Diffuse sound reflection by maximum-length sequences

M. R. Schroeder

Drittes Physikalisches Institut, Universität Göttingen, 34 Göttingen, Federal Republic of Germany

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Low-correlation sequences, such as "maximum-length" and Barker sequences and certain complex (magnitude one) sequences having flat power spectra, are ideally suited for designing surfaces of hard walls with highly diffuse reflections. Recent subjective-preference evaluations [M. R. Schroeder, D. Gottlob, and K. F. Siebrasse "Comparative Study of European Concert Halls," J. Acoust. Soc. Am. 56, 1195-1201 (1974)] indicate that low-loss, high-scatter surfaces may be required for better concert-hall acoustics. Surfaces shaped to give reflection coefficients according to such sequences show the expected high scatter in model experiments with microwaves.

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What wall shape has the highest possible sound diffusion in the sense that an incident wave from any direction is scattered evenly in all directions?

One answer to this problem is provided by "maximum-length sequences" $\sigma_n = \pm 1$ known from pseudorandom noise theory.[1] These periodic sequences, of period length $N = 2^M - 1$, have the property that their power spectrum is completely flat (except for a "dip" at dc). Thus, because of the relation between Fourier transform and directivity pattern, a wall with reflection coefficients alternating between ± 1, according to such a sequence, would scatter an incident plane wave evenly (except for a "dip" in the specular direction which corresponds to the dc component in the spectrum).

The basic relation between the spatial distribution of reflection coefficients $r(x)$ (along a wall) and its scattering amplitude $S(\alpha)$ is as follows:

$$S(\alpha) = \int r(x) \exp[2\pi j x(\sin\alpha - \sin\alpha_i)/\lambda]\, dx, \qquad (1)$$

where α is the scattering angle (measured with respect to the normal direction of the wall), α_i is the angle of incidence, and λ the wavelength of the incident wave. Equation 1 says that the scattering amplitude $S(\alpha)$ is the Fourier transform of the reflection coefficient $r(x)$ along the wall except for a change in variable—the relation between scattering angle α and "spatial frequency" k is as follows:

$$k = 2\pi(\sin\alpha - \sin\alpha_i)/\lambda\ . \qquad (2a)$$

The corresponding inverse relation is

$$\alpha = \arcsin(\sin\alpha_i + \lambda k/2\pi). \qquad (2b)$$

Thus, for example, in order to obtain scattered energy up to angles $\alpha_{max} = \pm 90°$ for $\alpha_i = 0$ (normal incidence), the highest spatial frequency in the Fourier transform of $r(x)$ must equal

$$k_{max} = 2\pi/\lambda. \qquad (3)$$

If $r(x)$ is designed according to a maximum-length sequence σ_n,

$$r(x) = \sum_{n=-\infty}^{\infty} \sigma_n \mathrm{rect}\left(\frac{x}{d} - n\right), \qquad (4)$$

where the rect function is equal to 1 between $\pm 1/2$ and to 0 elsewhere, the power spectrum of $r(x)$ equals

$$|R(k = 2\pi l/Nd)|^2 = (N+1)/N^2[\sin(\pi l/N)/(\pi l/N)]^2, \qquad (5)$$

for l equal any integer other than zero or multiples of the sequence length N.

With Eq. 2a and for normal incidence ($\alpha_i = 0$), the relation between l and scattering angle is $l = N \sin\alpha_l d/\lambda$. Thus, energy is scattered only in discrete directions

$$\alpha_l = \arcsin(\lambda l/Nd) \qquad (6)$$

in amounts given by Eq. 5, which can now be written in terms of scattering intensities:

$$|S(\alpha_l)|^2 = (N+1)/N^2[\sin(\pi \sin\alpha_l d/\lambda)/(\pi \sin\alpha_l d/\lambda)]^2.$$

If d is chosen no larger than $\lambda/2$, then the factor depending on α_l never drops below $(2/\pi)^2$, corresponding to -4 dB, even for $\alpha_l = \pm 90°$.

As a practical example, for $N = 2^4 - 1 = 15$ and $d = 7\lambda/15$, the near-equal-intensity scattering angles are $\pm 8°$, $\pm 17°$, $\pm 25°$, $\pm 35°$, $\pm 46°$, $\pm 59°$, and $90°$

Neglecting diffraction effects, reflection coefficients of ± 1 can be easily realized by hard walls with "grooves," a quarter-wavelength deep, in those wall areas where a reflection coefficient of -1 is called for. Figure 1 shows an amplitude scatter diagram for normal incidence from a wall representing *one* period of the maximum-length sequence for $N = 15$ (− ++ − + − ++++ − − − + −) and $d = \lambda/2$. The picture was obtained with electromagnetic waves of 3-cm wavelength and a piece of sheet metal shaped according to the above code.

Because of the finite width of this groovy "wall," the 14 discrete scattering angles become finite-width lobes, nine of which can actually be distinguished. By contrast, a plain piece of metal would exhibit only one narrow lobe. The expected pronounced "dip" at the specular reflection angle can also be seen in Fig. 1.

Other sequences (of magnitude one) with flat spectrum are certain number-theoretic periodic sequences based on "primitive roots of 1"[2] and the *finite*-length aperiodic "Barker sequences,"[3] the longest of which (presently known) has length 13 (+++++ − − ++ − + − +). A scatter diagram obtained from a reflector shaped according to this Barker code showed even better scattering than Fig. 1. However, for room-acoustical applications, maximum-length sequences may be preferable.

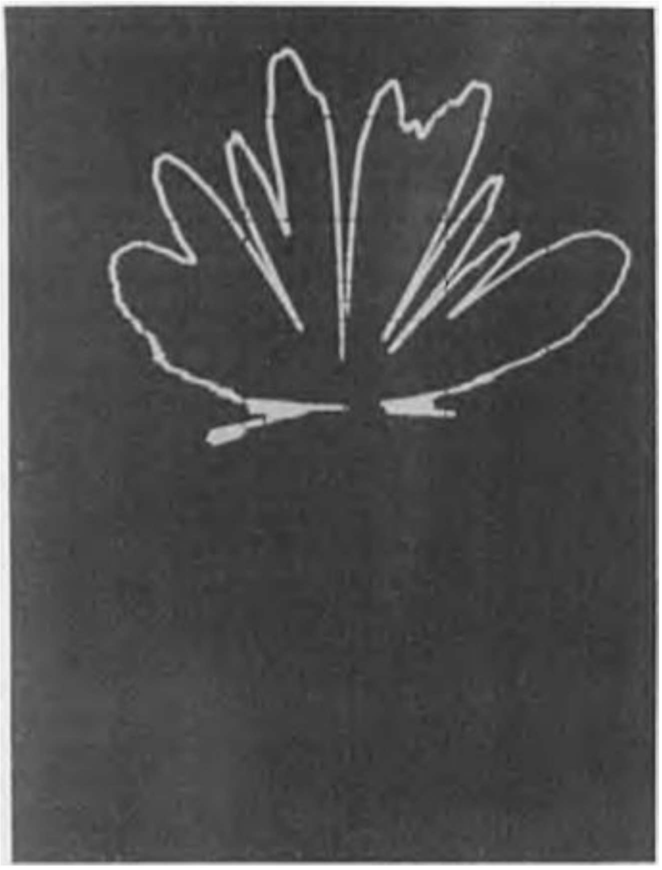

FIG. 1. Scatter diagram from surface with reflection coefficients alternating in x direction according to one period of maximum-length sequence $(-++-+-++++---+-)$. Result obtained in model experiment with electromagnetic microwaves.

For *two*-dimensional scattering, two sequences can be multiplied—

$$r(x,y)=r_1(x)\cdot r_2(y)$$

—by simply *adding* the two depth functions in the x and y directions. The corresponding scatter diagram is the product of the two individual scatter diagrams covering all directions [in polar coordinates ($0\le\phi\le 2\pi$; $0\le\theta\le\pi/2$)] with near-equal intensities.

Calculations show that a given sequence scatters well for a frequency range of about 1 octave (1/2 octave above and below the design frequency). Several octaves can be covered by scattering the first octave in the xz plane, the next octave in the yz plane, and so forth.

Potential applications are for better sound diffusion in reverberation chambers and concert halls. If "binaural coherence" is detrimental to good concert-hall acoustics,[4] then providing for efficient diffusion in the horizontal plane in a concert hall (which is expected to decrease binaural coherence) would improve the "acoustics" of the hall. Appropriate diffusors with high *lateral* scattering could be placed on the side walls *and* the ceiling.

ACKNOWLEDGMENT

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[1]S. W. Golomb, *Shift Register Sequences* (Holden Day, San Francisco, 1967).
[2]R. Turyn, "The Correlation Function of a Sequence of Roots of 1," IEEE Trans. Inf. Theory IT-13, 524–525 (1967).
[3]S. W. Golomb and R. A. Scholtz, "Generalized Barker Sequences," IEEE Trans. Inf. Theory IT-11, 533–537 (1965).
[4]M. R. Schroeder, D. Gottlob, and K. F. Siebrasse, "Comparative Study of European Concert Halls," J. Acoust. Soc. Am. 56, 1195–1201 (1974).