Operational Modal Analysis

or
Modal testing of structures subject to operational excitation forces

Structural Vibration Solutions

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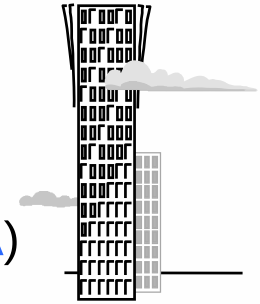
Contents

Operational Modal Analysis – *(OMA)*

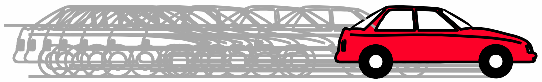
- Determination of Modal Model by response testing only
  - No measurement of input forces required
  - Measurement procedure similar to Operational Deflection Shapes (ODS)

- Determination of Modal Model under operational conditions
  - In-situ testing

- Used successfully in Civil Engineering applications (Ambient Modal)
  - Bridges and buildings
  - Off-shore platforms etc.

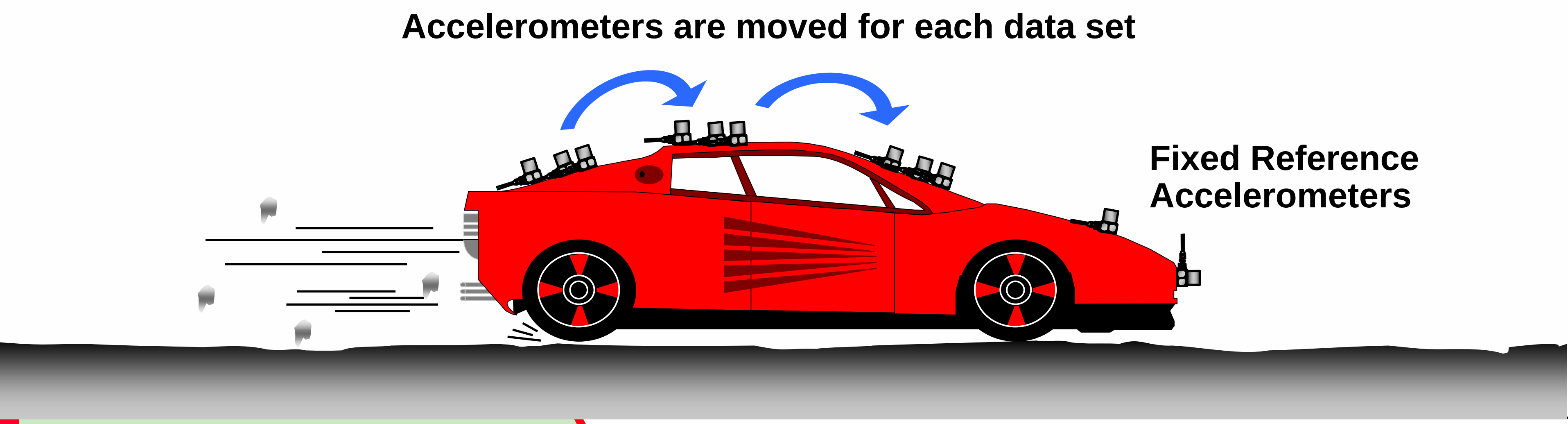


- Now introduced to Mechanical Engineering applications (OMA)
  - Rotating Machinery
  - On-road and in-flight testing etc.



Operational Modal Analysis *(measurement procedure)*

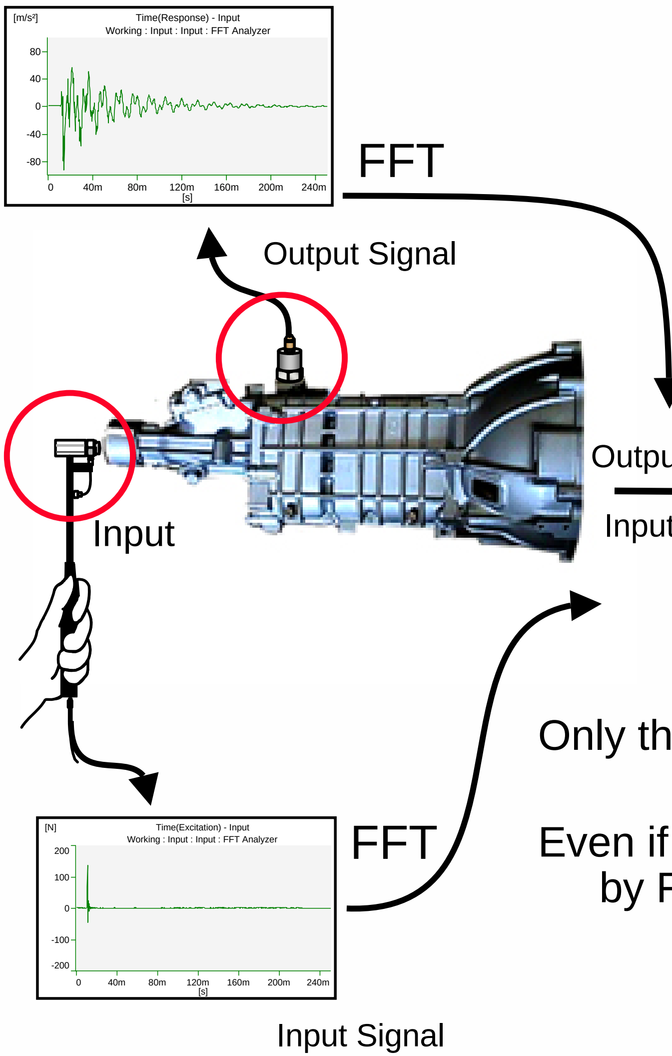
- Determination of Modal parameters based on natural excitation
- Measurement of reponses in a number of DOF's
  - simultaneously
  - by roving accelerometers with one or more fixed accelerometers as references



Mobility Measurements (*Traditional Modal Testing*)

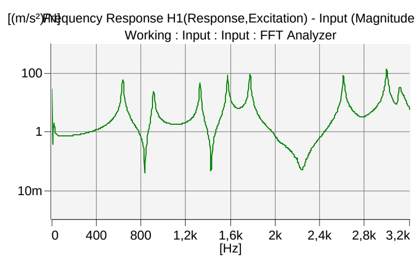
- Input Force is measured
- Output Response is measured
- Output is related to Input by FRF estimators
- FRF is independent of the input force

• Input/Output modal analysis



Output Spectrum

Input Spectrum



Structural System

Only the Modes of the System need to be identified

Even if the system is producing noise this is handled by FRF estimators as H_1, H_2 , H_3

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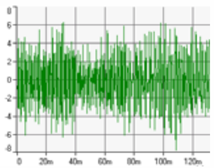
Operational Modal – *(The Combined Model)*

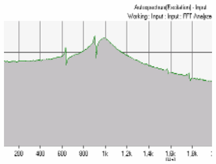
Rotating parts creates Harmonic vibrations



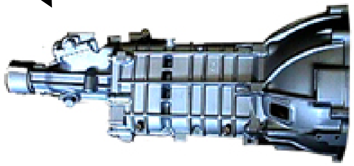
Computational Noise

Rotating Parts

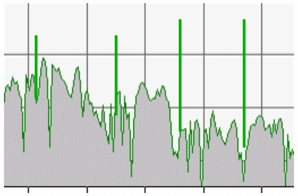
- Output only modal analysis



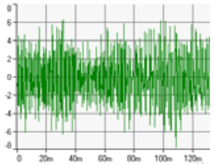
Force Spectrum



Structural System



Combined Spectrum



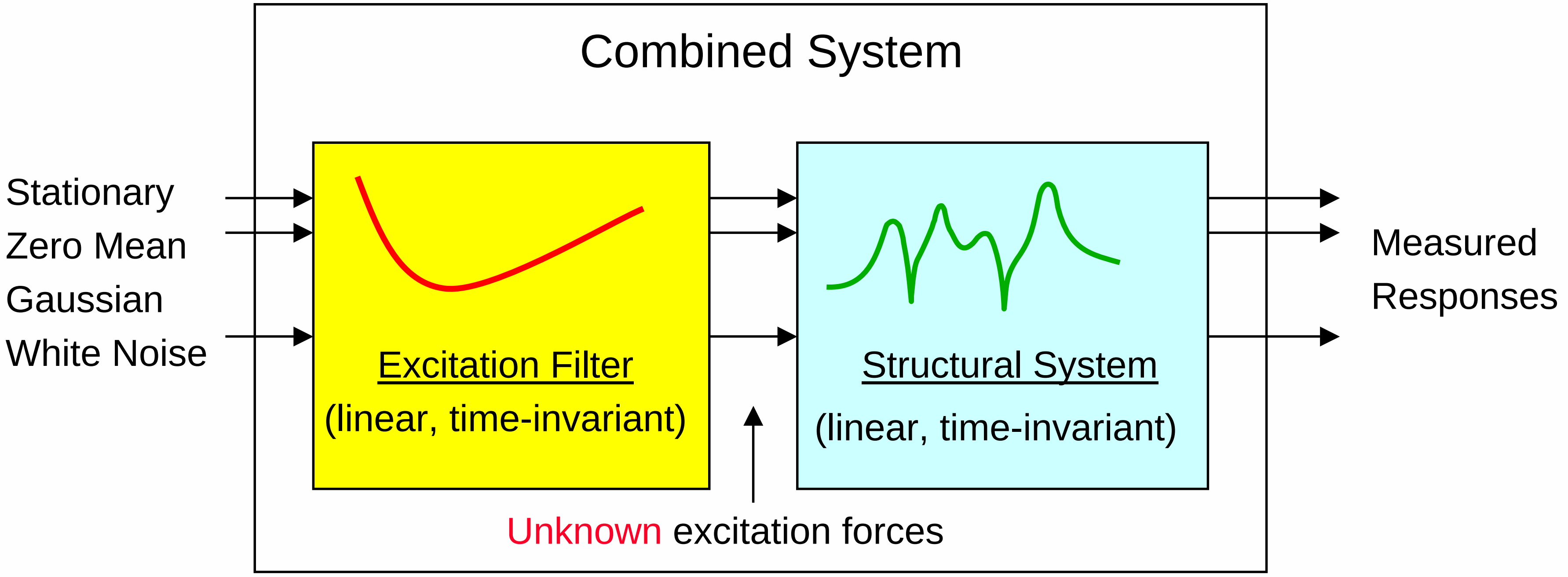
Measurement Noise

The “Modes” in the combined spectrum contains information of

- The system under test (Physical Modes)
- Input Force (Non-physical “Modes”)
- Noise (Non-physical “Modes”)
- Harmonics (Non-physical “Modes”)

Combined System Model *(analysis procedure)*

Model of the combined system is estimated from measured responses



Modal Model of Structural System
extracted from estimated model of Combined System

Assumptions

Mathematical

- Stationary input force signals can be approximated by filtered zero mean Gaussian white noise
  - Signals are completely described by their correlation functions or auto- & cross-spectra
  - Synthesized correlation functions or auto- & cross-spectra are similar to those obtained from experimental data

Practical

- Broadband excitation
- All modes must be excited

Does not mean that the physical excitation has to be white noise

Contents

Frequency Domain Decomposition (FDD)

Determination of complete Modal Model from Responses only

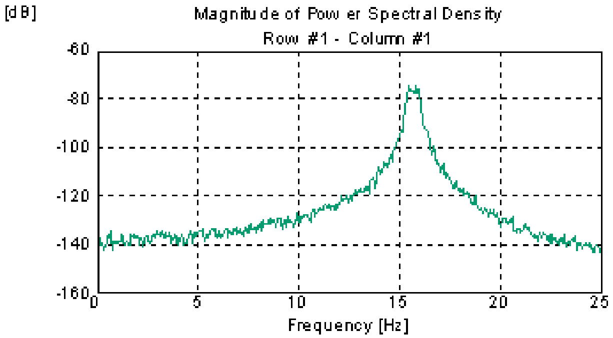
FDD procedure:

1. Power Spectral Density (PSD) matrix estimation
2. Singular Value Decomposition (SVD) of PSD matrix
3. Identification of Single Degree of Freedom (SDOF) models from SVD
4. Modal Parameter identification from SDOF models

Frequency Domain Decomposition (FDD)

Extracting Modal Parameters from PSD response matrix

- A number of modes can often not be found by simple peak-picking
- Modes may be coupled by small frequency difference or by high damping
- The number of modes equals the number of terms in the linear decomposition in the modal transformation
- The number of terms is the rank of the PSD matrix



$$G_{yy}(j\omega) = \sum_k \frac{d_k \phi_k \phi_k^T}{j\omega - \lambda_k} + \frac{\bar{d}_k \bar{\phi}_k \bar{\phi}_k^T}{j\omega - \bar{\lambda}_k}$$

The spectra can be used for Operational Deflection Shapes but do not contain modal information!

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Frequency Domain Decomposition (FDD)

Singular **V**alue **D**ecomposition of Hermitian matrices

$$[A] = [V]\,[S]\,[V]^H = s_1\, \boldsymbol{v_1}\, \boldsymbol{v_1}^{\boldsymbol{H}} + s_2\, \boldsymbol{v}_2\, \boldsymbol{v_2}^{\boldsymbol{H}} + ..$$

The Singular Value Decomposition
of the response matrices is
performed for each frequency

Singular values

$$[S] = \begin{bmatrix} s_1 & 0 & 0 & 0 & . & . & . & 0 \\ 0 & s_2 & 0 & . & . & & & . \\ 0 & 0 & . & . & & & & . \\ 0 & . & . & s_\rho & & & & . \\ . & . & & & 0 & & & . \\ . & & & & & 0 & & . \\ . & & & & & & 0 & . \\ 0 & . & . & . & . & . & . & 0 \end{bmatrix}$$

- A real diagonal matrix
- Number of non-zero elements in the diagonal equals the rank

Singular vectors

$$[V] = [\{v_1\} \quad \{v_2\} \quad \{v_3\} \quad . \quad . \quad . \quad \{v_n\}]$$

- Orthogonal columns
- Unity length columns
- Approximates the Mode shapes

Projection channels

- Reducing redundant information
  - Only a few independent row/columns exist
  - Many row/columns are linear combinations of the others
  - Much unnecessary (redundant) information exist

$$\begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} & \cdot & \cdot & \cdot & G_{1N} \\ G_{21} & G_{22} & G_{23} & \cdot & \cdot & & & \cdot \\ G_{31} & G_{32} & \cdot & \cdot & & & & \cdot \\ G_{41} & \cdot & \cdot & & & & & \cdot \\ \cdot & \cdot & & & & & & \cdot \\ \cdot & & & & & & & \cdot \\ \cdot & & & & & & & \cdot \\ G_{N1} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & G_{NN} \end{bmatrix}$$

- Reduction of linear dependent columns by proper choice of projection channels
  - $[G(:, [p_1\ p_2])]$
  - $[p_1\ p_2]$: projection channels

$$= \begin{bmatrix} G_{1p_1} & G_{1p_2} \\ G_{2p_1} & G_{2p_2} \\ \cdot & \cdot \\ G_{Np_1} & G_{Np_2} \end{bmatrix}$$

Contents

Stochastic Subspace Identification (SSI)

Classes of Identification

- Data Driven: Use of raw time data
- Covariance Driven: Use of Correlation functions

SSI procedure

- Generate compressed input format
  - Select total number of modes (structural, harmonics, noise) based on apriori knowledge
  - Select Identification Class
    - » Unweighted Principal Components (UPC); Principal Components (PC); Canonical Variate Analysis (CVA)
- Estimate Parameters from Stabilization diagram
  - Select interval of model order candidates (use SVD diagram)
  - Estimate models (adjust tolerance criteria)
  - Select the optimal model (use validation)
- Select and link modes across data sets

Discrete SDOF Models

Time Domain Equation of Motion: $m\ddot{x}(t)+c\dot{x}(t)+kx(t)= f(t)$

If ***x*** becomes discrete in time: $\dot{x}=\dfrac{x_t - x_{t-1}}{\Delta t}$

Time Equation of Motion becomes: $x_{t-2}+A_1x_{t-1}+A_2x_t=e_t$

Or formulated as an over-determined problem:

n is the arbitrary chosen order

Auto **R**egres.= **M**oving **A**verage $x_t + A_1x_{t-1} + A_2x_{t-2} + ... + A_nx_{t-n} = e_t + ... + B_ne_{t-n}$

As for the continuos models State Space Models

$$\begin{aligned} x_{t+1} &= Ax_t + w_t \\ y_t &= Cx_t + v_t \end{aligned}$$

Stochastic State Space Model

Stochastic Subspace Identification (SSI)

Modal parameter extraction from SSI

Discrete-time Stochastic State Space Model

$$x_{t+1} = Ax_t + w_t$$
$$y_t = Cx_t + v_t$$

***w**$_t$*: Process noise
***v**$_t$*: Measurement noise

Innovation form

$$\hat{x}_{t+1} = A\hat{x}_t + Ke_t$$
$$y_t = C\hat{x}_t + e_t$$

***e**$_t$*: Innovation (white noise)
K: Kalman gain (noise model)

Modal decomposition

$$z_{t+1} = [\mu_i]z_t + \Psi e_t$$
$$y_t = \Phi z_t + e_t$$

$$A = V[\mu_i]V^{-1}$$
$$z_t = V^{-1}\hat{x}_t$$

μ_i Eigenvalues
Modal frequency
and damping

Φ Left hand mode shapes
Physical Modes

Ψ Right hand mode shapes
Non-physical Modes
Modal distribution of ***e***
Initial modal amplitudes

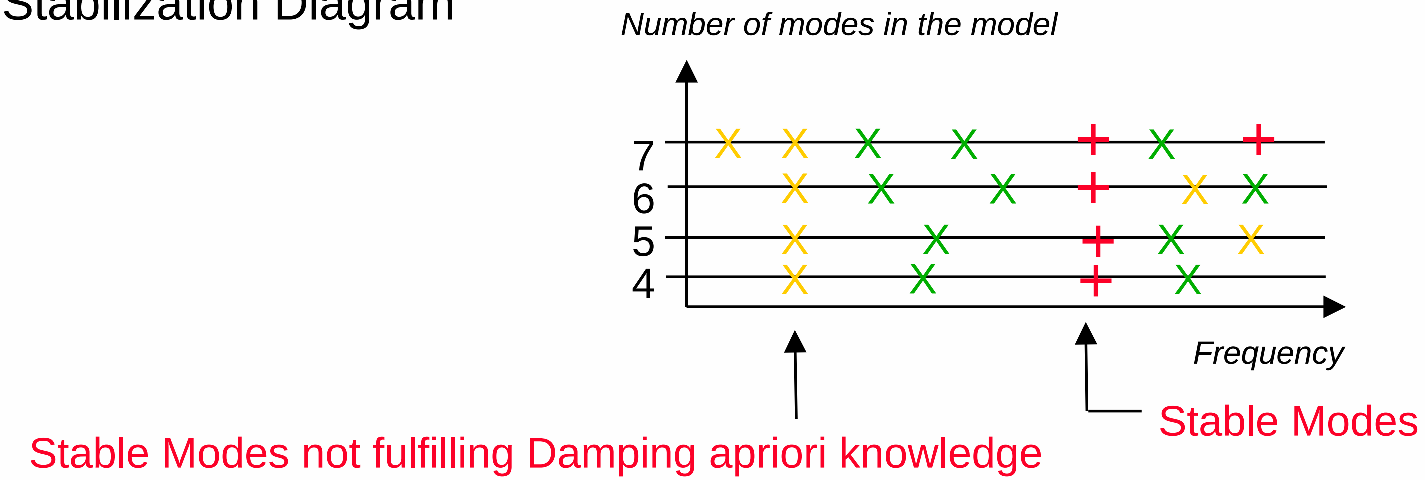
Stochastic Subspace Identification (SSI)

- Parametrical Modal estimation requiring **apriory knowledge** of Model Order
- **Physical Modes** as well as **Non-physical Modes** are estimated

How can we separate Physical Modes from Non-physical Modes?

Physical modes are repeated for multiple Model orders!

Stabilization Diagram



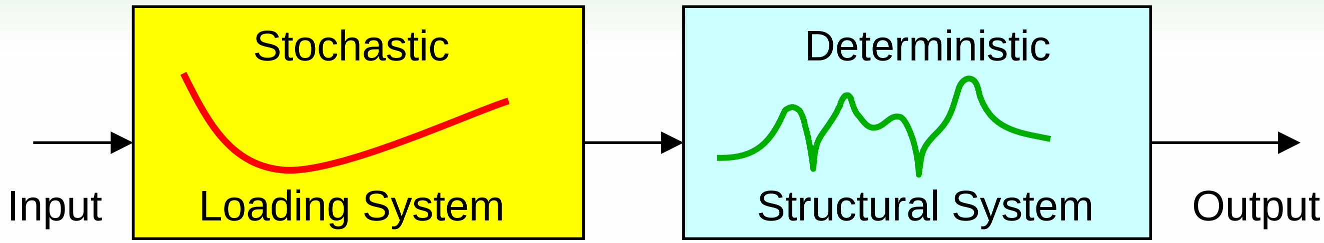
X Estimated parameters not fulfilling apriori knowledge of damping

\+ Stable modes are repeated in two consecutive models fulfilling user defined criteria

X Remaining modes are considered as unstable

Contents

Effects of time varying systems



		Structural System	
		Time Invariant	**Time Varying**
Loading System	**Time Invariant**	~ Spectral Density Correct ~ Clear Peaks ~ Good Results in FDD EFDD and SSI ~ Good SSI validation Covariance equivalance	~ Spectral Density Incorrect ~ Incorrect Modal parameters of all methods ~ Bad validation in SSI no Covariance equivalence AVOID!
	Time Varying	~ Spectral Density peaks correct, valleys not ~ Good modal parameters results in all methods ~ Bad validation in SSI no Covariance equivalence	**AVOID!**

- Run up/down tests gives good modal parameters, but might have bad SSI validation

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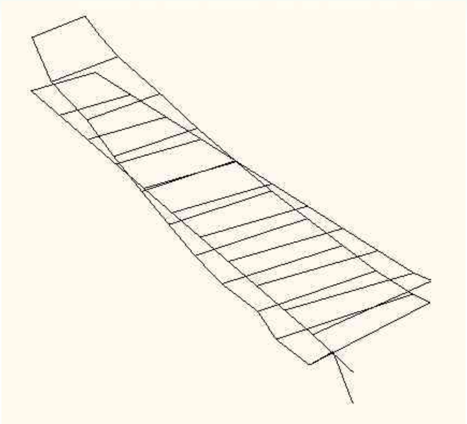
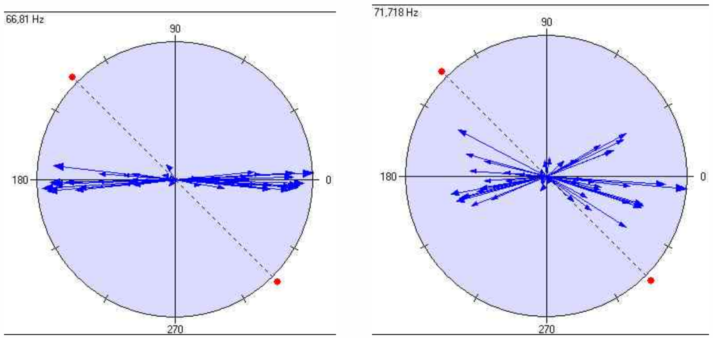
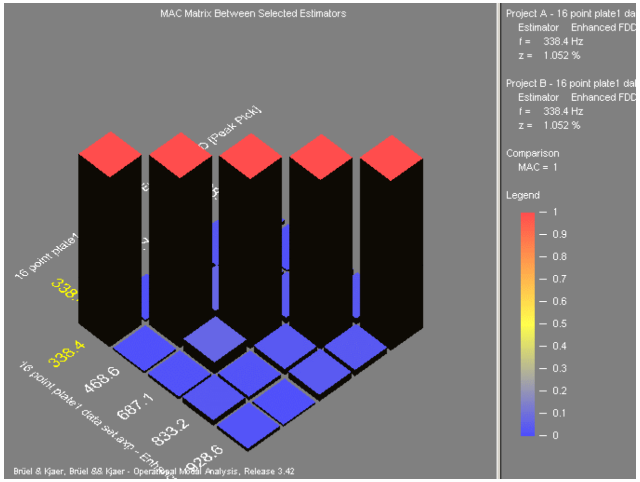
Validation



- Common sense (does it look like a mode shape?)
- AutoMAC (Modal Assurance Criterion)
- Complexity plots
  - » Normal vs. Complex shapes



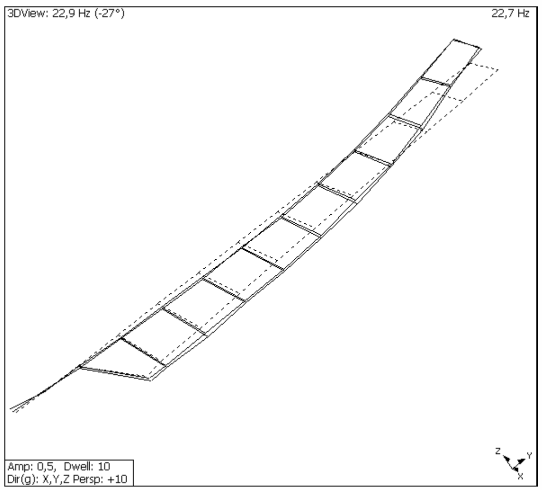


- Use several different methods
  - – Compare frequencies, damping and shapes (CrossMAC)

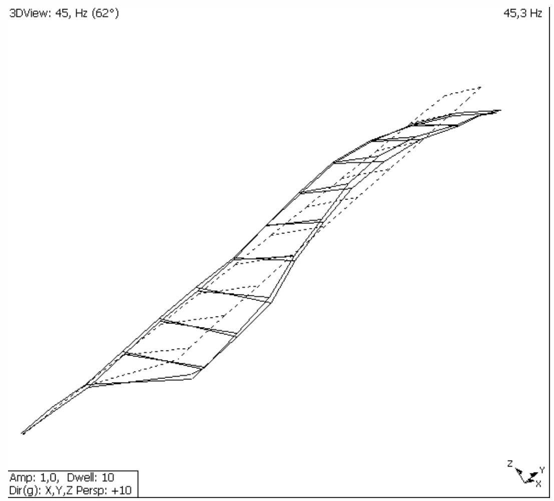
Comparison of Mode Shapes

- Mode shapes from Mobility Test and CVA

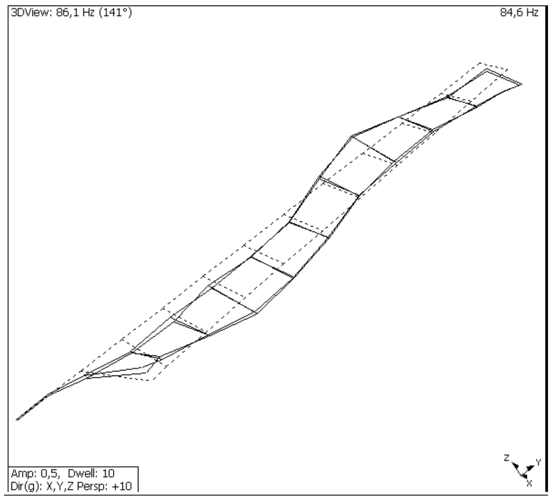
22Hz
Cross-MAC = 0,78



45Hz
Cross-MAC = 0,95



86Hz
Cross-MAC = 0,82



Example from:

APPLICATION NOTE
Operational Modal Analysis of a Wind Turbine Wing using Acoustical Excitation
H. Herlufsen and N. Møller, Brüel&Kjær, Denmark

To be downloaded from ***http://www.bksv.com/***

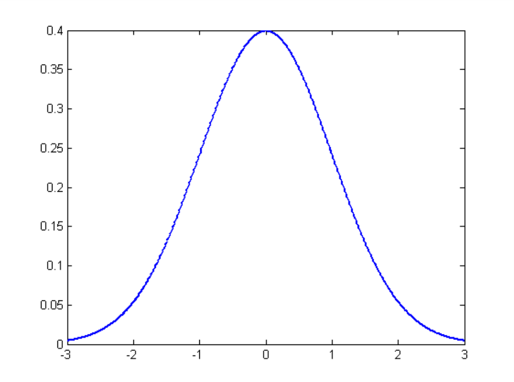
Consequences of Harmonic Components

Techniques	Consequences of Harmonic Components
All techniques	• Potentially mistaken for a structural mode • Potentially bias the estimation of structural modes (freq, damp, shape) • Potentially higher dynamic range required to extract “weak” modes
EFDD	• The identified SDOF used for modal parameter estimation may be biased by harmonic component(s) • Harmonics must be outside the SDOF bell thereby potentially narrowing the SDOF and resulting in poorer identification (leakage)
SSI (PC, UPC, CVA)	• The SSI methods will estimate both harmonics and modes. The modes are estimated correctly even for harmonics very close to the modes • Information in the time signal is used both to extract the harmonics and the modes, therefore the recording time should generally be longer

Probability Density Function (PDF)

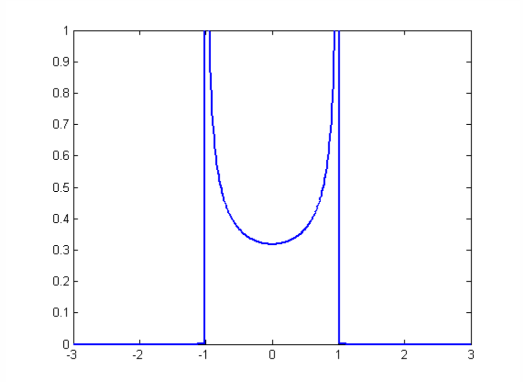
- Probability Density Function (PDF)
  - The statistical properties of a narrowband stochastic response of a structural mode and a harmonic component are significantly different

PDF for Pure Structural Mode



$$y = f(x|\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

PDF for Pure Harmonic



$$y = f(x|a) = (\pi \cos(\arcsin(x/a)))^{-1}$$

Automated Method

1. Band-pass filter each potential mode and calculate the PDF
2. Fit calculated PDF to both the PDF of a harmonic component and the PDF of a structural mode
3. Calculate the prediction error between fitted and measured data and use as harmonic indicator

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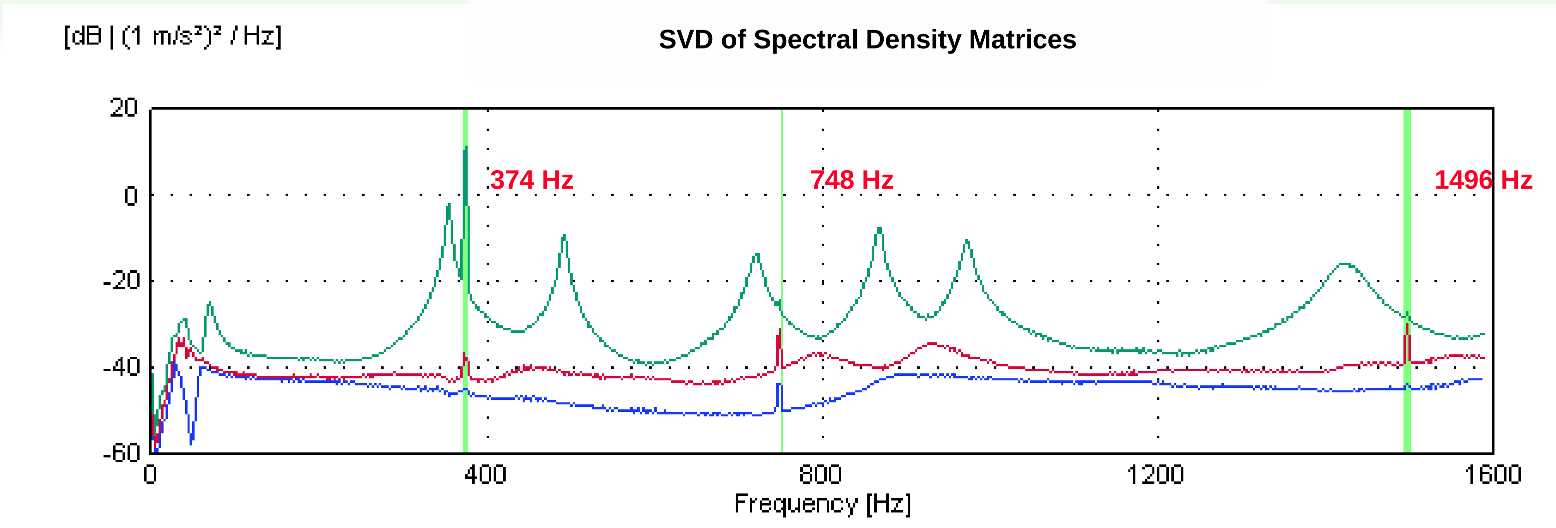
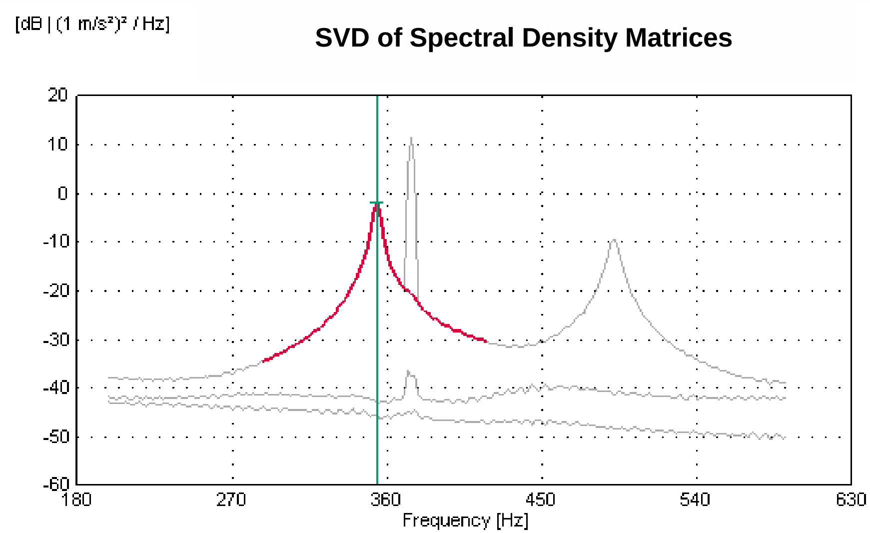
New Harmonic Indicator based on Kurtosis



- Green vertical lines indicates harmonic components
- Harmonic components removed by linear interpolation.
- Frequencies removed are not included in weighted mode shape estimation



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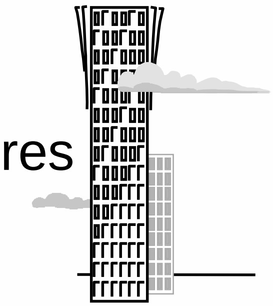
Contents

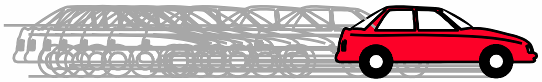
Conclusion *(PULSE-3560, MTC-7753 & OMA-7760 SoftWare)*

General Conclusions:

- Dedicated System for Operational Modal Analysis offering *reliable* estimation of modal parameters *without* known input force



- The method makes modal testing *easier* on *large* structures as *no* elaborate excitation is needed



- Use data acquired during operation for extraction of modal data

- The technique currently used in civil engineering *now* to be introduced into mechanical applications

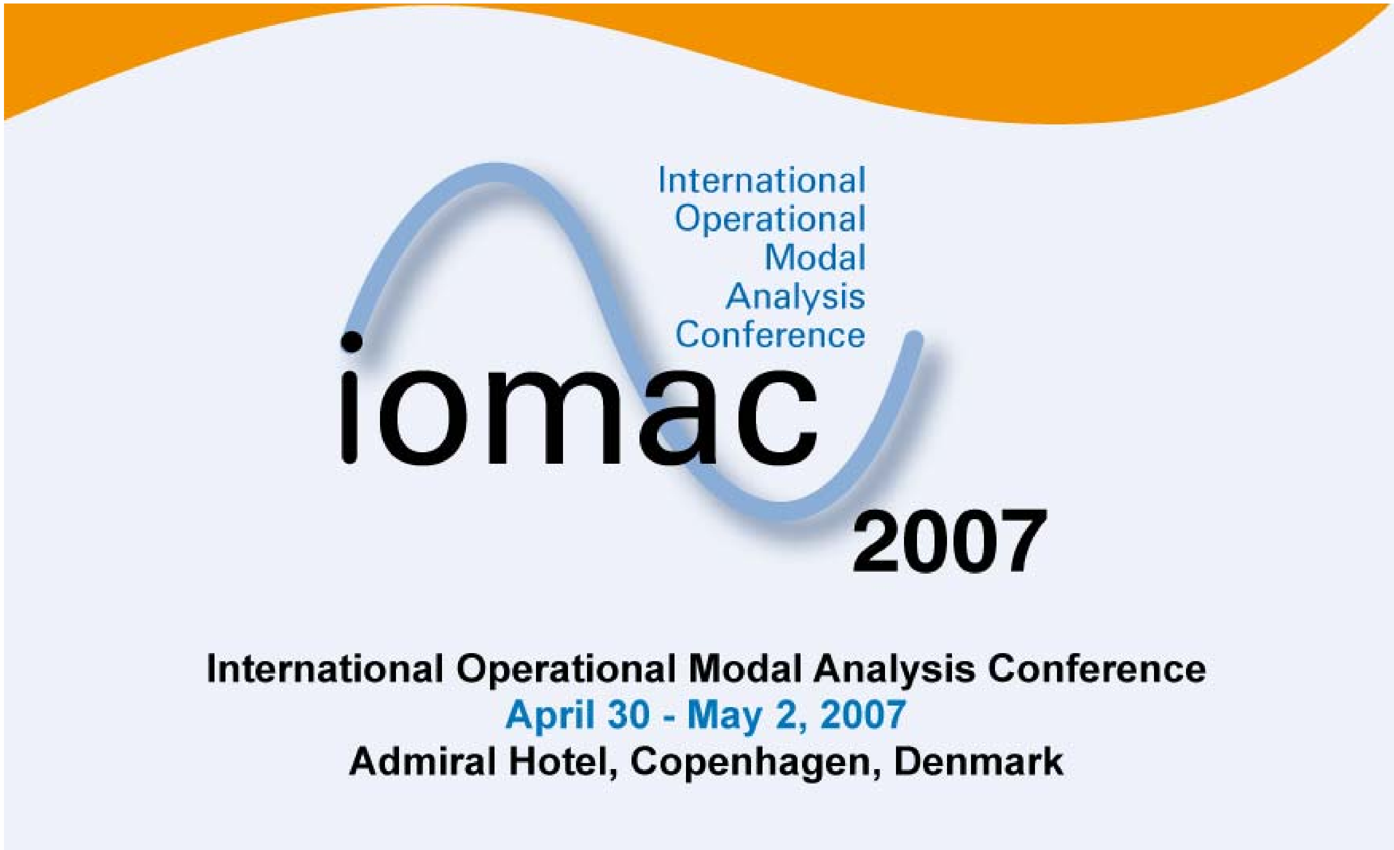
Conclusion (advantages)

- No elaborate fixturing of structures, shakers and force transducers
  - No test rigs needed
  - Short setup time
  - No dynamic loading from shakers and stingers
  - No crest factor problems as when using hammers
  - No potential destruction of structure
- Modal model can represent real operating conditions
  - True boundary conditions
  - Actual force and vibration levels
- Only natural random or unmeasured artificial excitation required
- No interference or interruption of daily use
- Modal testing can be applied in parallel with other applications

Conclusion (concerns)

- Unscaled (Non Calibrated) Modal Model
  – No Forced Response and Modification Simulations
- More Operator Skills required
  – Some apriori knowledge is advantageous
  – Pre-analysis is often needed
  – New technique to most engineers

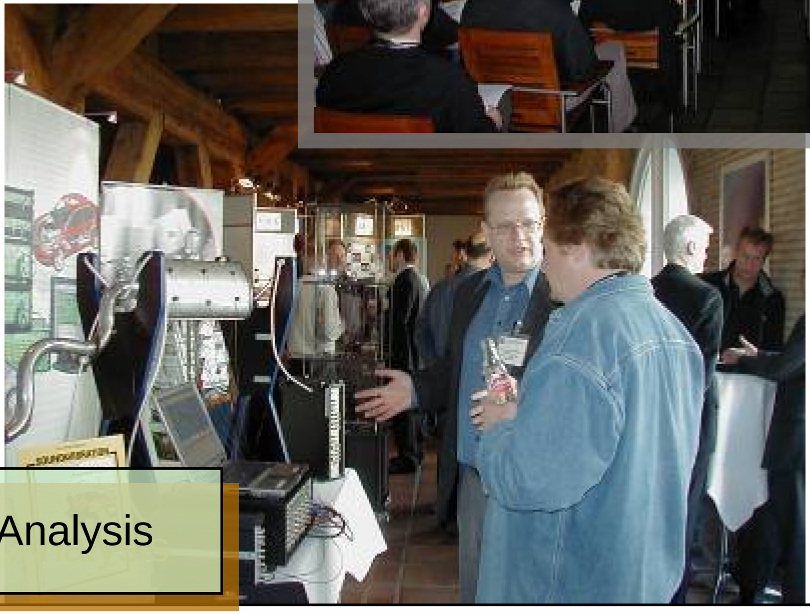
Like to learn more?



1st International Operational Modal Analysis Conference
History 2005

> 80 Papers Presented

>140 Participants



Exhibition presenting Equipment for Modal Analysis

Like to learn more?

Beginner?

A pre-conference course on Operational Modal Analysis is offered on Monday April 30, 2007 by Dr. Carlos Ventura, Prof. Univ. British Columbia, Canada and M.Sc. Svend Gade, Prof. Brüel & Kjær University, Denmark.

So what's on the drawing board?

Dealing with Harmonics

organized by Dr. Thomas Lagø and Dr. Anders Brandt, Acticut, Sweden.

Sensors, Types and Applications

organized by Prof. Thomas Schmidt, Univ. Magdeburg, Germany, and Mauricio Ciudad-Real, Kinemetrics, USA.

Best Practice for Testing and Identification

organized by Prof. Carlos Ventura, Univ. British Columbia, Canada.

Health Monitoring

organized by Prof. James Brownjohn, Univ. Sheffield, UK.

Automotive

organized by Bart Peeters, LMS International

Rotating Machinery

Organized by Nis Møller, Brüel & Kjær Sound & Vibration Measurement

and many other interesting sessions

Location & Accommodation

International Operational Modal Analysis Conference

April 30 - May 2, 2007

Admiral Hotel, Copenhagen, Denmark

www.iomac.dk

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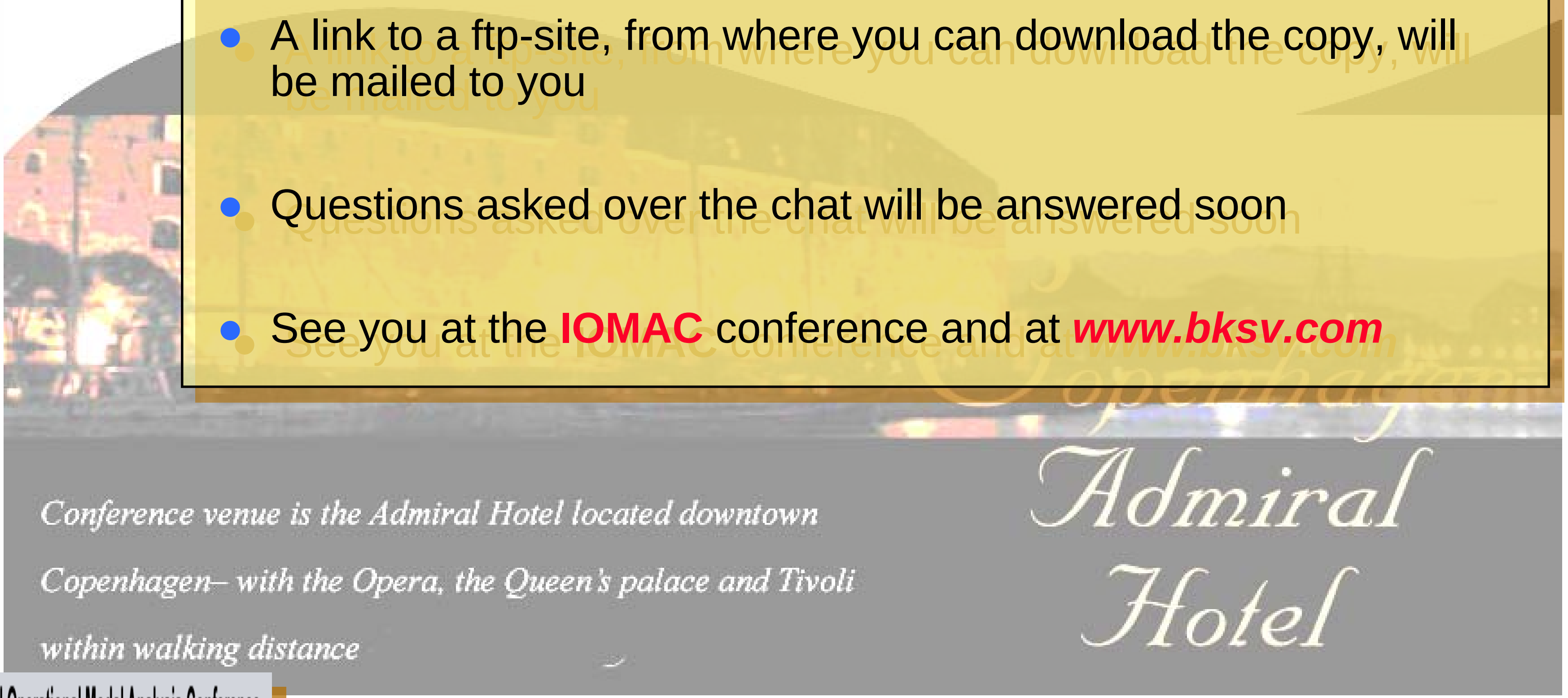
- A copy of the slides in .pdf format will be available
- A link to a ftp-site, from where you can download the copy, will be mailed to you
- Questions asked over the chat will be answered soon
- See you at the **IOMAC** conference and at ***www.bksv.com***



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