Representation of Noise in Linear Twoports*

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The compilation of standard methods of test often requires theoretical concepts that are not widely known nor readily available in the literature. While theoretical expositions are not properly part of a Standard they are necessary for its understanding and it seems desirable to make them easily accessible by simultaneous publication with the Standard. In such cases a technical committee report provides a convenient means of fulfilling this function.

The Standards Committee has decided to publish this technical committee report immediately following "Standards on Methods of Measuring Noise in Linear Twoports." A copy of this paper will be attached to all reprints of the Standards.

Summary—**This is a tutorial paper, written by the Subcommittee on Noise, IRE 7.9, to provide the theoretical background for some of the IRE Standards on Methods of Measuring Noise in Electron Tubes. The general-circuit-parameter representation of a linear twoport with internal sources and the Fourier representations of stationary noise sources are reviewed. The relationship between spectral densities and mean-square fluctuations is given and the noise factor of the linear twoport is expressed in terms of the mean-square fluctuations of the source current and the internal noise sources. The noise current is then split into two components, one perfectly correlated and one uncorrelated with the noise voltage. Expressed in terms of the noise voltage and these components of the noise current, the noise factor is then shown to be a function of four parameters which are independent of the circuit external to the twoport.**

I. Introduction

One of the basic problems in communication engineering is the distortion of weak signals by the ever-present thermal noise and by the noise of the devices used to process such signals. The transducers performing signal processing such as amplification, frequency mixing, frequency shifting, etc., may usually be classified as twoports. Since weak signals have amplitudes small compared with, say, the grid bias voltage of a vacuum tube, or emitter bias current of a transistor, etc., the amplitudes of the excitations at the ports can be linearly related. Consequently, the description and measurement of noise that is presented in this paper, and which forms the basis for the "Methods of Test" described in this same issue,[1] can be restricted to linear noisy twoports. Even if inherently nonlinear characteristics are involved, as in mixing, etc., linear relations still exist among the signal input and output amplitudes, although these may not be associated with the same frequency. Image-frequency components may be eliminated by proper filtering, but a generalization of our results to cases in which image frequencies are present is not difficult.

The effect of the noise originating in a twoport when the signal passes through the twoport is characterized at any particular frequency by the *(spot) noise factor* *(noise figure)*. Since this has meaning only if the source impedance used in obtaining the noise factor is also specified, the noise contribution of a twoport is often indicated by a minimum noise factor and the source impedance with which this minimum noise factor is achieved. However, the extent to which the noise factor depends upon the input source impedance, upon the amount of feedback, etc., can be indicated only by a more detailed representation of the linear twoport.[2,3]

As a basis for understanding the representation of networks containing (statistical) noise sources, we shall consider first the analysis of networks containing Fourier-transformable signal sources. We shall then describe noise in two ways: a) as a limit of Fourier-integral transforms, and b) as a limit of Fourier-series transforms. The reasons for the wider use of the latter description will be presented. With this background we shall be ready to show that the four noise parameters used in the standard "Methods of Test" completely characterize a noisy linear twoport.

II. Representations of Linear Twoports

The excitation at either port of a linear twoport can be completely described by a time-dependent voltage $v(t)$ and the time-dependent current $i(t)$. (For waveguide twoports, where no voltage or current can be identified uniquely, an equivalent voltage and current can always

* Original manuscript received by the IRE, September 15, 1958; revised manuscript received March 6, 1959.

[1] "IRE Standards on Methods of Measuring Noise in Linear Twoports, 1959," this issue, p. 60.

[2] H. Rothe and W. Dahlke, "Theory of noisy fourpoles," Proc. IRE, vol. 44, pp. 811–818; June, 1956. Also, "Theorie rauschender Vierpole," *Arch. elekt. Übertragung*, vol. 9, pp. 117–121; March, 1955.

[3] A. G. T. Becking, H. Groendijk, and K. S. Knol, "The noise factor of four-terminal networks," *Philips Res. Repts.*, vol. 10, pp. 349–357; October, 1955.

be used.) Let it be assumed that the voltage and current functions can be transformed from the time domain to the frequency domain, and that V and I stand for the Fourier transforms when the function is aperiodic and for the Fourier amplitudes when the function is periodic. The linearity of the twoport *without internal sources* then allows an impedance representation:

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2. \tag{1}$$

The subscripts 1 and 2 refer to the input and output ports, respectively, and the coefficients Z_{jk} are, in general functions of frequency. The currents are defined to be positive if the flow is into the network as shown in Fig. 1.

If the twoport contains internal sources, then (1) and the equivalent circuit must be modified. By a generalization of Thévenin's theorem, the twoport may be separated into a source-free network and two voltage generators, one in series with the input port and one in series with the output port. If the time-dependent functions $e_1(t)$ and $e_2(t)$ describing these equivalent generators can be transformed to functions of frequency E_1 and E_2, respectively, the impedance representation of the linear twoport *with internal sources* becomes

$$V_1 = Z_{11}I_1 + Z_{12}I_2 + E_1$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2 + E_2 \tag{2}$$

and the equivalent network is that shown in Fig. 2.

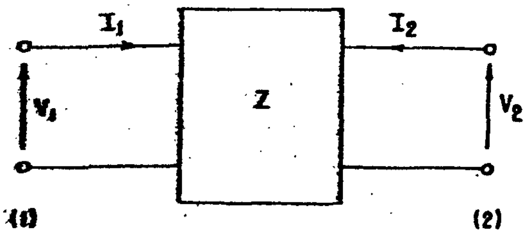

Fig. 1—Sign convention for impedance representation of linear twoport.

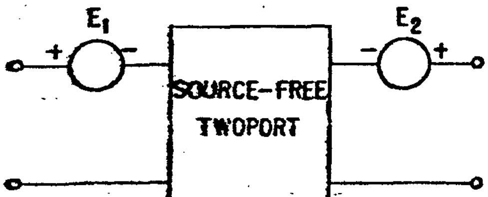

Fig. 2—Separation of twoport with internal sources into a source-free twoport and external voltage generators.

For the purpose of the analysis to follow, it should be emphasized again that such equations in general characterize the behavior of the twoport as a function of frequency. For practical purposes, it should be pointed out that in most cases the impedance parameters of a linear twoport can be measured at a particular frequency by applying sinusoidal voltages that produce outputs large compared with those caused by the internal sources.

The impedance representation of a linear twoport with internal sources has been reviewed because of its familiarity. However, it is well known that other representations, each leading to a different separation of the internal sources from the twoport, are possible. A particularly convenient one for the study of noise is the general-circuit-parameter representation[4]

$$V_1 = AV_2 + BI_2 + E$$
$$I_1 = CV_2 + DI_2 + I \tag{3}$$

where E and I are again functions of frequency which are the Fourier transforms of the time-dependent functions $e(t)$ and $i(t)$ describing the internal sources.

As shown in Fig. 3, the internal sources are now represented by a source of voltage acting in series with the input voltage and a source of current flowing in parallel with the input current. It will be seen that this particular representation of the internal sources leads to four noise parameters that can easily be derived from single-frequency measurements of the twoport noise factor as a function of input mismatch. It has the further advantage that such properties of the twoport as gain and input conductance do not enter into the noise factor expression in terms of these four parameters.

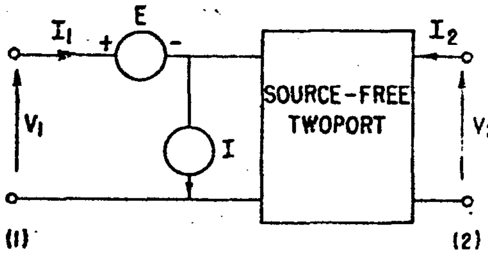

Fig. 3—Separation of twoport with internal sources into a source-free twoport and external input current and voltage sources.

III. Representations of Stationary Noise Sources

When a linear twoport contains stationary internal noise sources, the frequency functions E and I in (3) cannot be found by conventional Fourier methods from the time functions $e(t)$ and $i(t)$, since these are now random functions extending over all time and have infinite energy content. Two alternatives are possible. One may consider the substitute functions

$$e(t, T) = e(t) \quad \text{for } -\frac{T}{2} < t < \frac{T}{2}$$
$$= 0 \quad \text{for } \frac{T}{2} < |t|$$
$$i(t, T) = i(t) \quad \text{for } -\frac{T}{2} < t < \frac{T}{2}$$
$$= 0 \quad \text{for } \frac{T}{2} < |t| \tag{4}$$

[4] E. Guillemin, "Communication Networks," John Wiley and Sons, New York, N. Y., vol. 2, p. 138; 1935.

where T is some long but finite time interval, and use the Fourier transforms

$$E(\omega, T) = \frac{1}{2\pi}\int_{-T/2}^{T/2} e(t, T)e^{-j\omega t}dt$$

$$I(\omega, T) = \frac{1}{2\pi}\int_{-T/2}^{T/2} i(t, T)e^{-j\omega t}dt. \tag{5}$$

Alternatively, one may construct periodic functions,

$$e(t, T) = e(t) \quad \text{for } -\frac{T}{2} < t < \frac{T}{2}$$

$$e(t + nT, T) = e(t, T) \quad \text{with } n \text{ an integer,}$$

$$i(t, T) = i(t) \quad \text{for } -\frac{T}{2} < t < \frac{T}{2}$$

$$i(t + nT, T) = i(t, T) \quad \text{with } n \text{ an integer,} \tag{6}$$

and expand these functions into Fourier series with amplitudes

$$E_m(\omega, T) = \frac{1}{T}\int_{-T/2}^{T/2} e(t, T)e^{-j\omega t}dt$$

$$I_m(\omega, T) = \frac{1}{T}\int_{-T/2}^{T/2} i(t, T)e^{-j\omega t}dt \tag{7}$$

where $\omega = m2\pi/T$ with m an integer, and T is again finite. In either case, the substitute functions can be made to approach the actual functions as closely as desired by making the interval T larger and larger.

In either the Fourier-integral or the Fourier-series approach, we consider a set of substitute functions obtained in principle from a series of measurements a) on an ensemble of systems with identical statistical properties or b) on one and the same system at successive time intervals sufficiently separated so that no statistical correlation exists.[5]

Since noise is a statistical process, we are in general interested in statistical averages[6] rather than the exact details of any particular noise function. These averages are important since they relate to physically measurable stationary quantities. For example, in the Fourier-integral approach the spectral density of the noise voltage excitation is defined by

$$W_e(\omega) = \lim_{T\to\infty} \frac{\overline{|E(\omega, T)|^2}}{2\pi T} \tag{8}$$

where the bar indicates an arithmetic average of the Fourier transforms of an ensemble of functions described by (4). This spectral density is proportional to the noise power (in a narrow frequency band[7] around a given frequency) in a resistor across which the fluctuating voltage $e(t)$ appears.

Unfortunately, the notation developed in the literature for dealing with spectral densities is unwieldy, since the quantity with which the density is associated is relegated to a subscript. This is one of the reasons why researchers on noise have tended to use the historically older Fourier-series approach. The use of spectral densities is preferred in rigorous mathematical treatments of noise and in questions involving definitions of noise processes, since this assures that all points on the frequency axis within a narrow range are equivalent. For practical purposes, however, the noise amplitudes that are associated with discrete frequencies in the Fourier-series method can be as closely distributed as desired by choosing the time interval T sufficiently large.

IV. Relationship of Spectral Densities and Fourier Amplitudes to Mean-Square Fluctuations

If there is an open-circuit noise voltage $v(t)$ across a terminal pair, the mean-square value $\overline{v^2(t)}$ is related to the spectral density $W_v(\omega)$ and to the Fourier amplitudes $V_m(\omega)$ as follows:

$$\overline{v^2(t)} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} v^2(t)dt$$

$$= \int_{-\infty}^{+\infty} W_v(\omega)d\omega = \sum_{m=-\infty}^{\infty} \overline{|V_m(\omega)|^2} \tag{9}$$

where the amplitudes V_m are now those obtained as $T\to\infty$.

Suppose that the stationary time function $v(t)$ is passed through an ideal band-pass filter with the narrow bandwidth $\Delta f = \Delta\omega/2\pi$ centered at a frequency $f^0 = \omega^0/2\pi$. The mean-square value of the voltage appearing at the filter output, usually denoted by the symbol $\overline{v^2}$ and called the mean-square fluctuation of $v(t)$ within the frequency interval Δf" is

$$\overline{v^2} = 4\pi\Delta f W_v(\omega_0) = 2\overline{|V_m|^2}. \tag{10}$$

This equation relates the mean-square fluctuations, the spectral density, and the Fourier amplitude. If the filter is narrow-band but not ideal, the quantity Δf is the noise bandwidth.[1] The factor 2 in (10) arises because $W_v(\omega)$ and V_m have been defined on the negative as well as the positive frequency axis.

Eq. (10) shows that spectral densities and mean-square fluctuations are equivalent. Although mathematical limits are involved in the definitions, these

[5] The equivalence of the ensemble and time average is known as the ergodic hypothesis. An interesting discussion of the implications of the hypothesis can be found in Born, "Natural Philosophy of Cause and Chance," Oxford University Press, New York, N. Y., 1948. See also W. B. Davenport, Jr. and W. L. Root, "An Introduction to the Theory of Random Signals and Noise," McGraw-Hill Book Company, Inc., New York, N. Y., p. 66 ff.; 1958.

[6] W. R. Bennett, "Methods of solving noise problems," Proc. IRE, vol. 44, pp. 609–637; May, 1956.

[7] A frequency interval Δf is called narrow if the physical quantities under consideration are independent of frequency throughout the interval, and if $\Delta f \ll f_0$, where f_0 is the center frequency.

quantities can be measured to any desired degree of accuracy. For example, if one measures the power flowing into a termination connected to the terminals with which $v(t)$ is associated, one measures essentially $W_v(\omega_0)$, provided that the following requirements are met:

1) The termination is known and has a high impedance so that the voltage across the terminals remains essentially the open-circuit voltage $v(t)$, or the internal impedance associated with the two terminals is also known so that any change in voltage can be computed. The noise voltage contributed by the termination must either be negligible, or its statistical properties must be known so that its effect can be taken into account.

2) The termination is fed through a filter with a passband sufficiently narrow so that the spectral density and internal impedance are essentially constant throughout the band; or the termination is itself a resonant circuit with a high Q corresponding to a sufficiently narrow bandwidth.

3) The power measurement is made over a period of time that is long compared to the reciprocal bandwidth, $1/\Delta f$, of the filter. In this way the power-measuring device takes an average over many long time intervals that is equivalent to an ensemble average.

In the description of noise transformations by linear twoports that follows, the mean-square fluctuations will be used. Mean-square current fluctuations can be related to the Fourier amplitudes $I_m(\omega)$ by a procedure similar to the one just described. Since fluctuations of the cross products of statistical functions will also be used it should be noted that

$$\lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} v(t,T)i(t,T)dt = \sum_{m=-\infty}^{\infty} \overline{V_m I_m^*} \qquad (11)$$

where $i(t)$ is a noise current and V_m and I_m are again the amplitudes obtained as $T\to\infty$.

We may interpret the real part of the complex quantity $2\overline{V_m I_m^*}$ as the contribution to the average of vi from the frequency increment $\Delta f = 1/T$ at the angular frequency $\omega = m\Delta\omega$. However, the imaginary part of $V_m I_m^*$ also contains phase information that has to be used in noise computations. We shall, therefore, use the expression

$$\overline{vi^*} = 2\overline{V_m I_m^*} \qquad \text{for } m > 0, \omega > 0 \qquad (12)$$

as the complex cross-product fluctuations of $v(t)$ and $i(t)$ in the frequency increment Δf. They are related to the cross-spectral density by

$$\overline{vi^*} = 4\pi\Delta f W_{iv}(\omega) \qquad (13)$$

where

$$W_{iv}(\omega) = \lim_{T\to\infty} \frac{\overline{I^*(\omega,T)V(\omega,T)}}{2\pi T}. \qquad (14)$$

There are several ways of specifying the fluctuations (or the spectral densities) that characterize the internal noise sources. A mean-square voltage fluctuation can be given directly in units of volt² second. It is often convenient, however, to express this quantity in resistance units by using the Nyquist formula, which gives the mean-square fluctuation of the open-circuit noise voltage of a resistor R at temperature T as

$$\overline{e^2} = 4kTR\Delta f$$

where k is Boltzmann's constant. For any mean-square voltage fluctuation $\overline{e^2}$ within the frequency interval Δf, one defines the equivalent noise resistance R_n as

$$R_n = \frac{\overline{e^2}}{4kT_0\Delta f} \qquad (15)$$

where T_0 is the standard temperature, 290°K. The use of R_n has the advantage that a direct comparison can be made between the noise due to internal sources and the noise of resistances generally present in the circuit. Note that R_n is not the resistance of a physical resistor in the network in which e is a physical noise voltage and therefore does not appear as a resistance in the equivalent circuit of the network.

In a similar manner, a mean-square current fluctuation can be represented in terms of an equivalent noise conductance G_n which is defined by

$$G_n = \frac{\overline{i^2}}{4kT_0\Delta f}. \qquad (16)$$

V. Noise Transformations by Linear Twoports

The statistical averages discussed in Sections III and IV will now be used to describe the internal noise sources of a linear twoport. We may consider functions of the type given by (6) and represent the noise sources E and I in (3) by the Fourier amplitudes $E_m(\omega, T)$ and $I_m(\omega, T)$. Thus, if the circuit shown in Fig. 3 represents a separation of noise sources from a linear twoport, the noise-free circuit is preceded by a noise network. Since a noise-free network connected to a terminal pair does not change the signal-to-noise ratio (noise factor evaluated at that terminal pair) the noise factor of the over-all network is equal to that of the noise network.

To derive the noise factor, let us connect the noise network to a statistical source comprising an internal admittance Y_s and a current source again represented by a Fourier amplitude $I_s(\omega, T)$. The network to be used for the noise-factor computation is then as shown in Fig. 4. By definition,[8] *the spot noise factor (figure)* of a network at a specified frequency is given by the ratio

[8] "IRE Standards on Electron Tubes: Definitions of Terms, 1957," Proc. IRE, vol. 45, pp. 983–1010; July, 1957.

of 1) the total output noise power per unit bandwidth available at the output port to 2) that portion of 1) engendered by the input termination at the standard temperature T_0.

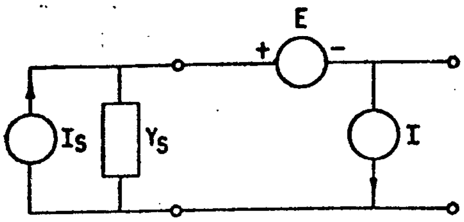

Fig. 4—Truncated network for noise-factor computation.

Now the total short-circuit noise current at the output of the network shown in Fig. 4 can be represented in terms of Fourier amplitudes by

$$I_s(\omega, T) + I_m(\omega, T) + Y_s E_m(\omega, T).$$

Let us assume that the internal noise of the twoport and the noise from the source are uncorrelated. If we then square the total short-circuit noise-current Fourier amplitudes, take ensemble averages and use equations similar to (10) and (12) to introduce mean-square fluctuations, we obtain a mean-square current fluctuation

$$\overline{i_s^2} + \overline{|i + Y_s e|^2} = \overline{i_s^2} + \overline{i^2} + |Y_s|^2\overline{e^2} + Y_s^*\overline{ie^*} + Y_s\overline{i^*e} \tag{17}$$

to which the total output noise power is proportional. It should be noted here that when e and i are complex, the symbols $\overline{e^2}$ and $\overline{i^2}$ denote, by convention, $\overline{|e|^2}$ and $\overline{|i|^2}$. Since the noise power due to the source alone is proportional to $\overline{i_s^2}$, the noise factor becomes

$$F = 1 + \frac{\overline{|i + Y_s e|^2}}{\overline{i_s^2}}. \tag{18}$$

In the denominator, the mean-square source noise current is related to the source conductance G_s by the Nyquist formula

$$\overline{i_s^2} = 4kT_0G_s\Delta f. \tag{19}$$

In the numerator of (18), the four real variables involved in $\overline{i^2}$, $\overline{e^2}$ and $\overline{ie^*}$, where e^* is the complex conjugate of e, describe the internal noise sources. If the Fourier transforms, (5), are used to characterize the noise, the self-spectral densities of noise current and voltage and the cross-spectral density of these quantities would have to be specified.

Before proceeding, let us review in general terms the methods employed and the conclusions reached. The representation of the internal noise sources of a noisy linear twoport by a voltage generator and a current generator lumps the effect of all internal noise sources into the two generators. A complete specification of these generators is thus equivalent to a complete description of the internal sources, as far as their contribution to terminal voltages and currents is concerned. Since the sources under consideration are noise sources, their description is confined to the methods applicable to noise. The extent and detail of the description depends on the amount of detail envisioned in the analysis. Since, in the case of the noise factor, only the mean square fluctuations of output currents are sought, the specification of self and cross-product fluctuations of the generator voltages and currents is adequate.

The expression for the noise factor given in (18) can be simplified if the noise current is split into two components, one perfectly correlated and one uncorrelated with the noise voltage. The uncorrelated noise current, designated by i_u, is defined at each frequency by the relations

$$\overline{ei_u^*} = 0 \tag{20}$$

$$\overline{(i - i_u)i_u^*} = 0. \tag{21}$$

The correlated noise current, $i - i_u$, can be written as $Y_\gamma e$, where the complex constant $Y_\gamma = G_\gamma + jB_\gamma$ has the dimensions of an admittance and is called the correlation admittance. The cross-product fluctuation $\overline{ei^*}$ may then be written

$$\overline{ei^*} = \overline{e(i - i_u)^*} = Y_\gamma^*\overline{e^2}. \tag{22}$$

The noise-voltage fluctuation can be expressed in terms of an equivalent noise resistance R_n as

$$\overline{e^2} = 4kT_0R_n\Delta f \tag{23}$$

and the uncorrelated noise-current fluctuation in terms of an equivalent noise conductance G_u as

$$\overline{i_u^2} = 4kT_0G_u\Delta f. \tag{24}$$

The fluctuations of the total noise current are then

$$\overline{i^2} = \overline{|i - i_u|^2} + \overline{i_u^2} = 4kT_0[|Y_\gamma|^2R_n + G_u]\Delta f. \tag{25}$$

From (18)–(24), the formula for the noise factor becomes

$$F = 1 + \frac{1}{4kT_0G_s\Delta f}\left[\overline{i_u^2} + |Y_s + Y_\gamma|^2\overline{e^2}\right] \tag{26}$$

$$= 1 + \frac{G_u}{G_s} + \frac{R_n}{G_s}\left[(G_s + G_\gamma)^2 + (B_s + B_\gamma)^2\right]. \tag{27}$$

Thus, the noise factor is a function of the four parameters G_u, R_n, G_γ and B_γ. These depend, in general, upon the operating point and operating frequency of the twoport, but not upon the external circuitry. In a vacuum tube triode the correlation susceptance B_γ is negligibly small at frequencies such that transit times are small compared to the period. Also, G_u and G_γ are vanishingly small when there is little grid loading. Thus, tube noise at low frequencies is adequately characterized by the single nonzero constant R_n. Tube noise at high frequencies and transistor noise at all frequencies have no such simple representation.

Since the noise factor is an explicit function of the source conductance and susceptance, it can readily be shown that the noise factor has an optimum (minimum) value at some optimum source admittance $Y_o = G_o + jB_o$ where

$$G_o = \left[\frac{G_u + R_n G_\gamma^2}{R_n}\right]^{1/2} \tag{28}$$

$$B_o = -B_\gamma \tag{29}$$

and the value of this minimum noise factor is

$$F_o = 1 + 2R_n(G_\gamma + G_o). \tag{30}$$

In terms of G_o, B_o and F_o, the noise factor for any arbitrary source impedance then becomes

$$F = F_o + \frac{R_n}{G_s}[(G_s - G_o)^2 + (B_s - B_o)^2]. \tag{31}$$

Eq. (31) shows that the four real parameters F_o, G_o, B_o and R_n give the noise factor of a twoport for every input termination of the twoport.[9] As shown in the methods of test that are published in this issue, a measurement of the minimum noise factor and of the source admittance Y_o with which F_o is achieved gives the first three parameters. The parameter R_n can be computed from an additional measurement of the noise factor for a source admittance Y_s other than Y_o.

From the given values, F_o, G_o, B_o and R_n, one can compute, if desired, the noise fluctuations $\overline{i^2}$, $\overline{e^2}$, and $\overline{ei^*}$ (or the corresponding spectral densities). To do this, one uses (28) to (30) to find Y_γ and G_u. From these one evaluates $\overline{e^2}$, $\overline{i^2}$, and $\overline{ei^*}$ from (23), (25), and (22). The fluctuations of any terminal voltage or current of the twoport produced with a given source and load can then be evaluated from the known coefficients, (A, B, C of (3), and the known noise fluctuations. Thus, the n[oise] in a twoport is completely characterized (with regar[d to] the fluctuations or the spectral densities at the [ter]minals) by the noise fluctuations $\overline{e^2}$, $\overline{i^2}$ and $\overline{ei^*}$, or al[ter]nately by the four noise parameters F_o, G_o, B_o [and R_n].

[9] Reasoning similar to that leading to (31) can be carried out in a dual representation, where impedances and admittances are interchanged. An equation similar to (31) results, again involving four noise parameters, some of which are different from the ones used here.

VI. Conclusion

The preceding discussion showed that, with lim[ited] objectives, the noise in a linear twoport can be c[har]acterized adequately by a limited number of parame[ters]. Thus, if one seeks only information concerning [the] mean-square fluctuations or the spectral densiti[es of] currents or voltages into or across the ports of [the] twoport at a particular frequency and for arbitrary [cir]cuit connections of the twoport, it is sufficient to spe[cify] two mean-square fluctuations and one product fluc[tua]tion or two spectral densities and one cross-spe[ctral] density. This involves the specification of four [real] numbers. If a band of frequencies is being conside[red], these quantities have to be given as functions of [fre]quency unless they are approximately constant over [the] band.

Different separations of internal noise sources lead, in general, to different frequency dependence[s of] the resulting fluctuations or spectral densities. Thus [a] particular separation of the noise sources may be p[ref]erable to another separation if it is found that fluc[tua]tions (spectral densities) are less frequency-depend[ent] in the band. In particular, if available info[rmation] about the physics of the noise in a particular dev[ice] suggests introducing, *inside the twoport*, appropri[ate] noise generators of fluctuations (spectral densiti[es]) with no frequency dependence, or with a simple f[re]quency dependence, it may be more advantageous [to] specify the device in terms of these physically sugg[es]tive generators.

However, usually one resorts to the characterizati[on] of noise presented in this paper, since they have the [ad]vantage that they do not require any knowledge of [the] physics of the internal noise. Furthermore, noise-fac[tor] measurements performed on the twoport yield (more [or] less) directly the noise parameters of the general cir[cuit] representation, namely the fluctuations (spectral den[si]ties) of the voltage and current generators attached [to] the input of the noise-free equivalent of the twop[ort]. This fact recommends the use of the noise paramet[ers] natural to the general-circuit-parameter representati[on].