# REVERBERATION TIME IN "DEAD" ROOMS

By Carl F. Eyring

*Bell Telephone Laboratories*

## Introduction

With the advent of radio broadcasting and sound pictures very "dead" rooms have been built, and the significant problem of just how much reverberation should be used in broadcasting and recording presents itself. The direct measurement of reverberation time or its calculation by the aid of a reliable formula, then, is an important aspect of applied acoustics. A reverberation time formula enables one to calculate the reverberation time once the volume, surface area and average absorption coefficient of the surface of the room are known; or if the reverberation time is measured it enables one to calculate the average coefficient of absorption of the surface treatment. A correct reverberation time formula is, therefore, much to be desired.

Theories of reverberation leading to Sabine's reverberation time equation have been given by W. C. Sabine (1900),[1] Franklin (1903),[2] Jaeger (1911),[3] Buckingham (1925).[4] Recently Schuster and Waetzmann (1929)[5] have pointed out that Sabine's formula is essentially a "live" room formula and they have shown as we also show that the reverberation time equation varies somewhat with the shape of the room. The present paper presents an analysis based on the assumption that image sources may replace the walls of a room in calculating the rate of decay of sound intensity after the sound source is cut off, which gives a form of reverberation time equation more general than Sabine's; it points out the difference between the basic assumptions leading to the two types of formulae; it adds experimental data which support the more general type; and it ends with the conclusion that no one formula without modification is essentially all inclusive.

## Reverberation Time Formulae

*Sabine's Formula.* If sound is emitted at a constant rate in a room, the sound energy density will build up till an equilibrium is reached

[1] W. C. Sabine, Collected Papers on Acoustics.

[2] Franklin, Phys. Rev. 16, 372 1903.

[3] Jaeger, Wiener Akad, Ber., Math.-Naturw, Klasse, Bd. 120 Abt. IIa, 1911.

[4] Buckingham, Bur. Standards, Sci. Paper, No. 506, 1925.

[5] Schuster and Waetzmann, Ann. d. Phys. March 1929; also Textbook by Muller-Pouillet, Vol. on Acoustics Chapter VII, pp. 456–460.

between the energy emitted and the energy absorbed. When the source is turned off, this energy density will drop off at a rate depending on the absorbing power of the walls and fixtures. Early in his researches Sabine found "the general applicability of the hyperbolic law of inverse proportionality" between reverberation time and absorbing power as given by the relation

$$T = \frac{KV}{a} \tag{1}$$

where $T$ is the duration of residual sound, $V$ the volume of the room, and $a$ the absorbing power of the walls. Standardizing reverberation time as the time required for the intensity of sound to drop to one millionth of its value, Sabine determined the constant $K$, and obtained in English units,

$$T = \frac{0.05V}{a} = \frac{0.05V}{S\alpha_a} \tag{2}$$

where $S$ is the surface of the room and $\alpha_a$ is the average coefficient of absorption defined by the relation

$$\alpha_a = \frac{s_1\alpha_1 + s_2\alpha_2 + s_3\alpha_3 + \cdots}{s_1 + s_2 + s_3 + \cdots} = \frac{\sum s\alpha}{S} \tag{3}$$

where $s_1, \alpha_1, s_2, \alpha_2$, etc., are the elements of surface and the corresponding absorption coefficients, and $S$ is the total surface. This method of averaging assigns equal weights to the elements of surface, which means that it tacitly assumes a perfectly diffuse condition of the energy density. If, on the other hand, an ordered condition exists, proper weights will need to be assigned to the elements of surface.

Although Sabine tested his reverberation time formula for rooms of various shapes and volumes, ranging from a "small committee room to a theatre having a seating capacity of nearly fifteen hundred," he did not increase the absorption power to the extent that the rooms became "dead," the majority of reverberation times ranging from 4 sec. to 1.5 sec. Thus Sabine did not test his formula in "dead" rooms and no doubt he did not expect it to have a meaning for the extreme case when the average coefficient is unity, for under this condition there can be no hang over of sound and hence no reverberation time, except as one wrongly calls the time for sound to travel from the source to the observer a reverberation time. Yet formula (2) does not become zero but

simply reduces to $T = 0.05V/S$ for this extreme case of absorption, and the reverberation time, strange as it may seem, becomes a function of the shape of the room, and reduces to zero only when $S$ becomes very much greater than $V$. What meaning the equation has for the out-of-doors is certainly very vague. It is evident that the formula fails for this extreme case, and it seems natural to test its validity also for very "dead" rooms.

Recent experiments in the Sound Stage, Sound Picture Laboratory, Bell Telephone Laboratories, indicate the failure of Sabine's formula when the average absorption coefficient for a room is rather high—above 0.5. The results are recorded in another part of this paper.

*The More General Formula.* Although Sabine's formula had its beginning in the experimental study of "live" rooms, it is also derivable from a theorectical study. A careful analysis of the theoretical basis of reverberation theory is, therefore, important in the development of a new formula.

This necessary analysis is aided by the method of images. Just as a plane mirror produces an image of a source of light, so also will a reflecting wall with dimensions large as compared with the wave length of the sound wave produce the image of a source of sound. An image will be produced at each reflection. In a rectangular room, the source images will be discretely located through space. This infinity of image sources may replace the walls of the room, for they will produce an energy density at a point in the room just as if they were absent and the walls were present.

One may picture the building up of the sound as follows. As soon as the source is turned on the infinity of image sources are at that instant all turned on. The walls are imagined removed, and hence at a given place in the former enclosure sound energy will begin to arrive first from the source, then from the first reflection image sources, then from the second reflection image sources, and so on till the energy arrives from the most distant sources.

The decay of sound may be pictured thus. When the source is stopped one may imagine that all the image sources are simultaneously stopped. The first drop will be heralded by the direct wave, then a series of drops by the waves from the first reflection image sources, then drops from the second reflection image sources and so on, and so on, till the contribution of all the remaining image sources is not sufficient to affect the hearing. Thus the decay ends. The effect of all the image

sources is a million times greater than the effect of all those located *beyond* a distance $cT$; this is the meaning of reverberation time $T$ in the new picture, $c$ being the speed of sound in air.

We wish to emphasize that this picture which gives the details of the decay, involving as it does the geometry of the room, the distribution of the absorbing material, the discontinuous nature of the energy decay, and interference phenomena, is fundamental in developing reverberation theory. Just how many details we shall represent by averages depends upon the degree of simplification desired. We shall ignore interference phenomena in the following considerations, and accordingly we shall attempt to make reverberation time meter measurements independent of this effect. We shall, following Sabine's experimental results, assume that $\alpha_a$ defined by equation (3) may be considered as the uniform absorption coefficient of the walls; yet we realize that for an ordered condition of the sound waves this procedure may need to be modified. We shall apply the method of images first to a few special cases involving rooms of simple geometrical form in order that later we may with better understanding approach the general case.

Consider a spherical room of diameter $D$ and volume $V$, with uniform absorption over the surface and with a sound source emitting $\overline{E}$ energy units per second located at its center. In a time $D/2c$ sec. after the source is turned on, the sphere will be filled with energy directly from the source of average density $\overline{E}D/2cV$, and then a first reflection will begin. The reflected wave will travel back to the center, then spread out and in a time $D/c$ sec. after the reflection first started a second reflection will begin. The average density due to the first reflected energy then is $\overline{E}DR/Vc$ where $R$ is the coefficient of reflection defined by the relation

$$R = (1 - \alpha_a). \tag{4}$$

After another $D/c$ sec. interval, the third reflection will begin and the average energy density of the second reflected energy is, $\overline{E}DR^2/Vc$. As time proceeds, at the end of each $D/c$ interval, a new reflection begins and a new increment will have been added to the total average density of the enclosure. Finally equilibrium is established and the total average energy density is given by the infinite sum

$$S_\infty = \frac{\overline{E}D}{Vc}\left(\frac{1}{2} + R + R^2 + R^3 + \cdots R^n + \cdots\right). \tag{5}$$

When the source is turned off, the decay begins. At a point of observa-

tion near the center the energy directly from the source stops, then in $D/c$ seconds the average energy represented by the second term of equation (5) vanishes; in $D/c$ seconds more the third term vanishes, etc., etc. The total average energy at any $D/c$ interval during the decay will be given by the equation
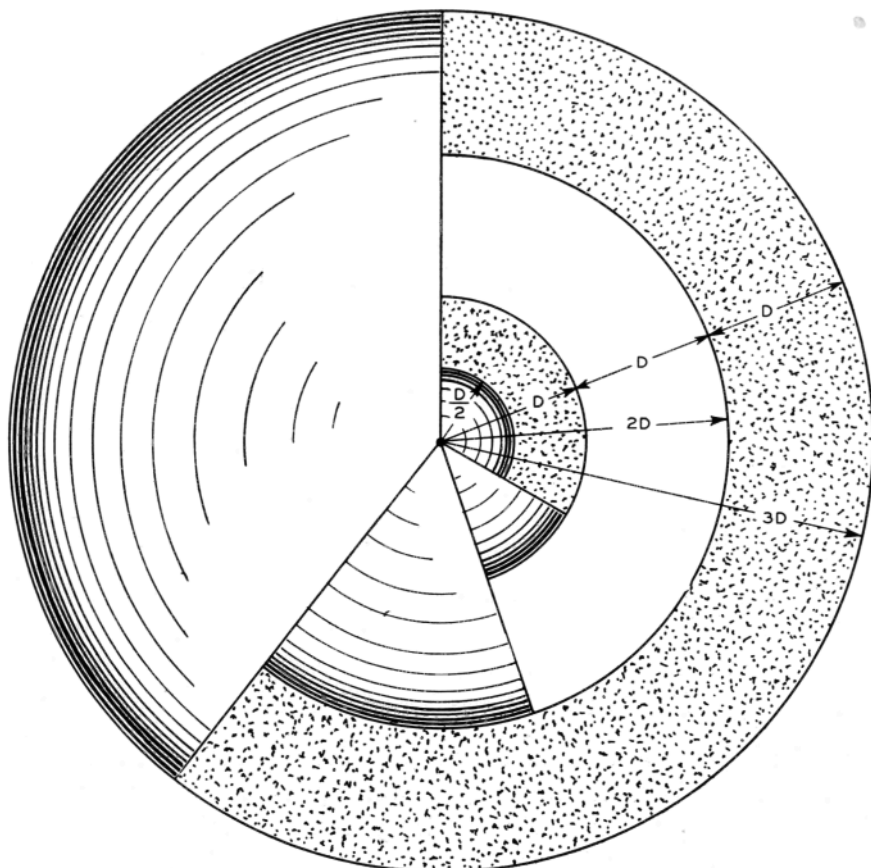


FIG. 1. *Illustrating a source at the center of a spherical room and a series of spherical concentric image sheets.*

$$S_\infty - S_n = \frac{\overline{E}DR^n}{Vc(1-R)} = \frac{6\overline{E}}{S\alpha_a c} \; e^{\dfrac{cS \log_e (1-\alpha_a)t}{6V}} \tag{6}$$

remembering that $n = ct/D$, that $R = (1-\alpha_a)$, and that for a sphere $D = 6V/S$. We must keep in mind that $t$ is not a continuous function but given by, $t = nD/c$ where $n$ is an integer, and also that the absorbing material is assumed uniformly distributed over the surface.

One may interpret the process just described by the aid of the method of images. The walls are imagined removed and image sources are substituted. These images are imagined to be spherical concentric sheets. (See Figure 1). They are separated by a distance $D$, the distance between two successive reflections. The first image sheet contributes to the enclosure the average energy density given by the second term of
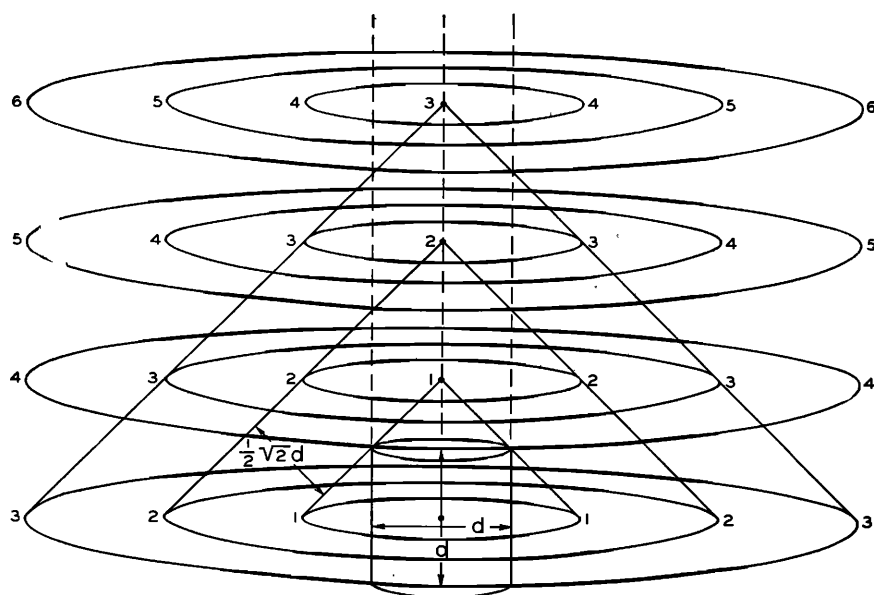


FIG. 2. *Illustrating a source at the center of a cylindrical room and a series of concentric circular line image sources.*

equation (5), the second one the amount given by the third term, etc., etc. When the source stops emitting energy all the image sheets do likewise. The persistence of sound is due to the fact that it takes time for the end of a sound wave which marks the termination of energy emission from a particular image sheet to reach the enclosure. The decay goes down in steps; due to the definition of reverberation time the end of the wave from a sheet located at a distance $cT$ from the source passes thru the enclosure at the instant the intensity reaches one millionth of its original value.

Next consider a cylindrical room with diameter $d$ equal to length, with uniform absorption over the surface and with a source at its center. Using the method of images we note that the source may be imagined imaged in the cylindrical walls as a series of concentric circular

line sources, and in the plane ends as a series of point sources, which in turn are imaged in the projected cylinder as concentric line image sources. (See Figure 2.) The numbers indicate the order of the reflection and if images of the same order are included in a surface, we shall have a series of concentric cones separated by a distance $\frac{1}{2}\sqrt{2}d$. If we identify this as the average distance between reflections[6] we may replace $D$
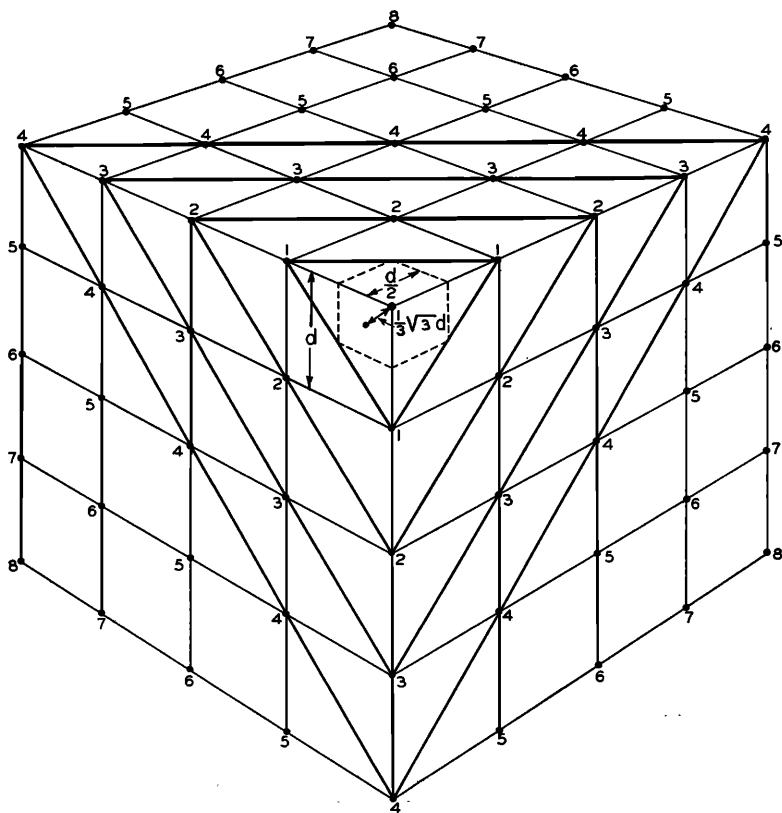


FIG. 3. *Illustrating a source at the center of a cubical room and a series of point image sources.*

of equation (6) by this value, and remembering that $d = 6V/S$ for an enclosure of this sort we rewrite (6) and obtain the following decay equation for a point near the center of this cylindrical room,

$$S_\infty - S_n = \frac{3\sqrt{2E}}{S\alpha_a c} e \frac{cS \log_e (1-\alpha_a)t}{3\sqrt{2}V} . \tag{7}$$

The same method may be applied to a cubical room with the source

[6] Schuster and Waetzman loc. cit. have obtained these values by a more rigorous method.

at its center. The images are point sources and are located discretely throughout space as shown in Figure 3. As before the numbers indicate the order of reflection, and if the image sources of the same order are included in a given surface we get a series of plane surfaces separated by the distance $\frac{1}{3}\sqrt{3}\,d$, where $d$ is the length of the cubical room. Identifying this distance as the average distance between reflections[6] and remembering that for a cube $d = 6V/S$, we have from equation (6) the
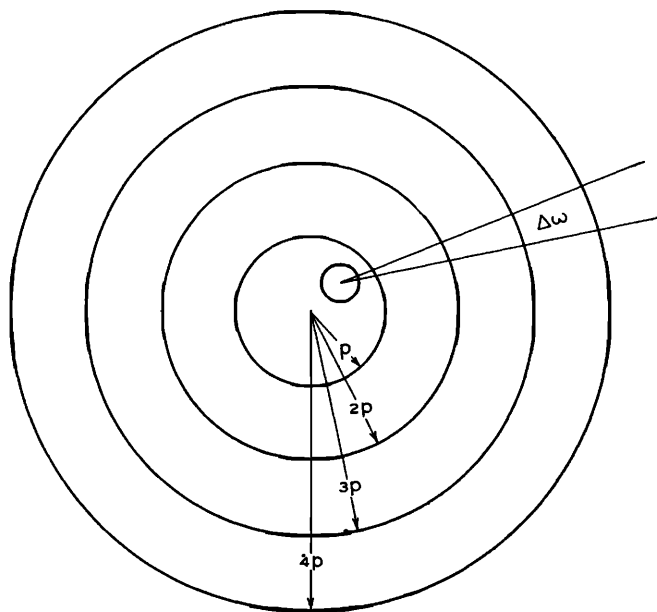


FIG. 4. *Illustrating zones of image sources.*

following decay equation for a point near the center of the cubical room,

$$S_\infty - S_n = \frac{2\sqrt{3E}}{S\alpha_a c}\, e^{\dfrac{cS\log_e(1-\alpha_a)t}{2\sqrt{3}\,V}}.\qquad(8)$$

Formula (7) is valid for a cubical room with a source at its center provided two opposite walls are completely absorbing. The $\alpha_a$ in the coefficient of the exponential term is the average value for the whole room, but the $\alpha_a$ in the exponent of this term is the average value for the four reflecting walls. Again if the cubical room has two pairs of opposite walls completely absorbing equation (6) is valid, but just as before the proper values for $\alpha_a$ must be used.

So far we have considered rooms of very simple geometrical shape and

we have located the source at the center in each case. We now consider the more general case, make the shape more complex, move the source from the center and thus approach a diffuse condition of the waves. In the interest of simplicity we shall be content with average effects and assume that the image sources are discretely located in zones—surfaces of concentric spheres with radii, $p$, $2p$, $3p$, $\cdots np$, $\cdots$ etc., where $p$ is the mean free path between reflections (See Figure 4). The mean free path is the average distance between reflections which the sound establishes on its many trips across the room from wall to wall, from wall to ceiling, from ceiling to floor, etc., as it travels in all conceivable directions. At present, the centers of these spheres are assumed located at the source of sound.

This means that we have replaced the source of sound and the walls of the room, by the source of sound surrounded by image sources located in evenly spaced discrete zones. To determine the energy density produced in the enclosure by any zone, we proceed as in the case of the spherical room and follow through the growth of sound in the room as follows.[7] The rate of energy emission is denoted by $\overline{E}$, $c$ is the speed of sound, $p/c$ is the average interval of time between reflections, and $R$ is the coefficient of reflection already defined. The amount of energy emitted into the room directly from the source during this interval is $p\overline{E}/c$; the amount left over after the first reflection, or the amount emitted into the enclosure by the first zone of image sources during the same interval of time, is $p\overline{E}R/c$; and that emitted in the same time into the enclosure by the $n$th zone is $p\overline{E}R^n/c$. Because of the meaning of the mean free path $p$, it takes just this interval of time for the energy flow to fill the enclosure once it starts to enter it. At once the energy density in the enclosure due to the $n$th zone is

$$E_n = \frac{p\overline{E}}{cV}R^n. \tag{9}$$

It now becomes interesting to determine whether $E_n$ is an average value or is constant for all parts of the enclosure. The sound intensity varies inversely with the square of the distance from the source or its images. The intensity of the direct sound, then simply obeys the inverse square law. To investigate the point to point variation within the enclosure of the sound energy density due to the various zones we proceed as follows. For simplicity neglect differences in phase and assume that

[7] Collected papers, Sabine, p. 43. Vibrating Systems and Sound, Crandall, p. 201.
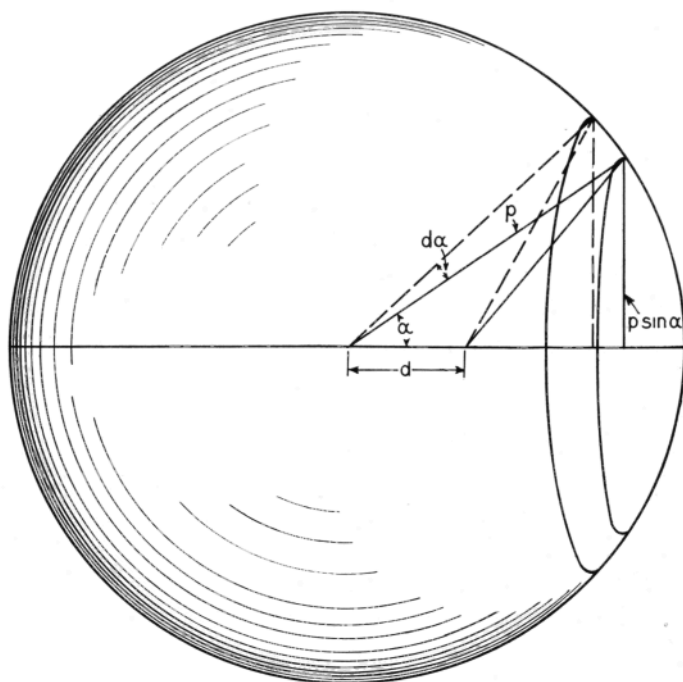
FIG. 5.

the image sources are evenly distributed over the zones. Call the surface density of image sources $i$. Then for the first zone, the radius of the sphere is $p$, and if we let $d$ be the distance of the point of observation from the center of the sphere, then the energy density at the point of observation is given by the relation (See Figure 5),

$$E_1 = M i \int_0^\pi \frac{2\pi p^2 \sin \alpha \, d\alpha}{p^2 - 2pd \cos \alpha + d^2} = \frac{Kp}{d} \log_e \frac{p+d}{p-d} \qquad (10)$$

where $M$ and $K$ are constants.

Now expanding $\log_e (p+d)/(p-d)$ we get

$$E_1 = 2K \left[ 1 + \frac{1}{3}\left(\frac{d}{p}\right)^2 + \cdots \right]$$

and for any zone

$$E_n = 2K_n \left[ 1 + \frac{1}{3}\left(\frac{d}{n_p^i}\right)^2 + \cdots \right]. \qquad (11)$$

For all but the first zone, $(d/np)^2$ is small and $E_n$ is essentially independent of $d$, the distance of the point of observation from the cen-

ter of the sphere. This means that neglecting interference any zone beyond the first, viz. the $r$th zone, contributes approximately the same energy to all points in the enclosure, and therefore, that the energy density produced by this zone is constant throughout the room, and of a magnitude $E_r$ as given by equation (9). However, $E_0$ and $E_1$ are average values, the actual point to point variation of the energy density being obtained for each case by the use of the inverse square law and the method of images. The actual energy density at a point in the enclosure, then, may be divided into three parts: (1) that which is produced by the source only and this varies inversely as the square of the distance from the source, (2) that which is produced by the first reflection image sources, and the point to point variation of intensity may be calculated for each particular enclosure; (3) that which is produced by all the remaining image sources, a term with a magnitude $\sum_2^\infty E_n$, which shows no point to point variation.

In the following development of the sound intensity decay equation, we assume that the average densities $E_0$ and $E_0R$ may be substituted for the actual densities produced at the point of observation by the source and the first reflection image sources respectively. Because of this the equation developed will not give an exact statement of the decay during the first two $(p/c)$ sec. intervals, but this is not important because from then on it will give the correct statement and in good experimental practice the rate of decay obtained need never depend upon the changes during these first two intervals.

Hence when sound is established in the enclosure, the average density due to the source and all the zones of image sources is $S_\infty = \sum_0^\infty E_n$. Remembering equation (9), we have $E_0 = p\overline{E}/cV$ and at once

$$S_\infty = E_0(1+R+R^2+R^3+ \cdots R^n+ \cdots ). \qquad (12)$$

When the source is stopped all the image sources stop, and, of course, the energy supply also. At the source, or very near it, the portion $E_0$ of the total energy at once vanishes. The intensity then remains constant for an interval $p/c$ seconds, the time it takes sound to travel from the first zone to the point of observation announcing the stopping of the energy supply of this zone. At the close of this interval the portion $E_0R$ abruptly vanishes. This is followed by another interval of constant intensity. Then a sudden disappearance of the portion $E_0R^2$ occurs. By this time the sound has traveled from the second zone to the point of observation and announces the stopping of the energy supply of that

zone. Thus the decay continues and remembering that equation (12) is a geometric series, the energy density during the $n$th interval is

$$S_\infty - S_n = \frac{E_0}{1-R} - \frac{E_0(1-R^n)}{1-R} = \frac{E_0 R^n}{1-R} \; . \tag{13}$$

We developed this equation on the assumption that the point of observation was very near the source, which was also the center of the zones. There seems to be no *a priori* reason why the center should be chosen at the source rather than at some other point in the enclosure, but once the center is located it does not seem legitimate to move it about without a careful investigation of what would happen if it were not moved to each point of observation.

Hence we select for study some point of observation not at the center of the zones. Waves leaving a given zone at the same instant will not arrive simultaneously at this new location. This means that unless one judges the energy density at a point by the flow of energy out in a *given direction*, equations (12) and (13) will not give a correct picture. But the decay history at a point as witnessed in a *given direction* is given exactly by equation (13). We propose to determine the decay history of the energy density by *adding up the complete decay histories of all the directions*; we shall not obtain the decay history by adding the instantaneous values of the directional histories, and then tracing the history of this instantaneous sum. Therefore, let $e_0$, $e_1$, $e_2$, etc. be the energy density due to the image sources, included within any solid angle $\Delta\omega$. (See Figure 4). Then

$$e_0 = \frac{\Delta\omega}{4\pi} E_0, \qquad e_1 = \frac{\Delta\omega}{4\pi} E_1, \; \text{etc.}$$

By equation (13)

$$s_\infty = \sum_0^\infty e_n = \frac{e_0}{1-R}$$

$$s_n = \sum_0^n e_n = \frac{e_0(1-R^n)}{1-R}$$

and

$$s_\infty - s_n = \frac{e_0 R^n}{1-R} = \frac{\Delta\omega}{4\pi} \frac{E_0 R^n}{1-R} \; . \tag{14}$$

Now "$n$" is an integer and simply denotes during which $p/c$ sec. interval the intensity is measured. Except in extreme cases, waves

which left simultaneously from a given zone will all arrive within the interval $p/c$ sec. after the first arrival takes place. Hence we are justified in adding up equations (14) for all directions keeping "$n$" constant. At once

$$\sum(s_\infty - s_n) = \frac{E_0 R^n}{4\pi} \frac{\sum\Delta\omega}{(1-R)} = \frac{E_0 R^n}{1-R} .$$ (15)

That this summation is legitimate, even for the extreme cases where a slightly greater interval than $p/c$ sec. is needed to have all the waves arrive, is illustrated by Fig. 6, in which it is clear that the staggering effect simply smooths out the distinct drops of the *total* energy but does not change the slope of the decay. From this it becomes clear that so far as the energy density at a point is concerned, "$n$" which was introduced
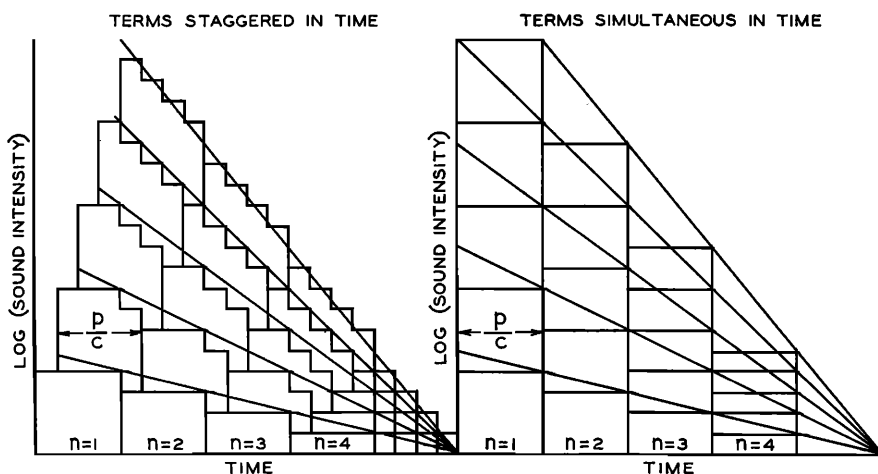


FIG. 6. *Illustrating that the staggering effect simply smooths out the distinct drops but does not change the slope.*

as an integer may after the summation is over, be considered, approximately at least, a continuous function. This method of tracing the "life history" (equation 14) for each particular direction and then adding the "life histories" is essentially the uniqueness of the new analysis.

In the old theory during the decay the energy density is assumed strictly continuous, but we have found this to be approximately true. The difference does not lie here. The old theory also assumes that the energy density at a point appears to have a continuous change as viewed from an element of surface. It tacitly assumes that an absorbing surface has the ability to register back into the oncoming wave the decrease in

intensity which it is causing. This is obviously not true; this particular absorption is registered in the outgoing wave, not in the oncoming wave. During decay the surface will receive *from a given direction* a constant energy flow for a time equal to the time it takes sound to travel the mean free path between reflections; then there will be an abrupt change. *This constant energy flow followed by an abrupt drop, rather than a continuous drop to this same level, means a greater absorption during the same interval of time and hence a more rapid decay of the sound.* (See Figures 7 and 8). This is in essence the physical difference between the foundation on which Sabine's formula has been erected, and the foundation on which we propose to erect a more general type of formula. The new type of equation must give Sabine's formula as a special case for "live" rooms and must do so simply because under these circumstances the two foundations become equivalent but not because the new picture ceases to hold for "live" rooms.

Rewriting equation (15) and remembering that

$$n = \frac{ct}{p} \tag{16}$$

we have

$$\sum (s_\infty - s_n) = \frac{E_0 R^{ct/p}}{1 - R}$$

and remembering that

$$E_0 = \frac{p\overline{E}}{cV}$$

we have

$$\sum (s_\infty - s_n) = \frac{p\overline{E}}{cV(1-R)} e^{\frac{c(\log_e R)t}{p}} \tag{17}$$

and from what we have said above we are justified in assuming that $t$ is a continuous function.

We now have to determine $p$, the mean free path between reflections. Sabine[1] obtained the experimental value, $p = 0.62(V)^{1/3}$, which reduces to $p = 3.7V/S$ for enclosures of reasonably compact proportions. Jaeger[3] using the theory of probability and assuming a perfect diffuse condition of the sound waves calculated the number of reflections per second in the same manner that the impacts of molecules are estimated

in the kinetic theory of gases, and found $p = 4V/S$. Recently Schuster and Waetzmann[5] have shown as we have also shown in the first part of this paper that for rooms of special shapes where an ordered not a diffuse condition of the sound waves is attained the mean free paths are as follows: cubical room, $p = 2\sqrt{3} V/S$; cylindrical room with length equal diameter, $p = 3\sqrt{2} V/S$; spherical room, $p = 6V/S$. Probably the formula which will be used most is one based on a perfect diffuse condition of the waves, a special formula being written down for each enclosure which cannot fulfill this condition. To obtain the formula for the diffuse condition we shall use the value obtained by Jaeger for the mean free path. That this value may be used for "dead" rooms follows from the fact that Jaeger's analysis does not in any way involve the absorption coefficients of the surfaces. Of course, we must demand that the "dead" room shall produce a diffuse condition among the sound waves; the assumption that the image sources are located on the surfaces of concentric spheres tacitly implies this diffuse state.

Substituting the value

$$p = \frac{4V}{S} \tag{18}$$

in equation (17) we get the decay equation

$$\rho = \rho_0 e^{\dfrac{cS \log_e (1-\alpha_a)t}{4V}} \tag{19}$$

where

$$\rho_0 = \frac{4\overline{E}}{cS\alpha_a} . \tag{20}$$

As we have already pointed out, for practical and experimental purposes, $t$ may be considered continuous, and hence (19) may be thought of as derivable from a differential equation.[8] In a very *"live"* *room* (and in this analysis we may make the room as "live" as we please) the absorption per reflection is so very small that the energy density as viewed from an element of surface can be considered continuous. This means that we may follow the old theory[4] and write down the differential equation for the sound intensity in a closed room as follows:

$$V\frac{\partial \rho}{\partial t} + \frac{1}{4}c\alpha_a S\rho = \overline{E} \tag{21}$$

[8] Schuster and Waelzmann, loc. cit., have made use of a differential equation involving velocity potentials.

where $V$ is the volume of the room, $S$ its surface, $\alpha_a$ the average coefficient of absorption, $c$ the speed of sound in air, $\rho$ the energy density and $\bar{E}$ the rate of sound energy emission, which is considered constant.[9]
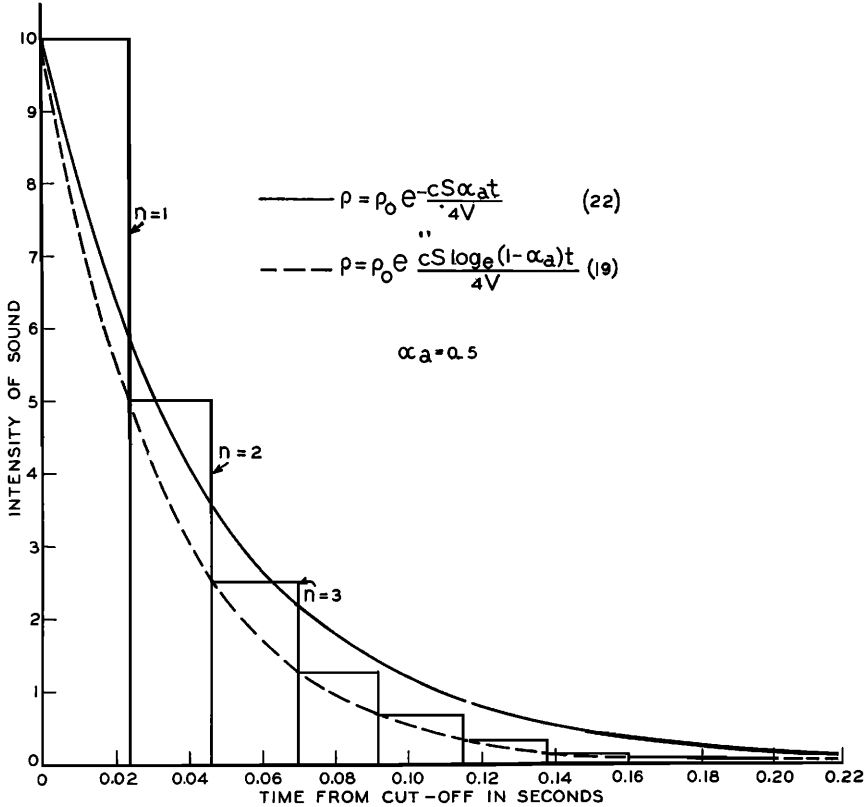


FIG. 7. *Illustrating rate of sound decay in a room with volume 80400 cu. ft., surface 12180 sq. ft. and walls of average coefficient of absorption 0.5.*

The solution of this equation under proper boundary conditions gives the following for the equation of decay

$$\rho = \rho_0 e - \frac{c\alpha_a S}{4V} t \tag{22}$$

and from this Sabine's reverberation time equation is obtained. Now

$$\log_e R = \log_e (1-\alpha_a) = -\left[\alpha_a + \frac{\alpha_a^2}{2} + \frac{\alpha_a^3}{3} + \cdots \right] \tag{23}$$

and when $\alpha_a$ is very small as is true for *very live* rooms,

$$\log_e (1-\alpha_a) = -\alpha_a.$$

[9] Vibrating Systems and Sound, Crandall, p. 207.

This means that (22) is a special case of (19). The value of $\rho_0$ is the same for both equations and may be derived from differential equation (21) as the maximum value reached by the energy density as a steady state is established. This agreement is natural since the steady state value depends simply on the conservation of energy principle, and not upon the rate of growth or decay when the sound source is turned on or cut off. This differential equation cannot give the correct damping factor,
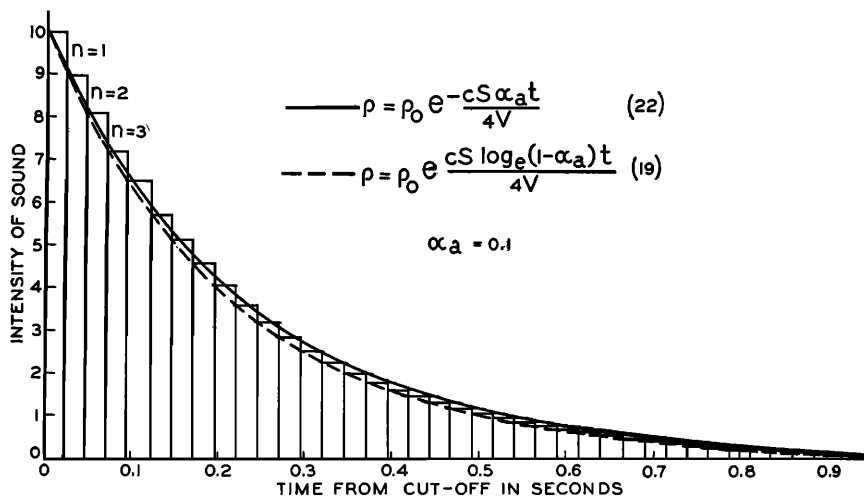


FIG. 8. *Illustrating rate of sound decay in a room with volume 80400 cu. ft., surface 12180 sq. ft. and walls of average coefficient of absorption 0.1.*

because as we have already pointed out, it does not describe the true nature of the absorption. However, if we put $\overline{E}=0$, substitute $-\log_e (1-\alpha_a)$ for $\alpha_a$ and set down the boundary condition, $\rho = \rho_0$ when $t=0$, we get a differential equation which gives (19) as its solution.

In Figures 7 and 8, we have plotted equation (19) first with $t$ considered continuous, then with $n = ct/p$ considered as an integer. The decay is shown to be greater than that for equation (22), the old decay formula, the difference becoming very marked for "dead" rooms.

To obtain the reverberation time formula involving the true nature of the absorption we make use of equation (19) and write

$$\frac{\rho_1}{\rho_2} = \frac{e^{\dfrac{cS \log_e (1-\alpha_a)t_1}{4V}}}{e^{\dfrac{cS \log_e (1-\alpha_a)t_2}{4V}}} = 10^6$$

and put $(t_2 - t_1) = T$, the reverberation time.

At once

$$\log_e 10^6 = \frac{-cS \log_e (1 - \alpha_a)}{4V} T$$

$$T = \frac{4 \log_e 10^6 V}{cS \log_e (1 - \alpha_a)} = \frac{0.05V}{-S \log_e (1 - \alpha_a)} \ . \tag{24}$$

From equation (23) it follows at once that for very small values of $\alpha_a$ the reverberation time formula (24) becomes

$$T = \frac{0.05V}{S \alpha_a} = \frac{0.05V}{a} \tag{25}$$

which is Sabine's formula—*a special case of the more general formula which we have developed.*
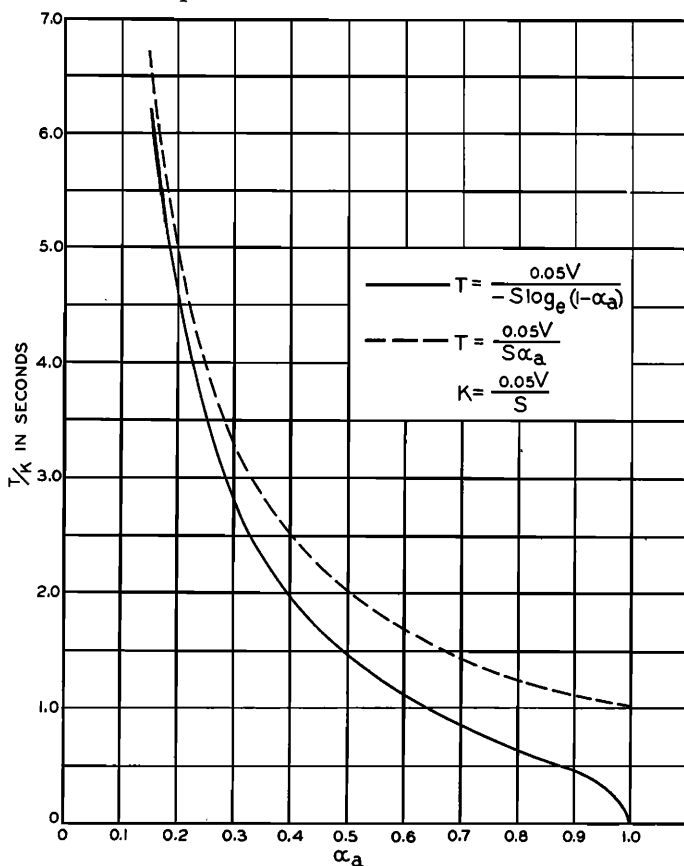


FIG. 9. *Comparison of reverberation time formulae.*

Although Sabine's formula had no meaning for $\alpha_a = 1$, complete absorption, the "new" formula does give the value zero, Curves comparing the new and old formulae are given as Fig. 9. These curves show a wide difference for large values of absorption but approach each other for small values of absorption. Thus we have established (24) as the general reverberation time equation for all types of rooms in which we can be certain that a diffuse condition of energy density is achieved. If we wish more generality, and certainly there will be cases as we have already pointed out where an ordered condition may exist among the sound waves, we shall not need to change the equation's form which is
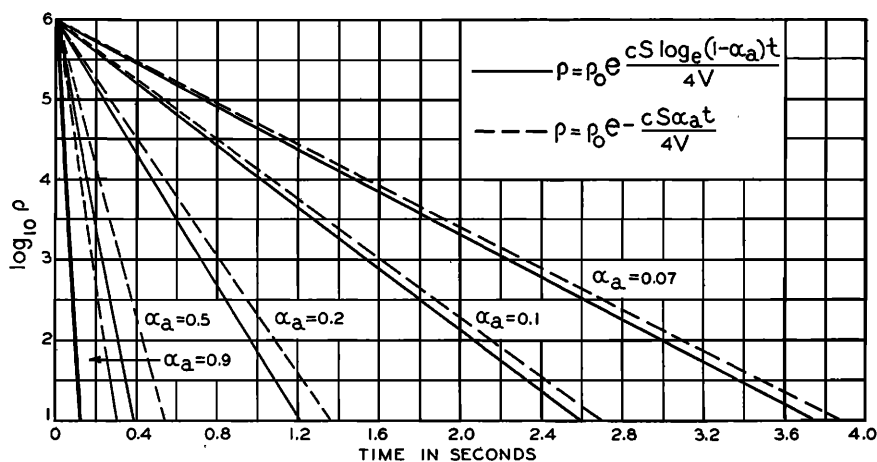


$$\rho = \rho_0 e^{\frac{cS \log_e(1-\alpha_a)t}{4V}}$$

$$\rho = \rho_0 e^{-\frac{cS\alpha_a t}{4V}}$$

FIG. 10. *Comparison of sound decay.*

intimately tied up with the nature of the absorption, but we will need to substitute a constant $k$ for 0.05 and then ascertain for each special problem the value of $k$ to be used, remembering that the mean free path between reflections is the physical quantity that controls its magnitude. How the averaging represented by equation (3) is modified to obtain the proper value of $\alpha_a$ will depend on the nature of the ordered condition which exists among the sound waves. The method of images as used and discussed especially in the first part of this paper may be an aid in determining these two constants as may also the following table.

|  | Mean free path | Constant k |
|---|---|---|
| Sabine's experimental value............ | 3.7 V/S | 0.045 |
| Statistical value...................... | 4.0 | 0.049 |
| Cubical Room........................ | 3.5 | 0.043 |
| Cylindrical room (length = dia.)......... | 4.2 | 0.051 |
| Spherical room....................... | 6.0 | 0.073 |

## EXPERIMENTAL RESULTS

The fact that the experimental results obtained by Sabine checked his formula does not detract from the validity of formula (24) since the two formulae become identical for very live rooms, the type in which Sabine worked. We now wish to present evidence for the applicability of the general formula to work with very "dead" rooms and the failure
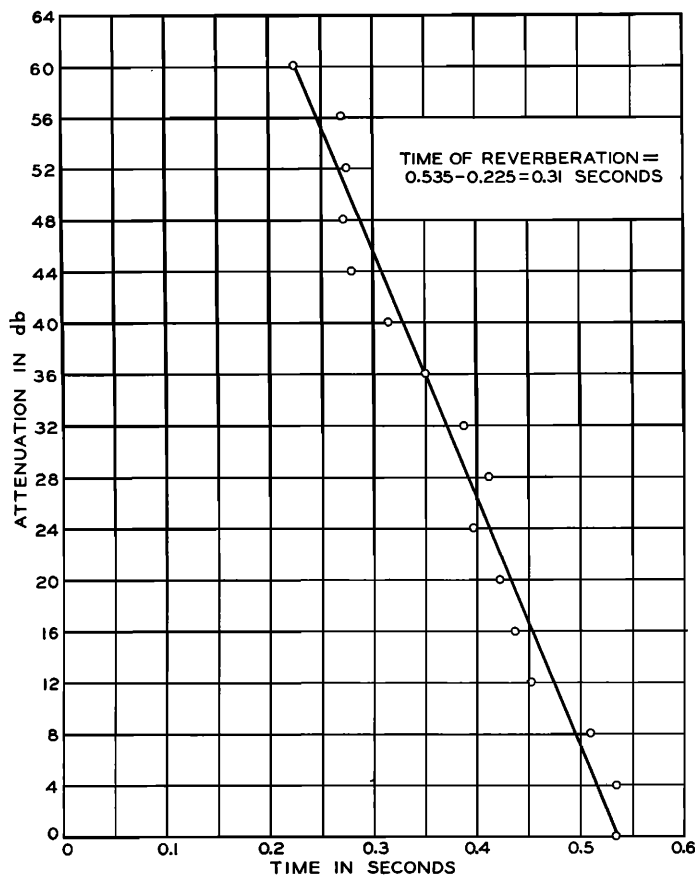


FIG. 11. *A typical experimental curve obtained in reverberation time measurements.*

of Sabine's formula for such cases. Formula (24) is used without modification because the shape of the room, and the location of the absorbing material, source and point of observation guarantee a diffuse condition of the sound waves. Equation (24) may be written thus

$$\log \rho = \frac{cS \log (1-\alpha_a)t}{4V} + \log \rho_0 \qquad (26)$$

which shows that the time plotted against the logarithm of the sound energy density is a straight line.  (See Figure 10).  Since in a "dead" room the energy steps are rather marked, a picture of a decay will not be a smooth curve.  The irregularity will also be greatly enhanced by interference phenomena which have not been considered in this paper. (See Figure 11).

Making use of equation (26) we may write

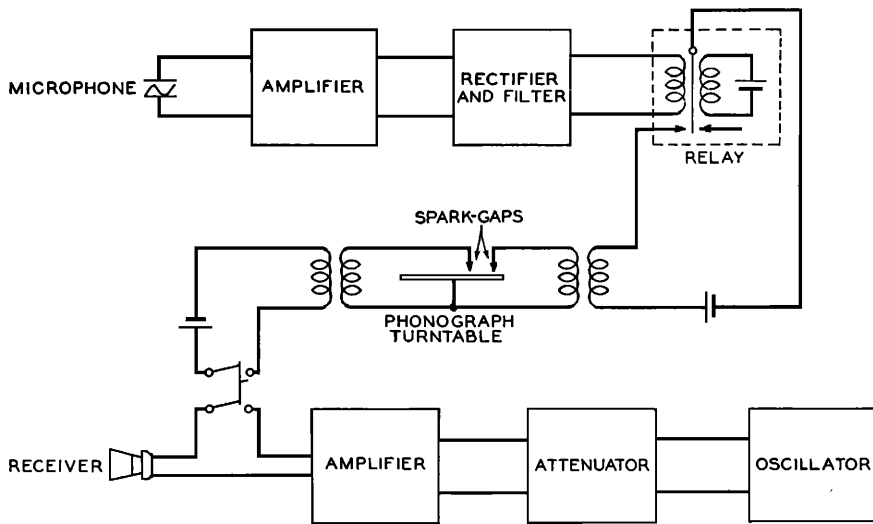$$\log \frac{\rho_1}{\rho_2} = - K(t_2 - t_1). \tag{27}$$



FIG. 12. *A block schematic of the reverberation time measuring apparatus used.*

If $\rho_1$ is the intensity of the sound at some specified instant, $t_1 = 0$, such as the time of sound cut-off, and if $\rho_2$ is a minimum intensity which marks a relay cut-off, then associated with each value of $\rho_1$ will be a certain time $t_2$.  If, therefore, we give $\rho_1$ various values by changing the output of a receiver and obtain the corresponding values of $t_2$, and if the power input measured in $db$ corresponding to each receiver output is plotted against each corresponding time between sound cut-off and relay action ($t_2$), we shall get a straight line with the same slope as equation (26).  From this line the time for a 60 $db$ drop may be obtained and this is the reverberation time.  A typical curve obtained for the sound stage is shown in Figure 11.  A block schematic of the apparatus is shown in Figure 12 and the plot of $db$ attenuation of the maximum input is used instead of

the power input measured in *db*. The slope of the line, which is all we wish, is not modified by this procedure. The details of the apparatus and the technique of its use have been described by Wente and Bedell in the paper, "Chronographic Method of Measuring Reverberation Time."

TABLE I

INSULATING MATERIAL ONLY

Volume of Room 73,475 cu. ft.  Total surface 11,553 sq. ft.
Ins. Material = 8401 sq. ft.  Concrete, glass, etc. = 3,152 sq. ft.

| Frequency | Reverberation Time | $\alpha_a$ | $\alpha$ for Insulating Material | |
|---|---|---|---|---|
| | | | New Formula | Sabine's Formula |
| 120 | 0.73 sec. | 0.35 | 0.48 | 0.59 |
| 240 | 0.49 | 0.47 | 0.65 | 0.89 |
| 500 | 0.38 | 0.56 | 0.77 | 1.14 |
| 1000 | 0.34 | 0.60 | 0.82 | 1.28 |
| 2000 | 0.34 | 0.60 | 0.82 | 1.28 |
| 4000 | 0.32 | 0.63 | 0.86 | 1.37 |

TABLE II

INSULATING MATERIAL AND MONK'S CLOTH CURTAINS

Curtain area = 1,670 sq. ft.  Ins. material only 6887 sq. ft.
Concrete, glass, etc. 2,996 sq. ft.

| Frequency | Reverberation Time | $\alpha_a$ | $\alpha$ for Combination |
|---|---|---|---|
| 120 | 0.52 | 0.45 | 1.10 |
| 240 | 0.40 | 0.54 | 1.03 |
| 500 | 0.32 | 0.62 | 1.00 |
| 1000 | 0.31 | 0.64 | 1.01 |
| 2000 | 0.28 | 0.67 | 1.13 |
| 4000 | 0.26 | 0.70 | 1.27[1] |

[1] No significance should be attributed to these values which are greater than unity since the additional absorption was introduced into a room which was already very "dead," and hence experimental errors are bound to be large.

We wish to emphasize the importance of making the time from sound cut-off to relay action sufficiently long so that enough reflections are included to give a good value of the mean free path between reflections. This means that for very high damping a time longer than the reverberation time will need to elapse between the stopping of the sound and the operation of a relay in the instrumental method of measuring reverberation time in order that "$n$" may be sufficiently large to insure the

use of a good number of terms of the geometric series. Only in this way will the walls have a chance to record their effect upon the sound decay. To make this more clear; suppose a source is 0.2 second from the point of observation and that the time of reverberation is 0.15 second. So long as the relay cuts off 60 *db* below the maximum intensity the only
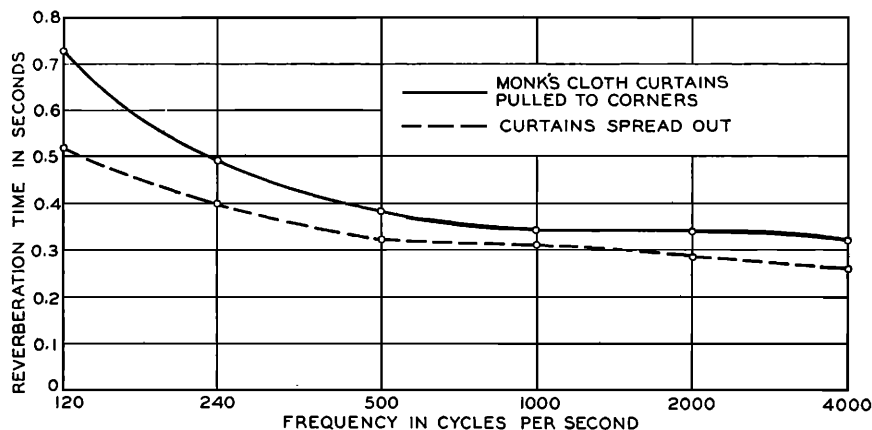


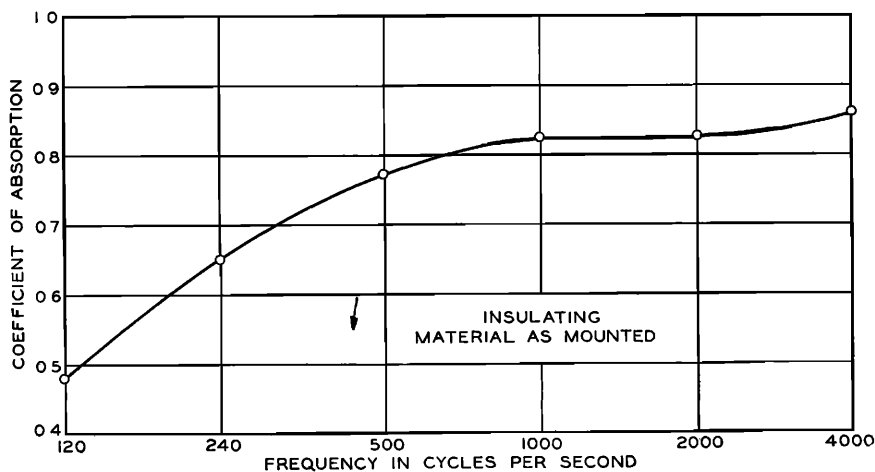FIG. 13. *Reverberation time-frequency curves for sound stage.*



FIG. 14. *Coefficient of absorption-frequency curve for insulating material on walls and ceiling of sound stage, as calculated by the more general formula.*

time possible to measure is 0.2 second, the time it takes the sound to travel from the source to the point of observation. However, if the intensity of the sound is increased and if the threshold of the relay cut-off is lowered so that the time measured is for a 120 *db* drop, then the
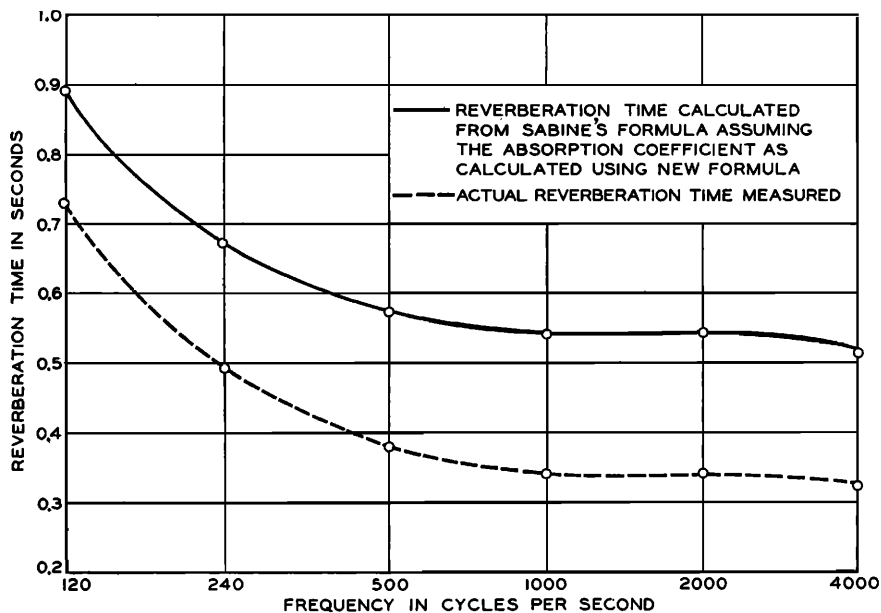
FIG. 15. *Comparison of reverberation times as measured and those calculated using Sabine's formula and coefficients of absorption shown in Figure 14.*
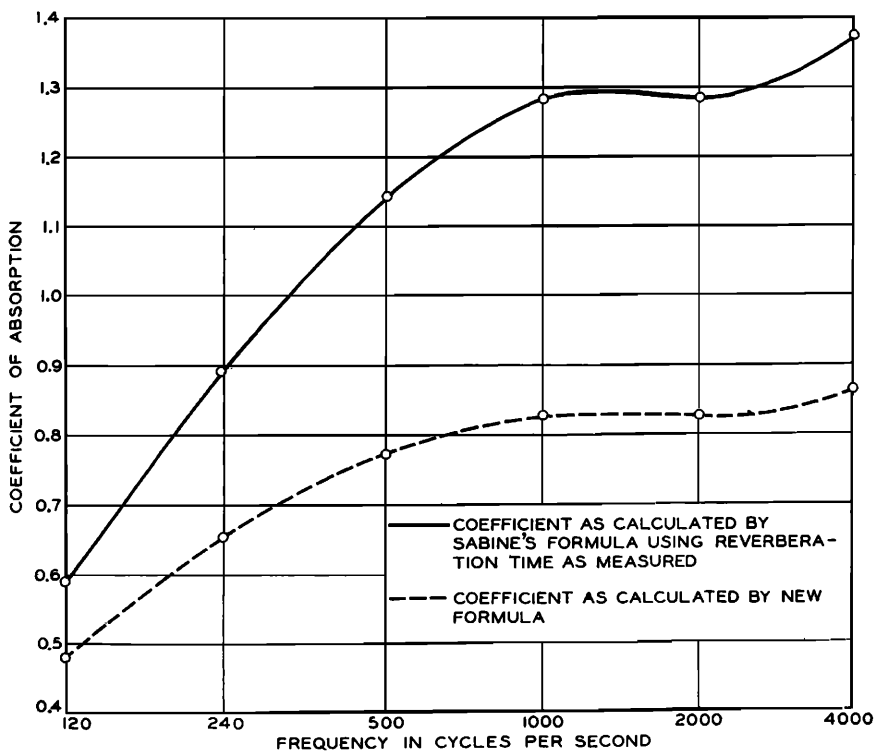


FIG. 16. *Comparison of coefficients of absorption as calculated by Sabine's and the more general formula using reverberation times as measured.*

time observed is 0.3 second and the reverberation time is 0.15 sec. Thus, a reverberation time approaching a zero value may be measured.

The results of measurements made in the Sound Stage, Sound Picture Laboratories, Bell Telephone Laboratories, Inc., are summarized in the preceding tables and illustrated in Figures 13 to 16.

## SUMMARY

We have shown that a correct picture of sound absorption in an enclosed room leads to a general reverberation formula of the form

$$T = \frac{kV}{-S \log_e (1 - \alpha_a)} . \tag{28}$$

The manner of the absorption, which has been described in detail in this paper, is responsible for the logarithm factor and therefore the new form; the mean free path between reflections which is independent of the nature of the absorption controls the magnitude of $k$; and $\alpha_a$ is obtained by assigning proper weights to the various elements of surface. For a diffuse condition of sound energy, probably the condition most frequently encountered, $k = 0.05$ and $\alpha_a$ is obtained from equation (3); but for each ordered condition of the waves a particular value of $k$ must be used and $\alpha_a$ must be obtained by proper averaging. It is thought that the method of images presented in this paper will be of aid in solving each particular problem at hand. The practical importance of the new type formula should be emphasized because it shows that a "dead" room may be obtained by the use of less absorbing material than that calculated by Sabine's formula. An example will suffice. To obtain a required reverberation time of 0.54 second, an engineer using Sabine's formula would have specified 8400 square feet of absorbing material for a room such as the Sound Stage, but without knowing it, he would have obtained instead a reverberation time of 0.34 second; using the new formula he would have specified 6200 square feet of absorbing material securing a saving of 26% in material and the reverberation time desired.