A new definition of boundary point between early reflections and late reverberation in room impulse responses

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The early reflections in the room impulse response are usually defined as those observed within the initial 80 ms after the arrival of the direct sound, after which time the sound field is called reverberant. This number was chosen from measurements of other functions in a limited number of halls. In order to give an objective foundation to this time separation and to establish a physical indicator for it, a new method is proposed that defines a "transition time t_L," which is the time at which the energy correlation between the direct plus initial sound and the subsequent decaying sound first achieves a specified low value. For various halls this number is shown and its relevance as a new parameter is discussed. [DOI: 10.1121/1.2743161]

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I. INTRODUCTION

In room acoustics a wave-related parameter (early reflections) and a statistical parameter (late reverberation) are utilized in the characterization of the sound fields in halls for the performance of music. It seems important that there should be a single physical measure for determining the point at which the early reflections have been overtaken by the late reverberant sound, i.e., the point at which the correlation between the two is at a low value.

Previously, several studies discussed the range of early reflections from a subjective point of view. Reichardt *et al.* (1974) showed that 80 ms gives the boundary in the room impulse response (RIR) between usable and useless transparency. Beranek (1992, Figs. 10 and 33) infers from two measured parameters that 80 ms is an approximate time separation between early reflections and late reverberation. Barron (1993) writes that individual reflections (in large halls) arriving after 100 ms are no longer distinguishable. Kuttruff (1993) indicates that the characteristic time that separates the early and late parts of the impulse response should be 100–150 ms. Hidaka *et al.* (1995) state "…the time that separates the early reflections from the late reflections exists in the range from 50 to 200 ms." This range in values indicates that a more meaningful separation time is needed, particularly for auralization (Heinz, 1993; Kuttruff, 2000), electronic reverberators (Schroeder, 1962; Gardner, 1998), and other sound field simulations (Krokstad *et al.*, 1983; Vorländer, 1989).

This paper investigates the possibility of a separation time t_L where the correlation between the direct plus initial sound and the subsequent sound first achieves a specified low value. This time t_L is a physical quantity that can be measured in actual halls and is not based on psychoacoustics or on geometrical acoustics. It will be shown that the value of t_L is related to the type of hall (symphony, chamber, opera), shape of the hall (shoebox versus non-shoebox), and reverberation time.

II. DEFINITION AND ANALYSIS

A. Early reflections

The early reflections (ER) are defined as the early part of the room impulse response (RIR) and are related to several of the subjective attributes perceived by audiences attending a concert (Beranek, 2004). The first reflection comes from a sidewall or ceiling and its arrival time is about 20–60 ms after the direct sound in typical halls (Beranek, 2004, Fig. 4.14; Hidaka and Beranek, 2000; Hidaka and Nishihara, 2004). Since the mean free path (MFP) of sound rays in any room is given by $4V/S$, the mean arrival time of each subsequent reflection MFP$/c$ is about 35–50 ms in large concert halls. (Here, V, S, and c are respectively room volume, total surface area, and sound speed.) When it is assumed that ER is no greater than 80 ms; those potential reflections included in ER are limited to at most those that have bounced once or twice at surfaces (Vorländer, 1995). In saying this, we are assuming that the reflecting surfaces are nearly rigid with dimensions large compared to a wave length so that the phases of those reflections are determined only by the total path length from a sound source to a receiving point. This means that that ER is a deterministic signal. Because the audience areas in halls are highly absorptive, the sound energy that impinges on these areas will not result in significant reflections.

B. RIR with increases in time

When a sound wave from a source reflects at interior surfaces repeatedly, the density of the number of reflections per unit time of the RIR increases rapidly as time increases. Concurrently, both the amplitude and the phase of the RIR are changed by the running phenomena so that the wave form of the RIR becomes complicated to a sizable degree. Basic assumptions to what follows in this chapter, including references to the literature, are the following:

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- Each surface in a hall is basically reflective, except for audience areas, and its size is finite so that the reflecting points that are geometrically determined on it are distributed randomly. Waveform deformation in the reflected sound occurs after undergoing several reflections because of the phase randomization effect (Schroeder, 1987), that is, the phase of the reflected sound becomes a sum of random variables.
- There are lots of edges, corners, and similar obstacles in a hall. When a sound wave encounters one of these objects, the diffracted wave that is generated propagates in every direction as a cylindrical wave (Bowman *et al.*, 1987). Accordingly, abundant multiple diffracted waves build up as time increases and nonspecular (incoherent) reflected sound is generated (Beckmann and Spizzichino, 1987). With the latter phenomenon, the random component in the RIR increases with time and, as near sound diffusion evolves, late reverberation LR is achieved. In other words, LR can be considered a stochastic signal.

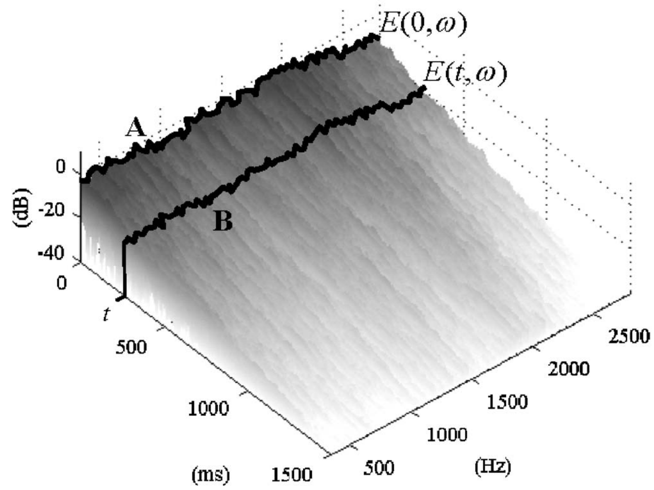

FIG. 1. Example of the time-frequency analysis of acoustic energy $E(t,\omega)$ by Eq. (1), calculated from the room impulse response, RIR, measured at Boston Symphony Hall (2625 seats). It shows the reverberant decay curves at each sampling frequency as well as the temporal variation of the amplitude of RIR. The measurement procedure of the RIR is given in a later section.

C. Late reverberation

Next, one can say that RIR is equal to the sum of ER, plus intermediate transition reflections, plus LR. When does LR become a diffuse sound field? Schroeder (1959) defines diffusion at a point as the angular distribution of sound energy flux in the plane wave expansion of the sound field and, if the distribution over the solid angle is uniform, he calls the sound field at this point "completely diffuse." Junius (1959) experimentally infers that diffuse reverberant sound begins after 100–150 ms. Even if the reverberant sound is completely diffuse, it is not clear how the measurement of RIR is affected when measured by a single omnidirectional microphone, i.e., with the results plotted on a time axis.

Kuttruff (1993) writes in his research on the digital synthesis of the LR in the artificial RIR that the "gross" temporal and spectrum characteristics are important rather than the fine structure and that the phase spectrum is subjectively insignificant. He states that it is only necessary to adjust the gross spectrum and temporal structure, neglecting its microscopic structures in order to achieve sufficient subjective impression. Kuttruff (1991) reports that phase randomization in the RIR occurs when the distance from the sound source exceeds twice the "reverberation distance" so that phase angles may safely be neglected. Thus, when evaluating LR, one may choose any phase function that is convenient for the processing.

Summarizing the above, one can infer that the LR is the posterior component of the RIR, which has gone through more than second or third order reflections; that the phase or microstructure of the RIR has negligible meaning; that the gross spectrum and temporal characteristics are meaningful; and that stochastic manipulation is applicable for analysis (Ebeling, 1984; Schroeder, 1987).

We must note also that the ensemble averages (defined as the intensities over many repeated measurements under identical physical conditions) of the rise in intensity of sound in a large room, starting with the instant at which the sound is turned on, is equal to the steady state intensity minus the intensity during the decay of sound, starting with the instant the sound is turned off (Schroeder, 1966). In other words, the longer the reverberation time, the longer the rise time.

D. Definition of transition time, t_L

By applying Levin distribution to evaluate a transient signal, the time-frequency distribution of the acoustic energy $E(t,\omega)$ is given by (Yamada and Hidaka, 2001)

$$E(t,\omega) = \left| \int_t^\infty p(\tau)\exp(i\omega\tau)d\tau \right|^2, \tag{1}$$

where $p(t)$ is the RIR between a sound source and a receiving position in the hall.

By taking the frequency average of Eq. (1), the next relation holds:

$$\begin{aligned}\langle E(t,\omega)\rangle &= \frac{1}{\omega_2-\omega_1}\int_{\omega_1}^{\omega_2} E(t,\omega)d\omega \\ &= \frac{2\pi}{\omega_2-\omega_1}\int_t^\infty \int_t^\infty p(\tau)p(\tau')\delta(\tau-\tau')d\tau d\tau' \\ &= \frac{1}{f_2-f_1}\int_t^\infty p^2(\tau)d\tau. \end{aligned}\tag{2}$$

Here, the integral in the last expression is identical to the Schroeder integration (Schroeder, 1965), and $t=0$ means the time when the direct sound arrives. Hereafter, the notation $\langle\ \rangle$ means the frequency averaging.

Figure 1 is a calculated example of Eq. (1) that shows the temporal variation of the amplitude of the RIR for a large concert hall, that is, the reverberant decay curves at each sampling frequency between frequency range (ω_1,ω_2). Because the response at $t=0$ (curve A) includes the direct sound and all reflections, it is equal to the integration of the squared

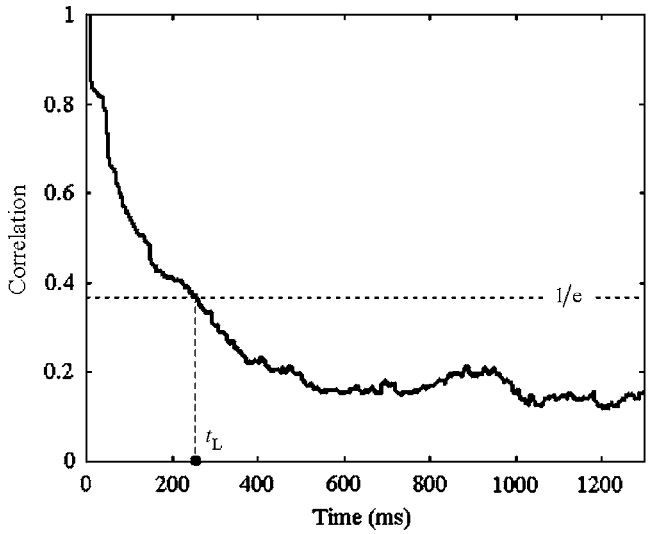

FIG. 2. Definition of the transition time t_L, the boundary point between the early reflections and the late reverberation on the RIR when the correlation function first becomes sufficiently low, i.e., $r(t)=1/e=0.367$. The vertical axis means normalized correlation between acoustic energies at $t=0$ and t. The frequency range of the RIR is from 354 to 2828 Hz that corresponds to 3 octave bands; 500 Hz, 1 kHz, and 2 kHz. Same data of Boston Symphony Hall in Fig. 1 were used.

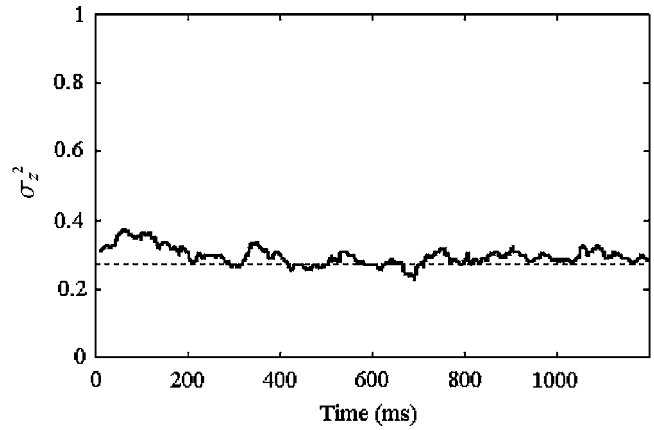

FIG. 3. Plot of the variance of normalized amplitude of the frequency response, $z=\rho/\langle\rho\rangle$ as a function of time. When z obeys the Rayleigh distribution, the theoretical value of σ_z^2 is $4/\pi-1=0.27$. The measurement result approaches the theoretical value early on. The frequency range is 500 Hz octave band. Same data as in Fig. 1 were used.

impulse response from $t=\infty$ to 0. Curve A contains all reflections that arrive after the time $t=0$, usually called the frequency response curve.

For t small, the frequency curve contains, even if the direct sound does not exist, lower order reflections with no phase randomization. Stated differently, the RIR is a sum of deterministic and stochastic components. Accordingly, Eq. (2) decreases stepwise with time as shown in Fig. 2.

If we consider the case where ER includes the direct sound plus the reflections within the initial 80 ms after the arrival of the direct sound, only second or third order reflections are involved. Measurements of C_{80} values in actual halls (Beranek, 2004) show that the sound energies before and after the 80 ms of the RIR are nearly the same order. If we start with this observation and decide on a time separation that includes more than second or third order reflections, then the deterministic component ER and stochastic component LR are, in curve A, about equal, i.e., we can also say that the ER and LR are stochastically independent from each other.

Now let us introduce a time dependant correlation function to evaluate the similarity between curves A and B by utilizing Pearson's correlation coefficient,

$$r(t) = \frac{\langle (E(0,\omega) - \mu(0))(E(t,\omega) - \mu(t))\rangle}{\sqrt{\langle (E(0,\omega) - \mu(0))^2\rangle\langle (E(t,\omega) - \mu(t))^2\rangle}}, \tag{3}$$

where $\mu(t)=\langle E(t,\omega)\rangle$ means a frequency average. This gives the correlation between the energy of the initial state ($t=0$ s) and the energy of state at any time t later for a particular frequency range.

At what point in time does Eq. (3) becomes sufficiently small for practical purposes? Certainly, ER should include more than one or two early reflections, and must not include sound that is diffused. In this paper we define a "transition time t_L" that is based on the historically low correlations commonly used when treating stochastic process in acoustics and other sciences (fluid dynamics, meteorology, and so on), namely $r(t)=1/e=0.367$ (Tatarskii, 1961; Skudrzyk, 1971; Ando, 1977; Bass and Fuks, 1979; Ishimaru, 1997). This definition is also an extension of the correlation length[1] in correlation analysis (Papoulis, 1984, Chap. 1). It seems definite that for this transition time the sound field has become incoherent compared to its initial state at $t=0$.

Considering that the definition of a reverberation decay curve assumes a diffuse sound field, t_L is the time when the total sound field (ER plus LR) of a hall changes to a sound field that is dominated by the energy of the reverberant sound alone.

E. Discussion

The time t for curve B in Fig. 1 is taken to be large enough that all the reflections involved have undergone multiple reflections in the hall. This frequency curve is equal to the steady state response of the RIR including only the LR. The amplitude $\rho=\sqrt{E(t,\omega)}$ obeys approximately the Rayleigh distribution because of the phase randomization (Schroeder, 1987; Kuttruff, 1991). Then, introducing the normalized variable $z=\rho/\langle\rho\rangle$, one obtains the variance $\sigma_z^2=\langle z^2\rangle-\langle z\rangle^2$ equal to $4/\pi-1=0.27$ (Papoulis, 1984, Chap. 5). A measurement example of σ_z^2 (Fig. 3) shows that the assumption of a Rayleigh distribution is likely true. This result was obtained for all the halls in this study. Thus, one can show that the squared amplitude $E(t,\omega)=\rho^2$ obeys the exponential distribution, where the probability density function is given by following equation:

$$p(E) = \frac{1}{2\sigma_E^2}\exp\left(-\frac{1}{2\sigma_E^2}E\right). \tag{4}$$

As discussed above, the initial part of the RIR decreases with increasing time. When t is larger than several times MFP/c, multiple reflections occur and the randomization in RIR is emphasized gradually. As a result, one can expect that

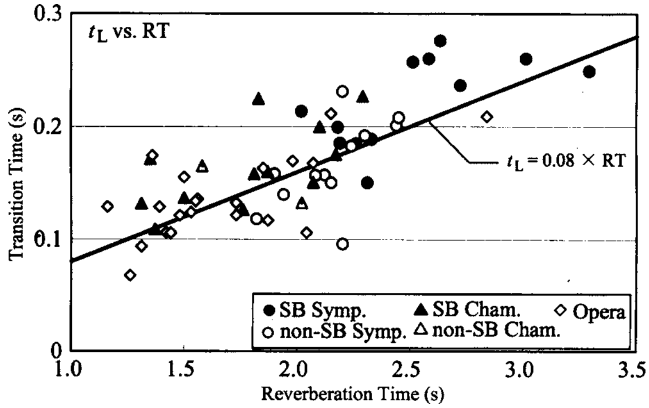

FIG. 4. Plot of the transition time versus the reverberation time. These values are the average of two receivers near the center positions at the main floor measured under the unoccupied state. The measured halls are divided into five categories: shoebox (SB) symphony hall, non-SB symphony hall, SB chamber hall, non-SB chamber hall, and opera house. The regression line (r=0.76) is shown. The frequency range is 500 Hz octave band.

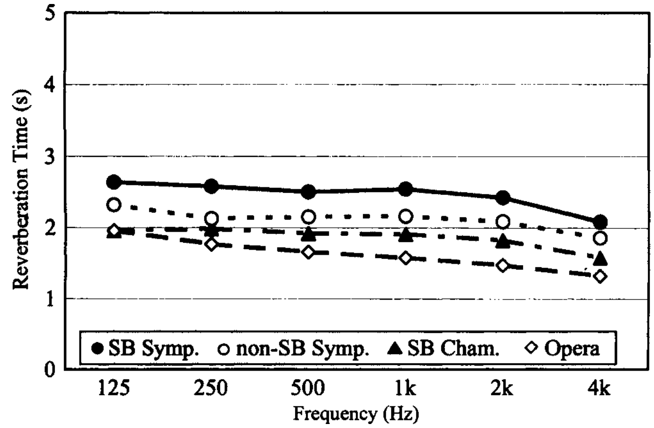

FIG. 6. Averaged values of the reverberation time for four category halls. Key is the same as Fig. 6.

$E(0,\omega)$ and $E(t,\omega)$ are stochastically independent of each other and the next approximation holds (Papoulis, 1984, Chap. 6),

$$\langle E(0,\omega)E(t,\omega)\rangle \cong \langle E(0,\omega)\rangle\langle E(t,\omega)\rangle. \tag{5}$$

One obtains from Eq. (4) calculating Nth moment $\langle E^N\rangle = \int_0^\infty E^N p(E)\, dE$,

$$\langle E\rangle = 2\sigma_E^2, \quad \langle E^2\rangle = 8\sigma_E^4, \tag{6}$$

so that the denominator of Eq. (3) converges to the finite value $4\sigma_{E(0)}^2\sigma_{E(t)}^2$. Then, from Eq. (2) it is concluded $r(t) \to 0$ for $t \to \infty$.

III. RESULTS OF ANALYSIS

The RIR was measured in each of a number of halls at receiving positions near the center of a hall, but off the centerline, with an omnidirectional source located on the centerline of the stage, usually located 3 m from the lip. The sampling frequency of the RIR was 44.1 kHz and the energy distribution of Eq. (1) was calculated for each 1/1 octave band.

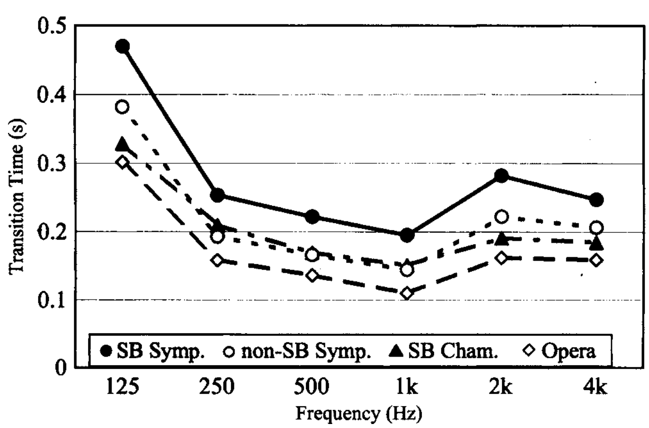

FIG. 5. Averaged values of the transition time for four category halls: 12 shoebox symphony halls, 12 non-shoebox symphony halls, 11 shoebox-chamber halls, and 22 opera houses. The measurement positions are the same as Fig. 4.

Transitions times t_L are plotted for the 500-Hz band in Fig. 4 against reverberation time RT for 24 symphony halls (Beranek, 2004), 13 chamber music halls (Hidaka and Nishihara, 2004), and 22 opera houses (Hidaka and Beranek, 2000). The first two types of halls are further divided into shoebox (rectangular) and non-shoebox (other) shapes. The correlation coefficients between t_L and RT for each octave band are 0.66 (125 Hz), 0.75 (250 Hz), 0.76 (500 Hz), 0.82 (1k Hz), 0.79 (2k Hz), and 0.66 (4k Hz). When RT value is known, it is seen that the transition time for the 500-Hz octave band is roughly estimated by the regression equation.

$$t_L = 0.08 \times \text{RT (in s)} \tag{7}$$

As shown in Fig. 4, there is a definite increase of t_L with regards to RT, which is consistent with Schroeder (1966), i.e., the longer the reverberation time, the later the LR will be achieved. Because the RT is determined from the simplified reverberation theory and the t_L is an actual measurement, the scatter of data from the regression line (±80 ms at maximum) is partially caused by that difference. In other words, t_L is not only determined by RT but also by that fact that the early part of RIR is influenced by the wave component.

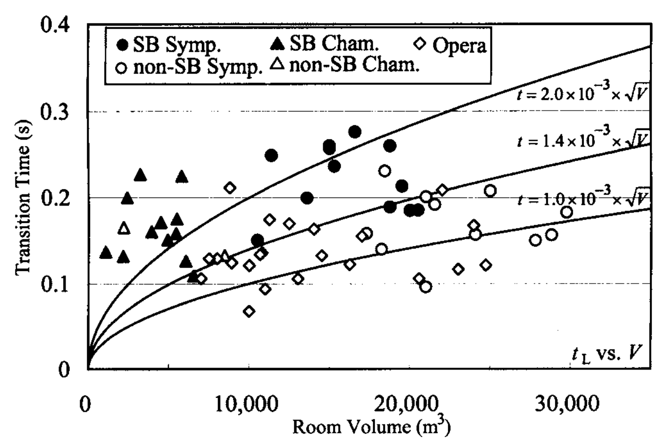

FIG. 7. The transition times for concert halls and opera houses are plotted against the room volume V. Key is the same as Fig. 4. Three solid curves are theoretical ones assuming Sabine's theory. The upper curve coincides with the *Nachhalleinsatzeit*.

TABLE I. Correlation coefficients between t_L and objective parameters. "ALL" means all the halls studied. The frequency range is 500 Hz octave band, but, for BQI and No. of reflections, 3 octave bands (500 Hz, and 1 kHz, and 2 kHz) are taken.

	V	S_A	RT	EDT	C_{80}	G	ITDG	BQI[a]	No. of reflections[a]
t_L (ALL)	0.14	−0.12	0.76	0.78	−0.77	0.21	−0.15	0.17	0.34
t_L (SB)	0.54	0.49	0.79	0.80	−0.76	−0.27	−0.07	−0.37	−0.64
t_L (non-SB)	0.15	0.05	0.48	0.51	−0.60	−0.28	0.08	−0.17	−0.31
t_L (Opera)	0.17	0.03	0.66	0.68	−0.54	−0.16	−0.05	−0.11	−0.12

3-octave band (500 Hz to 2 kHz)

Let us look at the ranges of t_L near an RT of 2.1 s. From Fig. 4 at 500 Hz, we see that the average values, given by the regression equation for each hall shape, plus/minus the residual standard deviation, are 190 ± 29 for shoebox, 163 ± 32 for non-shoebox, and 163 ± 28 for opera. This indicates that t_L is dependent on the hall shape and is about 17% longer for a shoebox hall than for the other two shapes.

Figure 5 shows frequency dependence of t_L. The four plots are averaged values for each category: 12 symphony halls (SB), 12 symphony halls (non-SB), 11 chamber halls (SB), and 22 opera houses. Except for the lowest frequency band, the transition times are nearly constant with frequency. But at 125 Hz, t_L is double its values at higher frequencies. The mean reverberation times versus frequency for the four types of halls are shown in Fig. 6. This is another indication of why t_L is influenced by both RT and the hall shape.

The number of reflections per unit time in a rectangular room, with uniform but not large absorption, is $4\pi c^3 t^2/V$. Hence, if the density of reflections relates directly to the transition time t_L, it should be proportional to $V^{1/2}$. In Fig. 7, t_L is plotted against room volume V. A general increase is seen if one considers the two categories: (1) shoebox symphony and shoebox chamber halls, 110 to 280 ms, and (2) non-shoebox halls and opera houses, 70 to 210 ms. The upper curve $2\times10^{-3}V^{1/2}$ (in s) corresponds to the well-known *Nachhalleinsatzeit* (literal translation, "time of onset of reverberation") (Cremer and Müller, 1982), but big scatter is seen.

Let us explore the range of t_L for the different hall types, considering that in each type the mean ratio of the room volume to the acoustical seating area, V/S_A, is 13.5 m. Calculation for this ratio shows that t_L is dependent on the hall shape and that the ranges are 193 ± 42 ms for SB, 158 ± 30 ms for non-SB, and 143 ± 34 ms for opera house, so that t_L for SB is 22% greater than that of non-SB. Beranek (2006) reports that RT for SB halls is 5% longer on average than that for non-SB halls when both V/S_A and the mean absorption coefficient α are the same, that is, similar types of seats are installed in each, which indicates that in a rectangular (SB) room with only one surface highly absorbent, i.e., the audience area, the early sound can persist longer in the upper space than in nonrectangular (non-SB) rooms where there is no upper space. From the Sabine equation, taking into account of the relation $V/S_A=\mathrm{RT}\alpha/0.161$, this result (22%) shows that t_L is more sensitive to the room shape than RT.

The correlations of t_L with other architectural and acoustical parameters are shown in Table I. High correlations with EDT and C_{80} are found because RT correlates highly with these two parameters (Hidaka, 2005). Additionally, the number of ER (with amplitudes greater than −20 dB relative to the direct sound) was counted for each hall (Hidaka and Nishihara, 2002) and this number shows little correlation with t_L. Also, the other orthogonal parameters (Beranek, 2004), i.e., audience area, binaural quality index BQI($=1-\mathrm{IACC}_{E3}$), ITDG, and G, do not contribute to t_L.

Finally, Fig. 8 is an example of spatial distribution of t_L. The transition time is virtually constant in the greater part of the seating area, i.e., $240 < t_L < 320$ (ms) and has smaller values in the front and rear seats, because the sound field at the front and rear areas is physically simpler. At front seats, direct sound weakens the influence of the successive reflections, and at rear seats, the times intervals between early reflections are short so that the onset time of the reverberation begins sooner.

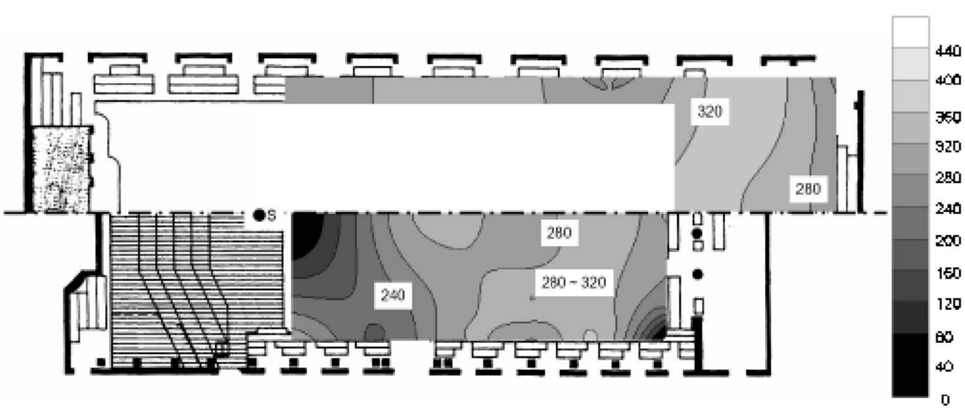

FIG. 8. Contour plots of the transition time measured in Musikvereinssaal Vienna (1680 seats). This quantity virtually takes constant value, except the front and rear seating areas in the hall. Each gray color means 40-ms step. The frequency range is 500 Hz octave band. S is a sound source on the centerline and at 3 m from the lip. Twenty-nine receiving points are used for the interpolation.

IV. CONCLUSION

The objective parameter t_L, proposed in this paper, is a definite means for separating the room impulse response RIR into two parts, that part which correlates meaningfully with the initial sound and that part which has low correlation ($r(t) \leq 1/e = 0.37$). If the classical separation of about 80 ms were used, $r(t)$ would equal 0.6 on average for large concert halls, indicating a substantial presence of ER in the later physical state. The time t_L is a scale value based on the correlation analysis in its own terms, which is in a similar situation as other room acoustical parameters. However, t_L is the only physical quantity that is derived from RIR measured in actual halls, from which one may judge that the sound field makes the transition from ER to LR.

For 57 actual halls for music performance t_L varies from 70 to 280 ms as shown in Fig. 4. Although there is wide scatter, the median for a shoebox symphony hall is 225 ms and the median for a shoebox chamber is 160 ms, while opera houses take about one-half of the former, 130 ms, as a median. The *Nachhalleinsatzeit* gives an intermediate value for shoebox halls but with big scatter (see the upper solid curve in Fig. 7). Transition time depends on RT, but does not show explicit dependence on simple architectural parameters like V or S_A, that is, 57% ($r^2 = 0.76^2$) of t_L is determined by RT but the remaining 43% is relevant to other factors. Transition time is related to room shape. In other words, one might advocate that t_L depends on not a simple number (or a density) of reflections but the intrinsic combination of the early reflections in RIR.

Transition time t_L is an independent parameter from basic architectural quantities, volume and audience area, and conventional objective parameters such as G, BQI, and so on. It is found that mean t_L takes constant value, except at the 125-Hz band, and shows small variation at most seating areas in the hall. It is interesting that the magnitude of t_L, 70 to 300 ms, is nearly equal to or slightly greater than the conventional values of 80 to 200 ms that are based on subjective or hybrid premises.

Transition time has no direct relation to subjective attribute in a hall, but gives one of the physical aspects of the sound field in it. By introducing the transition time, an objective premise can be established for numerical simulation or auralization of sound field in concert halls. Further research on the relationship between the transition time and subjective impressions of halls might demonstrate other aspects in room acoustics.

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[1]For irregular processes of natural phenomenon such as atmospheric fluctuations, the correlation function has its maximum at a time lag equal to 0 and decreases with time. Correlation length (sometimes called correlation time) is defined as a measure of average distance from a point beyond which there is no further correlation of a physical property associated with that point. Values for a given property at distances beyond the correlation length can be considered purely random. This length is only a characteristic temporal scale value, because the correlation analysis is a tool to cope with not deterministic but stochastic phenomenon.

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