# The "Schroeder frequency" revisited

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It is noted that the cross-over frequency that marks the transition from individual resonances of a multimode system to overlapping normal modes, when expressed as a cross-over wavelength, equals—within a numerical constant—the diffuse-field distance in both three- and two-dimensional resonators. © *1996 Acoustical Society of America.*

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## THE CROSS-OVER FREQUENCY

Many multimode resonance systems are characterized by a crossover frequency (''Schroeder frequency''[1]) that marks the transition from individual, well-separated resonances to many overlapping normal modes. For airborne sound in reverberant enclosures, this cross-over frequency is given by

$$f_c = 2000\sqrt{\frac{T}{V}}, \tag{1}$$

where $T$ is the 60-dB reverberation time in seconds and $V$ the volume of the enclosure in cubic meters.[2] Equation (1) results from equating the half-power bandwidth $B$ of the resonances

$$B = \frac{\log_e 10^6}{2\pi T} = \frac{2.2}{T}, \tag{2}$$

with three times the average asymptotic spacing $\Delta f$ between resonance frequencies[3]

$$\Delta f = \frac{c^3}{4\pi V f^2}. \tag{3}$$

The factor 2000 (which contains the velocity of sound $c$) in Eq. (1) guarantees that, on average, at least three resonances fall within the half-power bandwidth of one resonance at frequencies above $f_c$. As a result, the sound transfer function above $f_c$ between two ''distant'' points (i.e., points with negligible direct power transmission) can be considered an approximate complex Gaussian process with a number of well-known consequences (e.g., average frequency spacing between relative maxima of the power transfer function equal to $4/T$,[2,4] average range of the statistical fluctuations between maxima and minima equal to 10 dB, etc.).[2,5]

The awkward (and unit dependent) factor 2000 can be made to disappear if one converts the cross-over *frequency* to a cross-over *wavelength* $\lambda_c = c/f_c$. Using Sabine's formula for reverberation time

$$T = 13.8\frac{4V}{cA}, \tag{4}$$

where $A$ is the equivalent or ''open window'' absorption area, Eq. (1) takes on the following form[6]

$$\lambda_c = \sqrt{\frac{A}{6}}, \tag{5}$$

where the factor 6 is a *pure number* and $\lambda_c$ is given in the same unit as $\sqrt{A}$. In contrast to Eq. (1), Eq. (5) is independent of units. Threefold (or better) modal overlap obtains for wavelength *smaller* than $\lambda_c$.

## I. THE DIFFUSE-FIELD DISTANCE

The diffuse-field distance $r_c$ (also called reverberation distance) is defined as the distance $r$ from an omnidirectional source in a reverberant enclosure for which the direct energy density

$$\varepsilon_d = \frac{P}{4\pi r^2 c} \tag{6}$$

equals the reverberant energy density[7]

$$\varepsilon_r = \frac{PT}{\log_e 10^6 V}, \tag{7}$$

where $P$ is the power emitted by the source. Equating Eqs. (6) and (7) yields

$$r_c = \sqrt{\frac{\log_e 10^6 V}{4\pi c T}}. \tag{8a}$$

Replacing $T$ by $A$ via Eq. (4), one obtains

$$r_c = \sqrt{\frac{A}{16\pi}}. \tag{8b}$$

Comparison with Eq. (5) shows that the diffuse-field distance $r$ and the critical cross-over wavelength $\lambda_c$ are closely related. In fact, they differ only by a numerical constant:

$$r_c = \sqrt{\frac{3}{8\pi}}\lambda_c. \tag{9}$$

Is this a coincidence? Both $r_c$ and $\lambda_c$ describe transitions from a ''regular'' to a ''statistical'' regime, i.e., from a single source to many (image) sources (in the case of $r_c$) and from isolated resonances to many overlapping modes (in the case of $\lambda_c$). It is therefore tempting to look for some deeper meaning behind Eq. (9), especially since a similar relation holds in two dimensions:

$$r_c^{(2)} = \frac{1}{2\pi}\lambda_c^{(2)}. \tag{10}$$

The close correspondence between $r_c$ and $\lambda_c$ in both three- and two-dimensional resonators is all the more noteworthy because there is no dearth of other ''characteristic

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lengths'' that can be constructed from the relevant room-acoustical variables $c, V, T, A$, and the surface area $S$. Examples are $4V/S$, the mean-free path of the sound rays, and $\Delta\lambda = \lambda^4/4\pi V$ when the mode spacing, Eq. (3), is written in terms of wavelengths.

## II. DIFFUSE-FIELD DISTANCE AND ECHO DENSITY

For rectangular enclosures (and other simple shapes) reverberation is often analyzed in terms of image sources. In three dimensions, the number of image sources[7] within a radius $r$ of the origin is asymptotically equal to $(4\pi/3)r^3/V$. Equivalently we may say that the number of echos arriving at some selected point in the enclosure within time $t$ after a pistol shot was fired equals approximately $(4\pi/3)c^3t^3/V$. Thus the average distance between successive echos equals

$$\Delta t = \frac{V}{4\pi c^3 t^2}. \tag{11}$$

Equation (11) is the real-space analog of Eq. (3). Whereas Eq. (11) describes a phenomenon (echo density) in *real* (image-source) space, Eq. (3) describes a phenomenon (mode density) in *reciprocal* (wave-number) space.

If the average spacing $\Delta t$ between echos is smaller than the echo decay time $T/\log_e 10^6$, a selected point in the enclosure will receive many echos of comparable magnitude. Furthermore, if these echos arrive from many different directions, ''diffuse'' conditions at that point will prevail. Equating $\Delta t$ and $T/\log_e 10^6$ therefore defines a ''diffuse-field time interval'' $t_c$:

$$t_c = \sqrt{\frac{\log_e 10^6 V}{4\pi c^3 T}}$$

or a ''diffuse-field span''

$$d_c = c t_c = \sqrt{\frac{\log_e 10^6 V}{4\pi c T}}. \tag{12}$$

This diffuse-field span $d_c$, however, is none other than the diffuse-field distance $r_c$, see Eq. (8a)!

Thus we see that, given the concept of image sources, the cross-over frequency $f_c$ for modal overlap and the diffuse-field distance $r_c$ actually describe the same physical reality, albeit in two different spaces: real space in the case of $r_c$ and reciprocal (wave-number or frequency) space in the case of $f_c$.


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[1] A. D. Pierce, *Acoustics* (McGraw-Hill, New York, 1981), Chap. 6, Sec. 6-7.

[2] M. Schröder, ''Die statistischen Parameter der Frequenzkurve von großen Räumen,'' Acustica **4**, 594–600 (1954). Englishh translation: M. R. Schroeder, ''Statistical Parameters of the Frequency Response Curves of Large Rooms,'' J. Audio Eng. Soc. **35**, 299–306 (1987). *Note:* In my original (1954) paper I proposed a more ''conservative'' factor of 4000 (instead of 2000) in Eq. (1), corresponding to a tenfold modal overlap.

[3] M. Schröder, ''Eigenfrequenzstatistik und Anregungsstatistik in Räumen,'' Modellversuche mit elektrischen Wellen, Acustica **4**, 456–468 (1954). English translation: M. R. Schroeder, ''Normal Frequency and Excitation Statistics in Rooms: Model Experiments with Electric Waves,'' J. Audio Eng. Soc. **35**, 307–316 (1987).

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[7] H. Kuttruff, *Room Acoustics* (Applied Science, London, 1973), Chap. IV.