

Atmospheric Absorption of Sound: Analytical Expressions

H. E. BASS AND H.-J. BAUER*

University of Mississippi, Department of Physics and Astronomy, University, Mississippi 38677

AND

L. B. EVANS

Wyle Laboratories, 7800 Governors Drive, West, Huntsville, Alabama 35807

A set of analytical expressions has been developed which will predict the absorption of sound in air at 20°C. The results agree within 3% of previous work in the frequency range of 100 Hz–1 MHz for all humidities. Below 100 Hz accuracy cannot be judged because of lack of knowledge about the vibration–vibration energy transfer between CO₂ and O₂.

LIST OF SYMBOLS

A_i	$=\beta_i/\beta^\infty$, the relaxation strength	V	frequency-dependent sound velocity
a_d	$=[(M_{O_2}M_{N_2})/(X_{O_2}M_{O_2}+X_{N_2}M_{N_2})]^2$ $\times X_{O_2}X_{N_2}$, the molecular diffusion constant	V_0	low-frequency limit of sound velocity
C_p	specific heat at constant pressure	V_∞	high-frequency limit of sound velocity
C_v	specific heat at constant volume	X_i	concentration of molecular species ($i=O_2$, N_2 , etc.)
$C_p^{\text{dyn}}, C_v^{\text{dyn}}$	frequency-dependent complex specific heats	α	total amplitude sound absorption coefficient
C_p^∞, C_v^∞	high-frequency limits of specific heats (value of the specific heats at frequencies above the vibrational relaxation effects)	α_{cl}	amplitude sound absorption coefficient due to classical effects
C'	total vibrational specific heat, $\sum C'_i$	α_{rot}	amplitude sound absorption coefficient due to rotation
C'_i	vibrational specific heat of the i th vibrational mode	α_{vib}	amplitude sound absorption coefficient due to vibration
c	velocity of sound in 1000 ft/sec	α_{relax}	$=\alpha_{rot}+\alpha_{vib}$
D_{12}	diffusion coefficient for O ₂ and N ₂ mixture	β	$=(\beta^0-\beta^\infty)=\sum\beta_i$, the total adiabatic compressibility of vibrational modes
f	sound frequency	β^0	low-frequency limit of the adiabatic compressibility
H	percent relative humidity	β^∞	high-frequency limit of the adiabatic compressibility
h	mole fraction of water vapor	β_i	adiabatic compressibility of the i th vibrational process
j	$=\sqrt{-1}$	γ	$=C_p/C_v$
k	wavenumber	η	coefficient of shear viscosity
M	molecular mass	θ	absolute temperature
P	ambient pressure	κ	coefficient of heat conductivity
P_w	vapor pressure of water		
R	universal gas constant		
S	entropy		
T_i	$=2\pi\tau_i^{ps}$, the modified relaxation time		

ρ density
 τ_i^{pt} isobaric-isothermal relaxation time of the i th vibrational process

τ_i^{ps} isobaric-isentropic relaxation time of the i th vibrational process
 $\omega = 2\pi f$

INTRODUCTION

In a previous paper¹ a calculation of sound absorption in still air was discussed and results were presented in graphical form over the relative humidity range of 0%–100% at 20°C. The calculations were based on fundamental physical principles with as few empirical steps as possible. The method was quite successful and agreement between the predicted values and available experimental data was very good. However, in two oral presentations of those results, desires were expressed for a more usable format than the graphs of absorption in decibels/1000 ft as functions of frequency which were presented. To fulfill these needs a set of tables has been produced which cover an expanded frequency range,² and now in this paper an analytical expression for sound absorption is developed which can be easily programmed for computer computation or solved manually if necessary.

The absorption of sound in air as in all molecular gases is caused by two different mechanisms: classical effects and relaxation effects. The classical effects are a result of the transport processes which occur in a gas, i.e., internal friction, heat conduction, and molecular diffusion. Of these mechanisms the internal friction, or viscosity, and heat conduction are the largest contributors to the classical effects. Numerical inspection of the general equation for binary mixtures³ has shown that the largest of the diffusion terms contributes only 0.5% of the total classical absorption, so only that term has been included in the expression for air.¹ For the frequencies of interest, i.e., well below the collision frequency of the molecules, there is no translational dispersion and the transport properties of the gas obey the ordinary equations of continuum hydrodynamics, so

$$\alpha_{cl} = \frac{2\pi^2 f^2}{\gamma P V_0} \left(\frac{4}{3} \eta + \frac{\gamma - 1}{C_p} \kappa + \gamma a_d D_{12} \rho \right) \quad (1)$$

and

$$V^2 = \frac{P}{\rho} \frac{C_p}{C_v} = \frac{\gamma R \theta}{M} = \frac{1}{\rho \beta} \quad (2)$$

The relaxation effects are a result of an additional loss mechanism inherent in molecular gases. As a sound wave progresses in a molecular gas, part of the compressional energy is stored in the internal degrees of freedom of the molecules. Since this storage involves excitation and deexcitation of internal energy states which occur during collisions, it requires time, and a phase lag between the internal and translational energy modes of the gas results. This relaxation effect can be taken into account by introducing into Eq. 2 either frequency-dependent and complex dynamic heat capaci-

ties C_p^{dyn} and C_v^{dyn} or an adiabatic compressibility β . If the transport processes are neglected and only one internal storage mode for the energy with a single relaxation time is considered, then

$$C_p^{dyn} = C_p^\infty + \frac{C'}{1 + j\omega\tau^{pt}} \quad (3)$$

and the well-known bell-shaped absorption curve for $\alpha\lambda$ vs f would be obtained along with the S -shaped dispersion curve for the speed of sound.⁴

In the presence of internal relaxation, the classical absorption is altered in several ways³:

- The heat capacities appearing in Eq. 1 must be considered as frequency dependent and complex;
- The diffusion of excited molecules supplies an additional and frequency-dependent mechanism for the heat conduction, so that $\kappa = \kappa(\omega)$.
- For infrared-active modes the emission and reabsorption of radiation must be considered along with the diffusion outlined above. (However, it can be shown that there is no influence in the full frequency range in air owing to radiation effects. For high frequencies there is only small coupling into the internal modes due to the inefficient collision processes occurring in air, and for low frequencies the chance that the infrared photons are reabsorbed in essentially the same portion of the acoustic wavelength is very high.)
- If both the transport and relaxation parts of the absorption become very large, then after the above-mentioned alterations have been applied the sound absorption is no longer a simple sum of separate absorption processes.

Fortunately, numerical estimates show that all these complicated interdependences become negligible for the special case of air below a frequency of 1 MHz. This means that the dispersion can be calculated as if the transport processes are absent, i.e.,

$$V^2 = (P/\rho) \operatorname{Re}(C_p^{dyn}/C_v^{dyn}). \quad (4)$$

Also the absorption can be obtained by simple addition of the transport and relaxation contributions

$$\alpha = \alpha_{cl} + \alpha_{relax}, \quad (5)$$

where Eq. 1 for the classical absorption can be expressed as¹

$$\alpha_{cl} = (2\pi^2 f^2 / \gamma P V_0) (1.9) \eta. \quad (6)$$

I. THEORETICAL METHOD

The fact that in air there is more than one storage mode for the internal energy must be taken into account.

These modes are subdivided into: rotational degrees of freedom and vibrational degrees of freedom. The frequency range important for absorption in air is well below the rotational relaxation frequency and the contributions owing to rotation give a term which is linear in frequency for frequencies less than 1 MHz. There is also negligible velocity dispersion in this region as a result of rotation. To calculate the absorption due to rotation the rate constant for dry air can be taken from Greenspan,⁵ assuming that it is not humidity dependent. Unfortunately, there is no experimental verification of that assumption. In fact, very little is known about rotational relaxation in a mixture of rotators; however, it is thought that the error introduced by this assumption is small. When these assumptions are made the absorption due to relaxation effects is

$$\alpha_{\text{relax}} = \alpha_{\text{rot}} + \alpha_{\text{vib}}, \quad (7)$$

where

$$\alpha_{\text{rot}} = (2\pi^2 f^2 / \gamma P V_0) (0.6) \eta$$

at 20°C and the rotational relaxation rate has been taken from Greenspan.

For the vibrational relaxation process a simple extension of the addition technique applied above would lead to

$$C_{\text{vib}}^{\text{dyn}} = \sum \frac{C'_i}{1 + j\omega\tau_i^{ps}} \quad (8)$$

as a representation of the vibrational dynamic heat capacity where the C'_i are the vibrational heat capacities of O_2 , N_2 , CO_2 , and H_2O , and the τ_i^{ps} are closely connected to the lifetimes of the corresponding vibrational quantas. However, this interpretation of Eq. 8 is correct only if the vibrational energy is converted totally into translational energy (V - T process). If, instead, there is a vibration-to-vibration exchange of energy between the molecules (V - V process), then the heat capacities of these modes are coupled via that process. As a result of this coupling, each C'_i contains part of the heat capacities of all the vibrational degrees of freedom which are present in air and each τ_i^{ps} contains lifetimes of all quanta which depend upon the transition probabilities of all V - T and V - V processes. Furthermore, depending upon the number of excited levels involved, there can even be more than four vibrational relaxation times present in Eq. 8. Since at least three close resonances between vibrational states exist in air, namely those between N_2 and CO_2 ; O_2 and CO_2 ; and O_2 and H_2O , these V - V processes certainly must be considered. These processes totally dominate the sound absorption in the audible region, and there are no other means to predict the acoustic behavior except to investigate their influence and to determine their rates.

The method of calculation of the C'_i and τ_i^{ps} proceeds in a standard way. The coupled equations for the V - V processes owing to the excited vibrational levels are

decoupled by means of a main axis transformation.⁶ Each of the decoupled equations furnishes one term in Eq. 8. The significance of these terms is that they form the simplest mathematical representation of the dynamic heat capacities. Now, the dispersion and absorption can be given by

$$\frac{1}{V^2} = \frac{\rho}{P} \operatorname{Re} \left(\frac{C_v^{\infty} + C_{\text{vib}}^{\text{dyn}}}{C_p^{\infty} + C_{\text{vib}}^{\text{dyn}}} \right)$$

and

$$\alpha = \alpha_{\text{class}} + \alpha_{\text{rot}} + \frac{\omega}{2} \left(\frac{V}{V_{\infty}^2} \right) \operatorname{Im} \left(\frac{C_v^{\infty} + C_{\text{vib}}^{\text{dyn}}}{C_p^{\infty} + C_{\text{vib}}^{\text{dyn}}} \right), \quad (9)$$

where each contains a complex rational function of higher order in $(j\omega)$. The separation into real and imaginary parts would yield an even more complicated fraction; however, for the convenience of the user, we have expressed $(C_v^{\infty} + C_{\text{vib}})/(C_p^{\infty} + C_{\text{vib}})$ in partial fractions by using a second main axis transformation. This yields

$$\left(\frac{k}{\omega} \right)^2 = \frac{\rho}{P} \frac{C_v^{\infty}}{C_p^{\infty}} \left(1 + \sum \frac{A_i}{1 + j\omega\tau_i^{ps}} \right) + \text{classical effects} + \text{rotational effects}, \quad (10)$$

where the τ_i^{ps} differ slightly from the τ_i^{pt} , $A_i = \beta_i/\beta^{\infty}$, $\beta^{\infty} = C_v^{\infty}/PC_p^{\infty}$, and

$$\sum A_i = \frac{(\beta^0 - \beta^{\infty})}{\beta^{\infty}} = \sum \frac{\beta_i}{\beta^{\infty}} = \frac{\beta}{\beta^{\infty}}$$

is closely related to the so-called relaxation strength of a single relaxation process $(\beta^0 - \beta^{\infty})/\beta^{\infty} = \beta/\beta^0$. The A_i could be interpreted as telling which part of the vibrationally relaxing adiabatic compressibility β is connected to the particular τ_i^{ps} (called isobaric-adiabatic or isobaric-isentropic relaxation time in Ref. 6). Using this notation, the separation of the imaginary and real parts of the propagation constant k can be accomplished to obtain

$$\frac{V_{\infty}^2}{V^2} = 1 + \sum \frac{A_i}{1 + (\omega\tau_i^{ps})^2} \quad (11)$$

and

$$\alpha = \alpha_{\text{class}} + \alpha_{\text{rot}} + \frac{\omega}{4} \left(\frac{V}{V_{\infty}^2} \right) \sum \frac{A_i \omega \tau_i^{ps}}{1 + (\omega\tau_i^{ps})^2}. \quad (12)$$

II. RESULTS

The 24 binary energy transfer rates defined in Ref. 1 were used to obtain relaxation times and strengths which were in turn inserted into Eqs. 11 and 12 of this paper to obtain the absorption and dispersion of sound in still air. To facilitate these calculations the rate $MOP_1^1(5,2,0)$ was changed to $7 \times 10^5 \text{ sec}^{-1} \cdot \text{atm}^{-1}$. After this change the absorption results were within 3% of those published in Ref. 2 for all frequencies above

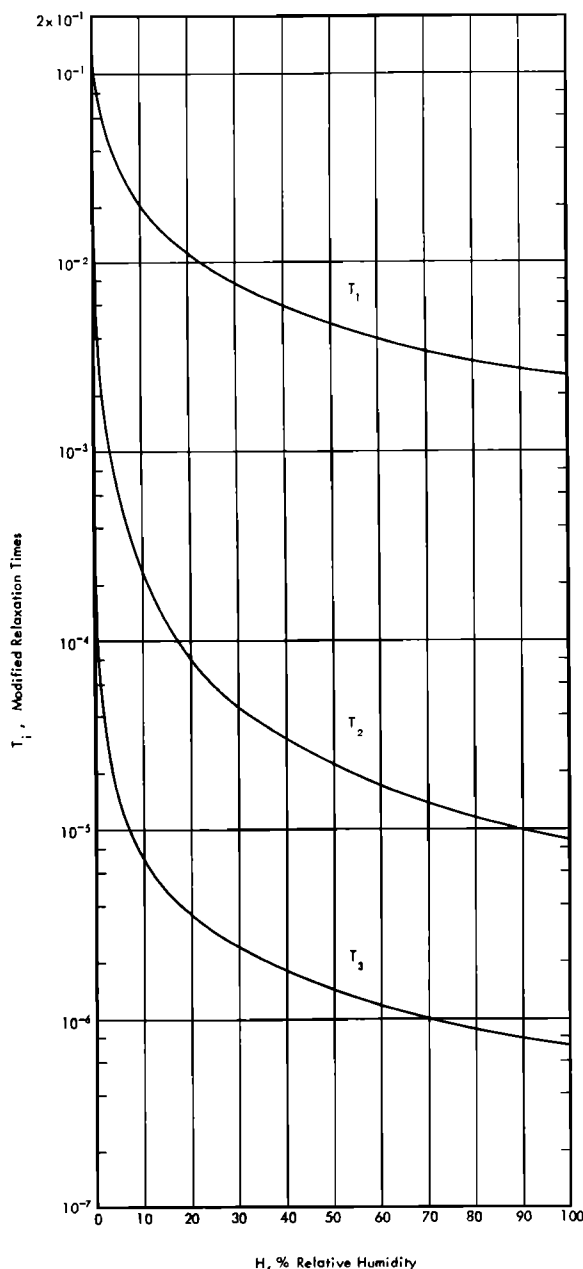


FIG. 1. Modified relaxation times T_1 , T_2 , and T_3 as a function of percent relative humidity.

100 Hz. Below 100 Hz the two differ by as much as 15% for some humidities; however, this is not considered serious since below 100 Hz the values in Ref. 2 may be in error by as much as 50%. The difference in the MOP_1^1 (5,2,0) rate will be discussed in a subsequent publication. The eigenvalue analysis using the 24 rates yielded several relaxation times; however, only three of these had an associated relaxation heat or strength large enough to influence the absorption. As a result, instead of requiring 24 rates to describe the vibrational relaxation absorption as in Ref. 1, only three relaxation times

TABLE I. Regression coefficients for Eq. 14.

	b_0	b_1	b_2	b_3
T_1	-2.357009	-0.5423307	-0.05253065	-0.0006430596
T_2	-5.388992	-1.231140	-0.04769421	0.004000068
T_3	-9.780594	-0.8459473	-0.03399849	0.002532959
A_1	-8.974335	-0.003204346	-0.0004720688	-0.0001525879
A_2	-7.397324	0.006179810	0.0001125336	-0.00001049042
A_3	-10.40355	0.01698303	-0.002468109	-0.0002794266

and strengths are needed at each humidity. In addition, the strengths are very nearly independent of humidity. For 20°C, the relaxation times and strengths are presented in Figs. 1 and 2. It is not possible to associate a relaxation time to a specific component of the atmosphere; however, examination of the contributions to the adiabatic compressibility shows that τ_i^{ps} is closely related to the relaxation of N_2 and τ_2^{ps} to O_2 . These relaxation times should be considered only as useful computational parameters, and no attempt should be made to ascribe fundamental physical significance to them. The fundamental quantities are the binary rate constants.

Using Eq. 12, adding the various absorption terms, and combining constants gives the results in decibels/

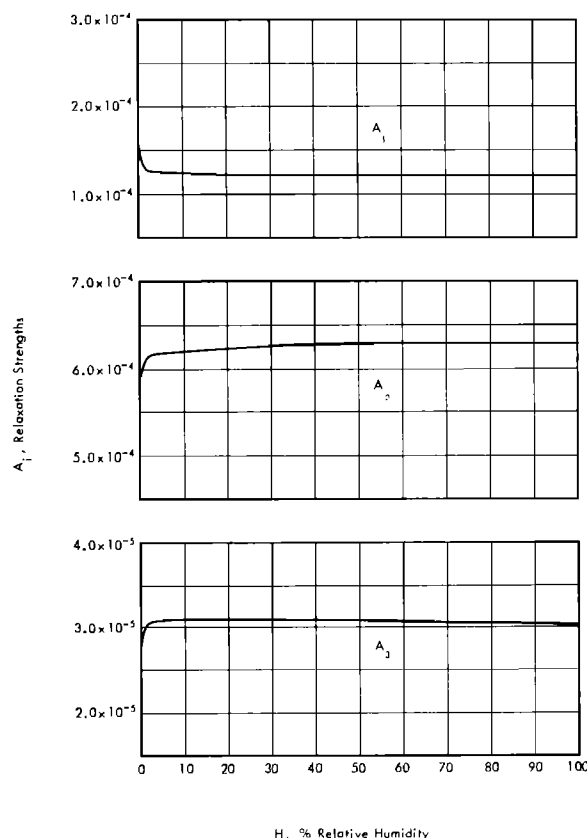


FIG. 2. Derived relaxation strengths A_1 , A_2 , and A_3 as a function of percent relative humidity.

1000 ft.

$$\alpha = 27.26 \left\{ \sum_{i=1}^4 \frac{f T_i A_i}{1 + f^2 T_i^2} + 1.525 \times 10^{-9} f \right\} \frac{f}{c} \quad (13)$$

The values of (T_1, T_2, T_3) and (A_1, A_2, A_3) can be taken from Figs. 1 and 2, respectively, or calculated using

$$\log_e(T_i \text{ or } A_i) = b_0 + b_1(\log_e H) + b_2(\log_e H)^2 + b_3(\log_e H)^3, \quad (14)$$

where the regression coefficients are given in Table I. A_4 is defined to be unity, $T_4 = 6.5 \times 10^{-10}$ sec, and c is calculated by

$$c = \frac{1}{30480} \left[\left(\frac{3.5 + 5h}{2.5 + 5h} \right) \frac{8.3166 \times 10^7 \theta}{28.966 - 10.95h} \right]^{\frac{1}{2}}, \quad (15)$$

where $h = P_w H / 100$.

For computational convenience c may be set equal to 1.13 if an empirically derived $T_4 = 5 \times 10^{-10}$ sec is used. This will also reproduce the tables of Ref. 2 within 3%. During any of these calculations it must be remembered that the frequency divided by the pressure, not the actual acoustic frequency, is the important variable. Hence, a decrease in pressure is equivalent to an increase in frequency. For calculations at pressures other than one standard atmosphere, f/p should be inserted at all places where f presently appears except outside the brackets in Eq. 13.

III. SUMMARY

A set of equations has been developed which will accurately predict the absorption of sound in still air at 20°C over the frequency range of 12 Hz–1 MHz and

over all humidities. The method of calculation presented was devised primarily as a computational convenience. The accuracy of the calculations is still limited, as was noted in Ref. 1, by limited knowledge of the binary energy transfer rates, but not by the computational method. Specifically, this lack of knowledge limits the accuracy of low-frequency (12–60 Hz) calculations which might be off as much as a factor of 2 due to the lack of knowledge about CO_2/O_2 V–V energy transfer. The ability to apply this formalism at other temperatures is also severely limited by lack of knowledge about the temperature dependence of the CO_2/O_2 rates and those rates involving H_2O as a collision partner. Nevertheless, the problem of sound absorption in still air has been simplified while maintaining a correct physical formalism. The temperature dependence and low-frequency problems will be solved as soon as binary rate data are available.

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* NSF Senior Foreign Scientist on leave from Universität Stuttgart, Germany.

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