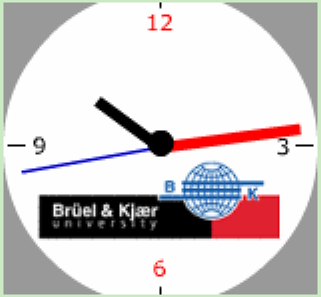
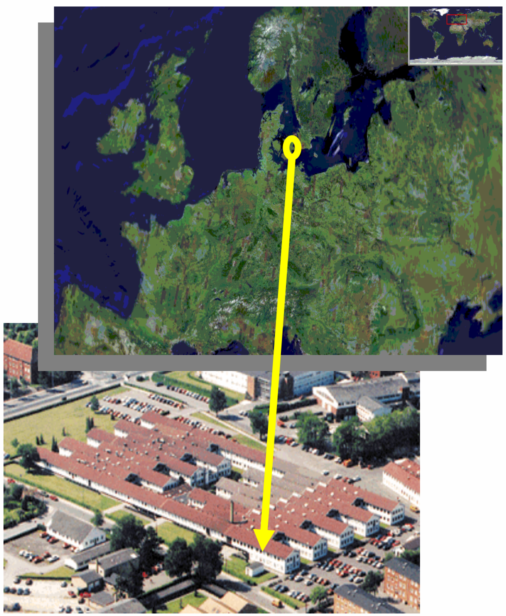
FFT analysis



Central European Time



Brüel & Kjær Headquarter, Nærum
Denmark

WebEx
21 August 2007

9:00 CET
&
14:00 CET

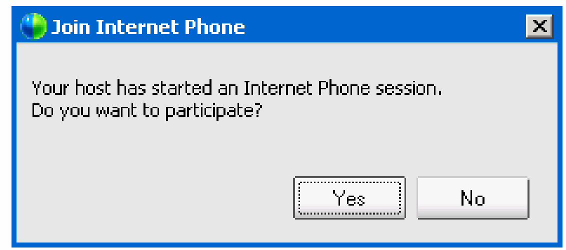
Svend Gade
Brüel & Kjær

To hear the sound:

Use headphones or loudspeakers connected to PC
(sound via Voice over Internet Phone - VoIP). Listening only.

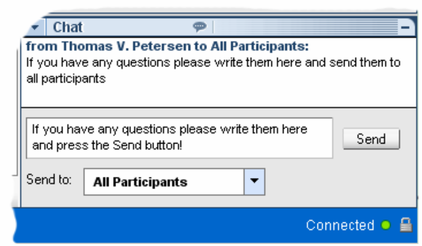
To join Internet Phone:

You will be prompted when the host has started the conference. Just click yes.



To ask questions or make comments

use the Chat window.
Note: I have limited time for online answers.



Copy of the slides will be available after a few days

Brüel & Kjær

Svend Gade

- M.Sc. In Acoustics, DTU 1973
- Brüel & Kjær since 1980
- Author of many articles & papers
- Application Specialist and Associate Professor at

Brüel & Kjær university

FFT Analysis

- Introduction
- Fourier Uncertainty Principle
- Discrete Fourier Transform, DFT
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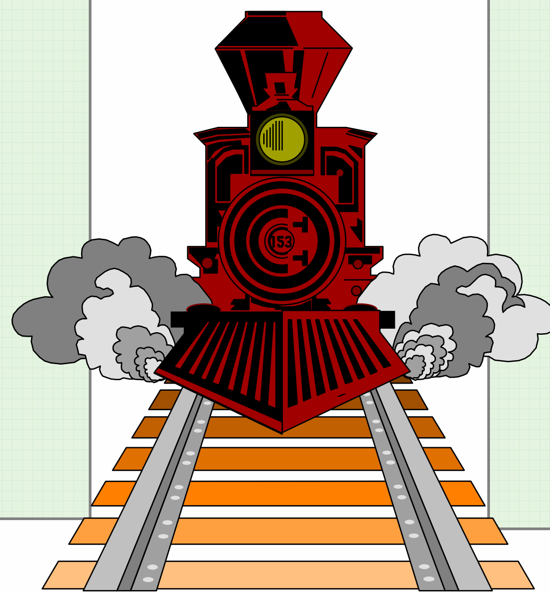
- Real-time Analysis
- Time Weighting
- Overlap Analysis
- Signal Types and Spectrum Units
- Triggering
- FFT Summary

Frequency Analysis Concept

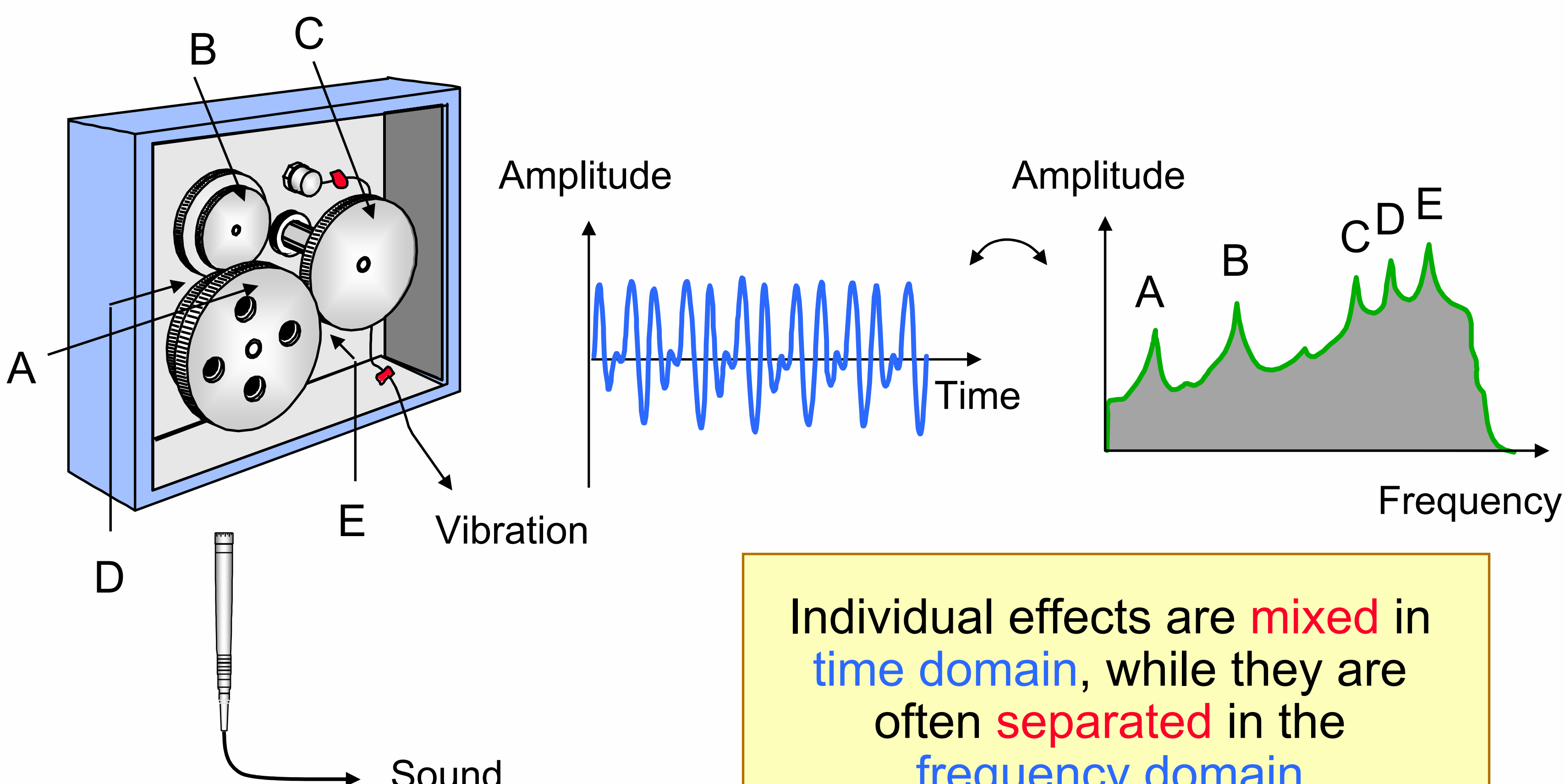
Train time table

Time (When?)	Frequency (How often?)
⋮ 06:10 06:30 06:50 07:10 07:30 07:50 08:10 ⋮	Three times per hour starting 10 minutes past the hour



Why Make a Frequency Analysis

Physical World ⟷ Numeric Description of sound and vibration



Individual effects are mixed in time domain, while they are often separated in the frequency domain

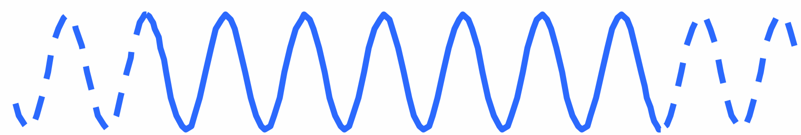
The Fourier Transform

$$G(f) = \int_{-\infty}^{+\infty} g(t)\, e^{-j2\pi f t}\, dt$$

$$g(t) = \int_{-\infty}^{+\infty} G(f)\, e^{j2\pi f t}\, df$$

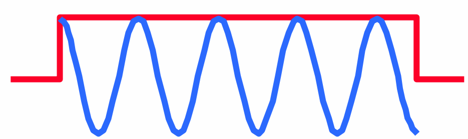
FFT Time Limitation

- Input signal

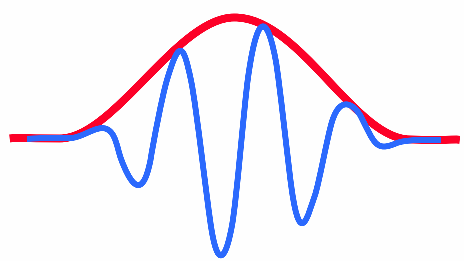


- Analyzed signal

Rectangular or Uniform weighting



Hanning weighting



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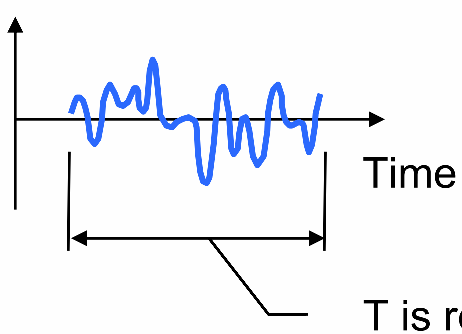
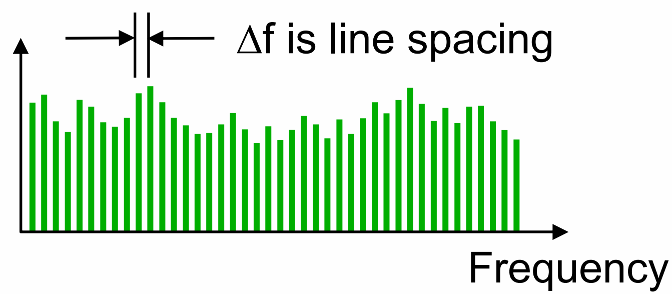
Uncertainty Principle

$$\boxed{\Delta t \cdot \Delta f \geq 1}$$

For FFT analysis $T \cdot \Delta f = 1$





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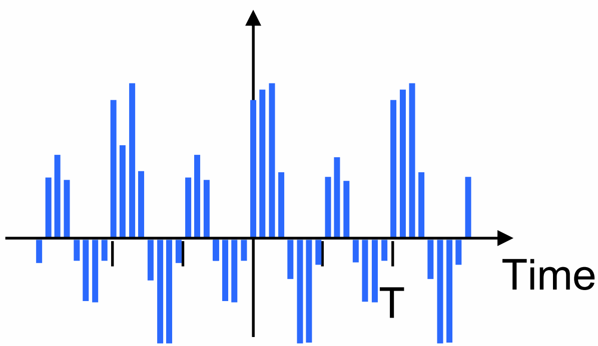
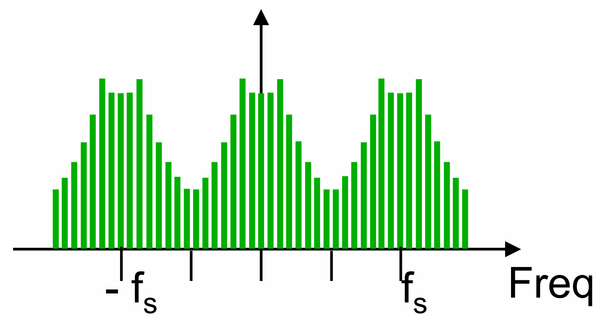
Discrete Fourier Transform

Discrete and periodic in both time and frequency domain





T is record length and

f_s is the sampling frequency

N time samples convert into

N frequency samples

Pitfalls in DFT

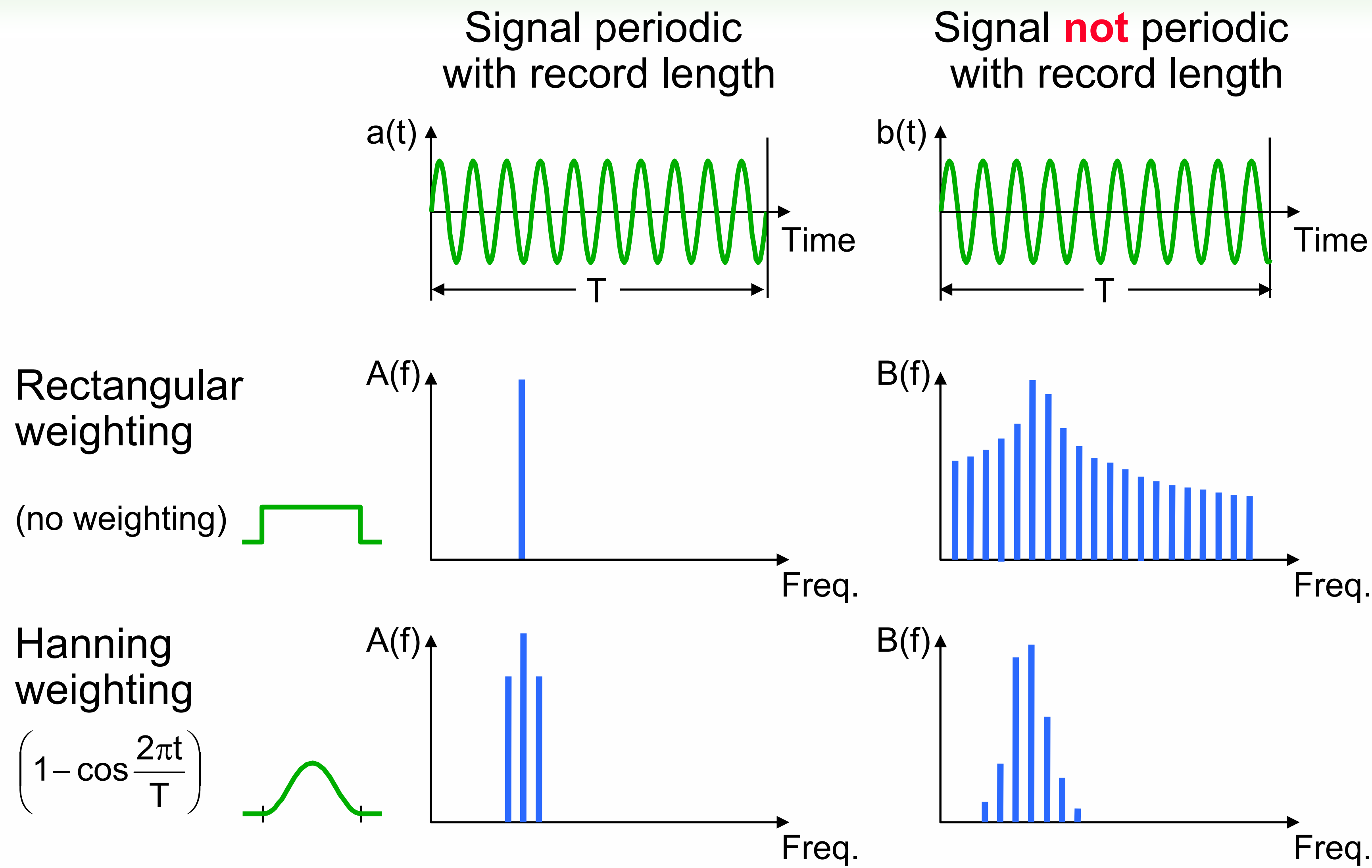
Time ⟷ Frequency

Sampling gives Aliasing

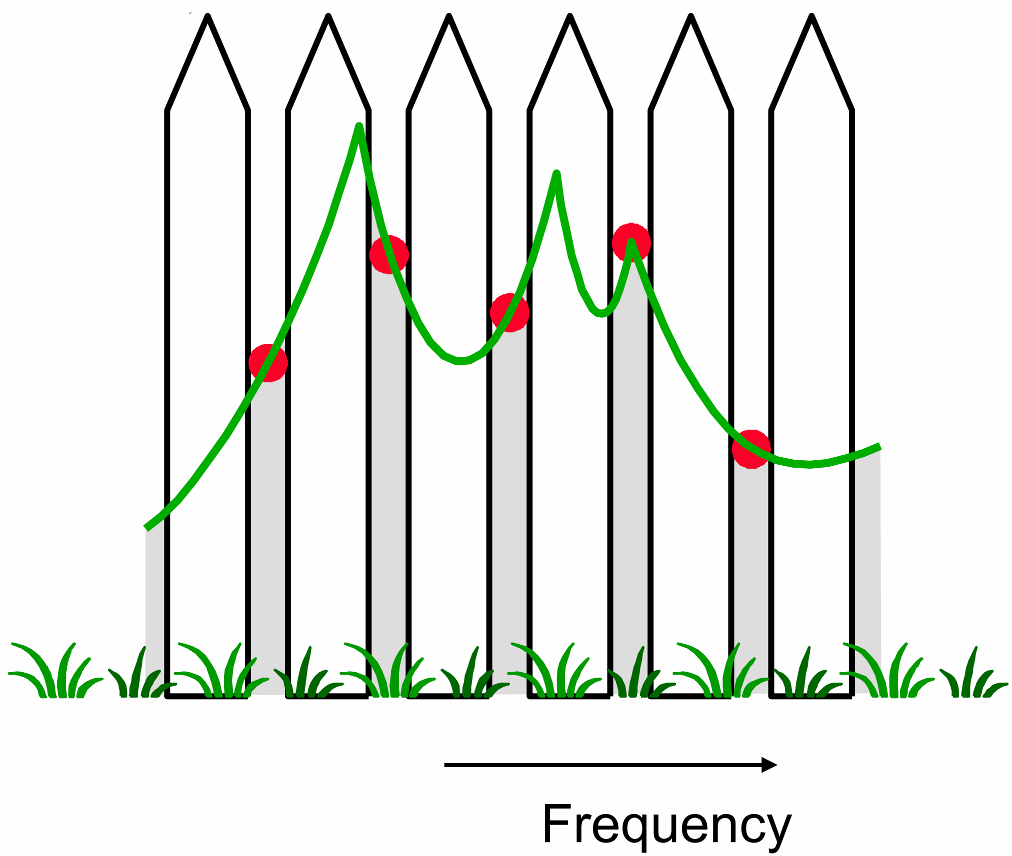
Time limitation gives Leakage

Freq. sampling gives Picket Fence Effect

Leakage


Signal periodic with record length
Signal **not** periodic with record length
a(t)
b(t)
Time
Time
T
T
Rectangular weighting
(no weighting)
Hanning weighting
$\left(1-\cos\frac{2\pi t}{T}\right)$
A(f)
B(f)
A(f)
B(f)
Freq.
Freq.
Freq.
Freq.

“Picket Fence” Effect



How to Avoid the Pitfalls of DFT

1. Aliasing: Caused by sampling in time

   Solution:
   - Use anti-aliasing filter (f_c) and sampling rate $f_s > 2\ f_c$

2. Leakage: Caused by time limitation

   Solutions:
   - Use correct weighting (signals)
   - Increase the frequency resolution (systems

3. Picket fence effect: Caused by sampling in frequency

   Solutions:
   - Use correct weighting (signals)
   - Increase the frequency resolution (systems

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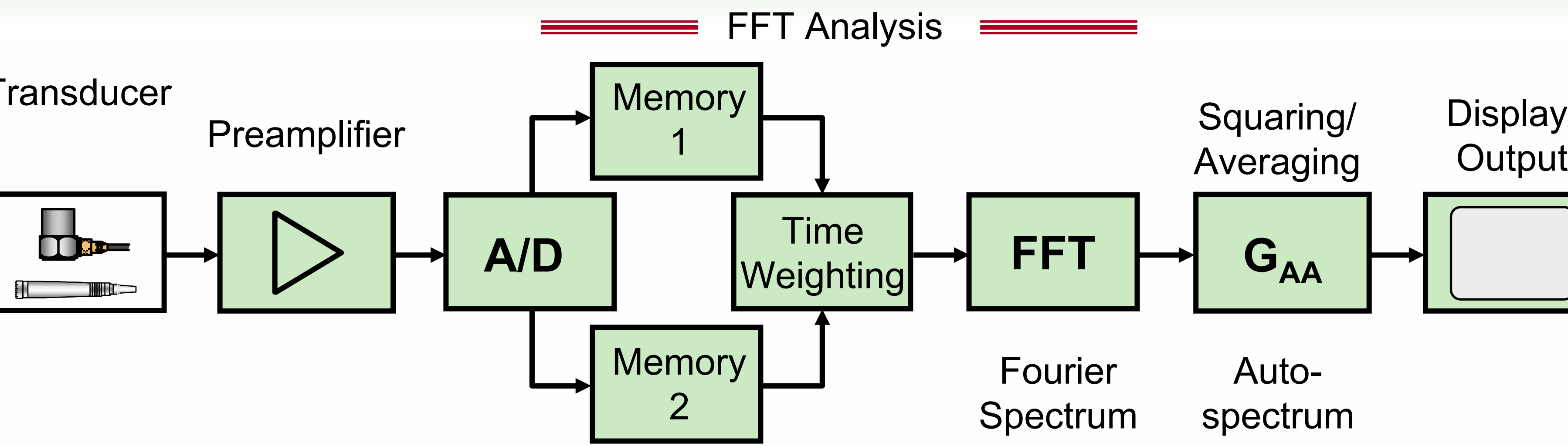
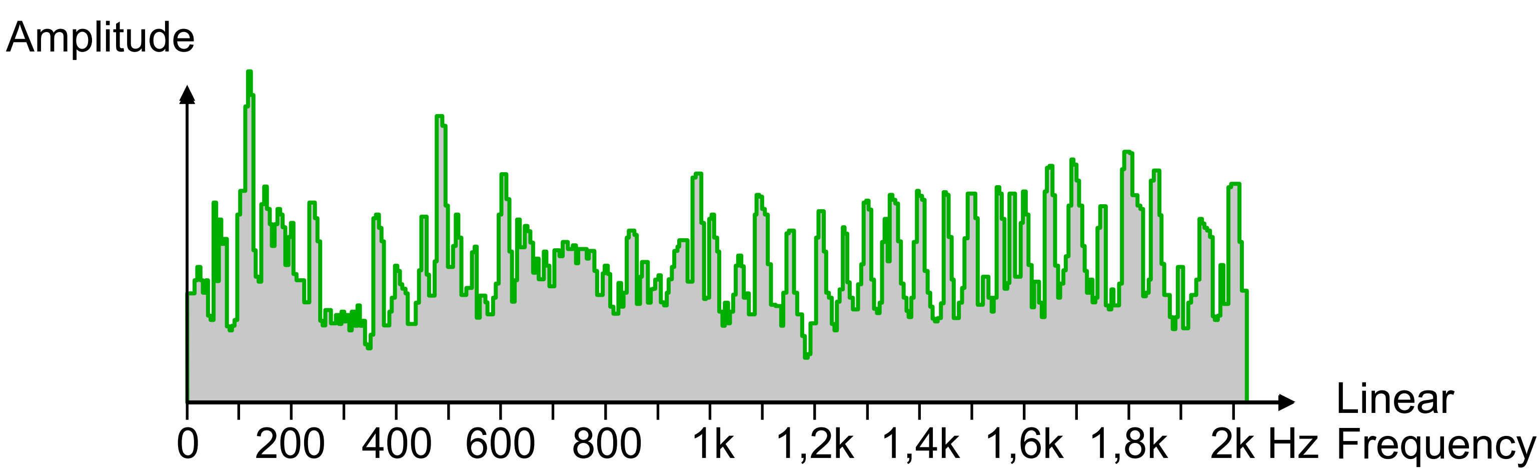
What is the Fast Fourier Transform

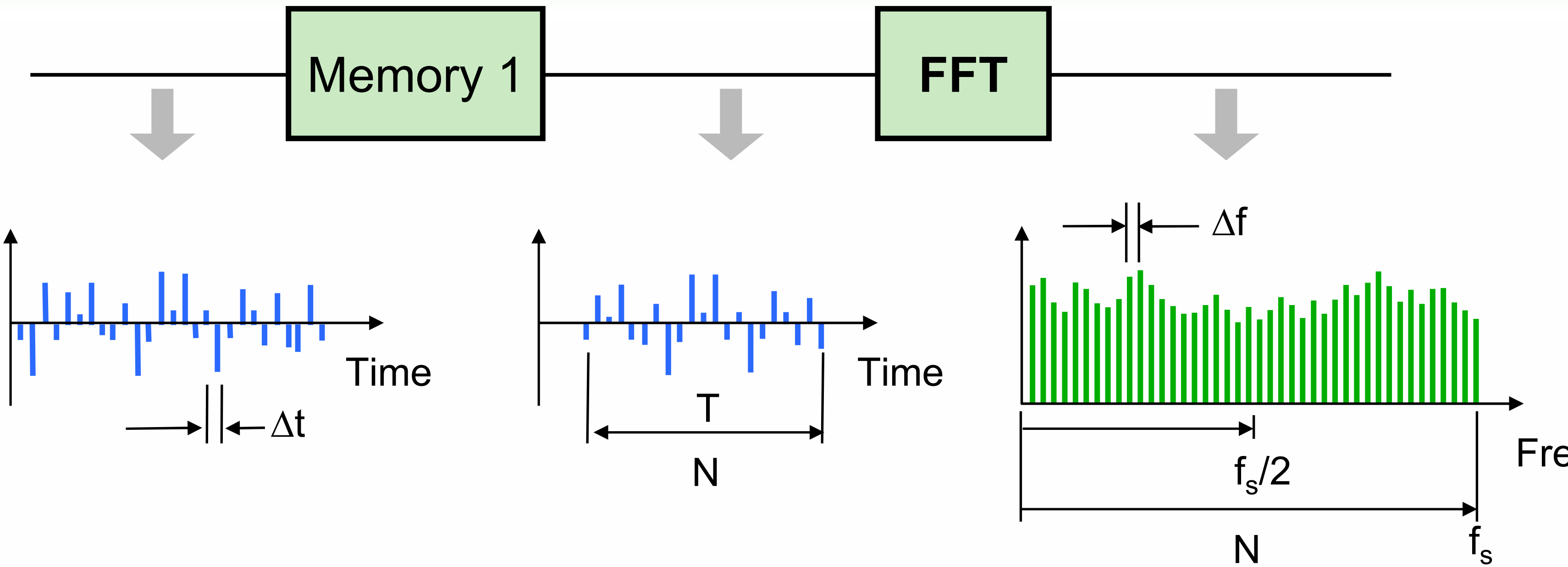
- An **algorithm** for increasing the speed of the computer calculation of the Discrete Fourier Transform.
  - Reduces the number of multiplications from N^2 to $(N/2)\log_2 N$
  - Computation speed increased by a factor of 372 for an 800 line FFT
- Block analysis of time data samples to provide equivalent frequency domain description
- Analysis with constant bandwidth filters
- “Rediscovered” in 1962 by Bell Lab scientists Cooley and Tukey

FFT Analyzer





Parameter Relations



$$T = N \times \Delta t$$

$$\Delta t = \frac{1}{f_s}$$

$$f_s = N \times \Delta f$$

$$\Delta f \times T = 1$$

N = Number of samples
T = Record time
Δt = Sampling interval
f_s = Sampling frequency
Δf = Frequency resolution

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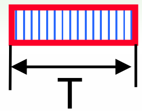
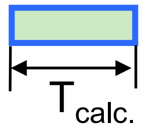
Real-time Analysis with FFT

Recording time:



Calculation time:

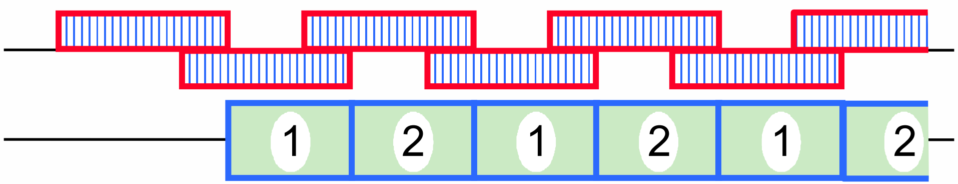


Real-time requirement

$T \geq T_{calc.}$

- Recording time $T \geq$ Calculation time $T_{calc.}$

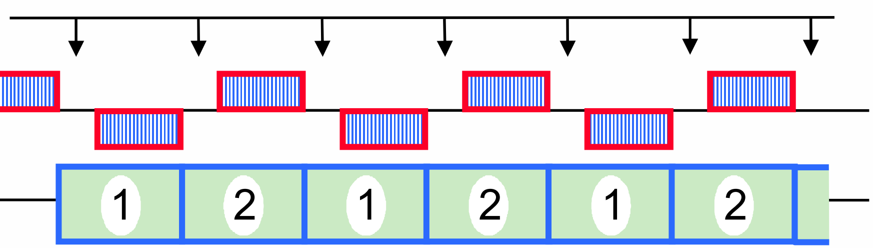
Memory 1
Memory 2
FFT analysis



No data loss = real-time processing

- Recording time $T <$ Calculation time $T_{calc.}$

Data loss
Memory 1
Memory 2
FFT analysis



Data loss = processing out of real-time

Use of Real-time Analysis

Real-time Analysis is **not** necessary for:

- Stationary signals
- Transients
  (if recording is in real-time)

Real-time Analysis is **is** necessary for:

- Non-stationary signals
  Examples:
  - Fast run-up/coast-down tests
  - Reverberation time measurements
  - Vehicle by-pass noise
  - Fly-over noise

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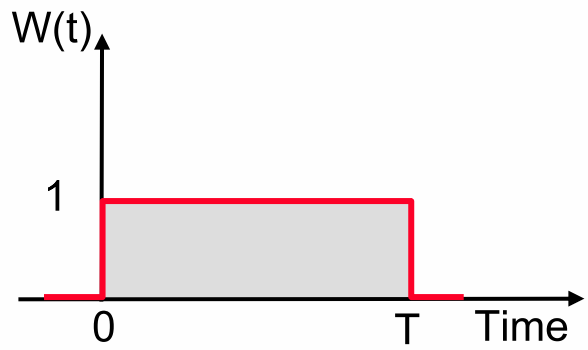
FFT Lines and Use of Weighting Functions

Interpretation

- The FFT memory represents one period of a periodically repeated signal. A smooth weighting function eliminates discontinuities at the beginning and end of the record
- Frequency components in the signal are convolved by the filter characteristics of the weighting functions. The spectrum is sampled at multiples of Δf from 1 to N
- Each line represents the output of a filter/detector centred at the FFT lines. The Filter characteristics are given by the weighting function

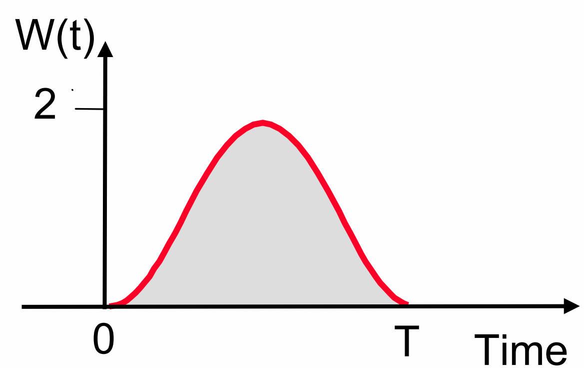
Time Weightings

Rectangular Weighing



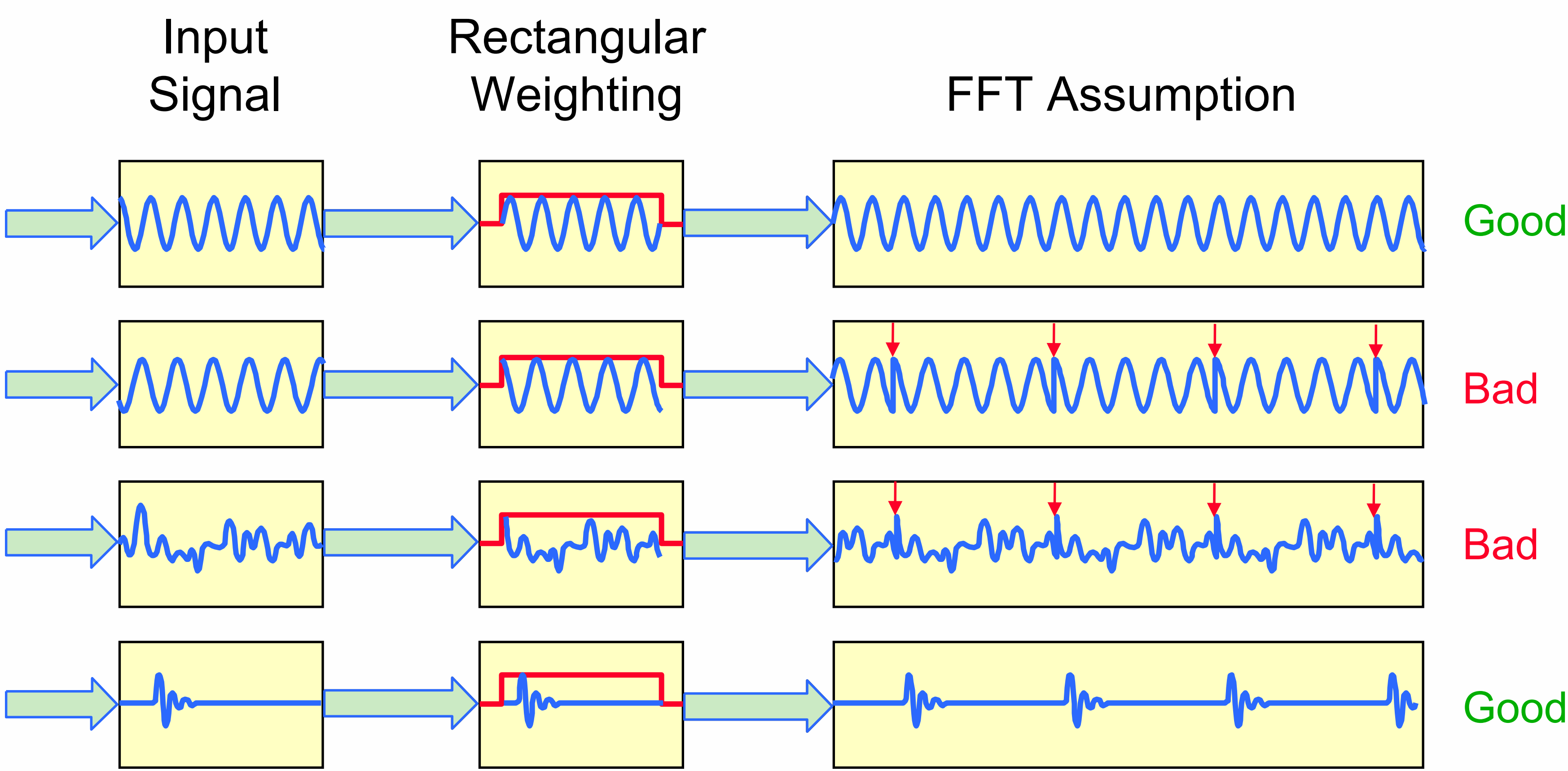
$$W(t) = 1 \; ; \; 0 \le t < T$$

Hanning Weighting



$$\begin{aligned} W(t) &= 1 - \cos\frac{2\pi t}{T}; \; 0 \le t \le T \\ &= 2\cos^2\frac{\pi t}{T}; \; 0 \le t \le T \end{aligned}$$

Rectangular Weighting in FFT



- Use Rectangular weighting when analyzing transients

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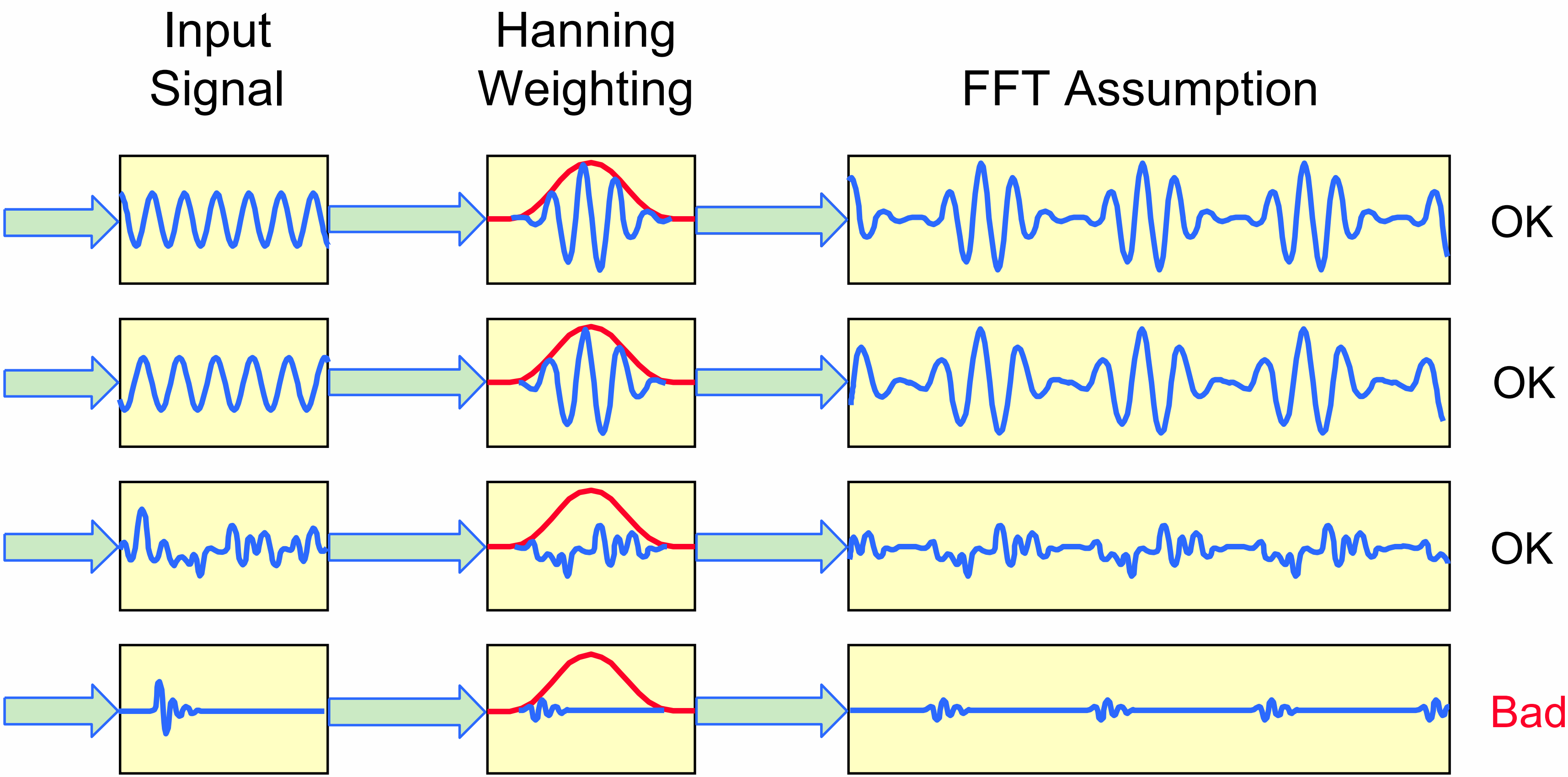
Hanning Weighting in FFT



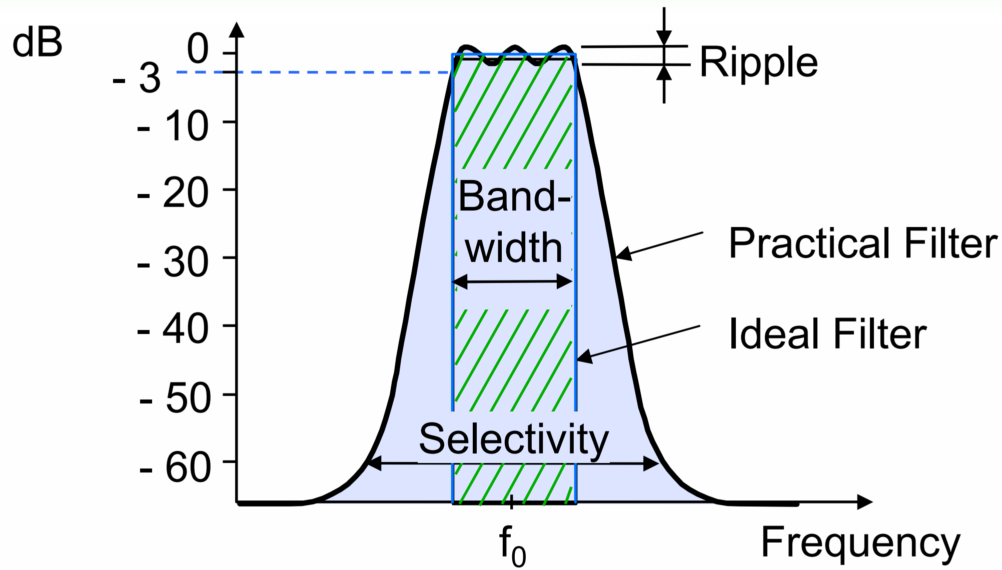
- Use Hanning weighting when analyzing continous signals

FFT Lines and Use of Weighting Functions

Interpretation

- The FFT memory represents one period of a periodically repeated signal. A smooth weighting function eliminates discontinuities at the beginning and end of the record
- Frequency components in the signal are convolved by the filter characteristics of the weighting functions. The spectrum is sampled at multiples of Δf from 1 to N
- Each line represents the output of a filter/detector centred at the FFT lines. The Filter characteristics are given by the weighting function

Filter Characteristics

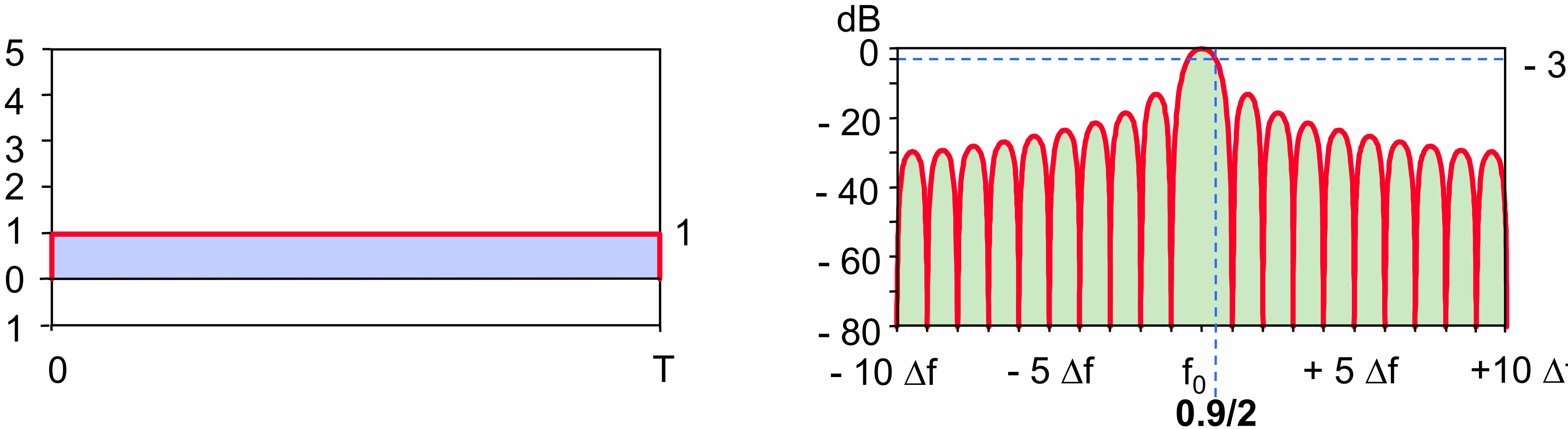


Four parameters:

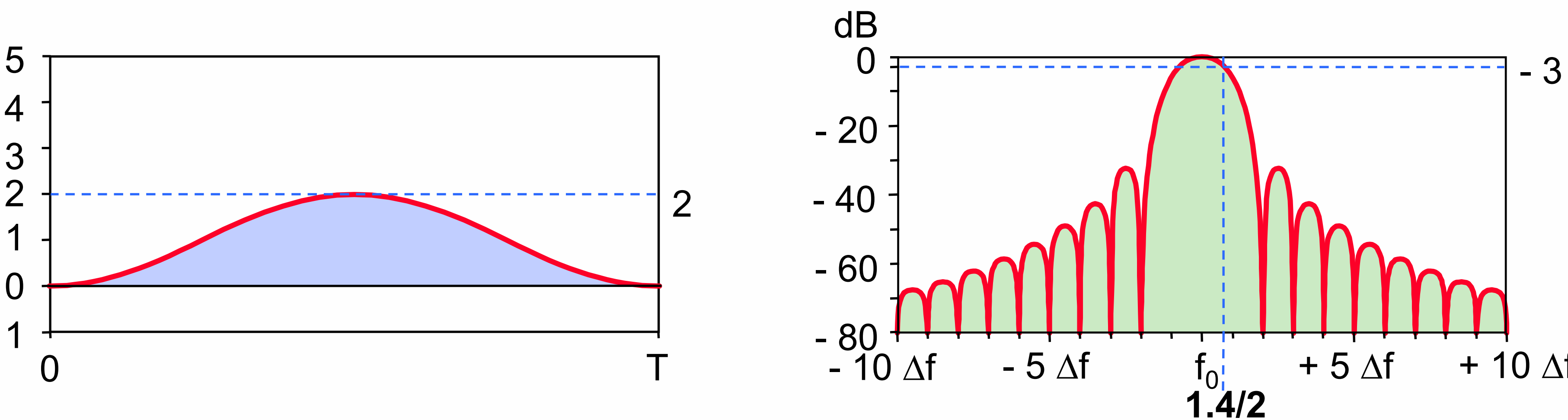
- Centre frequency: f_0
- In band ripple
- Bandwidth: B_3 or B_{noise}
- Selectivity: Shape factor = $\frac{B_{60}}{B_3}$

Weightings (1)

Rectangular Weighting



Hanning Weighting



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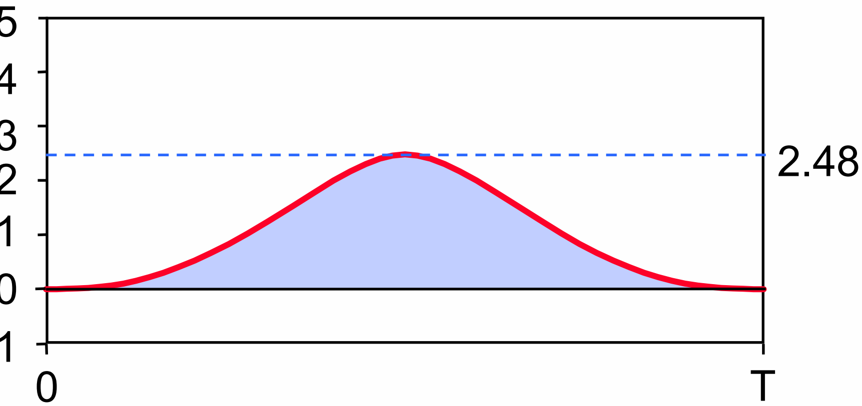
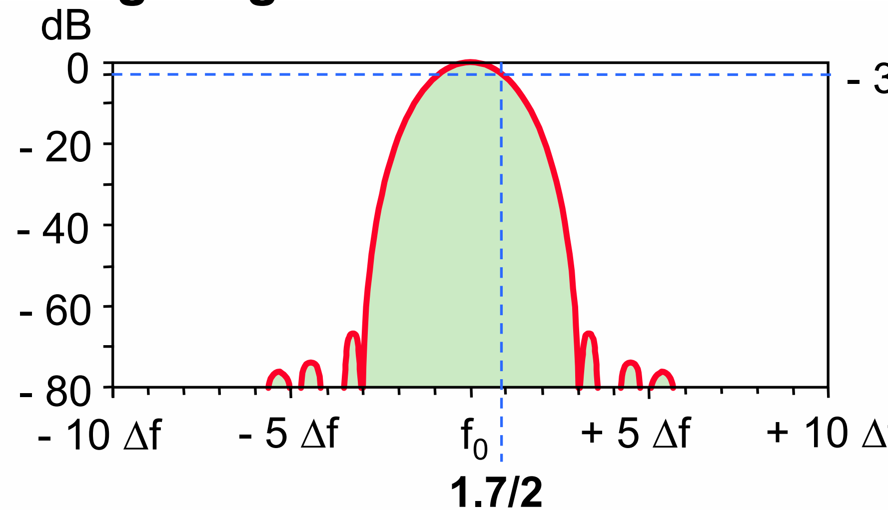
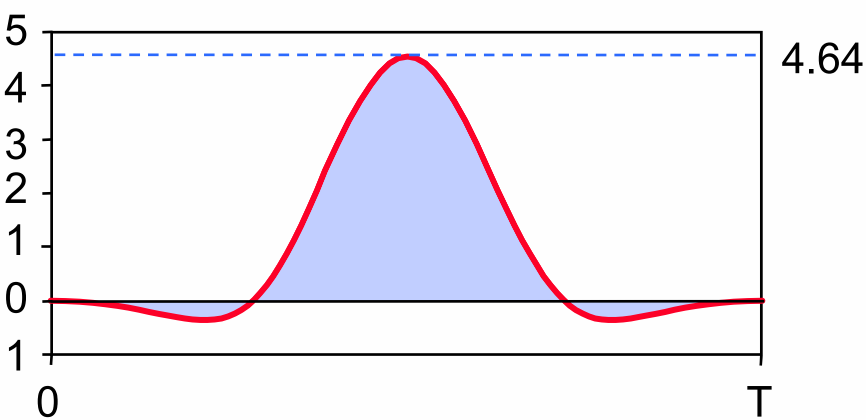
Weightings (2)

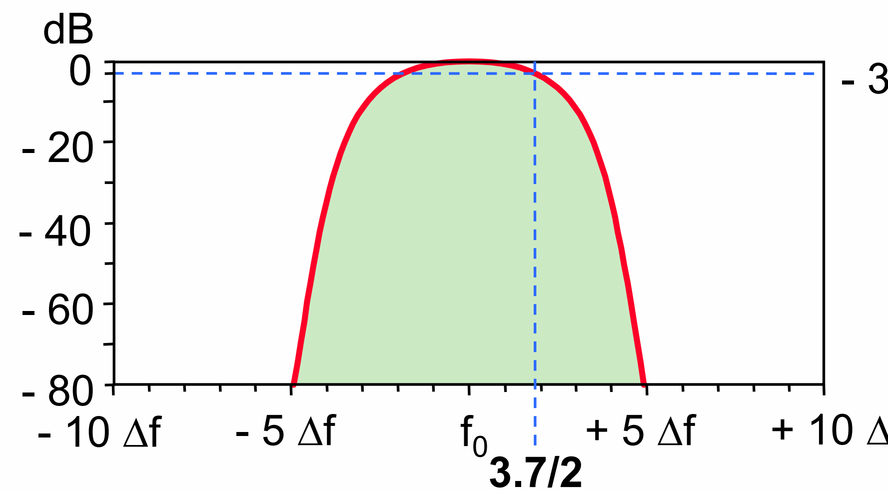
Kaiser-Bessel Weighting





Flat Top Weighting





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Weighting Function Summary

- Bandwidth, frequency accuracy
- Ripple, amplitude accuracy

Weighting	Noise Bandwidth	3 dB Bandwidth	First Zero	Ripple
Rectangular	1.0 Δf	0.9 Δf	1.0 Δf	3.9 dB
Hanning	1.5 Δf	1.4 Δf	2.0 Δf	1.4 dB
Kaiser-Bessel	1.8 Δf	1.7 Δf	3.1 Δf	1.0 dB
Flat Top	3.8 Δf	3.7 Δf	5.0 Δf	0.01 dB

- Selectivity, two-tone separation

Weighting	Highest sidelobe	Sidelobe fall-off rate per decade	60 dB Bandwidth	Shape Factor
Rectangular	- 13 dB	20 dB	665 Δf	750
Hanning	- 31 dB	60 dB	13.3 Δf	9.2
Kaiser-Bessel	- 68 dB	20 dB	6.1 Δf	3.6
Flat Top	- 93 dB	0 dB	9.1 Δf	2.5

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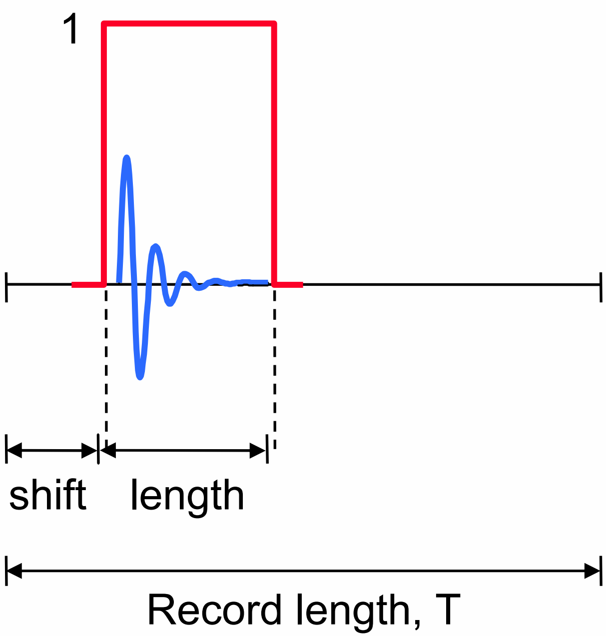
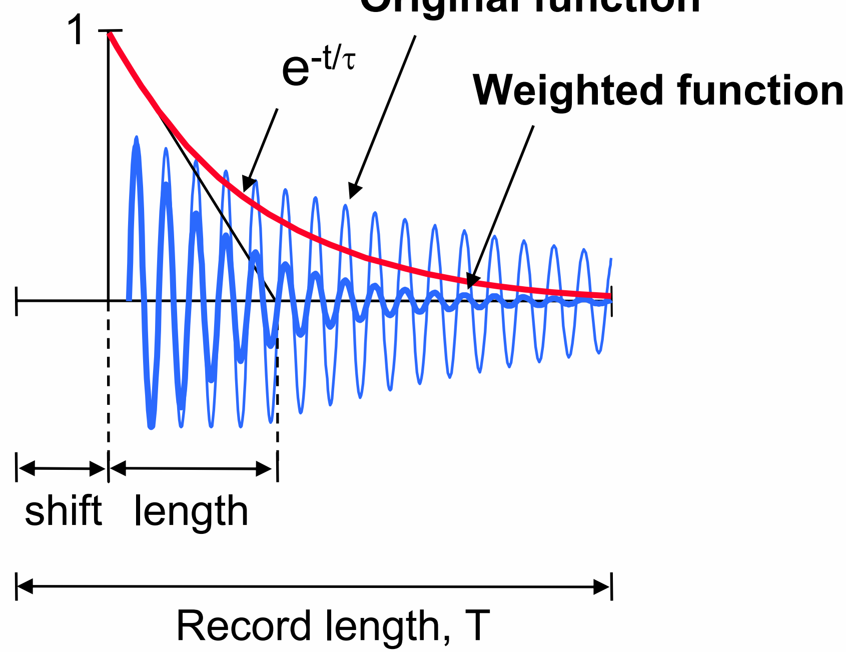
Weighting Functions for Transients

- Transient weighting for short transients



- Exponential weighting for long transients



Use of Weighting Functions in *Signal* Analysis

	Weighting					
	Rect-angular	Hanning	Transient	Expo-nential	Kaiser-Bessel	Flat Top
Transients:						
• General purpose	✓					
• Short transient			✓			
• Long decaying transients				✓		
• Very long transients		✓ + overlap				
Continuous signals:						
• General purpose, RTA		✓				
• Two-tone separation					✓	
• Calibration						✓
• Pseudo random	✓					

Use of Weighting Functions in *System* Analysis

Excitation	Weighting		
	Rectangular	Hanning	Transient and Exponential
• Impact			✓
• Burst random			✓
• Random impact		✓	
• Random		✓	
• Pseudo random	✓		

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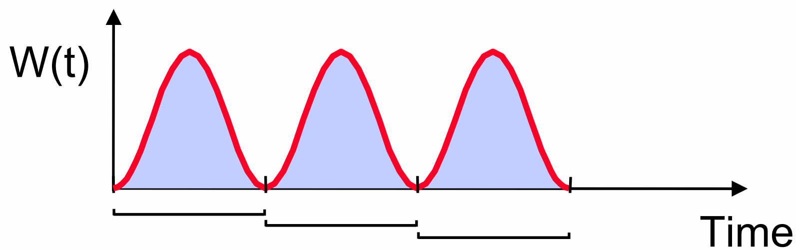
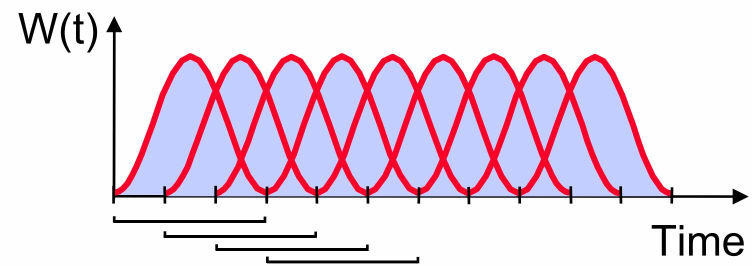
Overlap Analysis with Hanning Weighting (1)

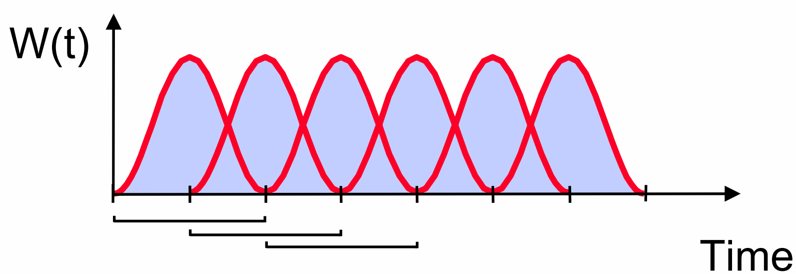
- No overlap



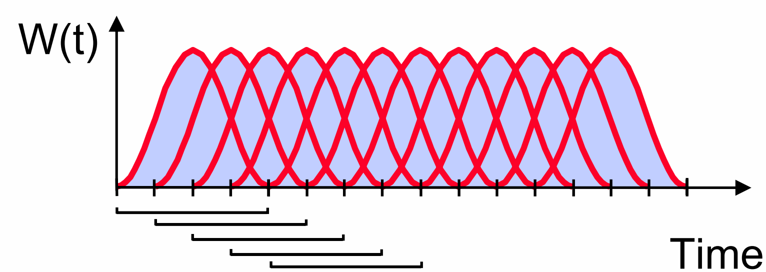
- $66^2/_3$% overlap



- 50% overlap



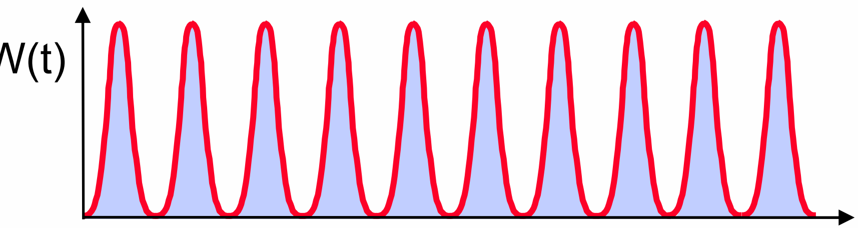
- 75% overlap



Overlap Analysis with Hanning Weighting (2)

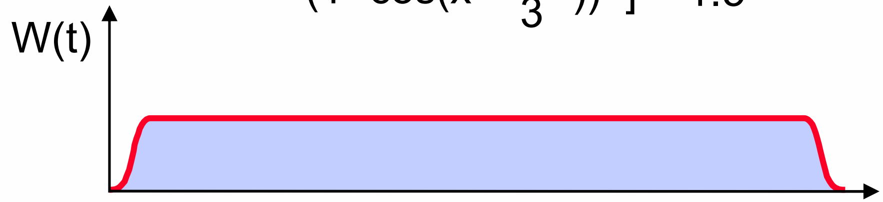
- No overlap

$(1 - \cos x)^2 = 1 - 2\cos x + \cos^2 x$



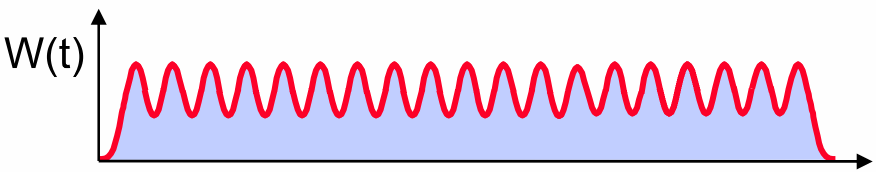
- $66^2/_3\%$ overlap

$$1/3 \,[\,(1 - \cos x)^2 + (1 - \cos(x - \frac{2\pi}{3}))^2 + (1 - \cos(x - \frac{4\pi}{3}))^2\,] = 1.5$$



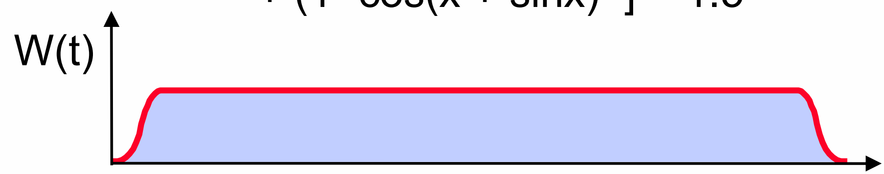
- 50% overlap

$1/2\,[(1 - \cos x)^2 = (1 + \cos x)^2\,]\ 1 = \cos^2 x$



- 75% overlap

$$1/4\,[\,(1 - \cos x)^2 + (1 - \sin x)^2 + \cos x)^2 + (1 - \cos(x + \sin x)^2\,] = 1.5$$



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Application of Overlap Analysis

- Overlap analysis is necessary in order to avoid loss of data when using other weightings than Rectangular
- Analysis with a certain accuracy of a random signal is faster with Hanning weighting and 50 % overlap than with Rectangular weighting
- Equal weighting of all time data is obtained with Hanning weighting and overlap of:
  - 2/3 ($66^2/_3$ %)
  - 3/4 (75 %)
  - 4/5 (80 %)
  - ....

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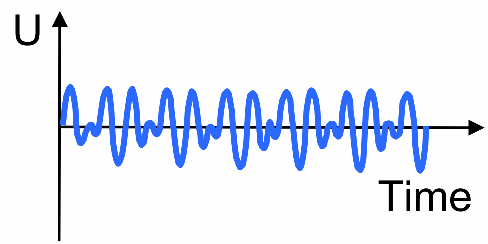
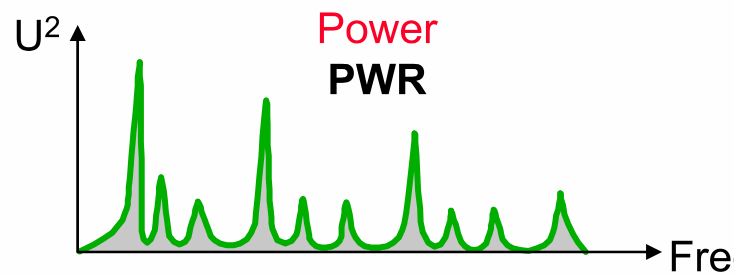
Signal Types and Spectrum Units

Correct use of Units

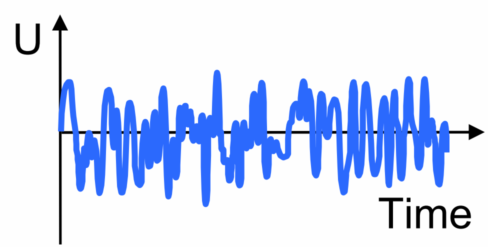
	Time	Frequency

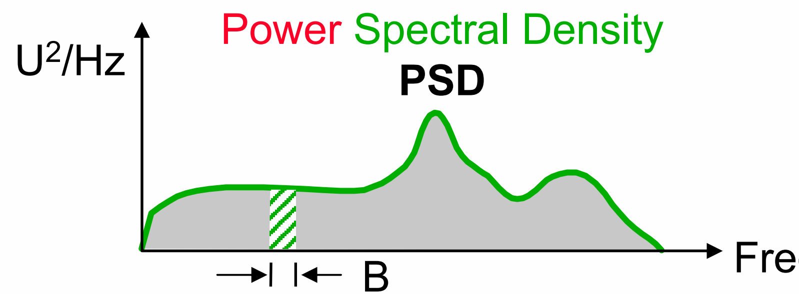
- Determistic, Periodic Signal



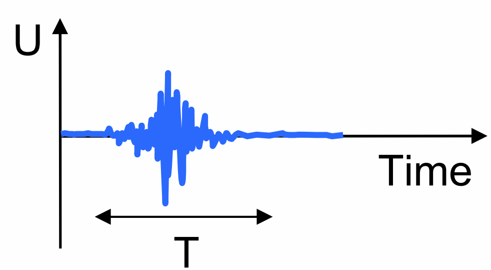


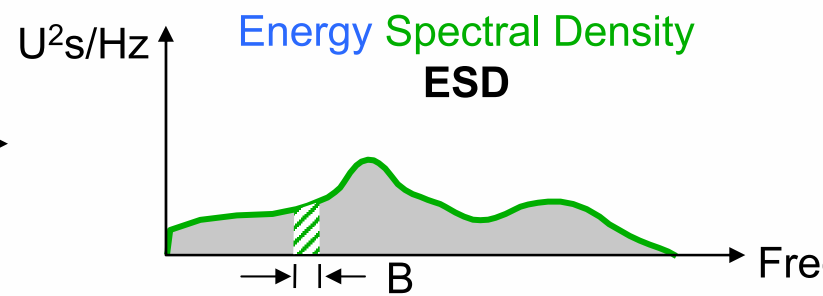
- Random Signal





- Transient



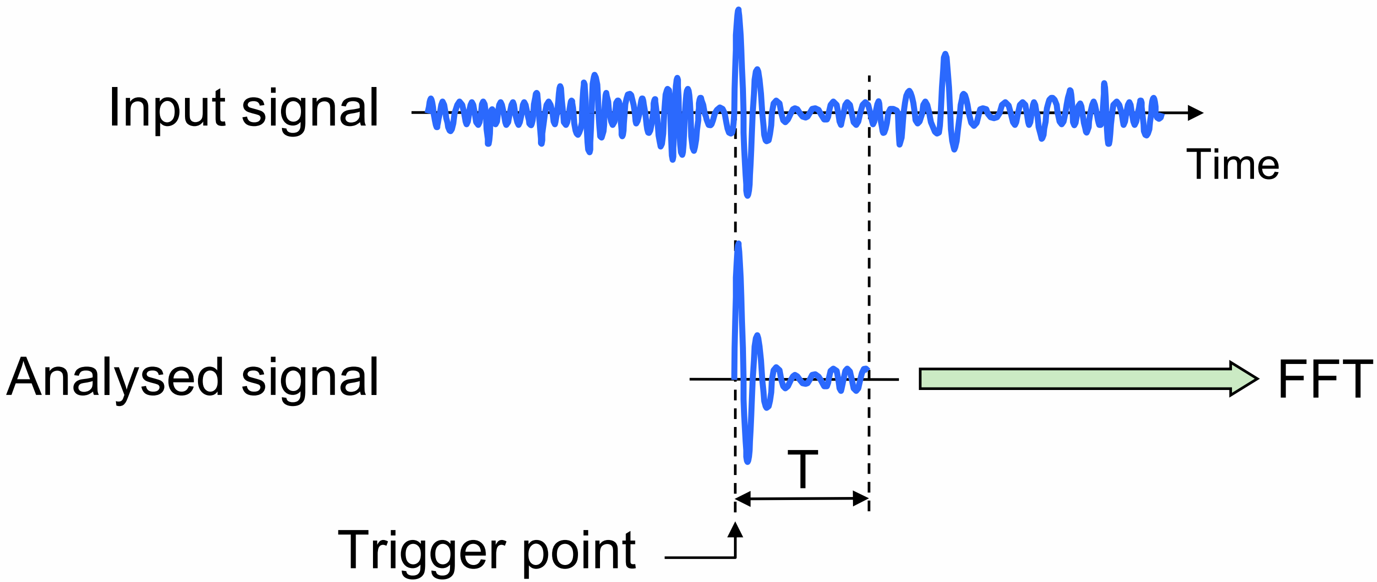


FFT Analysis

- Introduction
- Fourier Uncertainty Principle
- Discrete Fourier Transform, DFT
- Fast Fourier Transform, FFT
- Real-time Analysis
- Time Weighting
- Overlap Analysis
- Signal Types and Spectrum Units
- Triggering
- FFT Summary

Triggering — What is it used for?

— to select the part of the signal to be analysed



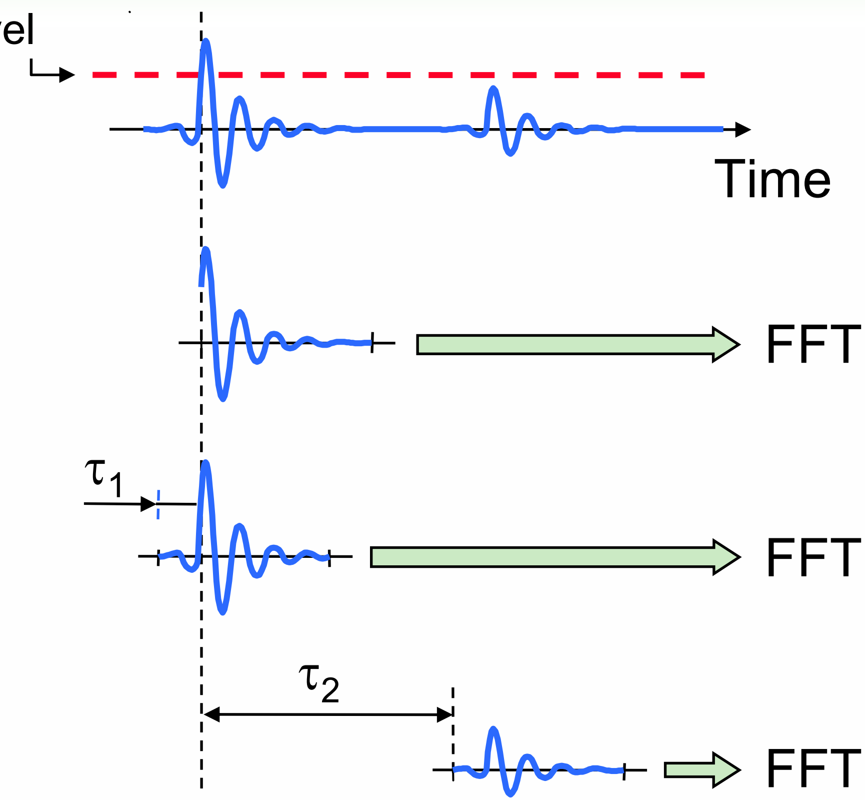
Types of triggers:

- Free Run
- Signal trigger
- Generator trigger
- External trigger
- Manual trigger

Signal Trigger

Input signal

- No delay
- Negative delay, τ_1
- Positive delay, τ_2



Applications:

- For analysis of single events
- For positioning the signal with respect to the time record.

FFT Analysis

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FFT - Summary

The Discrete Fourier Transform:

- The DFT has properties very similar to the integral Fourier Transform
- The DFT has certain pitfalls: aliasing, leakage and picket fence effect
- Recording time, sampling interval, sampling frequency, frequency span and frequency resolution are all related

Weighting functions, leakage and picket fence effect:

- Many different weightings exist for different purposes
- Use of the proper weighting can reduce leakage and picket fence effect errors
- Weightings can be regarded as filters

Real-time Analysis, Overlap Analysis and Triggering:

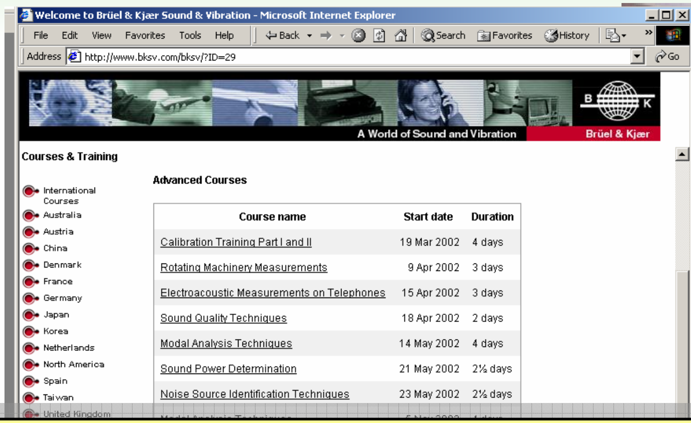
- Condition for real-time analysis: $T \geq T_{calc.}$
- Other weightings than rectangular may require overlap analysis to avoid loss of data or to get a flat overall weighting function
- Many different trigger functions exist for different purposes

Lecture material

- A link to a copy of the presentation will be sent to all participants by e-mail within a few days

Want to know more?

Brüel & Kjær Courses and Seminars



- **Integrated Noise Model:** 30 Oct. – 1 Nov. 07 - 2 days
  - **Objectives** To give an introduction to producing Noise Contours based on flight tracks for Airport Noise Monitoring
- **Advanced Acoustics:** 5 - 6 Nov. 07 - 2 days
  - **Objectives** To give an in-depth description of sound intensity measurements, calibration and its applications
- **Electroacoustic Measurements:** 27 - 28 Nov. 07 - 2 days
  - **Objectives** This course gives an overview of how to test electroacoustic devices such as receivers, loudspeakers etc.

http://www.bksv.com/courses

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