Application Note

Multi-reference Impact Testing for Modal Analysis using Type 3557 Four-channel Analyzer and CADA-PC

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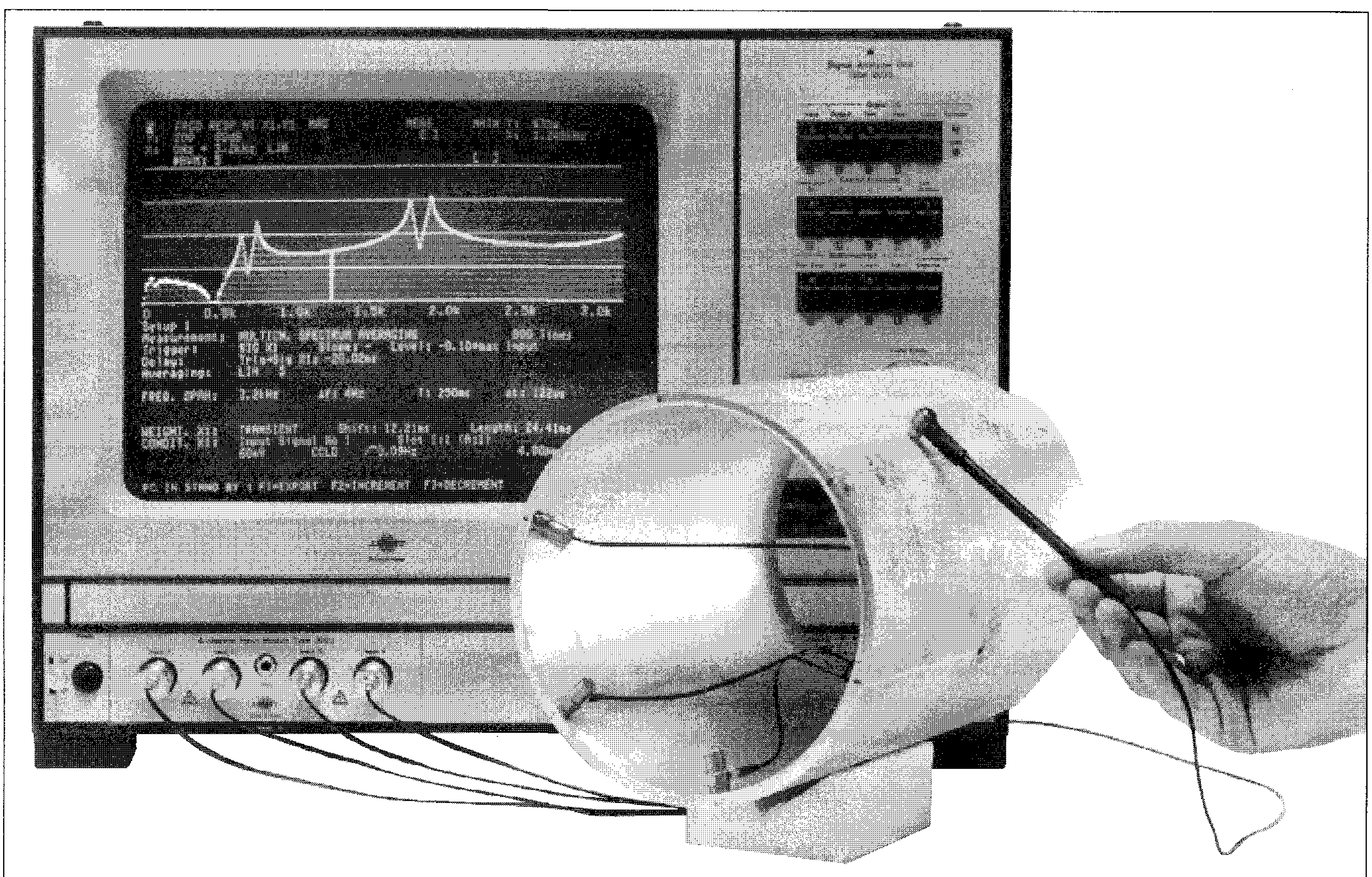
Modal testing using multi-reference data sets, is one way to solve the quandary of many cases where the classical mono-reference technique deteriorates. For instance, bi-symmetrical structures — and sometimes also structures of arbitrary geometries — exhibit repeated roots that can only be decomposed with an augmented data set of a multiplicity higher than, or equal to the multiplicity of the roots we want to extract. Also, complex structures in general may behave such that it can be impossible to find one adequate reference, but multi-reference techniques will most probably solve the problem.

Traditionally, multi-reference data sets were acquired using multiple-input/multiple-output (MIMO) tests. However, many new ideas from MIMO may be transferred directly to impact testing, to become multi-reference impact testing (MRIT), which is less demanding in terms of instrumentation and test setup complexity.

The features of MRIT are: multichannel data acquisition, possible with portable analyzers, using single-input/multiple-output (SIMO) classical FRF measurements, and Polyreference parameter estimation technique, now available on personal computers. The advantages are: in situ, field testing is possible, with most of the benefits that the MIMO technique offers in terms of good reliable parameter estimates — also with repeated roots, and on less expensive instrumentation and a simple test setup. And the benefits are: reliable modal parameters for minimum investments in transducers, analyzer, computer and software, and testing time.

This Application Note introduces the MRIT concept and demonstrates the techniques on a bi-symmetrical structure which has repeated roots in all the modal peaks. These are decomposed using the three-reference data set, measured with the Brüel & Kjær Type 3557 four-channel analyzer, and Polyreference curvefitting as implemented in the CADA-PC modal analysis package.



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Introduction

Using impact testing for measuring frequency response functions (FRF) is as old as FFT based experimental modal analysis itself. Through the last two decades, this technique has proved its practicability, notwithstanding the limitations the method also holds. The trends in experimental modal testing have evolved into very sophisticated multiple-input/ multiple-output (MIMO) techniques using two, or several, uncorrelated attached exciters and as many response transducers as affordable. Parameter estimation techniques, utilizing the resulting augmented data sets, have been successfully implemented, foremost, the Polyreference algorithm. Together, MIMO and Polyreference have proved to have many advantages over the classical minimum data set technique: the 'mono-reference'.

Many of the new ideas may be transferred directly to impact testing that is less demanding, in terms of instrumentation and test set-up complexity.

The features are: multichannel data acquisition, possible with portable analyzers, using single-input/ multiple-outputs (SIMO) classical FRF measurements, and the Polyreference parameter estimation technique, now available for personal computers. The advantages are: in situ field testing is possible, with all the benefits that the MIMO technique offers in terms of good reliable parameter estimates, also with repeated roots, and on less expensive instrumentation with simple test set-ups. And the benefits are reliable modal parameters for minimum investments in transducers, analyzer, computer and software.

Background

Traditionally, we know that for a linear structure only one full row, or one full column, of the FRF matrix need be measured; that is, only as many FRFs as there are defined DOFs in the test. From this minimum data set, a complete model identification is achievable. This, of course, assumes that sufficiently many DOFs are included in the test definition to describe the dynamics of the system in the frequency range of interest.

MIMO implies measuring as many minimum data sets as there are inputs in the test. Is the MIMO technique, then, only adding redundant information to the data set? Let us try to illustrate the whole matter graphically.

Fig. 1 shows one way to present the spatio-temporal data domain. The temporal domain is here represented by frequency — it might equally well have been time — the spatial domain represents the defined DOFs which are included in the measurement (model) and the last dimension, which is also spatial, represents the DOFs which are used for reference. In this space, the measured FRFs (the imaginary parts in Fig. 1) are arranged. Now comes the point: for a SISO test, all the measurements are found in one plane, while for a MIMO test, there are as many planes as there are inputs. In the figure, we find two planes of data as two references are present in this hypothetical measurement, effectively, doubling the amount of data.

Is this extra information surplus? No! In practice, a number of problems may arise in system identification, when only a minimum data set is available. Acquisition of an augmented data set, by adding one or more extra rows, or columns, of [H] can often surmount these.

The use of augmented data may be perceived as a kind of 'array' technique, where many, almost identical, data sets from a process are gathered. This may be interpreted as if the system or process, is observed

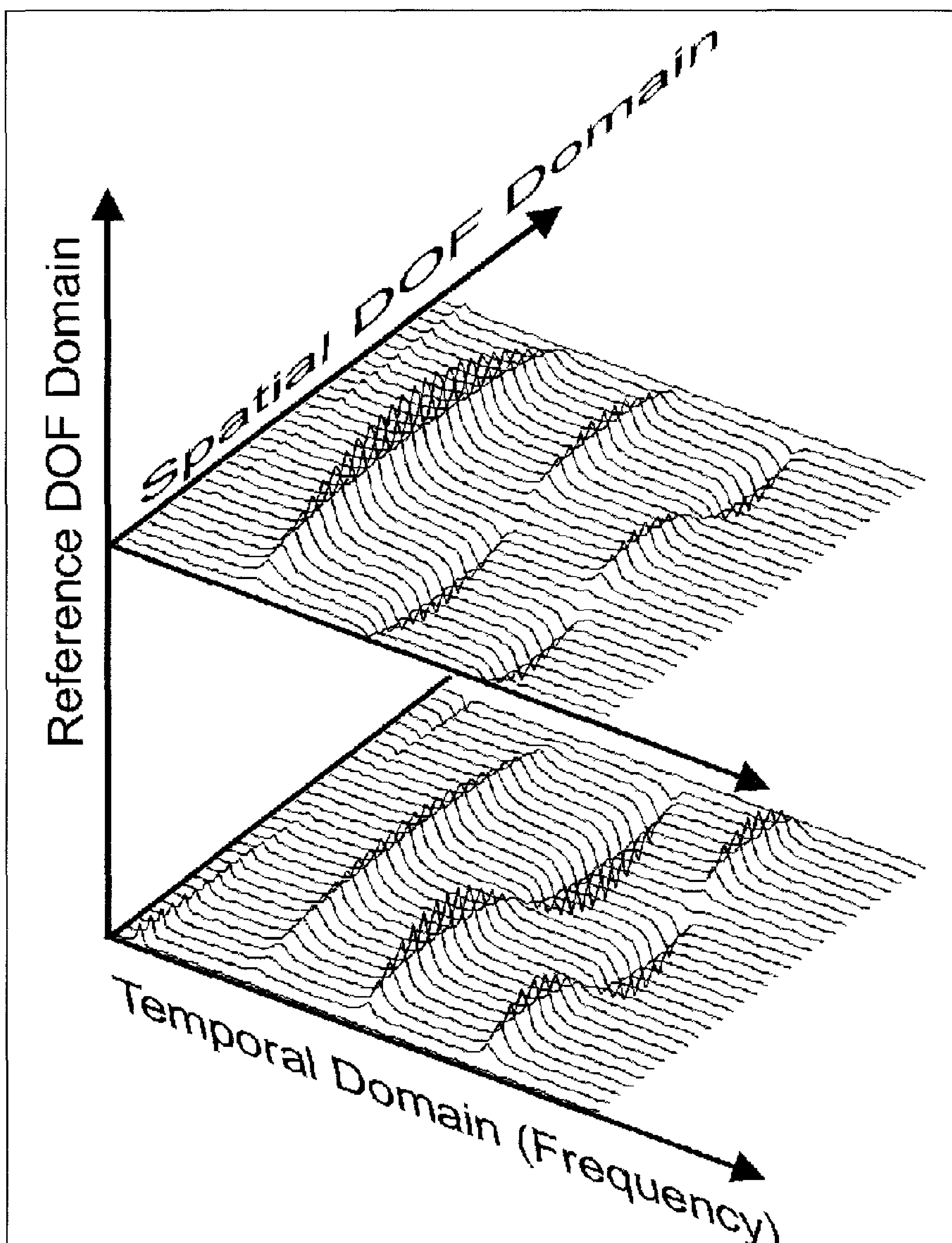


Fig. 1 Information space of the dimensions frequency, spatial DOFs and reference DOFs

and registered from different perspectives (looking around obstacles). This may sharpen the analysis and pinpoint fine details in the process, and extends the knowledge about the problem being studied. On the other hand, if the structure is simple or the augmented data is not gathered in such a way as to add new information, the extra information would merely be redundant, adding no extra value — only extra work.

A Generalization from MIMO to Multi-Reference-Test

'Multiple-Input/Multiple-Output' rigorously refers to a test with more than one active excitation source. However, we can use the advantages found in the MIMO technique with other types of data acquisition; for example, impact testing with more than one reference transducer. In an IMAC paper, scientists from the Structural Dynamics Research Lab at the University of Cincinnati termed this test: ***MRIT*** — Multiple Reference Impact Testing*. The essence of the whole matter is multiple references.

Where is a Multi-Reference Test Beneficial?

The kinds of problem that can be solved with multi-reference tests are, for instance, found in the following categories:

- On **Large Structures**, the sheer size may create problems with regard to the magnitude of that excitation energy it takes to ensure a sufficiently high signal-to-noise ratio. Using one exciter may require a force level so high that it drives the structure into non-linear behaviour. This phenomenon can, for instance, result in mode shapes where the amplitudes appear much higher at the shaker position than anywhere else on the structure. This cannot be true! As we may recall, mode shapes represent free response properties and, consequently, are independent of where and how the response was initiated. Distributed excitation, using several small shakers, scattered over the structure, can provide a sufficiently high signal-to-noise ratio at a relatively low response level that ensures linear behaviour of the structure.
- **Bi-symmetrical Structures** — and sometimes also structures of arbitrary geometries — exhibit repeated roots. That is, more than one mode shape is associated with a particular pole location (or in other words: apparently more modes are located exactly on top of each other as seen in the frequency domain). To solve that problem, that is, to decompose such repeated roots, the measurements must be augmented with a data set of a multiplicity higher than, or equal to, the multiplicity of the roots we want to extract.
- On **Complex Structures** in general, it can be impossible to find one adequate reference, that is, a DOF that has sufficient participation for all the modes in the frequency range of interest. For example, a structure may exhibit modes with predominant motion in only one plane, and other modes with predominant motion in an orthogonal plane to the first. Also, local modes may coincide in frequency with global modes and make parameter estimation difficult. The solution to this problem, again, may be the application of more (poly)references.

The Proof of the Pudding is in the Eating

Multiple Reference Impact Testing on a Cylindrical Structure

For a demonstration of the MRIT technique, we have chosen a geometrically simple, but dynamically complex structure: a cylindrical shell. This example demonstrates what we mean by 'complex', namely, that this bi-symmetrical structure has repeat-

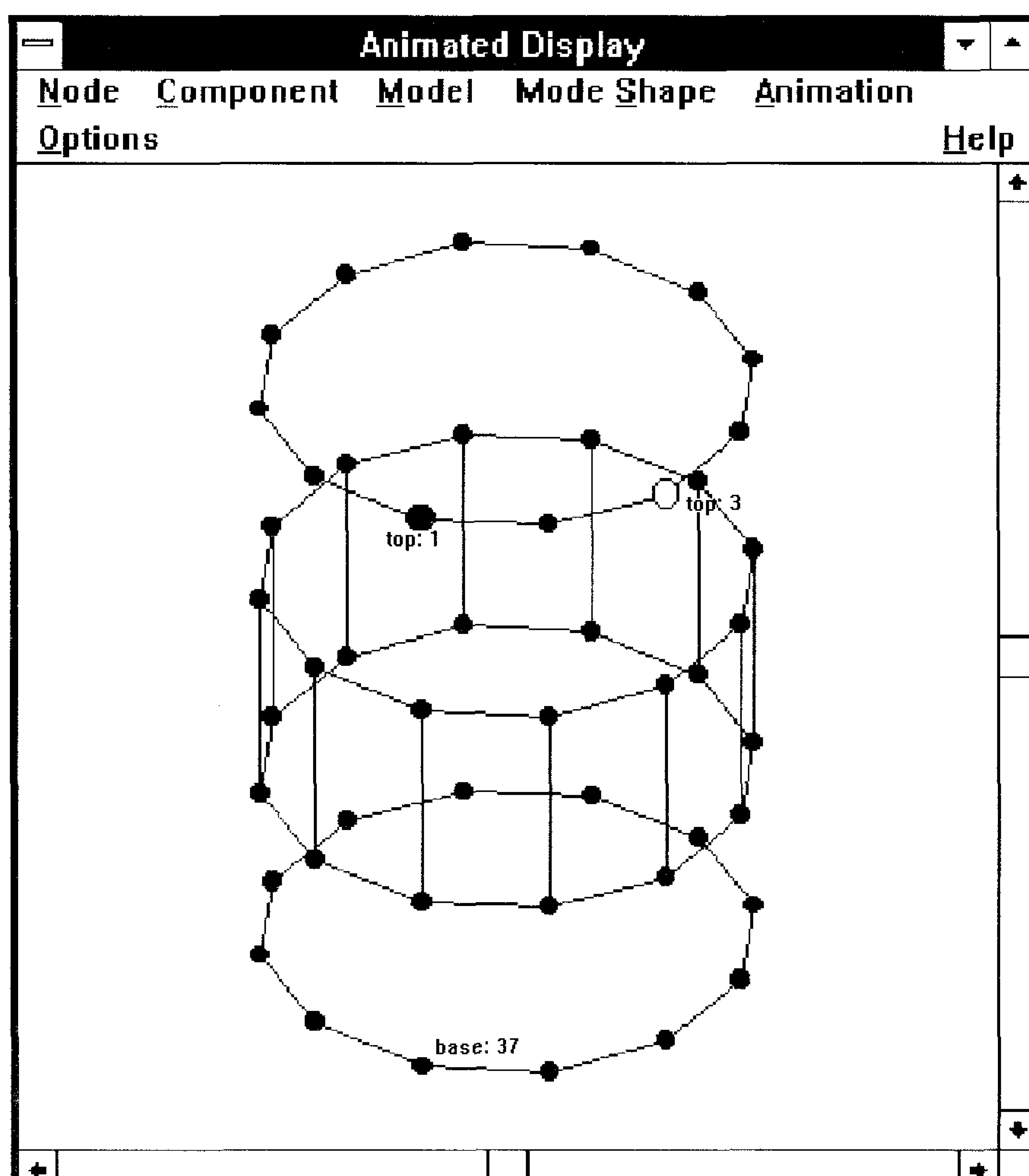


Fig.2 Geometrical model of test object with position of the three reference DOFs

* Fladung, W. A. & Brown, David L., **Multiple Reference Impact Testing**, 11th IMAC, 1993

ed roots which we cannot extract by classical mono-reference techniques.

Test Object and Set-up

As the test object, we use an aluminium cylinder with a diameter of 0.2 m and 0.3 m long. The wall thickness is 0.005 m.

Fig. 2 shows the geometric model of the structure, based on the defined test points. 12 equally spaced points around the periphery, repeated four times along the cylinder axis, give 48 points. One DOF, namely radially, is assigned for each point. The rationale is that, in the frequency range of the test, only motion perpendicular to the surface is anticipated. Three references are used in the set-up. Normally a pre-analysis is performed in order to determine the best reference DOF(s). However, from our *a priori* knowledge that mode shapes are orthogonal and the best set of references form another orthogonal subset of the mode shapes, we selected the three points as shown in Fig. 2. For practical reasons — access for the impact hammer — we mounted the accelerometers inside the cylinder. The accelerometers were the new Brüel & Kjær DeltaTron® Type 4394, and the impact hammer was assembled from the miniature force transducer kit, Type 8203.

Boundary conditions were chosen free-free, by placing the cylinder on soft pads of rubber-foam.

Data Acquisition

Signal conditioning and analysis was done by the Multichannel Measurement System Type 3557. Two particular issues of the data acquisition process must be discussed in a little more detail: namely, the characteristics of the Constant Current Line Drive (CCLD) input condition and the issue of weighting (windowing) of the signals.

Input Module Type 3023

Type 3557 is equipped with one 4-channel input module Type 3023. Besides the direct input connection, a CCLD, equivalent to ICP*, is available. The function of the input is to provide power to drive the integrated electronics in the accelerometers, which gives the benefit of saving external conditioning amplifiers, but more important, the gain and overload are controlled centrally from the analyzer, and increase data safety significantly. One trade-off, however, is that the cost of being able to measure down to 0.09 Hz (which is important for accurate measurements of transients) the recovery time of the high-pass-filter is correspondingly long. After an overload, for example a switch transient from changing gain, it may take several seconds before the offset has decayed completely. Therefore, particular attention should be paid to weighting the signals.

Signal Weighting: Choice of Windows

Excitation by an impact hammer implies a very short event in the acquired time record. Therefore, only that part of the signal that represents the contact phase of the impact is of interest. Consequently, we chose a window that zeros out the remaining part of the signal, which represents noise. The way we choose to zero out is very important. If an offset exists in the signal and this is abruptly cut with a rectangular window, a severe leakage problem probably arises. To circumvent this problem, we modify the window by adding a 'Half Hanning' to the leading and the trailing edge of a short rectangular window. The effect is a soft transition at both ends, reducing the leakage effect dramatically.

Characteristic for the response signals is that they have not fully decayed at the end of the record (time block). Consequently, if no weighting is applied before the fourier transformation, severe leakage emerges. The choice of weighting is the exponential window, that forces the signal to decay before the end of the record. We also want to remove the effect of a possible offset in the response channels. This, again, is obtained by adding a half-Hanning weight to the leading edge of the exponential window. Fig. 3 shows the force and one response signal with the appropriate weighting windows.

The effects of the windows are as follows:

For the force signal no effect of the transform is observed and no corrective action is required.

For the exponential window, however, the effect is an apparent increase in decay rate: moving the pole to the left. The effect is predictable and may subsequently be removed by correcting the modal damping parameters [see ref. B&K Technical Review No. 1 – 1984 regarding windowing].

The measurement set-up uses all four channels: one for the excitation force and the remaining three for response references. Each DOF is impacted 5 times, hence, for each averaging, three FRFs are produced: between the actual excitation DOF and each of the three reference DOFs. The number of measured FRFs totals 144.

Modal Analysis

For the sake of comparison, the modal analysis, or parameter extraction is done twice: one time utilizing only a subset of the data, a minimum data set, using only one reference, and another session using the full data set.

The data acquisition, and the following modal parameter estimation,

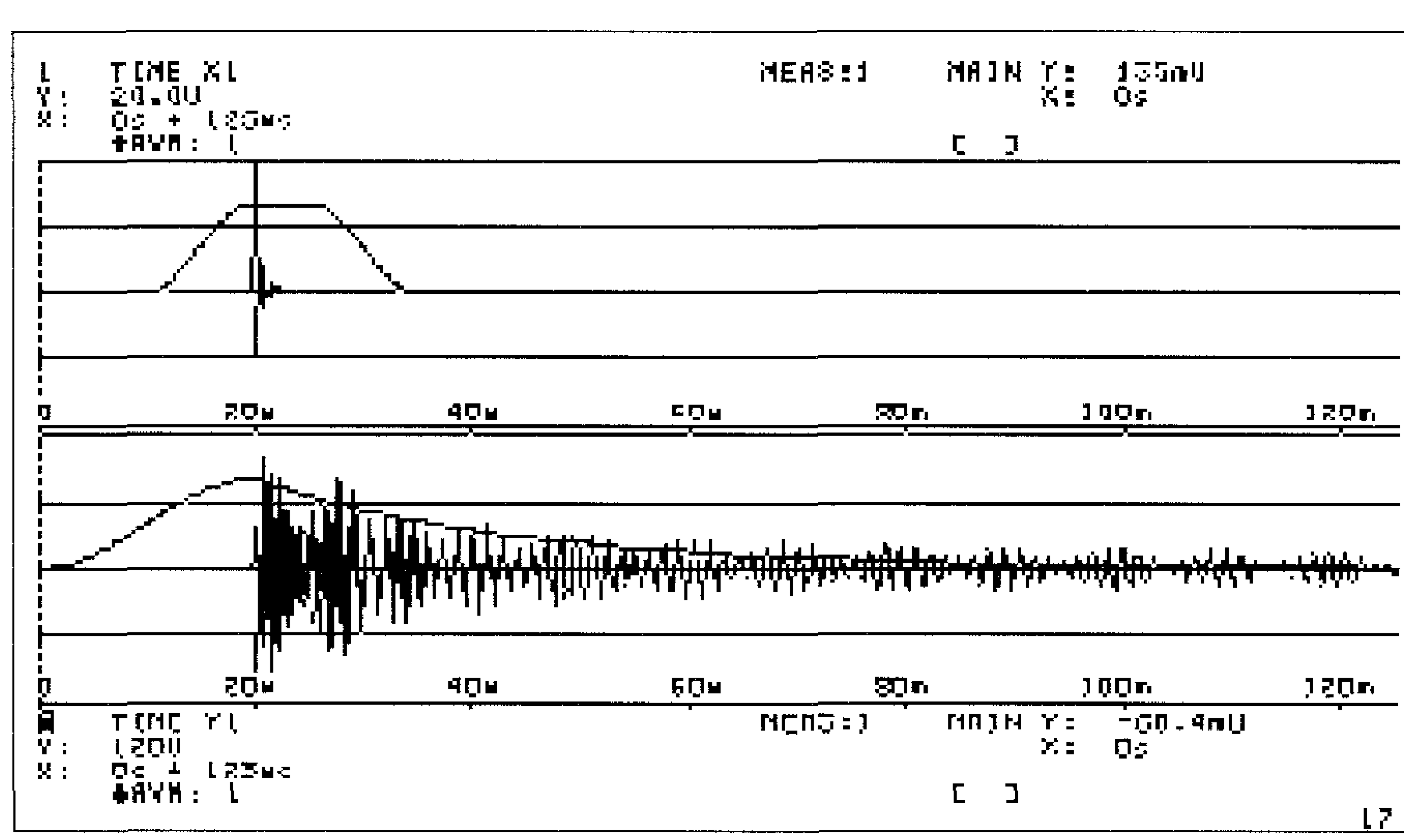
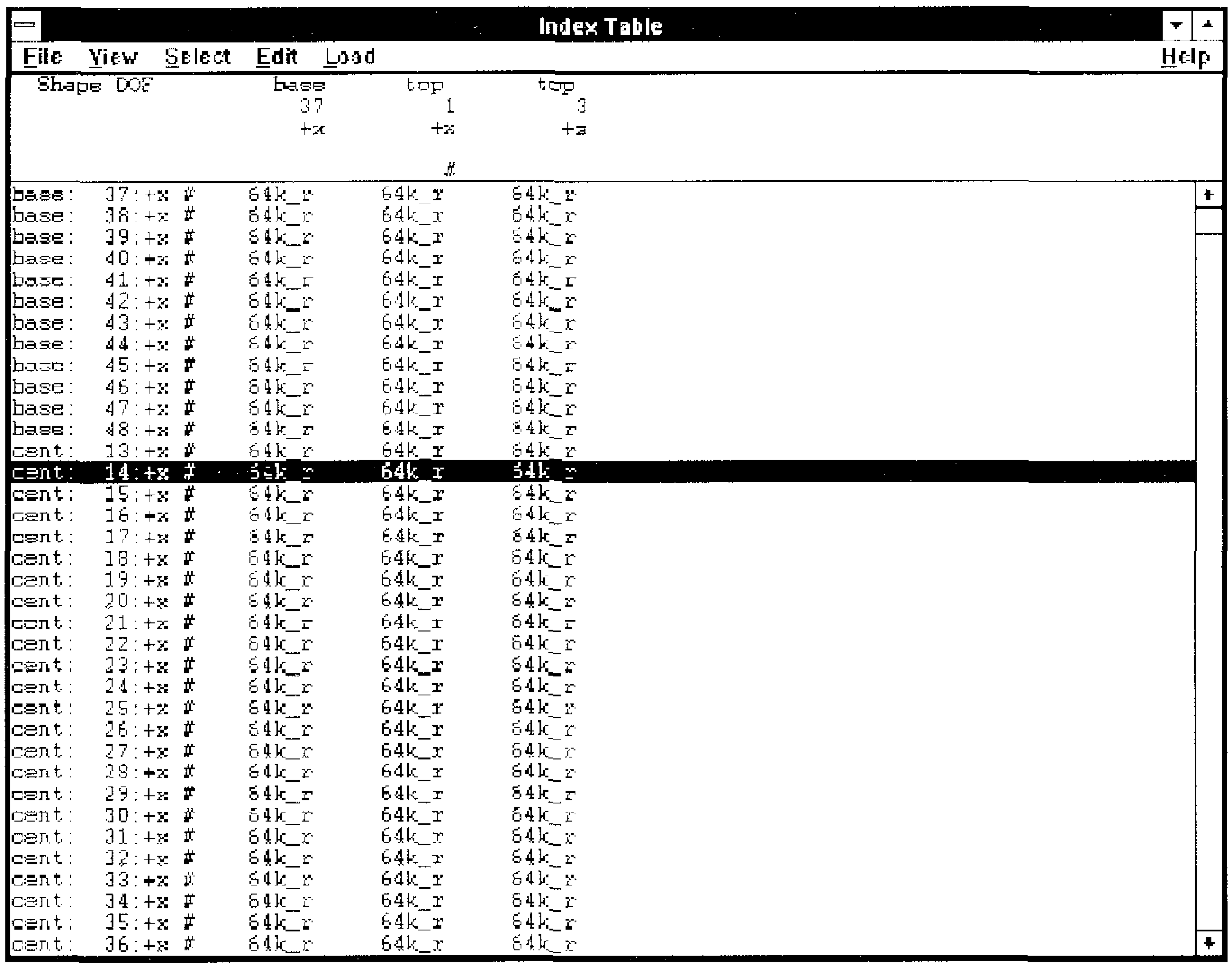


Fig.3 Signals and windows of the input and one of the outputs

* ICP is a trademark of PCB cooperation



Index Table				
Shape	DOF	base 37 +x	top 1 +x #	top 3 +z
base:	37 :+x #	64k_r	64k_r	64k_r
base:	38 :+x #	64k_r	64k_r	64k_r
base:	39 :+x #	64k_r	64k_r	64k_r
base:	40 :+x #	64k_r	64k_r	64k_r
base:	41 :+x #	64k_r	64k_r	64k_r
base:	42 :+x #	64k_r	64k_r	64k_r
base:	43 :+x #	64k_r	64k_r	64k_r
base:	44 :+x #	64k_r	64k_r	64k_r
base:	45 :+x #	64k_r	64k_r	64k_r
base:	46 :+x #	64k_r	64k_r	64k_r
base:	47 :+x #	64k_r	64k_r	64k_r
base:	48 :+x #	64k_r	64k_r	64k_r
cent:	13 :+x #	64k_r	64k_r	64k_r
cent:	14 :+x #	64k_r	64k_r	64k_r
cent:	15 :+x #	64k_r	64k_r	64k_r
cent:	16 :+x #	64k_r	64k_r	64k_r
cent:	17 :+x #	64k_r	64k_r	64k_r
cent:	18 :+x #	64k_r	64k_r	64k_r
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cent:	25 :+x #	64k_r	64k_r	64k_r
cent:	26 :+x #	64k_r	64k_r	64k_r
cent:	27 :+x #	64k_r	64k_r	64k_r
cent:	28 :+x #	64k_r	64k_r	64k_r
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cent:	31 :+x #	64k_r	64k_r	64k_r
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cent:	34 :+x #	64k_r	64k_r	64k_r
cent:	35 :+x #	64k_r	64k_r	64k_r
cent:	36 :+x #	64k_r	64k_r	64k_r

Fig.4 Index Table

is controlled from CADA-PC*. During the data acquisition the measurements are labelled according to the DOFs of the predefined geometric model.

* CADA-PC is a product of LMS, Leuven, and marketed as a Brüel & Kjær OEM product. The instrument interface is particularly developed for the Brüel & Kjær Type 3550/2148/2032 analyzers

Mono-reference Parameter Extraction

The parameter estimation phase is initiated by defining what data is to be included in the process. All the measurements are stored in a data base (the hard disk), and the data to be included is defined by editing the *Index Table*. Fig. 4 shows part of the index table. The #-character indicates which measurements are included. As indicated in the table, only measurements using TOP 1X as reference are included.

Graphically, we may review Fig. 1 and picture our information space. This data set consists of 48 spatial DOFs, for each of which we have 801 frequency points, and forms one plane of FRF data per spatial reference DOF. Hence, we have three information planes available, but for the mono-reference case, we only use one of them.

The curve fitting is done in two sessions:

Estimating poles

Firstly, the poles (modal frequency and modal damping) are found using *least squares complex exponential*, which, in simple terms, is a global MDOF curve fitter working on the inverse Fourier transform of the measured FRFs, representing impulse response functions. The curve fitter provides a stabilization diagram that suggests where the poles are found, as a function of model order (the number of modes in the fitted model), and the user chooses which poles to use. Fig. 5 shows the stability diagram and the chosen poles.

Estimation Residues

Secondly, the residues (mode shape components) are estimated, using a frequency domain algorithm that

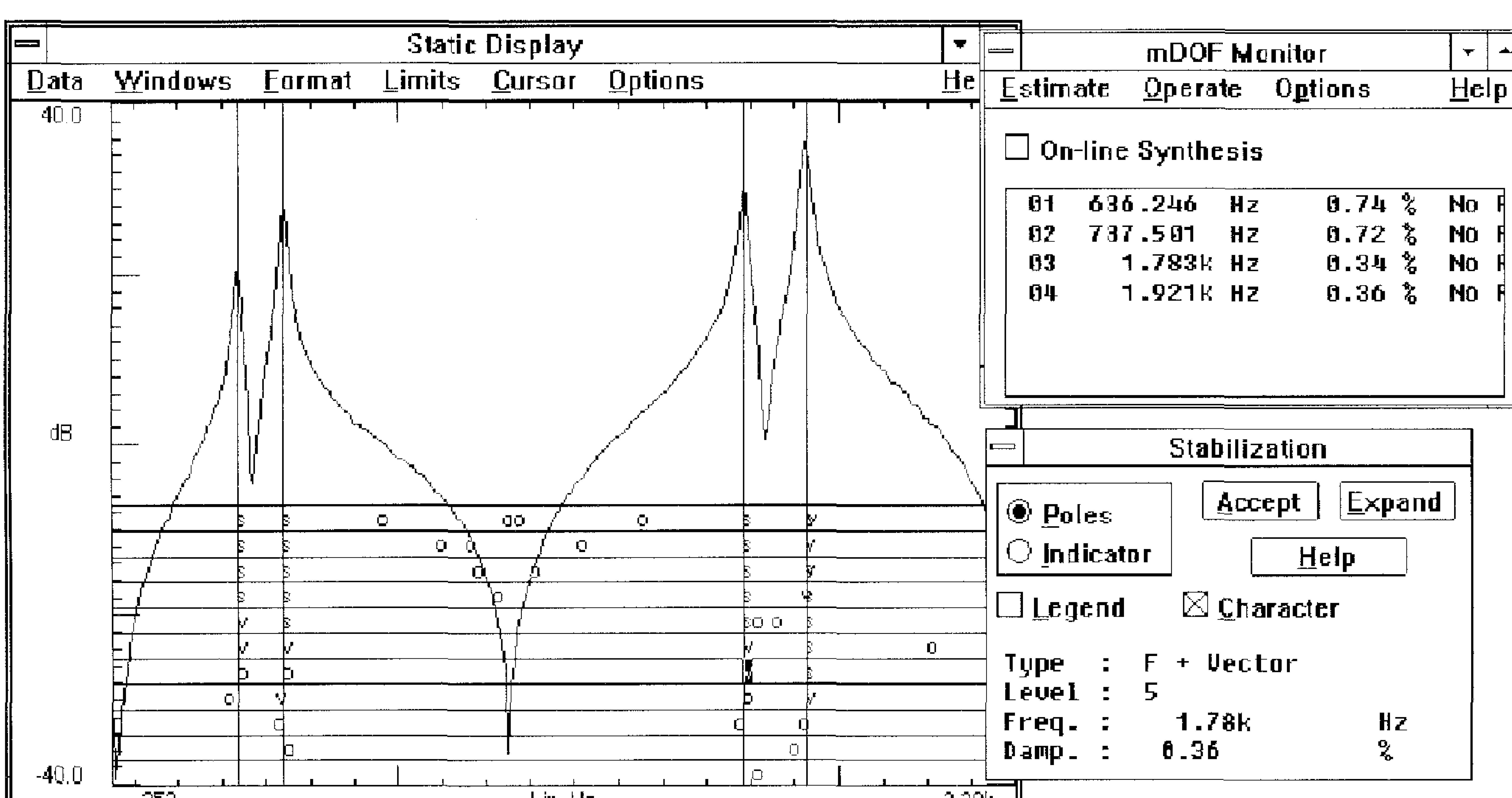

Fig.5 Stabilization diagram for mono-reference data set. One pole is found at each peak

uses the estimated poles as parameters in the algorithm.

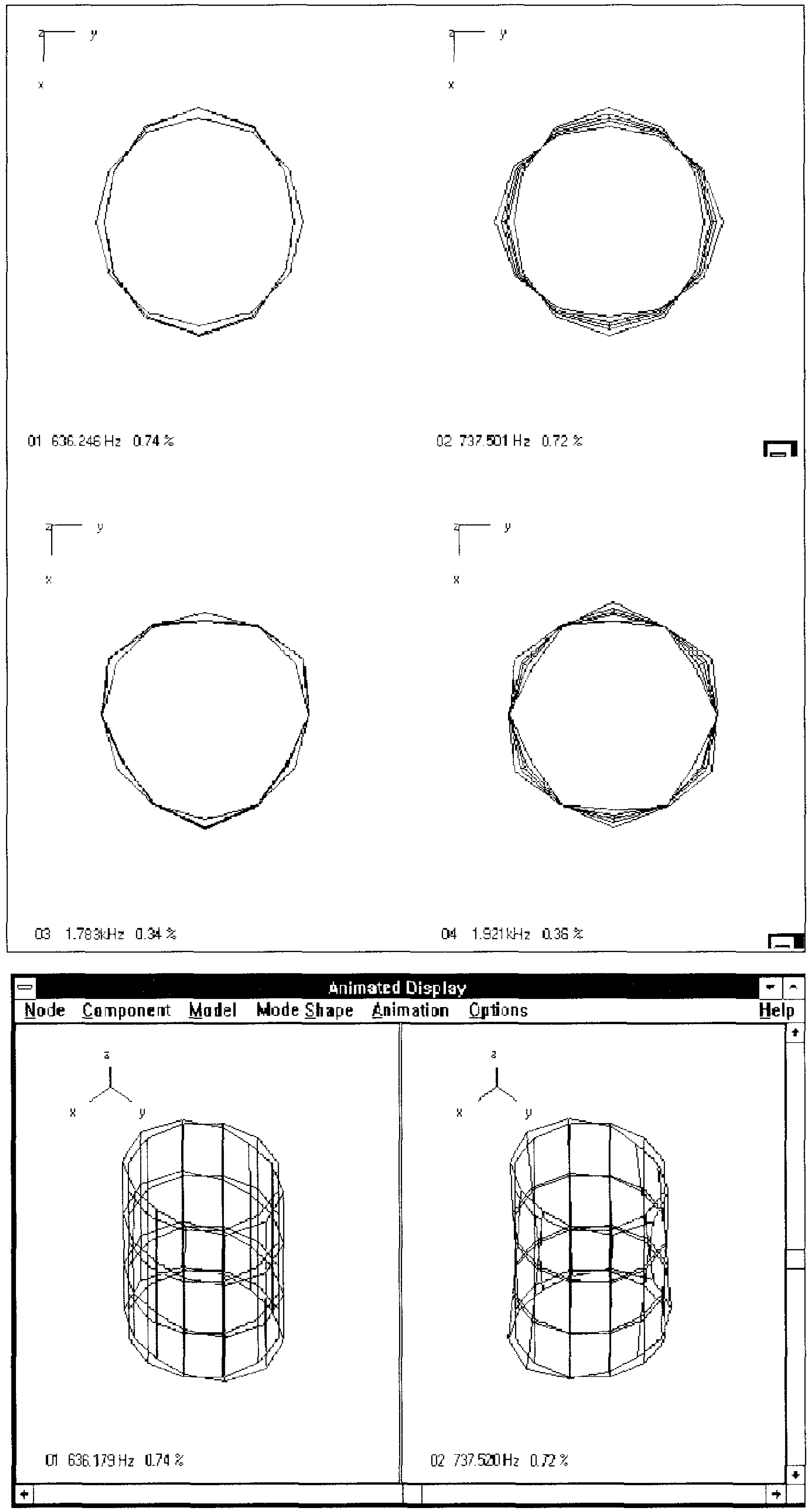

Fig. 6 Estimated mode shapes from mono-reference data. Note that mode #1 and #2 are 4:0 and 4:1 nodes, and modes 3 and 4 are 6:0 and 6:1

Results: Examining The Mode Shapes

Fig. 6 shows the mode shapes associated with the first 4 peaks in the FRFs. They all appear credible as they are confirmed by the standard checks, such as MAC (Modal Assurance Criterium). However, if the extracted parameters are used to synthesize the measured FRFs, we find some DOFs with strange deviations. *Something is wrong!*

Polyreference Parameter Extraction

We now repeat the analysis, however, including the full data set. Referring back to Fig. 1, we now include three planes of information. To initiate a multi-reference set, we edit the index table, and include all three reference DOFs. Then, we may utilize a very powerful tool, the Mode Indicator Function (MIF), to help indicate the number of modes in the data.

Mode Indicator Function

The MIF, as implemented in CADA-PC, includes all the measurements, as specified in the index table, and determines the minimum in-phase response (which is zero for normal modes) normalized by the total response. That is, the function equals unity for frequencies where the response is in phase with the driving force, and becomes zero for pure quadrature response — at the modal frequency for a normal mode — and dips to a level between zero and one for complex modes. If more than one reference is available, higher order MIFs are defined, and represent repeated roots. The order of multiplicity that can be determined equals the number of reference DOFs.

Fig. 7 shows a plot of one FRF and the 1st, 2nd and 3rd order MIFs. The plot reveals that all four peaks in the FRF represent repeated roots. As the 3rd order MIF remains 1 at all peaks, we conclude that the multiplicity is two. In other words, the MIFs show that the data contains 8 poles, and not 4 as found in the mono-reference estimation.

Modal Participation

Apparently, the parameter estimation follows exactly the same pattern as for the previous estimation process. However, transparent to the user, the algorithm finds a 'new' global parameter: *the modal participation.* For each pole the estimator finds, it also finds the modal participation, which is simply that part of the mode shape that represents the DOFs of the reference positions. For

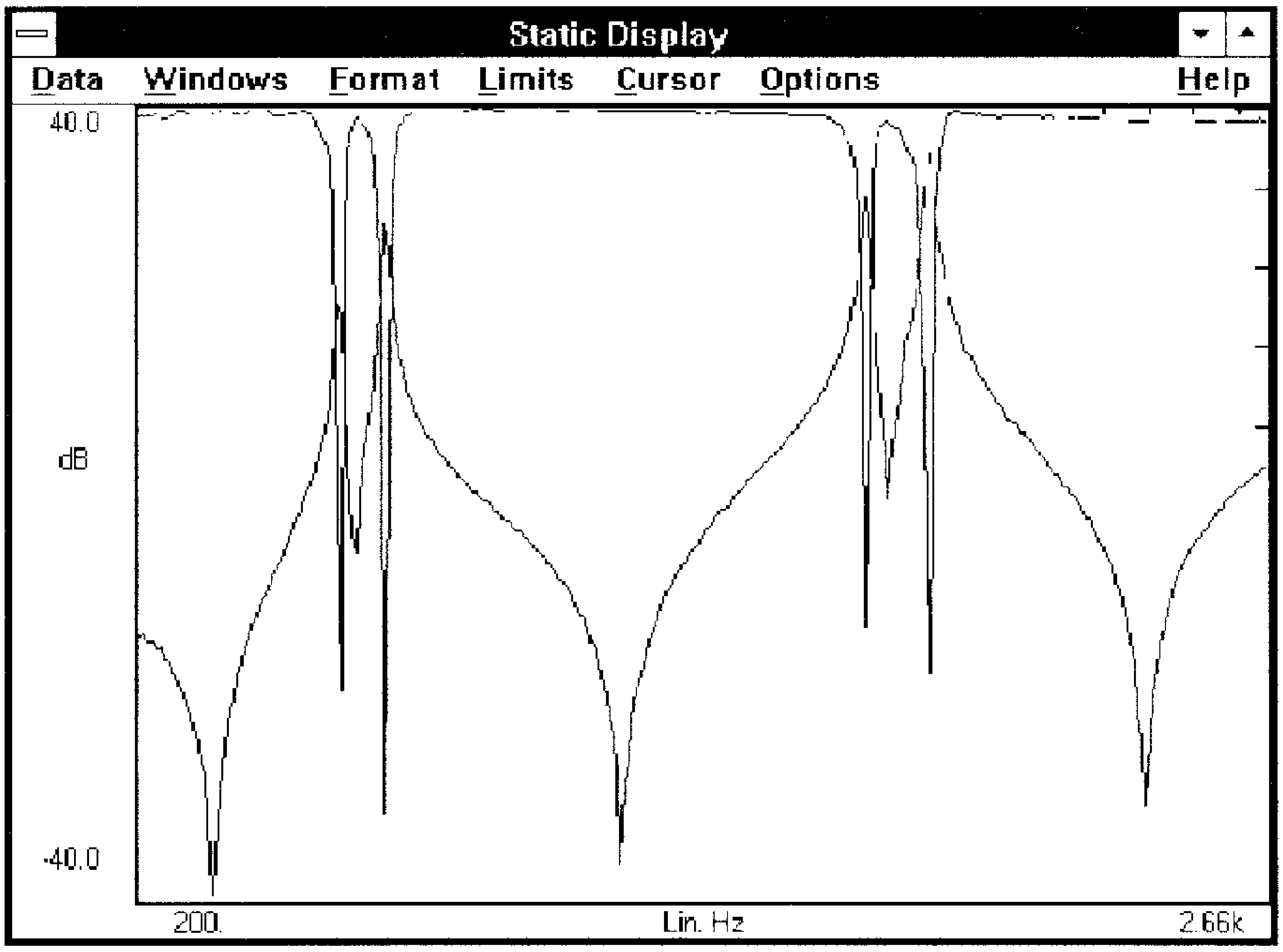

Fig. 7 One driving point FRF and MIF1, MIF2 and MIF3, suggests double poles in all four peaks

this example, we may think of the modal participation — one for each mode — as a 'short' mode shape vector that contains shape elements for the three reference DOFs. The superb point of the Polyreference technique is this: if more than one mode (mode shape) exists at one particular pole location, then there will be more modal participation for that pole. This is true provided that the references are chosen such that these modal participations are linearly independent. Or, in other words, provided that the difference in mode shapes appears in the reference positions.

Estimating Poles and Modal Participation

The estimation procedure is exactly the same as before. However, the difference appears in the stabilization diagram, that clearly indicates that two poles exist per peak.

Estimation Mode Shapes

Having selected the pole locations, the residue estimation follows the same scheme as for our first example. But, one very important point, which is transparent to the user, is that the mode shape estimates now also become global. That is, having three references, we can estimate the shapes three times. However, the augmented data is used in a least squares sense to estimate the one set of shapes that best fits the total data set.

Revisiting The Mode Shapes

So how do they look, the four extra mode shapes? Fig. 9 shows all eight mode shapes found in the multi-reference estimation. We see that the 'new' modes are of the same shape as the first modes found, but rotated 45 degrees. Although these couplets of shapes look like clones, they are, nevertheless, orthogonal — linearly independent — of each other, and orthogonal to all the others. Hence, they do belong to the dynamic model and are necessary for creating an accurate modal model.

The verification tools, MAC and FRF synthesis, prove that we now have identified a reliable representative model.

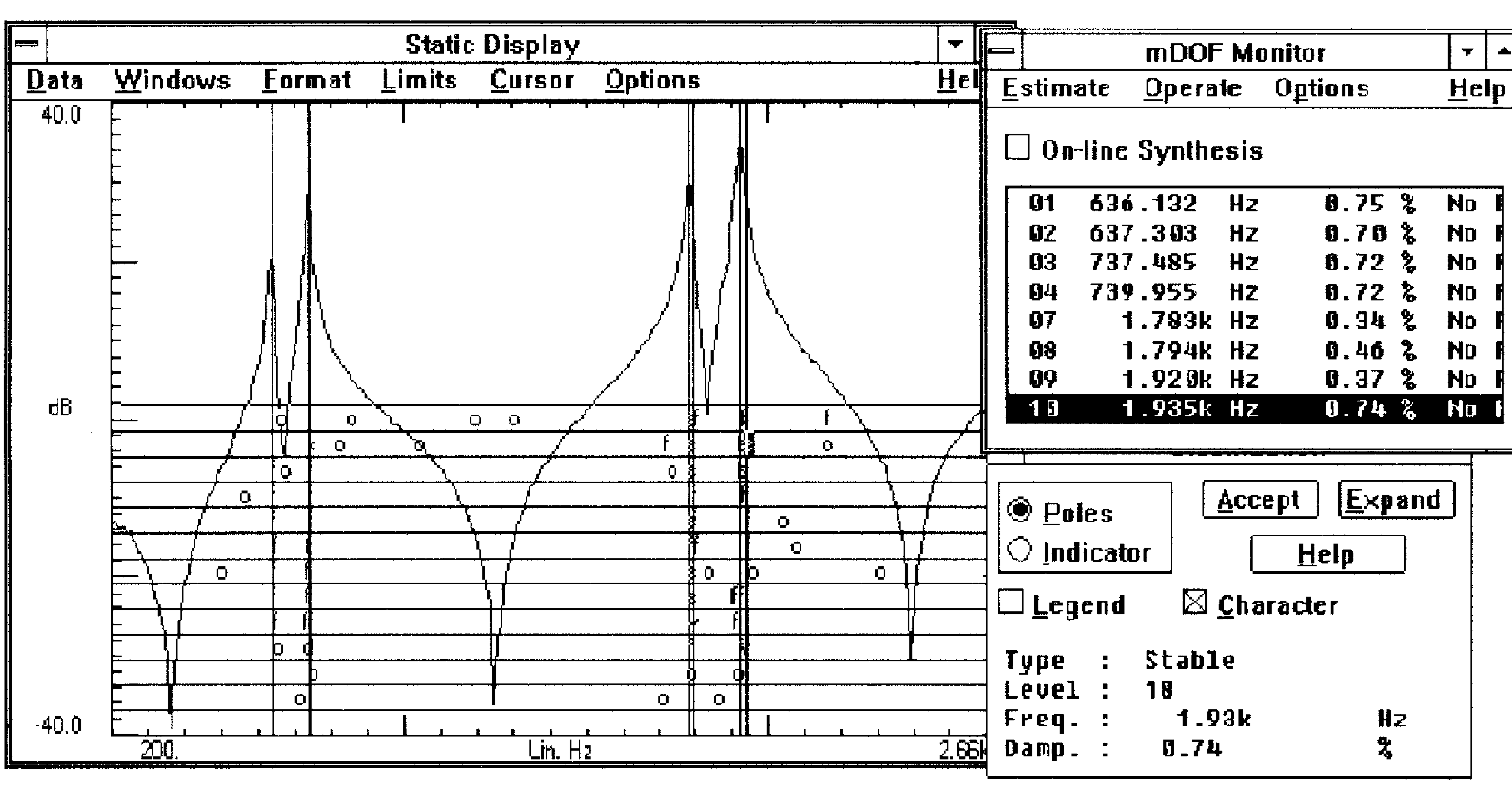

Fig. 8 Stabilization diagram for 3 reference data set. Two poles are found at each peak

Conclusions

Using the multiple-reference impact technique, we showed how to measure an augmented data set with multiple references — this did not require any multiple-input FRF estimation. From the data set, we extracted reliable modal parameters for a dynamically very complex structure. Two issues should be noted:

First, the decomposition of repeated roots cannot be done by increased frequency resolution. Only increasing the spatial resolution, by adding more references, makes this goal achievable.

Second, increased spatial resolution by adding more references is only achieved if these references provide augmented information to the data set. What is meant by this statement may be illustrated by taking another view at the two mode shapes of the first peak. The associated mode shapes, looking only at the rim of the tube, show four node-point modes. In our measurement we had the references offset 0 and 45 degrees. The associated participation factors are respectively [1,0] and [0,1]. Obviously, these two vectors are linearly independent, actually they are orthogonal. Visualize what would have happened, if we had chosen an offset of 90 degrees. Then the participation factors would had been [1,-1] and [0,0], which would not reveal two independent mode shapes. Therefore, we need, even with this powerful technique, to use our *a priori* knowledge, or fantasy, to think out good strategies for the choice of references. However, a shot-gun approach, throwing out a large number of more or less randomly chosen references would probably also have solved the problem, though less efficiently.

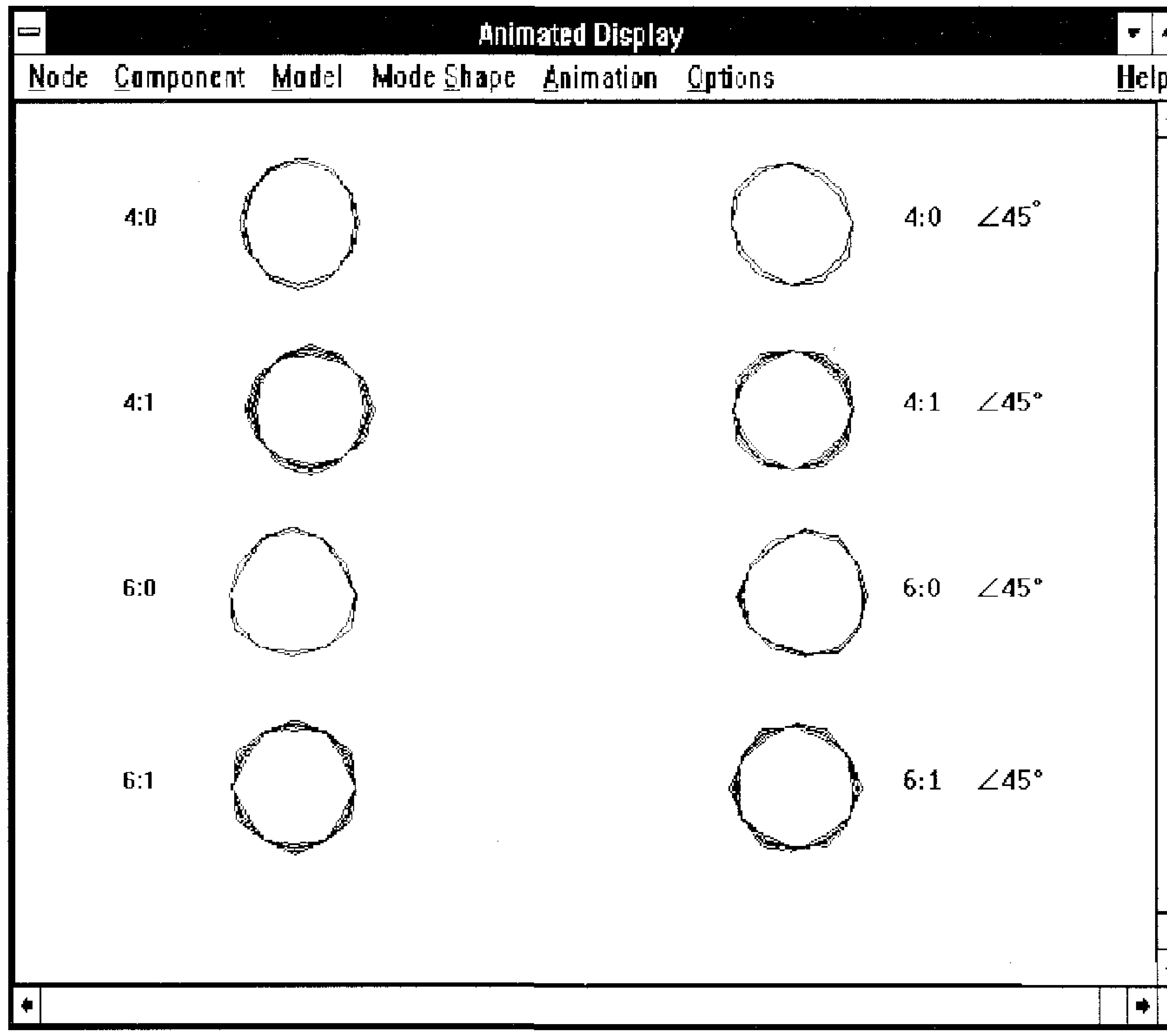

Fig.9 Mode shapes from MRIT

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