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Digital Simulation of Sound Transmission in Reverberant Spaces

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Digital computers, through their capability for accurate simulation of complex phenomena, have permitted new insights into a number of important problems arising in sound transmission in reverberant spaces: (1) Reverberation theories, based on the geometrical acoustics, are being refined by ray-tracing studies on digital computers. These studies have revealed significant discrepancies in existing reverberation-time formulas and unexpected large dependencies of the decay rate on the *shape* of the enclosure and the distribution of sound-absorbing materials. (2) Starting with reverberation-free speech or music signals as inputs, computers can add echoes and reverberation with specified delays, spectral content, and decay characteristics. The computer produces several output signals that—when radiated from loudspeakers in an anechoic chamber—produce, at a listener's ears, sound-pressure waves resembling those in real halls. To ensure "externalization" and proper directions of echo arrivals, the computer program is based on the measured sound diffraction around the listener's head. This digital simulation method is useful to "preaudit" architectural designs before construction and to investigate subjective correlates of a wide variety of reverberation processes. (3) Digital computers have made possible the simulation of frequency and space response of stationary sound fields and the calculation of their statistical properties. These properties are important for the design of electroacoustic systems and the evaluation of measurements in reverberant enclosures.

INTRODUCTION

Simulation is a powerful tool in helping us to understand complicated processes. It had been used to advantage long before the advent of computers. Flight simulators are a well-known instance of a successful application of simulation in training a task. In general, simulation has found particularly fruitful applications to problems characterized by the interaction of complex physical phenomena and human behavior. An example of such an interaction is the human listener in a concert hall. There are no simple "equations" to predict in detail the transmission of sound from a musical instrument on stage to the listener's ears, and there is even less quantitative information available to anticipate a listener's esthetic experience and his subjective preferences.

The art of simulation in architectural acoustics is not new. Acousticians have resorted to simulation and modeling in one form or another ever since Sabine's early experiments in 1913, in which he used ultrasonic waves and Schlieren photography to study reflections from the ceiling and walls of an acoustic scale model.[1] The art of acoustic model testing has since seen considerable advances, both in the Uniter States and abroad.[2–14]

[1] W. C. Sabine, "Theater Acoustics," reprinted in *Collected Papers on Acoustics* (Dover Publications, Inc., New York, 1964) pp. 180 ff.

[2] V. O. Knudsen, *Architectural Acoustics* (John Wiley & Sons, Inc., New York, 1932), pp. 132–143. Knudsen's book also describes early model work by R. F. Norris and T. Satow.

[3] F. Spandöck, Ann Phys. 20, 345 (1934).

[4] R. Vermeulen and J. de Boer, Philips Tech. Rev. 1, 46 (1936); see, also, R. Vermeulen, Philips Tech Rev. 5, 321 (1940).

[5] F. M. Oswald, Z. Tech. Physik 17, 561 (1936).

[6] H. Kuttruff, Acustica 8, 330 (1958).

[7] D. Bebreck, R. Bücklein, E. Krauth and F. Spandöck, Acustica 18, 213–226 (1967). (This paper by Prof. Spandöck and his students contains additional references to model work.)

[8] W. Burgtorf, Acustica 18, 323 (1967).

[9] E. Meyer, H. Kuttruff, and W. Lauterborn, Acustica 18, 21 (1967).

[10] E. Meyer, H. Kuttruff, and N. Roy, Acustica 19, 132 (1967-1968).

[11] V. O. Knudsen, J. Acoust. Soc. Amer. 47, 401–407 (1970).

[12] V. L. Jordan, J. Acoust. Soc. Amer. 47, 408–412 (1970)

[13] B. G. Watters, J. Acoust. Soc. Amer. 47, 413–418 (1970).

[14] P. S. Veneklasen, J. Acoust. Soc. Amer. 47, 419–423 (1970).

This paper reviews the application of digital computers to acoustic modeling, with emphasis on new approaches made possible by the precision and flexibility of digital computers. Digital simulation appears particularly attractive for solving some of the more difficult problems in architectural acoustics—notably the elusive interrelationship between the physical parameters of a reverberation process and its perceptual correlates. Another intriguing possibility is the simulation of ray propagation to elucidate the relation between the shape of an enclosure and its sound-decay characteristics.

I. RAY SIMULATION

Although reverberation is a wave phenomenon, the classical reverberation-time formulas, named after Sabine, Eyring, and Millington, are based on geometri-

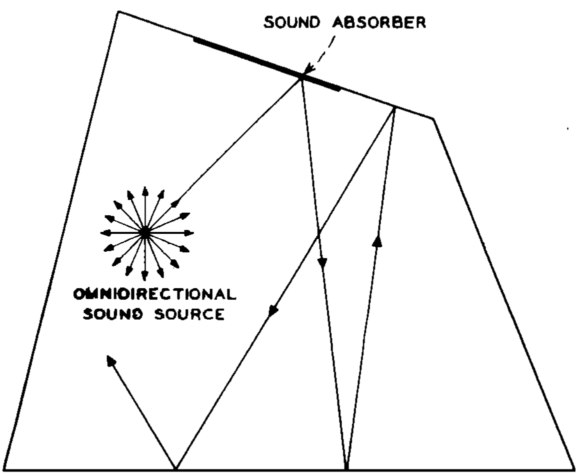

FIG. 1. Reverberation study, based on geometrical acoustics, by computer ray tracing. The computer calculates the paths of 300 rays from an omnidirectional source inside an (two-dimensional) enclosure. Every time a ray hits an absorbing material, its energy is reduced. The computer keeps a running account of the remaining energy..

cal acoustics. The computer is a superb tool for studying the validity of these reverberation-time formulas based on the ray approximation.

Shortcomings in the classical formulas to predict reverberation time correctly or to calculate absorption coefficients accurately have often been ascribed to "diffraction effects" and to a "lack of diffusion". However, there is increasing evidence that present reverberation-time formulas may be substantially in error, even when diffraction at the edges of absorbing areas is taken into account and good diffusion is provided. In fact, it appears that some of the basic assumptions that go into the most commonly used formulas are incorrect even within the geometrical acoustics approximation.[15]

[15] B. S. Atal and M. R. Schroeder, J. Acoust. Soc. Amer. 41, 1598(A) (1967).

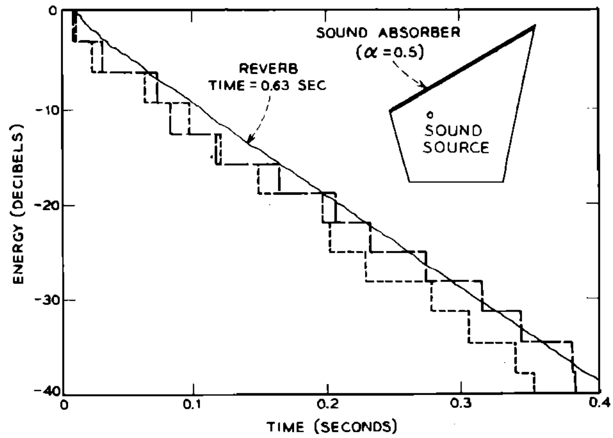

FIG. 2. The energy history of two individual rays and the average of all 300 rays for the configuration, shown in the upper right.

In the following, some results of ray simulation are described for two-dimensional spaces. Figure 1 shows such a two-dimensional "room" with a patch of absorbing material on one of its walls and an omnidirectional sound source emitting 300 equal-energy rays, one for every 1.2°. These rays are traced on the computer and are reflected from the walls either specularly or according to some specified random law. Each time a ray hits the absorber, its energy is diminished by a factor $(1-\alpha)$, where α is the absorption coefficient. In the present work, the absorption is considered to be independent of angle on incidence; this simplification can easily be removed.

Figure 2 shows the energy as a function of time of two individual rays for an enclosure with one wall completely covered with a material having an absorption coefficient $\alpha=0.5$. Each time a ray hits the absorber, its energy is reduced by 3 dB. The average energy of all 300 rays is also shown. It is a nearly straight line with small irregularities. The reverberation time (0.63 sec) can be unambiguously determined and, in this case, is independent of the portion of the decay considered.

Figure 3 shows another room with only partial covering, by asborbing material, of a single wall. The decay,

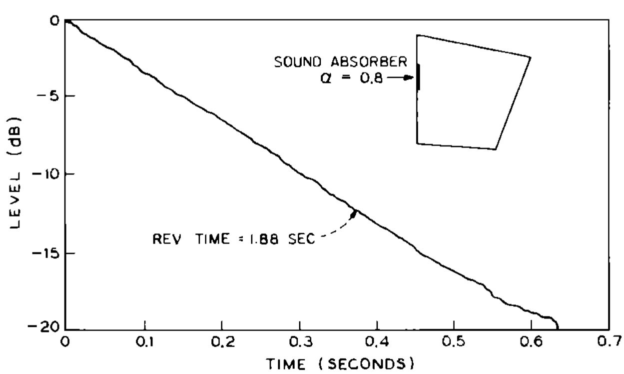

FIG. 3. The average energy of 300 rays for a different shape; only part of a single wall is covered by an absorber.

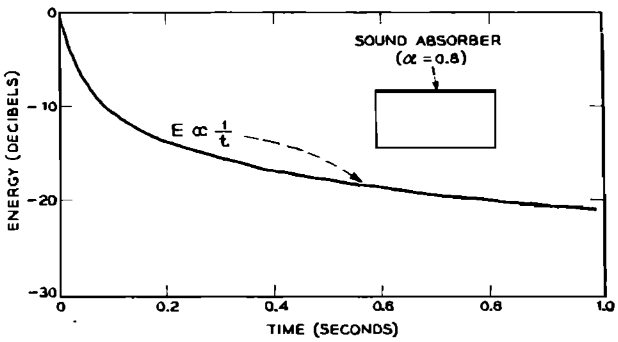

FIG. 4. Example of a nonexponential decay found in rectangular (and some other) enclosures.

given by the average energy of the 300 rays, is slightly more irregular. Again, a straight-line fit is nearly independent of the portion of the decay chosen, and it yields a reverberation time of 1.88 sec.

Figure 4 shows a rectangular room with one wall completely covered. This is one of the few cases that can be accurately treated by the solving of the wave equation. The well-known result is a nonexponential decay following a $(1/t)$ law. This theoretical prediction is well borne out by the computer solution.

It might be interesting to note that, in the course of this study, a number of shapes with nonexponential decays were found, most of which were not easily recognized as "pathological" cases. However, in all such cases, ray paths were found that closed on themselves, indicating a lack of ergodicity. These occurrences were, of course, limited to "rooms" with specularly reflecting walls.

Figure 5 illustrates the dependence of the decay on the source position. Although the two decays shown differ considerably in detail, the over-all slopes are remarkably similar. Straight-line fits by the computer to the first 40 dB of the decay yield reverberation times of 0.211 sec and 0.216 sec, respectively. In general, little

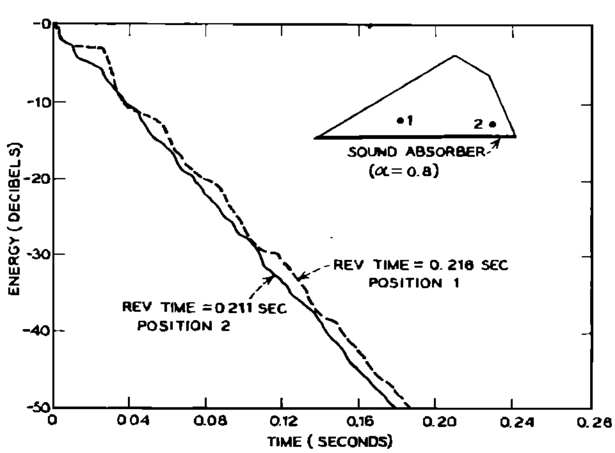

FIG. 5. Dependence of the decay on source position. Details of the decays differ, but over-all slopes are nearly the same.

influence of source position on reverberation time was found, except in some nonergodic cases.

Figure 6 shows a comparison between a decay obtained on the computer and the decays predicted by the two-dimensional forms of the Sabine, Eyring, and Millington formulas, respectively. The reverberation time predicted by the Sabine formula is 65% too large. The Millington value is 18% too small. The Eyring formula occupies an intermediate position, with an error of +45%.

Figure 7 shows the reverberation times of 32 different two-dimiensonal shapes that have specularly reflecting walls, as found by ray tracing on the computer (abscissa), and reverberation time calculated according to the Eyring formula (ordinate). The largest errors are nearly 70%; the average error is approximately 40%.

The discrepancies found for rooms with randomly reflecting walls and "suspended" diffusing elements were

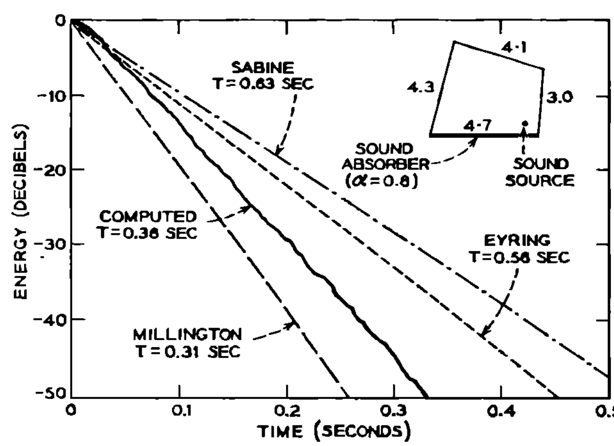

FIG. 6. Comparison of decay found by computer ray tracing with decay rates predicted by the Sabine, Eyring, and Millington formulas for two dimensions.

generally, although not always, somewhat smaller. In fact, in some respects the discrepancies between theory and ray simulation were more serious when the enclosure included diffusing elements.[16] This may be related to a recent theoretical finding by H. Kuttruff.[17]

The dependence of reverberation time on room shape (and other factors not considered by the classical formulas) found in the ray simulation is serious enough to affect concert-hall performance unless errors in the reverberation-time formula and the applied values of absorption coefficients have cancelled each other out.

Figure 8 shows absorption coefficients as calculated by the Eyring and Sabine formulas (ordinate) versus the absorption coefficient used in the ray simulation for the shape shown. The agreement is quite good for small values of α, but for $\alpha=90\%$, the Eyring and Sabine

[16] B. S. Atal and M. R. Schroeder, J. Acoust. Soc. Amer. (to be published).

[17] H. Kuttruff, Acustica 18, 131 (1967).

formulas yield absorption coefficients in excess of 160% and 200%, respectively.

Of course, the case illustrated in Fig. 8 deviates from established measuring practice in that a whole surface is covered by the absorbing material and that the average absorption coefficient approaches 0.3 for $\alpha=90\%$. What results would ray simulation give for the more realistic case of a partial covering of a surface and average absorption coefficients limited to 0.06, as recommended, for example, by ASTM C423, "Sound Absorption of Acoustical Materials In Reverberation Rooms"?

Such cases also have been studied by ray simulation. The configuration illustrated in Fig. 3 is one example. The ratio of the absorber surface to the total surface is 0.064, and the *average* absorption coefficient, for $\alpha=0.8$, is 0.051—well below the recommended value of 0.06. The mean-free path of the shape shown in Fig. 3, divided by the sound velocity, is precisely 10 msec. With the experimentally determined reverberation time of 1.88 sec, the absorption coefficient can then be calculated by means of Sabine's formula. The result is $\alpha=1.14$, for an error of more than 40%! Application of the Eyring formula would have yielded $\alpha=1.1$, an equally useless result.

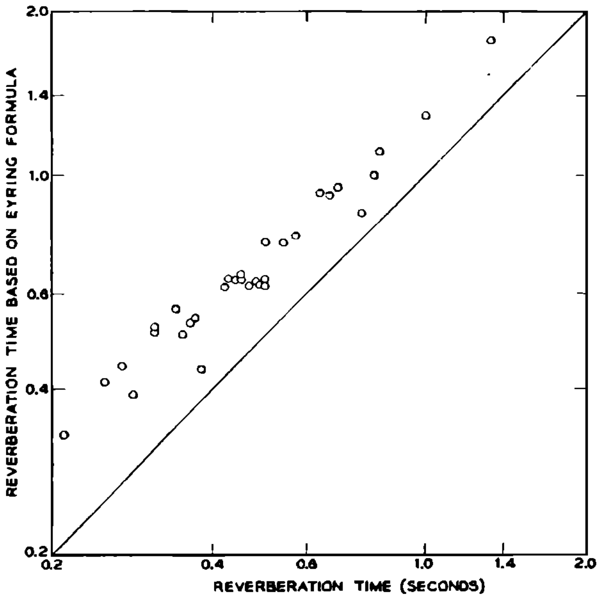

FIG. 7. Comparison of reverberation times, obtained by computer ray tracing (abscissa), with reverberation times computed from Eyring formula for 32 different shapes (ordinate). Average discrepancy is about 40%.

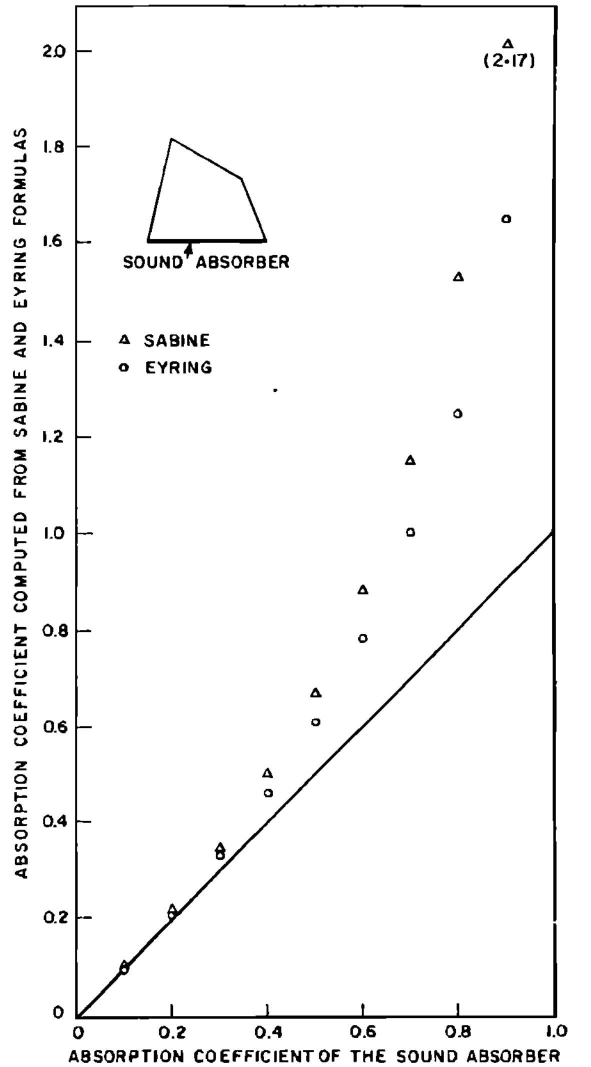

FIG. 8. Absorption coefficient, calculated according to the Sabine and Eyring formulas versus the actual absorption coefficient used in the ray-tracing program for the configuration shown.

These discrepancies are substantial, and one wonders which of the assumptions made in the derivation of the classical reverberation theories are responsible.

Is the problem connected with the mean-free path? The ray simulation can answer this question with little extra computation. Figure 9 shows the average number of reflections per second (ordinate) versus the theoretical value for two dimensions: $cS/\pi V$, where c is the velocity of sound, S the circumference, and V the area of the enclosed space. Except for small statistical deviations, the agreement is excellent. Thus, the mean-free path is not the problem.

However, if one examines the *distribution* of time intervals between successive reflections, one finds substantial discrepancies from what is implied in the classical formulas. Thus, Kutruff[18] has shown that the Eyring formula can be derived by assuming a *binomial distribution* of the number of reflections with a given wall section. Ray simulations show this assumption to be invalid for practically all cases examined. In fact, it is easy to see why the binomial distribution cannot be correct: it implies that the probability of a ray hitting

[18] H. Kuttruff, Acustica 8, 273, Beiheft 1 (1958).

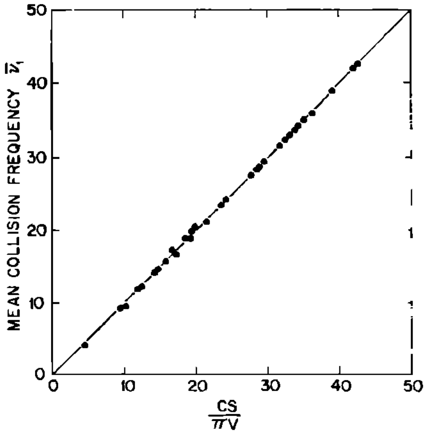

FIG. 9. Mean collision frequency of rays with walls (reciprocal mean-free path) found by the computer ray tracing (ordinate) versus the value commonly used in reverberation-time formulas for two dimensions. Agreement between theory and simulation results is excellent.

a given wall section is independent of the wall hit previously. This obviously cannot be true, however, because rays cannot hit the same wall section twice in succession. B. S. Atal is presently formulating a reverberation theory that takes these interactions into account. This approach yields more accurate formulas, which depend on the shape of the enclosure.[19]

II. SIMULATION OF SOUND TRANSMISSION IN REVERBERANT SPACES

Starting with reverberation-free speech or music signals as inputs, computers, programmed to act as "digital filters," can add echoes and reverberation with specified delays, spectral content, and decay characteristics. The computer produces several output signals which—when radiated from loudspeakers in an anechoic chamber—produce at a listener's ears sound-pressure waves resembling those in real halls. To ensure "externalization" and proper directions of echo arrivals, the computer program is based on the measured sound diffraction around the listener's head. This digital simulation method is useful to "preaudit" architectural designs before construction and to investigate subjective correlates of a wide variety of reverberation processes. If a design is found to be unsatisfactory, as a result of subjective evaluations of the simulation, modifications can be made in the architectural plans and the corresponding computer program, until a satisfactory compromise is reached.

Figure 10 illustrates, in block diagram form, the basic elements in this type of computer simulation.

A reverberation-free speech or music signal is sampled and quantized, by means of an analog-to-digital converter, and fed into the digital computer, where discrete reflections and reverberations are added.[20] Figure 11 shows deatils of the computer program, in which the blocks D represent the delay of the direct sound, and the delay differences between the direct sound and the early reflections. The blocks labeled A represent amplifiers with a frequency-dependent gain characteristic to account for the intensity and spectrum of the individual reflections.

Those reflections not individually perceived[21–23] and the later part of the reverberation process are simulated by parallel "combfilters," which represent the normal modes of the enclosure, and series "all-pass" filters, which simulate the high echo density encountered in three-dimensional spaces.[24]

The computer prepares two reverberated signals, one for each ear. The reverberated signals are then read out from the computer by means of a digital-to-analog converter and recorded on a two-channel tape recorder for presentation over two loudspeakers located in an anechoic chamber in front of the listener (see Fig. 10).

Other important aspects of reverberation are the direction of incidence at the listener's ears of the individual reflections and the angular diffusion of the reverberation. In electroacoustic modeling with analog equipment, this problem has been solved by using a number of loudspeakers suspended inside an anechoic chamber in as many different directions from the listener.[25]

However, for a given head orientation, the desired directionality can be obtained with as few as two loudspeakers. This approach requires fairly sophisticated signal processing of the signals going into the loudspeakers.

The simulation of an individual reflection from a forward direction (S) is illustrated in Fig. 12. A sound source at S on the right produces two distince pressure waves at the ears of the listener. If the source S emits a single pulse, then the two ears receive pulses that differ essentially in relative delay and amplitude, The same relative delay and amplitude can be achieved by the emission of one pulse from each loudspeader in front of the listener. However, the pulse from the loudspeaker on the right also goes to the left ear and has to be cancelled by a negative pulse from the left loudspeaker. Similarly, the "cross talk" from the left loudspeaker to the right ear has to be cancelled by appropriate pulses emitted from the right loudspeaker. Figure 12 illustrates the sequence of (exponentially decaying) pulses to be emit-

[19] B. S. Atal (to be published).

[20] M. R. Schroeder, B. S. Atal, and C. Bird, Proc. Intern. Congr. Acoust., 4th, Copenhagen (1962).
[21] H. P. Seraphim, Acustica 11, 80 (1961). See, also, Acustica 13, 75 (1963).
[22] W. Burgtorf, Acustica 11, 97 (1961). See, also, Acustica 13, 86 (1963).
[23] W. Burgtorf and H. K. Oehlschlägel, Acustica 14, 254 (1964).
[24] M. R. Schroeder, J. Acoust. Soc. Amer. 33, 1061 (1961). See, also, J. Audio Eng. Soc. 10, 219 (1962).
[25] E. Meyer, W. Burgtorf, and P. Damaske, Acustica 15, 339 (1965).

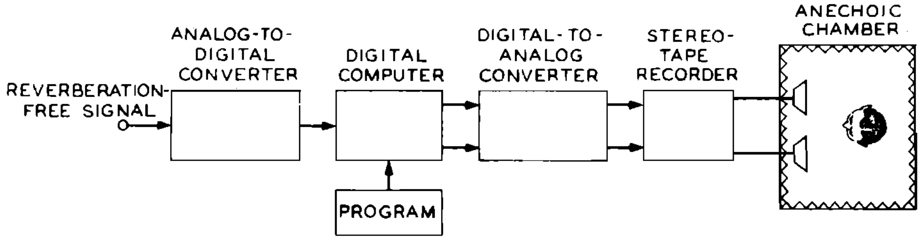

FIG. 10. Block diagram illustrating the use of computer simulation in subjective reverberation studies. The computer adds reflected sound and reverberation to a reverberation-free input signal. The reverberated signals are reproduced, via loudspeakers in an anechoic chamber, for subjective evaluation.

ted from the two loudspeakers to achieve the desired response from a source S on the right.

The explanation in terms of impulses also holds for continuous audio signals, which have to be filtered by networks with impulse responses, as shown. Such networks, which consist of delay lines in feedback loops, were constructed at Bell Telephone Laboratories, and the feasibility of creating realistic sound images over a wide angle was demonstrated.

Later, the experiment was transferred to a digital computer, which allowed an additional refinement. The difference between the responses at the two ears, even for a single direction of arrival, is of course, not just a frequency-independent amplitude and delay. Rather, sound waves are diffracted around the human head in a complicated way that depends in the direction of incidence. This diffraction was measured in amplitude and phase at both ears of a fully clothed mannequin. The measurement was repeated for a number of directions of incidence, including the directions of the loudspeakers (±22.5°) to be used in the simulation experiment.

These response measurements are then converted into filter specifications for computer simulation, The details of this calculation are contained in an earlier publication.[26]

Sound arrivals from about 10 different directions were then synthesized on the computer and reproduced in an anechoic chamber by means of two loudspeakers located several meters in front of the listener. Although the computations were based on an essentially fixed head position, the experiments showed that substantial head movements and rotations of ±10° did not affect the percept. Thus, it was not necessary to "clamp" the head; "externalization" (perceiving the sound as originating outside one's head) was perfect.

In fact, the synthesis of lateral arrivals, including arrivals from ±90°, with just two loudspeakers, was so realistic that subjects often turned their heads to *see* the (nonexistent) sound source—whereupon the percept of a well-defined lateral source vanished, because the computer program was based on a forward-oriented head position.

In addition to pretesting architectural designs of auditoriums still in the planning stage, the digital simulation method is useful for subjectively evaluating the effects of proposed design *changes* in existing halls. In this application, the impulse response of the hall is recorded and fed into the computer, where it can be modified in agreement with the proposed architectural alterations. Music or speech signals are then processed, both

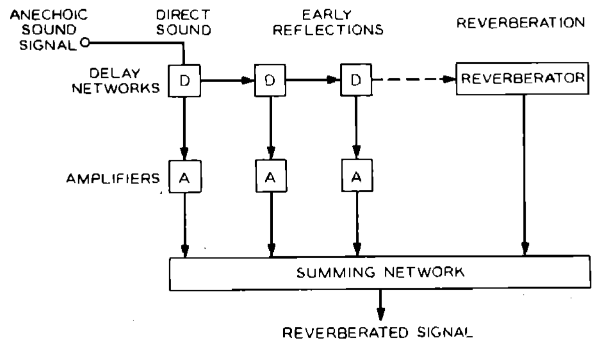

FIG. 11. Block diagram of computer program for adding reflected sound and reverberation to reverberation-free sound signals.

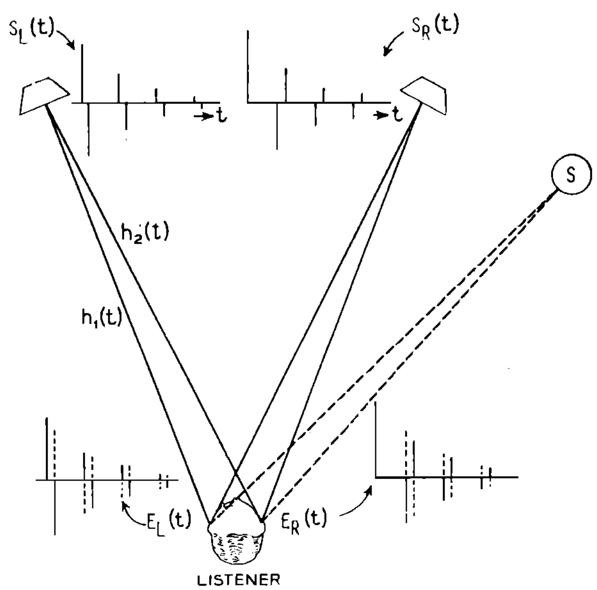

FIG. 12. Creation of virtual sound image at an arbitrary lateral angle by means of two loudspeakers in front of the listener. The signals from the two loudspeakers combine at the listener's ears to resemble those from a real lateral sound source.

[26] M. R. Schroeder and B. S. Atal, IEEE Intern. Conv. Rec. (1963).

according to the old and the new responses, and compared in paired comparison tests.

This method has been used to test the effectiveness of absorptive treatment of the rear wall of a large concert hall that had a disturbing echo. The simulation showed that the treatment would be effective, a conclusion later confirmed in the actual hall after the absorptive material was installed.[27]

III. SIMULATION FOR SUBJECTIVE STUDIES

Another equally important application of digital simulation is to answer one of the basic questions in architectural acoustics: "What does a given physically specified reverberation process sound like to a human listener?"

One way to increase our knowledge in this area is to listen to and compare music played in different halls and to correlate the perceived effects with the physical differences.

This method has its shortcomings, however. Even when one listens to the same musical piece, the stylistic differences between different orchestras and different conductors can make a reliable evaluation difficult. Furthermore, we cannot listen to two different halls within a sufficiently short time span. Days, weeks, or months may elapse between such comparisons. And finally, two halls differ not in one, but in many, respects. So, even when a significant difference can be perceived, one still does not know to what physical features to attribute these perceptual differences.

All these difficulties are avoided by the simulation method: one can compare identical program materials; one can switch from one condition to another instantly; and one can study the perceptual effect of a single physical parameter or the interaction of several well-defined parameters.

This technique has been applied to a study of subjective reverberation time[28] aimed at isolating those features of a physical decay process that correspond to our subjective perception of reverberation time. A variety of exponential and nonexponential decays with different direct-to-reverberant energy ratios were compared with each other.

On the basis of these comparison tests, a new criterion for reverberation time was formulated that is based on the *initial* portion of the decay measured with the integrated-impulse method.[29] Figure 13 shows the results for 30 different nonexponential decays.

The agreement between objective reverberation time (abscissa) and subjective reverberation time (ordinate) is generally quite good, particularly when compared with the definition of reverberation time based on the decay between −5 and −35 dB. One extreme result

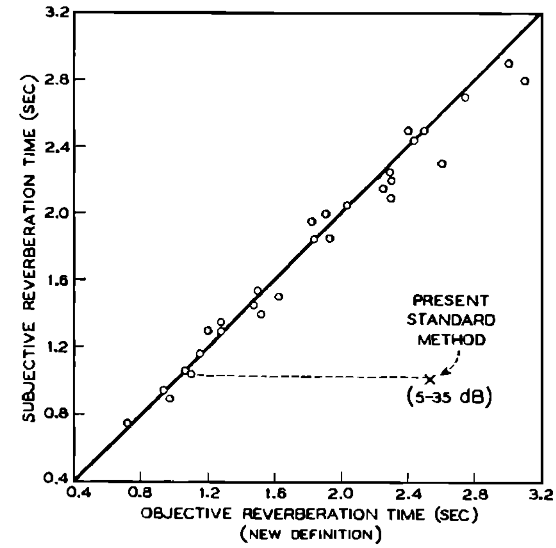

FIG. 13. *Subjective* reverberation times found in paired comparison tests of computer-simulated nonexponential decays versus *objective* reverberation time, according to a definition based on the early portions of the decay. The objective reverberation time, according to a widely used definition (based on the decay between −5 dB and −35 dB), is indicated by (×) for one extreme case.

using the old definition is included in Fig. 13 (indicated by X).

Many other aspects of reverberation can be studied in this way. There is little doubt that digital simulation is a most useful tool to advance our knowledge in one of the critical areas of architectural acoustics: the relation between the physical sound field and its perceptual correlates.

IV. MONTE CARLO SIMULATION OF FREQUENCY AND SPACE RESPONSES

The preceding sections were concerned with the digital simulation of *time* responses of reverberant enclosures. One can also simulate on the computer, however, the frequency and space response characteristics of reverberent rooms (and other multipath or multimode transmission media).

For, example, it can be shown theoretically[30] that the fluctuations of a steady-state single-frequency response in a reverberant field have a standard deviation of 5.6 dB (irrespective of whether frequency of transducer position is varied). How much are these fluctuations reduced by using bands of noise rather than sine waves as an excitation signal and by performing a spatial average? This question is important when one wants to measure the output power of a loudspeaker in a reverberation chamber. In some cases, questions such as this are difficult to answer analytically, but solutions can easily be obtained by simulating the responses on a computer and evaluating them for the desired answer following a Monte Carlo procedure.

[27] M. R. Schroeder, B. S. Atal, G. M. Sessler, and J. E. West, J. Acoust. Soc. Amer. **40**, 434 (1966).

[28] B. S. Atal, G. M. Sessler, and M. R. Schroeder, Proc. Intern. Congr. Acoust. 5th, Liège (1965).

[29] M. R. Schroeder, J. Acoust. Soc. Amer. **37**, 409 (1965).

[30] M. Schröder, Acustica **4**, 594 (Beiheft 2) (1954).

Many aspects of sound transmission in reverberant spaces and the performance of electroacoustic systems for sound reinforcement and artificial reverberation have been studied by this method.[31,32] However, these applications are not as fundamental as the ray and impulse-response simulations described in the preceding sections. Sometimes Monte Carlo simulations only fill a temporary gap until analytical approximations become available.

V. CONCLUSION

Digital simulation is a powerful research tool in architectural acoustics and is capable of resolving some of the most intricate theoretical questions related to reverberation processes and sound transmission in multipath media.

Equally important is the digital simulation of reverberation for subjective studies for a better understanding of the perceptual correlates of the physical parameters of reverberation processes.

Thus, the computer is forging a strong link between our theoretical concepts of reverberation and the realities of decay processes—both physical and perceptual.

[31] M. R. Schroeder and K. H. Kuttruff, J. Acoust. Soc. Amer. 34, 76 (1962).

[32] M. R. Schroeder, J. Acoust. Soc. Amer. 34, 1819 (1962).

ACKNOWLEDGMENT

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