Chapter 2
Motion in One Dimension

Conceptual Problems

1 •

Determine the Concept The "average velocity" is being requested as opposed to "average speed".

The average velocity is defined as the change in position or displacement divided by the change in time.

$$v_{av} = \frac{\Delta y}{\Delta t}$$

The change in position for any "round trip" is zero by definition. So the ***average velocity*** for any round trip must also be zero.

$$v_{av} = \frac{\Delta y}{\Delta t} = \frac{0}{\Delta t} = \boxed{0}$$

***2** •

Determine the Concept The important concept here is that "average speed" is being requested as opposed to "average velocity".

Under all circumstances, including **constant acceleration**, the definition of the average speed is the ratio of the total distance traveled ($H + H$) to the total time elapsed, in this case $2H/T$. $\boxed{(d) \text{ is correct.}}$

Remarks: **Because this motion involves a round trip, if the question asked for "average velocity," the answer would be zero.**

3 •

Determine the Concept Flying with the wind, the speed of the plane relative to the ground (v_{PG}) is the sum of the speed of the wind relative to the ground (v_{WG}) and the speed of the plane relative to the air ($v_{PG} = v_{WG} + v_{PA}$). Flying into or against the wind the speed relative to the ground is the difference between the wind speed and the true air speed of the plane ($v_g = v_w - v_t$). Because the ground speed landing against the wind is smaller than the ground speed landing with the wind, it is safer to land *against* the wind.

4 •

Determine the Concept The important concept here is that $a = dv/dt$, where a is the acceleration and v is the velocity. Thus, the acceleration is positive if dv is positive; the acceleration is negative if dv is negative.

(*a*) Let's take the direction a car is moving to be the positive direction:

Because the car is moving in the direction we've chosen to be positive, its velocity is positive ($dx > 0$). If the car is braking, then its velocity is decreasing ($dv < 0$) and its acceleration (dv/dt) is negative.

(*b*) Consider a car that is moving to

Because the car is moving in the direction the right but choose the positive direction to be to the left:

opposite to that we've chosen to be positive, its velocity is negative ($dx < 0$). If the car is braking, then its velocity is increasing ($dv > 0$) and its acceleration (dv/dt) is positive.

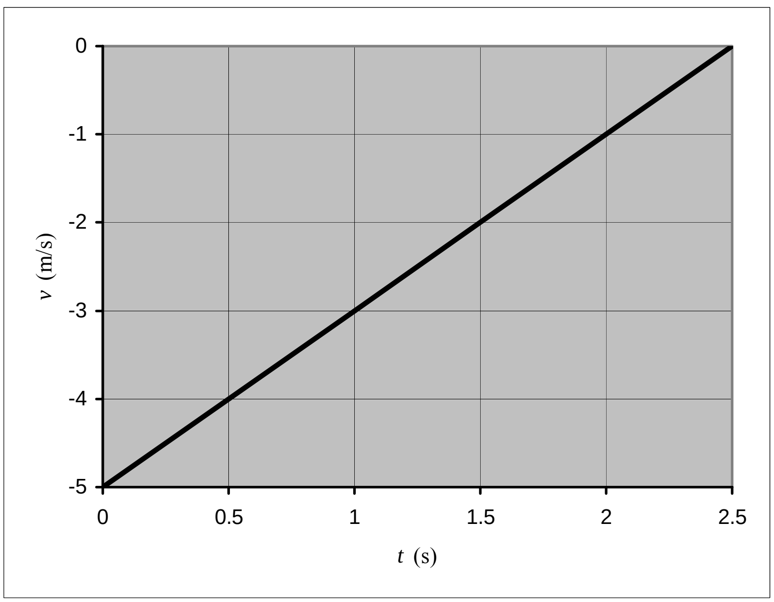
***5** •

Determine the Concept The important concept is that when both the acceleration and the velocity are in the same direction, the speed increases. On the other hand, when the acceleration and the velocity are in opposite directions, the speed decreases.

(*a*) $\boxed{\text{Because your velocity remains negative, your displacement must be } \textit{negative}.}$

(*b*) $\boxed{\text{Define the direction of your trip as the negative direction. During the last five steps gradually slow the speed of walking, until the wall is reached.}}$

(*c*) A graph of *v* as a function of *t* that is consistent with the conditions stated in the problem is shown below:



6 •

Determine the Concept True. We can use the definition of average velocity to express the displacement Δx as $\Delta x = v_{av}\Delta t$. Note that, if the acceleration is constant, the average velocity is also given by $v_{av} = (v_i + v_f)/2$.

7 •

Determine the Concept Acceleration is the slope of the velocity versus time curve, $a = dv/dt$; while velocity is the slope of the position versus time curve, $v = dx/dt$. The speed of an object is the magnitude of its velocity.

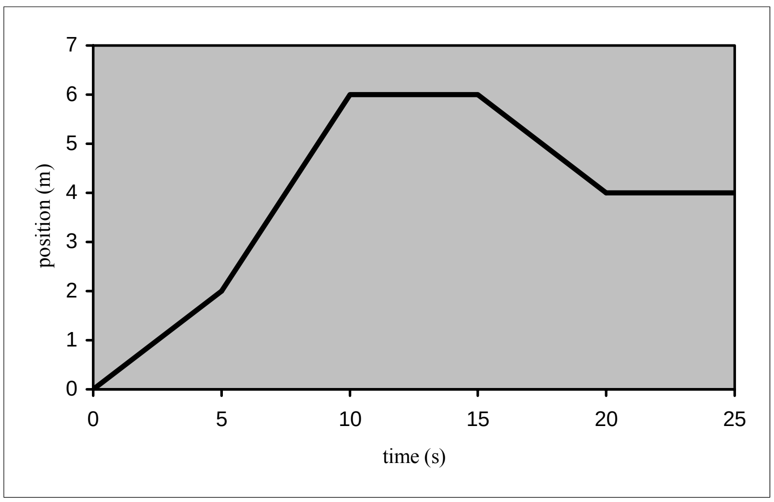
(*a*) True. Zero acceleration implies that the velocity is constant. If the velocity is constant (including zero), the speed must also be constant.

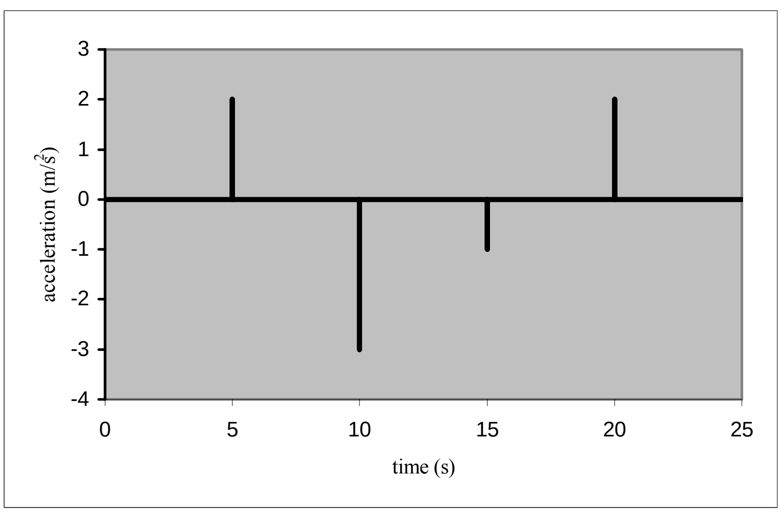
(*b*) True in one dimension.

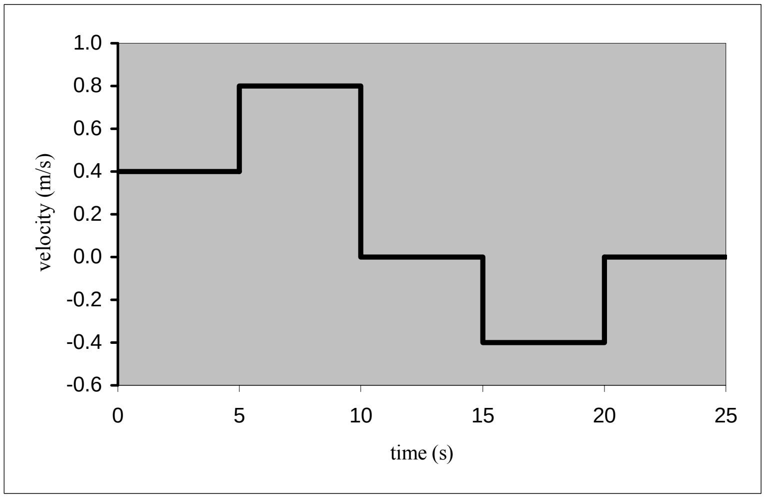
Remarks: The answer to (*b*) would be False in more than one dimension. In one dimension, if the speed remains constant, then the object cannot speed up, slow down, or reverse direction. Thus, if the speed remains constant, the velocity remains constant, which implies that the acceleration remains zero. (In more than one-dimensional motion, an object can change direction while maintaining constant speed. This constitutes a change in the direction of the velocity.) Consider a ball moving in a circle at a constant rotation rate. The speed (magnitude of the velocity) is constant while the velocity is tangent to the circle and always changing. The acceleration is always pointing inward and is certainly NOT zero.

***8** ••

Determine the Concept Velocity is the slope of the position versus time curve and acceleration is the slope of the velocity versus time curve. See the graphs below.







9 •

Determine the Concept False. The average velocity is defined (for any acceleration) as the change in position (the displacement) divided by the change in time $v_{\text{av}} = \Delta x/\Delta t$. It is always valid. If the acceleration remains constant the average velocity is also given by

$$v_{\text{av}} = \frac{v_{\text{i}} + v_{\text{f}}}{2}$$

Consider an engine piston moving up and down as an example of non-constant velocity. For one complete cycle, $v_{\text{f}} = v_{\text{i}}$ and $x_{\text{i}} = x_{\text{f}}$ so $v_{\text{av}} = \Delta x/\Delta t$ is zero. The formula involving the mean of v_{f} and v_{i} cannot be applied because the acceleration is not constant, and yields an incorrect nonzero value of v_{i}.

10 •

Determine the Concept This can occur if the rocks have different initial speeds. Ignoring air resistance, the acceleration is constant. Choose a coordinate system in which the origin is at the point of release and upward is the positive direction. From the constant-acceleration equation

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2$$

we see that the only way two objects can have the same acceleration ($-g$ in this case) and cover the same distance, $\Delta y = y - y_0$, in different times would be if the initial velocities of the two rocks were different. Actually, the answer would be the same whether or not the acceleration is constant. It is just easier to see for the special case of constant acceleration.

***11 ••**

Determine the Concept Neglecting air resistance, the balls are in free fall, each with the same free-fall acceleration, which is a constant.

At the time the second ball is released, the first ball is already moving. Thus, during any time interval their velocities will increase by exactly the same amount. What can be said about the speeds of the two balls? *The first ball will always be moving faster than the second ball.*

This being the case, what happens to the separation of the two balls while they are both falling? *Their separation increases*. $\boxed{(a)\text{ is correct.}}$

12 ••

Determine the Concept The slope of an $x(t)$ curve at any point in time represents the speed at that instant. The way the slope changes as time increases gives the sign of the acceleration. If the slope becomes less negative or more positive as time increases (as you move to the right on the time axis), then the acceleration is positive. If the slope becomes less positive or more negative, then the acceleration is negative. The slope of the slope of an $x(t)$ curve at any point in time represents the acceleration at that instant.

The slope of curve (a) is negative and becomes more negative as time increases. Therefore, the velocity is negative and the acceleration is negative.

The slope of curve (b) is positive and constant and so the velocity is positive and constant. Therefore, the acceleration is zero.

The slope of curve (c) is positive and decreasing. Therefore, the velocity is positive and the acceleration is negative.

The slope of curve (d) is positive and increasing. Therefore, the velocity and acceleration are positive. We need more information to conclude that a is constant.

The slope of curve (e) is zero. Therefore, the velocity and acceleration are zero.

$\boxed{(d)\text{ best shows motion with constant positive acceleration.}}$

***13 •**

Determine the Concept The slope of a $v(t)$ curve at any point in time represents the acceleration at that instant. Only one curve has a constant and positive slope.

$\boxed{(b)\text{ is correct.}}$

14 •

Determine the Concept No. The word average implies an interval of time rather than an instant in time; therefore, the statement makes no sense.

***15 •**

Determine the Concept Note that the ″average velocity″ is being requested as opposed to the ″average speed.″

Yes. In any roundtrip, A to B, and back to A, the average velocity is zero.

$$v_{\mathrm{av(A\to B\to A)}} = \frac{\Delta x}{\Delta t} = \frac{\Delta x_{\mathrm{AB}} + \Delta x_{\mathrm{BA}}}{\Delta t} = \frac{\Delta x_{\mathrm{AB}} + \left(-\Delta x_{\mathrm{BA}}\right)}{\Delta t} = \frac{0}{\Delta t} = \boxed{0}$$

On the other hand, the average velocity between A and B is not generally zero.

$$v_{\mathrm{av(A\to B)}} = \frac{\Delta x_{\mathrm{AB}}}{\Delta t} \neq \boxed{0}$$

Remarks: Consider an object launched up in the air. Its average velocity on the way up is NOT zero. Neither is it zero on the way down. However, over the round trip, it is zero.

16 •

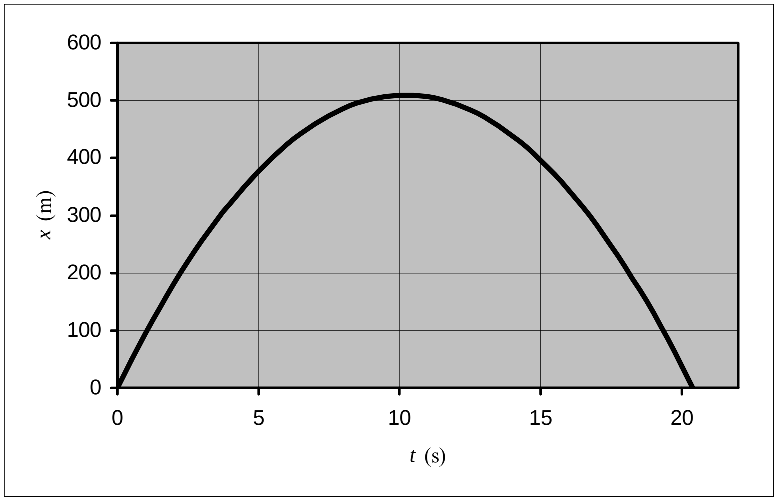
Determine the Concept An object is farthest from the origin when it is farthest from the time axis. In one-dimensional motion starting from the origin, the point located farthest from the time axis in a distance-versus-time plot is the farthest from its starting point. Because the object's initial position is at $x = 0$, point B represents the instant that the object is farthest from $x = 0$. $\boxed{(b)\text{ is correct.}}$

17 •

Determine the Concept No. If the velocity is constant, a graph of position as a function of time is linear with a constant slope equal to the velocity.

18 •

Determine the Concept Yes. The average velocity in a time interval is defined as the displacement divided by the elapsed time $v_{\mathrm{av}} = \Delta x/\Delta t$. The fact that $v_{\mathrm{av}} = 0$ for some time interval, Δt, implies that the displacement Δx over this interval is also zero. Because the instantaneous velocity is defined as $v = \lim_{\Delta t\to 0}(\Delta x/\Delta t)$, it follows that v must also be zero. As an example, in the following graph of x versus t, over the interval between $t = 0$ and $t \approx 21$ s, $\Delta x = 0$. Consequently, $v_{\mathrm{av}} = 0$ for this interval. Note that the instantaneous velocity is zero only at $t \approx 10$ s.



19 ••

Determine the Concept In the one-dimensional motion shown in the figure, the velocity is a minimum when the slope of a position-versus-time plot goes to zero (i.e., the curve becomes horizontal). At these points, the slope of the position-versus-time curve is zero; therefore, the speed is zero. $\boxed{(b) \text{ is correct.}}$

***20** ••

Determine the Concept In one-dimensional motion, the velocity is the slope of a position-versus-time plot and can be either positive or negative. On the other hand, the speed is the magnitude of the velocity and can only be positive. We'll use v to denote velocity and the word "speed" for how fast the object is moving.

(*a*)
curve a: $v(t_2) < v(t_1)$
curve b: $v(t_2) = v(t_1)$
curve c: $v(t_2) > v(t_1)$
curve d: $v(t_2) < v(t_1)$

(*b*)
curve a: $\text{speed}(t_2) < \text{speed}(t_1)$
curve b: $\text{speed}(t_2) = \text{speed}(t_1)$
curve c: $\text{speed}(t_2) < \text{speed}(t_1)$
curve d: $\text{speed}(t_2) > \text{speed}(t_1)$

21 •

Determine the Concept Acceleration is the slope of the velocity-versus-time curve, $a = dv/dt$, while velocity is the slope of the position-versus-time curve, $v = dx/dt$.

(*a*) False. Zero acceleration implies that the velocity is *not changing*. The velocity could be any constant (including zero). But, if the velocity is constant and nonzero, the particle must be moving.

(*b*) True. Again, zero acceleration implies that the velocity remains constant. This means that the x-versus-t curve has a constant slope (i.e., a straight line). Note: This does not necessarily mean a zero-slope line.

22 •

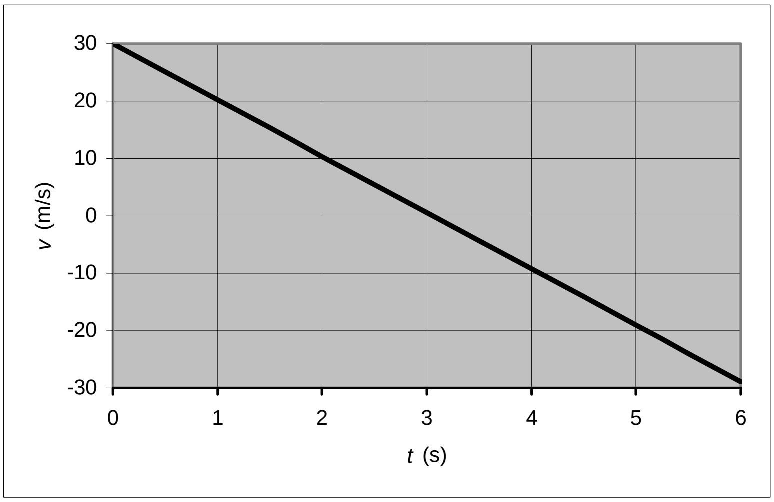
Determine the Concept Yes. If the velocity is changing the acceleration is not zero. The velocity is zero and the acceleration is nonzero any time an object is *momentarily* at rest. If the acceleration were also zero, the velocity would never change; therefore, the object would have to remain at rest.

Remarks: **It is important conceptually to note that when both the acceleration and the velocity have the same sign, the speed increases. On the other hand, when the acceleration and the velocity have opposite signs, the speed decreases.**

23 •

Determine the Concept In the absence of air resistance, the ball will experience a constant acceleration. Choose a coordinate system in which the origin is at the point of release and the upward direction is positive.

The graph shows the velocity of a ball that has been thrown straight upward with an initial speed of 30 m/s as a function of time. Note that the slope of this graph, the acceleration, is the same at every point, including the point at which $v = 0$ (at the top of its flight). Thus, $\boxed{v_{\text{top of flight}} = 0}$ and $\boxed{a_{\text{top of flight}} = -g}$.



The acceleration is the slope ($-g$).

24 •

Determine the Concept The "average speed" is being requested as opposed to "average velocity." We can use the definition of average speed as distance traveled divided by the elapsed time and expression for the average speed of an object when it is experiencing constant acceleration to express v_{av} in terms of v_0.

The average speed is defined as the total distance traveled divided by the change in time:

$$v_{\text{av}} = \frac{\text{total distance traveled}}{\text{total time}}$$

$$= \frac{H + H}{T} = \frac{2H}{T}$$

Find the average speed for the upward flight of the object:

$$v_{\text{av,up}} = \frac{v_0 + 0}{2} = \frac{H}{\frac{1}{2}T}$$

Solve for H to obtain:

$$H = \tfrac{1}{4}v_0 T$$

Find the average speed for the downward flight of the object:

$$v_{\text{av,down}} = \frac{0 + v_0}{2} = \frac{H}{\frac{1}{2}T}$$

Solve for H to obtain:

$$H = \tfrac{1}{4}v_0 T$$

Substitute in our expression for v_{av} to obtain:

$$v_{\text{av}} = \frac{2\left(\frac{1}{4}v_0 T\right)}{T} = \boxed{\frac{v_0}{2}}$$

Because $v_0 \neq 0$, the average speed is not zero.

Remarks: 1) Because this motion involves a roundtrip, if the question asked for ″average velocity″, the answer would be zero. 2) Another easy way to obtain this result is take the absolute value of the velocity of the object to obtain a graph of its speed as a function of time. A simple geometric argument leads to the result we obtained above.

25 •

Determine the Concept In the absence of air resistance, the bowling ball will experience constant acceleration. Choose a coordinate system with the origin at the point of release and upward as the positive direction. Whether the ball is moving upward and slowing down, is momentarily at the top of its trajectory, or is moving downward with ever increasing velocity, its acceleration is constant and equal to the acceleration due to gravity. $\boxed{(b)\text{ is correct.}}$

26 •

Determine the Concept Both objects experience the same constant acceleration. Choose a coordinate system in which downward is the positive direction and use a constant-acceleration equation to express the position of each object as a function of time.

Using constant-acceleration equations, express the positions of both objects as functions of time:

$$x_{\text{A}} = x_{0,\text{A}} + v_0 t + \tfrac{1}{2}gt^2$$

and

$$x_{\text{B}} = x_{0,\text{B}} + v_0 t + \tfrac{1}{2}gt^2$$

where $v_0 = 0$.

Express the separation of the two objects by evaluating $x_{\text{B}} - x_{\text{A}}$:

$$x_{\text{B}} - x_{\text{A}} = x_{0,\text{B}} - x_{0.\text{A}} = 10\,\text{m}$$

and $\boxed{(d)\text{ is correct.}}$

***27 ••**

Determine the Concept Because the Porsche accelerates uniformly, we need to look for a graph that represents constant acceleration. We are told that the Porsche has a constant acceleration that is positive (the velocity is increasing); therefore we must look for a velocity-versus-time curve with a positive constant slope and a nonzero intercept.

$\boxed{(c)\text{ is correct.}}$

***28 ••**

Determine the Concept In the absence of air resistance, the object experiences constant acceleration. Choose a coordinate system in which the downward direction is positive.

Express the distance D that an object, released from rest, falls in time t:

$$D = \tfrac{1}{2}gt^2$$

Because the distance fallen varies with the square of the time, during the first two seconds it falls four times the distance it falls during the first second.

$$\boxed{(a)\text{ is correct.}}$$

29 ••

Determine the Concept In the absence of air resistance, the acceleration of the ball is constant. Choose a coordinate system in which the point of release is the origin and upward is the positive y direction.

The displacement of the ball halfway to its highest point is:

$$\Delta y = \frac{\Delta y_{\max}}{2}$$

Using a constant-acceleration equation, relate the ball's initial and final velocities to its displacement and solve for the displacement:

$$v^2 = v_0^2 + 2a\Delta y = v_0^2 - 2g\Delta y$$

Substitute $v_0 = 0$ to determine the maximum displacement of the ball:

$$\Delta y_{\max} = -\frac{v_0^2}{2(-g)} = \frac{v_0^2}{2g}$$

Express the velocity of the ball at half its maximum height:

$$v^2 = v_0^2 - 2g\Delta y = v_0^2 - 2g\frac{\Delta y_{\max}}{2}$$

$$= v_0^2 - g\Delta y_{\max} = v_0^2 - g\frac{v_0^2}{2g} = \frac{v_0^2}{2}$$

Solve for v:

$$v = \frac{\sqrt{2}}{2}v_0 \approx 0.707v_0$$

and $\boxed{(c)\text{ is correct.}}$

30 •

Determine the Concept As long as the acceleration remains constant the following constant-acceleration equations hold. If the acceleration is not constant, they do not, in general, give correct results except by coincidence.

$$x = x_0 + v_0t + \tfrac{1}{2}at^2 \qquad v = v_0 + at \qquad v^2 = v_0^2 + 2a\Delta x \qquad v_{av} = \frac{v_i + v_f}{2}$$

(a) False. From the first equation, we see that (a) is true if and only if the acceleration is constant.

(b) False. Consider a rock thrown straight up into the air. At the "top" of its flight, the velocity is zero but it is changing (otherwise the velocity would remain zero and the rock would hover); therefore the acceleration is not zero.

(c) True. The definition of average velocity, $v_{av} = \Delta x/\Delta t$, requires that this always be true.

***31** •

Determine the Concept Because the acceleration of the object is constant, the constant-acceleration equations can be used to describe its motion. The special expression for average velocity for constant acceleration is $v_{av} = \dfrac{v_i + v_f}{2}$. $\boxed{(c)\,\text{is correct.}}$

32 •

Determine the Concept The constant slope of the x-versus-t graph tells us that the velocity is constant and the acceleration is zero. A linear position versus time curve implies a constant velocity. The negative slope indicates a constant negative velocity. The fact that the velocity is constant implies that the acceleration is also constant and zero. $\boxed{(e)\,\text{is correct.}}$

33 ••

Determine the Concept The velocity is the slope of the tangent to the curve, and the acceleration is the rate of change of this slope. Velocity is the slope of the position-versus-time curve. A parabolic $x(t)$ curve opening upward implies an increasing velocity. The acceleration is positive. $\boxed{(a)\,\text{is correct.}}$

34 ••

Determine the Concept The acceleration is the slope of the tangent to the velocity as a function of time curve. For constant acceleration, a velocity-versus- time curve must be a straight line whose slope is the acceleration. Zero acceleration means that slope of $v(t)$ must also be zero. $\boxed{(c)\,\text{is correct.}}$

35 ••

Determine the Concept The acceleration is the slope of the tangent to the velocity as a function of time curve. For constant acceleration, a velocity-versus- time curve must be a straight line whose slope is the acceleration. The acceleration and therefore the slope can be positive, negative, or zero. $\boxed{(d)\,\text{is correct.}}$

36 ••

Determine the Concept The velocity is positive if the curve is above the $v = 0$ line (the t axis), and the acceleration is negative if the tangent to the curve has a negative slope. Only graphs (a), (c), and (e) have positive v. Of these, only graph (e) has a negative slope. $\boxed{(e)\,\text{is correct.}}$

37 ••

Determine the Concept The velocity is positive if the curve is above the $v = 0$ line (the t axis), and the acceleration is negative if the tangent to the curve has a negative slope. Only graphs (b) and (d) have negative v. Of these, only graph (d) has a negative slope.

$\boxed{(d)\text{ is correct.}}$

38 ••

Determine the Concept A linear velocity-versus-time curve implies constant acceleration. The displacement from time $t = 0$ can be determined by integrating v-versus-t — that is, by finding the area under the curve. The initial velocity at $t = 0$ can be read directly from the graph of v-versus-t as the v-intercept; i.e., $v(0)$. The acceleration of the object is the slope of $v(t)$. The average velocity of the object is given by drawing a horizontal line that has the same area under it as the area under the curve. Because all of these quantities can be determined $\boxed{(e)\text{ is correct.}}$

***39** ••

Determine the Concept The velocity is the slope of a position versus time curve and the acceleration is the rate at which the velocity, and thus the slope, changes.

Velocity

(a) Negative at t_0 and t_1.
(b) Positive at t_3, t_4, t_6, and t_7.
(c) Zero at t_2 and t_5.

Acceleration

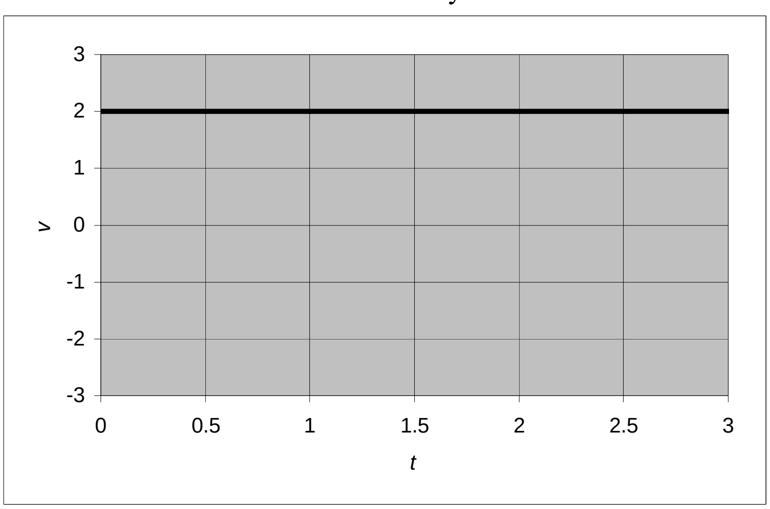
(a) Negative at t_4.
(b) Positive at t_2 and t_6.
(c) Zero at t_0, t_1, t_3, t_5, and t_7.

The acceleration is positive at points where the slope increases as you move toward the right.

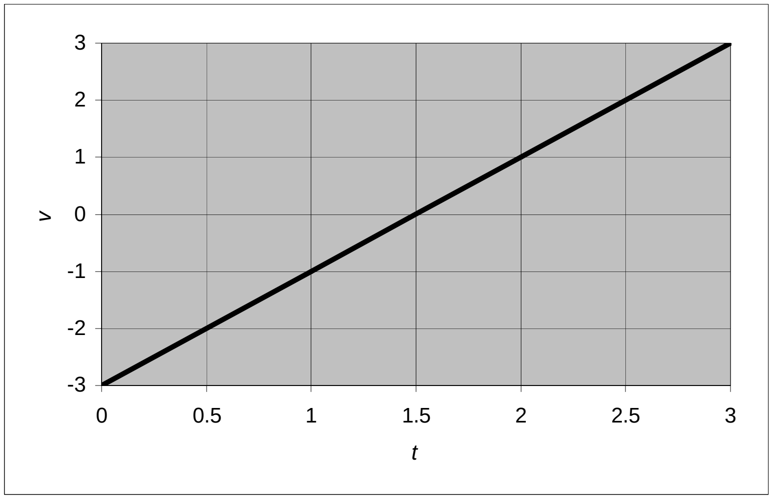
40 ••

Determine the Concept Acceleration is the slope of a velocity-versus-time curve.

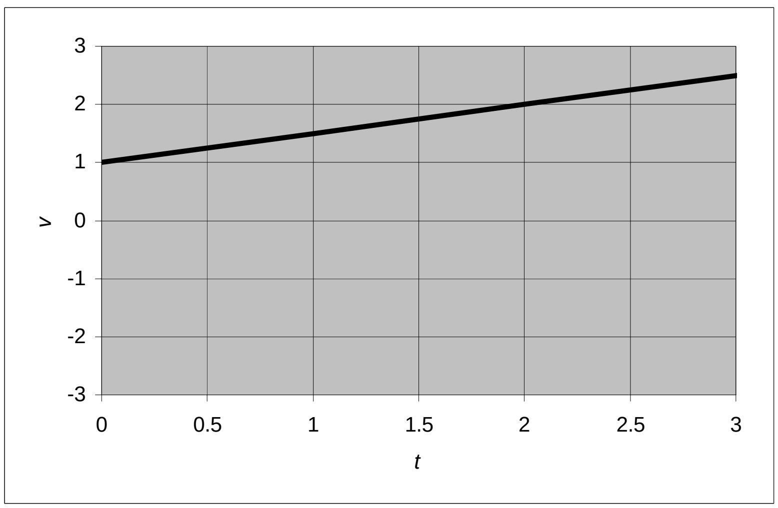
(a) Acceleration is zero and constant while velocity is not zero.



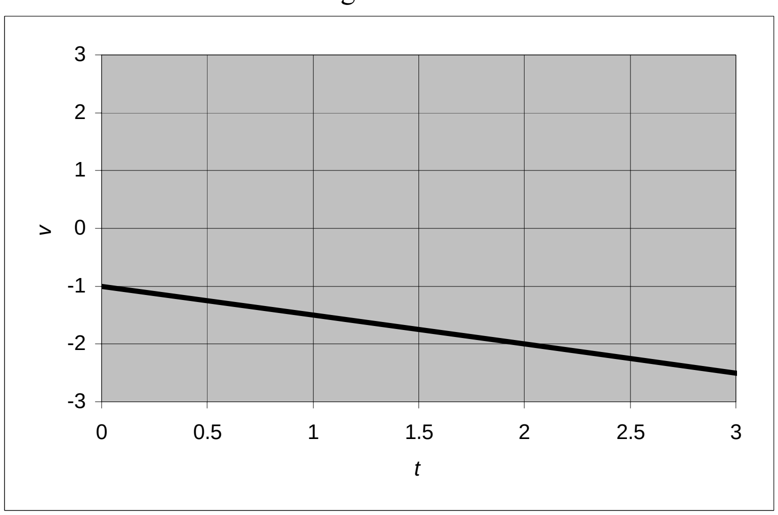
(b) Acceleration is constant but not zero.



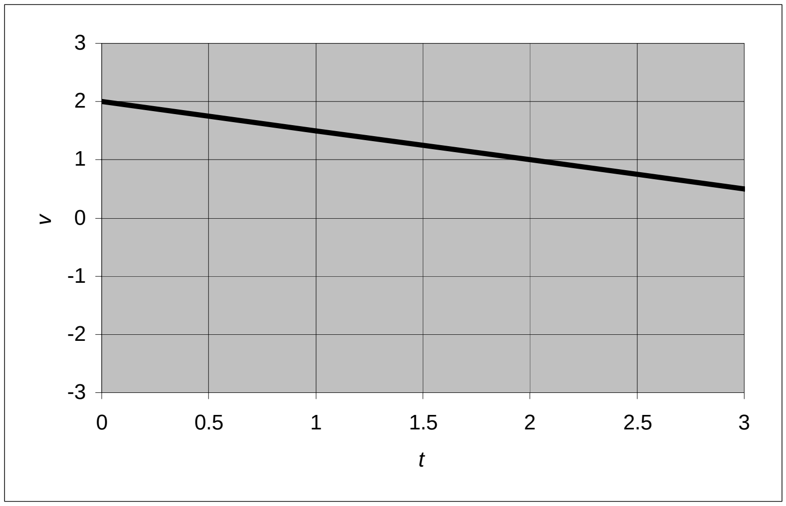
(c) Velocity and acceleration are both positive.



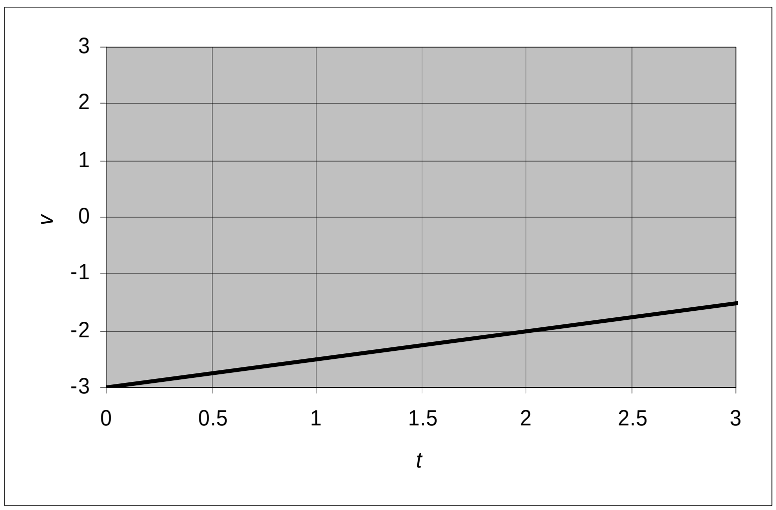
(d) Velocity and acceleration are both negative.



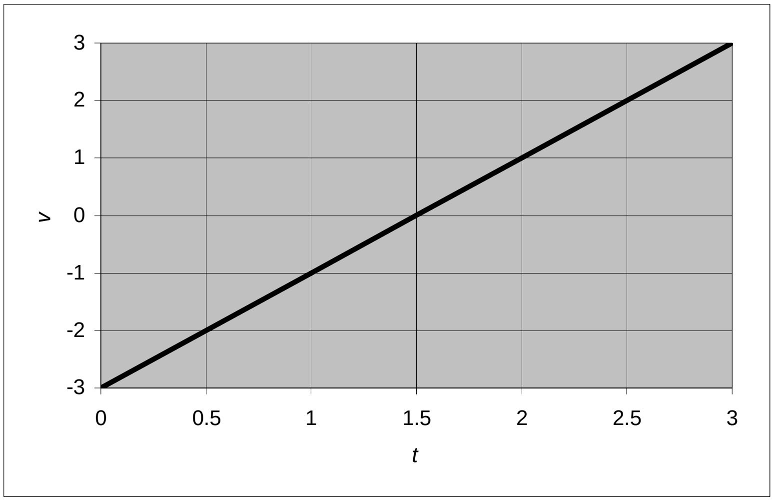
(*e*) Velocity is positive and acceleration is negative.



(*f*) Velocity is negative and acceleration is positive.



(g) Velocity is momentarily zero at the intercept with the t axis but the acceleration is not zero.



41 ••

Determine the Concept Velocity is the slope and acceleration is the slope of the slope of a position-versus-time curve. Acceleration is the slope of a velocity- versus-time curve.

(a) For constant velocity, x-versus-t must be a straight line; v-versus-t must be a horizontal straight line; and a-versus-t must be a straight horizontal line at $a = 0$.

(a), (f), and (i) are the correct answers.

(b) For velocity to reverse its direction x-versus-t must have a slope that changes sign and v-versus-t must cross the time axis. The acceleration cannot remain zero at all times.

(c) and (d) are the correct answers.

(c) For constant acceleration, x-versus-t must be a straight line or a parabola, v-versus-t must be a straight line, and a-versus-t must be a horizontal straight line.

(a), (d), (e), (f), (h), and (i) are the correct answers.

(d) For non-constant acceleration, x-versus-t must not be a straight line or a parabola; v-versus-t must not be a straight line, or a-versus-t must not be a horizontal straight line.

(b), (c), and (g) are the correct answers.

For two graphs to be mutually consistent, the curves must be consistent with the definitions of velocity and acceleration.

Graphs (*a*) and (*i*) are mutually consistent. Graphs (*d*) and (*h*) are mutually consistent. Graphs (*f*) and (*i*) are also mutually consistent.

Estimation and Approximation

42 •

Picture the Problem Assume that your heart beats at a constant rate. It does not, but the average is pretty stable.

(*a*) We will use an average pulse rate of 70 bpm for a seated (resting) adult. One's pulse rate is defined as the number of heartbeats per unit time:

$$\text{Pulse rate} = \frac{\#\text{ of heartbeats}}{\text{Time}}$$

and

$$\#\text{ of heartbeats} = \text{Pulse rate} \times \text{Time}$$

The time required to drive 1 mi at 60 mph is (1/60) h or 1 min:

$$\#\text{ of heartbeats} = (70\text{ beats/min})(1\text{ min}) = \boxed{70\text{ beats}}$$

(*b*) Express the number of heartbeats during a lifetime in terms of the pulse rate and the life span of an individual:

$$\#\text{of heartbeats} = \text{Pulse rate} \times \text{Time}$$

Assuming a 95-y life span, calculate the time in minutes:

$$\text{Time} = (95\text{ y})(365.25\text{ d/y})(24\text{ h/d})(60\text{ min/h}) = 5.00\times10^{7}\text{ min}$$

Substitute numerical values and evaluate the number of heartbeats:

$$\#\text{of heartbeats} = (70\text{ beats/min})(5.00\times10^{7}\text{ min}) = \boxed{3.50\times10^{9}\text{ beats}}$$

***43** ••

Picture the Problem In the absence of air resistance, Carlos' acceleration is constant. Because all the motion is downward, let's use a coordinate system in which downward is positive and the origin is at the point at which the fall began.

(*a*) Using a constant-acceleration equation, relate Carlos' final velocity to his initial velocity, acceleration, and distance fallen and solve for his final velocity:

$$v^2 = v_0^2 + 2a\Delta y$$

and, because $v_0 = 0$ and $a = g$,

$$v = \sqrt{2g\Delta y}$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{2(9.81\text{ m/s}^2)(150\text{ m})} = \boxed{54.2\text{ m/s}}$$

(*b*) While his acceleration by the snow is not constant, solve the same constant- acceleration equation to get an estimate of his average acceleration:

$$a = \frac{v^2 - v_0^2}{2\Delta y}$$

Substitute numerical values and evaluate *a*:

$$a = \frac{-\left(54\,\text{m/s}^2\right)^2}{2(1.22\,\text{m})} = -1.20\times10^3\,\text{m/s}^2$$
$$= \boxed{-123g}$$

Remarks: The final velocity we obtained in part (*a*), approximately 121 mph, is about the same as the terminal velocity for an "average" man. This solution is probably only good to about 20% accuracy.

44 ••

Picture the Problem Because we're assuming that the accelerations of the skydiver and the mouse are constant to one-half their terminal velocities, we can use constant-acceleration equations to find the times required for them to reach their ″upper-bound″ velocities and their distances of fall. Let's use a coordinate system in which downward is the positive *y* direction.

(*a*) Using a constant-acceleration equation, relate the upper-bound velocity to the free-fall acceleration and the time required to reach this velocity:

$$v_{\text{upper bound}} = v_0 + g\Delta t$$

or, because $v_0 = 0$,

$$v_{\text{upper bound}} = g\Delta t$$

Solve for Δt:

$$\Delta t = \frac{v_{\text{upper bound}}}{g}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{25\,\text{m/s}}{9.81\,\text{m/s}^2} = 2.55\,\text{s}$$

Using a constant-acceleration equation, relate the skydiver's distance of fall to the elapsed time Δt:

$$\Delta y = v_0\Delta t + \tfrac{1}{2}a(\Delta t)^2$$

or, because *v*0 = 0 and *a* = *g*,

$$\Delta y = \tfrac{1}{2}g(\Delta t)^2$$

Substitute numerical values and evaluate Δy:

$$\Delta y = \tfrac{1}{2}\left(9.81\,\text{m/s}^2\right)(2.55\,\text{s})^2 = \boxed{31.9\,\text{m}}$$

(*b*) Proceed as in (*a*) with $v_{\text{upper bound}}$ = 0.5 m/s to obtain:

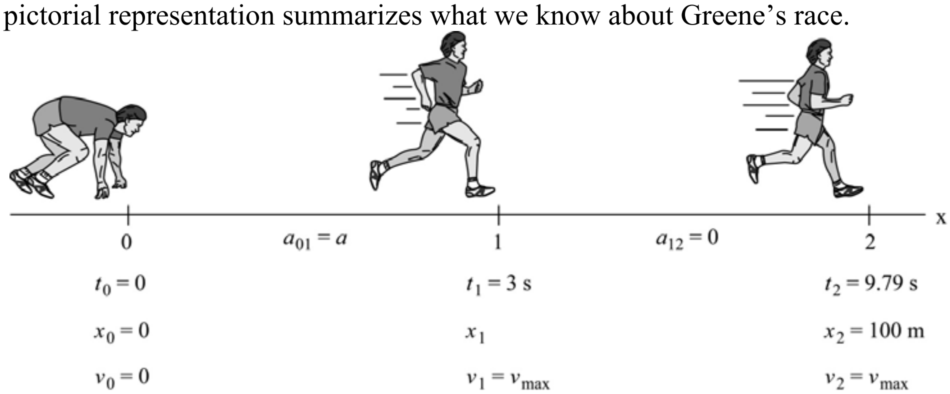
$$\Delta t = \frac{0.5\,\text{m/s}}{9.81\,\text{m/s}^2} = \boxed{0.0510\,\text{s}}$$

and

$$\Delta y = \tfrac{1}{2}\left(9.81\,\text{m/s}^2\right)(0.0510\,\text{s})^2 = \boxed{1.27\,\text{cm}}$$

45 ••

Picture the Problem This is a constant-acceleration problem. Choose a coordinate system in which the direction Greene is running is the positive x direction. During the first 3 s of the race his acceleration is positive and during the rest of the race it is zero. The pictorial representation summarizes what we know about Greene's race.



Express the total distance covered by Greene in terms of the distances covered in the two phases of his race:

$$100\,\text{m} = \Delta x_{01} + \Delta x_{12}$$

Express the distance he runs getting to his maximum velocity:

$$\Delta x_{01} = v_0 \Delta t_{01} + \tfrac{1}{2} a_{01} (\Delta t_{01})^2 = \tfrac{1}{2} a (3\,\text{s})^2$$

Express the distance covered during the rest of the race at the constant maximum velocity:

$$\begin{aligned}\Delta x_{12} &= v_{max} \Delta t_{12} + \tfrac{1}{2} a_{12} (\Delta t_{12})^2 \\ &= (a \Delta t_{01}) \Delta t_{12} \\ &= a (3\,\text{s})(6.79\,\text{s})\end{aligned}$$

Substitute for these displacements and solve for a:

$$100\,\text{m} = \tfrac{1}{2} a (3\,\text{s})^2 + a (3\,\text{s})(6.79\,\text{s})$$

and

$$a = \boxed{4.02\,\text{m/s}^2}$$

***46** ••

Determine the Concept This is a constant-acceleration problem with $a = -g$ if we take upward to be the positive direction.

At the maximum height the ball will reach, its speed will be near zero and when the ball has just been tossed in the air its speed is near its maximum value. What conclusion can you draw from the image of the ball near its maximum height?

Because the ball is moving slowly its blur is relatively short (i.e., there is less blurring).

To estimate the initial speed of the ball:

a) Estimate how far the ball being tossed moves in 1/30 s:

The ball moves about 3 ball diameters in 1/30 s.

b) Estimate the diameter of a tennis ball:

The diameter of a tennis ball is approximately 5 cm.

c) Now one can calculate the approximate distance the ball moved in 1/30 s:

$$\text{Distance traveled} = (3\text{ diameters}) \times (5\text{ cm/diameter}) = 15\text{ cm}$$

d) Calculate the average speed of the tennis ball over this distance:

$$\text{Average speed} = \frac{15\text{ cm}}{\frac{1}{30}\text{ s}} = 450\text{ cm/s} = 4.50\text{ m/s}$$

e) Because the time interval is very short, the average speed of the ball is a good approximation to its initial speed:

$\therefore v_0 = 4.5$ m/s

f) Finally, use the constant-acceleration equation $v^2 = v_0^2 + 2a\Delta y$ to solve for and evaluate Δy:

$$\Delta y = \frac{-v_0^2}{2a} = \frac{-(4.5\text{ m/s})^2}{2(-9.81\text{ m/s}^2)} = \boxed{1.03\text{ m}}$$

Remarks: **This maximum height is in good agreement with the height of the higher ball in the photograph.**

***47 ••**

Picture the Problem The average speed of a nerve impulse is approximately 120 m/s. Assume an average height of 1.7 m and use the definition of average speed to estimate the travel time for the nerve impulse.

Using the definition of average speed, express the travel time for the nerve impulse:

$$\Delta t = \frac{\Delta x}{v_{\text{av}}}$$

Substitute numerical values and evaluate Δ*t*:

$$\Delta t = \frac{1.7\text{ m}}{120\text{ m/s}} = \boxed{14.2\text{ ms}}$$

Speed, Displacement, and Velocity

48 •

Picture the Problem Think of the electron as traveling in a straight line at constant speed and use the definition of average speed.

(*a*) Using its definition, express the average speed of the electron:

$$\text{Average speed} = \frac{\text{distance traveled}}{\text{time of flight}} = \frac{\Delta s}{\Delta t}$$

Solve for and evaluate the time of flight:

$$\Delta t = \frac{\Delta s}{\text{Average speed}} = \frac{0.16\ \text{m}}{4\times10^{7}\ \text{m/s}}$$
$$= 4\times10^{-9}\,\text{s} = \boxed{4.00\ \text{ns}}$$

(*b*) Calculate the time of flight for an electron in a 16-cm long current carrying wire similarly.

$$\Delta t = \frac{\Delta s}{\text{Average speed}} = \frac{0.16\ \text{m}}{4\times10^{-5}\ \text{m/s}}$$
$$= 4\times10^{3}\,\text{s} = \boxed{66.7\ \text{min}}$$

***49** •

Picture the Problem In this problem the runner is traveling in a straight line but not at constant speed - first she runs, then she walks. Let's choose a coordinate system in which her initial direction of motion is taken as the positive *x* direction.

(*a*) Using the definition of average velocity, calculate the average velocity for the first 9 min:

$$v_{\text{av}} = \frac{\Delta x}{\Delta t} = \frac{2.5\ \text{km}}{9\ \text{min}} = \boxed{0.278\ \text{km/min}}$$

(*b*) Using the definition of average velocity, calculate her average speed for the 30 min spent walking:

$$v_{\text{av}} = \frac{\Delta x}{\Delta t} = \frac{-2.5\ \text{km}}{30\ \text{min}}$$
$$= \boxed{-0.0833\ \text{km/min}}$$

(*c*) Express her average velocity for the whole trip:

$$v_{\text{av}} = \frac{\Delta x_{\text{round trip}}}{\Delta t} = \frac{0}{\Delta t} = \boxed{0}$$

(*d*) Finally, express her average speed for the whole trip:

$$\text{Average speed} = \frac{\text{distance traveled}}{\text{elapsed time}} = \frac{2(2.5\ \text{km})}{30\ \text{min} + 9\ \text{min}}$$
$$= \boxed{0.128\ \text{km/min}}$$

50 •

Picture the Problem The car is traveling in a straight line but not at constant speed. Let the direction of motion be the positive *x* direction.

(*a*) Express the total displacement of the car for the entire trip:

$$\Delta x_{\text{total}} = \Delta x_1 + \Delta x_2$$

Find the displacement for each leg of the trip:

$$\Delta x_1 = v_{av,1}\Delta t_1 = (80\text{ km/h})(2.5\text{ h})$$
$$= 200\,\text{km}$$
and
$$\Delta x_2 = v_{av,2}\Delta t_2 = (40\text{ km/h})(1.5\text{ h})$$
$$= 60.0\,\text{km}$$

Add the individual displacements to get the total displacement:

$$\Delta x_{\text{total}} = \Delta x_1 + \Delta x_2 = 200\,\text{km} + 60.0\,\text{km}$$
$$= \boxed{260\,\text{km}}$$

(*b*) As long as the car continues to move in the same direction, the average velocity for the total trip is given by:

$$v_{av} \equiv \frac{\Delta x_{\text{total}}}{\Delta t_{\text{total}}} = \frac{260\,\text{km}}{2.5\text{ h} + 1.5\text{ h}}$$
$$= \boxed{65.0\,\text{km/h}}$$

51 •

Picture the Problem However unlikely it may seem, imagine that both jets are flying in a straight line at constant speed.

(*a*) The time of flight is the ratio of the distance traveled to the speed of the supersonic jet.

$$t_{\text{supersonic}} = \frac{s_{\text{Atlantic}}}{\text{speed}_{\text{supersonic}}}$$
$$= \frac{5500\text{ km}}{2(0.340\,\text{km/s})(3600\,\text{s/h})}$$
$$= \boxed{2.25\text{ h}}$$

(*b*) The time of flight is the ratio of the distance traveled to the speed of the subsonic jet.

$$t_{\text{subsonic}} = \frac{s_{\text{Atlantic}}}{\text{speed}_{\text{subsonic}}}$$
$$= \frac{5500\text{ km}}{0.9(0.340\,\text{km/s})(3600\,\text{s/h})}$$
$$= \boxed{4.99\,\text{h}}$$

(*c*) Adding 2 h on both the front and the back of the supersonic trip, we obtain the average speed of the supersonic flight.

$$\text{speed}_{\text{av, supersonic}} = \frac{5500\,\text{km}}{2.25\,\text{h} + 4.00\,\text{h}}$$
$$= \boxed{880\,\text{km/h}}$$

(*d*) Adding 2 h on both the front and the back of the subsonic trip, we obtain the average speed of the subsonic flight.

$$\text{speed}_{\text{av, subsonic}} = \frac{5500\,\text{km}}{5.00\,\text{h} + 4.00\,\text{h}}$$
$$= \boxed{611\,\text{km/h}}$$

***52 •**

Picture the Problem In free space, light travels in a straight line at constant speed, *c*.

(*a*) Using the definition of average speed, solve for and evaluate the time required for light to travel from the sun to the earth:

$$\text{average speed} = \frac{s}{t}$$

and

$$t = \frac{s}{\text{average speed}} = \frac{1.5\times10^{11}\text{ m}}{3\times10^{8}\text{ m/s}}$$

$$= 500\text{ s} = \boxed{8.33\text{ min}}$$

(*b*) Proceed as in (*a*) this time using the moon-earth distance:

$$t = \frac{3.84\times10^{8}\text{ m}}{3\times10^{8}\text{ m/s}} = \boxed{1.28\text{ s}}$$

(*c*) One light-year is the distance light travels in a vacuum in one year:

$$1\text{ light - year} = 9.48\times10^{15}\text{ m} = \boxed{9.48\times10^{12}\text{ km}}$$

$$= \left(9.48\times10^{12}\text{ km}\right)\left(1\text{ mi}/1.61\text{ km}\right)$$

$$= \boxed{5.89\times10^{12}\text{ mi}}$$

53 •

Picture the Problem In free space, light travels in a straight line at constant speed, *c*.

(*a*) Using the definition of average speed (equal here to the assumed constant speed of light), solve for the time required to travel the distance to Proxima Centauri:

$$t = \frac{\text{distance traveled}}{\text{speed of light}} = \frac{4.1\times10^{16}\text{ m}}{3\times10^{8}\text{ m/s}}$$

$$= 1.37\times10^{8}\text{ s} = \boxed{4.33\text{ y}}$$

(*b*) Traveling at $10^{-4}c$, the delivery time (t_{total}) will be the sum of the time for the order to reach Hoboken and the time for the pizza to be delivered to Proxima Centauri:

$$t_{\text{total}} = t_{\text{order to be sent to Hoboken}} + t_{\text{order to be delivered}}$$

$$= 4.33\,y + \frac{4.1\times10^{13}\text{ km}}{\left(10^{-4}\right)\left(3\times10^{8}\text{ m/s}\right)}$$

$$= 4.33\text{ y} + 4.33\times10^{6}\text{ y}$$

$$\approx 4.33\times10^{6}\text{ y}$$

$$\boxed{\text{Since } 4.33\times10^{6}\text{ y} >> 1000\text{ y, Gregor does not have to pay.}}$$

54 •

Picture the Problem The time for the second 50 km is equal to the time for the entire journey less the time for the first 50 km. We can use this time to determine the average speed for the second 50 km interval from the definition of average speed.

Using the definition of average speed, find the time required for the total journey:

$$t_{\text{total}} = \frac{\text{total distance}}{\text{average speed}} = \frac{100\text{ km}}{50\text{ km/h}} = 2\text{ h}$$

Find the time required for the first 50 km:

$$t_{\text{1st 50 km}} = \frac{50\,\text{km}}{40\,\text{km/h}} = 1.25\,\text{h}$$

Find the time remaining to travel the last 50 km:

$$t_{\text{2nd 50 km}} = t_{\text{total}} - t_{\text{1st 50 km}} = 2\,\text{h} - 1.25\,\text{h} = 0.75\,\text{h}$$

Finally, use the time remaining to travel the last 50 km to determine the average speed over this distance:

$$\text{Average speed}_{\text{2nd 50 km}} = \frac{\text{distance traveled}_{\text{2nd 50 km}}}{\text{time}_{\text{2nd 50 km}}} = \frac{50\,\text{km}}{0.75\,\text{h}} = \boxed{66.7\,\text{km/h}}$$

***55 ••**

Picture the Problem Note that both the arrow and the sound travel a distance *d*. We can use the relationship between distance traveled, the speed of sound, the speed of the arrow, and the elapsed time to find the distance separating the archer and the target.

Express the elapsed time between the archer firing the arrow and hearing it strike the target:

$$\Delta t = 1\,\text{s} = \Delta t_{\text{arrow}} + \Delta t_{\text{sound}}$$

Express the transit times for the arrow and the sound in terms of the distance, *d*, and their speeds:

$$\Delta t_{\text{arrow}} = \frac{d}{|v_{\text{arrow}}|} = \frac{d}{40\,\text{m/s}}$$

and

$$\Delta t_{\text{sound}} = \frac{d}{|v_{\text{sound}}|} = \frac{d}{340\,\text{m/s}}$$

Substitute these two relationships in the expression obtained in step 1 and solve for *d*:

$$\frac{d}{40\,\text{m/s}} + \frac{d}{340\,\text{m/s}} = 1\,\text{s}$$

and $d = \boxed{35.8\,\text{m}}$

56 ••

Picture the Problem Assume both runners travel parallel paths in a straight line along the track.

(*a*) Using the definition of average speed, find the time for Marcia:

$$t_{\text{Marcia}} = \frac{\text{distance run}}{\text{Marcia's speed}} = \frac{\text{distance run}}{1.15(\text{John's speed})} = \frac{100\,\text{m}}{1.15(6\,\text{m/s})} = 14.5\,\text{s}$$

Find the distance covered by John in 14.5 s and the difference between that distance and 100 m:

$$x_{\text{John}} = (6\,\text{m/s})(14.5\,\text{s}) = 87.0\,\text{m}$$

and Marcia wins by

$$100\,\text{m} - 87\,\text{m} = \boxed{13.0\,\text{m}}$$

(*b*) Using the definition of average speed, find the time required by John to complete the 100-m run:

$$t_{\text{John}} = \frac{\text{distance run}}{\text{John's speed}} = \frac{100\,\text{m}}{6\,\text{m/s}} = 16.7\,\text{s}$$

Marsha wins by 16.7 s – 14.5 s = 2.2 s

Alternatively, the time required by John to travel the last 13.0 m is

(13 m)/(6 m/s) = $\boxed{2.17\,\text{s}}$

57 •

Picture the Problem The average velocity in a time interval is defined as the displacement divided by the time elapsed; that is $v_{\text{av}} = \Delta x/\Delta t$.

(*a*) $\Delta x_a = 0$

$$v_{\text{av}} = \boxed{0}$$

(*b*) $\Delta x_b = 1$ m and $\Delta t_b = 3$ s

$$v_{\text{av}} = \boxed{0.333\,\text{m/s}}$$

(*c*) $\Delta x_c = -6$ m and $\Delta t_c = 3$ s

$$v_{\text{av}} = \boxed{-2.00\,\text{m/s}}$$

(*d*) $\Delta x_d = 3$ m and $\Delta t_d = 3$ s

$$v_{\text{av}} = \boxed{1.00\,\text{m/s}}$$

58 ••

Picture the Problem In free space, light travels in a straight line at constant speed *c*. We can use Hubble's law to find the speed of the two planets.

(*a*) Using Hubble's law, calculate the speed of the first galaxy:

$$v_{\text{a}} = \left(5\times10^{22}\,\text{m}\right)\left(1.58\times10^{-18}\,\text{s}^{-1}\right) = \boxed{7.90\times10^{4}\,\text{m/s}}$$

(*b*) Using Hubble's law, calculate the speed of the second galaxy:

$$v_{\text{b}} = \left(2\times10^{25}\,\text{m}\right)\left(1.58\times10^{-18}\,\text{s}^{-1}\right) = \boxed{3.16\times10^{7}\,\text{m/s}}$$

(*c*) Using the relationship between distance, speed, and time for both galaxies, determine how long ago they were both located at the same place as the earth:

$$t = \frac{r}{v} = \frac{r}{rH} = \frac{1}{H} = 6.33\times10^{17}\,\text{s} = 20.1\times10^{9}\,\text{y} = \boxed{20.1\,\text{billion years}}$$

***59 ••**

Picture the Problem Ignoring the time intervals during which members of this relay time get up to their running speeds, their accelerations are zero and their average speed can be found from its definition.

Using its definition, relate the average speed to the total distance traveled and the elapsed time:

$$|v_{av}| = \frac{\text{distance traveled}}{\text{elapsed time}}$$

Express the time required for each animal to travel a distance L:

$$t_{cheetah} = \frac{L}{v_{cheetah}},$$

$$t_{falcon} = \frac{L}{v_{falcon}},$$

and

$$t_{sailfish} = \frac{L}{v_{sailfish}}$$

Express the total time, Δt:

$$\Delta t = L\left(\frac{1}{v_{cheetah}} + \frac{1}{v_{falcon}} + \frac{1}{v_{sailfish}}\right)$$

Use the total distance traveled by the relay team and the elapsed time to calculate the average speed:

$$|v_{av}| = \frac{3L}{L\left(\frac{1}{113\,\text{km/h}} + \frac{1}{161\,\text{km/h}} + \frac{1}{105\,\text{km/h}}\right)} = \boxed{122\,\text{km/h}}$$

Calculate the average of the three speeds:

$$\text{Average}_{\text{three speeds}} = \frac{113\,\text{km/h} + 161\,\text{km/h} + 105\,\text{km/h}}{3} = \boxed{126\,\text{km/h} = 1.03v_{av}}$$

60 ••

Picture the Problem Perhaps the easiest way to solve this problem is to think in terms of the relative velocity of one car relative to the other. Solve this problem from the reference frame of car A. In this frame, car A remains at rest.

Find the velocity of car B relative to car A:

$$v_{rel} = v_B - v_A = (110 - 80)\text{ km/h} = 30\text{ km/h}$$

Find the time before car B reaches car A:

$$\Delta t = \frac{\Delta x}{v_{rel}} = \frac{45\,\text{km}}{30\,\text{km/h}} = 1.5\,\text{h}$$

Find the distance traveled, relative to the road, by car A in 1.5 h:

$$d = (1.5\,\text{h})(80\,\text{km/h}) = \boxed{120\,\text{km}}$$

***61 ••**

Picture the Problem One way to solve this problem is by using a graphing calculator to plot the positions of each car as a function of time. Plotting these positions as functions of time allows us to visualize the motion of the two cars relative to the (fixed) ground. More importantly, it allows us to see the motion of the two cars relative to each other. We can, for example, tell how far apart the cars are at any given time by determining the length of a vertical line segment from one curve to the other.

(*a*) Letting the origin of our coordinate system be at the intersection, the position of the slower car, $x_1(t)$, is given by:

$$x_1(t) = 20t$$

where x_1 is in meters if t is in seconds.

Because the faster car is also moving at a constant speed, we know that the position of this car is given by a function of the form:

$$x_2(t) = 30t + b$$

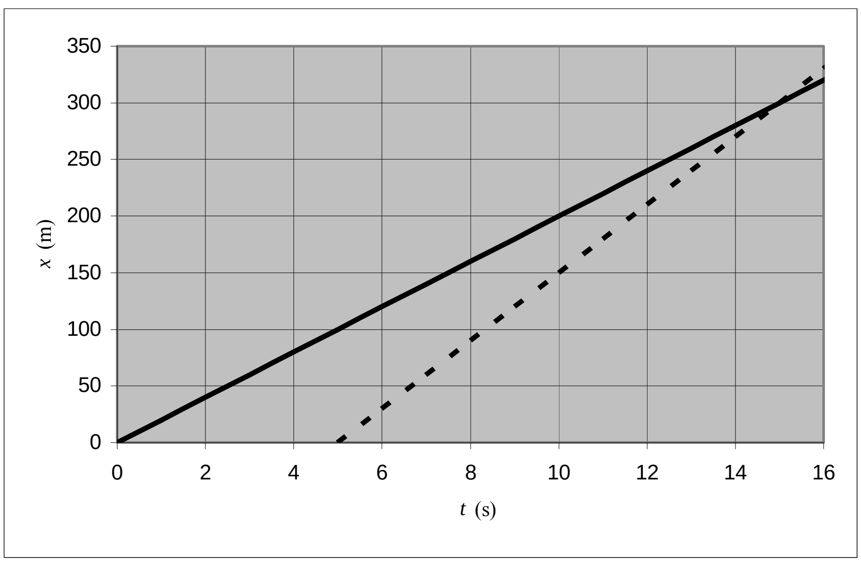
We know that when $t = 5$ s, this second car is at the intersection (i.e., $x_2(5\text{ s}) = 0$). Using this information, you can convince yourself that:

$$b = -150\text{ m}$$

Thus, the position of the faster car is given by:

$$x_2(t) = 30t - 150$$

One can use a graphing calculator, graphing paper, or a spreadsheet to obtain the graphs of $x_1(t)$ (the solid line) and $x_2(t)$ (the dashed line) shown below:



(*b*) Use the time coordinate of the intersection of the two lines to determine the time at which the second car overtakes the first:

From the intersection of the two lines, one can see that the second car will "overtake" (catch up to) the first car at $\boxed{t = 15\text{ s}}$.

(*c*) Use the position coordinate of the intersection of the two lines to determine the distance from the intersection at which the second car catches up to the first car:

From the intersection of the two lines, one can see that the distance from the intersection is $\boxed{300\text{ m}}$.

(*d*) Draw a vertical line from $t = 5$ s to the red line and then read the position coordinate of the intersection of this line and the red line to determine the position of the first car when the second car went through the intersection:

From the graph, when the second car passes the intersection, the first car was $\boxed{100\text{ m ahead}}$.

62 •

Picture the Problem Sally's velocity relative to the ground (v_{SG}) is the sum of her velocity relative to the moving belt (v_{SB}) and the velocity of the belt relative to the ground (v_{BG}). Joe's velocity relative to the ground is the same as the velocity of the belt relative to the ground. Let D be the length of the moving sidewalk.

Express D in terms of v_{BG} (Joe's speed relative to the ground):

$$D = (2\text{ min})v_{BG}$$

Solve for v_{BG}:

$$v_{BG} = \frac{D}{2\text{ min}}$$

Express D in terms of $v_{BG} + v_{SG}$ (Sally's speed relative to the ground):

$$D = (1\text{ min})(v_{BG} + v_{SG}) = (1\text{ min})\left(\frac{D}{2\text{ min}} + v_{SG}\right)$$

Solve for v_{SG}:

$$v_{SG} = \frac{D}{1\text{ min}} - \frac{D}{2\text{ min}} = \frac{D}{2\text{ min}}$$

Express D in terms of $v_{BG} + 2v_{SB}$ (Sally's speed for a fast walk relative to the ground):

$$D = t_f(v_{BG} + 2v_{SB}) = t_f\left(\frac{D}{2\text{ min}} + \frac{2D}{2\text{ min}}\right) = t_f\left(\frac{3D}{2\text{ min}}\right)$$

Solve for t_f as time for Sally's fast walk:

$$t_f = \frac{2\text{ min}}{3} = \boxed{40.0\text{ s}}$$

63 ••

Picture the Problem The speed of Margaret's boat relative to the riverbank ($|v_{BR}|$) is the sum or difference of the speed of her boat relative to the water ($|v_{BW}|$) and the speed of the water relative to the riverbank ($|v_{WR}|$), depending on whether she is heading with or against the current. Let D be the distance to the marina.

Express the total time for the trip:

$$t_{tot} = t_1 + t_2$$

Express the times of travel with the motor running in terms of $D, |v_{WR}|$ and $|v_{BW}|$:

$$t_1 = \frac{D}{|v_{BW}| - |v_{WR}|} = 4\,\text{h}$$

and

$$t_2 = \frac{D}{|v_{BW}| + |v_{WR}|}$$

Express the time required to drift distance D and solve for $|v_{WR}|$:

$$t_3 = \frac{D}{|v_{WR}|} = 8\,\text{h}$$

and

$$|v_{WR}| = \frac{D}{8\,\text{h}}$$

From $t_1 = 4$ h, find $|v_{BW}|$:

$$|v_{BW}| = \frac{D}{4\text{ h}} + |v_{WR}| = \frac{D}{4\text{ h}} + \frac{D}{8\text{ h}} = \frac{3D}{8\text{ h}}$$

Solve for t_2:

$$t_2 = \frac{D}{|v_{BW}| + |v_{WR}|} = \frac{D}{\frac{3D}{8\,\text{h}} + \frac{D}{8\,\text{h}}} = 2\,\text{h}$$

Add t_1 and t_2 to find the total time:

$$t_{tot} = t_1 + t_2 = \boxed{6\,\text{h}}$$

Acceleration

64 •

Picture the Problem In part (*a*), we can apply the definition of average acceleration to find a_{av}. In part (*b*), we can find the change in the car's velocity in one second and add this change to its velocity at the beginning of the interval to find its speed one second later.

(*a*) Apply the definition of average acceleration:

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{80.5\,\text{km/h} - 48.3\,\text{km/h}}{3.7\,\text{s}}$$

$$= 8.70\frac{\text{km}}{\text{h}\cdot\text{s}}$$

Convert to m/s^2:

$$a_{\text{av}} = \left(8.70\times10^3\,\frac{\text{m}}{\text{h}\cdot\text{s}}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right)$$
$$= \boxed{2.42\,\text{m/s}^2}$$

(*b*) Express the speed of the car at the end of 4.7 s:

$$v(4.7\,\text{s}) = v(3.7\,\text{s}) + \Delta v_{1\,\text{s}}$$
$$= 80.5\,\text{km/h} + \Delta v_{1\,\text{s}}$$

Find the change in the speed of the car in 1 s:

$$\Delta v = a_{\text{av}}\Delta t = \left(8.70\,\frac{\text{km}}{\text{h}\cdot\text{s}}\right)(1\,\text{s})$$
$$= 8.70\,\text{km/h}$$

Substitute and evaluate v(4.7 s):

$$v(4.7\,\text{s}) = 80.5\,\text{km/h} + 8.7\,\text{km/h}$$
$$= \boxed{89.2\,\text{km/h}}$$

65 •

Picture the Problem Average acceleration is defined as $a_{\text{av}} = \Delta v/\Delta t$.

The average acceleration is defined as the change in velocity divided by the change in time:

$$a_{\text{av}} = \frac{\Delta v}{\Delta t} = \frac{(-1\,\text{m/s}) - (5\,\text{m/s})}{(8\,\text{s}) - (5\,\text{s})}$$
$$= \boxed{-2.00\,\text{m/s}^2}$$

66 ••

Picture the Problem The important concept here is the difference between average acceleration and instantaneous acceleration.

(*a*) The average acceleration is defined as the change in velocity divided by the change in time:

$a_{\text{av}} = \Delta v/\Delta t$

Determine v at t = 3 s, t = 4 s, and t = 5 s:

v(3 s) = 17 m/s
v(4 s) = 25 m/s
v(5 s) = 33 m/s

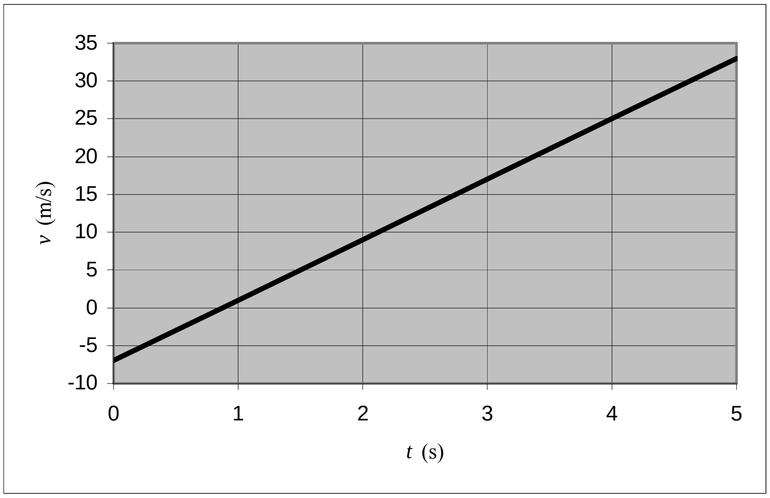
Find a_{av} for the two 1-s intervals:

a_{av}(3 s to 4 s) = (25 m/s – 17 m/s)/(1 s)
= 8 m/s^2
and
a_{av}(4 s to 5 s) = (33 m/s – 25 m/s)/(1 s)
= 8 m/s^2

The instantaneous acceleration is defined as the time derivative of the velocity or the slope of the velocity-versus-time curve:

$$a = \frac{dv}{dt} = \boxed{8.00\,\text{m/s}^2}$$

(*b*) The given function was used to plot the following spreadsheet-graph of *v*-versus-*t*:



67 ••

Picture the Problem We can closely approximate the instantaneous velocity by the average velocity in the limit as the time interval of the average becomes small. This is important because all we can ever obtain from any measurement is the average velocity, v_{av}, which we use to approximate the instantaneous velocity v.

(*a*) Find $x(4\text{ s})$ and $x(3\text{ s})$:

$x(4\text{ s}) = (4)^2 - 5(4) + 1 = -3\text{ m}$
and
$x(3\text{ s}) = (3)^2 - 5(3) + 1 = -5\text{ m}$

Find Δx:

$$\Delta x = x(4\text{ s}) - x(3\text{ s}) = (-3\text{ m}) - (-5\text{ m}) = \boxed{2\text{ m}}$$

Use the definition of average velocity:

$$v_{av} = \Delta x/\Delta t = (2\text{ m})/(1\text{ s}) = \boxed{2\text{ m/s}}$$

(*b*) Find $x(t + \Delta t)$:

$$x(t+\Delta t) = (t+\Delta t)^2 - 5(t+\Delta t) + 1 = (t^2 + 2t\Delta t + (\Delta t)^2) - 5(t+\Delta t) + 1$$

Express $x(t + \Delta t) - x(t) = \Delta x$:

$$\Delta x = \boxed{(2t-5)\Delta t + (\Delta t)^2}$$

where Δx is in meters if t is in seconds.

(*c*) From (*b*) find $\Delta x/\Delta t$ as $\Delta t \to 0$:

$$\frac{\Delta x}{\Delta t} = \frac{(2t-5)\Delta t + (\Delta t)^2}{\Delta t} = 2t - 5 + \Delta t$$

and

$$v = \lim_{\Delta t \to 0}(\Delta x / \Delta t) = \boxed{2t - 5}$$

where v is in m/s if t is in seconds.

Alternatively, we can take the derivative of $x(t)$ with respect to time to obtain the instantaneous velocity.

$$v(t) = dx(t)/dt = \frac{d}{dt}\left(at^2 + bt + 1\right)$$
$$= 2at + b = 2t - 5$$

***68 ••**

Picture the Problem The instantaneous velocity is dx/dt and the acceleration is dv/dt.

Using the definitions of instantaneous velocity and acceleration, determine v and a:

$$v = \frac{dx}{dt} = \frac{d}{dt}\left[At^2 - Bt + C\right] = 2At - B$$

and

$$a = \frac{dv}{dr} = \frac{d}{dt}\left[2At - B\right] = 2A$$

Substitute numerical values for A and B and evaluate v and a:

$$v = 2\left(8\text{m/s}^2\right)t - 6\text{ m/s}$$
$$= \boxed{\left(16\text{ m/s}^2\right)t - 6\text{m/s}}$$

and

$$a = 2\left(8\text{ m/s}^2\right) = \boxed{16.0\text{ m/s}^2}$$

69 ••

Picture the Problem We can use the definition of average acceleration ($a_{av} = \Delta v/\Delta t$) to find a_{av} for the three intervals of constant acceleration shown on the graph.

(*a*) Using the definition of average acceleration, find a_{av} for the interval AB:

$$a_{\text{av, AB}} = \frac{15\text{ m/s} - 5\text{ m/s}}{3\text{ s}} = \boxed{3.33\text{ m/s}^2}$$

Find a_{av} for the interval BC:

$$a_{\text{av, BC}} = \frac{15\text{ m/s} - 15\text{ m/s}}{3\text{ s}} = \boxed{0}$$

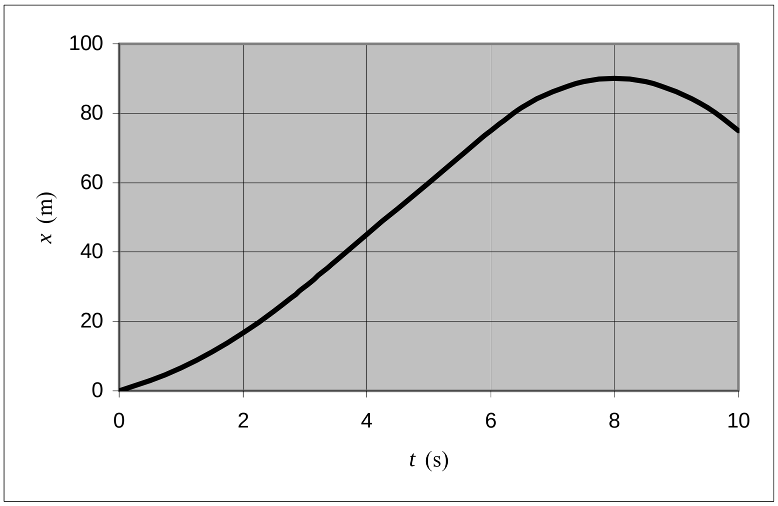
Find a_{av} for the interval CE:

$$a_{\text{av, CE}} = \frac{-15\text{ m/s} - 15\text{m/s}}{4\text{ s}} = \boxed{-7.50\text{m/s}^2}$$

(*b*) Use the formulas for the areas of trapezoids and triangles to find the area under the graph of v as a function of t.

$$\begin{aligned}\Delta x &= (\Delta x)_{A\to B} + (\Delta x)_{B\to C} + (\Delta x)_{C\to D} + (\Delta x)_{D\to E}\\ &= \tfrac{1}{2}(5\,\text{m/s} + 15\,\text{m/s})(3\,\text{s}) + (15\,\text{m/s})(3\,\text{s}) + \tfrac{1}{2}(15\,\text{m/s})(2\,\text{s}) + \tfrac{1}{2}(-15\,\text{m/s})(2\,\text{s})\\ &= \boxed{75.0\,\text{m}}\end{aligned}$$

(*c*) The graph of displacement, *x*, as a function of time, *t*, is shown in the following figure. In the region from B to C the velocity is constant so the *x*- versus-*t* curve is a straight line.



(*d*) Reading directly from the figure, we can find the time when the particle is moving the slowest.

$\boxed{\text{At point D, } t = 8\text{ s, the graph crosses the time axis; therefore, } v = 0.}$

Constant Acceleration and Free-Fall

***70** •

Picture the Problem Because the acceleration is constant (–*g*) we can use a constant-acceleration equation to find the height of the projectile.

Using a constant-acceleration equation, express the height of the object as a function of its initial velocity, the acceleration due to gravity, and its displacement:

$$v^2 = v_0^2 + 2a\Delta y$$

Solve for $\Delta y_{max} = h$:

Because $v(h) = 0$,

$$h = \frac{-v_0^2}{2(-g)} = \frac{v_0^2}{2g}$$

From this expression for *h* we see that the maximum height attained is proportional to the square of the launch speed:

$$h \propto v_0^2$$

Therefore, doubling the initial speed gives four times the height:

$$h_{2v_0} = \frac{(2v_0)^2}{2g} = 4\left(\frac{v_0^2}{2g}\right) = 4h_{v_0}$$

and $\boxed{(a)\text{ is correct.}}$

71 •

Picture the Problem Because the acceleration of the car is constant we can use constant-acceleration equations to describe its motion.

(*a*) Using a constant-acceleration equation, relate the velocity to the acceleration and the time:

$$v = v_0 + at = 0 + \left(8\,\text{m/s}^2\right)(10\,\text{s}) = \boxed{80.0\,\text{m/s}}$$

(*b*) sing a constant-acceleration equation, relate the displacement to the acceleration and the time:

$$\Delta x = x - x_0 = v_0 t + \frac{a}{2}t^2$$

Substitute numerical values and evaluate Δx:

$$\Delta x = \frac{1}{2}\left(8\,\text{m/s}^2\right)(10\,\text{s})^2 = \boxed{400\,\text{m}}$$

(*c*) Use the definition of v_{av}:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{400\,\text{m}}{10\,\text{s}} = \boxed{40.0\,\text{m/s}}$$

Remarks: Because the area under a velocity-versus-time graph is the displacement of the object, we could solve this problem graphically.

72 •

Picture the Problem Because the acceleration of the object is constant we can use constant-acceleration equations to describe its motion.

Using a constant-acceleration equation, relate the velocity to the acceleration and the displacement:

$$v^2 = v_0^2 + 2a\,\Delta x$$

Solve for and evaluate the displacement:

$$\Delta x = \frac{v^2 - v_0^2}{2a} = \frac{\left(15^2 - 5^2\right)\text{m}^2/\text{s}^2}{2\left(2\,\text{m/s}^2\right)} = \boxed{50.0\,\text{m}}$$

***73** •

Picture the Problem Because the acceleration of the object is constant we can use constant-acceleration equations to describe its motion.

Using a constant-acceleration equation, relate the velocity to the acceleration and the displacement:

$$v^2 = v_0^2 + 2a\,\Delta x$$

Solve for the acceleration:

$$a = \frac{v^2 - v_0^2}{2\Delta x}$$

Substitute numerical values and evaluate a:

$$a = \frac{(15^2 - 10^2)\mathrm{m^2/s^2}}{2(4\,\mathrm{m})} = \boxed{15.6\,\mathrm{m/s^2}}$$

74 •

Picture the Problem Because the acceleration of the object is constant we can use constant-acceleration equations to describe its motion.

Using a constant-acceleration equation, relate the velocity to the acceleration and the displacement:

$$v^2 = v_0^2 + 2a\Delta x$$

Solve for and evaluate v:

$$v = \sqrt{(1\,\mathrm{m/s})^2 + 2(4\,\mathrm{m/s^2})(1\,\mathrm{m})} = \boxed{3.00\,\mathrm{m/s}}$$

Using the definition of average acceleration, solve for the time:

$$t = \frac{\Delta v}{a_{\mathrm{av}}} = \frac{3\,\mathrm{m/s} - 1\,\mathrm{m/s}}{4\,\mathrm{m/s^2}} = \boxed{0.500\,\mathrm{s}}$$

75 ••

Picture the Problem In the absence of air resistance, the ball experiences constant acceleration. Choose a coordinate system with the origin at the point of release and the positive direction upward.

(*a*) Using a constant-acceleration equation, relate the displacement of the ball to the acceleration and the time:

$$\Delta y = v_0 t + \tfrac{1}{2}at^2$$

Setting $\Delta y = 0$ (the displacement for a round trip), solve for and evaluate the time required for the ball to return to its starting position:

$$t_{\text{round trip}} = \frac{2v_0}{g} = \frac{2(20\,\mathrm{m/s})}{9.81\,\mathrm{m/s^2}} = \boxed{4.08\,\mathrm{s}}$$

(*b*) Using a constant-acceleration equation, relate the final speed of the ball to its initial speed, the acceleration, and its displacement:

$$v_{\text{top}}^2 = v_0^2 + 2a\Delta y$$

or, because $v_{\text{top}} = 0$ and $a = -g$,

$$0 = v_0^2 + 2(-g)H$$

Solve for and evaluate H:

$$H = \frac{v_0^2}{2g} = \frac{(20\,\mathrm{m/s})^2}{2(9.81\,\mathrm{m/s^2})} = \boxed{20.4\,\mathrm{m}}$$

(*c*) Using the same constant-acceleration equation with which we began part (*a*), express the displacement as a function of time:

$$\Delta y = v_0 t + \tfrac{1}{2}at^2$$

Substitute numerical values to obtain:

$$15\,\text{m} = (20\,\text{m/s})t - \left(\frac{9.81\,\text{m/s}^2}{2}\right)t^2$$

Solve the quadratic equation for the times at which the displacement of the ball is 15 m:

The solutions are $t = \boxed{0.991\,\text{s}}$ (this corresponds to passing 15 m on the way up) and $t = \boxed{3.09\,\text{s}}$ (this corresponds to passing 15 m on the way down).

76 ••

Picture the Problem This is a multipart constant-acceleration problem using two different constant accelerations. We'll choose a coordinate system in which downward is the positive direction and apply constant-acceleration equations to find the required times.

(*a*) Using a constant-acceleration equation, relate the time for the slide to the distance of fall and the acceleration:

$$\Delta y = y - y_0 = h - 0 = v_0 t_1 + \tfrac{1}{2}at_1^2$$

or, because $v_0 = 0$,

$$h = \tfrac{1}{2}at_1^2$$

Solve for t_1:

$$t_1 = \sqrt{\frac{2h}{g}}$$

Substitute numerical values and evaluate t_1:

$$t_1 = \sqrt{\frac{2(460\,\text{m})}{9.81\,\text{m/s}^2}} = \boxed{9.68\,\text{s}}$$

(*b*) Using a constant-acceleration equation, relate the velocity at the bottom of the mountain to the acceleration and time:

$$v_1 = v_0 + a_1 t_1$$

or, because $v_0 = 0$ and $a_1 = g$,

$$v_1 = gt_1$$

Substitute numerical values and evaluate v_1:

$$v_1 = (9.81\,\text{m/s}^2)(9.68\,\text{s}) = \boxed{95.0\,\text{m/s}}$$

(*c*) Using a constant-acceleration equation, relate the time required to stop the mass of rock and mud to its average speed and the distance it slides:

$$\Delta t = \frac{\Delta x}{v_{\text{av}}}$$

Because the acceleration is constant:

$$v_{\text{av}} = \frac{v_1 + v_{\text{f}}}{2} = \frac{v_1 + 0}{2} = \frac{v_1}{2}$$

Substitute to obtain:

$$\Delta t = \frac{2\Delta x}{v_1}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{2(8000\,\text{m})}{95.0\,\text{m/s}} = \boxed{168\,\text{s}}$$

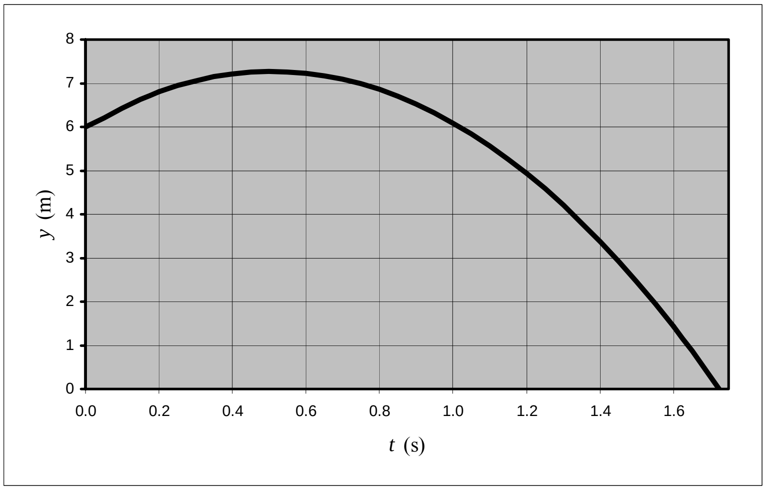
***77 ••**

Picture the Problem In the absence of air resistance, the brick experiences constant acceleration and we can use constant-acceleration equations to describe its motion. Constant acceleration implies a parabolic position-versus-time curve.

(*a*) Using a constant-acceleration equation, relate the position of the brick to its initial position, initial velocity, acceleration, and time into its fall:

$$y = y_0 + v_0 t + \tfrac{1}{2}(-g)t^2$$
$$= 6\,\text{m} + (5\,\text{m/s})t - (4.91\,\text{m/s}^2)t^2$$

The following graph of $y = 6\,\text{m} + (5\,\text{m/s})t - (4.91\,\text{m/s}^2)t^2$ was plotted using a spreadsheet program:



(*b*) Relate the greatest height reached by the brick to its height when it falls off the load and the additional height it rises Δy_{max}:

$$h = y_0 + \Delta y_{max}$$

Using a constant-acceleration equation, relate the height reached by the brick to its acceleration and initial velocity:

$$v_{top}^2 = v_0^2 + 2(-g)\Delta y_{max}$$

or, because $v_{top} = 0$,

$$0 = v_0^2 + 2(-g)\Delta y_{max}$$

Solve for Δy_{max}:

$$\Delta y_{max} = \frac{v_0^2}{2g}$$

Substitute numerical values and evaluate Δy_{max}:

$$\Delta y_{max} = \frac{(5\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} = 1.27\,\text{m}$$

Substitute to obtain:

$$h = y_0 + \Delta y_{max} = 6\,\text{m} + 1.27\,\text{m} = \boxed{7.27\,\text{m}}$$

Note: The graph shown above confirms this result.

(*c*) Using the quadratic formula, solve for *t* in the equation obtained in part (*a*):

$$t = \frac{-v_0 \pm \sqrt{v_0^2 - 4\left(\frac{-g}{2}\right)(-\Delta y)}}{2\left(\frac{-g}{2}\right)}$$

$$= \left(\frac{v_0}{g}\right)\left(1 \pm \sqrt{1 - \frac{2g(\Delta y)}{v_0^2}}\right)$$

With $y_{\text{bottom}} = 0$ and $y_o = 6$ m or $\Delta y = -6$ m, we obtain:

$t = \boxed{1.73\,\text{s}}$ and $t = -0.708$ s. Note: The second solution is nonphysical.

(*d*) Using a constant-acceleration equation, relate the speed of the brick on impact to its acceleration and displacement, and solve for its speed:

$$v = \sqrt{2gh}$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{2\left(9.81\,\text{m/s}^2\right)(7.27\,\text{m})} = \boxed{11.9\,\text{m/s}}$$

78 ••

Picture the Problem In the absence of air resistance, the acceleration of the bolt is constant. Choose a coordinate system in which upward is positive and the origin is at the bottom of the shaft ($y = 0$).

(*a*) Using a constant-acceleration equation, relate the position of the bolt to its initial position, initial velocity, and fall time:

$$y_{\text{bottom}} = 0 = y_0 + v_0 t + \tfrac{1}{2}(-g)t^2$$

Solve for the position of the bolt when it came loose:

$$y_0 = -v_0 t + \tfrac{1}{2}gt^2$$

Substitute numerical values and evaluate y_0:

$$y_0 = -(6\,\text{m/s})(3\,\text{s}) + \tfrac{1}{2}\left(9.81\,\text{m/s}^2\right)(3\,\text{s})^2 = \boxed{26.1\,\text{m}}$$

(*b*) Using a constant-acceleration equation, relate the speed of the bolt to its initial speed, acceleration, and fall time:

$$v = v_0 + at$$

Substitute numerical values and evaluate $|v|$:

$$v = 6\,\text{m/s} - \left(9.81\,\text{m/s}^2\right)\left(3\text{s}\right) = -23.4\,\text{m/s}$$

and

$$|v| = \boxed{23.4\,\text{m/s}}$$

***79 ••**

Picture the Problem In the absence of air resistance, the object's acceleration is constant. Choose a coordinate system in which downward is positive and the origin is at the point of release. In this coordinate system, $a = g$ and $y = 120$ m at the bottom of the fall.

Express the distance fallen in the last second in terms of the object's position at impact and its position 1 s before impact:

$$\Delta y_{\text{last second}} = 120\text{ m} - y_{1\,\text{s before impact}} \quad (1)$$

Using a constant-acceleration equation, relate the object's position upon impact to its initial position, initial velocity, and fall time:

$$y = y_0 + v_0 t + \tfrac{1}{2} g t^2$$

or, because $y_0 = 0$ and $v_0 = 0$,

$$y = \tfrac{1}{2} g t_{\text{fall}}^2$$

Solve for the fall time:

$$t_{\text{fall}} = \sqrt{\frac{2y}{g}}$$

Substitute numerical values and evaluate t_{fall}:

$$t_{\text{fall}} = \sqrt{\frac{2(120\,\text{m})}{9.81\,\text{m/s}^2}} = 4.95\,\text{s}$$

We know that, one second before impact, the object has fallen for 3.95 s. Using the same constant-acceleration equation, calculate the object's position 3.95 s into its fall:

$$y(3.95\text{ s}) = \tfrac{1}{2}\left(9.81\text{ m/s}^2\right)\left(3.95\,\text{s}\right)^2 = 76.4\,\text{m}$$

Substitute in equation (1) to obtain:

$$\Delta y_{\text{last second}} = 120\,\text{m} - 76.4\,\text{m} = \boxed{43.6\,\text{m}}$$

80 ••

Picture the Problem In the absence of air resistance, the acceleration of the object is constant. Choose a coordinate system with the origin at the point of release and downward as the positive direction.

Using a constant-acceleration equation, relate the height to the initial and final velocities and the acceleration; solve for the height:

$$v_{\text{f}}^2 = v_0^2 + 2a\Delta y$$

or, because $v_0 = 0$,

$$h = \frac{v_{\text{f}}^2}{2g} \quad (1)$$

Using the definition of average velocity, find the average velocity of the object during its final second of fall:

$$v_{\text{av}} = \frac{v_{\text{f-1s}} + v_{\text{f}}}{2} = \frac{\Delta y}{\Delta t} = \frac{38\,\text{m}}{1\,\text{s}} = 38\,\text{m/s}$$

Express the sum of the final velocity and the velocity 1 s before impact:

$$v_{\text{f-1s}} + v_{\text{f}} = 2(38\,\text{m/s}) = 76\,\text{m/s}$$

From the definition of acceleration, we know that the change in velocity of the object, during 1 s of fall, is 9.81 m/s:

$$\Delta v = v_{\text{f}} - v_{\text{f-1s}} = 9.81\ \text{m/s}$$

Add the equations that express the sum and difference of $v_{\text{f}-1\,\text{s}}$ and v_{f} and solve for v_{f}:

$$v_{\text{f}} = \frac{76\,\text{m/s} + 9.81\,\text{m/s}}{2} = 42.9\,\text{m/s}$$

Substitute in equation (1) and evaluate h:

$$h = \frac{(42.9\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} = \boxed{93.8\,\text{m}}$$

***81** •

Picture the Problem In the absence of air resistance, the acceleration of the stone is constant. Choose a coordinate system with the origin at the bottom of the trajectory and the upward direction positive. Let $v_{\text{f-1/2}}$ be the speed one-half second before impact and v_{f} the speed at impact.

Using a constant-acceleration equation, express the final speed of the stone in terms of its initial speed, acceleration, and displacement:

$$v_{\text{f}}^2 = v_0^2 + 2a\Delta y$$

Solve for the initial speed of the stone:

$$v_0 = \sqrt{v_{\text{f}}^2 + 2g\Delta y} \qquad (1)$$

Find the average speed in the last half second:

$$v_{\text{av}} = \frac{v_{\text{f-1/2}} + v_{\text{f}}}{2} = \frac{\Delta x_{\text{last half second}}}{\Delta t} = \frac{45\,\text{m}}{0.5\,\text{s}} = 90\,\text{m/s}$$

and

$$v_{\text{f-1/2}} + v_{\text{f}} = 2(90\,\text{m/s}) = 180\,\text{m/s}$$

Using a constant-acceleration equation, express the change in speed of the stone in the last half second in terms of the acceleration and the elapsed time; solve for the change in its speed:

$$\Delta v = v_{\text{f}} - v_{\text{f-1/2}} = g\Delta t = (9.81\,\text{m/s}^2)(0.5\,\text{s}) = 4.91\,\text{m/s}$$

Add the equations that express the sum and difference of $v_{f-½}$ and v_f and solve for v_f:

$$v_f = \frac{180\,\text{m/s} + 4.91\,\text{m/s}}{2} = 92.5\,\text{m/s}$$

Substitute in equation (1) and evaluate v_0:

$$v_0 = \sqrt{(92.5\text{m/s})^2 + 2(9.81\text{m/s}^2)(-200\text{m})} = \boxed{68.1\,\text{m/s}}$$

Remarks: The stone may be thrown either up or down from the cliff and the results after it passes the cliff on the way down are the same.

82 ••

Picture the Problem In the absence of air resistance, the acceleration of the object is constant. Choose a coordinate system in which downward is the positive direction and the object starts from rest. Apply constant-acceleration equations to find the average velocity of the object during its descent.

Express the average velocity of the falling object in terms of its initial and final velocities:

$$v_{av} = \frac{v_0 + v_f}{2}$$

Using a constant-acceleration equation, express the displacement of the object during the 1st second in terms of its acceleration and the elapsed time:

$$\Delta y_{1\text{st second}} = \frac{gt^2}{2} = 4.91\,\text{m} = 0.4\,h$$

Solve for the displacement to obtain:

$h = 12.3$ m

Using a constant-acceleration equation, express the final velocity of the object in terms of its initial velocity, acceleration, and displacement:

$$v_f^2 = v_0^2 + 2g\Delta y$$

or, because $v_0 = 0$,

$$v_f = \sqrt{2g\Delta y}$$

Substitute numerical values and evaluate the final velocity of the object:

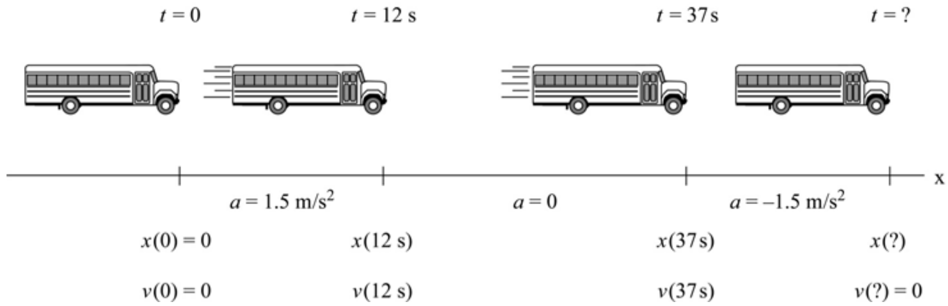
$$v_f = \sqrt{2(9.81\,\text{m/s}^2)(12.3\,\text{m})} = 15.5\,\text{m/s}$$

Substitute in the equation for the average velocity to obtain:

$$v_{av} = \frac{0 + 15.5\,\text{m/s}}{2} = \boxed{7.77\,\text{m/s}}$$

83 ••

Picture the Problem This is a three-part constant-acceleration problem. The bus starts from rest and accelerates for a given period of time, and then it travels at a constant velocity for another period of time, and, finally, decelerates uniformly to a stop. The pictorial representation will help us organize the information in the problem and develop our solution strategy.



(*a*) Express the total displacement of the bus during the three intervals of time.

$$\Delta x_{\text{total}} = \Delta x(0 \rightarrow 12\,\text{s}) + \Delta x(12\,\text{s} \rightarrow 37\,\text{s}) + \Delta x(37\,\text{s} \rightarrow \text{end})$$

Using a constant-acceleration equation, express the displacement of the bus during its first 12 s of motion in terms of its initial velocity, acceleration, and the elapsed time; solve for its displacement:

$$\Delta x(0 \rightarrow 12\,\text{s}) = v_0 t + \tfrac{1}{2}at^2$$

or, because $v_0 = 0$,

$$\Delta x(0 \rightarrow 12\,\text{s}) = \tfrac{1}{2}at^2 = 108\,\text{m}$$

Using a constant-acceleration equation, express the velocity of the bus after 12 seconds in terms of its initial velocity, acceleration, and the elapsed time; solve for its velocity at the end of 12 s:

$$v_{12\,\text{s}} = v_0 + a_{0\rightarrow 12\,\text{s}}\Delta t = (1.5\,\text{m/s}^2)(12\,\text{s}) = 18\,\text{m/s}$$

During the next 25 s, the bus moves with a constant velocity. Using the definition of average velocity, express the displacement of the bus during this interval in terms of its average (constant) velocity and the elapsed time:

$$\Delta x(12\,\text{s} \rightarrow 37\,\text{s}) = v_{12\,\text{s}}\Delta t = (18\,\text{m/s})(25\,\text{s}) = 450\,\text{m}$$

Because the bus slows down at the same rate that its velocity increased during the first 12 s of motion, we can conclude that its displacement during this braking period is the same as during its acceleration period and the time to brake to a stop is equal to the time that was required for the bus to accelerate to its cruising speed of 18 m/s. Hence:

$$\Delta x(37\,\text{s} \rightarrow 49\text{s}) = 108\,\text{m}$$

Add the displacements to find the distance the bus traveled:

$$\Delta x_{\text{total}} = 108\,\text{m} + 450\,\text{m} + 108\,\text{m} = \boxed{666\,\text{m}}$$

(*b*) Use the definition of average velocity to calculate the average velocity of the bus during this trip:

$$v_{\text{av}} = \frac{\Delta x_{\text{total}}}{\Delta t} = \frac{666\,\text{m}}{49\,\text{s}} = \boxed{13.6\,\text{m/s}}$$

Remarks: One can also solve this problem graphically. Recall that the area under a velocity as a function-of-time graph equals the displacement of the moving object.

***84** ••

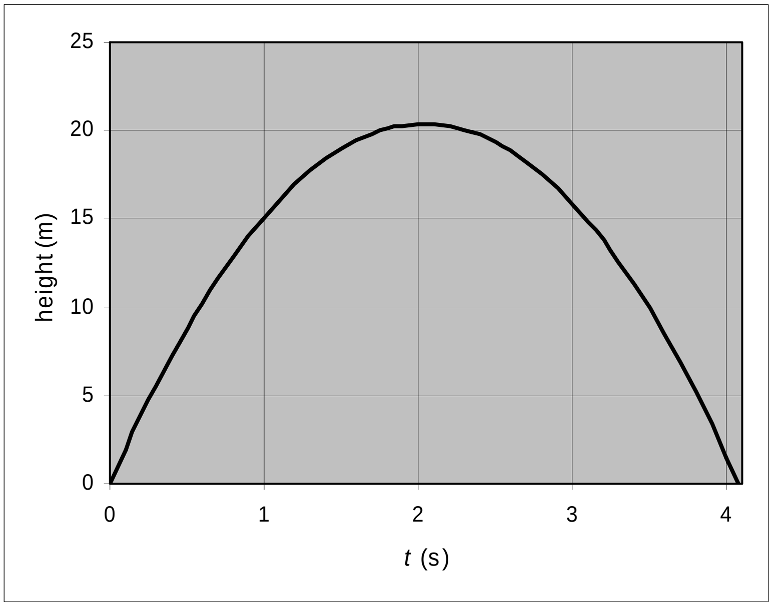
Picture the Problem While we can solve this problem analytically, there are many physical situations in which it is not easy to do so and one has to rely on numerical methods; for example, see the spreadsheet solution shown below. Because we're neglecting the height of the release point, the position of the ball as a function of time is given by $y = v_0 t - \frac{1}{2}gt^2$. The formulas used to calculate the quantities in the columns are as follows:

Cell	Content/Formula	Algebraic Form
B1	20	v_0
B2	9.81	g
B5	0	t
B6	B5 + 0.1	$t + \Delta t$
C6	B1*B6 – 0.5*B2*B6^2	$v_0 t - \frac{1}{2}gt^2$

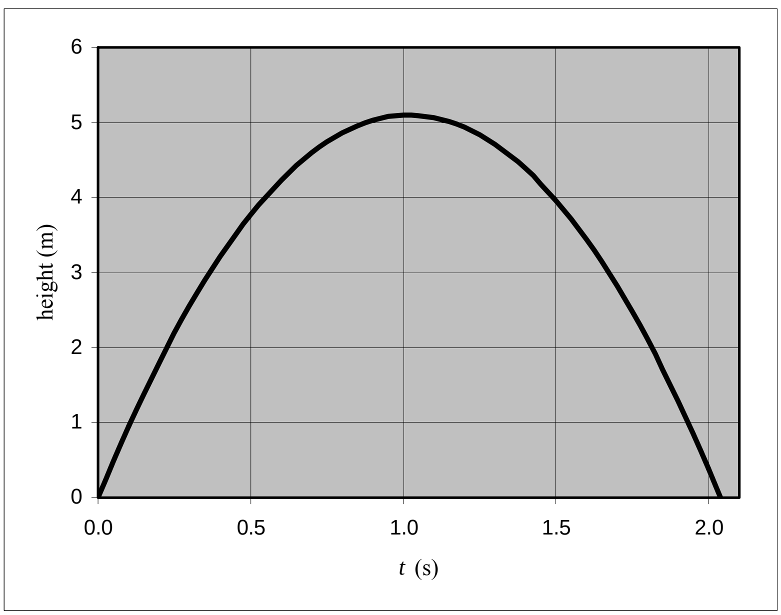
(*a*)

	A	B	C
1	v0 =	20	m/s
2	g =	9.81	m/s^2
3		t	height
4		(s)	(m)
5		0.0	0.00
6		0.1	1.95
7		0.2	3.80
44		3.9	3.39
45		4.0	1.52
46		4.1	–0.45

The graph shown below was generated from the data in the previous table. Note that the maximum height reached is a little more than 20 m and the time of flight is about 4 s.



(b) In the spreadsheet, change the value in cell B1 from 20 to 10. The graph should automatically update. With an initial velocity of 10 m/s, the maximum height achieved is approximately 5 m and the time-of-flight is approximately 2 s.



***85** ••

Picture the Problem Because the accelerations of both Al and Bert are constant, constant-acceleration equations can be used to describe their motions. Choose the origin of the coordinate system to be where Al decides to begin his sprint.

(a) Using a constant-acceleration equation, relate Al's initial velocity, his acceleration, and the time to reach the end of the trail to his

$$\Delta x = v_0 t + \tfrac{1}{2} a t^2$$

displacement in reaching the end of the trail:

Substitute numerical values to obtain:

$$35\,\text{m} = (0.75\,\text{m/s})t + \tfrac{1}{2}(0.5\,\text{m/s}^2)t^2$$

Solve for the time required for Al to reach the end of the trail:

$$t = \boxed{10.4\,\text{s}}$$

(*b*) Using constant-acceleration equations, express the positions of Bert and Al as functions of time. At the instant Al turns around at the end of the trail, $t = 0$. Also, $x = 0$ at a point 35 m from the end of the trail:

$$x_{\text{Bert}} = x_{\text{Bert},0} + (0.75\,\text{m/s})t$$

and

$$x_{\text{Al}} = x_{\text{Al},0} - (0.85\,\text{m/s})t$$
$$= 35\,\text{m} - (0.85\,\text{m/s})t$$

Calculate Bert's position at $t = 0$. At that time he has been running for 10.4 s:

$$x_{\text{Bert},0} = (0.75\,\text{m/s})(10.4\,\text{s}) = 7.80\,\text{m}$$

Because Bert and Al will be at the same location when they meet, equate their position functions and solve for t:

$$7.80\,\text{m} + (0.75\,\text{m/s})t = 35\,\text{m} - (0.85\,\text{m/s})t$$

and

$$t = 17.0\text{ s}$$

To determine the elapsed time from when Al began his accelerated run, we need to add 10.4 s to this time:

$$t_{\text{start}} = 17.0\,\text{s} + 10.4\,\text{s} = \boxed{27.4\,\text{s}}$$

(*c*) Express Bert's distance from the end of the trail when he and Al meet:

$$d_{\text{end of trail}} = 35\,\text{m} - x_{\text{Bert},0} - d_{\text{Bert runs until he meets Al}}$$

Substitute numerical values and evaluate $d_{\text{end of trail}}$:

$$d_{\text{end of trail}} = 35\,\text{m} - 7.80\,\text{m} - (17\,\text{s})(0.75\,\text{m/s}) = \boxed{14.5\,\text{m}}$$

86 ••

Picture the Problem Generate two curves on one graph with the first curve representing Al's position as a function of time and the second curve representing Bert's position as a function of time. Al's position, as he runs toward the end of the trail, is given by $x_{\text{Al}} = v_0 t + \tfrac{1}{2}a_{\text{Al}}t^2$ and Bert's position by $x_{\text{Bert}} = x_{0,\text{Bert}} + v_{\text{Bert}}t$. Al's position, once he's reached the end of the trail and is running back toward Bert, is given by $x_{\text{Al}} = x_{\text{Al},0} + v_{\text{Al}}(t - 10.5\,\text{s})$. The coordinates of the intersection of the two curves give the time and place where they meet. A spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

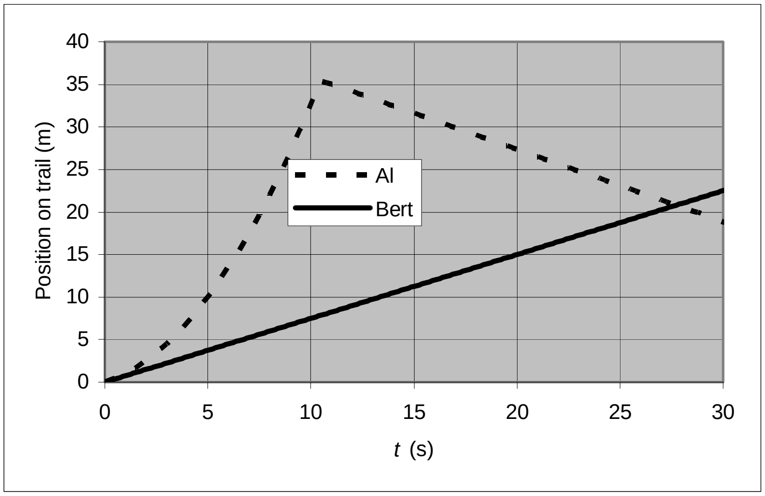
Cell	Content/Formula	Algebraic Form

B1	0.75	v_0
B2	0.50	a_{Al}
B3	–0.85	t
B10	B9 + 0.25	$t+\Delta t$
C10	B1*B10 + 0.5*B2*B10^2	$v_0 t+\frac{1}{2}a_{\text{Al}}t^2$
C52	C51 + B3*(B52 – B51)	$x_{\text{Al},0}+v_{\text{Al}}(t-10.5\,\text{s})$
F10	F9 + B1*B10	$x_{0,\text{Bert}}+v_{\text{Bert}}t$

(*b*) and (*c*)

	A	B	C	D	E	F
1	v0 =	0.75	m/s			
2	a(Al) =	0.5	m/s^2			
3	v(Al) =	–0.85	m/s			
4						
5		t (s)	x (m)			x (m)
6						
7						
8			Al			Bert
9		0.00	0.00			0.00
10		0.25	0.20			0.19
11		0.50	0.44			0.38
49		10.00	32.50			7.50
50		10.25	33.95			7.69
51		10.50	35.44	*Al reaches		7.88
52		10.75	35.23	end of trail		8.06
53		11.00	35.01	and starts		8.25
54		11.25	34.80	back toward		8.44
55		11.50	34.59	Bert		8.63
56		11.75	34.38			8.81
119		27.50	20.99			20.63
120		27.75	20.78			20.81
121		28.00	20.56			21.00
122		28.25	20.35			21.19
127		29.50	19.29			22.13
128		29.75	19.08			22.31
129		30.00	18.86			22.50

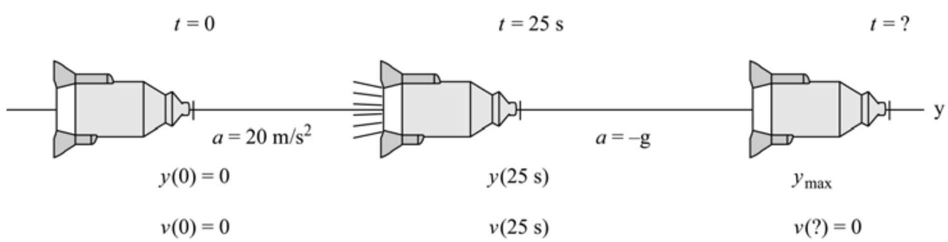
The graph shown below was generated from the spreadsheet; the positions of both Al and Bert were calculated as functions of time. The dashed curve shows Al's position as a function of time for the two parts of his motion. The solid line that is linear from the origin shows Bert's position as a function of time.



Note that the spreadsheet and the graph (constructed from the spreadsheet data) confirm the results in Problem 85 by showing Al and Bert meeting at about 14.5 m from the end of the trail after an elapsed time of approximately 28 s.

87 ••

Picture the Problem This is a two-part constant-acceleration problem. Choose a coordinate system in which the upward direction is positive. The pictorial representation will help us organize the information in the problem and develop our solution strategy.



(*a*) Express the highest point the rocket reaches, *h*, as the sum of its displacements during the first two stages of its flight:

$$h = \Delta x_{\text{1st stage}} + \Delta x_{\text{2nd stage}}$$

Using a constant-acceleration equation, express the altitude reached in the first stage in terms of the rocket's initial velocity, acceleration, and burn time; solve for the first stage altitude:

$$\begin{aligned} x_{\text{1st stage}} &= x_0 + v_0 t + \tfrac{1}{2} a_{\text{1st stage}} t^2 \\ &= \tfrac{1}{2}(20\ \text{m/s}^2)(25\ \text{s})^2 \\ &= 6250\ \text{m} \end{aligned}$$

Using a constant-acceleration equation, express the velocity of the rocket at the end of its first stage in terms of its initial velocity, acceleration, and displacement; calculate its end-of-first-stage velocity:

$$\begin{aligned} v_{\text{1st stage}} &= v_0 + a_{\text{1st stage}} t \\ &= (20\ \text{m/s}^2)(25\ \text{s}) \\ &= 500\ \text{m/s} \end{aligned}$$

Using a constant-acceleration equation, express the final velocity of the rocket during the remainder of its climb in terms of its shut-off velocity, free-fall acceleration, and displacement; solve for its displacement:

$$v_{\text{highest point}}^2 = v_{\text{shutoff}}^2 + 2a_{\text{2nd stage}}\Delta y_{\text{2nd stage}}$$

and, because $v_{\text{highest point}} = 0$,

$$\Delta y_{\text{2nd stage}} = \frac{-v_{\text{shutoff}}^2}{-2g} = \frac{(500\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} = 1.2742\times10^4\ \text{m}$$

Substitute in the expression for the total height to obtain:

$$h = 6250\,\text{m} + 1.27\times10^4\ \text{m} = \boxed{19.0\,\text{km}}$$

(*b*) Express the total time the rocket is in the air in terms of the three segments of its flight:

$$\Delta t_{\text{total}} = \Delta t_{\text{powered climb}} + \Delta t_{\text{2nd segment}} + \Delta t_{\text{descent}} = 25\,\text{s} + \Delta t_{\text{2nd segment}} + \Delta t_{\text{descent}}$$

Express $\Delta t_{\text{2nd segment}}$ in terms of the rocket's displacement and average velocity:

$$\Delta t_{\text{2nd segment}} = \frac{\text{Displacement}}{\text{Average velocity}}$$

Substitute numerical values and evaluate $\Delta t_{\text{2nd segment}}$:

$$\Delta t_{\text{2nd segment}} = \frac{1.2742\times10^4\ \text{m}}{\left(\frac{0+500\,\text{m/s}}{2}\right)} = 50.97\ \text{s}$$

Using a constant-acceleration equation, relate the fall distance to the descent time:

$$\Delta y = v_0 t + \tfrac{1}{2}g(\Delta t_{\text{descent}})^2$$

or, because $v_0 = 0$,

$$\Delta y = \tfrac{1}{2}g(\Delta t_{\text{descent}})^2$$

Solve for $\Delta t_{\text{descent}}$:

$$\Delta t_{\text{descent}} = \sqrt{\frac{2\Delta y}{g}}$$

Substitute numerical values and evaluate $\Delta t_{\text{descent}}$:

$$\Delta t_{\text{descent}} = \sqrt{\frac{2(1.90\times10^4\ \text{m})}{9.81\,\text{m/s}^2}} = 62.2\ \text{s}$$

Substitute and calculate the total time the rocket is in the air:

$$\Delta t = 25\,\text{s} + 50.97\,\text{s} + 62.2\,\text{s} = 138\,\text{s} = \boxed{2\,\text{min}\ 18\,\text{s}}$$

(*c*) Using a constant-acceleration equation, express the impact velocity of the rocket in terms of its initial downward velocity, acceleration under free-fall, and time of descent; solve for its impact velocity:

$$v_{\text{impact}} = v_0 + g\Delta t_{\text{descent}}$$

and, because $v_0 = 0$,

$$v_{\text{impact}} = g\Delta t = (9.81\,\text{m/s}^2)(62.2\,\text{s}) = \boxed{610\,\text{m/s}}$$

88 ••

Picture the Problem In the absence of air resistance, the acceleration of the flowerpot is constant. Choose a coordinate system in which downward is positive and the origin is at the point from which the flowerpot fell. Let t = time when the pot is at the top of the window, and $t + \Delta t$ the time when the pot is at the bottom of the window. To find the distance from the ledge to the top of the window, first find the time t_{top} that it takes the pot to fall to the top of the window.

Using a constant-acceleration equation, express the distance y below the ledge from which the pot fell as a function of time:

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2$$

Since $a = g$ and $v_0 = y_0 = 0$,

$$y = \tfrac{1}{2}gt^2$$

Express the position of the pot as it reaches the top of the window:

$$y_{top} = \tfrac{1}{2}gt_{top}^2$$

Express the position of the pot as it reaches the bottom of the window:

$$y_{bottom} = \tfrac{1}{2}g\left(t_{top} + \Delta t_{window}\right)^2$$

where $\Delta t_{window} = t_{top} - t_{bottom}$

Subtract y_{bottom} from y_{top} to obtain an expression for the displacement Δy_{window} of the pot as it passes the window:

$$\Delta y_{window} = \tfrac{1}{2}g\left[\left(t_{top} + \Delta t_{window}\right)^2 - t_{top}^2\right] = \tfrac{1}{2}g\left[2t_{top}\Delta t_{window} + \left(\Delta t_{window}\right)^2\right]$$

Solve for t_{top}:

$$t_{top} = \frac{\dfrac{2\Delta y_{window}}{g} - \left(\Delta t_{window}\right)^2}{2\Delta t_{window}}$$

Substitute numerical values and evaluate t_{top}:

$$t_{top} = \frac{\dfrac{2(4\,\text{m})}{9.81\,\text{m/s}^2} - (0.2\,\text{s})^2}{2(0.2\,\text{s})} = 1.839\,\text{s}$$

Substitute this value for t_{top} to obtain the distance from the ledge to the top of the window:

$$y_{top} = \tfrac{1}{2}(9.81\,\text{m/s}^2)(1.939\,\text{s})^2 = \boxed{18.4\,\text{m}}$$

***89** ••

Picture the Problem The acceleration of the glider on the air track is constant. Its average acceleration is equal to the instantaneous (constant) acceleration. Choose a coordinate system in which the initial direction of the glider's motion is the positive direction.

Using the definition of acceleration, express the average acceleration of the glider in terms of the glider's velocity change and the elapsed time:

$$a = a_{av} = \frac{\Delta v}{\Delta t}$$

Using a constant-acceleration equation, express the average velocity of the glider in terms of the displacement of the glider and the elapsed time:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{v_0 + v}{2}$$

Solve for and evaluate the initial velocity:

$$v_0 = \frac{2\Delta x}{\Delta t} - v = \frac{2(100\,\text{cm})}{8\,\text{s}} - (-15\,\text{cm/s})$$
$$= \boxed{40.0\,\text{cm/s}}$$

Substitute this value of v_0 and evaluate the average acceleration of the glider:

$$a = \frac{-15\,\text{cm/s} - (40\,\text{cm/s})}{8\,\text{s}}$$
$$= \boxed{-6.88\,\text{cm/s}^2}$$

90 ••

Picture the Problem In the absence of air resistance, the acceleration of the rock is constant and its motion can be described using the constant-acceleration equations. Choose a coordinate system in which the downward direction is positive and let the height of the cliff, which equals the displacement of the rock, be represented by *h*.

Using a constant-acceleration equation, express the height *h* of the cliff in terms of the initial velocity of the rock, acceleration, and time of fall:

$$\Delta y = v_0 t + \tfrac{1}{2}at^2$$

or, because $v_0 = 0$, $a = g$, and $\Delta y = h$,

$$h = \tfrac{1}{2}gt^2$$

Using this equation, express the displacement of the rock during the

a) first two-thirds of its fall, and

$$\tfrac{2}{3}h = \tfrac{1}{2}gt^2 \qquad (1)$$

b) its complete fall in terms of the time required for it to fall this distance.

$$h = \tfrac{1}{2}g(t + 1\,\text{s})^2 \qquad (2)$$

Substitute equation (2) in equation (1) to obtain a quadratic equation in *t*:

$$t^2 - (4\text{ s})t - 2\text{ s}^2 = 0$$

Solve for the positive root:

$$t = 4.45\,\text{s}$$

Evaluate $\Delta t = t + 1$ s:

$$\Delta t = 4.45\text{ s} + 1\text{ s} = 5.45\text{ s}$$

Substitute numerical values in equation (2) and evaluate *h*:

$$h = \tfrac{1}{2}(9.81\,\text{m/s}^2)(5.45\,\text{s})^2 = \boxed{146\,\text{m}}$$

91 •••

Picture the Problem Assume that the acceleration of the car is constant. The total distance the car travels while stopping is the sum of the distances it travels during the driver's reaction time and the time it travels while braking. Choose a coordinate system in which the positive direction is the direction of motion of the automobile and apply a constant-acceleration equation to obtain a quadratic equation in the car's initial speed v_0.

(*a*) Using a constant-acceleration equation, relate the velocity of the car to its initial velocity, acceleration, and displacement during braking:

$$v^2 = v_0^2 + 2a\Delta x_{\text{brk}}$$

or, because the final velocity is zero,

$$0 = v_0^2 + 2a\Delta x_{\text{brk}}$$

Solve for the distance traveled during braking:

$$\Delta x_{\text{brk}} = -\frac{v_0^2}{2a}$$

Express the total distance traveled by the car as the sum of the distance traveled during the reaction time and the distance traveled while slowing down:

$$\Delta x_{\text{tot}} = \Delta x_{\text{react}} + \Delta x_{\text{brk}} = v_0\Delta t_{\text{react}} - \frac{v_0^2}{2a}$$

Rearrange this quadratic equation to obtain:

$$v_0^2 - 2a\Delta t_{\text{react}}v_0 + 2a\Delta x_{\text{tot}} = 0$$

Substitute numerical values and simplify to obtain:

$$v_0^2 - 2\left(-7\ \text{m/s}^2\right)(0.5\ \text{s})v_0 + 2\left(-7\ \text{m/s}^2\right)(4\ \text{m}) = 0$$

or

$$v_0^2 + (7\ \text{m/s})v_0 - 56\ \text{m}^2/\text{s}^2 = 0$$

Solve the quadratic equation for the positive root to obtain:

$$v_0 = 4.7613558\ \text{m/s}$$

Convert this speed to mi/h::

$$v_0 = (4.7613558\ \text{m/s})\left(\frac{1\ \text{mi/h}}{0.477\ \text{m/s}}\right) = \boxed{10.7\ \text{mi/h}}$$

(*b*) Find the reaction-time distance:

$$\Delta x_{\text{react}} = v_0\Delta t_{\text{react}} = (4.76\ \text{m/s})(0.5\ \text{s}) = 2.38\ \text{m}$$

Express and evaluate the ratio of the reaction distance to the total distance:

$$\frac{\Delta x_{\text{react}}}{\Delta x_{\text{tot}}} = \frac{2.38\ \text{m}}{4\ \text{m}} = \boxed{0.595}$$

92 ••

Picture the Problem Assume that the accelerations of the trains are constant. Choose a coordinate system in which the direction of the motion of the train on the left is the positive direction. Take $x_o = 0$ as the position of the train on the left at $t = 0$.

Using a constant-acceleration equation, relate the distance the train on the left will travel before the trains pass to its acceleration and the time-to-passing:

$$x_L = \tfrac{1}{2}a_L t^2 = \tfrac{1}{2}\left(1.4\,\text{m/s}^2\right)t^2 = \left(0.7\,\text{m/s}^2\right)t^2$$

Using a constant-acceleration equation, relate the position of the train on the right to its initial velocity, position, and acceleration:

$$x_R = 40\,\text{m} - \tfrac{1}{2}a_R t^2 = 40\,\text{m} - \tfrac{1}{2}\left(2.2\,\text{m/s}^2\right)t^2$$

Equate x_L and x_R and solve for t:

$$0.7t^2 = 40 - 1.1t^2$$

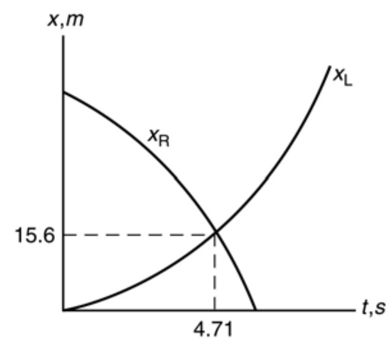
and

$$t = 4.71\,\text{s}$$

Find the position of the train initially on the left, x_L, as they pass:

$$x_L = \tfrac{1}{2}(1.4\,\text{m/s}^2)(4.71\,\text{s})^2 = \boxed{15.6\,\text{m}}$$

Remarks: One can also solve this problem by graphing the functions for x_L and x_R. The coordinates of the intersection of the two curves give one the time-to-passing and the distance traveled by the train on the left.



93 ••

Picture the Problem In the absence of air resistance, the acceleration of the stones is constant. Choose a coordinate system in which the downward direction is positive and the origin is at the point of release of the stones.

Using constant-acceleration equations, relate the positions of the two stones to their initial positions, accelerations, and time-of-fall:

$$x_1 = \tfrac{1}{2}gt^2$$

and

$$x_2 = \tfrac{1}{2}g(t - 1.6\,\text{s})^2$$

Express the difference between x_1 and x_2:

$$x_1 - x_2 = 36\text{ m}$$

Substitute for x_1 and x_2 to obtain:

$$36\,\text{m} = \tfrac{1}{2}gt^2 - \tfrac{1}{2}g\left(t - 1.6\,\text{s}\right)^2$$

Solve this equation for the time t at which the stones will be separated by 36 m:

$$t = 3.09\text{ s}$$

Substitute this result in the expression for x_2 and solve for x_2:

$$x_2 = \tfrac{1}{2}\left(9.81\,\text{m/s}^2\right)\left(3.09\,\text{s} - 1.6\,\text{s}\right)^2 = \boxed{10.9\,\text{m}}$$

***94 ••**

Picture the Problem The acceleration of the police officer's car is positive and constant and the acceleration of the speeder's car is zero. Choose a coordinate system such that the direction of motion of the two vehicles is the positive direction and the origin is at the stop sign.

Express the velocity of the car in terms of the distance it will travel until the police officer catches up to it and the time that will elapse during this chase:

$$v_{\text{car}} = \frac{d_{\text{caught}}}{t_{\text{car}}}$$

Letting t_1 be the time during which she accelerates and t_2 the time of travel at v_1 = 110 km/h, express the time of travel of the police officer:

$$t_{\text{officer}} = t_1 + t_2$$

Convert 110 km/h into m/s:

$$v_1 = \left(110\,\text{km/h}\right)(10^3\text{ m/km})(1\,\text{h}/3600\,\text{s}) = 30.6\,\text{m/s}$$

Express and evaluate t_1:

$$t_1 = \frac{v_1}{a_{\text{motorcycle}}} = \frac{30.6\text{ m/s}}{6.2\text{ m/s}^2} = 4.94\,\text{s}$$

Express and evaluate d_1:

$$d_1 = \tfrac{1}{2}v_1 t_1 = \tfrac{1}{2}(30.6\,\text{m/s})(4.94\,\text{s}) = 75.6\,\text{m}$$

Determine d_2:

$$d_2 = d_{\text{caught}} - d_1 = 1400\,\text{m} - 75.6\,\text{m} = 1324.4\,\text{m}$$

Express and evaluate t_2:

$$t_2 = \frac{d_2}{v_1} = \frac{1324.4\text{ m}}{30.6\text{ m/s}} = 43.3\,\text{s}$$

Express the time of travel of the car:

$$t_{\text{car}} = 2.0\text{ s} + 4.93\text{ s} + 43.3\text{ s} = 50.2\text{ s}$$

Finally, find the speed of the car:

$$v_{car} = \frac{d_{caught}}{t_{car}} = \frac{1400\text{ m}}{50.2\text{ s}} = 27.9\text{ m/s}$$
$$= (27.9\text{ m/s})\left(\frac{1\text{ mi/h}}{0.447\text{ m/s}}\right)$$
$$= \boxed{62.4\text{ mi/h}}$$

95 ••

Picture the Problem In the absence of air resistance, the acceleration of the stone is constant. Choose a coordinate system in which downward is positive and the origin is at the point of release of the stone and apply constant-acceleration equations.

Using a constant-acceleration equation, express the height of the cliff in terms of the initial position of the stones, acceleration due to gravity, and time for the first stone to hit the water:

$$h = \tfrac{1}{2}gt_1^2$$

Express the displacement of the second stone when it hits the water in terms of its initial velocity, acceleration, and time required for it to hit the water.

$$d_2 = v_{02}t_2 + \tfrac{1}{2}gt_2^2$$
where $t_2 = t_1 - 1.6$ s.

Because the stones will travel the same distances before hitting the water, equate h and d_2 and solve for t.

$$\tfrac{1}{2}gt_1^2 = v_{02}t_2 + \tfrac{1}{2}gt_2^2$$
or
$$\tfrac{1}{2}\left(9.81\text{m/s}^2\right)t_1^2 = (32\text{m/s})(t_1 - 1.6\text{s}) + \tfrac{1}{2}\left(9.81\text{m/s}^2\right)(t_1 - 1.6\text{s})^2$$

Solve for t_1 to obtain:

$$t_1 = 2.37\text{ s}$$

Substitute for t_1 and evaluate h:

$$h = \tfrac{1}{2}(9.81\text{ m/s}^2)(2.37\text{ s})^2 = \boxed{27.6\text{ m}}$$

96 •••

Picture the Problem Assume that the acceleration of the passenger train is constant. Let $x_p = 0$ be the location of the passenger train engine at the moment of sighting the freight train's end; let $t = 0$ be the instant the passenger train begins to slow (0.4 s after the passenger train engineer sees the freight train ahead). Choose a coordinate system in which the direction of motion of the trains is the positive direction and use constant-acceleration equations to express the positions of the trains in terms of their initial positions, speeds, accelerations, and elapsed time.

(*a*) Using constant-acceleration equations, write expressions for the positions of the front of the passenger train and the rear of the

$$x_p = (29\text{ m/s})(t + 0.4\text{ s}) - \tfrac{1}{2}at^2$$
$$x_f = (360\text{ m}) + (6\text{ m/s})(t + 0.4\text{ s})$$
where x_p and x_f are in meters if t is in

freight train, x_p and x_f, respectively: seconds.

Equate $x_f = x_p$ to obtain an equation for t:

$$\tfrac{1}{2}at^2 - (23\,\text{m/s})t + 350.8\,\text{m} = 0$$

Find the discriminant $\left(D = \text{B}^2 - 4\text{AC}\right)$ of this equation:

$$D = (23\,\text{m/s})^2 - 4\left(\frac{a}{2}\right)(350.8\,\text{m})$$

The equation must have real roots if it is to describe a collision. The necessary condition for real roots is that the discriminant be greater than or equal to zero:

If $(23\text{ m/s})^2 - a\,(701.6\text{ m}) \geq 0$, then

$$a \leq \boxed{0.754\,\text{m/s}^2}$$

(b) Express the relative speed of the trains:

$$v_{\text{rel}} = v_{\text{pf}} = v_{\text{p}} - v_{\text{f}} \qquad (1)$$

Repeat the previous steps with $a = 0.754\text{ m/s}^2$ and a 0.8 s reaction time. The quadratic equation that guarantees real roots with the longer reaction time is:

$$\tfrac{1}{2}\left(0.754\,\text{m/s}^2\right)t^2 - (23\,\text{m/s})t + 341.6\,\text{m} = 0$$

Solve for t to obtain the collision times:

$t = 25.6$ s and $t = 35.4$ s

Note that at $t = 35.4$ s, the trains have already collided; therefore this root is not a meaningful solution to our problem.

Note: In the graph shown below, you will see why we keep only the smaller of the two solutions.

Now we can substitute our value for t in the constant-acceleration equation for the passenger train and solve for the distance the train has moved prior to the collision:

$$\begin{aligned} x_p &= (29\text{ m/s})(25.6\text{ s} + 0.8\text{ s}) \\ &\quad - (0.377\text{ m/s}^2)(25.6\text{ s})^2 \\ &= 518\,\text{m} \end{aligned}$$

Find the speeds of the two trains:

$$\begin{aligned} v_p &= v_{op} + at \\ &= (29\text{ m/s}) + (-0.754\text{ m/s}^2)(25.5\text{ s}) \\ &= 9.77\text{ m/s} \end{aligned}$$

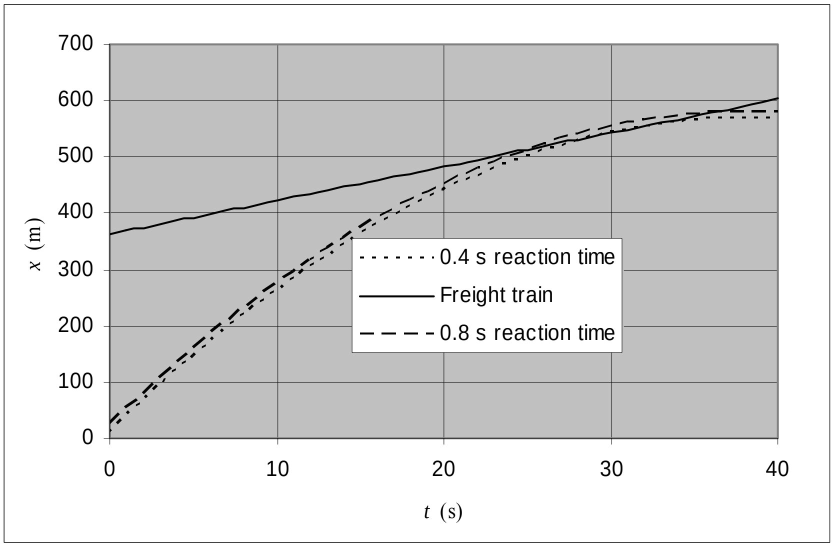
and

$$v_f = v_{of} = 6\text{ m/s}$$

Substitute in equation (1) and evaluate the relative speed of the trains:

$$v_{\text{rel}} = 9.77\,\text{m/s} - 6.00\,\text{m/s} = \boxed{3.77\,\text{m/s}}$$

The graph shows the location of both trains as functions of time. The solid straight line is for the constant velocity freight train; the dashed curves are for the passenger train, with reaction times of 0.4 s for the lower curve and 0.8 s for the upper curve.



Remarks: A collision occurs the first time the curve for the passenger train crosses the curve for the freight train. The smaller of two solutions will always give the time of the collision.

97 •

Picture the Problem In the absence of air resistance, the acceleration of an object near the surface of the earth is constant. Choose a coordinate system in which the upward direction is positive and the origin is at the surface of the earth and apply constant-acceleration equations.

Using a constant-acceleration equation, relate the velocity to the acceleration and displacement:

$$v^2 = v_0^2 + 2a\Delta y$$

or, because $v = 0$ and $a = -g$,

$$0 = v_0^2 - 2g\Delta y$$

Solve for the height to which the projectile will rise:

$$h = \Delta y = \frac{v_0^2}{2g}$$

Substitute numerical values and evaluate *h*:

$$h = \frac{(300\,\text{m/s})^2}{2(9.81\text{ m/s}^2)} = \boxed{4.59\,\text{km}}$$

***98** •

Picture the Problem This is a composite of two constant accelerations with the acceleration equal to one constant prior to the elevator hitting the roof, and equal to a different constant after crashing through it. Choose a coordinate system in which the upward direction is positive and apply constant-acceleration equations.

(*a*) Using a constant-acceleration equation, relate the velocity to the acceleration and displacement:

$$v^2 = v_0^2 + 2\,a\,\Delta y$$

or, because $v = 0$ and $a = -g$,

$$0 = v_0^2 - 2\,g\,\Delta y$$

Solve for v_0:

$$v_0 = \sqrt{2g\Delta y}$$

Substitute numerical values and evaluate v_0:

$$v_0 = \sqrt{2\left(9.81\,\text{m/s}^2\right)\left(10^4\,\text{m}\right)} = \boxed{443\,\text{m/s}}$$

(b) Find the velocity of the elevator just before it crashed through the roof:

$$v_f = 2 \times 443\text{ m/s} = 886\text{ m/s}$$

Using the same constant-acceleration equation, this time with $v_0 = 0$, solve for the acceleration:

$$v^2 = 2a\Delta y$$

Substitute numerical values and evaluate a:

$$a = \frac{\left(886\,\text{m/s}\right)^2}{2\left(150\,\text{m}\right)} = 2.62\times10^3\text{ m/s}^2$$
$$= \boxed{267g}$$

99 ••

Picture the Problem Choose a coordinate system in which the upward direction is positive. We can use a constant-acceleration equation to find the beetle's velocity as its feet lose contact with the ground and then use this velocity to calculate the height of its jump.

Using a constant-acceleration equation, relate the beetle's maximum height to its launch velocity, velocity at the top of its trajectory, and acceleration once it is airborne; solve for its maximum height:

$$v_{\text{highest point}}^2 = v_{\text{launch}}^2 + 2a\Delta y_{\text{free fall}}$$
$$= v_{\text{launch}}^2 + 2(-g)h$$

Because $v_{\text{highest point}} = 0$:

$$h = \frac{v_{\text{launch}}^2}{2g}$$

Now, in order to determine the beetle's launch velocity, relate its time of contact with the ground to its acceleration and push-off distance:

$$v_{\text{launch}}^2 = v_0^2 + 2a\Delta y_{\text{launch}}$$

or, because $v_0 = 0$,

$$v_{\text{launch}}^2 = 2a\Delta y_{\text{launch}}$$

Substitute numerical values and evaluate v_{launch}^2:

$$v_{\text{launch}}^2 = 2(400)\left(9.81\,\text{m/s}^2\right)\left(0.6\times10^{-2}\,\text{m}\right)$$
$$= 47.1\,\text{m}^2/\text{s}^2$$

Substitute to find the height to which the beetle can jump:

$$h = \frac{v_{\text{launch}}^2}{2g} = \frac{47.1\,\text{m}^2/\text{s}^2}{2\left(9.81\,\text{m/s}^2\right)} = \boxed{2.40\,\text{m}}$$

Using a constant-acceleration equation, relate the velocity of the beetle at its maximum height to its launch velocity, free-fall acceleration while in the air, and time-to-maximum height:

$$v = v_0 + at$$

or

$$v_{\text{max. height}} = v_{\text{launch}} - gt_{\text{max. height}}$$

and, because $v_{\text{max height}} = 0$,

$$0 = v_{\text{launch}} - gt_{\text{max. height}}$$

Solve for $t_{\text{max height}}$:

$$t_{\text{max height}} = \frac{v_{\text{launch}}}{g}$$

For zero displacement and constant acceleration, the time-of-flight is twice the time-to-maximum height:

$$t_{\text{flight}} = 2t_{\text{max. height}} = \frac{2v_{\text{launch}}}{g}$$

$$= \frac{2(6.86\,\text{m/s})}{9.81\,\text{m/s}^2} = \boxed{1.40\,\text{s}}$$

100 •

Picture the Problem Because its acceleration is constant we can use the constant-acceleration equations to describe the motion of the automobile.

Using a constant-acceleration equation, relate the velocity to the acceleration and displacement:

$$v^2 = v_0^2 + 2a\,\Delta x$$

or, because $v = 0$,

$$0 = v_0^2 + 2a\,\Delta x$$

Solve for the acceleration a:

$$a = \frac{-v_0^2}{2\Delta x}$$

Substitute numerical values and evaluate a:

$$a = \frac{-\left[(98\,\text{km/h})(10^3\,\text{m/km})(1\,\text{h}/3600\,\text{s})\right]^2}{2(50\text{m})}$$

$$= \boxed{-7.41\,\text{m/s}^2}$$

Express the ratio of a to g and then solve for a:

$$\frac{a}{g} = \frac{-7.41\,\text{m/s}^2}{9.81\,\text{m/s}^2} = -0.755$$

and $a = \boxed{-0.755g}$

Using the definition of average acceleration, solve for the stopping time:

$$a_{\text{av}} = \frac{\Delta v}{\Delta t} \Rightarrow \Delta t = \frac{\Delta v}{a_{\text{av}}}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{(-98\,\text{km/h})(10^3\,\text{m/km})(1\,\text{h}/3600\,\text{s})}{-7.41\,\text{m/s}^2}$$

$$= \boxed{3.67\,\text{s}}$$

***101** ••

Picture the Problem In the absence of air resistance, the puck experiences constant acceleration and we can use constant-acceleration equations to describe its position as a function of time. Choose a coordinate system in which downward is positive, the particle starts from rest ($v_o = 0$), and the starting height is zero ($y_0 = 0$).

Using a constant-acceleration equation, relate the position of the falling puck to the acceleration and the time. Evaluate the y-position at successive equal time intervals Δt, $2\Delta t$, $3\Delta t$, etc:

$$y_1 = \frac{-g}{2}(\Delta t)^2 = \frac{-g}{2}(\Delta t)^2$$

$$y_2 = \frac{-g}{2}(2\Delta t)^2 = \frac{-g}{2}(4)(\Delta t)^2$$

$$y_3 = \frac{-g}{2}(3\Delta t)^2 = \frac{-g}{2}(9)(\Delta t)^2$$

$$y_4 = \frac{-g}{2}(4\Delta t)^2 = \frac{-g}{2}(16)(\Delta t)^2$$

etc.

Evaluate the changes in those positions in each time interval:

$$\Delta y_{10} = y_1 - 0 = \left(\frac{-g}{2}\right)(\Delta t)^2$$

$$\Delta y_{21} = y_2 - y_1 = 3\left(\frac{-g}{2}\right)(\Delta t)^2 = 3\Delta y_{10}$$

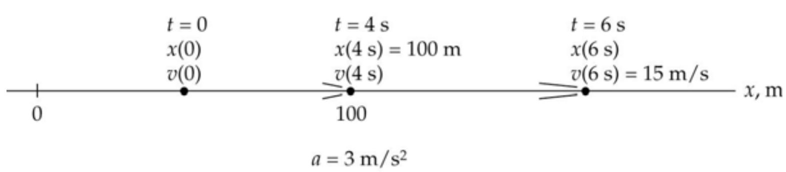
$$\Delta y_{32} = y_3 - y_2 = 5\left(\frac{-g}{2}\right)(\Delta t)^2 = 5\Delta y_{10}$$

$$\Delta y_{43} = y_4 - y_3 = 7\left(\frac{-g}{2}\right)(\Delta t)^2 = 7\Delta y_{10}$$

etc.

102 ••

Picture the Problem Because the particle moves with a constant acceleration we can use the constant-acceleration equations to describe its motion. A pictorial representation will help us organize the information in the problem and develop our solution strategy.



Using a constant-acceleration equation, find the position x at $t = 6$ s. To find x at $t = 6$ s, we first need to find v_0 and x_0:

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

Using the information that when $t = 4$ s, $x = 100$ m, obtain an equation in x_0 and v_0:

$$x(4\,\text{s}) = 100\,\text{m}$$
$$= x_0 + v_0(4\,\text{s}) + \tfrac{1}{2}(3\,\text{m/s}^2)(4\,\text{s})^2$$

or

$$x_0 + (4\,\text{s})v_0 = 76\,\text{m}$$

Using the information that when $t = 6$ s, $v = 15$ m/s, obtain a second equation in x_0 and v_0:

$$v(6\,\text{s}) = v_0 + (3\,\text{m/s}^2)(6\,\text{s})$$

Solve for v_0 to obtain:

$v_0 = -3$ m/s

Substitute this value for v_0 in the previous equation and solve for x_0:

$x_0 = 88$ m

Substitute for x_0 and v_0 and evaluate x at $t = 6$ s:

$$x(6\,\text{s}) = 88\,\text{m} + (-3\,\text{m/s})\,(6\,\text{s}) + \tfrac{1}{2}(3\,\text{m/s}^2)\,(6\,\text{s})^2 = \boxed{124\,\text{m}}$$

***103 ••**

Picture the Problem We can use constant-acceleration equations with the final velocity $v = 0$ to find the acceleration and stopping time of the plane.

(*a*) Using a constant-acceleration equation, relate the known velocities to the acceleration and displacement:

$$v^2 = v_0^2 + 2a\,\Delta x$$

Solve for a:

$$a = \frac{v^2 - v_0^2}{2\,\Delta x} = \frac{-v_0^2}{2\,\Delta x}$$

Substitute numerical values and evaluate a:

$$a = \frac{-(60\,\text{m/s})^2}{2(70\,\text{m})} = \boxed{-25.7\,\text{m/s}^2}$$

(*b*) Using a constant-acceleration equation, relate the final and initial speeds of the plane to its acceleration and stopping time:

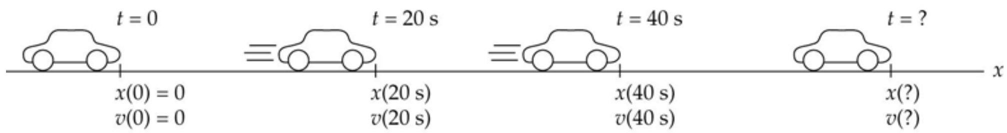
$$v = v_0 + a\Delta t$$

Solve for and evaluate the stopping time:

$$\Delta t = \frac{v - v_0}{a} = \frac{0 - 60\,\text{m/s}}{-25.7\,\text{m/s}^2} = \boxed{2.33\,\text{s}}$$

104 ••

Picture the Problem This is a multipart constant-acceleration problem using three different constant accelerations (+2 m/s^2 for 20 s, then zero for 20 s, and then –3 m/s^2 until the automobile stops). The final velocity is zero. The pictorial representation will help us organize the information in the problem and develop our solution strategy.



Add up all the displacements to get the total:

$$\Delta x_{03} = \Delta x_{01} + \Delta x_{12} + \Delta x_{23}$$

Using constant-acceleration formulas, find the first displacement:

$$\Delta x_{01} = v_0 t_1 + \tfrac{1}{2} a_{01} t_1^2$$
$$= 0 + \tfrac{1}{2}(2\text{ m/s}^2)(20\text{ s})^2 = 400\text{ m}$$

The speed is constant for the second displacement. Find the displacement:

$$\Delta x_{12} = v_1 (t_2 - t_1)$$
where $\mathrm{v}_1 = v_0 + a_{01} t_1 = 0 + a_{01} t_1$ and
$$\Delta x_{12} = a_{01} t_1 (t_2 - t_1)$$
$$= (2\text{ m/s}^2)(20\text{ s})(20\text{ s}) = 800\text{ m}$$

Find the displacement during the braking interval:

$$v_3^2 = v_2^2 + 2a_{23}\Delta x_{23}$$
where $v_2 = v_1 = a_{01} t_1$ and $v_3 = 0$ and
$$\Delta x_{23} = \frac{0^2 - (a_{01} t_1)^2}{2a_{23}} = \frac{-[(2\text{ m/s})(20\text{ s})]^2}{2(-3\text{m/s}^2)}$$
$$= 267\text{m}$$

Add the displacements to get the total:

$$\Delta x_{03} = \Delta x_{01} + \Delta x_{12} + \Delta x_{23} = 1467\text{ m}$$
$$= \boxed{1.47\text{ km}}$$

Remarks: Because the area under the curve of a velocity-versus-time graph equals the displacement of the object experiencing the acceleration, we could solve this problem by plotting the velocity as a function of time and finding the area bounded by it and the time axis.

***105** ••

Picture the Problem Note: No material body can travel at speeds faster than light. When one is dealing with problems of this sort, the kinematic formulae for displacement, velocity and acceleration are no longer valid, and one must invoke the special theory of relativity to answer questions such as these. For now, ignore such subtleties. Although the formulas you are using (i.e., the constant- acceleration equations) are not quite correct, your answer to part (*b*) will be wrong by about 1%.

(*a*) This part of the problem is an exercise in the conversion of units. Make use of the fact that 1 $c\cdot$y = 9.47×10^{15} m and 1 y = 3.16×10^{7} s:

$$g = (9.81\text{m/s}^2)\left(\frac{1c\cdot\text{y}}{9.47\times10^{15}\text{m}}\right)\left(\frac{(3.16\times10^{7}\text{s})^2}{(1\text{ y})^2}\right) = \boxed{1.03c\cdot\text{y}/\text{y}^2}$$

(*b*) Let $t_{1/2}$ represent the time it takes to reach the halfway point. Then the total trip time is:

$$t = 2\,t_{1/2} \qquad (1)$$

Use a constant- acceleration equation to relate the half-distance to Mars Δx to the initial speed, acceleration, and half-trip time $t_{1/2}$:

$$\Delta x = v_0 t + \tfrac{1}{2} a t_{1/2}^2$$

Because $v_0 = 0$ and $a = g$:

$$t_{1/2} = \sqrt{\frac{2\Delta x}{a}}$$

The distance from Earth to Mars at closest approach is 7.8×10^{10} m. Substitute numerical values and evaluate $t_{1/2}$:

$$t_{1/2} = \sqrt{\frac{2(3.9\times10^{10}\ \text{m})}{9.81\,\text{m/s}^2}} = 8.92\times10^4\ \text{s}$$

Substitute for $t_{1/2}$ in equation (1) to obtain:

$$t = 2(8.92\times10^4\ \text{s}) = 1.78\times10^5\ \text{s} \approx \boxed{2\,\text{d}}$$

Remarks: Our result in part (*b*) seems remarkably short, considering how far Mars is and how low the acceleration is.

106 •

Picture the Problem Because the elevator accelerates uniformly for half the distance and uniformly decelerates for the second half, we can use constant-acceleration equations to describe its motion

Let $t_{1/2} = 40$ s be the time it takes to reach the halfway mark. Use the constant-acceleration equation that relates the acceleration to the known variables to obtain:

$$\Delta y = v_0 t + \tfrac{1}{2} a t^2$$

or, because $v_0 = 0$,

$$\Delta y = \tfrac{1}{2} a t^2$$

Solve for a:

$$a = \frac{2\,\Delta y}{t_{1/2}^2}$$

Substitute numerical values and evaluate a:

$$a = \frac{2\left(\tfrac{1}{2}\right)(1173\,\text{ft})(1\,\text{m}/3.281\,\text{ft})}{(40\,\text{s})^2} = 0.223\,\text{m/s}^2$$

$$= \boxed{0.0228g}$$

107 ••

Picture the Problem Because the acceleration is constant, we can describe the motions of the train using constant-acceleration equations. Find expressions for the distances traveled, separately, by the train and the passenger. When are they equal? Note that the train is accelerating and the passenger runs at a constant minimum velocity (zero acceleration) such that she can just catch the train.

1. Using the subscripts "train" and "p" to refer to the train and the passenger and the subscript "c" to identify "critical" conditions, express the position of the train and the passenger:

$$x_{\text{train,c}}(t_c) = \frac{a_{\text{train}}}{2} t_c^2$$

and

$$x_{\text{p,c}}(t_c) = v_{\text{p,c}}(t_c - \Delta t)$$

Express the critical conditions that must be satisfied if the passenger is to catch the train:

$$v_{\text{train,c}} = v_{\text{p,c}}$$

and

$$x_{\text{train,c}} = x_{\text{p,c}}$$

2. Express the train's average velocity.

$$v_{\text{av}}(0 \text{ to } t_c) = \frac{0 + v_{\text{train,c}}}{2} = \frac{v_{\text{train,c}}}{2}$$

3. Using the definition of average velocity, express v_{av} in terms of $x_{\text{p,c}}$ and t_c.

$$v_{\text{av}} \equiv \frac{\Delta x}{\Delta t} = \frac{0 + x_{\text{p,c}}}{0 + t_c} = \frac{x_{\text{p,c}}}{t_c}$$

4. Combine steps 2 and 3 and solve for $x_{\text{p,c}}$.

$$x_{\text{p,c}} = \frac{v_{\text{train,c}} t_c}{2}$$

5. Combine steps 1 and 4 and solve for t_c.

$$v_{\text{p,c}}(t_c - \Delta t) = \frac{v_{\text{train,c}} t_c}{2}$$

or

$$t_c - \Delta t = \frac{t_c}{2}$$

and

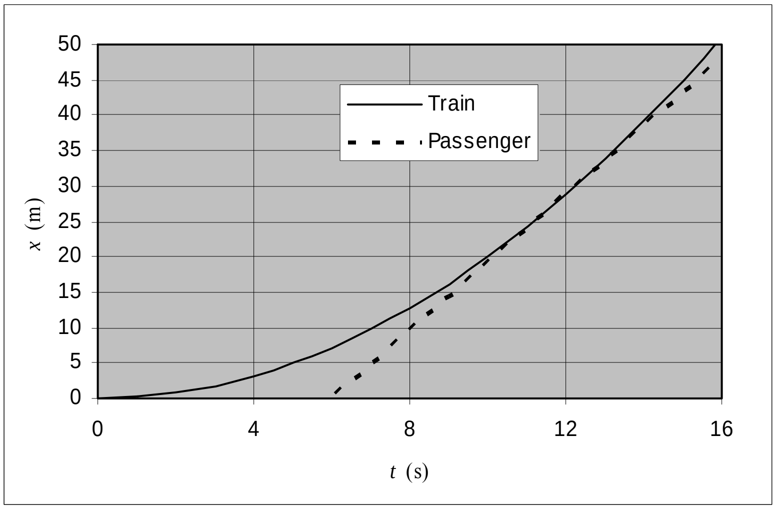
$t_c = 2\ \Delta t = 2\ (6 \text{ s}) = 12 \text{ s}$

6. Finally, combine steps 1 and 5 and solve for $v_{\text{train, c}}$.

$$v_{\text{p,c}} = v_{\text{train,c}} = a_{\text{train}} t_c = (0.4 \text{ m/s}^2)(12 \text{ s})$$

$$= \boxed{4.80 \text{ m/s}}$$

The graph shows the location of both the passenger and the train as a function of time. The parabolic solid curve is the graph of $x_{\text{train}}(t)$ for the accelerating train. The straight dashed line is passenger's position $x_{\text{p}}(t)$ if she arrives at $\Delta t = 6.0$ s after the train departs. When the passenger catches the train, our graph shows that her speed and that of the train must be equal ($v_{\text{train,c}} = v_{\text{p,c}}$). Do you see why?



108 •••

Picture the Problem Both balls experience constant acceleration once they are in flight. Choose a coordinate system with the origin at the ground and the upward direction positive. When the balls collide they are at the same height above the ground.

Using constant-acceleration equations, express the positions of both balls as functions of time. At the ground $y = 0$.

$$y_A = h - \tfrac{1}{2}gt^2$$
and
$$y_B = v_0 t - \tfrac{1}{2}gt^2$$

The conditions at collision are that the heights are equal and the velocities are related:

$$y_A = y_B$$
and
$$v_A = -2v_B$$

Express the velocities of both balls as functions of time:

$$v_A = -gt$$
and
$$v_B = v_0 - gt$$

Substituting the position and velocity functions into the conditions at collision gives:

$$h - \tfrac{1}{2}gt_c^2 = v_0 t_c - \tfrac{1}{2}gt_c^2$$
and
$$-gt_c = -2(v_0 - gt_c)$$
where t_c is the time of collision.

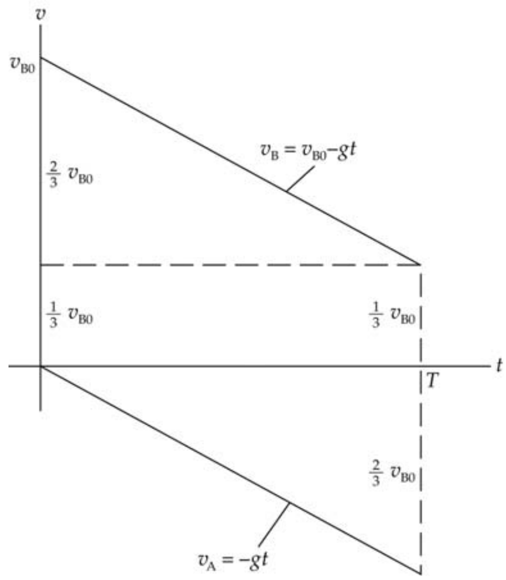
We now have two equations and two unknowns, t_c and v_0. Solving the equations for the unknowns gives:

$$t_c = \sqrt{\frac{2h}{3g}} \text{ and } v_0 = \sqrt{\frac{3gh}{2}}$$

Substitute the expression for t_c into the equation for y_A to obtain the height at collision:

$$y_A = h - \tfrac{1}{2}g\left(\frac{2h}{3g}\right) = \boxed{\frac{2h}{3}}$$

Remarks: We can also solve this problem graphically by plotting velocity- versus-time for both balls. Because ball A starts from rest its velocity is given by $v_A = -gt$. Ball B initially moves with an unknown velocity v_{B0} and its velocity is given by $v_B = v_{B0} - gt$. The graphs of these equations are shown below with T representing the time at which they collide.



The height of the building is the sum of the sum of the distances traveled by the balls. Each of these distances is equal to the magnitude of the area ″under″ the corresponding v-versus-t curve. Thus, the height of the building equals the area of the parallelogram, which is $v_{B0}T$. The distance that A falls is the area of the lower triangle, which is (1/3) $v_{B0}T$. Therefore, the ratio of the distance fallen by A to the height of the building is 1/3, so the collision takes place at 2/3 the height of the building.

109 •••

Picture the Problem Both balls are moving with constant acceleration. Take the origin of the coordinate system to be at the ground and the upward direction to be positive. When the balls collide they are at the same height above the ground. The velocities at collision are related by $v_A = 4v_B$.

Using constant-acceleration equations, express the positions of both balls as functions of time:

$$y_A = h - \tfrac{1}{2}gt^2$$
and
$$y_B = v_0 t - \tfrac{1}{2}gt^2$$

The conditions at collision are that the heights are equal and the velocities are related:

$$y_A = y_B$$
and
$$v_A = 4v_B$$

Express the velocities of both balls as functions of time:

$$v_A = -gt \text{ and } v_B = v_0 - gt$$

Substitute the position and velocity functions into the conditions at collision to obtain:

$$h - \tfrac{1}{2}gt_c^2 = v_0t_c - \tfrac{1}{2}gt_c^2$$

and

$$-gt_c = 4(v_0 - gt_c)$$

where t_c is the time of collision.

We now have two equations and two unknowns, t_c and v_0. Solving the equations for the unknowns gives:

$$t_c = \sqrt{\frac{4h}{3g}} \text{ and } v_0 = \sqrt{\frac{3gh}{4}}$$

Substitute the expression for t_c into the equation for y_A to obtain the height at collision:

$$y_A = h - \tfrac{1}{2}g\left(\frac{4h}{3g}\right) = \boxed{\frac{h}{3}}$$

***110 ••**

Determine the Concept The problem describes two intervals of constant acceleration; one when the train's velocity is increasing, and a second when it is decreasing.

(*a*) Using a constant-acceleration equation, relate the half-distance Δx between stations to the initial speed v_0, the acceleration a of the train, and the time-to-midpoint Δt:

$$\Delta x = v_0\Delta t + \tfrac{1}{2}a(\Delta t)^2$$

or, because $v_0 = 0$,

$$\Delta x = \tfrac{1}{2}a(\Delta t)^2$$

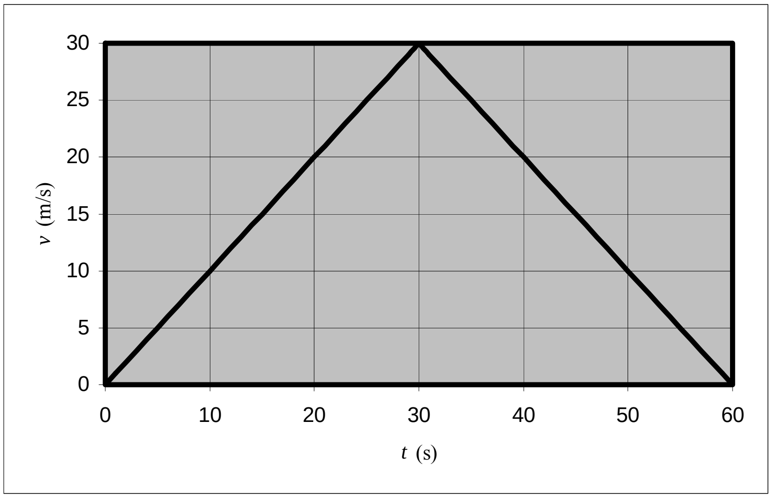
Solve for Δt:

$$\Delta t = \sqrt{\frac{2\Delta x}{a}}$$

Substitute numerical values and evaluate the time-to-midpoint Δt:

$$\Delta t = \sqrt{\frac{2(450\,\text{m})}{1\,\text{m/s}^2}} = 30.0\,\text{s}$$

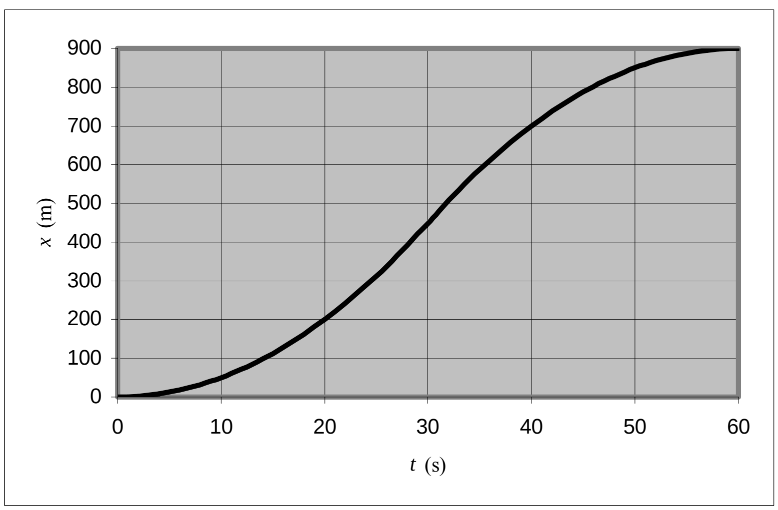
Because the train accelerates uniformly and from rest, the first part of its velocity graph will be linear, pass through the origin, and last for 30 s. Because it slows down uniformly and at the same rate for the second half of its journey, this part of its graph will also be linear but with a negative slope. The graph of v as a function of t is shown below.



(b) The graph of x as a function of t is obtained from the graph of v as a function of t by finding the area under the velocity curve. Looking at the velocity graph, note that when the train has been in motion for 10 s, it will have traveled a distance of

$$\tfrac{1}{2}(10\,\text{s})(10\,\text{m/s}) = 50\,\text{m}$$

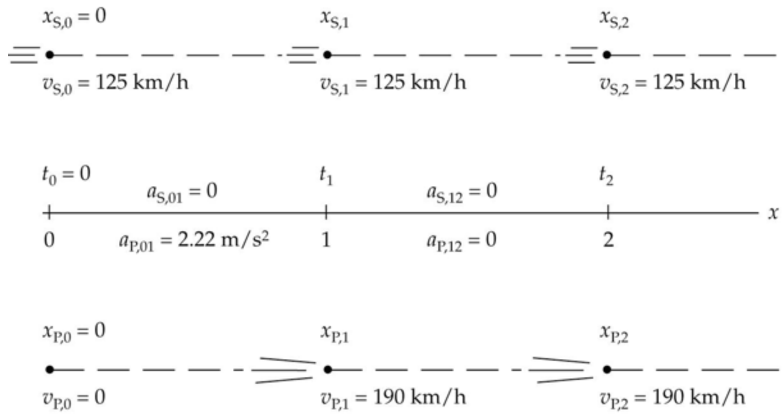
and that this distance is plotted above 10 s on the following graph.



Selecting additional points from the velocity graph and calculating the areas under the curve will confirm the graph of x as a function of t that is shown.

111 ••

Picture the Problem This is a two-stage constant-acceleration problem. Choose a coordinate system in which the direction of the motion of the cars is the positive direction. The pictorial representation summarizes what we know about the motion of the speeder's car and the patrol car.



Convert the speeds of the vehicles and the acceleration of the police car into SI units:

$$8\,\frac{\text{km}}{\text{h}\cdot\text{s}} = 8\,\frac{\text{km}}{\text{h}\cdot\text{s}} \times \frac{1\,\text{h}}{3600\,\text{s}} = 2.22\,\text{m/s}^2\,,$$

$$125\,\frac{\text{km}}{\text{h}} = 125\,\frac{\text{km}}{\text{h}} \times \frac{1\,\text{h}}{3600\,\text{s}} = 34.7\,\text{m/s}\,,$$

and

$$190\,\frac{\text{km}}{\text{h}} = 190\,\frac{\text{km}}{\text{h}} \times \frac{1\,\text{h}}{3600\,\text{s}} = 52.8\,\text{m/s}$$

(*a*) Express the condition that determines when the police car catches the speeder; i.e., that their displacements will be the same:

$$\Delta x_{P,02} = \Delta x_{S,02}$$

Using a constant-acceleration equation, relate the displacement of the patrol car to its displacement while accelerating and its displacement once it reaches its maximum velocity:

$$\begin{aligned}\Delta x_{P,02} &= \Delta x_{P,01} + \Delta x_{P,12} \\ &= \Delta x_{P,01} + v_{P,1}(t_2 - t_1)\end{aligned}$$

Using a constant-acceleration equation, relate the displacement of the speeder to its constant velocity and the time it takes the patrol car to catch it:

$$\begin{aligned}\Delta x_{S,02} &= v_{S,02}\Delta t_{02} \\ &= (34.7\,\text{m/s})t_2\end{aligned}$$

Calculate the time during which the police car is speeding up:

$$\begin{aligned}\Delta t_{P,01} &= \frac{\Delta v_{P,01}}{a_{P,01}} = \frac{v_{P,1} - v_{P,0}}{a_{P,01}} \\ &= \frac{52.8\,\text{m/s} - 0}{2.22\,\text{m/s}^2} = 23.8\,\text{s}\end{aligned}$$

Express the displacement of the patrol car:

$$\Delta x_{P,01} = v_{P,0}\Delta t_{P,01} + \tfrac{1}{2}a_{P,01}\Delta t_{P,01}^2 = 0 + \tfrac{1}{2}\left(2.22\,\text{m/s}^2\right)\left(23.8\,\text{s}\right)^2 = 629\,\text{m}$$

Equate the displacements of the two vehicles:

$$\Delta x_{P,02} = \Delta x_{P,01} + \Delta x_{P,12} = \Delta x_{P,01} + v_{P,1}\left(t_2 - t_1\right) = 629\,\text{m} + \left(52.8\,\text{m/s}\right)\left(t_2 - 23.8\,\text{s}\right)$$

Solve for the time to catch up to obtain:

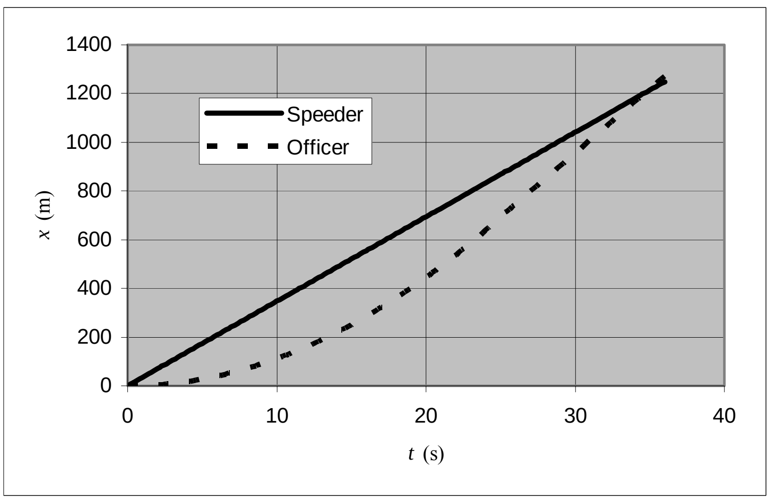
$$(34.7\text{ m/s})\, t_2 = 629\text{ m} + (52.8\text{ m/s})(t_2 - 23.8\text{ s})$$

$$\therefore\ t_2 = \boxed{34.7\,\text{s}}$$

(*b*) The distance traveled is the displacement, $\Delta x_{02,S}$, of the speeder during the catch:

$$\Delta x_{S,02} = v_{S,02}\Delta t_{02} = \left(34.7\,\text{m/s}\right)\left(34.7\,\text{s}\right) = \boxed{1.20\,\text{km}}$$

(*c*) The graphs of x_S and x_P are shown below. The straight line (solid) represents $x_S(t)$ and the parabola (dashed) represents $x_P(t)$.



112 ••

Picture the Problem The accelerations of both cars are constant and we can use constant-acceleration equations to describe their motions. Choose a coordinate system in which the direction of motion of the cars is the positive direction, and the origin is at the initial position of the police car.

(*a*) The collision will ***not*** occur if, during braking, the displacements of the two cars differ by less than 100 m.

$$\Delta x_P - \Delta x_S < 100\text{ m}.$$

Using a constant-acceleration equation, relate the speeder's initial and final speeds to its displacement and acceleration and solve for the displacement:

$$v_s^2 = v_{0,s}^2 + 2a_s\Delta x_s$$

or, because $v_s = 0$,

$$\Delta x_s = \frac{-v_{0,s}^2}{2a_s}$$

Substitute numerical values and evaluate Δx_S:

$$\Delta x_S = \frac{-(34.7\text{ m/s})^2}{2(-6\text{ m/s}^2)} = 100\text{ m}$$

Using a constant-acceleration equation, relate the patrol car's initial and final speeds to its displacement and acceleration and solve for the displacement:

$$v_p^2 = v_{0,p}^2 + 2a_p\Delta x_p$$

or, assuming $v_p = 0$,

$$\Delta x_p = \frac{-v_{0,p}^2}{2a_p}$$

Substitute numerical values and evaluate Δx_p:

$$\Delta x_P = \frac{-(52.8\text{ m/s})^2}{2(-6\text{ m/s}^2)} = 232\text{ m}$$

Finally, substitute these displacements into the inequality that determines whether a collision occurs:

232 m − 100 m = 132 m

Because this difference is greater than 100 m, $\boxed{\text{the cars collide}}$.

(*b*) Using constant-acceleration equations, relate the positions of both vehicles to their initial positions, initial velocities, accelerations, and time in motion:

$$x_S = 100\text{ m} + (34.7\text{ m/s})t - (3\text{ m/s}^2)t^2$$

and

$$x_P = (52.8\text{ m/s})t - (3\text{ m/s}^2)t^2$$

Equate these expressions and solve for t:

$$100\text{ m} + (34.7\text{ m/s})\,t - (3\text{ m/s}^2)\,t^2 = (52.8\text{ m/s})\,t - (3\text{m/s}^2)\,t^2$$

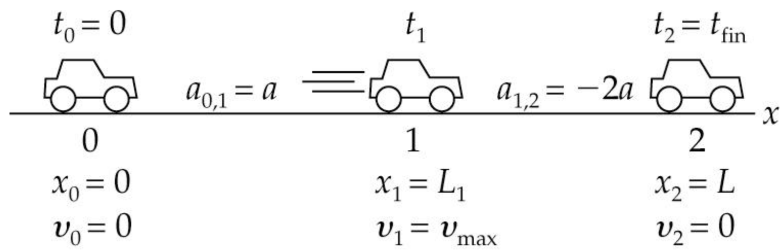
and

$$t = \boxed{5.52\text{ s}}$$

(*c*) $\boxed{\text{If you take the reaction time into account, the collision will occur sooner and be more severe.}}$

113 ••

Picture the Problem Lou's acceleration is constant during both parts of his trip. Let t_1 be the time when the brake is applied; L_1 the distance traveled from $t = 0$ to $t = t_1$. Let t_{fin} be the time when Lou's car comes to rest at a distance L from the starting line. A pictorial representation will help organize the given information and plan the solution.



(*a*) Express the total length, L, of the course in terms of the distance over which Lou will be accelerating, Δx_{01}, and the distance over which he will be braking, Δx_{12}:

$$L = \Delta x_{01} + \Delta x_{12}$$

Express the final velocity over the first portion of the course in terms of the initial velocity, acceleration, and displacement; solve for the displacement:

$$v_1^2 = v_0^2 + 2a_{01}\Delta x_{01}$$

or, because $v_0 = 0$, $\Delta x_{01} = L_1$, and $a_{01} = a$,

$$\Delta x_{01} = L_1 = \frac{v_1^2}{2a} = \frac{v_{max}^2}{2a}$$

Express the final velocity over the second portion of the course in terms of the initial velocity, acceleration, and displacement; solve for the displacement:

$$v_2^2 = v_1^2 + 2a_{12}\Delta x_{12}$$

or, because $v_2 = 0$ and $a_{12} = -2a$,

$$\Delta x_{12} = \frac{v_1^2}{4a} = \frac{L_1}{2}$$

Substitute for Δx_{01} and Δx_{12} to obtain:

$$L = \Delta x_{01} + \Delta x_{12} = L_1 + \tfrac{1}{2}L_1 = \tfrac{3}{2}L_1$$

and

$$L_1 = \boxed{\tfrac{2}{3}L}$$

(*b*) Using the fact that the acceleration was constant during both legs of the trip, express Lou's average velocity over each leg:

$$v_{av,01} = v_{av,12} = \frac{v_{max}}{2}$$

Express the time for Lou to reach his maximum velocity as a function of L_1 and his maximum velocity:

$$\Delta t_{01} = \frac{\Delta x_{01}}{v_{av,01}} = \frac{2L_1}{v_{max}}$$

and

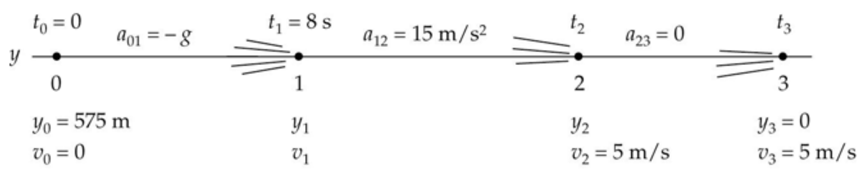
$$\Delta t_{01} \propto L_1 = \frac{2}{3}L$$

Having just shown that the time required for the first segment of the trip is proportional to the length of the segment, use this result to express Δt_{01} (= t_1) in terms t_{fin}:

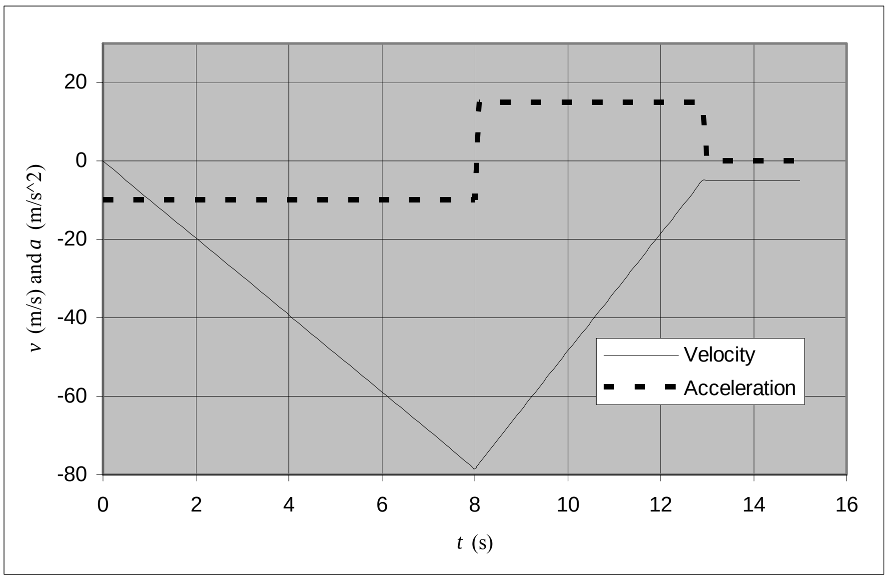
$$t = \boxed{\tfrac{2}{3}t_{fin}}$$

114 ••

Picture the Problem There are three intervals of constant acceleration described in this problem. Choose a coordinate system in which the upward direction (shown to the left below) is positive. A pictorial representation will help organize the details of the problem and plan the solution.



(a) The graphs of $a(t)$ (dashed lines) and $v(t)$ (solid lines) are shown below.



(b) Using a constant-acceleration equation, express her speed in terms of her acceleration and the elapsed time; solve for her speed after 8 s of fall:

$$v_1 = v_0 + a_{01}t_1$$
$$= 0 + (9.81\,\text{m/s}^2)(8\,\text{s})$$
$$= \boxed{78.5\,\text{m/s}}$$

(c) Using the same constant-acceleration equation that you used in part (b), determine the duration of her constant upward acceleration:

$$v_2 = v_1 + a_{12}\Delta t_{12}$$
$$\Delta t_{12} = \frac{v_2 - v_1}{a_{12}} = \frac{-5\,\text{m/s} - (-78.5\,\text{m/s})}{15\,\text{m/s}^2}$$
$$= \boxed{4.90\,\text{s}}$$

(d) Find her average speed as she slows from 78.5 m/s to 5 m/s:

$$v_{\text{av}} = \frac{v_1 + v_2}{2} = \frac{78.5\,\text{m/s} + 5\,\text{m/s}}{2}$$
$$= 41.8\,\text{m/s}$$

Use this value to calculate how far she travels in 4.90 s:

$$\Delta y_{12} = v_{av}\Delta t_{12} = (41.8\text{ m/s})(4.90\text{ s}) = 204\text{ m}$$

$$\boxed{\text{She travels } 204\text{ m while slowing down.}}$$

(*e*) Express the total time in terms of the times for each segment of her descent:

$$t_{total} = \Delta t_{01} + \Delta t_{12} + \Delta t_{23}$$

We know the times for the intervals from 0 to 1 and 1 to 2 so we only need to determine the time for the interval from 2 to 3. We can calculate Δt_{23} from her displacement and constant velocity during that segment of her descent.

$$\Delta y_{23} = \Delta y_{total} - \Delta y_{01} - \Delta y_{12} = 575\text{ m} - \left(\frac{78.5\text{ m/s}}{2}\right)(8\text{ s}) - 204\text{ m} = 57.0\text{ m}$$

Add the times to get the total time:

$$t_{total} = t_{01} + t_{12} + t_{23} = 8\text{ s} + 4.9\text{ s} + \frac{57.0\text{ m}}{5\text{ m/s}} = \boxed{24.3\text{ s}}$$

(*f*) Using its definition, calculate her average velocity:

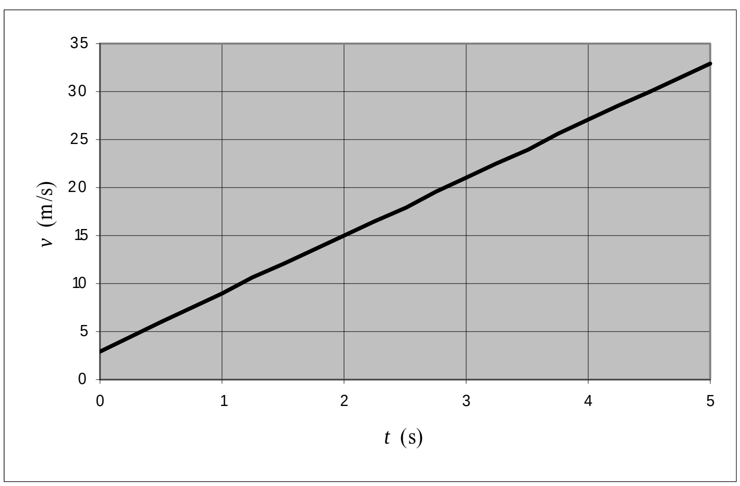
$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{-1500\text{ m}}{209\text{ s}} = \boxed{-7.18\text{ m/s}}$$

Integration of the Equations of Motion

***115** •

Picture the Problem The integral of a function is equal to the "area" between the curve for that function and the independent-variable axis.

(*a*) The graph is shown below:



The distance is found by determining the area under the curve. You can accomplish this easily because the shape of the area under the curve is a trapezoid.

A = (36 blocks)(2.5 m/block) = $\boxed{90\text{ m}}$

or

$$A=\left(\frac{33\,\text{m/s}+3\,\text{m/s}}{2}\right)(5\,\text{s}-0\,\text{s})=90\,\text{m}$$

Alternatively, we could just count the blocks and fractions thereof.

There are approximately 36 blocks each having an area of (5 m/s)(0.5 s) = 2.5 m.

(b) To find the position function $x(t)$, we integrate the velocity function $v(t)$ over the time interval in question:

$$x(t)=\int_0^t v(t')\,\text{d}t'$$

$$=\int_0^t\left[\left(6\,\text{m/s}^2\right)t'+\left(3\,\text{m/s}\right)\right]\text{d}t'$$

and

$$\boxed{x(t)=\left(3\,\text{m/s}^2\right)t^2+\left(3\,\text{m/s}\right)t}$$

Now evaluate $x(t)$ at 0 s and 5 s respectively and subtract to obtain Δx:

$$\Delta x = x(5\,\text{s})-x(0\,\text{s})=90\text{ m}-0\text{ m}$$

$$=\boxed{90.0\,\text{m}}$$

116 •

Picture the Problem The integral of $v(t)$ over a time interval is the displacement (change in position) during that time interval. The integral of a function is equivalent to the "area" between the curve for that function and the independent-variable axis. Count the grid boxes.

(a) Find the area of the shaded gridbox:

$$Area=(1\,\text{m/s})(1\,\text{s})=\boxed{1\,\text{m per box}}$$

(b) Find the approximate area under curve for 1 s ≤ t ≤ 2 s:

$$\Delta x_{1\text{ s to }2\text{ s}} = \boxed{1.2\text{ m}}$$

Find the approximate area under curve for 2 s ≤ t ≤ 3 s:

$$\Delta x_{2\text{ s to }3\text{ s}} = \boxed{3.2\text{ m}}$$

(c) Sum the displacements to obtain the total in the interval 1 s ≤ t ≤ 3 s:

$$\Delta x_{1\text{ s to }3\text{ s}} = 1.2\text{ m} + 3.2\text{ m} = 4.4\text{ m}$$

Using its definition, express and evaluate v_{av}:

$$v_{av}=\frac{\Delta x_{1\,\text{s to }3\,\text{s}}}{\Delta t_{1\,\text{s to }3\,\text{s}}}=\frac{4.4\,\text{m}}{2\,\text{s}}=\boxed{2.20\,\text{m/s}}$$

(d) Because the velocity of the particle is dx/dt, separate the

$$dx=\left(0.5\text{m/s}^3\right)dt$$

so

variables and integrate over the interval 1 s ≤ t ≤ 3 s to determine the displacement in this time interval:

$$\Delta x_{1s\to 3s} = \int_{x_0}^{x} dx' = \left(0.5\,\text{m/s}^3\right)\int_{1\,\text{s}}^{3\,\text{s}} t'^2\,dt'$$

$$= \left(0.5\,\text{m/s}^3\right)\left[\frac{t'^3}{3}\right]_{1\text{s}}^{3\text{s}} = \boxed{4.33\,\text{m}}$$

This result is a little smaller than the sum of the displacements found in part (*b*).

Calculate the average velocity over the 2-s interval from 1 s to 3 s:

$$v_{\text{av}\,(1\text{s}-3\text{s})} = \frac{\Delta x_{1\text{s}-3\text{s}}}{\Delta t_{1\text{s}-3\text{s}}} = \frac{4.33\,\text{m}}{2\,\text{s}} = 2.17\,\text{m/s}$$

Calculate the initial and final velocities of the particle over the same interval:

$$v(1\,\text{s}) = \left(0.5\,\text{m/s}^3\right)(1\,\text{s})^2 = 0.5\,\text{m/s}$$

$$v(3\,\text{s}) = \left(0.5\,\text{m/s}^3\right)(3\,\text{s})^2 = 4.5\,\text{m/s}$$

Finally, calculate the average value of the velocities at $t = 1$ s and $t = 3$ s:

$$\frac{v(1\,\text{s}) + v(3\,\text{s})}{2} = \frac{0.5\,\text{m/s} + 4.5\,\text{m/s}}{2}$$

$$= 2.50\,\text{m/s}$$

This average is not equal to the average velocity calculated above.

Remarks: The fact that the average velocity was not equal to the average of the velocities at the beginning and the end of the time interval in part (*d*) is a consequence of the acceleration not being constant.

***117** ••

Picture the Problem Because the velocity of the particle varies with the square of the time, the acceleration is not constant. The displacement of the particle is found by integration.

Express the velocity of a particle as the derivative of its position function:

$$v(t) = \frac{dx(t)}{dt}$$

Separate the variables to obtain:

$$dx(t) = v(t)dt$$

Express the integral of x from $x_o = 0$ to x and t from $t_0 = 0$ to t:

$$x(t) = \int_{t_0=0}^{x(t)} dx' = \int_{t_0=0}^{t} v(t')dt'$$

Substitute for $v(t')$ to obtain:

$$x(t) = \int_{t_0=0}^{t}\left[\left(7\,\text{m/s}^3\right)t'^2 - (5\,\text{m/s})\right]dt'$$

$$= \boxed{\left(\tfrac{7}{3}\,\text{m/s}^3\right)t^3 - (5\,\text{m/s})t}$$

118 ••

Picture the Problem The graph is one of constant negative acceleration. Because $v_x = v(t)$ is a linear function of t, we can make use of the slope-intercept form of the equation of a straight line to find the relationship between these variables. We can then differentiate $v(t)$ to obtain $a(t)$ and integrate $v(t)$ to obtain $x(t)$.

Find the acceleration (the slope of the graph) and the velocity at time 0 (the v-intercept) and use the slope-intercept form of the equation of a straight line to express $v_x(t)$:

$$\boxed{a = -10\,\text{m/s}^2}$$

$$\boxed{v_x(t) = 50\,\text{m/s} + (-10\,\text{m/s}^2)t}$$

Find $x(t)$ by integrating $v(t)$:

$$x(t) = \int \left[\left(-10\,\text{m/s}^2\right)t + 50\,\text{m/s}\right]dt$$
$$= (50\,\text{m/s})t - \left(5\,\text{m/s}^2\right)t^2 + \text{C}$$

Using the fact that $x = 0$ when $t = 0$, evaluate C:

$$0 = (50\,\text{m/s})(0) - \left(5\,\text{m/s}^2\right)(0)^2 + \text{C}$$
and
C = 0

Substitute to obtain:

$$\boxed{x(t) = (50\,\text{m/s})t - \left(5\,\text{m/s}^2\right)t^2}$$

Note that this expression is quadratic in t and that the coefficient of t^2 is negative and equal in magnitude to half the constant acceleration.

Remarks: We can check our result for $x(t)$ by evaluating it over the 10-s interval shown and comparing this result with the area bounded by this curve and the time axis.

119 •••

Picture the Problem During any time interval, the integral of $a(t)$ is the change in velocity and the integral of $v(t)$ is the displacement. The integral of a function equals the "area" between the curve for that function and the independent-variable axis.

(a) Find the area of the shaded grid box in Figure 2-37:

Area = $(0.5\ \text{m/s}^2)(0.5\ \text{s})$
$= \boxed{0.250\ \text{m/s per box}}$

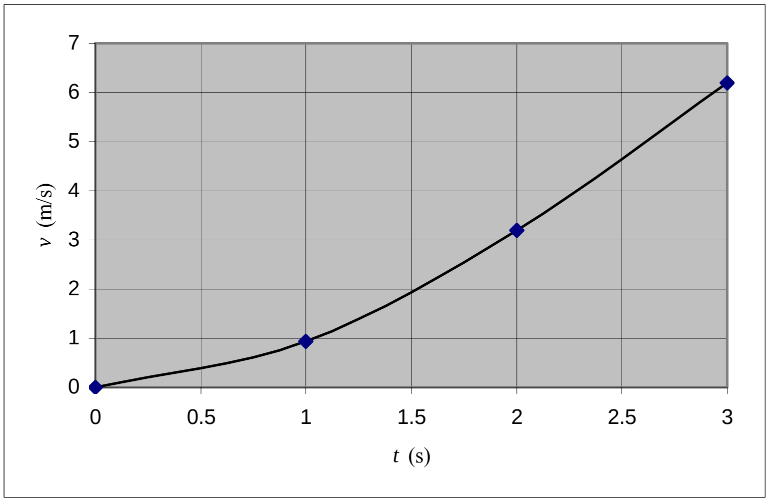
(b) We start from rest ($v_o = 0$) at $t = 0$. For the velocities at the other times, count boxes and multiply by the 0.25 m/s per box that we found in part (a):

Examples:
$v(1\ \text{s})$ = (3.7 boxes)[(0.25 m/s)/box]
$= \boxed{0.925\ \text{m/s}}$
$v(2\ \text{s})$ = (12.9 boxes)[(0.25 m/s)/box]
$= \boxed{3.22\ \text{m/s}}$

and

$v(3 \text{ s}) = (24.6 \text{ boxes})[(0.25 \text{ m/s})/\text{box}]$
$= \boxed{6.15 \text{ m/s}}$

(*c*) The graph of *v* as a function of *t* is shown below:



$Area = (1.0 \text{ m/s})(1.0 \text{ s}) = 1.0 \text{ m per box}$

Count the boxes under the $v(t)$ curve to find the distance traveled:

$$\begin{aligned} x(3\,\text{s}) &= \Delta x(0 \to 3\,\text{s}) \\ &= (7\,\text{boxes})[(1.0\,\text{m})/\,\text{box}] \\ &= \boxed{7.00\,\text{m}} \end{aligned}$$

120 ••

Picture the Problem The integral of $v(t)$ over a time interval is the displacement (change in position) during that time interval. The integral of a function equals the "area" between the curve for that function and the independent-variable axis. Because acceleration is the slope of a velocity versus time curve, this is a non-constant-acceleration problem. The derivative of a function is equal to the "slope" of the function at that value of the independent variable.

(*a*) To obtain the data for $x(t)$, we must estimate the accumulated area under the $v(t)$ curve at each time interval:

Find the area of a shaded grid box in Figure 2-38:

$A = (1 \text{ m/s})(0.5 \text{ s}) = 0.5 \text{ m per box.}$

We start from rest ($v_o = 0$) at $t_o = 0$. For the position at the other times, count boxes and multiply by the 0.5 m per box that we found above. Remember to add the offset from the origin, $x_o = 5$ m, and that boxes below the $v = 0$ line are counted as negative:

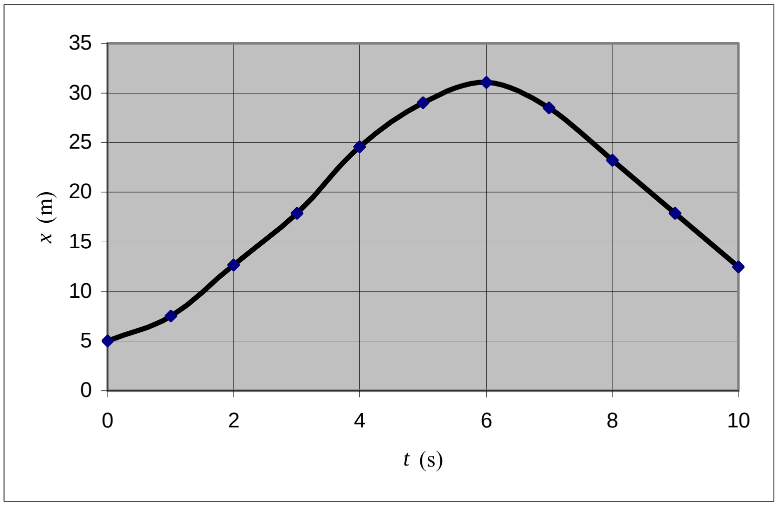
Examples:

$$\begin{aligned} x(3\,\text{s}) &= (25.8 \text{ boxes})\left(\frac{0.5\,\text{m}}{\text{box}}\right) + 5\,\text{m} \\ &= 17.9\,\text{m} \end{aligned}$$

$$\begin{aligned} x(5\,\text{s}) &= (48.0\,\text{boxes})\left(\frac{0.5\,\text{m}}{\text{box}}\right) + 5\,\text{m} \\ &= 29.0\,\text{m} \end{aligned}$$

$$x(10\,\text{s}) = (51.0\,\text{boxes})\left(\frac{0.5\,\text{m}}{\text{box}}\right) - (36.0\,\text{boxes})\left(\frac{0.5\,\text{m}}{\text{box}}\right) + 5\,\text{m}$$
$$= 12.5\,\text{m}$$

A graph of *x* as a function of *t* follows:

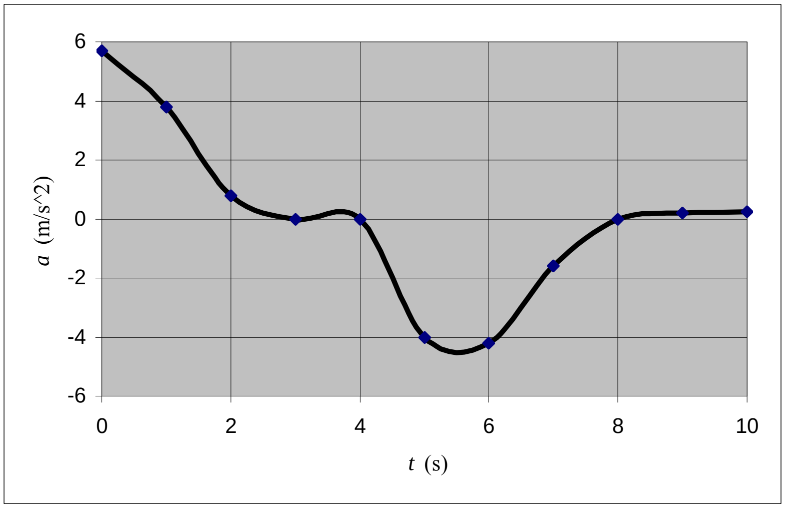


(*b*) To obtain the data for $a(t)$, we must estimate the slope ($\Delta v/\Delta t$) of the $v(t)$ curve at each time. A good way to get reasonably reliable readings from the graph is to enlarge several fold:

Examples:

$$a(1\,\text{s}) = \frac{v(1.25\,\text{s}) - v(0.75\,\text{s})}{0.5\,\text{s}}$$
$$= \frac{4.9\,\text{m/s} - 3.0\,\text{m/s}}{0.5\,\text{s}} = 3.8\,\text{m/s}^2$$
$$a(6\,\text{s}) = \frac{v(6.25\,\text{s}) - v(5.75\,\text{s})}{0.5\,\text{s}}$$
$$= \frac{-1.7\,\text{m/s} - 0.4\,\text{m/s}}{0.5\,\text{s}} = -4.2\,\text{m/s}^2$$

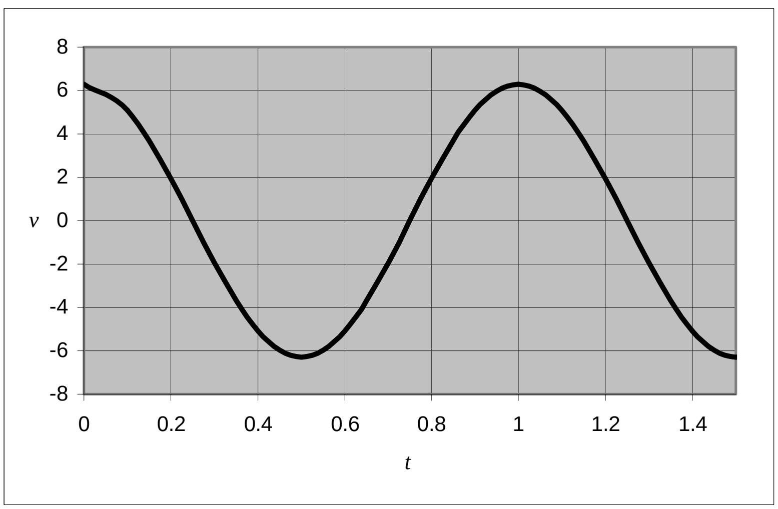
A graph of a as a function of t follows:



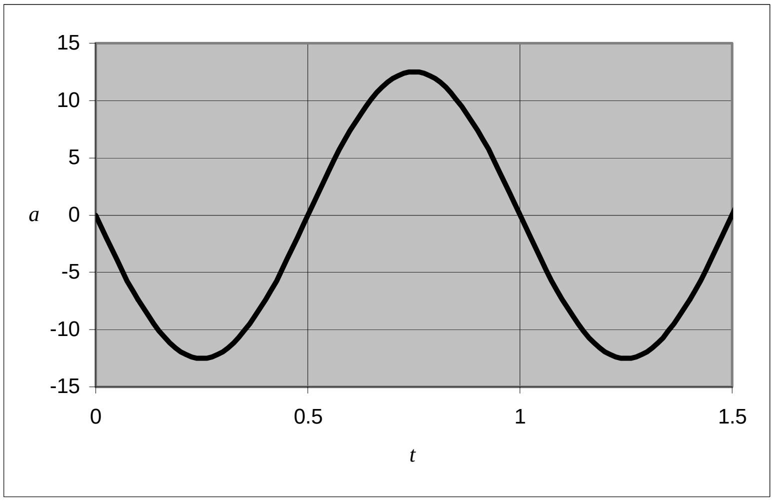
***121** ••

Picture the Problem Because the position of the body is not described by a parabolic function, the acceleration is not constant.

Select a series of points on the graph of $x(t)$ (e.g., at the extreme values and where the graph crosses the t axis), draw tangent lines at those points, and measure their slopes. In doing this, you are evaluating $v = dx/dt$ at these points. Plot these slopes above the times at which you measured the slopes. Your graph should closely resemble the following graph.



Select a series of points on the graph of $v(t)$ (e.g., at the extreme values and where the graph crosses the t axis), draw tangent lines at those points, and measure their slopes. In doing this, you are evaluating $a = dv/dt$ at these points. Plot these slopes above the times at which you measured the slopes. Your graph should closely resemble the graph shown below.



122 ••

Picture the Problem Because the acceleration of the rocket varies with time, it is not constant and integration of this function is required to determine the rocket's velocity and position as functions of time. The conditions on x and v at $t = 0$ are known as ***initial conditions***.

(*a*) Integrate $a(t)$ to find $v(t)$:

$$v(t) = \int a(t)\,dt = b\int t\,dt = \tfrac{1}{2}bt^2 + \mathrm{C}$$

where C, the constant of integration, can be determined from the initial conditions.

Integrate $v(t)$ to find $x(t)$:

$$x(t) = \int v(t)dt = \int\left[\tfrac{1}{2}bt^2 + \mathrm{C}\right]dt$$

$$= \tfrac{1}{6}bt^3 + \mathrm{C}t + \mathrm{D}$$

where D is a second constant of integration.

Using the initial conditions, find the constants C and D:

$$v(0) = 0 \;\Rightarrow\; \mathrm{C} = 0$$

and

$$x(0) = 0 \;\Rightarrow\; \mathrm{D} = 0$$

$$\therefore \boxed{x(t) = \tfrac{1}{6}bt^3}$$

(*b*) Evaluate $v(5\text{ s})$ and $x(5\text{ s})$ with $\mathrm{C} = \mathrm{D} = 0$ and $b = 3\text{ m/s}^2$:

$$v(5\,\mathrm{s}) = \frac{1}{2}\left(3\,\mathrm{m/s^2}\right)\left(5\,\mathrm{s}\right)^2 = \boxed{37.5\,\mathrm{m/s}}$$

and

$$x(5\,\mathrm{s}) = \frac{1}{6}\left(3\,\mathrm{m/s^2}\right)\left(5\,\mathrm{s}\right)^3 = \boxed{62.5\,\mathrm{m}}$$

123 ••

Picture the Problem The acceleration is a function of time; therefore it is not constant. The instantaneous velocity can be determined by integration of the acceleration and the average velocity from the displacement of the particle during the given time interval.

(*a*) Because the acceleration is the derivative of the velocity, integrate the acceleration to find the ***instantaneous velocity*** $v(t)$.

$$a(t)=\frac{dv}{dt}\ \Rightarrow\ v(t)=\int_{v_0=0}^{v(t)}dv'=\int_{t_0=0}^{t}a(t')dt'$$

Calculate the instantaneous velocity using the acceleration given.

$$v(t)=\left(0.2\,\text{m/s}^3\right)\int_{t_0=0}^{t}t'dt'$$

and

$$\boxed{v(t)=\left(0.1\,\text{m/s}^3\right)t^2}$$

(*b*) To calculate the ***average velocity***, we need the displacement:

$$v(t)\equiv\frac{dx}{dt}\Rightarrow x(t)=\int_{x_0=0}^{x(t)}dx'=\int_{t_0=0}^{t}v(t')dt'$$

Because the velocity is the derivative of the displacement, integrate the velocity to find Δx.

$$x(t)=\left(0.1\,\text{m/s}^3\right)\int_{t_0=0}^{t}t'^2\,dt'=\left(0.1\,\text{m/s}^3\right)\frac{t^3}{3}$$

and

$$\begin{aligned}\Delta x &= x(7\text{ s})-x(2\text{ s})\\ &=\left(0.1\,\text{m/s}^3\right)\left[\frac{(7\text{ s})^3-(2\text{ s})^3}{3}\right]\\ &=11.2\text{ m}\end{aligned}$$

Using the definition of the ***average velocity***, calculate v_{av}.

$$v_{\text{av}}=\frac{\Delta x}{\Delta t}=\frac{11.2\text{ m}}{5\text{ s}}=\boxed{2.23\text{ m/s}}$$

124 •

Determine the Concept Because the acceleration is a function of time, it is not constant. Hence we'll need to integrate the acceleration function to find the velocity as a function of time and integrate the velocity function to find the position as a function of time. The important concepts here are the definitions of velocity, acceleration, and average velocity.

(*a*) Starting from $t_o = 0$, integrate the instantaneous acceleration to obtain the instantaneous velocity as a function of time:

From $a=\dfrac{dv}{dt}$

it follows that

$$\int_{v_0}^{v}dv'=\int_0^t(a_0+bt')dt'$$

and

$$\boxed{v=v_0+a_0t+\tfrac{1}{2}bt^2}$$

(*b*) Now integrate the instantaneous velocity to obtain the position as a function of time:

From $v=\dfrac{dx}{dt}$

it follows that

$$\int_{x_0}^{x} dx' = \int_{t_0=0}^{t} v(t')dt'$$

$$= \int_{t_0}^{t}\left(v_9 + a_0 t' + \frac{b}{2}t'^2\right)dt'$$

and

$$\boxed{x = x_0 + v_0 t + \tfrac{1}{2}a_0 t^2 + \tfrac{1}{6}bt^3}$$

(*c*) The definition of the average velocity is the ratio of the displacement to the total time elapsed:

$$v_{\text{av}} \equiv \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - t_0} = \frac{v_0 t + \frac{1}{2}a_0 t^2 + \frac{1}{6}bt^3}{t}$$

and

$$\boxed{v_{\text{av}} = v_0 + \tfrac{1}{2}a_0 t + \tfrac{1}{6}bt^2}$$

Note that v_{av} is not the same as that due to constant acceleration:

$$\left(v_{\text{constant acceleration}}\right)_{\text{av}} = \frac{v_0 + v}{2}$$

$$= \frac{v_0 + \left(v_0 + a_0 t + \frac{1}{2}bt^2\right)}{2}$$

$$= v_0 + \tfrac{1}{2}a_0 t + \tfrac{1}{4}bt^2$$

$$\neq v_{\text{av}}$$

General Problems

125 •••

Picture the Problem The acceleration of the marble is constant. Because the motion is downward, choose a coordinate system with downward as the positive direction. The equation $g_{\text{exp}} = (1\text{ m})/(\Delta t)^2$ originates in the constant-acceleration equation $\Delta x = v_0 \Delta t + \frac{1}{2}a(\Delta t)^2$. Because the motion starts from rest, the displacement of the marble is 1 m, the acceleration is the experimental value g_{exp}, and the equation simplifies to $g_{\text{exp}} = (1\text{ m})/(\Delta t)^2$.

Express the percent difference between the accepted and experimental values for the acceleration due to gravity:

$$\%\text{ difference} = \frac{\left|g_{\text{accepted}} - g_{\text{exp}}\right|}{g_{\text{accepted}}}$$

Using a constant-acceleration equation, express the velocity of the marble in terms of its initial velocity, acceleration, and displacement:

$$v_{\text{f}}^2 = v_0^2 + 2a\Delta y$$

or, because $v_0 = 0$ and $a = g$,

$$v_{\text{f}}^2 = 2g\Delta y$$

Solve for v_{f}:

$$v_{\text{f}} = \sqrt{2g\Delta y}$$

Let v_1 be the velocity the ball has reached when it has fallen 0.5 cm,

$$v_1 = \sqrt{2\left(9.81\text{ m/s}^2\right)\left(0.005\text{ m}\right)} = 0.313\text{ m/s}$$

and v_2 be the velocity the ball has reached when it has fallen 0.5 m to obtain.

and

$$v_2 = \sqrt{2(9.81\,\text{m/s}^2)(0.5\,\text{m})} = 3.13\,\text{m/s}$$

Using a constant-acceleration equation, express v_2 in terms of v_1, g and Δt:

$$v_2 = v_1 + g\Delta t$$

Solve for Δt:

$$\Delta t = \frac{v_2 - v_1}{g}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{3.13\,\text{m/s} - 0.313\,\text{m/s}}{9.81\,\text{m/s}^2} = 0.2872\,\text{s}$$

Calculate the experimental value of the acceleration due to gravity from $g_{\text{exp}} = (1\text{ m})/(\Delta t)^2$:

$$g_{\text{exp}} = \frac{1\,\text{m}}{(0.2872\,\text{s})^2} = \boxed{12.13\,\text{m/s}^2}$$

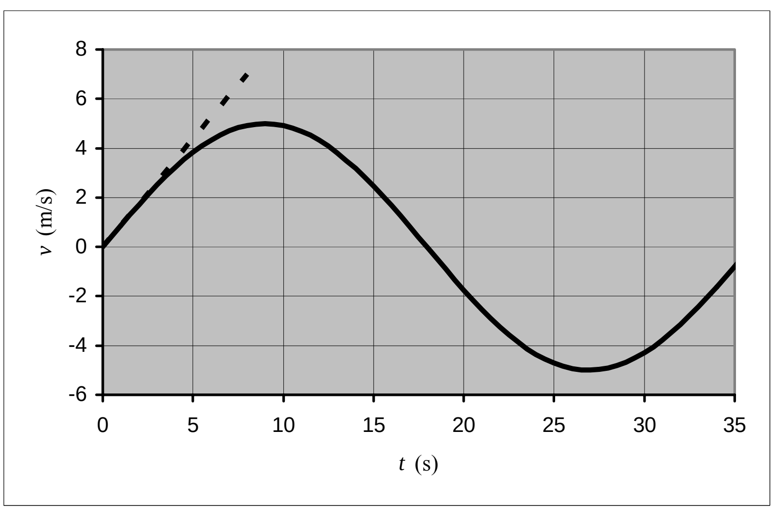
Finally, calculate the percent difference between this experimental result and the value accepted for g at sea level.

$$\%\text{ difference} = \frac{\left|9.81\,\text{m/s}^2 - 12.13\,\text{m/s}^2\right|}{9.81\,\text{m/s}^2}$$
$$= \boxed{23.6\,\%}$$

***126 •••**

Picture the Problem We can obtain an average velocity, $v_{\text{av}} = \Delta x/\Delta t$, over fixed time intervals. The instantaneous velocity, $v = dx/dt$ can only be obtained by differentiation.

(*a*) The graph of *x* versus *t* is shown below:



(*b*) Draw a tangent line at the origin and measure its rise and run. Use this ratio to obtain an approximate value for the slope at the origin:

The tangent line appears to, at least approximately, pass through the point (5, 4). Using the origin as the second point,

$$\Delta x = 4\text{ cm} - 0 = 4\text{ cm}$$

and

$$\Delta t = 5\text{ s} - 0 = 5\text{ s}$$

Therefore, the slope of the tangent line and the velocity of the body as it passes through the origin is approximately:

$$v(0) = \frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t} = \frac{4\,\text{cm}}{5\,\text{s}} = \boxed{0.800\,\text{cm/s}}$$

(*c*) Calculate the average velocity for the series of time intervals given by completing the table shown below:

t_0	t	Δt	x_0	x	Δx	$v_{av}=\Delta x/\Delta t$
(s)	(s)	(s)	(cm)	(cm)	(cm)	(m/s)
0	6	6	0	4.34	4.34	0.723
0	3	3	0	2.51	2.51	0.835
0	2	2	0	1.71	1.71	0.857
0	1	1	0	0.871	0.871	0.871
0	0.5	0.5	0	0.437	0.437	0.874
0	0.25	0.25	0	0.219	0.219	0.875

(*d*) Express the time derivative of the position:

$$\frac{dx}{dt} = A\omega\cos\omega t$$

Substitute numerical values and evaluate $\frac{dx}{dt}$ at $t = 0$:

$$\frac{dx}{dt} = A\omega\cos 0 = A\omega$$

$$= (0.05\,\text{m})(0.175\,\text{s}^{-1})$$

$$= \boxed{0.875\,\text{cm/s}}$$

(*e*) Compare the average velocities from part (*c*) with the instantaneous velocity from part (*d*):

As Δt, and thus Δx, becomes small, the value for the average velocity approaches that for the instantaneous velocity obtained in part (*d*). For $\Delta t = 0.25$ s, they agree to three significant figures.

127 •••

Determine the Concept Because the velocity varies nonlinearly with time, the acceleration of the object is not constant. We can find the acceleration of the object by differentiating its velocity with respect to time and its position function by integrating the velocity function. The important concepts here are the definitions of acceleration and velocity.

(*a*) The acceleration of the object is the derivative of its velocity with respect to time:

$$a = \frac{dv}{dt} = \frac{d}{dt}\left[v_{\max}\sin(\omega t)\right]$$

$$= \boxed{\omega v_{\max}\cos(\omega t)}$$

$$\boxed{\text{Because } a \text{ varies sinusoidally with time it is } \textit{not} \text{ constant.}}$$

(*b*) Integrate the velocity with respect to time from 0 to *t* to obtain the change in position of the body:

$$\int_{x_0}^{x} dx' = \int_{t_0}^{t} [v_{max}\sin(\omega t')]dt'$$

and

$$x - x_0 = \left[\frac{-v_{max}}{\omega}\cos(\omega t')\right]_0^t$$

$$= \frac{-v_{max}}{\omega}\cos(\omega t) + \frac{v_{max}}{\omega}$$

or

$$\boxed{x = x_0 + \frac{v_{max}}{\omega}[1-\cos(\omega t)]}$$

Note that, as given in the problem statement, $x(0\text{ s}) = x_0$.

128 •••

Picture the Problem Because the acceleration of the particle is a function of its position, it is not constant. Changing the variable of integration in the definition of acceleration will allow us to determine its velocity and position as functions of position.

(*a*) Because $a = dv/dt$, we must integrate to find $v(t)$. Because *a* is given as a function of *x*, we'll need to change variables in order to carry out the integration. Once we've changed variables, we'll separate them with *v* on the left side of the equation and *x* on the right:

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx} = (2\text{ s}^{-2})x$$

or, upon separating variables,

$$vdv = (2\text{ s}^{-2})xdx$$

Integrate from x_o and v_o to *x* and *v*:

$$\int_{v_0=0}^{v} v'dv' = \int_{x_0}^{x} (2\text{ s}^{-2})x'dx'$$

and

$$v^2 - v_0^2 = (2\text{ s}^{-2})(x^2 - x_0^2)$$

Solve for *v* to obtain:

$$v = \sqrt{v_0^2 + (2\text{ s}^{-2})(x^2 - x_0^2)}$$

Now set $v_o = 0$, $x_o = 1$ m, $x = 3$ m, $b = 2\text{ s}^{-2}$ and evaluate the speed:

$$v = \pm\sqrt{(2\text{ s}^{-2})[(3\text{ m})^2 - (1\text{ m})^2]}$$

and

$$|v| = \boxed{4.00\text{m/s}}$$

(*b*) Using the definition of *v*, separate the variables, and integrate to get an expression for *t*:

$$v(x)=\frac{dx}{dt}$$

and

$$\int_0^t dt'=\int_{x_0}^{x}\frac{dx'}{v(x')}$$

To evaluate this integral we first must find $v(x)$. Show that the acceleration is always positive and use this to find the sign of $v(x)$.

$a = (2\ \text{s}^{-2})x$ and $x_0 = 1$ m. x_0 is positive, so a_0 is also positive. v_0 is zero and a_0 is positive, so the object moves in the direction of increasing *x*. As *x* increases the acceleration remains positive, so the velocity also remains positive. Thus,

$$v=\sqrt{\left(2\,\text{s}^{-2}\right)\left(x^2-x_0^2\right)}.$$

Substitute $\sqrt{\left(2\,\text{s}^{-2}\right)\left(x^2-x_0^2\right)}$ for *v* and evaluate the integral. (It can be found in standard integral tables.)

$$t=\int_0^t dt'=\int_{x_0}^{x}\frac{dx'}{v(x')}$$

$$=\int_{x_0}^{x}\frac{dx'}{\sqrt{\left(2\,\text{s}^{-2}\right)\left(x'^2-x_0^2\right)}}$$

$$=\frac{1}{\sqrt{\left(2\,\text{s}^{-2}\right)}}\int_{x_0}^{x}\frac{dx'}{\sqrt{x'^2-x_0^2}}$$

$$=\frac{1}{\sqrt{\left(2\,\text{s}^{-2}\right)}}\ln\left(\frac{x+\sqrt{x^2-x_0^2}}{x_0}\right)$$

Evaluate this expression with $x_o = 1$ m and $x = 3$ m to obtain:

$$t=\boxed{1.25\,\text{s}}$$

129 •••

Picture the Problem The acceleration of this particle is not constant. Separating variables and integrating will allow us to express the particle's position as a function of time and the differentiation of this expression will give us the acceleration of the particle as a function of time.

(*a*) Write the definition of velocity:

$$v=\frac{dx}{dt}$$

We are given that $x = bv$, where $b = 1$ s. Substitute for *v* and separate variables to obtain:

$$\frac{dx}{dt}=\frac{x}{b}\Rightarrow dt=b\frac{dx}{x}$$

Integrate and solve for $x(t)$:

$$\int_{t_0}^{t}dt'=b\int_{x_0}^{x}\frac{dx'}{x'}\Rightarrow\left(t-t_0\right)=b\ln\left(\frac{x}{x_0}\right)$$

and

$$x(t) = \boxed{x_0 e^{(t-t_0)/b}}$$

(*b*) Differentiate twice to obtain *v*(*t*) and *a*(*t*):

$$v = \frac{dx}{dt} = \frac{1}{b} x_0 e^{(t-t_0)/b}$$

and

$$a = \frac{dv}{dt} = \frac{1}{b^2} x_0 e^{(t-t_0)/b}$$

Substitute the result in part (*a*) to obtain the desired results:

$$v(t) = \frac{1}{b} x(t)$$

and

$$a(t) = \frac{1}{b^2} x(t)$$

so

$$a(t) = \frac{1}{b} v(t) = \frac{1}{b^2} x(t)$$

$$\boxed{\begin{array}{l}\text{Because the numerical value of } b\text{, expressed in SI units, is one, the}\\ \text{numerical values of } a, v, \text{and } x \text{ are the same at each instant in time.}\end{array}}$$

130 •••

Picture the Problem Because the acceleration of the rock is a function of time, it is not constant. Choose a coordinate system in which downward is positive and the origin at the point of release of the rock.

Separate variables in $a(t) = dv/dt = ge^{-bt}$ to obtain:

$$dv = ge^{-bt} dt$$

Integrate from $t_o = 0$, $v_o = 0$ to some later time *t* and velocity *v*:

$$v = \int_0^v dv' = \int_0^t ge^{-bt'} dt' = \frac{g}{-b}\left[e^{-bt'}\right]_0^t$$

$$= \frac{g}{b}\left(1 - e^{-bt}\right) = v_{\text{term}}\left(1 - e^{-bt}\right)$$

where

$$v_{\text{term}} = \frac{g}{b}$$

Separate variables in $v = dy/dt = v_{\text{term}}\left(1 - e^{-bt}\right)$ to obtain:

$$dy = v_{\text{term}}\left(1 - e^{-bt}\right) dt$$

Integrate from $t_o = 0$, $y_o = 0$ to some later time *t* and position *y*:

$$\int_0^y dy' = \int_0^t v_{\text{term}}\left(1 - e^{-bt'}\right) dt'$$

$$y = v_{\text{term}}\left[t' + \frac{1}{b}e^{-bt'}\right]_0^t$$

$$= \boxed{v_{\text{term}}t - \frac{v_{\text{term}}}{b}\left(1 - e^{-bt}\right)}$$

This last result is very interesting. It says that throughout its free-fall, the object experiences drag; therefore it has not fallen as far at any given time as it would have if it were falling at the constant velocity, v_{term}.

On the other hand, just as the velocity of the object asymptotically approaches v_{term}, the distance it has covered during its free-fall as a function of time asymptotically approaches the distance it would have fallen if it had fallen with v_{term} throughout its motion.

$$y(t_{\text{large}}) \to v_{\text{term}}t - \frac{v}{b} \to v_{\text{term}}t$$

This should not be surprising because in the expression above, the first term grows linearly with time while the second term approaches a constant and therefore becomes less important with time.

***131** •••

Picture the Problem Because the acceleration of the rock is a function of its velocity, it is not constant. Choose a coordinate system in which downward is positive and the origin is at the point of release of the rock.

Rewrite $a = g - bv$ explicitly as a differential equation:

$$\frac{dv}{dt} = g - bv$$

Separate the variables, v on the left, t on the right:

$$\frac{dv}{g - bv} = dt$$

Integrate the left-hand side of this equation from 0 to v and the right-hand side from 0 to t:

$$\int_0^v \frac{dv'}{g - bv'} = \int_0^t dt'$$

and

$$-\frac{1}{b}\ln\left(\frac{g - bv}{g}\right) = t$$

Solve this expression for v.

$$v = \frac{g}{b}\left(1 - e^{-bt}\right)$$

Finally, differentiate this expression with respect to time to obtain an expression for the acceleration and

$$a = \frac{dv}{dt} = \boxed{ge^{-bt}}$$

complete the proof.

132 •••

Picture the Problem The skydiver's acceleration is a function of her velocity; therefore it is not constant. Expressing her acceleration as the derivative of her velocity, separating the variables, and then integrating will give her velocity as a function of time.

(*a*) Rewrite $a = g - cv^2$ explicitly as a differential equation:

$$\frac{dv}{dt} = g - cv^2$$

Separate the variables, with v on the left, and t on the right:

$$\frac{dv}{g - cv^2} = dt$$

Eliminate c by using $c = \dfrac{g}{v_T^2}$:

$$\frac{dv}{g - \dfrac{g}{v_T^2}v^2} = \frac{dv}{g\left[1 - \left(\dfrac{v}{v_T}\right)^2\right]} = dt$$

or

$$\frac{dv}{1 - \left(\dfrac{v}{v_T}\right)^2} = gdt$$

Integrate the left-hand side of this equation from 0 to v and the right-hand side from 0 to t:

$$\int_0^v \frac{dv'}{1 - \left(\dfrac{v'}{v_T}\right)^2} = g\int_0^t dt' = gt$$

The integral can be found in integral tables:

$$v_T \tanh^{-1}(v/v_T) = gt$$

or

$$\tanh^{-1}(v/v_T) = (g/v_T)t$$

Solve this equation for v to obtain:

$$v = v_T \tanh\left(\frac{g}{v_T}\right)t$$

Because c has units of m^{-1}, and g has units of m/s^2, $(cg)^{-1/2}$ will have units of time. Let's represent this expression with the time-scale factor T:

i.e., $T = (cg)^{-1/2}$

The skydiver falls with her terminal velocity when $a = 0$. Using this definition, relate her terminal velocity to the acceleration due to gravity and the constant c in the acceleration equation:

$$0 = g - cv_T^2$$

and

$$v_T = \sqrt{\frac{g}{c}}$$

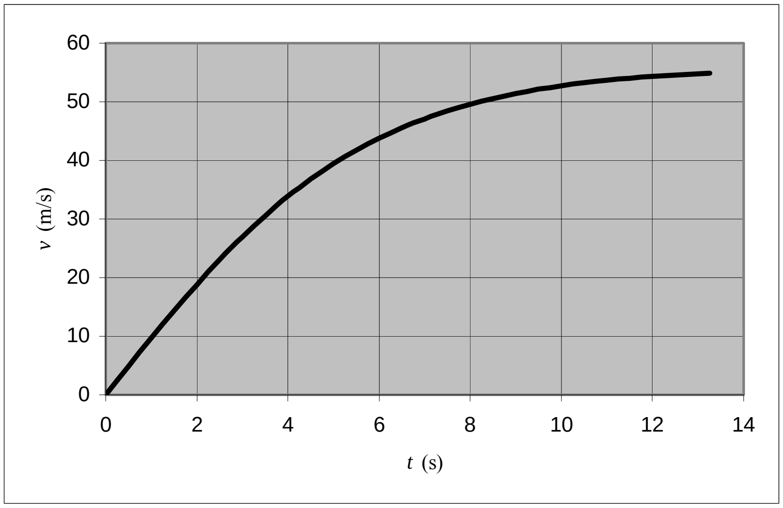
Convince yourself that T is also equal to v_T/g and use this relationship to eliminate g and v_T in the solution to the differential equation:

$$v(t) = \boxed{v_T \tanh\left(\frac{t}{T}\right)}$$

(*b*) The following table was generated using a spreadsheet and the equation we derived in part (*a*) for $v(t)$. The cell formulas and their algebraic forms are:

Cell	Content/Formula	Algebraic Form
D1	56	v_T
D2	5.71	T
B7	B6 + 0.25	$t + 0.25$
C7	B1*TANH(B7/B2)	$v_T \tanh\left(\frac{t}{T}\right)$

	A	B	C	D	E
1	vT =	56	m/s		
2	T=	5.71	s		
3					
4					
5		time (s)	*v* (m/s)		
6		0.00	0.00		
7		0.25	2.45		
8		0.50	4.89		
9		0.75	7.32		
10		1.00	9.71		
54		12.00	54.35		
55		12.25	54.49		
56		12.50	54.61		
57		12.75	54.73		
58		13.00	54.83		
59		13.25	54.93		



Note that the velocity increases linearly over time (i.e., with constant acceleration) for about time *T*, but then it approaches the terminal velocity as the acceleration decreases.