Análisis espectral con wavelets

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No existe !!!

Principales y permanentes áreas de investigación

- *Compresión (filtrado)*
- *Encriptación*

Convolución vs correlación

Se define la **convolución** (*g*) de la función *f* respecto a la función *h*:

- Para funciones unidimensionales continuas, como:

$$g(x) = h(x) * f(x) = \int_{i=-\infty}^{i=\infty} f(i)h(x-i)di$$

- Para funciones unidimensionales discretas, como:

$$g(x) = h(x) * f(x) = \sum_{i=-\infty}^{i=\infty} f(i)h(x-i)$$

Se define la **correlación** (*g*) de la función *f* respecto a la función *h*:

- Para funciones unidimensionales continuas, como:

$$f(x) \circ g(x) = h(x) = \int_{-\infty}^{+\infty} f(i)h(x+z)di$$

- Para funciones unidimensionales discretas, como:

$$f(x) \circ g(x) = \sum_{m=0}^{M-1} f(m)g(x+m)$$

Convolución vs correlación 2D

Correlación.

$$(h \odot I)(x,y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} h(u,v) I(x+u, y+v) \partial u \partial v$$

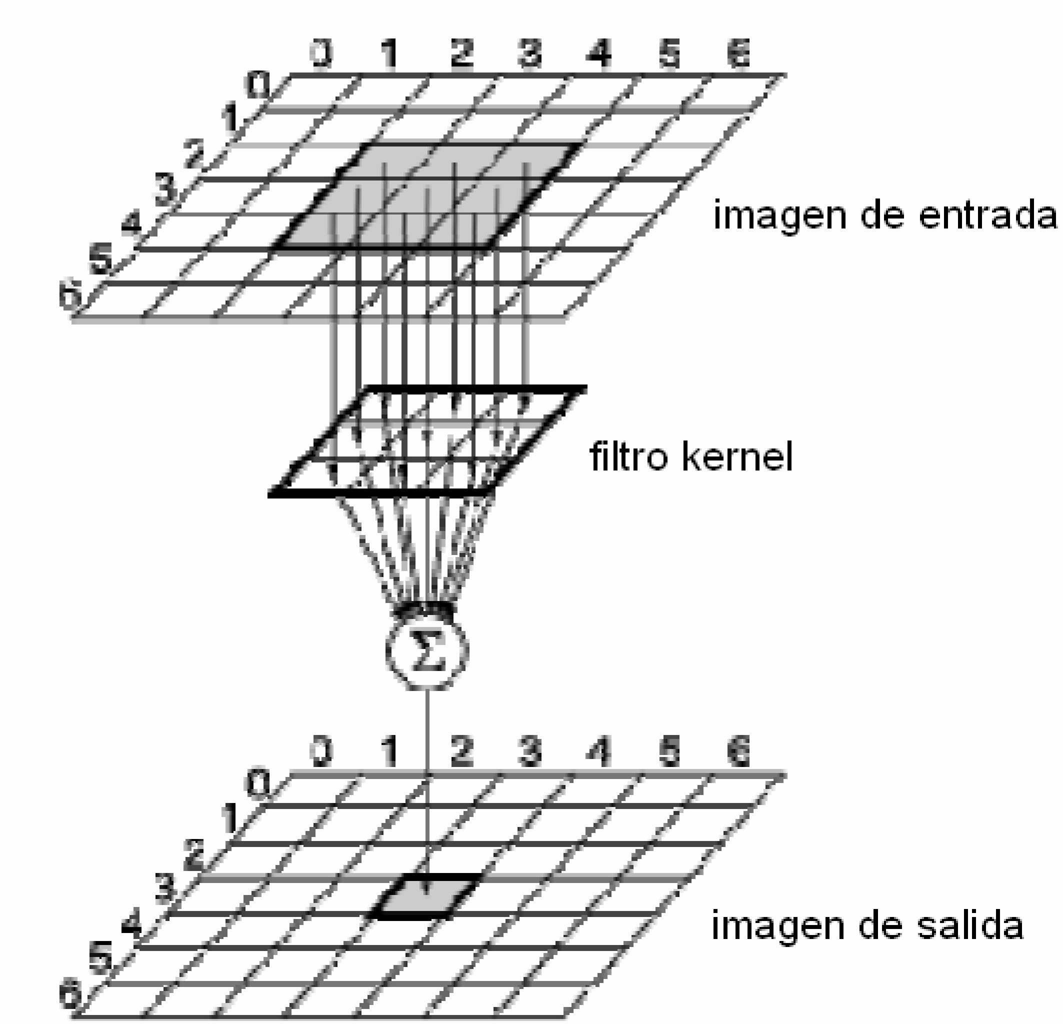
$$(h \odot I)(x,y) = \sum_i \sum_j h(i,j) I(x+i, y+j)$$

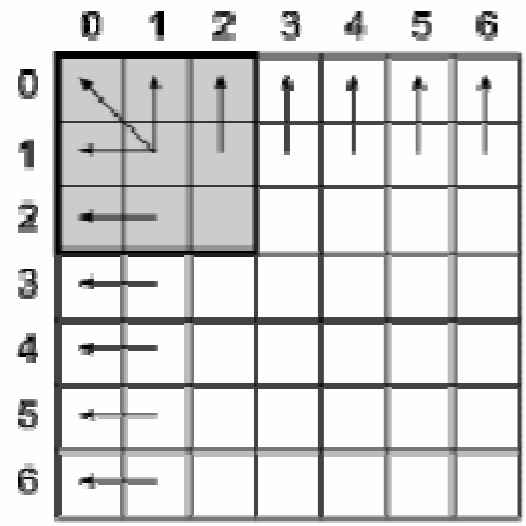
Convolución.

$$(h \otimes I)(x,y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} h(u,v) I(x-u, y-v) \partial u \partial v$$

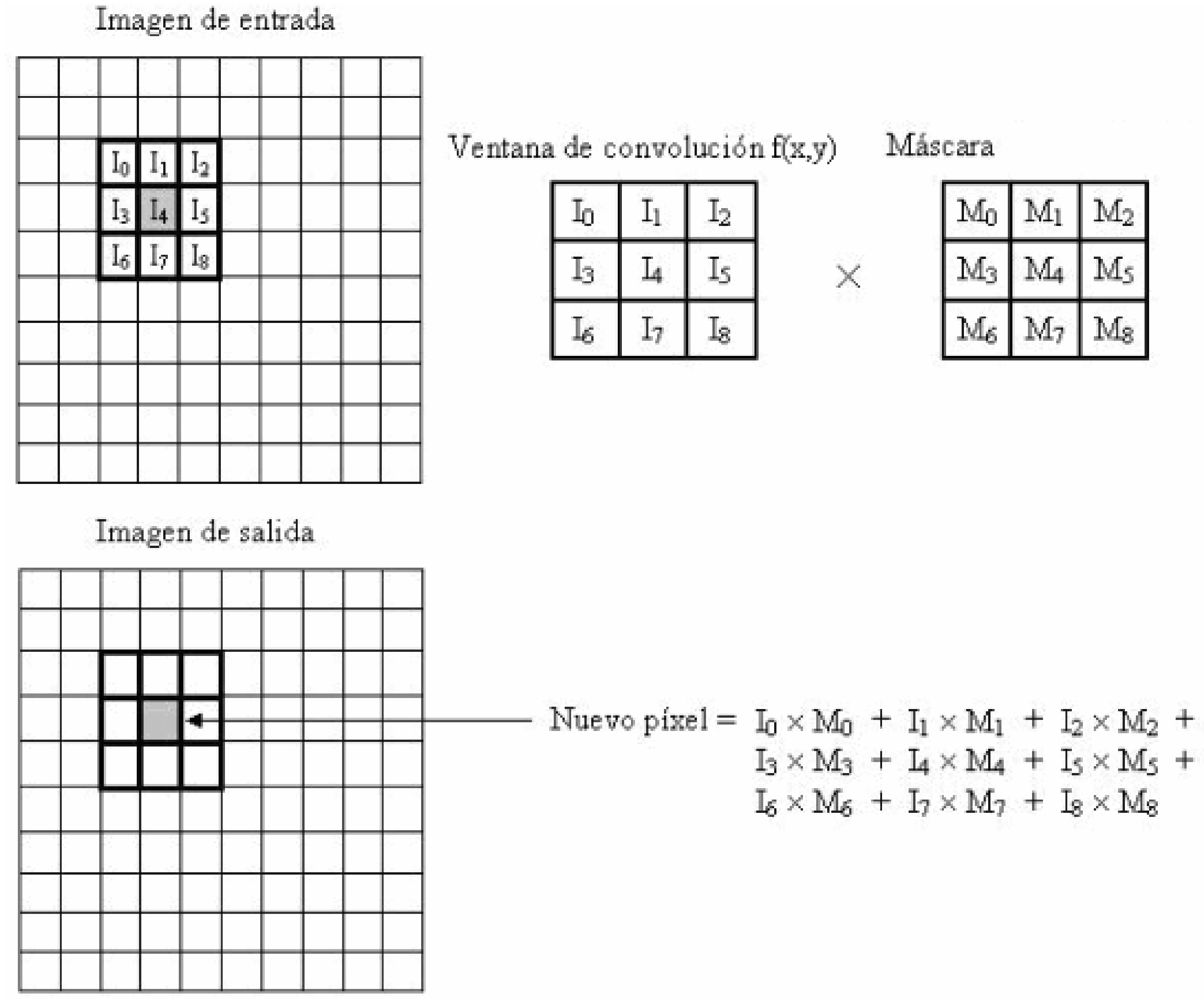
$$(h \otimes I)(x,y) = \sum_i \sum_j h(i,j) I(x-i, y-j)$$

Convolución 2D





Convolución 2D (cont.)



Noción de soporte compacto
DFT

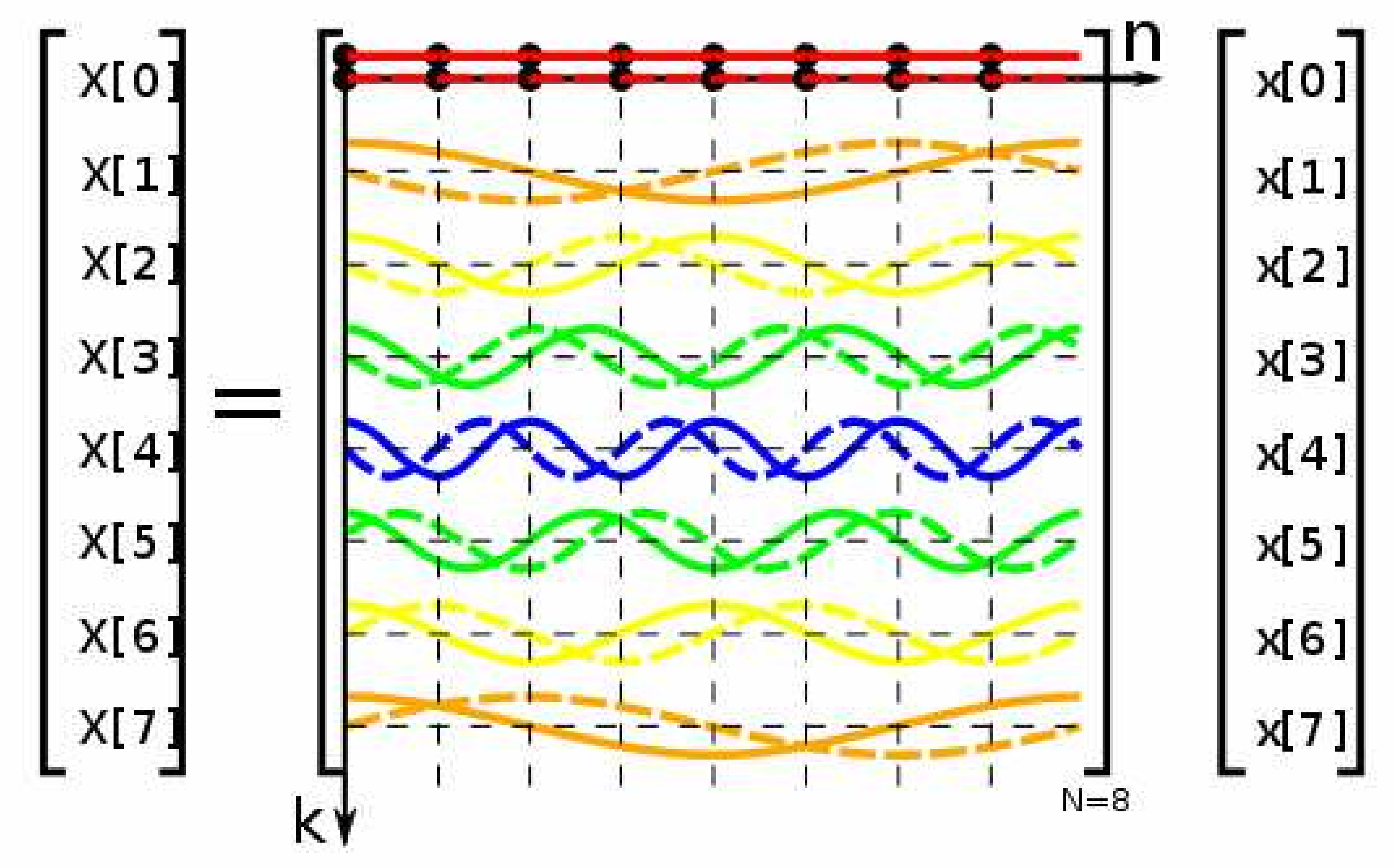
$$\begin{bmatrix} \omega_N^{0\cdot 0} & \omega_N^{0\cdot 1} & \ldots & \omega_N^{0\cdot(N-1)} \\ \omega_N^{1\cdot 0} & \omega_N^{1\cdot 1} & \ldots & \omega_N^{1\cdot(N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ \omega_N^{(N-1)\cdot 0} & \omega_N^{(N-1)\cdot 1} & \ldots & \omega_N^{(N-1)\cdot(N-1)} \end{bmatrix}$$

$$\omega_N = e^{-2\pi i/N}$$

$$W = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & \omega & \omega^2 & \omega^3 & \cdots & \omega^{N-1} \\ 1 & \omega^2 & \omega^4 & \omega^6 & \cdots & \omega^{2(N-1)} \\ 1 & \omega^3 & \omega^6 & \omega^9 & \cdots & \omega^{3(N-1)} \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 1 & \omega^{N-1} & \omega^{2(N-1)} & \omega^{3(N-1)} & \cdots & \omega^{(N-1)(N-1)} \end{bmatrix}$$

$$\omega = e^{-\frac{2\pi i}{N}},$$

Noción de soporte compacto
DFT (cont.)



Noción de soporte compacto
Haar Scaling Functions

$$\left[\begin{array}{c:c:c:c} 1/\sqrt{2} & & & \\ 1/\sqrt{2} & & & \\ & 1/\sqrt{2} & & \\ & 1/\sqrt{2} & & \\ & & 1/\sqrt{2} & \\ & & 1/\sqrt{2} & \\ & & & 1/\sqrt{2} \\ & & & 1/\sqrt{2} \end{array}\right] = \left[\begin{array}{c:c:c:c:c:c:c:c} 1 & & & & & & & \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & 1 & & & & \\ & & & & 1 & & & \\ & & & & & 1 & & \\ & & & & & & 1 & \\ & & & & & & & 1 \end{array}\right] \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & & & \\ 1 & & & \\ & 1 & & \\ & 1 & & \\ & & 1 & \\ & & 1 & \\ & & & 1 \\ & & & 1 \end{pmatrix}$$

Síntesis

Filtro P^3

Noción de soporte compacto
Haar Scaling Functions (cont.)

$$
\begin{pmatrix}
1/2 & \\
1/2 & \\
1/2 & \\
1/2 & \\
 & 1/2 \\
 & 1/2 \\
 & 1/2 \\
 & 1/2
\end{pmatrix}
=
\begin{bmatrix}
1/\sqrt{2} & & & \\
1/\sqrt{2} & & & \\
 & 1/\sqrt{2} & & \\
 & 1/\sqrt{2} & & \\
 & & 1/\sqrt{2} & \\
 & & 1/\sqrt{2} & \\
 & & & 1/\sqrt{2} \\
 & & & 1/\sqrt{2}
\end{bmatrix}
\frac{1}{\sqrt{2}}
\begin{pmatrix}
1 & \\
1 & \\
 & 1 \\
 & 1
\end{pmatrix}
\qquad
\begin{pmatrix}
1/2\sqrt{2} \\
1/2\sqrt{2} \\
1/2\sqrt{2} \\
1/2\sqrt{2} \\
1/2\sqrt{2} \\
1/2\sqrt{2} \\
1/2\sqrt{2} \\
1/2\sqrt{2}
\end{pmatrix}
=
\begin{pmatrix}
1/2 & \\
1/2 & \\
1/2 & \\
1/2 & \\
 & 1/2 \\
 & 1/2 \\
 & 1/2 \\
 & 1/2
\end{pmatrix}
\frac{1}{\sqrt{2}}
\begin{pmatrix}
1 \\
1
\end{pmatrix}
$$

Síntesis Filtro P^2

Síntesis Filtro P^1

Noción de soporte compacto
Haar Wavelets

$$\left[\begin{array}{c:c:c:c}
1/\sqrt{2} & & & \\
-1/\sqrt{2} & & & \\
 & 1/\sqrt{2} & & \\
 & -1/\sqrt{2} & & \\
 & & 1/\sqrt{2} & \\
 & & -1/\sqrt{2} & \\
 & & & 1/\sqrt{2} \\
 & & & -1/\sqrt{2}
\end{array}\right]
=
\left[\begin{array}{c:c:c:c:c:c:c:c}
1 & & & & & & & \\
 & 1 & & & & & & \\
 & & 1 & & & & & \\
 & & & 1 & & & & \\
 & & & & 1 & & & \\
 & & & & & 1 & & \\
 & & & & & & 1 & \\
 & & & & & & & 1
\end{array}\right]
\frac{1}{\sqrt{2}}
\begin{pmatrix}
1 & & & \\
-1 & & & \\
 & 1 & & \\
 & -1 & & \\
 & & 1 & \\
 & & -1 & \\
 & & & 1 \\
 & & & -1
\end{pmatrix}$$

Síntesis

Filtro Q^3

Noción de soporte compacto
Haar Wavelets (cont.)

$$
\begin{pmatrix}
1/2 & \\
1/2 & \\
-1/2 & \\
-1/2 & \\
 & 1/2 \\
 & 1/2 \\
 & -1/2 \\
 & -1/2
\end{pmatrix}
=
\begin{bmatrix}
1/\sqrt{2} & & & \\
1/\sqrt{2} & & & \\
 & 1/\sqrt{2} & & \\
 & 1/\sqrt{2} & & \\
 & & 1/\sqrt{2} & \\
 & & 1/\sqrt{2} & \\
 & & & 1/\sqrt{2} \\
 & & & 1/\sqrt{2}
\end{bmatrix}
\frac{1}{\sqrt{2}}
\begin{pmatrix}
1 & \\
-1 & \\
 & 1 \\
 & -1
\end{pmatrix}
$$

Síntesis Filtro Q²

$$
\begin{pmatrix}
1/2\sqrt{2} \\
1/2\sqrt{2} \\
1/2\sqrt{2} \\
1/2\sqrt{2} \\
-1/2\sqrt{2} \\
-1/2\sqrt{2} \\
-1/2\sqrt{2} \\
-1/2\sqrt{2}
\end{pmatrix}
=
\begin{pmatrix}
1/2 & \\
1/2 & \\
1/2 & \\
1/2 & \\
 & 1/2 \\
 & 1/2 \\
 & 1/2 \\
 & 1/2
\end{pmatrix}
\frac{1}{\sqrt{2}}
\begin{pmatrix}
1 \\
-1
\end{pmatrix}
$$

Síntesis Filtro Q¹

Análisis/Descomposición (Haar)

A^3

$$\begin{pmatrix} 5\sqrt{2} \\ 11\sqrt{2} \\ 7\sqrt{2} \\ 5\sqrt{2} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & & & & & & \\ & & 1 & 1 & & & & \\ & & & & 1 & 1 & & \\ & & & & & & 1 & 1 \end{bmatrix} \begin{pmatrix} 4 \\ 6 \\ 10 \\ 12 \\ 8 \\ 6 \\ 5 \\ 5 \end{pmatrix}$$

A^2

$$\begin{pmatrix} 16 \\ 12 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & & \\ & & 1 & 1 \end{bmatrix} \begin{pmatrix} 5\sqrt{2} \\ 11\sqrt{2} \\ 7\sqrt{2} \\ 5\sqrt{2} \end{pmatrix}$$

B^2

$$\begin{pmatrix} -6 \\ 2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 & & \\ & & 1 & -1 \end{bmatrix} \begin{pmatrix} 5\sqrt{2} \\ 11\sqrt{2} \\ 7\sqrt{2} \\ 5\sqrt{2} \end{pmatrix}$$

Análisis Filtro A^j

Análisis Filtro B^j

B^3

$$\begin{pmatrix} -\sqrt{2} \\ -\sqrt{2} \\ \sqrt{2} \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 & & & & & & \\ & & 1 & -1 & & & & \\ & & & & 1 & -1 & & \\ & & & & & & 1 & -1 \end{bmatrix} \begin{pmatrix} 4 \\ 6 \\ 10 \\ 12 \\ 8 \\ 6 \\ 5 \\ 5 \end{pmatrix}$$

A^1

$$\left(14\sqrt{2}\right) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{pmatrix} 16 \\ 12 \end{pmatrix}$$

$$\left(2\sqrt{2}\right) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{pmatrix} 16 \\ 12 \end{pmatrix}$$

B^1

Síntesis/Reconstrucción (Haar)

$$\begin{pmatrix}16\\12\end{pmatrix}=\underset{P^1}{\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}}14\sqrt{2}+\underset{Q^1}{\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}}2\sqrt{2}$$

$$\begin{pmatrix}5\sqrt{2}\\11\sqrt{2}\\7\sqrt{2}\\5\sqrt{5}\end{pmatrix}=\underset{P^2}{\frac{1}{\sqrt{2}}\begin{bmatrix}1&\\1&\\&1\\&1\end{bmatrix}}\begin{pmatrix}16\\12\end{pmatrix}+\underset{Q^2}{\frac{1}{\sqrt{2}}\begin{bmatrix}1&\\-1&\\&1\\&-1\end{bmatrix}}\begin{pmatrix}-6\\2\end{pmatrix}$$

$$\begin{pmatrix}4\\6\\10\\12\\8\\6\\5\\5\end{pmatrix}=\frac{1}{\sqrt{2}}\overset{P^3}{\begin{bmatrix}1&&&\\1&&&\\&1&&\\&1&&\\&&1&\\&&1&\\&&&1\\&&&1\end{bmatrix}}\begin{pmatrix}5\sqrt{2}\\11\sqrt{2}\\7\sqrt{2}\\5\sqrt{5}\end{pmatrix}+\frac{1}{\sqrt{2}}\overset{Q^3}{\begin{bmatrix}1&&&\\-1&&&\\&1&&\\&-1&&\\&&1&\\&&-1&\\&&&1\\&&&-1\end{bmatrix}}\begin{pmatrix}-\sqrt{2}\\-\sqrt{2}\\\sqrt{2}\\0\end{pmatrix}$$

Síntesis
Filtro P^j

Síntesis
Filtro Q^j

Análisis/Descomposición (dB4)

$$T_{D2} = \begin{pmatrix} h_0 & h_1 & h_2 & h_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_0 & h_1 & h_2 & h_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h_0 & h_1 & h_2 & h_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & h_0 & h_1 & h_2 & h_3 \\ h_2 & h_3 & 0 & 0 & 0 & 0 & 0 & 0 & h_0 & h_1 \\ g_0 & g_1 & g_2 & g_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & g_0 & g_1 & g_2 & g_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g_0 & g_1 & g_2 & g_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & g_0 & g_1 & g_2 & g_3 \\ g_2 & g_0 & 0 & 0 & 0 & 0 & 0 & 0 & g_0 & g_1 \end{pmatrix}$$

DFT, STFT y DWT

Frecuencia

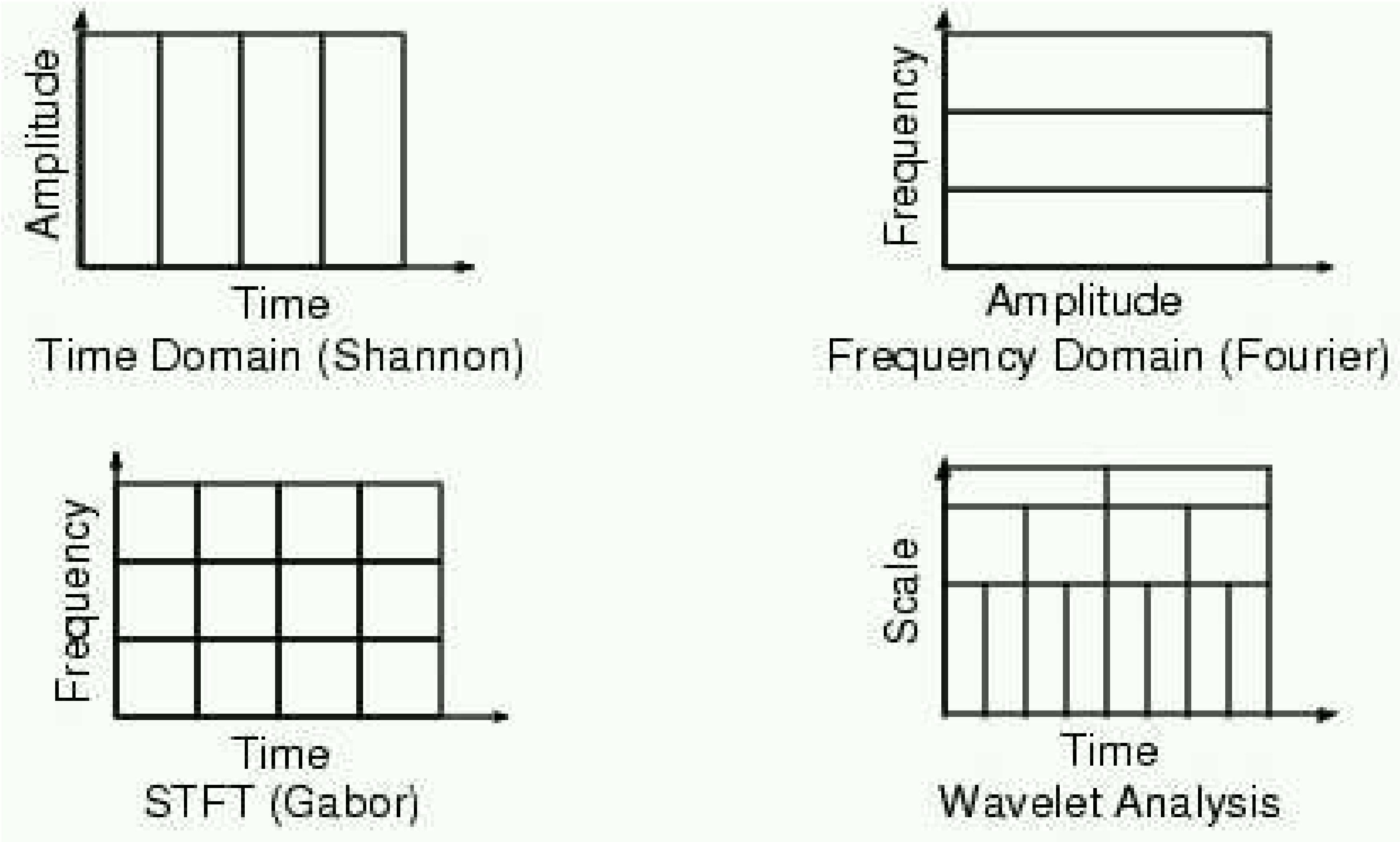
$$X(f)=\int_{-\infty}^{+\infty}x(t)e^{-j2\pi ft}\,df$$

Frecuencia + Tiempo
(intervalos de tiempo iguales)

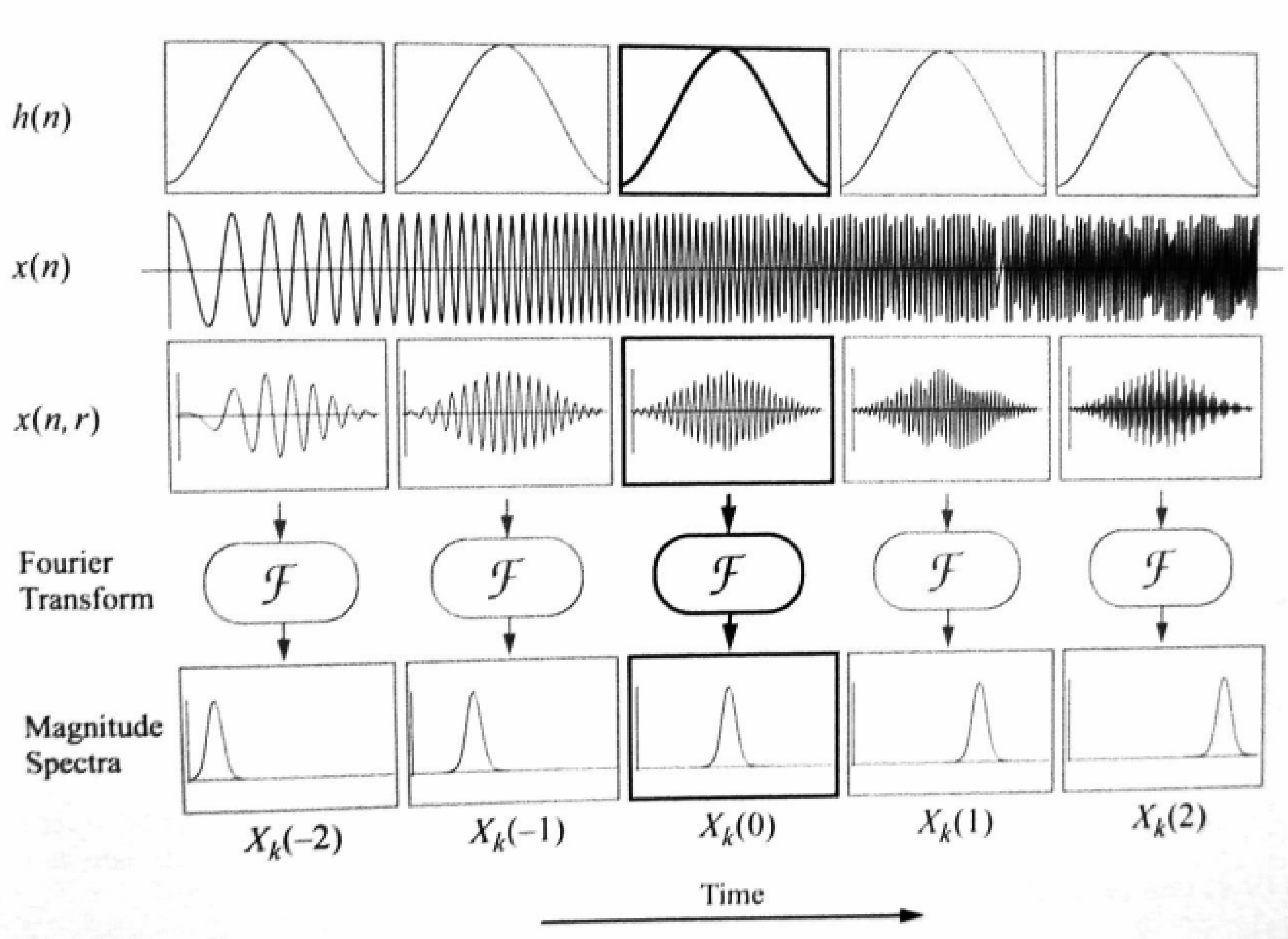
$$STFT(\tau,f)=\int_{-\infty}^{+\infty}x(t)\overline{w(t-\tau)}e^{-j2\pi ft}\,dt$$

Frecuencia + tiempo

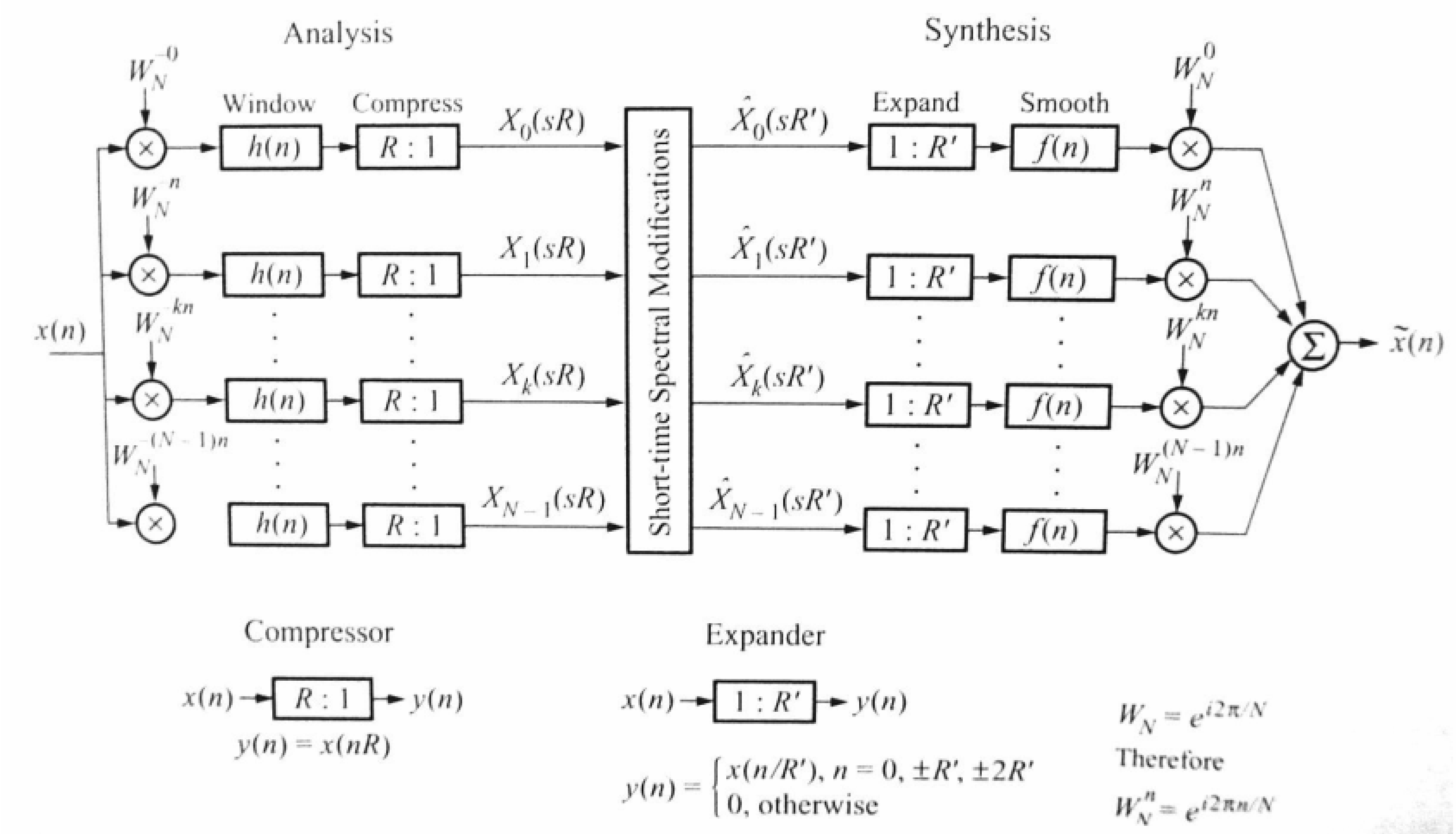
DFT, STFT y DWT (cont.)



Interpretación de la STFT



Interpretación de la STFT



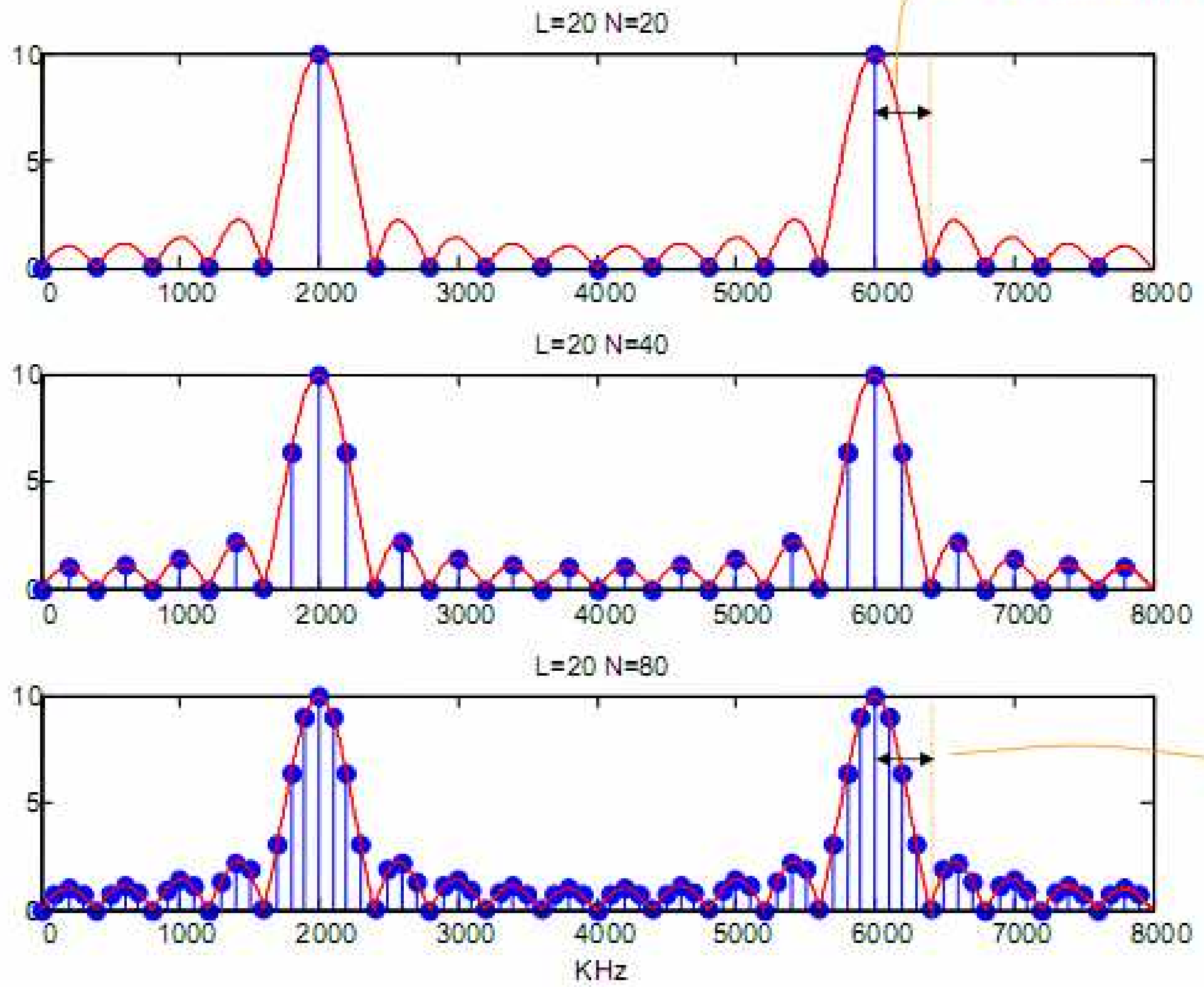
Ventanizado en STFT

Resolución espectral	
Ventana	Resolución
Rectangular	$2\pi/L$
Bartlett	$4\pi/L$
Hamming	$4\pi/L$
Hanning	$4\pi/L$
Blackman	$18\pi/L$

Mejor resolución
ventana rectangular

Resolución y DFT

$$x[n] = \text{sen}(\omega_o n) \cdot w[n]$$



Resolución 400 Hz

Si L=N (longitud de la ventana igual a número de puntos de la DFT) la resolución es igual a la separación entre muestras

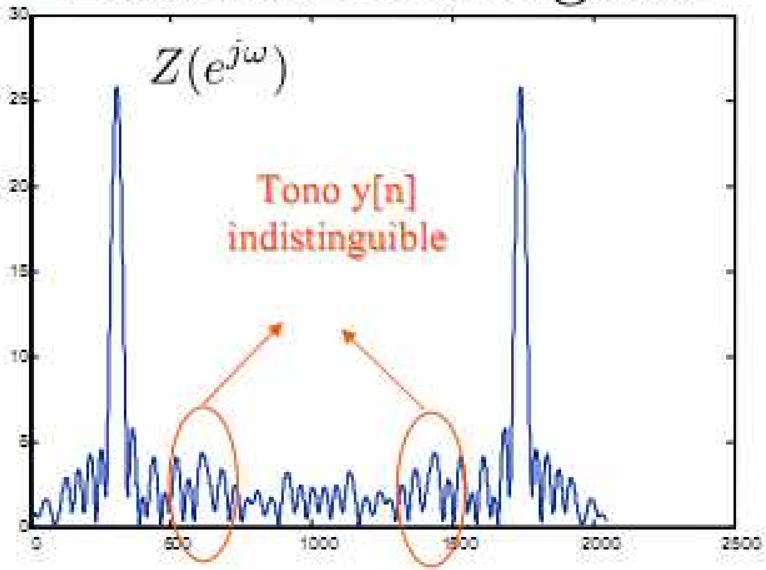
Si aumentamos N, no aumenta la resolución, que sigue fijada por el tamaño de la ventana

Resolución 400 Hz

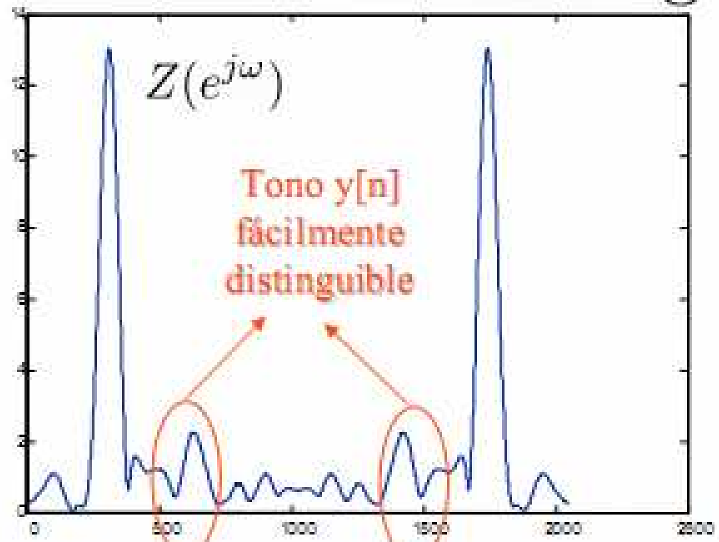
Ejemplo de Dispersión Espectral

$$x[n] = \mathrm{sen}(0,3\pi n)$$
$$y[n] = 0,1\mathrm{sen}(0,6\pi n) \qquad r[n] \text{ ruido blanco } \sigma_r = 0,2$$
$$z[n] = x[n] + y[n] + r[n]$$

Ventana rectangular



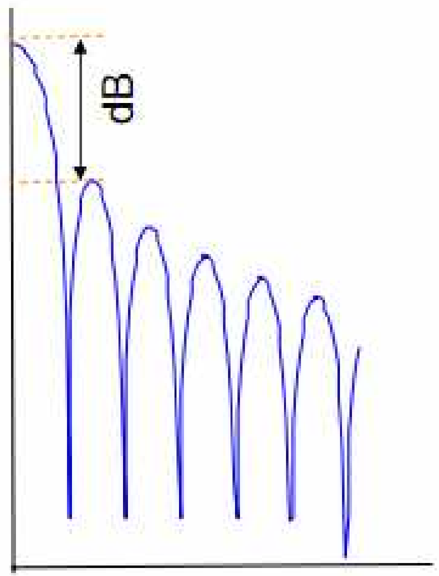
Ventana de Hanning



Resolución y dispersión

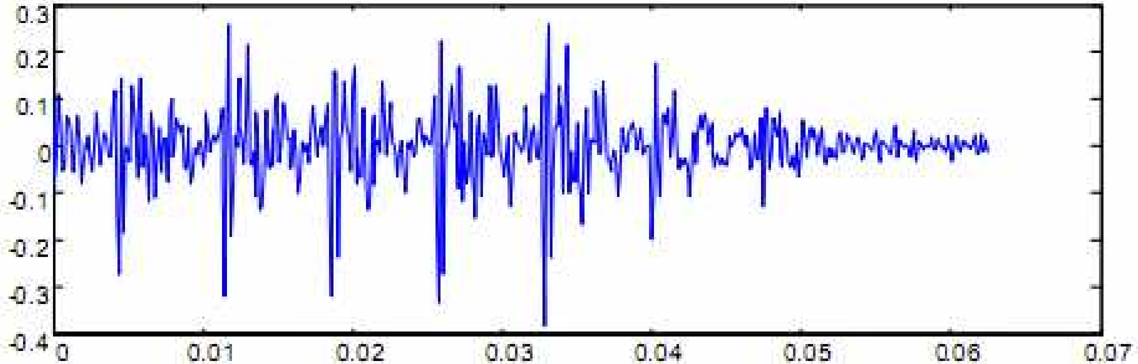
Lóbulo Secundario	
Tipo	Nivel
Rectangular	$-13dB$
Bartlett	$-25dB$
Hamming	$-41dB$
Hanning	$-31dB$
Blackman	$-57dB$

Mayor resolución
Mayor dispersión

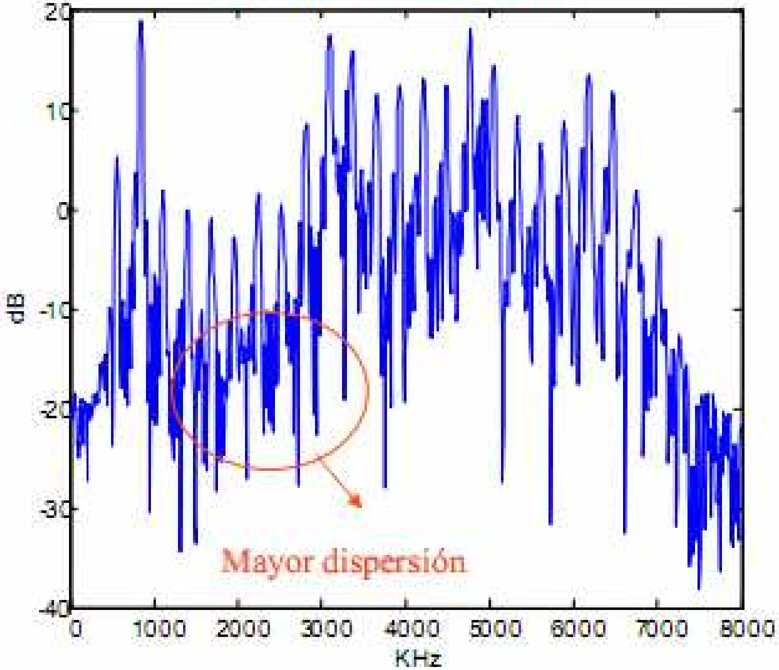


Compromiso Resolución-Dispersión

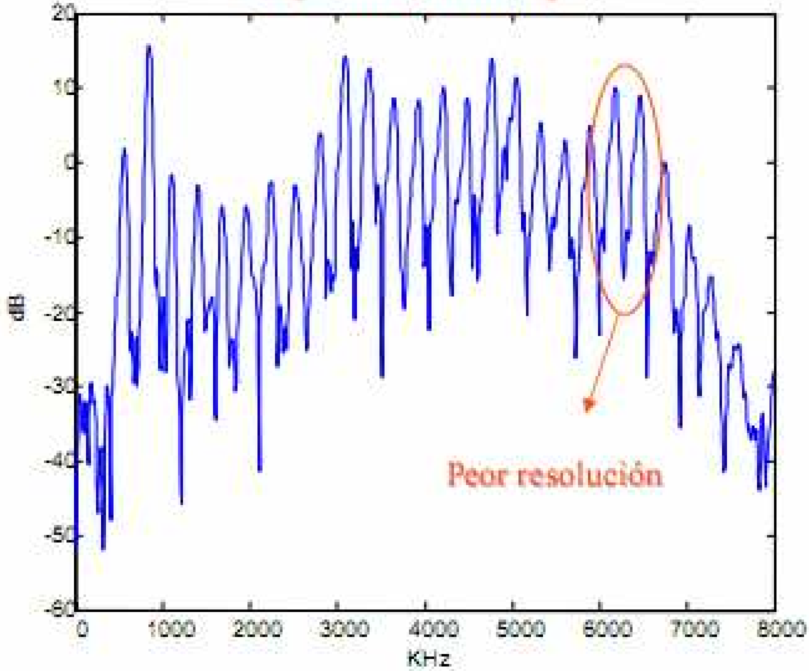
Secuencia de voz



Espectro Rectangular



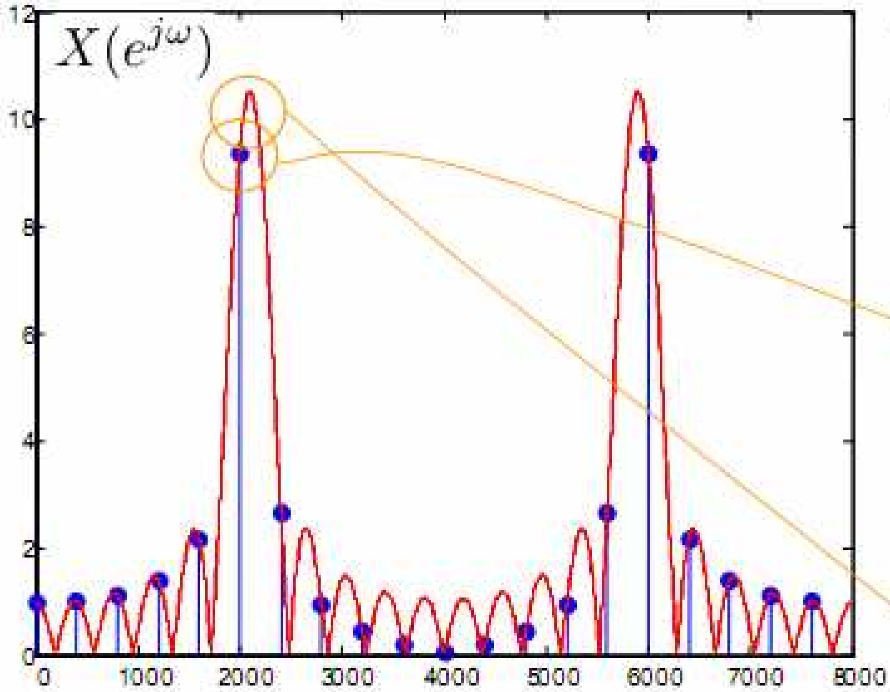
Espectro Hamming



Errores de Estimación en el Muestreo Frecuencial

$$x[n] = A\mathrm{sen}(\omega_o n)$$

$$\begin{aligned} L &= 20 \text{ muestras} \\ N &= 40 \text{ muestras} \\ f_s &= 8KHz \end{aligned}$$



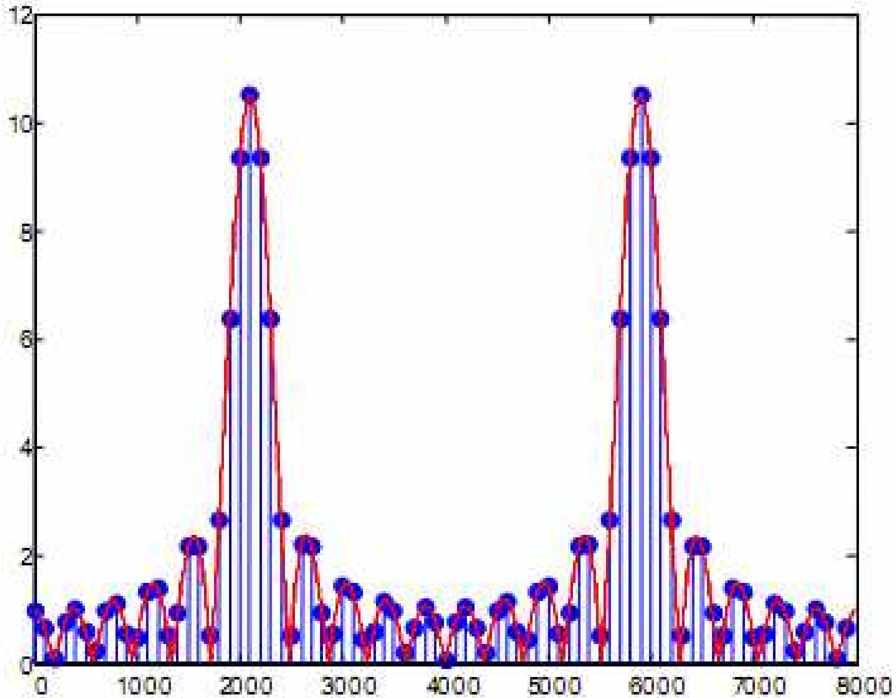
$$\begin{cases} A = 2\dfrac{9,3}{20} = 0,93 \\ k = 5 \qquad \omega_o = \dfrac{2\pi}{20}5 \\ f_0 = \dfrac{8000 \cdot 5}{20} = 2000Hz \end{cases}$$

$$\begin{cases} A = 1 \\ f_0 = 2100Hz \end{cases}$$

La precisión viene dada por el número de puntos de la DFT

precisión$= \pm\dfrac{\pi}{N}$ radianes.

Errores de Estimación en el Muestreo Frecuencial (II)



Si $\omega_o = 2\pi k/N$, el máximo coincide en una muestra de la DFT y no hay error.

Ejemplo: $f_s = 8KHz$, $\omega_o = 2.100Hz$. Obtener N para que no haya error.

$$\omega_o = \frac{2\pi}{N}k \qquad f_0 = \frac{f_s}{N}k$$

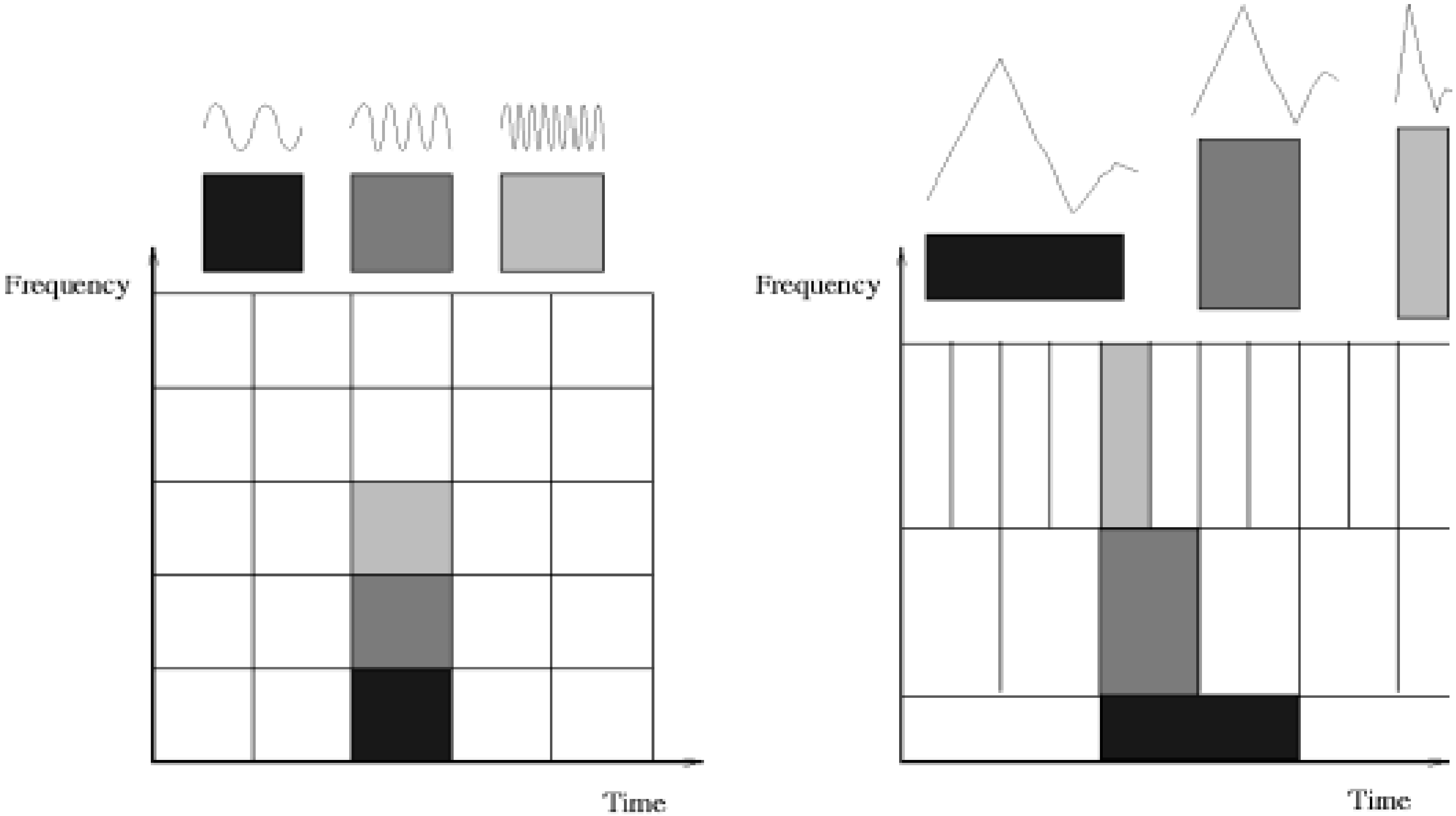
$$\frac{N}{k} = \frac{f_s}{f_0} = \frac{80}{21}$$

$$\boxed{N = 80 \quad k = 21}$$

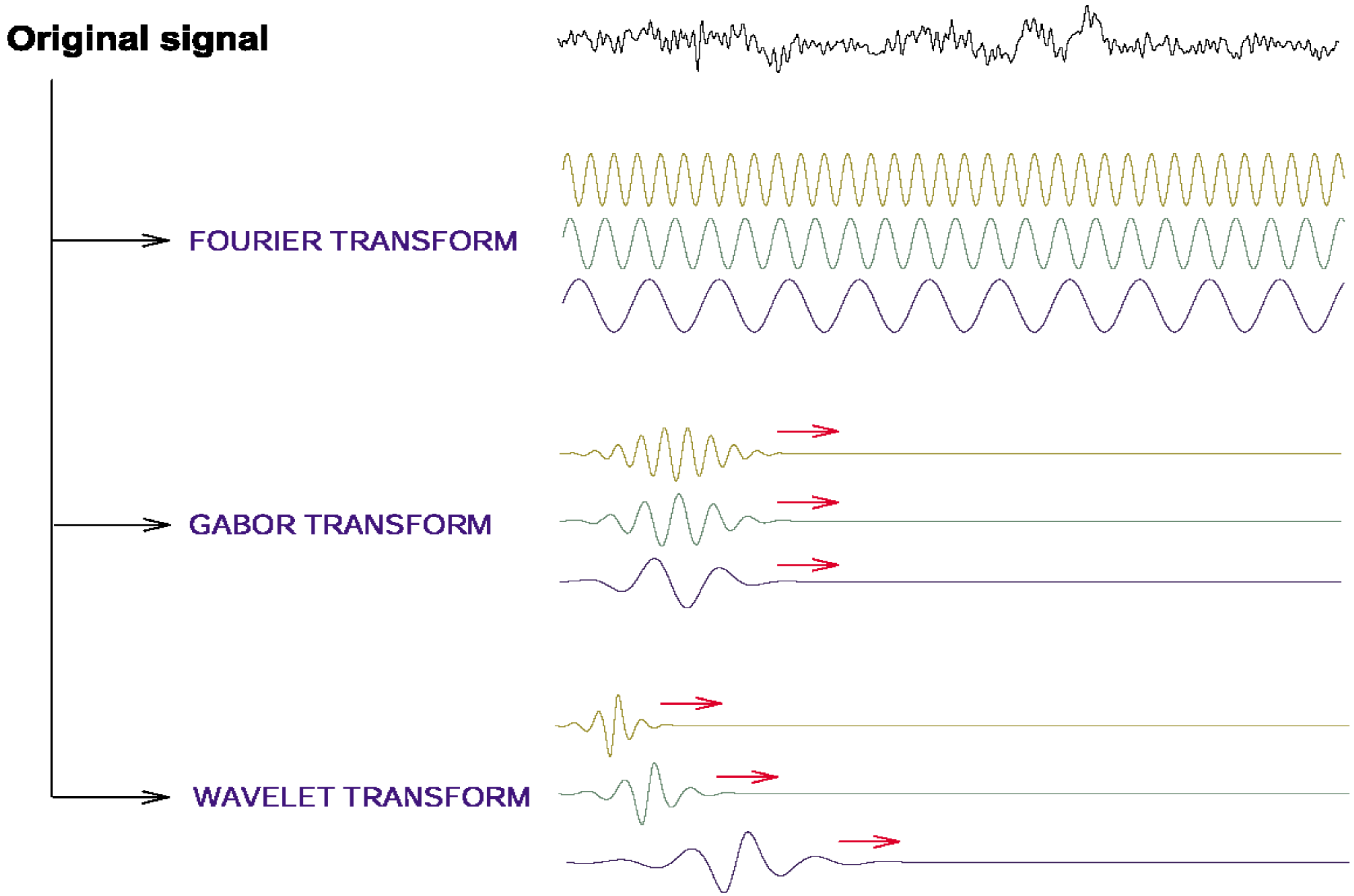
Skills

- FFT pierde info de fase pues no tiene soporte compacto
- STFT establece gracias al ventanizado una fijación simultánea en tiempo y frecuencia pero de resolución constante a lo largo de las muestras
- Las wavelets, hacen lo mismo que la STFT pero con resolución adaptable a las características espectrales de las muestras

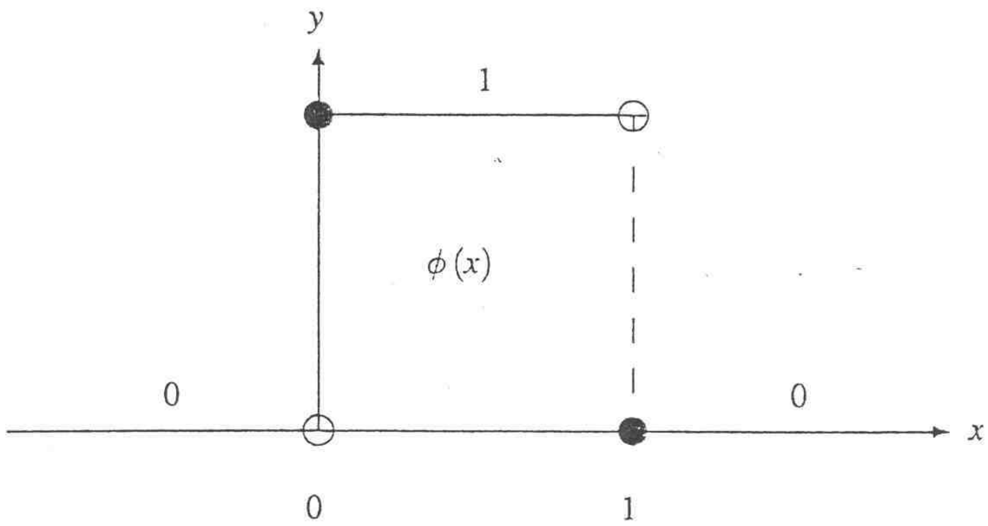
STFT vs DWT

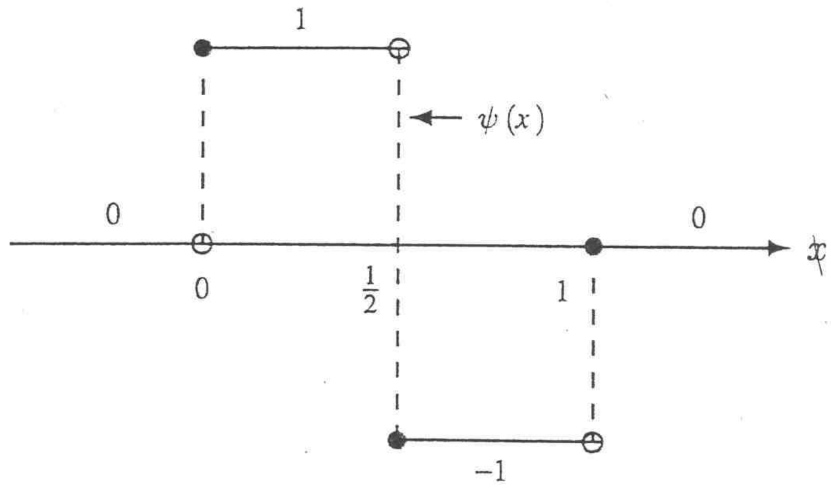


DFT, STFT y DWT (cont.)



Función de Scaling y Función Wavelet de Haar

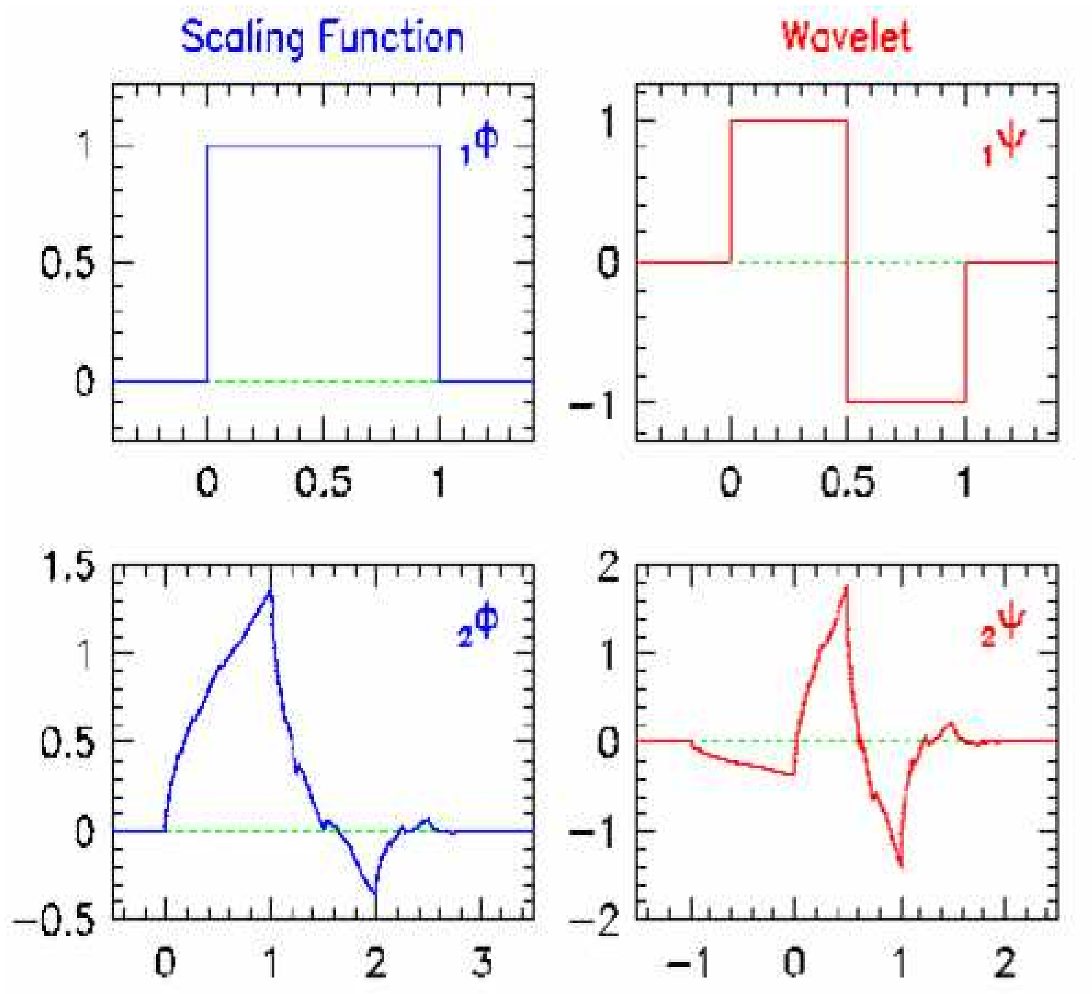




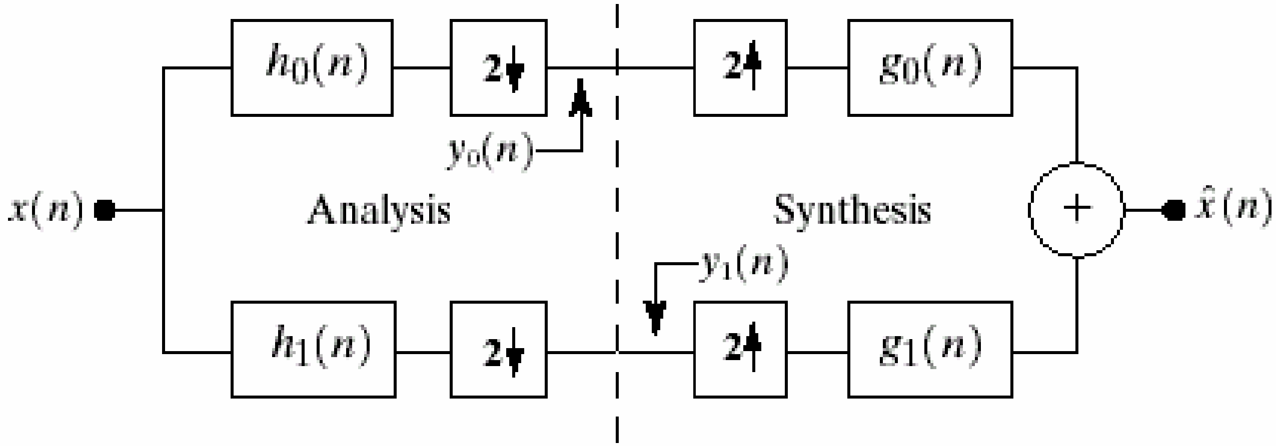
Función de Scaling y Función Wavelet de Haar y Daubechies

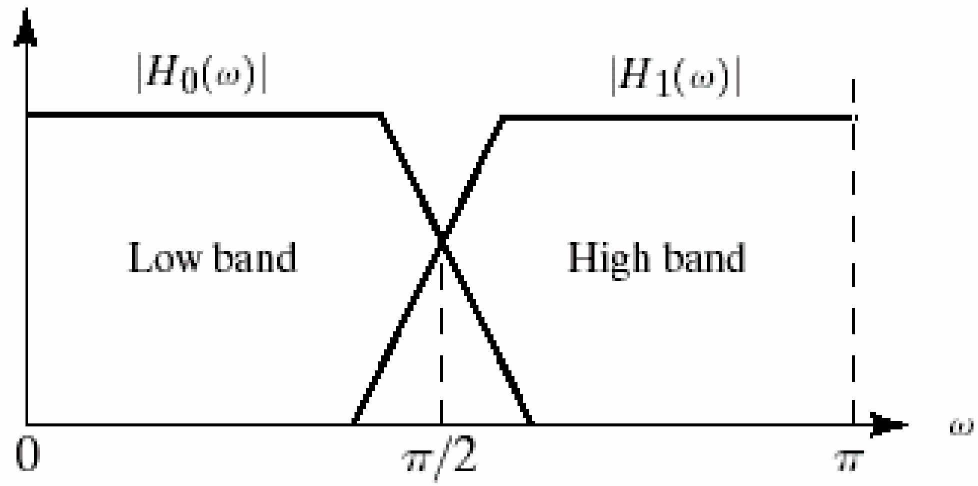
Haar function

Daubechies function

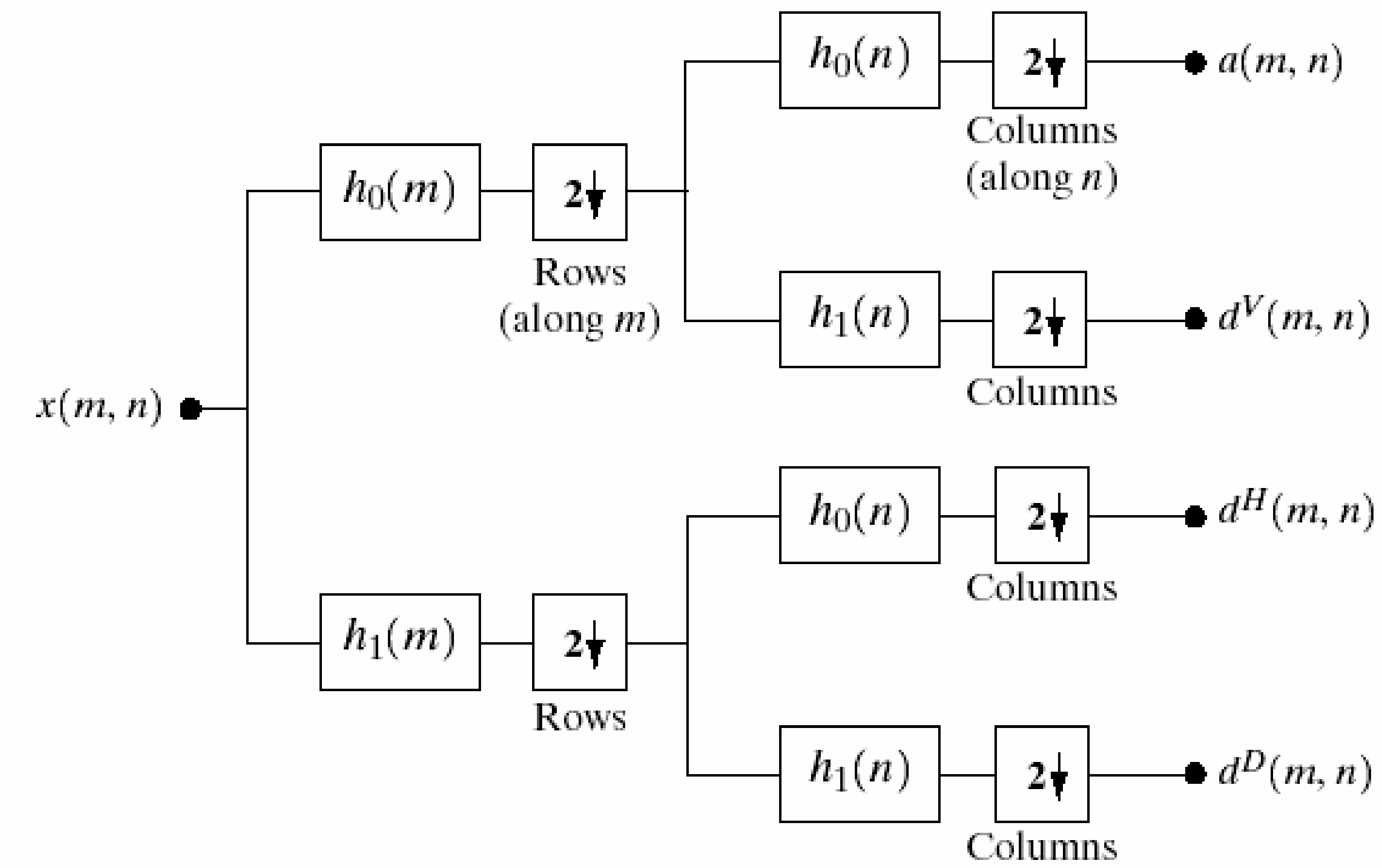


Wavelets y procesamiento de multiresolución

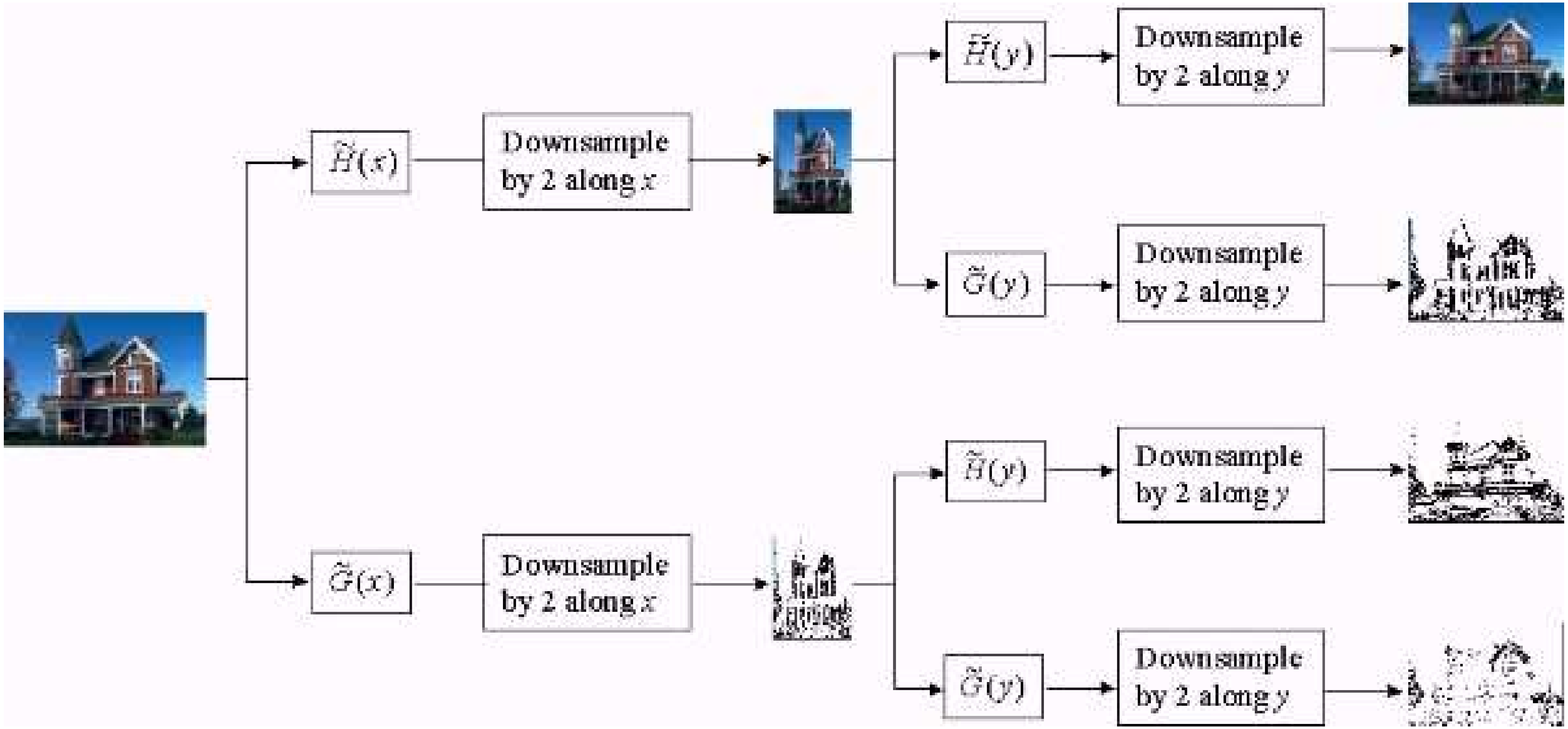




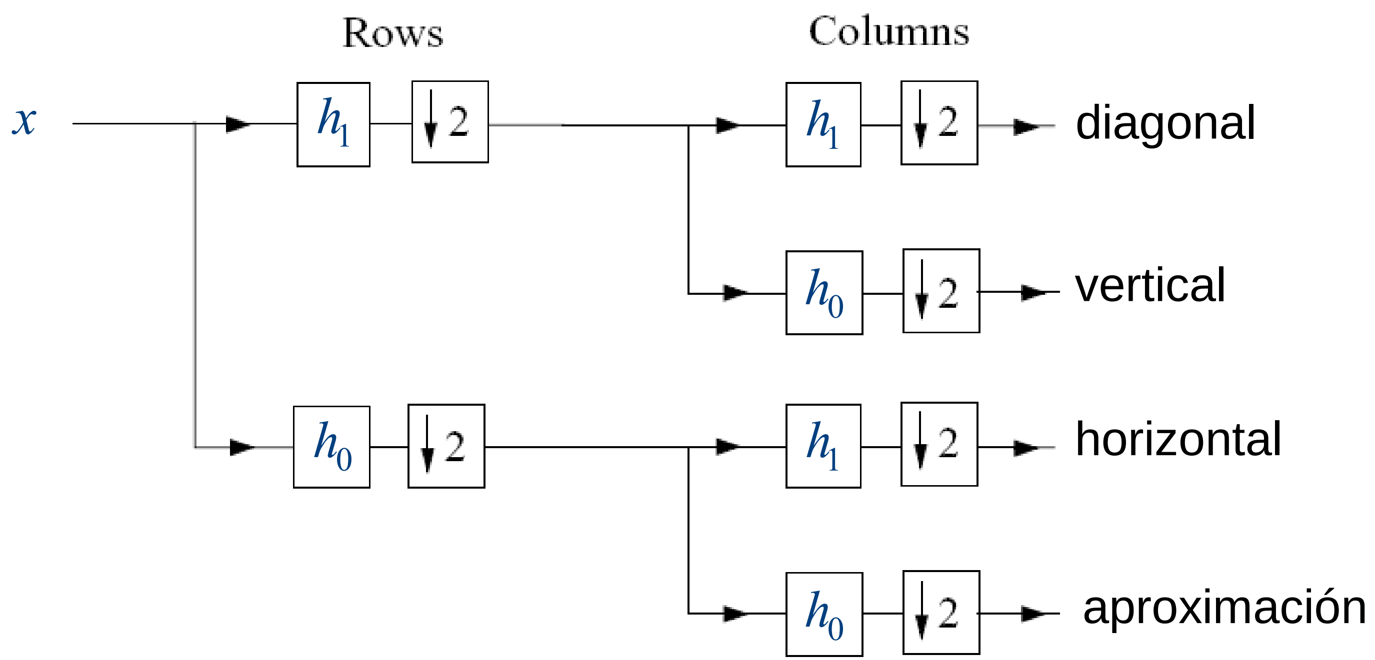
Wavelets y procesamiento de multiresolución (cont.)



Análisis en imágenes con wavelets



Análisis en imágenes con wavelets



Síntesis en imágenes con wavelets

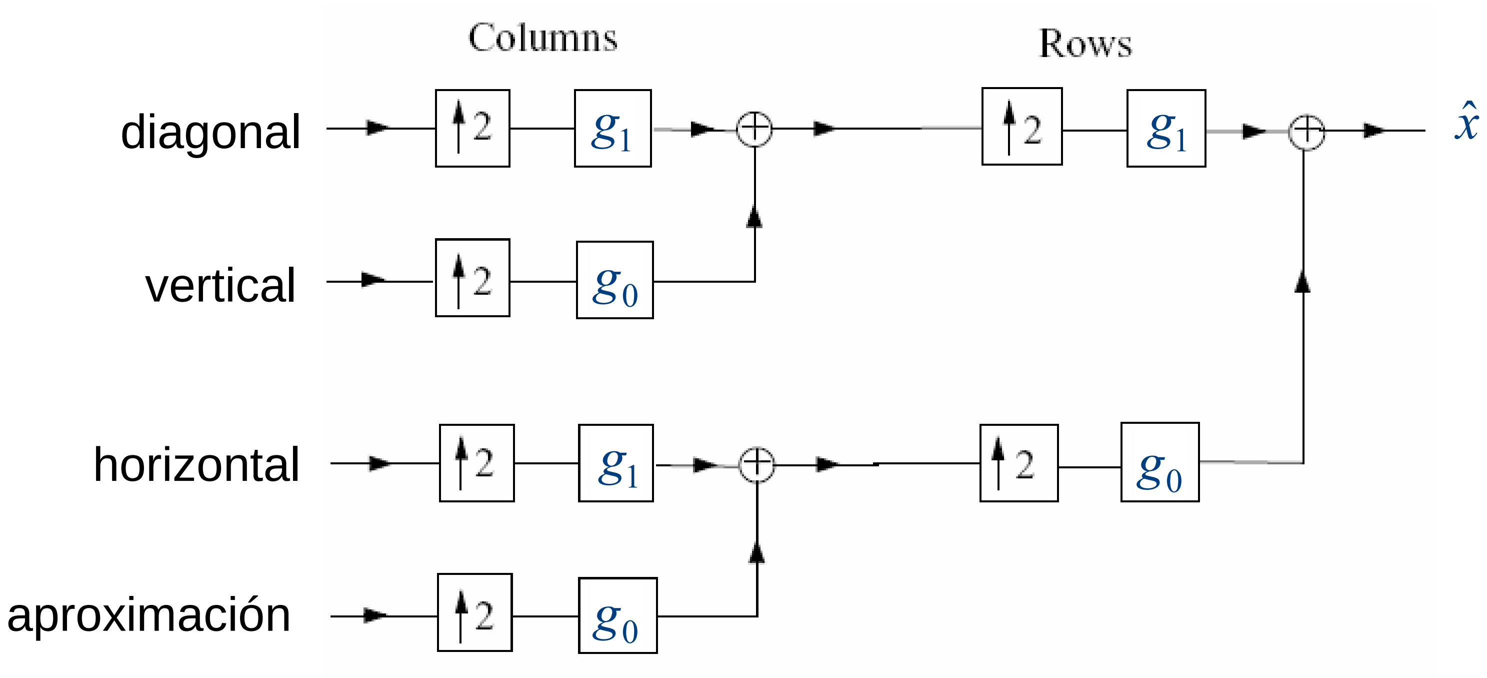


Diagrama del sistema de filtrado y compresión

señal ruidosa → Transformada directa → Operación de filtrado → Transformada inversa → señal filtrada

p.e.
KLT
DCT
WT

p.e.
Filtrado lineal de Wiener
Umbralizado no-lineal

IMPORTANTE:

- En el tratamiento mediante transformadas, cuando filtramos, comprimimos
- KLT:
  - Para Karaoke 2 (Misión Imposible), pero prohibitiva complejidad computacionl $O(N^3)$
  - En ciertos casos puede ser reemplazada por la DCT

Caso del empleo de wavelets

señal ruidosa $\longrightarrow$ DWT $\xrightarrow{Y}$ Umbralizado $\xrightarrow{\widetilde{X}}$ IWT $\longrightarrow$ señal filtrada

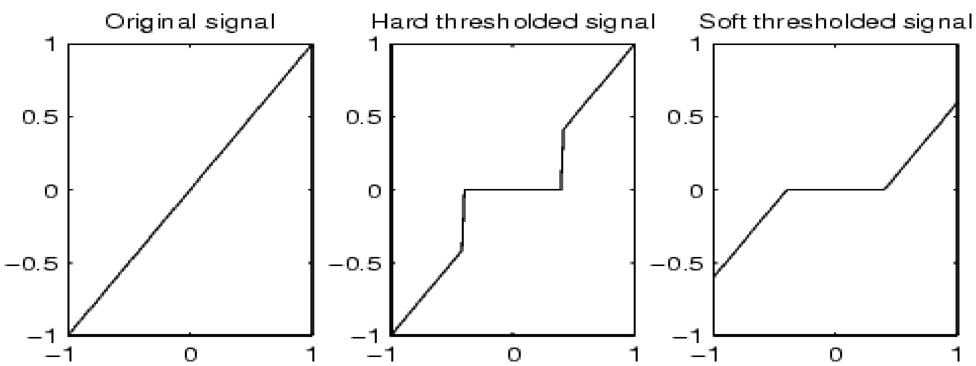
Hard thresholding
Umbralizado brusco

$$\widetilde{X}[n] = \begin{cases} Y[n] & if \ |Y[n]| \geq T \\ 0 & otherwise \end{cases}$$

$$T = 4 \cdot median\left\{ \frac{|x|}{0.6745} \right\}$$

Soft thresholding
Umbralizado suave

$$\widetilde{X}[n] = \begin{cases} Y[n] - T & Y[n] \geq T \\ Y[n] + T & Y[n] \leq -T \\ 0 & |Y[n]| < T \end{cases}$$



Elección del umbral

Donoho y Johnstone'1994

$$T = \sigma\sqrt{2\log_e N}$$

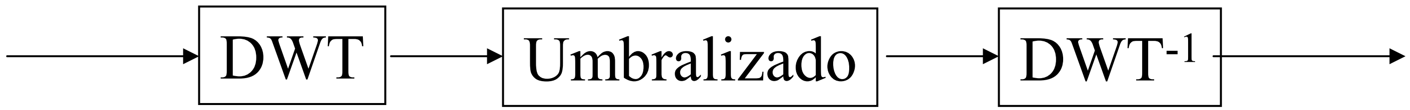
Dada una performance de filtrado cercana a la ponderación ideal

$$\tilde{X}[n] = \frac{\eta^2[n]}{\eta^2[n]+\sigma^2} Y[n]$$

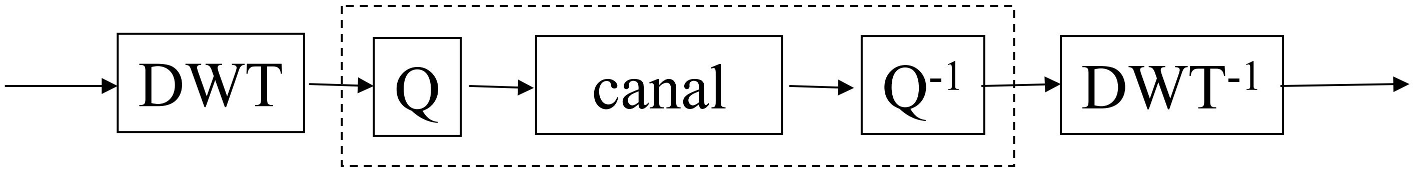
ideal $$\tilde{X}[n] = aY[n], a = \frac{\eta^2[n]}{\eta^2[n]+\sigma^2} < 1 \Rightarrow |\tilde{X}[n]| < |Y[n]|$$

soft $$\tilde{X}[n] = Y[n] - T, Y[n] > T \Rightarrow |\tilde{X}[n]| < |Y[n]|$$

Dualidad: filtrado-compresión

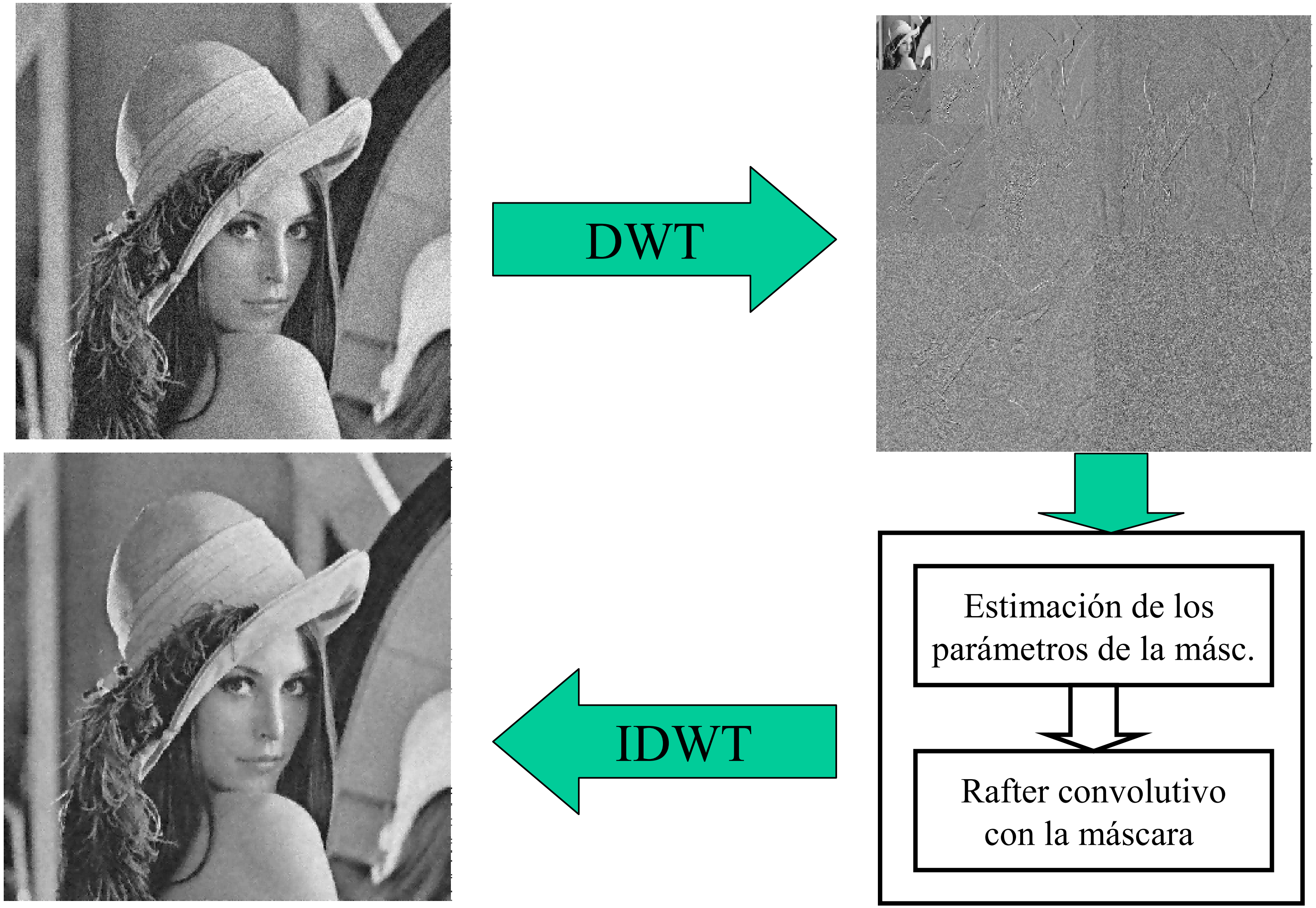


Sistema de filtrado



Sistema de codificación

Diagrama del sistema de filtrado con wavelets



Ejemplo de DWT-2D

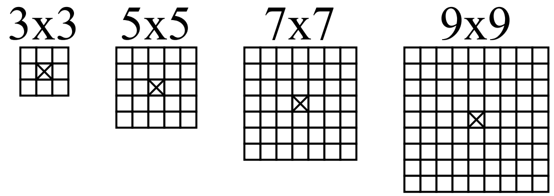
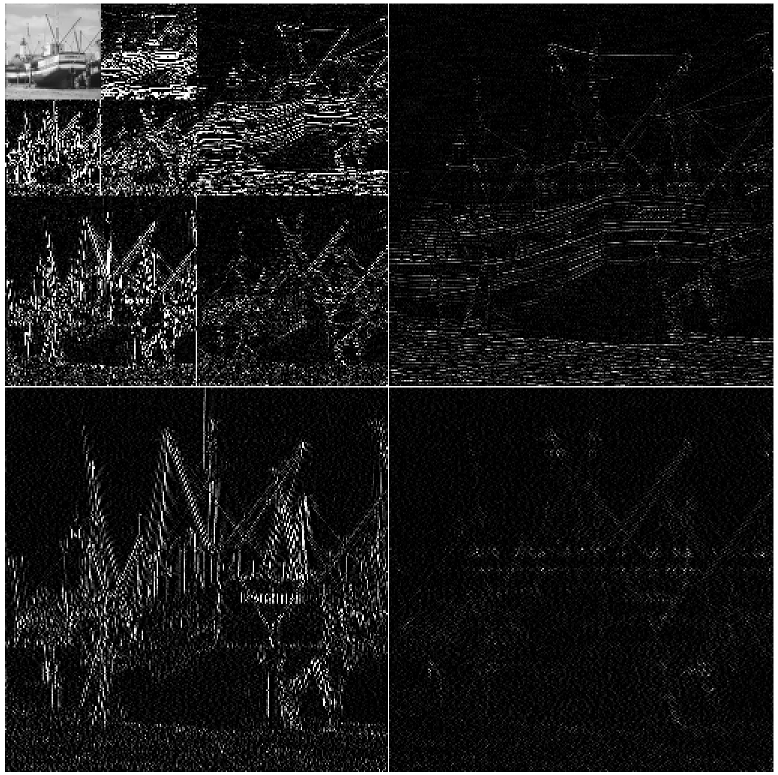
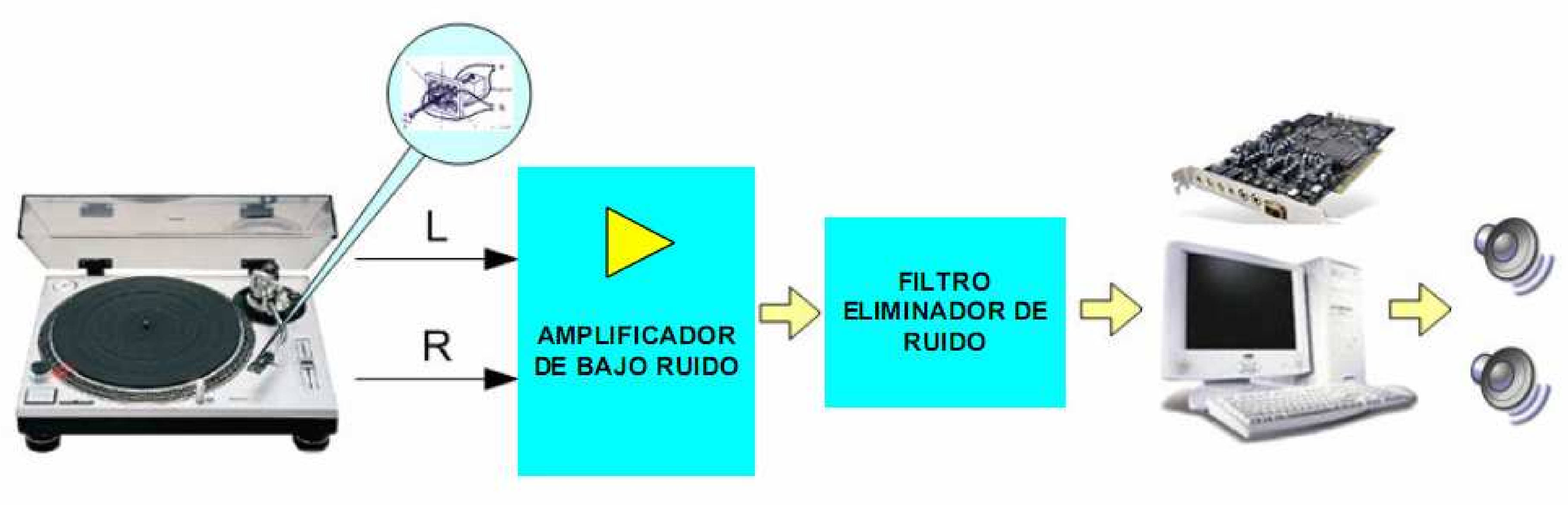


Imagen de bote

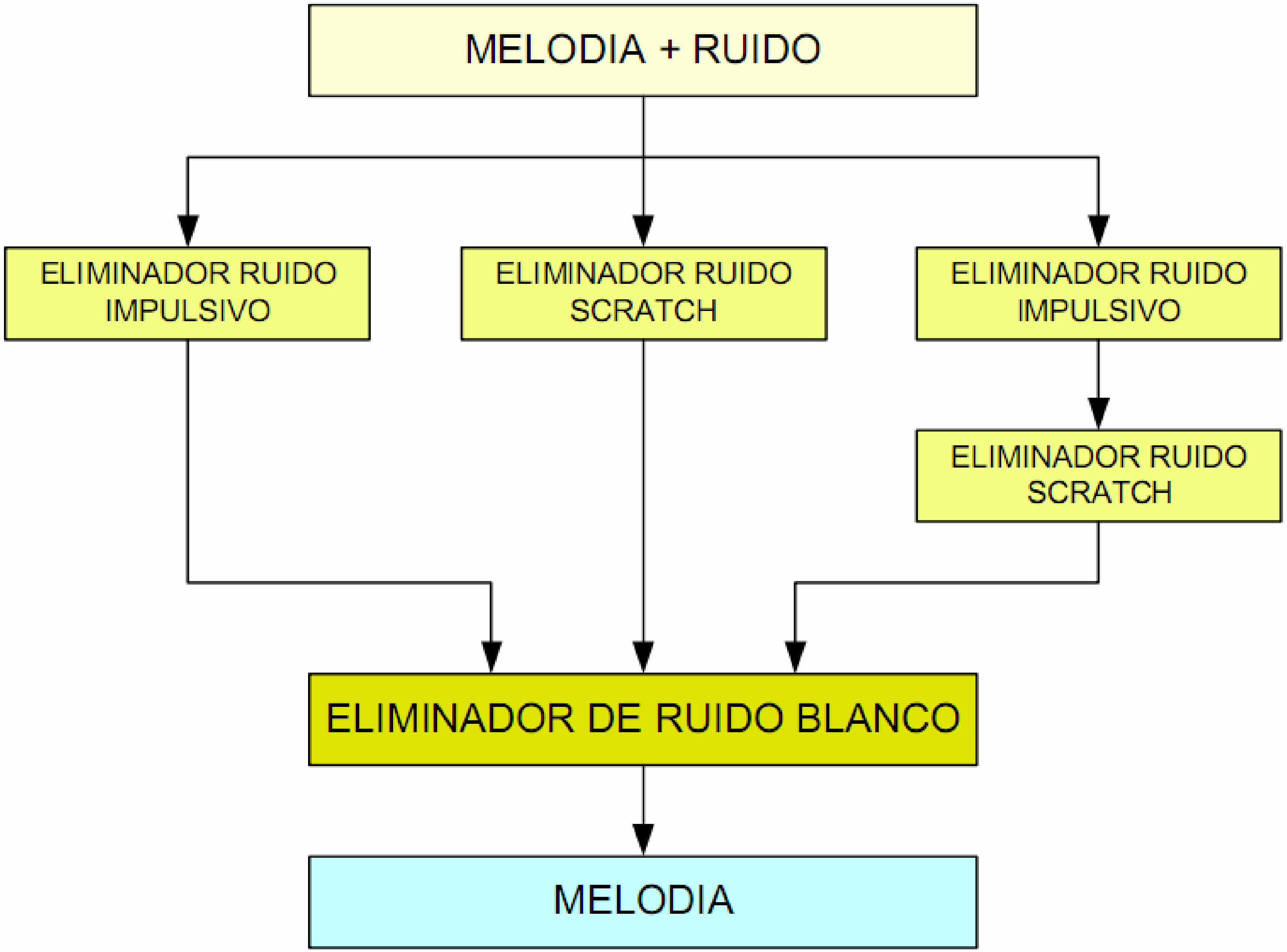


DWT en 3 niveles

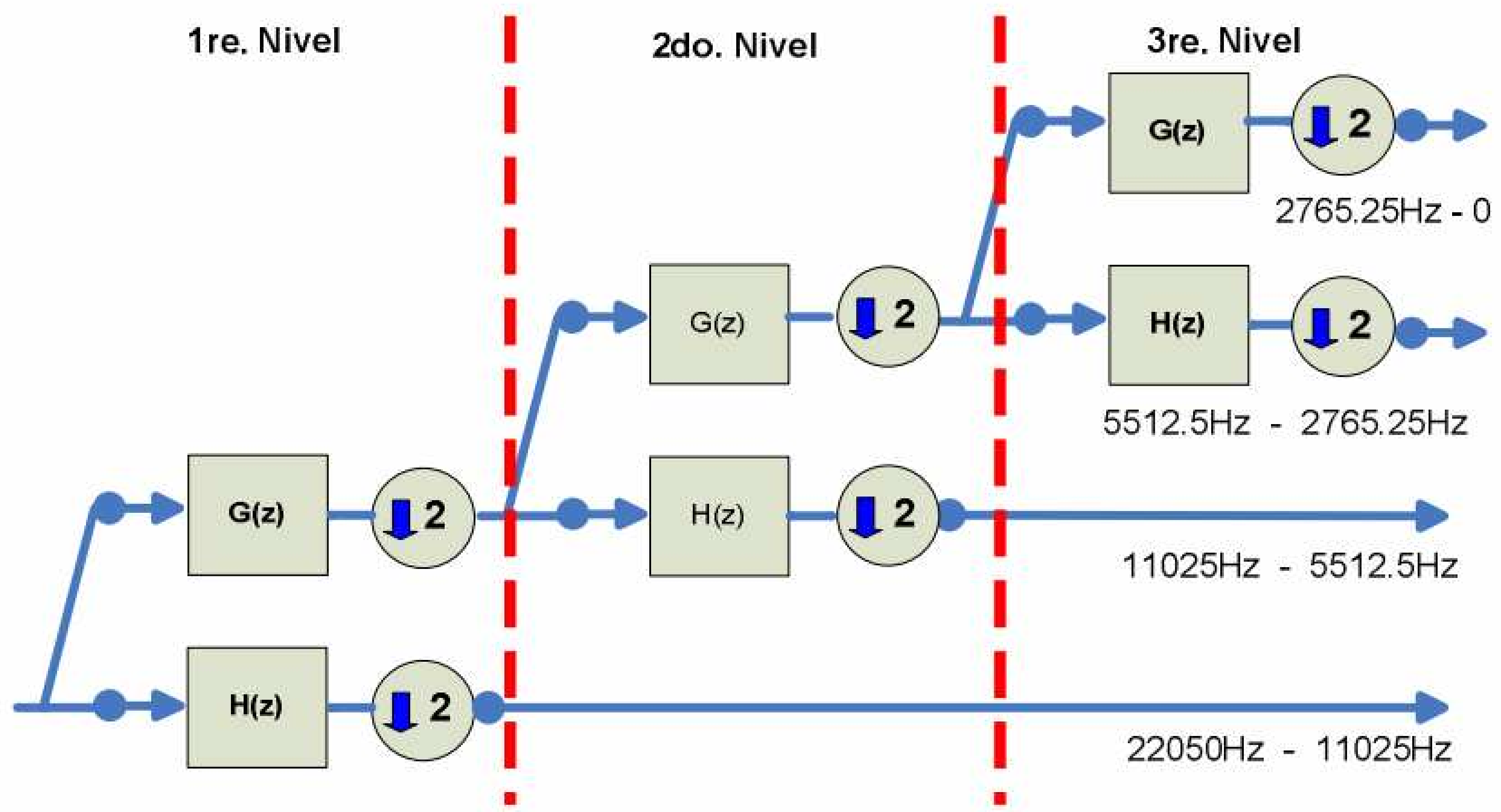
Copia de disco de vinilo



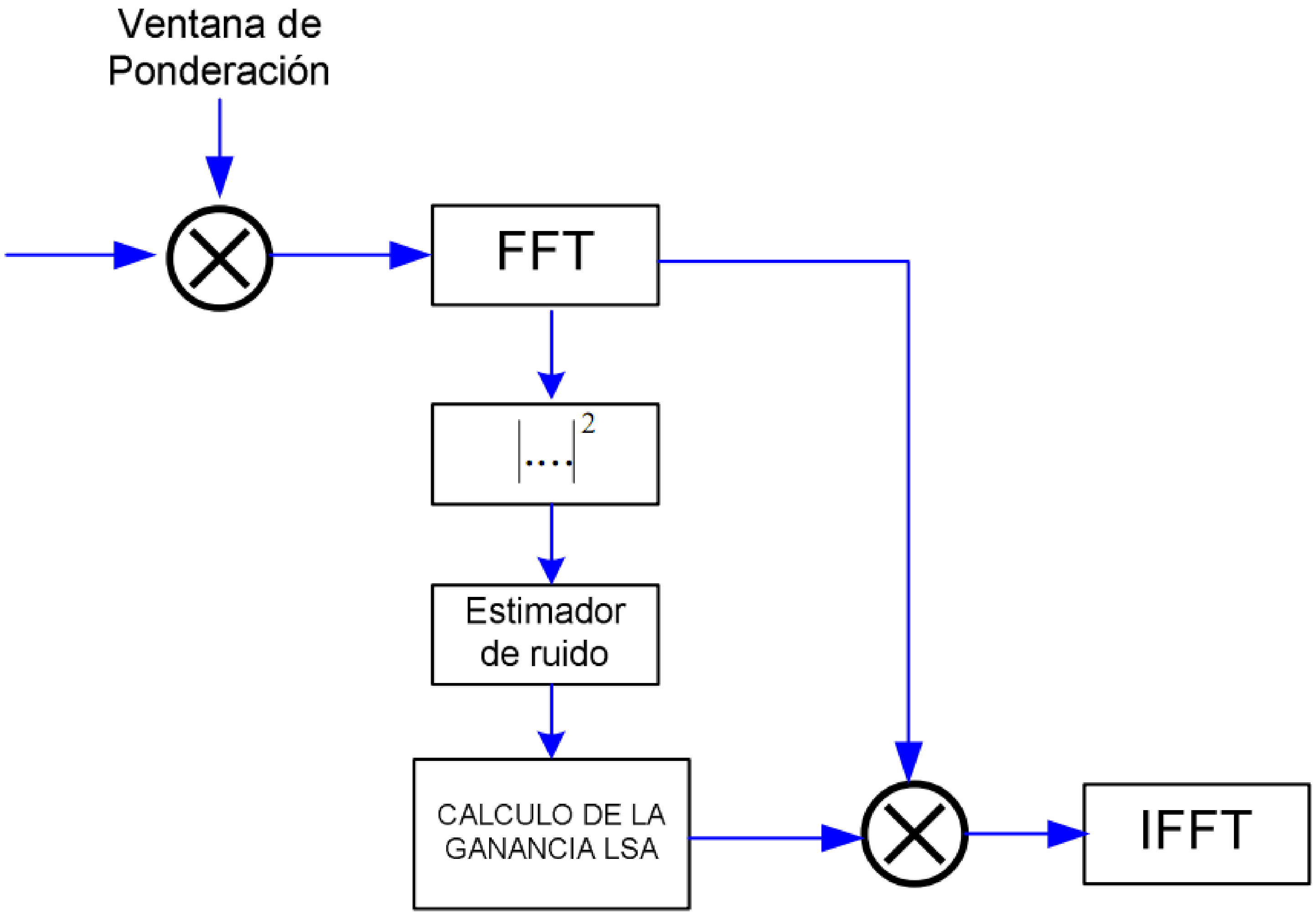
Copia de disco de vinilo



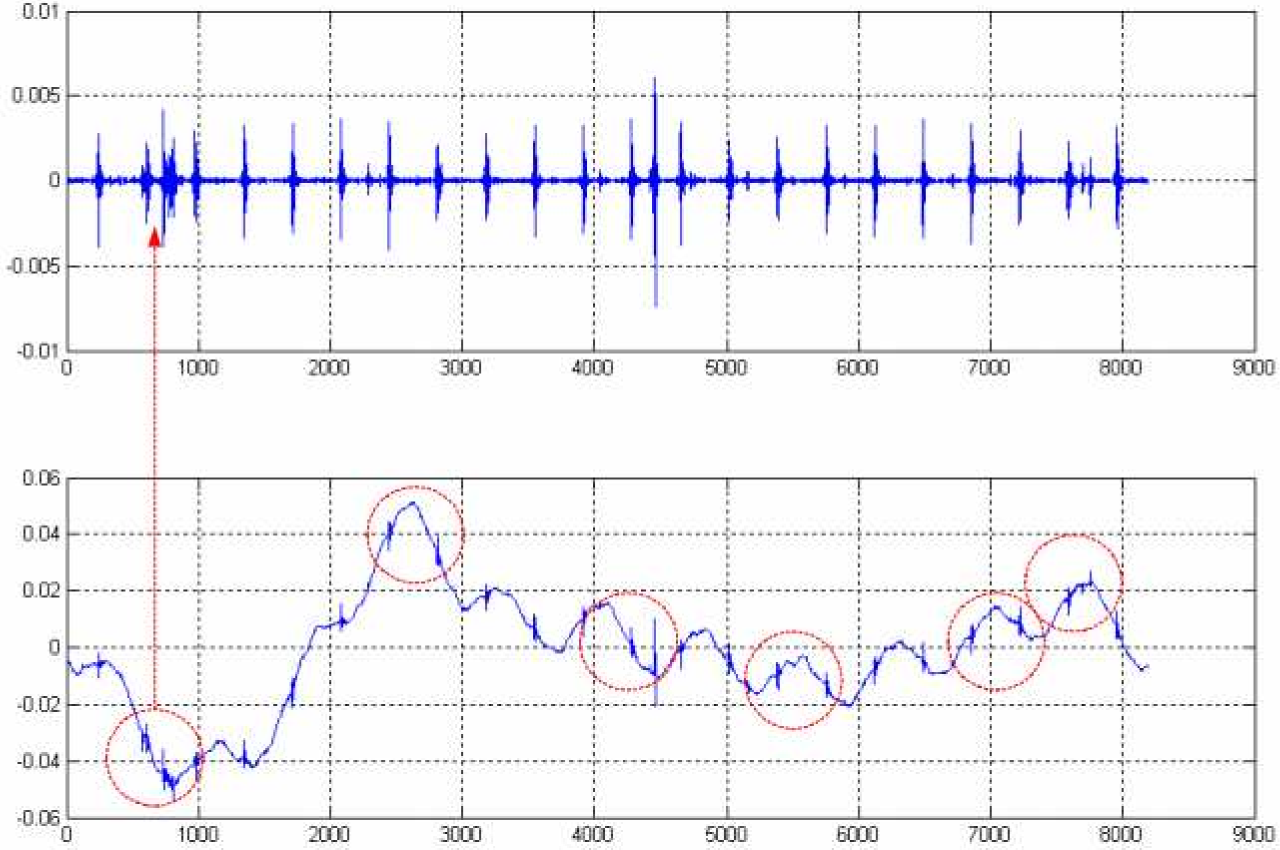
Copia de disco de vinilo



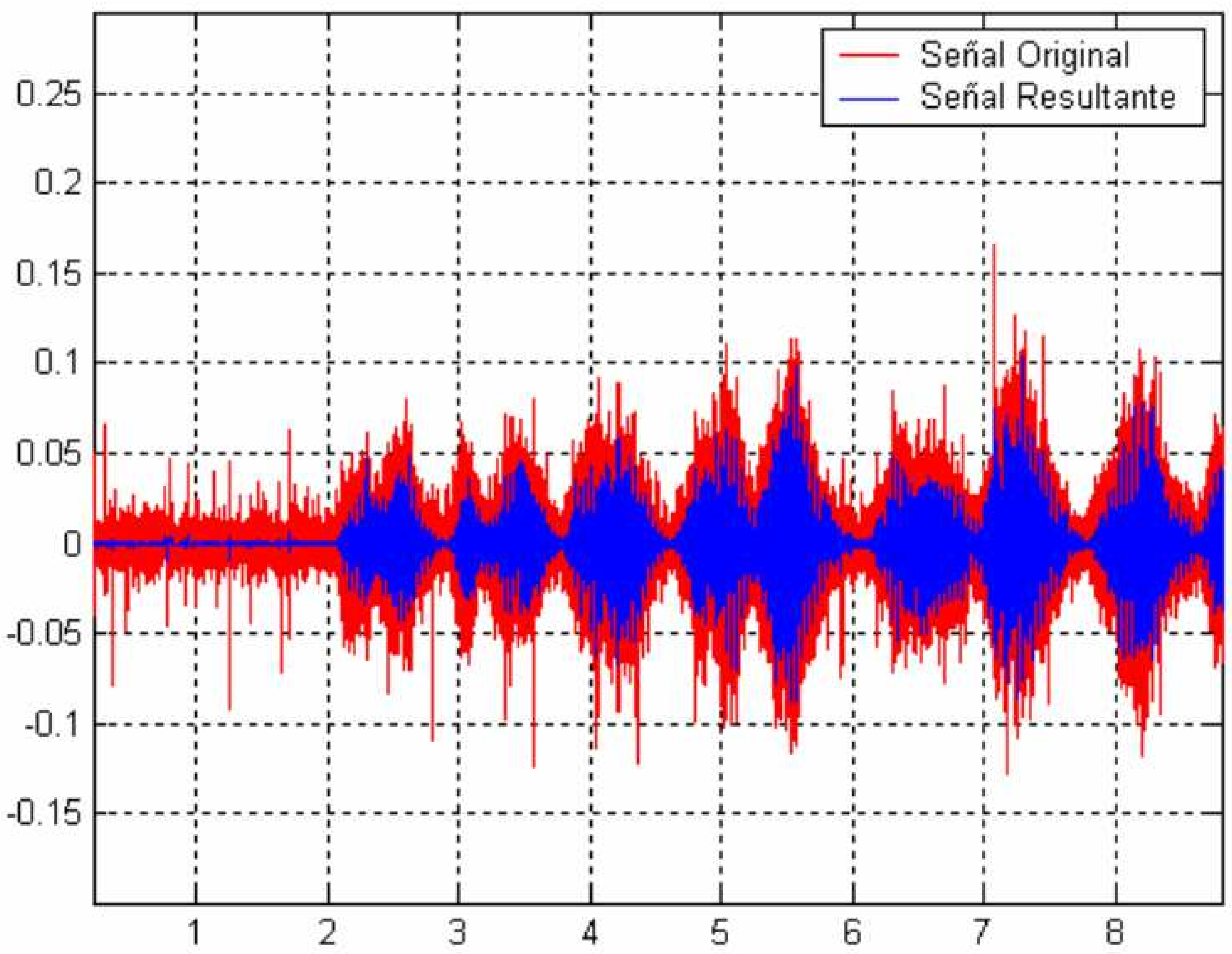
Copia de disco de vinilo



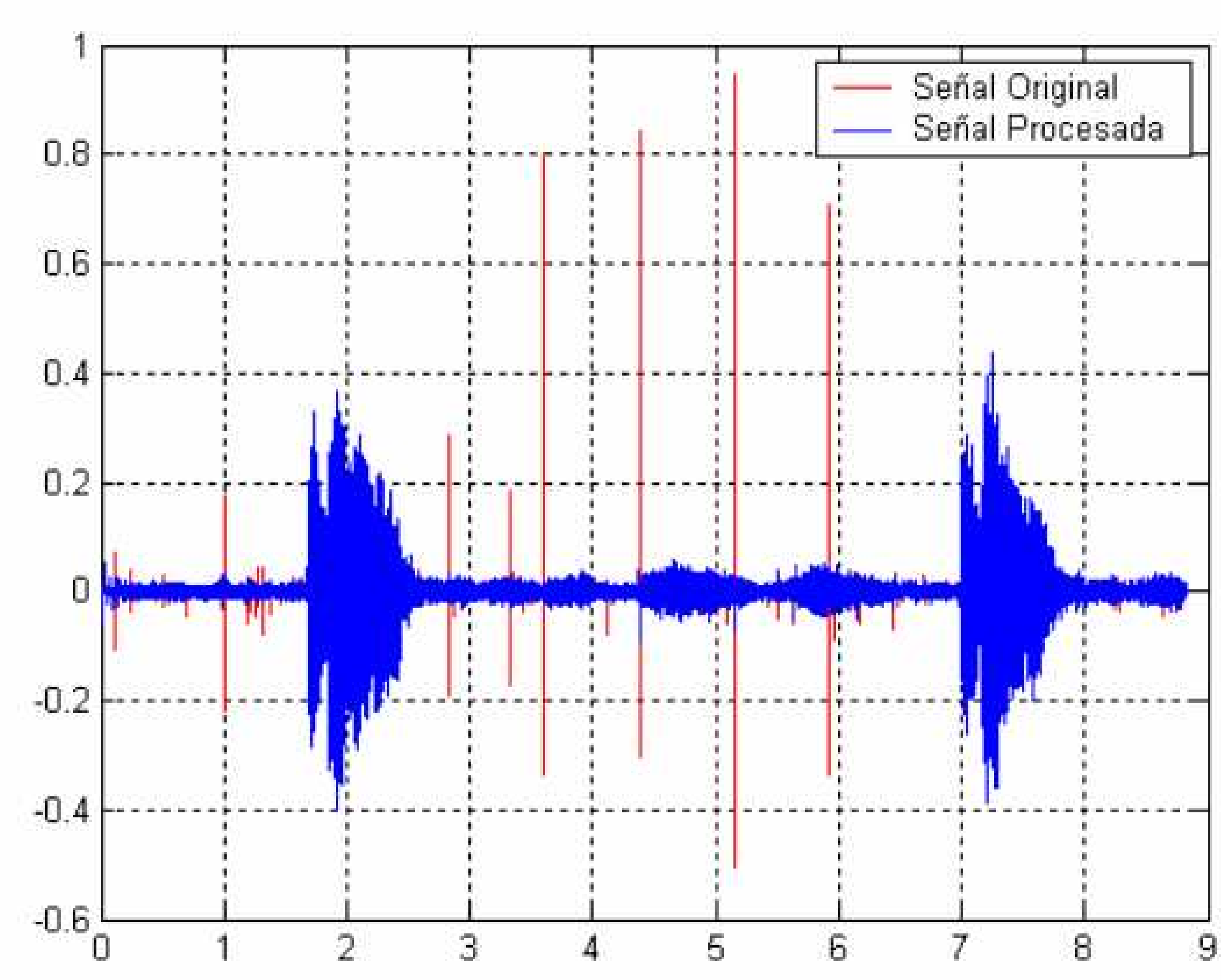
Copia de disco de vinilo



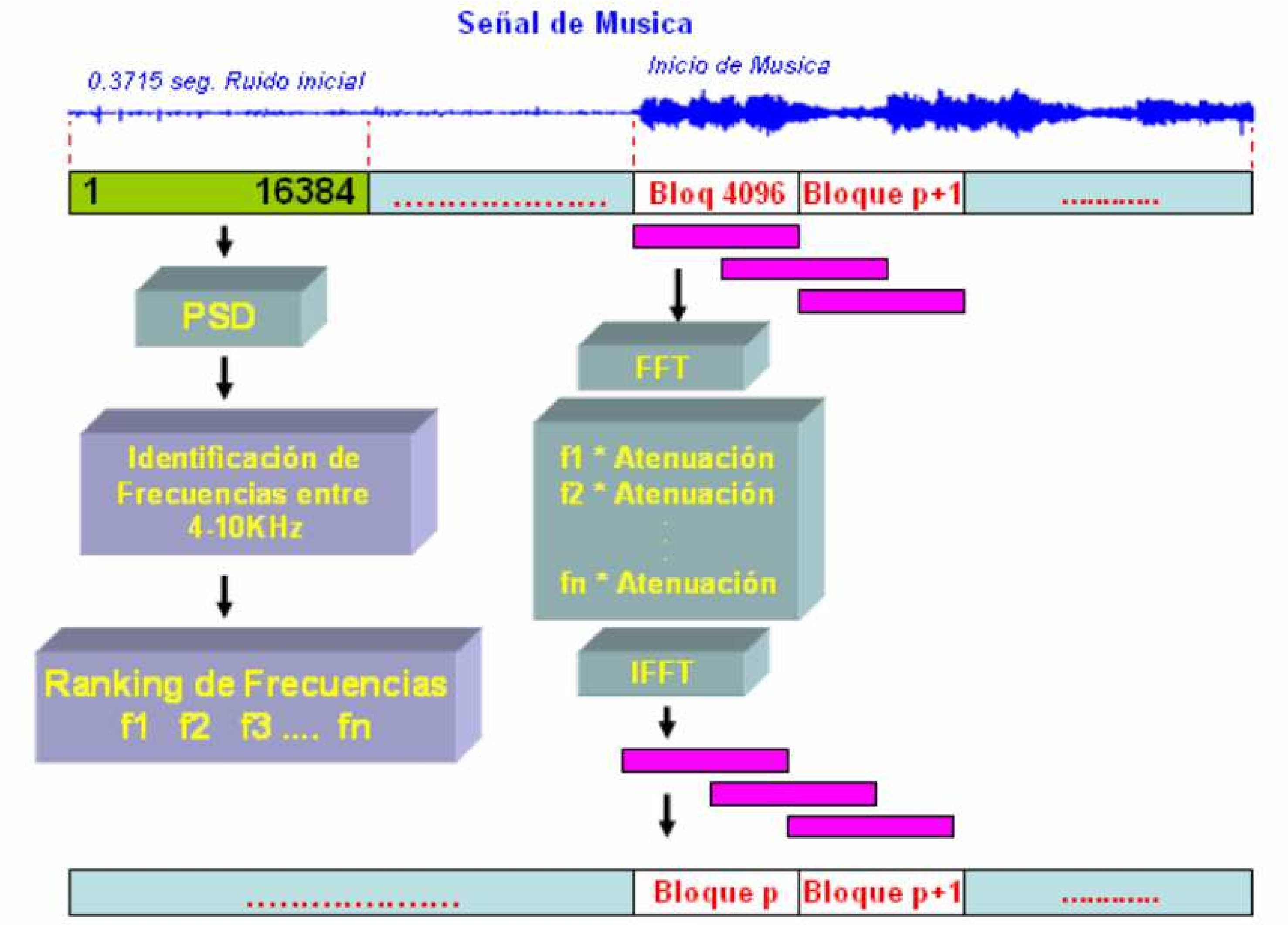
Copia de disco de vinilo



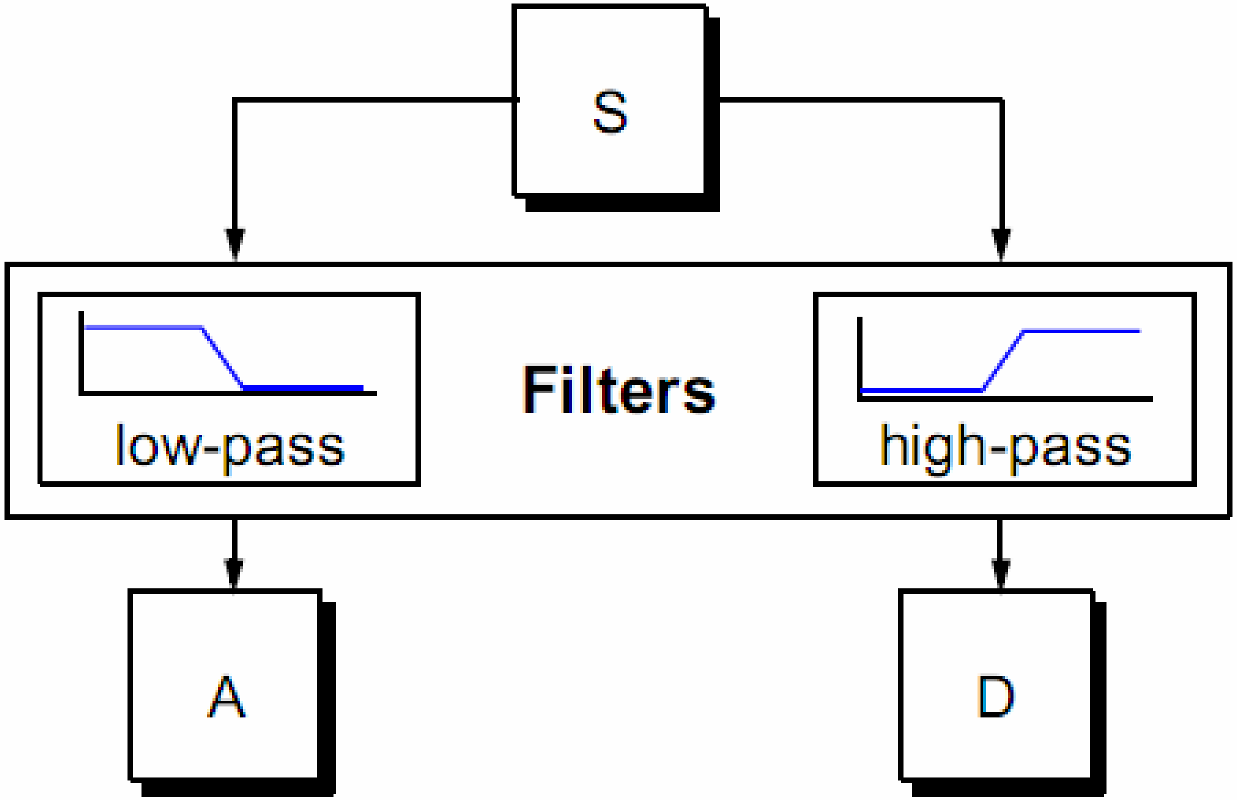
Copia de disco de vinilo



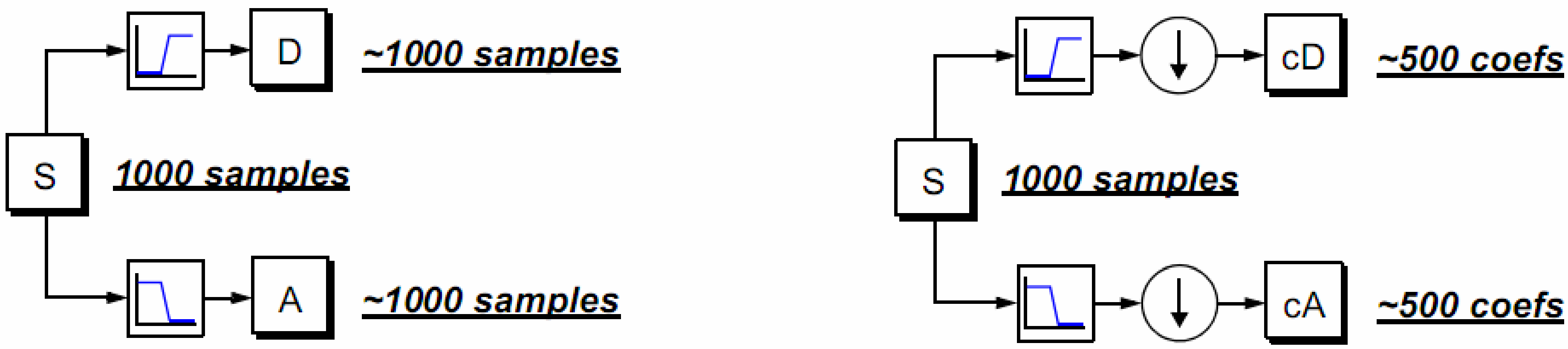
Copia de disco de vinilo



Subbandas

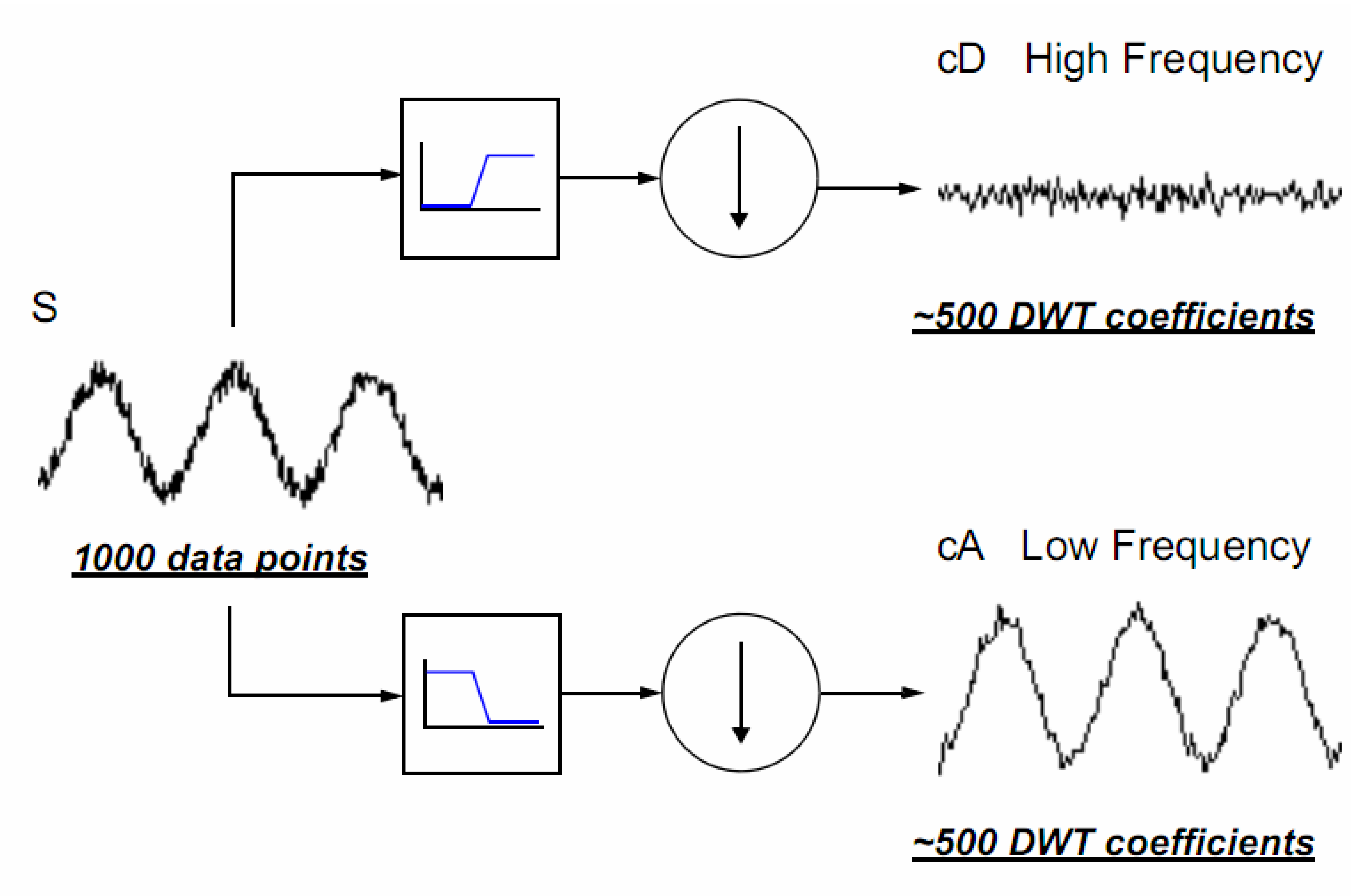


Subbandas

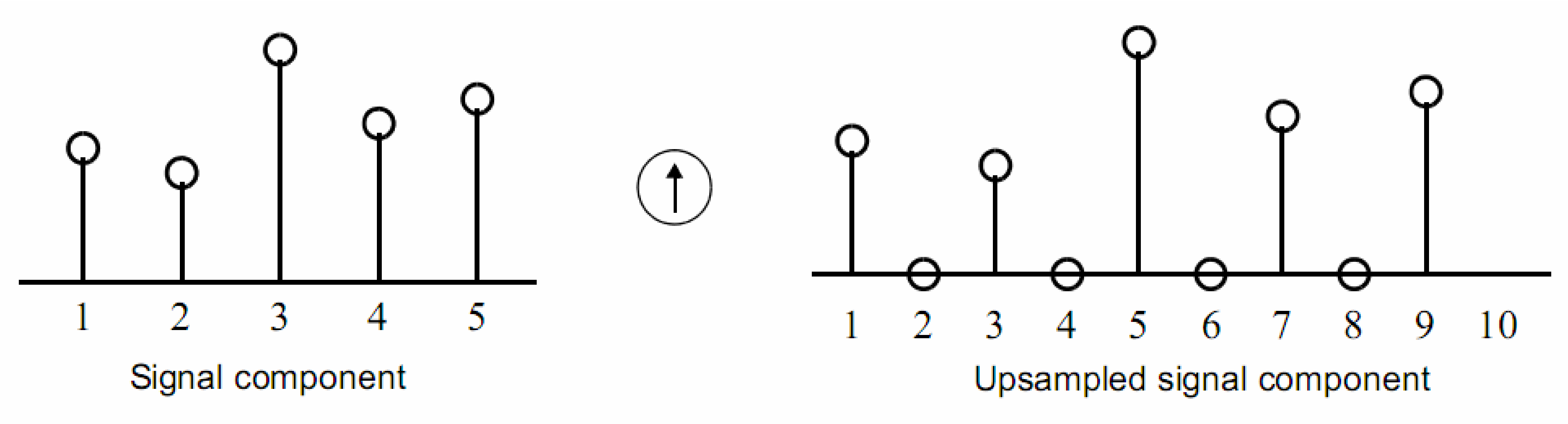


D
~1000 samples
S
1000 samples
A
~1000 samples
cD
~500 coefs
S
1000 samples
cA
~500 coefs

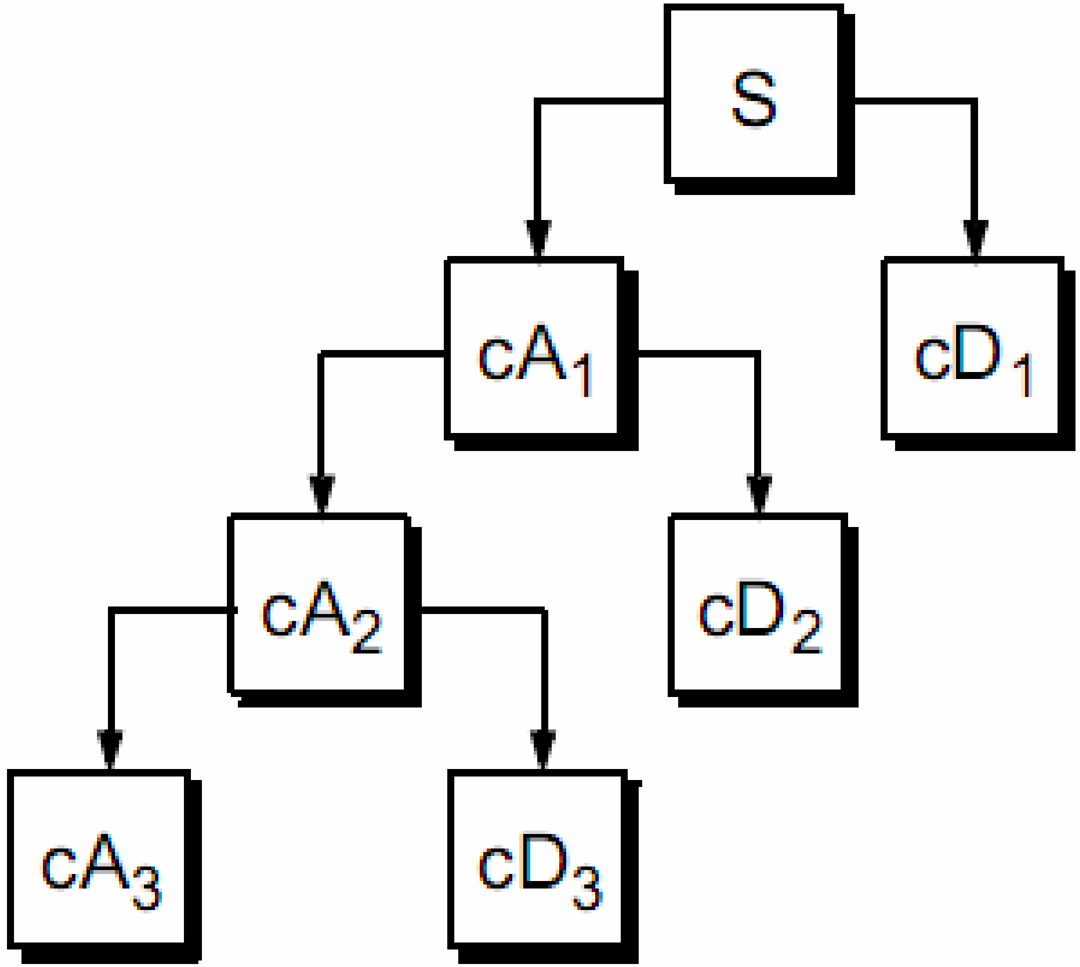
Subbandas


cD High Frequency
S
~500 DWT coefficients
1000 data points
cA Low Frequency
~500 DWT coefficients

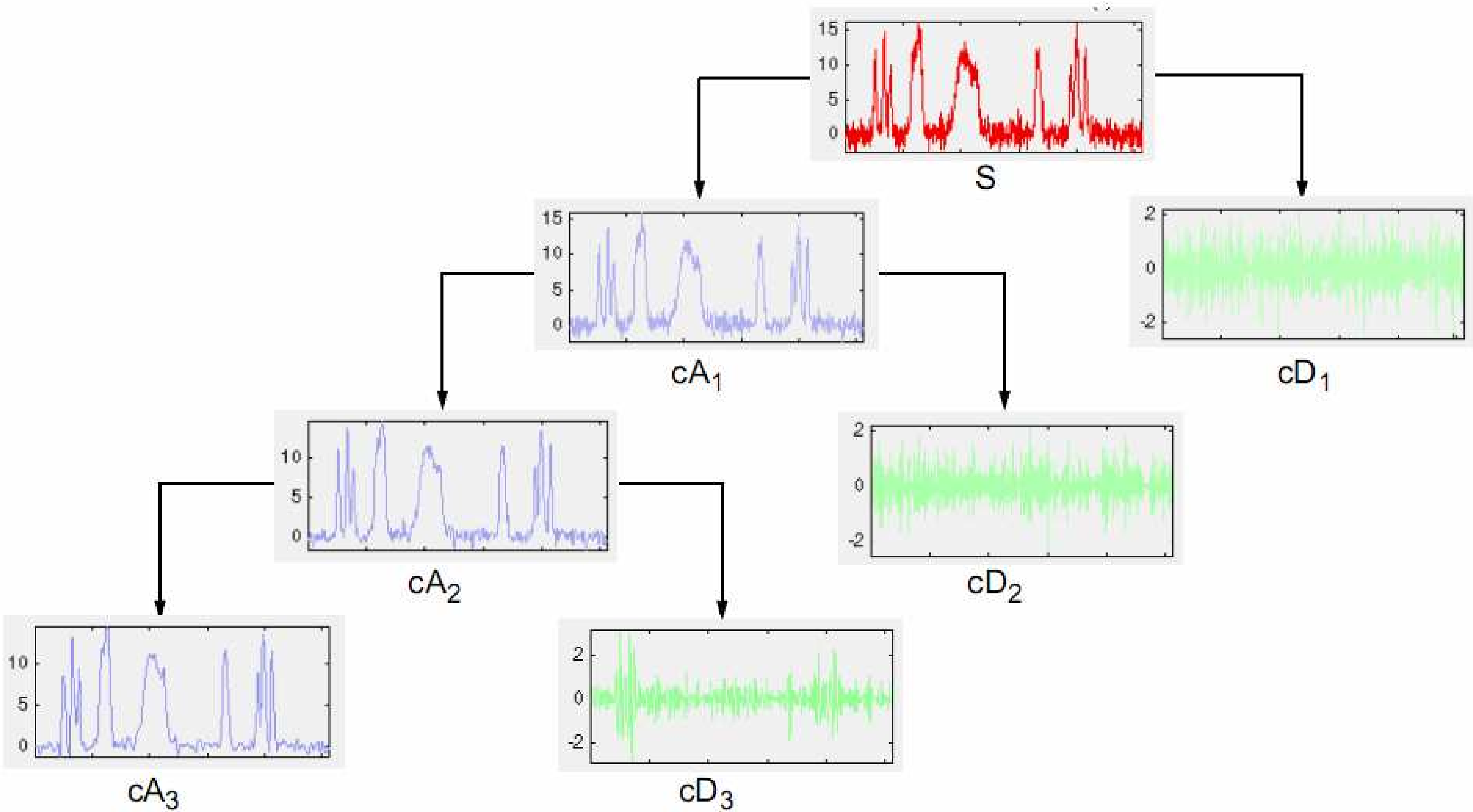
Subbandas


1
2
3
4
5
Signal component
1
2
3
4
5
6
7
8
9
10
Upsampled signal component

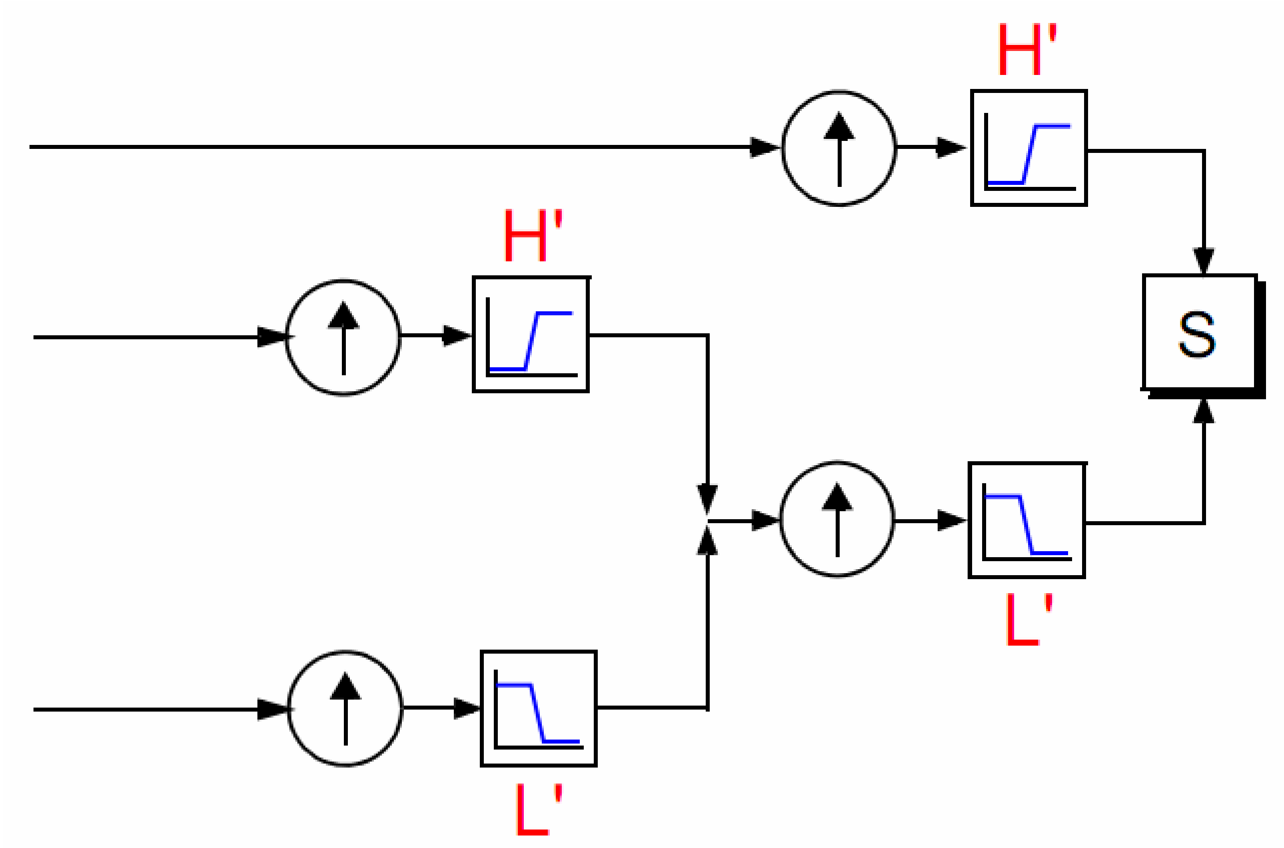
Subbandas



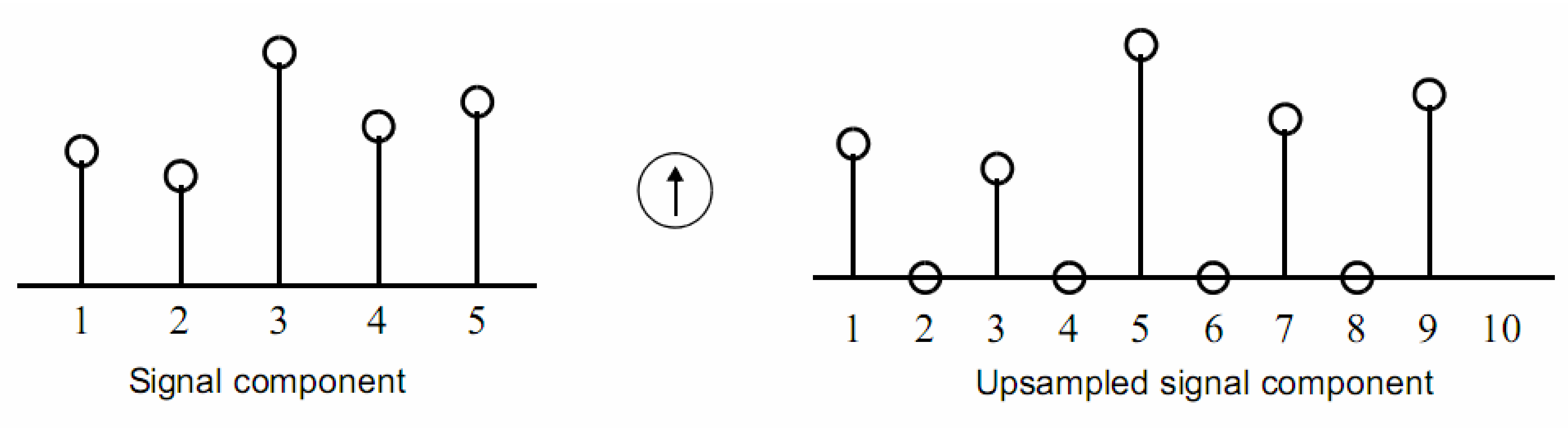
Subbandas



Subbandas

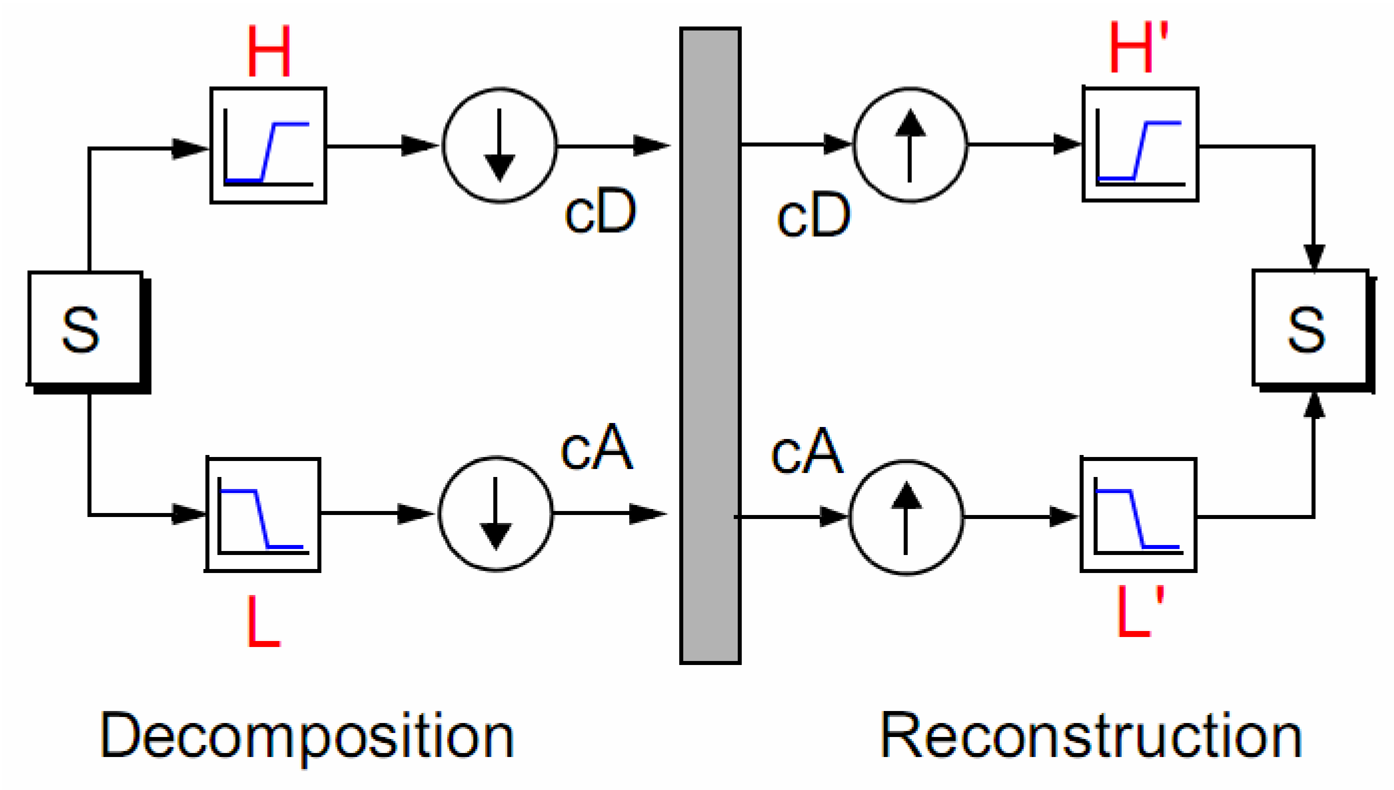


Subbandas

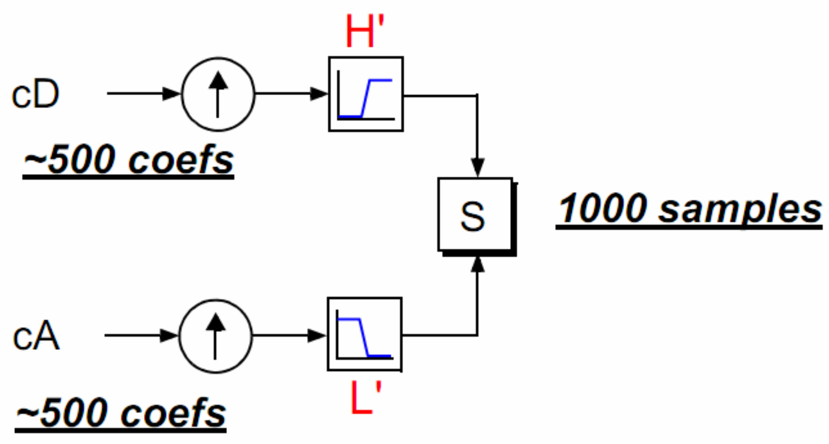


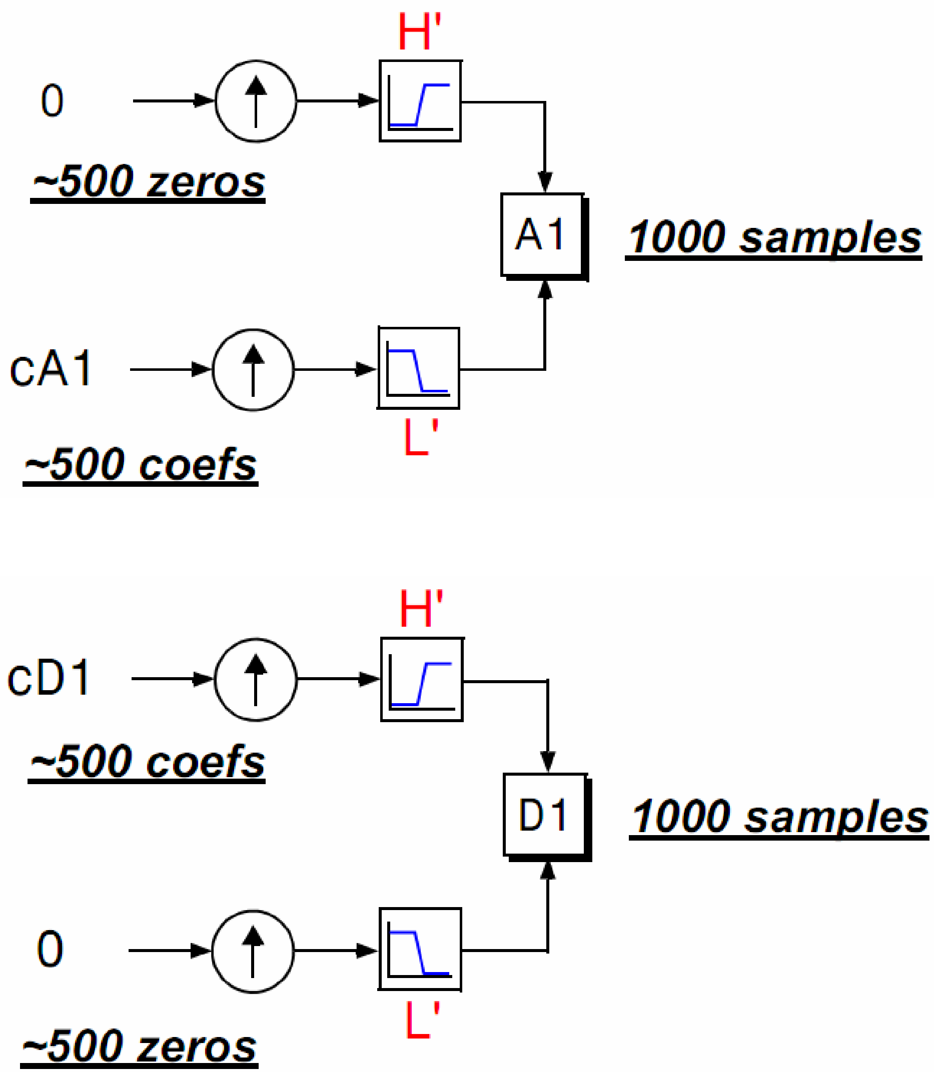
1
2
3
4
5
Signal component
1
2
3
4
5
6
7
8
9
10
Upsampled signal component

Subbandas

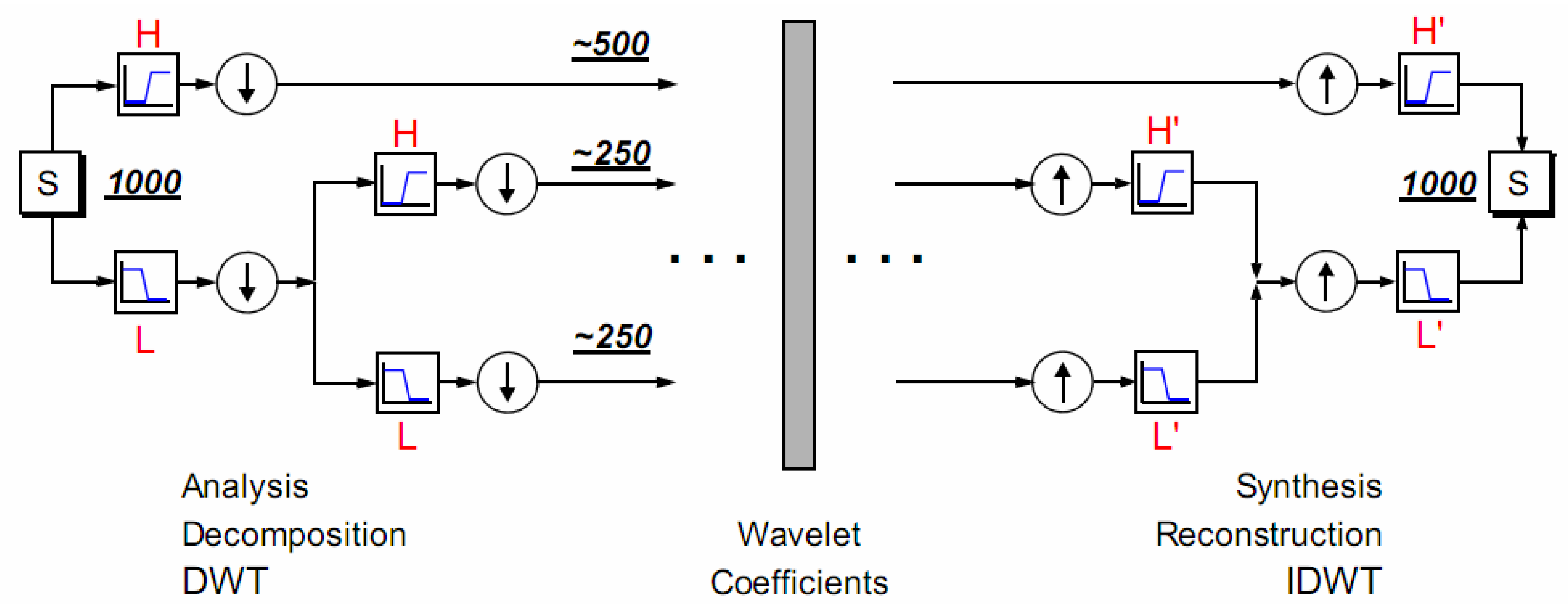


Subbandas


H'
cD
~500 coefs
S
1000 samples
cA
~500 coefs
L'

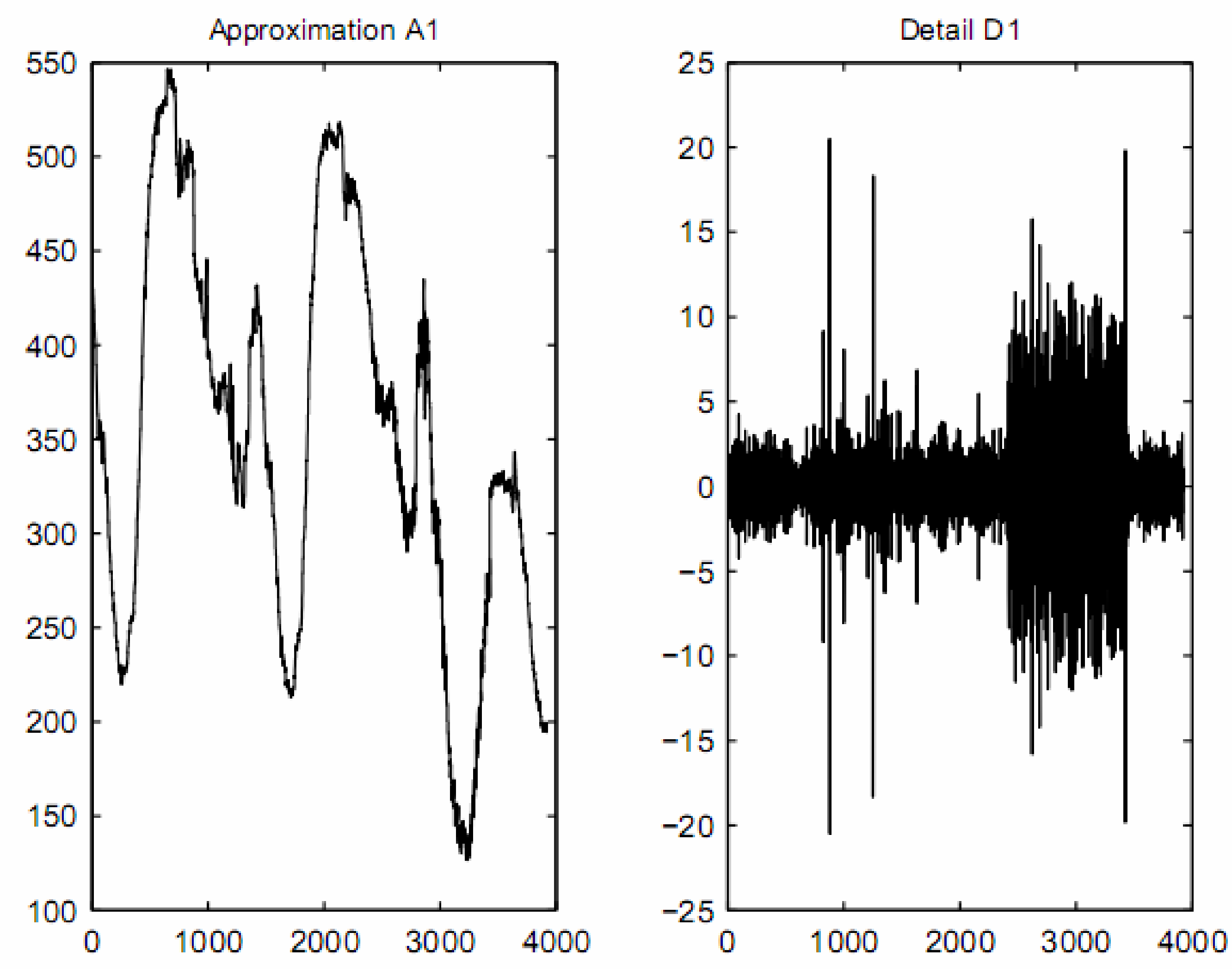

H'
0
~500 zeros
A1
1000 samples
cA1
~500 coefs
L'
H'
cD1
~500 coefs
D1
1000 samples
0
~500 zeros
L'

Subbandas


H
~500
S
1000
L
H
~250
~250
L
H'
H'
1000
S
L'
L'
Analysis
Decomposition
DWT
Wavelet
Coefficients
Synthesis
Reconstruction
IDWT

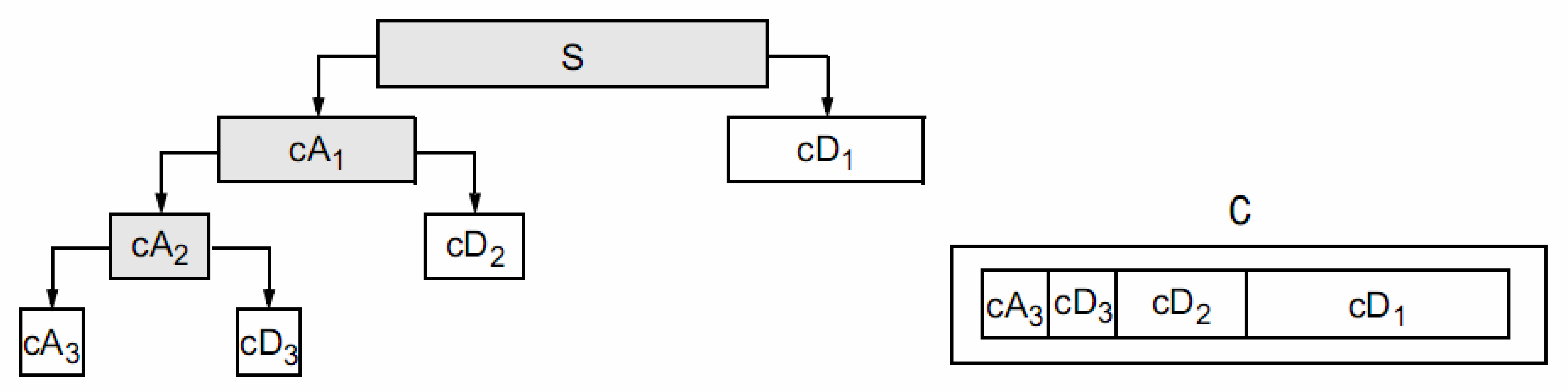
Subbandas

Subbandas

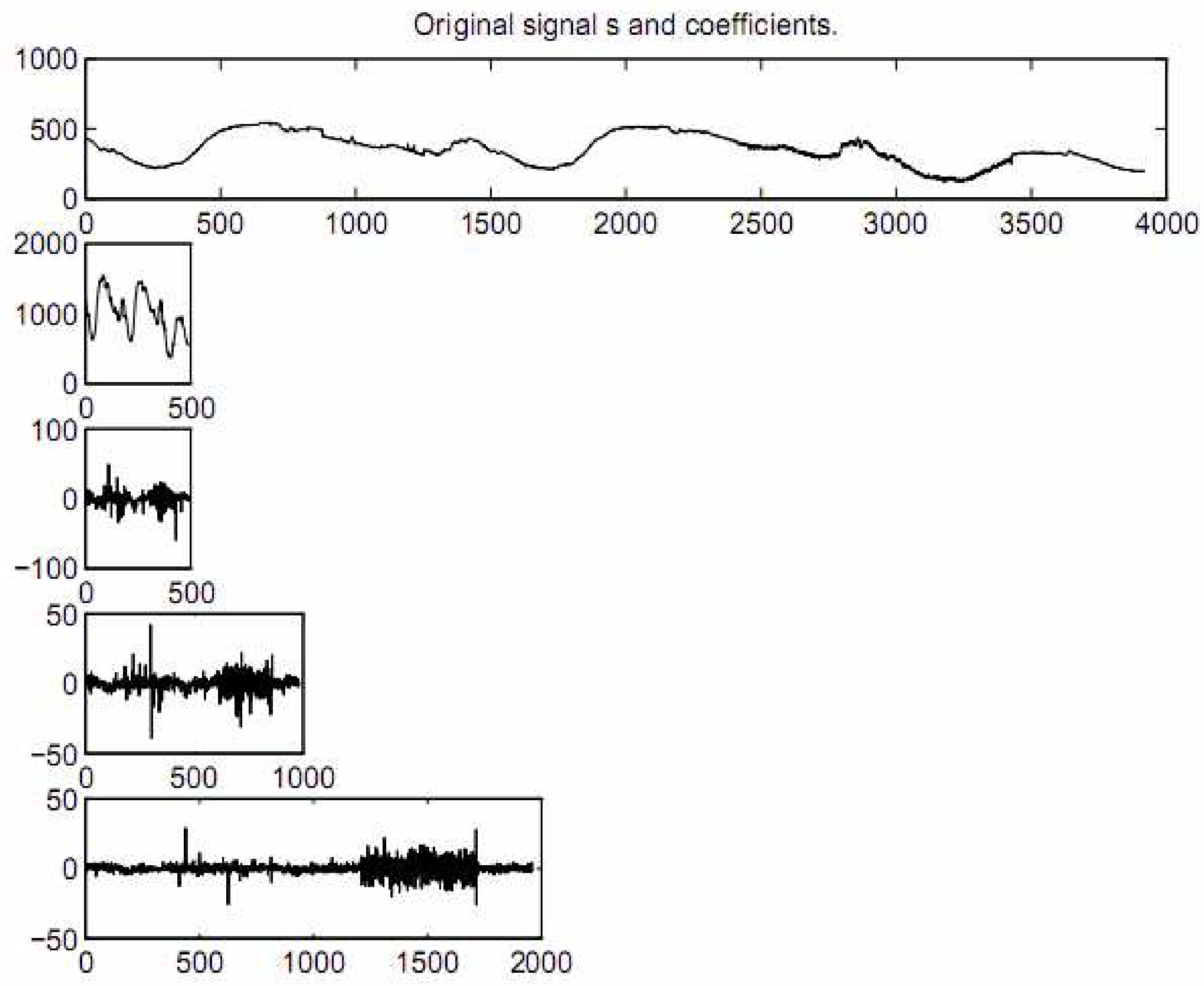


Approximation A1
550
500
450
400
350
300
250
200
150
100
0
1000
2000
3000
4000
Detail D1
25
20
15
10
5
0
−5
−10
−15
−20
−25
0
1000
2000
3000
4000

Subbandas

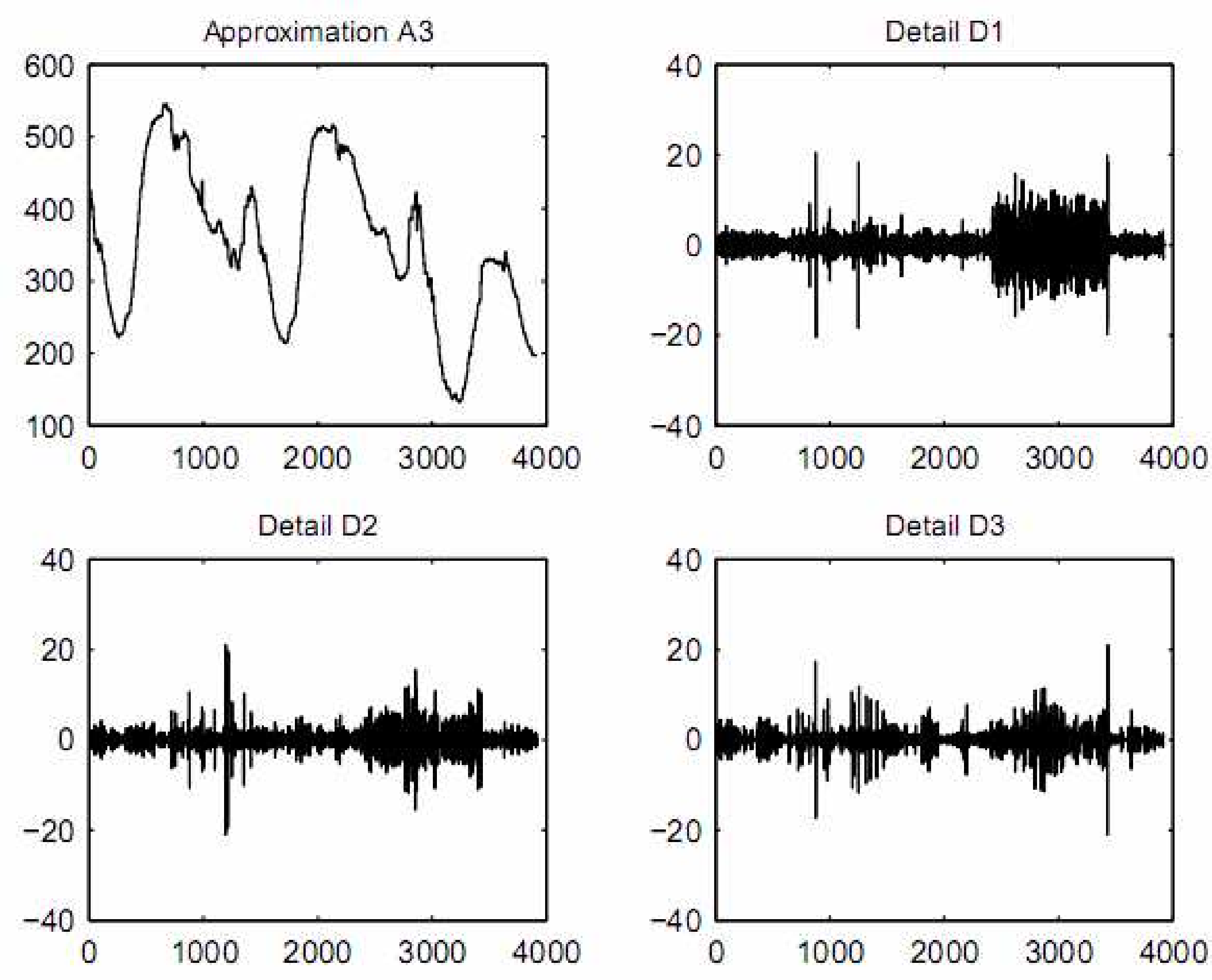


Subbandas



Original signal s and coefficients.
1000
500
0
0
500
1000
1500
2000
2500
3000
3500
4000
2000
1000
0
0
500
100
0
−100
0
500
50
0
−50
0
500
1000
50
0
−50
0
500
1000
1500
2000

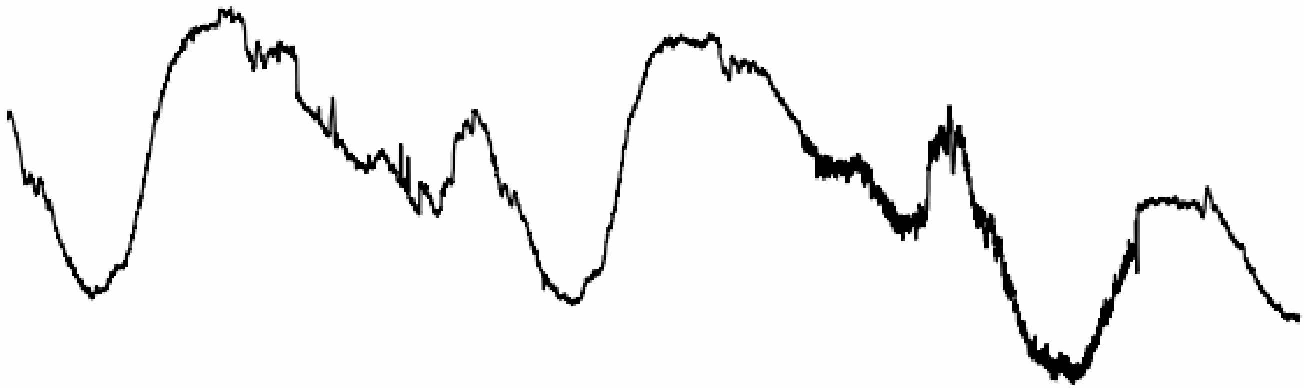
Subbandas



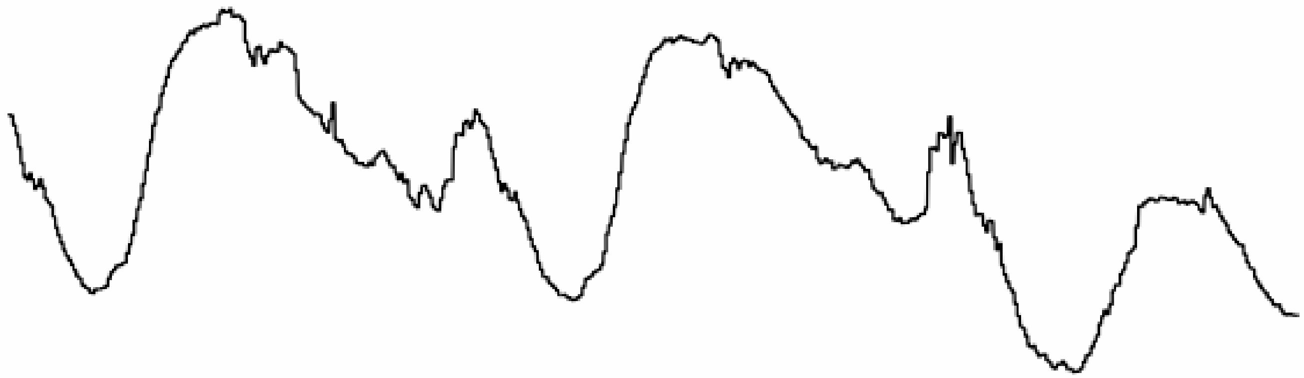
Approximation A3
600
500
400
300
200
100
0
1000
2000
3000
4000
Detail D1
40
20
0
−20
−40
0
1000
2000
3000
4000
Detail D2
40
20
0
−20
−40
0
1000
2000
3000
4000
Detail D3
40
20
0
−20
−40
0
1000
2000
3000
4000

Subbandas

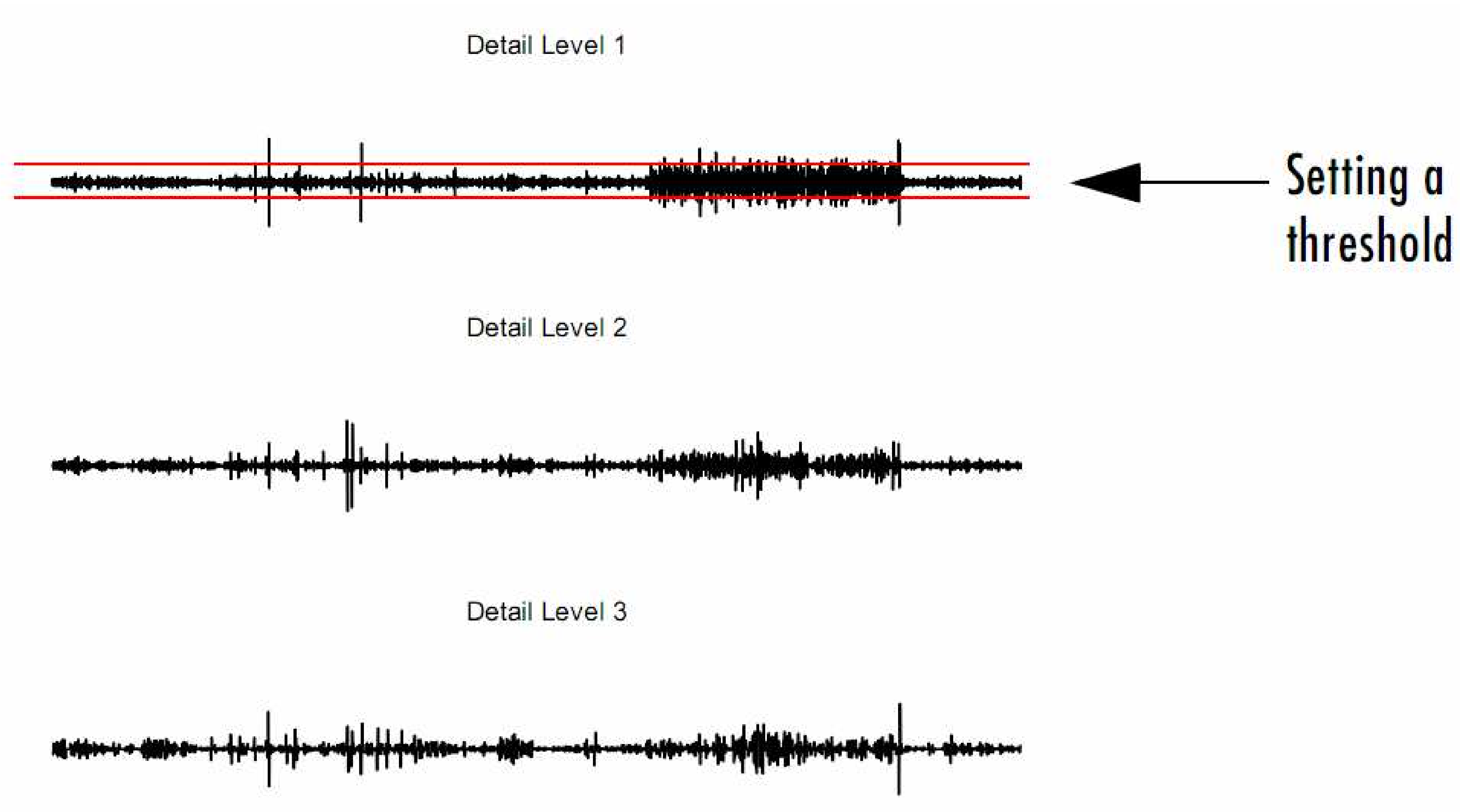
Original



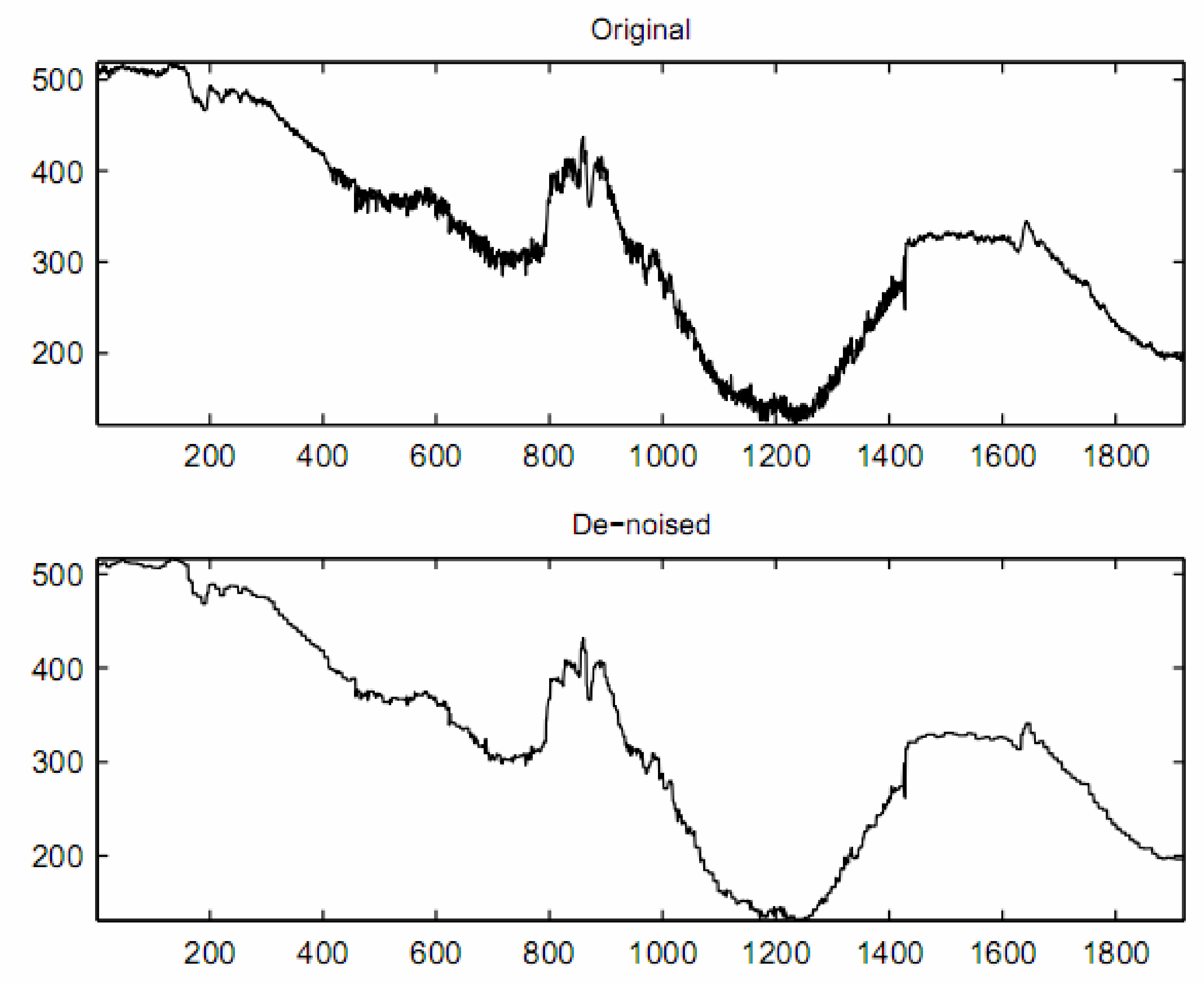
Level 3 Approximation



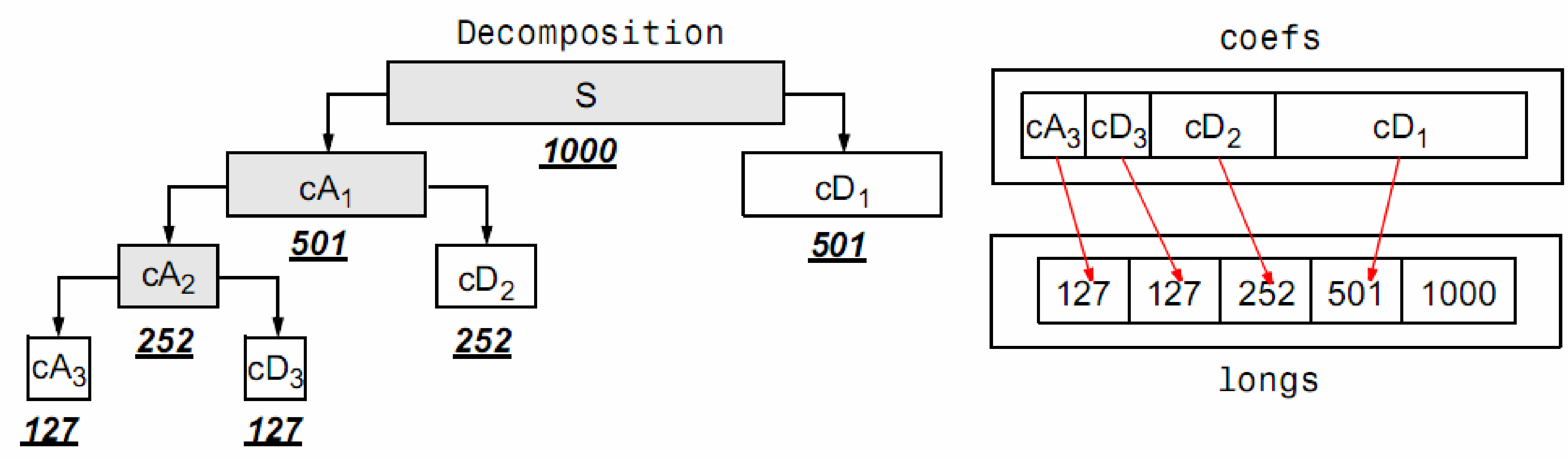
Subbandas


Detail Level 1
Setting a threshold
Detail Level 2
Detail Level 3

Subbandas

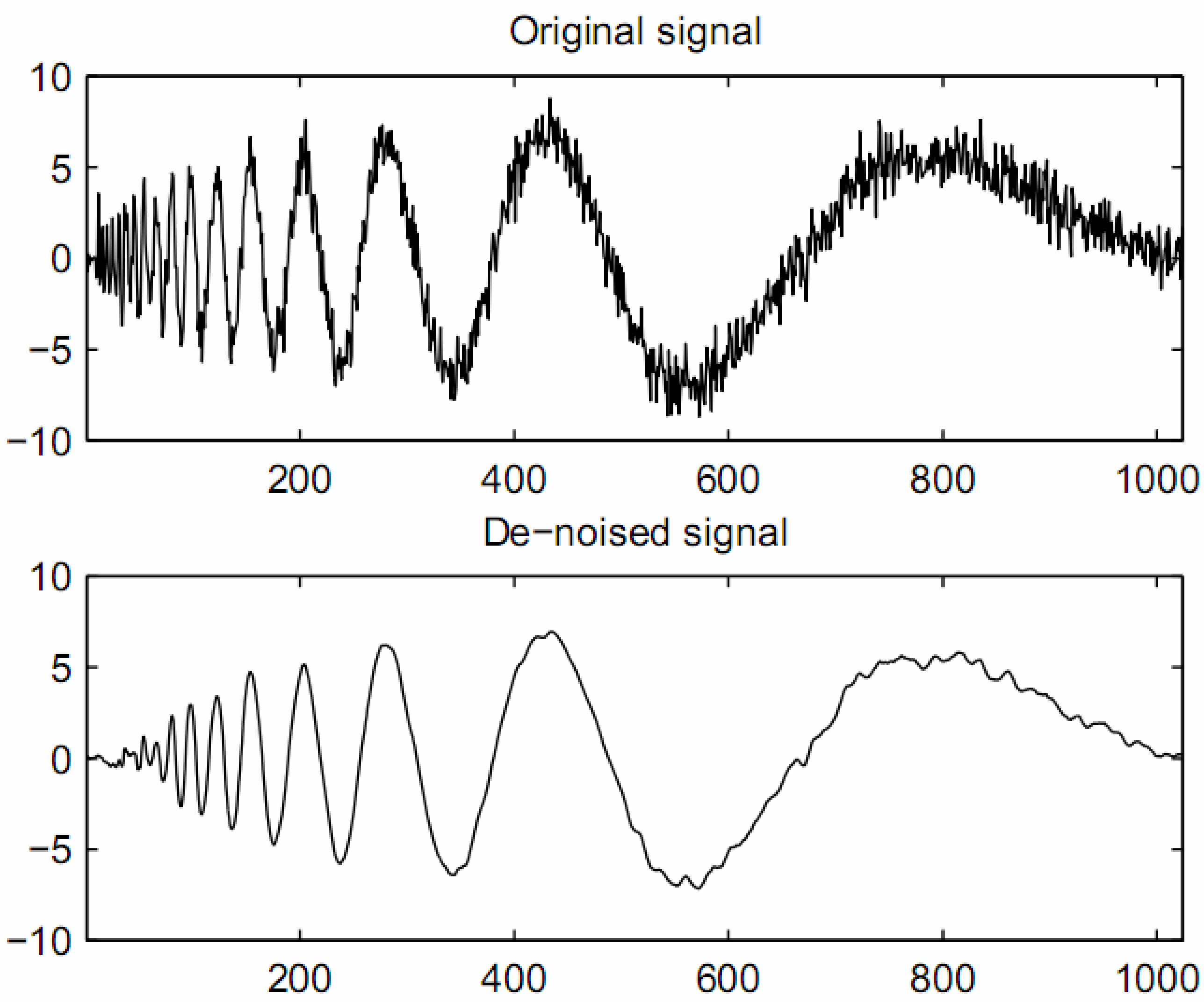

Original
500
400
300
200
200
400
600
800
1000
1200
1400
1600
1800
De-noised
500
400
300
200
200
400
600
800
1000
1200
1400
1600
1800

Subbandas

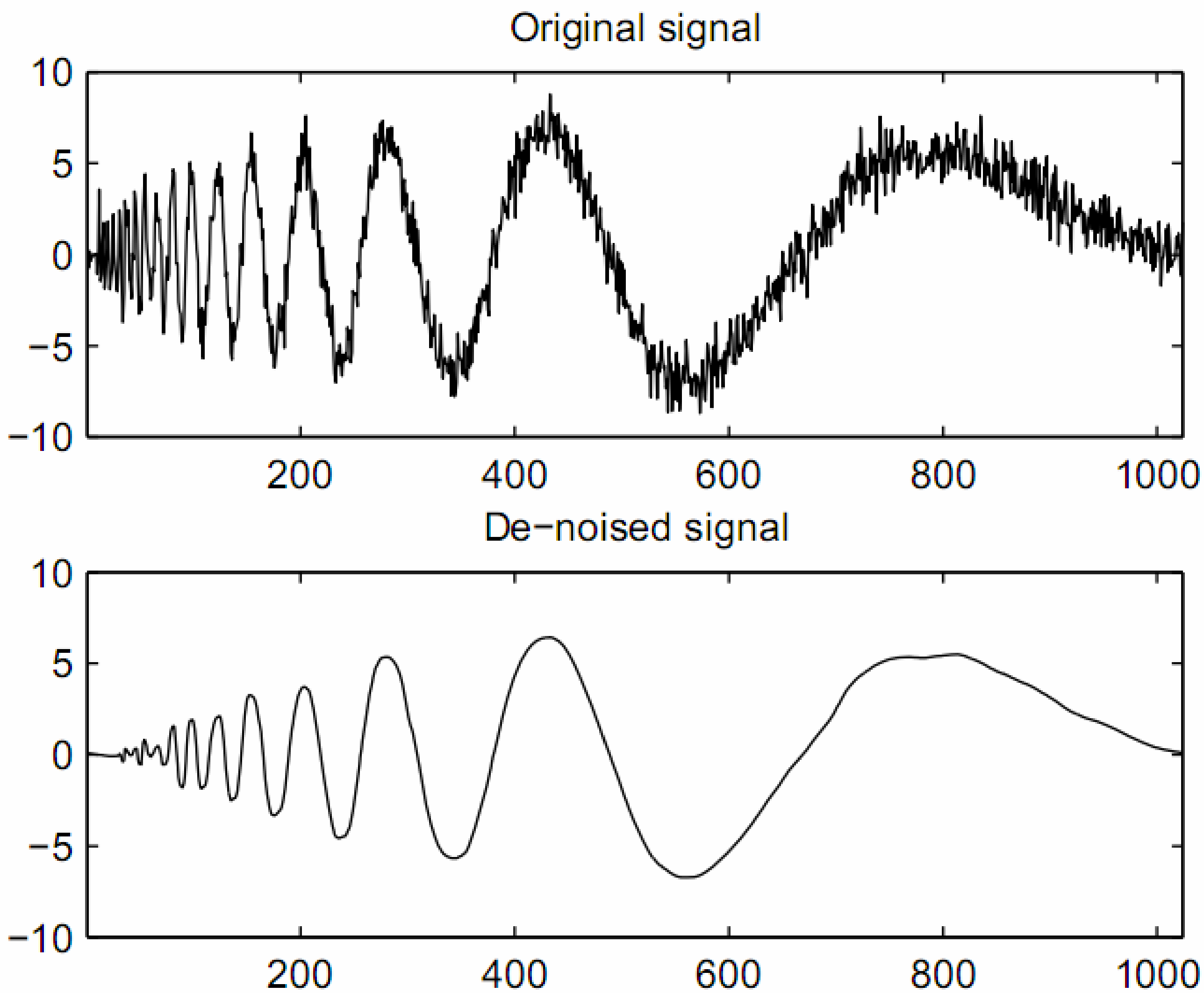


Decomposition
S
1000
cA1
501
cD1
501
cA2
252
cD2
252
cA3
127
cD3
127
coefs
cA3
cD3
cD2
cD1
127
127
252
501
1000
longs

Subbandas

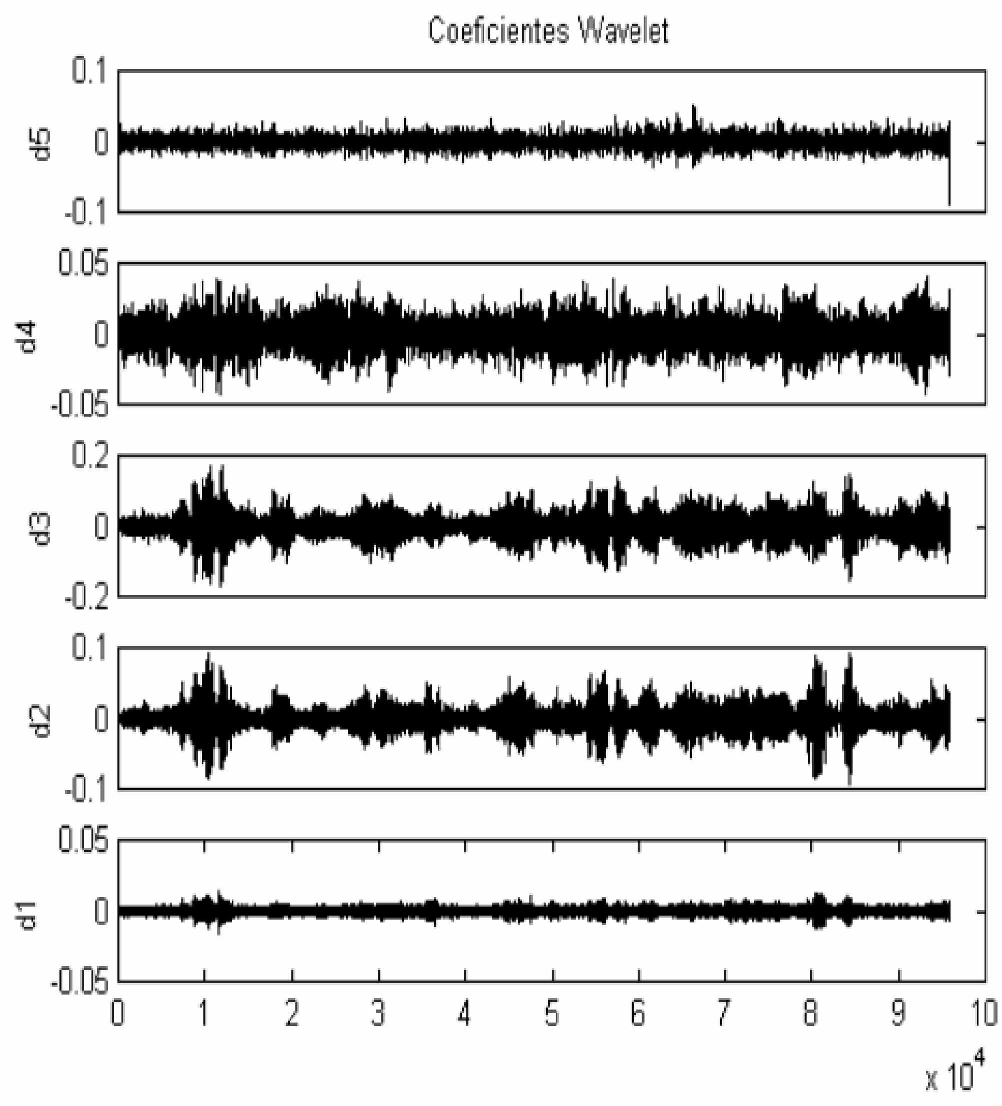

Original signal
10
5
0
−5
−10
200
400
600
800
1000
De−noised signal
10
5
0
−5
−10
200
400
600
800
1000

Subbandas



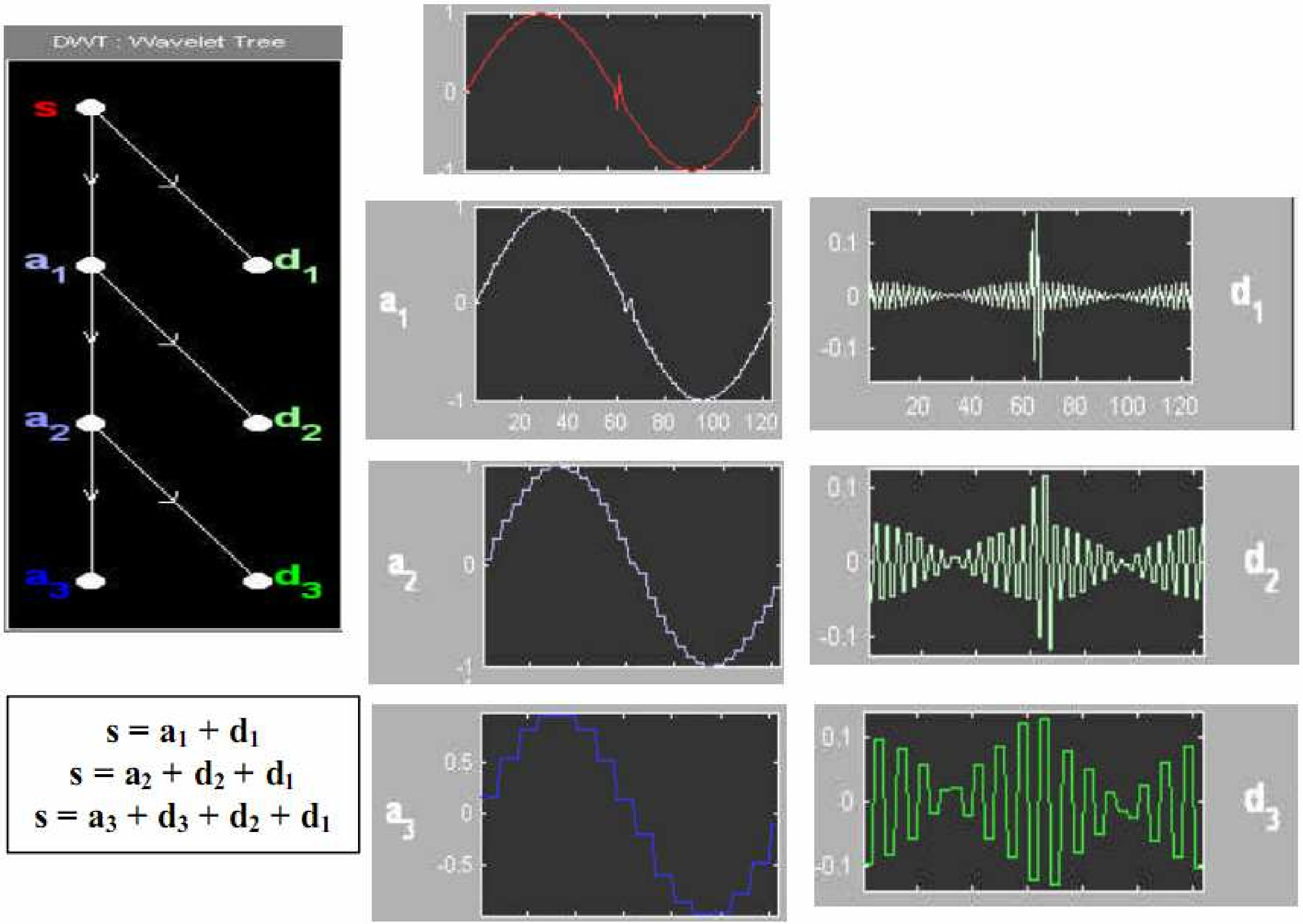
Original signal
10
5
0
−5
−10
200
400
600
800
1000
De−noised signal
10
5
0
−5
−10
200
400
600
800
1000

Subbandas



Coeficientes Wavelet
d5
0.1
0
-0.1
d4
0.05
0
-0.05
d3
0.2
0
-0.2
d2
0.1
0
-0.1
d1
0.05
0
-0.05
0 1 2 3 4 5 6 7 8 9 10
x 10^4

Splitting



$$s = a_1 + d_1$$
$$s = a_2 + d_2 + d_1$$
$$s = a_3 + d_3 + d_2 + d_1$$

Preguntas?

Muchas gracias!