Chapter 41
Elementary Particles and the Beginning of the Universe

Conceptual Problems

1 •

Similarities	Differences
Baryons and mesons are hadrons, i.e., they participate in the strong interaction. Both are composed of quarks.	Baryons consist of three quarks and are fermions. Mesons consist of two quarks and are bosons. Baryons have baryon number +1 or −1. Mesons have baryon number 0.

2 •

Determine the Concept The muon is a lepton. It is a spin-½ particle and is a fermion. It does not participate in strong interactions. It appears to be an elementary particle like the electron. The pion is a meson. Its spin is 0 and it is a boson. It does participate in strong interactions and is composed of quarks.

***3** •

Determine the Concept A decay process involving the strong interaction has a very short lifetime (~10^{-23} s), whereas decay processes that proceed via the weak interaction have lifetimes of order 10^{-10} s.

4 •

(*a*) True

(*b*) False. There are two kinds of hadrons-baryons, which have spin $\frac{1}{2}$ (or $\frac{3}{2}, \frac{5}{2}$, and so on), and mesons, which have zero or integral spin.

5 •

False. Mesons have zero or integral spins.

6 •

Determine the Concept A meson has 2 quarks, a baryon has 3 quarks.

7 •

Determine the Concept No; from Table 41-2 it is evident that any quark-antiquark combination always results in an integral or zero charge.

8 •

(*a*) False. Leptons are not made up of quarks.

(*b*) True

(*c*) False. Electrons are leptons and leptons interact via the weak interaction.

(*d*) True

(*e*) True

***9** •

Determine the Concept No. Such a reaction is impossible. A proton requires three quarks. Three quarks are not available because a pion is made of a quark and an antiquark and the antiproton consists of three antiquarks.

Estimation and Approximation

10 ••

Picture the Problem Assuming that the lifetime of a proton is 10^{32} y, one proton out of every 10^{32} protons should decay every year on average. Hence, we can estimate the expected time between proton-decays that occur in the water of a filled Olympic-size swimming pool by determining the number of protons N in the pool and dividing 10^{32} y by this number.

The mean time between disintegrations is the ratio of the lifetime of the protons to the number of protons N in the pool:

$$\Delta t_{\text{mean}} = \frac{10^{32}\ \text{y}}{N} \qquad (1)$$

The number of protons N in the pool is related to the mass of water in the pool M_{water}, the molar mass of water $m_{\text{molar, water}}$, and the number of protons per molecule n:

$$\frac{N}{M_{\text{water}}} = \frac{nN_{\text{A}}}{m_{\text{molar, water}}}$$

Solve for N to obtain:

$$N = \frac{nN_{\text{A}}M_{\text{water}}}{m_{\text{molar, water}}}$$

Because the mass of the water is the product of its density and the volume of the pool:

$$N = \frac{nN_{\text{A}}\rho_{\text{water}}V_{\text{pool}}}{m_{\text{molar, water}}}$$

Substituting for N in equation (1) yields:

$$\Delta t_{\text{mean}} = \frac{10^{32}\ \text{y}}{\dfrac{nN_{\text{A}}\rho_{\text{water}}V_{\text{pool}}}{m_{\text{molar, water}}}} = \frac{\left(10^{32}\ \text{y}\right)m_{\text{molar, water}}}{nN_{\text{A}}\rho_{\text{water}}V_{\text{pool}}}$$

Because each molecule of water has 10 protons:

$$n = 10\,\frac{\text{protons}}{\text{molecule}}$$

Substitute numerical values and evaluate Δt_{mean}:

$$\Delta t_{\text{mean}} = \frac{\left(10^{32}\ \text{y}\right)\left(18\,\dfrac{\text{g}}{\text{mol}}\times\dfrac{1\,\text{kg}}{10^{3}\,\text{g}}\right)}{\left(10\,\dfrac{\text{protons}}{\text{molecule}}\right)\left(6.02\times10^{23}\,\dfrac{\text{molecules}}{\text{mol}}\right)\left(10^{3}\,\dfrac{\text{kg}}{\text{m}^{3}}\right)(100\,\text{m})(25\,\text{m})(2\,\text{m})}$$

$$= 0.0598\,\text{y} = 0.0598\,\text{y}\times\frac{365.24\,\text{d}}{\text{y}} = \boxed{21.8\,\text{d}}$$

11 •

Picture the Problem We can use $F_{\text{em}} = kq_{\text{proton}}^{2}/r_{\text{nucleus}}^{2}$ and $F_{\text{grav}} = Gm_{\text{proton}}^{2}/r_{\text{nucleus}}^{2}$ to estimate the ratio of the electromagnetic and gravitational forces between two protons located in a nucleus.

The electromagnetic force between two protons located in a nucleus is given by:

$$F_{\text{em}} = \frac{kq_{\text{proton}}^{2}}{r_{\text{nucleus}}^{2}}$$

The gravitational force between these same protons is given by:

$$F_{\text{grav}} = \frac{Gm_{\text{proton}}^{2}}{r_{\text{nucleus}}^{2}}$$

Divide F_{em} by F_{grav} to obtain:

$$\frac{F_{\text{em}}}{F_{\text{grav}}} = \frac{\dfrac{kq_{\text{proton}}^{2}}{r_{\text{nucleus}}^{2}}}{\dfrac{Gm_{\text{proton}}^{2}}{r_{\text{nucleus}}^{2}}} = \frac{kq_{\text{proton}}^{2}}{Gm_{\text{proton}}^{2}}$$

Substitute numerical values and evaluate $F_{\text{em}}/F_{\text{grav}}$:

$$\frac{F_{\text{em}}}{F_{\text{grav}}}=\frac{\left(8.99\times10^{9}\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(1.60\times10^{-19}\,\text{C}\right)^2}{\left(6.67\times10^{-11}\,\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(1.67\times10^{-27}\,\text{kg}\right)^2}=\boxed{1.24\times10^{36}}$$

Spin and Antiparticles

***12 •**

Picture the Problem We can use both conservation of energy and momentum to explain why the energies of the two γ-rays must be equal. We can find the energy of each γ-ray in Table 41-1 and find their wavelengths using $\lambda = hc/E$.

(*a*)

The initial momentum is zero; therefore, the final momentum must be zero. The momentum of a photon is E/c. To conserve both momentum and energy the two photons must have the same momentum magnitude. Hence, they have the same energy.

(*b*) From Table 41-1:

$$E_\gamma=\boxed{139.6\,\text{MeV}}$$

(*c*) The wavelength of each γ ray is given by:

$$\lambda=\frac{hc}{E}=\frac{1240\,\text{MeV}\cdot\text{fm}}{E}$$

Substitute numerical values and evaluate λ:

$$\lambda=\frac{1240\,\text{MeV}\cdot\text{fm}}{139.6\,\text{MeV}}=\boxed{8.88\,\text{fm}}$$

13 •

Picture the Problem In each case, the required energy is given by $E = 2mc^2$ where m is mass of each particle produced in the pair-production reaction. These masses can be found in Tables 41-1 and 41-3.

(*a*) For $\gamma \rightarrow \pi^+ + \pi^-$:

$$E=2m_\pi c^2=2\left(139.6\,\text{MeV}/c^2\right)c^2=\boxed{279.2\,\text{MeV}}$$

(*b*) For $\gamma \rightarrow \text{p}+\text{p}^-$:

$$E=2m_\text{p}c^2=2\left(938.3\,\text{MeV}/c^2\right)c^2=\boxed{1877\,\text{MeV}}$$

(*c*) For $\gamma \rightarrow \mu^- + \mu^+$:

$$E=2m_\mu c^2=2\left(105.659\,\text{MeV}/c^2\right)c^2=\boxed{211.3\,\text{MeV}}$$

The Conservation Laws

14 •

Picture the Problem We need to check for conservation of energy, charge, baryon number, and lepton number.

(*a*) Energy conservation:

$\boxed{\text{Because } m_p < m_n\text{, energy conservation is violated.}}$

Charge conservation:
$+e \to 0 + e + 0 = +e$

$\boxed{\text{Because the net charge is } +e \text{ before and after the decay, charge is conserved.}}$

Baryon number:
$+1 \to +1 + 0 + 0 = +1$

$\boxed{\text{Because } B \text{ is } +1 \text{ before and after the decay, baryon number is conserved.}}$

Lepton number; electrons:
$0 \to 0 + 0 + 0 = 0$

$\boxed{\text{Because } L_e = 0 \text{ before and after the decay, the lepton number for electrons is conserved.}}$

$\boxed{\text{The process is not allowed because it violates conservation of energy.}}$

(*b*) Energy conservation:

$\boxed{\text{Because } m_n < m_p + m_{\pi^-}\text{, energy conservation is violated.}}$

Charge conservation:
$0 \to +e + (-e) = 0$

$\boxed{\text{Because the net charge is 0 before and after the decay, charge is conserved.}}$

Baryon number:
$1 \to 1 + 0 = 1$

$\boxed{\text{Because } B = 0 \text{ before and after the decay, baryon number is conserved.}}$

Lepton number; electrons:
$0 \to 0 + 0 = 0$

$\boxed{\text{Because L} = 0 \text{ before and after the decay, lepton number is conserved.}}$

$\boxed{\text{Because energy is not conserved, this decay is not allowed.}}$

(*c*) Momentum conservation is violated; two (or more) γ rays must be emitted to conserve momentum.

(*d*) Energy conservation: Energy is conserved.

Charge conservation:
$+1+(-1)\rightarrow 0+0 \ = 0$

Because the net charge is *zero* before and after the decay, charge is conserved.

Baryon number:
$+1+(-1)\rightarrow 0+0 \ = 0$

Because $B=0$ before and after the decay, baryon number is conserved.

Lepton number; electrons:
$0\rightarrow 0+0+0=0$

Because $L_e=0$ before and after the decay, the lepton number for electrons is conserved.

Because none of the conservation laws are violated, this is an allowed process.

(*e*) Energy conservation: Because $m_p > m_n + m_{e^+}$, energy is conserved.

Charge conservation:
$0+1\rightarrow 0+1 \ = 1$

Because the net charge is one before and after the decay, charge is conserved.

Baryon number:
$0+1\rightarrow 1+0 \ = 0$

Because $B=1$ before and after the decay, baryon number is conserved.

Lepton number; electrons:
$-1+0\rightarrow 0+(-1)=-1$

Because $L_e=-1$ before and after the decay, the lepton number for electrons is conserved.

Because none of the conservation laws are violated, this is an allowed process.

15 •

Picture the Problem The decay will occur via the strong interaction if strangeness is conserved. If $\Delta S = \pm 1$, it will occur via the weak interaction. If S changes by more than 1, the decay will not occur.

(*a*) List the strangeness of Ω^-, Ξ^0, and π^-:

Ω^-: $S = -3$
Ξ^0: $S = -2$
π^-: $S = 0$

Determine ΔS:

$$\Delta S = -2-(-3) = \boxed{+1}$$

$$\boxed{\text{Because } \Delta S = +1\text{, the reaction can proceed via the weak interaction.}}$$

(*b*) List the strangeness of Ξ^0, p, π^-, and π^0:

Ξ^0: $S = -2$
p: $S = 0$
π^-: $S = 0$
π^0: $S = 0$

Determine ΔS:

$$\Delta S = 0-(-2) = \boxed{+2}$$

$$\boxed{\text{Because } \Delta S = +2\text{, the reaction is not allowed.}}$$

(*c*) List the strangeness of Λ^0, p^+, and π^-:

Λ^0: $S = -1$
p^+: $S = 0$
π^-: $S = 0$

Determine ΔS:

$$\Delta S = 0-(-1) = \boxed{+1}$$

$$\boxed{\text{Because } \Delta S = +1\text{, the reaction can proceed via the weak interaction.}}$$

16 •

Picture the Problem The decay will occur via the strong interaction if strangeness is conserved. If $\Delta S = \pm 1$, it will occur via the weak interaction. If S changes by more than 1, the decay will not occur.

(*a*) List the strangeness of Ω^-, Λ^0, and K^-:

Ω^-: $S = -3$
Λ^0: $S = -1$
K^-: $S = -1$

Determine ΔS:

$$\Delta S = -1-1-(-3) = \boxed{+1}$$

$$\boxed{\text{Because } \Delta S = +1, \text{ the reaction can proceed via the weak interaction.}}$$

(*b*) List the strangeness of Ξ^0, p, and π^-:

Ξ^0: $S = -2$
p: $S = 0$
π^-: $S = 0$

Determine ΔS:

$$\Delta S = 0 - (-2) = \boxed{+2}$$

$$\boxed{\text{Because } \Delta S = +2, \text{ the reaction is not allowed.}}$$

17 •

Picture the Problem The decay will occur via the strong interaction if strangeness is conserved. If $\Delta S = \pm 1$, it will occur via the weak interaction. If S changes by more than 1, the decay will not occur.

(*a*) List the strangeness of Ω^-, Λ^0, $\overline{\nu}_e$, and e^-:

Ω^-: $S = -3$
Λ^0: $S = -1$
$\overline{\nu}_e$: $S = 0$
e^-: $S = 0$

Determine ΔS:

$$\Delta S = -1 - (-3) = \boxed{+2}$$

$$\boxed{\text{Because } \Delta S = +2, \text{ the reaction is not allowed.}}$$

(*b*) List the strangeness of Σ^+, p, and π^0:

Σ^+: $S = -1$
p: $S = 0$
π^0: $S = 0$

Determine ΔS:

$$\Delta S = 0 - (-1) = \boxed{+1}$$

$$\boxed{\text{Because } \Delta S = +1, \text{ the reaction can proceed via the weak interaction.}}$$

18 •

Picture the Problem We can decide whether the given decays of the τ particle are possible by determining whether energy conservation is satisfied and whether conservation of both the τ and μ lepton numbers is satisfied.

(*a*) $\boxed{\text{The first decay is allowed. It satisfies energy conservation and conservation of both the } \tau \text{ and } \mu \text{ lepton numbers.}}$

(*b*) $\boxed{\text{The second decay scheme is not allowed because it does not conserve } \tau \text{ and } \mu \text{ lepton numbers.}}$

(*c*) The total kinetic for the decay in (*a*) is:

$$K_{\text{tot}} = m_\tau c^2 - m_\mu c^2$$

From Table 41-3 we have:

$$m_\tau = 1784\,\text{MeV}/c^2$$

and

$$m_\mu = 105.659\,\text{MeV}/c^2$$

Substitute numerical values and evaluate K_{tot}:

$$K_{\text{tot}} = \left(1784\,\text{MeV}/c^2\right)c^2 - \left(106\,\text{MeV}/c^2\right)c^2 = \boxed{1678\,\text{MeV}}$$

Remarks: Note that the kinetic energy of the individual decay products cannot be determined from the decay scheme alone.

19 ••

Picture the Problem Examination of the decay products will reveal whether all the final products are stable. A decay process is allowed if energy, charge, baryon number, and lepton number are conserved.

(*a*) No; the neutron is not stable:

$$\boxed{n \to p^+ + e^- + \bar{\nu}_e}$$

(*b*) Add the reactions to obtain:

$$\boxed{\Omega^- \to p^+ + e^+ + 3e^- + \nu_e + 3\bar{\nu}_e + 2\bar{\nu}_\mu + 2\nu_\mu}$$

(*c*) Charge conservation:

$-1 \to 1 + 1 - 3 + 0 + 0 + 0 + 0 = -1$

$\boxed{\text{Because } Q = -1 \text{ before and after the decay, charge is conserved.}}$

Baryon number:

$1 \to 1 + 0 + 0 + 0 + 0 + 0 + 0 = 1$

$\boxed{\text{Because } B = 1 \text{ before and after the decay, baryon number is conserved.}}$

Lepton number:

$0 \to 0 - 1 + 3 + 1 - 3 - 2 + 2 = 0$

$\boxed{\text{Because } L_e = 0 \text{ before and after the decay, the lepton number for electrons is conserved.}}$

Strangeness:
$-3 \to 0$

Strangeness is not conserved. However, in each baryon decay $\Delta S = +1$, and each decay is allowed via the weak interaction.

***20** ••

Picture the Problem A decay process is allowed if energy, charge, baryon number, and lepton number are conserved.

(*a*) Energy conservation:

Because $m_{\mathrm{n}} > 2m_{\pi} + 2m_{\mu}$, energy conservation is not violated.

Charge conservation:
$0 \to +1 + (-1) + 0 + 0 = 0$

Because the total charge is 0 before and after the decay, charge is conserved.

Baryon number:
$1 \to 0 + 0 + 0 + 0 = 0$

Because baryon number changes from $+1$ to 0, conservation of baryon number is violated.

Lepton number:
$0 \to 0 + 0 + 1 + (-1) = 0$

Because $L_{\mu} = 0$ before and after the decay, the lepton number for muons is conserved.

The process is not allowed because it violates conservation of baryon number.

(*b*) Energy conservation:

Because $m_{\pi} > 2m_{\mathrm{e}}$, energy conservation is not violated.

Charge conservation:
$0 \to +1 + (-1) + 0 + 0 = 0$

Because the total charge is 0 before and after the decay, charge is conserved.

Baryon number:
$0 \to 0 + 0 + 0 + 0 = 0$

Because $B = 0$ before and after the decay, the baryon number is conserved.

Lepton number:
$0 \to 0 + 0 + 0 + 0 = 0$

$\boxed{\text{Because } L_e = 0 \text{ before and after the decay, the lepton number is conserved.}}$

$\boxed{\text{The decay satisfies all conservation laws and is allowed.}}$

Quarks

21 •

Picture the Problem For each quark combination we can determine the baryon number *B*, the charge *Q*, and the strangeness *S* and then use Table 41-1 to find a match and complete the following table.

	Combination	B	Q	S	hadron
(*a*)	*uud*	1	+1	0	p^+
(*b*)	*udd*	1	0	0	n
(*c*)	*uus*	1	+1	−1	Σ^+
(*d*)	*dds*	1	−1	−1	Σ^-
(*e*)	*uss*	1	0	−2	Ξ^0
(*f*)	*dss*	1	−1	−2	Ξ^-

22 •

Picture the Problem For each quark combination we can determine the baryon number *B*, the charge *Q*, and the strangeness *S* and then use Table 41-1 to find a match and complete the following table.

	Combination	B	Q	S	hadron
(*a*)	$u\bar{d}$	0	+1	0	π^+
(*b*)	$\bar{u}d$	0	−1	0	π^-
(*c*)	$u\bar{s}$	0	+1	+1	K^+
(*d*)	$\bar{u}s$	0	−1	−1	K^-

23 •

Determine the Concept From Table 41-2 we see that to satisfy the conditions of charge = +2 and zero strangeness, charm, topness, and bottomness, the quark combination must be *uuu*.

24 •

Picture the Problem Because K^+ and K^0 are mesons, they consist of a quark and an antiquark. We can use Tables 41-1 and 41-2 to find combinations of quarks with the correct values for electric charge, baryon number, and strangeness for these particles.

(a) For K^+ we need:

$$Q = +1$$
$$B = 0$$
$$S = +1$$

A combination of quarks with these properties is $\boxed{u\bar{s}.}$

(b) For K^0 we need:

$$Q = 0$$
$$B = 0$$
$$S = +1$$

A combination of quarks with these properties is $\boxed{d\bar{s}.}$

25 •

Determine the Concept Because D^+ and D^- are mesons, they consist of a quark and an antiquark.

(a) $B = 0$, so we must look for a combination of quark and antiquark. Because it has charm of +1, one of the quarks must be c. Because the charge is $+e$, the antiquark must be $\bar{d}$. The possible combination for D^+ is $\boxed{c\bar{d}.}$.

(b) Because D^- is the antiparticle of D^+, the quark combination is $\boxed{\bar{c}d.}$

26 •

Picture the Problem We can use Tables 41-1 and 41-2 to find combinations of quarks with the correct values for electric charge, baryon number, and strangeness for these particles. Because K^- and $\overline{K}^0$ are mesons, they consist of a quark and an antiquark.

(a) For K^- we need:

$$Q = -1$$
$$B = 0$$
$$S = -1$$

A combination of quarks with these properties is $\boxed{\bar{u}s.}$

(b) For $\overline{K}^0$ we need:

$$Q = 0$$
$$B = 0$$
$$S = -1$$

A combination of quarks with these properties is $\boxed{\bar{d}s.}$

Remarks: An alternative solution could take advantage of our results in Problem 20 for the antiparticles K^+ and K^0.

***27** ••

Picture the Problem Because Λ^0, $\bar{p}^-$, and Σ^- are baryons, they are made up of three quarks. We can use Tables 41-1 and 41-2 to find combinations of quarks with the correct values for electric charge, baryon number, and strangeness for these particles.

(*a*) For Λ^0 we need:

$Q = 0$
$B = +1$
$S = -1$

The quark combination that satisfies these conditions is $\boxed{uds.}$

(*b*) For $\bar{p}^-$ we need:

$Q = -1$
$B = -1$
$S = +1$

The quark combination that satisfies these conditions is $\boxed{\bar{u}\bar{u}\bar{d}.}$

(*c*) For Σ^- we need:

$Q = -1$
$B = +1$
$S = -1$

The quark combination that satisfies these conditions is $\boxed{dds.}$

28 ••

Picture the Problem Because $\bar{n}$, Ξ^0, and Σ^+ are baryons, they are made up of three quarks. We can use Tables 41-1 and 41-2 to find combinations of quarks with the correct values for electric charge, baryon number, and strangeness for these particles.

(*a*) For $\bar{n}$ we need:

$Q = 0$
$B = -1$
$S = 0$

The quark combination that satisfies these conditions is $\boxed{\bar{u}\bar{d}\bar{d}.}$

(*b*) For Ξ^0 we need:

$Q = 0$
$B = +1$
$S = -2$

The quark combination that satisfies these conditions is $\boxed{uss.}$

(c) For Σ^+ we need:

$$Q = +1$$
$$B = +1$$
$$S = -1$$

The quark combination that satisfies these conditions is $\boxed{uus.}$

29 ••

Picture the Problem Because Ω^- and Ξ^- are baryons, they are made up of three quarks. We can use Tables 41-1 and 41-2 to find combinations of quarks with the correct values for electric charge, baryon number, and strangeness for these particles.

(a) For Ω^- we need:

$$Q = -1$$
$$B = +1$$
$$S = -3$$

The quark combination that satisfies these conditions is $\boxed{sss.}$

(b) For Ξ^- we need:

$$Q = -1$$
$$B = +1$$
$$S = -2$$

The quark combination that satisfies these conditions is $\boxed{ssd.}$

30 ••

Picture the Problem We can use Table 41-2 to identify the properties of the particles made up of the given quarks.

(a) For ddd:

$$Q = \boxed{-1}$$
$$B = \boxed{+1}$$
$$S = \boxed{0}$$

(b) For $u\bar{c}$:

$$Q = \boxed{0}$$
$$B = \boxed{0}$$
$$S = \boxed{0}$$
$$\text{charm} = \boxed{-1}$$

(*c*) For $u\bar{b}$:

$$Q = \boxed{+1}$$
$$B = \boxed{0}$$
$$S = \boxed{0}$$
$$\text{bottomness} = \boxed{-1}$$

(*d*) For $\bar{s}\bar{s}\bar{s}$:

$$Q = \boxed{+1}$$
$$B = \boxed{-1}$$
$$S = \boxed{+3}$$

The Evolution of the Universe

***31** •

Picture the Problem We can use Hubble's law to find the distance from the earth to this galaxy.

The recessional velocity of galaxy is related to its distance by Hubble's law:

$$v = Hr$$

Solve for *r*:

$$r = \frac{v}{H}$$

Substitute numerical values and evaluate *r*:

$$r = \frac{(0.025)c}{\dfrac{23\,\text{km/s}}{10^6\,c\cdot\text{y}}} = \frac{(0.025)(3\times10^5\ \text{km/s})}{\dfrac{23\,\text{km/s}}{10^6\,c\cdot\text{y}}}$$
$$= \boxed{3.26\times10^8\,c\cdot\text{y}}$$

32 •

Picture the Problem We can use Hubble's law to find the speed of the galaxy.

The recessional velocity of galaxy is related to its distance by Hubble's law:

$$v = Hr$$

Substitute numerical values and evaluate *v*:

$$v = \left(\frac{23\,\text{km/s}}{10^6\,c\cdot\text{y}}\right)\left(12\times10^9\,c\cdot\text{y}\right)\left(\frac{c}{3.00\times10^5\ \text{km/s}}\right) = \boxed{0.920c}$$

33 ••

Picture the Problem We can substitute for f' and f_0, using $v = f\lambda$, in Equation 39-16*b* to show that the relativistic wavelength shift is $\lambda' = \lambda_0\sqrt{\frac{1+v/c}{1-v/c}}$.

From Equation 39-16*b*:

$$f' = f_0\sqrt{\frac{1-v/c}{1+v/c}}$$

Express f' and f_0 in terms of λ' and λ_0:

$$f' = \frac{c}{\lambda'} \text{ and } f_0 = \frac{c}{\lambda_0}$$

Substitute for f' and f_0 to obtain:

$$\frac{c}{\lambda'} = \frac{c}{\lambda_0}\sqrt{\frac{1-v/c}{1+v/c}}$$

Solve for λ':

$$\lambda' = \boxed{\lambda_0\sqrt{\frac{1+v/c}{1-v/c}}}$$

***34** ••

Picture the Problem Using Hubble's law, we can rewrite the equation from Problem 31 as $\lambda' = \lambda_0\sqrt{\frac{1+Hr/c}{1-Hr/c}}$.

From Problem 33 we have:

$$\lambda' = \lambda_0\sqrt{\frac{1+v/c}{1-v/c}}$$

Use Hubble's law to relate v to r:

$$v = Hr$$

Substitute for v to obtain:

$$\lambda' = \lambda_0\sqrt{\frac{1+Hr/c}{1-Hr/c}}$$

(*a*) For $r = 5\times10^6\ c\cdot\text{y}$:

$$\lambda' = 656.3\,\text{nm}\sqrt{\frac{1+\left(\frac{23\,\text{km/s}}{10^6\,\text{c}\cdot\text{y}}\right)\frac{(5\times10^6\,\text{c}\cdot\text{y})}{(3\times10^5\,\text{km/s})}}{1-\left(\frac{23\,\text{km/s}}{10^6\,\text{c}\cdot\text{y}}\right)\frac{(5\times10^6\,\text{c}\cdot\text{y})}{(3\times10^5\,\text{km/s})}}} = \boxed{656.6\,\text{nm}}$$

(*b*) For $r = 50\times10^6\ c\cdot\text{y}$:

$$\lambda' = 656.3\,\text{nm}\sqrt{\frac{1+\left(\dfrac{23\,\text{km/s}}{10^6\,\text{c}\cdot\text{y}}\right)\dfrac{\left(50\times10^6\,\text{c}\cdot\text{y}\right)}{\left(3\times10^5\,\text{km/s}\right)}}{1-\left(\dfrac{23\,\text{km/s}}{10^6\,\text{c}\cdot\text{y}}\right)\dfrac{\left(50\times10^6\,\text{c}\cdot\text{y}\right)}{\left(3\times10^5\,\text{km/s}\right)}}} = \boxed{658.8\,\text{nm}}$$

(*c*) For $r = 500\times10^6\ c{\cdot}\text{y}$:

$$\lambda' = 656.3\,\text{nm}\sqrt{\frac{1+\left(\dfrac{23\,\text{km/s}}{10^6\,\text{c}\cdot\text{y}}\right)\dfrac{\left(500\times10^6\,\text{c}\cdot\text{y}\right)}{\left(3\times10^5\,\text{km/s}\right)}}{1-\left(\dfrac{23\,\text{km/s}}{10^6\,\text{c}\cdot\text{y}}\right)\dfrac{\left(500\times10^6\,\text{c}\cdot\text{y}\right)}{\left(3\times10^5\,\text{km/s}\right)}}} = \boxed{682.0\,\text{nm}}$$

(*d*) For $r = 5\times10^9\ c{\cdot}\text{y}$:

$$\lambda' = 656.3\,\text{nm}\sqrt{\frac{1+\left(\dfrac{23\,\text{km/s}}{10^6\,\text{c}\cdot\text{y}}\right)\dfrac{\left(5\times10^9\,\text{c}\cdot\text{y}\right)}{\left(3\times10^5\,\text{km/s}\right)}}{1-\left(\dfrac{23\,\text{km/s}}{10^6\,\text{c}\cdot\text{y}}\right)\dfrac{\left(5\times10^9\,\text{c}\cdot\text{y}\right)}{\left(3\times10^5\,\text{km/s}\right)}}} = \boxed{983.0\,\text{nm}}$$

General Problems

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Determine the Concept

(*a*) It must be a meson, and it must consist of a quark and its antiquark.

(*b*) The π^0 is its own antiparticle.

(*c*) The Ξ^0 is a baryon; it cannot be its own antiparticle; the antiparticle is the $\overline{\Xi}^0 = \overline{uss}$.

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Picture the Problem Examination of the decay products will reveal whether all the final products are stable. A decay process is allowed if energy, charge, baryon number, and lepton number are conserved.

(*a*) $\boxed{\text{Yes, the final products are stable.}}$

(*b*) Add the reactions to obtain:

$$\boxed{\Xi^0 \to p^+ + e^- + \overline{\nu}_e + \nu_\mu + \overline{\nu}_\mu + 2\gamma}$$

(*c*) Charge conservation:
$0 \to e^+ + e^- = 0$

$\boxed{\text{Charge is conserved.}}$

Baryon number:
$1 \to 1 + 0 = 1$

$\boxed{\text{Baryon number is conserved.}}$

Lepton number:
$0 \to 0 + 1 - 1 + 1 - 1 = 0$

$\boxed{\text{Lepton number is conserved.}}$

Strangeness:
$-2 \to 0$

$\boxed{\text{Strangeness is not conserved. However, the reaction is allowed via the weak interaction because in the first two decays } \Delta S = +1.}$

(*d*) $\boxed{\text{No; the rest masses of the decay products would be greater than the rest mass of the } \Xi^0\text{, violating energy conservation.}}$

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Picture the Problem The π^0 particle is composed of two quarks, $u\overline{u}$. Hence, the reaction $\pi^0 \to \gamma + \gamma$ is equivalent to $u\overline{u} \to \gamma + \gamma$.

(*a*) $\boxed{\text{The u and } \overline{\text{u}} \text{ annihilate resulting in the photons.}}$

(*b*) $\boxed{\text{Two or more photons are required to conserve linear momentum.}}$

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Picture the Problem A decay process is allowed if energy, charge, baryon number, and lepton number are conserved.

(*a*) Energy conservation:

$\boxed{\text{Because } m_\Lambda > m_p + m_\pi\text{, energy conservation is not violated.}}$

Charge conservation:
$0 \to 1 - 1 = 0$

$\boxed{\text{Because the total charge is 0 before and after the decay, charge conservation is not violated.}}$

Baryon number:
$1 \to 1 + 0 = 1$

Because there is no change in baryon number, baryon number is conserved.

Lepton number:
$0 \to 0 + 0 = 0$

Because lepton number is 0 on both sides, lepton number is conserved.

The decay satisfied all conservation laws and is allowed.

(*b*) Energy conservation:

Because $m_\Sigma < m_n + m_p$, energy is not conserved.

Charge conservation:
$-1 \to 0 - 1 = -1$

Because the total charge does not change, charge is conserved.

Baryon number:
$+1 \to 1 - 1 = 0$

Because B changes from $+1$ to 0, baryon number is not conserved.

Lepton number:
$0 \to 0 + 0 = 0$

Because L is 0 on both sides, lepton number is conserved.

Because the decay violates both energy conservation and baryon number, it is not allowed.

(*c*) Energy conservation:

Energy is conserved.

Charge conservation:
$-1 \to -1 + 0 + 0 = -1$

Because the total charge does not change, charge is conserved.

Baryon number:
$0 \to 0 + 0 + 0 = 0$

Because B does not change, baryon number is conserved.

Lepton number:
$1 \to 1 - 1 + 1 = 1$

Because L does not change, lepton number is conserved.

The decay satisfied all conservation laws and is allowed.

Remarks: The decay in Part (*c*) is the decay process for the muon μ^- (see Example 41-2).

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Picture the Problem We can systematically determine *Q*, *B*, *S*, and s for each reaction and then use these values to identify the unknown particles.

(*a*) For the strong reaction: $\mathrm{p}+\pi^- \rightarrow \Sigma^o + ?$

Charge number: $+1-1=0+Q \Rightarrow Q=0$

Baryon number: $+1+0=+1+B \Rightarrow B=0$

Strangeness: $0+0=-1+S \Rightarrow S=+1$

Spin: $+\frac{1}{2}+0=+\frac{1}{2}+s \Rightarrow s=0$

$$\boxed{\text{These properties indicate that the particle is the kaon, } \mathrm{K}^0.}$$

(*b*) For the strong reaction: $\mathrm{p}+\mathrm{p} \rightarrow \pi^+ + \mathrm{n} + \mathrm{K}^+ + ?$

Charge number: $+1+1=+1+0+1+Q \Rightarrow Q=0$

Baryon number: $+1+1=0+1+0+B \Rightarrow B=+1$

Strangeness: $0+0=0+0+1+S \Rightarrow S=-1$

Spin: $+\frac{1}{2}+\frac{1}{2}=0+\frac{1}{2}+0+s \Rightarrow s=+\frac{1}{2}$

$$\boxed{\text{These properties indicate that the particle is either the } \Sigma^0 \text{ or the } \Lambda^0 \text{ baryon.}}$$

(*c*) For the strong reaction: $\mathrm{p}+\overline{\mathrm{K}}^- \rightarrow \Xi^- + ?$

Charge number: $+1-1=-1+Q \Rightarrow Q=+1$

Baryon number: $+1+0=+1+B \Rightarrow B=0$

Strangeness: $0-1=-2+S \Rightarrow S=-1$

Spin: $+\frac{1}{2}+0=+\frac{1}{2}+s \Rightarrow s=0$

$$\boxed{\text{These properties indicate that the particle is the kaon, } \mathrm{K}^+.}$$

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Picture the Problem We can systematically determine *Q*, *B*, *S*, and *s* for the reaction and then use these values to identify the unknown particle. The *Q* value for the reaction is given by $Q=-(\Delta m)c^2$ and the expression for the threshold energy for the reaction is given in the problem statement.

(*a*) For the strong reaction: $p+p \rightarrow \Lambda^o + K^o + p + ?$

Charge number: $+1+1=0+0+1+Q \Rightarrow Q=+1$

baryon number: $+1+1=+1+0+1+B \Rightarrow B=0$

strangeness: $0+0=-1+1+0+S \Rightarrow S=0$

spin: $+\frac{1}{2}+\frac{1}{2}=+\frac{1}{2}+0+\frac{1}{2}+s \Rightarrow s=0$

$$\boxed{\text{These properties indicate that the unknown particle is a pion, } \pi^+.}$$

(*b*) The reaction is: $p+p \rightarrow \Lambda^o + K^o + p + \pi^+$

The *Q*-value for the reaction is:

$$Q=\left[\left(m_p+m_p\right)-\left(M_{\Lambda^o}+M_{K^o}+M_p+M_{\pi^+}\right)\right]c^2$$

Use Table 41-1 to find the mass-energy values:

$$Q=\left[(938.3+938.3)-(1116+497.7+938.3+139.6)\right]\text{MeV}=\boxed{-815\,\text{MeV}}$$

Because $Q < 0$, the reaction is endothermic.

(c) The threshold energy for this reaction is:

$$K_{\text{th}}=-\frac{Q}{2m_p}\left(m_p+m_p+M_{\Lambda^o}+M_{K^o}+M_p+M_{\pi^+}\right)$$

Using Table 41-1 to find the mass-energy values, substitute numerical values and evaluate K_{th}:

$$K_{\text{th}}=-\frac{-815\,\text{MeV}}{2(938.3\,\text{MeV})}(938.3+938.3+1116+497.7+938.3+139.6)\text{MeV}$$
$$=1984\,\text{MeV}=\boxed{1.984\,\text{GeV}}$$

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Picture the Problem We can solve the equation derived in Problem 31 for the recessional velocity of the galaxy and then use Hubble’s equation to estimate the distance to the galaxy.

(*a*) From Problem 31 we have: $\lambda' = \lambda_0\sqrt{\frac{1+v/c}{1-v/c}}$

Solve for the radicand:

$$\frac{1+v/c}{1-v/c}=\left(\frac{\lambda'}{\lambda_0}\right)^2$$

Substitute numerical values for λ' and λ_0:

$$\frac{1+v/c}{1-v/c}=\left(\frac{1458\text{ nm}}{656.3\text{ nm}}\right)^2=4.935$$

Simplify to obtain:

$$4.953\left(1-\frac{v}{c}\right)=1+\frac{v}{c}$$

and

$$5.953\frac{v}{c}=3.953$$

Solve for v:

$$v=0.664c=0.664\left(3\times10^8\text{ m/s}\right)$$
$$=1.99\times10^8\text{ m/s}=\boxed{1.99\times10^5\text{ km/s}}$$

(*b*) From the Hubble equation we have:

$$r=\frac{v}{H}$$

Substitute numerical values and evaluate r:

$$r=\frac{1.99\times10^5\text{km/s}}{\frac{23\text{ km/s}}{10^6\,c\cdot\text{y}}}=\boxed{8.65\times10^9\,c\cdot\text{y}}$$

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Picture the Problem We can use the masses of the parent and daughters to find the total kinetic energy of the decay products under the assumption that the Λ^0 is initially at rest. Application of conservation of energy and the definition of kinetic energy will yield the ratio of the kinetic energy of the pion to the kinetic energy of the proton. Finally, we can use our results in (*a*) and (*b*) to find the kinetic energies of the proton and the pion for this decay.

(*a*) The total kinetic energy of the decay products is given by:

$$K_{\text{tot}}=\left(m_\Lambda-m_\text{p}-m_\pi\right)c^2$$

Substitute numerical values (see Table 41-1) and evaluate K_{tot}:

$$K_{\text{tot}}=\left(1116\frac{\text{MeV}}{c^2}-938.3\frac{\text{MeV}}{c^2}-139.6\frac{\text{MeV}}{c^2}\right)c^2=\boxed{38.1\text{MeV}}$$

(*b*) The ratio of the kinetic energies is given by:

$$\frac{K_\pi}{K_\text{p}}=\frac{\frac{1}{2}m_\pi v_\pi^2}{\frac{1}{2}m_\text{p}v_\text{p}^2}=\frac{m_\pi v_\pi^2}{m_\text{p}v_\text{p}^2}$$

Use conservation of momentum (nonrelativistic) to obtain:

$$m_\pi v_\pi = m_{\rm p} v_{\rm p} \Rightarrow \frac{v_\pi}{v_{\rm p}} = \frac{m_{\rm p}}{m_\pi}$$

Substitute for the ratio of the speeds to obtain:

$$\frac{K_\pi}{K_{\rm p}} = \frac{m_\pi}{m_{\rm p}}\left(\frac{m_{\rm p}}{m_\pi}\right)^2 = \frac{m_{\rm p}}{m_\pi}$$

Substitute numerical values and evaluate the ratio of the kinetic energies:

$$\frac{K_\pi}{K_{\rm p}} = \frac{938.3\,\frac{\rm MeV}{c^2}}{139.6\,\frac{\rm MeV}{c^2}} = \boxed{6.72}$$

(*c*) Express the total kinetic energy in terms of K_π and $K_{\rm p}$:

$$K_{\rm p} + K_\pi = K_{\rm tot} \qquad (1)$$

Use our results in (*a*) and (*b*) to obtain:

$$K_{\rm p} + 6.72K_{\rm p} = 38.1\,{\rm MeV}$$

Solve for $K_{\rm p}$:

$$K_{\rm p} = \boxed{4.94\,{\rm MeV}}$$

Substitute in equation (1) to obtain:

$$K_\pi = K_{\rm tot} - K_{\rm p} = \boxed{33.2\,{\rm MeV}}$$

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Picture the Problem The total kinetic energy of the decay products is the rest energy of the Σ^0 particle. We can find the momentum of the photon from its energy and use the conservation of momentum to calculate the kinetic energy of the Λ^0.

(*a*) The total kinetic energy of the decay products is given by:

$$K_{\rm tot} = (m_\Sigma)c^2$$

Substitute numerical values (see Table 41-1) and evaluate $K_{\rm tot}$:

$$K_{\rm tot} = \left(1193\,\frac{\rm MeV}{c^2}\right)c^2 = \boxed{1193\,{\rm MeV}}$$

(*b*) The momentum of the photon is given by:

$$p_\gamma = \frac{E_\gamma}{c} = \frac{E - m_\Lambda c^2}{c}$$

Substitute numerical values and evaluate p_γ:

$$p_\gamma = \frac{1193\,{\rm MeV} - \left(1116\,\frac{\rm MeV}{c^2}\right)c^2}{c} = \boxed{77.0\,\frac{\rm MeV}{c}}$$

(*c*) The kinetic energy of the Λ^0 is given by:

$$K_\Lambda = \frac{p_\Lambda^2}{2m_\Lambda}$$

or, because $p_\Lambda = p_\gamma$,

$$K_\Lambda = \frac{p_\gamma^2}{2m_\Lambda}$$

Substitute numerical values and evaluate K_Λ:

$$K_\Lambda = \frac{\left(77.0\,\frac{\text{MeV}}{c}\right)^2}{2\left(1116\,\frac{\text{MeV}}{c^2}\right)} = \boxed{2.66\,\text{MeV}}$$

(*d*) A better estimate of the energy of the photon is:

$$E_\gamma = E - m_\Lambda c^2 - K_\Lambda$$

Substitute numerical values and evaluate E_γ:

$$E_\gamma = 1193\,\text{MeV} - \left(1116\,\frac{\text{MeV}}{c^2}\right)c^2 - 2.66\,\text{MeV} = \boxed{74.3\,\text{MeV}}$$

The improved estimate of the momentum of the photon is:

$$p_\gamma = \frac{E_\lambda}{c} = \frac{74.3\,\text{MeV}}{c} = \boxed{74.3\,\frac{\text{MeV}}{c}}$$

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Picture the Problem The solution strategy is outlined in the problem statement.

(*a*) Express $\Delta t = t_2 - t_1$ in terms of u_2 and u_1:

$$\Delta t = t_2 - t_1 = \frac{x}{u_2} - \frac{x}{u_1} = \frac{x(u_1 - u_2)}{u_1 u_2}$$

Noting that $u_1 u_2 \approx c^2$, we have:

$$\Delta t \approx \boxed{\frac{x\Delta u}{c^2}} \qquad (1)$$

where $\Delta u = u_1 - u_2$

(*b*) From Equation 39-25 we have:

$$\frac{u}{c} = \sqrt{1 - \left(\frac{mc^2}{E}\right)^2} = \left(1 - \left(\frac{mc^2}{E}\right)^2\right)^{1/2}$$

Expand binomially to obtain:

$$\frac{u}{c} = \boxed{1 - \frac{1}{2}\left(\frac{mc^2}{E}\right)^2}$$

(c) Express $u_1 - u_2$ in terms of E_1, E_2, and mc^2:

$$u_1 - u_2 = \tfrac{1}{2}\left(mc^2\right)^2\left(\frac{1}{E_2^2} - \frac{1}{E_1^2}\right) = \frac{c\left(mc^2\right)^2\left(E_1^2 - E_2^2\right)}{2E_1^2E_2^2}$$

Substitute numerical values and evaluate Δu:

$$\Delta u = \frac{c\left(20\frac{\text{eV}}{c^2}c^2\right)^2\left[(20\,\text{MeV})^2 - (5\,\text{MeV})^2\right]}{2(20\,\text{MeV})^2(5\,\text{MeV})^2} = \boxed{7.50\times10^{-12}c}$$

Use equation (1) to evaluate Δt:

$$\Delta t \approx \frac{\left(1.7\times10^5\,c\cdot\text{y}\right)\left(7.5\times10^{-12}\,c\right)}{c^2} = 1.275\times10^{-6}\,\text{y}\times\frac{31.56\,\text{Ms}}{\text{y}} = \boxed{40.2\,\text{s}}$$

(d) Using mc^2 = 40 eV for the rest energy of a neutrino:

$$\Delta u = \frac{c\left(40\frac{\text{eV}}{c^2}c^2\right)^2\left[(20\,\text{MeV})^2 - (5\,\text{MeV})^2\right]}{2(20\,\text{MeV})^2(5\,\text{MeV})^2} = \boxed{3.00\times10^{-11}c}$$

Use equation (1) to evaluate Δt:

$$\Delta t \approx \frac{\left(1.7\times10^5\,c\cdot\text{y}\right)\left(3\times10^{-11}\,c\right)}{c^2} = 5.1\times10^{-6}\,\text{y}\times\frac{31.56\,\text{Ms}}{\text{y}} = \boxed{161\,\text{s}}$$

Remarks: Note that the spread in the arrival time for neutrinos from a supernova can be used to estimate the mass of a neutrino.