K_c(s + 1/T)}{(s + 1/\beta T)} \quad ; \beta > 1$$

$$\varphi_c = -180 + Mf\ \text{pedido} + 8° = -180° + 50° + 8° = -122° \Rightarrow \omega_n = 0,5\ rad/seg$$

$$-20 \log \frac{1}{\beta} = 16,5\,dB \qquad \frac{1}{\beta} = 10^{-\frac{16,5}{20}} \Rightarrow \beta = \frac{1}{10^{-\frac{16,5}{20}}} = 6,68$$

$\beta = 6,68$.

$$\frac{\omega_n}{10} < z_c < \frac{\omega_n}{2} \qquad 0,05 < z_c < 0,25 \Rightarrow z_c = 0,15$$

$$\Rightarrow P_c = \frac{z_c}{\beta} = 0,022$$

$$K = \beta K_c \Rightarrow K_c = \frac{K}{\beta} = \frac{1,66}{6,68} = 0,25$$

$$G_c = 0,25 \frac{(s + 0,15)}{(s + 0,022)}$$

$$G_cGM(s) = \frac{0,25(s + 0,15)}{(s + 0,022)} \cdot \frac{10}{s(s+1)(s+5)}$$

$$= \frac{2,5s + 0,375}{s^4 + 6,022 s^3 + 5,132 s^2 + 0,11 s + 0} \Rightarrow \text{Nos da } Mf = 44,4°$$

Si usamos $z_c = 0,06 \Rightarrow P_c = 0,01$. $\quad 0,25\frac{(s + 0,06)}{(s + 0,01)}$

$$= \frac{2,5s + 0,15}{s^4 + 6,01 s^3 + 5,06 s^2 + 0,05 s} \Rightarrow Mf = 54,4°$$

siempre elegir z_c lo mas bajo posible

Del Ogata A-96 compensar por adelanto sin especificación de kv

$M_f = 50°$

$M_G = 10dB$

$G(s) = \frac{1}{s^2(s+5)}$ $K_{tot} = 20 \log \frac{1}{5} = -14 dB$

Del bode

$M_f = -9°$

$M_G = -20dB$

$\varphi_c = M_{f\,req} - M_{f\,sist} + 5° = 50° + 9° + 5$

$\varphi_c = 64°$

$\operatorname{sen} \varphi_c = \frac{1-\alpha}{1+\alpha}$; $(1+\alpha) \operatorname{sen}\varphi_c = 1-\alpha$

$\operatorname{sen}\varphi_c + \alpha \operatorname{sen}\varphi_c - 1 + \alpha = 0$

$\alpha(\operatorname{sen}\varphi_c + 1) + \operatorname{sen}\varphi_c - 1 = 0$

$\alpha = \frac{1-\operatorname{sen}\varphi_c}{1+\operatorname{sen}\varphi_c} = 0{,}0533$

$$\boxed{\begin{aligned} G_c &= k_c \frac{(s+1/T)}{(s+1/\alpha T)} \\ G_c &= \alpha k_c \frac{(Ts+1)}{(\alpha Ts+1)} \end{aligned}}$$

$-20 \log \frac{1}{\sqrt{\alpha}} = -12{,}73\ dB \Rightarrow W_n = 0{,}9\ rad/seg$

$z_c = W_n \sqrt{\alpha} = 0{,}207\ rad/seg = \frac{1}{T}$, $P_c = \frac{z_c}{\alpha} = 3{,}89\ rad/seg$

$G_c = \frac{k_c(s+0{,}207)}{(s+3{,}89)}$

$G_c G(s) = k_c \frac{(s+0{,}207)}{(s+3{,}89)} \cdot \frac{1}{s^2(s+5)}$

$k_c \frac{(j0{,}9+0{,}207)}{(j0{,}9+3{,}89)} \cdot \frac{1}{-0{,}81(j0{,}9+5)} = \overbrace{0{,}0562}^{ABS} \cdot k_c = 1$

$\hookrightarrow ARG \Rightarrow -126 \Rightarrow M_{f\,W=0{,}9} = \underline{54°}$

$\Rightarrow k_c = \frac{1}{0{,}0562} = 17{,}8$ p/q a $W = 0{,}9 \Rightarrow G_c G(s) = 0dB$

$G_c G(s) = \frac{17{,}8s + 3{,}6846}{s^4 + 8{,}89s^3 + 19{,}45s^2 + 0s + 0}$

Cuando no se especifica kv y se pide compensación, la Kc debe ser tal que a la frecuencia W_n deseada, $|G_c G(s)| = 1 = 0dB$.

A 9-8) del Ogata: compensar por adelanto, si no funciona, probar de otra forma. (77)

$K_V = 20\ seg^{-1}$ $G(s) = \frac{10}{s(s+1)}$;

$M_f = 50°$

$M_g \geq 10 dB$

$K_V = \lim_{s \to 0} s\,G(s) = 10K = 20$; $K = 2$

$K\,G(s) = \frac{20}{s(s+1)}$ $K_{Tot} = 20 \log 20 = 26\ dB$

$M_f = 15{,}75°$ $\Rightarrow \varphi_c = 50° - 15{,}75 + 5° \approx 40°$

$\alpha = \frac{1 - \text{sen}\,\varphi_c}{1 + \text{sen}\,\varphi_c} = 0{,}217$

$-20 \log\left(\frac{1}{\sqrt{\alpha}}\right) = -6{,}62\ dB \Rightarrow \omega_n = 6{,}5\ rad/seg$

$z_c = \omega_n \sqrt{\alpha} = 3{,}02$, $P_c = \frac{z_c}{\alpha} = 13{,}95\ rad/seg$

$G_c = K_c \frac{(s + 3{,}02)}{(s + 13{,}95)} \Rightarrow K_c \alpha = K = 2 \Rightarrow K_c = \frac{K}{\alpha} = 9{,}21$

$G_c G(s) = \frac{92{,}1s + 278{,}142}{s^3 + 14{,}95 s^2 + 13{,}95 s + 0} \Rightarrow M_f = 48{,}3°$

Podemos calcular el Mf en este paso!
180 − Arg($G_cG(s)$, $\omega = \omega_n$)

otra forma de ver Kc: ABS $G_cG(s)|_{\omega = 6,5 = \omega_n}$

$K_c \left| \frac{(j6{,}5 + 3{,}02)}{(j6{,}5 + 13{,}95)} \cdot \frac{10}{j6{,}5\,(j6{,}5 + 1)} \right| = K_c\,\overbrace{0{,}108}$,

$M_f = 180 - 131{,}15 = 48{,}8°$

$\frac{1}{0{,}108} = K_c = 9{,}17$ p/ q' $|G_c G(s)|_{\omega = 6,5 = \omega_n} = 1 = 0\ dB$

$G_c G(s) = \frac{91{,}7s + 276{,}934}{s^3 + 14{,}95 s^2 + 13{,}95 s + 0} \Rightarrow M_f = 42{,}8$ (es muy poca diferencia) pero está bueno encararlo por este lado, es más conceptual.

A 8.9) Del grafo.

$$kv = 10\,seg^{-1}$$
$$Mf = 50^\circ$$
$$Mg \geq 10\,dB$$

Sea: $G(s) = \dfrac{K}{s(s+1)(s+4)}$

Se pide que la compensación sea tanto para la parte estática como para la parte dinámica.

$\Rightarrow$ Usamos un compensador de atraso adelanto.

$$Gc(s) = \underbrace{K_L}_{\text{Podemos tomarlo como 1 ya que K también varía.}} \underbrace{\frac{\left(s + \frac{1}{T_1}\right)}{\left(s + \frac{\beta}{T_1}\right)}}_{\substack{Lead \\ \beta = 1/\alpha}} \underbrace{\frac{\left(s + \frac{1}{T_2}\right)}{\left(s + \frac{1}{\beta T_2}\right)}}_{Lag} \qquad \beta > 1$$

Podemos tomarlo com 1 ya q' K también varía.

$$kv = \lim_{s \to 0} s\,K\,G(s) = \frac{K}{4} = 10 \Rightarrow K = 40 \quad ; K\,tot = 20\log\frac{40}{4} = 20\,dB$$

$Mf = -18^\circ \Rightarrow$ Inestable

$\omega_n = \angle G(180^\circ) = 2\,rad/seg = \omega_n \Rightarrow$ en esta ω_n deberán estar los 50° de Mf

Elegimos $\dfrac{1}{T_2} = \dfrac{\omega_n}{10} = 0{,}2\,rad/seg$

Para el polo $\dfrac{1}{\beta T_2}$ necesitamos conocer β. Lo calculamos con la expresión del máximo angulo del compensador por adelanto. reemplazando $\dfrac{1}{\alpha}$ por β

$$sen\,\varphi_c = \frac{1-\alpha}{1+\alpha} = \frac{1 - \frac{1}{\beta}}{1 + \frac{1}{\beta}} = \frac{\beta - 1}{\beta + 1} \quad ; \varphi_c = 50^\circ$$

$?/\beta = 10 \Rightarrow \varphi_c = 55^\circ \Rightarrow$ usamos este

$$\Rightarrow \frac{1}{\beta T_2} = 0{,}02\,rad/seg. \Rightarrow \frac{(s+0{,}2)}{(s+0{,}02)}$$

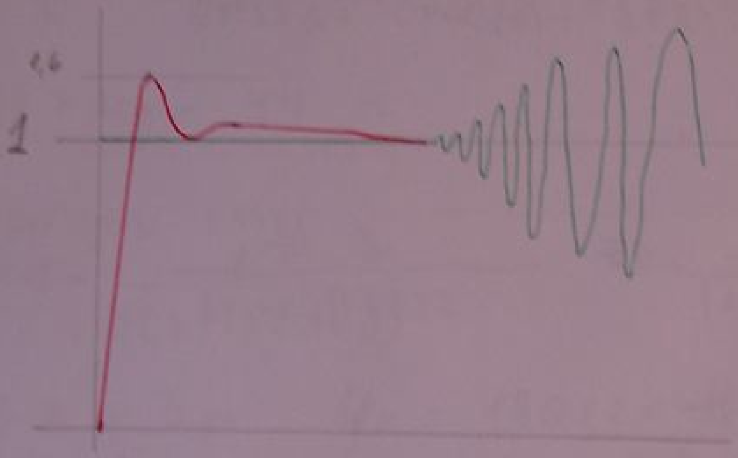
$\Rightarrow$ Para el cálculo de el compensador por adelanto, trazamos una recta con pendiente +20 dB q' va desde los -20 dB, pasa por el punto $-|G(j\omega n)|$ y llega hasta la línea de 0dB. Las corner frequencies son aquellas donde la recta corta la línea de -20 dB y la de 0dB.

$$\Rightarrow \frac{s+0{,}5}{s+5} \Rightarrow Gc\,G(s) = \frac{(s+0{,}5)}{(s+5)} \cdot \frac{(s+0{,}2)}{(s+0{,}02)} \cdot \frac{40}{s(s+1)(s+4)}$$

$$G_c\,G(s) = \frac{40s^2 + 28s + 4}{s^5 + 10{,}02s^4 + 29{,}2s^3 + 20{,}58s^2 + 0{,}4s + 0}$$

$$M_f = 59{,}2^\circ \; ; \; M_g = 14{,}4\ dB.$$

$$FT = \frac{40s^2 + 28s + 4}{s^5 + 10{,}02s^4 + 29{,}2s^3 + 60{,}58s^2 + 28{,}4s + 4}$$



El sistema sin compensar es inestable mientras que el compensado muestra la respuesta deseada.

Ejemplo 9.-3 del ogata.

Sea: $G(s) = \dfrac{K}{s(s+1)(s+2)}$

$$\left\{ \begin{array}{l} M_f = 50^\circ \\ K_V = 10\ seg^{-1} \\ M_g \geq 10 dB \end{array} \right\}$$ Compensar con atraso-adelanto.

$$G_c(s) = K_c \frac{(s + 1/T_1)}{(s + \beta/T_1)} \; \frac{(s + 1/T_2)}{(s + 1/\beta T_2)} \; ; \; \beta > 1.$$ Tomamos $K_c = 1$ y variamos con K.

$$K_V = \lim_{s \to 0} K\,s\,G(s) = \frac{K}{2} = 10 \Rightarrow K = 20$$

$$G(s) = \frac{20}{s(s+1)(s+2)} \; ; \; K_{Tot} = 20 \log 10 = 20\ dB$$

$$\omega_n = \angle G(-180) = 1{,}4\ rad/seg. \Rightarrow \frac{1}{T_2} = \frac{\omega_n}{10} = 0{,}14\ rad/seg = \frac{1}{T_2}$$

En ω_n es donde vamos a dar el nuevo $M_f = 50^\circ$

$$\operatorname{sen} \varphi_c = \frac{1-\alpha}{1+\alpha} = \frac{1 - 1/\beta}{1 + 1/\beta} = \frac{\beta - 1}{\beta + 1} \Rightarrow P/\beta = 10 \Rightarrow \varphi_c = 55^\circ$$

$$\Rightarrow 1/\beta T_2 = 0{,}014\ rad/seg \Rightarrow \frac{(s + 0{,}14)}{(s + 0{,}014)}$$

El comp. x adelanto => $\frac{(s+0{,}8)}{(s+8)}$
usando el método de la recta explicado en pg 74 al dorso

$$G_c G(s) = \frac{(s+0{,}14)}{(s+0{,}014)} \frac{(s+0{,}8)}{(s+8)} \frac{20}{s(s+1)(s+2)}$$

$$= \frac{20s^2 + 18{,}8s + 2{,}24}{s^5 + 11{,}014 s^4 + 26{,}154 s^3 + 16{,}364 s^2 + 0{,}224 s}$$

$M_f = 55{,}2°$; $M_g = 18{,}5\,dB$

Ejemplo 9-2 del ogata:

A. $G(s) = \frac{1\ K}{s(s+1)(0{,}5s+1)} = \frac{2\ K}{s(s+1)(s+2)}$

$\left.\begin{array}{l} k_v = 5\,sg^{-1} \\ M_f = 40° \\ M_g \geq 10dB \end{array}\right\}$ Compensar por otro Ej

10 - 45

$k_v = \lim_{s \to 0} s\,G(s) = \frac{\cancel{2}K}{\cancel{2}} = 5 \Rightarrow K = 5$

$G(s) = \frac{10}{(s+1)(s+2)s} \Rightarrow$ kte tot $= 20 \log 5 = 14\,dB$

$M_f = -18°$; $M_g = -9dB \Rightarrow$ Inestable.

$\varphi_c = -180 + M_f$ pedido $+ 8° = -132° \Rightarrow \omega_n = 0{,}4$ rad/seg.

$-20 \log 1/\beta = -22dB \qquad \left(10^{-\frac{22}{20}}\right)^{-1} = \beta = 12{,}58$

$\Rightarrow G_c = k_c \frac{(s+1/T)}{(s+1/\beta T)}$ $\beta > 1$;

$z_c \Rightarrow \frac{\omega_n}{10} < z_c < \frac{\omega_n}{2}$; $\Rightarrow z_c = 0{,}1$ rad/seg $= 1/T$

$P_c = \frac{z_c}{\beta} = 0{,}008$,

$k_c \beta = K \Rightarrow k_c = \frac{K}{\beta} = \frac{5}{12{,}58} = 0{,}4$

Ejemplo 17-7 Ogata (p. 449)

$$G_c G(s) = 0{,}4 \frac{(s+0{,}1)}{(s+0{,}008)} \cdot \frac{2}{(s+1)(s+2)s}$$

$$= \frac{0{,}8s + 0{,}08}{s^4 + 3{,}008s^3 + 2{,}024s^2 + 0{,}016s + 0}$$

$M_f = 44{,}9°$

$$\text{Arg}\left\{ 0{,}4 \frac{(j0{,}4+0{,}1)}{(j0{,}4+0{,}008)} \cdot \frac{2}{(j0{,}4+1)(j0{,}4+2)j0{,}4} \right\} = -136$$

$180 + \text{Arg} = 44$

A-7-8 Del Ogata (446)

$$G(s) = \frac{10}{s(s+2)(s+8)}$$

$PD = -2 \pm j2\sqrt{3}$; $K_V = 80 s^{-1}$

$N = p - z = 3$; $\varphi_K = 180 \frac{(2K-1)}{p-z}$ $\begin{cases} \varphi_1 = -60 \\ \varphi_2 = 60 \\ \varphi_3 = 180 \end{cases}$

$$\sigma_c = \frac{\sum Re\{P\} - \sum Re\{Z\}}{p-z} = \frac{10}{3} = \sigma_c$$

$G(s)H(s) + 1 = 0$

$K\,10 = K_T \Rightarrow s^3 + 10s^2 + 16s + K_T = 0$

$\frac{\partial K_T}{\partial s} = -(s^3 + 10s^2 + 16s) = 0 \Rightarrow 3s^2 + 20s + 16 = 0$

$\begin{cases} Pb_1 = -5{,}737 \times \Rightarrow \text{No} \in LR \\ Pb_2 = -0{,}929 \end{cases}$

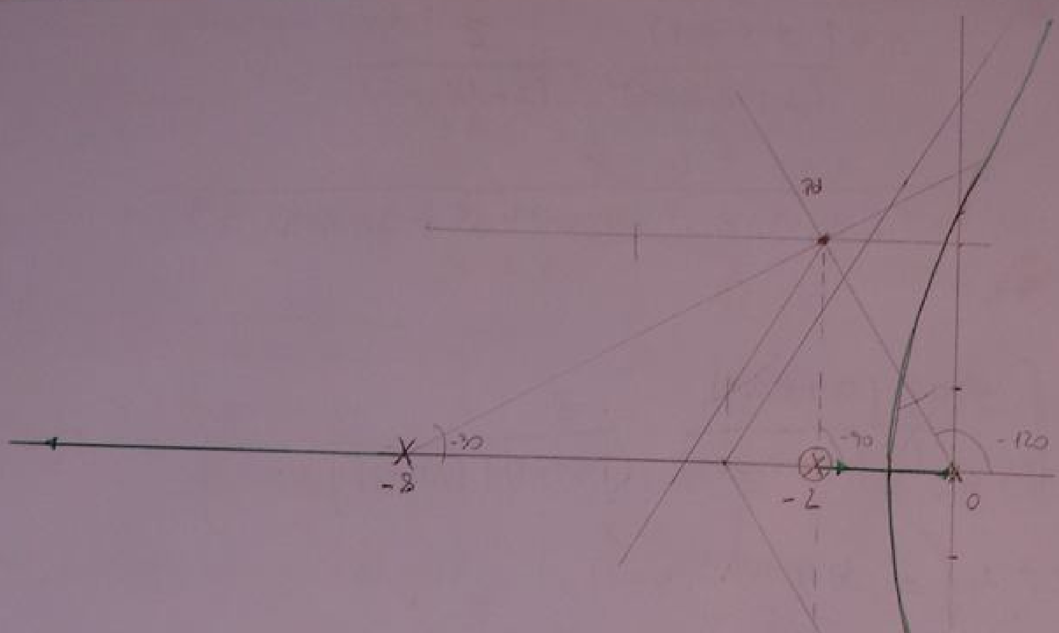
s^3	1	16
s^2	10	K_T
s^1	$\frac{160 - K_T}{10}$	
s^0	K_T	

$K_T < 160$

$\Rightarrow 0 < K < 16$

$10s^2 + 160 = \pm j4$

$-2 \pm j2\sqrt{3}$



$-(120+90+30) = -240 + \varphi_c = 180 \Rightarrow \varphi_c = 60°$

$$\left.\begin{array}{l} P_c = -8 \\ Z_c = -2 \end{array}\right\}$$

$$\Rightarrow K_c = \frac{7^2 \cdot 3{,}45 \cdot 4}{10 \cdot 3{,}45} = 19{,}6$$

196

$$G_c G(s) = 19{,}6 \frac{(s+2)}{(s+8)} \cdot \frac{10}{s(s+2)(s+8)} = \frac{196}{s(s+8)^2} = \frac{196}{s^3+16s^2+64s}$$

$$K_V = \lim_{s \to 0} s K G(s) = \frac{196}{8^2} K = 80 \; ; \; K = 26{,}12$$

$$\boxed{FTLC = \frac{196}{(s+12)(s+2+j2\sqrt{3})(s+2-j2\sqrt{3})}}$$

$K = K_c/\beta$; $K_c = 1 \Rightarrow \beta = 26{,}12$

$$G_c = K_c \frac{(s+1/T)}{(s+1/\beta T)} \; ; \; W_n = 4 \text{ rad/seg.}$$

$$1/T = \frac{W_n}{10} < Z_c < \frac{W}{2} \Rightarrow Z_c = 0{,}5$$

$$P_c = \frac{Z_c}{\beta} = 0{,}019$$

$$\frac{(s+0{,}5)}{(s+0{,}019)} \cdot \frac{196}{s(s+8)^2} =$$

$$\times \quad \angle \frac{s_1+0{,}5}{s_1+0{,}019} = -6{,}34°$$

$s_1 = -2 + 2\sqrt{3}$

No sirve.

$$FT = \frac{(s+0{,}5)\,196}{(s+0{,}019)\,s(s+8)^2 + (s+0{,}5)\,196} = \frac{196s + 98}{s^4 + 16{,}019s^3 + 64{,}304s^2 + 197{,}216s + 98}$$

[illegible]

$$\frac{K_c \cdot 10}{s(s+8)^2 + K_c 10} = \frac{10 K_c}{s^3 + 16s^2 + 64s + 10K_c}$$

$$
\begin{array}{l|l}
s^3 + 16s^2 + 64s + 10K_c & s^2 + 4s + 16 \\
\hline
-s^3 - 4s^2 - 16s & s + 12 \\
0 \quad 12s^2 + 48s + 10K_c & \\
\quad -12s^2 - 48s - 196 & \\
\hline
0 \quad 0 \quad 10K_c - 196 = 0 & \\
\end{array}
$$

$$10K_c - 196 = 0 \Rightarrow K_c = \frac{196}{10} = 19{,}6$$

$$z_c = 0{,}2 \text{ rad/seg} \Rightarrow p_c = \frac{z_c}{\beta} = 0{,}0076$$

$$\left.\angle \frac{s_1 + 0{,}2}{s_1 + 0{,}0076}\right|_{s_1 = -2 + 2\sqrt{3}} = -2{,}44°$$

El otro compensador por atraso aportaba $-6°$ lo cual modifica notablemente el lugar de raíces de FTLC. (los polos dom. quedaban a $-1{,}73 \pm 3{,}3$)

$$FTLC = \frac{(s+0{,}2)\,196}{(s+0{,}0076)\,s\,(s+2)^2 + (s+0{,}2)\,196} = \frac{196s + 39{,}2}{s^4 + 16{,}0076s^3 + 64{,}1216s^2 + 196{,}486s + 39{,}2}$$

con polos dominantes en: $-1{,}89 \pm 3{,}42$

se pedía: $-2 \pm 3{,}46$ → pequeño error

A-7-9) del Ogata p. 449

$K_v = 50 \text{ seg}^{-1}$

$\zeta = 0{,}5$

(El cero del comp. de adelanto debe estar en $s = -1$)

Diseñar con atraso adelanto.

$$G = \frac{1}{s(s+1)(s+5)}, \quad K_v = 50 \Rightarrow \frac{K}{5} = 50 \Rightarrow K = 250$$

$$N = p - z = 3, \quad \varphi_k = \frac{180(2k-1)}{p-z}, \quad \begin{cases} \varphi_0 = -60 \\ \varphi_1 = 60 \\ \varphi_2 = 180 \end{cases}$$

$$\sigma_c = \frac{\sum Re\{p\} - \sum Re\{z_c\}}{p-z} = \frac{6}{3} = 2 = \sigma_c$$

$G(s)H(s)+1=0$

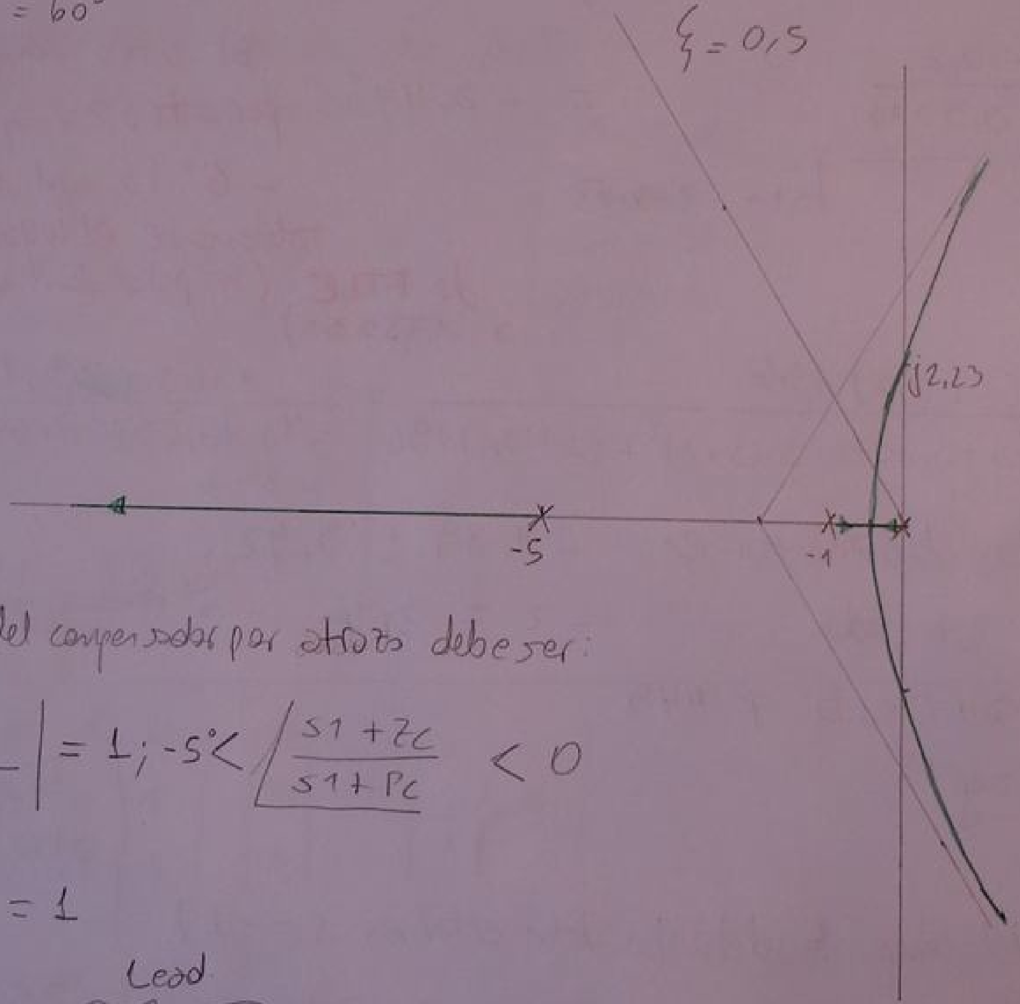
$s^3+6s^2+5s+K=0$

$\frac{\partial k}{\partial s}=-(3s^2+12s+5)=0 \quad \begin{cases} Pb_1=-3,52 \times \\ Pb_2=-0,472 \end{cases}$

$$\begin{array}{lll} s^3 & 1 & 5 \\ s^2 & 6 & k \\ s^1 & \frac{30-k}{6} & \\ s^0 & k & \end{array}$$

$, \quad 0<k<30, \quad 6s^2+30=\{\pm j2,23$

$A\cos\xi=\beta=60°$



El aporte del compensador por atraso debe ser:

$\left|\frac{s_1+z_c}{s_1+p_c}\right|=1; \; -5°< \angle\frac{s_1+z_c}{s_1+p_c}<0$

$\left|\frac{s_1+1}{s_1+\frac{1}{\beta}}\right|=1$

$\Rightarrow G_c(s_1)G(s_1)=\overbrace{\frac{\cancel{(s_1+1)}}{(s_1+\beta)}}^{\text{Lead}}\frac{1\ k_c}{\cancel{(s_1+1)}(s_1+5)s}=\frac{k_c}{(s_1+\beta)(s_1+5)s_1}$

$|G_c(s_1)G(s_1)|=1 \quad ; \quad \angle G_c(s_1)G(s_1)=180(2k+1)$

$Si\ \beta=60° \Rightarrow s_1=-x+j\sqrt{3}x \qquad \tan^{-1}\left(\frac{x\sqrt{3}}{x}\right)=60°$

$\Rightarrow G_c(s_1)G(s_1)=\left|\frac{k_c}{(-x+j\sqrt{3}x+\beta)(-x+j\sqrt{3}+5)(-x+j\sqrt{3})}\right|=1 \ldots$

No viene al caso...

Ejemplo 12-7 ogata (p. 845)

$$F(s) = \frac{1}{s(s+1)(s+2)}$$

Polos en: $-2 \pm j2\sqrt{3}$; -10

ess para escalón nulo.

$$F(s) = \frac{1}{s^3+3s^2+2s}$$

$a_1 = 3$	$b_0 = 0$
$a_2 = 2$	$b_1 = 0$
$a_3 = 0$	$b_2 = 0$
	$b_3 = 1$

$x_1 = y - \beta_0 U$

$x_2 = \dot{y} - \beta_0 \dot{U} - \beta_1 U = \dot{x}_1 - \beta_1 U$

$x_3 = \ddot{y} - \beta_0 \ddot{U} - \beta_1 \dot{U} - \beta_2 U = \dot{x}_2 - \beta_2 U$

$\dot{x}_3 = -a_3 x_1 - a_2 x_2 - a_1 x_1 + \beta_3 U$

$\beta_0 = b_0$

$\beta_1 = b_1 - a_1\beta_0$

$\beta_2 = b_2 - a_1\beta_1 - a_2\beta_0$

$\beta_3 = b_3 - a_1\beta_2 - a_2\beta_1 - a_3\beta_0$

$\beta_0 = 0$

$\beta_1 = 0$

$\beta_2 = 0$

$\beta_3 = 1$

$$\left.\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= -2x_2 - 3x_1 + U \\
x_1 &= y
\end{aligned}\right\}$$

$$\left.\begin{aligned}
\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{vmatrix} &= \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} U \\
y &= \begin{vmatrix} 1 & 0 & 0 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix}
\end{aligned}\right\}$$

$sx = Ax + BU$; $U = -Kx$

$sx = Ax - BKx$

$(sI - A + BK)x$

$$(sI - A + BK) = \begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ k_1 & 2+k_2 & s+3+k_3 \end{vmatrix}$$ 79

$$|(sI - A + BK)| = k_1 + sk_2 + s^2 k_3 + s^3 + 3s^2 + 2s$$
$$= s^3 + s^2(k_3+3) + s(k_2+2) + k_1$$
$$= s^3 + 14s^2 + 56s + 160$$

$$\left.\begin{matrix} k_3 + 3 = 14 \\ k_2 + 2 = 56 \\ k_1 = 160 \end{matrix}\right| \begin{matrix} k_3 = 11 \\ k_2 = 54 \\ k_1 = 160 \end{matrix} \Rightarrow K = [160 \quad 54 \quad 11]$$

$$\dot{x} = \underbrace{(A - BK)}_{AA} x + \underbrace{Bk_1}_{BB} U$$

$$AA = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -160 & -56 & -14 \end{vmatrix}$$

$$BB = \begin{vmatrix} 0 \\ 0 \\ 160 \end{vmatrix}$$

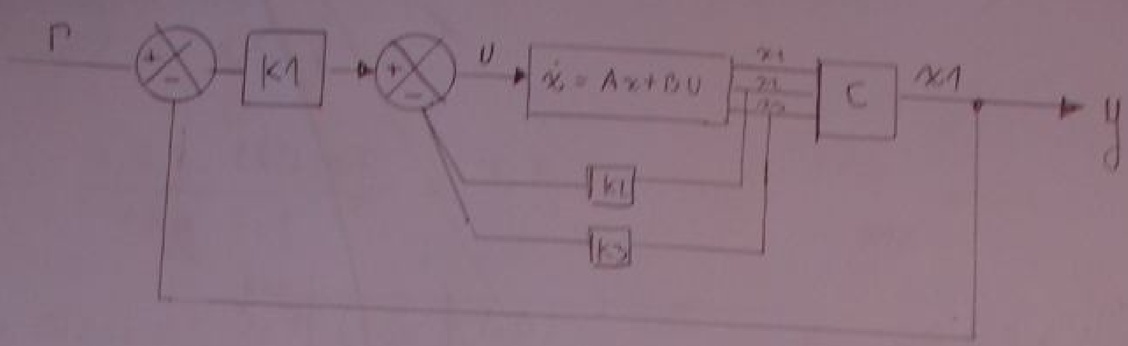
$$\left.\begin{matrix} \begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -160 & -56 & -14 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ 160 \end{vmatrix} . U \\ y = |1 \ 0 \ 0| \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} \end{matrix}\right\}$$

$$FT = C(sI - A)^{-1} B$$

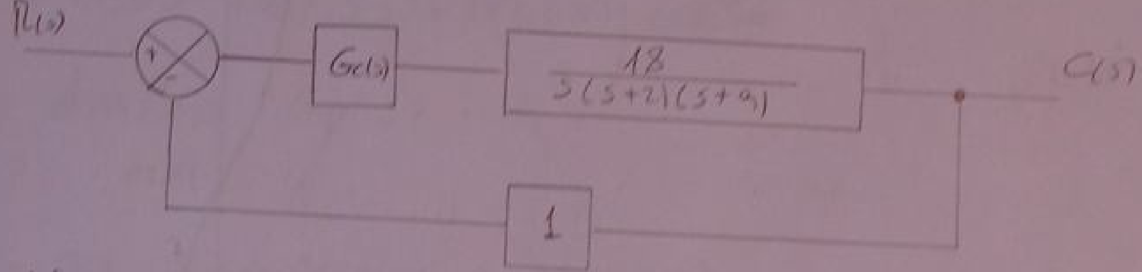
$$(sI - A) = \begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 160 & 56 & s-14 \end{vmatrix} \quad ; \quad det = s^3 - 14s^2 + 56s + 160$$

$$Adj = \begin{vmatrix} s^2 - 14s + 56 & s - 14 & 1 \\ -160 & s^2 - 14s & s \\ -160s & -(56s + 160) & s^2 \end{vmatrix}$$

$$FT = \frac{160}{s^3 - 14s^2 + 56s + 160}$$



De pág 45 al dorso:



1) Para $\omega_n = 2{,}828$ rad/seg ; $t_{s\,2\%} = 2\,seg^{-1}$

a) Compensar con un PD

b) Compensar en adelanto en cascada

a) $N = p - z = 3$, $\quad \varphi_k = 180\frac{(2k-1)}{p-z} = \begin{cases} \varphi_0 = 60 \\ \varphi_1 = -60 \\ \varphi_2 = 180 \end{cases}$

$\sigma_c = \frac{\sum Re\{P\} - \sum Re\{z\}}{p-z} = \frac{11}{3} = 3{,}66$

$G(s)H(s) + 1 = 0 \Rightarrow s^3 + 11s^2 + 18s + 18K = 0$

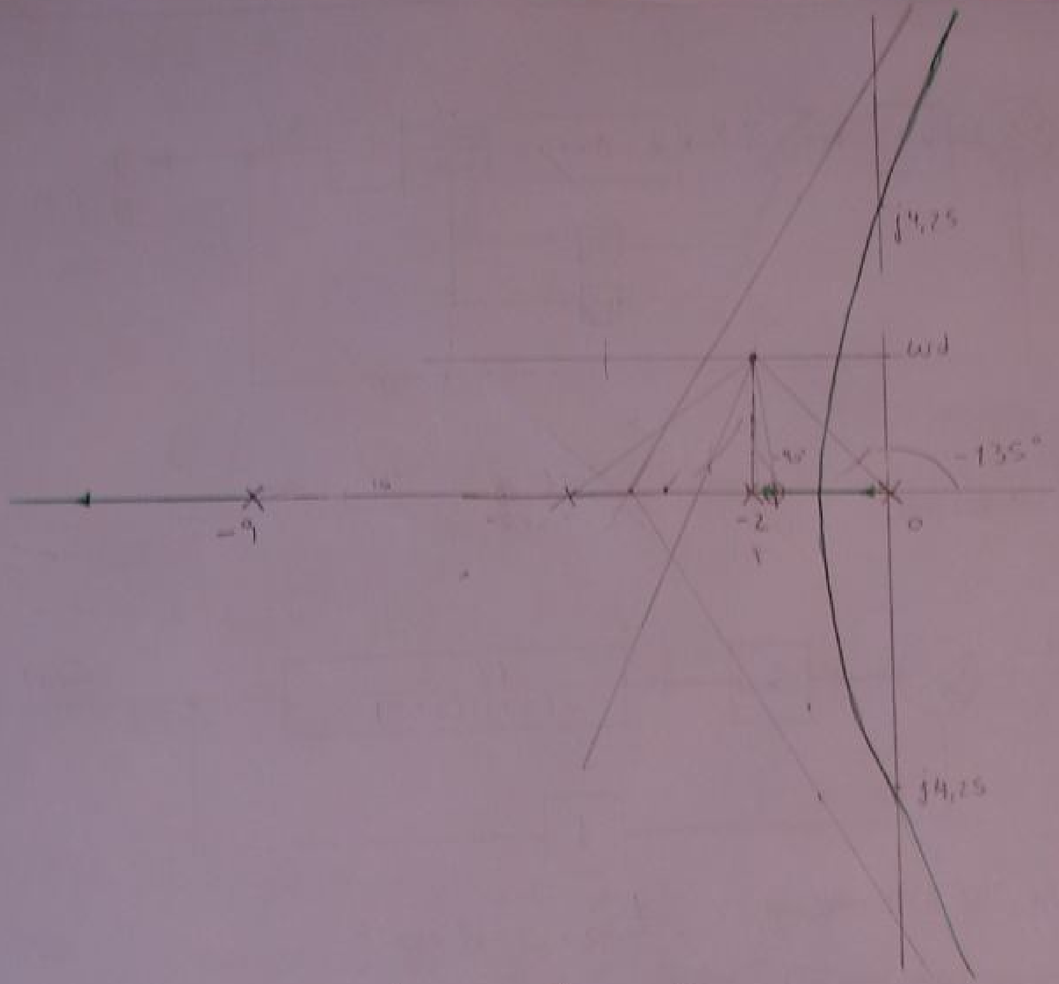
$K = -\frac{1}{18}(s^3 + 11s^2 + 18s)$

$\frac{dK}{ds} = -\frac{1}{18}(3s^2 + 22s + 18) = 0 \quad \begin{cases} Pb_1 = -6{,}4 \times \\ Pb_2 = -0{,}93 \end{cases}$

s^3	1	18
s^2	18	18K
s^1	324 − 18K	
s^0	18K	

$0 < K \leq 18$

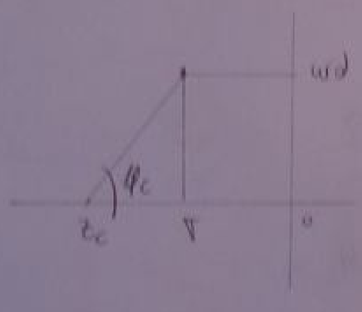
$18s^2 + 324 = \{ \pm j4{,}25$



$\omega_n = \overbrace{7,828}^{2\sqrt{2}}$, $t_{s_2} = 2\,seg^{-1} \Rightarrow t_{s_2} = \frac{4}{\sigma} \Rightarrow \sigma = -2$

$\omega d = \sqrt{\omega_n^2 - \sigma^2} = 2$

$-(45+90+135) = -240°$; $\varphi c = 240 - 180° = 60°$



$tg\,\varphi c = \frac{\omega d}{zc - \sigma} \Rightarrow zc = \frac{\omega d}{tg\,\varphi c} + \sigma$

$zc = \frac{2}{tg\,60°} + 2 = 3,15$

$K = \frac{7,5 \cdot 2 \cdot 2\sqrt{2}}{18 \cdot 2,4} = 0,78 \approx 1$

$\Rightarrow Gc(s)\,G(s) = \frac{s \cdot (s+3,15) \cdot 18}{s(s+2)(s+9)} = \frac{18s + 56,7}{s^3 + 11s^2 + 18s + 0}$

Siempre comprobar en la FTLC q' el pto de diseño sea polo dominante

b) $\left.\begin{array}{l} Z_c = -1,6 \\ P_c = -4,5 \end{array}\right\}$

$$k = \frac{7,5 \cdot 2 \cdot 2\sqrt{2} \cdot 3,3}{18 \cdot 2,1} = 3,7$$

$$G_c G(s) = \frac{3,7 (s+1,6) \cdot 18}{(s+4,5)(s+2)(s+9)s} = \frac{66,6s + 106,56}{s^4 + 15,5s^3 + 67,5s^2 + 81s + 0}$$

$$F_T = \frac{66,6s + 106,56}{s^4 + 15,5s^3 + 67,5s^2 + 147,6s + 106,56}$$

2) Para el mismo $G_p(s)$:

a) K_v?

b) $e_{ss} = 0,2$ (compensar en atraso)

c) Realizar un compensador PI q' cumpla con $e_{ss} = 0,2$

} e_{ss} para rampa.

d) Representar en VE, obtener $\phi(t)$ para FCD. Determinar el valor de salida p/ $x_1(t)$. Valor salida para impulso unitario y escalón unitario.

a)
$$K_V = \lim_{s \to 0} s\, G(s) = \frac{18\, \cancel{s}}{(s+9)(s+2)\cancel{s}} = 1$$

$$e_{ss_{|rampa}} = \frac{1}{K_V} = 1$$

b) $e_{ss} = 0,2 \Rightarrow K_V = 5$; $F_{LLC} = \dfrac{18}{(s+9)(s+2)s + 18} \Rightarrow P_d = -0,866 \pm j1$

$K = K_c \beta \Rightarrow p/ K_c = 1 \Rightarrow \beta = 5$

$Z_c = W_n/10 = 0,0866$

$P_c = \dfrac{Z_c}{\beta} = \dfrac{0,5}{5} = 0,01732$

$$\angle \frac{(-0,866 + j1) + 0,0866}{(-0,866 + j1) + 0,01732} = -2,38° ✓$$

$$\Rightarrow G_c G(s) = \frac{(s + 0,0866)}{(s + 0,01732)} \cdot \frac{18}{(s+9)(s+2)s}$$

c) $G_c = K_P + \frac{K_P}{T_i s} = K_P\left(1 + \frac{1}{sT_i}\right) = \frac{K_P}{s}\left(s + \frac{1}{T_i}\right)$

$\frac{K_P}{T_i}\frac{1}{s}\left(T_i s + 1\right)$ $\quad \frac{K_P}{T_i} = 5 \; ; \quad \frac{1}{T_i} = z_c = \frac{w_d}{10} = 0{,}0866$

$T_i = 11{,}54$

$\Rightarrow K_P = 5 \cdot 11{,}54 = 57{,}73$

$\Rightarrow G_c G(s) = \frac{57{,}73}{s}(s + 0{,}0866) \cdot \frac{18}{(s+2)(s+9)s} = \frac{(s+0{,}0866)\,1039{,}14}{s^2(s+2)(s+9)}$

$$FT = \frac{(s+0{,}0866)\,1039{,}14}{s^2(s+2)(s+9) + (s+0{,}0866)\,1039{,}14}$$

$$FT_{orig} = \frac{18}{s^3 + 11s^2 + 18s + 18}$$

$$FT = \frac{1039{,}14s + 90}{s^4 + 11s^3 + 18s^2 + 1039{,}14s + 90}$$

d) $G(s) = \frac{18}{s(s+2)(s+9)} \Rightarrow FT = \frac{18}{s(s+2)(s+9) + 18}$

$FT = \frac{18}{s^3 + 11s^2 + 18s + 18} = \frac{C(s)}{R(s)}$; $\quad C(s) = 18\,Q(s)$, $\quad R(s) = (s^3 + 11s^2 + 18s + 18)\,Q(s)$

uno $\phi(t)$ puede ser distinto para un mismo sistema dependiendo de su representación pero al momento de valuar las para una misma entrada DEBERIAMOS obtener la misma salida.

$$\underbrace{s^3 Q(s)}_{\dot{x}_3} = -11\overbrace{s^2 Q(s)}^{x_3} - 18\overbrace{s\,Q(s)}^{x_2} - 18\overbrace{Q(s)}^{x_1} + R(s)$$

$\dot{x}_3 = -11x_3 - 18x_2 - 18x_1 + 18u$

$x_1 = Q(s)$

$\dot{x}_1 = s\,Q(s) = x_2$

$\dot{x}_2 = s^2 Q(s) = x_3$

$$\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -18 & -18 & -11 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ 18 \end{vmatrix} u$$

(81)

$$C = \begin{vmatrix} 18 & 0 & 0 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix}$$

$$\lambda_1 = -0{,}866 + j1{,}1 \quad \lambda_3 = -9{,}26$$
$$\lambda_2 = -0{,}866 - j1{,}1$$

$$P = \begin{vmatrix} 1 & 1 & 1 \\ \lambda_1 & \lambda_2 & \lambda_3 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ -0{,}866 + j1{,}1 & -0{,}866 - j1{,}1 & -9{,}26 \\ -0{,}46 - j1{,}9 & -0{,}46 + j1{,}9 & 85{,}74 \end{vmatrix}$$

$$A_{FCD} = P^{-1} A P = \begin{vmatrix} -0{,}866 + j1{,}0917 & 0 & 0 \\ 0 & -0{,}866 - j1{,}097 & 0 \\ 0 & 0 & -9{,}26 \end{vmatrix}$$

$$B_{FCD} = P^{-1} B = \begin{vmatrix} -0{,}007 - j0{,}0536 \\ -0{,}007 + j0{,}0536 \\ 0{,}0139 \end{vmatrix}$$

$$C_{FCD} = C P = \begin{vmatrix} 18 & 18 & 18 \end{vmatrix}$$

$$(sI - A_{FCD}) = \begin{vmatrix} (s + 0{,}866 - j1{,}097) & 0 & 0 \\ 0 & (s + 0{,}866 + j1{,}097) & 0 \\ 0 & 0 & (s + 9{,}26) \end{vmatrix}$$

$$Adj = \begin{vmatrix} (s + 0{,}866 + j1{,}097)(s + 9{,}26) & 0 & 0 \\ 0 & (s + 0{,}866 - j1{,}097)(s + 9{,}26) & 0 \\ 0 & 0 & (s + 0{,}866 + j1{,}097)(s + 0{,}866 -) \end{vmatrix}$$

$$(sI - A)^{-1} = \begin{vmatrix} \frac{1}{(s + 0{,}866 - j1{,}097)} & 0 & 0 \\ 0 & \frac{1}{(s + 0{,}866 + j1{,}097)} & 0 \\ 0 & 0 & \frac{1}{s + 9{,}26} \end{vmatrix}$$

$$\mathcal{L}(sI - A)^{-1} = \phi(t) = \begin{vmatrix} e^{-(0{,}866 - j1{,}097)t} & 0 & 0 \\ & e^{-(0{,}866 + j1{,}097)t} & \\ 0 & 0 & e^{-9{,}26t} \end{vmatrix}$$

$$\dot{x}(s) = A x(s) + B U$$

$$s x(s) - x(0) = A x(s) + B U$$

$$(sI - A) x(s) - x(0) = B U$$

$$x(s) = (sI-A)^{-1} x(0) + (sI-A)^{-1} B U(s)$$

$$x(t) = \phi(t) x(0) + \int_0^t \phi(t-\tau) B U(\tau) d\tau$$

P/ escalón:

$x(0) = 0$

$$\Rightarrow \int_0^t \begin{vmatrix} e^{-(0{,}866-j1{,}097)(t-\tau)} & 0 & 0 \\ & e^{-(0{,}866+j1{,}097)(t-\tau)} & 0 \\ 0 & 0 & e^{-9{,}26(t-\tau)} \end{vmatrix} B\, U d\tau$$

$$\int_0^t e^{-a(t-\tau)} d\tau = \int_0^t e^{-at} e^{a\tau} d\tau = e^{-at} \int_0^t e^{a\tau} d\tau$$

$$= e^{-at} \left[\frac{1}{a} e^{a\tau} \right]_0^t = \frac{1}{a} e^{-at} \left[e^{at} - 1 \right] = \frac{1}{a}(1 - e^{-at})$$

$$\begin{vmatrix} \frac{1}{0{,}866-j1{,}097}\left(1 - e^{-(0{,}866-j1{,}097)t}\right) & 0 & 0 \\ 0 & \frac{1}{0{,}866+j1{,}097}\left(1 - e^{-(0{,}866+j1{,}097)t}\right) & 0 \\ 0 & 0 & \frac{1}{9{,}26}\left(1 - e^{-9{,}26 t}\right) \end{vmatrix}$$

$$| \quad | \cdot \underset{(s)}{B} \cdot U = | \quad |$$

$$\Rightarrow y(t) = \underset{(s)}{C}\, x(t)$$

P/ impulso $\rightarrow$ se trabaja con $\phi(s)$ y despues se transformar

se tiene:

(82)

$W_n^2 = W_d^2 + \sigma^2$

$$G(s) = \frac{63{,}21}{s(s+4)(s+6)}$$

a) Diseñar compensador para $T_{s_{2\%}} = 2$ seg; $M_p = 0{,}2$

$$t_{s_{2\%}} = \frac{4}{\sigma} \Rightarrow \sigma = \frac{4}{t_{s_{2\%}}} = 2$$

$$M_p = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} \qquad \ln M_p = \frac{-\xi\pi}{\sqrt{1-\xi^2}} \cdot \ln(e)$$

$$\sqrt{1-\xi^2}\,\ln M_p = -\xi\pi$$

$$\left(\ln M_p\sqrt{1-\xi^2}\right)^2 = (-\xi\pi)^2$$

$$(\ln M_p)^2(1-\xi^2) = \xi^2\pi^2$$

$$(\ln M_p)^2 = \xi^2(\ln M_p)^2 + \xi^2\pi^2$$

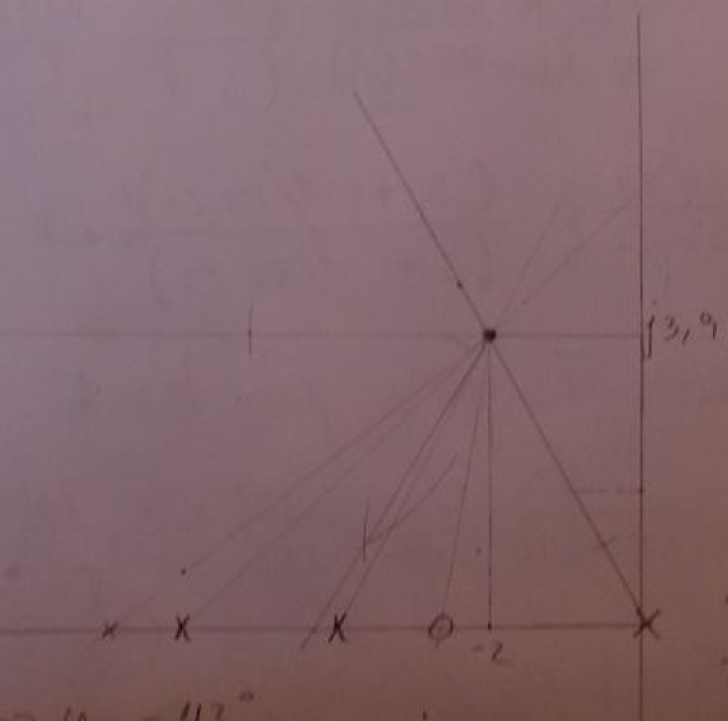
$$(\ln M_p)^2 = \xi^2(\ln M_p^2 + \pi^2)$$

$$\xi = \sqrt{\frac{\ln M_p^2}{\ln M_p^2 + \pi^2}} = 0{,}456$$

$\Rightarrow P_d \Rightarrow \xi = 0{,}456$; $\sigma = 2$; $\sigma = W_n\,\xi \Rightarrow W_n = \frac{\sigma}{\xi}$

$$W_d = \sqrt{\left(\frac{\sigma}{\xi}\right)^2 - \sigma^2} = 3{,}9 \qquad W_n = 4{,}38$$

$$K_c = \frac{6{,}3 \cdot 5{,}6 \cdot 4{,}3 \cdot 4{,}3}{63{,}21 \cdot 3{,}9} = 2{,}64$$



$P_c = -7$

$Z_c = -2{,}6$

$$-(43 + 62 + 118) = -223 \Rightarrow \varphi_c = 43°$$

$$GcG(s) = 2{,}64\,\frac{(s+2{,}6)}{(s+7)} \cdot \frac{63{,}21}{s(s+4)(s+6)}$$

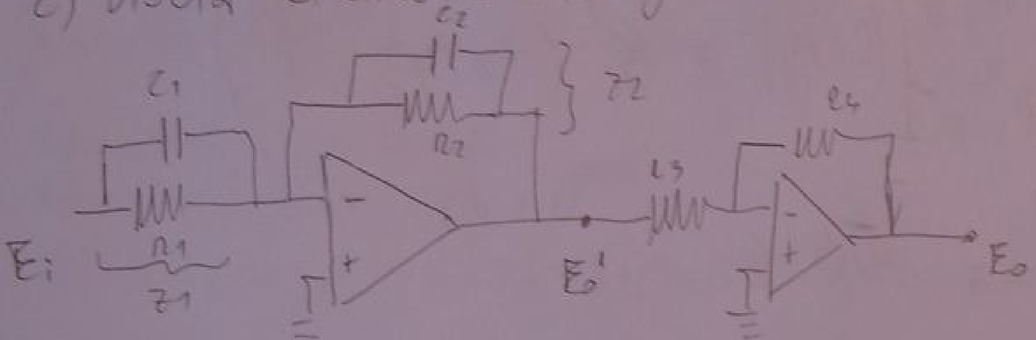
$$= \frac{166{,}874s + 433{,}873}{s^4 + 17s^3 + 94s^2 + 168s}$$

se espera ξ' en FTLC encontramos los polos dominantes en

$$s^2 + 2\xi\omega_n s + \omega_n^2 \Rightarrow s^2 + 4s + 19{,}184 \left\{ -2 \pm j3{,}896 \right.$$

y están en: $-2 \pm j3{,}814$; -10

c) Diseñar el circuito analógico:



$$\frac{R \cdot \frac{1}{sC}}{R + \frac{1}{sC}}$$

$$\frac{R}{1 + RsC}$$

$$\frac{E_o'}{E_i} = \frac{Z_2}{Z_1} = \frac{R_2 // 1/sC_2}{R_1 // 1/sC_1} = \frac{R_2(1 + sC_1R_1)}{R_1(1 + sC_2R_2)} =$$

$$= \frac{\cancel{R_2}\cancel{R_1}C_1}{\cancel{R_2}\cancel{R_1}C_2} \frac{\left(s + \frac{1}{R_1C_1}\right)}{\left(s + \frac{1}{R_2C_2}\right)}$$

$$\frac{E_o}{E_i} = \frac{R_4C_1}{R_3C_2} \cdot \frac{(s + 1/R_1C_1)}{(s + 1/R_2C_2)} \quad ; \quad R_4 = R_3 \quad \frac{1}{R_1C_1} = 2{,}6 \quad \Big| \quad \frac{1}{R_2C_2} = 7$$

d) Calcular T_s y M_p para $k = 1$ (del sist sin comp)

$$FTLC = \frac{63{,}21}{s(s+4)(s+6) + 63{,}21} \Rightarrow P_d = -1 \pm j2{,}62$$

$\sigma = -1$; $\omega_n = 2{,}8$; $\xi = \frac{\sigma}{\omega_n}$

$\xi = 0{,}357$

$\Rightarrow T_{s_{2\%}} = 4\,seg$; $M_p = 0{,}3$

Se tiene:

$$\frac{C(s)}{R(s)} = \frac{s+15}{s^3+6s^2+8s}$$

$a_1 = 6$ | $b_0 = 0$
$a_2 = 8$ | $b_1 = 0$
$a_3 = 0$ | $b_2 = 1$
| $b_3 = 15$

a) Representar en VE:

$x_1 = y - \beta_0 U$

$x_2 = \dot{y} - \beta_0 \dot{U} - \beta_1 U = \dot{x}_1 - \beta_1 U$

$x_3 = \ddot{y} - \beta_0 \ddot{U} - \beta_1 \dot{U} - \beta_2 U = \dot{x}_2 - \beta_2 U$

$\beta_0 = b_0$
$\beta_1 = b_1 - a_1\beta_0$
$\beta_2 = b_2 - \cancel{a_1} \beta_1 - a_2\beta_0$
$\beta_3 = b_3 - a_1\beta_2 - a_2\beta_1 - a_3\beta_0$

| $\beta_0 = 0$
| $\beta_1 = 0$
| $\beta_2 = 1$
| $\beta_3 = 9$

$x_1 = y$

$\dot{x}_1 = x_2 + \beta_1 U = x_2$

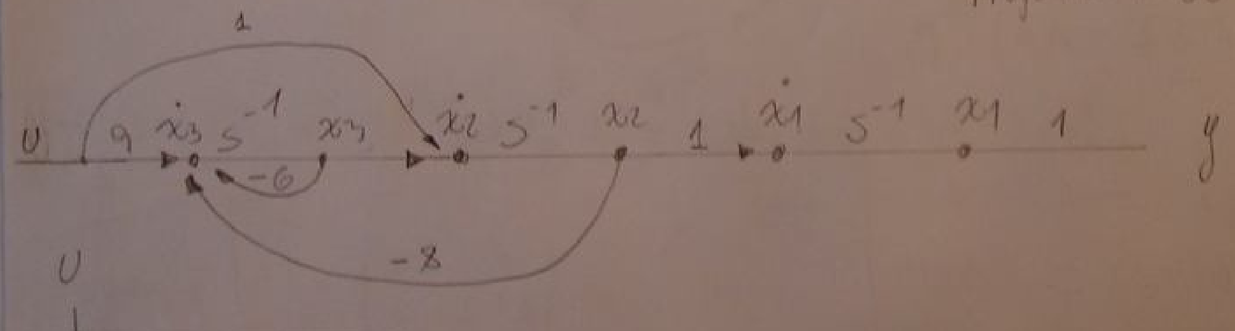
$\dot{x}_2 = x_3 + \beta_2 U = x_3 + U$

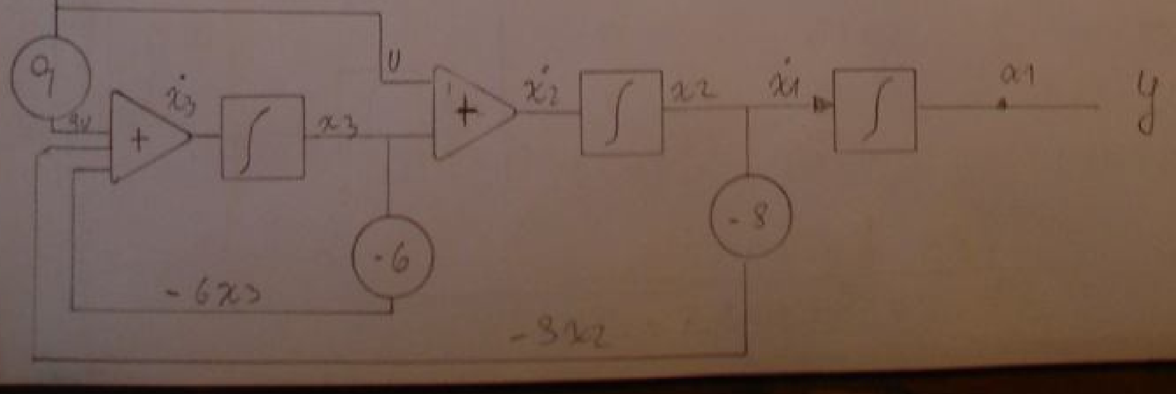
$\dot{x}_3 = -a_3 x_1 - a_2 x_2 - a_1 x_3 + \beta_3 U = -8x_2 - 6x_3 + 9U$

$$\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -8 & -6 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} + \begin{vmatrix} 0 \\ 1 \\ 9 \end{vmatrix} U$$

$$y = |1 \ 0 \ 0| \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix}$$

b) Diagrama de bloques y de flujo indicando VE.





c) Autovalores:

$$(\lambda I - A) = s^3 + 6s^2 + 8s \Rightarrow \begin{cases} \lambda_1 = 0 \\ \lambda_2 = -2 \\ \lambda_3 = -4 \end{cases}$$

e) Realimentar para $s_1 = -15,\ s_{2,3} = -3 \pm j3$

$$(sI - A + BK) = \begin{vmatrix} s & -1 & 0 \\ k_1 & s+k_2 & -1+k_3 \\ 9k_1 & 8+9k_2 & s+6+9k_3 \end{vmatrix}$$

$$\det = s^3 + (k_2 + 9k_3 + 6)s^2 + (k_1 + 15k_2 - 8k_3 + 8)s + 15k_1$$

$$= s^3 + 21s^2 + 108s + 270$$

$$\begin{aligned} 0k_1 + k_2 + 9k_3 + 6 &= 21 \\ k_1 + 15k_2 - 8k_3 + 8 &= 108 \\ 15k_1 + 0k_2 - 0k_3 + 0 &= 270 \end{aligned} \qquad \begin{vmatrix} 0 & 1 & 9 \\ 1 & 15 & -8 \\ 15 & 0 & 0 \end{vmatrix} \begin{vmatrix} k_1 \\ k_2 \\ k_3 \end{vmatrix} = \begin{vmatrix} 21 \\ 108 \\ 270 \end{vmatrix} - \begin{vmatrix} 6 \\ 8 \\ 0 \end{vmatrix}$$

$$\begin{vmatrix} k_1 \\ k_2 \\ k_3 \end{vmatrix} = \begin{vmatrix} 18 \\ 6 \\ 1 \end{vmatrix} \Rightarrow K = \begin{vmatrix} 18 & 6 & 1 \end{vmatrix}$$

$$\dot{x} = \overbrace{(A - Bk)}^{AA} + \overbrace{Bk_1}^{BB} r \Rightarrow AA = \begin{vmatrix} 0 & 1 & 0 \\ -18 & -6 & 0 \\ -162 & -62 & -15 \end{vmatrix};\ BB = \begin{vmatrix} 0 \\ 18 \\ 162 \end{vmatrix}$$

$$CC = C$$

$$FT = C(sI - AA)^{-1} B$$

$$\det = s^3 + 6s^2 + 8s = s(s+2)(s+$$

$$sI - AA = \begin{vmatrix} s & -1 & 0 \\ 18 & s+6 & 0 \\ 162 & 62 & s+15 \end{vmatrix}$$

$$\text{Adj} = \begin{vmatrix} s^2 + 21s + 90 & s+15 & 0 \\ -18s - 270 & s^2 + 15s & 0 \\ -162s + 144 & -62s - 162 & s^2 + 6s + 18 \end{vmatrix}$$

$$\frac{C\,\text{adj}(sI - AA)B}{\det(sI - AA)} = \frac{18s + 270}{s^3 + 21s^2 + 108s + 270}$$

$$\left.\begin{aligned} \begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \end{vmatrix} &= \begin{vmatrix} 0 & 1 \\ -6 & -5 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} + \begin{vmatrix} 0 \\ 1 \end{vmatrix} U \\ y &= \begin{vmatrix} 6 & 0 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} \end{aligned}\right\}$$

Diagonalizar:

$$(\lambda I - A) = \begin{vmatrix} s & -1 \\ 6 & s+5 \end{vmatrix} \Rightarrow Det = s^2 + 5s + 6 \begin{cases} -3 \\ -2 \end{cases}$$

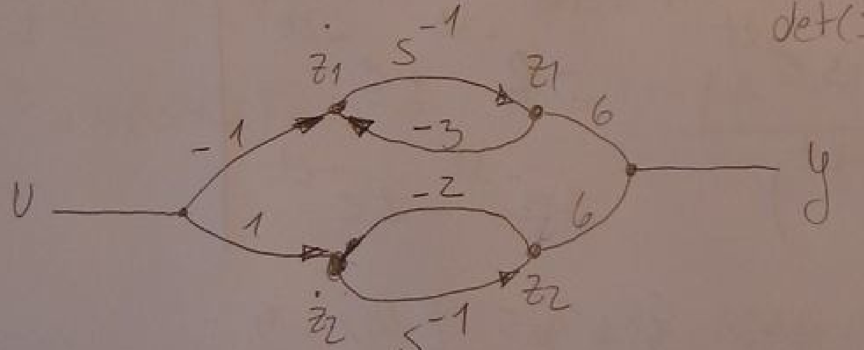
$$P = \begin{vmatrix} 1 & 1 \\ -3 & -2 \end{vmatrix} ; \quad P^{-1} = \begin{vmatrix} -2 & -1 \\ 3 & 1 \end{vmatrix}$$

$$A_{FCD} = \begin{vmatrix} -3 & 0 \\ 0 & -2 \end{vmatrix} ; \quad B_{FCD} = \begin{vmatrix} -1 \\ 1 \end{vmatrix} ; \quad C_{FCD} = \begin{vmatrix} 6 & 6 \end{vmatrix}$$

$$\left.\begin{aligned} \begin{vmatrix} \dot{z}_1 \\ \dot{z}_2 \end{vmatrix} &= \begin{vmatrix} -3 & 0 \\ 0 & -2 \end{vmatrix} \begin{vmatrix} z_1 \\ z_2 \end{vmatrix} + \begin{vmatrix} -1 \\ 1 \end{vmatrix} U \\ y &= \begin{vmatrix} 6 & 6 \end{vmatrix} \begin{vmatrix} z_1 \\ z_2 \end{vmatrix} \end{aligned}\right\}$$

$$(sI - A) = \begin{vmatrix} s+3 & 0 \\ 0 & s+2 \end{vmatrix}$$

$$\det(sI - A) = (s+3)(s+2)$$



$$FTLc = C(sI - A)^{-1} B = \frac{C \, Adj(sI - A) B}{\det(sI - A)}$$

$$Adj(sI - A) = \begin{vmatrix} s+2 & 0 \\ 0 & s+3 \end{vmatrix}$$

$$FTLC = \frac{6}{(s+3)(s+2)} = 6 \cdot \frac{-1}{s+3} + 6 \frac{1}{s+2}$$

$$\phi(t) = \mathcal{L}^{-1}(sI - A)^{-1}$$

$$(sI - A)^{-1} = \frac{Adj(sI-A)}{det(sI-A)} = \begin{vmatrix} \frac{(s+2)}{(s+2)(s+3)} & 0 \\ 0 & \frac{(s+3)}{(s+2)(s+3)} \end{vmatrix}$$

$$= \begin{vmatrix} \frac{1}{s+3} & 0 \\ 0 & \frac{1}{s+2} \end{vmatrix}$$

$$\phi(t)_{|FCD} = \begin{vmatrix} e^{-3t} & 0 \\ 0 & e^{-2t} \end{vmatrix} ; \phi(t)_{|FCC} = P\, \phi(t)_{|FCD} P^{-1}$$

$$(sI - A) = \begin{vmatrix} s & -1 \\ 6 & s+5 \end{vmatrix} ; Adj = \begin{vmatrix} s+5 & 1 \\ -6 & s \end{vmatrix} ; det = (s+3)(s+2)$$

$$\phi(s) = \begin{vmatrix} \frac{s+5}{(s+3)(s+2)} & \frac{1}{(s+3)(s+2)} \\ \frac{-6}{(s+3)(s+2)} & \frac{s}{(s+3)(s+2)} \end{vmatrix} = \begin{vmatrix} \frac{-2}{s+3} + \frac{3}{s+2} & \frac{-1}{s+3} + \frac{1}{s+2} \\ \frac{6}{s+3} - \frac{6}{s+2} & \frac{3}{s+3} - \frac{2}{s+2} \end{vmatrix}$$

$$\phi(t) = \begin{vmatrix} -2e^{-3t} + 3e^{-2t} & -e^{-3t} + e^{-2t} \\ 6(e^{-3t} - e^{-2t}) & 3e^{-3t} - 2e^{-2t} \end{vmatrix}$$

b) Realimentación para $\xi = 0,5$ y $\omega_n = 4$.

$$s^2 + 2\xi\omega_n s + \omega_n^2 = s^2 + 4s + 16 \left\{ -2 \pm j3,464 \right.$$

$$sI - A + BK = \begin{vmatrix} s & -1 \\ 6+k_1 & s+5+k_2 \end{vmatrix} \Rightarrow k_1 + sk_2 + s^2 + 5s + 6$$

$$s^2 + s(5+k_2) + 6 + k_1 = s^2 + 4s + 16$$

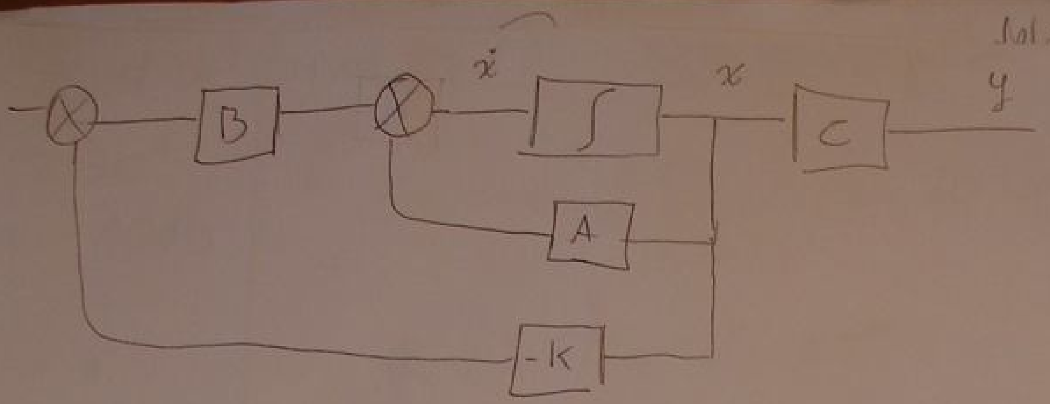
$5 + k_2 = 4 \quad ; \quad k_2 = -1$

$6 + k_1 = 16 \quad ; \quad k_1 = 10 \quad \Rightarrow k = |10 \quad -1|$

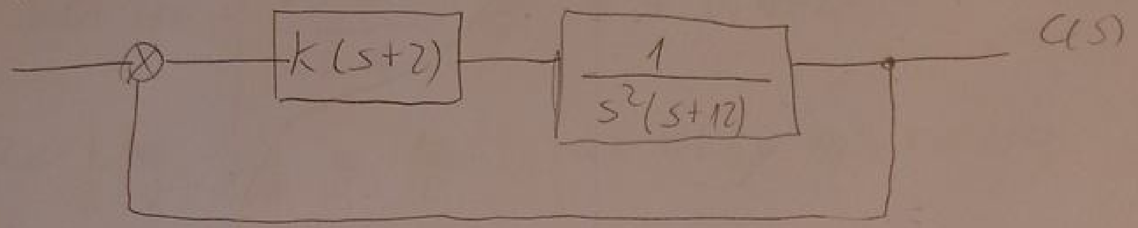
Sol. entr.

$\dot{x} = Ax + Bu$ 85

1)



2)



1) controlador PD $\Rightarrow k_p T_d \left(s + \frac{1}{T_d}\right)$

2) $G_c G(s) = \frac{s+2}{s^2(s+12)}$

$N = p - z = 3 - 1 = 2$, $\sigma_c = \frac{\sum Re\{P\} - \sum Re\{Z\}}{p-z} = \frac{12-2}{2} - 5$

$\varphi_k = 180\frac{(2k-1)}{p-z}$ $\begin{cases} \varphi_0 = -90 \\ \varphi_1 = +90 \end{cases}$

$G(s)H(s) + 1 = 0.$

$s^3 + 12s^2 + k(s+2) = 0$; $K = \frac{-(s^3+12s^2)}{(s+2)}$

$\frac{dK}{ds} = \qquad = 0$

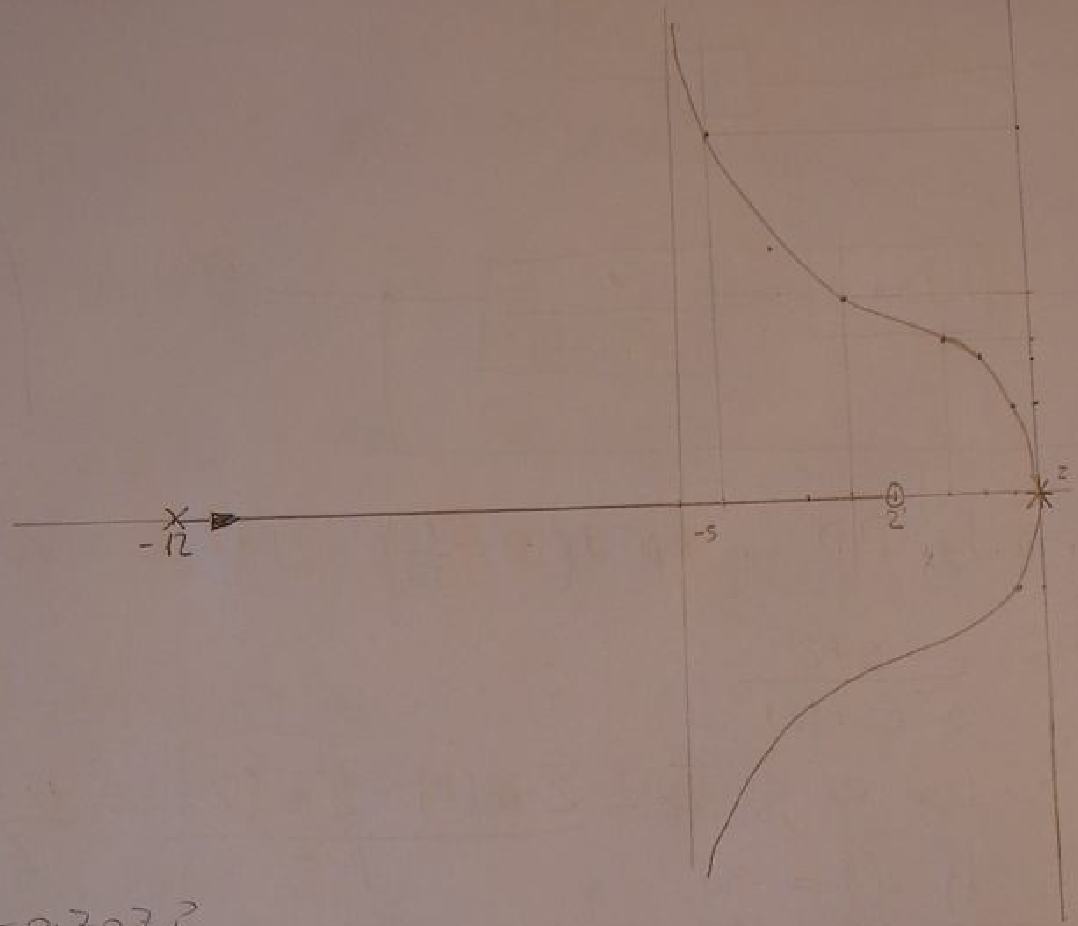
$pb = \begin{cases} 0 \\ -4,5 \pm j1,93 \end{cases}$

$s^3 + 12s^2 + ks + 2K$

$K(12-2) = 10K$

s^3	1	K
s^2	12	2K
s^1	12K	-2K
s^0	2K	

$0 < K$



K p/ $\xi = 0{,}707$?

$$s^2 + 2\frac{\sqrt{2}}{2}\omega_n + \omega_n^2 = s^2 + \sqrt{2}\omega_n + \omega_n^2$$

$$FTLC = \frac{K(s+2)}{s^2(s+12) + K(s+2)} = \frac{K(s+2)}{s^3 + 12s^2 + Ks + 2K}$$

$$s^3 + 12s^2 + Ks + 2K \quad \big|\ \underline{s^2 + \sqrt{2}\omega_n s + \omega_n^2}$$

$$\qquad\qquad\qquad\qquad\quad s + (12 - \sqrt{2}\omega_n)$$

$$\underline{-\sqrt{2}\omega_n s^2 - \omega_n^2 s}$$

$$s^2(12 - \sqrt{2}\omega_n) + s(K - \omega_n^2) + 2K$$

$$\underline{-s(12 - \sqrt{2}\omega_n)\sqrt{2}\omega_n - \omega_n^2(12 - \sqrt{2}\omega_n)}$$

$$s\left(K - \omega_n^2 - \sqrt{2}\omega_n(12 - \sqrt{2}\omega_n)\right) + \underbrace{2K - \omega_n^2(12 - \sqrt{2}\omega_n)}_{0}$$

Igual q' el del dorso pg 73 compensar por adelanto.

MF = 50°

Mg = 10dB

$$G(s) = \frac{1}{s^2(s+5)} \quad k = 20 \log \frac{1}{5} = -14dB$$

Mf = 0°

Mg = 3dB

$$\phi_c = 50° + 12° = 62°$$

$$\operatorname{sen} \varphi_c = \frac{1-\alpha}{1+\alpha} \quad ; \quad \alpha = 0{,}062$$

$$-20 \operatorname{Log} \frac{1}{\sqrt{\alpha}} = -12 \Rightarrow \omega_n = 0{,}9 \text{ rad/seg}$$

$$\omega_n \sqrt{\alpha} = z_c = 0{,}22 \quad ; \quad P_c = \frac{z_c}{\alpha} = 3{,}61$$

$$G(s) G_c = \left| K_c \frac{(s_1 + 0{,}22)}{(s_1 + 3{,}61)} \cdot \frac{1}{s_1^2 (s_1 + 5)} \right| = 1$$

P| $\omega_n = 0{,}8$

$$\Rightarrow \left| \frac{(j0{,}8 + 0{,}22)}{(j0{,}8 + 3{,}61)} \quad \frac{1}{(j0{,}8)^2 (j0{,}8 + 5)} \right| = 0{,}06$$

$$K_c = (0{,}063)^{-1} = 16{,}52$$

$$G(s) G_c = 16{,}52 \frac{(s + 0{,}22)}{(s + 3{,}61)} \cdot \frac{1}{s^2(s+5)}$$

Mf = 52°

$$2 \operatorname{sen} 10t = \quad 2 \operatorname{sen}\left(t - \frac{180\pi}{180}\right) \cdot \qquad 20 \operatorname{Log} x = -25$$

$$x = 10^{\frac{-25}{20}} = 0{,}056 \Rightarrow 0{,}11 \operatorname{sen}(t - \pi)$$

$k - \omega_n^2 - 12\sqrt{2}\omega_n + 2\omega_n^2 = 0$

$k = \omega_n^2 + 12\sqrt{2}\omega_n - 2\omega_n^2$

$2\omega_n^2 + 24\sqrt{2}\omega_n - 4\omega_n^2 - \omega_n^2(12 - \sqrt{2}\omega_n) = 0$

$$\omega_n = \begin{cases} 0 & \Rightarrow k \times \\ 4{,}24 & \Rightarrow k_1 = 54 \\ 5{,}65 & \Rightarrow k_2 = 64 \end{cases}$$

4) $e_{ss} = \frac{1}{k_v}$; $\quad k_v = \lim_{s \to 0} s\,G(s) = \frac{(s+2)\cancel{s}}{s^2(s+2)} = \frac{2}{0} = \infty$

$e_{ss} = \frac{1}{\infty} = 0$

5) S:

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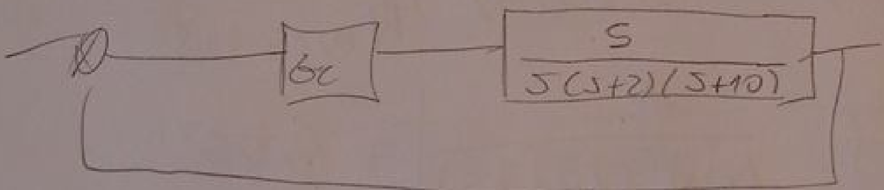
compensación por bode:



$M_f = 60°$
$k_v = 2$

$k_v = \lim_{s \to 0} s\,G(s) = \frac{5k}{2 \cdot 10} = 2 \Rightarrow k = 8$

$\Rightarrow k\,G(s) = \frac{40}{s(s+2)(s+10)}$; $\quad k_{Tot} = 20\log(2) = 6\,dB$

$M_f = 31{,}5°$;

$\varphi_c = -180 + 60° + 8 = -112° \Rightarrow \omega_n = 0{,}6\,rad \Rightarrow$

$-20\log\frac{1}{\beta} = 10\,dB \Rightarrow \left(10^{-10/20}\right)^{-1} = \beta = 3{,}16$ | $\beta k_c = 8$

$k_c = \frac{8}{\beta} = 2{,}53$

$z_c = \frac{\omega_n}{10} = 0{,}06\,rad/seg$
$p_c = \frac{z_c}{\beta} = 0{,}02\,rad/seg$ $\Big\}$ $-3{,}8°$ $\quad M_f = 66$

compensar en la realim p/ q' quede en $\xi = 0,5$ y
$\omega_n = 1,4142 = \sqrt{2}$.

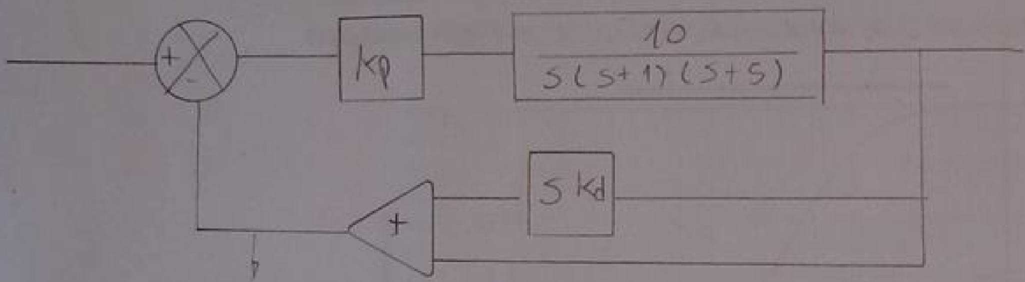
$$\Rightarrow s^2 + 2\xi\omega_n s + \omega_n^2 = s^2 + \sqrt{2}\,s + 2$$

$$G(s) = \frac{10}{s(s+1)(s+5)} \;;$$

$2 \log 0,5$

$\log(0,5^2)$

$[\log(0,5)]^2$



$$1 + s\,kd = kd\left(s + \frac{1}{kd}\right)$$

$$FTLC = \frac{kp\,10}{s(s+1)(s+5) + 10kp(1 + s\,kd)} = \frac{kp\,10}{s^3 + 6s^2 + (5 + 10kp\,kd)s + 10kp}$$

$$s^3 + 6s^2 + (5 + 10kp\,kd)s + 10kp \quad \Big| \quad 1s^2 + \sqrt{2}\,s + 2$$
$$-\sqrt{2}s^2 - 2s \qquad\qquad s + (6 - \sqrt{2}) \qquad\qquad -6\sqrt{2} + 2$$
$$(6 - \sqrt{2})s^2 + (3 + 10kp\,kd)s + 10kp$$
$$-\sqrt{2}(6 - \sqrt{2})s - 2(6 - \sqrt{2})$$
$$(5 - 6\sqrt{2} + 10kp\,kd)s + 10kp - 2(6 - \sqrt{2})$$

$$kp = \frac{2(6 - \sqrt{2})}{10} = 0,917 \;;\quad kd = \frac{6\sqrt{2} - 5}{10\,kp} = 0,38$$