MEDIOS DE ENLACE

3R1

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2025

1- $\nabla \times H e^{j\omega t} = \sigma E e^{j\omega t} + j\omega \varepsilon E e^{j\omega t}$

2- $\nabla \times E e^{j\omega t} = -j\omega\mu H e^{j\omega t}$

$E_x = \left(E_i e^{-\gamma z} + E_r e^{\gamma z}\right) e^{j\omega t}$

$$\nabla \times E = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & \underset{0}{\cancel{E_y}} & \underset{0}{\cancel{E_z}} \end{vmatrix} = -j\omega\mu H = a_x(0-0) - a_y\left(0 - \frac{\partial E_x}{\partial z}\right) + a_z\left(0 - \underset{0}{\cancel{\frac{\partial E_x}{\partial y}}}\right)$$

$$\frac{\partial E_x}{\partial z}\hat{a}_y = -j\omega\mu H$$

$$\frac{\partial E_x}{\partial z}\hat{a}_y = -j\omega\mu H_y$$

$$H_y = -\frac{1}{j\omega\mu}\frac{\partial E_x}{\partial z}$$

$$H_y e^{j\omega t} = -\frac{1}{j\omega\mu}\left[\frac{\partial}{\partial z}\left(E_i e^{-\gamma z} + E_r e^{\gamma z}\right)e^{j\omega t}\right]$$

$$H_y e^{j\omega t} = -\frac{1}{j\omega\mu}\left[\left(-\gamma E_i e^{-\gamma z} + \gamma E_r e^{\gamma z}\right)e^{j\omega t}\right]$$

$$H_y e^{j\omega t} = \frac{\gamma}{j\omega\mu}\left(E_i e^{-\gamma z} - E_r e^{\gamma z}\right)e^{j\omega t}$$

$$\boxed{\eta = \frac{j\omega\mu}{\gamma}}$$

impedancia intrínseca del medio

$$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

$$\eta = \frac{j\omega\mu}{\gamma} = \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}} = \sqrt{\frac{(j\omega\mu)^{\cancel{2}}}{\cancel{j\omega\mu}(\sigma + \underbrace{j\omega\varepsilon}_{f.e})}}$$

$$\eta = \sqrt{\frac{\cancel{j\omega}\mu}{\cancel{j\omega}\varepsilon\left(\frac{\sigma}{j\omega\varepsilon} + 1\right)}} = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\sqrt{1 + \frac{\sigma}{j\omega\varepsilon}}} = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\sqrt{1 - j\frac{\sigma}{\omega\varepsilon}}}$$

$$\frac{j\sigma}{-1\ \boxed{j.j}.\omega\varepsilon}$$

$$\eta = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\sqrt{1 - j\frac{\sigma}{\omega\varepsilon}}} \; [\Omega]$$

$$\mathrm{Mod}\left(1 - j\frac{\sigma}{\omega\varepsilon}\right) = \sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2}$$

$$\mathrm{Arg}\left(1 - j\frac{\sigma}{\omega\varepsilon}\right) = tg^{-1}\left(-\frac{\sigma}{\omega\varepsilon}\right)$$

$$|\eta| = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\sqrt[4]{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2}}$$

$$\arg(\eta) = \frac{1}{\sqrt{tg^{-1}\left(-\frac{\sigma}{\omega\varepsilon}\right)}} = \frac{1}{e^{j\frac{1}{2} tg^{-1}\left(-\frac{\sigma}{\omega\varepsilon}\right)}}$$

$$\arg(\eta) = e^{-j\frac{1}{2} tg^{-1}\left(-\frac{\sigma}{\omega\varepsilon}\right)}$$

$$\arg(\eta) = e^{j\frac{1}{2} tg^{-1}\left(\frac{\sigma}{\omega\varepsilon}\right)}$$

$$e^{-j\,tg^{-1}(-\theta)} = e^{j\,tg^{-1}(\theta)}$$

$$\gamma \simeq \alpha + j\beta$$

α — Cte de atenuacion

β — Cte de fase

γ — Cte de Propagación

$$\beta = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[1 + \sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2}\,\right]} \left[\frac{rad}{m}\right]$$

Ecuación Gral de la Cte de fase "β"

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[-1 + \sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2}\,\right]} \left[\frac{Neper}{m}\right]$$

$$|\eta| = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\sqrt[4]{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2}}$$

$$\arg(\eta) = e^{-j\frac{1}{2}tg^{-1}\left(-\frac{\sigma}{\omega\varepsilon}\right)}$$

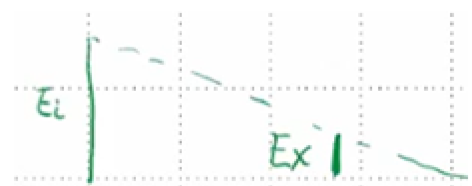
$$\ln e^{x} = x$$

$$E_x = E_i\, e^{-\alpha z} \underbrace{\cos(\omega t - \beta z + \phi_i)}_{=1}$$

$$\frac{E_x}{E_i} = e^{-\alpha z}$$

$$\alpha z = -\ln\left(\frac{E_x}{E_i}\right) \quad [\text{Neper}]$$

$$\alpha = -\frac{1}{z}\ln\left(\frac{E_x}{E_i}\right)\left[\frac{\text{Neper}}{m}\right]$$



$$\delta = \frac{1}{\alpha}\,[m]$$

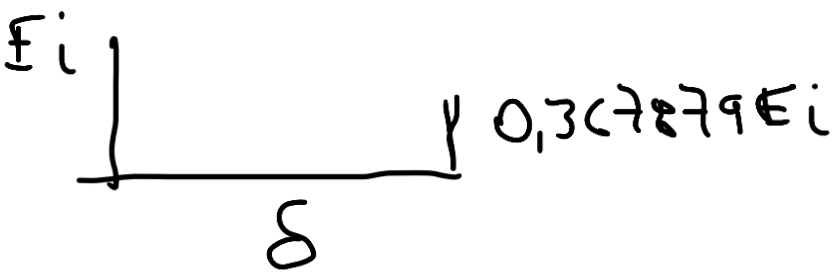
Delta

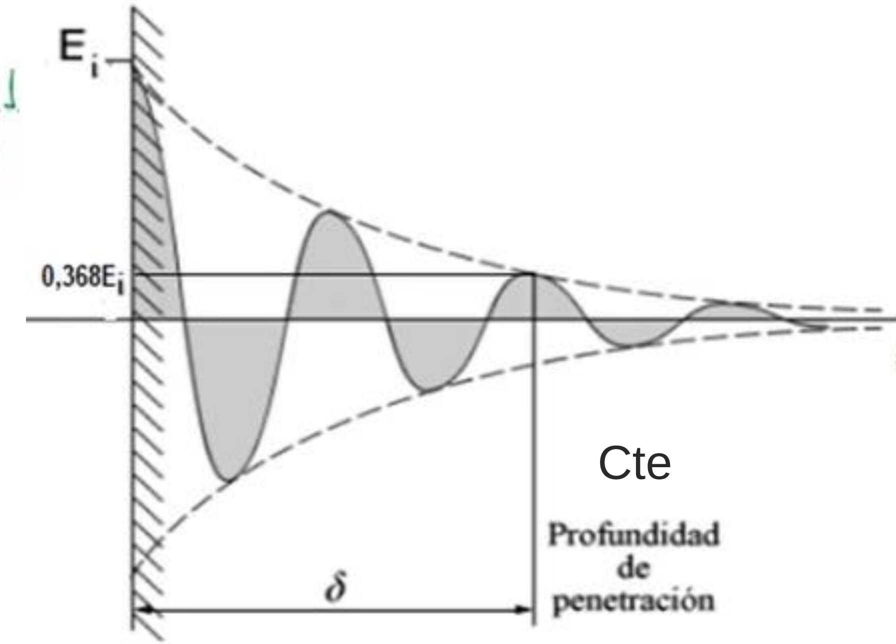
$$E_{x(z,t)} = E_i\, e^{-\alpha z}$$

si la dist $z = \delta$

$$E_x(z,t) = E_i\, e^{-\alpha\cdot\delta} = E_i\, e^{-\alpha\frac{1}{\alpha}} = E_i\, e^{-1}$$

$$E_x(z,t) = 0{,}367879\, E_i$$



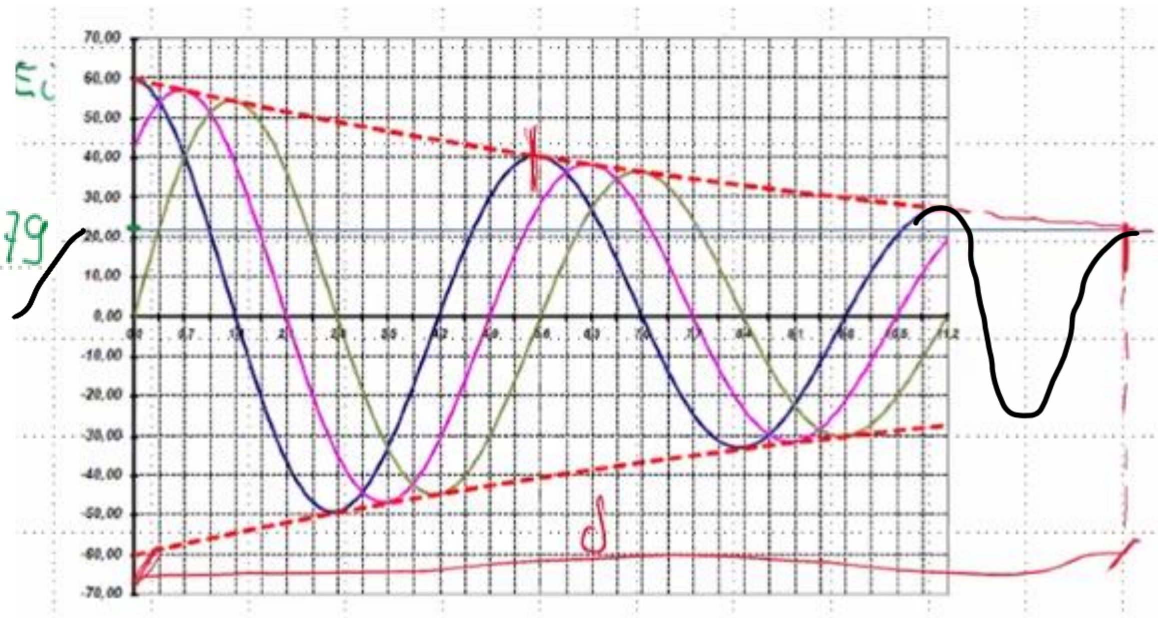


Para tener un dimension de la atenuacion

60 = Ei

0,367879

22



$\alpha = 0{,}00024 \left[\frac{N}{m}\right]$

$\delta = 4.166$ m

Mas distancia
Tarda en aten

$\alpha = 0{,}001 \frac{Neper}{m}$

$\delta = \frac{1}{0{,}001} = 1000$ metros