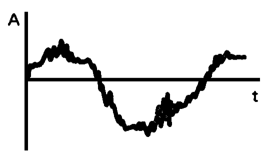
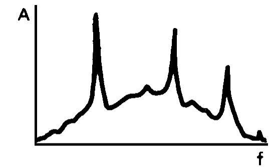
$$F(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

$$f(t) = \int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega$$



Baron Jean Baptiste Joseph Fourier



$$F(k) = \frac{1}{N}\sum_{n=0}^{N-1} f(n)e^{-j\frac{2\pi nk}{N}}$$

$$f(n) = \sum_{k=0}^{N-1} F(k)e^{j\frac{2\pi nk}{N}}$$

FFT Analysis 101

LDS

LOCHARD

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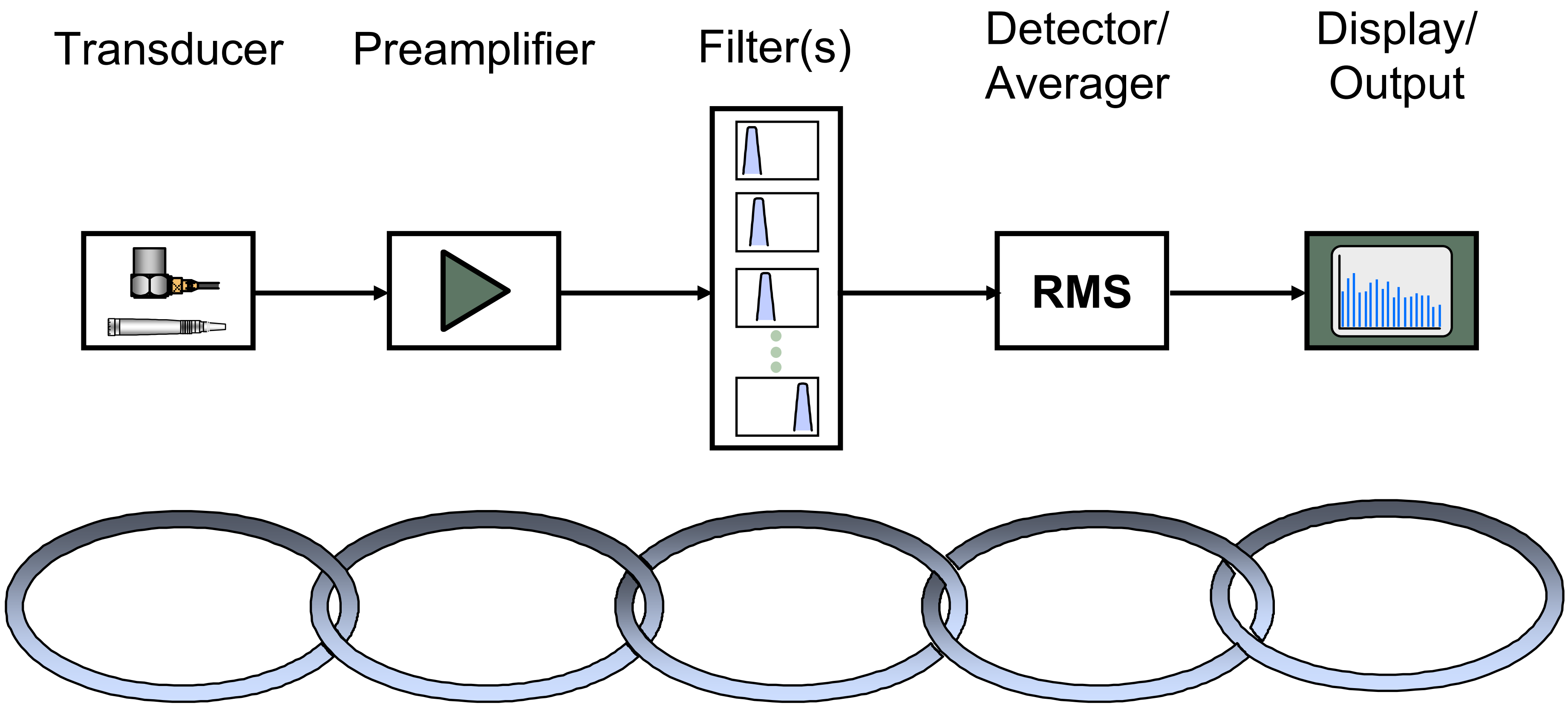
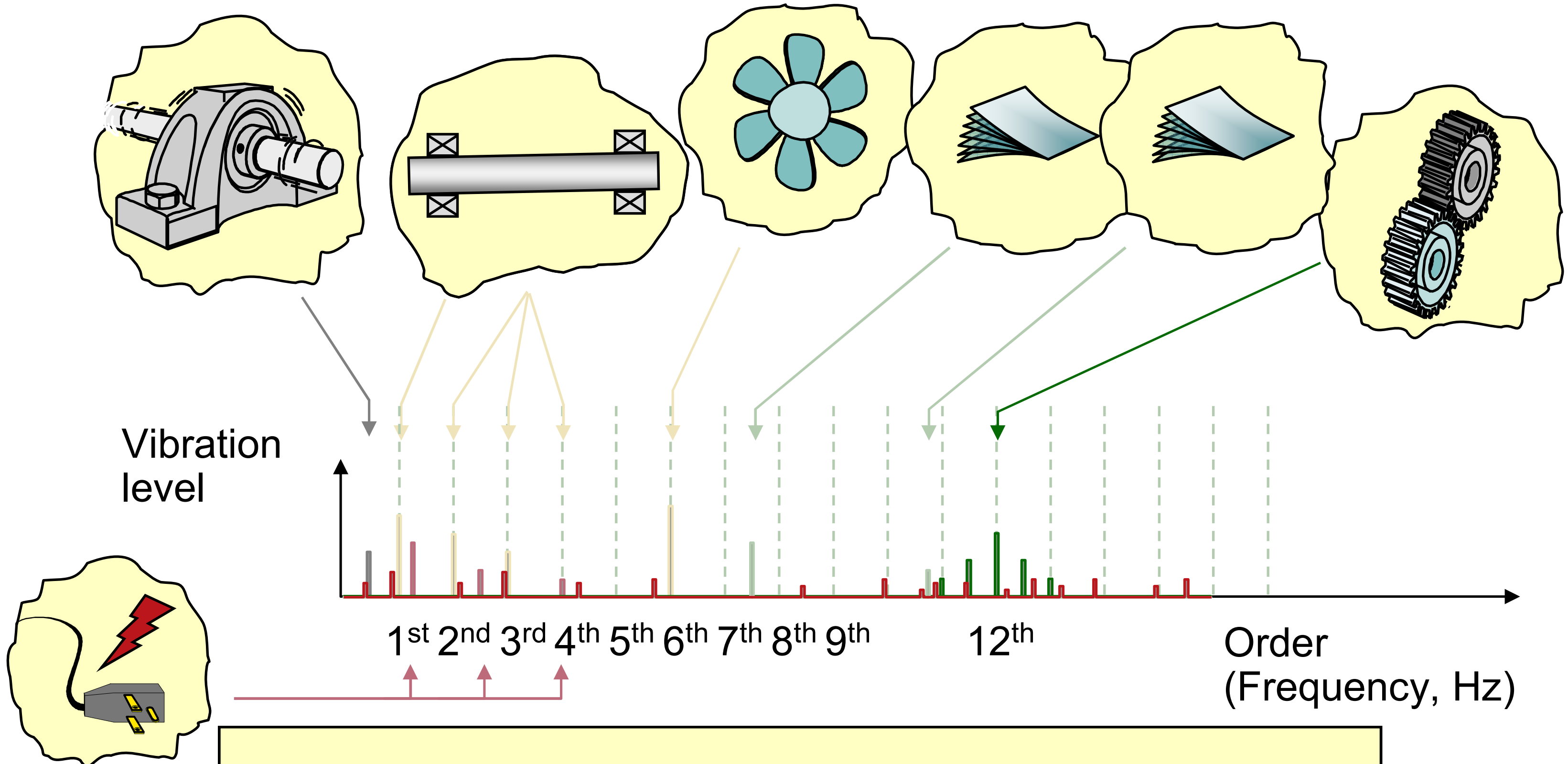
The Measurement Chain



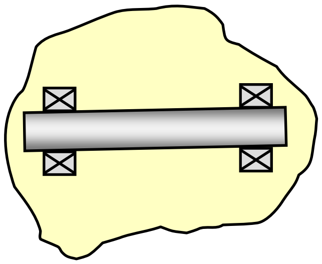
Sources of Machine Vibration

The moving parts of machines create vibration at different frequencies.

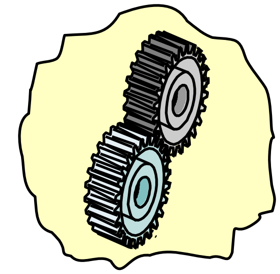


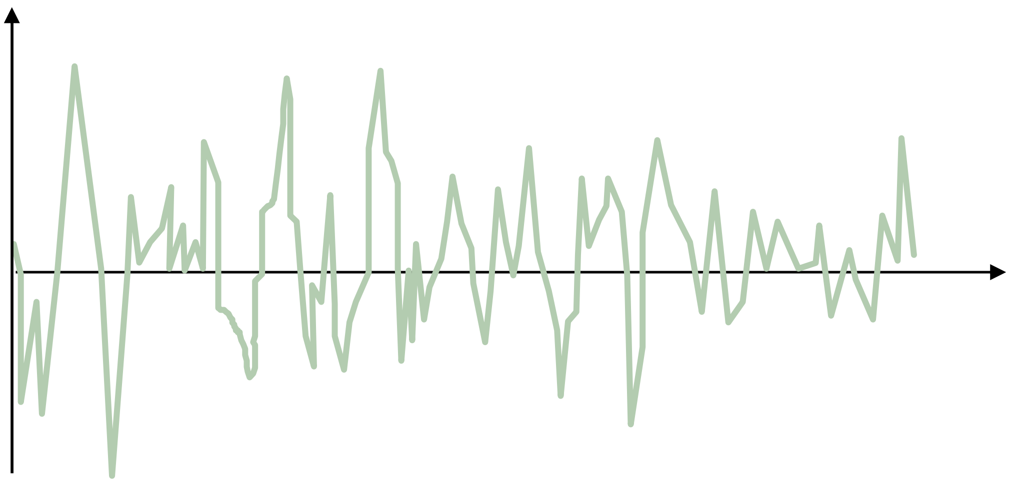
Many different vibrations compose a single frequency spectrum

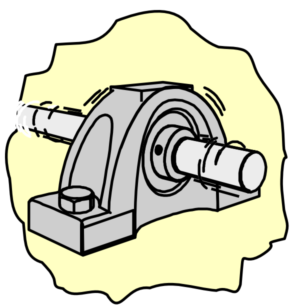
Easier Than This…

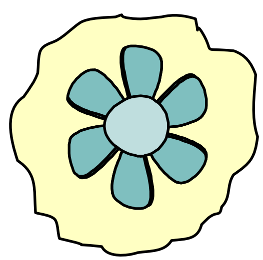


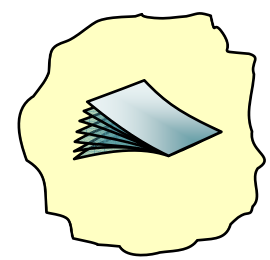
Imagine trying to determine all of the previous from just this!











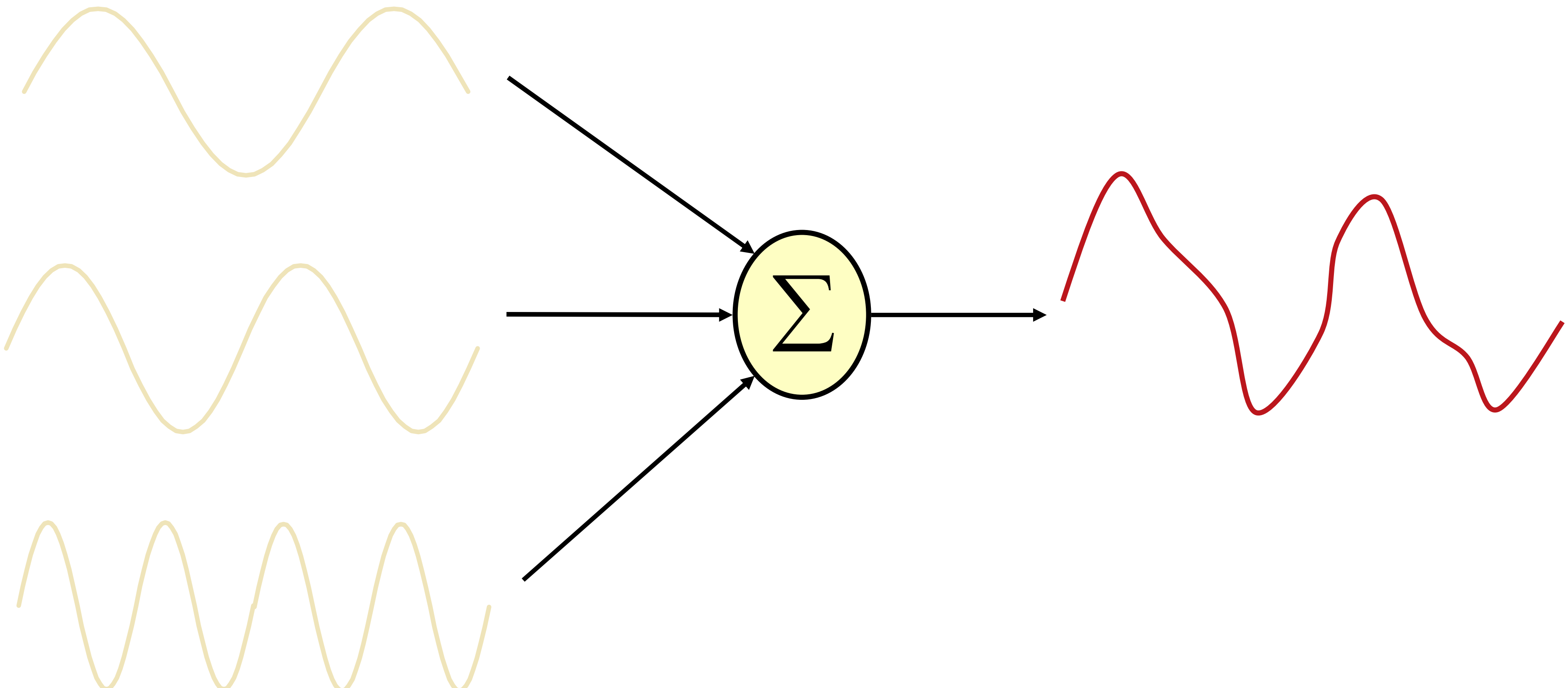
The Fourier Transform

$$G(f) = \int_{-\infty}^{+\infty} g(t)\, e^{-j2\pi f t} dt$$

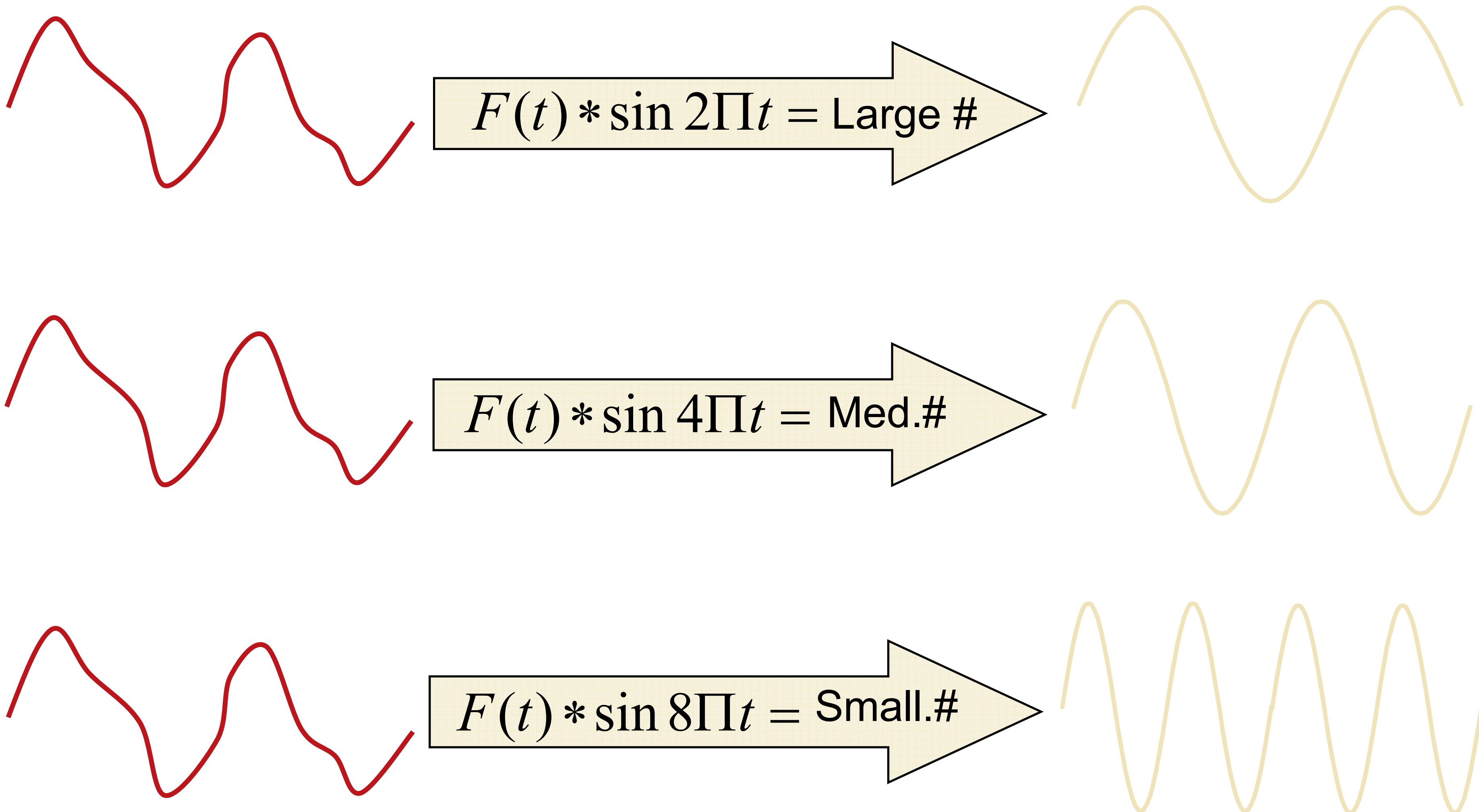
$$g(t) = \int_{-\infty}^{+\infty} G(f)\, e^{j2\pi f t} df$$

“All Complex Waves…”

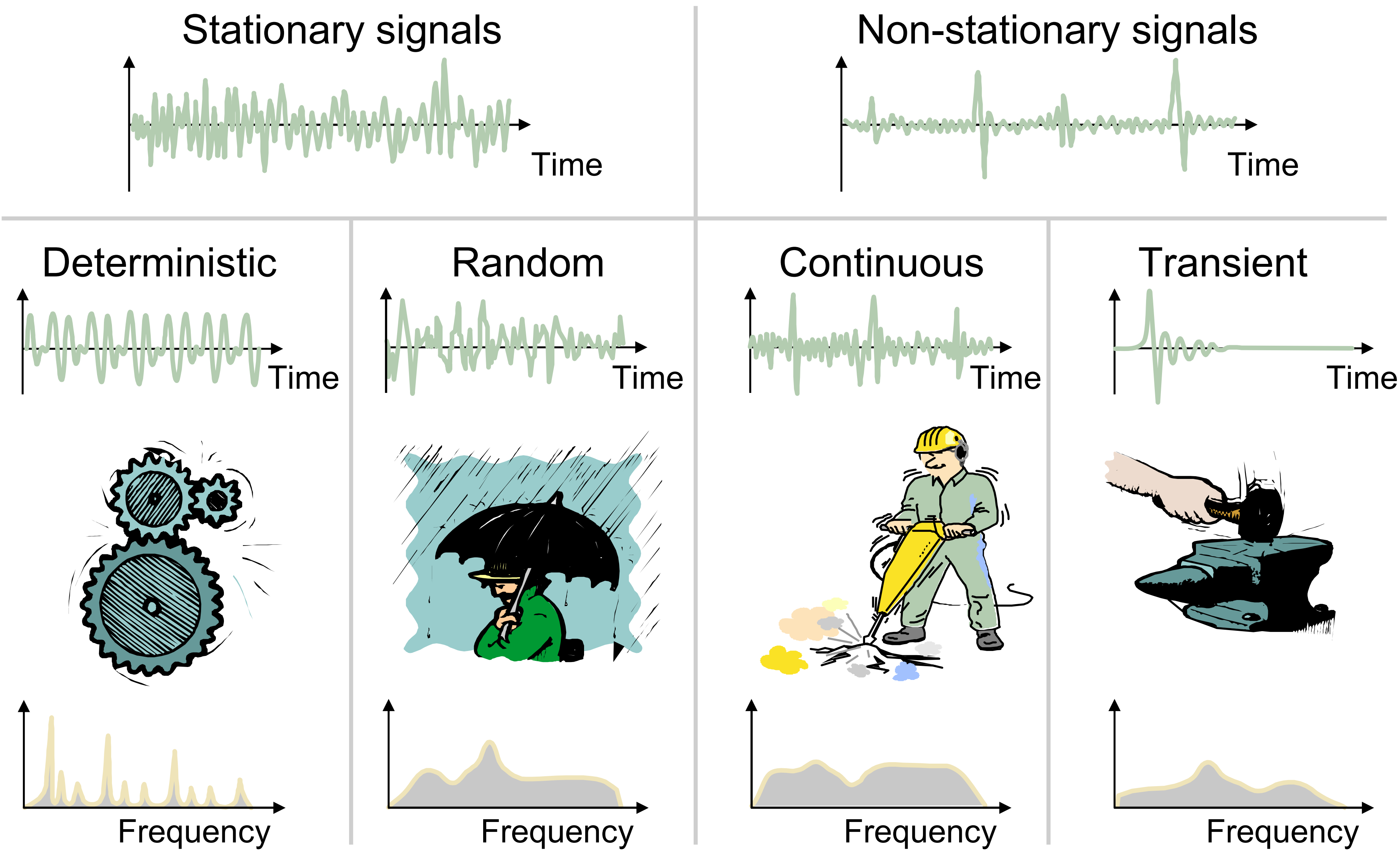
All Complex Waves are the Sum of Many Sine and Cosine Waves



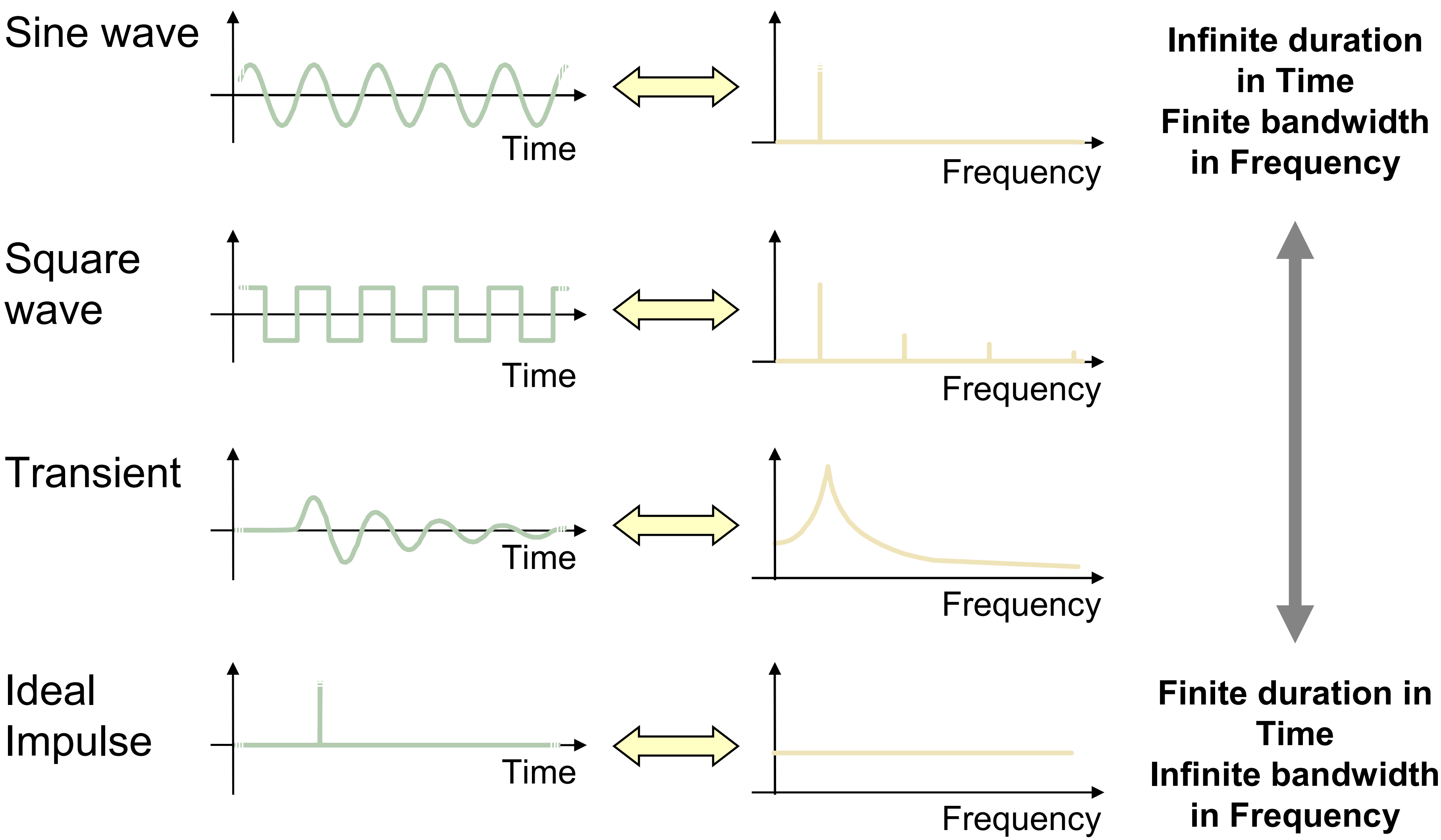
Fourier Transform is a Mathematical Filter!



Types of Signals



Types of Signals



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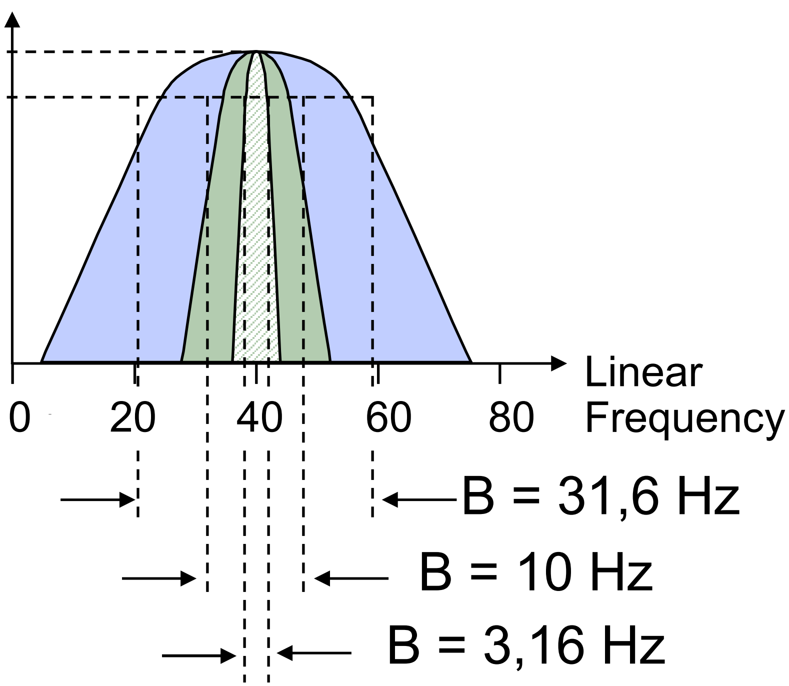
Filter Types

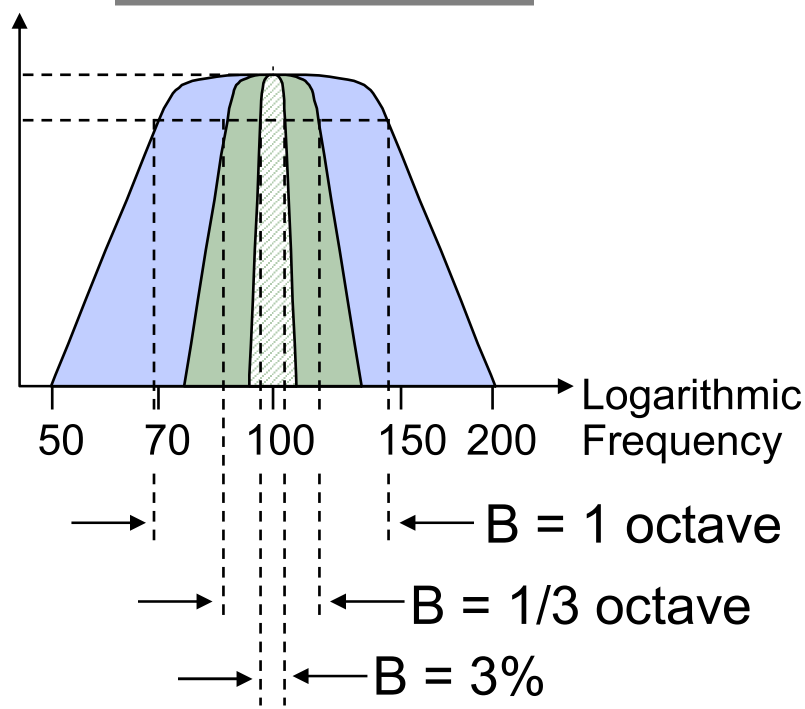
Constant Bandwidth

$$B = x \text{ Hz}$$



Constant Percentage Bandwidth (CPB)
or Relative Bandwidth

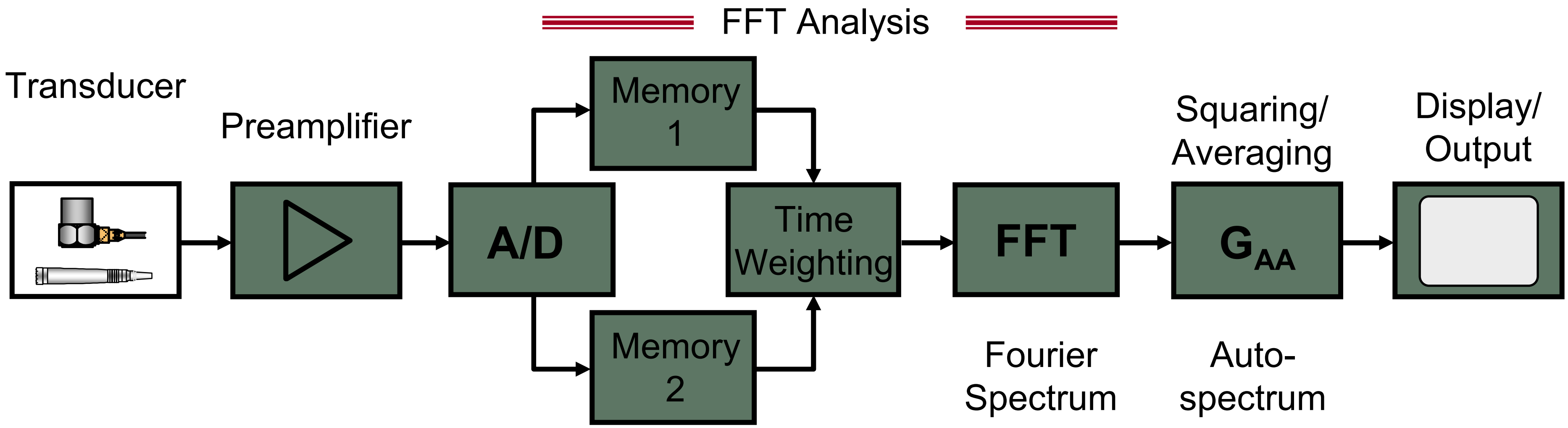
$$B = y\% = \frac{y \times f_0}{100}$$

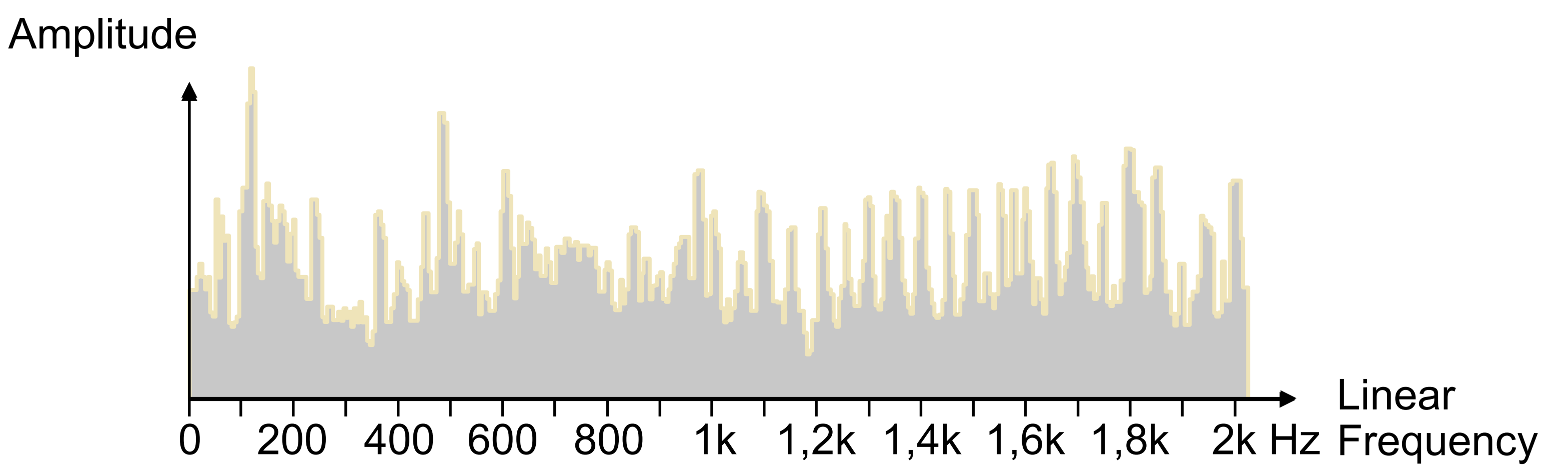


What is the Fast Fourier Transform?

- An algorithm for increasing the speed of the computer calculation of the Discrete Fourier transform.
  - Reduces the number of multiplications from N^2 to $(N/2)\log_2 N$
  - Computation speed increased by a factor of 372 for an 800 line FFT
- Block analysis of time data samples to provide equivalent frequency domain description
- Analysis with constant bandwidth filters
- “Rediscovered” in 1962 by Bell Lab scientists Cooley and Tukey

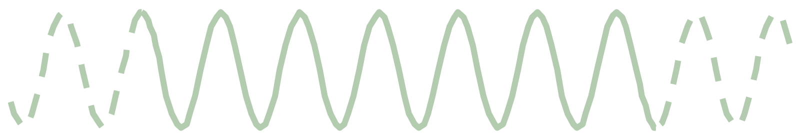
FFT Analyzer





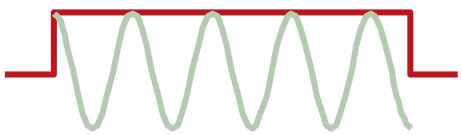
FFT Time Assumption

- Input signal

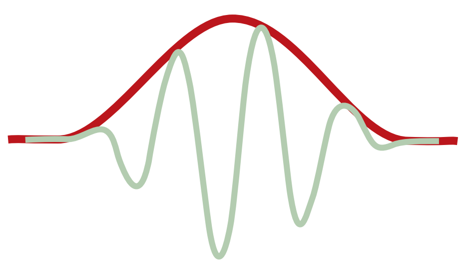


- Analysed signal

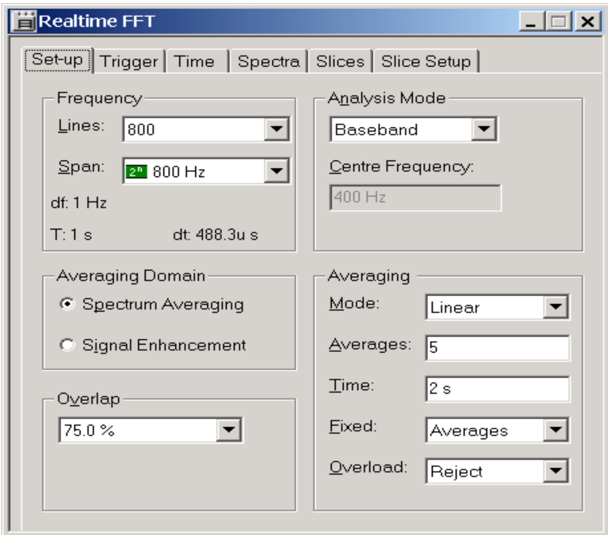
Rectangular or Uniform weighting



Hanning weighting

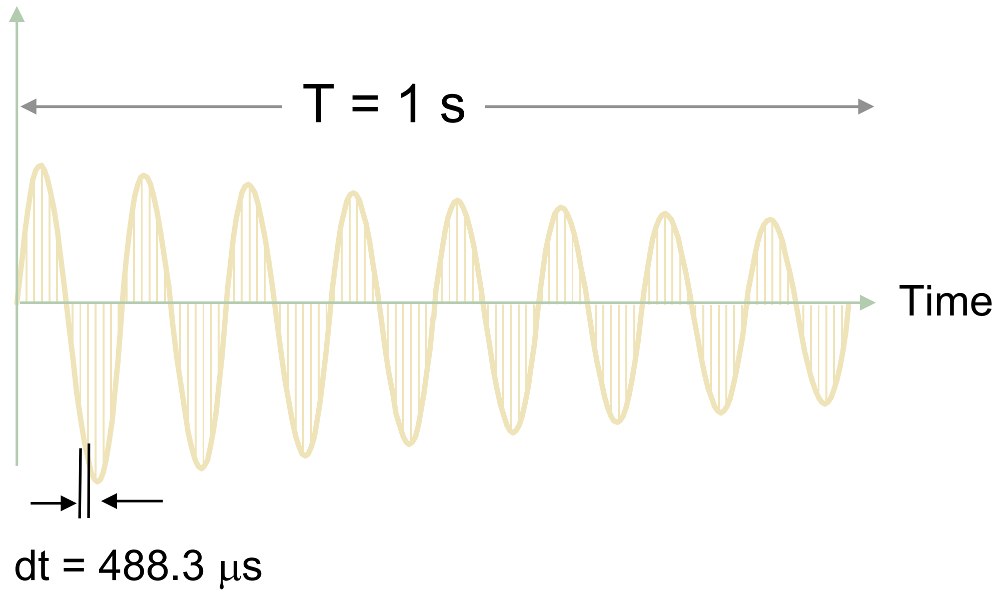


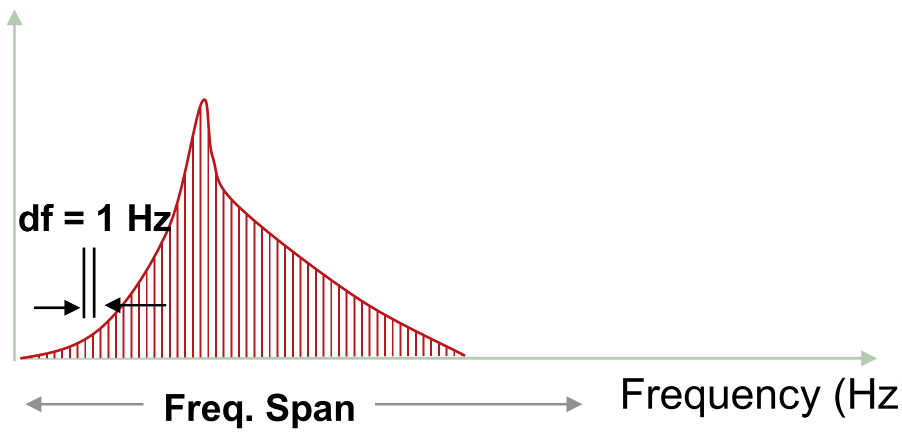
FFT Fundamentals



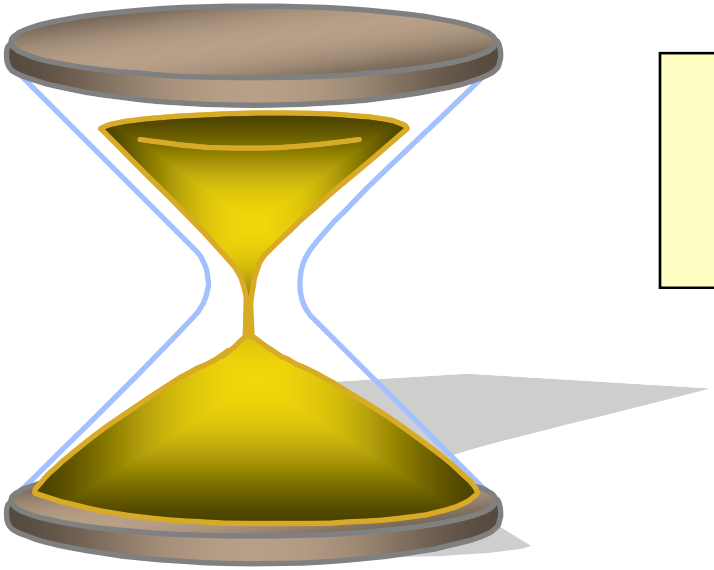
- Lines = resolution
- Span = upper freq. range

- df = Span/Lines
- T = 1/df
- dt = 1/(Span * 2.56)





Most important Law in Frequency Analysis



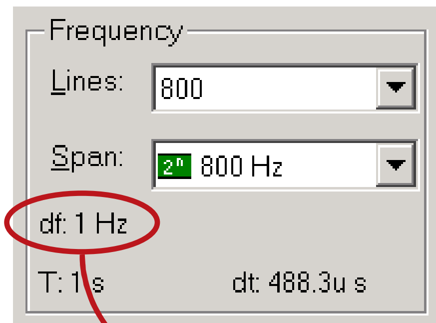
$$B \times T \geq 1$$

B = bandwidth

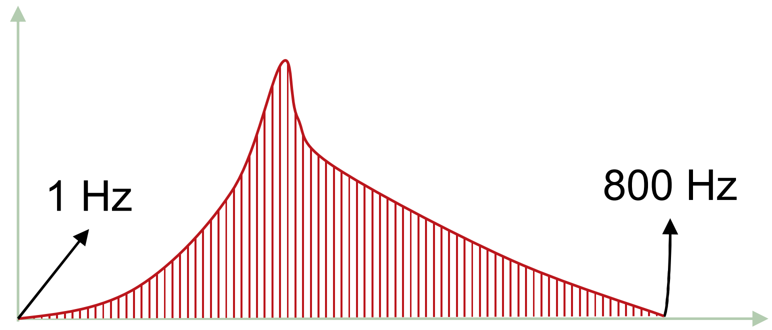
T = measurement time

Uncertainty Principle Example

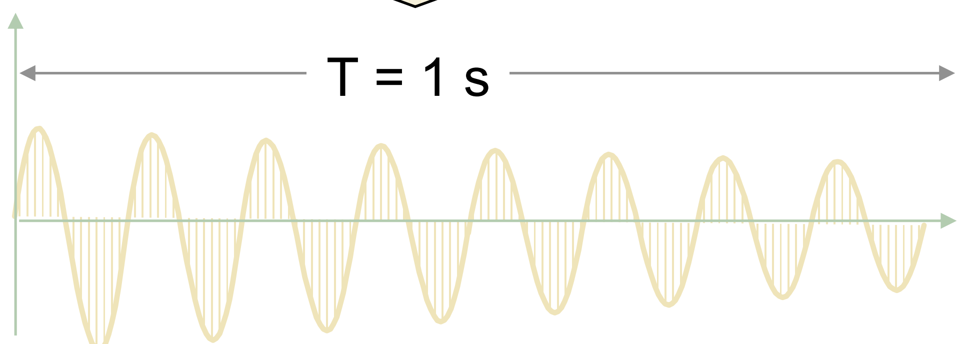
If the FFT Analyzer is:



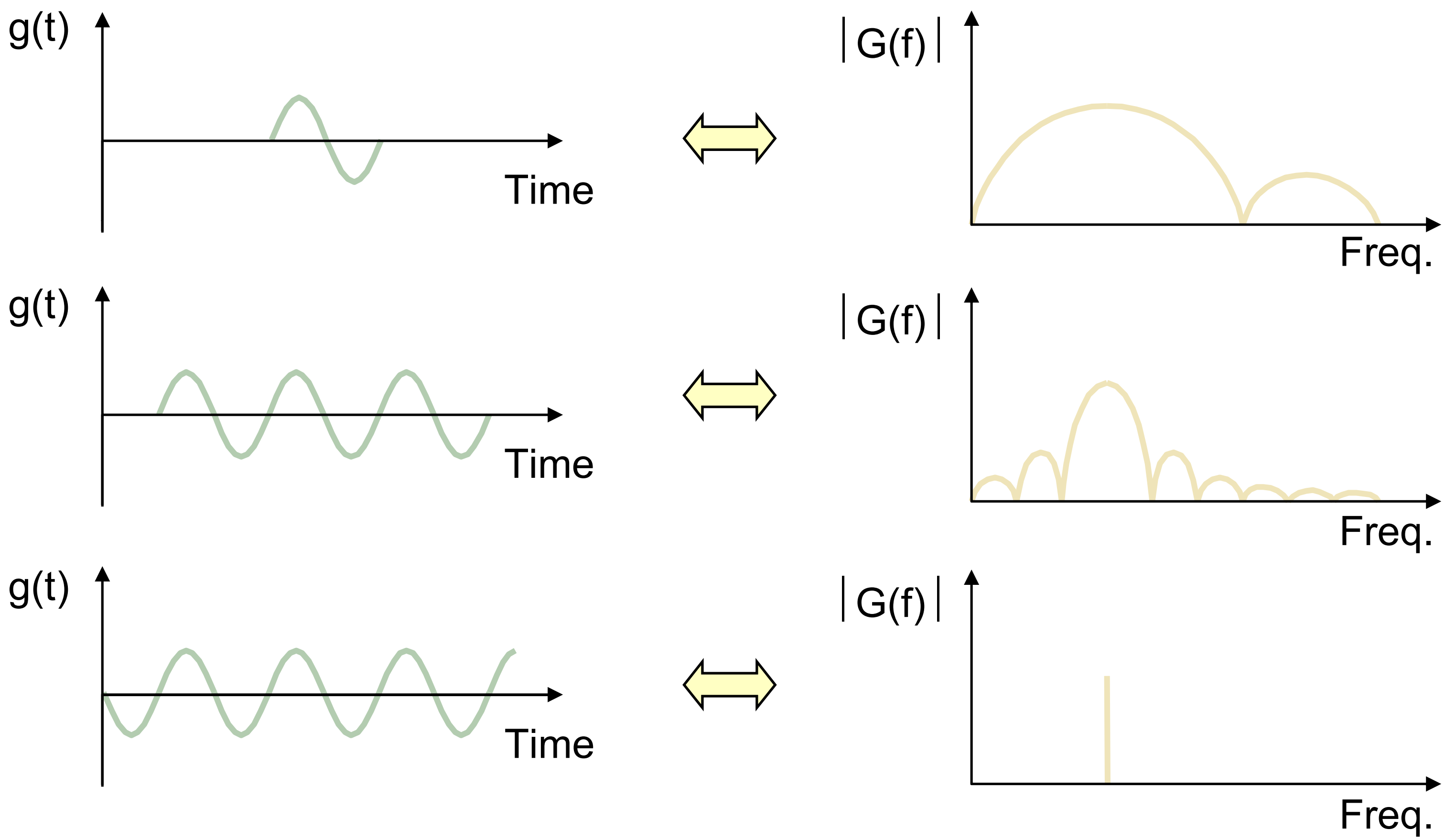
Then the lowest freq. is 1 Hz



To satisfy BT ≥ 1, then 1 Hz / 1 = 1s

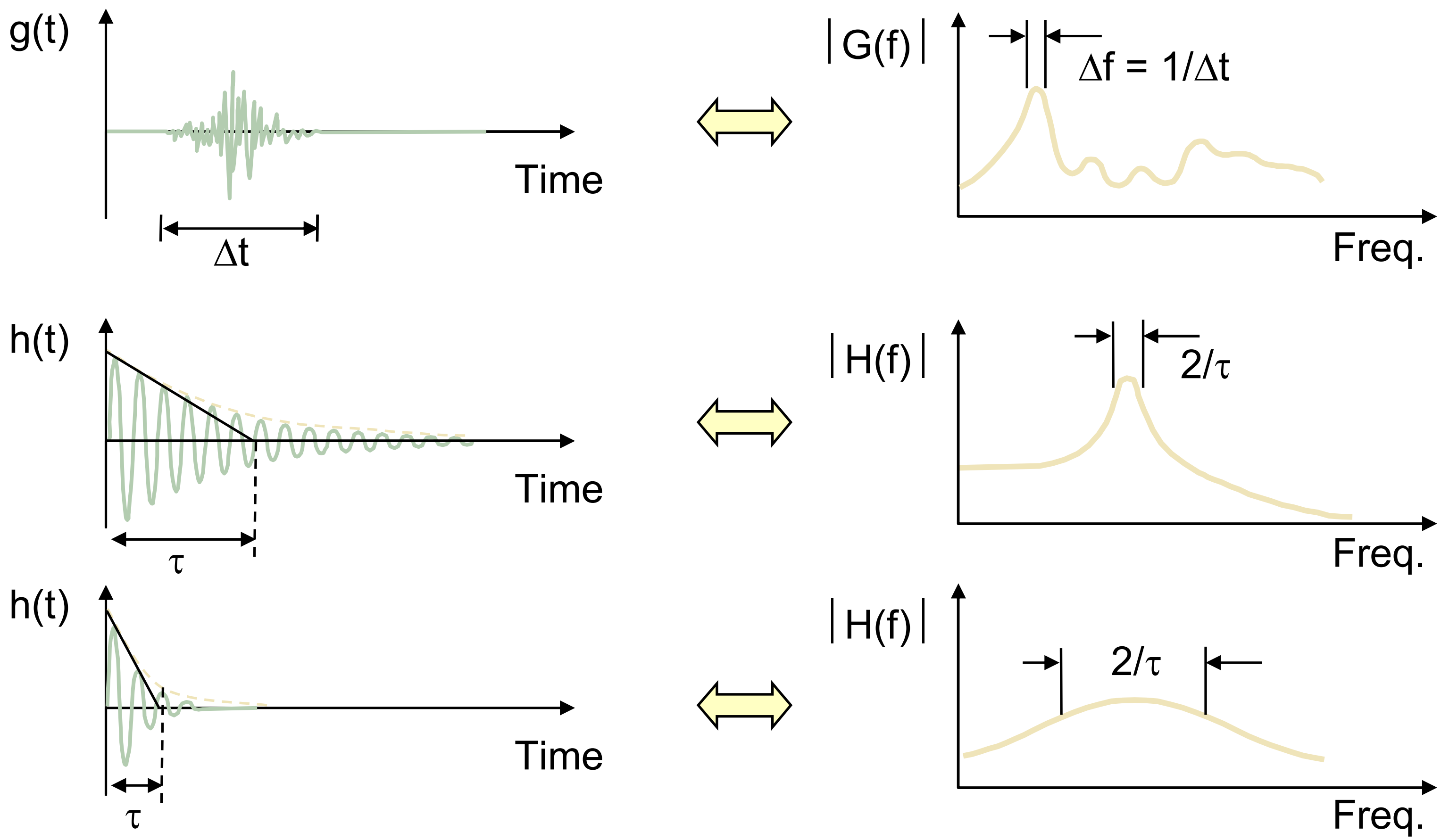


Uncertainty Principle – Stationary Signals



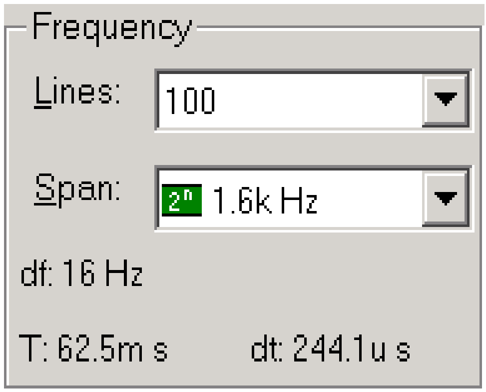
If BT = 1 then we are ‘Certain’…but then accuracy is not very high. If we obtain more cycles then accuracy will improve greatly.

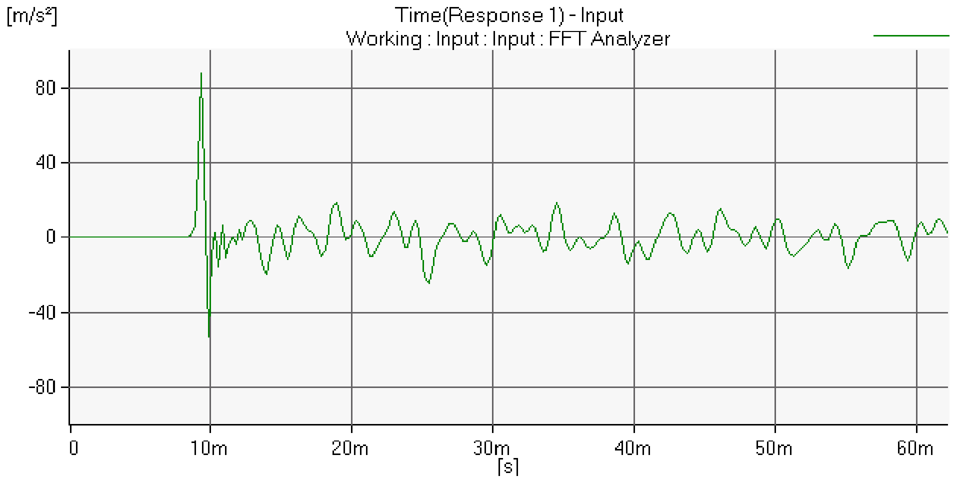
Uncertainty Principle – Non Stationary Signals

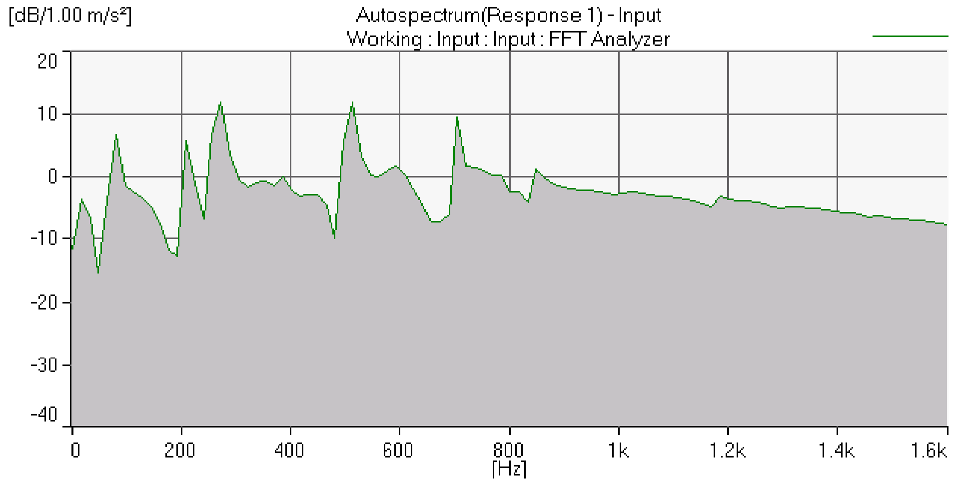


With transients things get more complicated.
More resolution is not always the best…

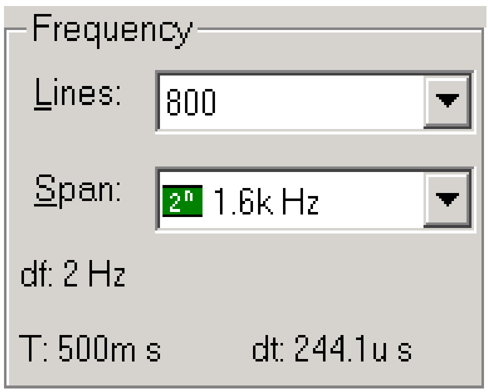
Are You Certain About Uncertainty (1)?

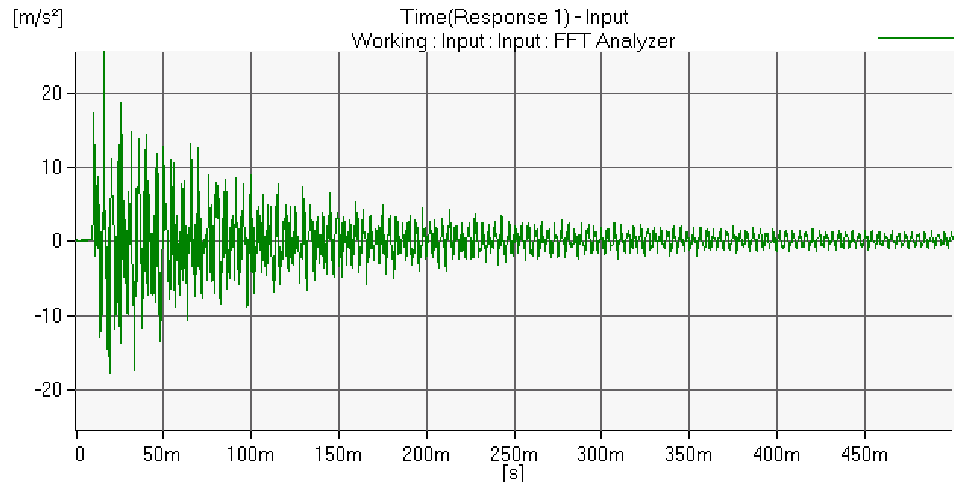


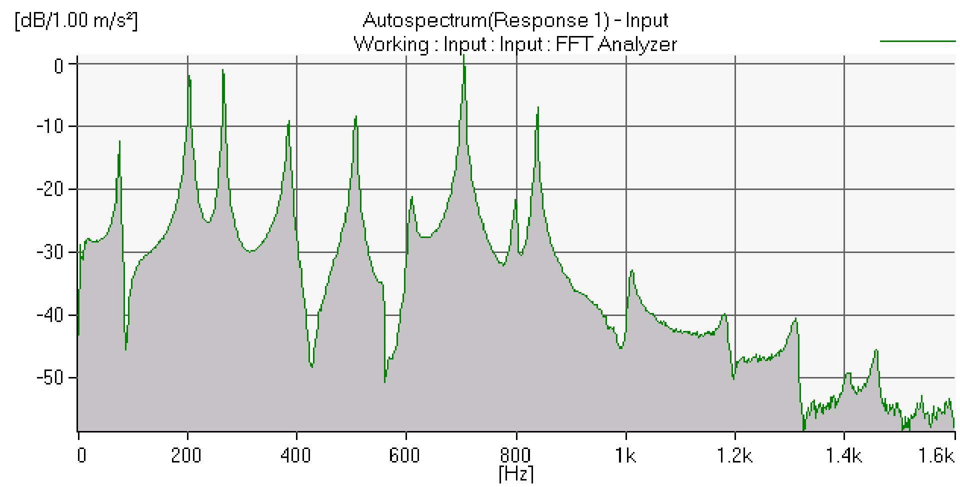




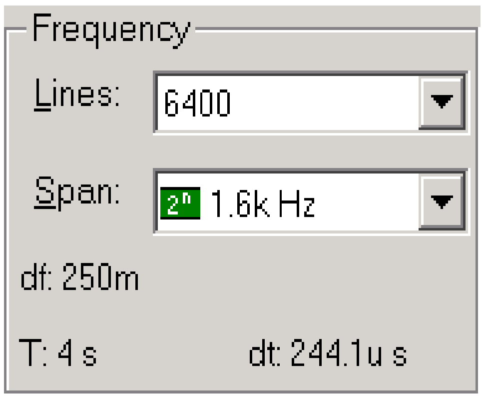
Are You Certain About Uncertainty (2)?

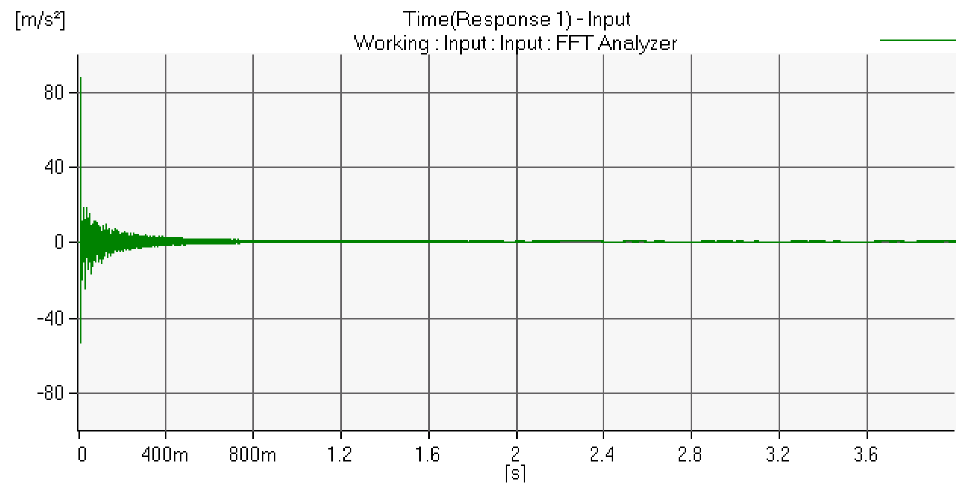


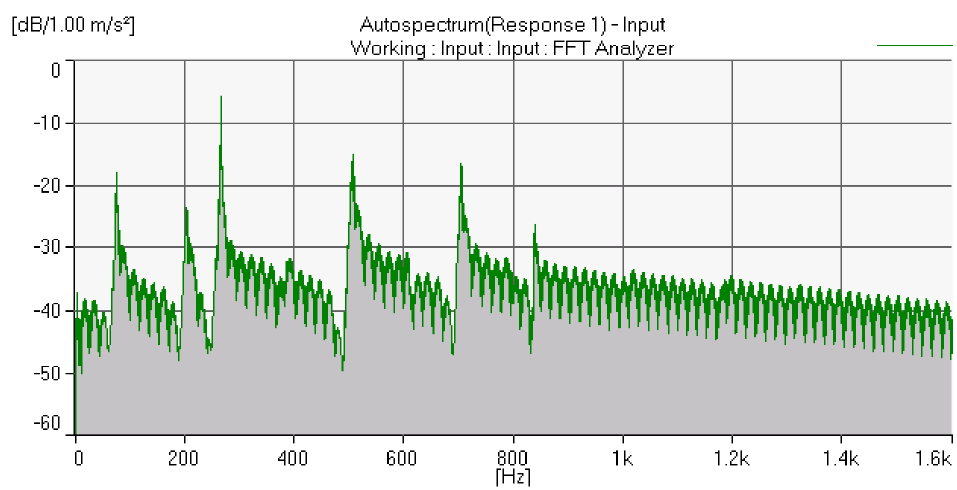




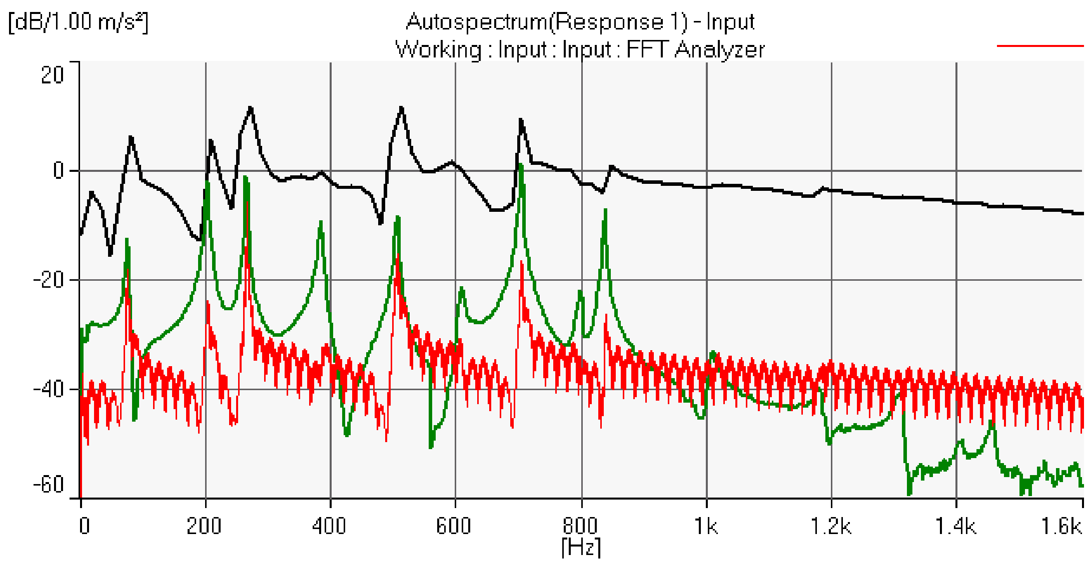
Are You Certain About Uncertainty (3)?



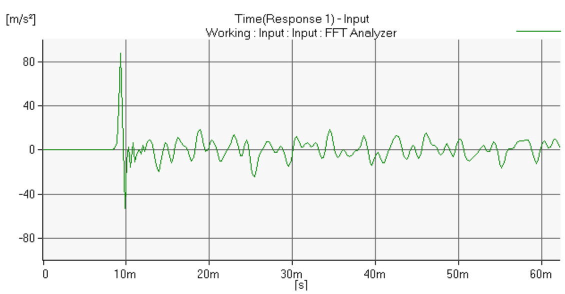


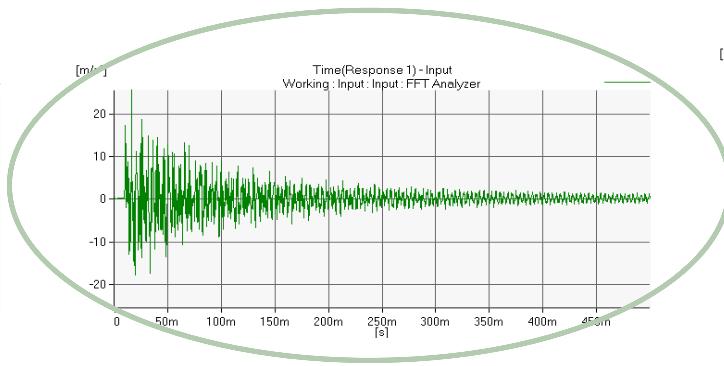


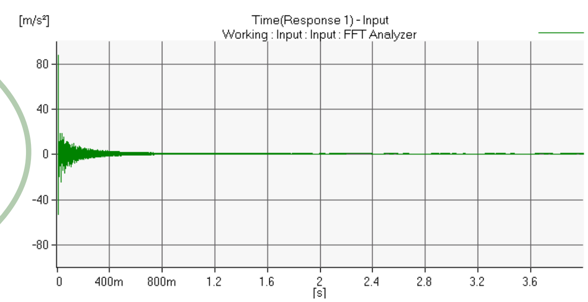
What Happened? Which Is Correct?



Whichever best fit the time block!







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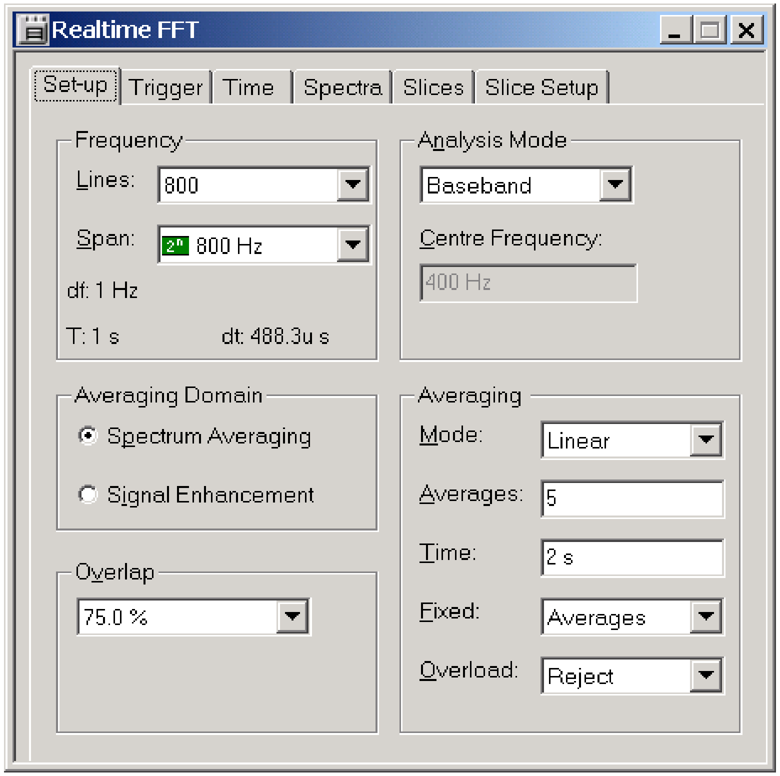
Ensuring Repeatable Measurements



- Think about the signal you are measuring
  - Stationary
  - Transient
  - Combination…
- Always report:
  - Lines
  - Span
  - Window used
  - Overlap
  - Averaging Type
  - # of Averages
  - Start Trigger?

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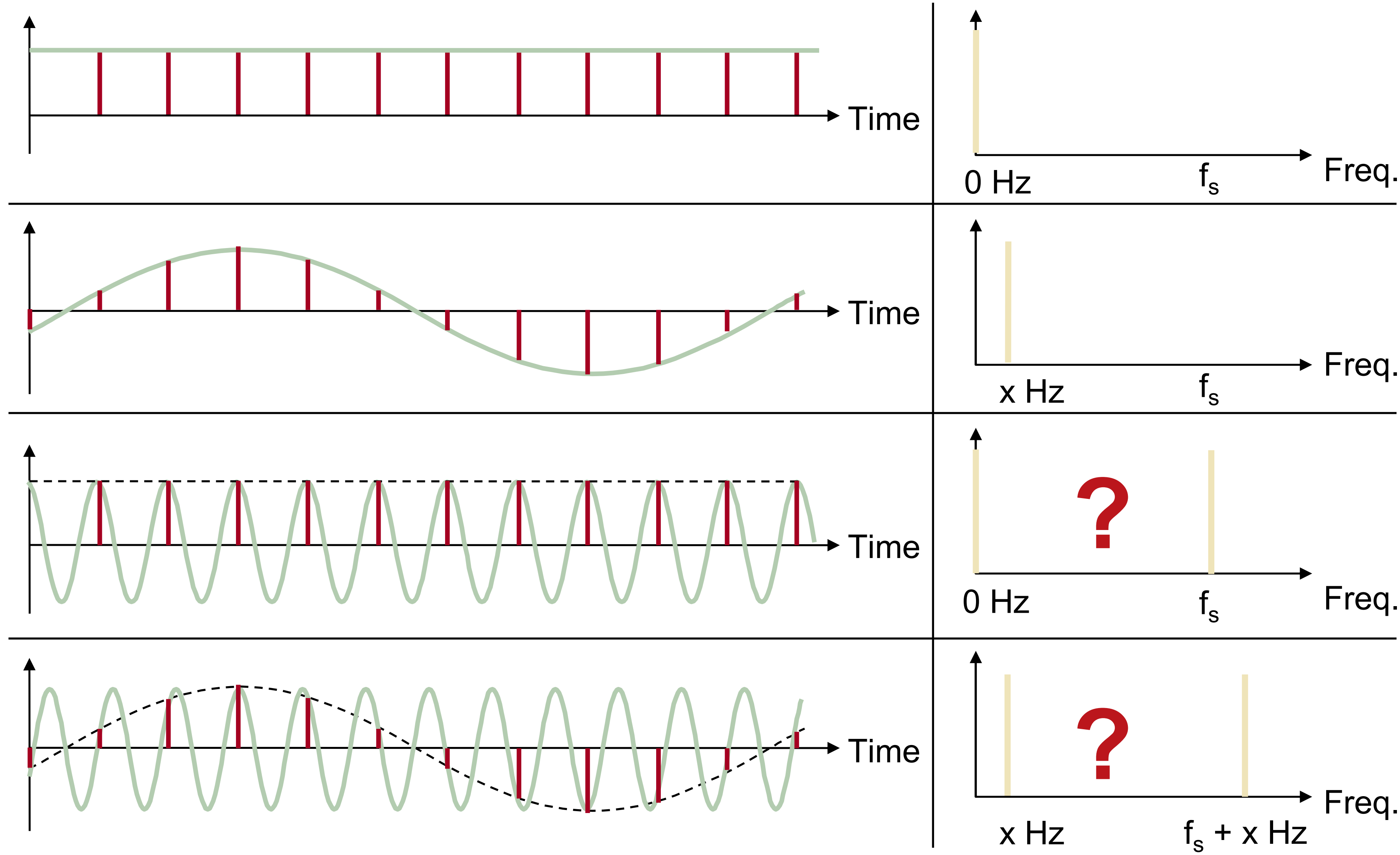
Pitfalls in DFT

Time		Frequency
Sampling	gives	Aliasing
Time limitation	gives	Leakage
Periodic repetition	gives	Picket Fence Effect

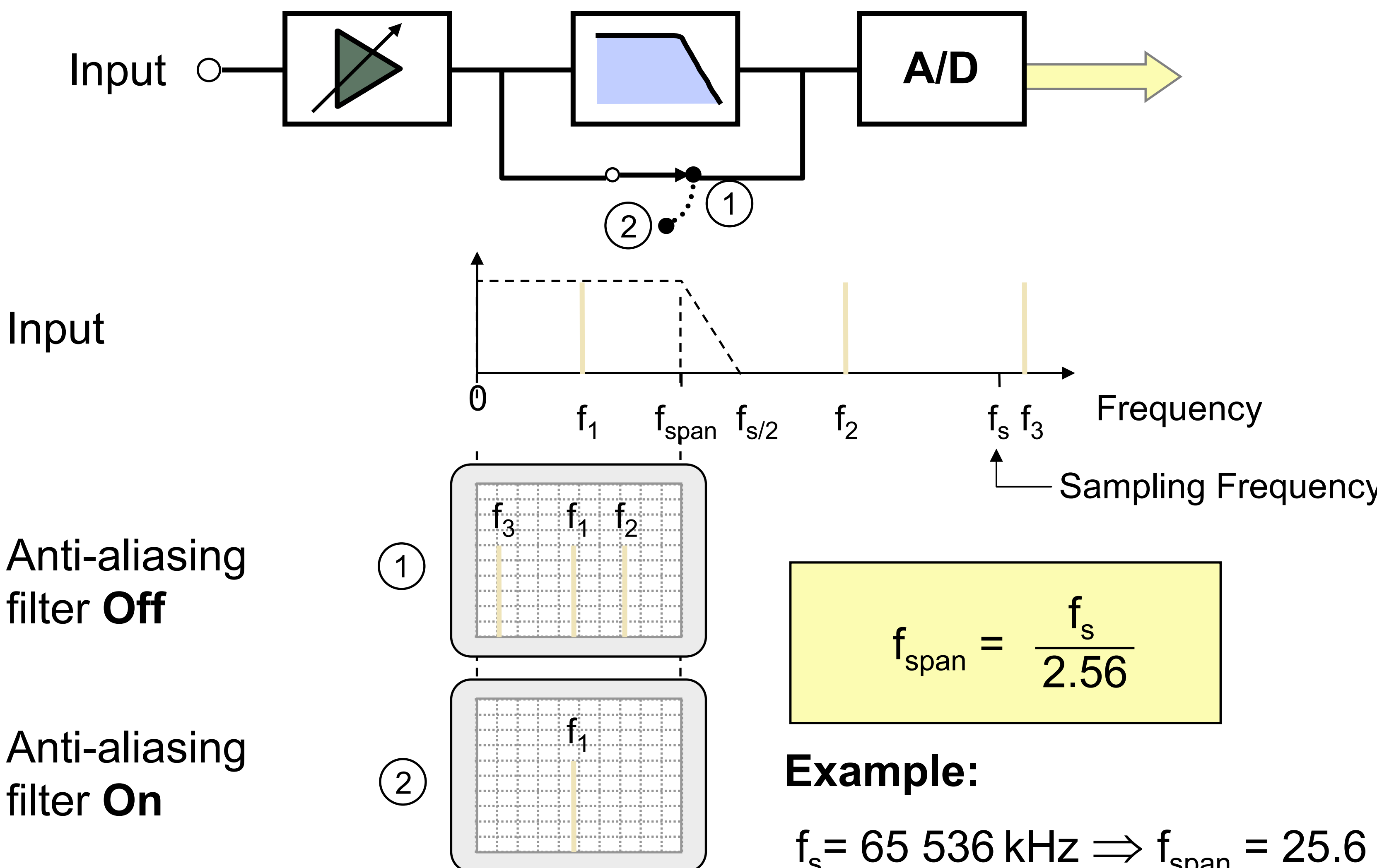
ALIASING

- Insufficient sampling allows a high frequency signal to masquerade under a low frequency “alias”

Aliasing


Time
0 Hz
f_s
Freq.
Time
x Hz
f_s
Freq.
Time
?
0 Hz
f_s
Freq.
Time
?
x Hz
f_s + x Hz
Freq.

Anti-aliasing Filter

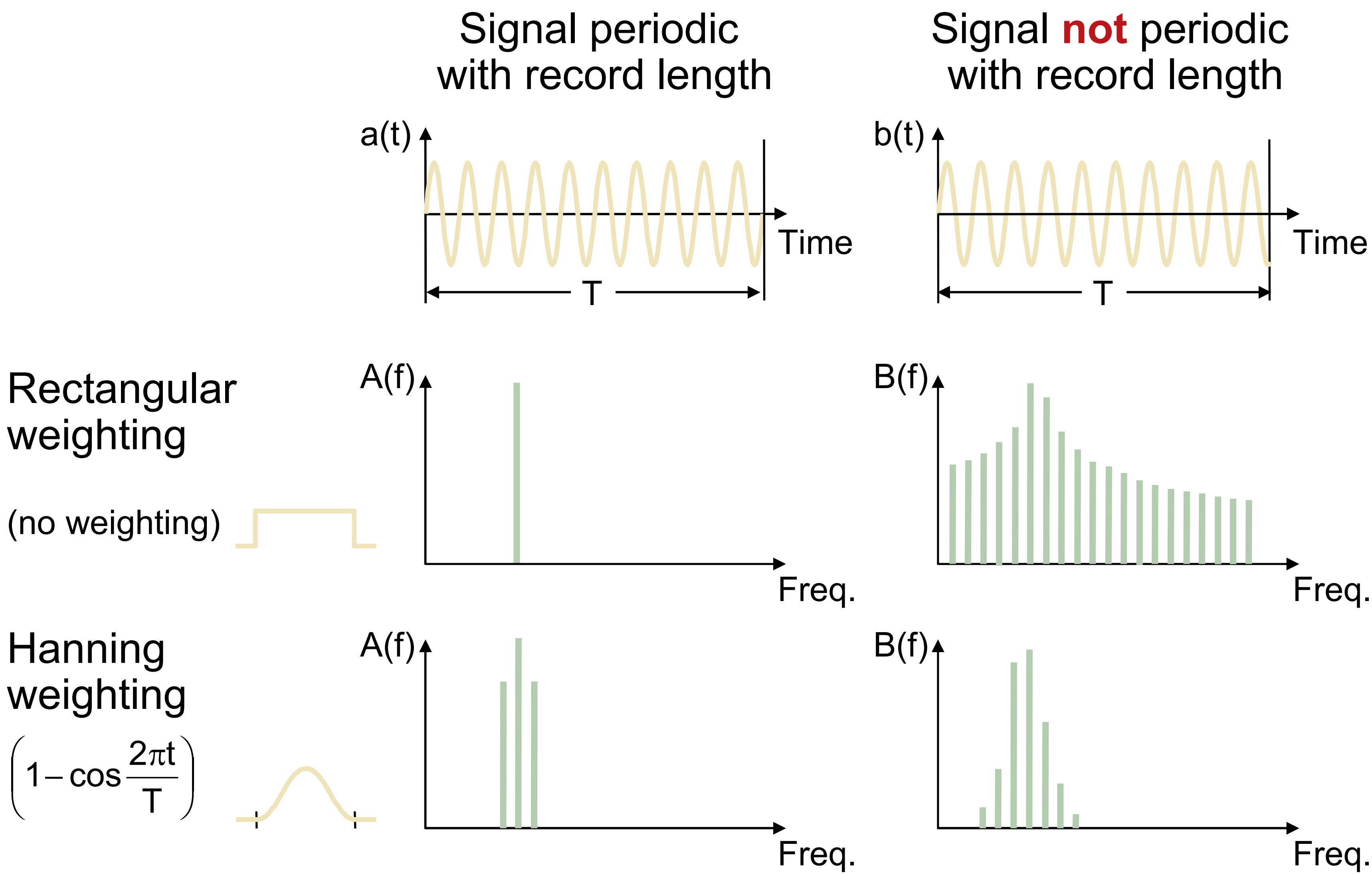


$$f_{span} = \frac{f_s}{2.56}$$

Example:

f_s= 65 536 kHz $\Rightarrow$ f_{span} = 25.6 kHz

Leakage



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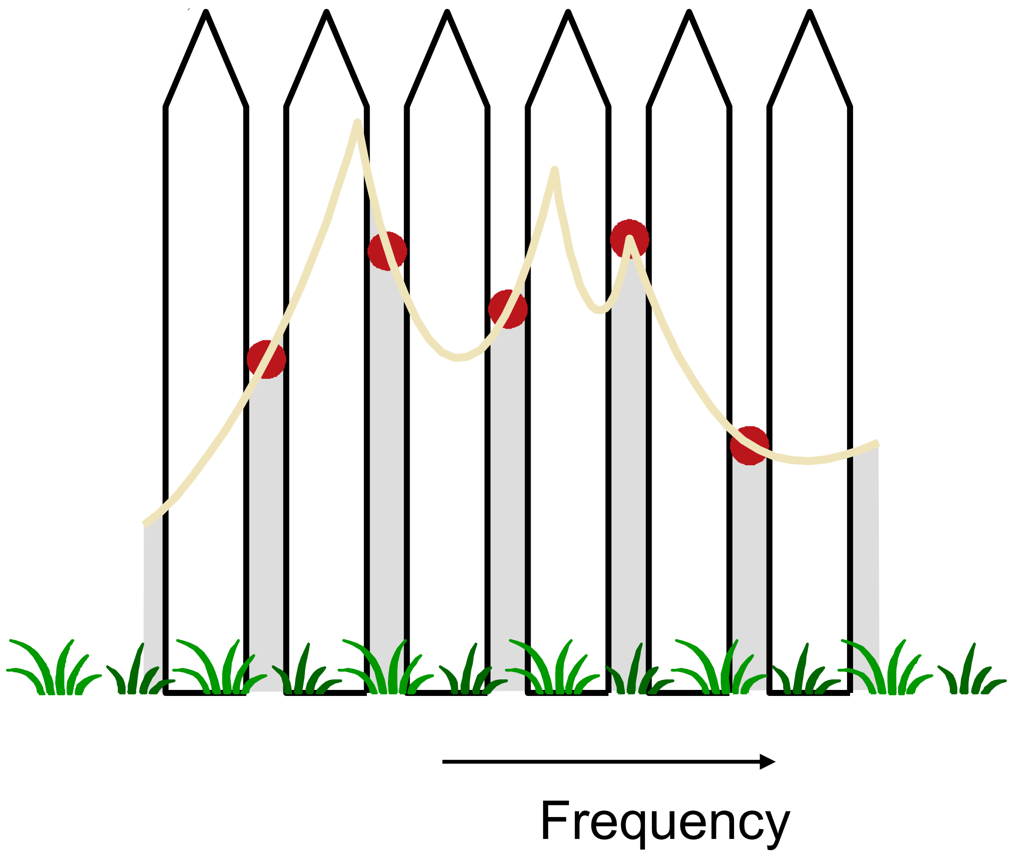
“Picket Fence” Effect



How to Avoid the Pitfalls of DFT

1. Aliasing: Caused by sampling in time

   Solution:
   - Use anti-aliasing filter (f_c) and sampling rate $f_s > 2\ f_c$

2. Leakage: Caused by time limitation

   Solutions:
   - Use correct weighting (signals)
   - Increase the frequency resolution (systems)

3. Picket fence effect: Caused by sampling in frequency

   Solutions:
   - Use correct weighting (signals)
   - Increase the frequency resolution (systems)

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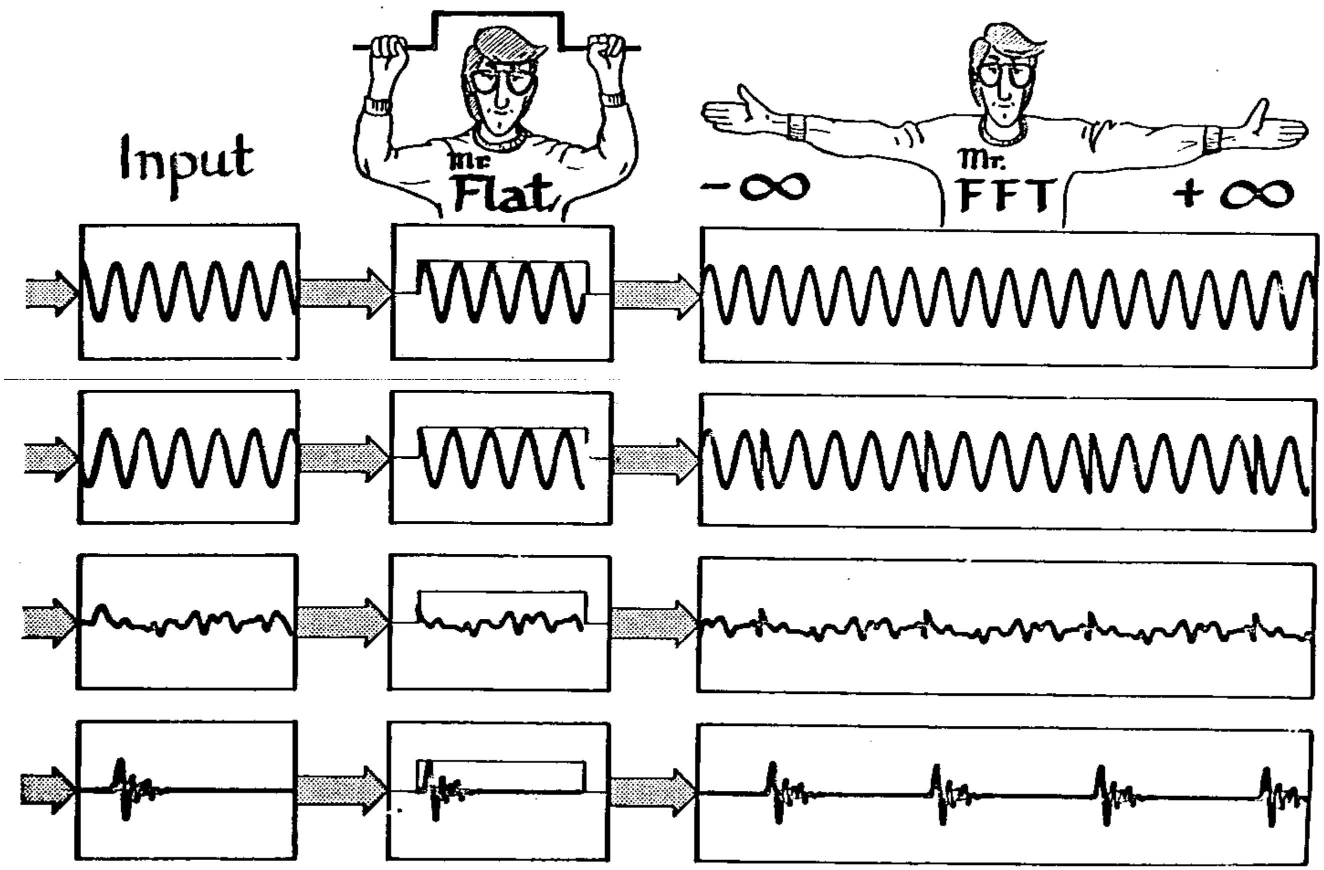
Discontinuities in the Time Record



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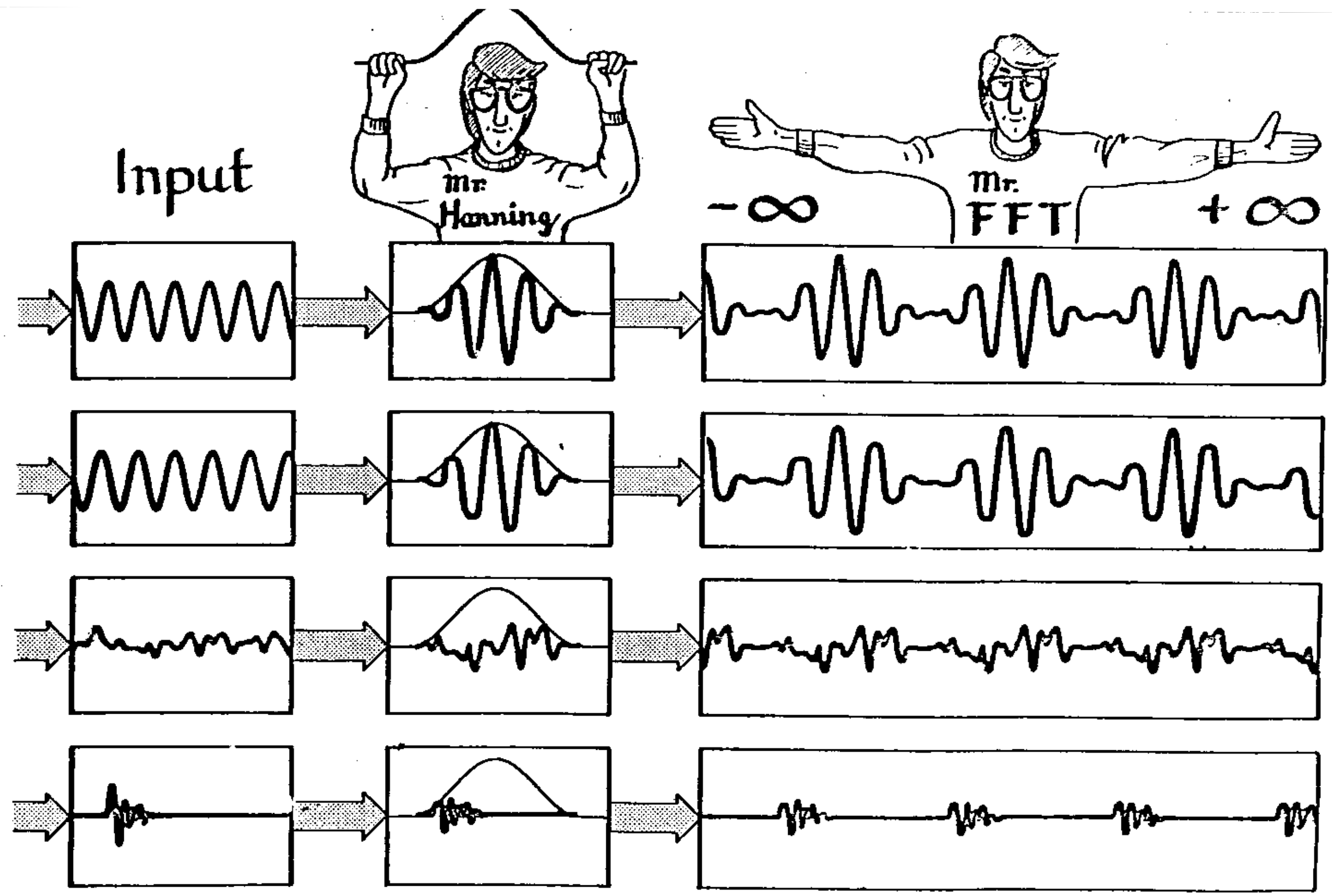
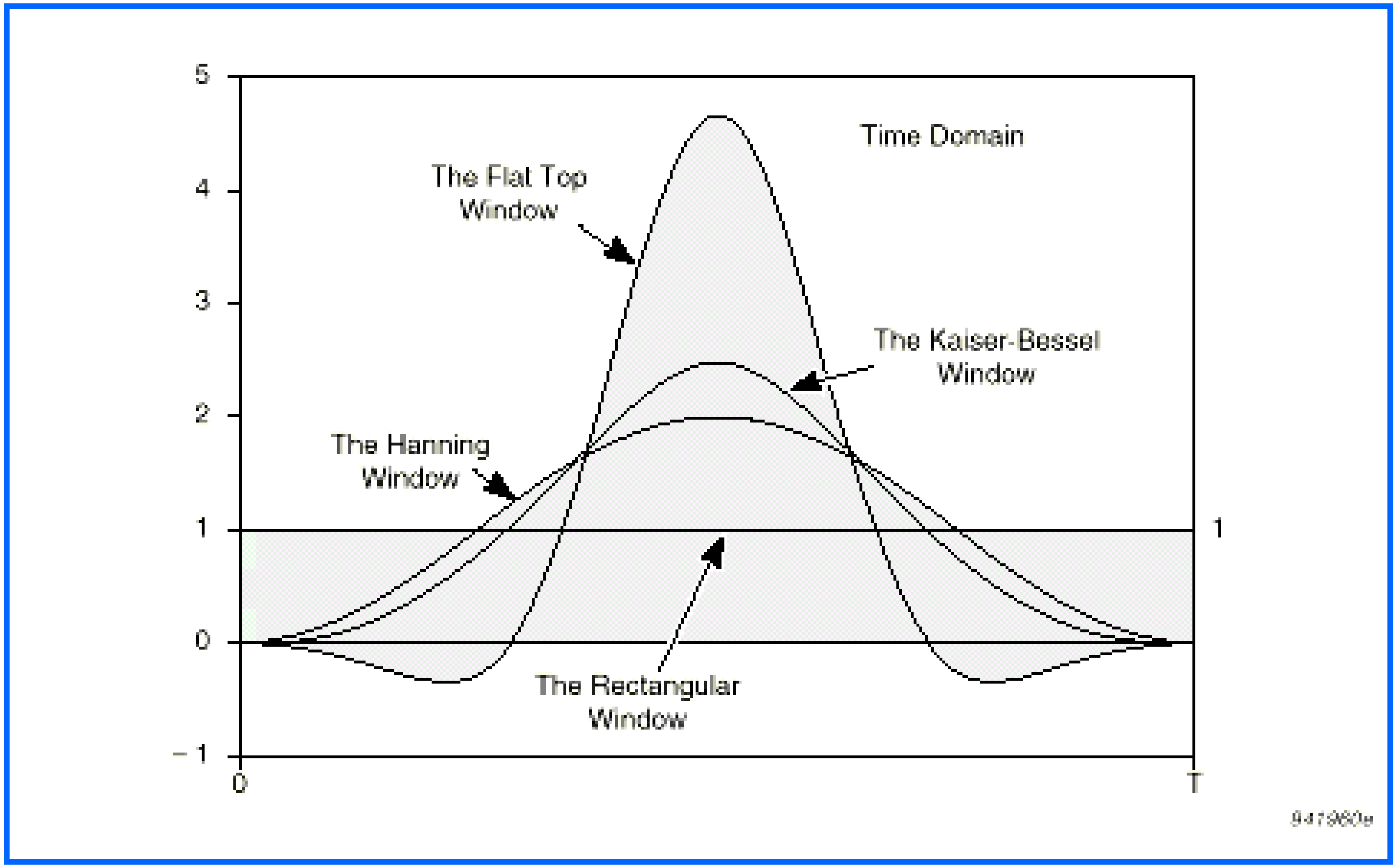
Windows “smooth” the discontinuity



Windows


5
4
3
2
1
0
– 1
0
T
1
Time Domain
The Flat Top Window
The Kaiser-Bessel Window
The Hanning Window
The Rectangular Window
941960e

Use of Weighting Functions in *Signal* Analysis

	Weighting					
	Rect-angular	Hanning	Transient	Expo-nential	Kaiser-Bessel	Flat Top
Transients:						
• General purpose		✓ + overlap				
• Short transient	✓		✓			
• Long decaying transients				✓		
• Very long transients		✓ + overlap				
Continuous signals:						
• General purpose, RTA		✓				
• Two-tone separation					✓	
• Calibration						✓
• Pseudo random	✓					

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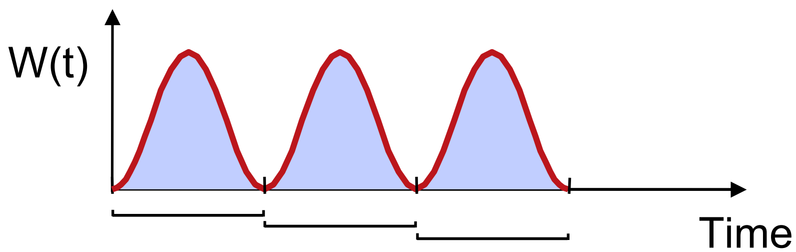
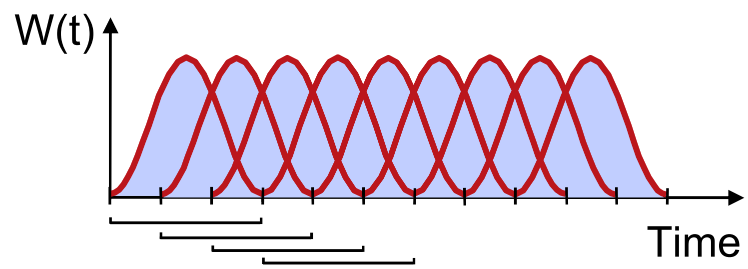
Overlap Analysis with Hanning Weighting (1)

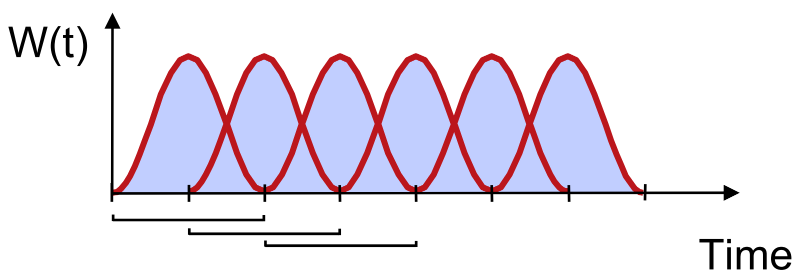
- No overlap



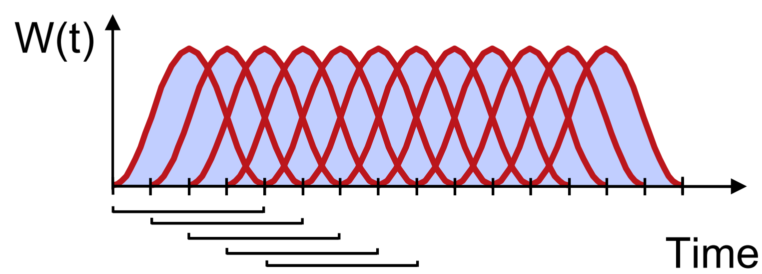
- 66²/₃% overlap



- 50% overlap



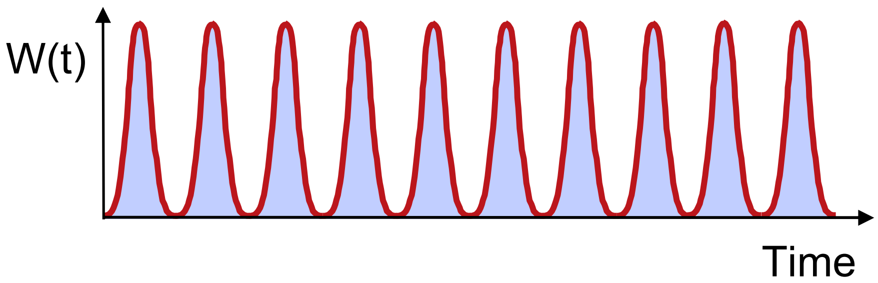
- 75% overlap



Overlap Analysis with Hanning Weighting (2)

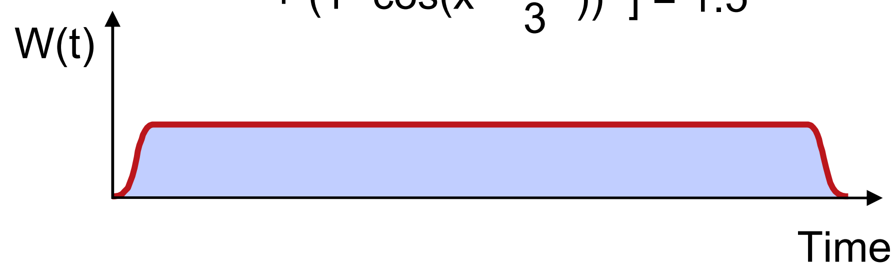
- No overlap

$$(1 - \cos x)^2 = 1 - 2\cos x + \cos^2 x$$



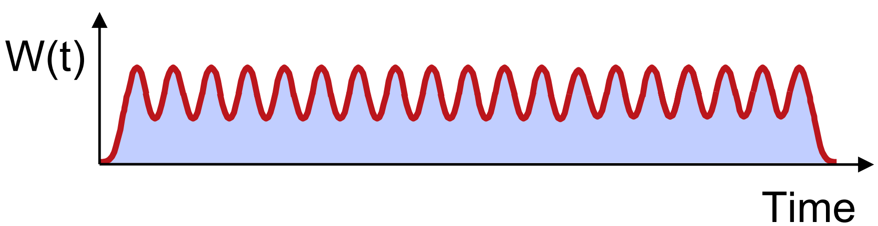
- $66^2/_3$% overlap

$$\frac{1}{3}\left[(1 - \cos x)^2 + (1 - \cos(x - \frac{2\pi}{3}))^2 + (1 - \cos(x - \frac{4\pi}{3}))^2 \right] = 1.5$$



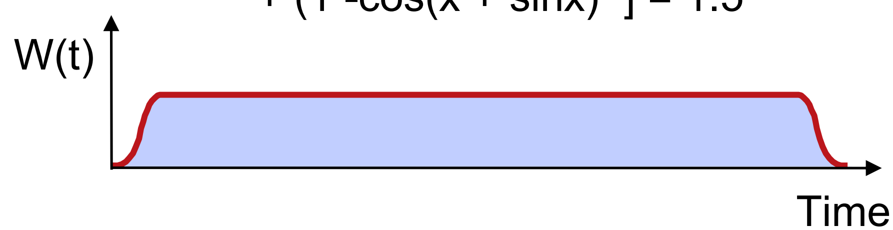
- 50% overlap

$$1/2\ [(1 - \cos x)^2 = (1 + \cos x)^2\ ]\ 1 = \cos^2 x$$



- 75% overlap

$$1/4\ [\ (1 - \cos x)^2 + (1 - \sin x)^2 + \cos x)^2 + (1 - \cos(x + \sin x)^2\ ] = 1.5$$



Window Summary

Window	Noise Bandwidth	Ripple	Highest Sidelobe
Rectangle	1.0 ΔF	3.92 dB	-13 dB
Hanning	1.5ΔF	1.42 dB	-31 dB
Kaiser-Bessel	1.8ΔF	0.98 dB	-68 dB
Flat Top	3.8ΔF	0.009 dB	-93 dB

FFT Analysis 101

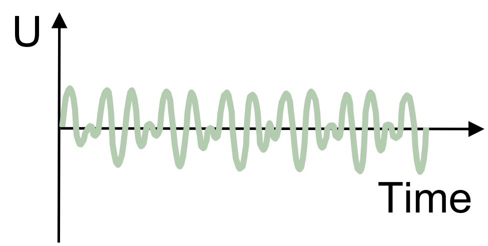
- Introduction
- Practical Set Up of FFT Analysers
- Pitfalls of an FFT Analyser
- Real-time Analysis
- Time Weighting
- Overlap Analysis
- Signal Types and Spectrum Units
- FFT Summary

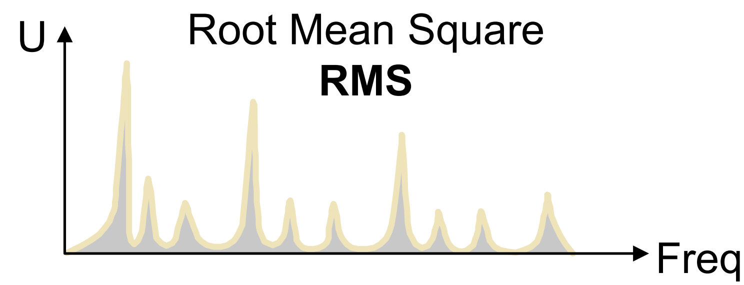
Signal Types and Spectrum Units

Correct use of Units

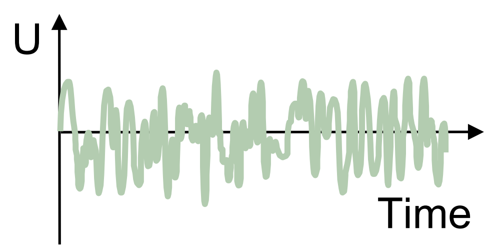
Time | **Frequency**

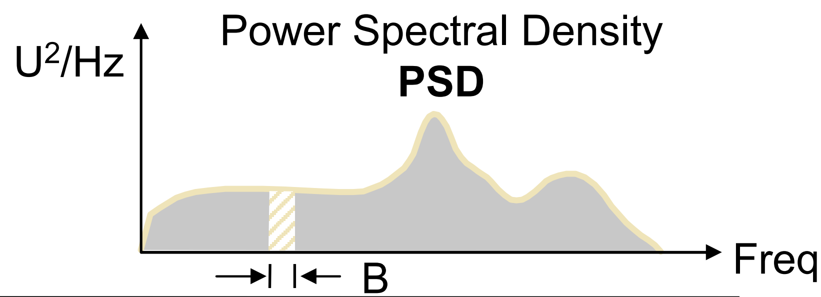
- Determistic, Periodic Signal



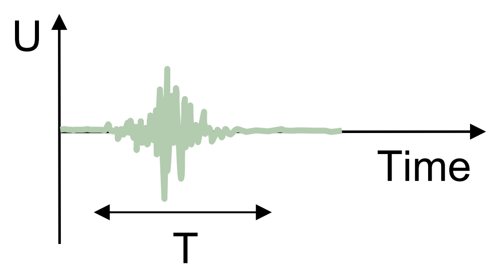


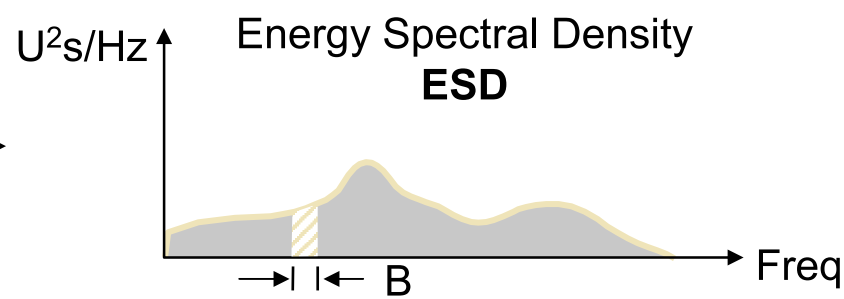
- Random Signal





- Transient





FFT - Summary

The Discrete Fourier Transform:

- The DFT has properties very similar to the integral Fourier Transform
- The DFT has certain pitfalls: aliasing, leakage and picket fence effect
- Recording time, sampling interval, sampling frequency, frequency span and frequency resolution are all related

Weighting functions, leakage and picket fence effect:

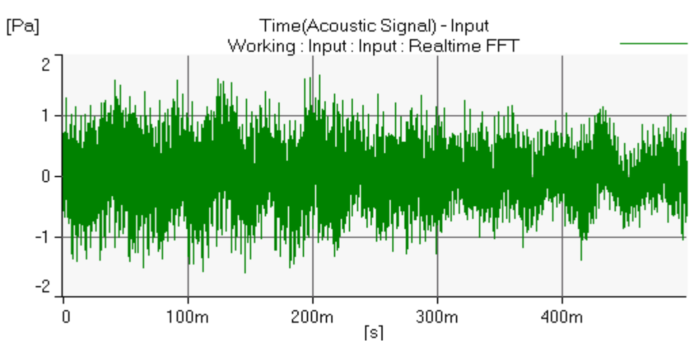
- Many different weightings exist for different purposes
- Use of the proper weighting can reduce leakage and picket fence effect errors
- Weightings can be regarded as filters

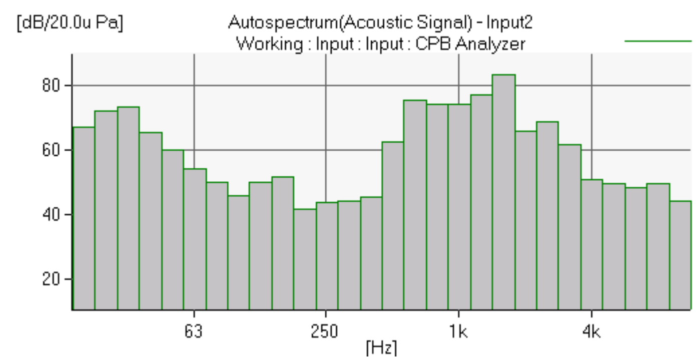
Real-time Analysis, Overlap Analysis and Triggering:

- Condition for real-time analysis: $T \geq T_{calc.}$
- Other weightings than rectangular may require overlap analysis to avoid loss of data or to get a flat overall weighting function
- Many different trigger functions exist for different purposes

CPB Advantages & Disadvantages

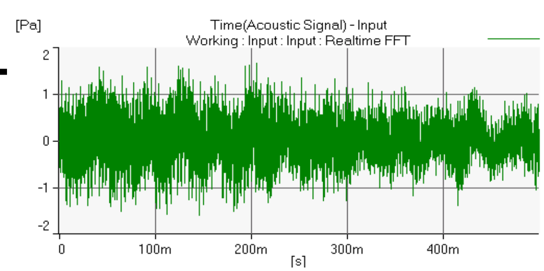
Pros	Cons
Excellent response time	Limited, but optimized freq. resolution (1/1,1/3,1/12,1/24)
Can measure ‘peak’, ‘RMS’	Stereotyped as an acoustics analyzer
Internationally standardized	Not as common as FFT in North America
Results are consistent	Higher Processor Demands
Optimized , but limited freq. resolution	

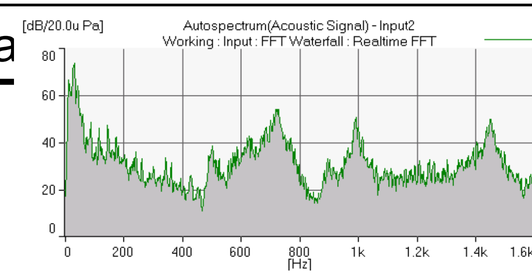




FFT Advantages & Disadvantages

Pros	Cons
High freq. resolution !ZOOM!	Poor response time
Wide selection of averaging types	Block time analysis
Constant bandwidth filters, make harmonic patterns obvious	Leakage
Very common	Windows
Great for ‘exact’ freq. determinations	No true ‘peak’ detection
Lower Processor Demands	“Ambiguous” results
	Not sta





Literature for Further Reading

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  (Brüel & Kjær Theory and Application Handbook BT 0007-11)
- The Fast Fourier Transform by E. Oran Brigham
  (Prentice-Hall, Inc. Englewood Cliffs, New Jersey)
- The Discrete Fourier Transform and FFT Analyzers by N. Thrane
  (Brüel & Kjær Technical Review No. 1, 1979)
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- Dual Channel FFT Analysis by H. Herlufsen
  (Brüel & Kjær Technical Review No. 1 & 2, 1984)
- Windows to FFT Analysis by S. Gade, H. Herlufsen

  (Brüel & Kjær Technical Review No. 3 & 4, 1987)
- Who is Fourier?

  (Transnational College of LEX, April 1995)

Questions

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