Chapter 12
Static Equilibrium and Elasticity

Conceptual Problems

1 •

(*a*) False. The conditions $\sum_i \vec{F}_i = 0$ and $\sum_i \vec{\tau}_i = 0$ must be satisfied.

(*b*) True. The necessary and sufficient conditions for static equilibrium are $\sum_i \vec{F}_i = 0$ and $\sum_i \vec{\tau}_i = 0$.

(*c*) True. The conditions $\sum_i \vec{F}_i = 0$ and $\sum_i \vec{\tau}_i = 0$ must be satisfied.

(*d*) False. An object is in equilibrium provided the conditions $\sum_i \vec{F}_i = 0$ and $\sum_i \vec{\tau}_i = 0$ are satisfied.

2 •

False. The location of the center of gravity depends on the mass distribution.

3 •

No. The definition of the center of gravity does not require that there be any material at its location.

4 •

Determine the Concept When the acceleration of gravity is not constant over an object, the center of gravity is the pivot point for balance.

5 ••

Determine the Concept This technique works because the center of mass must be directly under the balance point. Thus, a line drawn straight downward will pass through the center of mass, and another line drawn straight downward when the figure is hanging from another point will also pass through the center of mass. The center of mass is where the lines cross.

***6** •

Determine the Concept No. Because the floor can exert no horizontal force, neither can the wall. Consequently, the friction force between the wall and the ladder is zero regardless of the coefficient of friction between the wall and the ladder.

7 •

Determine the Concept We know that equal lengths of aluminum and steel wire of the same diameter will stretch different amounts when subjected to the same tension. Also, because we are neglecting the mass of the wires, the tension in them is independent of which one is closer to the roof and depends only on W. $\boxed{(b) \text{ is correct.}}$

8 •

Determine the Concept Yes; if it were otherwise, angular momentum conservation would depend on the choice of coordinates.

***9** •

Determine the Concept The condition that the bar is in rotational equilibrium is that the net torque acting on it be zero; i.e., $R_1M_1 = R_2M_2$. This condition is satisfied provided $R_1 = R_2$ and $M_1 = M_2$. $\boxed{(c) \text{ is correct.}}$

10 ••

Determine the Concept You cannot stand up because your body's center of gravity must be above your feet.

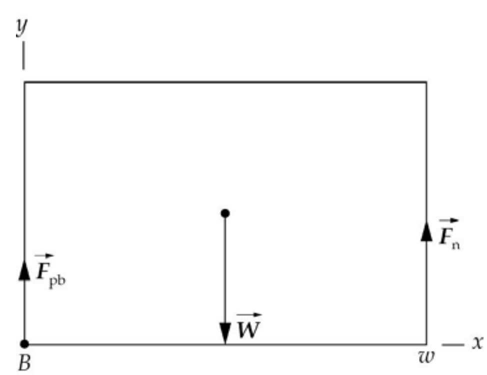
***11** ••

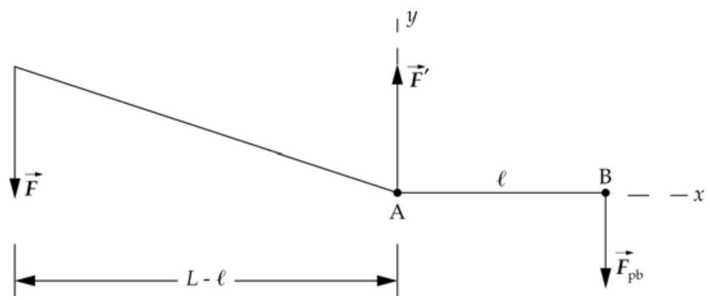
Determine the Concept The tensile strengths of stone and concrete are at least an order of magnitude lower than their compressive strengths, so you want to build compressive structures to match their properties.

Estimation and Approximation

12 ••

Picture the Problem The diagram to the right shows the forces acting on the crate as it is being lifted at its left end. Note that when the crowbar lifts the crate, only half the weight of the crate is supported by the bar. Choose the coordinate system shown and let the subscript ″pb″ refer to the pry bar. The diagram below shows the forces acting on the pry bar as it is being used to lift the end of the crate.





Assume that the maximum force F' you can apply is 500 N (about 110 lb). Let ℓ be the distance between the points of contact of the steel bar with the floor and the crate, and let L be the total length of the bar. Lacking information regarding the bend in pry bar at the fulcrum, we'll assume that it is small enough to be negligible. We can apply the condition for rotational equilibrium to the pry bar and a condition for translational equilibrium to the crate when its left end is on the verge of lifting.

Apply $\sum F_y = 0$ to the crate: $$F_{pb} - W + F_n = 0 \quad (1)$$

Apply $\sum \vec{\tau} = 0$ to the crate about an axis through point B and perpendicular to the plane of the page to obtain: $$wF_n - \tfrac{1}{2}wW = 0$$

Solve for F_n: $$F_n = \tfrac{1}{2}W$$ as noted in Picture the Problem.

Solve equation (1) for F_{pb} and substitute for F_n to obtain: $$F_{pb} = W - \tfrac{1}{2}W = \tfrac{1}{2}W$$

Apply $\sum \vec{\tau} = 0$ to the pry bar about an axis through point A and perpendicular to the plane of the page to obtain: $$F(L-\ell) - \ell F_{pb} = 0$$

Solve for L: $$L = \ell\left(1 + \frac{F_{pb}}{F}\right)$$

Substitute for F_{pb} to obtain: $$L = \ell\left(1 + \frac{W}{2F}\right)$$

Substitute numerical values and evaluate L:

$$L = (0.1\,\mathrm{m})\left(1 + \frac{4500\,\mathrm{N}}{2(500\,\mathrm{N})}\right) = \boxed{55.0\,\mathrm{cm}}$$

***13 ••**

Picture the Problem We can derive this expression by imagining that we pull on an area A of the given material, expressing the force each spring will experience, finding the fractional change in length of the springs, and substituting in the definition of Young's modulus.

(a) Express Young's modulus:

$$Y = \frac{F/A}{\Delta L/L} \qquad (1)$$

Express the elongation ΔL of each spring:

$$\Delta L = \frac{F_s}{k} \qquad (2)$$

Express the force F_s each spring will experience as a result of a force F acting on the area A:

$$F_s = \frac{F}{N}$$

Express the number of springs N in the area A:

$$N = \frac{A}{a^2}$$

Substitute to obtain:

$$F_s = \frac{Fa^2}{A}$$

Substitute in equation (2) to obtain, for the extension of one spring:

$$\Delta L = \frac{Fa^2}{kA}$$

Assuming that the springs extend/compress linearly, express the fractional extension of the springs:

$$\frac{\Delta L_{\mathrm{tot}}}{L} = \frac{\Delta L}{a} = \frac{1}{a}\frac{Fa^2}{kA} = \frac{Fa}{kA}$$

Substitute in equation (1) and simplify:

$$Y = \frac{\frac{F}{A}}{\frac{Fa}{kA}} = \boxed{\frac{k}{a}}$$

(b) From our result in part (a):

$$k = Ya$$

From Table 12-1:

$$Y = 200\mathrm{GN/m^2} = 2\times10^{11}\ \mathrm{N/m^2}$$

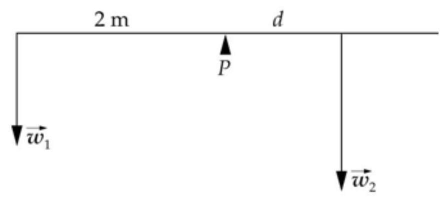
Assuming that a ~ 1 nm, evaluate k:

$$k = (2\times10^{11}\ \mathrm{N/m^2})(10^{-9}\ \mathrm{m}) = \boxed{200\,\mathrm{N/m}}$$

Conditions for Equilibrium

14 •

Picture the Problem Let w_1 represent the weight of the 28-kg child sitting at the left end of the board, w_2 the weight of the 40-kg child, and d the distance of the 40-kg child from the pivot point. We can apply the condition for rotational equilibrium to find d.



Apply $\sum \vec{\tau} = 0$ about an axis through the pivot point P:

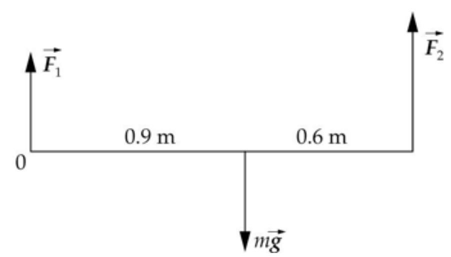
$$w_1(2\,m) - w_2 d = 0$$

Solve for and evaluate d:

$$d = \frac{w_1(2\,\text{m})}{w_2} = \frac{(28\,\text{kg})g(2\,\text{m})}{(40\,\text{kg})g} = \boxed{1.4\,\text{m}}$$

15 •

Picture the Problem Let F_1 represent the force exerted by the floor on Misako's feet, F_2 the force exerted on her hands, and m her mass. We can apply the condition for rotational equilibrium to find F_2.



Apply $\sum \vec{\tau} = 0$ about an axis through point 0:

$$F_2(1.5\,\text{m}) - mg(0.9\,\text{m}) = 0$$

Solve for F_2:

$$F_2 = \frac{mg(0.9\,\text{m})}{1.5\,\text{m}}$$

Substitute numerical values and evaluate F_2:

$$F_2 = \frac{(54\,\text{kg})(9.81\,\text{m/s}^2)(0.9\,\text{m})}{1.5\,\text{m}}$$

$$= \boxed{318\,\text{N}}$$

***16 •**

Picture the Problem Let F represent the force exerted by Misako's biceps. To find F we apply the condition for rotational equilibrium about a pivot chosen at the tip of her elbow.

Apply $\sum \vec{\tau} = 0$ about an axis

$$(5\,\text{cm})F - (28\,\text{cm})(18\,\text{N}) = 0$$

through the pivot:

Solve for F:

$$F = \frac{(28\,\text{cm})(18\,\text{N})}{5\,\text{cm}} = \boxed{101\,\text{N}}$$

17 •

Picture the Problem Choose a coordinate system in which upward is the positive y direction and to the right is the positive x direction and use the conditions for translational equilibrium.

(*a*) Apply $\sum \vec{F} = 0$ to the forces acting on the tip of the crutch:

$$\sum F_x = -f_s + F_c \sin\theta = 0 \quad (1)$$

and

$$\sum F_y = F_n - F_c \cos\theta = 0 \quad (2)$$

Solve equation (2) for F_n and assuming that $f_s = f_{s,\max}$, obtain:

$$f_s = f_{s,\max} = \mu_s F_n = \mu_s F_c \cos\theta$$

Substitute in equation (1) and solve for μ_s:

$$\mu_s = \boxed{\tan\theta}$$

(*b*) $\boxed{\text{Taking long strides requires a large coefficient of static friction because } \theta \text{ is large for long strides.}}$

(*c*) $\boxed{\text{If } \mu_s \text{ is small, i.e., there is ice on the surface, } \theta \text{ must be small to avoid slipping.}}$

The Center of Gravity

18 •

Picture the Problem Let the weight of the automobile be w. Choose a coordinate system in which the origin is at the point of contact of the front wheels with the ground and the positive x axis includes the point of contact of the rear wheels with the ground. Apply the definition of the center of gravity to find its location.

Use the definition of the center of gravity:

$$\begin{aligned} x_{cg}W &= \sum_i w_i x_i \\ &= 0.58w(0) + 0.42w(2\,\text{m}) \\ &= (0.84\,\text{m})w \end{aligned}$$

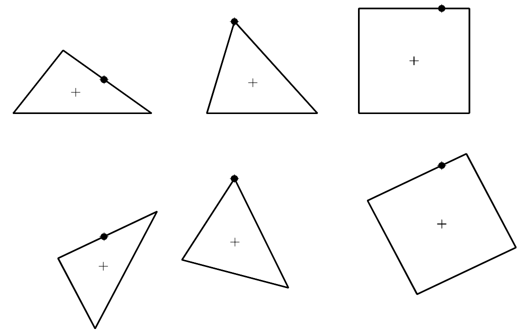
or, because $W = w$, $x_{cg}(w) = (0.84\,\text{m})w$

Solve for x_{cg}:

$$x_{cg} = \boxed{0.84\,\text{m}}$$

***19** •

Picture the Problem The figures are shown on the right. The center of mass for each is indicated by a small +. At static equilibrium, the center of gravity is directly below the point of support.



20 ••

Picture the Problem Using the coordinate system indicated in the figure, we can apply the definition of the center of gravity to determine x_{cg} and y_{cg}.

Apply the definition of the center of gravity to find x_{cg}:

$$\begin{aligned} x_{cg}W &= \sum_i w_i x_i \\ &= (40\,\text{N})\left(\tfrac{1}{2}a\right) + (60\,\text{N})\left(\tfrac{1}{2}a\right) \\ &\quad + (30\,\text{N})\left(\tfrac{3}{2}a\right) + (50\,\text{N})\left(\tfrac{3}{2}a\right) \\ &= (170\,\text{N})a \end{aligned}$$

or, because $W = 180$ N,

$$x_{cg}(180\,\text{N}) = (170\,\text{N})a$$

Solve for x_{cg}:

$$x_{cg} = \frac{170\,\text{N}}{180\,\text{N}}a = 0.944a$$

Apply the definition of the center of gravity to find y_{cg}:

$$\begin{aligned} y_{cg}W &= \sum_i w_i y_i \\ &= (40\,\text{N})\left(\tfrac{1}{2}a\right) + (60\,\text{N})\left(\tfrac{3}{2}a\right) \\ &\quad + (30\,\text{N})\left(\tfrac{3}{2}a\right) + (50\,\text{N})\left(\tfrac{1}{2}a\right) \\ &= (180\,\text{N})a \end{aligned}$$

or, because $W = 180$ N,

$$y_{cg}(180\,\text{N}) = (180\,\text{N})a$$

Solve for y_{cg}:

$$y_{cg} = a$$

The coordinates of the center of gravity are:

$$\left(x_{cg}, y_{cg}\right) = \boxed{(0.944a, a)}$$

21 ••

Picture the Problem Let the origin of the coordinate system be at the lower left corner of the plate and the positive x direction be to the right. Let a and b be the length and width of the plate. Let σ be the mass per unit area of the plate. Then the weight of the plate is given by $w = ab\sigma g$ and that of the matter missing from the hole is $-\pi R^2\sigma g$. Noting that, by symmetry, $y_{cg} = b/2$, we can apply the definition of the center of gravity to find x_{cg}.

Apply the definition of the center of gravity to find x_{cg}:

$$x_{cg}W = \sum_i w_i x_i$$
$$= (ab\sigma g)\left(\tfrac{1}{2}a\right) - \left(\pi R^2\sigma g\right)(a - R)$$

or, because

$$W = w_{plate} - w_{hole} = ab\sigma g - \pi R^2\sigma g,$$
$$x_{cg}\left(ab\sigma g - \pi R^2\sigma g\right) = (ab\sigma g)\left(\tfrac{1}{2}a\right) - \left(\pi R^2\sigma\right)(a - R)$$

Solve for x_{cg}:

$$x_{cg} = \frac{\frac{1}{2}a^2b - \pi aR^2 + \pi R^3}{ab - \pi R^2}$$

The coordinates of the center of gravity are:

$$\left(x_{cg}, y_{cg}\right) = \boxed{\left(\frac{\frac{1}{2}a^2b - \pi aR^2 + \pi R^3}{ab - \pi R^2}, \tfrac{1}{2}b\right)}$$

Some Examples of Static Equilibrium

22 •

Picture the Problem We can use the given definition of the mechanical advantage of a lever and the condition for rotational equilibrium to show that $M = x/X$.

(*a*) Express the definition of mechanical advantage for a lever:

$$M = \frac{F}{f}$$

Apply the condition for rotational equilibrium to the lever:

$$xf - XF = 0$$

Solve for the ratio of F to f to obtain:

$$\frac{F}{f} = \frac{x}{X}$$

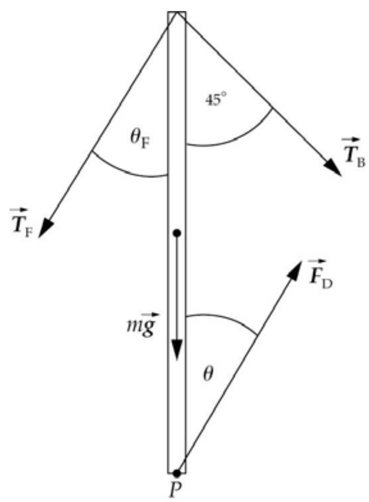
Substitute to obtain:

$$M = \boxed{\frac{x}{X}}$$

(*b*) A shorter moment arm for the applied force is useful when one wishes to move the load over a large distance using a short movement of the applied force.

23 •

Picture the Problem The force diagram shows the tension in the forestay, $\vec{T}_F$, the tension in the backstay, $\vec{T}_B$, the gravitational force on the mast $m\vec{g}$, and the force exerted by the deck, $\vec{F}_D$. Let the origin of the coordinate system be at the foot of the mast with the positive x direction to the right and the positive y direction upward. Because the mast is in equilibrium, we can apply the conditions for both translational and rotational equilibrium to find the tension in the backstay and the force that the deck exerts on the mast.



Apply $\sum\vec{\tau}=0$ to the mast about an axis through its foot and solve for T_B:

$$(4.88\,\text{m})(1000\,\text{N})\sin\theta_F - (4.88\,\text{m})T_B\sin 45° = 0$$

and

$$T_B = \frac{(1000\,\text{N})\sin\theta_F}{\sin 45°}$$

Find θ_F, the angle of the forestay with the vertical:

$$\theta_F = \tan^{-1}\left(\frac{2.74\,\text{m}}{4.88\,\text{m}}\right) = 29.3°$$

Substitute to obtain:

$$T_B = \frac{(1000\,\text{N})\sin 29.3°}{\sin 45°} = \boxed{692\,\text{N}}$$

Apply the condition for translational equilibrium in the x direction to the mast:

$$\sum F_x = F_D\cos\theta + T_B\sin 45° - T_F\sin\theta_F = 0$$

or

$$F_D\cos\theta = (1000\,\text{N})\sin 29.3° - (692\,\text{N})\sin 45° \approx 0$$

Apply the condition for translational equilibrium in the y direction to the mast:

$$\sum F_y = F_D\sin\theta - T_F\cos\theta_F - T_B\cos 45° - mg = 0$$

or

$$F_D \sin\theta = (1000\,\text{N})\cos 29.3° + (692\,\text{N})\cos 45° + (120\,\text{kg})(9.81\,\text{m/s}^2) = 2539\,\text{N}$$

Because $F_D\cos\theta = 0$:

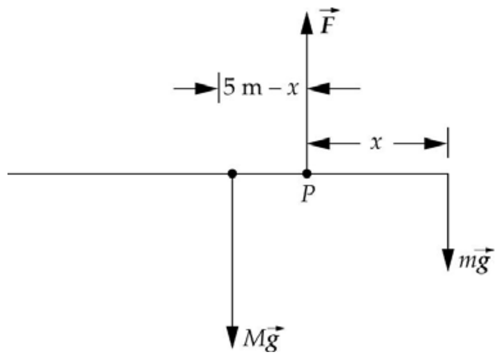
$$\theta = \boxed{90°},\ F_D = \boxed{2.54\,\text{kN}}$$

and

$\boxed{\text{no block is required to prevent the mast from moving.}}$

24 ••

Picture the Problem The diagram shows $M\vec{g}$, the weight of the beam, $m\vec{g}$, the weight of the student, and the force the ledge exerts $\vec{F}$, acting on the beam. Because the beam is in equilibrium, we can apply the condition for rotational equilibrium to the beam to find the location of the pivot point P that will allow the student to walk to the end of the beam.



Apply $\sum\vec{\tau} = 0$ about an axis through the pivot point P:

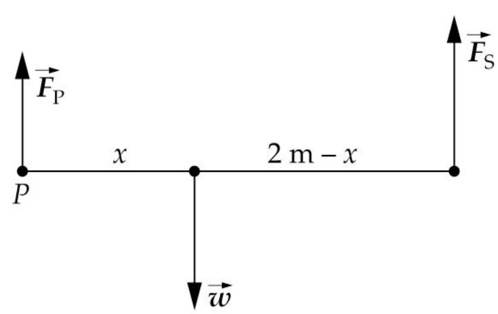
$$Mg(5\,\text{m} - x) - mgx = 0$$

Solve for x:

$$x = \frac{5M}{M+m} = \frac{5(300\,\text{kg})}{300\,\text{kg} + 60\,\text{kg}} = \boxed{4.17\,\text{m}}$$

***25** ••

Picture the Problem The diagram shows $\vec{w}$, the weight of the student, $\vec{F}_P$, the force exerted by the board at the pivot, and $\vec{F}_s$, the force exerted by the scale, acting on the student. Because the student is in equilibrium, we can apply the condition for rotational equilibrium to the student to find the location of his center of gravity.



Apply $\sum\vec{\tau} = 0$ about an axis

$$F_s(2\,\text{m}) - wx = 0$$

through the pivot point P:

Solve for x:

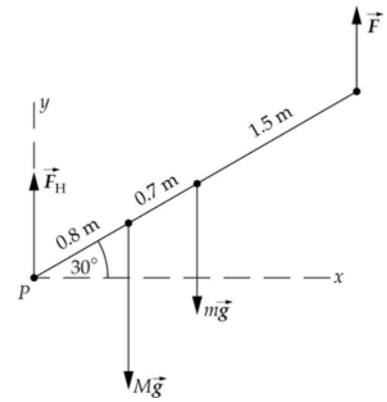
$$x = \frac{(2\,\text{m})F_s}{w}$$

Substitute numerical values and evaluate x:

$$x = \frac{(2\,\text{m})(250\,\text{N})}{(70\,\text{kg})(9.81\,\text{m/s}^2)} = \boxed{0.728\,\text{m}}$$

26 ••

Picture the Problem The diagram shows $m\vec{g}$, the weight of the board, $\vec{F}_H$, the force exerted by the hinge, $M\vec{g}$, the weight of the block, and $\vec{F}$, the force acting vertically at the right end of the board. Because the board is in equilibrium, we can apply the condition for rotational equilibrium to it to find the magnitude of $\vec{F}$.



(*a*) Apply $\sum \vec{\tau} = 0$ about an axis through the hinge:

$$F[(3\,\text{m})\cos 30°] - mg[(1.5\,\text{m})\cos 30°] - Mg[(0.8\,\text{m})\cos 30°] = 0$$

Solve for F:

$$F = \frac{m(1.5\,\text{m}) + M(0.8\,\text{m})}{3\,\text{m}}g$$

Substitute numerical values and evaluate F:

$$F = \frac{(5\,\text{kg})(1.5\,\text{m}) + (60\,\text{kg})(0.8\,\text{m})}{3\,\text{m}} \times (9.81\,\text{m/s}^2) = \boxed{181\,\text{N}}$$

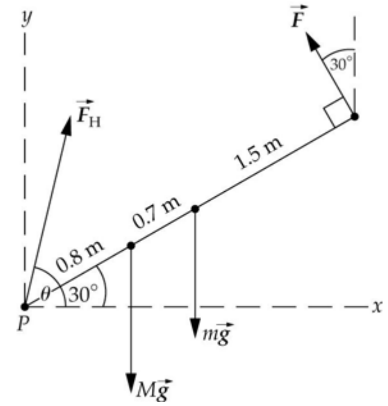
(*b*) Apply $\sum F_y = 0$ to the board:

$$F_H - Mg - mg + F = 0$$

Solve for and evaluate F_H:

$$F_H = Mg + mg - F = (M + m)g - F = (60\,\text{kg} + 5\,\text{kg})(9.81\,\text{m/s}^2) - 181\,\text{N} = \boxed{457\,\text{N}}$$

(*c*) The force diagram showing the force $\vec{F}$ acting at right angles to the board is shown to the right:



Apply $\sum \vec{\tau} = 0$ about the hinge:

$$F(3\,\mathrm{m}) - mg[(1.5\,\mathrm{m})\cos 30°] - Mg[(0.8\,\mathrm{m})\cos 30°] = 0$$

Solve for *F*:

$$F = \frac{m(1.5\,\mathrm{m}) + M(0.8\,\mathrm{m})}{3\,\mathrm{m}} g\cos 30°$$

Substitute numerical values and evaluate *F*:

$$F = \frac{(5\,\mathrm{kg})(1.5\,\mathrm{m}) + (60\,\mathrm{kg})(0.8\,\mathrm{m})}{3\,\mathrm{m}} \times (9.81\,\mathrm{m/s^2})\cos 30° = \boxed{157\,\mathrm{N}}$$

Apply $\sum F_y = 0$ to the board:

$$F_\mathrm{H}\sin\theta - Mg - mg + F\cos 30° = 0$$

or

$$F_\mathrm{H}\sin\theta = (M+m)g - F\cos 30° \quad (1)$$

Apply $\sum F_x = 0$ to the board:

$$F_\mathrm{H}\cos\theta - F\sin 30° = 0$$

or

$$F_\mathrm{H}\cos\theta = F\sin 30° \quad (2)$$

Divide the first of these equations by the second to obtain:

$$\frac{F_\mathrm{H}\sin\theta}{F_\mathrm{H}\cos\theta} = \frac{(M+m)g - F\cos 30°}{F\sin 30°}$$

Solve for θ:

$$\theta = \tan^{-1}\left[\frac{(M+m)g - F\cos 30°}{F\sin 30°}\right]$$

Substitute numerical values and evaluate θ:

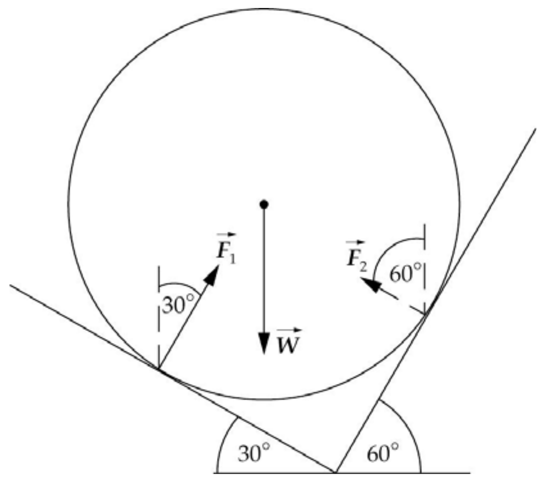
$$\theta = \tan^{-1}\left[\frac{(65\,\text{kg})(9.81\,\text{m/s}^2) - (157\,\text{N})\cos 30°}{(157\,\text{N})\sin 30°}\right] = 81.1°$$

Substitute numerical values in equation (2) and evaluate F_H:

$$F_H = \frac{(157\,\text{N})\sin 30°}{\cos 81.1°} = \boxed{507\,\text{N}}$$

***27 •**

Picture the Problem The planes are frictionless; therefore, the force exerted by each plane must be perpendicular to that plane. Let $\vec{F}_1$ be the force exerted by the 30° plane, and let $\vec{F}_2$ be the force exerted by the 60° plane. Choose a coordinate system in which the positive x direction is to the right and the positive y direction is upward. Because the cylinder is in equilibrium, we can use the conditions for translational equilibrium to find the magnitudes of $\vec{F}_1$ and $\vec{F}_2$.



Apply $\sum F_x = 0$ to the cylinder:

$$F_1 \sin 30° - F_2 \sin 60° = 0 \quad (1)$$

Apply $\sum F_y = 0$ to the cylinder:

$$F_1 \cos 30° + F_2 \cos 60° - W = 0 \quad (2)$$

Solve equation (1) for F_1:

$$F_1 = \sqrt{3}F_2 \quad (3)$$

Substitute in equation (2) to obtain:

$$\sqrt{3}F_2 \cos 30° + F_2 \cos 60° - W = 0$$

Solve for F_2:

$$\left(\sqrt{3}\cos 30° + \cos 60°\right)F_2 = W$$

or

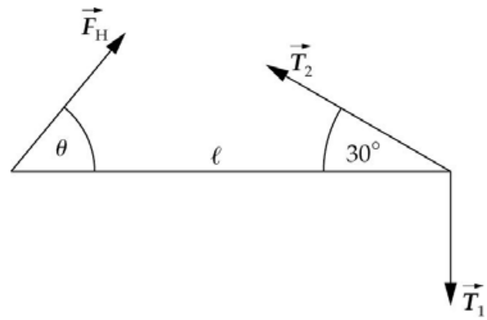
$$F_2 = \frac{W}{\sqrt{3}\cos 30° + \cos 60°} = \boxed{\tfrac{1}{2}W}$$

Substitute in equation (3):

$$F_1 = \sqrt{3}\left(\tfrac{1}{2}W\right) = \boxed{\tfrac{\sqrt{3}}{2}W}$$

28 ••

Picture the Problem The force diagram shows the forces $\vec{F}_H$, $\vec{T}_2$, and $\vec{T}_1$ acting on the strut. Choose a coordinate system in which the positive x direction is to the right and the positive y direction is upward. Because the strut is in equilibrium, we can apply the conditions for translational and rotational equilibrium to it.



(*a*) $\boxed{\text{The forces acting on the strut are the tensions } \vec{T}_1 \text{ and } \vec{T}_2 \text{ and } \vec{F}_H\text{, the force exerted on the strut by the hinge.}}$

(*b*) Apply $\sum \vec{\tau} = 0$ about an axis through the hinge:

$$T_2 \ell \sin 30° - T_1 \ell = 0$$

Solve for T_1:

$$T_{2v} = T_2 \sin 30° = T_1$$

or, because $T_1 = 80$ N,

$$T_{2v} = \boxed{80\text{ N}}$$

(*c*) Apply $\sum F_x = 0$ to the beam:

$$F_H \cos\theta - T_2 \cos 30° = 0$$

or

$$F_H \cos\theta = T_2 \cos 30° \quad (1)$$

Apply $\sum F_y = 0$ to the beam:

$$F_H \sin\theta + T_2 \sin 30° - T_1 = 0$$

or

$$F_H \sin\theta = T_1 - T_2 \sin 30° = 80\text{ N} - T_2 \sin 30° \quad (2)$$

Divide equation (2) by equation (1) to obtain:

$$\tan\theta = \frac{80\text{ N} - T_2 \sin 30°}{T_2 \cos 30°}$$

Solve for θ:

$$\theta = \tan^{-1}\left[\frac{80\text{ N} - T_2 \sin 30°}{T_2 \cos 30°}\right]$$

Express T_2 in terms of T_{2v}:

$$T_2 = \frac{T_{2v}}{\sin 30°} = \frac{80\text{ N}}{\sin 30°} = 160\text{ N}$$

Evaluate θ:

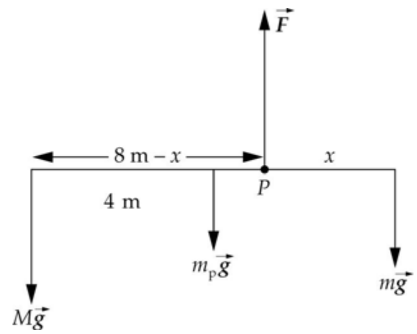
$$\theta = \tan^{-1}\left[\frac{80\,\text{N}-(160\,\text{N})\sin 30°}{(160\,\text{N})\cos 30°}\right] = 0°$$

Substitute numerical values in equation (1) and evaluate F_H:

$$F_H = \frac{(160\,\text{N})\cos 30°}{\cos 0°} = \boxed{139\,\text{N}}$$ to the right.

29 ••

Picture the Problem The force diagram shows the weight of the pirate, $M\vec{g}$, the weight of the victim, $m\vec{g}$, and the force the deck exerts at the edge of the ship, $\vec{F}$ acting at the fulcrum P. The diagram also shows, for part (b), the weight of the plank acting through the plank's center of gravity.



(a) Apply $\sum \vec{\tau} = 0$ at the pivot point P:

$$Mg(8\,\text{m}-x) - mgx = 0$$

or

$$M(8\,\text{m}-x) - mx = 0$$

Solve for x:

$$x = \frac{8M}{M+m} = \frac{8(105\,\text{kg})}{105\,\text{kg}+63\,\text{kg}} = \boxed{5.00\,\text{m}}$$

(b) Apply $\sum \vec{\tau} = 0$ about an axis through the pivot point P:

$$Mg(8\,\text{m}-x) + m_p g(4\,\text{m}-x) - mgx = 0$$

or

$$M(8\,\text{m}-x) + m_p(4\,\text{m}-x) - mx = 0$$

Solve for x:

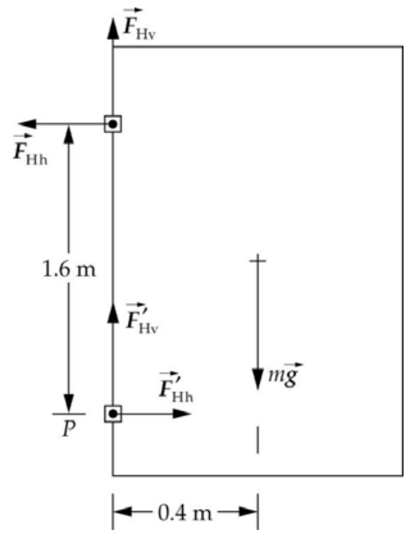
$$x = \frac{8M + 4m_p}{M+m+m_p}$$

Substitute numerical values and evaluate x:

$$x = \frac{8(105\,\text{kg}) + 4(25\,\text{kg})}{105\,\text{kg}+63\,\text{kg}+25\,\text{kg}} = \boxed{4.87\,\text{m}}$$

30 ••

Picture the Problem The drawing shows the door and its two supports. The center of gravity of the door is 0.8 m above (and below) the hinge, and 0.4 m from the hinges horizontally. Choose a coordinate system in which the positive x direction is to the right and the positive y direction is upward. Denote the horizontal and vertical components of the hinge force by F_{Hh} and F_{Hv}. Because the door is in equilibrium, we can use the conditions for translational and rotational equilibrium to determine the horizontal forces exerted by the hinges.



Apply $\sum \vec{\tau} = 0$ about an axis through the lower hinge:

$$F_{Hh}(1.6\,\text{m}) - mg(0.4\,\text{m}) = 0$$

Solve for F_{Hh}:

$$F_{Hh} = \frac{mg(0.4\,\text{m})}{1.6\,\text{m}}$$

Substitute numerical values and evaluate F_{Hh}:

$$F_{Hh} = \frac{(18\,\text{kg})(9.81\,\text{m/s}^2)(0.4\,\text{m})}{1.6\,\text{m}} = \boxed{44.1\,\text{N}}$$

Apply $\sum F_x = 0$ to the door and solve for F'_{Hh}:

$$F'_{Hh} - F_{Hh} = 0$$

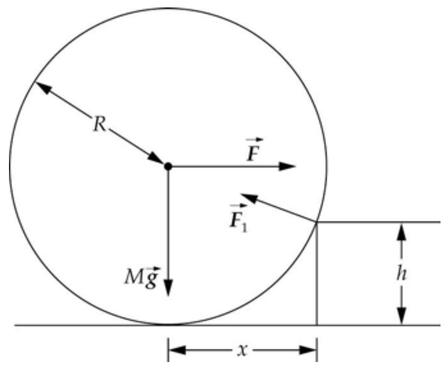
and

$$F'_{Hh} = \boxed{44.1\,\text{N}}$$

Note that the upper hinge pulls on the door and the lower hinge pushes on it.

31 ••

Picture the Problem The figure shows the wheel on the verge of rolling over the edge of the step. Note that, under this condition, the normal force the floor exerts on the wheel is zero. Choose the coordinate system shown in the figure and apply the conditions for translational equilibrium and the result for F from Example 12-4 to the wheel.



Apply $\sum \vec{F} = 0$ to the wheel:

$$\sum F_x = F - F_{1x} = 0$$

and

$$\sum F_y = F_{1y} - Mg = 0$$

Write $\vec{F}_1$ in vector form:

$$\vec{F}_1 = -F_{1x}\hat{\boldsymbol{i}} + F_{1y}\hat{\boldsymbol{j}} = -F\hat{\boldsymbol{i}} + Mg\hat{\boldsymbol{j}}$$

From Example 12-4 we have:

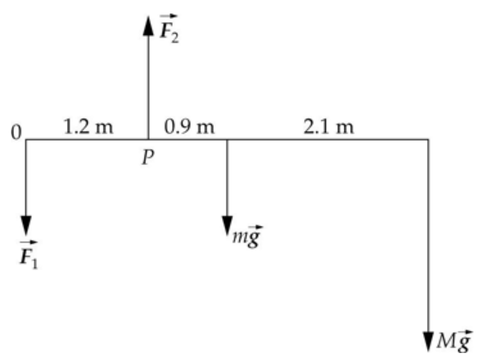
$$F = \frac{Mg\sqrt{h(2R-h)}}{R-h}$$

Substitute to obtain:

$$\vec{F}_1 = -\frac{Mg\sqrt{h(2R-h)}}{R-h}\hat{\boldsymbol{i}} + Mg\hat{\boldsymbol{j}} = \boxed{\frac{Mg\sqrt{h(2R-h)}}{h-R}\hat{\boldsymbol{i}} + Mg\hat{\boldsymbol{j}}}$$

32 ••

Picture the Problem The diagram shows the forces $\vec{F}_1$ and $\vec{F}_2$ acting at the supports, the weight of the board $m\vec{g}$, acting at its center of gravity, and the weight of the diver $M\vec{g}$ acting at the end of the diving board. Because the board is in equilibrium, we can apply the condition for rotational equilibrium to find the forces at the supports.



Apply $\sum \vec{\tau} = 0$ about an axis through the left support:

$$(1.2\,\text{m})F_2 - (2.1\,\text{m})mg - (4.2\,\text{m})Mg = 0$$

Solve for F_2:

$$F_2 = \frac{(2.1\,\text{m})m + (4.2\,\text{m})M}{(1.2\,\text{m})}g$$

Substitute numerical values and evaluate F_2:

$$F_2 = \frac{(2.1\,\text{m})(30\,\text{kg}) + (4.2\,\text{m})(70\,\text{kg})}{(1.2\,\text{m})}(9.81\,\text{m/s}^2) = \boxed{2.92\,\text{kN, compression}}$$

Apply $\sum \vec{\tau} = 0$ about an axis through the right support:

$$(1.2\,\text{m})F_1 - (0.9\,\text{m})mg - (3\,\text{m})Mg = 0$$

Solve for F_1:

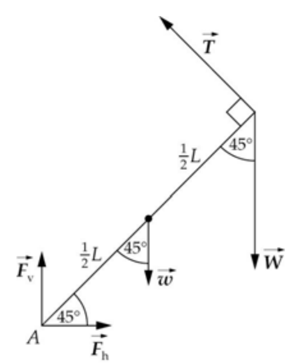
$$F_1 = \frac{(0.9\,\text{m})m + (3\,\text{m})M}{(1.2\,\text{m})}g$$

Substitute numerical values and evaluate F_1:

$$F_1 = \frac{(0.9\,\text{m})(30\,\text{kg}) + (3\,\text{m})(70\,\text{kg})}{(1.2\,\text{m})}(9.81\,\text{m/s}^2) = \boxed{1.94\,\text{kN, tension}}$$

33 ••

Picture the Problem Let T be the tension in the line attached to the wall and L be the length of the strut. The figure includes w, the weight of the strut, for part (b). Because the strut is in equilibrium, we can use the conditions for both rotational and translational equilibrium to find the force exerted on the strut by the hinge.



(a) Express the force exerted on the strut at the hinge:

$$\vec{F} = F_h\hat{i} + F_v\hat{j} \qquad (1)$$

Ignoring the weight of the strut, apply $\sum \vec{\tau} = 0$ at the hinge:

$$LT - (L\cos 45°)W = 0$$

Solve for the tension in the line:

$$T = W\cos 45° = (60\,\text{N})\cos 45° = 42.43\,\text{N}$$

Apply $\sum \vec{F} = 0$ to the strut:

$$\sum F_x = F_h - T\cos 45° = 0$$

and

$$\sum F_y = F_v + T\cos 45° - Mg = 0$$

Solve for F_h:

$$T_h = T\cos 45° = (42.43\,\text{N})\cos 45° = 30.0\,\text{N}$$

Solve for F_v:

$$F_v = Mg - T\cos 45° = 60\,\text{N} - (42.43\,\text{N})\cos 45° = 30.0\,\text{N}$$

Substitute in equation (1) to obtain:

$$\vec{F} = \boxed{(30.0\,\text{N})\hat{\boldsymbol{i}} + (30.0\,\text{N})\hat{\boldsymbol{j}}}$$

(b) Including the weight of the strut, apply $\sum \vec{\boldsymbol{\tau}} = 0$ at the hinge:

$$LT - (L\cos 45°)W - \left(\frac{L}{2}\cos 45\right)w = 0$$

Solve for the tension in the line:

$$T = (\cos 45°)W + \left(\frac{1}{2}\cos 45°\right)w = (\cos 45°)(60\,\text{N}) + \left(\frac{1}{2}\cos 45°\right)(20\,\text{N}) = 49.5\,\text{N}$$

Apply $\sum \vec{\boldsymbol{F}} = 0$ to the strut:

$$\sum F_x = F_h - T\cos 45° = 0$$

and

$$\sum F_y = F_v + T\cos 45° - W - w = 0$$

Solve for F_h:

$$T_h = T\cos 45° = (49.5\,\text{N})\cos 45° = 35.0\,\text{N}$$

Solve for F_v:

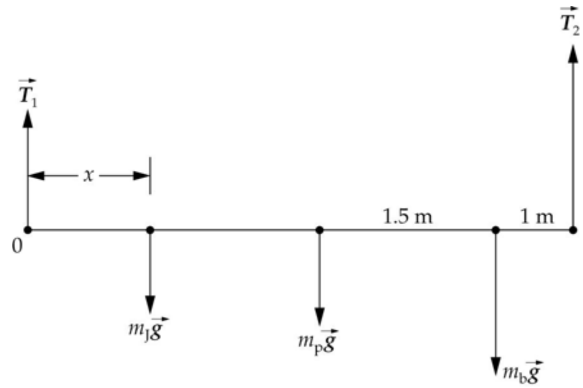
$$F_v = W + w - T\cos 45° = 60\,\text{N} + 20\,\text{N} - (49.5\,\text{N})\cos 45° = 45.0\,\text{N}$$

Substitute in equation (1) to obtain:

$$\vec{F} = \boxed{(35.0\,\text{N})\hat{\boldsymbol{i}} + (45.0\,\text{N})\hat{\boldsymbol{j}}}$$

34 ••

Picture the Problem Note that if the 60-kg mass is at the far left end of the plank, T_1 and T_2 are less than 1 kN. Let x be the distance of the 60-kg mass from T_1. Because the plank is in equilibrium, we can apply the condition for rotational equilibrium to relate the distance x to the other distances and forces.



Apply $\sum\vec{\tau} = 0$ about an axis through the left end of the plank:

$$(5\,\text{m})T_2 - (4\,\text{m})m_b g - (2.5\,\text{m})m_p g - m_J g x = 0$$

Solve for x:

$$x = \frac{(5\,\text{m})T_2 - (4\,\text{m})m_b g - (2\,\text{m})m_p g}{m_J g}$$

Substitute numerical values and simplify to obtain:

$$x = \frac{(5\,\text{m})T_2 - 3.63\,\text{kN}\cdot\text{m}}{0.5886\,\text{kN}}$$

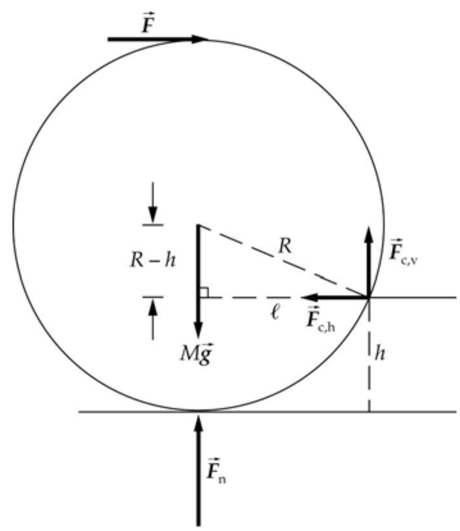
Set T_2 = 1 kN and evaluate x:

$$x = \frac{(5\,\text{m})(1\,\text{kN}) - 3.63\,\text{kN}\cdot\text{m}}{0.5886\,\text{kN}} = 2.33\,\text{m}$$

and $\boxed{\text{Julie is safe for } 0 < x < 2.33\,\text{m.}}$

35 ••

Picture the Problem The figure to the right shows the forces acting on the cylinder. Choose a coordinate system in which the positive x direction is to the right and the positive y direction is upward. Because the cylinder is in equilibrium, we can apply the conditions for translational and
rotational equilibrium to find F_n and the horizontal and vertical components of the force the corner of the step exerts on the cylinder.



(a) Apply $\sum\vec{\tau}=0$ to the cylinder about the step's corner:

$$Mg\ell - F_n\ell - F(2R-h) = 0$$

Solve for F_n:

$$F_n = Mg - \frac{F(2R-h)}{\ell}$$

Express ℓ as a function of R and h:

$$\ell = \sqrt{R^2-(R-h)^2} = \sqrt{2Rh-h^2}$$

$$F_n = Mg - \frac{F(2R-h)}{\sqrt{2Rh-h^2}}$$

$$= \boxed{Mg - F\sqrt{\frac{2R-h}{h}}}$$

(b) Apply $\sum F_x = 0$ to the cylinder:

$$-F_{c,h} + F = 0$$

Solve for $F_{c,h}$:

$$F_{c,h} = \boxed{F}$$

(c) Apply $\sum F_y = 0$ to the cylinder:

$$F_n - Mg + F_{c,v} = 0$$

Solve for $F_{c,v}$:

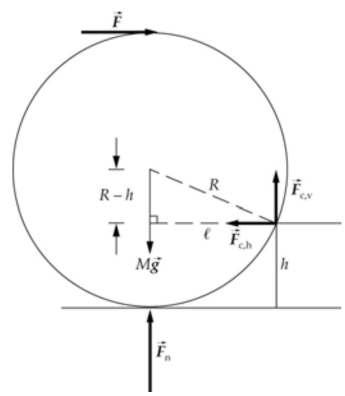
$$F_{c,v} = Mg - F_n$$

Substitute the result from part (*a*):

$$F_{c,v} = Mg - \left\{Mg - F\sqrt{\frac{2R-h}{h}}\right\} = \boxed{F\sqrt{\frac{2R-h}{h}}}$$

36 ••

Picture the Problem The figure to the right shows the forces acting on the cylinder. Because the cylinder is in equilibrium, we can use the condition for rotational equilibrium to express F_n in terms of F. Because, to roll over the step, the cylinder must lift off the floor, we can set $F_n = 0$ in our expression relating F_n and F and solve for F.



Apply $\sum\vec{\tau} = 0$ about the step's corner:

$$Mg\ell - F_n\ell - F(2R-h) = 0$$

Solve for F_n:

$$F_n = Mg - \frac{F(2R-h)}{\ell}$$

Express ℓ as a function of R and h:

$$\ell = \sqrt{R^2-(R-h)^2} = \sqrt{2Rh-h^2}$$

Substitute to obtain:

$$F_n = Mg - \frac{F(2R-h)}{\sqrt{2Rh-h^2}} = Mg - F\sqrt{\frac{2R-h}{h}}$$

To roll over the step, the cylinder must lift off the floor, i.e., $F_n = 0$:

$$0 = Mg - F\sqrt{\frac{2R-h}{h}}$$

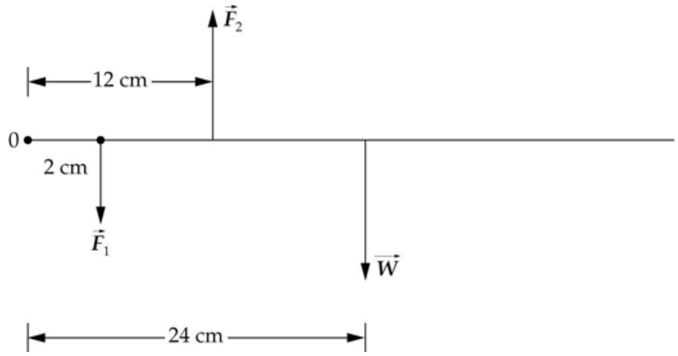
Solve for F:

$$F = \boxed{Mg\sqrt{\frac{h}{2R-h}}}$$

***37** ••

Picture the Problem The diagram shows the forces F_1 and F_2 that the fencer's hand exerts on the epee. We can use a condition for translational equilibrium to find the upward force the fencer must exert on the epee when it is in equilibrium and the definition of torque to determine the total torque exerted. In part (c) we can use the conditions for translational and rotational equilibrium to obtain two equations in F_1 and

F_2 that we can solve simultaneously. In part (*d*) we can apply Newton's 2nd law in rotational form and the condition for translational equilibrium to obtain two equations in F_1 and F_2 that, again, we can solve simultaneously.



(*a*) Letting the upward force exerted by the fencer's hand be *F*, apply $\sum F_y = 0$ to the epee to obtain:

$$F - W = 0$$

Solve for and evaluate *F*:

$$F = mg = (0.7\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{6.87\,\text{N}}$$

(*b*) Express the torque due to the weight about the left end of the epee:

$$\tau = \ell w = (0.24\,\text{m})(6.87\,\text{N}) = \boxed{1.65\,\text{N}\cdot\text{m}}$$

(*c*) Apply $\sum F_y = 0$ to the epee to obtain:

$$-F_1 + F_2 - 6.87\,\text{N} = 0 \qquad (1)$$

Apply $\sum \tau_0 = 0$ to obtain:

$$-(0.02\,\text{m})F_1 + (0.12\,\text{m})F_2 - 1.65\,\text{N}\cdot\text{m} = 0$$

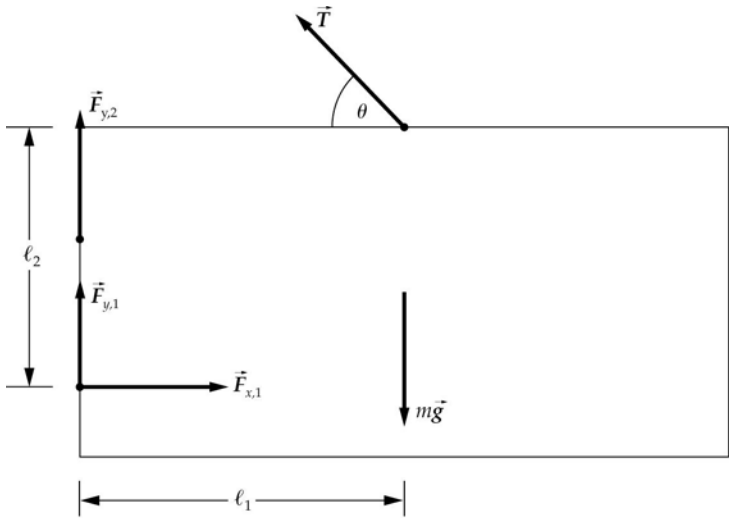
Solve these equations simultaneously to obtain:

$F_1 = \boxed{8.26\,\text{N}}$ and $F_2 = \boxed{15.1\,\text{N}}$.

Note that the force nearest the butt of the epee is directed downward and the force nearest the hand guard is directed upward.

38 ••

Picture the Problem In the force diagram, the forces exerted by the hinges are $\vec{F}_{y,2}$, $\vec{F}_{y,1}$, and $\vec{F}_{x,1}$ where the subscript 1 refers to the lower hinge. Because the gate is in equilibrium, we can apply the conditions for translational and rotational equilibrium to find the tension in the wire and the forces at the hinges.



(*a*) Apply $\sum \vec{\tau} = 0$ about an axis through the lower hinge and perpendicular to the plane of the page:

$$\ell_1 T \sin\theta + \ell_2 T \cos\theta - \ell_1 mg = 0$$

Solve for T:

$$T = \frac{\ell_1 mg}{\ell_1 \sin\theta + \ell_2 \cos\theta}$$

Substitute numerical values and evaluate T:

$$T = \frac{(1.5\,\text{m})(200\,\text{N})}{(1.5\,\text{m})\sin 45° + (1.5\,\text{m})\cos 45°} = \boxed{141\,\text{N}}$$

(*b*) Apply $\sum F_x = 0$ to the gate:

$$F_{x,1} - T\cos 45° = 0$$

Solve for and evaluate $F_{x,1}$:

$$F_{x,1} = T\cos 45° = (141\,\text{N})\cos 45° = \boxed{99.7\,\text{N}}$$

(*c*) Apply $\sum F_y = 0$ to the gate:

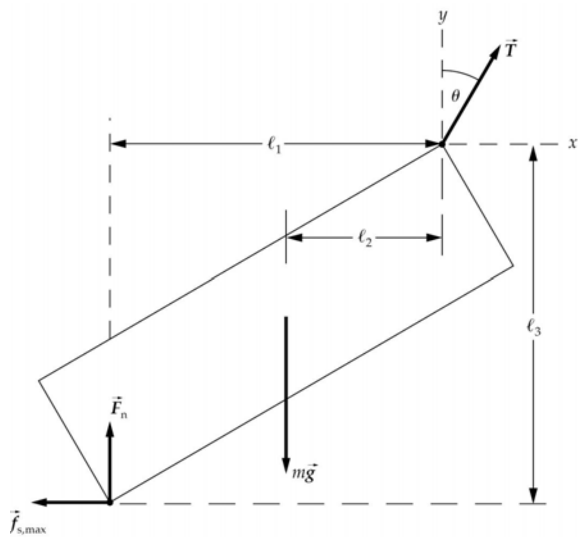
$$F_{y,1} + F_{y,2} + T\sin 45° - mg = 0$$

Because $F_{y,1}$ and $F_{y,2}$ cannot be determined independently, solve for and evaluate their sum:

$$F_{y,1} + F_{y,2} = mg - T\sin 45° = 200\,\text{N} - 99.7\,\text{N} = \boxed{100\,\text{N}}$$

39 •••

Picture the Problem Let T = the tension in the wire; F_n = the normal force of the surface; and $f_{s,max} = \mu_s F_n$ the maximum force of static friction. Letting the point at which the wire is attached to the log be the origin, the center of mass of the log is at (–1.838 m, –0.797 m) and the point of contact with the floor is at (–3.676 m, –1.594 m). Because the log is in equilibrium, we can apply the conditions for translational and rotational equilibrium.



Apply $\sum F_x = 0$ to the log:

$$T\sin\theta - f_{s,max} = 0$$

or

$$T\sin\theta = f_{s,max} = \mu_s F_n \qquad (1)$$

Apply $\sum F_y = 0$ to the log:

$$T\cos\theta + F_n - mg = 0$$

or

$$T\cos\theta = mg - F_n \qquad (2)$$

Divide equation (1) by equation (2) to obtain:

$$\frac{T\sin\theta}{T\cos\theta} = \frac{\mu_s F_n}{mg - F_n}$$

or

$$\theta = \tan^{-1}\frac{\mu_s}{\dfrac{mg}{F_n} - 1} \qquad (3)$$

Apply $\sum \vec{\tau} = 0$ about an axis through the origin:

$$\ell_2 mg - \ell_1 F_n - \ell_3 \mu_s F_n = 0$$

Solve for F_n:

$$F_n = \frac{\ell_2 mg}{\ell_1 + \ell_3 \mu_s}$$

Substitute numerical values and evaluate F_n:

$$F_n = \frac{1.838(100\,\text{kg})(9.81\,\text{m/s}^2)}{3.676 + 1.594(0.6)} = 389\,\text{N}$$

Substitute in equation (3) and evaluate θ:

$$\theta = \tan^{-1}\frac{0.6}{\dfrac{(100\,\text{kg})(9.81\,\text{m/s}^2)}{389\,\text{N}} - 1}$$

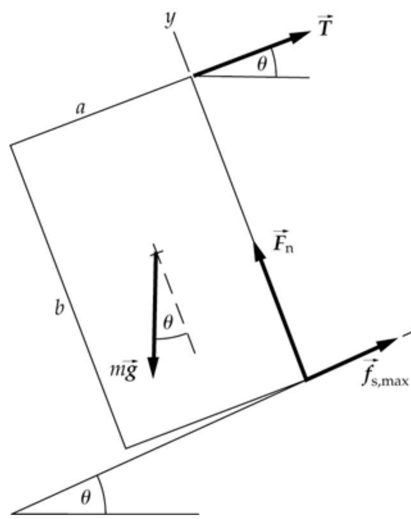
$$= \boxed{21.5°}$$

Substitute numerical values in equation (1) and evaluate T:

$$T = \frac{(0.6)(389\,\text{N})}{\sin 21.5°} = \boxed{636\,\text{N}}$$

40 •••

Picture the Problem Consider what happens just as θ increases beyond θ_{max}. Because the top of the block is fixed by the cord, the block will in fact rotate with only the lower right edge of the block remaining in contact with the plane. It follows that just prior to this slipping, F_n and $f_s = \mu_s F_n$ act at the lower right edge of the block. Choose a coordinate system in which up the incline is the positive x direction and the direction of $\vec{F}_n$ is the positive y direction. Because the block is in equilibrium, we can apply the conditions for translational and rotational equilibrium.



Apply $\sum F_x = 0$ to the block:

$$T + \mu_s F_n - mg\sin\theta = 0 \qquad (1)$$

Apply $\sum F_y = 0$ to the block:

$$F_n - mg\cos\theta = 0 \qquad (2)$$

Apply $\sum \vec{\tau} = 0$ about an axis through the lower right edge of the block:

$$\tfrac{1}{2}a(mg\cos\theta) + \tfrac{1}{2}b(mg\sin\theta) - bT = 0 \qquad (3)$$

Eliminate F_n between equations (1) and (2) and solve for T:

$$T = mg(\sin\theta - \mu_s\cos\theta)$$

Substitute for T in equation (3):

$$\tfrac{1}{2}a(mg\cos\theta)+\tfrac{1}{2}b(mg\sin\theta)-b[mg(\sin\theta-\mu_s\cos\theta)]=0$$

Substitute $4a$ for b:

$$\tfrac{1}{2}a(mg\cos\theta)+\tfrac{1}{2}(4a)(mg\sin\theta)-(4a)[mg(\sin\theta-\mu_s\cos\theta)]=0$$

Simplify to obtain:

$$(1+8\mu_s)\cos\theta-4\sin\theta=0$$

Solve for θ:

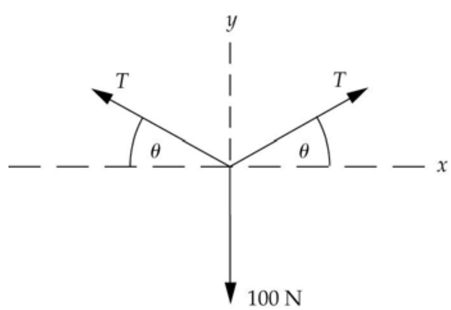
$$\theta=\tan^{-1}\frac{1+8\mu_s}{4}$$

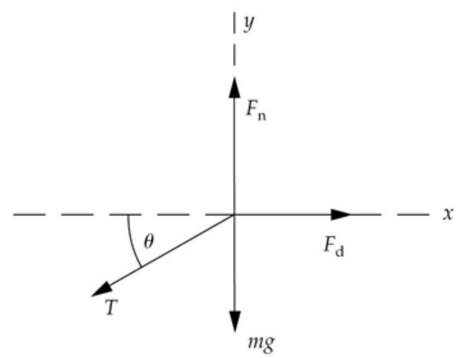
Substitute numerical values and evaluate θ:

$$\theta=\tan^{-1}\frac{1+8(0.8)}{4}=\boxed{61.6^\circ}$$

***41 ••**

Picture the Problem The free-body diagram shown to the left below is for the weight and the diagram to the right is for the boat. Because both are in equilibrium under the influences of the forces acting on them, we can apply a condition for translational equilibrium to find the tension in the chain.





(*a*) Apply $\sum F_x=0$ to the boat:

$$F_d-T\cos\theta=0$$

Solve for T:

$$T=\frac{F_d}{\cos\theta}$$

Apply $\sum F_y=0$ to the weight:

$$2T\sin\theta-100\text{ N}=0 \qquad (1)$$

Substitute for T to obtain:

$$2F_d\tan\theta-100\text{ N}=0$$

Solve for θ:

$$\theta=\tan^{-1}\frac{100\text{ N}}{2F_d}$$

Substitute for F_d and evaluate θ:

$$\theta = \tan^{-1}\frac{100\text{ N}}{2(50\text{ N})} = 45°$$

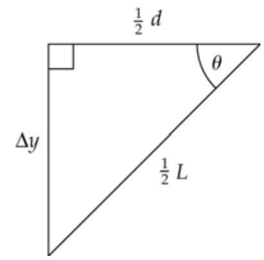
Solve equation (1) for T:

$$T = \frac{100\text{ N}}{2\sin\theta}$$

Substitute for θ and evaluate T:

$$T = \frac{100\text{ N}}{2\sin 45°} = \boxed{70.7\text{ N}}$$

(*b*) Use the diagram to the right to relate the sag Δy in the chain to the angle θ the chain makes with the horizontal:



$$\sin\theta = \frac{\Delta y}{\frac{1}{2}L}$$

where L is the length of the chain.

Solve for Δy:

$$\Delta y = \tfrac{1}{2}L\sin\theta$$

Because the horizontal and vertical forces in the chain are equal, $\theta =$ 45° and:

$$\Delta y = \tfrac{1}{2}(5\text{ m})\sin 45° = \boxed{1.77\text{ m}}$$

(*c*) Relate the distance d of the boat from the dock to the angle θ the chain makes with the horizontal:

$$\cos\theta = \frac{\frac{1}{2}d}{\frac{1}{2}L} = \frac{d}{L}$$

Solve for and evaluate d:

$$d = L\cos\theta = (5\text{ m})\cos 45° = \boxed{3.54\text{ m}}$$

(*d*) Relate the resultant tension in the chain to the vertical component of the tension F_v and the maximum drag force exerted on the boat by the water $F_{d,max}$:

$$F_v^2 + F_{d,max}^2 = (500\text{ N})^2$$

Solve for $F_{d,max}$:

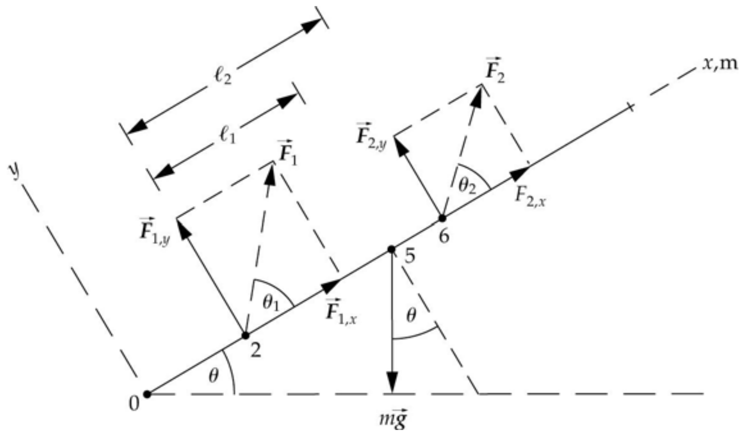
$$F_{d,max} = \sqrt{(500\text{ N})^2 - F_v^2}$$

Because the vertical component of the tension is 50 N:

$$F_{d,max} = \sqrt{(500\text{ N})^2 - (50\text{ N})^2} = \boxed{497\text{ N}}$$

42 ••

Picture the Problem Choose a coordinate system in which the positive x axis is along the rod and the positive y direction is normal to the rod. The rod and the forces acting on it are shown in the free-body diagram. The forces acting at the supports are denoted by the numerals 1 and 2. The resultant forces at the supports are shown as dashed lines. We'll assume that the rod is on the verge of sliding. Because the x components of the forces at the supports are friction forces, they are proportional to the normal, i.e., y, components of the forces at the supports. Because the rod is in equilibrium, we can apply the conditions for translational and rotational equilibrium.



Apply $\sum \vec{\tau} = 0$ about an axis through the support at $x = 2$ m:

$$\ell_2 F_{2,y} - \ell_1 mg\cos\theta = 0$$

Solve for $F_{2,y}$:

$$F_{2,y} = \frac{\ell_1 mg\cos\theta}{\ell_2}$$

Substitute numerical values and evaluate $F_{2,y}$:

$$F_{2,y} = \frac{(3\,\text{m})(20\,\text{kg})(9.81\,\text{m/s}^2)\cos 30°}{4\,\text{m}}$$
$$= 127.4\,\text{N}$$

Apply $\sum \vec{\tau} = 0$ about an axis through the support at $x = 6$ m:

$$(\ell_2 - \ell_1)mg\cos\theta - \ell_2 F_{1,y} = 0$$

Solve for $F_{1,y}$:

$$F_{1,y} = \frac{(\ell_2 - \ell_1)mg\cos\theta}{\ell_2}$$

Substitute numerical values and evaluate $F_{1,y}$:

$$F_{1,y} = \frac{(4\,\text{m} - 3\,\text{m})(20\,\text{kg})(9.81\,\text{m/s}^2)}{4\,\text{m}}$$
$$\times \cos 30°$$
$$= 42.48\,\text{N}$$

Apply $\sum F_x = 0$ to the rail:

$$F_{1,x} + F_{2,x} - mg\sin 30° = 0 \qquad (1)$$

Assuming that the rod is on the verge of sliding and that the coefficient of static friction is the same for both supports:

$$F_{1,\mathrm{x}} = \mu_{\mathrm{s}} F_{1,y}$$

and

$$F_{2,\mathrm{x}} = \mu_{\mathrm{s}} F_{2,y}$$

Divide the first of these equations by the second and evaluate this ratio to obtain:

$$\frac{F_{1,x}}{F_{2,x}} = \frac{F_{1,y}}{F_{2,y}} = \frac{42.48\,\mathrm{N}}{127.4\,\mathrm{N}} = \frac{1}{3}$$

Solve for $F_{2,x}$:

$$F_{2,x} = 3F_{1,x}$$

Substitute in equation (1):

$$F_{1,x} + 3F_{1,x} - mg\sin\theta = 0$$

Solve for $F_{1,x}$:

$$F_{1,x} = \tfrac{1}{4}mg\sin\theta$$

Substitute numerical values and evaluate $F_{1,x}$:

$$F_{1,x} = \tfrac{1}{4}(20\,\mathrm{kg})\left(9.81\,\mathrm{m/s^2}\right)\sin 30° = 24.53\,\mathrm{N}$$

Evaluate $F_{2,x}$:

$$F_{2,x} = 3(24.53\,\mathrm{N}) = 73.58\,\mathrm{N}$$

Find the angle θ_1 the force at support 1 (x = 2 m) makes with the rod:

$$\theta_1 = \tan^{-1}\frac{F_{1,y}}{F_{1,x}} = \tan^{-1}\frac{42.48\,\mathrm{N}}{24.53\,\mathrm{N}} = \boxed{60.0°}$$

Find the angle θ_2 the force at support 2 makes with the rod:

$$\theta_2 = \tan^{-1}\frac{F_{2,y}}{F_{2,x}} = \tan^{-1}\frac{127.4\,\mathrm{N}}{73.58\,\mathrm{N}} = \boxed{60.0°}$$

Find the magnitude of $\vec{F}_1$:

$$F_1 = \sqrt{F_{1,x}^2 + F_{1,y}^2} = \sqrt{(24.53\,\mathrm{N})^2 + (42.48\,\mathrm{N})^2} = \boxed{49.1\,\mathrm{N}}$$

Find the magnitude of $\vec{F}_2$:

$$F_2 = \sqrt{F_{2,x}^2 + F_{2,y}^2} = \sqrt{(73.58\,\text{N})^2 + (127.4\,\text{N})^2} = \boxed{147\,\text{N}}$$

43 •

Picture the Problem The forces shown in the figure constitute a couple and will cause the plate to experience a counterclockwise angular acceleration. We can find this net torque by expressing the torque about either of the corners of the plate.

Sum the torques about an axis through the upper left corner of the plate to obtain:

$$\tau_{\text{net}} = b[(80\,\text{N})\cos 30°] - a[(80\,\text{N})\sin 30°] = \boxed{(69.3\,\text{N})b - (40.0\,\text{N})a}$$

44 •

Picture the Problem We can use the condition for translational equilibrium and the definition of a couple to show that the force of static friction exerted by the surface and the applied force constitute a couple. We can use the definition of torque to find the torque exerted by the couple. We can use our result from (*b*) to find the effective point of application of the normal force when $F = Mg/3$ and the condition for rotational equilibrium to find the greatest magnitude of $\vec{F}$ for which the cube will not tip.

(*a*) Apply $\sum \vec{F}_x = 0$ to the stationary cube:

$$\vec{F} + \vec{f}_s = 0$$

$$\boxed{\therefore \vec{F} = -\vec{f}_s \text{ and this pair of equal, parallel, and oppositely directed forces constitute a couple.}}$$

The torque of the couple is:

$$\tau_{\text{couple}} = \boxed{Fa}$$

(*b*) Let x = the distance from the point of application of F_n to the center of the cube. Now, $F_n = Mg$, so applying $\sum \vec{\tau} = 0$ to the cube yields:

$$Mgx - Fa = 0 \qquad (1)$$

or

$$x = \frac{Fa}{Mg}$$

Substitute for $F = Mg/3$ to obtain:

$$x = \frac{\frac{Mg}{3}a}{Mg} = \boxed{\frac{a}{3}}$$

(*c*) Solve equation (1) for *F*:

$$F = \frac{Mgx}{a}$$

Noting that $x_{max} = a/2$, substitute to express the condition that the cube will tip:

$$F > \frac{Mgx_{max}}{a} = \frac{Mg\frac{a}{2}}{a} = \boxed{\frac{Mg}{2}}$$

45 ••

Picture the Problem We can find the perpendicular distance between the lines of action of the two forces by following the outline given in the problem statement.

Express the vertical components of the forces:

$$F\cos 30° = \frac{\sqrt{3}}{2}F$$

Express the horizontal components of the forces:

$$F\sin 30° = \frac{F}{2}$$

Express the net torque acting on the plate:

$$\tau_{net} = \frac{\sqrt{3}}{2}Fb - \frac{1}{2}Fa = \frac{1}{2}F\left(\sqrt{3}b - a\right)$$

Letting *D* be the moment arm of the couple, express the net torque acting on the plate:

$$\tau_{net} = FD$$

Equate these two expressions for τ_{net}:

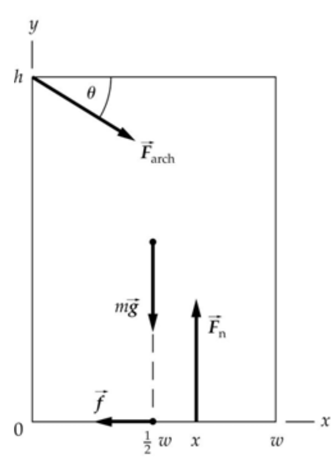
$$FD = \tfrac{1}{2}F\left(\sqrt{3}b - a\right)$$

Solve for *D*:

$$D = \boxed{\tfrac{1}{2}\left(\sqrt{3}b - a\right)}$$

***46** ••

Picture the Problem Choose the coordinate system shown in the diagram and let *x* be the coordinate of the thrust point. The diagram to the right shows the forces acting on the wall. The normal force must balance out the weight of the wall and the vertical component of the thrust from the arch and the frictional force must balance out the horizontal component of the thrust. We can apply the conditions for translational equilibrium to find *f* and F_n and the condition for rotational equilibrium to find the distance *x* from the origin of our coordinate system at which F_n acts.



(*a*) Apply the conditions for translational equilibrium to the wall to obtain:

$$\sum F_x = -f + F_{\text{arch}} \cos\theta = 0 \qquad (1)$$

and

$$\sum F_y = F_{\text{n}} - mg - F_{\text{arch}} \sin\theta = 0 \qquad (2)$$

Solve equation (1) for and evaluate *f*:

$$f = F_{\text{arch}} \cos\theta = \left(2\times10^4\ \text{N}\right)\cos 30° = \boxed{17.3\,\text{kN}}$$

Solve equation (2) for F_{n}:

$$F_{\text{n}} = mg + F_{\text{arch}} \sin\theta$$

Substitute numerical values and evaluate F_{n}:

$$F_{\text{n}} = \left(3\times10^4\ \text{kg}\right)\left(9.81\,\text{m/s}^2\right) + \left(2\times10^4\ \text{N}\right)\sin 30° = \boxed{304\,\text{kN}}$$

Apply $\sum \tau_{z\,\text{axis}} = 0$ to the to the wall:

$$xF_{\text{n}} - \tfrac{1}{2}wmg - hF_{\text{arch}} \cos\theta = 0$$

Solve for *x*:

$$x = \frac{\tfrac{1}{2}wmg + hF_{\text{arch}} \cos\theta}{F_{\text{n}}}$$

Substitute numerical values and evaluate *x*:

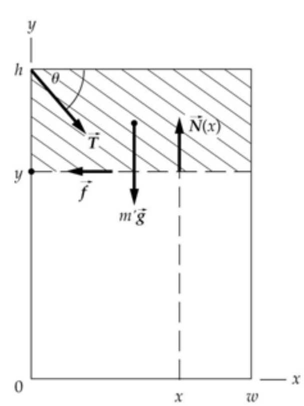
$$x = \frac{\tfrac{1}{2}(1.25\,\text{m})\left(3\times10^4\ \text{kg}\right)\left(9.81\,\text{m/s}^2\right) + (10\,\text{m})\left(2\times10^4\ \text{N}\right)\cos 30°}{304\,\text{kN}} = \boxed{0.570\,\text{m}}$$

(*b*)

$$\boxed{\begin{array}{l}\text{If there were no thrust on the side of the wall, the normal force would act}\\ \text{through the center of mass, so making the weight larger compared to the}\\ \text{thrust must move the point of action of the normal force closer to the center.}\end{array}}$$

47 ••

Picture the Problem Let *h* be the height of the structure, *T* be the thrust, θ the angle from the horizontal of the thrust, $m'g$ the weight of the wall above height *y*, *N*(*x*) the normal force, *f* the friction force the lower part of the wall exerts on the upper part, and *w* the width of the structure. We can apply the conditions for translational and rotational equilibrium to the portion of the wall above the point at which the thrust is applied to obtain two equations that we can solve simultaneously for *x*.



Apply $\sum F_y = 0$ to that fraction of the wall above height y:

$$N(x) - T\sin\theta - m'g = 0$$

Assuming the wall is of uniform density, express $m'g$ in terms of mg:

$$\frac{m'g}{h-y} = \frac{mg}{h}$$

and

$$m'g = mg\left(1 - \frac{y}{h}\right)$$

Substitute to obtain:

$$N(x) - T\sin\theta - mg\left(1 - \frac{y}{h}\right) = 0$$

Solve for $N(x)$:

$$N(x) = T\sin\theta + mg\left(1 - \frac{y}{h}\right)$$

Apply $\sum \vec{\tau} = 0$ about an axis through $(0,y)$ and perpendicular to the xy plane to obtain:

$$xN(x) - (h-y)T\cos\theta - \tfrac{1}{2}mgw\left(1 - \frac{y}{h}\right) = 0$$

Solve for x to obtain:

$$x = \frac{\tfrac{1}{2}mgw + hT\cos\theta}{N(x)}\left(1 - \frac{y}{h}\right)$$

Substitute for $N(x)$ to obtain:

$$x = \frac{\left(\tfrac{1}{2}mgw + hT\cos\theta\right)\left(1 - \frac{y}{h}\right)}{T\sin\theta + mg\left(1 - \frac{y}{h}\right)}$$

Substitute numerical values and simplify to obtain:

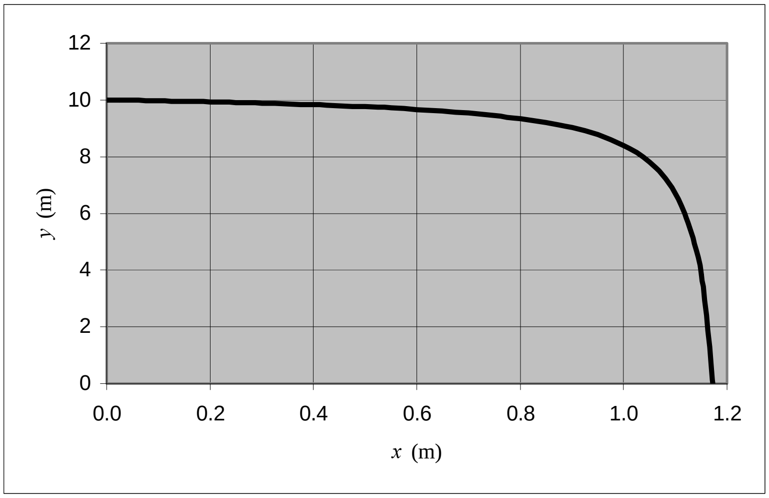
$$x = \frac{\left[\tfrac{1}{2}\left(3\times10^4\ \text{kg}\right)\left(9.81\ \text{m/s}^2\right)\left(1.25\ \text{m}\right) + \left(10\ \text{m}\right)\left(2\times10^4\ \text{N}\right)\cos30°\right]\left(1 - \frac{y}{10\ \text{m}}\right)}{\left(2\times10^4\ \text{N}\right)\sin30° + \left(3\times10^4\ \text{kg}\right)\left(9.81\ \text{m/s}^2\right)\left(1 - \frac{y}{10\ \text{m}}\right)}$$

$$= \frac{35.71\ \text{m} - 3.571y}{30.43 - \left(2.943\ \text{m}^{-1}\right)y}$$

Solve for y:

$$y = \boxed{\frac{35.71\ \text{m} - 30.43x}{3.571 - \left(2.943\ \text{m}^{-1}\right)x}}$$

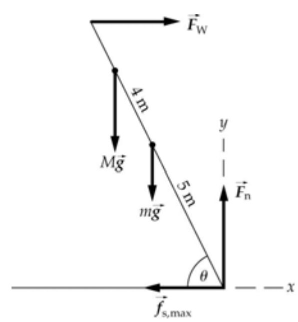
The graph shown below was plotted using a spreadsheet program:



Ladder Problems

***48** ••

Picture the Problem The ladder and the forces acting on it at the critical moment of slipping are shown in the diagram. Use the coordinate system shown. Because the ladder is in equilibrium, we can apply the conditions for translational and rotational equilibrium.



Using its definition, express μ_s:

$$\mu_s = \frac{f_{s,max}}{F_n} \qquad (1)$$

Apply $\sum \vec{\tau} = 0$ about the bottom of the ladder:

$$[(9\,\text{m})\cos\theta\,]Mg + [(5\,\text{m})\cos\theta]mg - [(10\,\text{m})\sin\theta]F_W = 0$$

Solve for F_W:

$$F_W = \frac{(9\,\text{m})M + (5\,\text{m})m}{(10\,\text{m})\sin\theta}\,g\cos\theta$$

Find the angle θ:

$$\theta = \cos^{-1}\frac{2.8\,\text{m}}{10\,\text{m}} = 73.74°$$

Evaluate F_W:

$$F_W = \frac{(9\,\text{m})(70\,\text{kg}) + (5\,\text{m})(22\,\text{kg})}{(10\,\text{m})\sin 73.74°} \times (9.81\,\text{m/s}^2)\cos 73.74° = 211.7\,\text{N}$$

Apply $\sum F_x = 0$ to the ladder and solve for $f_{s,max}$:

$$F_W - f_{s,max} = 0$$

and

$$f_{s,max} = F_W = 211.7\,\text{N}$$

Apply $\sum F_y = 0$ to the ladder:

$$F_n - Mg - mg = 0$$

Solve for F_n:

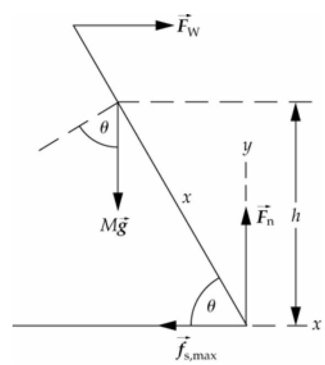
$$F_n = (M + m)g = (70\,\text{kg} + 22\,\text{kg})(9.81\,\text{m/s}^2) = 902.5\,\text{N}$$

Substitute numerical values in equation (1) and evaluate μ_s:

$$\mu_s = \frac{211.7\,\text{N}}{902.5\,\text{N}} = \boxed{0.235}$$

49 ••

Picture the Problem The ladder and the forces acting on it are shown in the diagram. Because the wall is smooth, the force the wall exerts on the ladder must be horizontal. Because the ladder is in equilibrium, we can apply the conditions for translational and rotational equilibrium to it.



Apply $\sum F_y = 0$ to the ladder and solve for F_n:

$$F_n - Mg = 0 \Rightarrow F_n = Mg$$

Apply $\sum F_x = 0$ to the ladder and solve for $f_{s,max}$:

$$F_W - f_{s,max} = 0 \Rightarrow f_{s,max} = F_W$$

Apply $\sum \vec{\tau} = 0$ about the bottom of the ladder:

$$Mgx\cos\theta - F_W L\sin\theta = 0$$

Solve for x:

$$x = \frac{F_W L\sin\theta}{Mg\cos\theta} = \frac{f_{s,max}L}{Mg}\tan\theta = \frac{\mu_s F_n L}{Mg}\tan\theta = \mu_s L\tan\theta$$

Referring to the figure, relate x to h and solve for h:

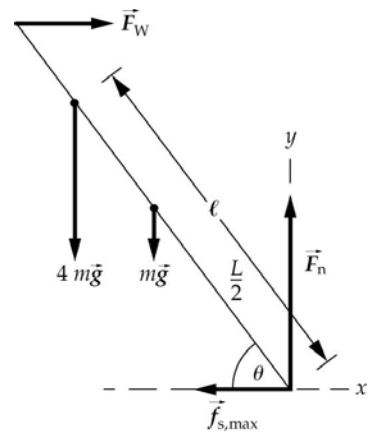
$$\sin\theta = \frac{h}{x}$$

and

$$h = x\sin\theta = \boxed{\mu_s L\tan\theta\sin\theta}$$

50 ••

Picture the Problem The ladder and the forces acting on it are shown in the drawing. Choose a coordinate system in which the positive x direction is to the right and the positive y direction is upward. Because the wall is smooth, the force the wall exerts on the ladder must be horizontal. Because the ladder is in equilibrium, we can apply the conditions for translational and rotational equilibrium.



Apply $\sum F_y = 0$ to the ladder and solve for F_n:

$$F_n - mg - 4mg = 0$$

and

$$F_n = 5mg$$

Apply $\sum F_x = 0$ to the ladder and solve for $f_{s,max}$:

$$F_W - f_{s,max} = 0$$

and

$$f_{s,max} = F_W$$

Apply $\sum \vec{\tau} = 0$ about an axis through the bottom of the ladder:

$$mg\frac{L}{2}\cos\theta + 4mg\ell\cos\theta - F_W L\sin\theta = 0$$

Substitute for F_W and then $f_{s,max}$ and solve for ℓ:

$$\ell = \frac{5\mu_s mgL\sin\theta - \frac{1}{2}mgL\cos\theta}{4mg\cos\theta}$$

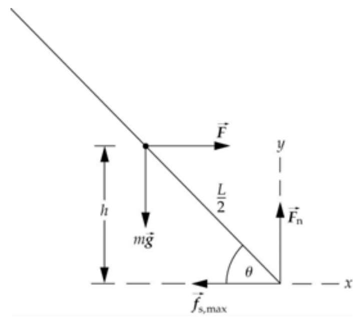
Simplify to obtain:

$$\ell = \left(\frac{5\mu_s}{4}\tan\theta - \frac{1}{8}\right)L = \left(\frac{5(0.45)}{4}\tan 60° - \frac{1}{8}\right)L = \boxed{0.849L}$$

i.e., you can climb about 85% of the way to the top of the ladder.

51 ••

Picture the Problem The ladder and the forces acting on it are shown in the figure. Because the ladder is separating from the wall, the force the wall exerts on the ladder is zero. Because the ladder is in equilibrium, we can apply the conditions for translational and rotational equilibrium.



To find the force required to pull the ladder away from the wall, apply $\sum\vec{\tau} = 0$ about an axis through the bottom of the ladder:

$$mg\frac{L}{2}\cos\theta - \frac{L}{2}F\sin\theta = 0$$

or, because $\frac{L}{2}\cos\theta = \frac{h}{\tan\theta}$,

$$\frac{mgh}{\tan\theta} - \frac{L}{2}F\sin\theta = 0$$

Solve for F:

$$F = \frac{2mgh}{L\tan\theta\sin\theta} \quad (1)$$

Apply $\sum F_x = 0$ to the ladder:

$$F - f_{s,max} = 0 \Rightarrow F = f_{s,max} = \mu_s F_n \quad (2)$$

Apply $\sum F_y = 0$ to the ladder:

$$F_n - mg = 0 \Rightarrow F_n = mg$$

Equate equations (1) and (2) and substitute for F_n to obtain:

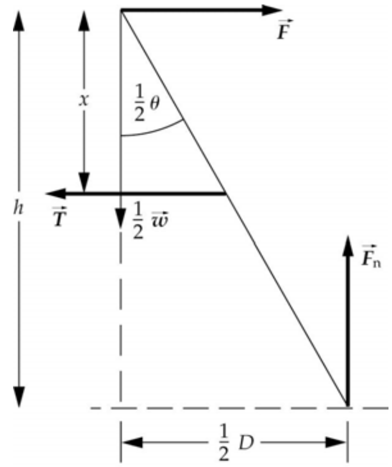
$$\mu_s mg = \frac{2mgh}{L\tan\theta\sin\theta}$$

Solve for μ_s:

$$\mu_s = \boxed{\frac{2h}{L\tan\theta\sin\theta}}$$

52 ••

Picture the Problem Assume that half the man's weight acts on each side of the ladder. The force exerted by the frictionless floor must be vertical. D is the separation between the legs at the bottom and x is the distance of the cross brace from the apex. Because each leg of the ladder is in equilibrium, we can apply the condition for rotational equilibrium the right leg to relate the tension in the cross brace to its distance from the apex.



(*a*) By symmetry, each leg carries half the total weight. So the force on each leg is:

$$\boxed{450\text{ N}}$$

(*b*) Consider one of the ladder's legs and apply $\sum\vec{\tau} = 0$ about the apex:

$$F_n\frac{D}{2} - Tx = 0$$

Solve for T:

$$T = \frac{F_n D}{2x}$$

Using trigonometry, relate h and θ through the tangent function:

$$\tan\tfrac{1}{2}\theta = \frac{D/2}{h}$$

Solve for D to obtain:

$$D = 2h\tan\tfrac{1}{2}\theta$$

Substitute and simplify to obtain:

$$T = \frac{2F_n h\tan\frac{1}{2}\theta}{2x} = \frac{F_n h\tan\frac{1}{2}\theta}{x}$$

Substitute numerical values and evaluate T:

$$T = \frac{F_n h\tan\frac{1}{2}\theta}{x}$$

Apply $\sum F_y = 0$ to the ladder and solve for F_n:

$$F_n - \tfrac{1}{2}w = 0 \text{ and } F_n = \tfrac{1}{2}w$$

Substitute to obtain:

$$T = \frac{wh\tan\frac{1}{2}\theta}{2x} \qquad (1)$$

Substitute numerical values and evaluate T:

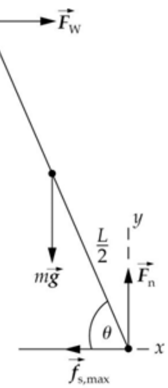
$$T = \frac{(900\,\text{N})(4\,\text{m})\tan 15°}{2(2\,\text{m})} = \boxed{241\,\text{N}}$$

(c) From equation (1) we can see that, if x is increased, i.e., the brace moved lower:

$$\boxed{T \text{ will decrease.}}$$

53 ••

Picture the Problem The figure shows the forces acting on the ladder. Because the wall is frictionless, the force the wall exerts on the ladder is perpendicular to the wall. Because the ladder is on the verge of slipping, the static friction force is $f_{\text{s,max}}$. Because the ladder is in equilibrium, we can apply the conditions for translational and rotational equilibrium.



Apply $\sum F_x = 0$ to the ladder:

$$F_\text{W} - f_{\text{s,max}} = 0 \Rightarrow F_\text{W} = f_{\text{s,max}} = \mu_\text{s} F_\text{n}$$

Apply $\sum F_y = 0$ to the ladder:

$$F_\text{n} - mg = 0 \Rightarrow F_\text{n} = mg$$

Apply $\sum \vec{\tau} = 0$ about an axis through the bottom of the ladder:

$$mg\frac{L}{2}\cos\theta - LF_\text{W}\sin\theta = 0$$

Substitute for F_W and F_n and simplify to obtain:

$$\tfrac{1}{2}\cos\theta - \mu_\text{s}\sin\theta = 0$$

Solve for and evaluate θ:

$$\theta = \tan^{-1}\frac{1}{2\mu_\text{s}} = \tan^{-1}\frac{1}{2(0.3)} = \boxed{59.0°}$$

Stress and Strain

***54** •

Picture the Problem L is the unstretched length of the wire, F is the force acting on it, and A is its cross-sectional area. The stretch in the wire ΔL is related to Young's modulus by $Y = (F/A)/(\Delta L/L)$. We can use Table 12-1 to find the numerical value of Young's modulus for steel.

Find the amount the wire is stretched from Young's modulus:

$$Y = \frac{F/A}{\Delta L/L}$$

Solve for ΔL:

$$\Delta L = \frac{FL}{YA}$$

Substitute for F and A to obtain:

$$\Delta L = \frac{mgL}{Y\pi r^2}$$

Substitute numerical values and evaluate ΔL:

$$\Delta L = \frac{(50\,\text{kg})(9.81\,\text{m/s}^2)(5\,\text{m})}{2\pi\times10^{11}\,\text{N/m}^2\left(2\times10^{-3}\,\text{m}\right)^2} = \boxed{0.976\,\text{mm}}$$

55 •

Picture the Problem L is the unstretched length of the wire, F is the force acting on it, and A is its cross-sectional area. The stretch in the wire ΔL is related to Young's modulus by $Y = \text{stress/strain} = (F/A)/(\Delta L/L)$.

(*a*) Express the maximum load in terms of the wire's breaking stress:

$$F_{\text{max}} = \text{breaking stress}\times A = \text{breaking stress}\times \pi r^2$$

Substitute numerical values and evaluate F_{max}:

$$F_{\text{max}} = \left(3\times10^{8}\,\text{N/m}^2\right)\pi\left(0.21\times10^{-3}\,\text{m}\right)^2 = \boxed{41.6\,\text{N}}$$

(*b*) Using the definition of Young's modulus, express the fractional change in length of the copper wire:

$$\Delta L/L = \frac{F/A}{Y} = \frac{1.5\times10^{8}\,\text{N/m}^2}{1.1\times10^{11}\,\text{N/m}^2} = 1.36\times10^{-3} = \boxed{0.136\%}$$

56 •

Picture the Problem L is the unstretched length of the wire, F is the force acting on it, and A is its cross-sectional area. The stretch in the wire ΔL is related to Young's modulus by $Y = (F/A)/(\Delta L/L)$. We can use Table 12-1 to find the numerical value of Young's modulus for steel.

Find the amount the wire is stretched from Young's modulus:

$$Y = \frac{F/A}{\Delta L/L}$$

Solve for ΔL:

$$\Delta L = \frac{FL}{YA}$$

Substitute for F and A to obtain:

$$\Delta L = \frac{mgL}{Y\pi r^2}$$

Substitute numerical values and evaluate ΔL:

$$\Delta L = \frac{(4\,\text{kg})(9.81\,\text{m/s}^2)(1.2\,\text{m})}{2\pi\times10^{11}\,\text{N/m}^2\,(0.3\times10^{-3}\,\text{m})^2} = \boxed{0.833\,\text{mm}}$$

***57** •

Picture the Problem The shear stress, defined as the ratio of the shearing force to the area over which it is applied, is related to the shear strain through the definition of the shear modulus; $M_\text{s} = \dfrac{\text{shear stress}}{\text{shear strain}} = \dfrac{F_\text{s}/A}{\tan\theta}$.

Using the definition of shear modulus, relate the angle of shear, θ to the shear force and shear modulus:

$$\tan\theta = \frac{F_\text{s}}{M_\text{s}A}$$

Solve for θ:

$$\theta = \tan^{-1}\frac{F_\text{s}}{M_\text{s}A}$$

Substitute numerical values and evaluate θ:

$$\theta = \tan^{-1}\frac{25\,\text{N}}{(1.9\times10^5\,\text{N/m}^2)(15\times10^{-4}\,\text{m}^2)} = \boxed{5.01°}$$

58 ••

Picture the Problem The stretch in the wire ΔL is related to Young's modulus by $Y = (F/A)/(\Delta L/L)$, where L is the unstretched length of the wire, F is the force acting on it, and A is its cross-sectional area. For a composite wire, the length under stress is the unstressed length plus the sum of the elongations of the components of the wire.

Express the length of the composite wire when it is supporting a mass of 5 kg:

$$L = 3.00\,\text{m} + \Delta L \qquad (1)$$

Express the change in length of the composite wire:

$$\Delta L = \Delta L_\text{steel} + \Delta L_\text{Al} = \frac{F}{A}\frac{L_\text{steel}}{Y_\text{steel}} + \frac{F}{A}\frac{L_\text{Al}}{Y_\text{Al}} = \frac{F}{A}\left(\frac{L_\text{steel}}{Y_\text{steel}} + \frac{L_\text{Al}}{Y_\text{Al}}\right)$$

Find the stress in each wire:

$$\frac{F}{A}=\frac{(5\,\text{kg})(9.81\,\text{m/s}^2)}{\pi(0.5\times10^{-3}\,\text{m})^2}=6.245\times10^7\ \text{N/m}^2$$

Substitute numerical values and evaluate ΔL:

$$\Delta L=(6.245\times10^7\ \text{N/m}^2)\left(\frac{1.5\,\text{m}}{2\times10^{11}\ \text{N/m}^2}+\frac{1.5\,\text{m}}{0.7\times10^{11}\ \text{N/m}^2}\right)=1.81\times10^{-3}\ \text{m}$$

Substitute in equation (1) and evaluate L:

$$L=3.00\,\text{m}+1.81\times10^{-3}\ \text{m}=\boxed{3.0018\,\text{m}}$$

59 ••

Picture the Problem We can use Hooke's law and Young's modulus to show that, if the wire is considered to be a spring, the force constant k is given by $k = AY/L$. By treating the wire as a spring we can show the energy stored in the wire is $U = \frac{1}{2}F\Delta L$.

Express the relationship between the stretching force, the stiffness constant, and the elongation of a spring:

$$F=k\Delta L$$

or

$$k=\frac{F}{\Delta L}$$

Using the definition of Young's modulus, express the ratio of the stretching force to the elongation of the wire:

$$\frac{F}{\Delta L}=\frac{AY}{L} \qquad (1)$$

Equate these two expressions for $F/\Delta L$ to obtain:

$$k=\boxed{\frac{AY}{L}}$$

Treating the wire as a spring, express its stored energy:

$$U=\tfrac{1}{2}k(\Delta L)^2=\tfrac{1}{2}\frac{AY}{L}(\Delta L)^2$$

Solve equation (1) for F:

$$F=\frac{AY\Delta L}{L}$$

Substitute in our expression for U to obtain:

$$U=\tfrac{1}{2}\frac{AY\Delta L}{L}\Delta L=\boxed{\tfrac{1}{2}F\Delta L}$$

60 ••

Picture the Problem Let L' represent the stretched and L the unstretched length of the wire. The stretch in the wire ΔL is related to Young's modulus by $Y = (F/A)/(\Delta L/L)$, where F is the force acting on it, and A is its cross-sectional area. In problem 58 we showed that the energy stored in the wire is $U = ½F\Delta L$, where Y is Young's modulus and ΔL is the amount the wire has stretched.

(*a*) Express the stretched length of the wire:

$$L' = L + \Delta L$$

Using the definition of Young's modulus, express ΔL:

$$\Delta L = \frac{LF}{AY}$$

Substitute and simplify:

$$L' = L + \frac{LF}{AY} = L\left(1 + \frac{F}{AY}\right)$$

Solve for L:

$$L = \frac{L'}{1 + \frac{F}{AY}}$$

Substitute numerical values and evaluate L:

$$L = \frac{0.35\,\text{m}}{1 + \frac{53\,\text{N}}{\pi\left(0.1\times10^{-3}\,\text{m}\right)^2\left(2\times10^{11}\,\text{N/m}^2\right)}} = \boxed{0.347\,\text{m}}$$

(*b*) Using the expression from Problem 59, express the work done in stretching the wire:

$$W = \Delta U = \tfrac{1}{2}F\Delta L = \tfrac{1}{2}(53\,\text{N})(0.35\,\text{m} - 0.347\,\text{m}) = \boxed{0.0795\,\text{J}}$$

***61** ••

Picture the Problem The table to the right summarizes the ratios $\Delta L/F$ for the student's data. Note that this ratio is constant, to three significant figures, for loads less than or equal to 200 g. We can use this ratio to calculate Young's modulus for the rubber strip.

Load (g)	F (N)	ΔL (m)	$\Delta L/F$ (m/N
100	0.981	0.006	6.12×10^{-3}
200	1.962	0.012	6.12×10^{-3}
300	2.943	0.019	6.46×10^{-3}
400	3.924	0.028	7.14×10^{-3}
500	4.905	0.05	10.2×10^{-3}

(*a*) Referring to the table, we see that for loads ≤ 200 g:

$$\frac{\Delta L}{F} = 6.12\times10^{-3}\ \text{m/N}$$

Use the definition of Young's modulus to express *Y*:

$$Y = \frac{FL}{A\Delta L} = \frac{L}{A\dfrac{\Delta L}{F}}$$

Substitute numerical values and evaluate *Y*:

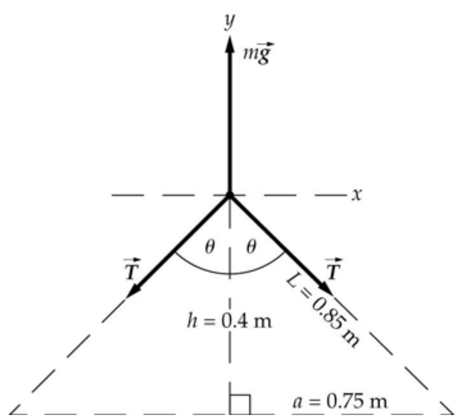
$$Y = \frac{5\times10^{-2}\ \text{m}}{\left(3\times10^{-3}\ \text{m}\right)\left(1.5\times10^{-3}\ \text{m}\right)\left(6.12\times10^{-3}\ \text{m/N}\right)} = \boxed{1.82\times10^{6}\ \text{N/m}^2}$$

(*b*) Interpolate to determine the stretch when the load is 150 g, and use the expression from Problem 58, to express the energy stored in the strip:

$$\begin{aligned} U &= \tfrac{1}{2}F\Delta L \\ &= \tfrac{1}{2}\left(0.15\,\text{kg}\right)\left(9.81\,\text{m/s}^2\right)\left(9\times10^{-3}\ \text{m}\right) \\ &= \boxed{6.62\,\text{mJ}} \end{aligned}$$

62 ••

Picture the Problem The figure shows the forces acting on the wire where it passes over the nail. *m* represents the mass of the mirror and *T* is the tension in the supporting wires. The figure also shows the geometry of the right triangle defined by the support wires and the top of the mirror frame. The distance *a* is fixed by the geometry while *h* and *L* will change as the mirror is suspended from the nail.



Express the distance between the nail and the top of the frame when the wire is under tension:

$$\begin{aligned} h' &= h + \Delta h \\ &= 0.4\,\text{m} + \Delta h \end{aligned} \qquad (1)$$

Apply $\sum F_y = 0$ to the wire where it passes over the supporting nail:

$$mg - 2T\cos\theta = 0$$

Solve for the tension in the wire:

$$T = \frac{mg}{2\cos\theta}$$

Substitute numerical values and evaluate T:

$$T = \frac{(2.4\,\text{kg})(9.81\,\text{m/s}^2)}{2\left(\frac{0.4\,\text{m}}{0.85\,\text{m}}\right)} = 25.0\,\text{N}$$

Using its definition, find the stress in the wire:

$$\text{stress} = \frac{T}{A} = \frac{25.0\,\text{N}}{\pi(0.1\times10^{-3}\,\text{m})^2} = 7.96\times10^8\ \text{N/m}^2$$

Using the definition of Young's modulus, find the strain in the hypotenuse of the right triangle shown in the figure:

$$\text{strain} = \frac{\Delta L}{L} = \frac{\text{stress}}{Y} = \frac{7.96\times10^8\ \text{N/m}^2}{2\times10^{11}\ \text{N/m}^2} = 3.98\times10^{-3}$$

Using the Pythagorean theorem, express the relationship between the sides of the right triangle in the figure:

$$a^2 + h^2 = L^2$$

Express the differential of this equation:

$$2a\Delta a + 2h\Delta h = 2L\Delta L$$

or, because $\Delta a = 0$,

$$h\Delta h = L\Delta L$$

Solve for and evaluate Δh:

$$\Delta h = \frac{L\Delta L}{h} = \frac{L^2}{h}\cdot\frac{\Delta L}{L}$$

Substitute numerical values and evaluate Δh:

$$\Delta h = \frac{(0.85\,\text{m})^2}{0.4\,\text{m}}(3.98\times10^{-3}) = 7.19\,\text{mm}$$

Substitute in equation (1) to obtain:

$$h' = 0.4\,\text{m} + 7.19\,\text{mm} = \boxed{40.72\,\text{cm}}$$

63 ••

Picture the Problem Let the numeral 1 denote the aluminum wire and the numeral 2 the steel wire. Because their initial lengths and amount they stretch are the same, we can use the definition of Young's modulus to express the change in the lengths of each wire and then equate these expressions to obtain an equation solvable for the ratio M_1/M_2.

Using the definition of Young's modulus, express the change in

$$\Delta L_1 = \frac{M_1 g L_1}{A_1 Y_{\text{Al}}}$$

length of the aluminum wire:

Using the definition of Young's modulus, express the change in length of the steel wire:

$$\Delta L_2 = \frac{M_2 g L_2}{A_2 Y_{\text{steel}}}$$

Because the two wires stretch by the same amount, equate ΔL_1 and ΔL_2 and simplify:

$$\frac{M_1}{A_1 Y_{\text{Al}}} = \frac{M_2}{A_2 Y_{\text{steel}}}$$

Solve for the ratio M_1/M_2:

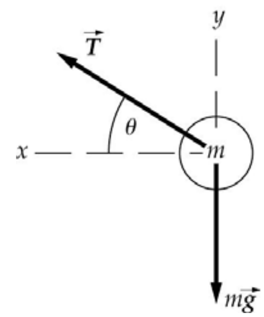
$$\frac{M_1}{M_2} = \frac{A_1 Y_{\text{Al}}}{A_2 Y_{\text{steel}}}$$

Substitute numerical values and evaluate M_1/M_2:

$$\frac{M_1}{M_2} = \frac{\frac{\pi}{4}(0.7\,\text{mm})^2(0.7\times10^{11}\ \text{N/m}^2)}{\frac{\pi}{4}(0.5\,\text{mm})^2(2\times10^{11}\ \text{N/m}^2)} = \frac{(0.7\,\text{mm})^2(0.7\times10^{11}\ \text{N/m}^2)}{(0.5\,\text{mm})^2(2\times10^{11}\ \text{N/m}^2)} = \boxed{0.686}$$

64 ••

Picture the Problem The free-body diagram shows the forces acting on the ball as it rotates around the post in a horizontal plane. We can apply Newton's 2nd law to find the tension in the wire and use the definition of Young's modulus to find the amount by which the aluminum wire stretches.



Express the length of the wire under tension to its unstretched length:

$$L = L_0 + \Delta L = 0.7\,\text{m} + \Delta L \qquad (1)$$

Apply $\sum F_y = 0$ to the ball:

$$T\sin\theta - mg = 0$$

Solve for the tension in the wire:

$$T = \frac{mg}{\sin\theta}$$

Substitute numerical values and evaluate T:

$$T = \frac{(0.5\,\text{kg})(9.81\,\text{m/s}^2)}{\sin 5°} = \boxed{56.3\,\text{N}}$$

Using the definition of Young's modulus, express ΔL:

$$\Delta L = \frac{FL}{AY}$$

Substitute numerical values and evaluate ΔL:

$$\Delta L = \frac{(56.3\,\text{N})(0.7\,\text{m})}{\frac{\pi}{4}(1.6\times10^{-3}\,\text{m})^2(0.7\times10^{11}\,\text{N/m}^2)} = 0.280\,\text{mm}$$

Substitute in equation (1) to obtain:

$$L = 0.7\,\text{m} + 0.280\,\text{mm} = \boxed{70.03\,\text{cm}}$$

***65 ••**

Picture the Problem We can use the definition of stress to calculate the failing stress of the cable and the stress on the elevator cable. Note that the failing stress of the composite cable is the same as the failing stress of the test sample.

Express the stress on the elevator cable:

$$\text{Stress}_{\text{cable}} = \frac{F}{A} = \frac{20\,\text{kN}}{1.2\times10^{-6}\,\text{m}^2} = 1.67\times10^{10}\,\text{N/m}^2$$

Express the failing stress of the sample:

$$\text{Stress}_{\text{failing}} = \frac{F}{A} = \frac{1\,\text{kN}}{0.2\times10^{-6}\,\text{m}^2} = 0.500\times10^{10}\,\text{N/m}^2$$

$$\boxed{\text{Because Stress}_{\text{failing}} < \text{Stress}_{\text{cable}}\text{, it will not support the elevator.}}$$

***66 •••**

Picture the Problem Let the length of the sides of the rectangle be x, y and z. Then the volume of the rectangle will be $V = xyz$ and we can express the new volume V' resulting from the pulling in the x direction and the change in volume ΔV in terms of Δx, Δy, and Δz. Discarding the higher order terms in ΔV and dividing our equation by V and using the given condition that $\Delta y/y = \Delta z/z$ will lead us to the given expression for $\Delta y/y$.

Express the new volume of the rectangular box when its sides change in length by Δx, Δy, and Δz:

$$\begin{aligned} V' = (x+\Delta x)(y+\Delta y)(z+\Delta z) &= xyz + \Delta x(yz) + \Delta y(xz) + \Delta z(xy) \\ &+ \{z\Delta x\Delta y + y\Delta x\Delta z + x\Delta y\Delta z + \Delta x\Delta y\Delta z\} \end{aligned}$$

where the terms in brackets are very small (i.e., second order or higher).

Discard the second order and higher terms to obtain:

$$V' = V + \Delta x(yz) + \Delta y(xz) + \Delta z(xy)$$

or

$$\Delta V = V' - V = \Delta x(yz) + \Delta y(xz) + \Delta z(xy)$$

Because $\Delta V = 0$:

$$\Delta x(yz) = -[\Delta y(xz) + \Delta z(xy)]$$

Divide both sides of this equation by $V = xyz$ to obtain:

$$\frac{\Delta x}{x} = -\left[\frac{\Delta y}{y} + \frac{\Delta z}{z}\right]$$

Because $\Delta y/y = \Delta z/z$, our equation becomes:

$$\frac{\Delta x}{x} = -2\frac{\Delta y}{y} \text{ or } \frac{\Delta y}{y} = \boxed{-\frac{1}{2}\frac{\Delta x}{x}}$$

67 ••

Picture the Problem We can evaluate the differential of the volume of the wire and, using the assumptions that the volume of the wire does not change under stretching and that the change in its length is small compared to its length, show that $\Delta r/r = -(1/2)\,\Delta L/L$.

Express the volume of the wire:

$$V = \pi r^2 L$$

Evaluate the differential of V to obtain:

$$dV = \pi r^2 dL + 2\pi rLdr$$

Because $dV = 0$:

$$0 = rdL + 2Ldr \Rightarrow \frac{dr}{r} = -\frac{1}{2}\frac{dL}{L}$$

Because $\Delta L << L$, we can approximate the differential changes dr and dL with small changes Δr and ΔL to obtain:

$$\frac{\Delta r}{r} = \boxed{-\frac{1}{2}\frac{\Delta L}{L}}$$

***68** •••

Picture the Problem Because the volume of the thread remains constant during the stretching process, we can equate the initial and final volumes to express r_0 in terms of r. We can also use Young's modulus to express the tension needed to break the thread in terms of Y and r_0.

(*a*) Express the conservation of volume during the stretching of the spider's silk:

$$\pi r^2 L = \pi r_0^2 L_0$$

Solve for r:

$$r = r_0\sqrt{\frac{L_0}{L}}$$

Substitute for L to obtain:

$$r = r_0\sqrt{\frac{L_0}{10L_0}} = \boxed{0.316r_0}$$

(*b*) Express Young's modulus in terms of the breaking tension T:

$$Y=\frac{T/A}{\Delta L/L}=\frac{T/\pi r^2}{\Delta L/L}=\frac{10T/\pi r_0^2}{\Delta L/L}$$

Solve for T to obtain:

$$T=\frac{1}{10}\pi r_0^2 Y\frac{\Delta L}{L}$$

Because $\Delta L/L = 9$:

$$T=\boxed{\frac{9\pi r_0^2 Y}{10}}$$

General Problems

69 •

Picture the Problem Because the board is in equilibrium, we can apply the conditions for translational and rotational equilibrium to determine the forces exerted by the supports.

Apply $\sum_i \vec{\tau}_i = 0$ about the right support:

$$(2\,\text{m})(360\,\text{N})+(5\,\text{m})(90\,\text{N})-(10\,\text{m})F_\text{L}=0$$

Solve for and evaluate F_L:

$$F_\text{L}=\frac{(2\,\text{m})(360\,\text{N})+(5\,\text{m})(90\,\text{N})}{10\,\text{m}}=\boxed{117\,\text{N}}$$

Apply $\sum F_y = 0$ to the board:

$$F_\text{L}+F_\text{R}-90\,\text{N}-360\,\text{N}=0$$

Solve for and evaluate F_R:

$$F_\text{R}=-F_\text{L}+90\,\text{N}+360\,\text{N}=-117\,\text{N}+90\,\text{N}+360\,\text{N}=\boxed{333\,\text{N}}$$

Remarks: **We could just as easily found F_R by applying $\sum \vec{\tau} = 0$ about the left support.**

70 •

Picture the Problem Because the man-and-board system is in equilibrium, we can apply the conditions for translational and rotational equilibrium to determine the forces exerted by the supports. Let d represent the distance from the man's feet to his center of gravity.

Apply $\sum \vec{\tau} = 0$ about an axis through the man's feet and perpendicular to the page:

$$(845\,\text{N})d-(1.88\,\text{m})(445\,\text{N})=0$$

Solve for and evaluate d:

$$d = \frac{(1.88\,\text{m})(445\,\text{N})}{845\,\text{N}} = 0.990\,\text{m}$$

$$= \boxed{99.0\,\text{cm}}$$

No. Holding his head slightly above the board would not change the location of his center of mass and so the scale readings would not change.

***71** •

Picture the Problem We can apply the balance condition $\sum \vec{\boldsymbol{\tau}} = 0$ successively, starting with the lowest part of the mobile, to find the value of each of the unknown weights.

Apply $\sum \vec{\boldsymbol{\tau}} = 0$ about an axis through the point of suspension of the lowest part of the mobile:

$$(3\,\text{cm})(2\,\text{N}) - (4\,\text{cm})w_1 = 0$$

Solve for and evaluate w_1:

$$w_1 = \frac{(3\,\text{cm})(2\,\text{N})}{4\,\text{cm}} = \boxed{1.50\,\text{N}}$$

Apply $\sum \vec{\boldsymbol{\tau}} = 0$ about an axis through the point of suspension of the middle part of the mobile:

$$(2\,\text{cm})w_2 - (4\,\text{cm})(2\,\text{N} + 1.5\,\text{N}) = 0$$

Solve for and evaluate w_2:

$$w_2 = \frac{(4\,\text{cm})(2\,\text{N} + 1.5\,\text{N})}{2\,\text{cm}} = \boxed{7.00\,\text{N}}$$

Apply $\sum \vec{\boldsymbol{\tau}} = 0$ about an axis through the point of suspension of the top part of the mobile:

$$(2\,\text{cm})(10.5\,\text{N}) - (6\,\text{cm})w_3 = 0$$

Solve for and evaluate w_3:

$$w_3 = \frac{(2\,\text{cm})(10.5\,\text{N})}{6\,\text{cm}} = \boxed{3.50\,\text{N}}$$

72 •

Picture the Problem We can determine the ratio of L to h by noting the number of ropes supporting the load whose mass is M.

(a) Noting that three ropes support the pulley to which the object whose mass is M is fastened we can

$$\frac{L}{h} = \boxed{3}$$

conclude that:

(*b*) Apply the work-energy principle to the block-tackle object to obtain:

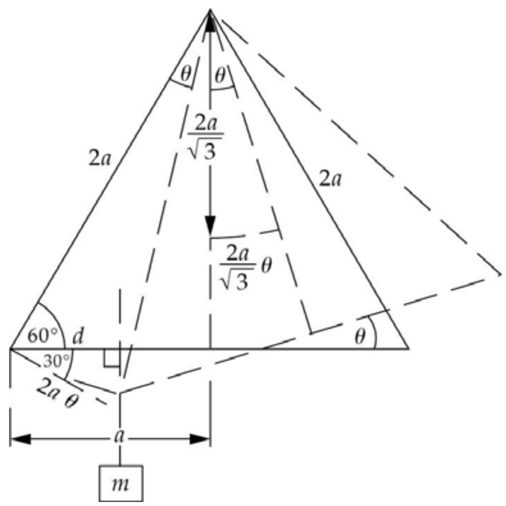
$$W_{ext} = \Delta E_{system} = \Delta U_{block\text{-}tackle}$$

or

$$\boxed{FL = Mgh}$$

73 ••

Picture the Problem The figure shows the equilateral triangle without the mass *m*, and then the same triangle with the mass *m* and rotated through an angle θ. Let the side length of the triangle to be $2a$. Then the center of mass of the triangle is at a distance of $\dfrac{2a}{\sqrt{3}}$ from each vertex. As the triangle rotates, its center of mass shifts by $\dfrac{2a}{\sqrt{3}}\theta$, for $\theta \ll 1$. Also, the vertex to which *m* is attached moves toward the plumb line by the distance $d = 2a\theta\cos 30°$ $= \sqrt{3}a\theta$ (see the drawing).



Apply $\sum \vec{\tau} = 0$ about an axis through the point of suspension:

$$mg\left(a - \sqrt{3}a\theta\right) - Mg\frac{2a}{\sqrt{3}}\theta = 0$$

Solve for m/M:

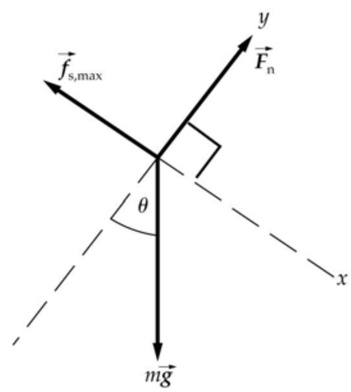
$$\frac{m}{M} = \frac{2\theta}{\sqrt{3}\left(1 - \sqrt{3}\theta\right)}$$

Substitute numerical values and evaluate m/M:

$$\frac{m}{M} = \frac{2(6°)\left(\dfrac{\pi\ \text{rad}}{180°}\right)}{\sqrt{3}\left[1 - \sqrt{3}(6°)\left(\dfrac{\pi\ \text{rad}}{180°}\right)\right]} = \boxed{0.148}$$

74 ••

Picture the Problem If the hexagon is to roll rather than slide, the incline's angle must be such that the center of mass falls just beyond the support base. From the geometry of the hexagon, one can see that the critical angle is 30°. The free-body diagram shows the forces acting on the hexagonal pencil when it is on the verge of sliding. We can use Newton's 2nd law to relate the coefficient of static friction to the angle of the incline for which rolling rather than sliding occurs.



Apply $\sum \vec{F} = 0$ to the pencil:

$$\sum F_x = mg\sin\theta - f_{s,max} = 0 \qquad (1)$$

and

$$\sum F_y = F_n - mg\cos\theta = 0 \qquad (2)$$

Substitute $f_{s,max} = \mu_s F_n$ in equation (1):

$$mg\sin\theta - \mu_s F_n = 0 \qquad (3)$$

Divide equation (3) by equation (2) to obtain:

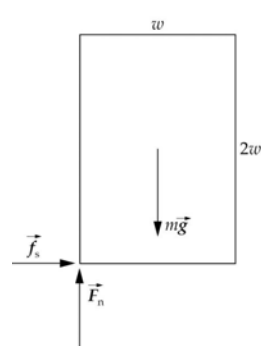
$$\tan\theta = \mu_s$$

Thus, if the pencil is to roll rather than slide when the pad is inclined:

$$\mu_s \geq \tan 30° = \boxed{0.577}$$

75 ••

Picture the Problem The box and the forces acting on it are shown in the figure. When the box is about to tip, F_n acts at its edge, as indicated in the drawing. We can use the definition of μ_s and apply the condition for rotational equilibrium in an accelerated frame to relate f_s to the weight of the box and, hence, to the normal force.



Using its definition, express μ_s:

$$\mu_s \geq \frac{f_s}{F_n}$$

Apply $\sum \vec{\tau} = 0$ about an axis through the box's center of mass:

$$wf_s - \tfrac{1}{2} wF_n = 0$$

Solve for the ratio $\frac{f_s}{F_n}$:

$$\frac{f_s}{F_n} = \frac{1}{2}$$

Substitute to obtain the condition for tipping:

$$\mu_s \geq 0.500$$

Therefore, if the box is to slide:

$$\mu_s < \boxed{0.500}$$

76 ••

Picture the Problem Because the balance is in equilibrium, we can use the condition for rotational equilibrium to relate the masses of the blocks to the lever arms of the balance in the two configurations described in the problem statement.

Apply $\sum \vec{\tau} = 0$ about an axis through the fulcrum:

$$(1.5\,\text{kg})L_1 = (1.95\,\text{kg})L_2$$

Solve for the ratio L_1/L_2:

$$\frac{L_1}{L_2} = \frac{1.95\,\text{kg}}{1.5\,\text{kg}} = 1.30$$

Apply $\sum \vec{\tau} = 0$ about an axis through the fulcrum with 1.5 kg at L_2:

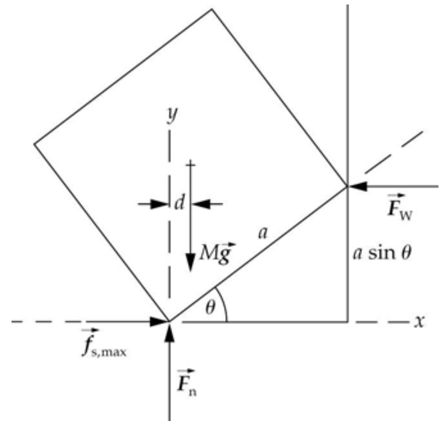
$$(1.5\,\text{kg})L_2 = ML_1$$

Solve for and evaluate M:

$$M = \frac{(1.5\,\text{kg})L_2}{L_1} = \frac{1.5\,\text{kg}}{L_1/L_2} = \frac{1.5\,\text{kg}}{1.30} = \boxed{1.15\,\text{kg}}$$

***77 ••**

Picture the Problem The figure shows the location of the cube's center of mass and the forces acting on the cube. The opposing couple is formed by the friction force $f_{\text{s,max}}$ and the force exerted by the wall. Because the cube is in equilibrium, we can use the condition for translational equilibrium to establish that $f_{\text{s,max}} = F_{\text{W}}$ and $F_{\text{n}} = Mg$ and the condition for rotational equilibrium to relate the opposing couples.



Apply $\sum \vec{F} = 0$ to the cube:

$$\sum F_y = F_{\text{n}} - Mg = 0 \Rightarrow F_{\text{n}} = Mg$$

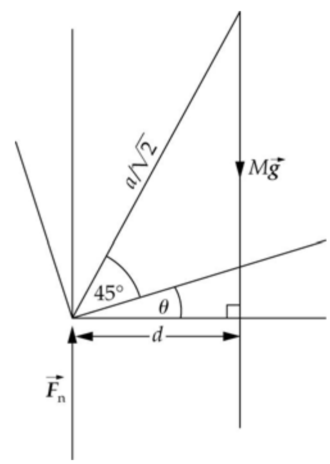
and

$$\sum F_x = f_{\text{s}} - F_W = 0 \Rightarrow F_{\text{W}} = f_{\text{s}}$$

Noting that $\vec{f}_{\text{s,max}}$ and $\vec{F}_{\text{W}}$ form a couple, as do $\vec{F}_{\text{n}}$ and $M\vec{g}$, apply $\sum \vec{\tau} = 0$ about an axis though the center of mass of the cube:

$$f_{\text{s,max}}\, a \sin\theta - Mgd = 0$$

Referring to the diagram to the right, note that $d = \frac{a}{\sqrt{2}}\sin(45° + \theta)$.



Substitute for d and $f_{\text{s,max}}$ to obtain:

$$\mu_{\text{s}} Mga \sin\theta - Mg\frac{a}{\sqrt{2}}\sin(45° + \theta) = 0$$

or

$$\mu_{\text{s}} \sin\theta - \frac{1}{\sqrt{2}}\sin(45° + \theta) = 0$$

Solve for μ_{s} and simplify to obtain:

$$\mu_s = \frac{1}{\sqrt{2}\sin\theta}\sin(45° + \theta) = \frac{1}{\sqrt{2}\sin\theta}(\sin 45°\cos\theta + \cos 45°\sin\theta)$$
$$= \frac{1}{\sqrt{2}\sin\theta}\left(\frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta\right) = \boxed{\frac{1}{2}(\cot\theta + 1)}$$

78 ••

Picture the Problem Because the meter stick is in equilibrium, we can apply the condition for rotational equilibrium to find the maximum distance from the hinge at which the block can be suspended.

Apply $\sum \vec{\tau} = 0$ about an axis through the hinge to obtain:

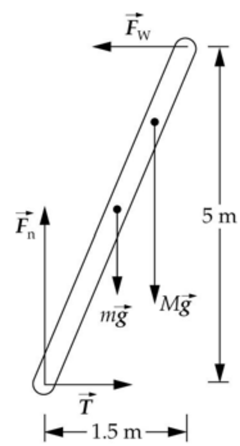
$$(1\,\text{m})(75\,\text{N}) - (0.5\,\text{m})(5\,\text{kg})(9.81\,\text{m/s}^2)\cos 45° - d(10\,\text{kg})(9.81\,\text{m/s}^2)\cos 45° = 0$$

Solve for and evaluate d:

$$d = \frac{(1\,\text{m})(75\,\text{N}) - (0.5\,\text{m})(5\,\text{kg})(9.81\,\text{m/s}^2)}{(10\,\text{kg})(9.81\,\text{m/s}^2)\cos 45°}\cos 45° = \boxed{0.831\,\text{m}}$$

79 ••

Picture the Problem Let m represent the mass of the ladder and M the mass of the person. The force diagram shows the forces acting on the ladder for part (b). From the condition for translational equilibrium, we can conclude that $T = F_W$, a result we'll need in part (b). Because the ladder is also in rotational equilibrium, summing the torques about the bottom of the ladder will eliminate both F_n and T.



(a) Apply $\sum_i \vec{\tau}_i = 0$ about an axis through the bottom of the ladder:

$$(5\,\text{m})F_W - (0.75\,\text{m})(20\,\text{kg})(9.81\,\text{m/s}^2) - (0.75\,\text{m})(80\,\text{kg})(9.81\,\text{m/s}^2) = 0$$

Solve for and evaluate F_W:

$$F_W = \frac{(0.75\,\text{m})(20\,\text{kg})(9.81\,\text{m/s}^2)}{5\,\text{m}} + \frac{(0.75\,\text{m})(80\,\text{kg})(9.81\,\text{m/s}^2)}{5\,\text{m}} = \boxed{147\,\text{N}}$$

(*b*) Solve for and evaluate f:

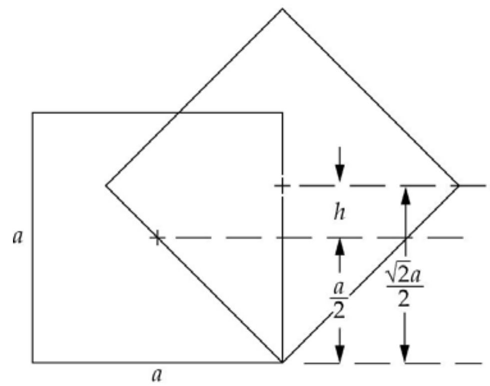
$$f = \frac{(5\,\text{m})(200\,\text{N}) - (0.75\,\text{m})(20\,\text{kg})(9.81\,\text{m/s}^2)}{(1.5\,\text{m})(80\,\text{kg})(9.81\,\text{m/s}^2)} = 0.724$$

Find the distance the 80-kg person can climb the ladder:

$$d = f(5\,\text{m}) = (0.724)(5\,\text{m}) = \boxed{3.62\,\text{m}}$$

***80 ••**

Picture the Problem To ″roll″ the cube one must raise its center of mass from $y = a/2$ to $y = \sqrt{2}a/2$, where a is the cube length. During this process the work done is the change in the gravitational potential energy of the cube. No additional work is done on the cube as it ″flops″ down. We can also use the definition of work to express the work done in sliding the cube a distance a along a horizontal surface and then equate the two expressions to determine μ_k.



Express the work done in moving the cube a distance a by raising its center of mass from $y = a/2$ to $y = \sqrt{2}a/2$ and then letting the cube flop down:

$$W = mg\left(\frac{\sqrt{2}a}{2} - \frac{a}{2}\right) = \frac{mga}{2}\left(\sqrt{2} - 1\right)$$
$$= 0.207mga$$

Letting f_k represent the kinetic friction force, express the work done in dragging the cube a distance a along the surface at constant speed:

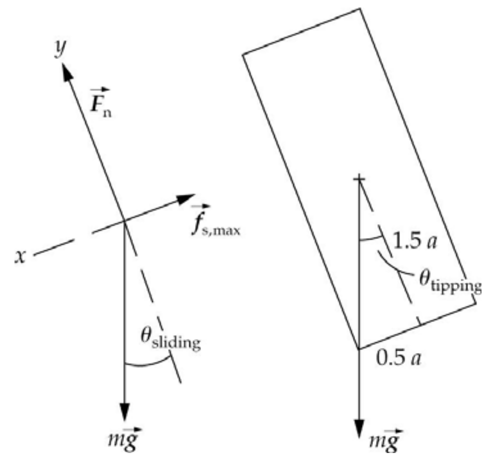
$$W = f_k a = \mu_k mga$$

Equate these two expressions to obtain:

$$\mu_k = \boxed{0.207}$$

81 ••

Picture the Problem The free-body diagram shows the forces acting on the block when it is on the verge of sliding. Because the block is in equilibrium, we can use the conditions for translational equilibrium to determine the minimum angle for which the block will slide. The diagram to the right of the FBD shows that the condition for tipping is that the plumb line from the center of mass pass outside of the base. We can determine the tipping angle from the geometry of the block under this condition.



Apply $\sum \vec{F} = 0$ to the block:

$$\sum F_x = mg\sin\theta_{sliding} - f_{s,max} \geq 0$$

if the block is to slide, and

$$\sum F_y = F_n - mg\cos\theta_{sliding} = 0$$

Substitute for $f_{s,max}$ and eliminate F_n between these equations to obtain:

$$\mu_s \leq \tan\theta_{sliding}$$

Solve for the condition for sliding:

$$\theta_{sliding} \geq \tan^{-1}(\mu_s) = \tan^{-1}(0.4) = 21.8°$$

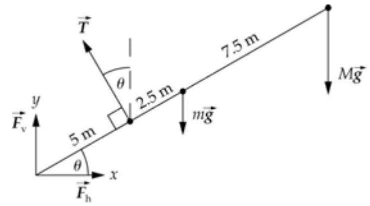
Using the geometry of the block, express the condition on θ that must be satisfied if the block is to tip:

$$\theta_{tipping} \geq \tan^{-1}\left(\frac{0.5a}{1.5a}\right) = \tan^{-1}\left(\frac{1}{3}\right) = 18.4°$$

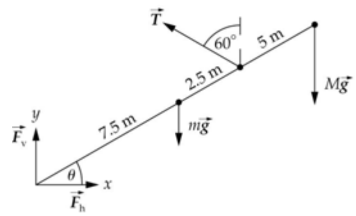
$$\boxed{\text{Because } \theta_{tipping} < \theta_{sliding}\text{, the block will tip before it slides.}}$$

82 ••

Picture the Problem Let m represent the mass of the bar, M the mass of the suspended object, F_v the vertical component of the force the wall exerts on the bar, F_h the horizontal component of the force exerted the wall exerts on the bar, and T the tension in the cable. The free-body diagrams show these forces and their points of application on the bar for parts (a) and (b) of the problem. Because the bar is in equilibrium, we can apply the conditions for translational and rotational equilibrium to relate the various forces and distances.



(a)



(b)

(a) Apply $\sum_i \vec{\tau}_i = 0$ about an axis through the hinge:

$$(5\,\text{m})T - (7.5\,\text{m})mg\cos 30° - (15\,\text{m})Mg\cos 30° = 0$$

Solve for T:

$$T = \frac{[(7.5\,\text{m})m + (15\,\text{m})M]g\cos 30°}{5\,\text{m}}$$

Substitute numerical values and evaluate T:

$$T = \frac{(7.5\,\text{m})(85\,\text{kg}) + (15\,\text{m})(360\,\text{kg})}{5\,\text{m}} \times (9.81\,\text{m/s}^2)\cos 30° = \boxed{10.3\,\text{kN}}$$

Apply $\sum_i \vec{F}_i = 0$ to the bar:

$$\sum F_y = F_v + T\sin 60° - mg - Mg = 0$$

and

$$\sum F_x = F_h - T\cos 60° = 0$$

Solve the y equation for F_v:

$$F_v = -T\sin 60° + (m + M)g = -(10.3\,\text{kN})\sin 60° + (85\,\text{kg} + 360\,\text{kg})(9.81\,\text{m/s}^2) = -4.55\,\text{kN}$$

Solve the x equation for F_h:

$$F_h = T\cos 60° = (10.3\,\text{kN})\cos 60° = 5.15\,\text{kN}$$

Find the magnitude of the force exerted by the wall on the bar:

$$F = \sqrt{F_v^2 + F_h^2} = \sqrt{(-4.55\,\text{kN})^2 + (5.15\,\text{kN})^2} = \boxed{6.87\,\text{kN}}$$

Find the direction of the force exerted by the wall on the bar:

$$\theta = \tan^{-1}\left(\frac{F_v}{F_h}\right) = \tan^{-1}\left(\frac{-4.55\,\text{kN}}{5.15\,\text{kN}}\right) = \boxed{-41.5^\circ}$$

i.e., 41.5° below the horizontal.

(*b*) Apply $\sum \vec{\tau} = 0$ about the hinge:

$$[(10\,\text{m})\sin 60^\circ]T - (7.5\,\text{m})mg\cos 30^\circ - (15\,\text{m})Mg\cos 30^\circ = 0$$

Solve for T:

$$T = \frac{(7.5\,\text{m})m + (15\,\text{m})M}{(10\,\text{m})\sin 60^\circ}\,g\cos 30^\circ$$

Substitute numerical values and evaluate T:

$$T = \frac{(7.5\,\text{m})(85\,\text{kg}) + (15\,\text{m})(360\,\text{kg})}{(10\,\text{m})\sin 60^\circ} \times (9.81\,\text{m/s}^2)\cos 30^\circ = \boxed{5.92\,\text{kN}}$$

Apply $\sum \vec{F} = 0$ to the bar:

$$\sum F_y = F_v + T\cos 60^\circ - (85\,\text{kg})g - (360\,\text{kg})g = 0$$

and

$$\sum F_x = F_h - T\sin 60^\circ = 0$$

Solve the y equation for F_v:

$$F_v = -(5.92\,\text{kN})\cos 60^\circ + (85\,\text{kg} + 360\,\text{kg})(9.81\,\text{m/s}^2) = 1.41\,\text{kN}$$

Solve the x equation for F_h:

$$F_h = T\sin 60^\circ = (5.92\,\text{kN})\sin 60^\circ = 5.13\,\text{kN}$$

Find the magnitude of the force exerted by the wall on the bar:

$$F = \sqrt{F_v^2 + F_h^2} = \sqrt{(1.41\,\text{kN})^2 + (5.13\,\text{kN})^2} = \boxed{5.32\,\text{kN}}$$

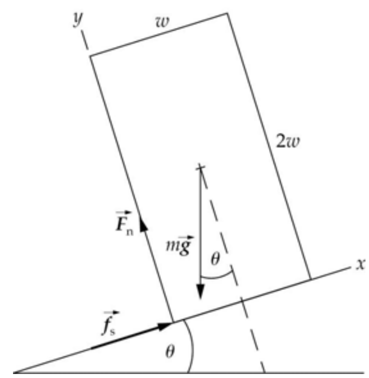
Find the direction of the force exerted by the wall on the bar:

$$\theta = \tan^{-1}\left(\frac{F_v}{F_h}\right) = \tan^{-1}\left(\frac{1.41\,\text{kN}}{5.13\,\text{kN}}\right)$$
$$= \boxed{15.4°}$$

i.e., 15.4° above the horizontal.

83 ••

Picture the Problem The box and the forces acting on it are shown in the figure. When the box is about to tip, F_n acts at its edge, as indicated in the drawing. We can use the definition of μ_s and apply the condition for rotational equilibrium in an accelerated frame to relate f_s to the weight of the box and, hence, to the normal force.



Using its definition, express μ_s:

$$\mu_s \geq \frac{f_s}{F_n}$$

Apply $\sum \vec{\tau} = 0$ about an axis through the box's center of mass:

$$wf_s - \tfrac{1}{2}wF_n = 0$$

Solve for the ratio $\frac{f_s}{F_n}$:

$$\frac{f_s}{F_n} = \frac{1}{2}$$

Substitute to obtain the condition for tipping:

$$\mu_s \geq 0.500$$

Therefore, if the box is to slide:

$\mu_s < \boxed{0.500}$, as in Problem 75.

Remarks: The difference between problems 75 and 83 is that in 75 the maximum acceleration before slipping is 0.5g, whereas in 88 it is (0.5 cos9°– sin9°) = 0.337g.

***84** ••

Picture the Problem Let the mass of the rod be represented by M. Because the rod is in equilibrium, we can apply the condition for rotational equilibrium to relate the masses of the objects placed on it to its mass.

Apply $\sum\vec{\tau}=0$ about an axis through the pivot for the initial condition:

$$(20\,\text{cm})(2m+2\,\text{g})-(40\,\text{cm})m-(10\,\text{cm})M=0$$

Solve for and evaluate M:

$$M=\frac{(20\,\text{cm})(2m+2\,\text{g})-(40\,\text{cm})m}{10\,\text{cm}}=\boxed{4.00\,\text{g}}$$

Apply $\sum\vec{\tau}=0$ about an axis through the pivot for the second condition:

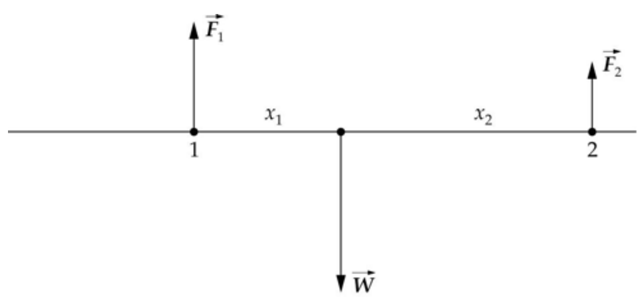
$$(20\,\text{cm})m-(10\,\text{cm})M=0$$

Solve for and evaluate m:

$$m=\frac{(10\,\text{cm})M}{20\,\text{cm}}=\tfrac{1}{2}M=\boxed{2.00\,\text{g}}$$

***85 ••**

Picture the Problem Let the distance from the center of the meterstick of either finger be x_1 and x_2 and W the weight of the stick. Because the meterstick is in equilibrium, we can apply the condition for rotational equilibrium to obtain expressions for the forces one's fingers exert on the meterstick as functions of the distances x_1 and x_2 and the weight of the meterstick W. We can then explain the stop-and-start motion of one's fingers as they are brought closer together by considering the magnitudes of these forces in relationship the coefficients of static and kinetic friction.



(*a*) $\boxed{\text{The stick remains balanced as long as the center of mass is between the two fingers. For a balanced stick the normal force exerted by the finger nearest the center of mass is greater than that exerted by the other finger. Consequently, a larger static-frictional force can be exerted by the finger closer to the center of mass, which means the slipping occurs at the other finger.}}$

(*b*) Apply $\sum \vec{\tau} = 0$ about an axis through point 1 to obtain:

$$F_2(x_1 + x_2) - Wx_1 = 0$$

Solve for F_2 to obtain:

$$F_2 = W\frac{x_1}{x_1 + x_2}$$

Apply $\sum \vec{\tau} = 0$ about an axis through point 2 to obtain:

$$-F_1(x_1 + x_2) + Wx_2 = 0$$

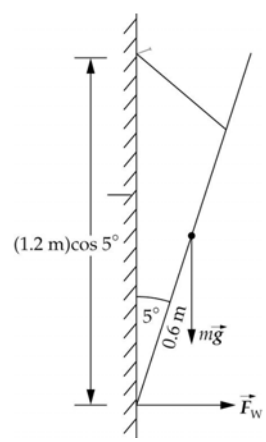
Solve for F_1 to obtain:

$$F_1 = W\frac{x_2}{x_1 + x_2}$$

$$\boxed{\begin{array}{l}\text{The finger farthest from the center of mass will slide inward until the normal}\\ \text{force it exerts on the stick is sufficiently large to produce a kinetic - frictional}\\ \text{force exceeding the maximum static - frictional force exerted by the other}\\ \text{finger. At that point the finger that was not sliding begins to slide, the finger}\\ \text{that was sliding stops sliding, and the process is reversed. When one finger is}\\ \text{is slipping the other is not.}\end{array}}$$

86 ••

Picture the Problem The drawing shows a side view of the wall-and-picture system. Because the frame's width is not specified, we assume it to be negligible. Note that 0.75, 0.4, and 0.85 form a Pythagorean triad. Thus, the nail will be at the same level as the top of the frame. We can apply the condition for rotational equilibrium to determine the force exerted by the wall.



(*a*) Because the center of gravity of the picture is in front of the wall, the torque due to *mg* about the nail must be balanced by an opposing torque due to the force of the wall on the picture, acting horizontally. So that $\sum F_x = 0$, the tension in the wire must have a horizontal component, and the picture must therefore tilt forward.

(*b*) Apply $\sum \vec{\tau} = 0$ about an axis through the nail and parallel to the

$$-[(0.6\,\text{m})\sin 5°](8\,\text{kg})(9.81\,\text{m/s}^2) + [(1.2\,\text{m})\cos 5°]F_W = 0$$

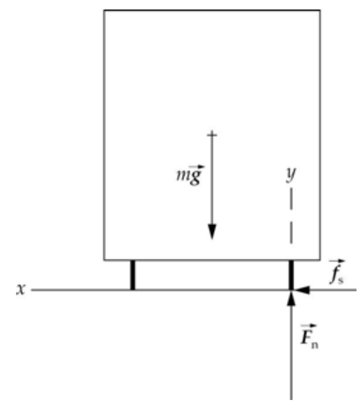
wall to obtain:

Solve for and evaluate F_W:

$$F_W = \frac{[(0.6\,\text{m})\sin 5°](8\,\text{kg})(9.81\,\text{m/s}^2)}{(1.2\,\text{m})\cos 5°} = \boxed{3.43\,\text{N}}$$

87 ••

Picture the Problem The box car and rail are shown in the drawing. At the critical speed, the normal force is entirely on the outside rail. The center of gravity is 0.775 m from that rail and 2.15 m above it. Choose the coordinate system shown in the figure. To find the speed at which this situation prevails, we can apply the conditions for static equilibrium in an accelerated frame.



Apply $\sum \vec{\tau} = 0$ about an axis through the center of gravity of the box car:

$$(0.775\,\text{m})F_n - (2.15\,\text{m})f_s = 0 \qquad (1)$$

Apply $\sum F_y = 0$ to the box car and solve for F_n:

$$F_n - mg = 0 \Rightarrow F_n = mg$$

Apply $\sum F_x = ma_{cm}$ to the box:

$$f_s = m\frac{v^2}{R}$$

Substitute in equation (1) to obtain:

$$(0.775\,\text{m})mg - (2.15\,\text{m})m\frac{v^2}{R} = 0$$

Solve for v:

$$v = \sqrt{0.360Rg}$$

(a) Evaluate v for R = 150 m:

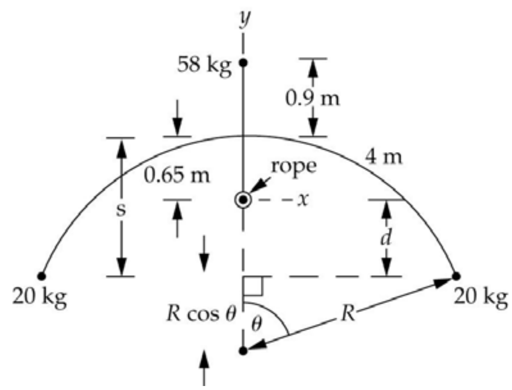
$$v = \sqrt{0.360(150\,\text{m})(9.81\,\text{m/s}^2)} = \boxed{23.0\,\text{m/s}}$$

(*b*) Evaluate v for R = 240 m:

$$v = \sqrt{0.360(240\,\text{m})(9.81\,\text{m/s}^2)} = \boxed{29.1\,\text{m/s}}$$

88 ••

Picture the Problem For neutral equilibrium, the center of mass of the system must be at the same height as the feet of the tightrope walker. The system is shown in the drawing. Let the origin of the coordinate system be at the rope. We'll determine the distance d such that y_{cm} = 0. We'll then determine the angle θ subtended by one half the long rod.



Express the y coordinate of the center of mass of the system:

$$y_{cm} = \frac{(58\,\text{kg})(0.9\,\text{m}) - 2(20\,\text{kg})d}{58\,\text{kg} + 40\,\text{kg}}$$

Set y_{cm} = 0 and solve for d:

d = 1.305 m

Relate the distances s and d and solve for s:

s = 0.65 m + d = 1.955 m

Relate s to R and θ:

$$s = R(1 - \cos\theta) \qquad (1)$$

Relate R and θ to the half-length of the rod:

$$R\theta = 4\,\text{m} \qquad (2)$$

Substitute in equation (1) to obtain:

$$1.955\,\text{m} = (4\,\text{m})\frac{1-\cos\theta}{\theta}$$

or

$$\frac{1-\cos\theta}{\theta} = 0.489$$

Use graphical or trial-and-error methods to solve for θ:

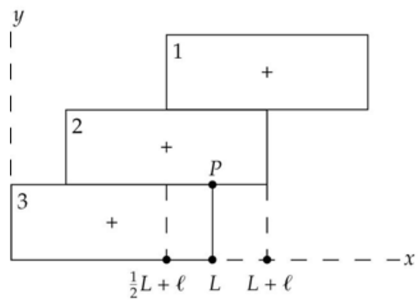
$$\theta = 1.08\,\text{rad}$$

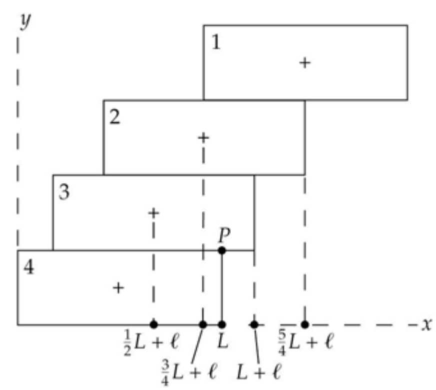
Substitute in equation (2) to obtain:

$$R = \frac{4\,\text{m}}{1.08\,\text{rad}} = \boxed{3.70\,\text{m}}$$

***89** •••

Picture the Problem Let the mass of each brick be m and number them as shown in the diagrams for 3 bricks and 4 bricks below. Let ℓ denote the maximum offset of the nth brick. Choose the coordinate system shown and apply the condition for rotational equilibrium about an axis parallel to the z axis and passing through the point P at the supporting edge of the nth brick.





(*a*) Apply $\sum\vec{\tau}=0$ about an axis through P and parallel to the z axis to bricks 1 and 2 for the 3-brick arrangement shown above on the left:

$$mg\left[L-\left(\tfrac{1}{2}L+\ell\right)\right]-mg\ell=0$$

Solve for ℓ to obtain:

$$\ell=\boxed{\tfrac{1}{4}L}$$

(*b*) Apply $\sum\vec{\tau}=0$ about an axis through P and parallel to the z axis to bricks 1 and 2 for the 4-brick arrangement shown above on the right:

$$mg\left[L-\left(\tfrac{1}{2}L+\ell\right)\right]+mg\left[L-\left(\tfrac{3}{4}L+\ell\right)\right]-mg\left(\tfrac{5}{4}L+\ell-L\right)=0$$

Solve for ℓ to obtain:

$$\ell=\tfrac{1}{6}L$$

Continuing in this manner we obtain, as the successive offsets, the sequence:

$$\boxed{\frac{L}{2},\frac{L}{4},\frac{L}{6},\frac{L}{8},\ldots\frac{L}{2n}}$$

where n = 1, 2, 3, … N.

(c) Express the offset of the (n +1)st brick in terms of the offset of the nth brick:

$$\ell_{n+1}=\ell_n+\frac{L}{2n}$$

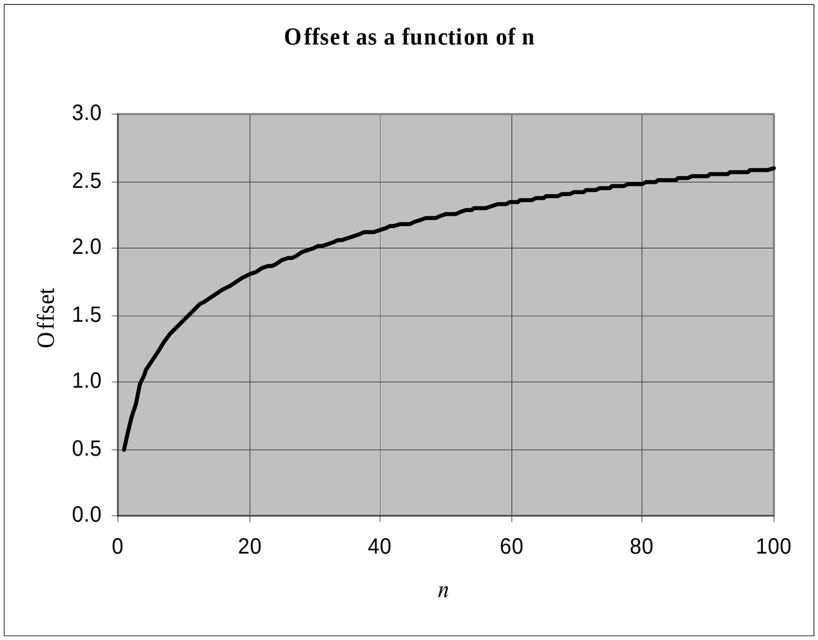
A spreadsheet program to calculate the sum of the offsets as a function of n is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
B5	B4+1	$n+1$
C5	C4+B1/(2*B5)	$\ell_n + \dfrac{L}{2n}$

	A	B	C	D
1	L=	1	m	
2				
3		n	offset	
4		1	0.500	
5		2	0.750	
6		3	0.917	
7		4	1.042	
8		5	1.142	
9		6	1.225	
10		7	1.296	
11		8	1.359	
12		9	1.414	
13		10	1.464	
98		95	2.568	
99		96	2.573	
100		97	2.579	
101		98	2.584	
102		99	2.589	
103		100	2.594	

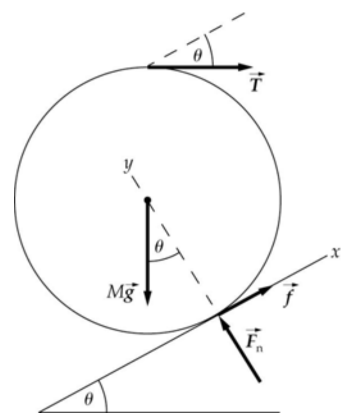
From the table we see that $\ell_5 = \boxed{1.142\,\text{m},}$ $\ell_{10} = \boxed{1.464\,\text{m},}$ and $\ell_{100} = \boxed{2.594\,\text{m}.}$

(*d*) Increasing N in the spreadsheet solution suggests that the sum of the individual offsets continues to grow as N increases without bound. The series is, in fact, divergent and the stack of bricks has no maximum offset or length.



90 •••

Picture the Problem The four forces acting on the sphere: its weight, *mg*; the normal force of the plane, F_n; the frictional force, *f*, acting parallel to the plane; and the tension in the string, *T*, are shown in the figure. Choose the coordinate system shown. Because the sphere is in equilibrium, we can apply the conditions for translational and rotational equilibrium to find *f*, F_n, and *T*.



(*a*) Apply $\sum \vec{\tau} = 0$ about an axis through the center of the sphere:

$$fR - TR = 0 \Rightarrow T = f$$

Apply $\sum F_x = 0$ to the sphere:

$$f + T\cos\theta - Mg\sin\theta = 0$$

Substitute for *f* and solve for *T*:

$$T = \frac{Mg\sin\theta}{1+\cos\theta}$$

Substitute numerical values and evaluate *T*:

$$T = \frac{(3\text{ kg})(9.81\text{ m/s}^2)\sin 30°}{1+\cos 30°} = \boxed{7.89\text{ N}}$$

(*b*) Apply $\sum F_y = 0$ to the sphere:

$$F_n - T\sin\theta - Mg\cos\theta = 0$$

Solve for F_n:

$$F_n = T\sin\theta + Mg\cos\theta$$

Substitute numerical values and evaluate F_n:

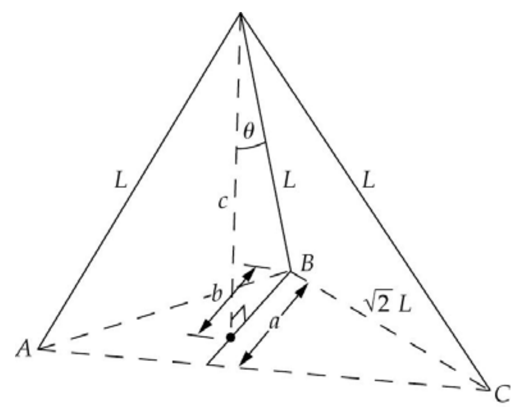
$$F_n = (7.89\text{ N})\sin 30° + (3\text{ kg})(9.81\text{ m/s}^2)\cos 30° = \boxed{29.4\text{ N}}$$

(c) In part (a) we showed that $f = T$:

$$f = \boxed{7.89\text{ N}}$$

91 •••

Picture the Problem Let L be the length of each leg of the tripod. Applying the Pythagorean theorem leads us to conclude that the distance a shown in the figure is $\sqrt{\frac{3}{2}}L$ and the distance b, the distance to the centroid of the triangle ABC is $\frac{2}{3}\sqrt{\frac{3}{2}}L$, and the distance c is $\frac{L}{\sqrt{3}}$. These results allow us to conclude that $\cos\theta = \frac{L}{\sqrt{3}}$. Because the tripod is in equilibrium, we can apply the condition for translational equilibrium to find the compressional forces in each leg.



Letting F_C represent the compressional force in a leg of the tripod, apply $\sum \vec{F} = 0$ to the apex of the tripod:

$$3F_C\cos\theta - mg = 0$$

Solve for F_C:

$$F_C = \frac{mg}{3\cos\theta}$$

Solve for F_C:

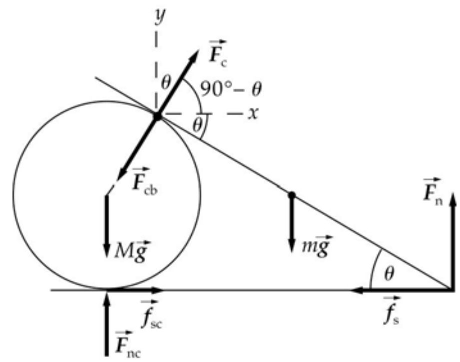
$$F_C = \frac{mg}{3\times\frac{1}{\sqrt{3}}} = \frac{\sqrt{3}}{3}mg$$

Substitute numerical values and evaluate F_C:

$$F_C = \frac{\sqrt{3}}{3}(100\text{ kg})(9.81\text{ m/s}^2) = \boxed{566\text{ N}}$$

92 ••

Picture the Problem The forces that act on the beam are its weight, mg; the force of the cylinder, F_c, acting along the radius of the cylinder; the normal force of the ground, F_n; and the friction force $f_s = \mu_s F_n$. The forces acting on the cylinder are its weight, Mg; the force of the beam on the cylinder, $F_{cb} = F_c$ in magnitude, acting radially inward; the normal force of the ground on the cylinder, F_{nc}; and the force of friction, $f_{sc} = \mu_{sc} F_{nc}$. Choose the coordinate system shown in the figure and apply the conditions for rotational and translational equilibrium.



Express $\mu_{s,\text{beam-floor}}$ in terms of f_s and F_n:

$$\mu_{s,\text{beam-floor}} = \frac{f_s}{F_n} \qquad (1)$$

Express $\mu_{s,\text{cylinder-floor}}$ in terms of f_{sc} and F_{nc}:

$$\mu_{s,\text{cylinder-floor}} = \frac{f_{sc}}{F_{nc}} \qquad (2)$$

Apply $\sum \vec{\tau} = 0$ about an axis through the right end of the beam:

$$[(10\,\text{cm})\cos\theta]mg - (15\,\text{cm})F_c = 0$$

Solve for and evaluate F_c:

$$F_c = \frac{[(10\,\text{cm})\cos\theta]mg}{15\,\text{cm}} = \frac{[10\cos 30°](5\,\text{kg})(9.81\,\text{m/s}^2)}{15} = 28.3\,\text{N}$$

Apply $\sum F_y = 0$ to the beam:

$$F_n + F_c\cos(90° - \theta) - mg = 0$$

Solve for F_n:

$$F_n = mg - F_c\cos\theta = (5\,\text{kg})(9.81\,\text{m/s}^2) - (28.3\,\text{N})\cos 30° = 24.5\,\text{N}$$

Apply $\sum F_x = 0$ to the beam:

$$-f_s + F_c\cos(90° - \theta) = 0$$

Solve for and evaluate f_s:

$$f_s = F_c \cos(90° - \theta) = (28.3\,\text{N})\cos 60° = 14.2\,\text{N}$$

$\vec{F}_{cb}$ is the reaction force to $\vec{F}_c$:

$F_{cb} = F_c = 28.3\,\text{N}$ radially inward.

Apply $\sum F_y = 0$ to the cylinder:

$$F_{nc} - F_{cb}\cos\theta - Mg = 0$$

Solve for and evaluate F_{nc}:

$$F_{nc} = F_{cb}\cos\theta + Mg = (28.3\,\text{N})\cos 30° + (8\,\text{kg})(9.81\,\text{m/s}^2) = 103\,\text{N}$$

Apply $\sum F_x = 0$ to the cylinder:

$$f_{sc} - F_{cb}\cos(90° - \theta) = 0$$

Solve for and evaluate f_{sc}:

$$f_{sc} = F_{cb}\cos(90° - \theta) = (28.3\,\text{N})\cos 60° = 14.2\,\text{N}$$

Substitute numerical values in equations (1) and (2) and evaluate $\mu_{\text{s,beam-floor}}$ and $\mu_{\text{s,cylinder-floor}}$:

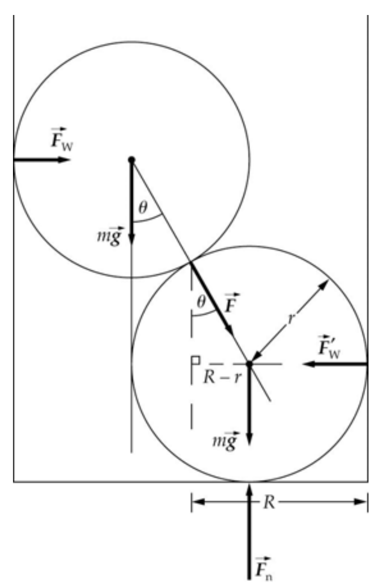
$$\mu_{\text{s,beam-floor}} = \frac{14.2\,\text{N}}{24.5\,\text{N}} = \boxed{0.580}$$

and

$$\mu_{\text{s,cylinder-floor}} = \frac{14.2\,\text{N}}{103\,\text{N}} = \boxed{0.138}$$

93 •••

Picture the Problem The geometry of the system is shown in the drawing. Let upward be the positive y direction and to the right be the positive x direction. Let the angle between the vertical center line and the line joining the two centers be θ. Then $\sin\theta = \dfrac{R-r}{r}$ and $\tan\theta = \dfrac{R-r}{\sqrt{R(2r-R)}}$.



The force exerted by the bottom of the cylinder is just $2mg$. Let F be the force that the top sphere exerts on the lower sphere. Because the spheres are in equilibrium, we can apply the condition for translational equilibrium.

Apply $\sum F_y = 0$ to the spheres:

$$F_n - mg - mg = 0$$

Solve for F_n: $F_n = \boxed{2mg}$

Because the cylinder wall is smooth, $F\cos\theta = mg$, and: $F = \boxed{\dfrac{mg}{\cos\theta}}$

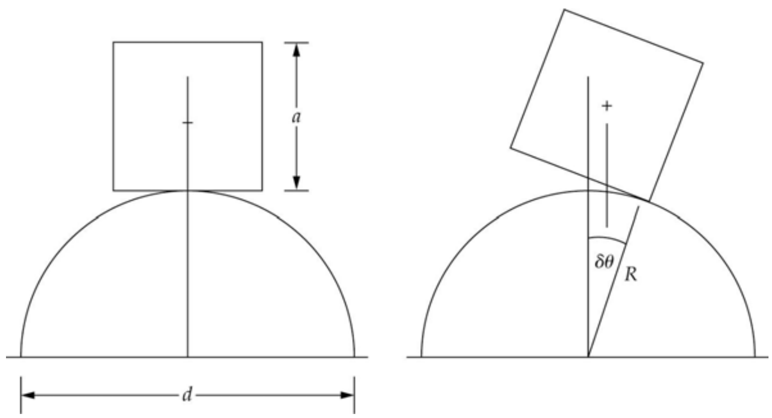
Express the x component of F: $F_x = F\sin\theta = mg\tan\theta$

Express the force that the wall of the cylinder exerts: $F_W = \boxed{mg\dfrac{R-r}{\sqrt{R(2r-R)}}}$

Remarks: Note that as *r* approaches *R*/2, $F_w \to \infty$.

***94 •••**

Picture the Problem Consider a small rotational displacement, $\delta\theta$ of the cube from equilibrium. This shifts the point of contact between cube and cylinder by $R\delta\theta$, where $R = d/2$. As a result of that motion, the cube itself is rotated through the same angle $\delta\theta$, and so its center is shifted in the same direction by the amount $(a/2)\delta\theta$, neglecting higher order terms in $\delta\theta$.



If the displacement of the cube's center of mass is less than that of the point of contact, the torque about the point of contact is a restoring torque, and the cube will return to its equilibrium position. If, on the other hand, $(a/2)\delta\theta > (d/2)\delta\theta$, then the torque about the point of contact due to mg is in the direction of $\delta\theta$, and will cause the displacement from equilibrium to increase. We see that the minimum value of d/a for stable equilibrium is $d/a = 1$.