# MEDIOS DE ENLACE

3R1

Ing. Luis Contrera

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UNIDAD TEMATICA 4

ECUACIÓN DE ONDA

EN UN MEDIO CONTINUO

## ECUACION DE ONDA

1: $\nabla \times H = \nabla E + \varepsilon \frac{\partial E}{\partial t}$

2: $\nabla \times E = -\mu \frac{\partial H}{\partial t}$

forma Vectorial

$$\nabla \times \nabla \times E = \nabla \times \left(-\mu \frac{\partial H}{\partial t}\right)$$

$$\nabla \times \nabla \times E = -\mu \frac{\partial}{\partial t} \nabla \times H$$

$$\nabla \times \nabla \times E = -\mu \frac{\partial}{\partial t}\left(\cancel{\nabla E} + \varepsilon \frac{\partial E}{\partial t}\right)$$

dieléctrico perfecto

$\nabla = 0$

$$\nabla \times \nabla \times E = -\mu \varepsilon \frac{\partial^2 E}{\partial t^2}$$

Identidad Vectorial

$$\nabla\times\nabla\times E = \nabla\left(\underbrace{\nabla\cdot E}\right) - \nabla^2 E$$

Como $\nabla\cdot E = 0$

# Identidad vectorial Nº 21

$$\nabla\times\nabla\times E = -\nabla^2 E$$

$$\nabla\times\nabla\times E = -\mu\varepsilon\frac{\partial^2 E}{\partial t^2}$$

$$-\nabla^2 E = -\mu\varepsilon\frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 E - \mu\varepsilon\frac{\partial^2 E}{\partial t^2} = 0$$

Ec. de Helmholtz

Ec. Dif de 2 orden
Movimientos ondulatorios

Identidad Vectorial

$$\nabla \times \nabla \times E = \nabla.(\underbrace{\nabla . E}) - \nabla^2 E$$

Como $\nabla . E = 0$

$$\nabla \times \nabla \times E = -\nabla^2 E$$

$$\nabla^2 \bar{E} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) \cdot \left(E_x\, \hat{a}_x + E_y . \hat{a}_y + E_z . a_z\right)$$

$$\nabla^2 E = \cancel{\frac{\partial^2 E_x}{\partial x^2} a_x} + \cancel{\frac{\partial^2 E_y}{\partial x^2} \hat{a}_y} + \cancel{\frac{\partial^2 E_z}{\partial x^2} \hat{a}_z} + \cancel{\frac{\partial^2 E_x}{\partial y^2} \hat{a}_x} + \cancel{\frac{\partial^2 E_y}{\partial y^2} \hat{a}_y} + \cancel{\frac{\partial^2 E_z}{\partial y^2} a_z} + \underline{\frac{\partial^2 E_x}{\partial z^2} \hat{a}_x} + \cancel{\frac{\partial^2 E_y}{\partial z^2} \hat{a}_y} + \cancel{\frac{\partial^2 E_z}{\partial z^2} \hat{a}_z}$$

$$E_x = f(z, t)$$

"Arbitrario"

Tomando solo una sola dirección y componente

Para Onda Plana

$$\nabla^2 E = \frac{\partial^2 E_x}{\partial z^2} \hat{a}_x$$

2 ec Maxwell fasorial

$$\nabla \times E\, e^{j\omega t} = -j\omega\mu H\, e^{j\omega t}$$

$$\nabla \times \nabla \times E = -\nabla^2 E$$ Identidad vectorial

$$\nabla \times \nabla \times E\, e^{j\omega t} = -j\omega\mu\, \nabla \times H\, e^{j\omega t}$$

$$\nabla \times H\, e^{j\omega t} = (\sigma + j\omega\varepsilon) E\, e^{j\omega t}$$

$$-\nabla^2 E\, e^{j\omega t} = -j\omega\mu(\sigma + j\omega\varepsilon) E\, e^{j\omega t}$$

$$\nabla^2 E\, e^{j\omega t} = j\omega\mu(\sigma + j\omega\varepsilon) E\, e^{j\omega t}$$

$$\nabla^2 E = \frac{\partial^2 E_x}{\partial z^2}\hat{a}_x$$

$$\frac{\partial^2 E\, e^{j\omega t}}{\partial z^2} = \underbrace{j\omega\mu(\sigma + j\omega\varepsilon)}_{\gamma^2} E\, e^{j\omega t}$$

gamma

$$\frac{\partial^2 E\, e^{j\omega t}}{\partial z^2} = \gamma^2 E\, e^{j\omega t}$$

$$\frac{\partial^2 E\, e^{j\omega t}}{\partial z^2} - \gamma^2 E\, e^{j\omega t} = 0$$

Ec. Helmholtz

$$\frac{\partial^2 E e^{j\omega t}}{\partial z^2} - \gamma^2 E e^{j\omega t} = 0$$

Ec. Helmholtz

$$E_x(z,t) = C_1 e^{-\gamma z} + C_2 e^{\gamma z}$$

Solución de la ecuación diferencial

$$\gamma = \alpha + j\beta$$

Cte de atenuacion

Cte de fase

Cte de Propagación

$$E_x(z,t) = \left(C_1 e^{-\alpha z} \cdot e^{-j\beta z} + C_2 e^{\alpha z} \cdot e^{j\beta z}\right) e^{j\omega t}$$

$$E_x(z,t) = \left(C_1 e^{-\alpha z} \cdot e^{-j\beta z} + C_2 e^{\alpha z} \cdot e^{j\beta z}\right) e^{j\omega t}$$

$$E_x(z,t) = \underbrace{C_1 e^{-\alpha z} e^{j(\omega t - \beta z)}}_{\text{onda incidente}} + \underbrace{C_2 e^{\alpha z} e^{j(\omega t + \beta z)}}_{\text{onda reflejada}}$$

$C_1 = E_i$

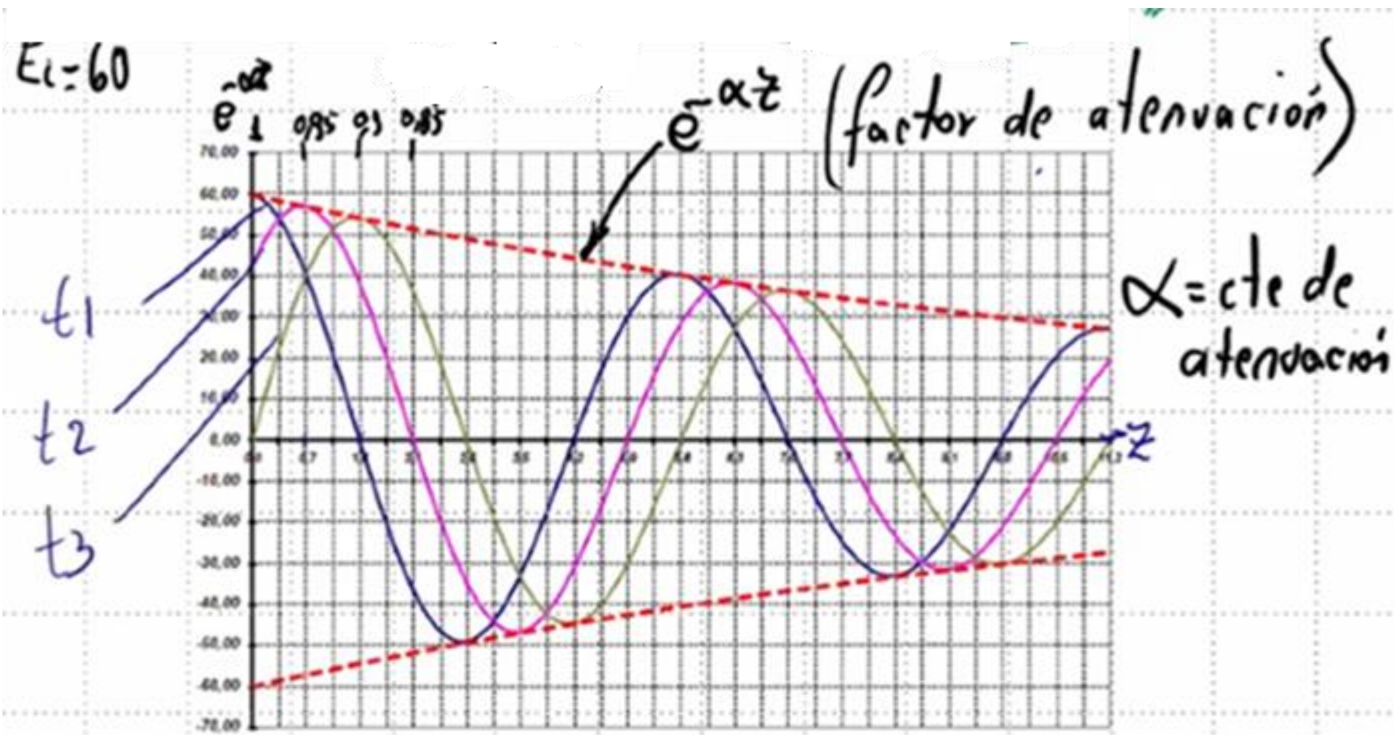
$C_2 = E_r$

Euler $e^{j\theta} = \underbrace{\cos\theta}_{\text{Real}} + j\underbrace{\operatorname{sen}\theta}_{\text{Imag}}$

Solo trabajaremos con la parte "Real"

$$E_x(z,t) = \underbrace{E_i e^{-\alpha z} \cos(\omega t - \beta z)}_{\text{onda incidente}} + \underbrace{E_r e^{\alpha z} \cos(\omega t + \beta z)}_{\text{Onda Reflejada}}$$

$$E_x(z,t) = \underbrace{E_i\, e^{-\alpha z}}_{\text{Amplitud total}} \cos(\omega t - \beta z + \phi_i)$$



$$E_x(z,t) = E_i\, e^{-\alpha z} \cos(\omega t - \beta z + \phi_i)$$

$\sigma \neq 0$
"General"

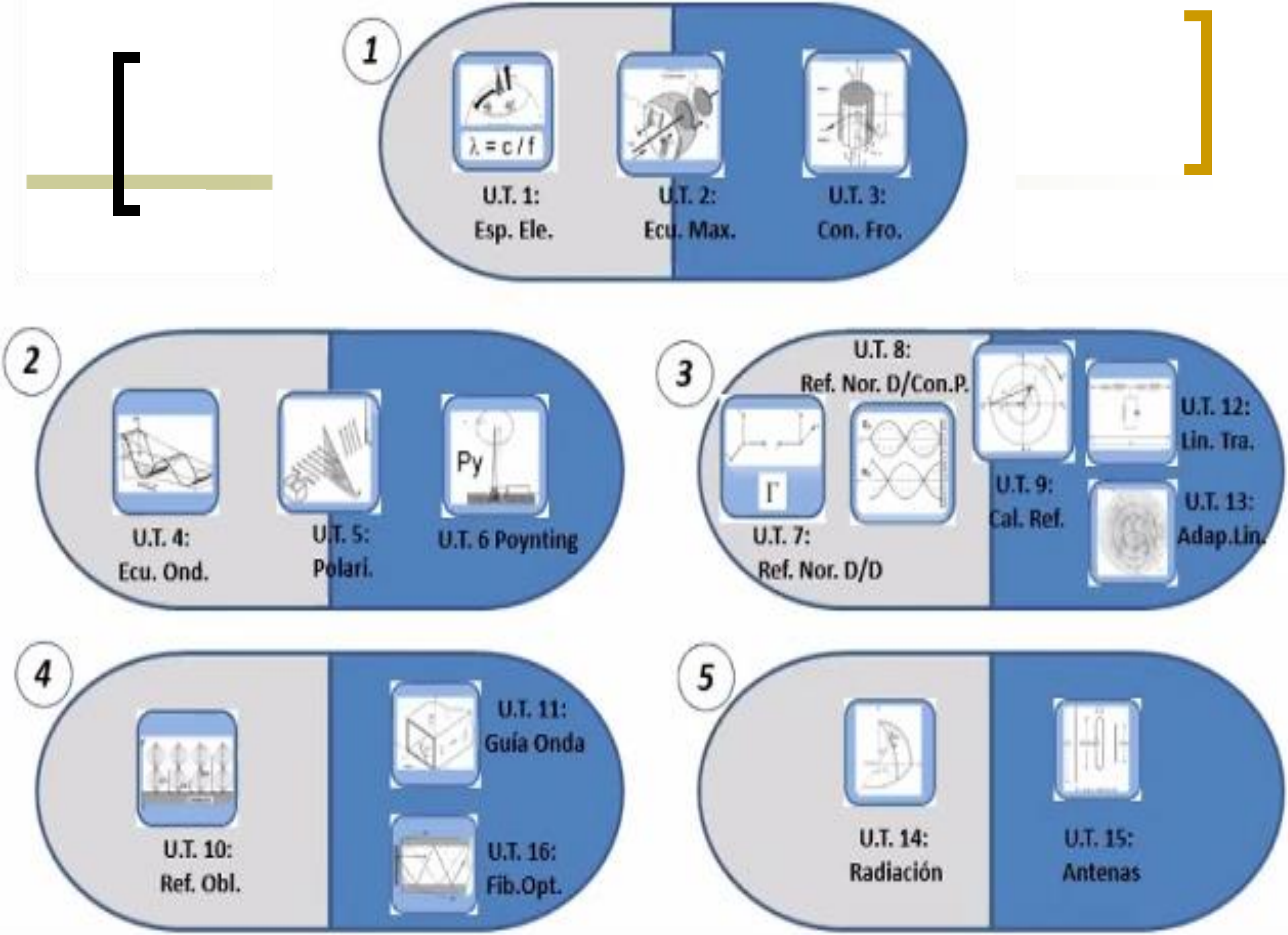
$$E_x(z,t) = E_i \cos(\omega t - \beta z + \phi_i)$$

$\sigma = 0$
Dieléctrico Perfecto

$$\beta = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[1 + \sqrt{1 + \underbrace{\left(\frac{\sigma}{\omega\varepsilon}\right)^2}_{0}}\right]} \quad \left[\frac{rad}{m}\right]$$

Ecuación Gral de la Cte de fase "$\beta$"

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[-1 + \sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2}\right]} \quad \left[\frac{Neper}{m}\right]$$



1
λ = c / f
U.T. 1:
Esp. Ele.
U.T. 2:
Ecu. Max.
U.T. 3:
Con. Fro.
2
U.T. 4:
Ecu. Ond.
U.T. 5:
Polari.
Py
U.T. 6 Poynting
3
U.T. 8:
Ref. Nor. D/Con.P.
Γ
U.T. 7:
Ref. Nor. D/D
U.T. 9:
Cal. Ref.
U.T. 12:
Lin. Tra.
U.T. 13:
Adap.Lin.
4
U.T. 10:
Ref. Obl.
U.T. 11:
Guía Onda
U.T. 16:
Fib.Opt.
5
U.T. 14:
Radiación
U.T. 15:
Antenas