THE EFFECT OF POSITION ON THE ABSORPTION OF MATERIALS FOR THE CASE OF A CUBICAL ROOM

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Much work has been done in determining the absorption coefficient of various materials. Most of this work involves the use of the following formula for the decay of sound in a room:

$$E_t = E_0 e^{-\bar{\alpha} v s t / 4V}$$

where $\bar{\alpha}$ represents the "average" absorption coefficient of the room and S the surface of the room. Now it is customary in this decay formula to set $\bar{\alpha} = \Sigma_i \alpha_i S_i / \Sigma S_i$ where α_i is the absorption coefficient of any uniform patch of area S_i. The value of $\bar{\alpha}$ may be determined by measurements of the period of reverberation of the room. In order to determine α_i it is necessary to determine $\bar{\alpha}$ with various areas of the absorbing materials exposed in the room. In this way, a set of simultaneous equations will be obtained in which the values of $\bar{\alpha}$ will be known and from which the values of α_i may be determined.

The justification for this method of averaging the individual absorption coefficients to determine the average absorption coefficient arises from experimental evidence by Wallace Sabine which indicates that the effect of a given amount of absorbing material is independent of its position in the room. The implications behind this method of averaging have, so far as the writer knows, never been carefully discussed.

A simple illustration will show that this method of averaging will at times lead to inconsistent results. Consider a room whose six walls are made of material with an absorption coefficient of .5. The average absorption coefficient will then be $\bar{\alpha} = .5$, and the total absorption will be $1/2\ S$. Now consider a room of the same dimensions, three adjacent walls of which have an absorption coefficient of unity, and the other three walls of which have an absorption coefficient of zero: i.e., three walls would be perfect reflectors, while the other three would be open to free space. Such a room would, according to the formula under discussion, again have an average absorption coefficient of $\bar{\alpha} = .5$ and the total absorption would be $1/2S$. Thus although the physical situations are sufficiently dissimilar to make it apparent that these two rooms have different periods of reverberation, the formula would show them identical. It is of course true that the distribution of sound energy in these rooms would not be homogeneous and therefore these rooms do not satisfy the conditions which must be accepted in order to arrive at the above decay formulae.

Nevertheless, acoustical engineers frequently need to estimate the period of reverberation in rooms in which conditions approach those just cited and it would be a decided advantage to have a formula which would be valid under such circumstance. If such a formula cannot be developed, it should at least be desirable to present clearly the limitations of the present formula.

To this end, let us consider the following simple cases. Let a sound wave, Fig. 1, strike simultaneously surfaces S_1 and S_2 having absorption

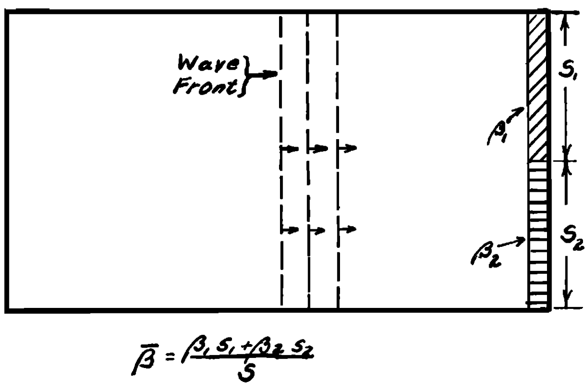

FIG. 1.

coefficients respectively of α_1 and α_2 or reflections coefficient of β_1 and β_2 where $\beta = (1-\alpha)$. If E equals the total energy striking the surface and E_1 that remaining after the first reflection, we have:

$$E_1 = \frac{\beta_1 S_1}{S} E + \frac{\beta_2 S_2}{S} E, \text{ where } S_1 + S_2 = S.$$

If $\bar{\beta}$ equals the average reflection coefficient it should have such a value that

$$E_1 = \bar{\beta} E$$

or

$$E_1 = \bar{\beta} E = \frac{\beta_1 S_1 E + \beta_2 S_2 E}{S}.$$

From which

$$\bar{\beta} = \frac{\beta_1 S_1 + \beta_2 S_2}{S}.$$

This value corresponds to that obtained by the customary method of averaging.

Consider, however, the case where sound strikes first surface S_1 and then all the sound that is reflected strikes surface S_2, (Fig. 2.) The energy remaining after the first reflection is $E_1=\beta_1 E$ and after the second reflection we have $E_2=\beta_1\beta_2 E$.

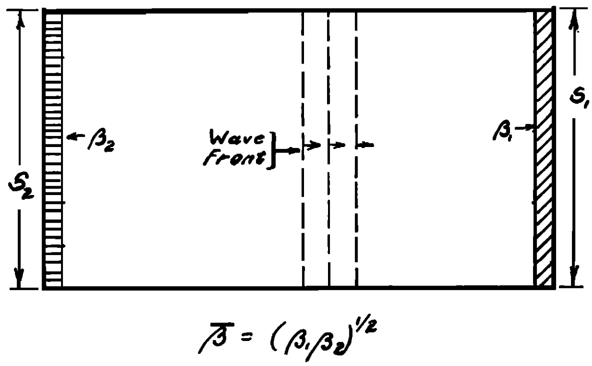

FIG. 2.

Now the average reflection coefficient should be such that for the two reflections we have

$$E_2 = \bar{\beta}^2 E$$

and therefore

$$E_2 = \bar{\beta}^2 E = \beta_1\beta_2 E$$

or

$$\bar{\beta} = (\beta_1\beta_2)^{1/2}$$

Here $\bar{\beta}$ is no longer the weighted arithmetic average of β_1 and β_2, but rather the geometric average.

Let us now consider a combination of these two cases as shown in Fig. 3

The energy remaining after the first reflection will be:

$$E_1 = \beta_1 E$$

and after the second reflection:

$$E_2 = \frac{\beta_1\beta_2 S_2 E}{S_2 + S_3} + \frac{\beta_1\beta_3 S_3 E}{S_2 + S_3}\,.$$

Now again we have:

$$\bar{\beta}^2 E = E_2 = \frac{\beta_1\beta_2 S_2 E}{S_2 + S_3} + \frac{\beta_1\beta_3 S_3 E}{S_2 + S_3}\cdot$$

From which

$$\bar{\beta}^2 = \beta_1\left(\frac{\beta_2 S_2 + \beta_3 S_3}{S_2 + S_3}\right).$$

It is to be noted that:

$$\left(\frac{\beta_2 S_2 + \beta_3 S_3}{S_2 + S_3}\right)$$

represents an average as obtained by consideration of Figure 1, and that this value is averaged geometrically with β_1 as indicated by the consideration of Figure 2.

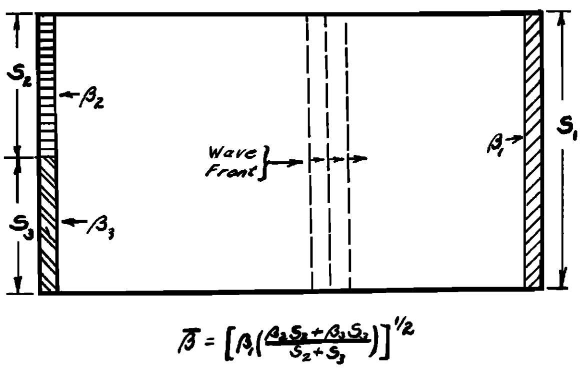

FIG. 3.

Let us consider now, in an approximate way, the effect of dispersion on these formulae. If we assume that sound travels in straight lines, the angle of incidence always being equal to the angle of reflection, we shall thereby imply the dispersion effect to be zero. On the other hand, if, after each reflection, we assume that the sound is traveling equally in all directions and is uniformly distributed, we shall have a case in which the lines of sound propagation suffer more bending than in the most severe case of dispersion.* Obviously the correct assumption lies somewhere between these extremes. Consider again our second illustration and, solely to make our problem more simple at the start, assume a one dimensional space so far as the direction of sound travel is concerned,

* The writer is using the word "dispersion" in a special sense. It includes all those phenomena which cause the angle of reflection to differ from the angle of incidence and those which cause the propagation of the sound wave to depart from a straight line.

i.e., the sound can travel straight back and forth between the two absorbing surfaces specified. If we take the case of no dispersion we can formulate the following table (Table 1) assuming the sound to be uniformly distributed at the start, i.e., the same amount of sound is assumed to be going from left to right as from right to left. It should be

TABLE I

No. of reflections	Remaining Energy "E"
1	$\frac{1}{2}\beta_1 E+\frac{1}{2}\beta_2 E$
2	$\frac{1}{2}\beta_1\beta_2 E+\frac{1}{2}\beta_1\beta_2 E$ or $\beta_1\beta_2 E$
4	$\beta_1^2 \quad \beta_2^2 \quad E$
. . . n	. . . . $\beta_1^{n/2} \quad \beta_2^{n/2} \quad E$

noted that the general formula for n reflections is not correct unless n is even. However, the error introduced by assuming the formulae to hold for odd values of n becomes very small for reasonably large values of n.

From the table we then have: $\bar{\beta}^n = \beta_1^{n/2}\beta_2^{n/2}$ or $\bar{\beta} = (\beta_1\beta_2)^{1/2}$ which is the geometric mean of β_1 and β_2.

Now, however, let us take the extreme case of dispersion where the energy is thoroughly "mixed" after each reflection. We shall have after the first reflection:

$$E_1 = (\tfrac{1}{2}\beta_1 E + \tfrac{1}{2}\beta_2 E) = \tfrac{1}{2}(\beta_1 E + \beta_2 E) = \bar{\beta}E.$$

Now, after mixing, half of this energy will go to the left and half to the right and therefore after the second reflection we have:

$$E_2 = \tfrac{1}{2}\left[\tfrac{1}{2}\beta_1(\beta_1 E + \beta_2 E) + \tfrac{1}{2}\beta_2(\beta_1 E + \beta_2 E)\right] = \tfrac{1}{2}^2 E(\beta_1 + \beta_2)^2 = \bar{\beta}^2 E$$

For the energy remaining after the third reflection, we have:

$$E_3 = \tfrac{1}{2}^3(\beta_1 + \beta_2)^3 E = \bar{\beta}^3 E$$

or in general

$$\bar{\beta}^n = \tfrac{1}{2}^n(\beta_1 + \beta_2)^n$$

from which $\bar{\beta} = 1/2(\beta_1+\beta_2)$ which is the arithmetic average of β_1 and β_2.

We note, therefore, in this simple illustration, that where we assume no dispersion we use the geometric mean and where we assume thorough mixing we use the customary arithmetic average.

Let us now consider a very special two dimensional case. Assume a square array of absorbing surfaces. Let the sides of the square "S" be parallel to the x and y axes. Let sound travel only in the x direction and y direction, and repeat the above considerations. For the sound which

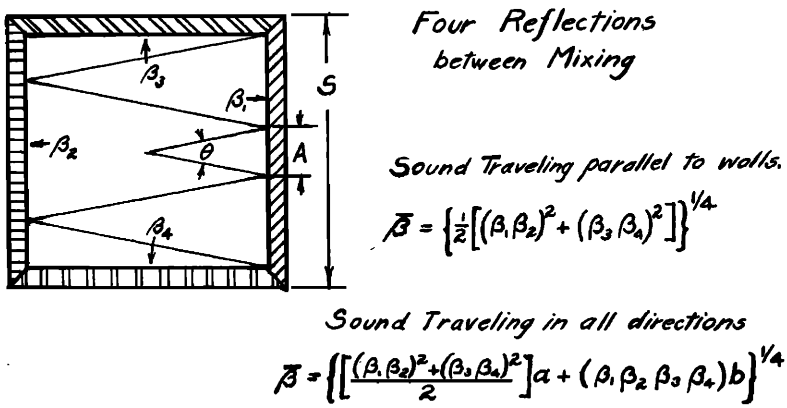

FIG. 4.

is traveling in the x direction we will have after n reflections, if there is no mixing,

$$E_x = \tfrac{1}{2}(\beta_1{}^{n/2}\beta_2{}^{n/2})E$$

and for the sound in the y direction

$$E_y = \tfrac{1}{2}(\beta_3{}^{n/2}\beta_4{}^{n/2})E\,.$$

From which

$$\bar{\beta}^n E = (\tfrac{1}{2}\beta_1{}^{n/2}\beta_2{}^{n/2} + \tfrac{1}{2}\beta_3{}^{n/2}\beta_4{}^{n/2})E$$

or

$$\bar{\beta} = (\tfrac{1}{2}\beta_1{}^{n/2}\beta_2{}^{n/2} + \tfrac{1}{2}\beta_3{}^{n/2}\beta_4{}^{n/2})^{1/n}.$$

It is to be noted that this formulae makes $\bar{\beta}$ a function of n, the number of reflections, and while this may be consistent with theory, any formula which makes $\bar{\beta}$ a function of n would be a very unfortunate formula to have to use in calculating the period of reverberation of an auditorium. If, however, we again consider an extreme case of dispersion where the sound energy is assumed thoroughly mixed after each reflection, we again have

$$\bar{\beta}^n E = (\tfrac{1}{4})^n(\beta_1 + \beta_2 + \beta_3 + \beta_4)^n E$$

or

$$\bar{\beta} = \tfrac{1}{4}(\beta_1 + \beta_2 + \beta_3 + \beta_4)$$

which is the customary arithmetic average.

Now, if instead of assuming the sound to be "mixed" after *each* reflection, we assume this to be done after each *alternate* reflection, the resulting formulae will be:

$$\bar{\beta} = \left[\frac{\beta_1\beta_2 + \beta_3\beta_4}{2}\right]^{1/2}.$$

Where large moving reflecting panels for the purpose of mixing up the sound are not installed, it appears to the writer that this last assumption would more nearly account for the effects of dispersion than either of the other two assumptions.

If we assume the sound to be thoroughly mixed after every four reflections we should be assuming a still milder case of dispersion and should have:

$$\bar{\beta} = \left[\frac{(\beta_1\beta_2)^2 + (\beta_3\beta_4)^2}{2}\right]^{1/4}.$$

We note, therefore, that by varying our assumptions as to the number of reflections between mixing we can account for varying degrees of dispersion.

Consider now a more general two dimensional set up, where sound would travel not only in the x and y directions but in all other possible directions. We will arbitrarily assume that all sound which is not confined to opposite pairs of sides will strike all four sides in succession. We have then somewhat arbitrarily divided the total energy into two parts. The fraction of the total energy which we assume confined to opposite pairs of sides we will designate by "a", while the fraction of the total energy which is assumed to strike all four sides we will designate by "b". Then, if E represents the total energy, aE will represent the energy confined to opposite pairs of sides while bE represents that which strikes all four sides.

After four successive reflections, the energy which remains of the bE will be

$$E_{4b} = \beta_1\beta_2\beta_3\beta_4 bE.$$

After n reflections we can write for this portion of the energy:

$$E_{nb} = (\beta_1\beta_2\beta_3\beta_4)^{n/4} bE.$$

We have already seen that the aE portion of energy after n reflections may be written as

$$E_{na} = \left[\frac{(\beta_1\beta_2)^{n/2} + (\beta_3\beta_4)^{n/2}}{2}\right] aE.$$

Before attempting to combine this formulae with the preceding one, we must give some consideration to the difference in mean length of path between reflections for the energy aE and bE. If the mean length of path for the "b" portion of energy is 1/2 that for the "a" portion, it is then obvious that in a given length of time it will suffer twice as many reflections as the "a" portion. It would, therefore, be necessary to include a weighting factor to take this into account. In the case of a square array of sound absorbing surfaces, sound which travels parallel with the sides has the maximum path of length, S, while sound which travels at an angle of 45° has the minimum path of $.707S$. The difference between the mean length of path for the "a" and "b" portion of energy cannot be greater than the difference between S and $.707S$. In view of the other approximations which must be made we shall neglect this difference in the mean length of path and assume that both the "a" and "b" portion of energy travel the same distance between reflections. With these assumptions the energy remaining after four reflections would be:

$$E_4 = \bar{\beta}^4 E = \left[\frac{(\beta_1\beta_2)^2 + (\beta_3\beta_4)^2}{2}\right] aE + (\beta_1\beta_2\beta_3\beta_4) bE.$$

Now let us assume thorough mixing. Then, after four more reflections, we would have:

$$E_8 = \bar{\beta}^8 E = \left\{\left[\frac{(\beta_1\beta_2)^2 + (\beta_3\beta_4)^2}{2}\right] a + (\beta_1\beta_2\beta_3\beta_4) b\right\}^2 E$$

or in general:

$$\bar{\beta}^n = \left\{\left[\frac{(\beta_1\beta_2)^2 + (\beta_3\beta_4)^2}{2}\right] a + (\beta_1\beta_2\beta_3\beta_4) b\right\}^{n/4}$$

from which:

$$\bar{\beta} = \left\{\left[\frac{(\beta_1\beta_2)^2 + (\beta_3\beta_4)^2}{2}\right] a + (\beta_1\beta_2\beta_3\beta_4) b\right\}^{1/4}.$$

We desire now to ascribe suitable values to "a" and "b". Consider first the case of a sound source at the center. We have assumed that for the first four reflections the (a) portion of sound will be very nearly all confined to opposite walls. Now in Fig. 4, if $A = 1/5S$, all sound within the angle θ would strike walls 1, 2 at least three times before striking walls 3,4 (if there were no dispersion). For an angle $\theta/2$ the sound would strike walls 1,2 at least six times before striking walls 3,4. The angle θ includes approximately 1/5 of the sound and "a" will be assumed to be 1/5 and "b" to be 4/5. If we assume a source at any one of the corners, the values of "a" and "b" would not differ widely from those just chosen.

We may now consider the more general case of a cube having sides 1,2, — 3,4, and 5,6. (Fig. 5) We will assume that some energy will strike all six sides in succession. Let this portion of the total energy be c. There will be three possible paths for the "a" portion of energy which strikes opposite sides. They are between 1,2, — 3,4, and 5,6. There will also be three possible paths for the "b" portion of energy which strikes four sides. They will be 1,2,3,4 — 1,2,5,6 and 3,4,5,6. The "c" portion of energy will strike 1,2,3,4,5,6. To assume thorough mixing after every four reflections is inconsistent with the assumption of six successive reflections for the "c" portion of energy. If we assume thorough mixing after every two reflections it would be impossible to have either four or six successive reflections. Let us assume, however, that we have thorough mixing after every two reflections and that the b and c portion merely starts out as though to strike four or six walls but that thorough mixing occurs after two reflections. The formula for $\bar{\beta}$ would then be:

$$\bar{\beta} = \left\{\frac{(\beta_1\beta_2 + \beta_3\beta_4 + \beta_5\beta_6)}{3} a + \frac{1}{12}(\beta_1\beta_3 + \beta_1\beta_4 + \beta_1\beta_5 + \beta_1\beta_6 + \beta_2\beta_3 + \beta_2\beta_4 + \beta_2\beta_5 + \beta_2\beta_6 + \beta_3\beta_5 + \beta_3\beta_6 + \beta_4\beta_5 + \beta_4\beta_6)(b + c)\right\}^{1/2} \quad (2)$$

If, in the same manner, we assume thorough mixing after every four reflections the formula becomes:

$$\bar{\beta} = \Bigg\{\left[\frac{(\beta_1\beta_2)^2 + (\beta_3\beta_4)^2 + (\beta_5\beta_6)^2}{3}\right]a + \left(\frac{\beta_1\beta_2\beta_3\beta_4 + \beta_1\beta_2\beta_5\beta_6 + \beta_3\beta_4\beta_5\beta_6}{3}\right)b + \frac{1}{12}(\beta_1\beta_2\beta_3\beta_5 + \beta_1\beta_2\beta_3\beta_6 + \beta_1\beta_2\beta_4\beta_5 + \beta_1\beta_2\beta_4\beta_6 + \beta_1\beta_3\beta_4\beta_5 + \beta_1\beta_3\beta_4\beta_6 + \beta_1\beta_3\beta_5\beta_6 + \beta_1\beta_4\beta_5\beta_6 + \beta_2\beta_3\beta_4\beta_5 + \beta_2\beta_3\beta_4\beta_6 + \beta_2\beta_3\beta_5\beta_6 + \beta_2\beta_4\beta_5\beta_6)c\Bigg\}^{1/4} \tag{3}$$

where the c portion is assumed to start off as though to strike all six sides but is thoroughly mixed after striking four sides.

Now, if we assume thorough mixing after every six reflections, a slight complication arises relative to the b portion of energy. For thorough mixing after every four reflections the formulae for just the b portion of energy produces:

$$\frac{(\beta_1\beta_2\beta_3\beta_4 + \beta_1\beta_2\beta_5\beta_6 + \beta_3\beta_4\beta_5\beta_6)}{3}\,b.$$

While if we assume eight reflections between mixing we have for just this portion of energy:

$$\frac{[(\beta_1\beta_2\beta_3\beta_4)^2 + (\beta_1\beta_2\beta_5\beta_6)^2 + (\beta_3\beta_4\beta_5\beta_6)^2]b}{3}.$$

Now for six reflections the energy would strike four sides and then there would be two reflections left over which might come in any order. This would make the formulae unnecessarily involved. If we observe that for four reflections we have $\beta_1\beta_2\beta_3\beta_4$ raised to the first power, and that for eight reflections we have it raised to the second power, we may assume that for six reflections it will be weighted reasonably accurately if we raise it to the three-halves power. Our formula then becomes:

$$\bar{\beta} = \Bigg\{\left[\frac{(\beta_1\beta_2)^3 + (\beta_3\beta_4)^3 + (\beta_5\beta_6)^3}{3}\right]a + \left[\frac{(\beta_1\beta_2\beta_3\beta_4)^{3/2} + (\beta_1\beta_2\beta_5\beta_6)^{3/2} + (\beta_3\beta_4\beta_5\beta_6)^{3/2} + b}{3}\right] + [\beta_1\beta_2\beta_3\beta_4\beta_5\beta_6]c\Bigg]^{1/6}.$$

Now, referring back to the case of mixing after every four reflections, if in order to obtain a simpler formulae, we assume mixing after every four reflections for the a and b portion of energy and after every six reflections for the c portion of energy; but then weight this portion for every four reflections in a fashion similar to that just employed above, we shall then have:

$$\bar{\beta} = \left\{\left[\frac{(\beta_1\beta_2)^2 + (\beta_3\beta_4)^2 + (\beta_5\beta_6)^2}{3}\right]a + \left[\frac{\beta_1\beta_2\beta_3\beta_4+\beta_1\beta_2\beta_5\beta_6+\beta_3\beta_4\beta_5\beta_6}{3}\right]b + (\beta_1\beta_2\beta_3\beta_4\beta_5\beta_6)c^{4/6}\right\}^{1/4}. \quad (4)$$

If we assume thorough mixing after every single reflection, our formula reduces as we have seen in previous cases to that of the customary arithmetic average or

$$\bar{\beta} = \frac{\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6}{6}. \quad (1)$$

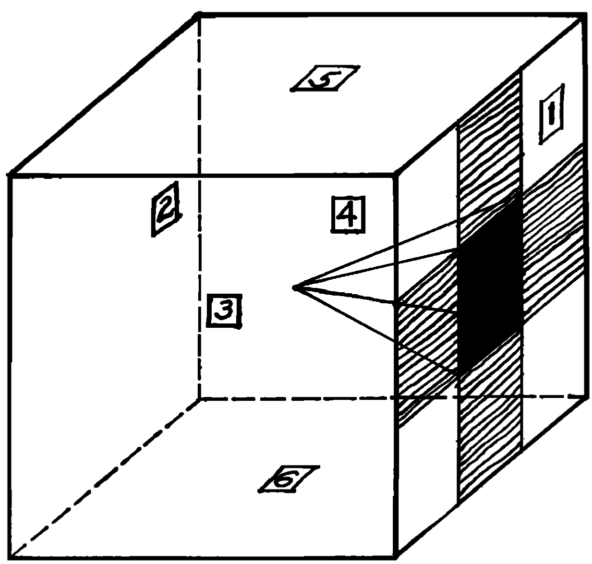

FIG. 5.

We now seek suitable values for a, b, c for the case of a cube. Very much the same reasoning as in the preceding case may be applied. Where we assume four reflections, the angle θ will be assumed the same as that in the two dimensional case so that A is again $1/5$ S. Fig. 5. The energy striking the small black square on the first reflection will be confined principally to surfaces 1 and 2, for the succeeding three reflections. This portion of the total energy will be (for all sides) 1/25. The energy striking the shaded rectangles will be principally confined to four sides and its portion of the total will be 8/25. The remaining energy 16/25 will strike all six sides. For the case where we assume but two reflections, it is apparent that the "a" portion of energy should be somewhat larger. If A/S is 1/2 then about one-half of the energy striking the small black square on the first reflection would strike the opposite walls at least once before striking adjacent walls. Let us take $A/S=1/2$. Then the "a" portion of energy will be 1/4 while the b and c portions will total 3/4.

For the case where we are using 6 reflections, it is apparant that the "a" portion of energy should be somewhat smaller. We will take $A/S=1/10$. Then $a=1/100$, $b=18/100$ and $c=81/100$.

Let us now calculate a few values of $\bar{\beta}$ by means of the five preceding formulae, remembering that formula 1 is the customary method of averaging.

We shall then have the following table (Table II):

Before discussing this table let us consider what is implied by the statement of "thorough mixing." Where this is assumed to occur for every reflection, as in equation (1), it means that for any element of sound reflected from a given area the probability is just as great that the sound will, on the next reflection, return and be reflected from that same area as that it will be reflected from any other equal area. While this is the assumption behind the statement of "thorough mixing" it is obvious that a milder statement will suffice to procure equation (1). So far as the average reflection coefficient is concerned, it can make no difference whether the sound returns to the area from which it was reflected or whether it returns to some other area having the same reflection coefficient. We therefore conclude that behind equation (1) lies the assumption that sound which is reflected from material having a given reflection coefficient has a probability of encountering, on its next reflection, material with a like coefficient which is equal to the ratio of the area of that material to the total area present.* To fulfill this requirement it would be preferable to distribute the absorbing materials, in so far as possible, uniformly over the six walls of the room although the "edge effect" of the material, under these conditions, would proba-

* This assumption may also be arrived at directly from the assumption that we have "diffuse sound" in a room.

TABLE II

Case No.	Individual Reflection Coefficients						Average Reflection Coefficients by formula					Average Absorption Coefficients by formula					Average Absorption Values	% difference between
	β_1	β_2	β_3	β_4	β_5	β_6	1	2	3	4	5	1	2	3	4	5	of 1, 2, 3, 4, 5	1 and the average
1	1	1	1	1	1	1	1.00	1.00	1.00	1.00	1.00	0	0	0	0	0	0	0%
2	0	1	1	1	1	1	.83	.820	.765	.602	.635	.170	.180	.235	.398	.365	.270	37.0
3	0	0	1	1	1	1	.67	.646	.602	.602	.635	.330	.354	.398	.398	.365	.369	10.6
4	0	0	0	1	1	1	.50	.455	.340	.340	.386	.500	.545	.660	.660	.614	.596	16.1
5	0	0	0	0	1	1	.33	.288	.340	.340	.386	.670	.712	.660	.660	.614	.663	1.0
6	0	0	0	0	0	1	.17	0	0	0	0	.830	1.00	1.00	1.00	1.00	.966	14.1
7	0	1	0	1	1	1	.67	.630	.507	.340	.386	.330	.370	.493	.660	.614	.493	33.1
8	0	1	0	1	0	1	.50	.432	0	0	0	.500	.568	1.00	1.00	1.00	.938	46.7
9	.5	.5	.5	.5	.5	.5	.50	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	0.0
10	.5	1	1	1	1	1	.92	.910	.900	.895	.891	.083	.090	.100	.105	.109	.097	14.8
11	.5	.5	1	1	1	1	.83	.830	.820	.814	.808	.170	.170	.180	.186	.192	.180	5.6
12	.5	.5	.5	1	1	1	.75	.741	.725	.720	.715	.250	.259	.275	.280	.285	.270	7.4
13	.5	.5	.5	.5	1	1	.67	.660	.652	.645	.640	.330	.340	.348	.355	.360	.347	4.9
14	.5	1	.5	1	.5	1	.75	.739	.718	.707	.707	.250	.261	.282	.293	.293	.276	9.4
15	.2	.2	.2	.8	.8	.8	.50	.486	.450	.436	.431	.500	.514	.550	.564	.569	.539	7.3
16	.2	.8	.2	.8	.2	.8	.50	.477	.433	.400	.400	.500	.523	.567	.600	.600	.558	10.4

bly be serious. It is apparent that this requirement is far from satisfied in reverberation chambers when the material is all placed on one wall as is customary. The probability that the sound which leaves this material will strike it again on the next reflection will probably be zero unless mixing panels of some sort are employed. The argument that this requirement might be satisfied after the second reflection instead of the first, leads one to equation two instead of to equation one.

Let us now turn our attention to a study of the table. We shall see that, with few exceptions, for a given room condition the values of the absorption coefficients either increase or remain constant with the different formulae in the following order, 1,2,3,4,5. Now we will recall that the number of reflections between "mixings" for the various formulae was as follows:

Formulae	Reflections
1	1
2	2
3	4) different
4	4) forms
5	6

This means that if the value of the individual absorption coefficients were given, and if from these the average absorption coefficients were calculated for a cube, formulae 1 would give values smaller than would any of the other formulae, and formula 5 would give the largest values. In those cases where the absorption coefficients over the six walls of the cubical room are the same, all formulae give the same results. As we depart from this condition of uniformity, the discrepancies between formula 1 and the other formulae increase until we have the maximum discrepancy in case 8 where non-uniformity in the distribution and magnitude of the absorption coefficients is greatest. This case, being in effect a room of but three walls, is so extreme that intuition serves as a reasonable check and we may safely say that neither formula 1 nor formula 2 applies. In the other cases, the accuracy of the various formulae must await experimental verification. However, it may be of interest to study these values, somewhat more in detail.

Consider cases 3 and 7 in which the distribution of the absorbing material is varied but the total amount remains fixed. In case 3 the absorbing material is placed on opposite walls while in 7 it is placed on adjacent walls. For all formulae except 1, case 7 shows a higher average absorption than case 3. Formula 1 shows the same average absorption for these two cases. Cases 4 and 8, cases 12 and 14, and cases 15 and 16 all show, in the same way, a higher absorption for absorbing material placed on adjacent walls as compared with opposite walls. The conclusion to be derived from all formulae except 1, therefore, is that absorbing material is more effective when placed on adjacent walls than when placed on opposite walls.

While this is the conclusion directly to be inferred from the data, it appears that the statement may be rephrased to bring out more clearly the physical situation which accounts for this conclusion. We observe, for example, that if two highly reflecting surfaces are so disposed in a room as to have opposite them two highly absorbing surfaces, the sound for the most part strikes alternately absorbing and reflecting surfaces. If, on the other hand, these four surfaces are so disposed in the room as to have the two highly reflecting surfaces opposite each other, and the two absorbing surfaces opposite each other, then a portion of the sound will strike, for the most part, reflecting surfaces and will not be readily absorbed, and another portion will strike, for the most part, absorbing surfaces and will be readily absorbed. The calculations indicate that in the second case the material is less effectively distributed due to the slow decrease in the energy from the two reflecting surfaces and that, to obtain the most effective disposition, absorbing material should be placed opposite reflecting material so the possibility that a portion of the sound energy will be confined to reflecting surfaces is reduced to a minimum.

So far, we have considered only the situation in which the individual absorption coefficients were given and the average was to be determined. It may be well, for completeness, to consider also the situation in which the periods of reverberation are given for a room under certain conditions and the value of the individual absorption coefficients are to be determined.

This calculation involves the use of the formulae for the decay of sound. Two such formulae are in use. That due to Wallace Sabine is:

$$E_t = E_0 e^{-tv(s\bar{\alpha})/4V}.$$

In this formulae $\bar{\alpha} = 1 - \bar{\beta}$ and $\bar{\beta}$, the average value of the reflection coefficient, may be determined from any one of the formulae 1,2,3,4,5, that have been developed in this paper. The second decay formulae was first brought to the author's attention by R. F. Norris, of the Burgess Laboratory, and was subsequently developed at great length by C. F.

Eyring* of the Bell Telephone Laboratories. This formula takes the form:

$$E_t = E_0 e^{-tv(-s \log \bar{\beta})/4V}$$

where $\bar{\beta}$ may again be determined by any one of the formulae 1,2,3,4, or 5, that have been developed. It is to be noted that $(-S \log \bar{\beta})$ in Eyring's formula is equivalent to $(S\bar{\alpha})$ in Sabine's formula. Let us suppose that the period of reverberation of a cubical room has been determined under three conditions. Let each dimension of the cubical room be 20 feet.

Condition 1 represents the empty sound chamber. The walls are supposedly concrete, having an absorption coefficient of α_0 and the period of reverberation is determined to be 6 seconds.

Condition 2 represents the sound chamber with a test sample having an absorption coefficient α_1 completely covering one wall. The period of reverberation is now determined to be 1.0 seconds.

Condition 3 represents an installation using this test sample. To simplify things this room is assumed to have the dimensions of the sound chamber. The sample is assumed to cover the three adjacent walls. The other three walls are assumed to have the same absorption coefficient as the empty sound chamber.

If now we calculate the absorption coefficients $\bar{\alpha}_0$ and $\bar{\alpha}_1$ from conditions 1 and 2 and if, from these values, we determine the period of the room under condition 3 we will arrive at the values t_p indicated in Table 3. In this table the numerals 1, 2, 3, 4, 5, appearing in the column headed

TABLE III

Formulae	α_0	α_1	t_p	t_p'
Sabine 1	.0123	.383	.376	.176
Eyring 1	.0280	.803	.318	.318
Eyring 2	.0280	.752	.316	.285
Eyring 3	.0280	.676	.322	.235
Eyring 4	.0280	.615	.346	.206
Eyring 5	.0280	.607	.355	.206

Formulae, refer to the methods of averaging which have been developed in this paper.

It is interesting to note in considering table 3 that the Sabine formulae

* Reverberation time in "dead" rooms. C. F. Eyring, J. Acoustical Soc. Am. 1: 217–241 (1930).

and the weighted arithmetic average give a value of α_1 which is low and a calculated reverberation time which is high compared with the other formulae. Just the reverse is true of the Eyring formulae and the arithmetic average. The other formulae result in values between these extremes. It is also interesting to note that, although the absorption coefficients calculated on this basis differ widely, the periods of reverberation do not differ very much. Thus there is a certain partial compensation attained when the same formula is used both for determining the coefficient of the material and for determining the resultant time of reverberation of the room. If the absorption coefficient had been assumed the same for each case the periods of reverberation would have differed by much larger amounts. This calculation is given in the last column of Table 3. Here the period of reverberation is calculated using the various formulae but keeping the values of α_0 and α_1 fixed. The values of α_0 and α_1 are respectively .0280 and .803, the values obtained in the second row of Table 3.

In conclusion, it may be well to emphasize the obvious limitations of the present paper. A method of attack has been developed yielding a number of formulae for the particular case of a cube. It is obvious that the same method of attack may be employed for other simple geometric shapes. In particular, it is hoped that the method of attack will be applicable to sound chambers, since they are generally of simple geometric shape. The various formulae purport to account, in varying degrees, for the effect of dispersion. In addition, they contain other arbitrary constants such as a, b, c. It is hoped that these constants may turn out to be useful parameters and that the accumulation of data will assign reasonable values to these parameters, which will permit the extension of these formulae to rooms of more complicated geometrical shape. It would be quite feasible to adopt now such assumptions as would permit of greater generalization and simplification of the preceding formulae. For example, the discussion pertaining to Fig. 3 indicates that where the walls of the cube are made up of various materials it is permissible to obtain an average coefficient for this wall by adopting the ordinary arithmetic average for the materials composing the wall. However, it appears to the writer that such generalizations may far better await the accumulation of more extensive and more precise data. It is largely with the hope of showing the need for such data and of stimulating the search for it that this paper has been presented.