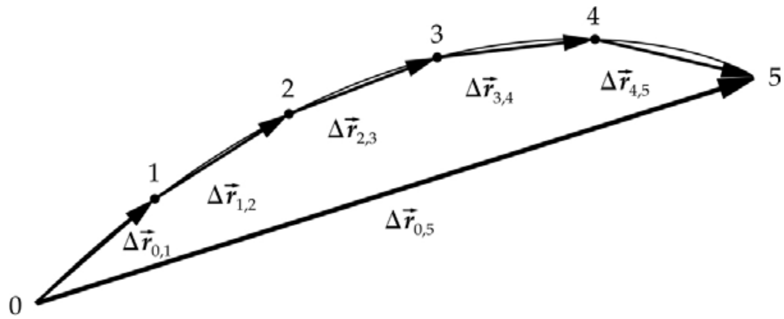
Chapter 3
Motion in Two and Three Dimensions

Conceptual Problems

***1** •

Determine the Concept The distance traveled along a path can be represented as a sequence of displacements.



Suppose we take a trip along some path and consider the trip as a sequence of many very small displacements. The net displacement is the vector sum of the very small displacements, and the total distance traveled is the sum of the magnitudes of the very small displacements. That is,

$$\textit{total distance} = \left|\Delta\vec{r}_{0,1}\right| + \left|\Delta\vec{r}_{1,2}\right| + \left|\Delta\vec{r}_{2,3}\right| + \ldots + \left|\Delta\vec{r}_{N-1,N}\right|$$

where N is the number of very small displacements. (For this to be exactly true we have to take the limit as N goes to infinity and each displacement magnitude goes to zero.) Now, using "the shortest distance between two points is a straight line," we have

$$\left|\Delta\vec{r}_{0,N}\right| \le \left|\Delta\vec{r}_{0,1}\right| + \left|\Delta\vec{r}_{1,2}\right| + \left|\Delta\vec{r}_{2,3}\right| + \ldots + \left|\Delta\vec{r}_{N-1,N}\right|,$$

where $\left|\Delta\vec{r}_{0,N}\right|$ is the magnitude of the net displacement.

Hence, we have shown that the magnitude of the displacement of a particle is less than or equal to the distance it travels along its path.

2 •

Determine the Concept The displacement of an object is its final position vector minus its initial position vector ($\Delta\vec{r} = \vec{r}_\text{f} - \vec{r}_\text{i}$). The displacement can be less but never more than the distance traveled. Suppose the path is one complete trip around the earth at the equator. Then, the displacement is 0 but the distance traveled is $2\pi R_\text{e}$.

3 •

Determine the Concept The important distinction here is that ***average velocity*** is being requested, as opposed to ***average speed***.

The average velocity is defined as the displacement divided by the elapsed time.

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{0}{\Delta t} = 0$$

The displacement for any trip around the track is zero. Thus we see that no matter how fast the race car travels, the average velocity is always zero at the end of each complete circuit.

What is the correct answer if we were asked for *average speed*?

The average speed is defined as the distance traveled divided by the elapsed time.

$$v_{av} \equiv \frac{\text{total distance}}{\Delta t}$$

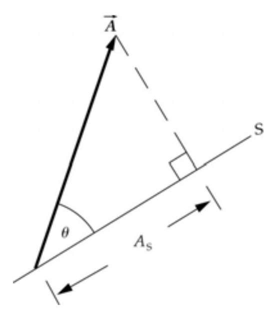
For one complete circuit of any track, the total distance traveled will be greater than zero and the average is not zero.

4 •

False. Vectors are quantities with magnitude and direction that can be added and subtracted like displacements. Consider two vectors that are equal in magnitude and oppositely directed. Their sum is zero, showing by counterexample that *the statement is false.*

5 •

Determine the Concept We can answer this question by expressing the relationship between the magnitude of vector $\vec{\boldsymbol{A}}$ and its component A_S and then using properties of the cosine function.



Express A_S in terms of A and θ:

$$A_S = A\cos\theta$$

Take the absolute value of both sides of this expression:

$$|A_S| = |A\cos\theta| = A|\cos\theta|$$

and

$$|\cos\theta| = \frac{|A_S|}{A}$$

Using the fact that $0 < |\cos\theta| \le 1$, substitute for $|\cos\theta|$ to obtain:

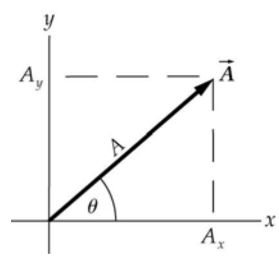
$$0 < \frac{|A_S|}{A} \le 1 \text{ or } 0 < |A_S| \le A$$

$\boxed{\text{No. The magnitude of a component of a vector must be less than or equal to the magnitude of the vector.}}$

$\boxed{\text{If the angle } \theta \text{ shown in the figure is equal to } 0° \text{ or multiples of } 180°\text{, then the magnitude of the vector and its component are equal.}}$

***6** •

Determine the Concept The diagram shows a vector $\vec{A}$ and its components A_x and A_y. We can relate the magnitude of $\vec{A}$ is related to the lengths of its components through the Pythagorean theorem.



Suppose that $\vec{A}$ is equal to zero. Then $A^2 = A_x^2 + A_y^2 = 0$.

But $A_x^2 + A_y^2 = 0 \Rightarrow A_x = A_y = 0$.

$\boxed{\text{No. If a vector is equal to zero, each of its components must be zero too.}}$

7 •

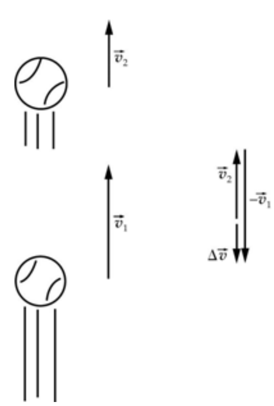
Determine the Concept No. Consider the special case in which $\vec{B} = -\vec{A}$. If $\vec{B} = -\vec{A} \ne 0$, then $\vec{C} = 0$ and the magnitudes of the components of $\vec{A}$ and $\vec{B}$ are larger than the components of $\vec{C}$.

***8** •

Determine the Concept The *instantaneous acceleration* is the limiting value, as Δt approaches zero, of $\Delta\vec{v}/\Delta t$. Thus, the acceleration vector is in the same direction as $\Delta\vec{v}$.

False. Consider a ball that has been thrown upward near the surface of the earth and is slowing down. *The direction of its motion is upward.*

The diagram shows the ball's velocity vectors at two instants of time and the determination of $\Delta\vec{v}$. Note that because $\Delta\vec{v}$ is downward so is the *acceleration* of the ball.



9 •

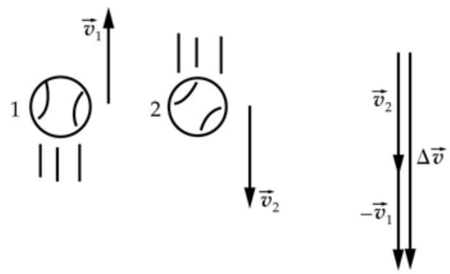
Determine the Concept The *instantaneous acceleration* is the limiting value, as Δt approaches zero, of $\Delta\vec{v}/\Delta t$ and is in the same direction as $\Delta\vec{v}$.

Other than through the definition of $\vec{a}$, the instantaneous velocity and acceleration vectors are unrelated. Knowing the direction of the velocity at one instant tells one nothing about how the velocity is changing at that instant. $\boxed{(e)\text{ is correct.}}$

10 •

Determine the Concept The changing velocity of the golf ball during its flight can be understood by recognizing that it has both horizontal and vertical components. The nature of its acceleration near the highest point of its flight can be understood by analyzing the vertical components of its velocity on either side of this point.

At the highest point of its flight, the ball is still *traveling horizontally* even though its vertical velocity is momentarily zero. The figure to the right shows the vertical components of the ball's velocity just before and just after it has reached its highest point. The change in velocity during this short interval is a non-zero, downward-pointing vector. Because the acceleration is proportional to the change in velocity, it must also be nonzero.



$\boxed{(d)\text{ is correct.}}$

Remarks: Note that v_x is nonzero and v_y is zero, while a_x is zero and a_y is nonzero.

11 •

Determine the Concept The change in the velocity is in the same direction as the acceleration. Choose an *x*-*y* coordinate system with east being the positive *x* direction and north the positive *y* direction.

Given our choice of coordinate system, the *x* component of $\vec{a}$ is negative and so $\vec{v}$ will decrease. The *y* component of $\vec{a}$ is positive and so $\vec{v}$ will increase toward the north.

$\boxed{(c)\text{ is correct.}}$

***12 •**

Determine the Concept The average velocity of a particle, $\vec{v}_{av}$, is the ratio of the particle's displacement to the time required for the displacement.

(*a*) We can calculate $\Delta\vec{r}$ from the given information and Δt is known. $\boxed{(a)\text{ is correct.}}$

(*b*) We do not have enough information to calculate $\Delta\vec{v}$ and cannot compute the particle's average acceleration.

(*c*) We would need to know how the particle's velocity varies with time in order to compute its instantaneous velocity.

(*d*) We would need to know how the particle's velocity varies with time in order to compute its instantaneous acceleration.

13 ••

Determine the Concept The velocity vector is always in the direction of motion and, thus, tangent to the path.

(*a*) $\boxed{\text{The velocity vector, as a consequence of always being in the direction of motion, is tangent to the path.}}$

(*b*) A sketch showing two velocity vectors for a particle moving along a path is shown to the right.

14 •

Determine the Concept An object experiences acceleration whenever either its speed changes or it changes direction.

The acceleration of a car moving in a straight path at constant speed is zero. In the other examples, either the magnitude or the direction of the velocity vector is changing and, hence, the car is accelerated. $\boxed{(b)\text{ is correct.}}$

***15** •

Determine the Concept The velocity vector is defined by $\vec{v} = d\vec{\mathbf{r}}/dt$, while the acceleration vector is defined by $\vec{a} = d\vec{\mathbf{v}}/dt$.

(*a*) A car moving along a straight road while braking.

(*b*) A car moving along a straight road while speeding up.

(*c*) A particle moving around a circular track at constant speed.

16 •

Determine the Concept A particle experiences accelerated motion when either its speed or direction of motion changes.

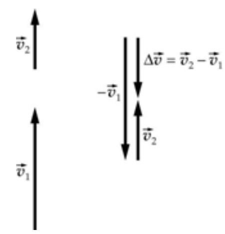
A particle moving at constant speed in a circular path is accelerating because the direction of its velocity vector is changing.

If a particle is moving at constant velocity, it is not accelerating.

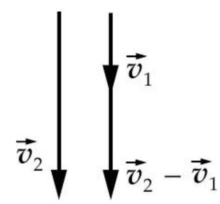
17 ••

Determine the Concept The acceleration vector is in the same direction as the *change in velocity vector,* $\Delta\vec{v}$.

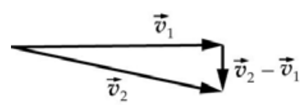
(*a*) The sketch for the dart thrown upward is shown to the right. The acceleration vector is in the direction of the *change* in the velocity vector $\Delta\vec{v}$.



(*b*) The sketch for the falling dart is shown to the right. Again, the acceleration vector is in the direction of the *change* in the velocity vector $\Delta\vec{v}$.



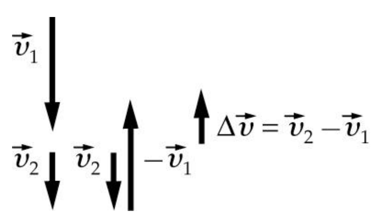
(*c*) The acceleration vector is in the direction of the *change* in the velocity vector … and hence is downward as shown the right:



***18** ••

Determine the Concept The acceleration vector is in the same direction as the *change in velocity vector*, $\Delta\vec{v}$.

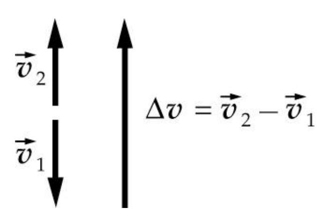
The drawing is shown to the right.



19 ••

Determine the Concept The acceleration vector is in the same direction as the *change in velocity vector,* $\Delta\vec{v}$.

The sketch is shown to the right.



20 •

Determine the Concept We can decide what the pilot should do by considering the speeds of the boat and of the current.

Give up. The speed of the stream is equal to the maximum speed of the boat in still water. The best the boat can do is, while facing directly upstream, maintain its position relative to the bank. $\boxed{(d)\text{ is correct.}}$

***21 •**

Determine the Concept True. In the absence of air resistance, both projectiles experience the same downward acceleration. Because both projectiles have initial vertical velocities of zero, their vertical motions must be identical.

22 •

Determine the Concept In the absence of air resistance, the horizontal component of the projectile's velocity is constant for the duration of its flight.

At the highest point, the speed is the horizontal component of the initial velocity. The vertical component is zero at the highest point. $\boxed{(e)\text{ is correct.}}$

23 •

Determine the Concept In the absence of air resistance, the acceleration of the ball depends only on the *change in its velocity* and is independent of its velocity.

As the ball moves along its trajectory between points A and C, the vertical component of its velocity decreases and the *change* in its velocity is a downward pointing vector. Between points C and E, the vertical component of its velocity increases and the *change* in its velocity is also a downward pointing vector. There is no change in the horizontal component of the velocity. $\boxed{(d)\text{ is correct.}}$

24 •

Determine the Concept In the absence of air resistance, the horizontal component of the velocity remains constant throughout the flight. The vertical component has its maximum values at launch and impact.

(*a*) The speed is greatest at A and E.

(*b*) The speed is least at point C.

(*c*) The speed is the same at A and E. The horizontal components are equal at these points but the vertical components are oppositely directed.

25 •

Determine the Concept Speed is a scalar quantity, whereas acceleration, equal to the *rate of change* of velocity, is a vector quantity.

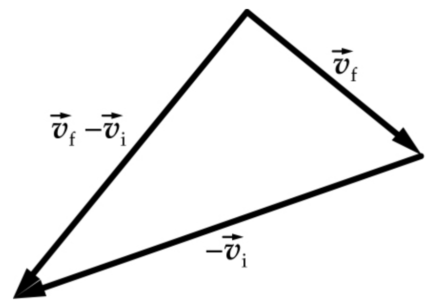
(*a*) False. Consider a ball on the end of a string. The ball can move with constant *speed* (a scalar) even though its *acceleration* (a vector) is always changing direction.

(*b*) True. From its definition, if the acceleration is zero, the velocity must be constant and so, therefore, must be the speed.

26 •

Determine the Concept The average acceleration vector is defined by $\vec{a}_{\text{av}} = \Delta\vec{v}/\Delta t$.

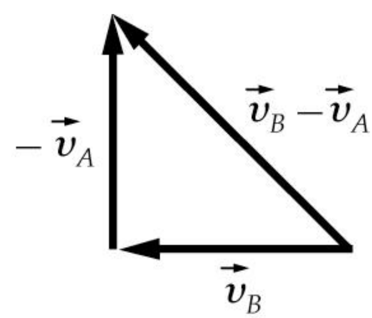
The direction of $\vec{a}_{\text{av}}$ is that of $\Delta\vec{v} = \vec{v}_{\text{f}} - \vec{v}_{\text{i}}$, as shown to the right.



27 •

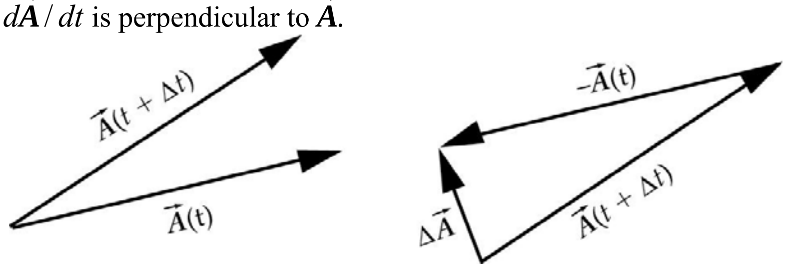
Determine the Concept The velocity of B relative to A is $\vec{v}_{\text{BA}} = \vec{v}_{\text{B}} - \vec{v}_{\text{A}}$.

The direction of $\vec{v}_{\text{BA}} = \vec{v}_{\text{B}} - \vec{v}_{\text{A}}$ is shown to the right.



***28 ••**

(*a*) The vectors $\vec{A}(t)$ and $\vec{A}(t+\Delta t)$ are of equal length but point in slightly different directions. $\Delta\vec{A}$ is shown in the diagram below. Note that $\Delta\vec{A}$ is nearly perpendicular to $\vec{A}(t)$. For very small time intervals, $\Delta\vec{A}$ and $\vec{A}(t)$ are perpendicular to one another. Therefore, $d\vec{A}/dt$ is perpendicular to $\vec{A}$.



(*b*) If $\vec{A}$ represents the position of a particle, the particle must be undergoing circular motion (i.e., it is at a constant distance from some origin). The velocity vector is tangent to the particle's trajectory; in the case of a circle, it is perpendicular to the circle's radius.

(*c*) Yes, it could in the case of uniform circular motion. The speed of the particle is constant, but its heading is changing constantly. The acceleration vector in this case is always perpendicular to the velocity vector.

29 ••

Determine the Concept The velocity vector is in the same direction as *the change in the position vector* while the acceleration vector is in the same direction as *the change in the velocity vector*. Choose a coordinate system in which the y direction is north and the x direction is east.

(*a*)

Path	Direction of velocity vector
AB	north
BC	northeast
CD	east
DE	southeast
EF	south

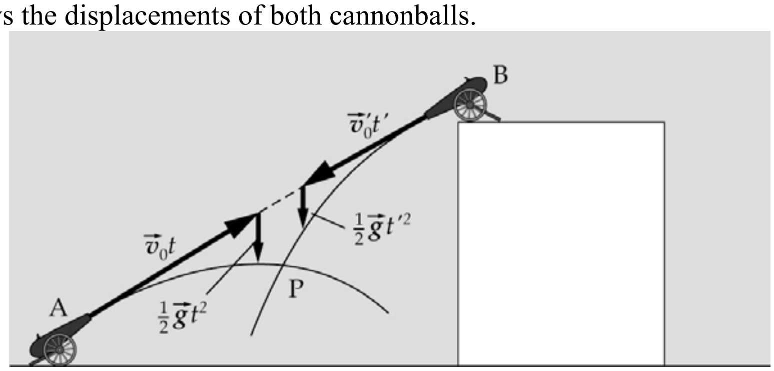
(*b*)

Path	Direction of acceleration vector
AB	north
BC	southeast
CD	0
DE	southwest
EF	north

(*c*) The magnitudes are comparable, but larger for DE since the radius of the path is smaller there.

***30** ••

Determine the Concept We'll assume that the cannons are identical and use a constant-acceleration equation to express the displacement of each cannonball as a function of time. Having done so, we can then establish the condition under which they will have the same vertical position at a given time and, hence, collide. The modified diagram shown below shows the displacements of both cannonballs.



Express the displacement of the cannonball from cannon A at any time t after being fired and before any collision:

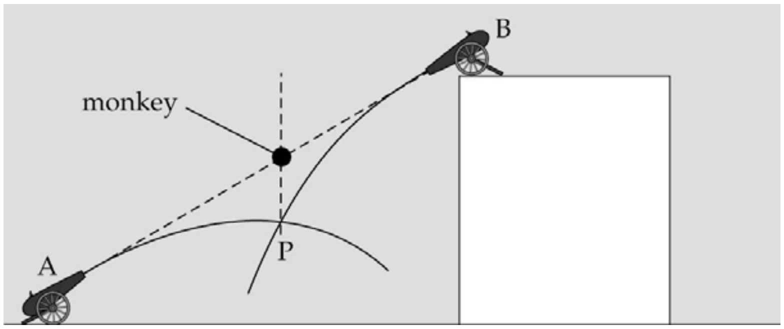
$$\Delta\vec{r} = \vec{v}_0t + \tfrac{1}{2}\vec{g}t^2$$

Express the displacement of the cannonball from cannon A at any time t' after being fired and before any collision:

$$\Delta\vec{r}' = \vec{v}_0't' + \tfrac{1}{2}\vec{g}t'^2$$

If the guns are fired simultaneously, $t = t'$ and the balls are the same distance $\frac{1}{2}gt^2$ below the line of sight at all times. Therefore, they should fire the guns simultaneously.

Remarks: This is the "monkey and hunter" problem in disguise. If you imagine a monkey in the position shown below, and the two guns are fired simultaneously, and the monkey begins to fall when the guns are fired, then the monkey and the two cannonballs will all reach point P at the same time.



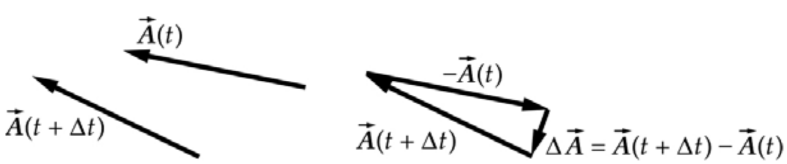
31 ••

Determine the Concept The droplet leaving the bottle has the same horizontal velocity as the ship. During the time the droplet is in the air, it is also moving horizontally with the same velocity as the rest of the ship. Because of this, it falls into the vessel, which has the same horizontal velocity. Because you have the same horizontal velocity as the ship does, you see the same thing as if the ship were standing still.

32 •

Determine the Concept

(*a*) Because $\vec{A}$ and $\vec{D}$ are tangent to the path of the stone, either of them could represent the velocity of the stone.



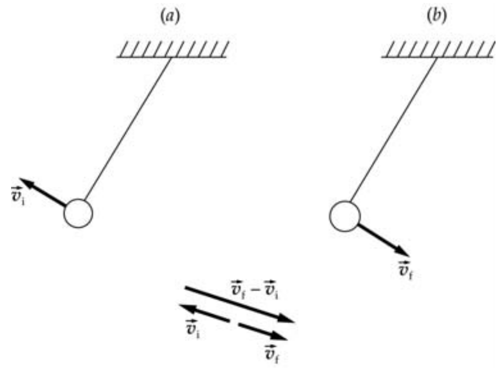
(*b*) Let the vectors $\vec{A}(t)$ and $\vec{B}(t+\Delta t)$ be of equal length but point in slightly different directions as the stone moves around the circle. These two vectors and $\Delta\vec{A}$ are shown in the diagram above. Note that $\Delta\vec{A}$ is nearly perpendicular to $\vec{A}(t)$. For very small time intervals, $\Delta\vec{A}$ and $\vec{A}(t)$ are perpendicular to one another. Therefore, $d\vec{A}/dt$ is perpendicular to $\vec{A}$ and only the vector $\vec{E}$ could represent the acceleration of the stone.

33 •

Determine the Concept True. An object accelerates when its velocity changes; that is, when either its speed or its direction changes. When an object moves in a circle the direction of its motion is continually changing.

34 ••

Picture the Problem In the diagram, (*a*) shows the pendulum just before it reverses direction and (*b*) shows the pendulum just after it has reversed its direction. The acceleration of the bob is in the direction of the *change* in the velocity $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$ and is tangent to the pendulum trajectory at the point of reversal of direction. This makes sense because, at an extremum of motion, $v = 0$, so there is no centripetal acceleration. However, because the velocity is reversing direction, the tangential acceleration is nonzero.



35 •

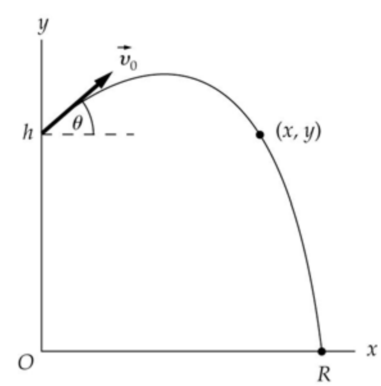
Determine the Concept The principle reason is aerodynamic drag. When moving through a fluid, such as the atmosphere, the ball's acceleration will depend strongly on its velocity.

Estimation and Approximation

***36 ••**

Picture the Problem During the flight of the ball the acceleration is constant and equal to 9.81 m/s^2 directed downward. We can find the flight time from the vertical part of the motion, and then use the horizontal part of the motion to find the horizontal distance. We'll assume that the release point of the ball is 2 m above your feet.

Make a sketch of the motion. Include coordinate axes, initial and final positions, and initial velocity components:



Obviously, how far you throw the ball will depend on how fast you can throw it. A major league baseball pitcher can throw a fastball at 90 mi/h or so. Assume that you can throw a ball at two-thirds that speed to obtain:

$$v_0 = 60\,\text{mi/h} \times \frac{0.447\,\text{m/s}}{1\,\text{mi/h}} = 26.8\,\text{m/s}$$

There is no acceleration in the x direction, so the horizontal motion is one of constant velocity. Express the horizontal position of the ball as a function of time:

$$x = v_{0x}t \qquad (1)$$

Assuming that the release point of the ball is a distance h above the ground, express the vertical position of the ball as a function of time:

$$y = h + v_{0y}t + \tfrac{1}{2}a_y t^2 \qquad (2)$$

(a) For $\theta = 0$ we have:

$$v_{0x} = v_0 \cos\theta_0 = (26.8\,\text{m/s})\cos 0° = 26.8\,\text{m/s}$$

and

$$v_{0y} = v_0 \sin\theta_0 = (26.8\,\text{m/s})\sin 0° = 0$$

Substitute in equations (1) and (2) to obtain:

$$x = (26.8\,\text{m/s})t$$

and

$$y = 2\,\text{m} + \tfrac{1}{2}\left(-9.81\,\text{m/s}^2\right)t^2$$

Eliminate t between these equations to obtain:

$$y = 2\,\text{m} - \frac{4.91\,\text{m/s}^2}{(26.8\,\text{m/s})^2}x^2$$

At impact, $y = 0$ and $x = R$:

$$0 = 2\,\text{m} - \frac{4.91\,\text{m/s}^2}{(26.8\,\text{m/s})^2}R^2$$

Solve for R to obtain:

$$R = \boxed{17.1\,\text{m}}$$

(b) Using trigonometry, solve for v_{0x} and v_{0y}:

$$v_{0x} = v_0 \cos\theta_0 = (26.8\,\text{m/s})\cos 45° = 19.0\,\text{m/s}$$

and

$$v_{0y} = v_0 \sin\theta_0 = (26.8\,\text{m/s})\sin 45° = 19.0\,\text{m/s}$$

Substitute in equations (1) and (2) to obtain:

$$x = (19.0\,\text{m/s})t$$

and

$$y = 2\,\text{m} + (19.0\,\text{m/s})t + \tfrac{1}{2}\left(-9.81\,\text{m/s}^2\right)t^2$$

Eliminate t between these equations to obtain:

$$y = 2\,\text{m} + x - \frac{4.905\,\text{m/s}^2}{(19.0\,\text{m/s})^2}x^2$$

At impact, $y = 0$ and $x = R$. Hence:

$$0 = 2\,\text{m} + R - \frac{4.905\,\text{m/s}^2}{(19.0\,\text{m/s})^2}R^2$$

or

$$R^2 - (73.60\,\text{m})R - 147.2\,\text{m}^2 = 0$$

Solve for R (you can use the "solver" or "graph" functions of your calculator) to obtain:

$$R = \boxed{75.6\,\text{m}}$$

(*c*) Solve for v_{0x} and v_{0y}:

$$v_{0x} = v_0 = 26.8\,\text{m/s}$$

and

$$v_{0y} = 0$$

Substitute in equations (1) and (2) to obtain:

$$x = (26.8\,\text{m/s})t$$

and

$$y = 14\,\text{m} + \tfrac{1}{2}\left(-9.81\,\text{m/s}^2\right)t^2$$

Eliminate t between these equations to obtain:

$$y = 14\,\text{m} - \frac{4.905\,\text{m/s}^2}{(26.8\,\text{m/s})^2}x^2$$

At impact, $y = 0$ and $x = R$:

$$0 = 14\,\text{m} - \frac{4.905\,\text{m/s}^2}{(26.8\,\text{m/s})^2}R^2$$

Solve for R to obtain:

$$R = \boxed{45.3\,\text{m}}$$

(*d*) Using trigonometry, solve for v_{0x} and v_{0y}:

$$v_{0x} = v_{0y} = 19.0\,\text{m/s}$$

Substitute in equations (1) and (2) to obtain:

$$x = (19.0\,\text{m/s})t$$

and

$$y = 14\,\text{m} + (19.0\,\text{m/s})t + \tfrac{1}{2}\left(-9.81\,\text{m/s}^2\right)t^2$$

Eliminate t between these equations to obtain:

$$y = 14\,\text{m} + x - \frac{4.905\,\text{m/s}^2}{(19.0\,\text{m/s})^2}x^2$$

At impact, $y = 0$ and $x = R$:

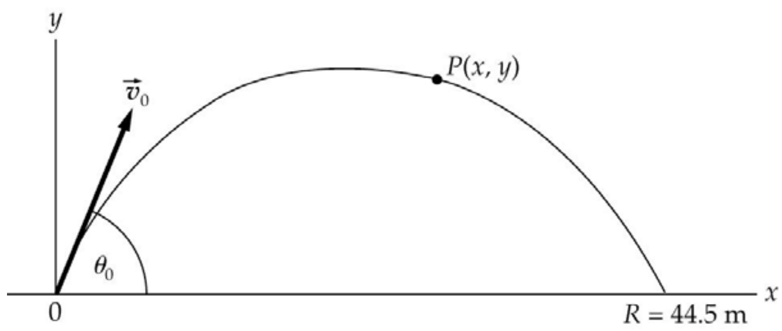
$$0 = 14\,\text{m} + R - \frac{4.905\,\text{m/s}^2}{(19.0\,\text{m/s})^2}R^2$$

Solve for R (you can use the "solver" or "graph" function of your calculator) to obtain:

$$R = \boxed{85.6\,\text{m}}$$

37 ••

Picture the Problem We'll ignore the height of Geoff's release point above the ground and assume that he launched the brick at an angle of 45°. Because the velocity of the brick at the highest point of its flight is equal to the horizontal component of its initial velocity, we can use constant-acceleration equations to relate this velocity to the brick's x and y coordinates at impact. The diagram shows an appropriate coordinate system and the brick when it is at point P with coordinates (x, y).



Using a constant-acceleration equation, express the x coordinate of the brick as a function of time:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

or, because $x_0 = 0$ and $a_x = 0$,

$$x = v_{0x}t$$

Express the y coordinate of the brick as a function of time:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

or, because $y_0 = 0$ and $a_y = -g$,

$$y = v_{0y}t - \tfrac{1}{2}gt^2$$

Eliminate the parameter t to obtain:

$$y = (\tan\theta_0)x - \frac{g}{2v_{0x}^2}x^2$$

Use the brick's coordinates when it strikes the ground to obtain:

$$0 = (\tan\theta_0)R - \frac{g}{2v_{0x}^2}R^2$$

where R is the range of the brick.

Solve for v_{0x} to obtain:

$$v_{0x} = \sqrt{\frac{gR}{2\tan\theta_0}}$$

Substitute numerical values and evaluate v_{0x}:

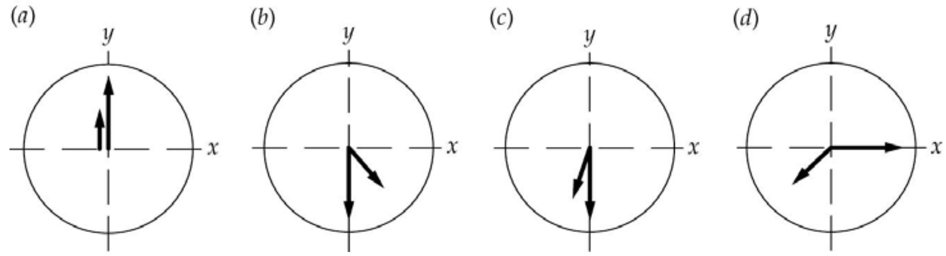
$$v_{0x} = \sqrt{\frac{(9.81\,\text{m/s}^2)(44.5\,\text{m})}{2\tan 45°}} = \boxed{14.8\,\text{m/s}}$$

Note that, at the brick's highest point, $v_y = 0$.

Vectors, Vector Addition, and Coordinate Systems

38 •

Picture the Problem Let the positive y direction be straight up, the positive x direction be to the right, and $\vec{A}$ and $\vec{B}$ be the position vectors for the minute and hour hands. The pictorial representation below shows the orientation of the hands of the clock for parts (*a*) through (*d*).



(*a*) The position vector for the minute hand at12:00 is:

$$\vec{A}_{12:00} = \boxed{(0.5\,\text{m})\hat{j}}$$

The position vector for the hour hand at 12:00 is:

$$\vec{B}_{12:00} = \boxed{(0.25\,\text{m})\hat{j}}$$

(*b*) At 3:30, the minute hand is positioned along the $-y$ axis, while the hour hand is at an angle of (3.5 h)/12 h × 360° = 105°, measured clockwise from the top.

The position vector for the minute hand is:

$$\vec{A}_{3:30} = \boxed{-(0.5\,\text{m})\hat{j}}$$

Find the x-component of the vector representing the hour hand:

$$B_x = (0.25\,\text{m})\sin 105° = 0.241\,\text{m}$$

Find the y-component of the vector representing the hour hand:

$$B_y = (0.25\,\text{m})\cos 105° = -0.0647\,\text{m}$$

The position vector for the hour hand is:

$$\vec{B}_{3:30} = \boxed{(0.241\,\text{m})\hat{i} - (0.0647\,\text{m})\hat{j}}$$

(*c*) At 6:30, the minute hand is positioned along the $-y$ axis, while the hour hand is at an angle of (6.5 h)/12 h × 360° = 195°, measured clockwise from the top.

The position vector for the minute hand is:

$$\vec{A}_{6:30} = \boxed{-(0.5\,\text{m})\hat{j}}$$

Find the x-component of the vector representing the hour hand:

$$B_x = (0.25\,\text{m})\sin 195° = -0.0647\,\text{m}$$

Find the y-component of the vector representing the hour hand:

$$B_y = (0.25\,\text{m})\cos 195° = -0.241\,\text{m}$$

The position vector for the hour hand is:

$$\vec{B}_{6:30} = \boxed{-(0.0647\,\text{m})\hat{i} - (0.241\,\text{m})\hat{j}}$$

(*d*) At 7:15, the minute hand is positioned along the $+x$ axis, while the hour hand is at an angle of (7.25 h)/12 h × 360° = 218°, measured clockwise from the top.

The position vector for the minute hand is:

$$\vec{A}_{7:15} = \boxed{(0.5\,\text{m})\hat{i}}$$

Find the x-component of the vector representing the hour hand:

$$B_x = (0.25\,\text{m})\sin 218° = -0.154\,\text{m}$$

Find the y-component of the vector representing the hour hand:

$$B_y = (0.25\,\text{m})\cos 218° = -0.197\,\text{m}$$

The position vector for the hour hand is:

$$\vec{B}_{7:15} = \boxed{-(0.154\,\text{m})\hat{i} - (0.197\,\text{m})\hat{j}}$$

(*e*) Find $\vec{A} - \vec{B}$ at 12:00:

$$\vec{A} - \vec{B} = (0.5\,\text{m})\hat{j} - (0.25\,\text{m})\hat{j} = \boxed{(0.25\,\text{m})\hat{j}}$$

Find $\vec{A} - \vec{B}$ at 3:30:

$$\begin{aligned}\vec{A} - \vec{B} &= -(0.5\,\text{m})\hat{j} \\ &\quad - \left[(0.241\,\text{m})\hat{i} - (0.0647\,\text{m})\hat{j}\right] \\ &= \boxed{-(0.241\,\text{m})\hat{i} - (0.435\,\text{m})\hat{j}}\end{aligned}$$

Find $\vec{A} - \vec{B}$ at 6:30:

$$\begin{aligned}\vec{A} - \vec{B} &= -(0.5\,\text{m})\hat{j} \\ &\quad - \left[(0.0647\,\text{m})\hat{i} - (0.241\,\text{m})\hat{j}\right] \\ &= \boxed{-(0.0647\,\text{m})\hat{i} - (0.259\,\text{m})\hat{j}}\end{aligned}$$

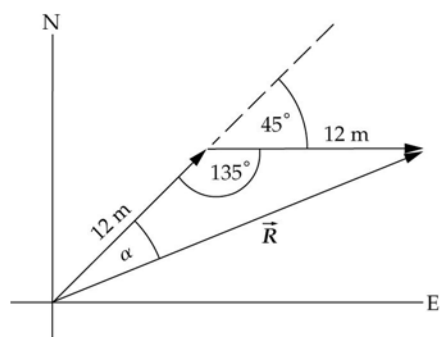
Find $\vec{A} - \vec{B}$ at 7:15:

$$\begin{aligned}\vec{A} - \vec{B} &= (0.5\,\text{m})\hat{j} \\ &\quad - \left[-(0.152\,\text{m})\hat{i} - (0.197\,\text{m})\hat{j}\right] \\ &= \boxed{(0.152\,\text{m})\hat{i} + (0.697\,\text{m})\hat{j}}\end{aligned}$$

***39** •

Picture the Problem The resultant displacement is the vector sum of the individual displacements.

The two displacements of the bear and its resultant displacement are shown to the right:



Using the law of cosines, solve for the resultant displacement:

$$R^2 = (12\,\text{m})^2 + (12\,\text{m})^2 - 2(12\,\text{m})(12\,\text{m})\cos 135°$$

and

$$R = \boxed{22.2\,\text{m}}$$

Using the law of sines, solve for α:

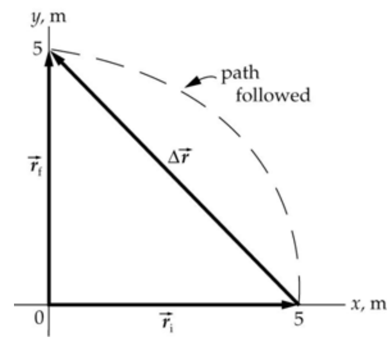
$$\frac{\sin\alpha}{12\,\text{m}} = \frac{\sin 135°}{22.2\,\text{m}}$$

$\therefore$ α = 22.5° and the angle with the horizontal is 45° – 22.5° = $\boxed{22.5°}$

40 •

Picture the Problem The resultant displacement is the vector sum of the individual displacements.

(*a*) Using the endpoint coordinates for her initial and final positions, draw the student's initial and final position vectors and construct her displacement vector.



Find the magnitude of her displacement and the angle this displacement makes with the positive x-axis:

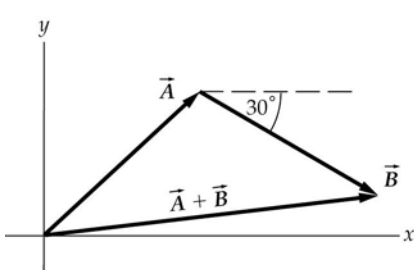
$\boxed{\text{Her displacement is } 5\sqrt{2}\text{ m @ }135°.}$

(*b*) $\boxed{\text{His initial and final positions are the same as in } (a)\text{, so his displacement is also } 5\sqrt{2}\text{ @ }135°.}$

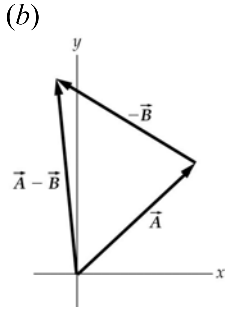
***41** •

Picture the Problem Use the standard rules for vector addition. Remember that changing the sign of a vector reverses its direction.

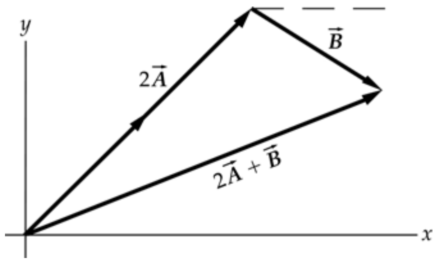
(*a*)



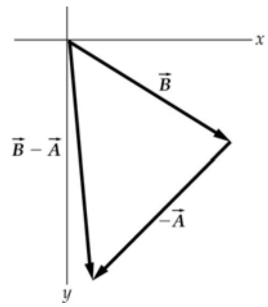
(*b*)



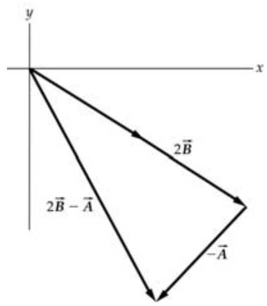
(*c*)



(*d*)

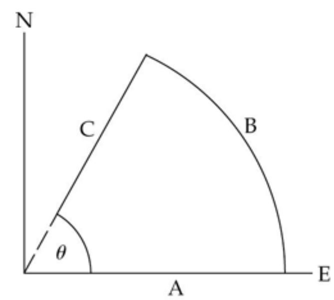


(*e*)



42 •

Picture the Problem The figure shows the paths walked by the Scout. The length of path A is 2.4 km; the length of path B is 2.4 km; and the length of path C is 1.5 km:



(*a*) Express the distance from the campsite to the end of path C:

$$2.4\text{ km} - 1.5\text{ km} = \boxed{0.9\,\text{km}}$$

(*b*) Determine the angle θ subtended by the arc at the origin (campsite):

$$\theta_{\text{radians}} = \frac{\text{arc length}}{\text{radius}} = \frac{2.4\,\text{km}}{2.4\,\text{km}}$$

$$= 1\,\text{rad} = 57.3°$$

$$\boxed{\begin{array}{l}\text{His direction from camp}\\ \text{is 1 rad north of east.}\end{array}}$$

(*c*) Express the total distance as the sum of the three parts of his walk:

$$d_{\text{tot}} = d_{\text{east}} + d_{\text{arc}} + d_{\text{toward camp}}$$

Substitute the given distances to find the total:

$$d_{\text{tot}} = 2.4\text{ km} + 2.4\text{ km} + 1.5\text{ km}$$
$$= 6.3\text{ km}$$

Express the ratio of the magnitude of his displacement to the total distance he walked and substitute to obtain a numerical value for this ratio:

$$\frac{\text{Magnitude of his displacement}}{\text{Total distance walked}} = \frac{0.9\,\text{km}}{6.3\,\text{km}} = \boxed{\frac{1}{7}}$$

43 •

Picture the Problem The direction of a vector is determined by its components.

$$\theta = \tan^{-1}\left(\frac{-3.5\,\text{m/s}}{5.5\,\text{m/s}}\right) = -32.5°$$

The vector is in the fourth quadrant and $\boxed{(b) \text{ is correct.}}$

44 •

Picture the Problem The components of the resultant vector can be obtained from the components of the vectors being added. The magnitude of the resultant vector can then be found by using the Pythagorean Theorem.

A table such as the one shown to the right is useful in organizing the information in this problem. Let $\vec{D}$ be the sum of vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$.

Vector	x-component	y-component
$\vec{A}$	6	–3
$\vec{B}$	–3	4
$\vec{C}$	2	5
$\vec{D}$		

Determine the components of $\vec{D}$ by adding the components of $\vec{A}$, $\vec{B}$, and $\vec{C}$.

$D_x = 5$ and $D_y = 6$

Use the Pythagorean Theorem to calculate the magnitude of $\vec{D}$:

$$D = \sqrt{D_x^2 + D_y^2} = \sqrt{(5)^2 + (6)^2} = 7.81$$

and $\boxed{(d) \text{ is correct.}}$

45 •

Picture the Problem The components of the given vector can be determined using right-triangle trigonometry.

Use the trigonometric relationships between the magnitude of a vector and its components to calculate the x- and y-components of each vector.

	A	θ	A_x	A_y
(a)	10 m	30°	8.66 m	5 m
(b)	5 m	45°	3.54 m	3.54 m
(c)	7 km	60°	3.50 km	6.06 km
(d)	5 km	90°	0	5 km

(*e*)	15 km/s	150°	–13.0 km/s	7.50 km/s
(*f*)	10 m/s	240°	–5.00 m/s	–8.66 m/s
(*g*)	8 m/s^2	270°	0	–8.00 m/s^2

***46 •**

Picture the Problem Vectors can be added and subtracted by adding and subtracting their components.

Write $\vec{A}$ in component form:

$A_x = (8\text{ m})\cos 37° = 6.4\text{ m}$

$A_y = (8\text{ m})\sin 37° = 4.8\text{ m}$

$\therefore \vec{A} = (6.4\,\text{m})\hat{i} + (4.8\,\text{m})\hat{j}$

(*a*), (*b*), (*c*) Add (or subtract) *x*- and *y*-components:

$$\vec{D} = \boxed{(0.4\text{m})\hat{i} + (7.8\text{m})\hat{j}}$$

$$\vec{E} = \boxed{(-3.4\text{m})\hat{i} - (9.8\text{m})\hat{j}}$$

$$\vec{F} = \boxed{(-17.6\text{m})\hat{i} + (23.8\text{m})\hat{j}}$$

(*d*) Solve for $\vec{G}$ and add components to obtain:

$$\vec{G} = -\frac{1}{2}\left(\vec{A} + \vec{B} + 2\vec{C}\right)$$

$$= \boxed{(1.3\,\text{m})\hat{i} - (2.9\,\text{m})\hat{j}}$$

47 ••

Picture the Problem The magnitude of each vector can be found from the Pythagorean theorem and their directions found using the inverse tangent function.

(*a*) $\vec{A} = 5\hat{i} + 3\hat{j}$

$$A = \sqrt{A_x^2 + A_y^2} = \boxed{5.83}$$

and, because $\vec{A}$ is in the 1st quadrant,

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \boxed{31.0°}$$

(*b*) $\vec{B} = 10\hat{i} - 7\hat{j}$

$$B = \sqrt{B_x^2 + B_y^2} = \boxed{12.2}$$

and, because $\vec{B}$ is in the 4th quadrant,

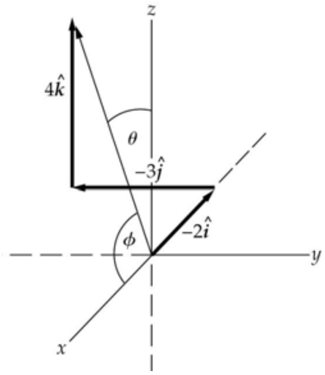
$$\theta = \tan^{-1}\left(\frac{B_y}{B_x}\right) = \boxed{-35.0°}$$

(*c*) $\vec{C} = -2\hat{i} - 3\hat{j} + 4\hat{k}$

$$C = \sqrt{C_x^2 + C_y^2 + C_z^2} = \boxed{5.39}$$

$$\theta = \cos^{-1}\left(\frac{C_z}{C}\right) = \boxed{42.1°}$$

where θ is the polar angle measured from the positive *z*-axis and



$$\phi = \cos^{-1}\left(\frac{C_x}{C}\right) = \cos^{-1}\left(\frac{-2}{\sqrt{29}}\right) = \boxed{112°}$$

48 •

Picture the Problem The magnitude and direction of a two-dimensional vector can be found by using the Pythagorean Theorem and the definition of the tangent function.

(*a*) $\vec{A} = -4\hat{i} - 7\hat{j}$

$$A = \sqrt{A_x^2 + A_y^2} = \boxed{8.06}$$

and, because $\vec{A}$ is in the 3rd quadrant,

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \boxed{240°}$$

$\vec{B} = 3\hat{i} - 2\hat{j}$

$$B = \sqrt{B_x^2 + B_y^2} = \boxed{3.61}$$

and, because $\vec{B}$ is in the 4th quadrant,

$$\theta = \tan^{-1}\left(\frac{B_y}{B_x}\right) = \boxed{-33.7°}$$

$\vec{C} = \vec{A} + \vec{B} = -\hat{i} - 9\hat{j}$

$$C = \sqrt{C_x^2 + C_y^2} = \boxed{9.06}$$

and, because $\vec{C}$ is in the 3rd quadrant,

$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \boxed{264°}$$

(*b*) Follow the same steps as in (*a*).

$$A = \boxed{4.12}\,;\ \theta = \boxed{-76.0°}$$

$$B = \boxed{6.32}\,;\ \theta = \boxed{71.6°}$$

$$C = \boxed{3.61}\,;\ \theta = \boxed{33.7°}$$

49 •

Picture the Problem The components of these vectors are related to the magnitude of each vector through the Pythagorean Theorem and trigonometric functions. In parts (*a*) and (*b*), calculate the rectangular components of each vector and then express the vector in rectangular form.

(*a*) Express $\vec{v}$ in rectangular form:

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

Evaluate v_x and v_y:

$v_x = (10\text{ m/s})\cos 60° = 5\text{ m/s}$
and
$v_y = (10\text{ m/s})\sin 60° = 8.66\text{ m/s}$

Substitute to obtain:

$$\vec{v} = \boxed{(5\,\text{m/s})\hat{i} + (8.66\,\text{m/s})\hat{j}}$$

(*b*) Express $\vec{v}$ in rectangular form:

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

Evaluate A_x and A_y:

$A_x = (5\text{ m})\cos 225° = -3.54\text{ m}$
and
$A_y = (5\text{ m})\sin 225° = -3.54\text{ m}$

Substitute to obtain:

$$\vec{A} = \boxed{(-3.54\,\text{m})\hat{i} + (-3.54\,\text{m})\hat{j}}$$

(*c*) There is nothing to calculate as we are given the rectangular components:

$$\vec{r} = \boxed{(14\text{m})\hat{i} - (6\text{m})\hat{j}}$$

50 •

Picture the Problem While there are infinitely many vectors $\vec{B}$ that can be constructed such that $A = B$, the simplest are those which lie along the coordinate axes.

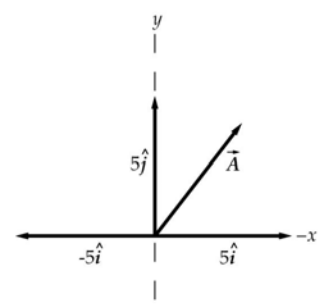
Determine the magnitude of $\vec{A}$:

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{3^2 + 4^2} = 5$$

Write three vectors of the same magnitude as $\vec{A}$:

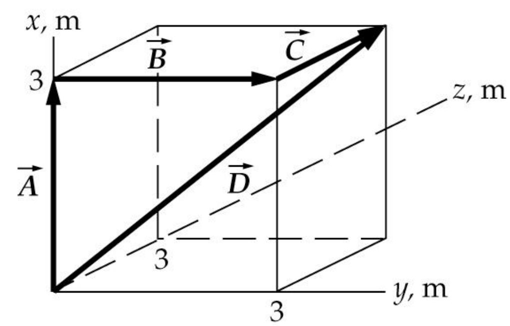
$$\vec{B}_1 = 5\hat{i},\ \vec{B}_2 = -5\hat{i},\text{ and }\vec{B}_3 = 5\hat{j}$$

The vectors are shown to the right:



***51 ••**

Picture the Problem While there are several walking routes the fly could take to get from the origin to point C, its displacement will be the same for all of them. One possible route is shown in the figure.



Express the fly's displacement $\vec{D}$ during its trip from the origin to point C and find its magnitude:

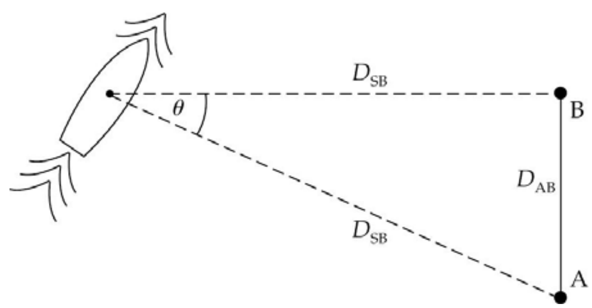
$$\vec{D} = \vec{A} + \vec{B} + \vec{C} = (3\,\text{m})\hat{i} + (3\,\text{m})\hat{j} + (3\,\text{m})\hat{k}$$

and

$$D = \sqrt{(3\,\text{m})^2 + (3\,\text{m})^2 + (3\,\text{m})^2} = \boxed{5.20\,\text{m}}$$

***52 •**

Picture the Problem The diagram shows the locations of the transmitters relative to the ship and defines the distances separating the transmitters from each other and from the ship. We can find the distance between the ship and transmitter B using trigonometry.



Relate the distance between A and B to the distance from the ship to A and the angle θ:

$$\tan\theta = \frac{D_{AB}}{D_{SB}}$$

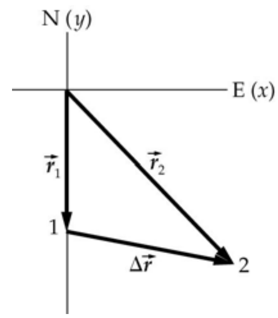
Solve for and evaluate the distance from the ship to transmitter B:

$$D_{SB} = \frac{D_{AB}}{\tan\theta} = \frac{100\,\text{km}}{\tan 30°} = \boxed{173\,\text{km}}$$

Velocity and Acceleration Vectors

53 •

Picture the Problem For constant speed and direction, the instantaneous velocity is identical to the average velocity. Take the origin to be the location of the stationary radar and construct a pictorial representation.



Express the average velocity:

$$\vec{\boldsymbol{v}}_{\rm av} = \frac{\Delta\vec{r}}{\Delta t}$$

Determine the position vectors:

$$\vec{\boldsymbol{r}}_1 = (-10\,\text{km})\hat{\boldsymbol{j}}$$

and

$$\vec{\boldsymbol{r}}_2 = (14.1\,\text{km})\hat{\boldsymbol{i}} + (-14.1\,\text{km})\hat{\boldsymbol{j}}$$

Find the displacement vector:

$$\Delta\vec{\boldsymbol{r}} = \vec{\boldsymbol{r}}_2 - \vec{\boldsymbol{r}}_1 = (14.1\,\text{km})\hat{\boldsymbol{i}} + (-4.1\,\text{km})\hat{\boldsymbol{j}}$$

Substitute for $\Delta\vec{\boldsymbol{r}}$ and Δt to find the average velocity.

$$\vec{\boldsymbol{v}}_{\rm av} = \frac{(14.1\,\text{km})\hat{\boldsymbol{i}} + (-4.1\,\text{km})\hat{\boldsymbol{j}}}{1\,\text{h}} = \boxed{(14.1\,\text{km/h})\hat{\boldsymbol{i}} + (-4.1\,\text{km/h})\hat{\boldsymbol{j}}}$$

54 •

Picture the Problem The average velocity is the change in position divided by the elapsed time.

(*a*) The average velocity is:

$$v_{\rm av} = \frac{\Delta r}{\Delta t}$$

Find the position vectors and the displacement vector:

$$\vec{\boldsymbol{r}}_0 = (2\text{m})\hat{\boldsymbol{i}} + (3\text{m})\hat{\boldsymbol{j}}$$

$$\vec{\boldsymbol{r}}_2 = (6\text{m})\hat{\boldsymbol{i}} + (7\text{m})\hat{\boldsymbol{j}}$$

and

$$\Delta\vec{\boldsymbol{r}} = \vec{\boldsymbol{r}}_2 - \vec{\boldsymbol{r}}_1 = (4\,\text{m})\hat{\boldsymbol{i}} + (4\,\text{m})\hat{\boldsymbol{j}}$$

Find the magnitude of the displacement vector for the interval between $t = 0$ and $t = 2$ s:

$$\Delta r_{02} = \sqrt{(4\text{m})^2 + (4\text{m})^2} = 5.66\text{m}$$

Substitute to determine v_{av}:

$$v_{av} = \frac{5.66\text{ m}}{2\text{ s}} = \boxed{2.83\text{ m/s}}$$

and

$$\theta = \tan^{-1}\left(\frac{4\text{ m}}{4\text{ m}}\right) = \boxed{45.0°} \text{ measured}$$

from the positive x axis.

(b) Repeat (a), this time using the displacement between $t = 0$ and $t = 5$ s to obtain:

$$\vec{r}_5 = (13\text{ m})\hat{i} + (14\text{ m})\hat{j},$$

$$\Delta\vec{r}_{05} = \vec{r}_5 - \vec{r}_0 = (11\text{ m})\hat{i} + (11\text{ m})\hat{j},$$

$$\Delta r_{05} = \sqrt{(11\text{ m})^2 + (11\text{ m})^2} = 15.6\text{ m},$$

$$v_{av} = \frac{15.6\text{ m}}{5\text{ s}} = \boxed{3.11\text{ m/s}},$$

and

$$\theta = \tan^{-1}\left(\frac{11\text{ m}}{11\text{ m}}\right) = \boxed{45.0°} \text{ measured}$$

from the positive x axis.

***55** •

Picture the Problem The magnitude of the velocity vector at the end of the 2 s of acceleration will give us its speed at that instant. This is a constant-acceleration problem.

Find the final velocity vector of the particle:

$$\begin{aligned}\vec{v} &= v_x\hat{i} + v_y\hat{j} = v_{x0}\hat{i} + a_y t\hat{j} \\ &= (4.0\text{ m/s})\hat{i} + (3.0\text{ m/s}^2)(2.0\text{ s})\hat{j} \\ &= (4.0\text{ m/s})\hat{i} + (6.0\text{ m/s})\hat{j}\end{aligned}$$

Find the magnitude of $\vec{v}$:

$$v = \sqrt{(4.0\text{ m/s})^2 + (6.0\text{ m/s})^2} = 7.21\text{ m/s}$$

and $\boxed{(b)\text{ is correct.}}$

56 •

Picture the Problem Choose a coordinate system in which north coincides with the positive y direction and east with the positive x direction. Expressing the west and north velocity vectors is the first step in determining $\Delta\vec{v}$ and $\vec{a}_{av}$.

(a) The magnitudes of $\vec{v}_W$ and $\vec{v}_N$ are 40 m/s and 30 m/s, respectively. The change in the magnitude of the particle's velocity during this time is:

$$\begin{aligned}\Delta v &= v_N - v_W \\ &= \boxed{-10\text{ m/s}}\end{aligned}$$

(*b*) The change in the direction of the velocity is from west to north.

The change in direction is $\boxed{90°}$

(*c*) The change in velocity is:

$$\Delta\vec{v} = \vec{v}_{\rm N} - \vec{v}_{\rm W} = (30\,\text{m/s})\hat{j} - (-40\,\text{m/s})\hat{i}$$
$$= (40\,\text{m/s})\hat{i} + (30\,\text{m/s})\hat{j}$$

Calculate the magnitude and direction of $\Delta\vec{v}$:

$$|\Delta v| = \sqrt{(40\,\text{m/s})^2 + (30\,\text{m/s})^2} = \boxed{50\,\text{m/s}}$$

and

$$\theta_{+x\,\text{axis}} = \tan^{-1}\frac{30\,\text{m/s}}{40\,\text{m/s}} = \boxed{36.9°}$$

(*d*) Find the average acceleration during this interval:

$$\vec{a}_{\rm av} \equiv \Delta\vec{v}/\Delta t = \frac{(40\,\text{m/s})\hat{i} + (30\,\text{m/s})\hat{j}}{5\,\text{s}}$$
$$= (8\,\text{m/s}^2)\hat{i} + (6\,\text{m/s}^2)\hat{j}$$

The magnitude of this vector is:

$$a_{\rm av} = \sqrt{(8\,\text{m/s}^2)^2 + (6\,\text{m/s}^2)^2} = \boxed{10\,\text{m/s}^2}$$

and its direction is

$$\theta = \tan^{-1}\left(\frac{6\,\text{m/s}^2}{8\,\text{m/s}^2}\right) = \boxed{36.9°}$$ measured from the positive x axis.

57 •

Picture the Problem The initial and final positions and velocities of the particle are given. We can find the average velocity and average acceleration using their definitions by first calculating the given displacement and velocities using unit vectors $\hat{i}$ and $\hat{j}$.

(*a*) The average velocity is:

$$\vec{v}_{\rm av} \equiv \Delta\vec{r}/\Delta t$$

The displacement of the particle during this interval of time is:

$$\Delta\vec{r} = (100\,\text{m})\hat{i} + (80\,\text{m})\hat{j}$$

Substitute to find the average velocity:

$$\vec{v}_{\rm av} = \frac{(100\,\text{m})\hat{i} + (80\,\text{m})\hat{j}}{3\,\text{s}}$$
$$= \boxed{(33.3\,\text{m/s})\hat{i} + (26.7\,\text{m/s})\hat{j}}$$

(*b*) The average acceleration is:

$$\vec{a}_{\rm av} = \Delta\vec{v}/\Delta t$$

Find $\vec{v}_1$, $\vec{v}_2$, and $\Delta\vec{v}$:

$$\vec{v}_1 = (28.3\,\text{m/s})\hat{i} + (28.3\,\text{m/s})\hat{j}$$

and

$$\vec{v}_2 = (19.3\,\text{m/s})\hat{i} + (23.0\,\text{m/s})\hat{j}$$

$$\therefore\ \Delta\vec{v} = (-9.00\,\text{m/s})\hat{i} + (-5.30\,\text{m/s})\hat{j}$$

Using $\Delta t = 3$ s, find the average acceleration:

$$\vec{a}_{\text{av}} = \boxed{(-3.00\,\text{m/s}^2)\hat{i} + (-1.77\,\text{m/s}^2)\hat{j}}$$

***58 ••**

Picture the Problem The acceleration is constant so we can use the constant-acceleration equations in vector form to find the velocity at $t = 2$ s and the position vector at $t = 4$ s.

(*a*) The velocity of the particle, as a function of time, is given by:

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

Substitute to find the velocity at $t = 2$ s:

$$\begin{aligned}\vec{v} &= (2\,\text{m/s})\hat{i} + (-9\,\text{m/s})\hat{j} \\ &\quad + \left[(4\,\text{m/s}^2)\hat{i} + (3\,\text{m/s}^2)\hat{j}\right](2\text{s}) \\ &= \boxed{(10\,\text{m/s})\hat{i} + (-3\,\text{m/s})\hat{j}}\end{aligned}$$

(*b*) Express the position vector as a function of time:

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2$$

Substitute and simplify:

$$\begin{aligned}\vec{r} &= (4\,\text{m})\hat{i} + (3\,\text{m})\hat{j} \\ &\quad + \left[(2\,\text{m/s})\hat{i} + (\text{-}9\,\text{m/s})\hat{j}\right](4\,\text{s}) \\ &\quad + \tfrac{1}{2}\left[(4\,\text{m/s}^2)\hat{i} + (3\,\text{m/s}^2)\hat{j}\right](4\,\text{s})^2 \\ &= \boxed{(44\,\text{m})\hat{i} + (-9\,\text{m})\hat{j}}\end{aligned}$$

Find the magnitude and direction of $\vec{r}$ at $t = 4$ s:

$$r(4\,\text{s}) = \sqrt{(44\,\text{m})^2 + (-9\,\text{m})^2} = \boxed{44.9\,\text{m}}$$

and, because $\vec{r}$ is in the 4$^{\text{th}}$ quadrant,

$$\theta = \tan^{-1}\left(\frac{-9\,\text{m}}{44\,\text{m}}\right) = \boxed{-11.6^\circ}$$

59 ••

Picture the Problem The velocity vector is the time-derivative of the position vector and the acceleration vector is the time-derivative of the velocity vector.

Differentiate $\vec{r}$ with respect to time:

$$\begin{aligned}\vec{v} &= \frac{d\vec{r}}{dt} = \frac{d}{dt}\left[(30t)\hat{i} + (40t - 5t^2)\hat{j}\right] \\ &= \boxed{30\hat{i} + (40 - 10t)\hat{j}}\end{aligned}$$

where $\vec{v}$ has units of m/s if t is in seconds.

Differentiate $\vec{v}$ with respect to time:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left[30\hat{i} + (40 - 10t)\hat{j}\right] = \boxed{\left(-10\,\text{m/s}^2\right)\hat{j}}$$

60 ••

Picture the Problem We can use the constant-acceleration equations in vector form to solve the first part of the problem. In the second part, we can eliminate the parameter t from the constant-acceleration equations and express y as a function of x.

(*a*) Use $\vec{v} = \vec{v}_0 + \vec{a}t$ with $\vec{v}_0 = 0$ to find $\vec{v}$:

$$\boxed{\vec{v} = \left[\left(6\text{m/s}^2\right)\hat{i} + \left(4\text{m/s}^2\right)\hat{j}\right]t}$$

Use $\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$ with $\vec{r}_0 = (10\,\text{m})\hat{i}$ to find $\vec{r}$:

$$\boxed{\vec{r} = \left[(10\text{m}) + \left(3\text{m/s}^2\right)t^2\right]\hat{i} + \left[\left(2\text{m/s}^2\right)t^2\right]\hat{j}}$$

(*b*) Obtain the x and y components of the path from the vector equation in (*a*):

$$x = 10\,\text{m} + \left(3\,\text{m/s}^2\right)t^2$$

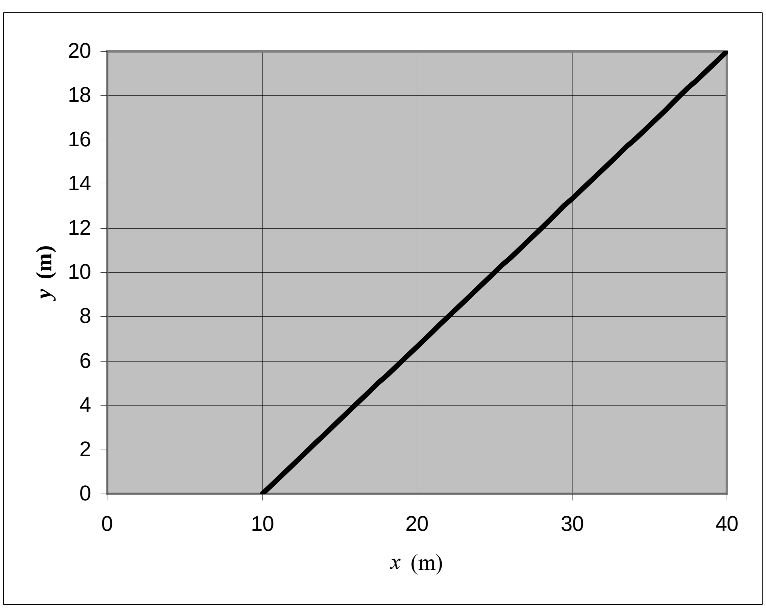
and

$$y = \left(2\,\text{m/s}^2\right)t^2$$

Eliminate the parameter t from these equations and solve for y to obtain:

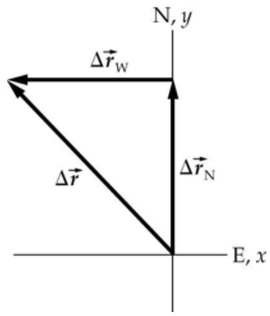
$$\boxed{y = \frac{2}{3}x - \frac{20}{3}\text{m}}$$

Use this equation to plot the graph shown below. Note that the path in the xy plane is a straight line.



61 •••

Picture the Problem The displacements of the boat are shown in the figure. We need to determine each of the displacements in order to calculate the average velocity of the boat during the 30-s trip.



(*a*) Express the average velocity of the boat:

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t}$$

Express its total displacement:

$$\Delta\vec{r} = \Delta\vec{r}_N + \Delta\vec{r}_W = \tfrac{1}{2}a_N(\Delta t_N)^2\,\hat{j} + v_W\Delta t_W(-\hat{i})$$

To calculate the displacement we first have to find the speed after the first 20 s:

$$v_W = v_{N,\,f} = a_N\Delta t_N = 60\ \text{m/s}$$

so

$$\Delta\vec{r} = \tfrac{1}{2}a_N(\Delta t_N)^2\,\hat{j} - (60\ \text{m/s})\Delta t_W\hat{i} = (600\text{m})\hat{j} - (600\text{m})\hat{i}$$

Substitute to find the average velocity:

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{(600\text{m})(-\hat{i}+\hat{j})}{30\text{s}} = \boxed{(20\text{m/s})(-\hat{i}+\hat{j})}$$

(*b*) The average acceleration is given by:

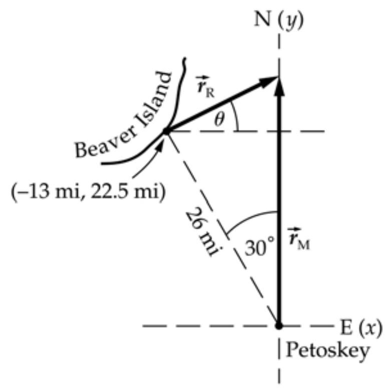
$$\vec{a}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} = \frac{(-60\,\text{m/s})\hat{i} - 0}{30\,\text{s}} = \boxed{(-2\,\text{m/s}^2)\hat{i}}$$

(*c*) The displacement of the boat from the dock at the end of the 30-s trip was one of the intermediate results we obtained in part (*a*).

$$\Delta\vec{r} = (600\text{m})\hat{j} + (-600\text{m})\hat{i} = \boxed{(600\text{m})(-\hat{i}+\hat{j})}$$

***62** •••

Picture the Problem Choose a coordinate system with the origin at Petoskey, the positive x direction to the east, and the positive y direction to the north. Let $t = 0$ at 9:00 a.m. and θ be the angle between Robert's velocity vector and the easterly direction and let ″M″ and ″R″ denote Mary and Robert, respectively. You can express the positions of Mary and Robert as functions of time and then equate their north (y) and east (x) coordinates at the time they rendezvous.



Express Mary's position as a function of time:

$$\vec{r}_M = v_M t\,\hat{j} = (8t)\hat{j}$$

where $\vec{r}_M$ is in miles if t is in hours.

Note that Robert's initial position coordinates (x_i, y_i) are:

$(x_i, y_i) = (-13\text{ mi}, 22.5\text{ mi})$

Express Robert's position as a function of time:

$$\begin{aligned}\vec{r}_R &= [x_i + (v_R\cos\theta)(t-1)])\hat{i} + [y_i + (v_R\sin\theta)(t-1)]\hat{j}\\ &= [-13 + \{6(t-1)\cos\theta\}]\hat{i} + [22.5 + \{6(t-1)\sin\theta\}]\hat{j}\end{aligned}$$

where $\vec{r}_R$ is in miles if t is in hours.

When Mary and Robert rendezvous, their coordinates will be the same. Equating their north and east coordinates yields:

East: $-13 + 6t\cos\theta - 6\cos\theta = 0$ (1)

North: $22.5 + 6t\sin\theta - 6\sin\theta = 8t$ (2)

Solve equation (1) for $\cos\theta$:

$$\cos\theta = \frac{13}{6(t-1)} \tag{3}$$

Solve equation (2) for $\sin\theta$:

$$\sin\theta = \frac{8t - 22.5}{6(t-1)} \tag{4}$$

Square and add equations (3) and (4) to obtain:

$$\sin^2\theta + \cos^2\theta = 1 = \left[\frac{8t-22.5}{6(t-1)}\right]^2 + \left[\frac{13}{6(t-1)}\right]^2$$

Simplify to obtain a quadratic equation in t:

$$28t^2 - 288t + 639 = 0$$

Solve (you could use your calculator's ″solver″ function) this

$$t = \boxed{3.24\,\text{h} = 3\,\text{h}\ 15\,\text{min}}$$

equation for the smallest value of t (both roots are positive) to obtain:

Now you can find the distance traveled due north by Mary:

$$r_M = v_M t = (8\,\text{mi/h})(3.24\,\text{h}) = \boxed{25.9\,\text{mi}}$$

Finally, solving equation (3) for θ and substituting 3.24 h for t yields:

$$\theta = \cos^{-1}\left[\frac{13}{6(t-1)}\right] = \cos^{-1}\left[\frac{13}{6(3.24-1)}\right] = 14.7°$$

and so Robert should head $\boxed{14.7° \text{ north of east.}}$

Remarks: Another solution that does not depend on the components of the vectors utilizes the law of cosines to find the time *t* at which Mary and Robert meet and then uses the law of sines to find the direction that Robert must head in order to rendezvous with Mary.

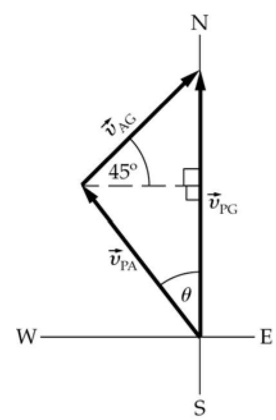
Relative Velocity

63 ••

Picture the Problem Choose a coordinate system in which north is the positive y direction and east is the positive x direction. Let θ be the angle between north and the direction of the plane's heading. The velocity of the plane relative to the ground, $\vec{v}_{PG}$, is the sum of the velocity of the plane relative to the air, $\vec{v}_{PA}$, and the velocity of the air relative to the ground, $\vec{v}_{AG}$. i.e.,

$$\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$$

The pilot must head in such a direction that the east-west component of $\vec{v}_{PG}$ is zero in order to make the plane fly due north.



(*a*) From the diagram one can see that:

$$v_{AG} \cos 45° = v_{PA} \sin\theta$$

Solve for and evaluate θ:

$$\theta = \sin^{-1}\left(\frac{56.6\,\text{km/h}}{250\,\text{km/h}}\right) = \boxed{13.1° \text{ west of north}}$$

(*b*) Because the plane is headed due north, add the north components of

$$\left|\vec{v}_{PG}\right| = (250\text{ km/h}) \cos 13.1° + (80\text{ km/h}) \sin 45°$$

$\vec{v}_{PA}$ and $\vec{v}_{AG}$ to determine the plane's ground speed:

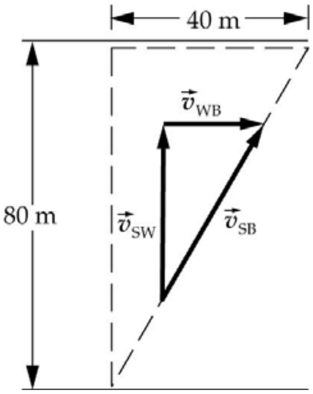
$$= \boxed{300\,\text{km/h}}$$

64 ••

Picture the Problem Let $\vec{v}_{SB}$ represent the velocity of the swimmer relative to the bank; $\vec{v}_{SW}$ the velocity of the swimmer relative to the water; and $\vec{v}_{WB}$ the velocity of the water relative to the shore; i.e.,

$$\vec{v}_{SB} = \vec{v}_{SW} + \vec{v}_{WB}$$

The current of the river causes the swimmer to drift downstream.



(*a*) The triangles shown in the figure are similar right triangles. Set up a proportion between their sides and solve for the speed of the water relative to the bank:

$$\frac{v_{WB}}{v_{SW}} = \frac{40\,\text{m}}{80\,\text{m}}$$

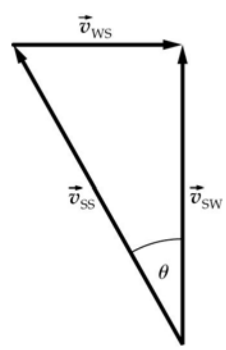
and

$$v_{WB} = \tfrac{1}{2}(1.6\,\text{m/s}) = \boxed{0.800\,\text{m/s}}$$

(*b*) Use the Pythagorean Theorem to solve for the swimmer's speed relative to the shore:

$$v_{SB} = \sqrt{v_{SW}^2 + v_{WS}^2}$$
$$= \sqrt{(1.6\,\text{m/s})^2 + (0.8\,\text{m/s})^2}$$
$$= \boxed{1.79\,\text{m/s}}$$

(*c*) The swimmer should head in a direction such that the upstream component of her velocity is equal to the speed of the water relative to the shore:



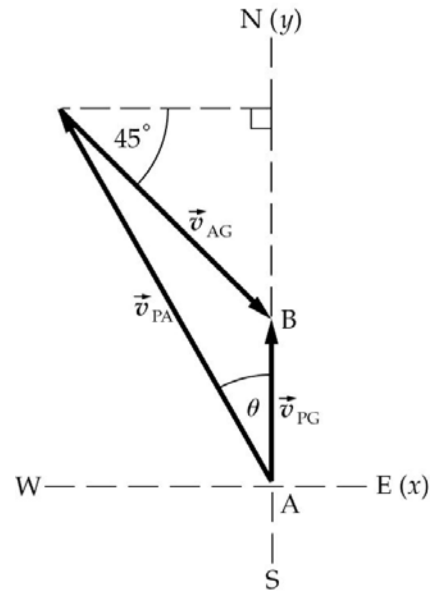
Use a trigonometric function to evaluate θ:

$$\theta = \sin^{-1}\left(\frac{0.8\,\text{m/s}}{1.6\,\text{m/s}}\right) = \boxed{30.0°}$$

***65 ••**

Picture the Problem Let the velocity of the plane relative to the ground be represented by $\vec{v}_{PG}$; the velocity of the plane relative to the air by $\vec{v}_{PA}$, and the velocity of the air relative to the ground by $\vec{v}_{AG}$. Then

$$\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG} \quad (1)$$



Choose a coordinate system with the origin at point A, the positive x direction to the east, and the positive y direction to the north. θ is the angle between north and the direction of the plane's heading. The pilot must head so that the east-west component of $\vec{v}_{PG}$ is zero in order to make the plane fly due north.

Use the diagram to express the condition relating the eastward component of $\vec{v}_{AG}$ and the westward component of $\vec{v}_{PA}$. This must be satisfied if the plane is to stay on its northerly course. [Note: this is equivalent to equating the x-components of equation (1).]

$$(50\text{ km/h})\cos 45° = (240\text{ km/h})\sin\theta$$

Now solve for θ to obtain:

$$\theta = \sin^{-1}\left[\frac{(50\,\text{km/h})\cos 45°}{240\,\text{km/h}}\right] = \boxed{8.47°}$$

Add the north components of $\vec{v}_{PA}$ and $\vec{v}_{AG}$ to find the velocity of the plane relative to the ground:

$$v_{PG} + v_{AG}\sin 45° = v_{PA}\cos 8.47°$$

and

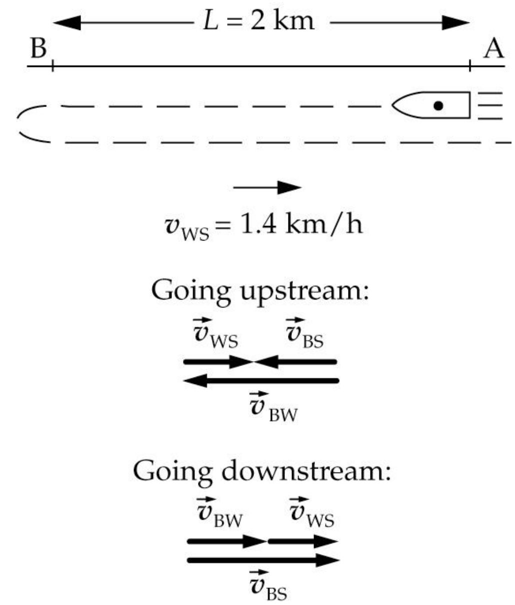
$$\begin{aligned} v_{PG} &= (240\text{ km/h})\cos 8.47° \\ &\qquad - (50\text{ km/h})\sin 45° \\ &= 202\text{ km/h} \end{aligned}$$

Finally, find the time of flight:

$$t_{\text{flight}} = \frac{\text{distance travelled}}{v_{PG}} = \frac{520\,\text{km}}{202\,\text{km/h}} = \boxed{2.57\,\text{h}}$$

66 ••

Picture the Problem Let $\vec{v}_{BS}$ be the velocity of the boat relative to the shore; $\vec{v}_{BW}$ be the velocity of the boat relative to the water; and $\vec{v}_{WS}$ represent the velocity of the water relative to the shore. Independently of whether the boat is going upstream or downstream:



$$\vec{v}_{BS} = \vec{v}_{BW} + \vec{v}_{WS}$$

Going upstream, the speed of the boat relative to the shore is reduced by the speed of the water relative to the shore.

Going downstream, the speed of the boat relative to the shore is increased by the same amount.

For the upstream leg of the trip: $v_{BS} = v_{BW} - v_{WS}$

For the downstream leg of the trip: $v_{BS} = v_{BW} + v_{WS}$

Express the total time for the trip in terms of the times for its upstream and downstream legs:

$$t_{total} = t_{upstream} + t_{downstream} = \frac{L}{v_{BW} - v_{WS}} + \frac{L}{v_{BW} + v_{WS}}$$

Multiply both sides of the equation by $(v_{BW} - v_{WS})(v_{BW} + v_{WS})$ (the product of the- denominators) and rearrange the terms to obtain:

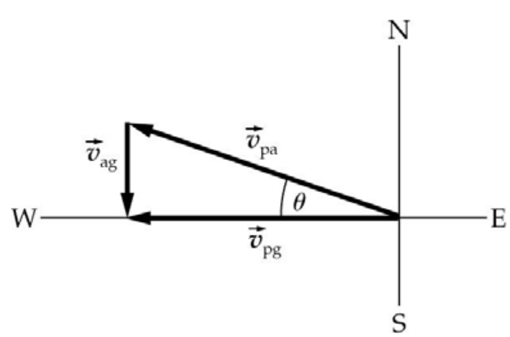
$$v_{BW}^2 - \frac{2L}{t_{total}} v_{BW} - v_{WS}^2 = 0$$

Solve the quadratic equation for v_{BW}. (Only the positive root is physically meaningful.)

$$v_{BW} = \boxed{5.18\,\text{km/h}}$$

67 ••

Picture the Problem Let $\vec{v}_{pg}$ be the velocity of the plane relative to the ground; $\vec{v}_{ag}$ be the velocity of the air relative to the ground; and $\vec{v}_{pa}$ the velocity of the plane relative to the air. Then, $\vec{v}_{pg} = \vec{v}_{pa} + \vec{v}_{ag}$. The wind will affect the flight times differently along these two paths.



The velocity of the plane, relative to the ground, on its eastbound leg is equal to its velocity on its westbound leg. Using the diagram, find the velocity of the plane relative to the ground for both directions:

$$v_{pg} = \sqrt{v_{pa}^2 - v_{ag}^2} = \sqrt{(15\,\text{m/s})^2 - (5\,\text{m/s})^2} = 14.1\,\text{m/s}$$

Express the time for the east-west roundtrip in terms of the distances and velocities for the two legs:

$$t_{\text{roundtrip,EW}} = t_{\text{eastbound}} + t_{\text{westbound}} = \frac{\text{radius of the circle}}{v_{\text{pg,eastbound}}} + \frac{\text{radius of the circle}}{v_{\text{pg,westbound}}} = \frac{2\times10^3\,\text{m}}{14.1\,\text{m/s}} = 141\,\text{s}$$

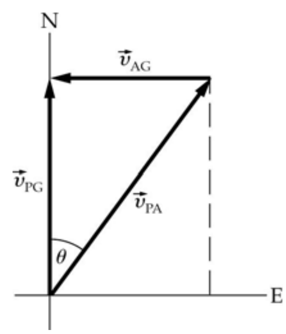
Use the distances and velocities for the two legs to express and evaluate the time for the north-south roundtrip:

$$t_{\text{roundtrip,NS}} = t_{\text{northbound}} + t_{\text{southbound}} = \frac{\text{radius of the circle}}{v_{\text{pg,northbound}}} + \frac{\text{radius of the circle}}{v_{\text{pg,southbound}}} = \frac{10^3\,\text{m}}{(15\,\text{m/s})-(5\,\text{m/s})} + \frac{10^3\,\text{m}}{(15\,\text{m/s})+(5\,\text{m/s})} = 150\,\text{s}$$

$$\boxed{\text{Because } t_{\text{roundtrip,EW}} < t_{\text{roundtrip,NS}}, \text{ you should fly your plane across the wind.}}$$

68 •

Picture the Problem This is a relative velocity problem. The given quantities are the direction of the velocity of the plane relative to the ground and the velocity (magnitude and direction) of the air relative to the ground. Asked for is the direction of the velocity of the air relative to the ground. Using $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$, draw a vector addition diagram and solve for the unknown quantity.



Calculate the heading the pilot must take:

$$\theta = \sin^{-1}\frac{30\text{ kts}}{150\text{ kts}} = \boxed{11.5°}$$

Because this is also the angle of the plane's heading clockwise from north, it is also its azimuth or the required true heading:

Az = (011.5°)

***69** ••

Picture the Problem The position of B relative to A is the vector from A to B; i.e.,

$$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$$

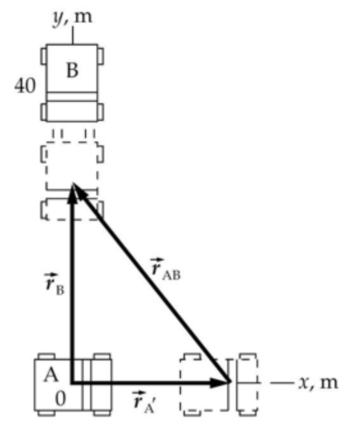
The velocity of B relative to A is

$$\vec{v}_{AB} = d\vec{r}_{AB}/dt$$

and the acceleration of B relative to A is

$$\vec{a}_{AB} = d\vec{v}_{AB}/dt$$

Choose a coordinate system with the origin at the intersection, the positive x direction to the east, and the positive y direction to the north.



(a) Find $\vec{r}_B$, $\vec{r}_A$, and $\vec{r}_{AB}$:

$$\vec{r}_B = \left[40\text{m} - \tfrac{1}{2}\left(2\text{m/s}^2\right)t^2\right]\hat{j}$$

$$\vec{r}_A = \left[(20\text{m/s})t\right]\hat{i}$$

and

$$\begin{aligned}\vec{r}_{AB} &= \vec{r}_B - \vec{r}_A \\ &= \left[(-20\text{m/s})t\right]\hat{i} \\ &\quad + \left[40\text{m} - \tfrac{1}{2}\left(2\text{m/s}^2\right)t^2\right]\hat{j}\end{aligned}$$

Evaluate $\vec{r}_{AB}$ at $t = 6$ s:

$$\vec{r}_{AB}(6\,\text{s}) = \boxed{(120\text{ m})\,\hat{i} + (4\text{ m})\,\hat{j}}$$

(b) Find $\vec{v}_{AB} = d\vec{r}_{AB}/dt$:

$$\begin{aligned}\vec{v}_{AB} &= \frac{d\vec{r}_{AB}}{dt} = \frac{d}{dt}\Big[\left\{(-20\text{ m/s})t\right\}\vec{i} \\ &\quad + \left\{40\text{ m} - \tfrac{1}{2}\left(2\text{ m/s}^2\right)t^2\right\}\hat{j}\Big] \\ &= (-20\text{ m/s})\,\hat{i} + (-2\text{ m/s}^2)\,t\,\hat{j}\end{aligned}$$

Evaluate $\vec{v}_{AB}$ at $t = 6$ s:

$$\vec{v}_{AB}(6\,\text{s}) = \boxed{(-20\text{ m/s})\hat{i} - (12\text{ m/s})\hat{j}}$$

(c) Find $\vec{a}_{AB} = d\vec{v}_{AB}/dt$:

$$\begin{aligned}\vec{a}_{AB} &= \frac{d}{dt}\left[(-20\text{ m/s})\,\hat{i} + (-2\text{ m/s}^2)\,t\,\hat{j}\right] \\ &= \boxed{\left(-2\text{ m/s}^2\right)\hat{j}}\end{aligned}$$

Note that $\vec{a}_{AB}$ is independent of time.

***70** •••

Picture the Problem Let h and h' represent the heights from which the ball is dropped and to which it rebounds, respectively. Let v and v' represent the speeds with which the ball strikes the racket and rebounds from it. We can use a constant-acceleration equation to relate the pre- and post-collision speeds of the ball to its drop and rebound heights.

(*a*) Using a constant-acceleration equation, relate the impact speed of the ball to the distance it has fallen:

$$v^2 = v_0^2 + 2gh$$

or, because $v_0 = 0$,

$$v = \sqrt{2gh}$$

Relate the rebound speed of the ball to the height to which it rebounds:

$$v^2 = v'^2 - 2gh'$$

or because $v = 0$,

$$v' = \sqrt{2gh'}$$

Divide the second of these equations by the first to obtain:

$$\frac{v'}{v} = \frac{\sqrt{2gh'}}{\sqrt{2gh}} = \sqrt{\frac{h'}{h}}$$

Substitute for h' and evaluate the ratio of the speeds:

$$\frac{v'}{v} = \sqrt{\frac{0.64h}{h}} = 0.8 \Rightarrow v' = \boxed{0.8v}$$

(*b*) Call the speed of the racket V. In a reference frame where the racket is unmoving, the ball initially has speed V, moving *toward* the racket. After it "bounces" from the racket, it will have speed 0.8 V, moving *away* from the racket.

In the reference frame where the racket is moving and the ball initially unmoving, we need to add the speed of the racket to the speed of the ball in the racket's rest frame. Therefore, the ball's speed is:

$$v' = V + 0.8V = 1.8V = 45\,\text{m/s}$$

$$\approx \boxed{100\,\text{mi/h}}$$

This speed is close to that of a tennis pro's serve. Note that this result tells us that the ball is moving significantly faster than the racket.

(*c*) $\boxed{\text{From the result in part } (b)\text{, the ball can never move more than twice as fast as the racket.}}$

Circular Motion and Centripetal Acceleration

71 •

Picture the Problem We can use the definition of centripetal acceleration to express a_c in terms of the speed of the tip of the minute hand. We can find the tangential speed of the tip of the minute hand by using the distance it travels each revolution and the time it takes to complete each revolution.

Express the acceleration of the tip of the minute hand of the clock as a function of the length of the hand and the speed of its tip:

$$a_c = \frac{v^2}{R}$$

Use the distance the minute hand travels every hour to express its speed:

$$v = \frac{2\pi R}{T}$$

Substitute to obtain:

$$a_c = \frac{4\pi^2 R}{T^2}$$

Substitute numerical values and evaluate a_c:

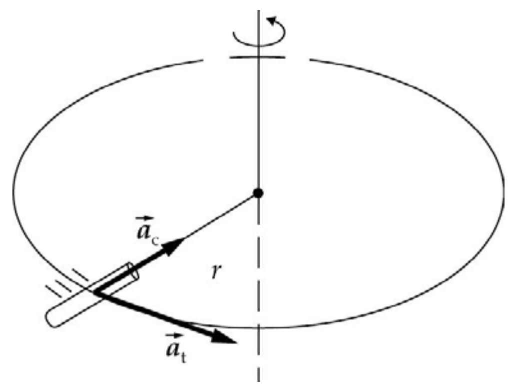
$$a_c = \frac{4\pi^2(0.5\,\text{m})}{(3600\,\text{s})^2} = \boxed{1.52\times10^{-6}\ \text{m/s}^2}$$

Express the ratio of a_c to g:

$$\frac{a_c}{g} = \frac{1.52\times10^{-6}\ \text{m/s}^2}{9.81\,\text{m/s}^2} = \boxed{1.55\times10^{-7}}$$

72 •

Picture the Problem The diagram shows the centripetal and tangential accelerations experienced by the test tube. The tangential acceleration will be zero when the centrifuge reaches its maximum speed. The centripetal acceleration increases as the tangential speed of the centrifuge increases. We can use the definition of centripetal acceleration to express a_c in terms of the speed of the test tube. We can find the tangential speed of the test tube by using the distance it travels each revolution and the time it takes to complete each revolution. The tangential acceleration can be found from the change in the tangential speed as the centrifuge is spinning up.



(*a*) Express the acceleration of the centrifuge arm as a function of the length of its arm and the speed of the test tube:

$$a_c = \frac{v^2}{R}$$

Use the distance the test tube travels every revolution to express its speed:

$$v = \frac{2\pi R}{T}$$

Substitute to obtain:

$$a_c = \frac{4\pi^2 R}{T^2}$$

Substitute numerical values and evaluate a_c:

$$a_c = \frac{4\pi^2(0.15\,\text{m})}{\left(\frac{1\,\text{min}}{15000\,\text{rev}}\times\frac{60\,\text{s}}{\text{min}}\right)^2}$$

$$= \boxed{3.70\times10^5\ \text{m/s}^2}$$

(*b*) Express the tangential acceleration in terms of the difference between the final and initial tangential speeds:

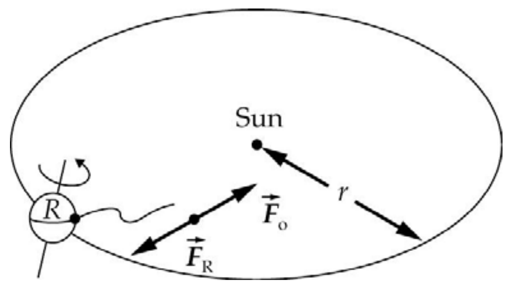
$$a_t = \frac{v_f - v_i}{\Delta t} = \frac{\frac{2\pi R}{T} - 0}{\Delta t} = \frac{2\pi R}{T\Delta t}$$

Substitute numerical values and evaluate a_T:

$$a_t = \frac{2\pi(0.15\,\text{m})}{\left(\frac{1\,\text{min}}{15000\,\text{rev}} \times \frac{60\,\text{s}}{\text{min}}\right)(75\,\text{s})} = \boxed{3.14\,\text{m/s}^2}$$

73 •

Picture the Problem The diagram includes a pictorial representation of the earth in its orbit about the sun and a force diagram showing the force on an object at the equator that is due to the earth's rotation, $\vec{F}_R$, and the force on the object due to the orbital motion of the earth about the sun, $\vec{F}_o$. Because these are centripetal forces, we can calculate the accelerations they require from the speeds and radii associated with the two circular motions.



Express the radial acceleration due to the rotation of the earth:

$$a_R = \frac{v_R^2}{R}$$

Express the speed of the object on the equator in terms of the radius of the earth R and the period of the earth's rotation T_R:

$$v_R = \frac{2\pi R}{T_R}$$

Substitute for v_R in the expression for a_R to obtain:

$$a_R = \frac{4\pi^2 R}{T_R^2}$$

Substitute numerical values and evaluate a_R:

$$a_R = \frac{4\pi^2(6370\times10^3\,\text{m})}{\left[(24\,\text{h})\left(\frac{3600\,\text{s}}{1\,\text{h}}\right)\right]^2} = 3.37\times10^{-2}\,\text{m/s}^2 = \boxed{3.44\times10^{-3}\,g}$$

Note that this effect gives rise to the well-known latitude correction for g.

Express the radial acceleration due to the orbital motion of the earth:

$$a_o = \frac{v_o^2}{r}$$

Express the speed of the object on the equator in terms of the earth-sun distance r and the period of the earth's motion about the sun T_o:

$$v_o = \frac{2\pi r}{T_o}$$

Substitute for v_o in the expression for a_o to obtain:

$$a_o = \frac{4\pi^2 r}{T_o^2}$$

Substitute numerical values and evaluate a_o:

$$a_o = \frac{4\pi^2 (1.5\times 10^{11}\ \text{m})}{\left[(365\ \text{d})\left(\frac{24\ \text{h}}{1\ \text{d}}\right)\left(\frac{3600\ \text{s}}{1\ \text{h}}\right)\right]^2}$$
$$= 5.95\times 10^{-3}\ \text{m/s}^2 = \boxed{6.07\times 10^{-4}\ g}$$

74 ••

Picture the Problem We can relate the acceleration of the moon toward the earth to its orbital speed and distance from the earth. Its orbital speed can be expressed in terms of its distance from the earth and its orbital period. From tables of astronomical data, we find that the sidereal period of the moon is 27.3 d and that its mean distance from the earth is 3.84×10^8 m.

Express the centripetal acceleration of the moon:

$$a_c = \frac{v^2}{r}$$

Express the orbital speed of the moon:

$$v = \frac{2\pi r}{T}$$

Substitute to obtain:

$$a_c = \frac{4\pi^2 r}{T^2}$$

Substitute numerical values and evaluate a_c:

$$a_c = \frac{4\pi^2 (3.84\times 10^8\ \text{m})}{\left(27.3\ \text{d}\times\frac{24\ \text{h}}{\text{d}}\times\frac{3600\ \text{s}}{\text{h}}\right)^2}$$
$$= 2.72\times 10^{-3}\ \text{m/s}^2$$
$$= \boxed{2.78\times 10^{-4}\ g}$$

Remarks: Note that $\frac{a_c}{g} = \frac{\textit{radius of earth}}{\textit{distance from earth to moon}}$ **(a_c is just the acceleration due to the earth's gravity evaluated at the moon's position). This is Newton's famous ″falling apple″ observation.**

75 •

Picture the Problem We can find the number of revolutions the ball makes in a given period of time from its speed and the radius of the circle along which it moves. Because the ball's centripetal acceleration is related to its speed, we can use this relationship to express its speed.

Express the number of revolutions per minute made by the ball in terms of the circumference c of the circle and the distance x the ball travels in time t:

$$n = \frac{x}{c} \qquad (1)$$

Relate the centripetal acceleration of the ball to its speed and the radius of its circular path:

$$a_c = g = \frac{v^2}{R}$$

Solve for the speed of the ball:

$$v = \sqrt{Rg}$$

Express the distance x traveled in time t at speed v:

$$x = vt$$

Substitute to obtain:

$$x = \sqrt{Rg}\,t$$

The distance traveled per revolution is the circumference c of the circle:

$$c = 2\pi R$$

Substitute in equation (1) to obtain:

$$n = \frac{\sqrt{Rg}\,t}{2\pi R} = \frac{1}{2\pi}\sqrt{\frac{g}{R}}\,t$$

Substitute numerical values and evaluate n:

$$n = \frac{1}{2\pi}\sqrt{\frac{9.81\,\text{m/s}^2}{0.8\,\text{m}}}(60\,\text{s}) = \boxed{33.4\,\text{min}^{-1}}$$

Remarks: The ball will oscillate at the end of this string as a simple pendulum with a period equal to 1/*n*.

Projectile Motion and Projectile Range

76 •

Picture the Problem Neglecting air resistance, the accelerations of the ball are constant and the horizontal and vertical motions of the ball are independent of each other. We can use the horizontal motion to determine the time-of-flight and then use this information to determine the distance the ball drops. Choose a coordinate system in which the origin is at the point of release of the ball, downward is the positive y direction, and the horizontal direction is the positive x direction.

Express the vertical displacement of the ball:

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

or, because $v_{0y} = 0$ and $a_y = g$,

$$\Delta y = \tfrac{1}{2}g(\Delta t)^2$$

Find the time of flight from $v_x = \Delta x/\Delta t$:

$$\Delta t = \frac{\Delta x}{v_x} = \frac{(18.4\,\text{m})(3600\,\text{s/h})}{(140\,\text{km/h})(1000\,\text{m/km})} = 0.473\,\text{s}$$

Substitute to find the vertical displacement in 0.473 s:

$$\Delta y = \tfrac{1}{2}(9.81\,\text{m/s}^2)(0.473\,\text{s})^2 = \boxed{1.10\,\text{m}}$$

77 •

Picture the Problem In the absence of air resistance, the maximum height achieved by a projectile depends on the vertical component of its initial velocity.

The vertical component of the projectile's initial velocity is:

$$v_{0y} = v_0 \sin\theta_0$$

Use the constant-acceleration equation:

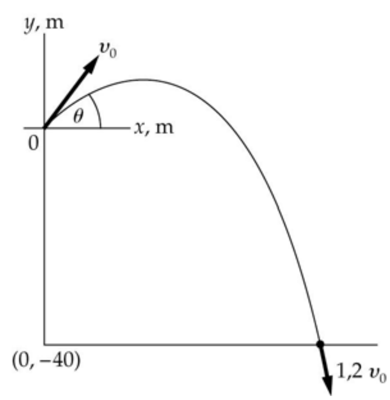
$$v_y^2 = v_{0y}^2 + 2a_y\Delta y$$

Set $v_y = 0$, $a = -g$, and $\Delta y = h$ to obtain:

$$h = \boxed{\frac{(v_0 \sin\theta_0)^2}{2g}}$$

***78** ••

Picture the Problem Choose the coordinate system shown to the right. Because, in the absence of air resistance, the horizontal and vertical speeds are independent of each other, we can use constant-acceleration equations to relate the impact speed of the projectile to its components.



The horizontal and vertical velocity components are:

$$v_{0x} = v_x = v_0\cos\theta$$

and

$$v_{0y} = v_0\sin\theta$$

Using a constant-acceleration equation, relate the vertical

$$v_y^2 = v_{0y}^2 + 2a_y\Delta y$$

or, because $a_y = -g$ and $\Delta y = -h$, component of the velocity to the vertical displacement of the projectile:

$$v_y^2 = (v_0 \sin\theta)^2 + 2gh$$

Express the relationship between the magnitude of a velocity vector and its components, substitute for the components, and simplify to obtain:

$$\begin{aligned} v^2 &= v_x^2 + v_y^2 = (v_0 \cos\theta)^2 + v_y^2 \\ &= v_0^2(\sin^2\theta + \cos^2\theta) + 2gh \\ &= v_0^2 + 2gh \end{aligned}$$

Substitute for v:

$$(1.2v_0)^2 = v_0^2 + 2gh$$

Set $v = 1.2\ v_0$, $h = 40$ m and solve for v_0:

$$v_0 = \boxed{42.2\,\text{m/s}}$$

Remarks: Note that v is independent of θ. This will be more obvious once conservation of energy has been studied.

79 ••

Picture the Problem Example 3-12 shows that the dart will hit the monkey unless the dart hits the ground before reaching the monkey's line of fall. What initial speed does the dart need in order to just reach the monkey's line of fall? First, we will calculate the fall time of the monkey, and then we will calculate the horizontal component of the dart's velocity.

Using a constant-acceleration equation, relate the monkey's fall distance to the fall time:

$$h = \tfrac{1}{2}gt^2$$

Solve for the time for the monkey to fall to the ground:

$$t = \sqrt{\frac{2h}{g}}$$

Substitute numerical values and evaluate t:

$$t = \sqrt{\frac{2(11.2\,\text{m})}{9.81\,\text{m/s}^2}} = 1.51\,\text{s}$$

Let θ be the angle the barrel of the dart gun makes with the horizontal. Then:

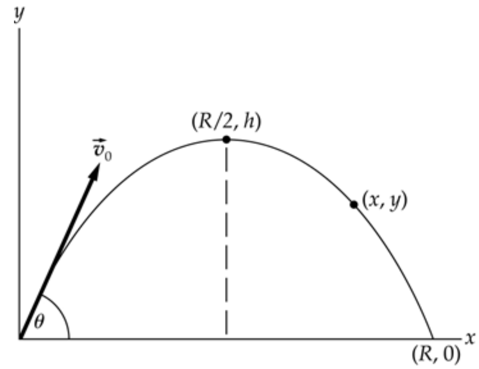
$$\theta = \tan^{-1}\left(\frac{10\,\text{m}}{50\,\text{m}}\right) = 11.3°$$

Use the fact that the horizontal velocity is constant to determine v_0:

$$v_0 = \frac{v_x}{\cos\theta} = \frac{(50\,\text{m}/1.51\,\text{s})}{\cos 11.3°} = \boxed{33.8\,\text{m/s}}$$

80 ••

Picture the Problem Choose the coordinate system shown in the figure to the right. In the absence of air resistance, the projectile experiences constant acceleration in both the x and y directions. We can use the constant-acceleration equations to express the x and y coordinates of the projectile along its trajectory as functions of time. The elimination of the parameter t will yield an expression for y as a function of x that we can evaluate at $(R, 0)$ and $(R/2, h)$. Solving these equations simultaneously will yield an expression for θ.



Express the position coordinates of the projectile along its flight path in terms of the parameter t:

$$x = (v_0 \cos\theta)t$$

and

$$y = (v_0 \sin\theta)t - \tfrac{1}{2}gt^2$$

Eliminate the parameter t to obtain:

$$y = (\tan\theta)x - \frac{g}{2v_0^2\cos^2\theta}x^2 \qquad (1)$$

Evaluate equation (1) at $(R, 0)$ to obtain:

$$R = \frac{2v_0^2 \sin\theta\cos\theta}{g}$$

Evaluate equation (1) at $(R/2, h)$ to obtain:

$$h = \frac{(v_0 \sin\theta)^2}{2g}$$

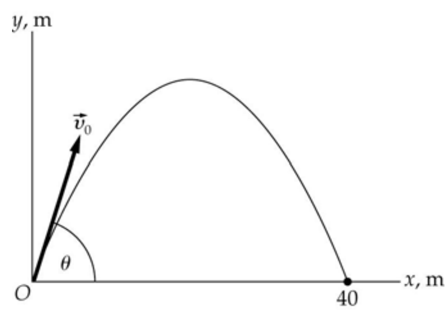
Equate R and h and solve the resulting equation for θ:

$$\theta = \tan^{-1}(4) = \boxed{76.0°}$$

Remarks: Note that this result is independent of v_0.

81 ••

Picture the Problem In the absence of air resistance, the motion of the ball is uniformly accelerated and its horizontal and vertical motions are independent of each other. Choose the coordinate system shown in the figure to the right and use constant-acceleration equations to relate the x and y components of the ball's initial velocity.



Use the components of v_0 to express θ in terms of v_{0x} and v_{0y}:

$$\theta = \tan^{-1}\frac{v_{0y}}{v_{0x}} \qquad (1)$$

Use the Pythagorean relationship between the velocity and its components to express v_0:

$$v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} \qquad (2)$$

Using a constant-acceleration equation, express the vertical speed of the projectile as a function of its initial upward speed and time into the flight:

$$v_y = v_{0y} + a_y t$$

Because $v_y = 0$ halfway through the flight (at maximum elevation):

$$v_{0y} = (9.81\ \text{m/s}^2)(1.22\ \text{s}) = 12.0\ \text{m/s}$$

Determine v_{0x}:

$$v_{0x} = \frac{\Delta x}{\Delta t} = \frac{40\,\text{m}}{2.44\,\text{s}} = 16.4\,\text{m/s}$$

Substitute in equation (2) and evaluate v_0:

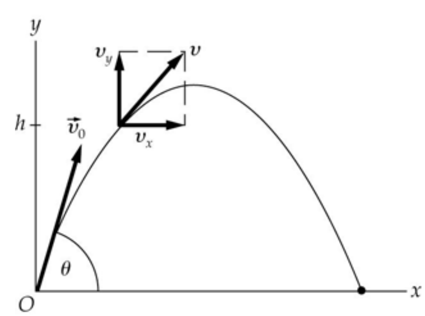
$$v_0 = \sqrt{(16.4\,\text{m/s})^2 + (12.0\,\text{m/s})^2} = \boxed{20.3\,\text{m/s}}$$

Substitute in equation (1) and evaluate θ:

$$\theta = \tan^{-1}\left(\frac{12.0\,\text{m/s}}{16.4\,\text{m/s}}\right) = \boxed{36.2°}$$

***82 ••**

Picture the Problem In the absence of friction, the acceleration of the ball is constant and we can use the constant-acceleration equations to describe its motion. The figure shows the launch conditions and an appropriate coordinate system. The speeds v, v_x, and v_y are related through the Pythagorean Theorem.



The squares of the vertical and horizontal components of the object's velocity are:

$$v_y^2 = v_0^2 \sin^2\theta - 2gh$$

and

$$v_x^2 = v_0^2 \cos^2\theta$$

The relationship between these variables is:

$$v^2 = v_x^2 + v_y^2$$

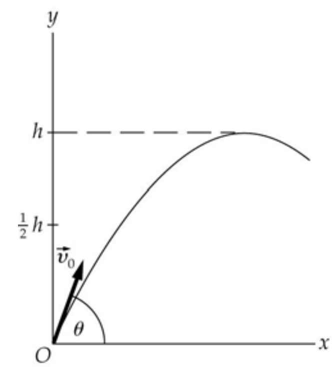
Substitute and simplify to obtain:

$$v^2 = \boxed{v_0^2 - 2gh}$$

Note that v is independent of θ ... as was to be shown.

83 ••

Picture the Problem In the absence of air resistance, the projectile experiences constant acceleration during its flight and we can use constant-acceleration equations to relate the speeds at half the maximum height and at the maximum height to the launch angle θ of the projectile.



The angle the initial velocity makes with the horizontal is related to the initial velocity components.

$$\tan\theta = \frac{v_{0y}}{v_{0x}}$$

Write the equation $v_y^2 = v_{0y}^2 + 2a\Delta y$, for $\Delta y = h$ and $v_y = 0$:

$$\Delta y = h \Rightarrow 0 = v_{0y}^2 - 2gh \qquad (1)$$

Write the equation $v_y^2 = v_{0y}^2 + 2a\Delta y$, for $\Delta y = h/2$:

$$\Delta y = \frac{h}{2} \Rightarrow v_y^2 = v_{0y}^2 - 2g\frac{h}{2} \qquad (2)$$

We are given $v_y = (3/4)v_0$. Square both sides and express this using the components of the velocity. The x component of the velocity remains constant.

$$v_{0x}^2 + v_y^2 = \left(\frac{3}{4}\right)^2 \left(v_{0x}^2 + v_{0y}^2\right) \qquad (3)$$

where we have used $v_x = v_{0x}$.

(Equations 1, 2, and 3 constitute three equations and four unknowns v_{0x}, v_{0y}, v_y, and h. To solve for any of these unknowns, we first need a fourth equation. However, to solve for the ratio (v_{0y}/v_{0x}) of two of the unknowns, the three equations are sufficient. That is because dividing both sides of each equation by v_{0x}^2 gives three equations and three unknowns v_y/v_{0x}, v_{0y}/v_{0x}, and h/v_{0x}^2.

Solve equation 2 for gh and substitute in equation 1:

$$v_{0y}^2 = 2\left(v_{0y}^2 - v_h^2\right) \Rightarrow v_y^2 = \frac{v_{0y}^2}{2}$$

Substitute for v_y^2 in equation 3:

$$v_{0x}^2 + \frac{1}{2}v_{0y}^2 = \left(\frac{3}{4}\right)^2 \left(v_{0x}^2 + v_{0y}^2\right)$$

Divide both sides by v_{0x}^2 and solve for v_{0y}/v_{0x} to obtain:

$$1+\frac{1}{2}\frac{v_{0y}^2}{v_{0x}^2}=\frac{9}{16}\left(1+\frac{v_{0y}^2}{v_{0x}^2}\right)$$

and

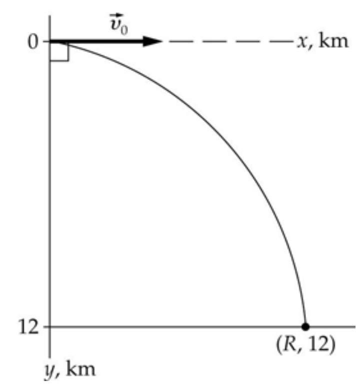
$$\frac{v_{0y}}{v_{0x}}=\sqrt{7}$$

Using $\tan\theta = v_{0y}/v_{0x}$, solve for θ:

$$\theta=\tan^{-1}\left(\frac{v_{0y}}{v_{0x}}\right)=\tan^{-1}\left(\sqrt{7}\right)=\boxed{69.3°}$$

84 •

Picture the Problem The horizontal speed of the crate, in the absence of air resistance, is constant and equal to the speed of the cargo plane. Choose a coordinate system in which the direction the plane is moving is the positive *x* direction and downward is the positive *y* direction and apply the constant-acceleration equations to describe the crate's displacements at any time during its flight.



(*a*) Using a constant-acceleration equation, relate the vertical displacement of the crate Δy to the time of fall Δt:

$$\Delta y = v_{0y}\Delta t+\tfrac{1}{2}g(\Delta t)^2$$

or, because $v_{0y}=0$,

$$\Delta y=\tfrac{1}{2}g(\Delta t)^2$$

Solve for Δt:

$$\Delta t=\sqrt{\frac{2\Delta y}{g}}$$

Substitute numerical values and evaluate Δt:

$$\Delta t=\sqrt{\frac{2\left(12\times10^3\text{ m}\right)}{9.81\text{m/s}^2}}=\boxed{49.5\text{s}}$$

(*b*) The horizontal distance traveled in 49.5 s is:

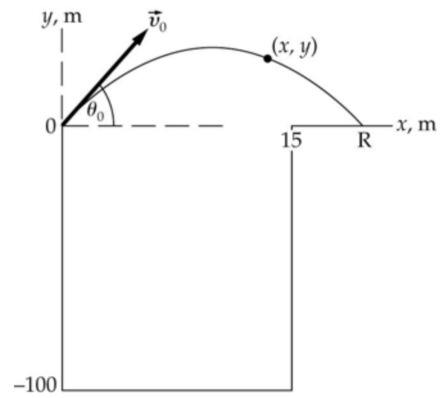
$$R=\Delta x=v_{0x}\Delta t$$
$$=(900\,\text{km/h})\left(\frac{1\text{h}}{3600\text{s}}\right)(49.5\text{s})$$
$$=\boxed{12.4\,\text{km}}$$

(*c*) Because the velocity of the plane is constant, it will be directly over the crate when it hits the ground; i.e., the distance to the aircraft will be the elevation of the aircraft.

$$\Delta y=\boxed{12.0\,\text{km}}$$

***85** ••

Picture the Problem In the absence of air resistance, the accelerations of both Wiley Coyote and the Roadrunner are constant and we can use constant-acceleration equations to express their coordinates at any time during their leaps across the gorge. By eliminating the parameter t between these equations, we can obtain an expression that relates their y coordinates to their x coordinates and that we can solve for their launch angles.



(*a*) Using constant-acceleration equations, express the x coordinate of the Roadrunner while it is in flight across the gorge:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

or, because $x_0 = 0$, $a_x = 0$ and $v_{0x} = v_0 \cos\theta_0$,

$$x = (v_0 \cos\theta_0)t$$

Using constant-acceleration equations, express the y coordinate of the Roadrunner while it is in flight across the gorge:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

or, because $y_0 = 0$, $a_y = -g$ and $v_{0y} = v_0 \sin\theta_0$,

$$y = (v_0 \sin\theta_0)t - \tfrac{1}{2}gt^2$$

Eliminate the parameter t to obtain:

$$y = (\tan\theta_0)x - \frac{g}{2v_0^2\cos^2\theta_0}x^2 \qquad (1)$$

Letting R represent the Roadrunner's range and using the trigonometric identity $\sin 2\theta = 2\sin\theta\cos\theta$, solve for and evaluate its launch speed:

$$v_0 = \sqrt{\frac{Rg}{\sin 2\theta_0}} = \sqrt{\frac{(16.5\,\text{m})(9.81\,\text{m/s}^2)}{\sin 30°}} = \boxed{18.0\,\text{m/s}}$$

(*b*) Letting R represent Wiley's range, solve equation (1) for his launch angle:

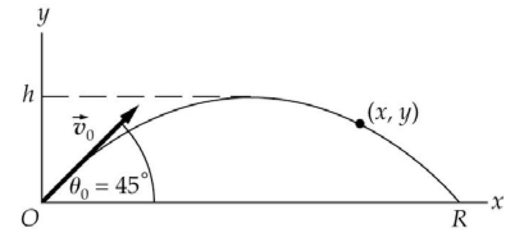
$$\theta_0 = \frac{1}{2}\sin^{-1}\left(\frac{Rg}{v_0^2}\right)$$

Substitute numerical values and evaluate θ_0:

$$\theta_0 = \frac{1}{2}\sin^{-1}\left[\frac{(14.5\,\text{m})(9.81\,\text{m/s}^2)}{(18.0\,\text{m/s})^2}\right] = \boxed{13.0°}$$

86 •

Picture the Problem Because, in the absence of air resistance, the vertical and horizontal accelerations of the cannonball are constant, we can use constant-acceleration equations to express the ball's position and velocity as functions of time and acceleration. The maximum height of the ball and its time-of-flight are related to the components of its launch velocity.



(*a*) Using a constant-acceleration equation, relate *h* to the initial and final speeds of the cannonball:

$$v^2 = v_{0y}^2 + 2a_y\Delta y$$

or, because $v = 0$ and $a_y = -g$,

$$0 = v_{0y}^2 - 2g\Delta y$$

Find the vertical component of the firing speed:

$$v_{0y} = v_0 \sin\theta = (300\text{ m/s})\sin 45° = 212\text{ m/s}$$

Solve for and evaluate *h*:

$$h = \frac{v_{0y}^2}{2g} = \frac{(212\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} = \boxed{2.29\,\text{km}}$$

(*b*) The total flight time is:

$$\Delta t = t_{\text{up}} + t_{\text{dn}} = 2t_{\text{up}}$$

$$= 2\frac{v_{0y}}{g} = \frac{2(212\,\text{m/s})}{9.81\,\text{m/s}^2} = \boxed{43.2\,\text{s}}$$

(*c*) Express the *x* coordinate of the ball as a function of time:

$$x = v_{0x}\Delta t = (v_0\cos\theta)\Delta t$$

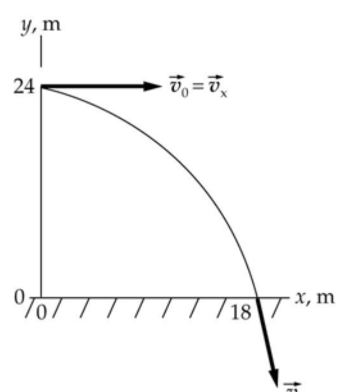
Evaluate *x* (= *R*) when $\Delta t = 43.2$ s:

$$x = [(300\,\text{m/s})\cos 45°](43.2\,\text{s})$$

$$= \boxed{9.16\,\text{km}}$$

87 ••

Picture the Problem Choose a coordinate system in which the origin is at the base of the tower and the *x*- and *y*-axes are as shown in the figure to the right. In the absence of air resistance, the horizontal speed of the stone will remain constant during its fall and a constant-acceleration equation can be used to determine the time of fall. The final velocity of the stone will be the vector sum of its *x* and *y* components.



(*a*) Using a constant-acceleration equation, express the vertical displacement of the stone (the height of the tower) as a function of the fall time:

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

or, because $v_{0y} = 0$ and $a = -g$,

$$\Delta y = -\tfrac{1}{2}g(\Delta t)^2$$

Solve for and evaluate the time of fall:

$$\Delta t = \sqrt{-\frac{2\Delta y}{g}} = \sqrt{-\frac{2(-24\,\text{m})}{9.81\,\text{m/s}^2}} = 2.21\,\text{s}$$

Use the definition of average velocity to find the velocity with which the stone was thrown from the tower:

$$v_x = v_{0x} \equiv \frac{\Delta x}{\Delta t} = \frac{18\,\text{m}}{2.21\,\text{s}} = \boxed{8.14\,\text{m/s}}$$

(*b*) Find the *y* component of the stone's velocity after 2.21 s:

$$\begin{aligned} v_y &= v_{0y} - gt \\ &= 0 - (9.81\,\text{m/s2})(2.21\,\text{s}) \\ &= -21.7\,\text{m/s} \end{aligned}$$

Express v in terms of its components:

$$v = \sqrt{v_x^2 + v_y^2}$$

Substitute numerical values and evaluate v:

$$\begin{aligned} v &= \sqrt{(8.14\,\text{m/s})^2 + (-21.7\,\text{m/s})^2} \\ &= \boxed{23.2\,\text{m/s}} \end{aligned}$$

88 ••

Picture the Problem In the absence of air resistance, the acceleration of the projectile is constant and its horizontal and vertical motions are independent of each other. We can use constant-acceleration equations to express the horizontal and vertical displacements of the projectile in terms of its time-of-flight.

Using a constant-acceleration equation, express the horizontal displacement of the projectile as a function of time:

$$\Delta x = v_{0x}\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$

or, because $v_{0x} = v_0\cos\theta$ and $a_x = 0$,

$$\Delta x = (v_0\cos\theta)\Delta t$$

Using a constant-acceleration equation, express the vertical displacement of the projectile as a function of time:

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

or, because $v_{0y} = v_0\sin\theta$ and $a_y = -g$,

$$\Delta y = (v_0\cos\theta)\Delta t - \tfrac{1}{2}g(\Delta t)^2$$

Substitute numerical values to obtain the quadratic equation:

$$-200\,\text{m} = (60\text{m/s})(\sin 60°)\Delta t - \tfrac{1}{2}(9.81\,\text{m/s}^2)(\Delta t)^2$$

Solve for Δt:

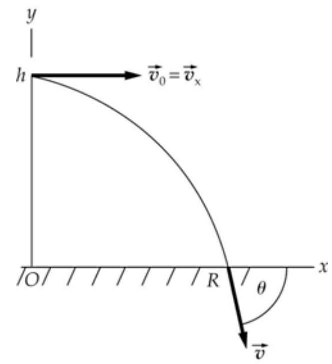
$$\Delta t = 13.6\ \text{s}$$

Substitute for Δt and evaluate the horizontal distance traveled by the projectile:

$$\Delta x = (60\text{ m/s})(\cos 60°)(13.6\text{ s}) = \boxed{408\text{ m}}$$

89 ••

Picture the Problem In the absence of air resistance, the acceleration of the cannonball is constant and its horizontal and vertical motions are independent of each other. Choose the origin of the coordinate system to be at the base of the cliff and the axes directed as shown and use constant- acceleration equations to describe both the horizontal and vertical displacements of the cannonball.



Express the direction of the velocity vector when the projectile strikes the ground:

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Express the vertical displacement using a constant-acceleration equation:

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

or, because $v_{0y} = 0$ and $a_y = -g$,

$$\Delta y = -\tfrac{1}{2}g(\Delta t)^2$$

Set $\Delta x = -\Delta y$ ($R = -h$) to obtain:

$$\Delta x = v_x\Delta t = \tfrac{1}{2}g(\Delta t)^2$$

Solve for v_x:

$$v_x = \frac{\Delta x}{\Delta t} = \tfrac{1}{2}g\Delta t$$

Find the y component of the projectile as it hits the ground:

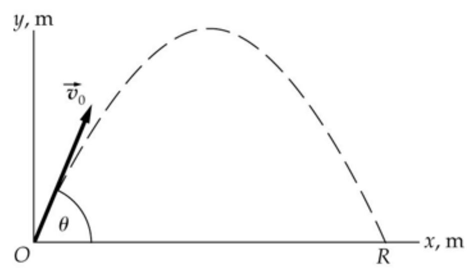
$$v_y = v_{0y} + a\Delta t = -g\Delta t = -2v_x$$

Substitute and evaluate θ:

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}(-2) = \boxed{-63.4°}$$

90 •

Picture the Problem In the absence of air resistance, the vertical and horizontal motions of the projectile experience constant accelerations and are independent of each other. Use a coordinate system in which up is the positive y direction and horizontal is the positive x direction and use constant-acceleration equations to describe the horizontal and vertical displacements of the projectile as functions of the time into the flight.



(*a*) Use a constant-acceleration equation to express the horizontal displacement of the projectile as a function of time:

$$\Delta x = v_{0x}\Delta t = (v_0 \cos\theta)\Delta t$$

Evaluate this expression when $\Delta t = 6$ s:

$$\Delta x = (300\,\text{m/s})(\cos 60°)(6\,\text{s}) = \boxed{900\,\text{m}}$$

(*b*) Use a constant-acceleration equation to express the vertical displacement of the projectile as a function of time:

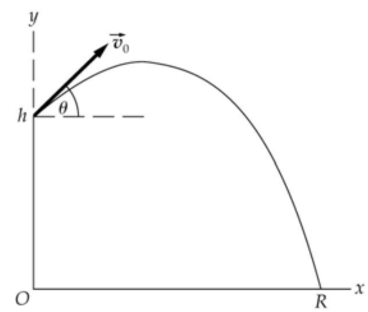
$$\Delta y = (v_0 \sin\theta)\Delta t - \tfrac{1}{2}g(\Delta t)^2$$

Evaluate this expression when $\Delta t = 6$ s:

$$\Delta y = (300\,\text{m/s})(\sin 60°)(6\,\text{s}) - \tfrac{1}{2}(9.81\,\text{m/s}^2)(6\,\text{s})^2 = \boxed{1.38\,\text{km}}$$

91 ••

Picture the Problem In the absence of air resistance, the acceleration of the projectile is constant and the horizontal and vertical motions are independent of each other. Choose the coordinate system shown in the figure with the origin at the base of the cliff and the axes oriented as shown and use constant-acceleration equations to find the range of the cannonball.



Using a constant-acceleration equation, express the horizontal displacement of the cannonball as a function of time:

$$\Delta x = v_{0x}\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$

or, because $v_{0x} = v_0\cos\theta$ and $a_x = 0$,

$$\Delta x = (v_0 \cos\theta)\Delta t$$

Using a constant-acceleration equation, express the vertical displacement of the cannonball as a function of time:

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

or, because $y = -40$ m, $a = -g$, and $v_{0y} = v_0\sin\theta$,

$$-40\,\text{m} = (42.2\,\text{m/s})(\sin 30°)\Delta t - \tfrac{1}{2}(9.81\,\text{m/s}^2)(\Delta t)^2$$

Solve the quadratic equation for Δt:

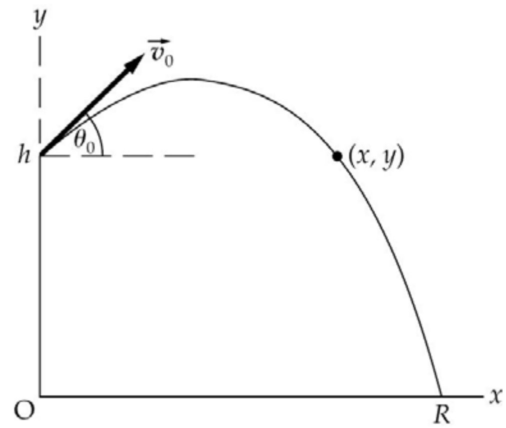
$\Delta t = 5.73$ s

Calculate the range:

$$R = \Delta x = (42.2\,\text{m/s})(\cos 30°)(5.73\,\text{s}) = \boxed{209\,\text{m}}$$

***92** ••

Picture the Problem Choose a coordinate system in which the origin is at ground level. Let the positive *x* direction be to the right and the positive *y* direction be upward. We can apply constant-acceleration equations to obtain parametric equations in time that relate the range to the initial horizontal speed and the height *h* to the initial upward speed. Eliminating the parameter will leave us with a quadratic equation in *R,* the solution to which will give us the range of the arrow. In (*b*), we'll find the launch speed and angle as viewed by an observer who is at rest on the ground and then use these results to find the arrow's range when the horse is moving at 12 m/s.



(*a*) Use constant-acceleration equations to express the horizontal and vertical coordinates of the arrow's motion:

$$R = \Delta x = x - x_0 = v_{0x}t$$

and

$$y = h + v_{0y}t + \tfrac{1}{2}(-g)t^2$$

where

$$v_{0x} = v_0 \cos\theta \text{ and } v_{0y} = v_0 \sin\theta$$

Solve the *x*-component equation for time:

$$t = \frac{R}{v_{0x}} = \frac{R}{v_0 \cos\theta}$$

Eliminate time from the *y*-component equation:

$$y = h + v_{0y}\frac{R}{v_{0x}} - \frac{1}{2}g\left(\frac{R}{v_{0x}}\right)^2$$

and, at (R, 0),

$$0 = h + (\tan\theta)R - \frac{g}{2v_0^2\cos^2\theta}R^2$$

Solve for the range to obtain:

$$R = \frac{v_0^2}{2g}\sin 2\theta\left(1 + \sqrt{1 + \frac{2gh}{v_0^2 \sin^2\theta}}\right)$$

Substitute numerical values and evaluate R:

$$R = \frac{(45\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)}\sin 20°\left(1 + \sqrt{1 + \frac{2(9.81\,\text{m/s}^2)(2.25\,\text{m})}{(45\,\text{m/s})^2(\sin^2 10°)}}\right) = \boxed{81.6\,\text{m}}$$

(*b*) Express the speed of the arrow in the horizontal direction:

$$v_x = v_{\text{arrow}} + v_{\text{archer}} = (45\,\text{m/s})\cos 10° + 12\,\text{m/s} = 56.3\,\text{m/s}$$

Express the vertical speed of the arrow:

$$v_y = (45\,\text{m/s})\sin 10° = 7.81\,\text{m/s}$$

Express the angle of elevation from the perspective of someone on the ground:

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{7.81\,\text{m/s}}{56.3\,\text{m/s}}\right) = 7.90°$$

Express the arrow's speed relative to the ground:

$$v_0 = \sqrt{v_x^2 + v_y^2} = \sqrt{(56.3\,\text{m/s})^2 + (7.81\,\text{m/s})^2} = 56.8\,\text{m/s}$$

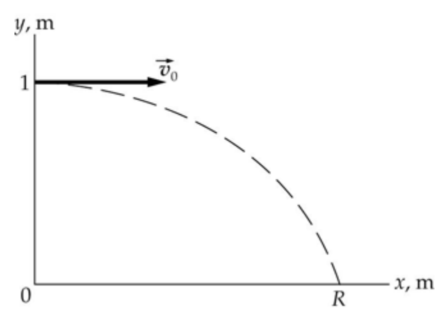
Substitute numerical values and evaluate R:

$$R = \frac{(56.8\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)}\sin 15.8°\left(1 + \sqrt{1 + \frac{2(9.81\,\text{m/s}^2)(2.25\,\text{m})}{(56.8\,\text{m/s})^2(\sin^2 7.9°)}}\right) = \boxed{104\,\text{m}}$$

Remarks: An alternative solution for part (*b*) is to solve for the range in the reference frame of the archer and then add to it the distance the frame travels, relative to the earth, during the time of flight.

93 •

Picture the Problem In the absence of air resistance, the horizontal and vertical motions are independent of each other. Choose a coordinate system oriented as shown in the figure to the right and apply constant-acceleration equations to find the time-of-flight and the range of the spud-plug.



(*a*) Using a constant-acceleration equation, express the vertical displacement of the plug:

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

or, because $v_{0y} = 0$ and $a_y = -g$,

$$\Delta y = -\tfrac{1}{2}g(\Delta t)^2$$

Solve for and evaluate the flight time Δt:

$$\Delta t = \sqrt{-\frac{2\Delta y}{g}} = \sqrt{-\frac{2(-1.00\,\text{m})}{9.81\,\text{m/s}^2}} = \boxed{0.452\,\text{s}}$$

(*b*) Using a constant-acceleration equation, express the horizontal displacement of the plug:

$$\Delta x = v_{0x}\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$

or, because $a_x = 0$ and $v_{0x} = v_0$,

$$\Delta x = v_0\Delta t$$

Substitute numerical values and evaluate R:

$$\Delta x = R = (50\,\text{m/s})(0.452\,\text{s}) = \boxed{22.6\,\text{m}}$$

94 ••

Picture the Problem An extreme value (i.e., a maximum or a minimum) of a function is determined by setting the appropriate derivative equal to zero. Whether the extremum is a maximum or a minimum can be determined by evaluating the second derivative at the point determined by the first derivative.

Evaluate $dR/d\theta_0$:

$$\frac{dR}{d\theta_0} = \frac{v_0^2}{g}\frac{d}{d\theta_0}[\sin(2\theta_0)] = \frac{2v_0^2}{g}\cos(2\theta_0)$$

Set $dR/d\theta_0 = 0$ for extrema and solve for θ_0:

$$\frac{2v_0^2}{g}\cos(2\theta_0) = 0$$

and

$$\theta_0 = \tfrac{1}{2}\cos^{-1}0 = 45°$$

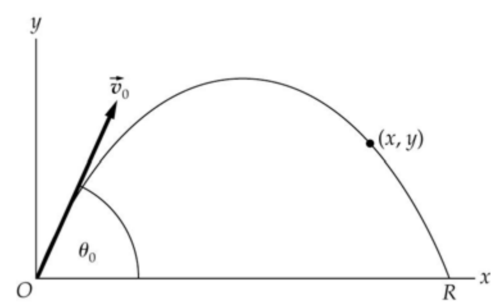
Determine whether 45° is a maximum or a minimum:

$$\left.\frac{d^2R}{d\theta_0^{\,2}}\right|_{\theta_0=45°} = \left[-4\left(v_0^2/g\right)\sin 2\theta_0\right]_{\theta_0=45°} < 0$$

$\therefore$ R is a maximum at $\theta_0 = 45°$

95 •

Picture the Problem We can use constant-acceleration equations to express the x and y coordinates of a bullet in flight on the moon as a function of t. Eliminating this parameter will yield an expression for y as a function of x that we can use to find the range of the bullet. The necessity that the centripetal acceleration of an object in orbit at the surface of a body equal the acceleration due to gravity at the surface will allow us to determine the required muzzle velocity for orbital motion.



(*a*) Using a constant-acceleration equation, express the x coordinate of a bullet in flight on the moon:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

or, because $x_0 = 0$, $a_x = 0$ and $v_{0x} = v_0\cos\theta_0$,

$$x = (v_0\cos\theta_0)t$$

Using a constant-acceleration equation, express the y coordinate of a bullet in flight on the moon:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

or, because $y_0 = 0$, $a_y = -g_{\text{moon}}$ and $v_{0y} = v_0 \sin\theta_0$,

$$y = (v_0 \sin\theta_0)t - \tfrac{1}{2}g_{\text{moon}}t^2$$

Eliminate the parameter t to obtain:

$$y = (\tan\theta_0)x - \frac{g_{\text{moon}}}{2v_0^2\cos^2\theta_0}x^2$$

When $y = 0$ and $x = R$:

$$0 = (\tan\theta_0)R - \frac{g_{\text{moon}}}{2v_0^2\cos^2\theta_0}R^2$$

and

$$R = \frac{v_0^2}{g_{\text{moon}}}\sin 2\theta_0$$

Substitute numerical values and evaluate R:

$$R = \frac{(900\,\text{m/s})^2}{1.67\,\text{m/s}^2}\sin 90° = 4.85\times10^5\,\text{m}$$

$$= \boxed{485\,\text{km}}$$

This result is probably not very accurate because it is about 28% of the moon's radius (1740 km). This being the case, we can no longer assume that the ground is "flat" because of the curvature of the moon.

(*b*) Express the condition that the centripetal acceleration must satisfy for an object in orbit at the surface of the moon:

$$a_c = g_{\text{moon}} = \frac{v^2}{r}$$

Solve for and evaluate v:

$$v = \sqrt{g_{\text{moon}}r} = \sqrt{(1.67\,\text{m/s}^2)(1.74\times10^6\,\text{m})} = \boxed{1.70\,\text{km/s}}$$

96 •••

Picture the Problem We can show that $\Delta R/R = -\Delta g/g$ by differentiating R with respect to g and then using a differential approximation.

Differentiate the range equation with respect to g:

$$\frac{dR}{dg} = \frac{d}{dg}\left(\frac{v_0^2}{g}\sin 2\theta_0\right) = -\frac{v_0^2}{g^2}\sin 2\theta_0 = -\frac{R}{g}$$

Approximate dR/dg by $\Delta R/\Delta g$:

$$\frac{\Delta R}{\Delta g} = -\frac{R}{g}$$

Separate the variables to obtain:

$$\frac{\Delta R}{R} = -\frac{\Delta g}{g}$$

i.e., for small changes in gravity ($g \approx g \pm \Delta g$), the fractional change in R is linearly opposite to the fractional change in g.

Remarks: This tells us that as gravity increases, the range will decrease, and vice versa. This is as it must be because *R* is inversely proportional to *g*.

97 •••

Picture the Problem We can show that $\Delta R/R = 2\Delta v_0/ v_0$ by differentiating R with respect to v_0 and then using a differential approximation.

Differentiate the range equation with respect to v_0:

$$\frac{dR}{dv_0} = \frac{d}{dv_0}\left(\frac{v_0^2}{g}\sin 2\theta_0\right) = \frac{2v_0}{g}\sin 2\theta_0$$
$$= 2\frac{R}{v_0}$$

Approximate dR/dv_0 by $\Delta R/\Delta v_0$:

$$\frac{\Delta R}{\Delta v_0} = 2\frac{R}{v_0}$$

Separate the variables to obtain:

$$\frac{\Delta R}{R} = 2\frac{\Delta v_0}{v_0}$$

i.e., for small changes in the launch velocity ($v_0 \approx v_0 \pm \Delta v_0$), the fractional change in R is twice the fractional change in v_0.

Remarks: This tells us that as launch velocity increases, the range will increase twice as fast, and vice versa.

98 •••

Picture the Problem Choose a coordinate system in which the origin is at the base of the surface from which the projectile is launched. Let the positive x direction be to the right and the positive y direction be upward. We can apply constant-acceleration equations to obtain parametric equations in time that relate the range to the initial horizontal speed and the height h to the initial upward speed. Eliminating the parameter will leave us with a quadratic equation in R, the solution to which is the result we are required to establish.

Write the constant-acceleration equations for the horizontal and vertical parts of the projectile's

$$x = v_{0x}t$$

and

motion:

$$y = h + v_{0y}t + \tfrac{1}{2}(-g)t^2$$

where

$v_{0x} = v_0\cos\theta$ and $v_{0y} = v_0\sin\theta$

Solve the x-component equation for time:

$$t = \frac{x}{v_{0x}} = \frac{x}{v_0\cos\theta}$$

Using the x-component equation, eliminate time from the y-component equation to obtain:

$$y = h + (\tan\theta)x - \frac{g}{2v_0^2\cos^2\theta}x^2$$

When the projectile strikes the ground its coordinates are (R, 0) and our equation becomes:

$$0 = h + (\tan\theta)R - \frac{g}{2v_0^2\cos^2\theta}R^2$$

Using the plus sign in the quadratic formula to ensure a physically meaningful root (one that is positive), solve for the range to obtain:

$$\boxed{R = \left(1 + \sqrt{1 + \frac{2gh}{v_0^2\sin^2\theta_0}}\right)\frac{v_0^2}{2g}\sin 2\theta_0}$$

***99 ••**

Picture the Problem We can use trigonometry to relate the maximum height of the projectile to its range and the sighting angle at maximum elevation and the range equation to express the range as a function of the launch speed and angle. We can use a constant-acceleration equation to express the maximum height reached by the projectile in terms of its launch angle and speed. Combining these relationships will allow us to conclude that $\tan\phi = \tfrac{1}{2}\tan\theta$.

Referring to the figure, relate the maximum height of the projectile to its range and the sighting angle ϕ:

$$\tan\phi = \frac{h}{R/2}$$

Express the range of the rocket and use the trigonometric identity $\sin 2\theta = 2\sin\theta\cos\theta$ to rewrite the expression as:

$$R = \frac{v^2}{g}\sin(2\theta) = 2\frac{v^2}{g}\sin\theta\cos\theta$$

Using a constant-acceleration equation, relate the maximum height of a projectile to the vertical component of its launch speed:

$$v_y^2 = v_{0y}^2 - 2gh$$

or, because $v_y = 0$ and $v_{0y} = v_0\sin\theta$,

$$v_0^2\sin^2\theta = 2gh$$

Solve for the maximum height h:

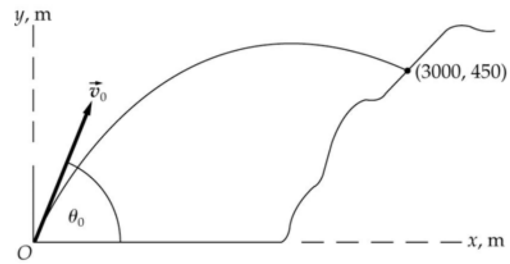
$$h = \frac{v^2}{2g}\sin^2\theta$$

Substitute for R and h and simplify to obtain:

$$\tan\phi = \frac{2\dfrac{v^2}{2g}\sin^2\theta}{2\dfrac{v^2}{g}\sin\theta\cos\theta} = \boxed{\tfrac{1}{2}\tan\theta}$$

100 •

Picture the Problem In the absence of air resistance, the horizontal and vertical displacements of the projectile are independent of each other and describable by constant-acceleration equations. Choose the origin at the firing location and with the coordinate axes as shown in the figure and use constant-acceleration equations to relate the vertical displacement to vertical component of the initial velocity and the horizontal velocity to the horizontal displacement and the time of flight.



(*a*) Using a constant-acceleration equation, express the vertical displacement of the projectile as a function of its time of flight:

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

or, because $a_y = -g$,

$$\Delta y = v_{0y}\Delta t - \tfrac{1}{2}g(\Delta t)^2$$

Solve for v_{0y}:

$$v_{0y} = \frac{\Delta y + \tfrac{1}{2}g(\Delta t)^2}{\Delta t}$$

Substitute numerical values and evaluate v_{0y}:

$$v_{0y} = \frac{450\,\text{m} + \tfrac{1}{2}(9.81\,\text{m/s}^2)(20\,\text{s})^2}{20\,\text{s}}$$

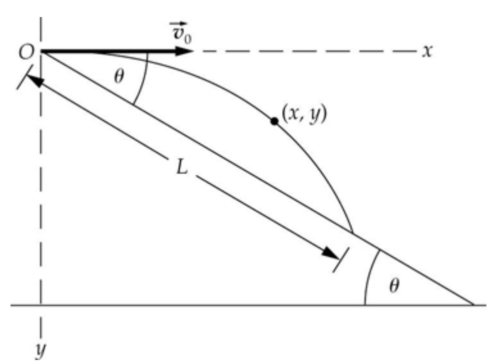
$$= \boxed{121\,\text{m/s}}$$

(*b*) The horizontal velocity remains constant, so:

$$v_{0x} = v_x = \frac{\Delta x}{\Delta t} = \frac{3000\ \text{m}}{20\ \text{s}} = \boxed{150\ \text{m/s}}$$

***101** ••

Picture the Problem In the absence of air resistance, the acceleration of the stone is constant and the horizontal and vertical motions are independent of each other. Choose a coordinate system with the origin at the throwing location and the axes oriented as shown in the figure and use constant- acceleration equations to express the x and y coordinates of the stone while it is in flight.



Using a constant-acceleration equation, express the x coordinate of the stone in flight:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

or, because $x_0 = 0$, $v_{0x} = v_0$ and $a_x = 0$,

$$x = v_0 t$$

Using a constant-acceleration equation, express the y coordinate of the stone in flight:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

or, because $y_0 = 0$, $v_{0y} = 0$ and $a_y = g$,

$$y = \tfrac{1}{2}gt^2$$

Referring to the diagram, express the relationship between θ, y and x at impact:

$$\tan\theta = \frac{y}{x}$$

Substitute for x and y and solve for the time to impact:

$$\tan\theta = \frac{gt^2}{2v_0 t} = \frac{g}{2v_0}t$$

Solve for t to obtain:

$$t = \frac{2v_0}{g}\tan\theta$$

Referring to the diagram, express the relationship between θ, L, y and x at impact:

$$x = L\cos\theta = \frac{y}{\tan\theta}$$

Substitute for y to obtain:

$$\frac{gt^2}{2g} = L\cos\theta$$

Substitute for t and solve for L to obtain:

$$L = \boxed{\frac{2v_0^2\tan\theta}{g\cos\theta}}$$

102 •••

Picture the Problem The equation of a particle's trajectory is derived in the text so we'll use it as our starting point in this derivation. We can relate the coordinates of the point of impact (x, y) to the angle ϕ and use this relationship to eliminate y from the equation for the cannonball's trajectory. We can then solve the resulting equation for x and relate the horizontal component of the point of impact to the cannonball's range.

The equation of the cannonball's trajectory is given in the text:

$$y(x) = (\tan\theta_0)x - \left(\frac{g}{2v_0^2\cos^2\theta_0}\right)x^2$$

Relate the x and y components of a point on the ground to the angle ϕ:

$$y(x) = (\tan\phi)x$$

Express the condition that the cannonball hits the ground:

$$(\tan\phi)x = (\tan\theta_0)x - \left(\frac{g}{2v_0^2\cos^2\theta_0}\right)x^2$$

Solve for x to obtain:

$$x=\frac{2v_0^2\cos^2\theta_0(\tan\theta_0-\tan\phi)}{g}$$

Relate the range of the cannonball's flight R to the horizontal distance x:

$$x=R\cos\phi$$

Substitute to obtain:

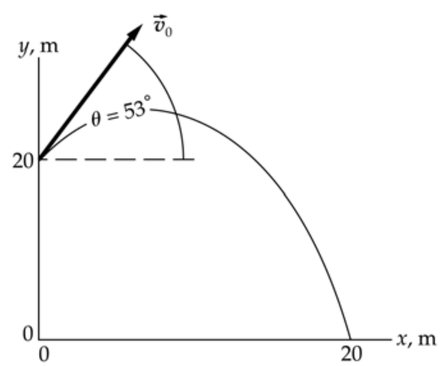
$$R\cos\phi=\frac{2v_0^2\cos^2\theta_0(\tan\theta_0-\tan\phi)}{g}$$

Solve for R:

$$R=\boxed{\frac{2v_0^2\cos^2\theta_0(\tan\theta_0-\tan\phi)}{g\cos\phi}}$$

103 ••

Picture the Problem In the absence of air resistance, the acceleration of the rock is constant and the horizontal and vertical motions are independent of each other. Choose the coordinate system shown in the figure with the origin at the base of the building and the axes oriented as shown and apply constant-acceleration equations to relate the horizontal and vertical displacements of the rock to its time of flight.



Find the horizontal and vertical components of v_0:

$$v_{0x}=v_0\cos 53°=0.602v_0$$
$$v_{0y}=v_0\sin 53°=0.799v_0$$

Using a constant-acceleration equation, express the horizontal displacement of the projectile:

$$\Delta x=20\,\text{m}=v_{0x}\Delta t=(0.602v_0)\Delta t$$

Using a constant-acceleration equation, express the vertical displacement of the projectile:

$$\Delta y=-20\,\text{m}=v_{0y}\Delta t-\tfrac{1}{2}g(\Delta t)^2$$
$$=(0.799v_0)\Delta t-\tfrac{1}{2}g(\Delta t)^2$$

Solve the x-displacement equation for Δt:

$$\Delta t=\frac{20\,\text{m}}{0.602v_0}$$

Substitute Δt into the expression for Δy:

$$-20\text{ m}=(0.799v_0)\Delta t-(4.91\text{ m/s}^2)(\Delta t)^2$$

Solve for v_0 to obtain:

$$v_0=\boxed{10.8\,\text{m/s}}$$

Find Δt at impact:

$$\Delta t=\frac{20\,\text{m}}{(10.8\,\text{m/s})\cos 53°}=3.08\,\text{s}$$

Using constant-acceleration equations, find v_y and v_x at impact:

$$v_x = v_{0x} = 6.50\,\text{m/s}$$

and

$$v_y = v_{0y} - g\Delta t = -21\,\text{m/s}$$

Express the velocity at impact in vector form:

$$\vec{v} = \boxed{(6.50\,\text{m/s})\hat{i} + (-21.6\,\text{m/s})\hat{j}}$$

104 ••

Picture the Problem The ball experiences constant acceleration, except during its collision with the wall, so we can use the constant-acceleration equations in the analysis of its motion. Choose a coordinate system with the origin at the point of release, the positive x axis to the right, and the positive y axis upward.

Using a constant-acceleration equation, express the vertical displacement of the ball as a function of Δt:

$$\Delta y = v_{0y}\Delta t - \tfrac{1}{2}g(\Delta t)^2$$

When the ball hits the ground, $\Delta y = -2$ m:

$$-2\,\text{m} = (10\,\text{m/s})\Delta t - \tfrac{1}{2}(9.81\,\text{m/s}^2)(\Delta t)^2$$

Solve for the time of flight:

$$t_{\text{flight}} = \Delta t = 2.22\text{ s}$$

Find the horizontal distance traveled in this time:

$$\Delta x = (10\text{ m/s})\,(2.22\text{ s}) = 22.2\text{ m}$$

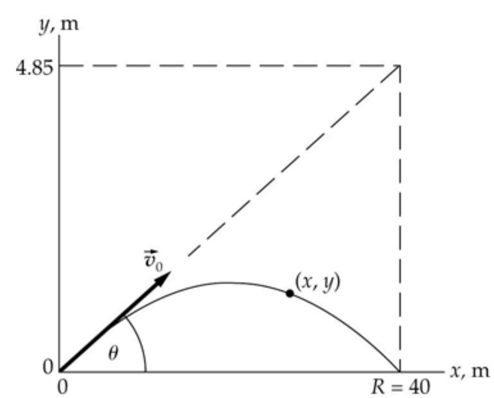
The distance from the wall is:

$$\Delta x - 4\text{ m} = \boxed{18.2\,\text{m}}$$

Hitting Targets and Related Problems

105 •

Picture the Problem In the absence of air resistance, the acceleration of the pebble is constant. Choose the coordinate system shown in the diagram and use constant-acceleration equations to express the coordinates of the pebble in terms of the time into its flight. We can eliminate the parameter t between these equations and solve for the launch velocity of the pebble. We can determine the launch angle from the sighting information and, once the range is known, the time of flight can be found using the horizontal component of the initial velocity.



Referring to the diagram, express θ in terms of the given distances:

$$\theta = \tan^{-1}\left(\frac{4.85\,\text{m}}{40\,\text{m}}\right) = 6.91°$$

Use a constant-acceleration equation to express the horizontal position of the pebble as a function of time:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

or, because $x_0 = 0$, $v_{0x} = v_0\cos\theta$, and $a_x = 0$,

$$x = (v_0\cos\theta)t \qquad (1)$$

Use a constant-acceleration equation to express the vertical position of the
the
pebble as a function of time:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

or, because $y_0 = 0$, $v_{0y} = v_0\sin\theta$, and $a_y = -g$,

$$y = (v_0\sin\theta)t - \tfrac{1}{2}gt^2$$

Eliminate the parameter t to obtain:

$$y = (\tan\theta)x - \frac{g}{2v_0^2\cos^2\theta}x^2$$

At impact, $y = 0$ and $x = R$:

$$0 = (\tan\theta)R - \frac{g}{2v_0^2\cos^2\theta}R^2$$

Solve for v_0 to obtain:

$$v_0 = \sqrt{\frac{Rg}{\sin 2\theta}}$$

Substitute numerical values and evaluate v_0:

$$v_0 = \sqrt{\frac{(40\,\text{m})(9.81\,\text{m/s}^2)}{\sin 13.8°}} = \boxed{40.6\,\text{m/s}}$$

Substitute in equation (1) to relate R to t_{flight}:

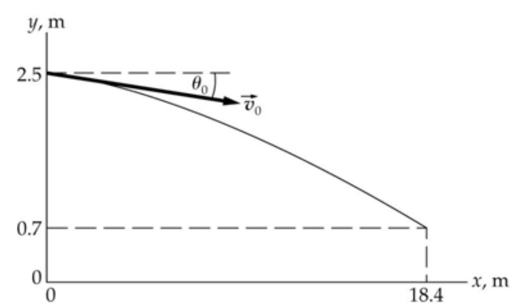
$$R = (v_0\cos\theta)t_{\text{flight}}$$

Solve for and evaluate the time of flight:

$$t_{\text{flight}} = \frac{40\,\text{m}}{(40.6\,\text{m/s})\cos 6.91°} = \boxed{0.992\,\text{s}}$$

***106** ••

Picture the Problem The acceleration of the ball is constant (zero horizontally and –g vertically) and the vertical and horizontal components are independent of each other. Choose the coordinate system shown in the figure and assume that v and t are unchanged by throwing the ball slightly downward.



Express the horizontal displacement of the ball as a function of time:

$$\Delta x = v_{0x}\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$

or, because $a_x = 0$,
$$\Delta x = v_{0x}\Delta t$$

Solve for the time of flight if the ball were thrown horizontally:

$$\Delta t = \frac{\Delta x}{v_{0x}} = \frac{18.4\text{ m}}{37.5\text{ m/s}} = 0.491\text{ s}$$

Using a constant-acceleration equation, express the distance the ball would drop (vertical displacement) if it were thrown horizontally:

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$
or, because $v_{0y} = 0$ and $a_y = -g$,
$$\Delta y = -\tfrac{1}{2}g(\Delta t)^2$$

Substitute numerical values and evaluate Δy:

$$\Delta y = -\tfrac{1}{2}(9.81\text{ m/s}^2)(0.491\text{ s})^2 = -1.18\text{ m}$$

The ball must drop an additional 0.62 m before it gets to home plate.

$y = (2.5 - 1.18)$ m
$= 1.32$ m above ground

Calculate the initial downward speed the ball must have to drop 0.62 m in 0.491 s:

$$v_y = \frac{-0.62\text{ m}}{0.491\text{s}} = -1.26\text{ m}$$

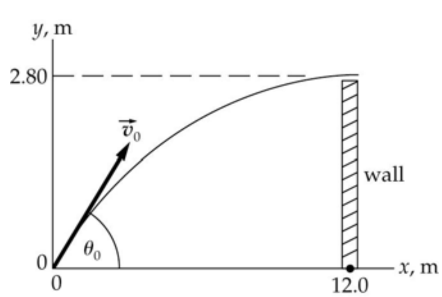
Find the angle with horizontal:

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-1.26\text{ m/s}}{37.5\text{ m/s}}\right) = \boxed{-1.92°}$$

Remarks: One can readily show that $\sqrt{v_x^2 + v_y^2}$ = 37.5 m/s to within 1%; so the assumption that *v* and *t* are unchanged by throwing the ball downward at an angle of 1.93° is justified.

107 ••

Picture the Problem The acceleration of the puck is constant (zero horizontally and $-g$ vertically) and the vertical and horizontal components are independent of each other. Choose a coordinate system with the origin at the point of contact with the puck and the coordinate axes as shown in the figure and use constant-acceleration equations to relate the variables v_{0y}, the time t to reach the wall, v_{0x}, v_0, and θ_0.



Using a constant-acceleration equation for the motion in the y direction, express v_{0y} as a function of the puck's displacement Δy:

$$v_y^2 = v_{0y}^2 + 2a_y\Delta y$$
or, because $v_y = 0$ and $a_y = -g$,
$$0 = v_{0y}^2 - 2g\Delta y$$

Solve for and evaluate v_{0y}:

$$v_{0y} = \sqrt{2g\Delta y} = \sqrt{2(2.80\,\text{m})(9.81\,\text{m/s}^2)} = \boxed{7.41\,\text{m/s}}$$

Find t from the initial velocity in the y direction:

$$t = \frac{v_{0y}}{g} = \frac{7.41\,\text{m/s}}{9.81\,\text{m/s}^2} = \boxed{0.756\,\text{s}}$$

Use the definition of average velocity to find v_{0x}:

$$v_{0x} = v_x = \frac{\Delta x}{t} = \frac{12.0\,\text{m}}{0.756\,\text{s}} = \boxed{15.9\,\text{m/s}}$$

Substitute numerical values and evaluate v_0:

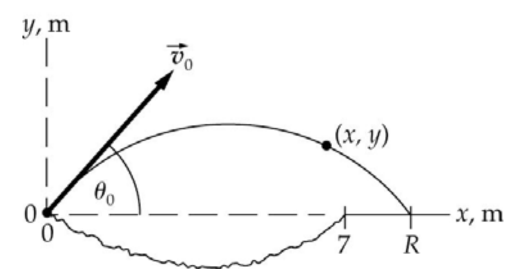
$$v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{(15.9\,\text{m/s})^2 + (7.41\,\text{m/s})^2} = \boxed{17.5\,\text{m/s}}$$

Substitute numerical values and evaluate θ:

$$\theta = \tan^{-1}\left(\frac{v_{0y}}{v_{0x}}\right) = \tan^{-1}\left(\frac{7.41\,\text{m/s}}{15.9\,\text{m/s}}\right) = \boxed{25.0^\circ}$$

108 ••

Picture the Problem In the absence of air resistance, the acceleration of Carlos and his bike is constant and we can use constant-acceleration equations to express his x and y coordinates as functions of time. Eliminating the parameter t between these equations will yield y as a function of x … an equation we can use to decide whether he can jump the creek bed as well as to find the minimum speed required to make the jump.



(a) Use a constant-acceleration equation to express Carlos' horizontal position as a function of time:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

or, because $x_0 = 0$, $v_{0x} = v_0\cos\theta$, and $a_x = 0$,

$$x = (v_0 \cos\theta)t$$

Use a constant-acceleration equation to express Carlos' vertical position as a function of time:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

or, because $y_0 = 0$, $v_{0y} = v_0\sin\theta$, and $a_y = -g$,

$$y = (v_0 \sin\theta)t - \tfrac{1}{2}gt^2$$

Eliminate the parameter t to obtain:

$$y = (\tan\theta)x - \frac{g}{2v_0^2\cos^2\theta}x^2$$

Substitute $y = 0$ and $x = R$ to obtain:

$$0 = (\tan\theta)R - \frac{g}{2v_0^2\cos^2\theta}R^2$$

Solve for and evaluate R:

$$R = \frac{v_0^2}{g}\sin(2\theta_0) = \frac{(11.1\,\text{m/s})^2}{9.81\,\text{m/s}^2}\sin 20° = 4.30\,\text{m}$$

$$\boxed{\text{He should apply the brakes!}}$$

(*b*) Solve the equation we used in the previous step for $v_{0,\text{min}}$:

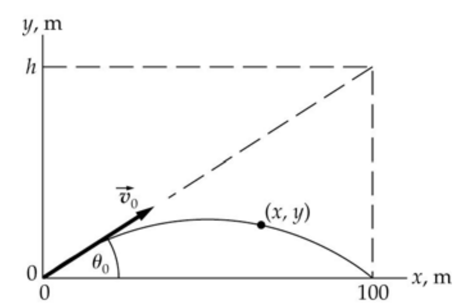
$$v_{0,\text{min}} = \sqrt{\frac{Rg}{\sin(2\theta_0)}}$$

Letting $R = 7$ m, evaluate $v_{0,\text{min}}$:

$$v_{0,\text{min}} = \sqrt{\frac{(7\,\text{m})(9.81\,\text{m/s}^2)}{\sin 20°}} = \boxed{14.2\,\text{m/s} = 51.0\,\text{km/h}}$$

109 •••

Picture the Problem In the absence of air resistance, the bullet experiences constant acceleration along its parabolic trajectory. Choose a coordinate system with the origin at the end of the barrel and the coordinate axes oriented as shown in the figure and use constant-acceleration equations to express the x and y coordinates of the bullet as functions of time along its flight path.



Use a constant-acceleration equation to express the bullet's horizontal position as a function of time:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_xt^2$$

or, because $x_0 = 0$, $v_{0x} = v_0\cos\theta$, and $a_x = 0$,

$$x = (v_0\cos\theta)t$$

Use a constant-acceleration equation to express the bullet's vertical position as a function of time:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_yt^2$$

or, because $y_0 = 0$, $v_{0y} = v_0\sin\theta$, and $a_y = -g$,

$$y = (v_0\sin\theta)t - \tfrac{1}{2}gt^2$$

Eliminate the parameter t to obtain:

$$y = (\tan\theta)x - \frac{g}{2v_0^2\cos^2\theta}x^2$$

Let $y = 0$ when $x = R$ to obtain:

$$0 = (\tan\theta)R - \frac{g}{2v_0^2\cos^2\theta}R^2$$

Solve for the angle above the horizontal that the rifle must be fired to hit the target:

$$\theta_0 = \tfrac{1}{2}\sin^{-1}\left(\frac{Rg}{v_0^2}\right)$$

Substitute numerical values and evaluate θ_0:

$$\theta_0 = \tfrac{1}{2}\sin^{-1}\left[\frac{(100\,\text{m})(9.81\,\text{m/s}^2)}{(250\,\text{m/s})^2}\right] = 0.450°$$

Note: A second value for θ_0, 89.6° is physically unreasonable.

Referring to the diagram, relate h to θ_0 and solve for and evaluate h:

$$\tan\theta_0 = \frac{h}{100\,\text{m}}$$

and

$$h = (100\,\text{m})\tan(0.450°) = \boxed{0.785\,\text{m}}$$

General Problems

110 •

Picture the Problem The sum and difference of two vectors can be found from the components of the two vectors. The magnitude and direction of a vector can be found from its components.

(a) The table to the right summarizes the components of $\vec{A}$ and $\vec{B}$.

Vector	x component	y component
	(m)	(m)
$\vec{A}$	0.707	0.707
$\vec{B}$	0.866	–0.500

(b) The table to the right shows the components of $\vec{S}$.

Vector	x component	y component
	(m)	(m)
$\vec{A}$	0.707	0.707
$\vec{B}$	0.866	–0.500
$\vec{S}$	1.57	0.207

Determine the magnitude and direction of $\vec{S}$ from its components:

$$S = \sqrt{S_x^2 + S_y^2} = \boxed{1.59\,\text{m}}$$

and, because $\vec{S}$ is in the 1st

$$\theta_S = \tan^{-1}\left(\frac{S_y}{S_x}\right) = \boxed{7.50°}$$

(*c*) The table to the right shows the components of $\vec{D}$:

Vector	x component (m)	y component (m)
$\vec{A}$	0.707	0.707
$\vec{B}$	0.866	−0.500
$\vec{D}$	−0.159	1.21

Determine the magnitude and direction of $\vec{D}$ from its components:

$$D = \sqrt{D_x^2 + D_y^2} = \boxed{1.22\,\text{m}}$$

and, because $\vec{D}$ is in the 2nd quadrant,

$$\theta_D = \tan^{-1}\left(\frac{D_y}{D_x}\right) = \boxed{97.5°}$$

***111 •**

Picture the Problem A vector quantity can be resolved into its components relative to any coordinate system. In this example, the axes are orthogonal and the components of the vector can be found using trigonometric functions.

The x and y components of $\vec{g}$ are related to g through the sine and cosine functions:

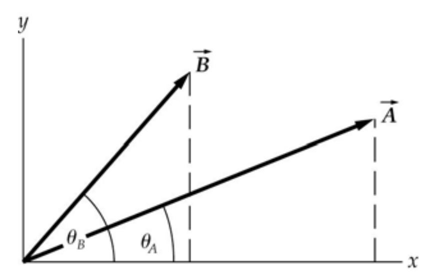
$$g_x = g\sin 30° = \boxed{4.91\,\text{m/s}^2}$$

and

$$g_y = g\cos 30° = \boxed{8.50\,\text{m/s}^2}$$

112 •

Picture the Problem The figure shows two arbitrary, co-planar vectors that (as drawn) do not satisfy the condition that $A/B = A_x/B_x$.



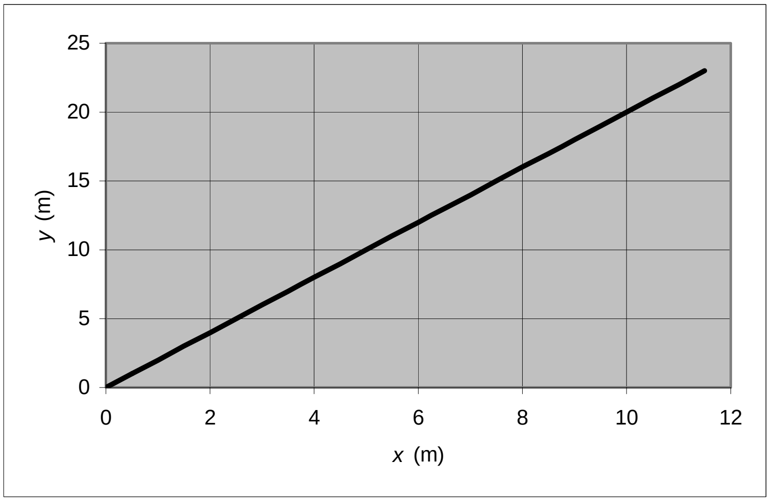
Because $A_x = A\cos\theta_A$ and $B_x = B\cos\theta_B$, $\dfrac{\cos\theta_A}{\cos\theta_B} = 1$ for the condition to be satisfied.

$\therefore$ $A/B = A_x/B_x$ if and only if $\vec{A}$ and $\vec{B}$ are parallel ($\theta_A = \theta_B$) or on opposite sides of the x-axis ($\theta_A = -\theta_B$).

113 •

Picture the Problem We can plot the path of the particle by substituting values for t and evaluating r_x and r_y coordinates of $\vec{r}$. The velocity vector is the time derivative of the position vector.

(*a*) We can assign values to t in the parametric equations $x = (5\text{ m/s})t$ and $y = (10\text{ m/s})t$ to obtain ordered pairs (x, y) that lie on the path of the particle. The path is shown in the following graph:



(*b*) Evaluate $d\vec{\mathbf{r}}/dt$:

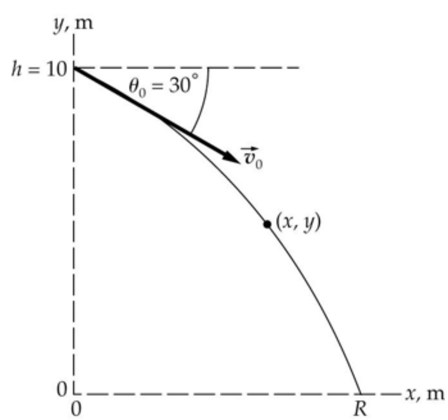
$$\vec{\boldsymbol{v}} = \frac{d\vec{\mathbf{r}}}{dt} = \frac{d}{dt}\left[(5\,\text{m/s})t\,\hat{\boldsymbol{i}} + (10\text{m/s})t\,\hat{\boldsymbol{j}}\right]$$
$$= \boxed{(5\,\text{m/s})\hat{\boldsymbol{i}} + (10\text{m/s})\hat{\boldsymbol{j}}}$$

Use its components to find the magnitude of $\vec{\boldsymbol{v}}$:

$$v = \sqrt{v_x^2 + v_y^2} = \boxed{11.2\,\text{m/s}}$$

114 ••

Picture the Problem In the absence of air resistance, the hammer experiences constant acceleration as it falls. Choose a coordinate system with the origin and coordinate axes as shown in the figure and use constant-acceleration equations to describe the *x* and *y* coordinates of the hammer along its trajectory. We'll use the equation describing the vertical motion to find the time of flight of the hammer and the equation describing the horizontal motion to determine its range.



Using a constant-acceleration equation, express the *x* coordinate of the hammer as a function of time:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_xt^2$$

or, because $x_0 = 0$, $v_{0x} = v_0\cos\theta_0$, and $a_x = 0$,

$$x = (v_0\cos\theta_0)t$$

Using a constant-acceleration equation, express the *y* coordinate of the hammer as a function of time:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_yt^2$$

or, because $y_0 = h$, $v_{0y} = v_0\sin\theta$, and $a_y = -g$,

$$y = h + (v_0\sin\theta)t - \tfrac{1}{2}gt^2$$

Substitute numerical values to obtain:

$$y = 10\,\text{m} + (4\,\text{m/s})(\sin 30°)t - \tfrac{1}{2}(9.81\,\text{m/s}^2)t^2$$

Substitute the conditions that exist when the hammer hits the ground:

$$0 = 10\,\text{m} - (4\,\text{m/s})\sin 30°\,t - \tfrac{1}{2}(9.81\,\text{m/s}^2)\,t^2$$

Solve for the time of fall to obtain:

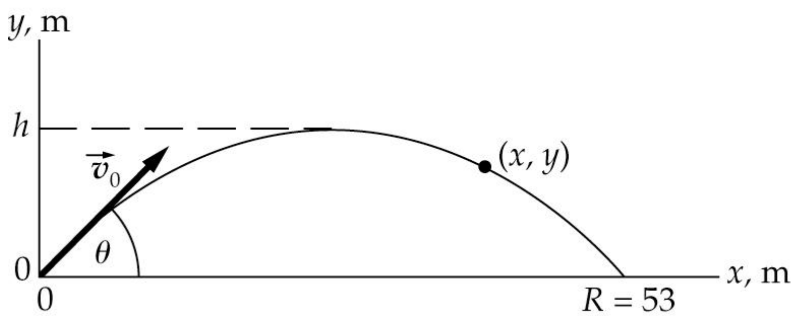
$t = 1.24$ s

Use the x-coordinate equation to find the horizontal distance traveled by the hammer in 1.24 s:

$$R = (4\,\text{m/s})(\cos 30°)(1.24\,\text{s}) = \boxed{4.29\,\text{m}}$$

115 ••

Picture the Problem We'll model Zacchini's flight as though there is no air resistance and, hence, the acceleration is constant. Then we can use constant- acceleration equations to express the x and y coordinates of Zacchini's motion as functions of time. Eliminating the parameter t between these equations will leave us with an equation we can solve for θ. Because the maximum height along a parabolic trajectory occurs (assuming equal launch and landing elevations) occurs at half range, we can use this same expression for y as a function of x to find h.



Use a constant-acceleration equation to express Zacchini's horizontal position as a function of time:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

or, because $x_0 = 0$, $v_{0x} = v_0\cos\theta$, and $a_x = 0$,

$$x = (v_0 \cos\theta)t$$

Use a constant-acceleration equation to express Zacchini's vertical position as a function of time:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

or, because $y_0 = 0$, $v_{0y} = v_0\sin\theta$, and $a_y = -g$,

$$y = (v_0 \sin\theta)t - \tfrac{1}{2}gt^2$$

Eliminate the parameter t to obtain:

$$y = (\tan\theta)x - \frac{g}{2v_0^2\cos^2\theta}x^2$$

Use Zacchini's coordinates when he lands in a safety net to obtain:

$$0 = (\tan\theta)R - \frac{g}{2v_0^2\cos^2\theta}R^2$$

Solve for his launch angle θ:

$$\theta = \tfrac{1}{2}\sin^{-1}\left(\frac{Rg}{v_0^2}\right)$$

Substitute numerical values and evaluate θ:

$$\theta = \tfrac{1}{2}\sin^{-1}\left[\frac{(53\,\text{m})(9.81\,\text{m/s}^2)}{(24.2\,\text{m/s})^2}\right] = \boxed{31.3°}$$

Use the fact that his maximum height was attained when he was halfway through his flight to obtain:

$$h = (\tan\theta)\frac{R}{2} - \frac{g}{2v_0^2\cos^2\theta}\left(\frac{R}{2}\right)^2$$

Substitute numerical values and evaluate h:

$$h = (\tan 31.3°)\frac{53\,\text{m}}{2} - \frac{9.81\,\text{m/s}^2}{2(24.2\,\text{m/s})^2\cos^2 31.3°}\left(\frac{53\,\text{m}}{2}\right)^2 = \boxed{8.06\,\text{m}}$$

116 ••

Picture the Problem Because the acceleration is constant; we can use the constant-acceleration equations in vector form and the definitions of average velocity and average (instantaneous) acceleration to solve this problem.

(*a*) The average velocity is given by:

$$\vec{\boldsymbol{v}}_{\text{av}} = \frac{\Delta\vec{\boldsymbol{r}}}{\Delta t} = \frac{\vec{\boldsymbol{r}}_2 - \vec{\boldsymbol{r}}_1}{\Delta t}$$
$$= (3\,\text{m/s})\hat{\boldsymbol{i}} + (-2.5\,\text{m/s})\hat{\boldsymbol{j}}$$

The average velocity can also be expressed as:

$$\vec{\boldsymbol{v}}_{\text{av}} = \frac{\vec{\boldsymbol{v}}_1 + \vec{\boldsymbol{v}}_2}{2}$$

and

$$\vec{v}_1 = 2\vec{v}_{\text{av}} - \vec{v}_2$$

Substitute numerical values to obtain:

$$\vec{\boldsymbol{v}}_1 = \boxed{(1\,\text{m/s})\hat{\boldsymbol{i}} + (1\,\text{m/s})\hat{\boldsymbol{j}}}$$

(*b*) The acceleration of the particle is given by:

$$\vec{\boldsymbol{a}} = \frac{\Delta\vec{\boldsymbol{v}}}{\Delta t} = \frac{\vec{\boldsymbol{v}}_2 - \vec{\boldsymbol{v}}_1}{\Delta t}$$
$$= \boxed{(2\,\text{m/s}^2)\hat{\boldsymbol{i}} + (-3.5\,\text{m/s}^2)\hat{\boldsymbol{j}}}$$

(*c*) The velocity of the particle as a function of time is:

$$\vec{\boldsymbol{v}}(t) = \vec{\boldsymbol{v}}_1 + \vec{\boldsymbol{a}}t = \boxed{[(1\,\text{m/s}) + (2\,\text{m/s}^2)t]\hat{\boldsymbol{i}} + [(1\,\text{m/s}) + (-3.5\,\text{m/s}^2)t]\hat{\boldsymbol{j}}}$$

(*d*) Express the position vector as a function of time:

$$\vec{\boldsymbol{r}}(t) = \vec{\boldsymbol{r}}_1 + \vec{\boldsymbol{v}}_1 t + \tfrac{1}{2}\vec{\boldsymbol{a}}t^2$$

Substitute numerical values and evaluate $\vec{r}(t)$:

$$\vec{r}(t) = \boxed{[(4\text{ m}) + (1\text{ m/s})t + (1\text{ m/s}^2)t^2]\hat{i} + [(3\text{ m}) + (1\text{ m/s})t + (-1.75\text{ m/s}^2)t^2]\hat{j}}$$

***117 ••**

Picture the Problem In the absence of air resistance, the steel ball will experience constant acceleration. Choose a coordinate system with its origin at the initial position of the ball, the x direction to the right, and the y direction downward. In this coordinate system $y_0 = 0$ and $a = g$. Letting (x, y) be a point on the path of the ball, we can use constant-acceleration equations to express both x and y as functions of time and, using the geometry of the staircase, find an expression for the time of flight of the ball. Knowing its time of flight, we can find its range and identify the step it strikes first.

The angle of the steps, with respect to the horizontal, is:

$$\theta = \tan^{-1}\left(\frac{0.18\text{ m}}{0.3\text{ m}}\right) = 31.0°$$

Using a constant-acceleration equation, express the x coordinate of the steel ball in its flight:

$$x = x_0 + v_0 t + \tfrac{1}{2}a_y t^2$$

or, because $x_0 = 0$ and $a_y = 0$,

$$x = v_0 t$$

Using a constant-acceleration equation, express the y coordinate of the steel ball in its flight:

$$y = y_0 + v_{0y} t + \tfrac{1}{2}a_y t^2$$

or, because $y_0 = 0$, $v_{0y} = 0$, and $a_y = g$,

$$y = \tfrac{1}{2}gt^2$$

The equation of the dashed line in the figure is:

$$\frac{y}{x} = \tan\theta = \frac{gt}{2v_0}$$

Solve for the flight time:

$$t = \frac{2v_0}{g}\tan\theta$$

Find the x coordinate of the landing position:

$$x = \frac{y}{\tan\theta} = \frac{2v_0^2}{g}\tan\theta$$

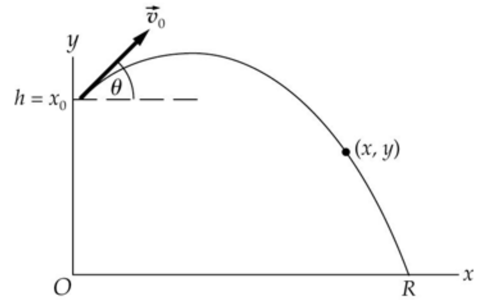
Substitute the angle determined in the first step:

$$x = \frac{2(3\text{ m/s})^2}{9.81\text{ m/s}^2}\tan 31° = 1.10\text{ m}$$

$$\boxed{\text{The first step with } x > 1.10\text{ m is the 4th step.}}$$

118 ••

Picture the Problem Ignoring the influence of air resistance, the acceleration of the ball is constant once it has left your hand and we can use constant-acceleration equations to express the x and y coordinates of the ball. Elimination of the parameter t will yield an equation from which we can determine v_0. We can then use the y equation to express the time of flight of the ball and the x equation to express its range in terms of x_0, v_0, θ and the time of flight.



Use a constant-acceleration equation to express the ball's horizontal position as a function of time:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

or, because $x_0 = 0$, $v_{0x} = v_0\cos\theta$, and $a_x = 0$,

$$x = (v_0\cos\theta)t \qquad (1)$$

Use a constant-acceleration equation to express the ball's vertical position as a function of time:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

or, because $y_0 = x_0$, $v_{0y} = v_0\sin\theta$, and $a_y = -g$,

$$y = x_0 + (v_0\sin\theta)t - \tfrac{1}{2}gt^2 \qquad (2)$$

Eliminate the parameter t to obtain:

$$y = x_0 + (\tan\theta)x - \frac{g}{2v_0^2\cos^2\theta}x^2$$

For the throw while standing on level ground we have:

$$0 = (\tan\theta)x_0 - \frac{g}{2v_0^2\cos^2\theta}x_0^2$$

and

$$x_0 = \frac{v_0^2}{g}\sin 2\theta = \frac{v_0^2}{g}\sin 2(45°) = \frac{v_0^2}{g}$$

Solve for v_0:

$$v_0 = \sqrt{gx_0}$$

At impact equation (2) becomes:

$$0 = x_0 + \left(\sqrt{gx_0}\sin\theta\right)t_{\text{flight}} - \tfrac{1}{2}gt_{\text{flight}}^2$$

Solve for the time of flight:

$$t_{\text{flight}} = \sqrt{\frac{x_0}{g}}\left(\sin\theta + \sqrt{\sin^2\theta + 2}\right)$$

Substitute in equation (1) to express the range of the ball when thrown from an elevation x_0 at an angle θ with the horizontal:

$$R = \left(\sqrt{gx_0}\cos\theta\right)t_{\text{flight}}$$
$$= \left(\sqrt{gx_0}\cos\theta\right)\sqrt{\frac{x_0}{g}}\left(\sin\theta + \sqrt{\sin^2\theta + 2}\right)$$
$$= x_0\cos\theta\left(\sin\theta + \sqrt{\sin^2\theta + 2}\right)$$

Substitute $\theta = 0°$, 30°, and 45°:

$$x(0°) = \boxed{1.41x_0}$$

$$x(30°) = \boxed{1.73x_0}$$

and

$$x(45°) = \boxed{1.62x_0}$$

119 •••

Picture the Problem Choose a coordinate system with its origin at the point where the motorcycle becomes airborne and with the positive x direction to the right and the positive y direction upward. With this choice of coordinate system we can relate the x and y coordinates of the motorcycle (which we're treating as a particle) using Equation 3-21.

(a) The path of the motorcycle is given by:

$$y(x) = (\tan\theta)x - \left(\frac{g}{2v_0^2\cos^2\theta}\right)x^2$$

For the jump to be successful, $h < y(x)$. Solving for v_0, we find:

$$v_{\text{min}} > \boxed{\frac{x}{\cos\theta}\sqrt{\frac{g}{2(x\tan\theta - h)}}}$$

(b) Use the values given to obtain:

$$v_{\text{min}} > \boxed{26.0\,\text{m/s or } 58.0\,\text{mph}}$$

(c) In order for our expression for v_{min} to be real valued; i.e., to predict values for v_{min} that are physically meaningful, $x\tan\theta - h > 0$.

$$\therefore\ h_{\text{max}} < x\tan\theta$$

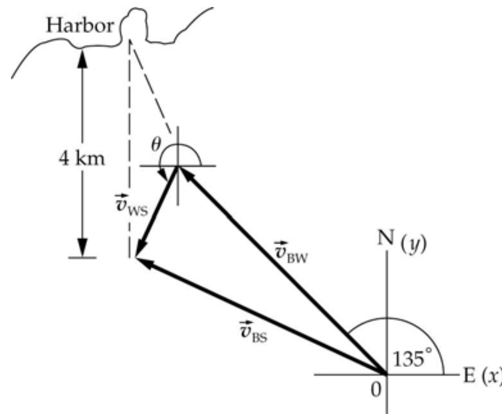
The interpretation is that the bike "falls away" from traveling on a straight-line path due to the free-fall acceleration downwards. No matter what the initial speed of the bike, it must fall a little bit before reaching the other side of the pit.

120 •••

Picture the Problem Let the origin be at the position of the boat when it was engulfed by the fog. Take the x and y directions to be east and north, respectively. Let $\vec{v}_{BW}$ be the velocity of the boat relative to the water, $\vec{v}_{BS}$ be the velocity of the boat relative to the shore, and $\vec{v}_{WS}$ be the velocity of the water with respect to the shore. Then

$$\vec{v}_{BS} = \vec{v}_{BW} + \vec{v}_{WS}.$$

θ is the angle of $\vec{v}_{WS}$ with respect to the x (east) direction.



(*a*) Find the position vector for the boat at $t = 3$ h:

$$\vec{r}_{boat} = \{(32\text{ km})(\cos 135°)t\}\hat{i} + \{(32\text{ km})(\sin 135°)t - 4\text{ km}\}\hat{j}$$
$$= \{(-22.6\text{ km})t\}\hat{i} + \{(22.6\text{ km})t - 4\text{ km}\}\hat{j}$$

Find the coordinates of the boat at $t = 3$ h:

$$r_x = [(10\text{ km/h})\cos 135° + v_{WS}\cos\theta](3\text{ h})$$
and
$$r_y = [(10\text{ km/h})\sin 135° + v_{WS}\sin\theta](3\text{ h})$$

Simplify the expressions involving r_x and r_y and equate these simplified expressions to the x and y components of the position vector of the boat:

$$3v_{WS}\cos\theta = -1.41\text{ km/h}$$
and
$$3v_{WS}\sin\theta = -2.586\text{ km/h}$$

Divide the second of these equations by the first to obtain:

$$\tan\theta = \frac{-2.586\text{ km}}{-1.41\text{ km}}$$
or
$$\theta = \tan^{-1}\left(\frac{-2.586\text{ km}}{-1.41\text{ km}}\right) = 61.4° \text{ or } 241.4°$$

Because the boat has drifted south, use $\theta = 241.4°$ to obtain:

$$v_{WS} = \frac{v_x}{\cos\theta} = \frac{-\frac{1.41\text{ km/h}}{3}}{\cos(241.4°)} = \boxed{0.982\text{ km/h at }\theta = 241.4°}$$

(*b*) Letting ϕ be the angle between east and the proper heading for the boat, express the components of the velocity of the boat with respect to the shore:

$$v_{BS,x} = (10 \text{ km/h}) \cos\phi + (0.982 \text{ km/h}) \cos(241.3°)$$

$$v_{BS,y} = (10 \text{ km/h}) \sin\phi + (0.982 \text{ km/h}) \sin(241.3°)$$

For the boat to travel northwest:

$$v_{BS,x} = -v_{BS,y}$$

Substitute the velocity components, square both sides of the equation, and simplify the expression to obtain the equations:

$\sin\phi + \cos\phi = 0.133$,
$\sin^2\phi + \cos^2\phi + 2\sin\phi\cos\phi = 0.0177$,
and
$1 + \sin(2\phi) = 0.0177$

Solve for ϕ:

$$\phi = 129.6° \text{ or } 140.4°$$

Because the current pushes south, the boat must head more northerly than 135°:

Using 129.6°, the correct heading is $\boxed{39.6° \text{ west of north}}$.

(*c*) Find v_{BS}:

$v_{BS,x} = -6.84$ km/h
and
$v_{BS} = v_{Bx}/\cos 135° = 9.68$ km/h

To find the time to travel 32 km, divide the distance by the boat's actual speed:

$$t = (32 \text{ km})/(9.68 \text{ km/h}) = \boxed{3.31\text{ h} = 3\text{ h } 18\text{ min}}$$

***121 ••**

Picture the Problem In the absence of air resistance, the acceleration of the projectile is constant and the equation of a projectile for equal initial and final elevations, which was derived from the constant-acceleration equations, is applicable. We can use the equation giving the range of a projectile for equal initial and final elevations to evaluate the ranges of launches that exceed or fall short of 45° by the same amount.

Express the range of the projectile as a function of its initial speed and angle of launch:

$$R = \frac{v_0^2}{g}\sin 2\theta_0$$

Let $\theta_0 = 45° \pm \theta$:

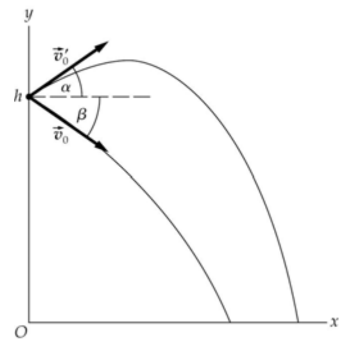
$$R = \frac{v_0^2}{g}\sin(90° \pm 2\theta) = \frac{v_0^2}{g}\cos(\pm 2\theta)$$

Because $\cos(-\theta) = \cos(+\theta)$ (the cosine function is an *even* function):

$$\boxed{R(45° + \theta) = R(45° - \theta)}$$

122 ••

Picture the Problem In the absence of air resistance, the acceleration of both balls is that due to gravity and the horizontal and vertical motions are independent of each other. Choose a coordinate system with the origin at the base of the cliff and the coordinate axes oriented as shown and use constant-acceleration equations to relate the x and y components of the ball's speed.



Independently of whether a ball is thrown upward at the angle α or downward at β, the vertical motion is described by:

$$v_y^2 = v_{0y}^2 + 2a\Delta y$$
$$= v_{0y}^2 - 2gh$$

The horizontal component of the motion is given by:

$$v_x = v_{0x}$$

Find v at impact from its components:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_{0x}^2 + v_{0y}^2 - 2gh}$$
$$= \boxed{\sqrt{v_0^2 - 2gh}}$$