MEDIOS DE ENLACE

3R1

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Algebra Vectorial

$$\nabla = \text{Nabla} = \frac{\partial}{\partial x}\hat{a_x} + \frac{\partial}{\partial y}\hat{a_y} + \frac{\partial}{\partial z}\hat{a_z} \quad \text{Operador Diferencial}$$

Vectores
- Dirección
- sentido
- Módulo

Versor
- Dirección
- sentido
- Módulo 1

GRADIENTE

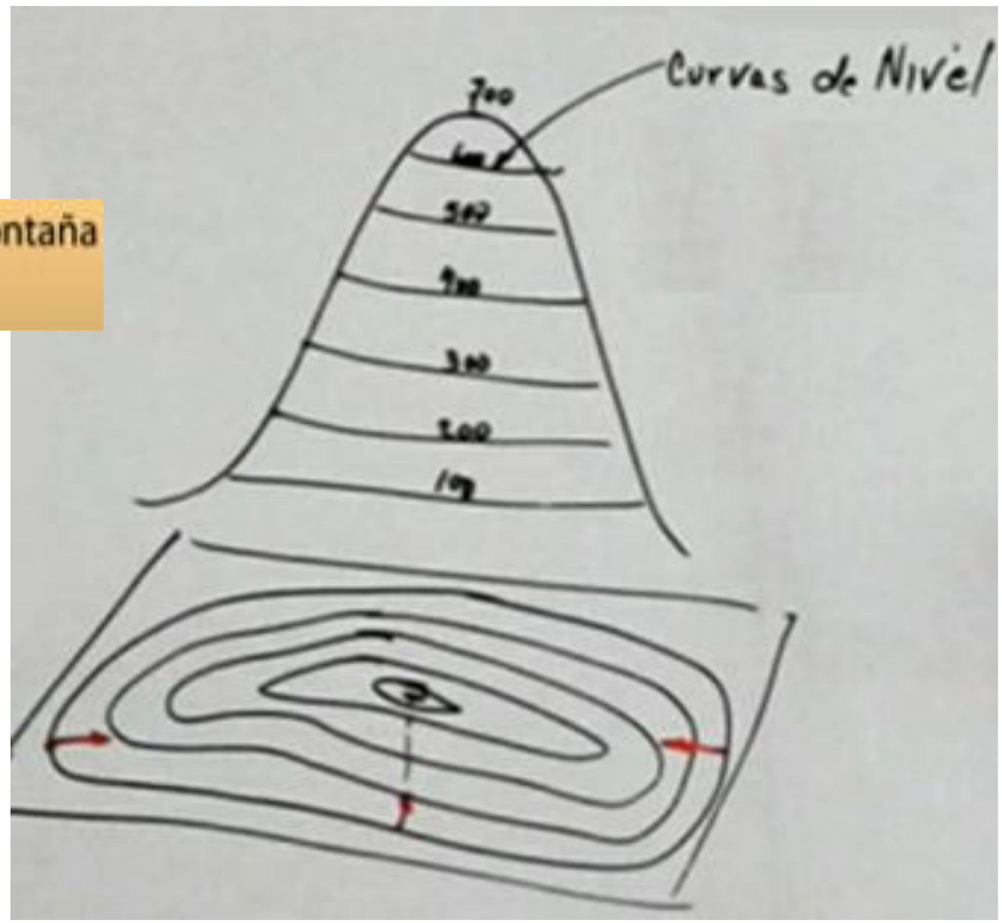
$$\nabla \cdot V = \overline{V}$$

Producto Escalar

Valor Escalar

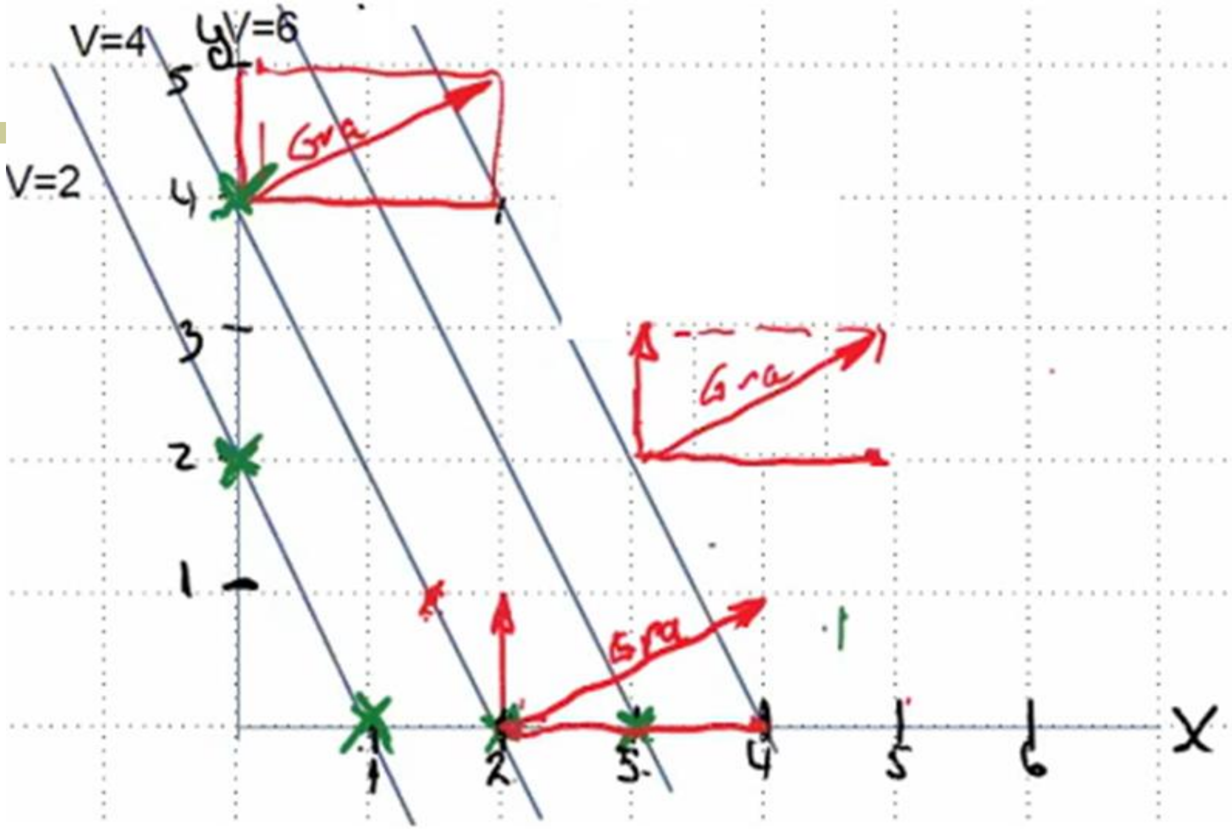
Altura de cada punto de la montaña

$$\boxed{h(x,y) = 4 - 3x^2 - 3y^2}$$



$V = 2x + y$

$V = 2;\ 4;\ 6;\ 8$



Hallar el gradiente

P(0,2); P(0,4); P(0,6); P(1,0); P(2,0); P(3,0)

$$\nabla \cdot V = \left(\frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z\right) \cdot (2X + y) = 2\hat{a}_x + 1\hat{a}_y$$

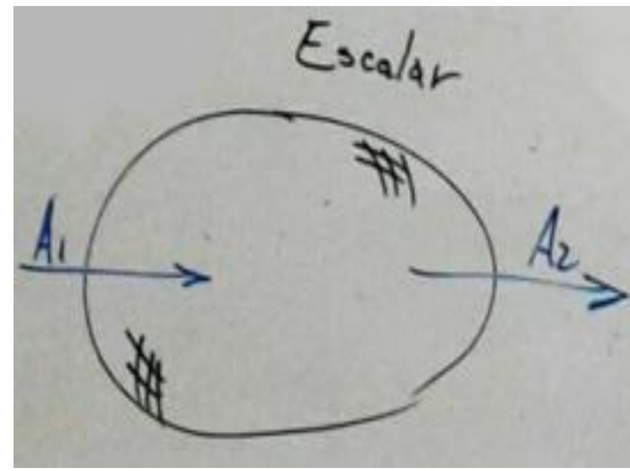
DIVERGENCIA

$$\nabla \cdot \bar{D} = \bar{D}$$

Prod Escalar

Valor Vectorial

$$\nabla \bar{D} = \left(\frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_x + \frac{\partial}{\partial z}\hat{a}_z\right)\left(D_x\hat{a}_x + D_y\hat{a}_y + D_z\hat{a}_z\right)$$



$$\nabla \bar{D} = \left(\frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z\right)\left(D_x\hat{a}_x + D_y\hat{a}_y + D_z\hat{a}_z\right)$$

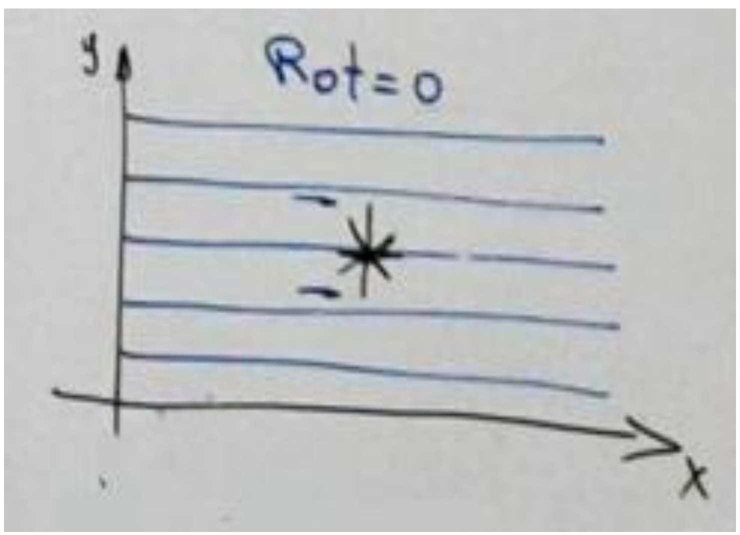
$$a_x \cdot a_x = \cos 0 = 1 \qquad \hat{a}_x \cdot \hat{a}_y = \hat{a}_y \cdot \hat{a}_z = \ldots = 0$$

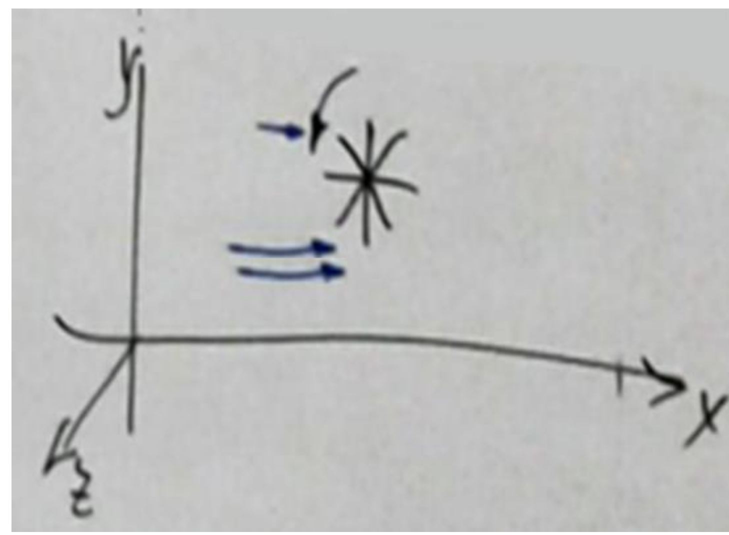
$$\nabla \bar{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$ Escalar

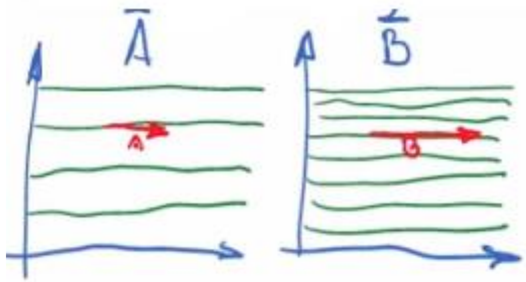
Si $A_1 = A_2$ $Div = 0$

ROTOR

$$\underset{\text{Rot Vectorial}}{\nabla \times} \underset{\text{Valor Vectorial}}{\bar{H}} = \bar{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = \hat{a}_x\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) - \hat{a}_y\left(\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right) + \hat{a}_z\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)$$







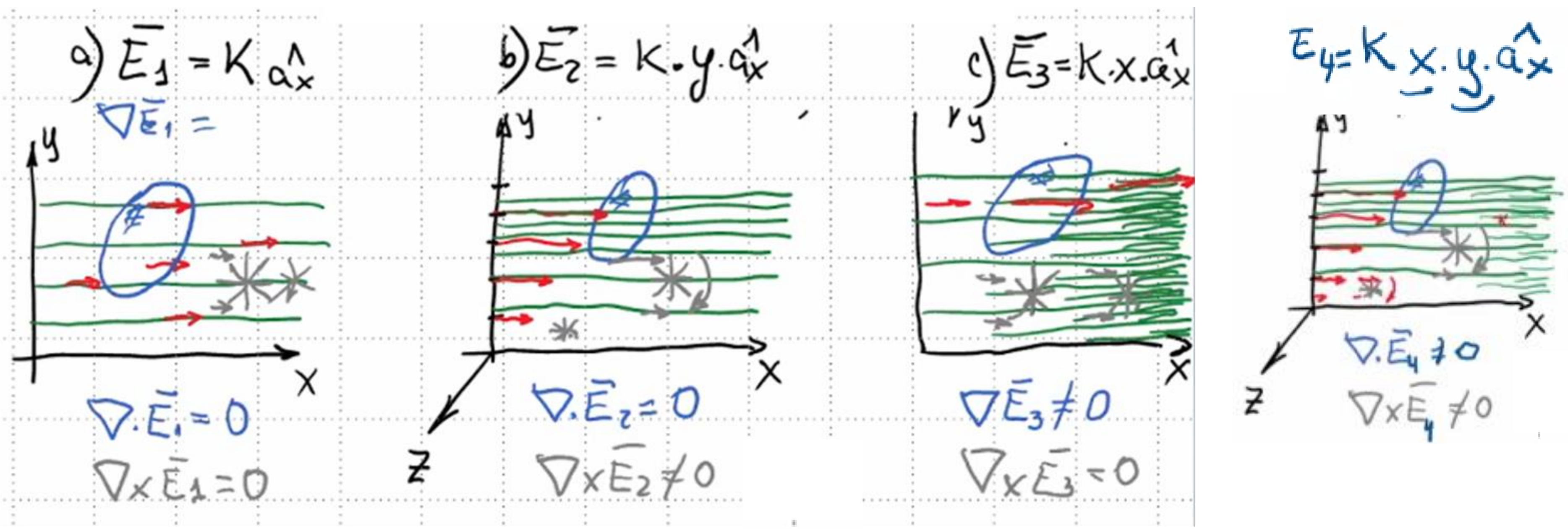
$$|B| > |A|$$

Lineas de flujo

1) EL vector es tg a las lineas de flujo

2) EL Nº de Lineas de flujo o densidad es proporcional a la magnitud del vector

Hallar el Rotor y Div en los siguientes campos



$$\nabla . V(GEN) = \frac{1}{h_1}\frac{\partial V}{\partial \mu_1}\hat{a}_1 + \frac{1}{h_2}\frac{\partial V}{\partial \mu_2}\hat{a}_2 + \frac{1}{h_3}\cdot\frac{\partial V}{\partial \mu_3}\hat{a}_3$$

$$\nabla . V(REC) = \frac{\partial V}{\partial x}\hat{a_x} + \frac{\partial V}{\partial y}\hat{a_y} + \frac{\partial V}{\partial z}\hat{a_z} \qquad (x, y, z)$$

$$\nabla . V(CIL) = \frac{\partial V}{\partial \rho}\hat{a_\rho} + \frac{1}{\rho}\cdot\frac{\partial V}{\partial \phi}\hat{a_\phi} + \frac{\partial V}{\partial z}\hat{a_z} \qquad (\rho, \phi, z)$$

$$\nabla . V(ESF) = \frac{\partial V}{\partial r}\hat{a_r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a_\theta} + \frac{1}{r . sen\theta}\cdot\frac{\partial V}{\partial \phi}\hat{a_\phi} \quad (r, \theta, \phi)$$

Identidades Vectoriales

$$\underbrace{\nabla \cdot}_{\substack{\text{Prod} \\ \text{Esc.} \\ \text{Div}}} \underbrace{\left(\nabla \times \bar{H}\right)}_{\substack{\text{Prod} \\ \text{Vect} \\ \text{Rotor}}} = 0$$

$$\nabla \times H = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = \hat{a_x}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) - \hat{a_y}\left(\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right) + \hat{a_z}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)$$

$$\nabla = \frac{\partial}{\partial x}\hat{a_x} + \frac{\partial}{\partial y}\vec{a_y} + \frac{\partial}{\partial z}\cdot\hat{a_z}$$

$$\hat{a_x} \cdot \hat{a_x} = 1$$

$$a_x \cdot a_y = 0$$

$$a_x \cdot a_z = 0$$

$$\cancel{\frac{\partial^2 H_z}{\partial x \partial y}} - \cancel{\frac{\partial^2 H_y}{\partial x \partial z}} - \cancel{\frac{\partial^2 H_z}{\partial x \partial y}} + \cancel{\frac{\partial^2 H_x}{\partial z \partial y}} + \cancel{\frac{\partial^2 H_y}{\partial z \partial x}} - \cancel{\frac{\partial^2 H_x}{\partial z \partial y}} = 0$$

$$\underbrace{\nabla \times}_{\text{Rot}} \left(\underbrace{\nabla \cdot V}_{\text{GraV}}\right) = 0$$

Prod Esc. Escalar

$$\nabla \cdot V = \frac{\partial V}{\partial x}\hat{\partial}_x + \frac{\partial V}{\partial y}\hat{\partial}_y + \frac{\partial V}{\partial z}\hat{\partial}_z$$

$$\nabla \times (\nabla \cdot V) = \begin{vmatrix} \hat{\partial}_x & \hat{\partial}_y & \hat{\partial}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial V}{\partial x}\hat{\partial}_x & \frac{\partial V}{\partial y}\hat{\partial}_y & \frac{\partial V}{\partial z}\hat{\partial}_z \end{vmatrix} = \hat{\partial}_x\left(\frac{\partial^2 V}{\partial y \partial z}\hat{\partial}_z - \frac{\partial^2 V}{\partial z \partial y}\hat{\partial}_y\right) - \hat{\partial}_y\left(\frac{\partial^2 V}{\partial x \partial z}\hat{\partial}_z - \frac{\partial^2 V}{\partial z \partial x}\hat{\partial}_x\right)$$

$$+ \hat{\partial}_z\left(\frac{\partial^2 V}{\partial x \partial y}\hat{\partial}_y - \frac{\partial^2 V}{\partial x \partial y}\hat{\partial}_x\right)$$

0 0 0

$$\left.\begin{matrix} \hat{\partial}_x \hat{\partial}_x \\ \hat{\partial}_y \hat{\partial}_y \\ \hat{\partial}_z \hat{\partial}_z \end{matrix}\right\} = 1$$

cualquier convinacion = 0

$$\nabla . \nabla = \nabla^2 = \text{Laplaciano} = \frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\nabla . (\nabla . V) = \nabla^2 V$$

$$\nabla = \frac{\partial}{\partial x} \hat{\partial}_x + \frac{\partial}{\partial y} \hat{\partial}_y + \frac{\partial}{\partial z} \hat{\partial}_z$$

$$\nabla . V = \frac{\partial V}{\partial x} \hat{\partial}_x + \frac{\partial V}{\partial y} \hat{\partial}_y + \frac{\partial V}{\partial z} \hat{\partial}_z$$

$$\nabla . (\nabla V) = \frac{\partial^2 V}{\partial x^2} \hat{\partial}_x \hat{\partial}_x + \frac{\partial^2 V}{\partial y^2} \hat{\partial}_y \hat{\partial}_y + \frac{\partial^2 V}{\partial z^2} \hat{\partial}_z \hat{\partial}_z$$

$$= \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \nabla^2 V$$