Some remarks on electro-mechano-acoustical circuits[a]

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The advanced development of electro-mechano-acoustical circuits dates in the US from Wente (1917), Firestone (1933), Mason (1941), Olson (1943), LeCorbeillier and Yueng (1952), Bauer (1953), Beranek (1954), and Hunt (1954). Earlier significant use of electrical (radio) analogies was made by Darrieus (France, 1929) and Haehnle (Germany, 1932). This paper treats the period in which Olson's work was centered and cites some examples of the use of the electro-mechano-acoustical circuits in microphones, loudspeakers, and rotational devices.

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INTRODUCTION

Electrical circuit diagrams were developed early in the history of telephony and radio because they resembled exactly the inductors, capacitors, resistors, transformers, and the physical configuration of the devices of that day being portrayed. Anyone with an understanding of simple algebra and the rules of connected meshes, could write by visual inspection the equations relating the flow of electrical current and the applied voltages, whether direct or alternating. Kirchhoff's laws and the principal of reciprocity were easily visualized. It was natural, therefore, for designers of electroacoustic transducers to adopt electrical circuit diagrams to portray the electrical and mechanical characteristics of their devices.

I. BACKGROUND

This paper is being presented at the "Harry F. Olson Memorial Session" and a little later I shall discuss his work. At this point I wish to mention that Olson was very proud of his contributions to the field of electro-mechano-acoustical analogies, principally because of his text *Dynamical Analogies*,[1] which he first published in 1943 and updated in 1958. But first, a little history:

The condenser microphone was described by Wente[2] in 1917. He showed an analogous circuit (see Fig. 1) in which velocity "flows" in the mesh, force is analogous to voltage and mechanical compliance C_m is analogous to electrical capacitance. Because condenser microphones generally operate into almost open-circuit inputs to amplifiers, no electrical circuit was needed—one could simply write:

$$e_0 = -(v/j\omega)(E/x_0),$$

where E/x_0 is a constant for a given microphone.

Wente and Thuras[3] demonstrated the moving coil microphone with the same simplicity (see Fig. 2). Here, the total compliance in the circuit is a series combination of the mechanical compliance of the diaphragm C_D and the acoustical compliance C_{air} of the airspace behind the diaphragm and the mass and mechanical resistance are indicated by an inductance and a resistor, respectively. The open circuit voltage is given by

$$e_0 = Blv,$$

where, B is the magnetic flux density in the air gap in which a conductor of length l is located.

In both cases above, force is analogous to voltage and velocity to electrical current. This type of electro-mechanical circuit was dubbed the impedance analogy because mechanical "impedance" was defined f/v, analogous to e/i. Thus, a mechanical device that offers high impedance to movement has a high ratio of driving force to resulting velocity.

Floyd Firestone[4] knew that the great value of electrical circuits to the early development of radio resulted from the one-to-one appearance of the circuit elements to the electrical devices that they represented, including the joining points of the elements. He saw that his students could not draw with equal ease a mechanical impedance circuit from visual inspection of a mechanical device. One difficulty is that a mass seems to have only one terminal, while the impedance analogy shows it (an inductance) with two. Also, a mechanical compliance does not resemble two plates separated by an air space (a condenser). He also was keenly aware that many of the early developments in radio had been made by amateurs (radio "hams"). From the electrical circuit alone they could visualize the results of changes in element sizes or the addition of meshes, without having the faintest idea of the differential equations needed to analyze the results. Some of those early inventors were high school students or hobbyists. He reasoned, "Wouldn't it be great to make the behavior of mechanical systems as easy to visualize as the behavior of electrical devices without any knowledge of the underlying differential equations?"

Firestone observed that a mechanical spring resembles an inductance, and that the distinguishable thing about the

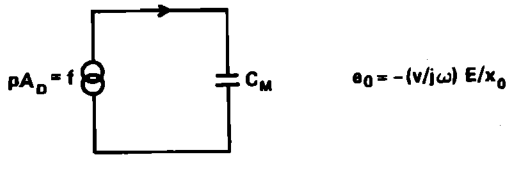

FIG. 1. The condenser microphone described by Wente[2] in 1917. p = sound pressure, C_M = mechanical compliance, A_D = area of diaphragm, $(v/j\omega)$ = rms displacement, e_0 = open-circuit voltage, and E/x_0 = polarizing voltage divided by diaphragm backspace. The force is analogous to voltage.

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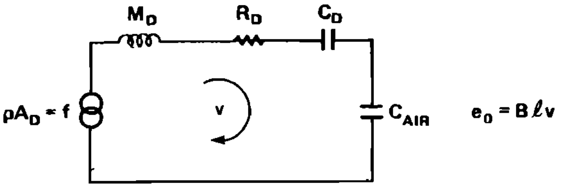

FIG. 2. The moving coil microphone described by Wente and Thuras[3] in 1931. M_D, R_D, and C_D = mechanical mass, resistance, and compliance, respectively; C_{air} = mechanical compliance of airspace behind diaphragm; e_0 = open-circuit voltage; and Bl = constant. The force is analogous to voltage.

two ends of a spring is the difference in velocity, just as there is a difference in voltage at the two ends (not in the current) of an inductance (see Fig. 3). Further, one can determine the joining points between elements in a mechanical circuit because they are points of the same velocity. Velocity and voltage are both simple to measure, while force and electrical current are both "buried" in the circuit. Fortunately, a dashpot looks something like a resistor (see Fig. 4), which leaves a capacitance to represent a mass.

The suitability of these representations can be seen by comparing the steady-state equations for each circuit element:

$$v = f/(j\omega M_m), \quad e = i/(j\omega C),$$

$$v = j\omega C_m f, \quad e = j\omega Li,$$

$$v = r_m f, \quad e = Ri,$$

where v is analogous to e, f is analogous to i, M_m is analogous to C, C_m is analogous to L, and r_m (called mechanical responsiveness, the inverse of R_m, the mechanical resistance) is analogous to R.

The problem of the second terminal for mass M_m is solved by *always grounding* one side of the analogous capacitance, because one "side" of a mass is the inertial frame or "ground" (see lower part of Fig. 3).

Firestone described the ratio of velocity v to force f as "mechanical mobility," an example of which is shown in Fig. 5. A mechanical constant-velocity generator drives the joining point of a mass, spring, and dashpot with the same velocity. The mass is represented in the analogous circuit as a grounded capacitance with value M, the spring as an inductance with value C_m, and the friction of the dashpot as a responsiveness with value r_m. Kirchhoff's equations say that,

$$f = f_1 + f_2 + f_3.$$

All three elements have the same velocity across the two ends.

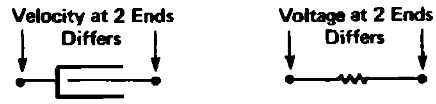

FIG. 4. Firestone's[4] 1933 observation of mechanical mobility, where the dashpot looks like a resistor. Therefore, velocity is analogous to voltage, force is analogous to current, and mobility = velocity/force = v/f.

Harry Olson used the impedance analogy freely in his books, the first of which, *Applied Acoustics*,[5] was published in 1934 with Frank Massa as co-author. In Fig. 6, taken from his book *Elements of Acoustical Engineering*,[6] first published in 1940, we see a typical Olson handling of analogous circuits, this for a bass reflex loudspeaker. Oslon drew the analogous circuit in the impedance analogy with the driving force shown as f_m, the velocity of the diaphragm v_D flowing in the upper part of the first mesh and the velocity of the port v_p flowing in the second mesh. The electrical circuit contains the internal resistance of the amplifier and the inductance and resistance of the loudspeaker voice coil. The impedance introduced into the electrical circuit by the motion of the loudspeaker cone is labeled Z_{em}, the "motional impedance." This term was first introduced by Kennelly and Pierce in 1912.[7]

Kennelly, Pierce, and Hunt, their successor at Harvard University, spent much teaching effort in computing and drawing what were called motional impedance diagrams, which expressed the ratio of e_0 to i in the upper circuit of Fig. 6. Near resonance, Z_{em} became large and profoundly changed the loading on the loudspeaker. As a student of theirs, I cannot say that this kind of diagramming helped me understand the basic operation of the loudspeaker nor did it suggest developmental changes in an electromagnetic loudspeaker that might have led to improvements in performance. Clearly, what was needed was an analogous circuit diagram that would allow one to see by inspection the important variables in a particular loudspeaker design and how to affect loudspeaker performance by modification of them.

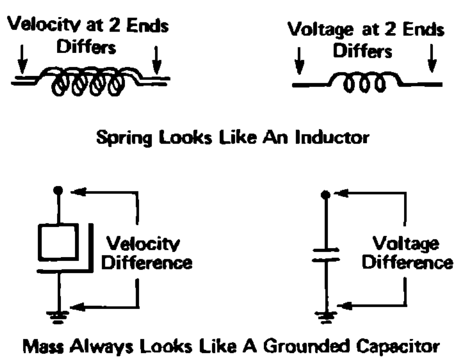

FIG. 3. Firestone's[4] 1933 observation of mechanical mobility.

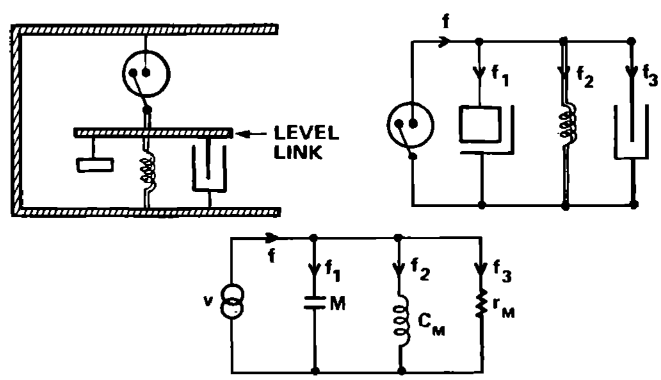

FIG. 5. An example of mechanical mobility as described by Firestone,[4] where M = mass, C_M = compliance, r_M = responsiveness, and $f = f_1 + f_2 + f_3$.

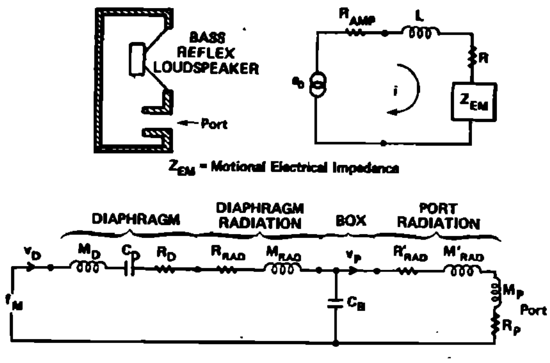

FIG. 6. The analogous circuit in the impedance analogy described by Olson[6] in 1940, for a bass reflex loudspeaker.

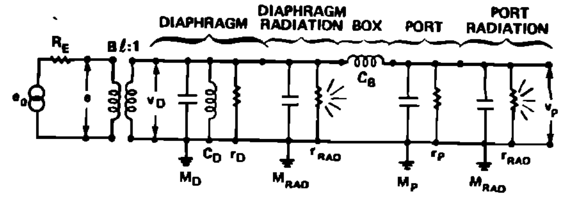

FIG. 7. The moving coil loudspeaker direct radiator with a ported box described by Beranek[9] in 1954. The electrical, mechanical, and acoustical components are combined. The rule is to draw every mass as a grounded capacitor, and locate the other elements by attaching points of equal velocity. The transformer $e = Blv_D$; — means radiation into air.

II. IMPROVEMENTS

Having studied Firestone's paper and a later monograph along the same lines by Gehlshoj of Copenhagen,[8] I introduced the concept of a "Bl" transformer into my acoustics courses at MIT.[9] The point here is simply that the voice coil acts as a transformer between the mechanical and the electrical circuits, and if the mobility analogy is used on the mechanical side, the whole can be drawn as one circuit. Inspection of the electro-mechano-acoustic diagram of Fig. 7, shows that a change in the mass of the diaphragm M_D acts like a change in electrical capacitance. One clearly sees that there will be two principal resonances: (1) the anti-resonance of the $(M_D + M_{\rm rad})$ plus C_D circuit and (2) the resonance of the $(M_p + M_{\rm rad'})$ plus C_B circuit. The secret to good low-frequency response is to plan the element sizes, the frequencies of resonances and the resistances so as to obtain optimal efficiency and uniform response at the lower frequencies. Hunt[10] does not refer to my work, but he shows my type of circuit for an electromagnetic transducer. He then removes the transformer by multiplying the compliances (inductances) and responsiveness (resistances) by B^2l^2 and dividing the masses by B^2l^2, which I did also, and exclaiming, "Note that the electro-mechanical coupling transformer has now disappeared entirely."

My text[9] also shows the mechano-acoustic side of an electrostatic transducer couples to the electric side through a transformer that requires the use of the mechanical impedance analogy on the right-hand side (see Fig. 8). In this type of transducer, a change in mass on the mechanical side shows up through the transformer as a change in inductance.

As my teachings show, a mechanical mobility type of analogous circuit must be used for electromagnetic transducers and a mechanical impedance type for electrostatic transducers. Thus we need an easy way to transfer from one to another for the same device. This transfer is possible through the theory of duality. However, I must warn students of electric circuit theory, that the duals taken here are different from those usually taught in electrical engineering. Here, if force is analogous to voltage and velocity to current in one circuit, the opposite will be true in the dual. Thus, velocity "flows" in one circuit and force "flows" in the dual. In electrical circuit theory, current flows in both the first and the dual circuits.

One procedure for taking the dual of a circuit is the "dot" method. At least one other, more cumbersome, procedure is possible.[8] The origin of the "dot" method is not easy to trace. I first learned about it exactly as presented here in 1938 from the late John L. Barnes, then Professor of Mathematics at Tufts University. It was later published by Gardner and Barnes.[11]

The "dot" procedure is shown in Fig. 9. In the upper circuit, a dot is put in the center of each mesh and one dot is put outside all meshes. The dots are connected by drawing a single dotted line through each element in the circuit terminating either at a dot in an adjoining mesh or at an outside dot. As seen, dot No. 1 connects to dot No. 3 through three elements and to dot No. 2 through one element. Similarly, dot No. 2 connects to dot No. 3 through two elements.

In the lower circuit, the dual, the bottom line becomes No. 3. The top line to the left of C_m becomes No. 1 and that to the right No. 2. Inductances (masses) in the upper circuit become capacitances (also masses) in the lower circuit and connect between the points that were connected in the upper circuit by corresponding dotted lines. Similarly, capacitances (compliances) in the upper circuit become inductances (also compliances) in the lower. The constant force generator in the upper circuit becomes a constant velocity generator in the lower. Note that the element r_{m1} in the lower circuit is the reciprocal of the mechanical resistance R_{m1} in the upper circuit. However, in using electric circuit theory in analyzing the lower analogous circuit, r_m is treated as a resistance, not a conductance. Thus, the parallel mobility (analogous to "impedance" in electrical circuit theory) of the lower circuit is $r_{m1} j\omega M_1/(r_{m1} + j\omega M_1)$.

III. HUNT'S CONTRIBUTION

At this point it is important to introduce Hunt's major contribution to the representation of electroacoustic trans-

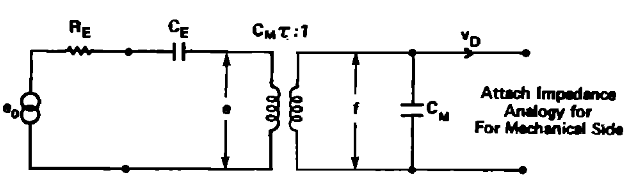

FIG. 8. An electrostatic transducer as described by Beranek[9] in 1954, which shows that the mechano-acoustic side couples to the electric side through a transformer which requires an attached impedance analogy. C_E: measured with $f = 0$ (shorted terminals); C_M: measured with $i = 0$ (open terminals).

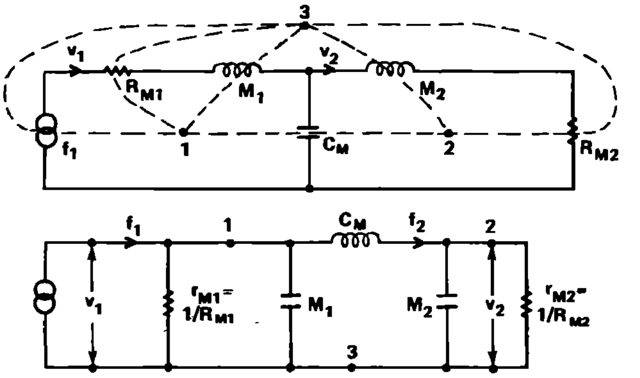

FIG. 9. A transducer, as described by Beranek[9] in 1954, which shows transfer between a mechanical impedance and a mechanical mobility type of analogous circuit, through the theory of duality. The "dot" procedure is shown (see text). v flows in the impedance analogy (upper), and f flows in the mobility analogy (lower).

ducers by analogous circuits. From the preface of his book,[10] I quote:

"It has been standard practice to say that one type of analog recommends itself for use with one type of coupling, and that the "other" type must be used with the other—but never the twain could be connected back to back....

"I have been experimenting...with a method for resolving this dilemma by using a *space operator* to import analytical symmetry into electromechanical-coupling equations for the antireciprocal cases involving magnetic fields.... It became possible to establish, on sound physical grounds, the validity of using such a space operator to restore symmetry in the analysis of electromagnetic coupling.

"The ability to represent all transducer types with a single form of equivalent circuit makes it relatively more useful to invoke the methods of electric-impedance analysis for the study of transducer performance."

Hunt's major contribution, therefore, is to permit the simultaneous driving of a mechanical device by an electromagnetic transducer and an electrostatic transducer, which is certainly not possible if the analogous circuit conventions of Figs. 7 and 8 must be used. Hunt says that the fundamental difficulty lies in (a) the quadrature relations among force, current, and magnetic field in the case of an electromagnetic transducer driven by a current, and (b) in the quadrature relations among induced voltage, velocity, and magnetic field in the case of the same electromagnetic transducer driven by a velocity. He defines a factor "k" as a space operator, indeed a *versor* as defined in the theory involving Hamilton's quaternions.

In case (a) above, "k" is defined as a space operator that rotates the positive direction of the vector that follows it, namely the positive direction of the current I, by 90° in space causing it to point in the same direction as the positive direction of the force F. Thus the force equation is simply written,

$$F = BlkI.$$

B, of course, will remain in quadrature to both F and kI. In case (b) above, K is defined as a space vector that rotates the positive direction of the velocity vector v into the same space direction as the positive direction of the induced voltage E. Thus the induced voltage equation is written,

$$E = Blkv.$$

B remains in quadrature to both E and kv, and its positive space direction is the same as for B in situation (a).

In practical electromechanical systems, and I quote Hunt, the mechanical construction usually constrains each of the quantities E, I, B, F, and v to a fixed line of action. Then if the positive directions of E and I, and of F and v, coincide, the scalar relations become

$$F = BlI \text{ and } E = -Blv,$$

and put boldly into evidence the need for the operator k, wherein the negative sign is produced by the square of k, i.e.

$$k^2 = -1.$$

The only confusion that results is that we also have in the steady-state the time operator j, which represents a time shift between two sine wavefunctions. We know that,

$$j^2 = -1.$$

Unfortunately, k does not commute with j, so that one must define,

$$jk = -kj.$$

This is standard behavior for quaternion space operators and it turns out to be just what is needed, says Hunt, to deal systematically and successfully with interconnected and coupled systems involving both electrostatic and electromagnetic coupling. Thus, in crossing or eliminating "transformers" in dual-coupled systems, the relations between space and time operators must be preserved.

IV. HARRY OLSON

Now let us return to Harry Olson. In his book, *Dynamical Analogies*[1] (1958) he presents both the impedance analogy and, for the first time in his publications, the mobility analogy. He treats them much the same as described here and in clear detail (Fig. 10). It is obvious from this figure that the second side of a mass is ground and that the reference for the velocity driver also is ground. This figure illustrates as clearly as any the ease with which an analogous circuit can be drawn from visual inspection of a mechanical device.

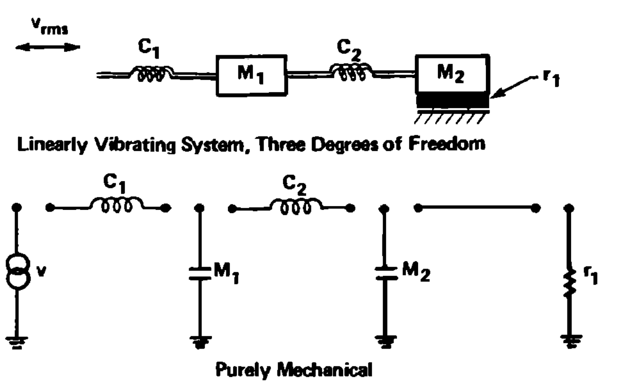

FIG. 10. An analogous circuit as described by Olson[1] in 1958, which presents both the impedance and mobility analogies. Both the second side of a mass and the reference for the velocity driver are ground.

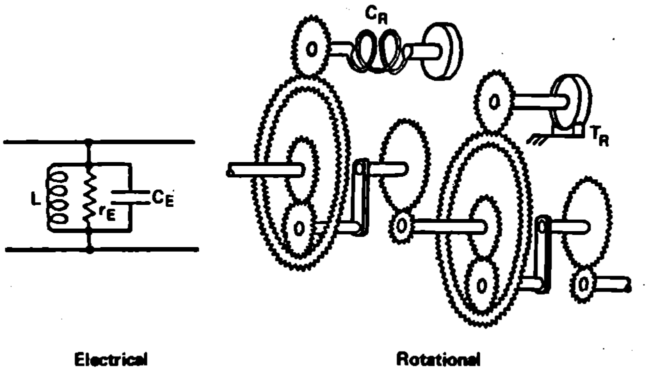

FIG. 11. A rotational analogous circuit as described by Olson[1] in 1943, which shows damped parallel resonance.

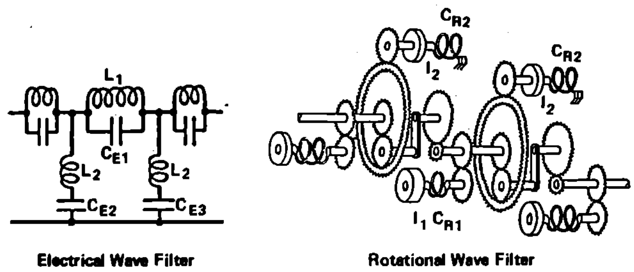

FIG. 12. A rotational analogous circuit as described by Olson[1] in 1943, which shows a wave filter.

Olson's unique contribution in *Dynamical Analogies* is the application of analogous circuits to rotational devices. In every part of the book, he carries on a parallel development of representations for masses and moments of inertia, for rectilineal resistance and rotational resistance and for rectilineal compliances and rotational compliance. He develops ingenious ways of illustrating many types of rotational analogous cicuits, e.g., damped parallel resonance in Fig. 11, and a wave filter in Fig. 12. Although the device of Fig. 12 might never be built, it serves its purpose as a source of ideas for representing complex rotational devices in the familiar analogous electrical circuit.

V. SUMMARY

In summary, the use of analogous circuits that clearly represent electro-mechano-acoustical devices has become widespread. At present, adequate procedures exist for developing such circuits for devices that are electromagnetically or electrostatically driven, or both, and that are rectilineal or rotational in operation.

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