Integrated-impulse method measuring sound decay without using impulses

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A method of measuring linear-system responses (such as room responses) is described using "maximum-length" pseudorandom noise as the test signal. In this manner, high signal-to-noise ratios can be achieved, even for measurements in noisy environments and for low-power test signals. *Pseudo*random noise has also been used successfully as the test signal in the "integrated-impulse" method of measuring sound decay and reverberation time. Thus, the need to radiate a short pulse of high peak energy for impulse type measurements is completely avoided. Improvements in signal-to-noise ratios are equal to the period length of the pseudorandom noise, typically 40 dB in room acoustical applications. The necessary digital processing to realize these gains in signal-to-noise ratio and accuracy of response can be performed on available minicomputers. Apart from maximum-length sequences, another type of periodic binary sequency, called the "Legendre sequence," can be used as a test signal. Like maximum-length sequences, Legendre sequences have flat power spectra, but their discrete Fourier components have only two phase angles ($\simeq \pm 90°$), thus simplifying their digital representation (for storage and transmission). In fact, the discrete Fourier transform of a Legendre sequence is equal (within a constant factor) to the sequence itself! Legendre sequences exist for all period lengths equal to a prime number of the form $4k-1$, where k is an integer. Thus, there are many more period lengths to choose from than for maximum-length sequences.

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INTRODUCTION

One problem of the integrated-impulse method of measuring sound decay[1-5] and other impulse test methods is the requirement of radiating a sufficiently powerful *impulse* with a flat spectrum. Electrical sparks, popping balloons, and pistol and cannon shots, although powerful sources of sound, may not have sufficiently flat spectra for some applications (although this may be of small concern if measurements are made in octaves or third-octave bands). Loudspeakers, on the other hand, usually do not radiate sufficient power for the required signal-to-noise ratio. To overcome these difficulties, the author, several years ago, suggested the use of pseudorandom noise as an excitatory signal for room-acoustical measurements. Pseudorandom noise,[6] conveniently generated by shift registers, has a perfectly flat power spectrum and a peak amplitude that is typically 100 times smaller than that of a single impulse of the same energy. As an additional advantage, for binary (±1) pseudorandom noise, memoryless nonlinear distortion of the loudspeaker radiating the noise is immaterial. However, the use of pseudorandom noise, instead of a single impulse, requires digital processing to recover the desired single-impulse response.

Pseudorandom noise based on "maximum-length" sequences has been used routinely for room-acoustical measurements by the author and his collaborators at Göttingen[7] for a number of years. Methods of generating the noise and processing the received signal are described in this paper.

I. MAXIMUM-LENGTH SEQUENCES

Maximum-length (ML) sequences are *periodic* sequences of integers a_k. In the case of *binary* sequences, the integers are restricted to having two values only, say $+1$ and -1. They are generated by n-stage shift registers and the period length is $N=2^n-1$. Their most important property for our purposes is that their Fourier transform (DFT) has the *same magnitude for all frequency components* (except the dc component). Thus, their power spectrum is like that of a *single* impulse, namely independent of frequency.

An equivalent statement[6] is that their periodic autocorrelation

$$r_m = \frac{1}{N}\sum_{k=1}^{N} a_k a_{k+m} \tag{1}$$

is two-valued:

$$r_0 = 1$$

and

$$r_m = -\frac{1}{N} \text{ for } m \not\equiv 0 \ (\mathrm{mod}\, N) \,. \tag{2}$$

This property immediately suggests that response measurements on linear systems (such as rooms) can be made with a ML sequence with N constant-magnitude pulses per period, instead of a single pulse that, for equal energy, must have a $(N)^{1/2}$ times larger magnitude. It is only necessary to cross correlate the system response with the same ML sequence that is used for exciting the system to obtain the desired system impulse response. (This operation is also referred to in the signal-processing literature as "deconvolution.")

For room-acoustical investigations at frequencies below 10 kHz, a minimum sampling rate $f_s = 20$ kHz is required. Furthermore, for reverberation times of up to 2.4 s, the period of the excitation signal should be at least 1.6 s long (to cover a 40-dB decay range). Thus, the period length N must be greater than 32 000. The nearest larger value of n, so that $2^n - 1 > 32\,000$, is $n = 15$. Thus, a possible period length equals

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$N = 2^{15} - 1 = 32\,767$.

The corresponding realizable gain in signal-to-noise ratio is $10 \log_{10} 32\,767 = 45$ dB.

A ML sequence of that period length is generated by a 15-step shift register with the following "recursion relation": "The outputs of the last and last-but-one stage are multiplied and fed back into the input of the first stage." (In the 0, 1 notation usually employed with shift registers and other digital circuits, where 0 corresponds to our +1 and 1 corresponds to our −1, multiplication is replaced by the "*EXCLUSIVE-OR*" operation.)

It is always possible to generate ML sequences for any n with a recursion relation of not more than four terms. The recursion relation for ML sequences is directly related to the existence of irreducible polynomials over the Galois Field GF(2). Irreducible polynomials over GF(2) have been calculated for n up to 168 (and perhaps larger values by now). This allows the generation of ML sequences of period length N up to $2^{168} - 1 = 3.74 \times 10^{50}$. (At a sampling rate of 1 billion THz the period length of such a sequence exceeds the estimated age of the universe by a factor of 600 billion. The fact that precise statements can be made about "objects" of such awesome dimensions attests to the power of mathematical reasoning, aided in this case by highly efficient methods of numerical testing—the same combination of intellect and machine power that has recently cracked the four-color map problem.)

The required computing power to generate an ML sequence, once the recursion relation is known, is fortunately much smaller. All that is needed is an n-stage shift register with a feedback loop. ML sequences can also be generated easily by programmable hand-held calculators and the author can attest to the fun of trying and (eventually) succeeding.

For small values of n, ML sequences can be generated "by hand." Thus, for $n = 4$, one of two possible recursion relations is the same as that given above for $n = 15$, which together with the "initial condition" −1, −1, −1, −1 leads to the sequence

$$-1, -1, -1, -1, 1, 1, 1, -1, 1, 1, -1, -1, 1, -1, 1, \ldots,$$

repeated with a period of $N = 2^4 - 1 = 15$.

II. FAST ALGORITHMS

The fastest method of computing the cross correlation to obtain the system impulse response consists of Fourier transforming the ML sequence and the system response, multiplication of the two Fourier transforms and an inverse Fourier transformation. In order to exploit the computational economics of fast Fourier transformation (FFT), which are highest for period lengths that are powers of 2, the period length N of the ML sequence must be augmented by one sample to yield a period length that is a power of 2: $N + 1 = 2^n$. The extra sample per period is obtained by interpolation.[8]

The search for ML sequences whose period lengths are a power of 2, thus obviating the need for interpolation, has so far yielded no nontrivial result. Such sequences, of course, cannot be binary (because their lengths are always one less than a power of 2). For ternary ML sequences, the period lengths are $3^n - 1$. But, except for $n = m = 1$, and $n = 2$ and $n = 3$, no relation of the form

$$3^n - 1 = 2^m \tag{3}$$

exists for integers n and m.[9]

III. STORAGE REQUIREMENTS

ML sequences are easily generated on the computer. However, instead of generating the sequence and recomputing its Fourier transform each time it is needed, one could simply compute it once and store the Fourier coefficients. Since the magnitudes of all Fourier coefficients are equal (except for the irrelevant dc component), it suffices to store their phase angles. If the sequences are shifted such that $a_{2k} = a_k$ (which is always possible, see Ref. 6), the number of different phase angles is only of order N/n—which could still be a large number, however (e.g., 2191 for $n = 15$).

IV. FLAT-SPECTRUM SEQUENCES WITH ONLY TWO PHASE ANGLES

Are there other periodic binary sequences that have both a flat spectrum and much fewer different phase angles? The answer, surprisingly, is yes. So-called Legendre sequences of period length p, where p is a prime of the form $4m - 1$, have flat spectra and only *two* different phase angles, namely $+90°$ and $-90°$. In fact, the Fourier transform of such a Legendre sequence is identical with the sequence itself [except for a factor $(-p)^{1/2}$] (See Appendix). Legendre sequences are defined as follows: $a_n = 1$, if n is a quadratic residue modulo p; $a_n = -1$, if n is a quadratic nonresidue modulo p; a_0 can be chosen arbitrarily +1 or −1.

For $p = 7$ and $a_0 = -1$, the Legendre sequence is identical with an ML sequence for $N = 7$: −1, 1, 1, −1, 1, −1, −1.

But for larger p, several ML sequences have to be added (mod 2) to yield a Legendre sequence.

An efficient method of generating Legendre sequences is based on one of Gauss' famous number-theoretic theorems:

$$a_n = n^{(p-1)/2} \text{ (taken as the least absolute remainder modulo } p\text{)} . \tag{4}$$

For typical room-acoustical applications, a prime number around 32 000 may be useful, for example $p = 32\,771$, which is of the form $4m - 1$. To speed up the horrendous calculations implied by Eq. (4), the method of "repeated squaring" is suggested. Since, for $p = 32\,771$, $(p-1)/2$ equals $2^{14} + 1$, a_n can be obtained by squaring n 14 times and multiplying the result by n. After each squaring, the intermediate result should be reduced to the least absolute remainder modulo p to avoid the creation of astronomical intermediate results for a final result of +1 or −1.

For periods twice as long, $p = 65\,539$ is recommended, because $(p-1)/2=2^{15}+1$, which leads again to a simple calculation of a_n.

IV. IMPROVEMENT IN SIGNAL-TO-NOISE RATIO

The use of periodic pseudorandom noise, such as maximum-length or Legendre sequences, not only permits making measurements at excellent signal-to-noise ratios in "quiet," but looks promising for making measurements in halls during actual performances! Because ML and Legendre sequences are periodic, the system's response is likewise periodic (after an initial build-up time of a few seconds). If the period length is, say 1.64 s, 200 responses can be averaged during a 55-min performance. The resulting improvement in signal-to-noise ratio is 33 dB—provided the room responses can be added coherently and the music or speech signal is incoherent with the pseudorandom test signal. Thus, making full use of auditory masking, the pseudorandom test signal can be played at an inaudible level during a performance yielding room responses with signal-to-noise ratios sufficient to determine (at least) the subjectively most important *initial* reverberation time. For the coherent addition to work properly, room responses must be sufficiently stable during the measurement. This means that the audience should not "mill around" during the performance. Slow variations in room temperature, causing changes in the speed of sound can be compensated by appropriate signal processing. If analog tape recording is used, a synchronizing frequency should be recorded simultaneously with the room response. In this manner, any tape stretch and "wow" can also be compensated.

The author routinely demonstrates the principle of this method of measuring room responses during an ongoing "performance" in the course of his lectures on acoustics at the University of Göttingen: A relatively stable impulse response of the lecture hall accumulates (and is shown on a large TV screen) within a few minutes. A soft (but audible) pseudorandom noise is used for excitation, while the lecture proceeds uninterrupted.

V. AN EXPERIMENTAL RESULT

As an example of the application of ML sequences as test signals in room acoustics, Fig. 1 shows an impulse response obtained in the Glockensaal in Bremen. The sampling rate was 25 kHz and the ML-sequence test signal had a period length of $2^{15}-1=32\,767$ samples or 1.31 s. The analog response shown in Fig. 1 was lowpass filtered to 10 kHz.

Although the Glockensaal was not particularly quiet during the measurement and only a medium power loudspeaker was used to radiate the test signal, the initial-peak signal-to-noise ratio of this response is 48 dB—a very good value indeed, especially for a *wide-band* (0–10 kHz) response. Signal-to-noise ratios as high as this are characteristic of measurements with ML sequences.

Figure 2 shows the integrated-decay curve obtained from the impulse response of Fig. 1 by squaring and integrating. Only above 950 ms is there any indication of

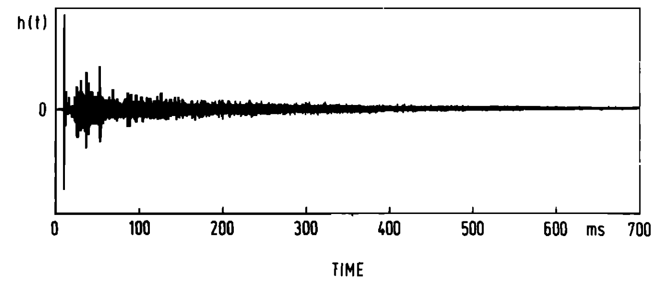

FIG. 1. Wide-band impulse response of concert hall obtained with "maximum-length" pseudorandom noise as test signal. Initial-peak signal-to-noise ratio is 48 dB, although the hall was not very quiet during the measurement and an ordinary loudspeaker was used to radiate the test signal. Bandwidth of the test signal and the response: 10 kHz.

the effect of the background noise. Thus, the problem of low signal-to-noise ratio discussed by Chu,[5] if pulses are used as test signals, appears to have been overcome.

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APPENDIX

Let a_n be a "Legendre sequence," i.e., $a_n=1$ if there is an x such that $x^2 \equiv n \pmod{p}$ and $a_n=-1$ otherwise, where p is a prime. For simplicity, we shall first consider sequences with $a_0=0$.

The discrete Fourier transform A_m of a_n is defined by

$$A_m=\sum_{n=0}^{p-1} a_n \exp(-2\pi i nm/p)\,. \tag{A1}$$

Because there is an equal number of quadratic residues and nonresidues in the range $n=1$ to $n=p-1$, one has

$$A_0=\sum_{n=1}^{p-1} a_n = 0\,. \tag{A2}$$

Because of the well-known multiplicative relations for quadratic residues (residue times residue equals residue; residue times nonresidue equals nonresidue; and nonresidue times nonresidue equals residue), one can write

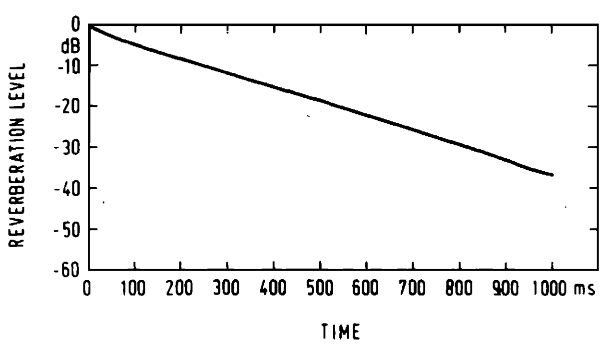

FIG. 2. Integrated decay curve obtained from the impulse response of Fig. 1 by squaring and integrating. Effect of background noise becomes visible only above 950 ms. No smoothing has been applied to the integrated decay curve shown here!

$$a_n = a_{nm} a_m (m \neq 0) . \quad \text{(A3)}$$

Thus,

$$A_m = a_m \sum_{n=1}^{p-1} a_{nm} \exp(-2\pi i n m/p) .$$

Here, the sum does not depend on m for $m \neq 0$ because the value of m determines only the *order* of terms in the sum. Consequently, setting $m=1$,

$$A_m = a_m \sum_{n=1}^{p-1} a_n \exp(-2\pi i n/p) = a_m A_1 . \quad \text{(A4)}$$

Thus, except for a constant factor A_1, the A_m equal the original sequence a_m, and the $|A_m|^2 (m \neq 0)$ do not depend on m, i.e., the power spectrum of the sequence a_n is *flat*. From Parseval's theorem it follows that

$$|A_1|^2 = p . \quad \text{(A5)}$$

For a prime of the form $p = 4k-1$,

$$a_{-1} = (-1)^{(p-1)/2} = (-1)^{2k-1} = -1 ,$$

and, with (A3)

$$a_{-m} = -a_m , \quad \text{(A6)}$$

i.e., the sequence a_n has odd symmetry. Thus, the A_m are purely imaginary. (In fact, Gauss has shown that A_1 has a *positive* imaginary part.) Hence, with Eqs. (A4) and (A5),

$$A_m = i a_m (p)^{1/2}, \quad (a_0 = 0, p = 4k-1) . \quad \text{(A7)}$$

If we now let $a_0 = +1$ or $a_0 = -1$ (instead of $a_0 = 0$), a real constant, namely a_0, is added to each Fourier coefficient. Thus,

$$A_0 = a_0 ,$$

and (A8)

$$A_m = a_0 + i a_m (p)^{1/2} (m \neq 0) .$$

The magnitudes and phase angles of the ac components are therefore

$$|A_m|^2 = 1 + p \quad (m \neq 0) ,$$
$$\tan\Phi_m = i a_m (p)^{1/2} a_0 . \quad \text{(A9)}$$

With $p = 32\,771$, for example, the phase angles are $\pm 89.68°$; where the sign is determined by the sign of a_m.

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