Chapter 4
Newton's Laws

Conceptual Problems

***1** ••

Determine the Concept A reference frame in which the law of inertia holds is called an inertial reference frame.

If an object with no net force acting on it is at rest or is moving with a constant speed in a straight line (i.e., with constant velocity) relative to the reference frame, then the reference frame is an inertial reference frame. Consider sitting at rest in an accelerating train or plane. The train or plane is not an inertial reference frame even though you are at rest relative to it. In an inertial frame, a dropped ball lands at your feet. You are in a noninertial frame when the driver of the car in which you are riding steps on the gas and you are pushed back into your seat.

2 ••

Determine the Concept A reference frame in which the law of inertia holds is called an inertial reference frame. A reference frame with acceleration *a* relative to the initial frame, and with *any* velocity relative to the initial frame, is inertial.

3 •

Determine the Concept No. If the net force acting on an object is zero, its acceleration is zero. The only conclusion one can draw is that the *net* force acting on the object is zero.

***4** •

Determine the Concept An object accelerates when a *net* force acts on it. The fact that an object is accelerating tells us nothing about its velocity other than that it is always changing.

Yes, the object must have an acceleration relative to the inertial frame of reference. According to Newton's 1st and 2nd laws, an object must accelerate, relative to any inertial reference frame, in the direction of the net force. If there is ″only a single nonzero force,″ then this force is the net force.

Yes, the object's velocity may be momentarily zero. During the period in which the force is acting, the object may be momentarily at rest, but its velocity cannot remain zero because it must continue to accelerate. Thus, its velocity is always changing.

5 •

Determine the Concept No. Predicting the direction of the subsequent motion correctly requires knowledge of the initial velocity as well as the acceleration. While the acceleration can be obtained from the net force through Newton's 2nd law, the velocity can only be obtained by integrating the acceleration.

6 •

Determine the Concept An object in an inertial reference frame accelerates if there is a *net* force acting on it. Because the object is moving at constant velocity, the net force acting on it is zero. $\boxed{(c)\text{ is correct.}}$

7 •

Determine the Concept The mass of an object is an intrinsic property of the object whereas the weight of an object depends directly on the local gravitational field. Therefore, the mass of the object would not change and $w_{\text{grav}} = mg_{\text{local}}$. Note that if the gravitational field is zero then the gravitational force is also zero.

***8** •

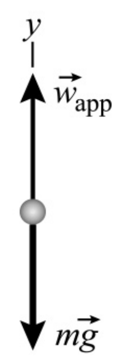
Determine the Concept If there is a force on her in addition to the gravitational force, she will experience an additional acceleration relative to her space vehicle that is proportional to the net force required producing that acceleration and inversely proportional to her mass.

She could do an experiment in which she uses her legs to push off from the wall of her space vehicle and measures her acceleration and the force exerted by the wall. She could calculate her mass from the ratio of the force exerted by the wall to the acceleration it produced.

***9** •

Determine the Concept One's apparent weight is the reading of a scale in one's reference frame.

Imagine yourself standing on a scale that, in turn, is on a platform accelerating upward with an acceleration *a*. The free-body diagram shows the force the gravitational field exerts on you, $m\vec{g}$, and the force the scale exerts on you, $\vec{w}_{\text{app}}$. The scale reading (the force the scale exerts on you) is your apparent weight.



Choose the coordinate system shown in the free-body diagram and apply $\sum \vec{F} = m\vec{a}$ to the scale:

$$\sum F_y = w_{\text{app}} - mg = ma_y$$

or

$$w_{\text{app}} = mg + ma_y$$

So, your apparent weight would be greater than your true weight when observed from a reference frame that is accelerating upward. That is, when the surface on which you are standing has an acceleration *a* such that a_y is positive: $\boxed{a_y > 0}$.

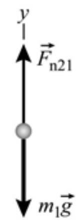
10 ••

Determine the Concept Newton's 2nd law tells us that forces produce *changes* in the velocity of a body. If two observers pass each other, each traveling at a constant velocity, each will experience no net force acting on them, and so each will feel as if he or she is standing still.

11 •

Determine the Concept Neither block is accelerating so the net force on each block is zero. Newton's 3rd law states that objects exert equal and opposite forces on each other.

(a) and (b) Draw the free-body diagram for the forces acting on the block of mass m_1:



Apply $\sum \vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$ to the block 1:

$$\sum F_y = F_{n21} - m_1 g = m_1 a_1$$

or, because $a_1 = 0$,

$$F_{n21} - m_1 g = 0$$

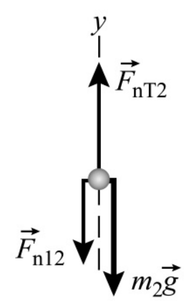
Therefore, the magnitude of the force that block 2 exerts on block 1 is given by:

$$F_{n21} = \boxed{m_1 g}$$

From Newton's 3rd law of motion we know that the force that block 1 exerts on block 2 is equal to, but opposite in direction, the force that block 2 exerts on block 1.

$$\vec{\boldsymbol{F}}_{n21} = -\vec{\boldsymbol{F}}_{n12} \Rightarrow F_{n12} = \boxed{m_1 g}$$

(c) and (d) Draw the free-body diagram for the forces acting on block 2:



Apply $\sum \vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$ to block 2:

$$\sum F_{2y} = F_{nT2} - F_{n12} - m_2 g = m_2 a_2$$

or, because $a_2 = 0$,

$$F_{nT2} = F_{nT2} + m_2 g = m_1 g + m_2 g$$
$$= (m_1 + m_2)g$$

and the normal force that the table exerts on body 2 is

$$F_{nT2} = \boxed{(m_1 + m_2)g}$$

From Newton's 3rd law of motion we know that the force that block 2 exerts on the table is equal to, but opposite in direction, the force that the table exerts on block 2.

$$\vec{\boldsymbol{F}}_{nT2} = -\vec{\boldsymbol{F}}_{n2T} \Rightarrow F_{n2T} = \boxed{(m_1 + m_2)g}$$

***12** •

(*a*) True. By definition, action-reaction force pairs cannot act on the same object.

(*b*) False. Action equals reaction independent of any motion of the two objects.

13 •

Determine the Concept Newton's 3rd law of motion describes the interaction between the man and his less massive son. According to the 3rd law description of the interaction of two objects, these are action-reaction forces and therefore must be equal in magnitude. $\boxed{(b)\text{ is correct.}}$

14 •

Determine the Concept According to Newton's 3rd law the reaction force to a force exerted by object A on object B is the force exerted by object B on object A. The bird's weight is a gravitational field force exerted by the earth on the bird. Its reaction force is the gravitational force the bird exerts on the earth. $\boxed{(b)\text{ is correct.}}$

15 •

Determine the Concept We know from Newton's 3rd law of motion that the reaction to the force that the bat exerts on the ball is the force the ball exerts on the bat and is equal in magnitude but oppositely directed. The action-reaction pair consists of the force with which the bat hits the ball and the force the ball exerts on the bat. These forces are equal in magnitude, act in opposite directions. $\boxed{(c)\text{ is correct.}}$

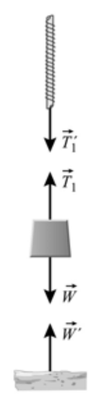
16 •

Determine the Concept The statement of Newton's 3rd law given in the problem is not complete. It is important to remember that the action and reaction forces act on different bodies. The reaction force does not cancel out because it does not act on the same body as the external force.

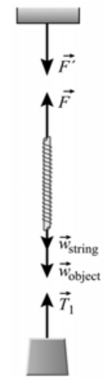
***17** •

Determine the Concept The force diagrams will need to include the ceiling, string, object, and earth if we are to show all of the reaction forces as well as the forces acting on the object.

(*a*) The forces acting on the 2.5-kg object are its weight $\vec{W}$, and the tension $\vec{T}_1$, in the string. The reaction forces are $\vec{W}'$ acting on the earth and $\vec{T}_1'$ acting on the string.



(*b*) The forces acting on the string are its weight, the weight of the object, and $\vec{\boldsymbol{F}}$, the force exerted by the ceiling. The reaction forces are $\vec{\boldsymbol{T}}_1$ acting on the string and $\vec{\boldsymbol{F}}\,'$ acting on the ceiling.



18 •

Determine the Concept Identify the objects in the block's environment that are exerting forces on the block and then decide in what directions those forces must be acting if the block is sliding *down* the inclined plane.

Because the incline is frictionless, the force the incline exerts on the block must be normal to the surface. The second object capable of exerting a force on the block is the earth and its force; the weight of the block acts directly downward. The magnitude of the normal force is less than that of the weight because it supports only a portion of the weight. $\boxed{\text{The forces shown in FBD } (c) \text{ satisfy these conditions.}}$

19 •

Determine the Concept In considering these statements, one needs to decide whether they are consistent with Newton's laws of motion. A good strategy is to try to think of a counterexample that would render the statement false.

(*a*) True. If there are no forces acting on an object, the *net* force acting on it must be zero and, hence, the acceleration must be zero.

(*b*) False. Consider an object moving with constant velocity on a frictionless horizontal surface. While the *net* force acting on it is zero (it is not accelerating), gravitational and normal forces are acting on it.

(*c*) False. Consider an object that has been thrown vertically upward. While it is still rising, the direction of the gravitational force acting on it is downward.

(*d*) False. The mass of an object is an intrinsic property that is independent of its location (the gravitational field in which it happens to be situated).

20 •

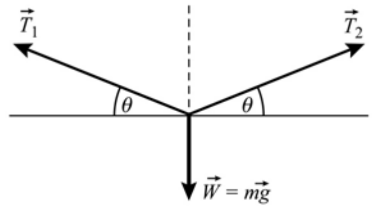
Determine the Concept In considering these alternatives, one needs to decide which alternatives are consistent with Newton's 3rd law of motion. According to Newton's 3rd law, the magnitude of the gravitational force exerted by her body on the earth is equal and opposite to the force exerted by the earth on her. $\boxed{(a) \text{ is correct.}}$

***21 •**

Determine the Concept In considering these statements, one needs to decide whether they are consistent with Newton's laws of motion. In the absence of a *net* force, an object moves with constant velocity. $\boxed{(d)\text{ is correct.}}$

22 •

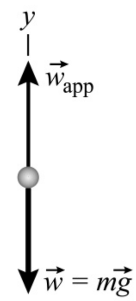
Determine the Concept Draw the free-body diagram for the towel. Because the towel is hung at the center of the line, the magnitudes of $\vec{T}_1$ and $\vec{T}_2$ are the same.



No. To support the towel, the tension in the line must have a vertical component equal to the towel's weight. Thus $\theta > 0$.

23 •

Determine the Concept The free-body diagram shows the forces acting on a person in a descending elevator. The upward force exerted by the scale on the person, $\vec{w}_{app}$, is the person's apparent weight.



Apply $\sum F_y = ma_y$ to the person and solve for w_{app}:

$w_{app} - mg = ma_y$

or

$w_{app} = mg + ma_y = m(g + a_y)$

$\boxed{\text{Because } w_{app} \text{ is independent of } v, \text{ the velocity of the elevator has no effect on the person's apparent weight.}}$

Remarks: Note that a nonconstant velocity will alter the apparent weight.

Estimation and Approximation

24 ••

Picture the Problem Assuming a stopping distance of 25 m and a mass of 80 kg, use Newton's 2[nd] law to determine the force exerted by the seat belt.

The force the seat belt exerts on the driver is given by:

$F_{net} = ma$, where m is the mass of the driver.

Using a constant-acceleration equation, relate the velocity of the car to its stopping distance and acceleration:

$$v^2 = v_0^2 + 2a\Delta x$$

or, because $v = 0$,

$$-v_0^2 = 2a\Delta x$$

Solve for a:

$$a = \frac{-v_0^2}{2\Delta x}$$

Substitute numerical values and evaluate a:

$$a = -\frac{\left(90\,\frac{\text{km}}{\text{h}} \times \frac{1\,\text{h}}{3600\,\text{s}} \times \frac{10^3\,\text{m}}{\text{km}}\right)^2}{2(25\,\text{m})}$$
$$= -12.5\,\text{m/s}^2$$

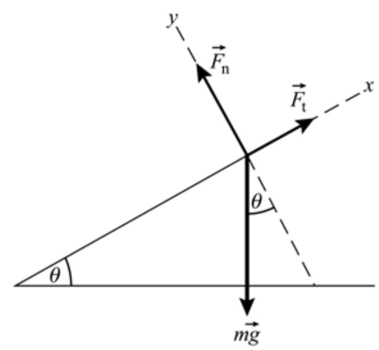
Substitute for a and evaluate F_{net}:

$$F_{\text{net}} = (80\,\text{kg})(-12.5\,\text{m/s}^2)$$
$$= \boxed{-1.00\,\text{kN}}$$

F_{net} is negative because it is opposite the direction of motion.

***25** •••

Picture the Problem The free-body diagram shows the forces acting on you and your bicycle as you are either ascending or descending the grade. The magnitude of the normal force acting on you and your bicycle is equal to the component of your weight in the *y* direction and the magnitude of the tangential force is the *x* component of your weight. Assume a combined mass (you plus your bicycle) of 80 kg.



(*a*) Apply $\sum F_y = ma_y$ to you and your bicycle and solve for F_n:

$F_n - mg\cos\theta = 0$, because there is no acceleration in the *y* direction.

$\therefore\ F_n = mg\cos\theta$

Determine θ from the information concerning the grade:

$\tan\theta = 0.08$

and

$\theta = \tan^{-1}(0.08) = 4.57°$

Substitute to determine F_n:

$F_n = (80\text{ kg})(9.81\text{ m/s}^2)\cos 4.57°$

$= \boxed{782\,\text{N}}$

Apply $\sum F_x = ma_x$ to you and your bicycle and solve for F_t, the tangential force exerted by the road on the wheels:

$F_t - mg\sin\theta = 0$, because there is no acceleration in the *x* direction.

Evaluate F_t:

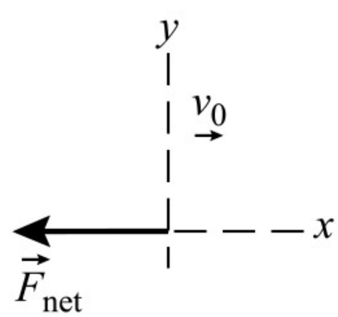
$$F_t = (80\text{ kg})(9.81\text{ m/s}^2)\sin 4.57°$$
$$= \boxed{62.6\text{ N}}$$

(*b*) $\boxed{\text{Because there is no acceleration, the forces are the same going up and going down the incline.}}$

Newton's First and Second Laws: Mass, Inertia, and Force

26 •

Picture the Problem The acceleration of the particle can be found from the stopping distance by using a constant-acceleration equation. The mass of the particle and its acceleration are related to the net force through Newton's second law of motion. Choose a coordinate system in which the direction the particle is moving is the positive *x* direction and apply $\vec{F}_{net} = m\vec{a}$.



Use Newton's 2nd law to relate the mass of the particle to the net force acting on it and its acceleration:

$$m = \frac{F_{net}}{a_x}$$

Because the force is constant, use a constant-acceleration equation with $v_x = 0$ to determine a:

$$v_x^2 = v_{0x}^2 + 2a_x\Delta x$$
and
$$a_x = \frac{-v_{0x}^2}{2\Delta x}$$

Substitute to obtain:

$$m = \frac{2\Delta x F_{net}}{v_{0x}^2}$$

Substitute numerical values and evaluate m:

$$m = \frac{2(62.5\text{ m})(15.0\text{ N})}{(25.0\text{ m/s})^2} = 3.00\text{ kg}$$
and $\boxed{(b)\text{ is correct.}}$

27 •

Picture the Problem The acceleration of the object is related to its mass and the *net* force acting on it by $F_{net} = F_0 = ma$.

(*a*) Use Newton's 2nd law of motion to calculate the acceleration of the object:

$$a = \frac{F_{net}}{m} = \frac{2F_0}{m}$$
$$= 2(3\text{ m/s}^2) = \boxed{6.00\text{ m/s}^2}$$

(*b*) Let the subscripts 1 and 2 distinguish the two objects. The ratio of the two masses is found from Newton's 2nd law:

$$\frac{m_2}{m_1}=\frac{F_0/a_2}{F_0/a_1}=\frac{a_1}{a_2}=\frac{3\,\text{m/s}^2}{9\,\text{m/s}^2}=\boxed{\frac{1}{3}}$$

(*c*) The acceleration of the two-mass system is the net force divided by the total mass $m = m_1 + m_2$:

$$a=\frac{F_{\text{net}}}{m}=\frac{F_0}{m_1+m_2}$$
$$=\frac{F_0/m_1}{1+m_2/m_1}=\frac{a_1}{1+1/3}$$
$$=\tfrac{3}{4}a_1=\boxed{2.25\,\text{m/s}^2}$$

28 •

Picture the Problem The acceleration of an object is related to its mass and the *net* force acting on it by $F_{\text{net}}=ma$. Let m be the mass of the ship, a_1 be the acceleration of the ship when the net force acting on it is F_1, and a_2 be its acceleration when the net force is $F_1 + F_2$.

Using Newton's 2nd law, express the net force acting on the ship when its acceleration is a_1:

$$F_1 = ma_1$$

Express the net force acting on the ship when its acceleration is a_2:

$$F_1 + F_2 = ma_2$$

Divide the second of these equations by the first and solve for the ratio F_2/F_1:

$$\frac{F_1+F_2}{F_1}=\frac{ma_1}{ma_2}$$
and
$$\frac{F_2}{F_1}=\frac{a_2}{a_1}-1$$

Substitute for the accelerations to determine the ratio of the accelerating forces and solve for F_2:

$$\frac{F_2}{F_1}=\frac{(16\,\text{km/h})/(10\,\text{s})}{(4\,\text{km/h})/(10\,\text{s})}-1=3$$
or
$$F_2=\boxed{3F_1}$$

***29 ••**

Picture the Problem Because the deceleration of the bullet is constant, we can use a constant-acceleration equation to determine its acceleration and Newton's 2nd law of motion to find the average resistive force that brings it to a stop.

Apply $\sum\vec{F}=m\vec{a}$ to express the force exerted on the bullet by the wood:

$$F_{\text{wood}} = ma$$

Using a constant-acceleration

$$v^2=v_0^2+2a\Delta x$$

equation, express the final velocity of the bullet in terms of its acceleration and solve for the acceleration:

and

$$a=\frac{v^2-v_0^2}{2\Delta x}=\frac{-v_0^2}{2\Delta x}$$

Substitute to obtain:

$$F_{\text{wood}}=-\frac{mv_0^2}{2\Delta x}$$

Substitute numerical values and evaluate F_{wood}:

$$F_{\text{wood}}=-\frac{(1.8\times10^{-3}\,\text{kg})(500\,\text{m/s})^2}{2(0.06\,\text{m})}$$

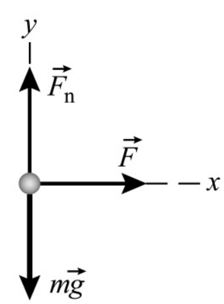
$$=\boxed{-3.75\,\text{kN}}$$

where the negative sign means that the direction of the force is opposite the velocity.

***30 ••**

Picture the Problem The pictorial representation summarizes what we know about the motion. We can find the acceleration of the cart by using a constant-acceleration equation.

The free-body diagram shows the forces acting on the cart as it accelerates along the air track. We can determine the net force acting on the cart using Newton's 2nd law and our knowledge of its acceleration.



(*a*) Apply $\sum F_x = ma_x$ to the cart to obtain an expression for the net force F:

$$F = ma$$

Using a constant-acceleration equation, relate the displacement of the cart to its acceleration, initial speed, and travel time:

$$\Delta x = v_0\Delta t+\tfrac{1}{2}a(\Delta t)^2$$

or, because $v_0 = 0$,

$$\Delta x = \tfrac{1}{2}a(\Delta t)^2$$

Solve for a:

$$a = \frac{2\Delta x}{(\Delta t)^2}$$

Substitute for a in the force equation to obtain:

$$F = m\frac{2\Delta x}{(\Delta t)^2} = \frac{2m\Delta x}{(\Delta t)^2}$$

Substitute numerical values and evaluate F:

$$F = \frac{2(0.355\,\text{kg})(1.5\,\text{m})}{(4.55\,\text{s})^2} = \boxed{0.0514\,\text{N}}$$

(*b*) Using a constant-acceleration equation, relate the displacement of the cart to its acceleration, initial speed, and travel time:

$$\Delta x = v_0\Delta t + \tfrac{1}{2}a'(\Delta t)^2$$

or, because $v_0 = 0$,

$$\Delta x = \tfrac{1}{2}a'(\Delta t)^2$$

Solve for Δt:

$$\Delta t = \sqrt{\frac{2\Delta x}{a'}}$$

If we assume that air resistance is negligible, the net force on the cart is still 0.0514 N and its acceleration is:

$$a' = \frac{0.0514\,\text{N}}{0.722\,\text{kg}} = 0.0713\,\text{m/s}^2$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \sqrt{\frac{2(1.5\,\text{m})}{0.0713\,\text{m/s}^2}} = \boxed{6.49\,\text{s}}$$

31 •

Picture the Problem The acceleration of an object is related to its mass and the *net* force acting on it according to $\vec{F}_{\text{net}} = m\vec{a}$. Let m be the mass of the object and choose a coordinate system in which the direction of $2F_0$ in (*b*) is the positive x and the direction of the left-most F_0 in (a) is the positive y direction. Because both force and acceleration are vector quantities, find the resultant force in each case and then find the resultant acceleration.

(*a*) Calculate the acceleration of the object from Newton's 2nd law of motion:

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$$

Express the net force acting on the object:

$$\vec{F}_{\text{net}} = F_x\hat{i} + F_y\hat{j} = F_0\hat{i} + F_0\hat{j}$$

and

Find the magnitude and direction of this net force:

$$F_{\text{net}} = \sqrt{F_x^2 + F_y^2} = \sqrt{2}F_0$$

and

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{F_0}{F_0}\right) = 45°$$

Use this result to calculate the magnitude and direction of the acceleration:

$$a = \frac{F_{\text{net}}}{m} = \frac{\sqrt{2}F_0}{m} = \sqrt{2}a_0$$
$$= \sqrt{2}\left(3\,\text{m/s}^2\right)$$
$$= \boxed{4.24\,\text{m/s}^2 \text{ @ } 45.0° \text{ from each force.}}$$

(*b*) Calculate the acceleration of the object from Newton's 2nd law of motion:

$$\vec{a} = \vec{F}_{\text{net}}/m$$

Express the net force acting on the object:

$$\vec{F}_{\text{net}} = F_x\hat{i} + F_y\hat{j}$$
$$= \left(-F_0 \sin 45°\right)\hat{i} + \left(2F_0 + F_0 \cos 45°\right)\hat{j}$$

Find the magnitude and direction of this net force:

$$F_{\text{net}} = \sqrt{F_x^2 + F_y^2} = \sqrt{\left(-F_0 \sin 45°\right)^2 + \left(2F_0 + F_0 \cos 45°\right)^2} = 2.80F_0$$

and

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{2F_0 + F_0 \cos 45°}{-F_0 \sin 45°}\right) = -75.4°$$
$$= 14.6° \text{ from } 2\vec{F}_0$$

Use this result to calculate the magnitude and direction of the acceleration:

$$a = \frac{F_{\text{net}}}{m} = 2.80\frac{F_0}{m} = 2.80a_0$$
$$= 2.80\left(3\,\text{m/s}^2\right)$$
$$= \boxed{8.40\,\text{m/s}^2 \text{ @ } 14.6° \text{ from } 2\vec{F}_0}$$

32 •

Picture the Problem The acceleration of an object is related to its mass and the *net* force acting on it according to $\vec{a} = \vec{F}_{\text{net}}/m$.

Apply $\vec{a} = \vec{F}_{\text{net}}/m$ to the object to obtain:

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} = \frac{(6\,\text{N})\hat{i} - (3\,\text{N})\hat{j}}{1.5\,\text{kg}}$$
$$= \boxed{\left(4.00\,\text{m/s}^2\right)\hat{i} - \left(2.00\,\text{m/s}^2\right)\hat{j}}$$

Find the magnitude of $\vec{a}$:

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{\left(4.00\,\text{m/s}^2\right)^2 + \left(2.00\,\text{m/s}^2\right)^2} = \boxed{4.47\,\text{m/s}^2}$$

33 •

Picture the Problem The mass of the particle is related to its acceleration and the *net* force acting on it by Newton's 2nd law of motion. Because the force is constant, we can use constant-acceleration formulas to calculate the acceleration. Choose a coordinate system in which the positive *x* direction is the direction of motion of the particle.

The mass is related to the net force and the acceleration by Newton's 2nd law:

$$m = \frac{\sum \vec{F}}{\vec{a}} = \frac{F_x}{a_x}$$

Because the force is constant, the acceleration is constant. Use a constant-acceleration equation to find the acceleration:

$\Delta x = v_{0x}t + \frac{1}{2}a_x(\Delta t)^2$, where $v_{0x} = 0$,

so

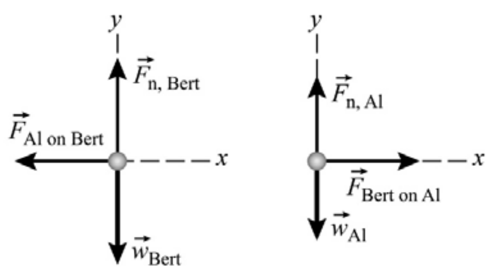
$$a_x = \frac{2\Delta x}{(\Delta t)^2}$$

Substitute this result into the first equation and solve for and evaluate the mass *m* of the particle:

$$m = \frac{F_x}{a_x} = \frac{F_x(\Delta t)^2}{2\Delta x} = \frac{(12\,\text{N})(6\,\text{s})^2}{2(18\,\text{m})} = \boxed{12.0\,\text{kg}}$$

***34** •

Picture the Problem The speed of either Al or Bert can be obtained from their accelerations; in turn, they can be obtained from Newtons 2nd law applied to each person. The free-body diagrams to the right show the forces acting on Al and Bert. The forces that Al and Bert exert on each other are action-and-reaction forces.



(*a*) Apply $\sum F_x = ma_x$ to Bert and solve for his acceleration:

$$-F_{\text{Al on Bert}} = m_{\text{Bert}}a_{\text{Bert}}$$

$$a_{\text{Bert}} = \frac{-F_{\text{Al on Bert}}}{m_{\text{Bert}}} = \frac{-20\,\text{N}}{100\,\text{kg}} = -0.200\,\text{m/s}^2$$

Using a constant-acceleration equation, relate Bert's speed to his initial speed, speed after 1.5 s, and acceleration and solve for his speed at the end of 1.5 s:

$$v = v_0 + a\Delta t = 0 + (-0.200\,\text{m/s}^2)(1.5\,\text{s}) = \boxed{-0.300\,\text{m/s}}$$

(*b*) From Newton's 3rd law, an equal but oppositely directed force acts on Al while he pushes Bert. Because the ice is frictionless, Al speeds off in the opposite direction. Apply Newton's 2nd law to the forces acting on Al and solve for his acceleration:

$$\sum F_{x,\text{Al}} = F_{\text{Bert on Al}} = m_{\text{Al}} a_{\text{Al}}$$

and

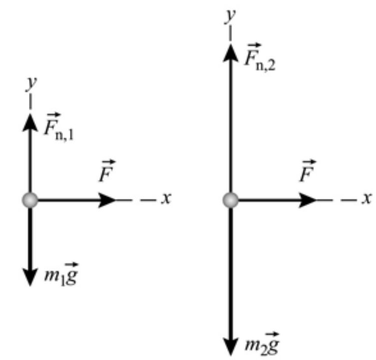
$$a_{\text{Al}} = \frac{F_{\text{Bert on Al}}}{m_{\text{Al}}} = \frac{20\,\text{N}}{80\,\text{kg}} = 0.250\,\text{m/s}^2$$

Using a constant-acceleration equation, relate Al's speed to his initial speed, speed after 1.5 s, and acceleration; solve for his speed at the end of 1.5 s:

$$v = v_0 + a\Delta t = 0 + (0.250\text{ m/s}^2)(1.5\text{ s}) = \boxed{0.375\,\text{m/s}}$$

35 •

Picture the Problem The free-body diagrams show the forces acting on the two blocks. We can apply Newton's second law to the forces acting on the blocks and eliminate F to obtain a relationship between the masses. Additional applications of Newton's 2nd law to the sum and difference of the masses will lead us to values for the accelerations of these combinations of mass.



(*a*) Apply $\sum F_x = ma_x$ to the two blocks:

$$\sum F_{x,1} = F = m_1 a_1$$

and

$$\sum F_{x,2} = F = m_2 a_2$$

Eliminate F between the two equations and solve for m_2:

$$m_2 = \frac{a_1}{a_2} m_1 = \frac{12\,\text{m/s}^2}{3\,\text{m/s}^2} m_1 = 4m_1$$

Express and evaluate the acceleration of an object whose mass is $m_2 - m_1$ when the net force acting on it is F:

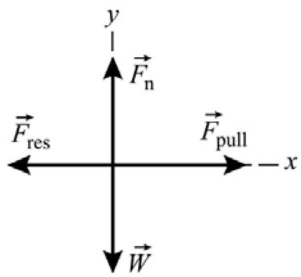
$$a = \frac{F}{m_2 - m_1} = \frac{F}{4m_1 - m_1} = \frac{F}{3m_1} = \tfrac{1}{3} a_1 = \tfrac{1}{3}\left(12\,\text{m/s}^2\right) = \boxed{4.00\,\text{m/s}^2}$$

(*b*) Express and evaluate the acceleration of an object whose mass is $m_2 + m_1$ when the net force acting on it is F:

$$a = \frac{F}{m_2 + m_1} = \frac{F}{4m_1 + m_1} = \frac{F}{5m_1} = \tfrac{1}{5} a_1 = \tfrac{1}{5}\left(12\,\text{m/s}^2\right) = \boxed{2.40\,\text{m/s}^2}$$

36 •

Picture the Problem Because the velocity is constant, the net force acting on the log must be zero. Choose a coordinate system in which the positive x direction is the direction of motion of the log. The free-body diagram shows the forces acting on the log when it is accelerating in the positive x direction.



(*a*) Apply $\sum F_x = ma_x$ to the log when it is moving at constant speed:

$$F_{pull} - F_{res} = ma_x = 0$$

Solve for and evaluate F_{res}:

$$F_{res} = F_{pull} = \boxed{250\,\text{N}}$$

(*b*) Apply $\sum F_x = ma_x$ to the log when it is accelerating to the right:

$$F_{pull} - F_{res} = ma_x$$

Solve for and evaluate F_{pull}:

$$\begin{aligned} F_{pull} &= F_{res} + ma_x \\ &= 250\text{ N} + (75\text{ kg})\,(2\text{ m/s}^2) \\ &= \boxed{400\,\text{N}} \end{aligned}$$

37 •

Picture the Problem The acceleration can be found from Newton's 2nd law. Because both forces are constant, the net force and the acceleration are constant; hence, we can use the constant-acceleration equations to answer questions concerning the motion of the object at various times.

(*a*) Apply Newton's 2nd law to the object to obtain:

$$\begin{aligned} \vec{a} &= \frac{\vec{F}_{net}}{m} = \frac{\vec{F}_1 + \vec{F}_2}{m} \\ &= \frac{(6\,\text{N})\hat{i} + (-14\,\text{N})\hat{j}}{4\,\text{kg}} \\ &= \boxed{(1.50\,\text{m/s}^2)\hat{i} + (-3.50\,\text{m/s}^2)\hat{j}} \end{aligned}$$

(*b*) Using a constant-acceleration equation, express the velocity of the object as a function of time and solve for its velocity when t = 3 s:

$$\begin{aligned} \vec{v} &= \vec{v}_0 + \vec{a}t \\ &= 0 + \left[(1.50\,\text{m/s}^2)\hat{i} + (-3.50\,\text{m/s}^2)\hat{j}\right](3\,\text{s}) \\ &= \boxed{(4.50\,\text{m/s})\hat{i} + (-10.5\,\text{m/s})\hat{j}} \end{aligned}$$

(*c*) Express the position of the object in terms of its average velocity and evaluate this expression at t = 3 s:

$$\begin{aligned} \vec{r} &= \vec{v}_{av}t \\ &= \tfrac{1}{2}\vec{v}t \\ &= \boxed{(6.75\,\text{m})\hat{i} + (-15.8\,\text{m})\hat{j}} \end{aligned}$$

Mass and Weight

***38** •

Picture the Problem The mass of the astronaut is independent of gravitational fields and will be the same on the moon or, for that matter, out in deep space.

Express the mass of the astronaut in terms of his weight on earth and the gravitational field at the surface of the earth:

$$m = \frac{w_{\text{earth}}}{g_{\text{earth}}} = \frac{600\,\text{N}}{9.81\,\text{N/kg}} = 61.2\,\text{kg}$$

and $\boxed{(c)\text{ is correct.}}$

39 •

Picture the Problem The weight of an object is related to its mass and the gravitational field through $w = mg$.

(*a*) The weight of the girl is:

$$w = mg = (54\,\text{kg})(9.81\,\text{N/kg}) = \boxed{530\,\text{N}}$$

(*b*) Convert newtons to pounds:

$$w = \frac{530\,\text{N}}{4.45\,\text{N/lb}} = \boxed{119\,\text{lb}}$$

40 •

Picture the Problem The mass of an object is related to its weight and the gravitational field.

Find the weight of the man in newtons:

$$165\,\text{lb} = (165\,\text{lb})(4.45\,\text{N/lb}) = 734\,\text{N}$$

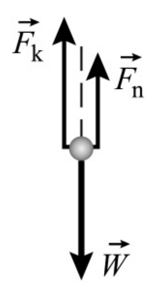
Calculate the mass of the man from his weight and the gravitational field:

$$m = \frac{w}{g} = \frac{734\,\text{N}}{9.81\,\text{N/kg}} = \boxed{74.8\,\text{kg}}$$

Contact Forces

***41** •

Picture the Problem Draw a free-body diagram showing the forces acting on the block. $\vec{F}_k$ is the force exerted by the spring, $\vec{W} = m\vec{g}$ is the weight of the block, and $\vec{F}_{\text{n}}$ is the normal force exerted by the horizontal surface. Because the block is resting on a surface, $F_{\text{k}} + F_{\text{n}} = W$.



(*a*) Calculate the force exerted by the spring on the block:

$$F_x = kx = (600\,\text{N/m})(0.1\,\text{m}) = \boxed{60.0\,\text{N}}$$

(*b*) Choosing the upward direction to be positive, sum the forces acting on the block and solve for F_n:

$$\sum \vec{F} = 0 \Rightarrow F_k + F_n - W = 0$$

and

$$F_n = W - F_k$$

Substitute numerical values and evaluate F_n:

$$F_n = (12\text{ kg})(9.81\text{ N/kg}) - 60\text{ N} = \boxed{57.7\text{ N}}$$

42 •

Picture the Problem Let the positive *x* direction be the direction in which the spring is stretched. We can use Newton's 2nd law and the expression for the force exerted by a stretched (or compressed) spring to express the acceleration of the box in terms of its mass *m*, the stiffness constant of the spring *k*, and the distance the spring is stretched *x*.

Apply Newton's 2nd law to the box to obtain:

$$a = \frac{\sum F}{m}$$

Express the force exerted on the box by the spring:

$$F = -kx$$

Substitute to obtain:

$$a = \frac{-kx}{m}$$

Substitute numerical values and evaluate *a*:

$$a = -\frac{(800\text{ N/m})(0.04\text{ m})}{6\text{ kg}} = \boxed{-5.33\text{ m/s}^2}$$

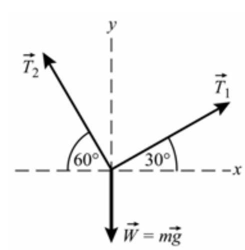
where the minus sign tells us that the box's acceleration is toward its equilibrium position.

Free-Body Diagrams: Static Equilibrium

43 •

Picture the Problem Because the traffic light is not accelerating, the *net* force acting on it must be zero; i.e., $\vec{T}_1 + \vec{T}_2 + m\vec{g} = 0$.

Construct a free-body diagram showing the forces acting on the knot and choose the coordinate system shown:



Apply $\sum F_x = ma_x$ to the knot: $T_1\cos30° - T_2\cos60° = ma_x = 0$

Solve for T_2 in terms of T_1:

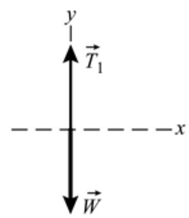
$$T_2 = \frac{\cos 30°}{\cos 60°}T_1 = 1.73T_1$$

$$\therefore \boxed{T_2 \text{ is greater than } T_1}$$

44 •

Picture the Problem Draw a free-body diagram showing the forces acting on the lamp and apply $\sum F_y = 0$.

From the FBD, it is clear that T_1 supports the full weight mg = 418 N.



Apply $\sum F_y = 0$ to the lamp to obtain:

$$T_1 - w = 0$$

Solve for T_1:

$$T_1 = w = mg$$

Substitute numerical values and evaluate T_1:

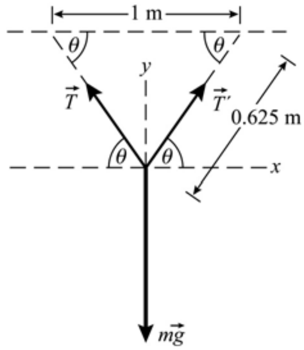
$$T_1 = (42.6\,\text{kg})(9.81\,\text{m/s}^2) = 418\,\text{N}$$

and $\boxed{(b) \text{ is correct.}}$

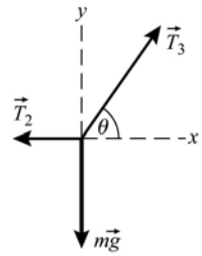
***45** ••

Picture the Problem The free-body diagrams for parts (*a*), (*b*), and (*c*) are shown below. In both cases, the block is in equilibrium under the influence of the forces and we can use Newton's 2nd law of motion and geometry and trigonometry to obtain relationships between θ and the tensions.

(*a*) and (*b*)



(*c*)



(*a*) Referring to the FBD for part (*a*), use trigonometry to determine θ:

$$\theta = \cos^{-1}\frac{0.5\,\text{m}}{0.625\,\text{m}} = \boxed{36.9°}$$

(*b*) Noting that $T = T'$, apply $\sum F_y = ma_y$ to the 0.500-kg block and solve for the tension *T*:

$$2T\sin\theta - mg = 0 \text{ since } a = 0$$

and

$$T = \frac{mg}{2\sin\theta}$$

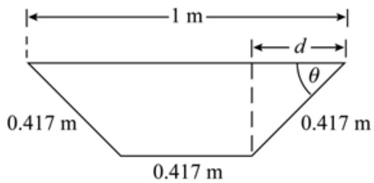
Substitute numerical values and evaluate *T*:

$$T = \frac{(0.5\,\text{kg})(9.81\,\text{m/s}^2)}{2\sin 36.9°} = \boxed{4.08\,\text{N}}$$

(*c*) The length of each segment is:

$$\frac{1.25\,\text{m}}{3} = 0.417\,\text{m}$$

Find the distance *d*:



$$d = \frac{1\,\text{m} - 0.417\,\text{m}}{2} = 0.2915\,\text{m}$$

Express θ in terms of *d* and solve for its value:

$$\theta = \cos^{-1}\left(\frac{d}{0.417\,\text{m}}\right) = \cos^{-1}\left(\frac{0.2915\,\text{m}}{0.417\,\text{m}}\right) = 45.7°$$

Apply $\sum F_y = ma_y$ to the 0.250-kg block and solve for the tension T_3:

$$T_3\sin\theta - mg = 0 \text{ since } a = 0.$$

and

$$T_3 = \frac{mg}{\sin\theta}$$

Substitute numerical values and evaluate T_3:

$$T_3 = \frac{(0.25\,\text{kg})(9.81\,\text{m/s}^2)}{\sin 45.7°} = \boxed{3.43\,\text{N}}$$

Apply $\sum F_x = ma_x$ to the 0.250-kg block and solve for the tension T_2:

$$T_3\cos\theta - T_2 = 0 \text{ since } a = 0.$$

and

$$T_2 = T_3\cos\theta$$

Substitute numerical values and evaluate T_2:

$$T_2 = (3.43\,\text{N})\cos 45.7° = \boxed{2.40\,\text{N}}$$

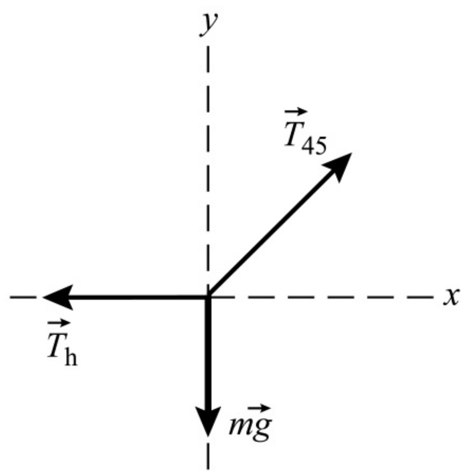
By symmetry:

$$T_1 = T_3 = \boxed{3.43\,\text{N}}$$

46 •

Picture the Problem The suspended body is in equilibrium under the influence of the forces $\vec{T}_h$, $\vec{T}_{45}$, and $m\vec{g}$; i.e., $\vec{T}_h + \vec{T}_{45} + m\vec{g} = 0$

Draw the free-body diagram of the forces acting on the knot just above the 100-N body. Choose a coordinate system with the positive x direction to the right and the positive y direction upward. Apply the conditions for translational equilibrium to determine the tension in the horizontal cord.



If the system is to remain in static equilibrium, the vertical component of T_{45} must be exactly balanced by, and therefore equal to, the tension in the string suspending the 100-N body:

$$T_v = T_{45}\sin 45° = mg$$

Express the horizontal component of T_{45}:

$$T_h = T_{45}\cos 45°$$

Because $T_{45}\sin 45° = T_{45}\cos 45°$:

$$T_h = mg = \boxed{100\,\text{N}}$$

47 •

Picture the Problem The acceleration of *any* object is directly proportional to the *net* force acting on it. Choose a coordinate system in which the positive x direction is the same as that of $\vec{F}_1$ and the positive y direction is to the right. Add the two forces to determine the net force and then use Newton's 2nd law to find the acceleration of the object. If $\vec{F}_3$ brings the system into equilibrium, it must be true that $\vec{F}_3 + \vec{F}_1 + \vec{F}_2 = 0$.

(*a*) Find the components of $\vec{F}_1$ and $\vec{F}_2$:

$$\vec{F}_1 = (20\,\text{N})\hat{\boldsymbol{i}}$$

$$\begin{aligned}\vec{F}_2 &= \{(-30\,\text{N})\sin 30°\}\hat{\boldsymbol{i}} \\ &\quad + \{(30\,\text{N})\cos 30°\}\hat{\boldsymbol{j}} \\ &= (-15\,\text{N})\hat{\boldsymbol{i}} + (26\,\text{N})\hat{\boldsymbol{j}}\end{aligned}$$

Add $\vec{F}_1$ and $\vec{F}_2$ to find $\vec{F}_{tot}$:

$$\vec{F}_{tot} = (5\,\text{N})\hat{\boldsymbol{i}} + (26\,\text{N})\hat{\boldsymbol{j}}$$

Apply $\sum \vec{F} = m\vec{a}$ to find the acceleration of the object:

$$\vec{a} = \frac{\vec{F}_{\text{tot}}}{m}$$

$$= \boxed{(0.500\,\text{m/s}^2)\hat{i} + (2.60\,\text{m/s}^2)\hat{j}}$$

(*b*) Because the object is in equilibrium under the influence of the three forces, it must be true that:

$$\vec{F}_3 + \vec{F}_1 + \vec{F}_2 = 0$$

and

$$\vec{F}_3 = -\left(\vec{F}_1 + \vec{F}_2\right)$$

$$= \boxed{(-5.00\,\text{N})\hat{i} + (-26.0\,\text{N})\hat{j}}$$

***48 •**

Picture the Problem The acceleration of the object equals the net force, $\vec{T} - m\vec{g}$, divided by the mass. Choose a coordinate system in which upward is the positive *y* direction. Apply Newton's 2nd law to the forces acting on this body to find the acceleration of the object as a function of *T*.

(*a*) Apply $\sum F_y = ma_y$ to the object:

$$T - w = T - mg = ma_y$$

Solve this equation for *a* as a function of *T*:

$$a_y = \frac{T}{m} - g$$

Substitute numerical values and evaluate a_y:

$$a_y = \frac{5\,\text{N}}{5\,\text{kg}} - 9.81\,\text{m/s}^2 = \boxed{-8.81\,\text{m/s}^2}$$

(*b*) Proceed as in (*a*) with $T = 10$ N:

$$a = \boxed{-7.81\,\text{m/s}^2}$$

(*c*) Proceed as in (*a*) with $T = 100$ N:

$$a = \boxed{10.2\,\text{m/s}^2}$$

49 ••

Picture the Problem The picture is in equilibrium under the influence of the three forces shown in the figure. Due to the symmetry of the support system, the vectors $\vec{T}$ and $\vec{T}'$ have the same magnitude *T*. Choose a coordinate system in which the positive *x* direction is to the right and the positive *y* direction is upward. Apply the condition for translational equilibrium to obtain an expression for *T* as a function of θ and *w*.

(*a*) Referring to Figure 4-37, apply the condition for translational equilibrium in the vertical direction and solve for *T*:

$$\sum F_y = 2T\sin\theta - w = 0$$

and

$$T = \boxed{\frac{w}{2\sin\theta}}$$

T_{min} occurs when $\sin\theta$ is a maximum:

$$\theta = \sin^{-1} 1 = \boxed{90°}$$

T_{max} occurs when $\sin\theta$ is a minimum. Because the function is undefined when $\sin\theta$ = 0, we can conclude that:

$$\boxed{T \to T_{max} \text{ as } \theta \to 0°}$$

(*b*) Substitute numerical values in the result in (*a*) and evaluate *T*:

$$T = \frac{(2\,\text{kg})(9.81\,\text{m/s}^2)}{2\sin 30°} = \boxed{19.6\,\text{N}}$$

Remarks: θ = 90° requires wires of infinite length; therefore it is not possible. As θ gets small, *T* gets large without limit.

***50** •••

Picture the Problem In part (*a*) we can apply Newton's 2nd law to obtain the given expression for *F*. In (*b*) we can use a symmetry argument to find an expression for tan θ_0. In (*c*) we can use our results obtained in (*a*) and (*b*) to express x_i and y_i.

(*a*) Apply $\sum F_y = 0$ to the balloon:

$$F + T_i \sin\theta_i - T_{i-1}\sin\theta_{i-1} = 0$$

Solve for *F* to obtain:

$$F = \boxed{T_{i-1}\sin\theta_{i-1} - T_i \sin\theta_i}$$

(*b*) By symmetry, each support must balance half of the force acting on the entire arch. Therefore, the vertical component of the force on the support must be *NF*/2. The horizontal component of the tension must be T_H. Express tanθ_0 in terms of *NF*/2 and T_H:

$$\tan\theta_0 = \frac{NF/2}{T_H} = \frac{NF}{2T_H}$$

By symmetry, $\theta_{N+1} = -\theta_0$. Therefore, because the tangent function is odd:

$$\tan\theta_0 = \boxed{-\tan\theta_{N+1} = \frac{NF}{2T_H}}$$

(*c*) Using $T_H = T_i \cos\theta_i = T_{i-1}\cos\theta_{i-1}$, divide both sides of our result in (*a*) by T_H and simplify to obtain:

$$\frac{F}{T_H} = \frac{T_{i-1}\sin\theta_{i-1}}{T_{i-1}\cos\theta_{i-1}} - \frac{T_i \sin\theta_i}{T_i\cos\theta_i}$$

$$= \tan\theta_{i-1} - \tan\theta_i$$

Using this result, express tan θ_1:

$$\tan\theta_1 = \tan\theta_0 - \frac{F}{T_H}$$

Substitute for tan θ_0 from (*a*):

$$\tan\theta_1 = \frac{NF}{2T_H} - \frac{F}{T_H} = (N-2)\frac{F}{2T_H}$$

Generalize this result to obtain:

$$\boxed{\tan\theta_{\text{i}} = (N-2i)\frac{F}{2T_{\text{H}}}}$$

Express the length of rope between two balloons:

$$\ell_{\text{between balloons}} = \frac{L}{N+1}$$

Express the horizontal coordinate of the point on the rope where the ith balloon is attached, x_i, in terms of x_{i-1} and the length of rope between two balloons:

$$x_{\text{i}} = x_{\text{i}-1} + \frac{L}{N+1}\cos\theta_{\text{i}-1}$$

Sum over all the coordinates to obtain:

$$\boxed{x_{\text{i}} = \frac{L}{N+1}\sum_{j=0}^{\text{i}-1}\cos\theta_j}$$

Proceed similarly for the vertical coordinates to obtain:

$$\boxed{y_{\text{i}} = \frac{L}{N+1}\sum_{j=0}^{\text{i}-1}\sin\theta_j}$$

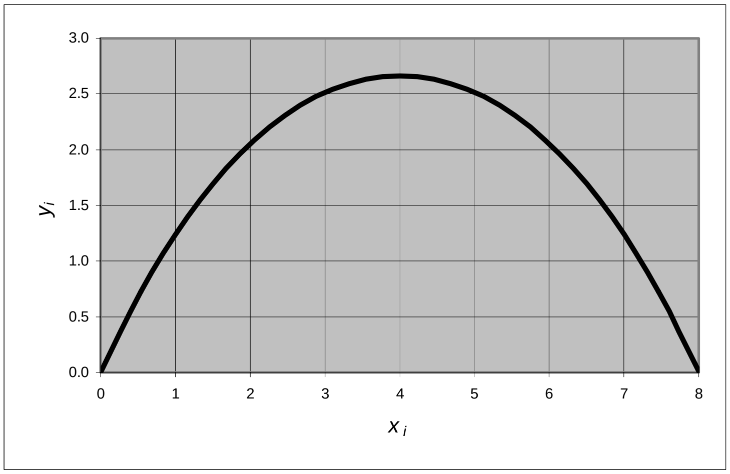
(d) A spreadsheet program is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Content/Formula	Algebraic Form
C9	(\$B\$2-2*B9)/(2*\$B\$4)	$(N-2\text{i})\frac{F}{2T_{\text{H}}}$
D9	SIN(ATAN(C9))	$\sin(\tan^{-1}\theta_{\text{i}})$
E9	COS(ATAN(C9))	$\cos(\tan^{-1}\theta_{\text{i}})$
F10	F9+\$B\$1/(\$B\$2+1)*E9	$x_{\text{i}-1} + \frac{L}{N+1}\cos\theta_{\text{i}-1}$
G10	G9+\$B\$1/(\$B\$2+1)*D9	$y_{\text{i}-1} + \frac{L}{N+1}\cos\theta_{\text{i}-1}$

	A	B	C	D	E	F	G
1	L =	10	m				
2	N =	10					
3	F =	1	N				
4	TH=	3.72	N				
5							
6							
7							
8		I	tan(thetai)	sin(thetai)	cos(thetai)	xi	yi
9		0	1.344	0.802	0.597	0.000	0.000
10		1	1.075	0.732	0.681	0.543	0.729
11		2	0.806	0.628	0.778	1.162	1.395
12		3	0.538	0.474	0.881	1.869	1.966
13		4	0.269	0.260	0.966	2.670	2.396

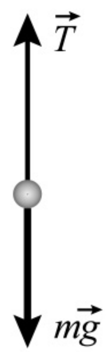
14		5	0.000	0.000	1.000	3.548	2.632
15		6	–0.269	–0.260	0.966	4.457	2.632
16		7	–0.538	–0.474	0.881	5.335	2.396
17		8	–0.806	–0.628	0.778	6.136	1.966
18		9	–1.075	–0.732	0.681	6.843	1.395
19		10	–1.344	–0.802	0.597	7.462	0.729
20		11				8.005	0.000

(*e*) A horizontal component of tension 3.72 N gives a spacing of 8 m. At this spacing, the arch is 2.63 m high, tall enough for someone to walk through.



51 ••

Picture the Problem We know, because the speed of the load is changing in parts (*a*) and (*c*), that it is accelerating. We also know that, if the load is accelerating in a particular direction, there must be a *net* force in that direction. A free-body diagram for part (*a*) is shown to the right. We can apply Newton's 2nd law of motion to each part of the problem to relate the tension in the cable to the acceleration of the load. Choose the upward direction to be the positive *y* direction.



(*a*) Apply $\sum F_y = ma_y$ to the load and solve for *T*:

$$T - mg = ma$$

and

$$T = ma_y + mg = m(a_y + g) \qquad (1)$$

Substitute numerical values and evaluate *T*:

$$T = (1000\,\text{kg})(2\,\text{m/s}^2 + 9.81\,\text{m/s}^2)$$

$$= \boxed{11.8\,\text{kN}}$$

(*b*) Because the crane is lifting the

$$T = \text{mg} = \boxed{9.81\,\text{kN}}$$

load at constant speed, $a = 0$:

(*c*) Because the acceleration of the load is downward, a is negative. Apply $\sum F_y = ma_y$ to the load:

$$T - mg = ma_y$$

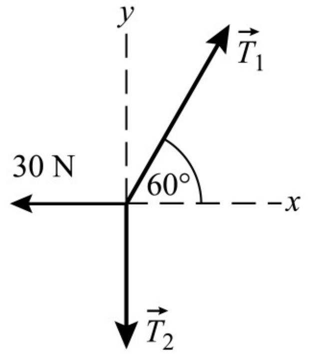
Substitute numerical values in equation (1) and evaluate T:

$$T = (1000\text{ kg})(9.81\text{ m/s}^2 - 2\text{ m/s}^2) = \boxed{7.81\text{ kN}}$$

52 ••

Picture the Problem Draw a free-body diagram for each of the depicted situations and use the conditions for translational equilibrium to find the unknown tensions.

(*a*)



$$\Sigma F_x = T_1\cos 60° - 30\text{ N} = 0$$

and

$$T_1 = (30\text{ N})/\cos 60° = \boxed{60.0\text{ N}}$$

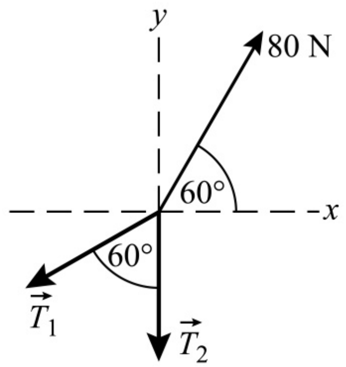
$$\Sigma F_y = T_1\sin 60° - T_2 = 0$$

and

$$T_2 = T_1\sin 60° = \boxed{52.0\text{ N}}$$

$$\therefore\ m = T_2/g = \boxed{5.30\text{ kg}}$$

(*b*)



$$\Sigma F_x = (80\text{ N})\cos 60° - T_1\sin 60° = 0$$

and

$$T_1 = (80\text{ N})\cos 60°/\sin 60° = \boxed{46.2\text{ N}}$$

$$\Sigma F_y = (80\text{ N})\sin 60° - T_2 - T_1\cos 60° = 0$$

$$T_2 = (80\text{ N})\sin 60° - (46.2\text{ N})\cos 60° = \boxed{46.2\text{ N}}$$

$$m = T_2/g = \boxed{4.71\text{ kg}}$$

(*c*)

$$\Sigma F_x = -T_1\cos 60° + T_3\cos 60° = 0$$

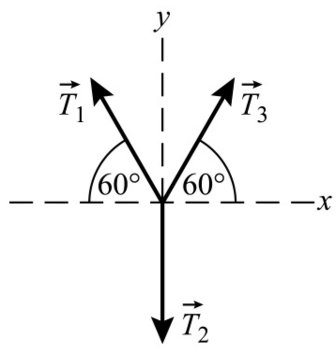
and

$$T_1 = T_3$$

$$\Sigma F_y = 2T_1\sin 60° - mg = 0$$

and

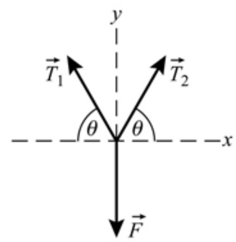
$$T_1 = T_3 = (58.9\text{ N})/(2\sin 60°) = \boxed{34.0\text{ N}}$$



$\therefore\ m = T_1/g = \boxed{3.46\ \text{kg}}$

53 ••

Picture the Problem Construct the free-body diagram for that point in the rope at which you exert the force $\vec{F}$ and choose the coordinate system shown on the FBD. We can apply Newton's 2nd law to the rope to relate the tension to F.



(*a*) Noting that $T_1 = T_2 = T$, apply $\sum F_y = ma_y$ to the car:

$2T\sin\theta - F = ma_y = 0$ because the car's acceleration is zero.

Solve for and evaluate T:

$$T = \frac{F}{2\sin\theta} = \frac{400\ \text{N}}{2\sin 3°} = \boxed{3.82\ \text{kN}}$$

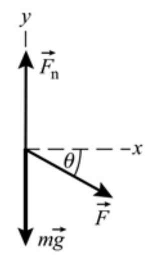
(*b*) Proceed as in part (*a*):

$$T = \frac{600\ \text{N}}{2\sin 4°} = \boxed{4.30\ \text{kN}}$$

Free-Body Diagrams: Inclined Planes and the Normal Force

***54** •

Picture the Problem The free-body diagram shows the forces acting on the box as the man pushes it across the frictionless floor. We can apply Newton's 2nd law of motion to the box to find its acceleration.



Apply $\sum F_x = ma_x$ to the box:

$$F\cos\theta = ma_x$$

Solve for a_x:

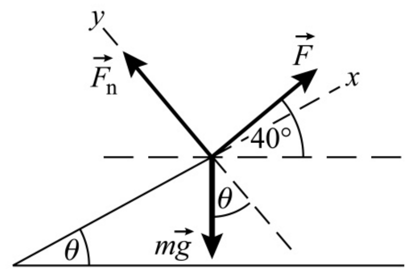
$$a_x = \frac{F\cos\theta}{m}$$

Substitute numerical values and evaluate a_x:

$$a_x = \frac{(250\ \text{N})\cos 35°}{20\ \text{kg}} = \boxed{10.2\ \text{m/s}^2}$$

55 •

Picture the Problem The free-body diagram shows the forces acting on the box as the man pushes it up the frictionless incline. We can apply Newton's 2nd law of motion to the box to determine the smallest force that will move it up the incline at constant speed.



Apply $\sum F_x = ma_x$ to the box as it moves up the incline with constant speed:

$$F_{\text{min}}\cos(40° - \theta) - mg\sin\theta = 0$$

Solve for F_{min}:

$$F_{\text{min}} = \frac{mg\sin\theta}{\cos(40° - \theta)}$$

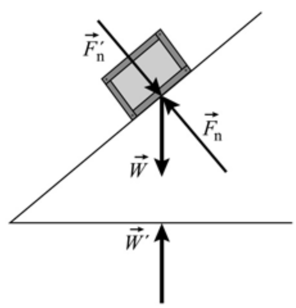
Substitute numerical values and evaluate F_{min}:

$$F_{\text{min}} = \frac{(20\,\text{kg})(9.81\,\text{m/s}^2)}{\cos 25°} = \boxed{56.0\,\text{N}}$$

56 •

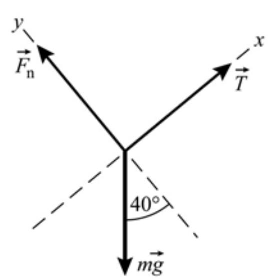
Picture the Problem Forces always occur in equal and opposite pairs. If object *A* exerts a force, $\vec{F}_{B,A}$ on object *B*, an equal but opposite force, $\vec{F}_{A,B} = -\vec{F}_{B,A}$ is exerted by object *B* on object *A*.

The forces acting on the box are its weight, $\vec{W}$, and the normal reaction force of the inclined plane on the box, $\vec{F}_n$. The reaction forces are the forces the box exerts on the inclined plane and the gravitational force the box exerts on the earth. The reaction forces are indicated with primes.



57 •

Picture the Problem Because the block whose mass is *m* is in equilibrium, the sum of the forces $\vec{F}_n$, $\vec{T}$, and $m\vec{g}$ must be zero. Construct the free-body diagram for this object, use the coordinate system shown on the free-body diagram, and apply Newton's 2nd law of motion.



Apply $\sum F_x = ma_x$ to the block on the incline:

$T - mg\sin 40° = ma_x = 0$ because the system is in equilibrium.

Solve for m:

$$m = \frac{T}{g\sin 40°}$$

The tension must equal the weight of the 3.5-kg block because that block is also in equilibrium:

$T = (3.5\text{ kg})g$

and

$$m = \frac{(3.5\,\text{kg})g}{g\sin 40°} = \frac{3.5\,\text{kg}}{\sin 40°}$$

Because this expression is not included in the list of solution candidates,

$\boxed{(d)\text{ is correct.}}$

Remarks: Because the object whose mass is *m* does not hang vertically, its mass must be greater than 3.5 kg.

***58** •

Picture the Problem The balance(s) indicate the tension in the string(s). Draw free-body diagrams for each of these systems and apply the condition(s) for equilibrium.

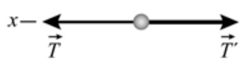
(*a*)

$$\sum F_y = T - mg = 0$$

and

$$T = mg = (10\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{98.1\,\text{N}}$$

(*b*)



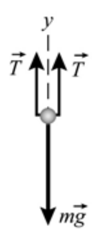
$$\sum F_x = T - T' = 0$$

or, because $T' = mg$,

$$T = T' = mg$$

$$= (10\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{98.1\,\text{N}}$$

(*c*)



$$\sum F_y = 2T - mg = 0$$

and

$$T = \tfrac{1}{2}mg$$

$$= \tfrac{1}{2}(10\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{49.1\,\text{N}}$$

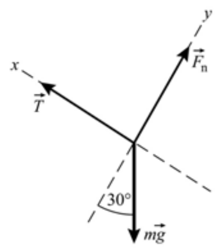
(*d*)

$$\sum F_x = T - mg\sin 30° = 0$$

and

$$T = mg\sin 30°$$

$$= (10\,\text{kg})(9.81\,\text{m/s}^2)\sin 30° = \boxed{49.1\,\text{N}}$$



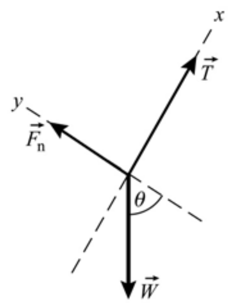
Remarks: Note that (*a*) and (*b*) give the same answers … a rather surprising result until one has learned to draw FBDs and apply the conditions for translational equilibrium.

59 ••

Picture the Problem Because the box is held in place (is in equilibrium) by the forces acting on it, we know that

$$\vec{\boldsymbol{T}} + \vec{\boldsymbol{F}}_n + \vec{\boldsymbol{W}} = 0$$

Choose a coordinate system in which the positive x direction is in the direction of $\vec{\boldsymbol{T}}$ and the positive y direction is in the direction of $\vec{\boldsymbol{F}}_n$. Apply Newton's 2nd law to the block to obtain expressions for $\vec{\boldsymbol{T}}$ and $\vec{\boldsymbol{F}}_n$.



(*a*) Apply $\sum F_x = ma_x$ to the box: $T - mg\sin\theta = 0$

Solve for T: $T = mg\sin\theta$

Substitute numerical values and evaluate T: $T = (50\,\text{kg})(9.81\,\text{m/s}^2)\sin 60° = \boxed{425\,\text{N}}$

Apply $\sum F_y = ma_y$ to the box: $F_n - mg\cos\theta = 0$

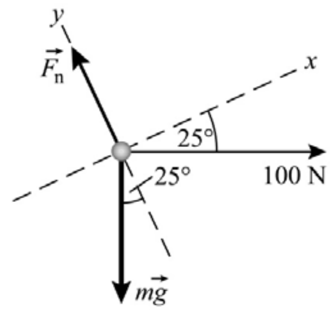
Solve for F_n: $F_n = mg\cos\theta$

Substitute numerical values and evaluate F_n: $F_n = (50\,\text{kg})(9.81\,\text{m/s}^2)\cos 60° = \boxed{245\,\text{N}}$

(*b*) Using the result for the tension from part (*a*) to obtain: $T_{90°} = mg\sin 90° = \boxed{mg}$ and $T_{0°} = mg\sin 0° = \boxed{0}$

60 ••

Picture the Problem Draw a free-body diagram for the box. Choose a coordinate system in which the positive x-axis is parallel to the inclined plane and the positive y-axis is in the direction of the normal force the incline exerts on the block. Apply Newton's 2nd law of motion to both the x and y directions.



(a) Apply $\sum F_y = ma_y$ to the block:

$$F_n - mg\cos 25° - (100\,\text{N})\sin 25° = 0$$

Solve for F_n:

$$F_n = mg\cos 25° + (100\,\text{N})\sin 25°$$

Substitute numerical values and evaluate F_n:

$$F_n = (12\,\text{kg})(9.81\,\text{m/s}^2)\cos 25° + (100\,\text{N})\sin 25° = \boxed{149\,\text{N}}$$

(b) Apply $\sum F_x = ma_x$ to the block:

$$(100\,\text{N})\cos 25° - mg\sin 25° = ma$$

Solve for a:

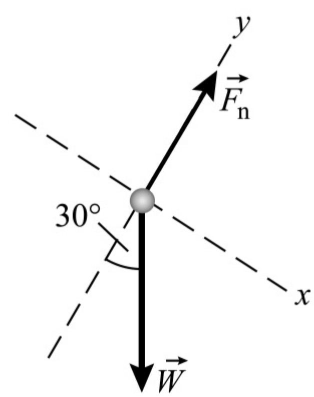
$$a = \frac{(100\,\text{N})\cos 25°}{m} - g\sin 25°$$

Substitute numerical values and evaluate a:

$$a = \frac{(100\,\text{N})\cos 25°}{12\,\text{kg}} - (9.81\,\text{m/s}^2)\sin 25° = \boxed{3.41\,\text{m/s}^2}$$

***61** ••

Picture the Problem The scale reading (the boy's apparent weight) is the force the scale exerts on the boy. Draw a free-body diagram for the boy, choosing a coordinate system in which the positive x-axis is parallel to and down the inclined plane and the positive y-axis is in the direction of the normal force the incline exerts on the boy. Apply Newton's 2nd law of motion in the y direction.



Apply $\sum F_y = ma_y$ to the boy to find F_n. Remember that there is no acceleration in the y direction:

$$F_n - W\cos 30° = 0$$

Substitute for W to obtain:

$$F_n - mg\cos 30° = 0$$

Solve for F_n:

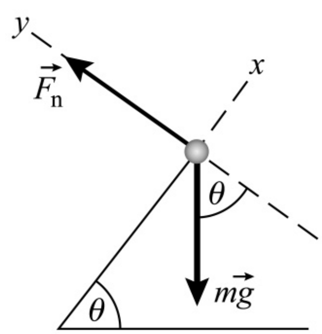
$$F_n = mg\cos 30°$$

Substitute numerical values and evaluate F_n:

$$F_n = (65\,\text{kg})(9.81\,\text{m/s}^2)\cos 30° = \boxed{552\,\text{N}}$$

62 ••

Picture the Problem The free-body diagram for the block sliding up the incline is shown to the right. Applying Newton's 2nd law to the forces acting in the x direction will lead us to an expression for a_x. Using this expression in a constant-acceleration equation will allow us to express h as a function of v_0 and g.



The height h is related to the distance Δx traveled up the incline:

$$h = \Delta x \sin\theta$$

Using a constant-acceleration equation, relate the final speed of the block to its initial speed, acceleration, and distance traveled:

$$v^2 = v_0^2 + 2a_x\Delta x$$

or, because $v = 0$,

$$0 = v_0^2 + 2a_x\Delta x$$

Solve for Δx to obtain:

$$\Delta x = \frac{-v_0^2}{2a_x}$$

Apply $\sum F_x = ma_x$ to the block and solve for its acceleration:

$$-mg\sin\theta = ma_x$$

and

$a_x = -g\sin\theta$

Substitute these results in the equation for h and simplify:

$$h = \Delta x\sin\theta = \left(\frac{v_0^2}{2g\sin\theta}\right)\sin\theta = \boxed{\frac{v_0^2}{2g}}$$

which is independent of the ramp's angle θ.

Free-Body Diagrams: Elevators

63 •

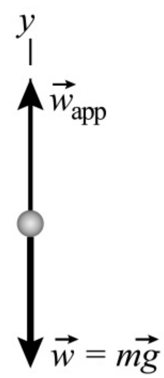
Picture the Problem Because the elevator is descending at constant speed, the object is in equilibrium and $\vec{T} + m\vec{g} = 0$. Draw a free-body diagram of the object and let the upward direction be the positive y direction. Apply Newton's 2nd law with $a = 0$.

Because the downward speed is constant, the acceleration is zero. Apply $\sum F_y = ma_y$ and solve for T:

$T - mg = 0 \Rightarrow T = mg$
and
$\boxed{(a)\text{ is correct.}}$

64 •

Picture the Problem The sketch to the right shows a person standing on a scale in a descending elevator. To its right is a free-body diagram showing the forces acting on the person. The force exerted by the scale on the person, $\vec{w}_{\text{app}}$, is the person's apparent weight. Because the elevator is slowing down while descending, the acceleration is directed upward.



Apply $\sum F_y = ma_y$ to the person:

$w_{\text{app}} - mg = ma_y$

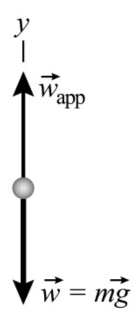
Solve for w_{app}:

$w_{\text{app}} = mg + ma_y > mg$

$$\boxed{\begin{array}{l}\text{The apparent weight will be higher. Because an upward acceleration is required}\\ \text{to "slow" a downward velocity, the normal force exerted on you by the scale}\\ \text{(your } \textit{apparent weight}\text{) must be greater than your weight.}\end{array}}$$

***65 •**

Picture the Problem The sketch to the right shows a person standing on a scale in the elevator immediately after the cable breaks. To its right is the free-body diagram showing the forces acting on the person. The force exerted by the scale on the person, $\vec{w}_{\text{app}}$, is the person's apparent weight.



From the free-body diagram we can see that $\vec{w}_{\text{app}} + m\vec{g} = m\vec{a}$ where $\vec{g}$ is the local gravitational field and $\vec{a}$ is the acceleration of the reference frame (elevator). When the elevator goes into free fall ($\vec{a} = \vec{g}$), our equation becomes $\vec{w}_{\text{app}} + m\vec{g} = m\vec{a} = m\vec{g}$. This tells us that $\vec{w}_{\text{app}} = 0$. $\boxed{(e) \text{ is correct.}}$

66 •

Picture the Problem The free-body diagram shows the forces acting on the 10-kg block as the elevator accelerates upward. Apply Newton's 2nd law of motion to the block to find the minimum acceleration of the elevator required to break the cord.

Apply $\sum F_y = ma_y$ to the block: $T - mg = ma_y$

Solve for a_y to determine the minimum breaking acceleration:

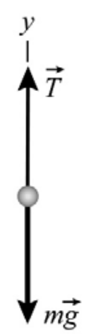
$$a_y = \frac{T - mg}{m} = \frac{T}{m} - g$$

Substitute numerical values and evaluate a_y:

$$a_y = \frac{150\,\text{N}}{10\,\text{kg}} - 9.81\,\text{m/s}^2 = \boxed{5.19\,\text{m/s}^2}$$

67 ••

Picture the Problem The free-body diagram shows the forces acting on the 2-kg block as the elevator ascends at a constant velocity. Because the acceleration of the elevator is zero, the block is in equilibrium under the influence of $\vec{T}$ and $m\vec{g}$. Apply Newton's 2nd law of motion to the block to determine the scale reading.



(*a*) Apply $\sum F_y = ma_y$ to the block to obtain:

$$T - mg = ma_y \qquad (1)$$

For motion with constant velocity, $a_y = 0$:

$$T - mg = 0 \text{ and } T = mg$$

Substitute numerical values and evaluate T:

$$T = (2\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{19.6\,\text{N}}$$

(*b*) As in part (*a*), for constant velocity, $a = 0$:

$$T - mg = ma_y$$

and

$$T = (2\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{19.6\,\text{N}}$$

(*c*) Solve equation (1) for T and simplify to obtain:

$$T = mg + ma_y = m(g + a_y) \qquad (2)$$

Because the elevator is ascending and its speed is increasing, we have $a_y = 3\ \text{m/s}^2$. Substitute numerical values and evaluate T:

$$T = (2\,\text{kg})(9.81\,\text{m/s}^2 + 3\,\text{m/s}^2) = \boxed{25.6\,\text{N}}$$

(*d*) For $0 < t < 5$ s: $a_y = 0$ and

$$T_{0\to 5\,\text{s}} = \boxed{19.6\ \text{N}}$$

Using its definition, calculate a for $5\ \text{s} < t < 9\ \text{s}$:

$$a = \frac{\Delta v}{\Delta t} = \frac{0 - 10\,\text{m/s}}{4\,\text{s}} = -2.5\,\text{m/s}^2$$

Substitute in equation (2) and evaluate T:

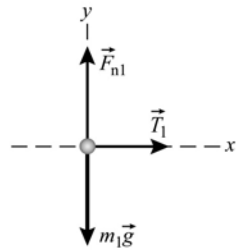
$$T_{5\,\text{s}\to 9\,\text{s}} = (2\,\text{kg})(9.81\,\text{m/s}^2 - 2.5\,\text{m/s}^2)$$
$$= \boxed{14.6\,\text{N}}$$

Free-Body Diagrams: Ropes, Tension, and Newton's Third Law

68 •

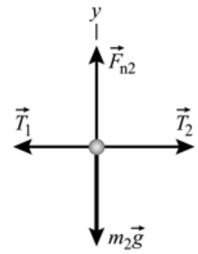
Picture the Problem Draw a free-body diagram for each object and apply Newton's 2nd law of motion. Solve the resulting simultaneous equations for the ratio of T_1 to T_2.

Draw the FBD for the box to the left and apply $\sum F_x = ma_x$:



$$T_1 = m_1 a_1$$

Draw the FBD for the box to the right and apply $\sum F_x = ma_x$:



$$T_2 - T_1 = m_2 a_2$$

The two boxes have the same acceleration:

$$a_1 = a_2$$

Divide the second equation by the first:

$$\frac{T_1}{T_2 - T_1} = \frac{m_1}{m_2}$$

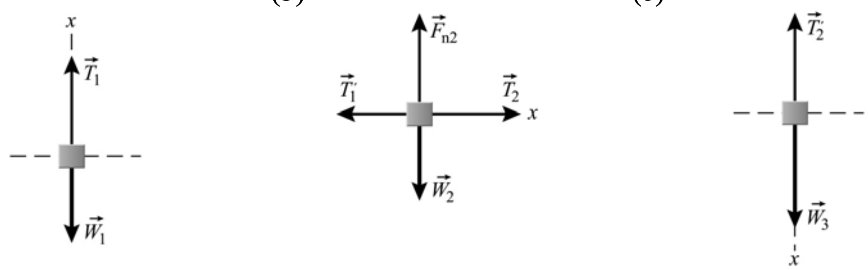
Solve for the ratio T_1/T_2 :

$$\frac{T_1}{T_2} = \frac{m_1}{m_1 + m_2} \text{ and } \boxed{(d) \text{ is correct.}}$$

69 ••

Picture the Problem Call the common acceleration of the boxes a. Assume that box 1 moves upward, box 2 to the right, and box 3 downward and take this direction to be the positive x direction. Draw free-body diagrams for each of the boxes, apply Newton's 2nd law of motion, and solve the resulting equations simultaneously.

(a) (b) (c)



(a) Apply $\sum F_x = ma_x$ to the box whose mass is m_1:

$$T_1 - w_1 = m_1 a$$

Apply $\sum F_x = ma_x$ to the box whose mass is m_2:

$$T_2 - T_1 = m_2 a$$

Noting that $T_2 = T_2'$, apply $\sum F_x = ma_x$ to the box whose mass is m_3:

$$w_3 - T_2 = m_3 a$$

Add the three equations to obtain:

$$w_3 - w_1 = (m_1 + m_2 + m_3)a$$

Solve for a:

$$a = \frac{(m_3 - m_1)g}{m_1 + m_2 + m_3}$$

Substitute numerical values and evaluate a:

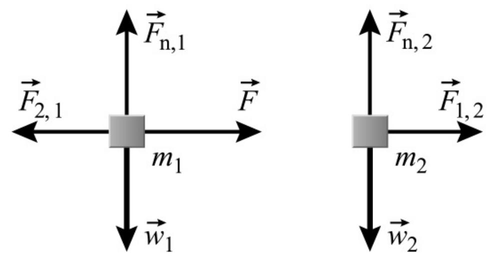
$$a = \frac{(2.5\,\text{kg} - 1.5\,\text{kg})(9.81\,\text{m/s}^2)}{1.5\,\text{kg} + 3.5\,\text{kg} + 2.5\,\text{kg}} = \boxed{1.31\,\text{m/s}^2}$$

(b) Substitute for the acceleration in the equations obtained above to find the tensions:

$$T_1 = \boxed{16.7\,\text{N}} \text{ and } T_2 = \boxed{21.3\,\text{N}}$$

***70** ••

Picture the Problem Choose a coordinate system in which the positive x direction is to the right and the positive y direction is upward. Let $\vec{F}_{2,1}$ be the contact force exerted by m_2 on m_1 and $\vec{F}_{1,2}$ be the force exerted by m_1 on m_2. These forces are equal and opposite so $\vec{F}_{2,1} = -\vec{F}_{1,2}$. The free-body diagrams for the blocks are shown to the right. Apply Newton's 2nd law to each block separately and use the fact that their accelerations are equal.



(*a*) Apply $\sum F_x = ma_x$ to the first block:

$$F - F_{2,1} = m_1 a_1 = m_1 a$$

Apply $\sum F_x = ma_x$ to the second block:

$$F_{1,2} = m_2 a_2 = m_2 a \qquad (1)$$

Add these equations to eliminate $F_{2,1}$ and $F_{1,2}$ and solve for $a = a_1 = a_2$:

$$a = \boxed{\frac{F}{m_1 + m_2}}$$

Substitute your value for a into equation (1) and solve for $F_{1,2}$:

$$F_{1,2} = \boxed{\frac{Fm_2}{m_1 + m_2}}$$

(*b*) Substitute numerical values in the equations derived in part (*a*) and evaluate a and $F_{1,2}$:

$$a = \frac{3.2\,\text{N}}{2\,\text{kg} + 6\,\text{kg}} = \boxed{0.400\,\text{m/s}^2}$$

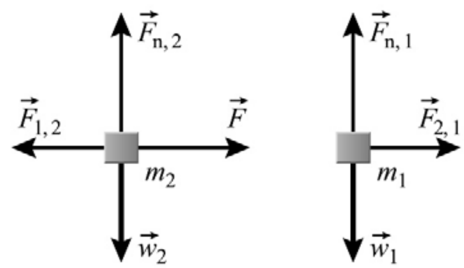
and

$$F_{1,2} = \frac{(3.2\,\text{N})(6\,\text{kg})}{2\,\text{kg} + 6\,\text{kg}} = \boxed{2.40\,\text{N}}$$

Remarks: Note that our results for the acceleration are the same as if the force *F* had acted on a single object whose mass is equal to the sum of the masses of the two blocks. In fact, because the two blocks have the same acceleration, we can consider them to be a single system with mass $m_1 + m_2$.

71 •

Picture the Problem Choose a coordinate system in which the positive x direction is to the right and the positive y direction is upward. Let $\vec{\boldsymbol{F}}_{2,1}$ be the contact force exerted by m_2 on m_1 and $\vec{\boldsymbol{F}}_{1,2}$ be the force exerted by m_1 on m_2. These forces are equal and opposite so $\vec{\boldsymbol{F}}_{2,1} = -\vec{\boldsymbol{F}}_{1,2}$. The free-body diagrams for the blocks are shown. We can apply Newton's 2nd law to each block separately and use the fact that their accelerations are equal.



(*a*) Apply $\sum F_x = ma_x$ to the first block:

$$F - F_{1,2} = m_2 a_2 = m_2 a$$

Apply $\sum F_x = ma_x$ to the second block:

$$F_{2,1} = m_1 a_1 = m_1 a \qquad (1)$$

Add these equations to eliminate $F_{2,1}$ and $F_{1,2}$ and solve for $a = a_1 = a_2$:

$$a = \boxed{\frac{F}{m_1 + m_2}}$$

Substitute your value for a into equation (1) and solve for $F_{2,1}$:

$$F_{2,1} = \boxed{\frac{Fm_1}{m_1 + m_2}}$$

(*b*) Substitute numerical values in the equations derived in part (*a*) and evaluate a and $F_{2,1}$:

$$a = \frac{3.2\,\text{N}}{2\,\text{kg} + 6\,\text{kg}} = \boxed{0.400\,\text{m/s}^2}$$

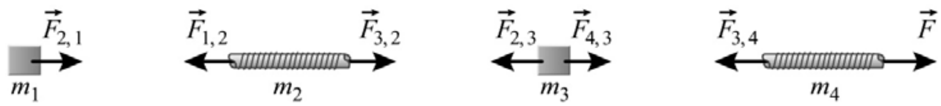
and

$$F_{2,1} = \frac{(3.2\,\text{N})(2\,\text{kg})}{2\,\text{kg} + 6\,\text{kg}} = \boxed{0.800\,\text{N}}$$

Remarks: Note that our results for the acceleration are the same as if the force *F* had acted on a single object whose mass is equal to the sum of the masses of the two blocks. In fact, because the two blocks have the same acceleration, we can consider them to be a single system with mass $m_1 + m_2$.

72 ••

Picture the Problem The free-body diagrams for the boxes and the ropes are below. Because the vertical forces have no bearing on the problem they have not been included. Let the numeral 1 denote the 100-kg box to the left, the numeral 2 the rope connecting the boxes, the numeral 3 the box to the right and the numeral 4 the rope to which the force $\vec{F}$ is applied. $\vec{\boldsymbol{F}}_{3,4}$ is the tension force exerted by m_3 on m_4, $\vec{\boldsymbol{F}}_{4,3}$ is the tension force exerted by m_4 on m_3, $\vec{\boldsymbol{F}}_{2,3}$ is the tension force exerted by m_2 on m_3, $\vec{\boldsymbol{F}}_{3,2}$ is the tension force exerted by m_3 on m_2, $\vec{\boldsymbol{F}}_{1,2}$ is the tension force exerted by m_1 on m_2, and $\vec{\boldsymbol{F}}_{2,1}$ is the tension force exerted by m_2 on m_1. The equal and opposite pairs of forces are $\vec{\boldsymbol{F}}_{2,1} = -\vec{\boldsymbol{F}}_{1,2}$, $\vec{\boldsymbol{F}}_{3,2} = -\vec{\boldsymbol{F}}_{2,3}$, and $\vec{\boldsymbol{F}}_{4,3} = -\vec{\boldsymbol{F}}_{3,4}$. We can apply Newton's 2nd law to each box and rope separately and use the fact that their accelerations are equal.



Apply $\sum\vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$ to the box whose mass is m_1:

$$F_{2,1} = m_1 a_1 = m_1 a \qquad (1)$$

Apply $\sum\vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$ to the rope whose mass is m_2:

$$F_{3,2} - F_{1,2} = m_2 a_2 = m_2 a \qquad (2)$$

Apply $\sum\vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$ to the box whose mass is m_3:

$$F_{4,3} - F_{2,3} = m_3 a_3 = m_3 a \qquad (3)$$

Apply $\sum\vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$ to the rope whose mass is m_4:

$$F - F_{3,4} = m_4 a_4 = m_4 a$$

Add these equations to eliminate $F_{2,1}$, $F_{1,2}$, $F_{3,2}$, $F_{2,3}$, $F_{4,3}$, and $F_{3,4}$ and solve for F:

$$\begin{aligned} F &= (m_1 + m_2 + m_3 + m_4)a \\ &= (202\,\text{kg})(1.0\,\text{m/s}^2) = \boxed{202\,\text{N}} \end{aligned}$$

Use equation (1) to find the tension at point A:

$$F_{2,1} = (100\,\text{kg})(1.0\,\text{m/s}^2) = \boxed{100\,\text{N}}$$

Use equation (2) to find the tension at point B:

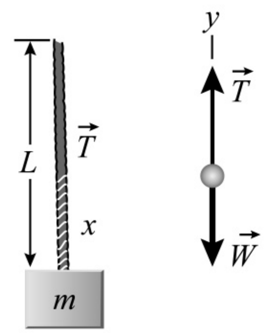
$$\begin{aligned} F_{3,2} &= F_{1,2} + m_2 a \\ &= 100\,\text{N} + (1\,\text{kg})(1.0\,\text{m/s}^2) \\ &= \boxed{101\,\text{N}} \end{aligned}$$

Use equation (3) to find the tension at point C:

$$\begin{aligned} F_{4,3} &= F_{2,3} + m_3 a \\ &= 101\,\text{N} + (100\,\text{kg})(1.0\,\text{m/s}^2) \\ &= \boxed{201\,\text{N}} \end{aligned}$$

73 ••

Picture the Problem Because the distribution of mass in the rope is uniform, we can express the mass m' of a length x of the rope in terms of the total mass of the rope M and its length L. We can then express the total mass that the rope must support at a distance x above the block and use Newton's 2nd law to find the tension as a function of x.



Set up a proportion expressing the mass m' of a length x of the rope as a function of M and L and solve for m':

$$\frac{m'}{x}=\frac{M}{L}\Rightarrow m'=\frac{M}{L}x$$

Express the total mass that the rope must support at a distance x above the block:

$$m+m'=m+\frac{M}{L}x$$

Apply $\sum F_y = ma_y$ to the block and a length x of the rope:

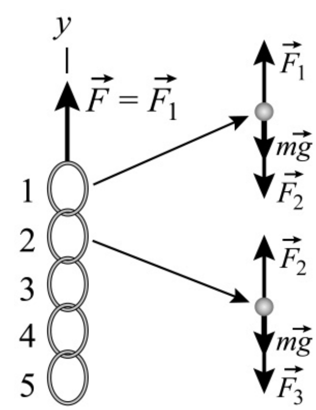
$$T-w=T-\left(m+\frac{M}{L}x\right)g$$
$$=\left(m+\frac{M}{L}x\right)a$$

Solve for T to obtain:

$$T=\boxed{(a+g)\left(m+\frac{M}{L}x\right)}$$

***74 ••**

Picture the Problem Choose a coordinate system with the positive y direction upward and denote the top link with the numeral 1, the second with the numeral 2, etc.. The free-body diagrams show the forces acting on links 1 and 2. We can apply Newton's 2nd law to each link to obtain a system of simultaneous equations that we can solve for the force each link exerts on the link below it. Note that the net force on each link is the product of its mass and acceleration.



(*a*) Apply $\sum F_y = ma_y$ to the top link and solve for F:

$$F-5mg=5ma$$
and
$$F=5m(g+a)$$

Substitute numerical values and evaluate F:

$$F = 5(0.1\,\text{kg})(9.81\,\text{m/s}^2 + 2.5\,\text{m/s}^2) = \boxed{6.16\,\text{N}}$$

(*b*) Apply $\sum F_y = ma_y$ to a single link:

$$F_{1\,\text{link}} = m_{1\,\text{link}}a = (0.1\,\text{kg})(2.5\,\text{m/s}^2) = \boxed{0.250\,\text{N}}$$

(*c*) Apply $\sum F_y = ma_y$ to the 1st through 5th links to obtain:

$$F - F_2 - mg = ma\,, \quad (1)$$
$$F_2 - F_3 - mg = ma\,, \quad (2)$$
$$F_3 - F_4 - mg = ma\,, \quad (3)$$
$$F_4 - F_5 - mg = ma\,, \text{ and} \quad (4)$$
$$F_5 - mg = ma \quad (5)$$

Add equations (2) through (5) to obtain:

$$F_2 - 4mg = 4ma$$

Solve for F_2 to obtain:

$$F_2 = 4mg + 4ma = 4m(a + g)$$

Substitute numerical values and evaluate F_2:

$$F_2 = 4(0.1\,\text{kg})(9.81\,\text{m/s}^2 + 2.5\,\text{m/s}^2) = \boxed{4.92\,\text{N}}$$

Substitute for F_2 to find F_3, and then substitute for F_3 to find F_4:

$$F_3 = \boxed{3.69\,\text{N}} \text{ and } F_4 = \boxed{2.46\,\text{N}}$$

Solve equation (5) for F_5:

$$F_5 = m(g + a)$$

Substitute numerical values and evaluate F_5:

$$F_5 = (0.1\,\text{kg})(9.81\,\text{m/s}^2 + 2.5\,\text{m/s}^2) = \boxed{1.23\,\text{N}}$$

75 •

Picture the Problem A *net* force is required to accelerate the object. In this problem the net force is the difference between $\vec{T}$ and $\vec{W}(= m\vec{g})$. The free-body diagram of the object is shown to the right. Choose a coordinate system in which the upward direction is positive.

Apply $\sum \vec{F} = m\vec{a}$ to the object to obtain:

$$F_{\text{net}} = T - W = T - mg$$

Solve for the tension in the lower portion of the rope:

$$T = F_{\text{net}} + mg = ma + mg = m(a + g)$$

Using its definition, find the acceleration of the object:

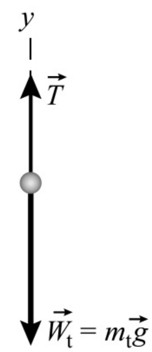
$a \equiv \Delta v/\Delta t = (3.5\text{ m/s})/(0.7\text{ s})$
$= 5.00\text{ m/s}^2$

Substitute numerical values and evaluate T:

$T = (40\text{ kg})(5.00\text{ m/s}^2 + 9.81\text{ m/s}^2)$
$= 592\text{ N}$ and $\boxed{(a)\text{ is correct.}}$

76 •

Picture the Problem A net force in the downward direction is required to accelerate the truck downward. The net force is the difference between $\vec{\boldsymbol{W}}_\text{t}$ and $\vec{\boldsymbol{T}}$. A free-body diagram showing these forces acting on the truck is shown to the right. Choose a coordinate system in which the downward direction is positive.



Apply $\sum F_y = ma_y$ to the truck to obtain:

$$T - m_\text{t}g = m_\text{t}a_y$$

Solve for the tension in the lower portion of the cable:

$$T = m_\text{t}g + m_\text{t}a_y = m_\text{t}\left(g + a_y\right)$$

Substitute to find the tension in the rope:

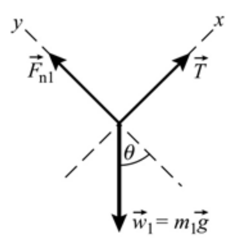
$$T = m_\text{t}\left(g - 0.1g\right) = 0.9m_\text{t}g$$

and $\boxed{(c)\text{ is correct.}}$

77 ••

Picture the Problem Because the string does not stretch or become slack, the two objects must have the same speed and therefore the magnitude of the acceleration is the same for each object. Choose a coordinate system in which up the incline is the positive x direction for the object of mass m_1 and downward is the positive x direction for the object of mass m_2. This idealized pulley acts like a piece of polished pipe; i.e., its only function is to change the direction the tension in the massless string acts. Draw a free-body diagram for each of the two objects, apply Newton's 2nd law of motion to both objects, and solve the resulting equations simultaneously.

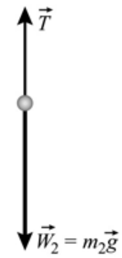
(*a*) Draw the FBD for the object of mass m_1:



Apply $\sum F_x = ma_x$ to the object whose mass is m_1:

$$T - m_1 g\sin\theta = m_1 a$$

Draw the FBD for the object of mass m_2:



Apply $\sum F_x = ma_x$ to the object whose mass is m_2:

$$m_2 g - T = m_2 a$$

Add the two equations and solve for a:

$$a = \boxed{\frac{g(m_2 - m_1 \sin\theta)}{m_1 + m_2}}$$

Substitute for a in either of the equations containing the tension and solve for T:

$$T = \boxed{\frac{g m_1 m_2 (1 + \sin\theta)}{m_1 + m_2}}$$

(b) Substitute the given values into the expression for a:

$$a = \boxed{2.45\,\text{m/s}^2}$$

Substitute the given data into the expression for T:

$$T = \boxed{36.8\,\text{N}}$$

78 •

Picture the Problem The magnitude of the accelerations of Peter and the counterweight are the same. Choose a coordinate system in which up the incline is the positive x direction for the counterweight and downward is the positive x direction for Peter. The pulley changes the direction the tension in the rope acts. Let Peter's mass be m_P. Ignoring the mass of the rope, draw free-body diagrams for the counterweight and Peter, apply Newton's 2[nd] law to each of them, and solve the resulting equations simultaneously.

(a) Using a constant-acceleration equation, relate Peter's displacement to her acceleration and descent time:

$$\Delta x = v_0 \Delta t + \tfrac{1}{2} a (\Delta t)^2$$

or, because $v_0 = 0$,

$$\Delta x = \tfrac{1}{2} a (\Delta t)^2$$

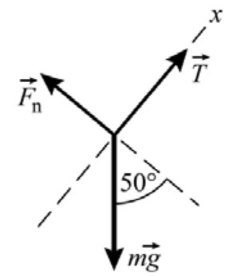
Solve for the common acceleration of Peter and the counterweight:

$$a = \frac{2\Delta x}{(\Delta t)^2}$$

Substitute numerical values and evaluate a:

$$a = \frac{2(3.2\,\text{m})}{(2.2\,\text{s})^2} = 1.32\,\text{m/s}^2$$

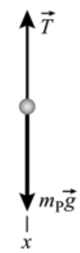
Draw the FBD for the counterweight:



Apply $\sum F_x = ma_x$ to the counterweight:

$T - mg\sin 50° = ma$

Draw the FBD for Peter:



Apply $\sum F_x = ma_x$ to Peter:

$m_Pg - T = m_Pa$

Add the two equations and solve for m:

$$m = \frac{m_P(g-a)}{a + g\sin 50°}$$

Substitute numerical values and evaluate m:

$$m = \frac{(50\,\text{kg})(9.81\,\text{m/s}^2 - 1.32\,\text{m/s}^2)}{1.32\,\text{m/s}^2 + (9.81\,\text{m/s}^2)\sin 50°} = \boxed{48.0\,\text{kg}}$$

(*b*) Substitute for m in the force equation for the counterweight and solve for T:

$$T = m(a + g\sin 50°)$$

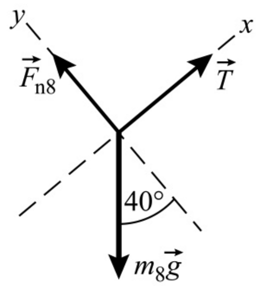
(*b*) Substitute numerical values and evaluate T:

$$T = (48.0\,\text{kg})\left[1.32\,\text{m/s}^2 + (9.81\,\text{m/s}^2)\sin 50°\right] = \boxed{424\,\text{N}}$$

79 ••

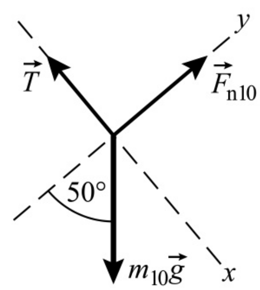
Picture the Problem The magnitude of the accelerations of the two blocks are the same. Choose a coordinate system in which up the incline is the positive x direction for the 8-kg object and downward is the positive x direction for the 10-kg object. The peg changes the direction the tension in the rope acts. Draw free-body diagrams for each object, apply Newton's 2nd law of motion to both of them, and solve the resulting equations simultaneously.

(a) Draw the FBD for the 3-kg object:



Apply $\sum F_x = ma_x$ to the 3-kg block: $T - m_8 g \sin 40° = m_3 a$

Draw the FBD for the 10-kg object:



Apply $\sum F_x = ma_x$ to the 10-kg block: $m_{10} g \sin 50° - T = m_{10} a$

Add the two equations and solve for and evaluate a:

$$a = \frac{g\left(m_{10}\sin 50° - m_8 \sin 40°\right)}{m_8 + m_{10}}$$
$$= \boxed{1.37\,\text{m/s}^2}$$

Substitute for a in the first of the two force equations and solve for T:

$$T = m_8 g \sin 40° + m_8 a$$

Substitute numerical values and evaluate T:

$$T = (8\,\text{kg})\left[\left(9.81\,\text{m/s}^2\right)\sin 40° + 1.37\,\text{m/s}^2\right]$$
$$= \boxed{61.4\,\text{N}}$$

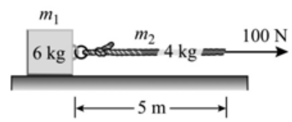
(b) Because the system is in equilibrium, set $a = 0$, express the force equations in terms of m_1 and m_2, add the two force equations, and solve for and evaluate the ratio m_1/m_2:

$T - m_1 g \sin 40° = 0$
$m_2 g \sin 50° - T = 0$
$\therefore\ m_2 g \sin 50° - m_1 g \sin 40° = 0$
and

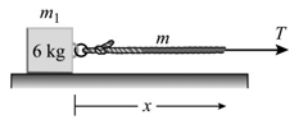
$$\frac{m_1}{m_2} = \frac{\sin 50°}{\sin 40°} = \boxed{1.19}$$

80 ••

Picture the Problem The pictorial representations shown to the right summarize the information given in this problem. While the mass of the rope is distributed over its length, the rope and the 6-kg block have a common acceleration. Choose a coordinate system in which the direction of the 100-N force is the positive *x* direction. Because the surface is horizontal and frictionless, the only force that influences our solution is the 100-N force.



Part (a)



Part (b)

(*a*) Apply $\sum F_x = ma_x$ to the objects shown for part (*a*):

$100\text{ N} = (m_1 + m_2)a$

Solve for *a* to obtain:

$$a = \frac{100\,\text{N}}{m_1 + m_2}$$

Substitute numerical values and evaluate *a*:

$$a = \frac{100\,\text{N}}{10\,\text{kg}} = \boxed{10.0\,\text{m/s}^2}$$

(*b*) Let *m* represent the mass of a length *x* of the rope. Assuming that the mass of the rope is uniformly distributed along its length:

$$\frac{m}{x} = \frac{m_2}{L_{\text{rope}}} = \frac{4\,\text{kg}}{5\,\text{m}}$$

and

$$m = \left(\frac{4\,\text{kg}}{5\,\text{m}}\right)x$$

Let *T* represent the tension in the rope at a distance *x* from the point at which it is attached to the 6-kg block. Apply $\sum F_x = ma_x$ to the system shown for part (*b*) and solve for *T*:

$$T = (m_1 + m)a$$
$$= \left[6\,\text{kg} + \left(\frac{4\,\text{kg}}{5\,\text{m}}\right)x\right]\left(10\,\text{m/s}^2\right)$$
$$= \boxed{60\,\text{N} + (8\,\text{N/m})x}$$

***81** ••

Picture the Problem Choose a coordinate system in which upward is the positive *y* direction and draw the free-body diagram for the frame-plus- painter. Noting that $\vec{F} = -\vec{T}$, apply Newton's 2[nd] law of motion.

(*a*) Letting $m_{\text{tot}} = m_{\text{frame}} + m_{\text{painter}}$,

$2T - m_{\text{tot}}g = m_{\text{tot}}a$

and

apply $\sum F_y = ma_y$ to the frame-plus-painter and solve T:

$$T = \frac{m_{\text{tot}}(a+g)}{2}$$

Substitute numerical values and evaluate T:

$$T = \frac{(75\,\text{kg})(0.8\,\text{m/s}^2 + 9.81\,\text{m/s}^2)}{2} = 398\,\text{N}$$

Because $F = T$:

$$F = \boxed{398\,\text{N}}$$

(b) Apply $\sum F_y = ma_y$ with $a = 0$ to obtain:

$$2T - m_{\text{tot}}g = 0$$

Solve for T:

$$T = \tfrac{1}{2} m_{\text{tot}} g$$

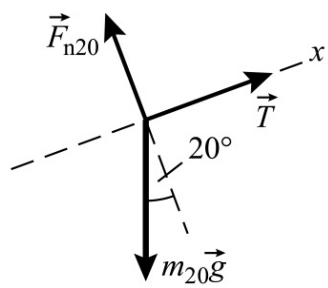
Substitute numerical values and evaluate T:

$$T = \tfrac{1}{2}(75\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{368\,\text{N}}$$

82 •••

Picture the Problem Choose a coordinate system in which up the incline is the positive x direction and draw free-body diagrams for each block. Noting that $\vec{a}_{20} = -\vec{a}_{10}$, apply Newton's 2nd law of motion to each block and solve the resulting equations simultaneously.

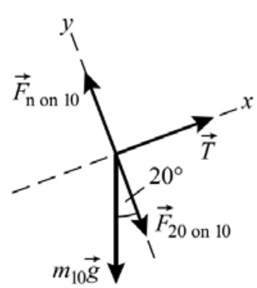
Draw a FBD for the 20-kg block:



Apply $\sum F_x = ma_x$ to the block to obtain:

$$T - m_{20}g\sin 20° = m_{20}a_{20}$$

Draw a FBD for the 10-kg block. Because all the surfaces, including the surfaces between the blocks, are frictionless, the force the 20-kg block exerts on the 10-kg block must be normal to their surfaces as shown to the right.



Apply $\sum F_x = ma_x$ to the block to obtain:

$$T - m_{10}g\sin 20° = m_{10}a_{10}$$

Because the blocks are connected by a taut string: $a_{20} = -a_{10}$

Substitute for a_{20} and eliminate T between the two equations to obtain:

$$a_{10} = \boxed{1.12\,\text{m/s}^2}$$

and

$$a_{20} = \boxed{-1.12\,\text{m/s}^2}$$

Substitute for either of the accelerations in the force equations and solve for T:

$$T = \boxed{44.8\,\text{N}}$$

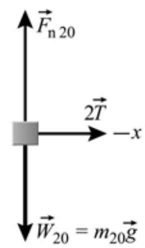
83 •••

Picture the Problem Choose a coordinate system in which the positive x direction is to the right and draw free-body diagrams for each block. Because of the pulley, the string exerts a force of $2T$. Apply Newton's 2nd law of motion to both blocks and solve the resulting equations simultaneously.

(a) Noting the effect of the pulley, express the distance the 20-kg block moves in a time Δt:

$$\Delta x_{20} = \tfrac{1}{2}\Delta x_5 = \tfrac{1}{2}(10\,\text{cm}) = \boxed{5.00\,\text{cm}}$$

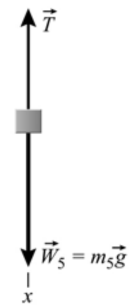
(b) Draw a FBD for the 20-kg block:



Apply $\sum F_x = ma_x$ to the block to obtain:

$$2T = m_{20}a_{20}$$

Draw a FBD for the 5-kg block:



Apply $\sum F_x = ma_x$ to the block to obtain:

$$m_5 g - T = m_5 a_5$$

Using a constant-acceleration equation, relate the displacement of the 5-kg block to its acceleration

$$\Delta x_5 = \tfrac{1}{2}a_5(\Delta t)^2$$

and the time during which it is accelerated:

Using a constant-acceleration equation, relate the displacement of the 20-kg block to its acceleration and the time during which it is accelerated:

$$\Delta x_{20} = \tfrac{1}{2}a_{20}(\Delta t)^2$$

Divide the first of these equations by the second to obtain:

$$\frac{\Delta x_5}{\Delta x_{20}} = \frac{\tfrac{1}{2}a_5(\Delta t)^2}{\tfrac{1}{2}a_{20}(\Delta t)^2} = \frac{a_5}{a_{20}}$$

Use the result of part (*a*) to obtain:

$$a_5 = 2a_{20}$$

Let $a_{20} = a$. Then $a_5 = 2a$ and the force equations become:

$$2T = m_{20}a$$

and

$$m_5 g - T = m_5(2a)$$

Eliminate T between the two equations to obtain:

$$a = a_{20} = \frac{m_5 g}{2m_5 + \tfrac{1}{2}m_{20}}$$

Substitute numerical values and evaluate a_{20} and a_5:

$$a_{20} = \frac{(5\,\text{kg})(9.81\,\text{m/s}^2)}{2(5\,\text{kg}) + \tfrac{1}{2}(20\,\text{kg})} = \boxed{2.45\,\text{m/s}^2}$$

and

$$a_5 = 2(2.45\,\text{m/s}^2) = \boxed{4.91\,\text{m/s}^2}$$

Substitute for either of the accelerations in either of the force equations and solve for T:

$$T = \boxed{24.5\,\text{N}}$$

Free-Body Diagrams: The Atwood's Machine

***84 ••**

Picture the Problem Assume that $m_1 > m_2$. Choose a coordinate system in which the positive y direction is downward for the block whose mass is m_1 and upward for the block whose mass is m_2 and draw free-body diagrams for each block. Apply Newton's 2nd law of motion to both blocks and solve the resulting equations simultaneously.

Draw a FBD for the block whose mass is m_2:

Apply $\sum F_y = ma_y$ to this block: $T - m_2g = m_2a_2$

Draw a FBD for the block whose mass is m_1:

Apply $\sum F_y = ma_y$ to this block: $m_1g - T = m_1a_1$

Because the blocks are connected by a taut string, let a represent their common acceleration: $a = a_1 = a_2$

Add the two force equations to eliminate T and solve for a:

$$m_1g - m_2g = m_1a + m_2a$$

and

$$\boxed{a = \frac{m_1 - m_2}{m_1 + m_2}g}$$

Substitute for a in either of the force equations and solve for T:

$$\boxed{T = \frac{2m_1m_2g}{m_1 + m_2}}$$

85 ••

Picture the Problem The acceleration can be found from the given displacement during the first second. The ratio of the two masses can then be found from the acceleration using the first of the two equations derived in Problem 89 relating the acceleration of the Atwood's machine to its masses.

Using a constant-acceleration equation, relate the displacement of the masses to their acceleration and solve for the acceleration:

$$\Delta y = v_0t + \tfrac{1}{2}a(\Delta t)^2$$

or, because $v_0 = 0$,

$$\Delta y = \tfrac{1}{2}a(\Delta t)^2$$

Solve for and evaluate a:

$$a = \frac{2\Delta y}{(\Delta t)^2} = \frac{2(0.3\,\text{m})}{(1\,\text{s})^2} = 0.600\,\text{m/s}^2$$

Solve for m_1 in terms of m_2 using the first of the two equations given in Problem 84:

$$m_1 = m_2\frac{g + a}{g - a} = \frac{10.41\,\text{m/s}^2}{9.21\,\text{m/s}^2}m_2 = 1.13m_2$$

Find the second mass for m_2 or $m_1 = 1.2$ kg:

$$m_{\text{2nd mass}} = \boxed{1.36\,\text{kg or }1.06\,\text{kg}}$$

86 ••

Picture the Problem Let F_{nm} be the force the block of mass m_2 exerts on the pebble of mass m. Because $m_2 < m_1$, the block of mass m_2 accelerates upward. Draw a free-body diagram for the pebble and apply Newton's 2nd law and the acceleration equation given in Problem 84.

Apply $\sum F_y = ma_y$ to the pebble: $F_{nm} - mg = ma$

Solve for F_{nm}: $F_{nm} = m(a+g)$

From Problem 84: $$a = \frac{m_1 - m_2}{m_1 + m_2} g$$

Substitute for a and simplify to obtain: $$F_{nm} = m\left(\frac{m_1 - m_2}{m_1 + m_2} g + g\right) = \boxed{\frac{2m_1 m}{m_1 + m_2} g}$$

87 ••

Picture the Problem Note from the free-body diagrams for Problem 89 that the net force exerted by the accelerating blocks is $2T$. Use this information, together with the expression for T given in Problem 84, to derive an expression for $F = 2T$.

From Problem 84 we have: $$T = \frac{2m_1 m_2 g}{m_1 + m_2}$$

The net force, F, exerted by the Atwood's machine on the hanger is: $$F = 2T = \boxed{\frac{4m_1 m_2 g}{m_1 + m_2}}$$

If $m_1 = m_2 = m$, then: $$F = \frac{4m^2 g}{2m} = 2mg \text{ … as expected.}$$

If either m_1 or $m_2 = 0$, then: $F = 0$ … also as expected.

88 •••

Picture the Problem Use a constant-acceleration equation to relate the displacement of the descending (or rising) mass as a function of its acceleration and then use one of the results from Problem 84 to relate a to g. Differentiation of our expression for g will allow us to relate uncertainty in the time measurement to uncertainty in the measured value for g … and to the values of m_2 that would yield an experimental value for g that is good to within 5%.

(*a*) Use the result given in Problem 84 to express g in terms of a:

$$g = a\frac{m_1 + m_2}{m_1 - m_2} \qquad (1)$$

Using a constant-acceleration equation, express the displacement, L, as a function of t and solve for the acceleration:

$$\Delta y = L = v_0\Delta t + \tfrac{1}{2}a(\Delta t)^2$$

or, because $v_0 = 0$ and $\Delta t = t$,

$$a = \frac{2L}{t^2} \qquad (2)$$

Substitute this expression for a:

$$g = \boxed{\frac{2L}{t^2}\left(\frac{m_1 + m_2}{m_1 - m_2}\right)}$$

(*b*) Evaluate dg/dt to obtain:

$$\frac{dg}{dt} = -4Lt^{-3}\left(\frac{m_1 + m_2}{m_1 - m_2}\right) = \frac{-2}{t}\left[\frac{2L}{t^2}\right]\left(\frac{m_1 + m_2}{m_1 - m_2}\right) = \frac{-2g}{t}$$

Divide both sides of this expression by g and multiply both sides by dt:

$$\boxed{\frac{dg}{g} = -2\frac{dt}{t}}$$

(*c*) We have:

$$\frac{dg}{g} = \pm 0.05 \text{ and } \frac{dt}{t} = \pm 0.025$$

Solve the second of these equations for t to obtain:

$$t = \frac{dt}{0.025} = \frac{1\,\text{s}}{0.025} = 4\,\text{s}$$

Substitute in equation (2) to obtain:

$$a = \frac{2(3\,\text{m})}{(4\,\text{s})^2} = 0.375\,\text{m/s}^2$$

Solve equation (1) for m_2 to obtain:

$$m_2 = \frac{g - a}{g + a}m_1$$

Evaluate m_2 with $m_1 = 1$ kg:

$$m_2 = \frac{9.81\,\text{m/s}^2 - 0.375\,\text{m/s}^2}{9.81\,\text{m/s}^2 + 0.375\,\text{m/s}^2}(1\,\text{kg}) = 0.926\,\text{kg}$$

Solve equation (1) for m_1 to obtain:

$$m_1 = m_2\frac{g + a}{g - a}$$

Substitute numerical values to obtain:

$$m_1 = (0.926\,\text{kg})\frac{9.81\,\text{m/s}^2 + 0.375\,\text{m/s}^2}{9.81\,\text{m/s}^2 - 0.375\,\text{m/s}^2} = 1.08\,\text{kg}$$

Because the masses are interchangeable: $m_2 = \boxed{0.926\,\text{kg or } 1.08\,\text{kg}}$

***89** ••

Picture the Problem We can reason to this conclusion as follows: In the two extreme cases when the mass on one side or the other is zero, the tension is zero as well, because the mass is in free-fall. By symmetry, the maximum tension must occur when the masses on each side are equal. An alternative approach that is shown below is to treat the problem as an extreme-value problem.

Express m_2 in terms of M and m_1: $m_2 = M - m_1$

Substitute in the equation from Problem 84 and simplify to obtain:

$$T = \frac{2gm_1(M - m_1)}{m_1 + M - m_1} = 2g\left(m_1 - \frac{m_1^2}{M}\right)$$

Differentiate this expression with respect to m_1 and set the derivative equal to zero for extreme values:

$$\frac{dT}{dm_1} = 2g\left(1 - \frac{2m_1}{M}\right) = 0 \text{ for extreme values}$$

Solve for m_1 to obtain: $m_1 = \frac{1}{2}M$

Show that $m_1 = M/2$ is a maximum value by evaluating the second derivative of T with respect to m_1 at $m_1 = M/2$:

$$\frac{d^2T}{dm_1^2} = -\frac{4g}{M} < 0, \text{ independently of } m_1$$

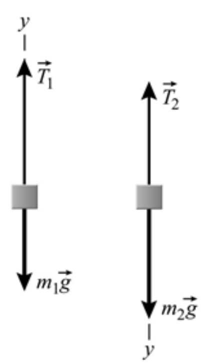
and we have shown that

$\boxed{T \text{ is a maximum when } m_1 = m_2 = \tfrac{1}{2}M.}$

Remarks: An alternative solution is to use a graphing calculator to show that *T* as a function of m_1 is concave downward and has its maximum value when $m_1 = m_2 = M/2$.

90 •••

Picture the Problem The free-body diagrams show the forces acting on the objects whose masses are m_1 and m_2. The application of Newton's 2nd law to these forces and the accelerations the net forces are responsible for will lead us to an expression for the tension in the string as a function of m_1 and m_2. Examination of this expression as for $m_2 >> m_1$ will yield the predicted result.



(*a*) Apply $\sum F_y = ma_y$ to the objects whose masses are m_1 and m_2 to obtain:

$$T_1 - m_1g = m_1a_1$$

and

$$m_2g - T_2 = m_2a_2$$

Assume that the role of the pulley is simply to change the direction the tension acts. Then $T_1 = T_2 = T$. Because the two objects have a common acceleration, let $a = a_1 = a_2$. Eliminate a between the two equations and solve for T to obtain:

$$T = \frac{2m_1 m_2}{m_1 + m_2} g$$

Divide the numerator and denominator of this fraction by m_2:

$$T = \frac{2m_1 g}{1 + \frac{m_1}{m_2}}$$

Take the limit of this fraction as $m_2 \to \infty$ to obtain:

$$T = \boxed{2m_1 g}$$

(*b*) Imagine the situation when $m_2 >> m_1$:

Under these conditions, the object whose mass is m_2 is essentially in free-fall, so the object whose mass is m_1 is accelerating *upward* with an acceleration of magnitude g.

Under these conditions, the net force acting on the object whose mass is m_1 is $m_1 g$ and:

$T - m_1 g = m_1 g \Rightarrow T = 2m_1 g$.
Note that this result agrees with that obtained using more analytical methods.

General Problems

91 •

Picture the Problem Choose a coordinate system in which the force the tree exerts on the woodpecker's head is in the negative-x direction and determine the acceleration of the woodpecker's head from Newton's 2nd law of motion. The depth of penetration, under the assumption of constant acceleration, can be determined using a constant- acceleration equation. Knowing the acceleration of the woodpecker's head and the depth of penetration of the tree, we can calculate the time required to bring the head to rest.

(*a*) Apply $\sum F_x = ma_x$ to the woodpecker's head to obtain:

$$a_x = \frac{\sum F_x}{m} = \frac{-6\,\text{N}}{0.060\,\text{kg}} = \boxed{-100\,\text{m/s}^2}$$

(*b*) Using a constant-acceleration equation, relate the depth-of-penetration into the bark to the acceleration of the woodpecker's head:

$$v^2 = v_0^2 + 2a\Delta x$$

or, because $v = 0$,

$$0 = v_0^2 + 2a\Delta x$$

Solve for and evaluate Δx:

$$\Delta x = \frac{-v_0^2}{2a} = \frac{-(3.5\,\text{m/s})^2}{2(-100\,\text{m/s}^2)} = \boxed{6.13\,\text{cm}}$$

(*c*) Use the definition of acceleration to express the time required for the woodpecker's head to come to rest:

$$\Delta t = \frac{v - v_0}{a}$$

or, because $v = 0$,

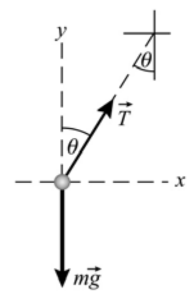
$$\Delta t = \frac{v - v_0}{a}$$

Substitute numerical values and evaluate Δt:

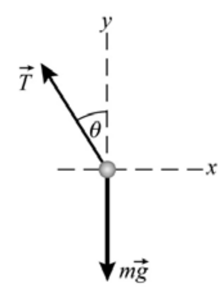
$$\Delta t = \frac{-v_0}{a} = \frac{-3.5\,\text{m/s}}{-100\,\text{m/s}^2} = \boxed{35.0\,\text{ms}}$$

***92 ••**

Picture the Problem The free-body diagram shown to the right shows the forces acting on an object suspended from the ceiling of a car that is accelerating to the right. Choose the coordinate system shown and use Newton's laws of motion and constant- acceleration equations in the determination of the influence of the forces on the behavior of the suspended object.



The second free-body diagram shows the forces acting on an object suspended from the ceiling of a car that is braking while it moves to the right.



(*a*) $\boxed{\text{In accordance with Newton's law of inertia, the object's displacement will be in the direction opposite that of the acceleration.}}$

(*b*) Resolve the tension, T, into its components and apply $\sum \vec{F} = m\vec{a}$ to the object:

$$\Sigma F_x = T\sin\theta = ma$$

and

$$\Sigma F_y = T\cos\theta - mg = 0$$

Take the ratio of these two equations to eliminate T and m:

$$\frac{T\sin\theta}{T\cos\theta} = \frac{ma}{mg}$$

or

$$\tan\theta = \frac{a}{g} \Rightarrow \boxed{a = g\tan\theta}$$

(*c*) $\boxed{\text{Because the acceleration is opposite the direction the car is moving, the accelerometer will swing forward.}}$

Using a constant-acceleration equation, express the velocity of the car in terms of its acceleration and solve for the acceleration:

$$v^2 = v_0^2 + 2a\Delta x$$

or, because $v = 0$,

$$0 = v_0^2 + 2a\Delta x$$

Solve for a:

$$a = \frac{-v_0^2}{2\Delta x}$$

Substitute numerical values and evaluate a:

$$a = \frac{-(50\,\text{km/h})^2}{2(60\,\text{m})} = \boxed{-1.61\,\text{m/s}^2}$$

Solve the equation derived in (b) for θ:

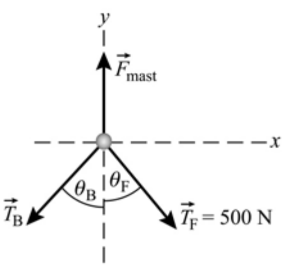
$$\theta = \tan^{-1}\left(\frac{a}{g}\right)$$

Substitute numerical values and evaluate θ:

$$\theta = \tan^{-1}\left(\frac{1.61\,\text{m/s}^2}{9.81\,\text{m/s}^2}\right) = \boxed{9.32°}$$

93 ••

Picture the Problem The free-body diagram shows the forces acting at the top of the mast. Choose the coordinate system shown and use Newton's 2nd and 3rd laws of motion to analyze the forces acting on the deck of the sailboat.



Apply $\sum F_x = ma_x$ to the top of the mast:

$$T_F \sin\theta_F - T_B \sin\theta_B = 0$$

Find the angles that the forestay and backstay make with the vertical:

$$\theta_F = \tan^{-1}\left(\frac{3.6\,\text{m}}{12\,\text{m}}\right) = 16.7°$$

and

$$\theta_B = \tan^{-1}\left(\frac{6.4\,\text{m}}{12\,\text{m}}\right) = 28.1°$$

Solve the x-direction equation for T_B:

$$T_B = T_F \frac{\sin\theta_F}{\sin\theta_B} = (500\,\text{N})\frac{\sin 16.7°}{\sin 28.1°}$$
$$= \boxed{305\,\text{N}}$$

Find the downward forces that T_B and T_F exert on the mast:

$$\sum F_y = F_{\text{mast}} - T_F \cos\theta_F - T_B \cos\theta_B = 0$$

Solve for F_{mast} to obtain:

$$F_{\text{mast}} = T_F \cos\theta_F + T_B \cos\theta_B$$

Substitute numerical values and evaluate F_{mast}:

$$F_{\text{mast}} = (500\,\text{N})\cos 16.7° + (305\,\text{N})\cos 28.1° = 748\,\text{N}$$

The force that the mast exerts on the deck is the sum of its weight and the downward forces exerted on it by the forestay and backstay:

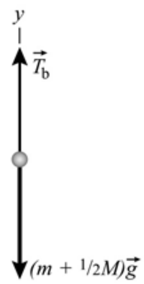
$$F_{\text{mast on the deck}} = 748\,\text{N} + 800\,\text{N} = \boxed{1.55\,\text{kN}}$$

94 ••

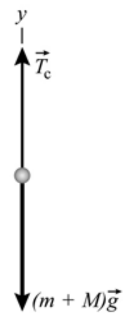
Picture the Problem Let m be the mass of the block and M be the mass of the chain. The free-body diagrams shown below display the forces acting at the locations identified in the problem. We can apply Newton's 2nd law with $a_y = 0$ to each of the segments of the chain to determine the tensions.

(*a*)

(*b*)



(*c*)



(*a*) Apply $\sum F_y = ma_y$ to the block and solve for T_a:

$$T_a - mg = ma_y$$

or, because $a_y = 0$,

$$T_a = mg$$

Substitute numerical values and evaluate T_a:

$$T_a = (50\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{491\,\text{N}}$$

(*b*) Apply $\sum F_y = ma_y$ to the block and half the chain and solve for T_b:

$$T_b - \left(m + \frac{M}{2}\right)g = ma_y$$

or, because $a_y = 0$,

$$T_b = \left(m + \frac{M}{2}\right)g$$

Substitute numerical values and evaluate T_b:

$$T_b = (50\,\text{kg} + 10\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{589\,\text{N}}$$

(*c*) Apply $\sum F_y = ma_y$ to the block and chain and solve for T_c:

$$T_c - (m + M)g = ma_y$$

or, because $a_y = 0$,

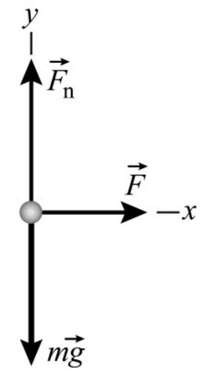
$$T_c = (m + M)g$$

Substitute numerical values and evaluate T_c:

$$T_c = (50\,\text{kg} + 20\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{687\,\text{N}}$$

***95** •••

Picture the Problem The free-body diagram shows the forces acting on the box as the man pushes it across a frictionless floor. Because the force is time-dependent, the acceleration will be, too. We can obtain the acceleration as a function of time from the application of Newton's 2nd law and then find the velocity of the box as a function of time by integration. Finally, we can derive an expression for the displacement of the box as a function of time by integration of the velocity function.



(*a*) The velocity is related to the acceleration according to:

$$\frac{dv}{dt} = a(t) \qquad (1)$$

Apply $\sum F_x = ma_x$ to the box and solve for its acceleration:

$$F = ma$$

and

$$a = \frac{F}{m} = \frac{(8\,\text{N/s})t}{24\,\text{kg}} = \left(\tfrac{1}{3}\,\text{m/s}^3\right)t$$

Because the box's acceleration is a function of time, separate variables in equation (1) and integrate to find v as a function of time:

$$v(t) = \int_0^t a(t')dt' = \left(\tfrac{1}{3}\,\text{m/s}^3\right)\int_0^t t'\,dt' = \left(\tfrac{1}{3}\,\text{m/s}^3\right)\frac{t^2}{2} = \left(\tfrac{1}{6}\,\text{m/s}^3\right)t^2$$

Evaluate v at $t = 3$ s:

$$v(3\,\text{s}) = \left(\tfrac{1}{6}\,\text{m/s}^3\right)(3\,\text{s})^2 = \boxed{1.50\,\text{m/s}}$$

(*b*) Integrate $v = dx/dt$ between 0 and 3 s to find the displacement of the box during this time:

$$\Delta x = \int_0^{3\,\text{s}} v(t')dt' = \left(\tfrac{1}{6}\,\text{m/s}^3\right)\int_0^{3\,\text{s}} t'^2\,dt' = \left[\left(\tfrac{1}{6}\,\text{m/s}^3\right)\frac{t'^3}{3}\right]_0^{3\,\text{s}} = \boxed{1.50\,\text{m}}$$

(*c*) The average velocity is given by:

$$v_\text{ave} = \frac{\Delta x}{\Delta t} = \frac{1.5\,\text{m}}{3\,\text{s}} = \boxed{0.500\,\text{m/s}}$$

(*d*) Use Newton's 2nd law to express the average force exerted on the box by the man:

$$F_\text{av} = ma_\text{av} = m\frac{\Delta v}{\Delta t}$$

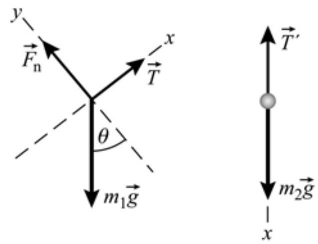
Substitute numerical values and evaluate F_{av}:

$$F_{av} = (24\,\text{kg})\frac{1.5\,\text{m/s} - 0\,\text{m/s}}{3\,\text{s}} = \boxed{12.0\,\text{N}}$$

96 ••

Picture the Problem The application of Newton's 2nd law to the glider and the hanging weight will lead to simultaneous equations in their common acceleration *a* and the tension *T* in the cord that connects them. Once we know the acceleration of this system, we can use a constant-acceleration equation to predict how long it takes the cart to travel 1 m from rest. Note that the magnitudes of $\vec{T}$ and $\vec{T}'$ are equal.

(*a*) The free-body diagrams are shown to the right. m_1 represents the mass of the cart and m_2 the mass of the hanging weight.



(*b*) Apply $\sum F_x = ma_x$ to the cart and the suspended mass:

$$T - m_1 g\sin\theta = m_1 a_1$$

and

$$m_2 g - T = m_2 a_2$$

Letting *a* represent the common accelerations of the two objects, eliminate *T* between the two equations and solve *a*:

$$a = \frac{m_2 - m_1\sin\theta}{m_1 + m_2}g$$

Substitute numerical values and evaluate *a*:

$$a = \frac{0.075\,\text{kg} - (0.270\,\text{kg})\sin 30°}{0.075\,\text{kg} + 0.270\,\text{kg}} \times (9.81\,\text{m/s}^2) = \boxed{-1.71\,\text{m/s}^2}$$

i.e., the acceleration is down the incline.

Substitute for *a* in either of the force equations to obtain:

$$T = \boxed{0.863\,\text{N}}$$

(*c*) Using a constant-acceleration equation, relate the displacement of the cart down the incline to its initial speed and acceleration:

$$\Delta x = v_0\Delta t + \tfrac{1}{2}a(\Delta t)^2$$

or, because $v_0 = 0$,

$$\Delta x = \tfrac{1}{2}a(\Delta t)^2$$

Solve for Δt:

$$\Delta t = \sqrt{\frac{2\Delta x}{a}}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \sqrt{\frac{2(1\,\text{m})}{1.71\,\text{m/s}^2}} = \boxed{1.08\,\text{s}}$$

97 ••

Picture the Problem Note that, while the mass of the rope is distributed over its length, the rope and the block have a common acceleration. Because the surface is horizontal and smooth, the only force that influences our solution is $\vec{F}$. The figure misrepresents the situation in that each segment of the rope experiences a gravitational force; the combined effect of which is that the rope must sag.

(*a*) Apply $\vec{a} = \vec{F}_{\text{net}}/m_{\text{tot}}$ to the rope-block system to obtain:

$$a = \boxed{\frac{F}{m_1 + m_2}}$$

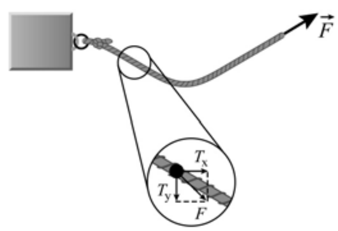
(*b*) Apply $\sum \vec{F} = m\vec{a}$ to the rope, substitute the acceleration of the system obtained in (*a*), and simplify to obtain:

$$F_{\text{net}} = m_2 a = m_2\left(\frac{F}{m_1 + m_2}\right) = \boxed{\frac{m_2}{m_1 + m_2}F}$$

(*c*) Apply $\sum \vec{F} = m\vec{a}$ to the block, substitute the acceleration of the system obtained in (*a*), and simplify to obtain:

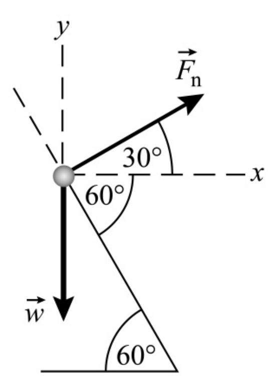
$$T = m_1 a = m_1\left(\frac{F}{m_1 + m_2}\right) = \boxed{\frac{m_1}{m_1 + m_2}F}$$

(*d*) The rope sags and so $\vec{F}$ has both vertical and horizontal components; with its horizontal component being less than $\vec{F}$. Consequently, *a* will be somewhat smaller.



***98** ••

Picture the Problem The free-body diagram shows the forces acting on the block. Choose the coordinate system shown on the diagram. Because the surface of the wedge is frictionless, the force it exerts on the block must be normal to its surface.



(*a*) Apply $\sum F_y = ma_y$ to the block to obtain:

$$F_n \sin 30° - w = ma_y$$

or, because $a_y = 0$ and $w = mg$,

$$F_n \sin 30° - mg = 0$$

or

$$F_n \sin 30° = mg \qquad (1)$$

Apply $\sum F_x = ma_x$ to the block:

$$F_n \cos 30° = ma_x \qquad (2)$$

Divide equation (2) by equation (1) to obtain:

$$\frac{a_x}{g} = \cot 30°$$

Solve for and evaluate a_x:

$$a_x = g\cot 30° = (9.81\,\text{m/s}^2)\cot 30°$$
$$= \boxed{17.0\,\text{m/s}^2}$$

(*b*) An acceleration of the wedge greater than g cot30° would require that the normal force exerted on the body by the wedge be greater than that given in part (*a*); i.e., $F_n > mg/\sin 30°$.

Under this condition, there would be a net force in the y direction and the block would accelerate up the wedge.

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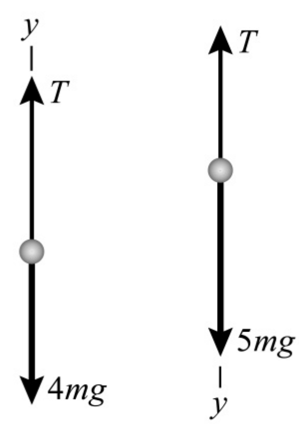
Picture the Problem Because the system is initially in equilibrium, it follows that $T_0 = 5mg$. When one washer is removed on the left side, the remaining washers will accelerate upward (and those on the right side downward) in response to the net force that results. The free-body diagrams show the forces under this unbalanced condition. Applying Newton's 2nd law to each collection of washers will allow us to determine both the acceleration of the system and the mass of a single washer.



(*a*) Apply $\sum F_y = ma_y$ to the rising masses:

$$T - 4mg = (4m)a \qquad (1)$$

Apply $\sum F_y = ma_y$ to the descending masses:

$$5mg - T = (5m)a \qquad (2)$$

Eliminate T between these equations to obtain:

$$a = \tfrac{1}{9}g$$

Use this acceleration in equation (1) or equation (2) to obtain:

$$T = \frac{40}{9}mg$$

Express the difference between T_0 and T and solve for m:

$$T_0 - T = 5mg - \frac{40}{9}mg = 0.3\,\text{N}$$

and

$$m = \boxed{0.0550\,\text{kg} = 55.0\,\text{g}}$$

(*b*) Proceed as in (*a*) to obtain:

$T - 3mg = 3ma$
and
$5mg - T = 5ma$

Eliminate T and solve for a:

$$a = \tfrac{1}{4}g = \tfrac{1}{4}\left(9.81\,\text{m/s}^2\right) = \boxed{2.45\,\text{m/s}^2}$$

Eliminate a in either of the motion equations and solve for T to obtain:

$$T = \frac{15}{4}mg$$

Substitute numerical values and evaluate T:

$$T = \frac{15}{4}(0.0550\,\text{kg})\left(9.81\,\text{m/s}^2\right) = \boxed{2.03\,\text{N}}$$

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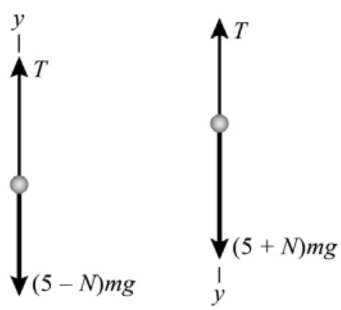
Picture the Problem The free-body diagram represents the Atwood's machine with N washers moved from the left side to the right side. Application of Newton's 2nd law to each collection of washers will result in two equations that can be solved simultaneously to relate N, a, and g. The acceleration can then be found from the given data.



Apply $\sum F_y = ma_y$ to the rising washers:

$T - (5 - N)mg = (5 - N)ma$

Apply $\sum F_y = ma_y$ to the descending washers:

$(5 + N)mg - T = (5 + N)ma$

Add these equations to eliminate T:

$$(5 + N)mg - (5 - N)mg = (5 - N)ma + (5 + N)ma$$

Simplify to obtain:

$$2Nmg = 10ma$$

Solve for N:

$N = 5a/g$

Using a constant-acceleration equation, relate the distance the washers fell to their time of fall:

$$\Delta y = v_0\Delta t + \tfrac{1}{2}a(\Delta t)^2$$

or, because $v_0 = 0$,

$$\Delta y = \tfrac{1}{2}a(\Delta t)^2$$

Solve for the acceleration:

$$a = \frac{2\Delta y}{(\Delta t)^2}$$

Substitute numerical values and evaluate a:

$$a = \frac{2(0.471\,\text{m})}{(0.40\,\text{s})^2} = 5.89\,\text{m/s}^2$$

Substitute in the expression for N:

$$N = 5\left(\frac{5.89\,\text{m/s}^2}{9.81\,\text{m/s}^2}\right) = \boxed{3}$$

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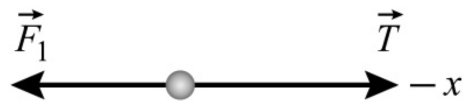
Picture the Problem Draw the free-body diagram for the block of mass m and apply Newton's 2nd law to obtain the acceleration of the system and then the tension in the rope connecting the two blocks.



(a) Letting T be the tension in the connecting string, apply $\sum F_x = ma_x$ to the block of mass m:

$$T - F_1 = ma$$

Apply $\sum F_x = ma_x$ to both blocks to determine the acceleration of the system:

$$F_2 - F_1 = (m + 2m)a = (3m)a$$

Substitute and solve for a:

$$a = (F_2 - F_1)/3m$$

Substitute for a in the first equation and solve for T:

$$T = \boxed{\tfrac{1}{3}(F_2 + 2F_1)}$$

(b) Substitute for F_1 and F_2 in the equation derived in part (a):

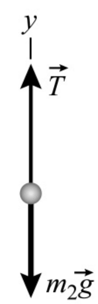
$$T = (2Ct + 2Ct)/3 = 4Ct/3$$

Evaluate this expression for $T = T_0$ and $t = t_0$ and solve for t_0:

$$t_0 = \boxed{\frac{3T_0}{4C}}$$

***102** •••

Picture the Problem Because a constant-upward acceleration has the same effect as an increase in the acceleration due to gravity, we can use the result of Problem 89 (for the tension) with a replaced by $a + g$. The application of Newton's 2nd law to the object whose mass is m_2 will connect the acceleration of this body to tension from Problem 84.



In Problem 84 it is given that, when the support pulley is not accelerating, the tension in the rope and the acceleration of the masses are related according to:

$$T = \frac{2m_1m_2}{m_1 + m_2}g$$

Replace a with $a + g$:

$$T = \frac{2m_1m_2}{m_1 + m_2}(a + g)$$

Apply $\sum F_y = ma_y$ to the object whose mass is m_2 and solve for a_2:

$$T - m_2g = m_2a_2$$

and

$$a_2 = \frac{T - m_2g}{m_2}$$

Substitute for T and simplify to obtain:

$$a_2 = \boxed{\frac{(m_1 - m_2)g + 2m_1a}{m_1 + m_2}}$$

The expression for a_1 is the same as for a_2 with all subscripts interchanged (note that a positive value for a_1 represents acceleration upward):

$$a_1 = \boxed{\frac{(m_2 - m_1)g + 2m_2a}{m_1 + m_2}}$$