

length of the window is an integer multiple of the period of the sinusoid. The cylinder pressure waveform, however, has cycle-to-cycle variations. Thus, it cannot be treated as an ideal periodic waveform. Consequently, care must be exercised to prevent leakage error.

<sup>1</sup>J. Y. Chung, M. J. Crocker, and J. F. Hamilton, "Measurement of frequency responses and the multiple coherence

function of the noise-generation system of a diesel engine," J. Acoust. Soc. Am. 58, 635-642 (1975).

<sup>2</sup>P. J. Holmes and C. A. Mercer, "Comments on 'Measurement of frequency responses and the multiple coherence function of the noise-generation system of a diesel engine,'" [J. Acoust. Soc. Am. 58, 635-642 (1975)], J. Acoust. Soc. Am. 60, 951-952 (1976).

<sup>3</sup>J. S. Bendat and A. G. Piersal, *Random Data: Analysis and Measurement Procedures* (Wiley-Interscience, New York, 1971).

## Response to "Comments on 'Diffuse sound reflection by maximum length sequences' " [J. Acoust. Soc. Am. 60, 268-268(1976)]

M. R. Schroeder and R. Gerlach

Drittes Physikalisches Institut, Universität Göttingen, D-3400 Göttingen, Federal Republic of Germany  
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Complementary maximum length sequences, recently proposed by Moharir, do not give as good sound diffusors as those originally proposed by Schroeder. A rule for constructing two-dimensional sequences is given.

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Moharir<sup>1</sup> has proposed "complementary" maximum length sequences which are claimed to provide better sound diffusion than those originally proposed by Schroeder. The sum of the correlation function of such sequences has ideal properties. However, what is additive are not correlation functions but sound pressure. Thus, the correlation function that counts is the one of the sum of the two complementary sequences  $x_1$ ,  $x_2$ :

$$P(k) = \sum_{i=0}^{N-i-k} [x_1(i) + x_2(i)][x_1(i+k) + x_2(i+k)]$$

$$= \rho_1(k) + \rho_2(k) + \sum x_1(i)x_2(i+k) + \sum x_1(i+k)x_2(i). \quad (1)$$

Here  $\rho_1(k) + \rho_2(k) = 0$  for  $k \neq 0$ , but unfortunately the two cross terms are not well behaved. For

$$x_1 = +++-+-++ \quad \text{and} \quad x_2 = +++-+---, \quad (2)$$

one has

$$P(k) = 16, 4, 0, -4, 0, 0, 0, 0. \quad (3)$$

As a result of the two nonzero values of  $P(k)$  (4 and -4), the corresponding reflection pattern is very uneven with deep and broad minima. For normal incidence and a half-wavelength spacing of the ridges, the reflection minima occur at reflection angles of  $\pm 44^\circ$ , i. e., right in the center of the angular range.

The existence of two (and higher)-dimensional generalizations of maximum length sequences was recently pointed out to us by Mrs. F. J. MacWilliams of Bell Laboratories who is writing a book on coding theory with N. J. A. Sloane. The prescription for constructing a two-dimensional array is as follows: take a maxi-

imum length sequence and factor its length ( $2^m - 1$ ) into two relative-prime factors. Example for  $m = 4$ :  $15 = 3 \times 5$ . Then fill the  $(3 \times 5)$  array with the sequence starting along the main diagonal, "jumping across" the array whenever an array boundary is encountered. Thus, for a sequence  $a_0, a_1, \dots, a_{14}$ , the corresponding array will look as follows:

$$\begin{array}{ccccc} a_0 & a_8 & a_{12} & a_3 & a_9 \\ a_{10} & a_1 & a_7 & a_{13} & a_4 \\ a_5 & a_{11} & a_2 & a_6 & a_{14} \end{array} \quad (4)$$

Hence the maximum length sequence

$$0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 0 \ 1 \ 1 \ 1 \ 1 \quad (5)$$

produces the two-dimensional array

$$\begin{array}{ccccc} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{array} \quad (6)$$

The two-dimensional autocorrelation function of such (periodically repeated) arrays is  $-1/(2^m - 1)$  everywhere in the plane (except at the origin and multiples of the periods). The reflection properties of such arrays should be superior to those generated by multiplying two arrays as suggested in Ref. 2.

<sup>1</sup>P. S. Moharir, "Comments on diffuse sound reflection by maximum length sequences," J. Acoust. Soc. Am. 60, 268-268 (1976).

<sup>2</sup>M. R. Schroeder, "Diffuse sound reflection by maximum-length sequences," J. Acoust. Soc. Am. 57, 149-150 (1975).