Nis Bjørn Møller

M.Sc. Danish Technical University

- Acoustics
- Servo Techniques

Professional Sound Reproduction

Brüel & Kjær since 1969

- Structural Dynamics
- Machine Diagnostics
- Vibration testing

Author of many Conference Papers

- International Modal Analysis Conference (IMAC)
- SAE – USA – Brazil
- …….

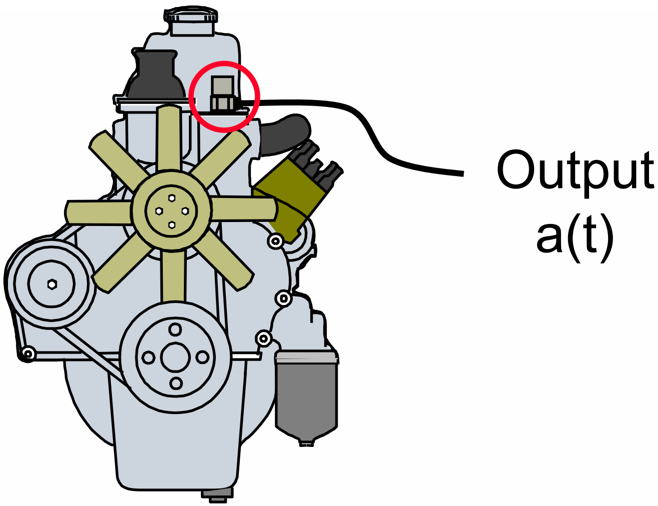
System Analysis

System Analysis

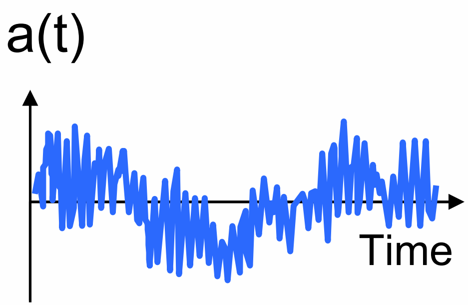
- Introduction
- System Descriptors
- Dual-channel FFT Analysis
- Cross Spectrum
- Coherence Function
- Frequency Response Function

Signal Analysis

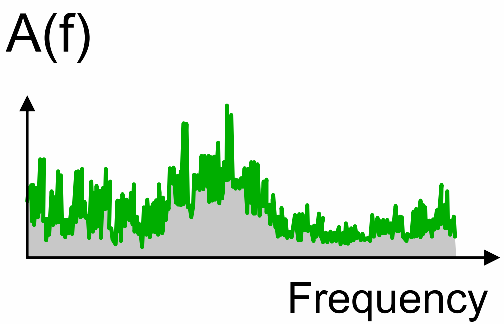
Definition



Time signal

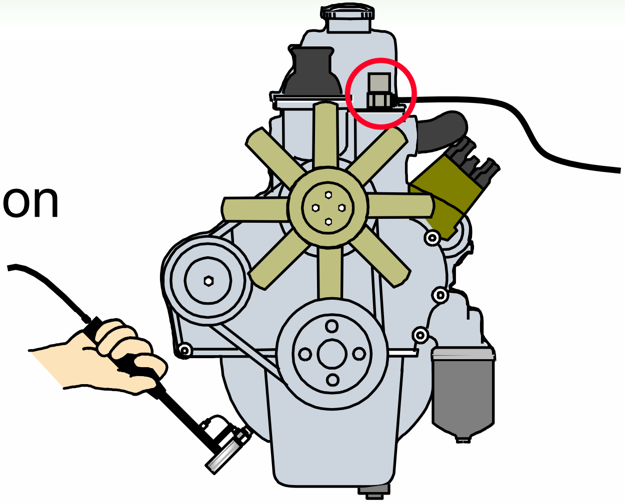


Frequency spectrum



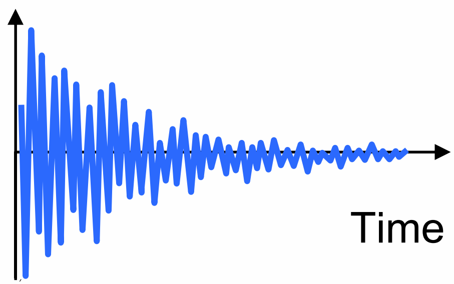
System Analysis

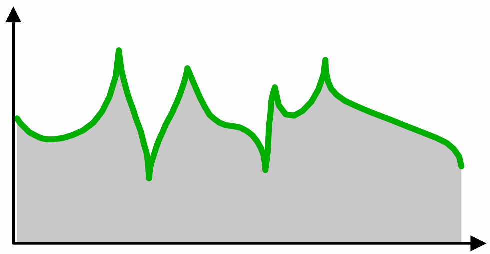
Definition



Impulse Response Function

Frequency Response Function





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System Descriptors

Impulse Response Function

$$a(t) \longrightarrow \boxed{h(\tau)} \longrightarrow b(t)$$

Convolution

$$b(t) = \int_{-\infty}^{\infty} h(\tau) \cdot a(t-\tau)d\tau = h(t) * a(t)$$

Frequency Response Function

$$A(f) \longrightarrow \boxed{H(f)} \longrightarrow B(f)$$

Multiplication

$$B(f) = H(f) \cdot A(f)$$

$h(\tau)$ and $H(f)$ are system descriptors *independent* of the signals involved

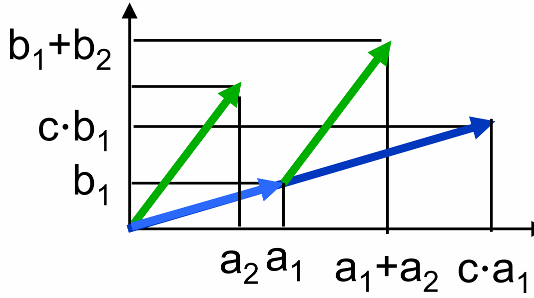
The Ideal Physical System

$$a(t),\ A(f) \longrightarrow \boxed{h(\tau),\ H(f)} \longrightarrow b(t),\ B(f)$$

Constant parameters $h(\tau, t) = h(\tau)$

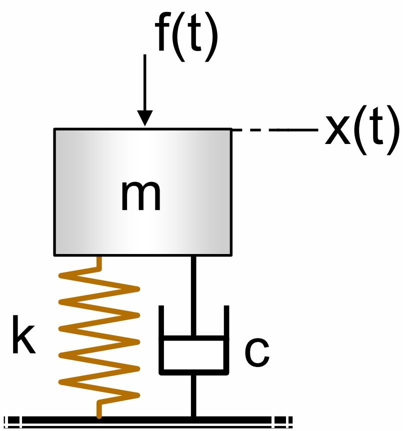
$$H(f,t) = H(f)$$

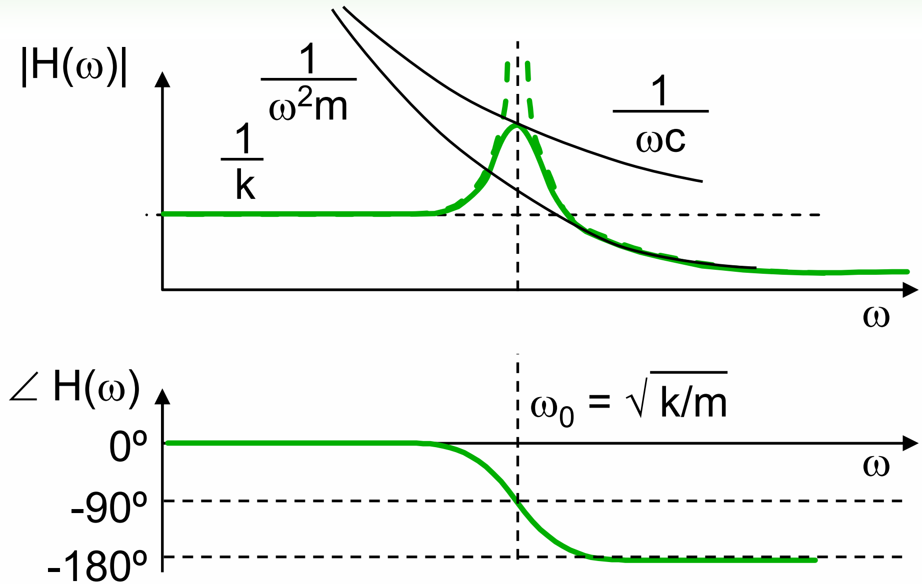
Linearity	If	$a_1(t)$	produces	$b_1(t)$
	and	$a_2(t)$	produces	$b_2(t)$
	then	$a_1(t)+ a_2(t)$	produces	$b_1(t)+ b_2(t)$
	and	$c \cdot a_1(t)$	produces	$c \cdot b_1(t)$



Additive and homogeneous in time and frequency domain

Frequency Response Function for SDOF system





Time domain $m\,a = f - cv - kx \Leftrightarrow m\ddot{x} + c\dot{x} + kx = f$

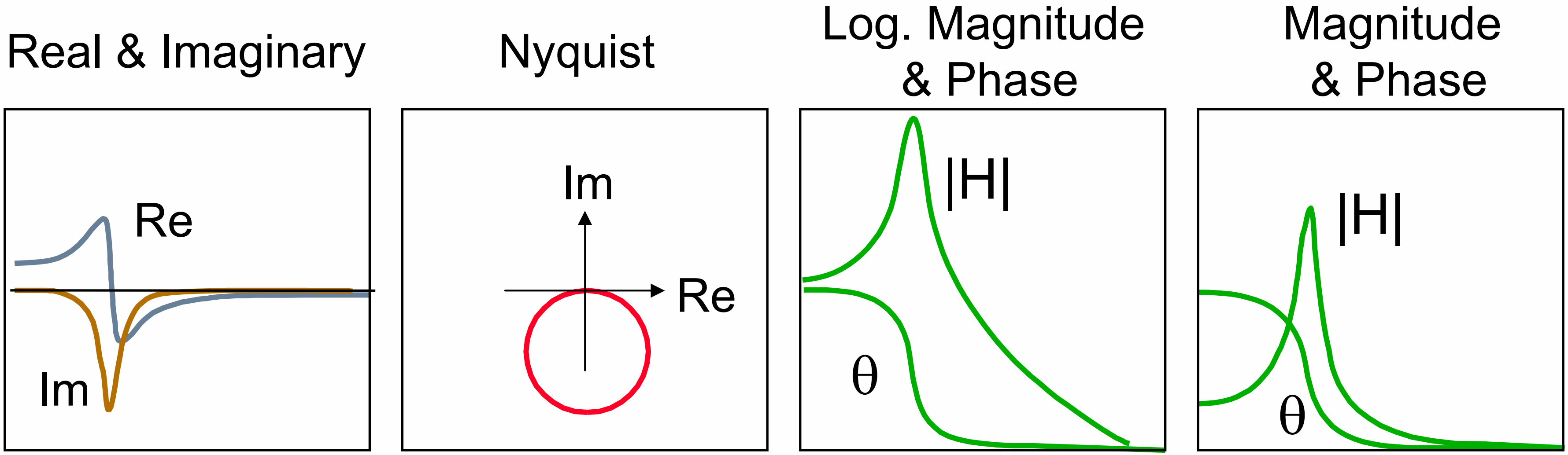
Frequency domain $$H(\omega) = \frac{X(\omega)}{F(\omega)} = \frac{1}{-\omega^2 m + j\omega c + k} = \frac{R}{j\omega - (-\sigma + j\omega_d)} + \frac{R^*}{j\omega - (-\sigma - j\omega_d)}$$

where $R = \frac{1}{j2m\omega_d}$ $\omega_d = \sqrt{\frac{k}{m} - \frac{c^2}{4m^2}}$ $\sigma = \frac{c}{2m}$

Frequency Response Function for SDOF system

$$H(\omega) = \frac{R}{j\omega - p} + \frac{R^*}{j\omega - p^*} \qquad \text{where } p = -\sigma + j\omega_d$$

$$\text{At resonance:} \quad |H(\omega)| \approx \frac{R}{\sigma} \qquad R = \text{Residue}$$



Damping Parameters

3 dB bandwidth	$\Delta f_{3dB} = \frac{2\sigma}{2\pi}$, $\Delta\omega_{3dB} = 2\sigma$
Loss factor	$\eta = \frac{1}{Q} = \frac{\Delta f_{3db}}{f_0} = \frac{\Delta\omega_{3dB}}{\omega_0}$
Damping ratio	$\zeta = \frac{\eta}{2} = \frac{\Delta f_{3dB}}{2f_0} = \frac{\Delta\omega_{3dB}}{2\omega_0}$
Decay constant	$\sigma = \zeta\,\omega_0 = \pi\,\Delta f_{3dB} = \frac{\Delta\omega_{3dB}}{2}$
Quality factor	$Q = \frac{f_0}{\Delta f_{3dB}} = \frac{\omega_0}{\Delta\omega_{3dB}}$

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Dual-channel FFT Analysis

- Simultaneous measurements at the input and output are performed
- The input and output autospectra, and the cross spectrum between the input and output are measured
- Many other functions can be calculated from the basic, measured data

Advantages of Dual-channel FFT Analysis

- Phase information is available
- Effects of noise are minimised
- A controlled input signal is not needed
- Easy to use
- Extension, for instance to Modal Analysis

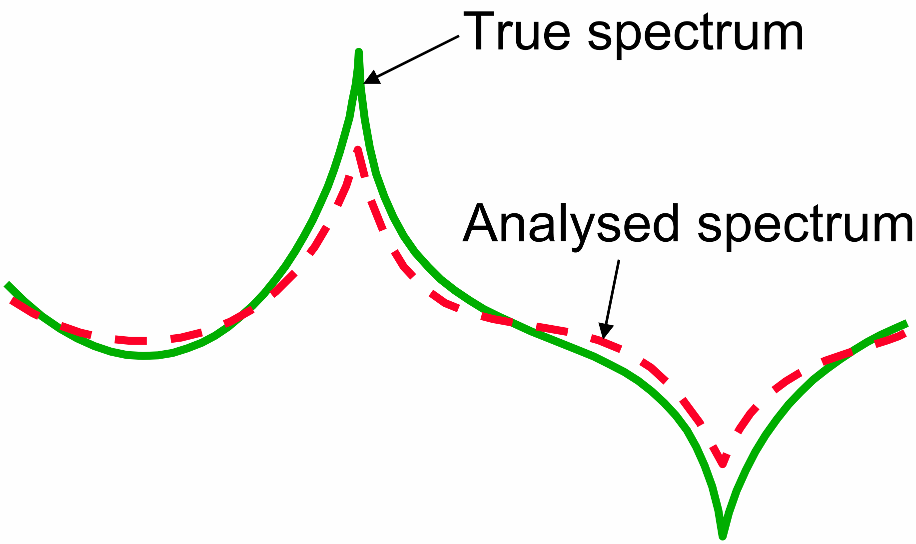
Pitfalls of Dual-channel FFT Analysis

Pitfalls of Dual-channel Analysis include:

- Leakage
- A linear system is assumed
- Compensation for system delays necessary

Leakage

- Leakage occurs because FFT analyzers operate on truncated time signals
- Leakage means that measured peaks can be too low and measured valleys too high
- To combat leakage in system analysis:
  - increase the resolution
  - use optimum time weighting function
  - choose optimum excitation

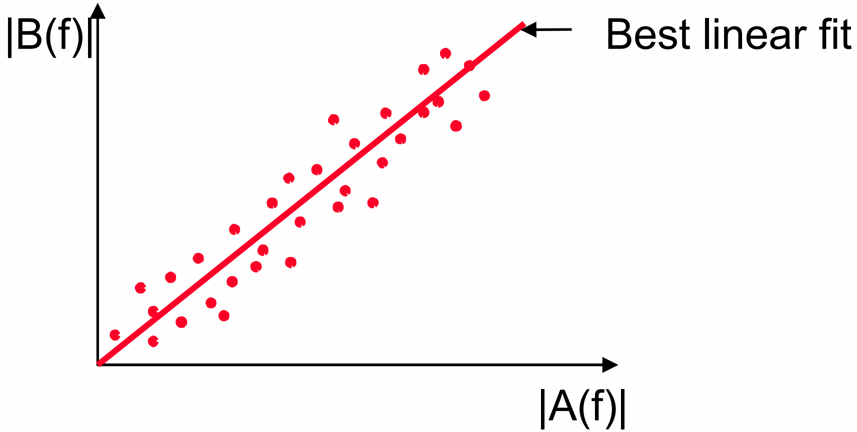


Linearisation

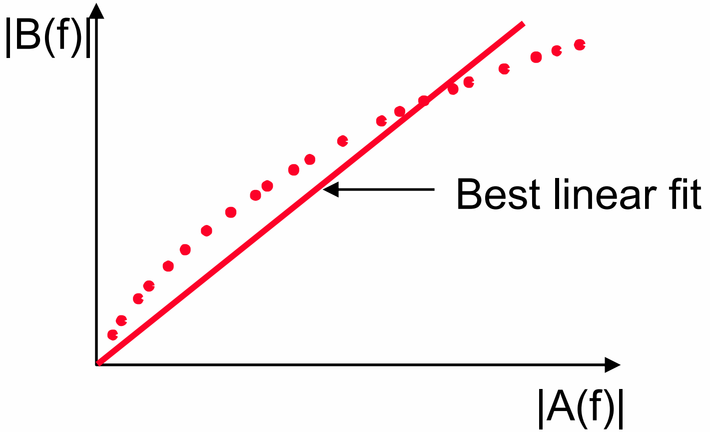
All FFT calculations assume a linear system

Using random excitation, the best linear estimate of the response functions can be obtained with a FFT analyzer

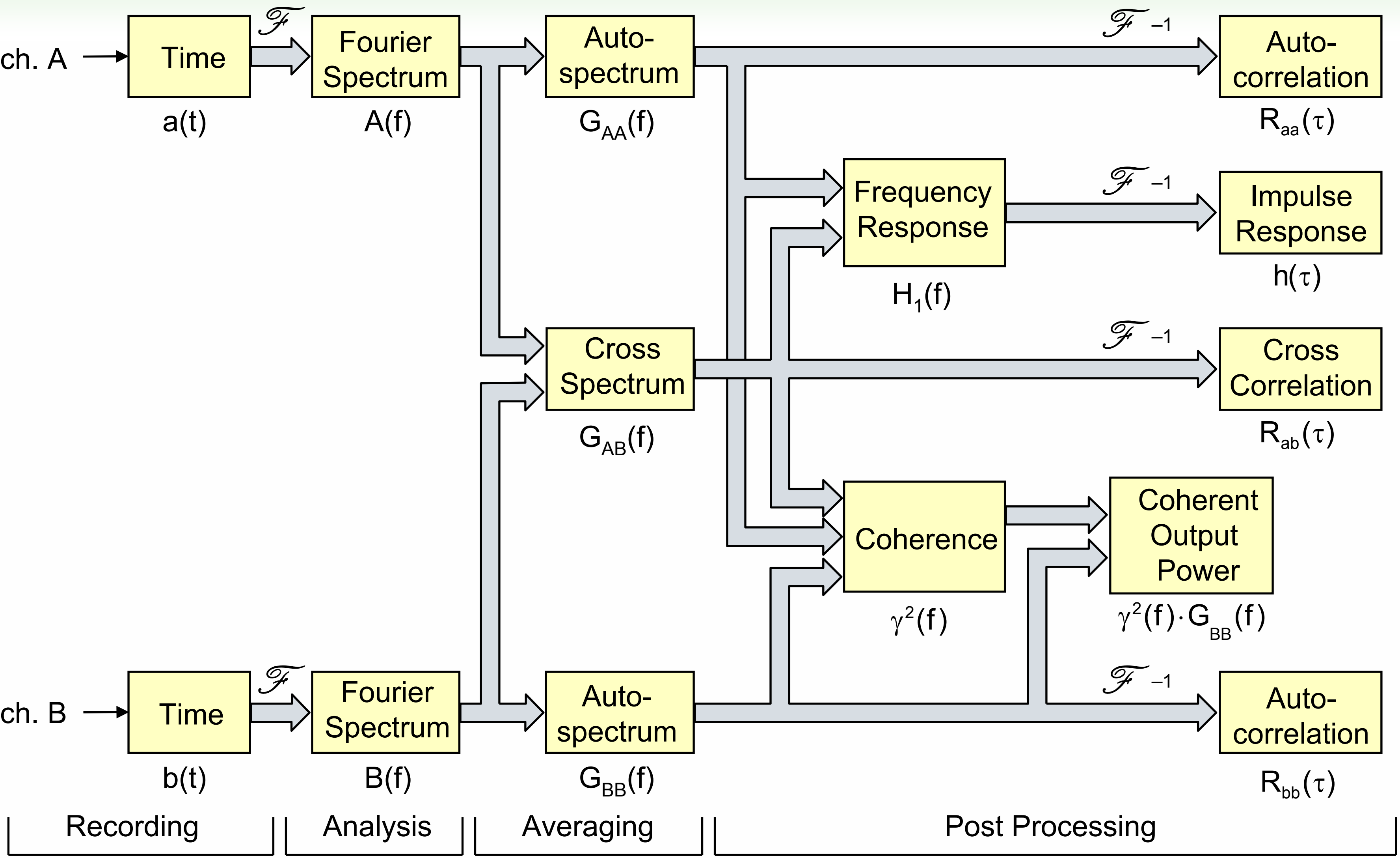
Noise



Non-linear system



Dual Channel Spectrum Averaging



System Analysis

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- Impulse Response Function
- Summary

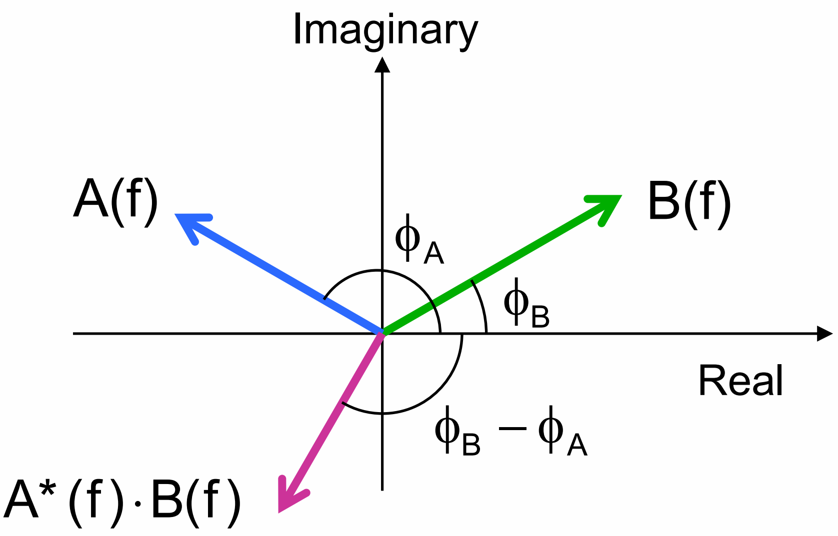
Cross Spectrum

$$S_{AB}(f) = E\left[A^*(f) \cdot B(f)\right]$$

$$A(f) = |A(f)| e^{i\phi_A(f)}$$

$$B(f) = |B(f)| e^{i\phi_B(f)}$$

$$S_{AB}(f) = E\left[|A(f)| \cdot |B(f)| e^{i(\phi_B(f) - \phi_A(f))}\right]$$



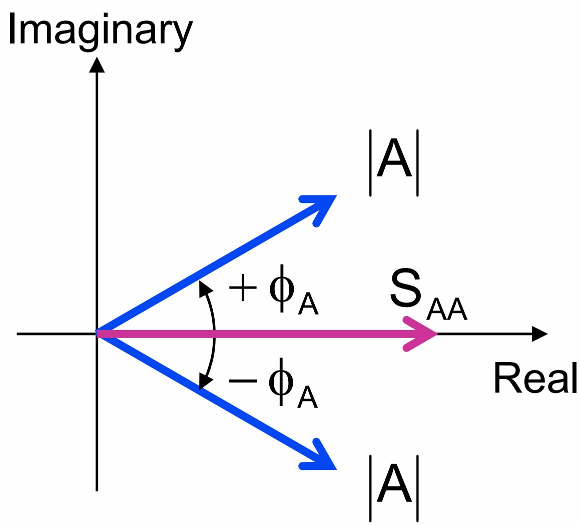
Phase of Cross Spectrum is *phase* of system

Auto Spectrum

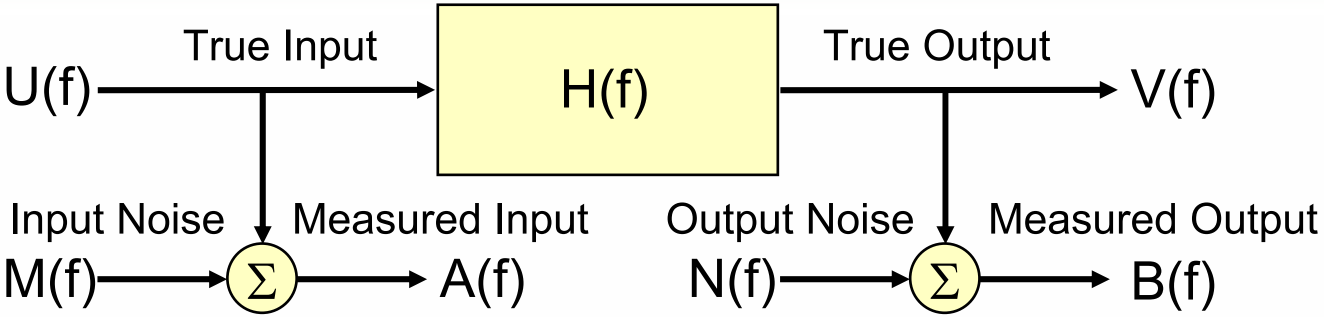
$$S_{AA}(f) = E\left[A(f)\cdot A^*(f)\right]$$
$$= \mathscr{F}\ E\left[a(t) * a(-t)\right]$$
$$= \mathscr{F}\ \left[R_{aa}(\tau)\right]$$

$$A(f) = |A(f)|e^{i\phi_A(f)} \quad A^*(f) = |A(f)|e^{-i\phi_A(f)}$$

$$S_{AA}(f) = E\left[|A(f)|\cdot|A^*(f)|\cdot e^{i0}\right]$$



Influence of Noise



Autospectrum Ch.A

$$
\begin{aligned}
S_{AA} &= E\left[(U+M)^* \cdot (U+M)\right] \\
&= E\left[(U^* \cdot U)\right] + E\left[(\cancel{U^* \cdot M})\right] + E\left[(\cancel{M^* \cdot U})\right] + E\left[(M^* \cdot M)\right] \\
&= S_{UU} + S_{MM}
\end{aligned}
$$

Autospectrum Ch.B

$$S_{AA} = S_{VV} + S_{NN}$$

Cross Spectrum

$$
\begin{aligned}
S_{AB} &= E\left[(U+M)^* \cdot (V+N)\right] \\
&= E\left[(U^* \cdot V)\right] + E\left[(\cancel{U^* \cdot N})\right] + E\left[(\cancel{M^* \cdot V})\right] + E\left[(\cancel{M^* \cdot N})\right] \\
&= S_{UV}
\end{aligned}
$$

Summary

Cross Spectrum

$$S_{AB}(f) = E\left[A^*(f) \cdot B(f)\right]$$

- Measures phase difference
- The effect of noise is reduced through averaging

Any

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The Coherence Function

- Input-Output Relationship

$$\left[\text{Cross Power}\right]^2 \leq \left[\text{Input Power}\right] \cdot \left[\text{Output Power}\right]$$

$$\left|G_{AB}(f)\right|^2 \leq \left|G_{AA}(f)\right| \cdot \left|G_{BB}(f)\right|$$

- Definition

$$\boxed{\gamma^2(f) \equiv \frac{\left|G_{AB}(f)\right|^2}{\left|G_{AA}(f)\right| \cdot \left|G_{BB}(f)\right|}}$$

- It expresses degree of *linear* relationship between A(f) and B(f)

$$\boxed{0 \leq \gamma^2(f) \leq 1}$$

Coherence vs Correlation Coefficient

- Coherence

$$\gamma^2(f) \equiv \frac{\left|G_{AB}(f)\right|^2}{G_{AA}(f)\cdot G_{BB}(f)} \qquad 0 \leq \gamma^2(f) \leq 1$$

- Correlation Coefficient

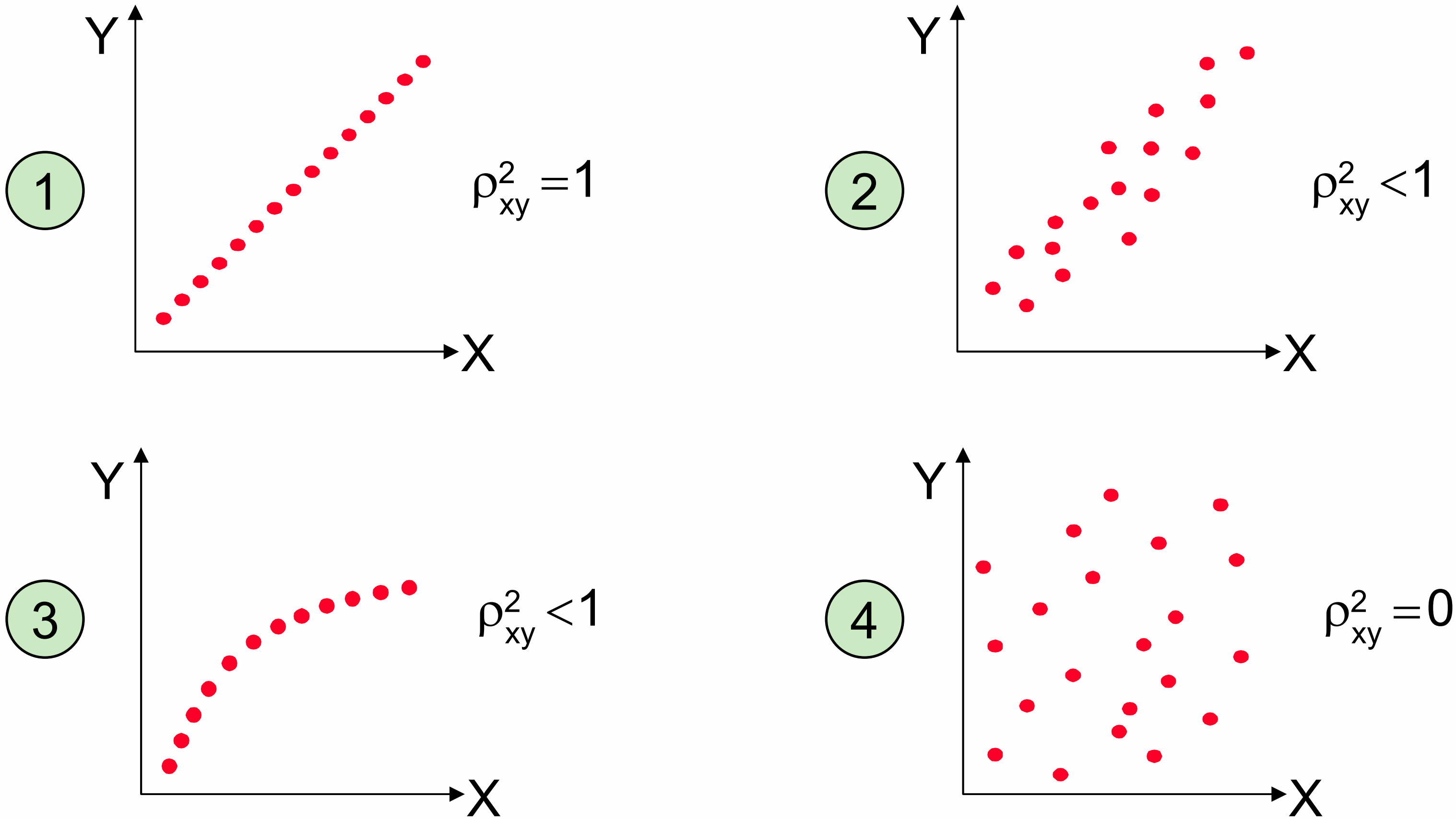
$$\rho_{xy}^2 \equiv \frac{\sigma_{xy}^2}{\sigma_x^2 \cdot \sigma_y^2} \qquad 0 \leq \rho_{xy}^2 \leq 1$$

Variance ~ Autospectrum

Covariance ~ Cross Spectrum

The Correlation Coefficient

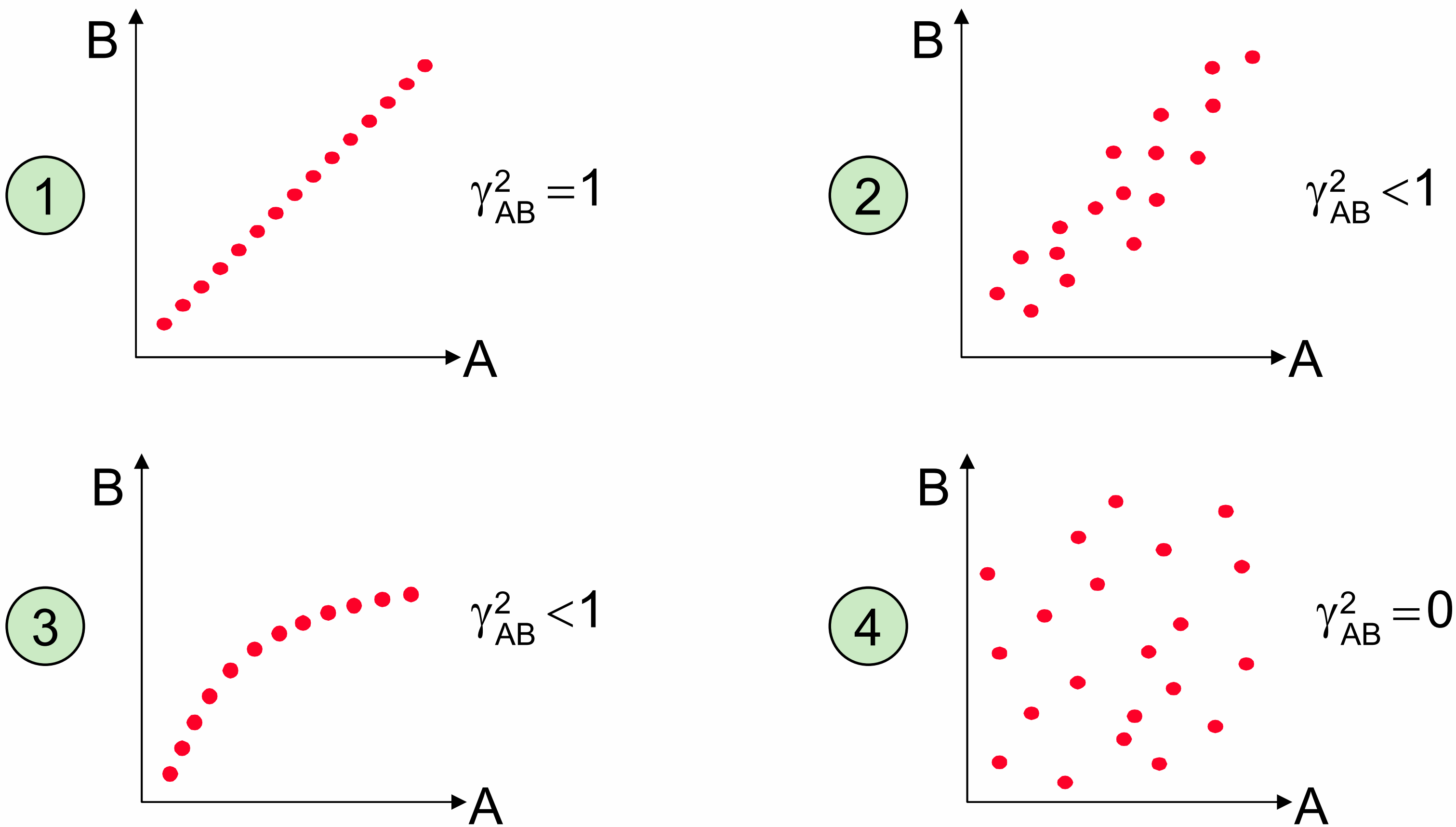
$$\rho_{xy}^2 = \frac{\sigma_{xy}^2}{\sigma_x^2 \cdot \sigma_y^2}$$



The Coherence Function

$$\gamma^2(f) \equiv \frac{\left|G_{AB}(f)\right|^2}{G_{AA}(f)\cdot G_{BB}(f)}$$

For each frequency



Averaging of The Coherence Function

$$\gamma^2(f) \equiv \frac{\left|G_{AB}(f)\right|^2}{G_{AA}(f) \cdot G_{BB}(f)}$$

- The coherence only provides useful information when $G_{AB}(f)$, $G_{AA}(f)$ and $G_{BB}(f)$ are estimates, i.e averaged over many records
- For one record only (no averaging):

$$\gamma^2(f) = 1$$

Reasons for Low Coherence

Difficult measurements:

- Noise in measured output signal
- Noise in measured input signal
- Other inputs not correlated with measured input signal

Bad measurements:

- Leakage
- Time varying systems
- Non-linearities of system
- DOF-jitter
- Propagation time not compensated for

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Definition of the Frequency Response Function

- Definition for the **ideal** system:

$$A(f) \longrightarrow \boxed{H(f)} \longrightarrow B(f)$$

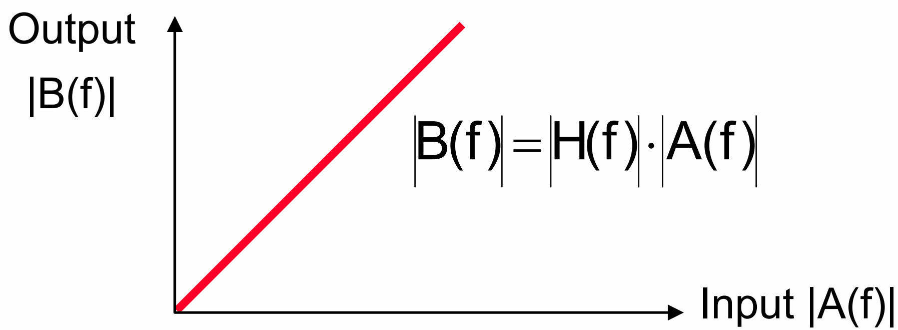
Output:

$B(f) = H(f) \cdot A(f)$

- Frequency Response Function:

$$H(f) \equiv \frac{B(f)}{A(f)}$$

- For each frequency:



H is the slope of the straight line describing the output as function of input

Alternative Estimators

$$a(t) \longrightarrow \boxed{\begin{array}{c} h(\tau) \\ H(f) \end{array}} \longrightarrow b(t)$$

$$H(f) \equiv \frac{B(f)}{A(f)}$$

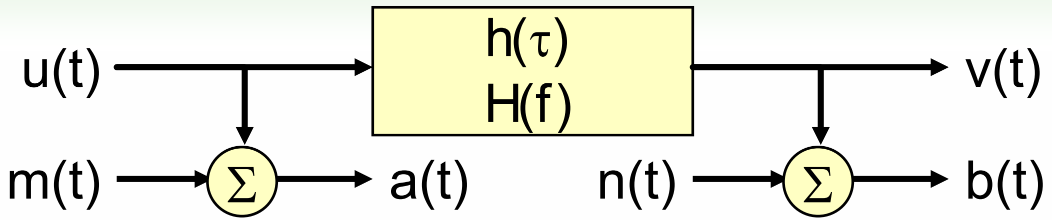
$$H_1(f) = \frac{G_{AB}(f)}{G_{AA}(f)}$$

$$H_2(f) = \frac{G_{BB}(f)}{G_{BA}(f)}$$

$$H_3(f) = \sqrt{\frac{G_{BB}}{G_{AA}}} \cdot \frac{G_{AB}}{|G_{AB}|} = \sqrt{H_1 \cdot H_2}$$

$$\gamma^2(f) = \frac{|G_{AB}|^2}{G_{AA} \cdot G_{BB}} = \frac{G_{AB}}{G_{AA}} \cdot \frac{G^*_{AB}}{G_{BB}} = \frac{H_1}{H_2}$$

Noise at Input and Output



$$\varepsilon_i = G_{MM}/G_{UU} \qquad \varepsilon_o = G_{NN}/G_{VV}$$

$$H_1 = \frac{G_{AB}}{G_{AA}} = H\frac{1}{1+\varepsilon_i}$$

$$H_2 = \frac{G_{BB}}{G_{BA}} = H\left[1+\varepsilon_o\right]$$

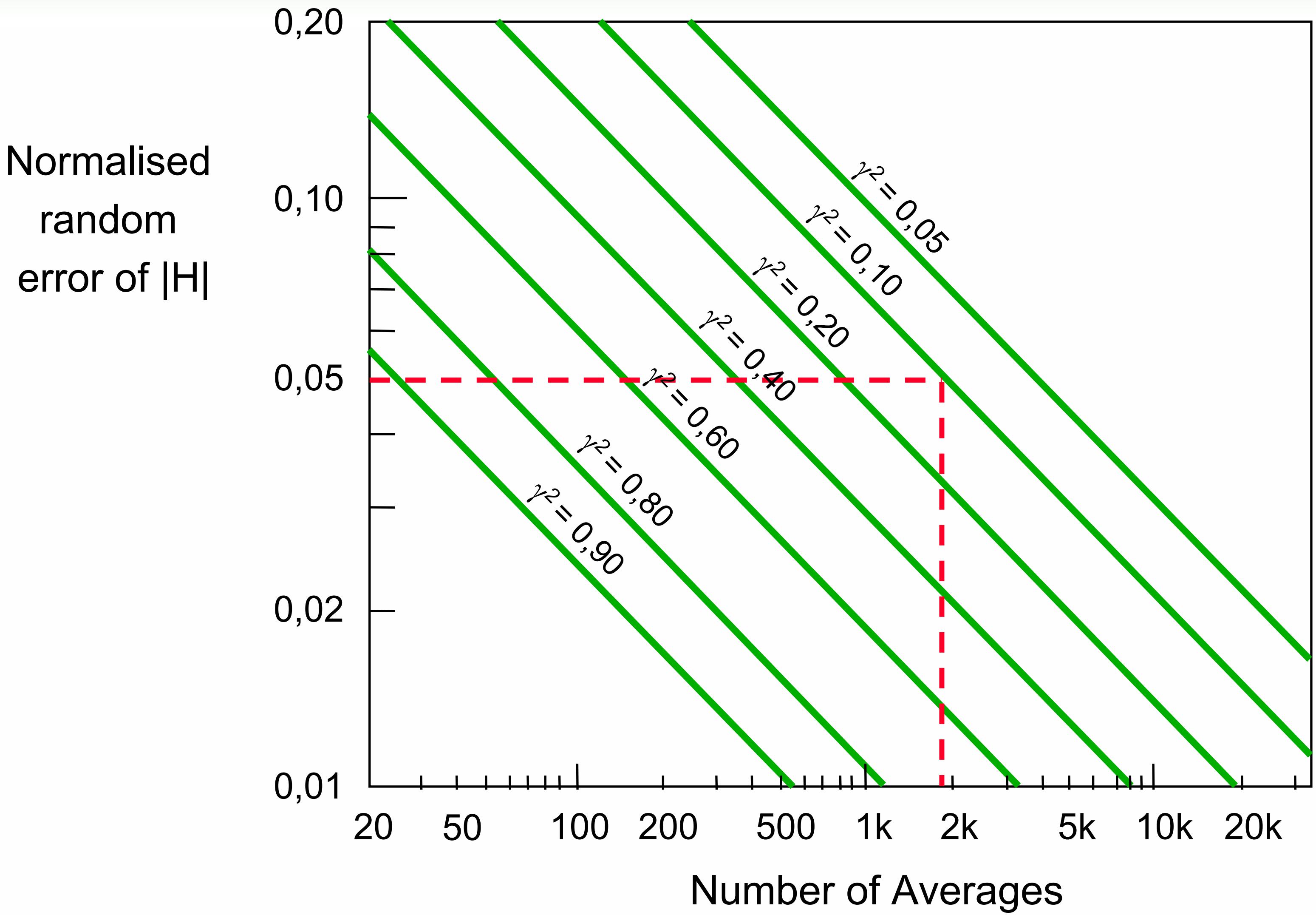
$$\left|H_3\right|^2 = \frac{G_{BB}}{G_{AA}} = \left|H\right|^2 \frac{1+\varepsilon_o}{1+\varepsilon_i}$$

$$\gamma^2 = \frac{H_1}{H_2} = \frac{1}{\left(1+\varepsilon_i\right)\left(1+\varepsilon_o\right)}$$

$$\left|H_3\right|^2 = \left|H_1\right| \cdot \left|H_2\right|$$

$$\boxed{\left|H_1\right| \le \left|H\right| \le \left|H_2\right|}$$

Noise at Input and Output



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Leakage

Signal periodic with record length

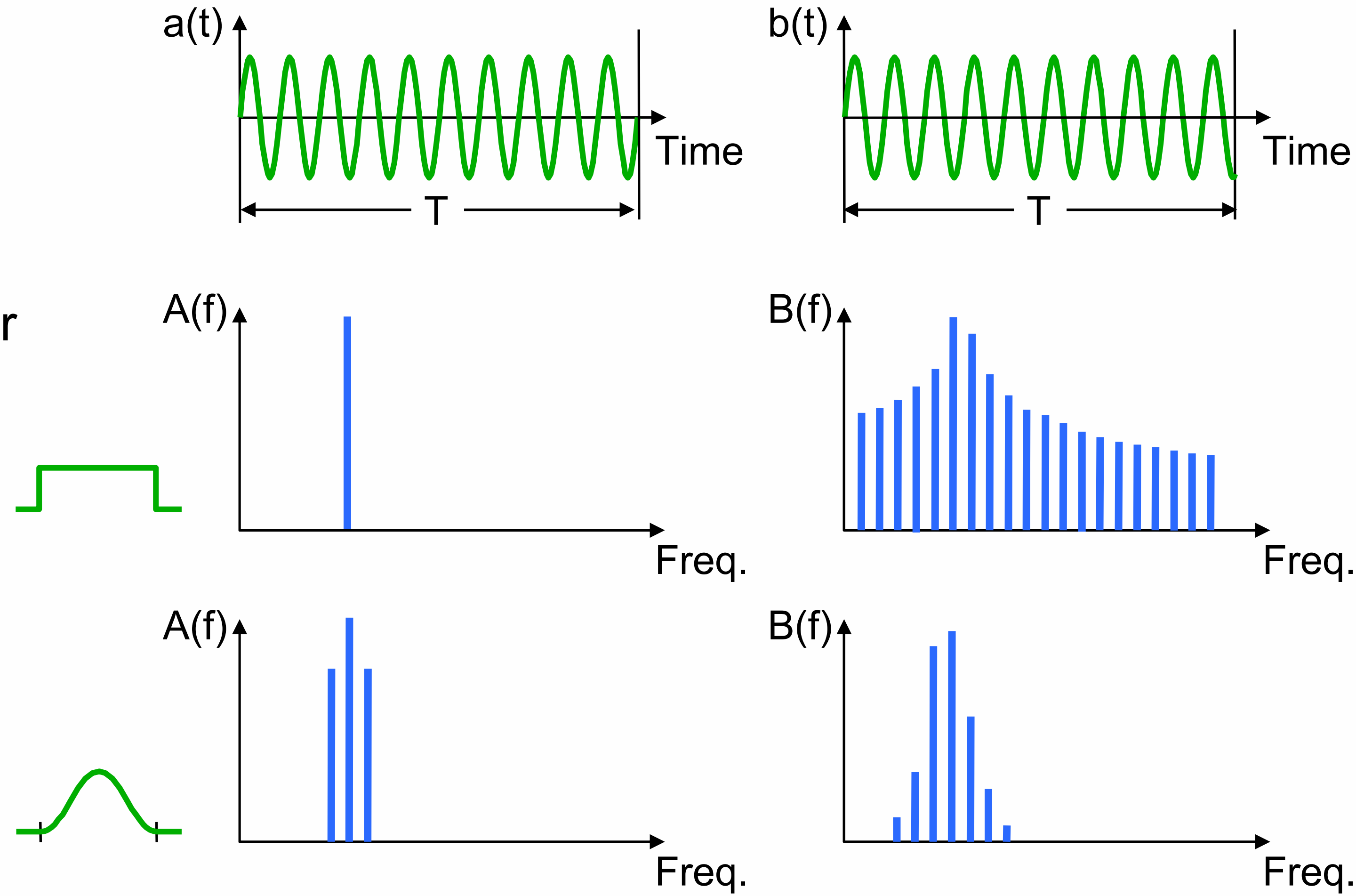
Signal **not** periodic with record length

Rectangular weighting

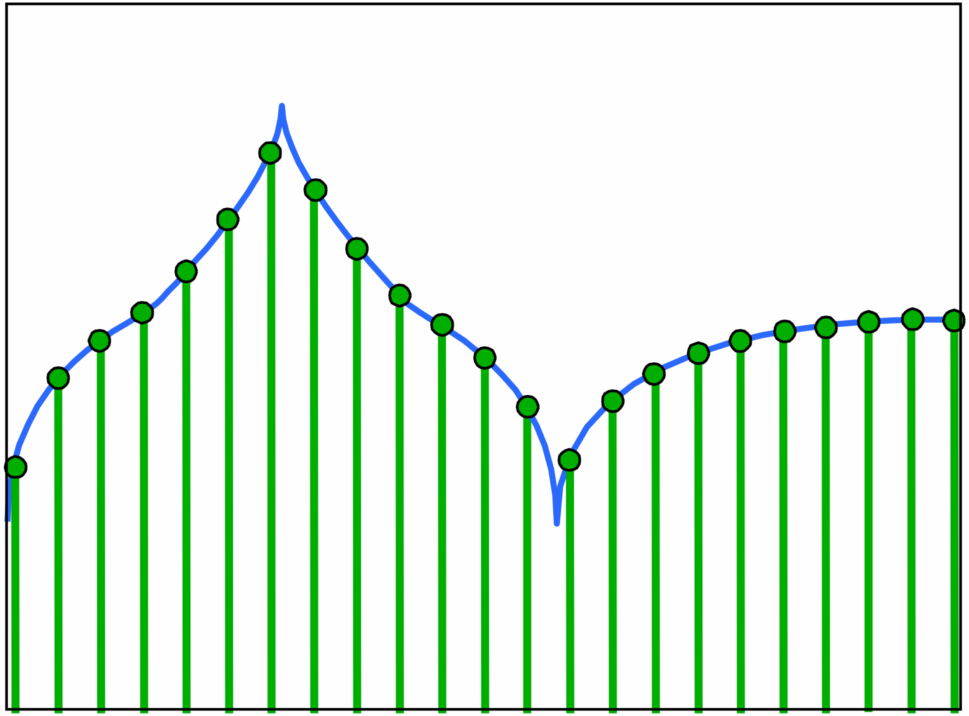
(no weighting)

Hanning weighting

$$\left(1 - \cos\frac{2\pi t}{T}\right)$$



FFT Fundamentals - Picket Fence Effect



The measured points give the correct values, but the exact frequency and magnitude of the resonance may not be found.

Frequency Response Function Estimates

Accuracy

Definitions: $H_1 \equiv \dfrac{G_{AB}}{G_{AA}}$ $\quad H_2 \equiv \dfrac{G_{BB}}{G_{BA}}$ $\quad H_3 \equiv \sqrt{\dfrac{G_{BB}}{G_{AA}}}\,\dfrac{G_{AB}}{|G_{AB}|}$

Accuracy for systems with:	H_1	H_2	H_3
Input noise	-	**Best**	-
Output noise	**Best**	-	-
Input + output noise	-	-	**Best**
Peaks (leakage)	-	**Best**	-
Valleys (leakage)	**Best**	-	-

User can choose H_1, H_2 or H_3 after measurement

Summary

- Measure both Input and Output
- Using Dual (Multi-) channel FFT
- Averaging - improving accuracy
- Leakage
  - Increase Frequency Resolution
  - Use Zoom
  - Use Time weighting
- Noise at
  - Input
  - Output

Any

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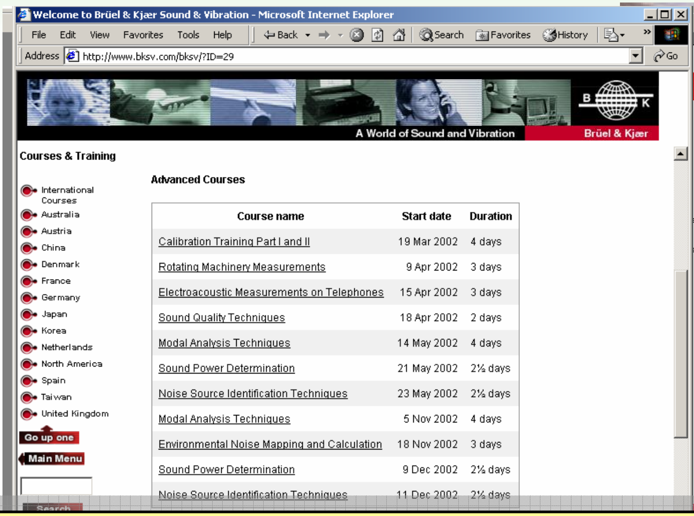
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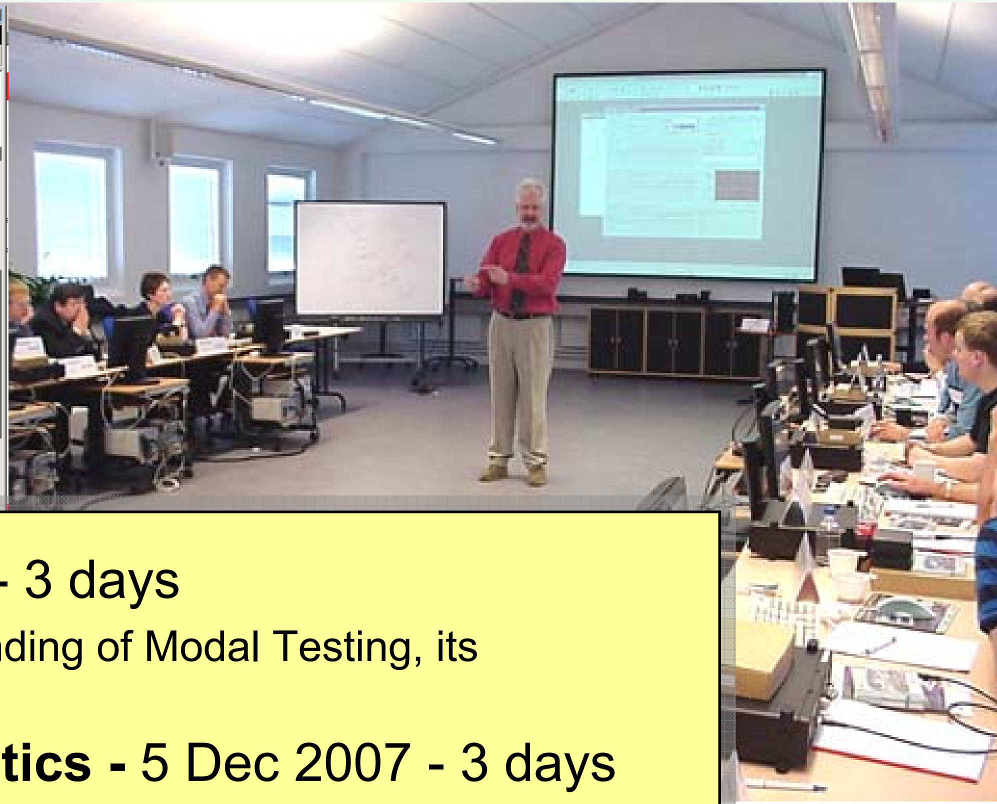

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  - **Objectives** To give an understanding of Modal Testing, its possibilities and limitations.
- **Rotating Machinery Diagnostics -** 5 Dec 2007 - 3 days
  - **Objectives** To teach and explain about the different vibration sources of rotating machinery.

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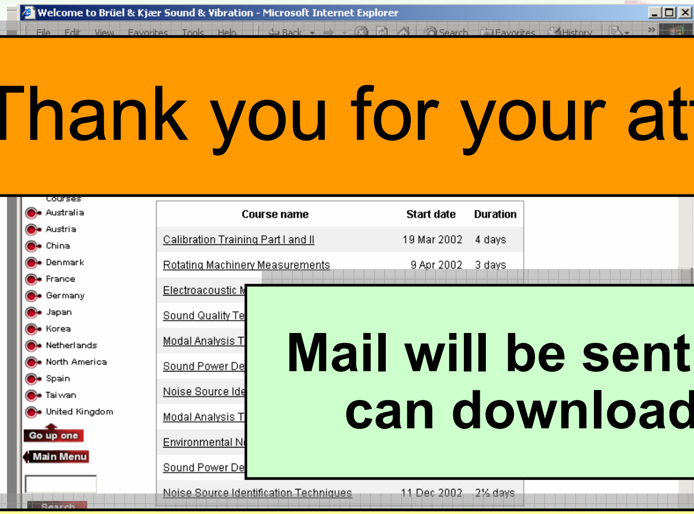

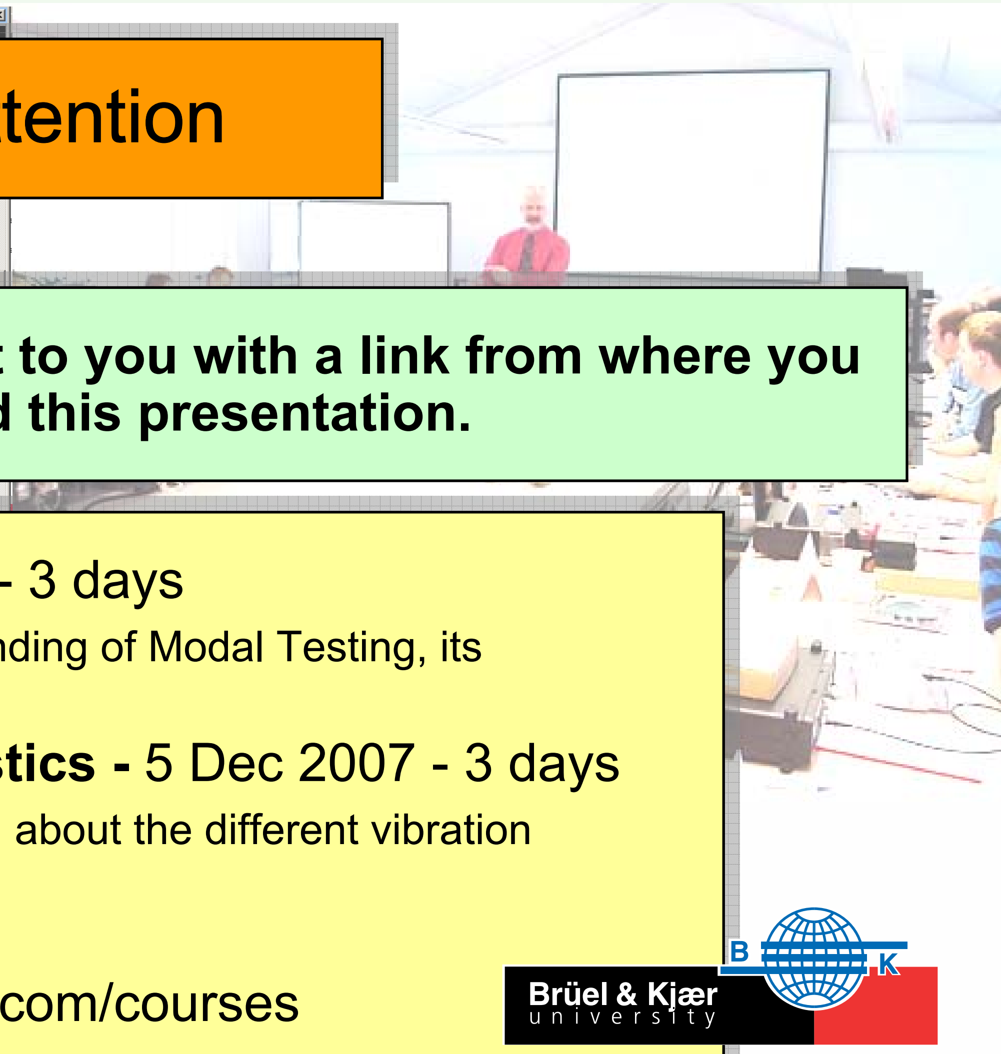

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