Improvement of Acoustic-Feedback Stability by Frequency Shifting

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(Received 22 May 1964)

The tendency of public-address systems to become unstable, when operated in reverberant spaces, can be reduced by shifting all frequency components of the audio signal by about 5 cps. Frequency shifts of this magnitude will raise the gain at which a public-address system remains stable by about 10 dB. However, beating the direct-sound signal and the frequency-shifted signals limits the usable additional gain to about 6 dB. Electronic devices for effecting frequency shifts of the desired magnitude and stability have been described previously and are now available commercially. In this paper, a theory of the stability of public-address systems with and without frequency shifting is presented and compared with experimental results obtained in rooms of various dimensions. Good agreement is found between theory and experiment. In addition, frequency responses of rooms are simulated by a Monte Carlo method on a digital computer and the permissible gains for stable operations of public-address systems are evaluated on the computer. The Monte Carlo results fall within 0.6 dB of the theory.

INTRODUCTION

INSTABILITY in public-address systems occurs because some of the acoustic energy radiated by the loudspeaker(s) is fed back into the microphone. If the feedback signal is in phase and greater in amplitude than the original signal at one or more frequencies, sustained oscillations will occur. Conventional methods of minimizing feedback include the use of directional microphones and loudspeakers and their proper placement. If the loudspeaker(s) or other components of the public-address system exhibit resonances, substantial gains in stability may be realized by equalizing these resonances with electrical networks. Further improvements may be obtained in some cases by limiting the frequency band of the public-address system to those frequencies that are essential for speech intelligibility. This method is particularly successful if the room has a rising response at low frequencies. In this case, suppression of the low frequencies actually benefits speech perception.

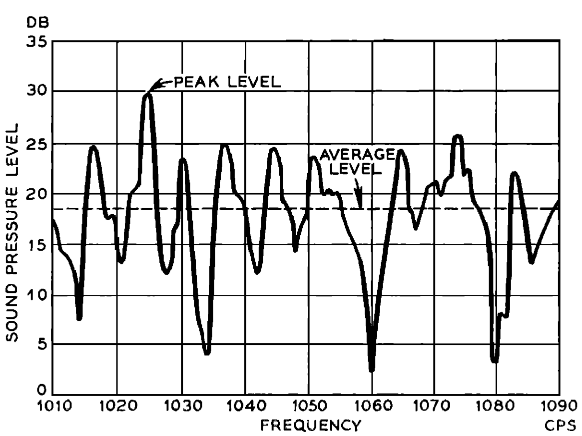

FIG. 1. Section of frequency response of a large room between two points, with negligible direct-sound transmission. Peak level exceeds average level by more than 10 dB. Spacing between peaks and adjacent valley averages about 5 cps.

After eliminating feedback due to *direct* sound transmission between loudspeaker and microphone and due to resonances of audio components or excessive bass response of the room, there remains one major cause of instability in reverberant spaces: feedback via reflections from the walls, the ceiling, and other sound-reflecting obstacles—in other words: feedback through the reverberant sound field.

In this paper, the effect of the reverberant sound field on feedback stability is discussed in terms of the steady-state frequency response of the room.

I. FREQUENCY-SHIFTING IDEA

Figure 1 shows a typical section of the frequency response of a room between two locations with neg-

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ligible direct-sound transmission. Major peaks occur at an average frequency spacing of about 10 cps. The average fluctuation between peaks and valleys is about 10 dB and extreme variations cover a range of 30 dB or more. The highest peak exceeds the average level by more than 10 dB.

According to Nyquist's absolute-stability criterion[1] for feedback loops, a public-address system operating over a room frequency response, such as shown in Fig. 1, is unstable if, for any frequency, the phase shift around the feedback loop is a multiple of 2π and the amplitude gain exceeds unity (0 dB). Thus, the peaks in the frequency response of a room are particularly apt to give rise to public-address-system instability.

A second look at Fig. 1 suggests a possible method of interrupting the continued feedback at a frequency of high gain without reducing the gain: by inserting an electronic frequency shifter between the microphone and the loudspeaker, no signal component will travel around the feedback loop more than once at the same frequency. Thus, buildup at a single frequency is avoided.[2]

Figure 1 also suggests an optimum amount for the frequency shift: namely, one corresponds to the average spacing between peaks and adjacent valleys, or about 5 cps in the example. In this manner, any excessive gain of a signal component at a response peak is quickly compensated by the low gain that it encounters in the adjacent valley of the response during its next trip around the feedback loop. Fortunately, frequency shifts of this mangitude are hardly perceptible for speech[3] and many types of music, too.

In the following sections, it is shown that the stability of a public-address system employing frequency shifting in the feedback loop is given by the *average* (decibel) level of the frequency response.

In a farther section, a formula for the stability of a public-address system *without* frequency shifting is derived on the basis of Nyquist's stability criterion for linear passive systems. The difference of the stability levels obtained with and without frequency shifting is the additional stable gain due to frequency shifting. This theoretical result (approximately 10 dB of additional stable gain for most rooms and public-address systems) is compared with results obtained by simulating room behavior on the digital computer according to a Monte Carlo method.

In another section, experimental results obtained in rooms of various sizes are presented. Measurements with variable-frequency shifts confirm the desirability of frequency shifts of about 5 cps for most rooms.

In the final section, two early applications of the frequency-shifting idea are described.

II. FEEDBACK STABILITY OF PUBLIC-ADDRESS SYSTEMS WITH FREQUENCY SHIFTING

It may seem preposterous to propose to theorize about the stability of a feedback loop closed through a reverberant room, with many thousands of normal modes in the audio-frequency band. However, it is exactly this high degree of complexity of sound waves traveling in rooms that makes a theory, a statistical one to be sure, possible.

It appears that the stability of a public-address system with frequency shifting is much easier to compute than without frequency shifting. Therefore, this case is considered first.

Figure 2 shows a block diagram of a simple public-address system, including a frequency shifter that shifts all frequency components of its input signal by a constant amount Δf. It is also assumed that the frequency shifter has unity gain in the audiofrequency band and that all other audio components have a flat frequency response in the audiofrequency band. Thus, the "open-loop" gain (the gain around the feedback loop when it is opened at any point) is given, apart from a constant factor that we shall assume to be unity, by the frequency response of the room.

A continuous sinusoidal signal of frequency f_1 applied to any point in the feedback loop will have its frequency shifted to $f_1+\Delta f$ and its power increased (or decreased) by a factor $|g(f_1)|^2$, where $g(f)$ is the complex open-loop gain. After N trips around the feedback loop, the signal power has been multiplied by

$$M_N=|g(f_1)|^2\cdot|g(f_1+\Delta f)|^2\cdot|g(f_1+2\Delta f)|^2\cdots \times|g(f_1+(N-1)\Delta f|^2. \quad (1)$$

If the public-address system is to be stable, M_N must converge to zero as N goes to infinity.

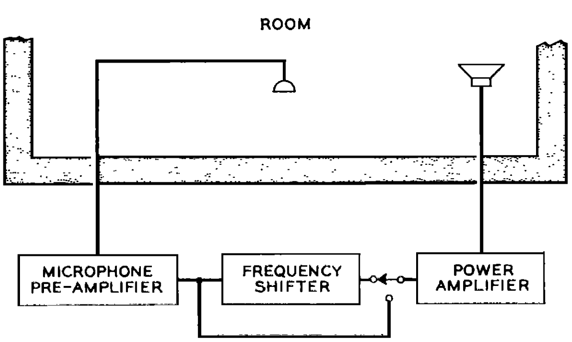

FIG. 2. Insertion of frequency shifter in public-address system feedback loop. Provision to bypass shifter (which has unity gain) for comparison measurements.

[1] H. Nyquist, Bell System Tech. J. **11**, 126–147 (1932).

[2] M. R. Schroeder, "Improvement of Acoustics Feedback Stability in Public Address Systems," in *Proceedings of the Third International Congress on Acoustics, Stuttgart, 1959*, L. Cremer, Ed. (Elsevier Publishing Co., Amsterdam, 1961), Vol. 2, pp. 771–775.

[3] H. Fletcher, *Speech and Hearing in Communications* (D. Van Nostrand Co., Inc., New York, 1953), p. 352.

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Expressing gains in decibels, $l(f)\equiv 10\log|g(f)|^2$, the above stability criterion requires that

$$10\log M_N = l(f)_1 + l(f_1+\Delta f) + l(f_1+2\Delta f) + \cdots + l(f_1+[N-1]\Delta f) \quad (2)$$

approach minus infinity as N goes to infinity.

For large N, one may write in good approximation

$$10\log M_N = N\cdot\bar{l}, \quad (3)$$

where $\bar{l}$ is the average level of the room's frequency response.

It follows from Eq. (3) that stability is guaranteed if

$$\bar{l}<0, \quad (4)$$

while instability occurs for $\bar{l}\geq 0$.

In the remainder of this section, the value of $\bar{l}$ is computed for a given average power gain around the feedback loop.

It has been shown previously[4] that the squared modulus of the transmission function between two points in a room (with negligible direct-sound transmission and for frequencies where substantial overlapping of the normal modes occur) is exponentially distributed. Assuming flat frequency responses of the audio components, the open-loop power gain $|g(f)|^2$ is also exponentially distributed:

$$p(|g|^2) = G^{-1}\exp(-G^{-1}|g|^2), \quad (5)$$

where G is the frequency-average power gain around the feedback loop. From Eq. (5), one obtains the distribution of the gain in decibels, $l=10\log|g|^2$, by a simple transformation of the variable:

$$p(l) = aG^{-1}\exp[-G^{-1}\exp(al)+al], \quad (6)$$

where $a=(10\log e)^{-1}=0.2303$. The mean of this distribution is[5]

$$\bar{l} = 10\log G - C/a, \quad (7)$$

where C is Euler's constant $(0.5772\cdots)$.

By combining Eq. (7) and the stability criterion (4), one obtains the following critical value for the average power gain of the public-address system:

$$10\log G_0 = C/a = 2.5\text{ dB}. \quad (8)$$

Equation (8) states that a public-address system with frequency shifting will be stable if the average power gain around the feedback loop is less than 2.5 dB, which, as Eq. (7) tells us, corresponds to 0 dB average logarithmic gain.

In the Sec. III, a corresponding stability relation for the average power gain of a public-address system *not* benefiting from frequency shifting is derived. By comparing the two results, one will know how much additional gain the frequency shifting permits without incurring instability.

III. FEEDBACK STABILITY OF PUBLIC-ADDRESS SYSTEMS WITHOUT FREQUENCY SHIFTING

Figure 3 shows a typical section of the complex open-loop gain $g(f)$ of a public-address system in a room. Whether or not its real part $r(f)$ will stay left of the point ± 1 cannot be predicted with precision. However, a fairly accurate forecast can be made of the *probability* that the system will be stable for given amplifier gains and room conditions. This limitation is analogous to those encountered in statistical mechanics with which sound propagation in irregular rooms has much in common.

In order to proceed with a statistical analysis of the stability problem, one has to know the distribution of the real part of the open-loop gain and how often $g(f)$ crosses the real axis. In an earlier paper,[4] it was shown that the real (and imaginary) part $r(f)$ of the transmission function of a room is Gaussian-distributed. This is a consequence of the central-limit theorem of probability theory applied to the overlapping normal modes excited simultaneously at every frequency. Thus, the probability that $r(f)$ is smaller than a given value x at any one frequency is

$$P_1(x) = (\pi G)^{-\frac{1}{2}}\int_{-\infty}^{x}\exp(-G^{-1}r^2)dr. \quad (9)$$

where G is, as before, the average power gain of the open feedback loop. The expected number of crossings of $g(f)$ with the real axis in a bandwidth W is[6]

$$n = WT_{60}/4.9, \quad (10)$$

where T_{60} is the reverberation time of the room and W is the bandwidth of the public-address system over which its frequency response is essentially flat.

If, at all of the crossings of $g(f)$ with the real axis, its real part $r(f)$ is smaller than $+1$, the system will be stable. For a given average power gain, the probability of stability can be calculated. It is the probability that the largest value of $r(f)$ is less than $+1$.

It appears that it is impossible to derive a closed expression for the extremal distribution of $r(f)$ unless one assumes that the values of $r(f)$ are independent. Fortunately, this is approximately true. It can be shown that the normalized frequency autocorrelation function of $r(f)$ is[7]

$$\varphi_r(f) = [1+(fT_{60}/2.2)^2]^{-1}. \quad (11)$$

Since, according to Eq. (10), the average spacing between successive crossings of $g(f)$ with the real axis is

[4] M. R. Schroeder, Acustica 4, Beih. 2, 594–600 (1954).

[5] H. Cramer, *Mathematical Methods of Statistics* (Princeton University Press, Princeton, N. J., 1946), pp. 374–376.

[6] M. R. Schroeder, "Measurement of Reverberation Time by Counting Phase Coincidences," in *Proceedings of the Third International Congress on Acoustics, Stuttgart, 1959*, L. Cremer, Ed. (Elsevier Publishing Co., Amsterdam, 1961), Vol. 2, p. 987.

[7] M. R. Schroeder, J. Acoust. Soc. Am. 34, 1819–1823 (1962).

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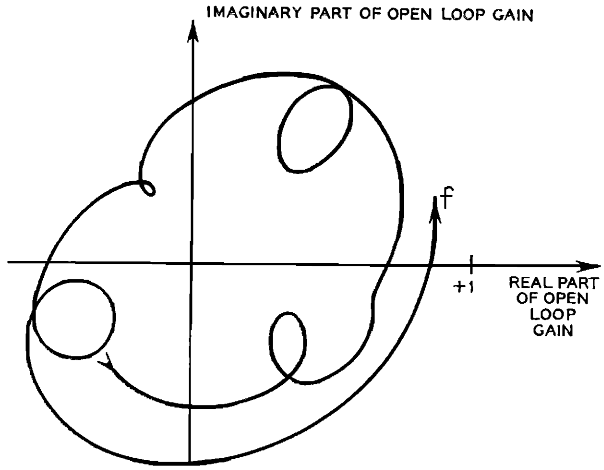

FIG. 3. Nyquist diagram (locus of complex open-loop gain) of public-address systems and room. For stability, point +1 on the real axis must not be encircled by the locus of the complex open-loop gain.

$4.9/T_{60}$, the correlation of two samples of $r(f)$ taken at this average frequency difference is less than $\frac{1}{5}$. Thus, we proceed as if the values of $r(f)$ at crossings of $g(f)$ with the real axis were independent.

The probability that *each* of n independent values of $r(f)$ is smaller than x is given by

$$P_n(x)=P_1{}^n(x), \tag{12}$$

where $P_1(x)$ is the probability that a single value of $r(f)$ is smaller than x. By differentiating with respect to x, one obtains the probability density distribution of the largest of n independent samples of $r(f)$:

$$p_n(x)=nP_1{}^{n-1}(x)p_1(x), \tag{13}$$

where, with Eq. (9),

$$p_1(x)=(\pi G)^{-\frac{1}{2}}\exp(-G^{-1}x^2). \tag{14}$$

The most probable value of x, i.e., the mode x_m of distribution (13), can be found by equating the logarithmic derivative of $p_n(x)$ with zero. Thus,

$$(n-1)p_1(x_m)/P_1(x_m)+p_1'(x_m)/p_1(x_m)=0. \tag{15}$$

For large n, x_m will be considerably larger than $G^{\frac{1}{2}}$. Thus, $P_1(x_m)$ will be approximately one. This simplification and replacing $n-1$ by n yields

$$np_1{}^2(x_m)=-p_1'(x_m). \tag{16}$$

An *implicit* solution of this equation for $x_m{}^2$ is

$$x_m{}^2=\tfrac{1}{2}G\ln(n^2G/\pi)-\tfrac{1}{2}G\ln(4x_m{}^2), \tag{17}$$

where $\ln(\;)$ denotes logarithm to the base $e=2.718\cdots$. By inserting $x_m{}^2$ from the left side into the right side of Eq. (17), one obtains

$$x_m{}^2=\tfrac{1}{2}G\ln(n^2/2\pi)-\tfrac{1}{2}G\ln[\ln(n^2G/\pi)-2G\ln(4x_m{}^2)]. \tag{18}$$

An exact solution for $x_m{}^2$ for $n=1000$, corresponding to $WT_{60}=4900$, is $x_m{}^2=4.85G$. Inserting this value for $x_m{}^2$ into Eq. (18) yields

$$x_m{}^2=\tfrac{1}{2}G\ln(n^2/2\pi)-\tfrac{1}{2}G\ln[\ln(n^2/61.5)],$$

or

$$x_m{}^2=\tfrac{1}{2}G\ln(n^2/2\pi)-\tfrac{1}{2}G\ln[9.7+\ln(n^2/10^6)].$$

Here, $\ln(n^2/10^6)$ is smaller than one-half of 9.7 in the range of interest ($100<10\,000$ corresponding to $490<WT_{60}<49\,000$). Thus, the second logarithm may be expanded into a Taylor series. Breaking off after one term gives $x_m{}^2=\frac{1}{2}G\ln(n^2/61)-\frac{1}{2}G(9.7)^{-1}\ln(n^2/10^6)$. By writing the factor $-(9.7)^{-1}$ as an exponent inside the second logarithm and then combining the two logarithms, one obtains the following simple explicit form for $x_m{}^2$:

$$x_m{}^2=0.9G\ln(n/4.5)=0.9G\ln(WT_{60}/22). \tag{19}$$

This solution for $x_m{}^2$ deviates from the exact one, obtained by iterating Eq. (17), by less than 1% (0.04 dB) in the range $500<WT_{60}<50\,000$.

According to Gumbel,[8] the *median* of the largest of n independent Gaussian samples for $n>100$ exceeds the mode by about 0.1 times the standard deivation or $0.1(\frac{1}{2}G)^{\frac{1}{2}}$. Thus, the median of the largest of n values of $r(f)$ is

$$x_{\text{med}}=G^{\frac{1}{2}}[0.9\ln(WT_{60}/22)+0.07]^{\frac{1}{2}}. \tag{20}$$

For $x_{\text{med}}=+1$, the probability that the public-address system will be stable (or unstable) is $\frac{1}{2}$. The corresponding average power gain is $G_{\text{med}}=0.96/\ln(WT_{60}/22)=0.417/\log(WT_{60}/22)$, or, in decibels,

$$10\log G_{\text{med}}=-10\log[\log(WT_{60}/22)]-3.8\text{ dB}. \tag{21}$$

Subtracting this result from the critical gain G_0 obtained with frequency shifting [see Eq. (8)], we obtain the additional stable gain in decibels due to frequency shifting:

$$\Delta l=10\log[\log(WT_{60}/22)]+6.3\text{ dB},\quad (WT_{60}>500). \tag{22}$$

Because the extremal distribution is relatively narrow for large n, a gain increase of a few decibels above the value given by Eq. (22) will result in almost certain instability; a gain decrease of a few decibels will assure almost certain stability.

Because of the double logarithm in Eq. (22), the additional stable gain does not depend critically on the bandwidth W of the public-address system and the reverberation time T_{60} of the room. Examples: for $W=5000$ cps and $T_{60}=1$ sec, $\Delta l=10.0$ dB. For $W=10\,000$ cps and $T=1.6$ sec, $\Delta l=10.9$ dB.

In two previous papers[2,9] the stability of a public-address system was derived by considering only the

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[8] E. J. Gumbel, *Statistics of Extremes* (Columbia University Press, New York, 1958), p. 134, Graph 4.2.2.(4).

[9] M. R. Schroeder, Radio Electron. **31**, 40–42 (1960).

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magnitude of the open-loop gain, neglecting its phase. As a result, the stability of public-address systems without frequency shifting was underestimated by about 2 dB. The predictions for the additional stable gain were accordingly about 2 dB too high (for example, 11.8 instead of 10.0 dB for $WT_{60}=5000$).

IV. COMPARISON WITH MONTE CARLO METHOD

The above predictions for the additional stable gain owing to frequency shifting are confirmed by measurements in many different rooms (see Sec. V). However, in view of the simplifying assumptions that went into the derivation of Eq. (22), it was considered desirable to check it by an independent mathematical method. For the kind of problem at hand, the Monte Carlo method is particularly suited.

In a Monte Carlo computation, the statistical properties of the physical situation are simulated (for instance, on a digital computer) and the unknown quantities are observed on the simulated data. By making the computation long enough, any desired accuracy, within, the limitations of the simulated model, can be achieved.

Accordingly, two Gaussian processes with the proper spectral densities and covariance, simulating the real and imaginary parts of the complex open-loop gain through a room, were generated on an IBM-7090 computer. (Further details of how this was accomplished can be found in an earlier paper.[10]) The values of the real part, for the vanishing imaginary part, were stored in the memory and the largest of these values in a bandwidth $W=4000/T_{60}$ was printed out.

The individual values in decibels, relative to the average level, are recorded in Table I in decreasing rank order for 20 computer runs. The median value is 10.5 dB, which is in fair agreement with 9.8 dB, the value given by Eq. (22) for $WT_{60}=4000$.

As can be seen the additional stable gain for these 20 cases varies between 8.9 and 11.8 dB or about ± 1.5 dB about the median value. The same uncertainty applies

TABLE I. Twenty values of the additional stable gain obtained by Monte Carlo computation.

Rank order	Gain (dB)	Rank order	Gain (dB)
1	11.8	11	10.4
2	11.8	12	9.8
3	11.7	13	9.8
4	11.1	14	9.7
5	10.8	15	9.6
6	10.7	16	9.6
7	10.7	17	9.5
8	10.6	18	9.5
9	10.6	19	9.2
10	10.6	20	8.9

[10] M. R. Schroeder and K. H. Kuttruff, J. Acoust. Soc. Am. 34, 76–80 (1962).

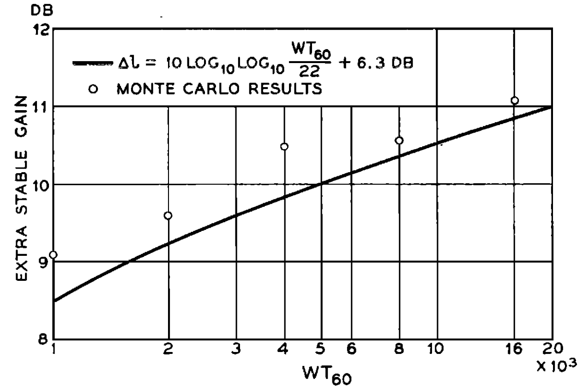

FIG. 4. Comparison of Monte Carlo results (small circles) with theoretical formula (solid curve) for the additional stable gain due to frequency shifting.

also to the theoretical Eq. (22). As was pointed out above, the prediction of the additional stable gain is probabilistic; Eq. (22) gives the median value from which the actual value in any physical situation may deviate by several decibels in either direction.

More Monte Carlo results were obtained for other values of WT_{60}. Their medians are shown as circles in Fig. 4, together with the theoretical curve for Δl according to Eq. (22). The Monte Carlo results deviate by less than 0.6 dB from the theory.

V. MEASUREMENTS OF THE ADDITIONAL STABLE GAIN AS A FUNCTION OF FREQUENCY SHIFT

Measurements of the additional stable gain as a function of the magnitude and sign of the frequency shift have been made in a large auditorium (Fig. 5), a medium-size room (Fig. 6), and, for curiosity, in a small soundproof booth (Fig. 7).

From a theoretical point of view, the stability should not depend on the magnitude of the frequency shift. However, for very small shifts, the amplifiers overload

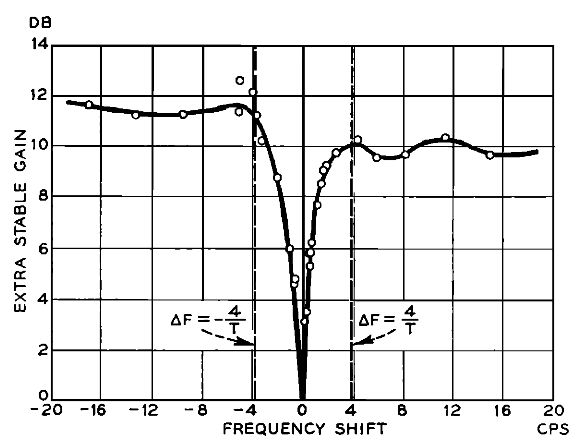

FIG. 5. Additional stable gain as a function of the frequency shift in a large auditorium (volume 2500 m^3, average reverberation time $T_{60}=1$ sec).

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before stability is reached. An "optimum" frequency shift, as pointed out above, is equal to the average spacing between maxima and minima of the room's frequency response or about $4/T_{60}$. This is borne out by the measurements shown in Figs. 5–7, where the frequency shifts corresponding to $\pm 4/T_{60}$ are indicated by dashed vertical lines. Larger shifts do not give any significant improvement and, in some cases, are even less effective. There is also no significant *consistent* difference between positive and negative shifts. The somewhat larger gain for negative shifts seen in Fig. 5 may be explained by the presence of a deep valley in the frequency response just *below* the major peak. For slightly different physical conditions in the room, this asymmetry would vanish and possibly favor positive shifts.

According to Eq. (22), the additional stable gain of public-address systems with a bandwidth of 8000 cps is 10.4 dB for the auditorium, 10.1 dB for the small room, and 9.5 dB for the booth. These predictions are in good agreement with the measured results, except for the small booth, where somewhat higher additional stable gains are measured. [It should be mentioned that the booth falls outside the range of validity of the statistical theory. The lowest frequency for which the statistical theory is valid,[4] $f=4000\ (T_{60}/V)^{\frac{1}{2}}$ is about 600 cps in the case of the booth. Since the instability occurred at a frequency below this limit, i.e., in the frequency range of isolated normal modes, somewhat higher gains must be expected because the frequency response is more irregular in this frequency range.[11]]

VI. PRACTICALLY REALIZABLE, ADDITIONAL STABLE GAIN

The additional stable gain, for which a theoretical formula was derived above, is a mathematical quantity, defined as the difference in gains for 'stable" operation with and without frequency shifting. From the viewpoint of subjective acceptability, the limitations on the gain are as follows.

1. *Without* frequency shifting. In order to avoid audible ringing effects, the permissible gain is about 2 dB below the gain at which instability (singing) occurs.

2. *With* frequency shifting. The subjectively acceptable gain is limited to about 6 dB below the corresponding instability level because of audible beating effects of the multiply shifted signals with each other. Thus, using subjective acceptability as a criterion, the net realizable additional gain due to frequency shifting is about 10 dB−6 dB+2 dB=6 dB.

The two stability levels and the regions in which subjectively objectionable effects occur are illustrated in Fig. 8. As can be seen, the usable additional gain due to frequency shifting is about 4 dB less than the differ-

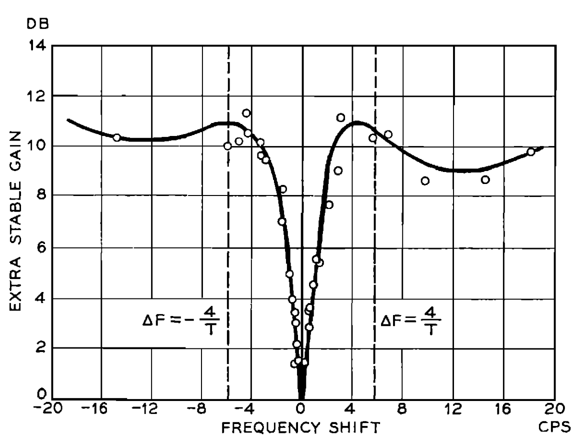

FIG. 6. Additional stable gain in a medium-size room (volume 100 m³, average reverberation time $T_{60}=0.7$ sec).

[11] R. H. Bolt and R. W. Roop, J. Acoust. Soc. Am. **22**, 280–289 (1950).

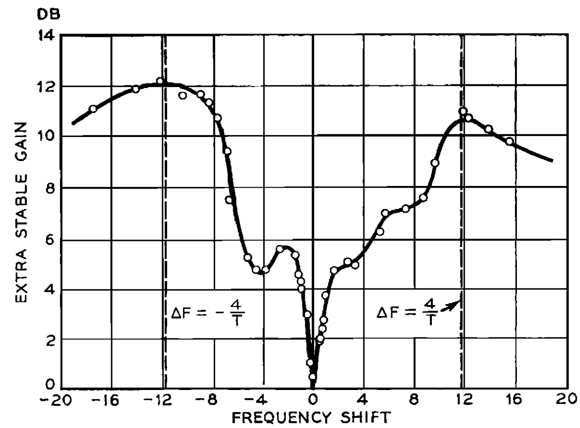

FIG. 7. Additional stable gain in soundproof booth (volume 14 m³, reverberation time $T_{60}=0.34$ sec).

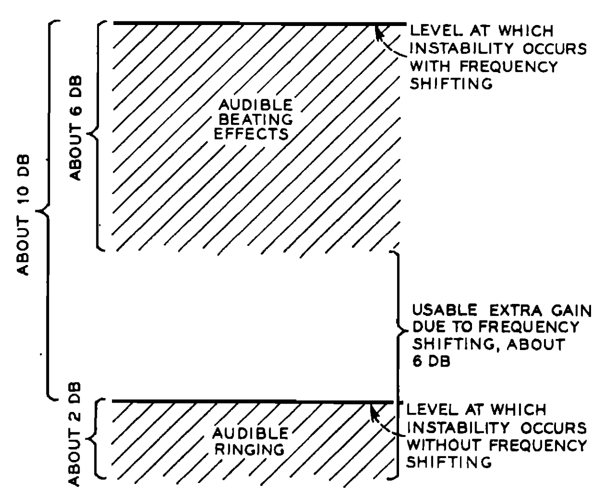

FIG. 8. Stability levels with and without frequency shifting and unusable regions (shaded). Difference in stability levels is typically about 10 dB. Usable additional gain may vary between 5 and 8 dB (see text).

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ence between the two instability levels. Because both the region of audible ringing and the region of audible beating are not precisely defined and because the difference between the two stability levels has an uncertainty of several decibles, the following approximate relations hold:

USABLE ADDITIONAL STABLE GAIN

$$=10\log[\log(WT_{60}/22)]+\Delta\ \mathrm{dB}, \quad (24)$$

where $1\ \mathrm{dB}\leq\Delta\leq 4\ \mathrm{dB}$. For a typical choice, $WT_{60}=5000$, the usable extra gain lies between about 5 and 8 dB according to Eq. (24).

In addition, a public-address system with frequency shifting is able to "absorb" additional short-time gain increases of between 3 and 6 dB before becoming physically unstable. This is particularly important in complex public-address systems (as are being used, for example, at large conventions) in which microphones and other audio components have to be switched rapidly and where the proper stable gain cannot always be maintained.

VII. FIELD APPLICATIONS

Since the spring of 1959, when the first experimental frequency shifter was completed, frequency shifting has been applied to a variety of public-address systems in the laboratory, in local auditoriums, and in very large halls. Particularly noteworthy are the applications at the 78th annual stockholders meeting of the Standard Oil Company (New Jersey) and the shareowners' meeting of the American Telephone and Telegraph Company in McCormick Hall in Chicago in 1961. The latter meeting was attended by over 20 000 people.

There were many instances during these meetings when the small voice of a speaker from the floor had to be brought up several decibels above the normal instability level. Thanks to the frequency shifter, this could be done without any signs of instability.

Equally important was the additional *safety margin* afforded by the frequency shifter against accidental singing due to unexpected increases in gain during switching of microphones and amplifier warmup.

The circuit complexity of the frequency shifter, originally a considerable obstacle to a widespread application of the frequency-shifting principle, has been reduced substantially in the past four years. Prestigiacomo and MacLean[12] have developed a transistorized frequency shifter that utilizes 8 transistors, two quartz crystals, and one single-sideband filter as its main components. Frequency shifters based on this circuit are now available commerically from several manufactures.

In conclusion, it may be said that the electronic antifeedback system based on frequency shifting has proven its usefulness both in small auditoriums and in large halls where feedback and instability are particularly obstinate problems.

ACKNOWLEDGMENTS

The author is indebted to H. W. Hines for constructing the first frequency shifter and to J. E. West for performing the measurements of the additional stable gain as a function of frequency shift. F. K. Harvey, D. J. MacLean, and A. J. Prestigiacomo have rendered valuable assistance in operating and evaluating numerous demonstrations of public-address systems with frequency shifters.

[12] A. J. Prestigiacomo and D. J. MacLean, J. Audio Eng. Soc. **10**, 110–113 (1962).

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