Characterization of a diffuse field in a reverberant room

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(Received 30 April 1996; revised 22 November 1996; accepted 31 December 1996)

An efficient modal approach to characterize the diffuseness of the sound field in a rectangular room is presented. Using two simple descriptors, the correlation function and the spatial uniformity of the pressure field, a practical and convenient tool is proposed to study the diffuse field in the room. A precise criterion has been given in terms of the least permissible number of room modes to achieve an adequate diffusion. It has been shown that the criterion is in great accordance with the well-known ''Schroeder frequency'' limit for the diffuse field. Detailed calculations of the correlation function are presented to show the importance of the $\Delta k/k$ correction terms to the well-known sin(kR)/kR prediction. A new closed form for the correlation function is thus derived. A discussion about the importance of having more than one descriptors is also presented. It is shown that a diffuse field can be established in a room with strong modal behavior under certain assumptions. © *1997 Acoustical Society of America.* [S0001-4966(97)02105-X]

PACS numbers: 43.55.Br, 43.55.Cs, 43.55.Nd [JDQ]

INTRODUCTION

The degree of diffuseness of the acoustical field in a reverberant room has been widely investigated in the last five decades. However, even if the concept of diffuse field is well-understood, there are few practical and meaningful theoretical tools to characterize the sound field in a room. Moreover, many definitions of diffuseness and descriptors used to quantify the degree of diffuseness are often misused, if not misunderstood. As an example, the knowledge of only the uniformity of the pressure field in the room gives no sufficient information about the degree of diffuseness of the field. Generally, more than one descriptor have to be used to correctly define the sound field.

When dealing with reverberant test rooms, many questions can be raised by both experimentalists and theoreticians. For a given reverberant room, is it possible to create or simulate a diffuse field? If so, what are the frequency limits or what is the spatial extent of such a field? What are the basic assumptions that have been made to establish the diffuse field and what is the ''quality'' of the diffuseness? Since the room is used to conduct material testings, what will be the effect of these materials on the diffuseness? This paper tries to give some answers to these questions for an empty rectangular room, keeping in mind that the room is designed to contain testing structures. The main objective is to provide to acousticians concrete criterion of diffuseness using simple and well-known descriptors as well as a simple theoretical tool to compute these descriptors.

A simple analytical modal approach is presented to compute descriptors of interest in a rectangular reverberant room. Two indicators have been used: the correlation function and the spatial uniformity. The results are compared with the ''perfect'' diffuse field model, the so-called plane-wave model. The modal approach provides a useful tool to characterize the acoustic field in the empty chamber as well as in the one containing structures.

I. DEFINITION OF A DIFFUSE SOUND FIELD

A generally accepted definition of a diffuse field[1–3] can be given by: *An acoustic field is considered to be perfectly diffuse in a volume V if the energy density is the same on all points of this volume V*. It is possible to build such a field by a superposition of an infinite number of freely propagating plane waves, such that all directions of propagation are equally probable and the phase relations of the waves are random. This ''construction'' is usually called the plane-wave model (PWM). From a conceptual point of view this definition is quite adequate but it gives no practical help or criterion to determine the degree of diffuseness of an acoustical field.

II. METHODS TO CHARACTERIZE A DIFFUSE FIELD

Many experimental and theoretical methods have been proposed for evaluating diffuseness, each of these having their pros and cons. Brief reviews of various descriptors are presented by Schultz[4] and more recently by Abdou.[5] Bodlund[6] has proposed an estimator ϵ_d based on the standard deviation of the correlation coefficients in regard to the theoretical predictions for a diffuse field. A stochastic model is used to compute the theoretical predictions. Property of the estimator ϵ_d has been experimentally verified. A complete statistical study of the diffuse field has been performed by Jacobsen[7] using a similar stochastic model than the one used by Bodlund. Quantities such as distribution, mean and variance of the energy density or spatial autocovariance, and correlation function are studied. Comparisons between theoretical predictions and measurements are also presented.

Two of the most commonly used techniques are the cross-correlation and the spatial uniformity of the pressure field. These two have been chosen here as they can be both easily measured and, for simple rooms, easily computed.

Experimental works have been performed by many authors. Among all of these, Cook *et al.*[8] have presented some results for the correlation function. Good agreements with the sin(kR)/kR predictions are obtained for a high-frequency white-noise excitation. In the same spirit, Balachandran[9] has presented one-third octave bands results for a reverberant room excited by a white noise. Once again, good agreements with the sin(kR)/kR form are found for high frequencies. Schroeder[10] has computed, using an orthonormal modes expansion, the angular distribution of the energy density and some correlation functions for incident diffuse field on measuring walls. Pressure and pressure gradient are computed for both single frequencies and finite frequency bands. Recommendations and precautions are presented for the evaluation of the degree of diffuseness. Using the traversing microphone spectroscopy (TM), Lubman[11] has presented a technique to measure the autocorrelation function and the directivity function with an omnidirectional microphone for a 2-D and 3-D diffuse field. More recently, sound propagation and sound decay in reverberation rooms have been studied experimentally by Hodgson[12,13] where results are compared with the method of images predictions. Zeng[14] has computed the acoustic intensity and spatial distribution of the pressure field with a ray tracing approach. Comparisons with experimental results are also shown. It is shown that the field in the room is not perfectly diffuse, even for a high frequency excitation. Experimental results for the correlation function in a reverberation room has also been given by Koyasu and Yamashita[15] as well as results for the directional pattern in the room. It is pointed out that it is important to observe the correlation coefficients for all directions in the field. Correlation coefficients along a line and directivity power spectra have been both computed and measured in a rectangular room by Tohyama *et al.*[16,17] Good agreements between measurements and theoretical predictions are found but no quantitative assessment of sound field has been proposed.

On the theoretical side, Morrow[18] has computed the correlation function in a cavity. A high-modal density is assumed and cross terms in the correlation function are neglected. Blake and Waterhouse[19] have calculated the correlation function for an isotropic and anisotropic diffuse field. It is shown that anisotropy has no effect on the real part of the correlation but strongly modifies the imaginary part. Chien and Soroka[20] have calculated the correlation function in a high-modal cavity at high frequency for a stationary and a decaying state. In the stationary case, the sin(kR)/kR predictions are obtained. These calculations, as well as those of Morrow,[18] for the correlation function are discussed by Chu.[21,22] A discussion on the cross terms for the correlation function is presented. Computations of the spatial uniformity have been performed by Waterhouse[23] and Chu.[24] Their main conclusions are that spatial uniformity cannot be established for a monochromatic source. More recently, Kubota and Dowell[25] have proposed an asymptotic modal analysis (AMA) to study the spatial uniformity at high frequencies in a cavity. It is shown that AMA gives better results than the ray tracing technique.

This brief review reveals that, while a great amount of work has been done, no simple and clear criterion have been stated in terms of modal density. Generally speaking, a ‘‘Schroeder frequency’’-type limit is given but it gives no clear insight on the number of modes per band required to achieve an adequate diffusion. Most of the studies have been dedicated to high frequencies and there is no precise indication on how to characterize the acoustic field when strong modal behavior is involved, i.e., at lower frequencies. In this sense, this paper is addressed to the study of cavity and frequency range for which modal behavior is important. Among all the descriptors commonly proposed to characterize the sound field, two of them have retained our attention in this paper: the correlation function and the spatial uniformity. These two descriptors are reliable, easy to measure and easy to compute in the framework of the PWM model and the modal approach.

A. Correlation function

The spatial cross-correlation function of the pressure field between two points, or simply the correlation function, is defined by

$$C(\mathbf{r},\mathbf{r}+\mathbf{r}')=\frac{\overline{P(\mathbf{r})P(\mathbf{r}+\mathbf{r}')}}{\sqrt{\overline{P^2(\mathbf{r})}\,\overline{P^2(\mathbf{r}+\mathbf{r})}}}. \tag{1}$$

The acoustic pressure at $\mathbf{r}$ is given by $P(\mathbf{r})$ while the horizontal bar means time average.

It will be shown later that the plane-wave model leads to

$$C(\mathbf{r},\mathbf{r}+\mathbf{r}')=\frac{\sin(kR)}{kR}, \tag{2}$$

with $R=|\mathbf{r}-\mathbf{r}'|$.

This function has been widely studied theoretically[18,23,19,21,20] and experimentally.[8,9,15–17] Generally, the plane-wave prediction sin(kR)/kR is obtained if a narrow-band containing enough chamber modes is used. In most of the theoretical studies, certain assumptions have been made to simplify the calculations of the correlation function: (i) in the modal expression of the correlation function, cross terms are neglected using a high-frequency approximation; (ii) high-frequency approximation allows to change the discrete summation over chamber modes to a continuous frequency integration. This assumption is valid only if the modal density is high; (iii) Detailed calculation of the correlation function shows that a $\Delta k/k$ correction applies to the well-known sin(kR)/kR form but it is always neglected.

One of the goals in this paper is to compute the correlation function without approximations, in order to extend the previous calculations to lower frequencies range and to take into account cross terms in the modal expansions.

B. Spatial uniformity

To study the spatial uniformity of the pressure field in a volume V, the standard deviation σ of the acoustic pressure field, which is related to the variance, is used. If one considers N points for which the sound-pressure levels SPL($\mathbf{r}$) are computed in the central volume of the room, one defines the standard deviation to be given by

$$\sigma^2(f)=\frac{1}{N-1}\sum_{i=1}^{N}[\mathrm{SPL}(\mathbf{r}_i,f)-\overline{\mathrm{SPL}}(f)]^2, \tag{3}$$

TABLE I. Maximum admissible standard deviation for third-octave band as prescribed by the ISO standard #3741.

Central frequency (one-third-octave band)	Maximum standard deviation (dB)
100–160	1.5
200–630	1.0
800–2500	0.5
3150–10 000	1.0

where $\overline{\mathrm{SPL}}(f)$ is the mean sound-pressure level in the volume V at the frequency f. At the point $\mathbf{r}_i$ in V, the sound-pressure level $\mathrm{SPL}(\mathbf{r}_i,f)$ is defined by

$$\mathrm{SPL}(\mathbf{r}_i,f)=10\log(|P(\mathbf{r}_i,f)|^2). \tag{4}$$

It has been accepted that an acoustic field in a qualified reverberation room exhibits adequate diffuseness if the standard deviation σ remains under 1.5 dB. To be more precise, the ISO standard[26] states that the maximum admissible standard deviation must not exceed the values given in Table I. Naturally, the plane-wave model gives $\sigma=0$ dB.

C. Modal approach

To compute the acoustic pressure generated in an empty reverberant room that exhibits strong modal behavior, a modal expansion model, noted MODAP, has been used. A rigid-walled rectangular room (dimensions $L_x \times L_y \times L_z$) is considered, as shown in Fig. 1. The room can be excited by a vibrating surface S_s lying in the $z=0$ plane. It is to be noted that only one acoustic source has been used to help better understand the physical phenomena. However, the present model can be easily adapted to take into account many sources. The wall surfaces are considered to be perfectly reflective. The acoustic pressure in the room can be computed with a modal approach that uses a two-indices empty cavity Green's function.[27,28] It allows to reduce the number of summations by one which is a great improvement for algorithm efficiency. Appendix A draws a summary of this modal approach. With this expansion the acoustic pressure reads

$$P(\mathbf{r},f)=\sum_{mn} P_{mn}(z,f)\,\psi_{mn}(x,y) \tag{5}$$

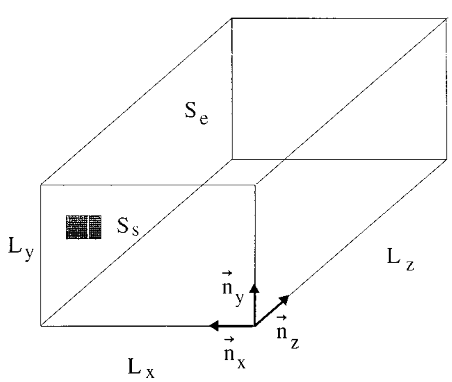

FIG. 1. Geometrical representation of an empty rectangular room excited acoustically.

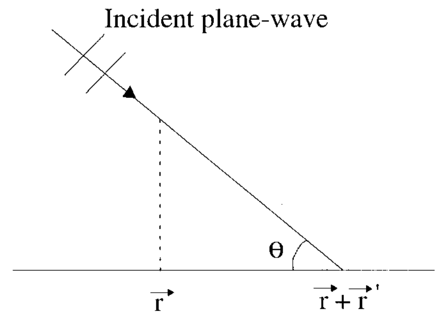

FIG. 2. Representation of an incident plane wave.

where the coefficients $P_{mn}(z,f)$ and the basis functions $\psi_{mn}(x,y)$ are given in Appendix A.

To include losses in the fluid, an acoustic damping factor η is used. It relates the wave number k to the piston driving frequency ω by the relation

$$k^2=\frac{\omega^2}{c^2}\frac{1}{(1+j\eta)}. \tag{6}$$

The damping factor η can be related to the reverberation time of the room if an exponential decay is assumed after the source has been cut off.

The correlation function and the spatial uniformity has been computed with this model and compared with the PWM predictions. The results are presented in Sec. IV.

III. PWM PREDICTIONS FOR THE CORRELATION FUNCTION

One wishes to compute the correlation function [Eq. (1)] for the plane-wave model. Consider the points $\mathbf{r}$ and $\mathbf{r}+\mathbf{r}'$ as shown in Fig. 2. The pressure at these points is given by

$$P(\mathbf{r})=Ae^{i\mathbf{k}\cdot\mathbf{r}} \quad \text{and} \quad P(\mathbf{r}+\mathbf{r}')=Ae^{i\mathbf{k}\cdot(\mathbf{r}+\mathbf{r}')}. \tag{7}$$

If a harmonic state is assumed for the pressure field, the correlation function becomes

$$C(kR)=\mathrm{Re}\left(\frac{P(\mathbf{r})P^*(\mathbf{r}+\mathbf{r}')}{\sqrt{|P(\mathbf{r})|^2|P(\mathbf{r}+\mathbf{r}')|^2}}\right)$$
$$=\mathrm{Re}(e^{-ikR\cos\theta})=\cos(kR\cos\theta). \tag{8}$$

It is to be noted that the correlation function only depends on the distance R. For a diffuse field an integration over all the possible directions (θ,ϕ) of the plane waves has to be performed. It gives the following results for 1, 2, and 3 dimensions:

1D

$$C(kR)=\cos(kR). \tag{9}$$

2D

$$C(kR)=\frac{1}{2\pi}\int_0^{2\pi}\cos(kR\cos\theta)d\theta=J_0(kR), \tag{10}$$

where $J_0(kR)$ is the first-order Bessel function of the first kind.

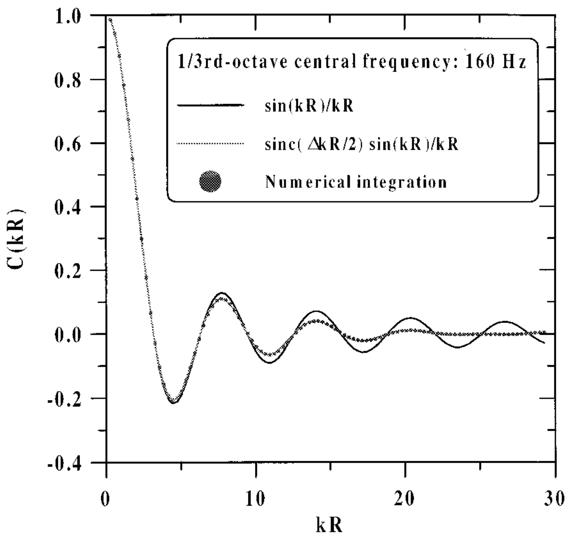

FIG. 3. Correlation function $\sin(k_cR)/k_cR$ (bold line), $C(kR)$ from Eq. (13) (gray line), and from the numerical integration of Eq. (11) (gray circles) as a function of kR.

3D

$$C(kR)=\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}\cos(kR\cos\theta)\sin\theta\,d\theta\,d\phi$$

$$=\frac{\sin(kR)}{kR}. \tag{11}$$

If one is interested in a frequency-band response, one must integrate the correlation function over the frequency in the bandwidth $\Delta f = f_2 - f_1$. Appendix B gives the calculation of the correlation function in the 3D case. It leads to:

$$C_{\Delta f}(kR)\sim \operatorname{sinc}\left(\frac{\Delta kR}{2}\right)\left[\frac{\sin(kR)}{kR} - 2\frac{\cos(kR)}{(kR)^2}\sin^2\left(\frac{\Delta kR}{4}\right)\right], \tag{12}$$

where k is now defined by $k=(k_2+k_1)/2$ and where $\operatorname{sinc}(x)=\sin(x)/x$.

The above form for $C_{\Delta f}(kr)$ is quite different than those one can find in the literature for which only additional correction terms are found from a Taylor expansion. It is to be noted that for almost all practical situations, the second term of (12) can be neglected so that Eq. (12) can be written:

$$C_{\Delta f}(kR)\sim \operatorname{sinc}\left(\frac{\Delta kR}{2}\right)\frac{\sin(kR)}{kR}, \tag{13}$$

It then gives a simple closed form for the influence of the $\Delta k/k$ correction.

To validate the last relation for $C_{\Delta f}(kR)$ [Eq. (13)], the frequency integrated equation (11) has been computed numerically using a simple Simpson's rule algorithm for the wave number integral. Figure 3 presents the results from the simple form $\sin(kR)/kR$ (bold line), from Eq. (13) (gray line) and from the numerical integration of Eq. (11) (gray circles) as a function of kR. The figure shows that Eq. (13) is a very good approximation for the finite frequency band correlation

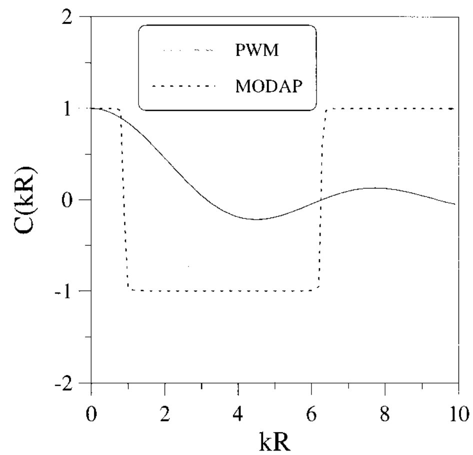

FIG. 4. Correlation function for the 121.6–121.7 Hz band.

function. The importance of the $\Delta k/k$ correction to the usual $\sin(kR)/kR$ form clearly appears for high values of kR.

IV. RESULTS

A. Correlation results

To compute the correlation with the MODAP model, one has used

$$C(kR)_{\Delta f}=\frac{\mathrm{Re}\,\Sigma_{f'=f_1}^{f_2}P(\mathbf{r})P^*(\mathbf{r}+\mathbf{r}')}{\sqrt{\Sigma_{f'=f_1}^{f_2}|P(\mathbf{r})|^2\Sigma_{f'=f_1}^{f_2}|P(\mathbf{r}+\mathbf{r}')|^2}}, \tag{14}$$

where the acoustic pressures is given by Eq. (5).

If there is only one chamber mode in the bandwidth or for a monochromatic source, one can show that the correlation function becomes 0 or $\pm$ 1. This fact is well illustrated in Fig. 4. A $8.0\times6.9\times9.75$ m^3 chamber with a damping $\eta = 5\times10^{-3}$ has been used to compute the correlation function for the 121.6–121.7 Hz frequency band as a function of kR. For this room, only the (3,3,4) mode has its natural frequency in the frequency band of interest ($f_{334} = 121.65$ Hz). All the different points used to compute the correlation have been chosen in order to avoid the effect of the walls on the acoustic pressure.[18,24] The damping has been chosen in order to simulate losses in real cavity.[24] The dashed line represents the MODAP predictions while the solid line gives the PWM predictions. A value of ±1 is correctly obtained for the correlation function with the MODAP technique. Similar results have been obtained for higher frequencies. It suggests that a diffuse field cannot be established in a room for a monochromatic source, a result not predicted by the plane-wave model. Chu[22] has presented similar experimental results for which the same conclusion has been drawn.

Figure 5 shows the correlation function for the same chamber and damping with, now, a frequency band ranging from $f_1=90$ Hz to $f_2=112$ Hz. For the test room used here, there exist 49 modes in this bandwidth. Three cases have been shown: the MODAP predictions and the PWM with and without the $\Delta k/k$ correction. One can see that the MODAP model gives satisfactory results, in particular for low values

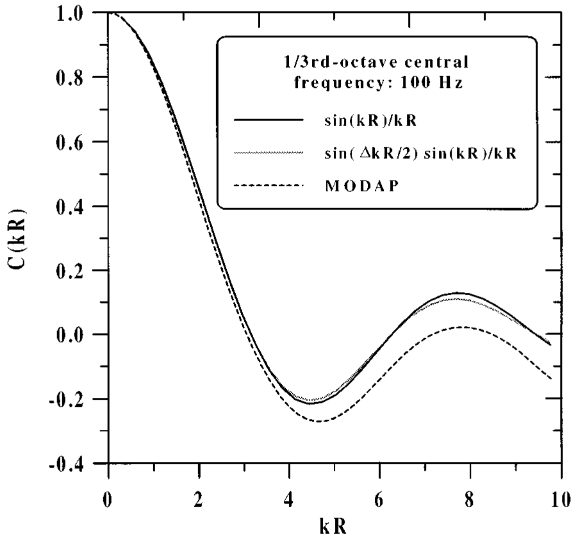

FIG. 5. Correlation function for the 90–112 Hz band.

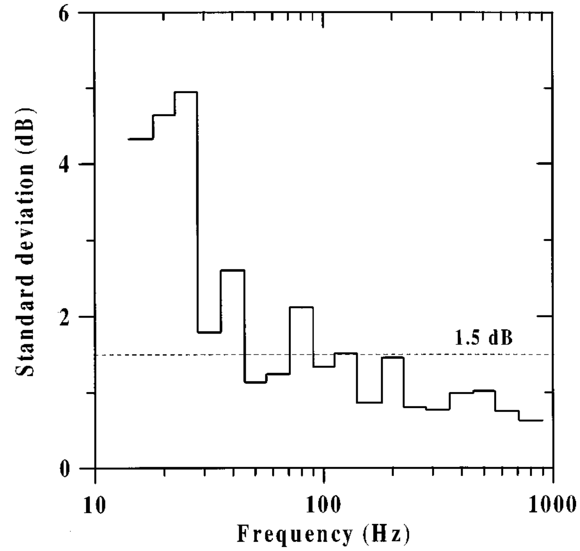

FIG. 6. Descriptor $\sigma(f)$ as a function of the frequency for one-third-octave bands.

of kR. Above $kR=4$, discrepancies between MODAP and PWM are observed. It suggests that the field is not perfectly diffuse (in the sense of the PWM). However, since the MODAP predictions are far from the single-frequency results discussed above $[C(kR)=\pm 1]$, it is believed that these results indicate that a diffuse field can be obtained for a measurement bandwidth containing enough chamber modes. By the way, it gives no indication on the least permissible number of room modes to achieve adequate diffuseness. To do so, one can use the spatial uniformity as an indicator. Moreover, as pointed out by Schultz,[4] the correlation function is not very sensitive to deviations from perfect diffuseness.

B. Spatial uniformity results

To study the spatial uniformity of the sound field, the relations (3) and (4) have been used. If one is interested in a frequency-band response one has:

$$\mathrm{SPL}(\mathbf{r},f)=10\log\left(\sum_{f'=f_1}^{f_2}|P(\mathbf{r},f')|^2\delta f'\right), \qquad (15)$$

with $f=(f_2+f_1)/2$, the center of the band.

A number of 20 points have been chosen in a volume V far from the walls (at least $\lambda/2$ from the nearest wall).[24] The indicator $\sigma(f)$ is shown in Fig. 6 for the $8.0\times 6.9\times 9.75$ m^3 chamber with a damping $\eta=5\times 10^{-3}$. One-third octave bands have been used. It has been verified that a sufficient number of frequency points has been chosen to ensure convergence. In the same way, Table II gives the number of room modes in each of the one-third-octave band used for the computation of $\sigma(f)$. It is observed that the value $\sigma(f)$ begins to be in accordance with the prescribed values of Table I, less than 1.5 dB, at the 90–112 Hz band. For this band, one has $\sigma(f)\sim 1.3$ dB, which is in accordance with the previous results for the correlation function (Fig. 5). From these results, a tentative criterion can be stated: A diffuse field can be established in a rectangular room if there is at least 20–30 modes in the measurement bandwidth. For a bandwidth which contains less modes there is no adequate diffuseness, in particular at a given frequency (monochromatic source). For lower dimensions (1D and 2D rooms), different values have been obtained: 2 modes/band at 1D and 6 modes/band at 2D.

Similar simulations have been performed for different room dimensions to examine if a possible ''critical'' frequency above which the field is assumed diffuse can be estimated (analogous to the ''Schroeder frequency''). Figure 7 shows the standard deviation for three rooms with different volumes using the MODAP as a function of kL where L

TABLE II. Number of modes in the one-third-octave bands for the $8.0\times 6.9\times 9.75$ m^3 room.

f_1-f_2	Number of modes/band
5.6−7.1	0
7.1−9	0
9−11.2	0
11.2−14	0
14−18	1
18−22.4	1
22.4−28	2
28−35.5	3
35.5−45	4
45−56	9
56−71	14
71−90	32
90−112	49
112−140	95
140−180	206
180−224	347
224−280	678
280−355	1380
355−450	2773
450−560	4975
560−710	10617
710−900	21371
900−1120	38612

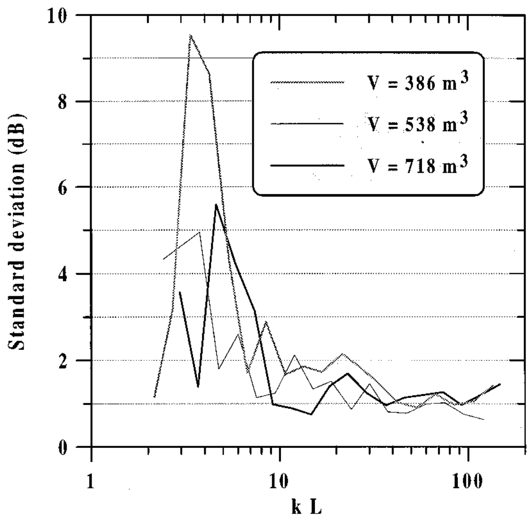

FIG. 7. Descriptor $\sigma(f)$ as a function of the frequency for three different rooms' volumes.

$=\sqrt[3]{V}$ is a characteristic dimension of the room. If one approximates the critical wave number $k_c = 2\pi f_c / c$ to be the one for which the standard deviation remains stable and under 2 dB, one has

$$k_c \approx \frac{22}{\sqrt[3]{V}}. \tag{16}$$

which gives, with $c = 340$ m/s

$$f_c \approx \frac{1190}{\sqrt[3]{V}}. \tag{17}$$

For the room with $V = 538\ \mathrm{m}^3$ it gives $f_c = 146$ Hz.

From our damping model [see Eq. (6)] it can be shown that the ''Schroeder frequency''[29] is given by

$$f_s \approx \sqrt[3]{\frac{\alpha c^3}{4\pi\eta V}}, \tag{18}$$

where α is the modal overlap. Schroeder has proposed a modal overlap $\alpha = 3$. For a damping of $\eta = 5\times 10^{-3}$ one then obtains $f_s = 152$ Hz. This is in great accordance with the ''critical'' frequency f_c found above. Meanwhile, it has to be noted that the choice of the factor 22 in (16) (and by the way the modal overlap of 3) is a quite subjective one. However, it gives a good approximation for the ''critical'' frequency and shows how the present model can be related to the well-known ''Schroeder frequency.''

From the above results it becomes clear that a frequency limit for which the diffuse field exists can be estimated under certain conditions. In the limit where there is enough modes in the measurements bands the usual ''Schroeder frequency'' relation can be applied to estimate the diffuse field frequency limit. For very narrow bands the frequency limit will be higher than for broadbands ones and no clear relation can be given for this ''critical frequency.'' This frequency depends on the room dimensions, the frequency, and the width of the band. With this information, the spatial uniformity is a practical tool to determine from which band the field begins to be considered as diffuse, in terms of modal density. Meanwhile, the use of the correlation function assures whether or not if the field really presents adequate diffuseness. It is thought that the two descriptors must be used together to insure the quality of the diffuseness. The plane-wave model gives a good example of a case where only the use of the spatial uniformity is not sufficient to characterize a sound field.

V. CONCLUSION

The present paper has been dedicated to the study of the characterization of the sound field in a rectangular cavity. Even if the concept of diffuse field is well-understood, thanks to the numerous papers on this topic, there is a need for practical tools and in-depth analysis of diffuseness in the sense of the plane-wave model in relation with field that exhibits strong modal behavior.

By the use of two descriptors, the correlation function and the spatial uniformity, this paper has presented a simple and convenient model to characterize the sound field in a rectangular cavity. The main advantage of the proposed approach is the ability to simply implement it, both experimentally and analytically. The use of a two-indices Green's function allows to obtain a fast computation code. Emphasis has been put on the fact that only one descriptor is not sufficient to define correctly the diffuse field. Results have shown that a diffuse field can be established in a volume V if there is at least 20–30 room modes in the measurement band. This condition applies for all types of frequency bands. Relation between the present model and the well-known ''Schroeder frequency'' has been discussed. Moreover, a computation of the correlation function in the framework of the plane-wave model has been presented for 1, 2, and 3 dimensions. The importance of the $\Delta k/k$ correction to the generally used form for the correlation function has been discussed. A new simple closed form for the finite-frequency band correlation function has been given. It has been validated by the modal approach.

An important conclusion of the paper lies in the fact that a diffuse field can be established in a room that exhibits strong modal behavior. Even if it seems simple, this assertion indicates that an acoustic field without modal characteristics can be simulated by an a priori different approach with less computation costs. It also validates the generally used approximation which consists in neglecting the walls effects on the pressure by generating the diffuse field with a infinite number of plane waves. The present model allows to give the limitations of the plane-wave model with precise criterion.

While being a good starting basis, the present approach would benefit from experimental validations. It would help experimentalists to conduct practical works with insightful indications such as orientation and location of microphones in the room, spatial and/or frequency averaging, and so on. In this sense, a parametric analysis of the present model, with the help of experimental results, could provide this type of information.

ACKNOWLEDGMENTS

The authors would like to thanks the reviewers of this paper for very helpful suggestions, in particular for the computation of the correlation function. This work was supported by grants from the National Sciences and Engineering Research Council of Canada (NSERC).

APPENDIX A: MODAL-CAVITY APPROACH FOR THE ACOUSTIC PRESSURE

One has to compute the acoustic pressure in a rectangular cavity. Consider first the cavity shown in Fig. 1. The acoustic source is given by a vibrating surface S_s while the surface of the walls is noted S_e. The ''Kirchoff–Helmholtz'' equation for this problem reads:

$$P(\mathbf{r}_0)=\int\int_{S_s} dS\ G(\mathbf{r},\mathbf{r}_0)\boldsymbol{\nabla}_{\mathbf{r}}P(\mathbf{r})\cdot\mathbf{n}_s \tag{A1}$$

where the walls have been considered to be perfectly rigid. This condition can be written

$$\nabla_{\mathbf{r}}G(\mathbf{r},\mathbf{r}_0)\cdot\mathbf{n}_e=0\forall\mathbf{r}\in S_e. \tag{A2}$$

Instead of expanding the Green's function on cavity modes, which implies a triple summation, the function is expanded on a set of two-dimensional functions, reducing the number of summations by one. Such an expansion is written, as shown by Bruneau,[27] in the following form:

$$G(\mathbf{r},\mathbf{r}_0)=\sum_{tu} g_{tu}(z,z_0)\psi_{tu}(x,y)\psi_{tu}(x_0,y_0). \tag{A3}$$

The orthonormal basis functions ψ_{tu} are given by

$$\psi_{tu}(x,y)=\frac{4}{(1+\delta_{t0})(1+\delta_{u0})L_xL_y}\cos\left(\frac{t\pi x}{L_x}\right)\cos\left(\frac{u\pi y}{L_y}\right), \tag{A4}$$

where δ_{tu} is the Kronecker delta and the function $g_{tu}(z,z_0)$ is

$$g_{tu}(z,z_0)=\begin{cases} -\dfrac{\cos[k_{ztu}(L_z-z_0)]\cos(k_{ztu}z)}{k_{ztu}\sin(k_{ztu}L_z)}, & \text{if } z\leqslant z_0,\\ -\dfrac{\cos[k_{ztu}(L_z-z)]\cos(k_{ztu}z_0)}{k_{ztu}\sin(k_{ztu}L_z)}, & \text{if } z\geqslant z_0,\end{cases} \tag{A5}$$

where

$$k_{ztu}^2=k^2-\left[\left(\frac{t\pi}{L_x}\right)^2+\left(\frac{u\pi}{L_y}\right)^2\right]. \tag{A6}$$

With this Green's function, all the modes along the z axis are implicitly present so faster convergence can be achieved comparing to the classical three-indices Green's function.

If one considers that the surface S_s vibrates harmonically with the frequency ω and with the velocity V_ω, one has:

$$P(\mathbf{r})=i\omega\rho V_\omega\int\int_{S_s} dS\ G(\mathbf{r},\mathbf{r}_0). \tag{A7}$$

Using the above Green's function (A3), the acoustic pressure in the room reads

$$P(\mathbf{r})=-i\omega\rho\sum_{tu}\frac{V_{tu}\cos[k_{ztu}(L_z-z)]}{k_{ztu}\sin(k_{ztu}L_z)}\psi_{tu}(x,y). \tag{A8}$$

The term V_{tu} is obtained from the integration over the piston surface and is given by

$$V_{tu}=V_\omega\int_{S_s} dS\ \psi_{tu}(x,y). \tag{A9}$$

APPENDIX B: CALCULATION OF THE CORRELATION FUNCTION

One wants to compute the correlation function $G_{\Delta f}(kR)$ given by

$$C_{\Delta f}(kR)=\frac{1}{\Delta k}\int_{k_1}^{k_2}\frac{\sin(kR)}{kR}\,dk. \tag{B1}$$

By a simple change of variables, the integral can be rewritten

$$C_{\Delta f}(kR)=\frac{1}{\Delta k}\int_{-\Delta k/2}^{\Delta k/2}\frac{\sin[(k+k_c)R]}{(k+k_c)R}\,dk, \tag{B2}$$

where $k_c=(k_1+k_2)/2$ and $\Delta k=k_2-k_1$.

One first uses the following trigonometric identity

$$\sin[(k+k_c)R]=\sin(kR)\cos(k_cR)+\cos(kR)\sin(k_cR) \tag{B3}$$

to write

$$C_{\Delta f}(kR)=\frac{1}{\Delta k}\int_{-\Delta k/2}^{\Delta k/2}\left\{\frac{\sin(kR)\cos(k_cR)}{(k+k_c)R}+\frac{\cos(kR)\sin(k_cR)}{(k+k_c)R}\right\}dk. \tag{B4}$$

Since k varies in the range of Δk, the denominator of the two above integrals can be Taylor expanded as power of k/k_c to give:

$$\frac{1}{kR+k_cR}=\frac{1}{k_cR}\left[1-\frac{k}{k_c}+\left(\frac{k}{k_c}\right)^2-\cdots\right]. \tag{B5}$$

Using this expansion in (B4) and neglecting high $\Delta k/k$ orders, it can be shown after few manipulations that the correlation function is given by

$$C_{\Delta f}(kR)\sim\operatorname{sinc}\left(\frac{\Delta kR}{2}\right)\left[\frac{\sin(kR)}{kR}-2\frac{\cos(kR)}{(kR)^2}\sin^2\left(\frac{\Delta kR}{4}\right)\right], \tag{B6}$$

where the function $\operatorname{sinc}(x)$ is defined by

$$\operatorname{sinc}(x)=\frac{\sin(x)}{x} \tag{B7}$$

and where one has now $k\equiv k_c=(k_1+k_2)/2$.

The second term of (B6) decreases as R^3 as R increases while its maximum value is at $R=0$ and is given by

$1/8(\Delta k/k)^2$. Therefore, for typical finite frequency bands, the second terms in (B6) can be neglected so the correlation function reads

$$C_{\Delta f}(kR) \sim \mathrm{sinc}\left(\frac{\Delta kR}{2}\right) \frac{\sin(kR)}{kR}. \tag{B8}$$

Numerical integrations of (B1) have shown that this approximation remains valid for frequency bands as large as octave bands.

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