MEDIOS DE ENLACE

3R1

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Campos Estáticos

$$\nabla \times \bar{H} = J \qquad \oint \bar{H}\, dr = I$$

$$\nabla \times \bar{E} = 0 \qquad \oint \bar{E}\, dr = 0$$

$$\nabla \cdot \bar{B} = 0 \qquad \oint \bar{B}\, ds = 0$$

$$\nabla \cdot \bar{D} = \rho \qquad \oint \bar{D}\, ds = Q$$

forma vectorial

forma integral

Leyes de Maxwell

1) $\nabla \times H = J + \dfrac{\partial D}{\partial t} \qquad \oint H\, dr = \int \left(J + \dfrac{\partial D}{\partial t} \right) ds$

2) $\nabla \times E = -\mu \dfrac{\partial H}{\partial t} \qquad \oint E\, dr = -\mu \int \dfrac{\partial H}{\partial t}\, ds$

3) $\nabla \cdot \bar{B} = 0 \qquad \oint B\, ds = 0$

4) $\nabla \cdot \bar{D} = \rho \qquad \oint D\, ds = Q$

Forma Fasorial

1) $$\boxed{\nabla \times H = J + \frac{\partial D}{\partial t}}$$

forma Vectorial

$$H = H_0 e^{j\omega t}$$

$$E = E_0 e^{j\omega t} \qquad D = \varepsilon E \qquad J = \sigma E$$

$$\nabla \times H_0 e^{j\omega t} = J + \varepsilon j\omega E_0 e^{j\omega t}$$

$$\nabla \times H_0 e^{j\omega t} = \sigma . E_0 e^{j\omega t} + j\omega\varepsilon E_0 e^{j\omega t}$$

1) $$\boxed{\nabla \times H_0 e^{j\omega t} = (\sigma + j\omega\varepsilon) E_0 e^{j\omega t}}$$

2) $$\boxed{\nabla \times E = -\mu \frac{\partial H}{\partial t}}$$

2) $$\boxed{\nabla \times E_0 e^{j\omega t} = -j\omega\mu H_0 e^{j\omega t}}$$

Densidad de corriente conducción

Densidad de corriente desplazamiento

1º Ec Maxwell
Forma Fasorial

$$\boxed{\nabla \times H e^{j\omega t} = \overbrace{\sigma E_0 e^{j\omega t}}^{J_C} + \overbrace{j\omega\varepsilon E_0 e^{j\omega t}}^{J_D}}$$

Factor de disipación

$$F.D = \frac{J_C}{J_D} = \frac{\sigma \cancel{E_0 e^{j\omega t}}}{\omega\varepsilon \cancel{E_0 e^{j\omega t}}} = \frac{\sigma}{\omega\varepsilon}$$

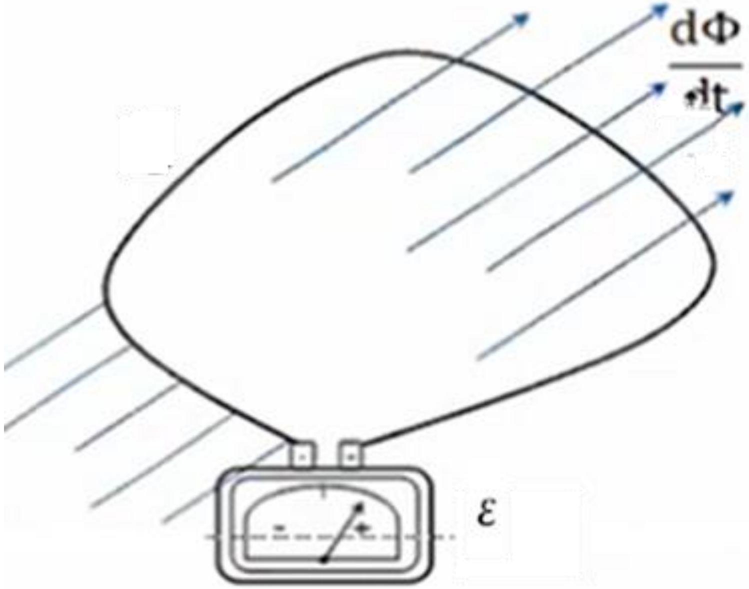
Comportamiento de los Medios

$$\frac{\sigma}{\underbrace{\omega}_{2\pi f}\varepsilon} = \begin{cases} >1 & \text{Conductor} \\ <1 & \text{Dieléctrico} \end{cases}$$

Tierra

$$\left.\begin{matrix} \mu_T \\ \varepsilon_T \\ \sigma_T \end{matrix}\right\} \text{Medio}$$

Frec →



$$\varepsilon = -\frac{d\phi}{dt}$$ Ley de Faraday

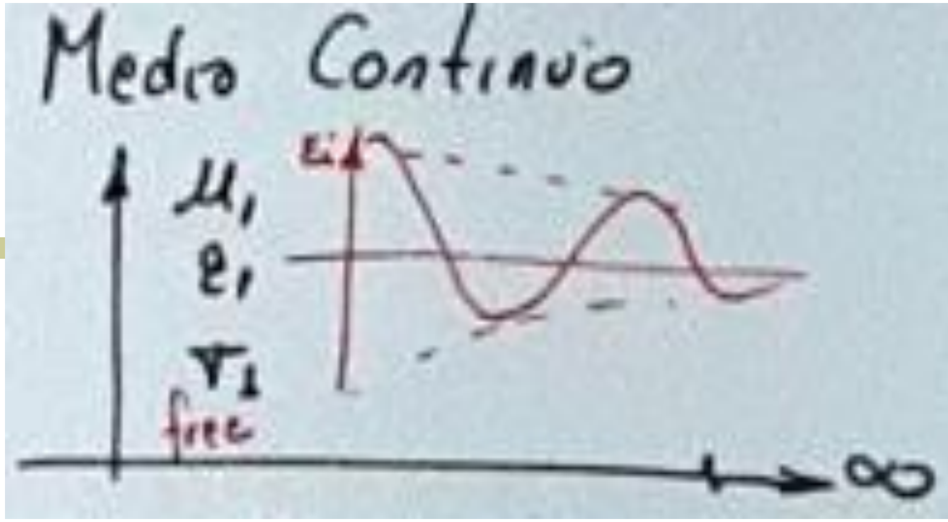
$$\phi = B\,S$$

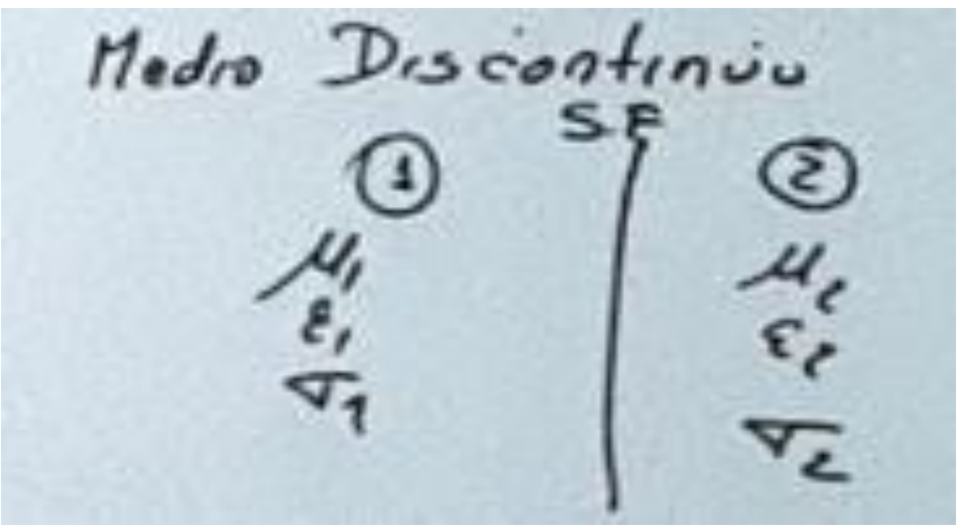
$$B = \mu \cdot H$$ H = Ho cos (w t)

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d\,B\,S}{dt}$$

UNIDAD TEMATICA 3

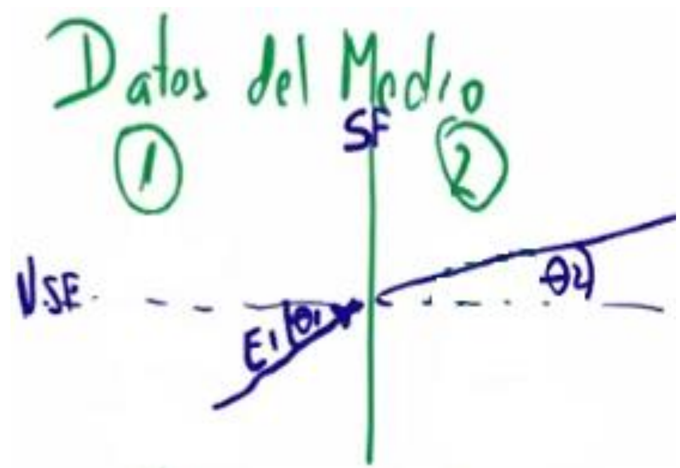
CONDICIONES DE CONTORNO

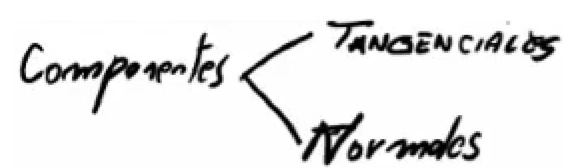




Dieléctrico / Dieléctrico

Dieléctrico / Conductor perfecto



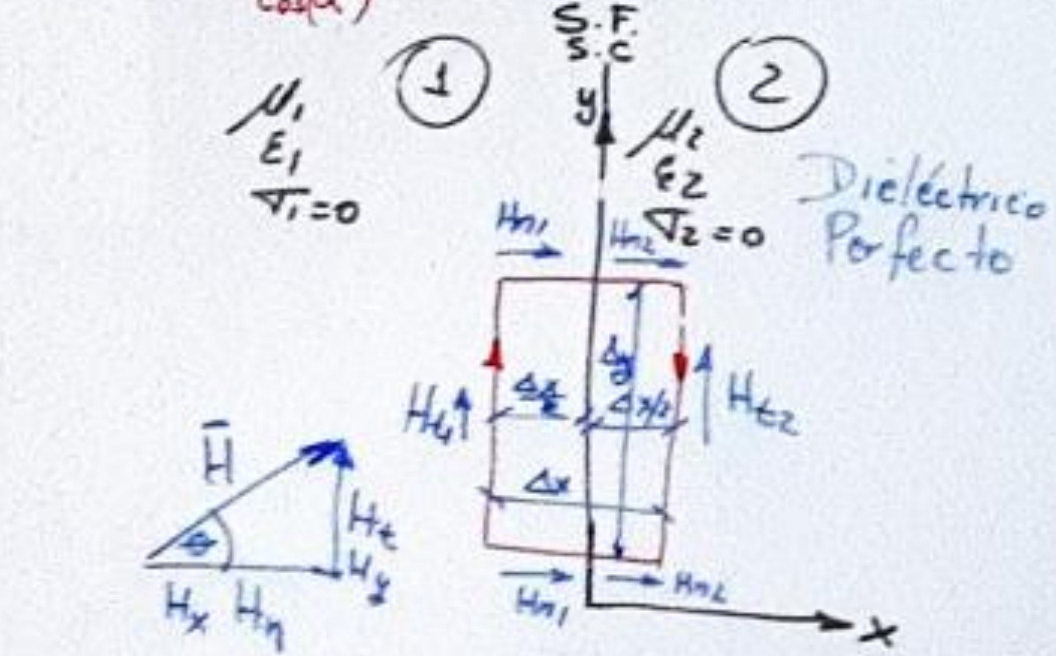


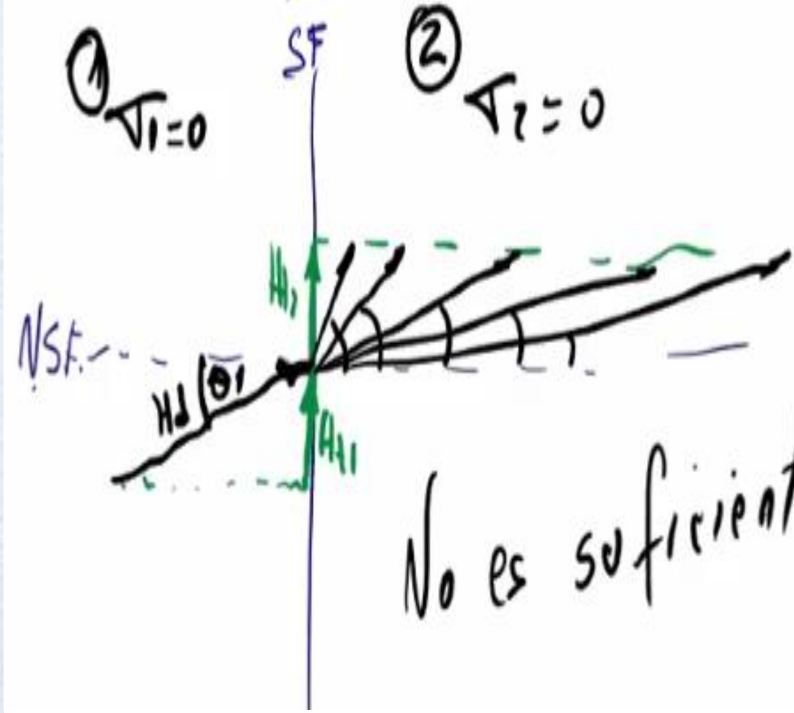
CONDICIONES DE CONTORNO o FRONTERA

Dos Medios Dieléctricos Perfectos

$\sigma = 0 \qquad J = \sigma E = 0$

1·) $\oint_c \underbrace{H.dl}_{\cos(\alpha)} = \int \left(\cancelto{0}{J} + \varepsilon \frac{\partial E}{\partial t} \right) ds$





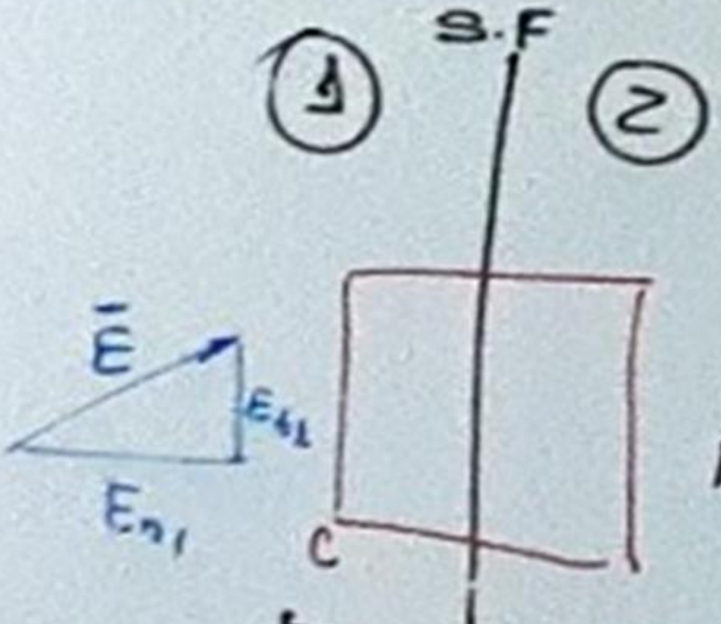
No es suficiente

$H_{t1} \cdot \Delta y + H_{n1} \cancelto{0}{\frac{\Delta x}{2}} + H_{n2} \cancelto{0}{\frac{\Delta x}{2}} - H_{t2} \cdot \Delta y - H_{n2} \cancelto{0}{\frac{\Delta x}{2}} - H_{n1} \cancelto{0}{\frac{\Delta x}{2}} = \varepsilon \frac{\partial E}{\partial t} \cancelto{0}{\Delta x} \cdot \Delta y$

Si hacemos $\Delta x \to 0$

$H_{t1} \Delta y - H_{t2} \Delta y = 0 \therefore \boxed{H_{t1} = H_{t2}}$

2.) $\oint_C E\,dl = -\int \mu \frac{\partial H}{\partial t}\, ds$



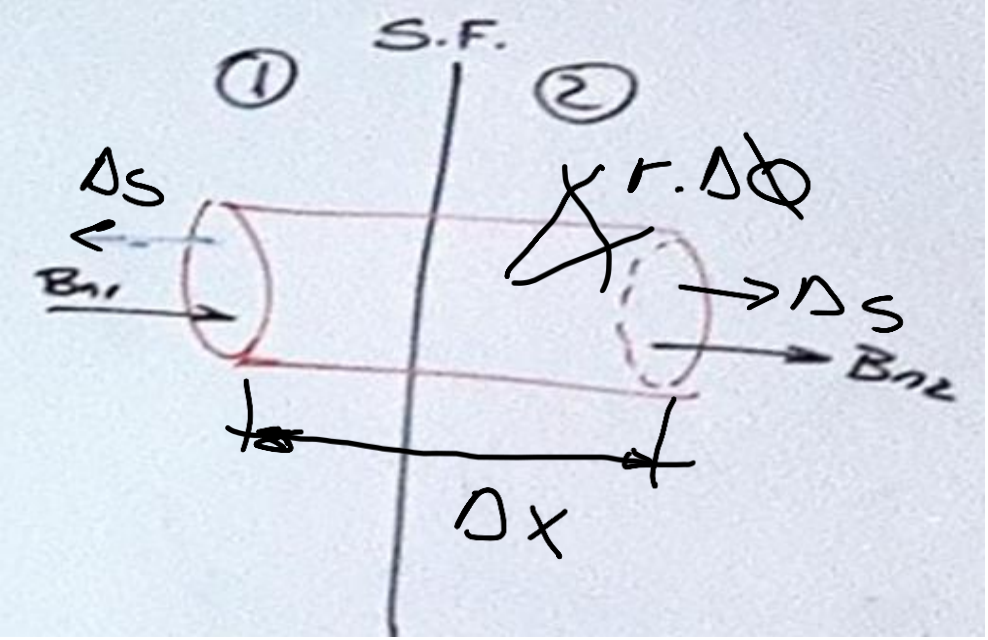
$$\boxed{E_{t1} = E_{t2}}$$

Mismo Prodedimiento Anterior

3) $\oint B\,ds = 0$ $\oint \bar{B}\,ds = \int \nabla . \bar{B}\,dv$

$$-B_{n_1}\,\Delta S + B_r\, r\,\Delta\phi\,\Delta X + B_{n_2}\,\Delta S = 0$$

$$\boxed{B_{n1} = B_{n2}}$$



4) $\oint D \, ds = Q$

Desplazamiento eléctrico

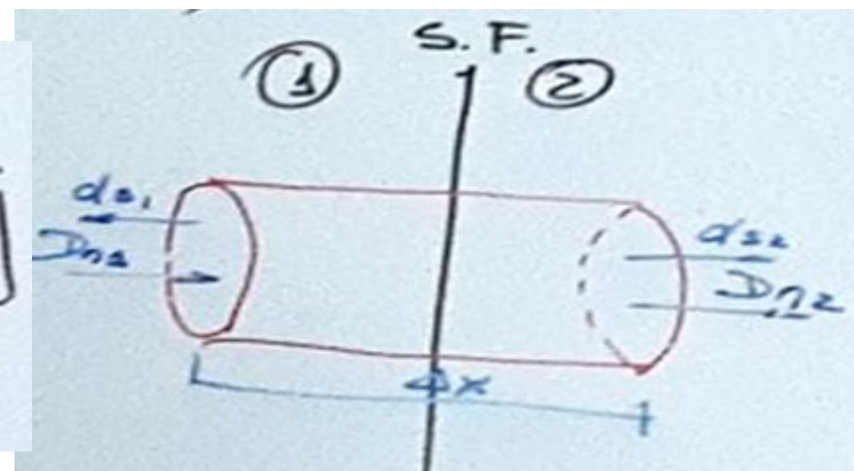
$$\underbrace{\int_{Sup} \overline{D}\cdot\overline{ds}} = \int_{Vol} \rho\cdot \underbrace{dv}$$

$$\oint_{Sup} \vec{D}\cdot\vec{dS} = -D_{n_1}\Delta S + \cancel{D_r r d\phi \Delta x} + D_{n_2}\Delta S = \rho \underbrace{\cancel{\Delta S \Delta X}}_{dv}$$

si $\Delta x = 0$

$$\boxed{-D_{n1}\, ds_1 + D_{n2}\, ds_2 = 0}$$

$$\boxed{D_{n1} = D_{n2}}$$

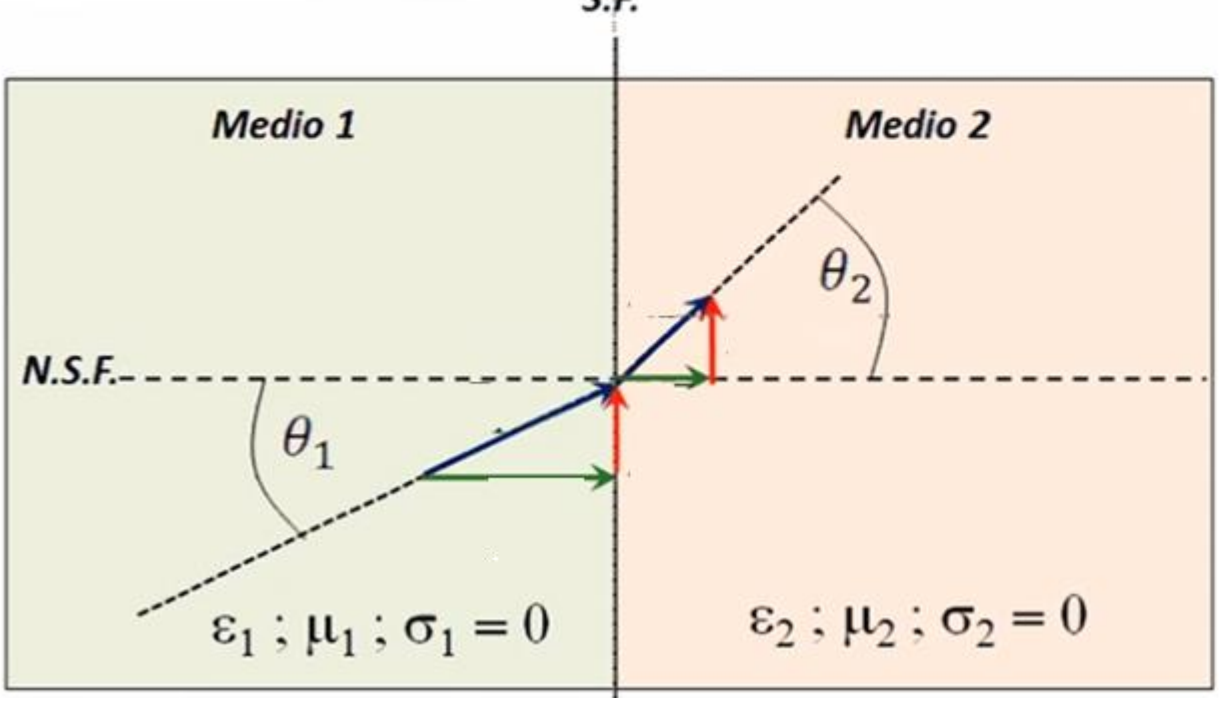


Condiciones de Contorno D/D

$$\boxed{\begin{aligned} H_{t_1} &= H_{t_2} \\ E_{t_1} &= E_{t_2} \\ D_{n_1} &= D_{n_2} \\ B_{n_1} &= B_{n_2} \end{aligned}}$$

$$D = \varepsilon E$$

$$B = \mu H.$$



$$tg\,\theta_1 = \frac{H_{t1}}{H_{n1}} \quad ; \quad tg\,\theta_2 = \frac{H_{t2}}{H_{n2}}$$

$$H_{n1} \cdot tg\,\theta_1 = H_{n2} \cdot tg\,\theta_2$$

$$B = \mu H$$

$$B_n = \mu \cdot H_n$$

$$H_n = \frac{B_n}{\mu}$$

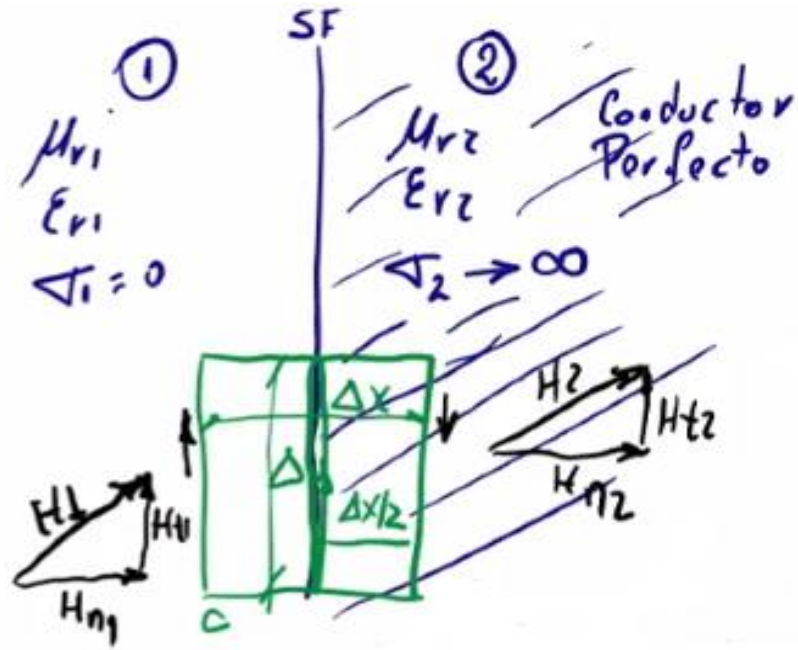
$$\frac{B_{n1}}{\mu_1}\, tg\,\theta_1 = \frac{B_{n2}}{\mu_2}\, tg\,\theta_2$$

$$tg\,\theta_1 = \frac{\mu_1}{\mu_2} \cdot tg\,\theta_2$$

$$B_{1n} = B_{2n}$$

$$\theta_1 = tg^{-1}\left(\frac{\mu_1}{\mu_2} \cdot tg\,\theta_2 \right)$$

Diel / Cond Perfecto



$$\underbrace{\oint H\,dr}_{*} = \int \left(\underbrace{\sigma E}_{J} + \varepsilon \frac{\partial E}{\partial t}\right) ds$$

Densidad Superficial de corriente

$$J\left[\frac{A}{m^2}\right] = \frac{I}{\Delta x \Delta y} \qquad J_{Ls}\left[\frac{A}{m}\right]$$

Densidad corriente lineal sup

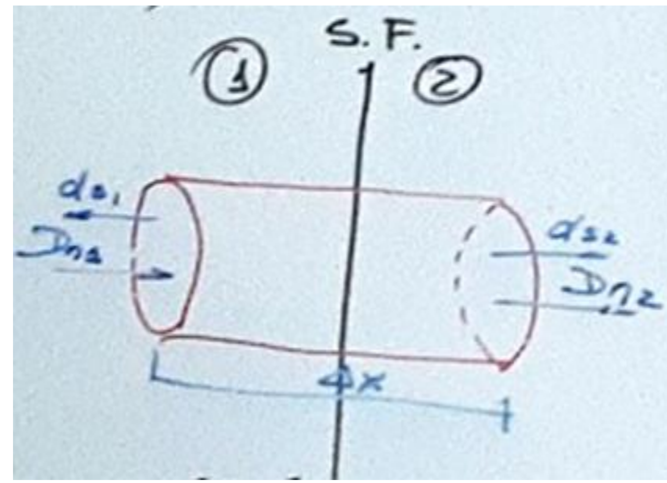
$$H_{t1}\cdot\Delta y + \cancel{H_{n1}\frac{\Delta x}{2}} + \cancel{H_{n2}\frac{\Delta x}{2}} - H_{t2}\Delta y - \cancel{H_{n2}\frac{\Delta x}{2}}^{0} - \cancel{H_{n1}\frac{\Delta x}{2}}^{0} = \overset{*}{J}\cdot\Delta x\cdot\Delta y = \frac{J_{Ls}}{\cancel{\Delta x}}\cdot\cancel{\Delta x}\cdot\Delta y$$

Si $\Delta x \to 0$ $\quad H_{t1}\cancel{\Delta y} - H_{t2}\cancel{\Delta y} = J_{Ls}\cancel{\Delta y}$

$$\boxed{H_{t1} = J_{Ls}} \qquad \boxed{H_{t2} = 0}$$

$$\int_{Sup} \overline{D} \cdot \overline{ds} = \int_{Vol} \rho \cdot dv$$

$$\rho = \frac{C}{m^3} = \frac{C}{\Delta x \Delta y} \qquad \rho_s = \frac{C}{m^2}$$



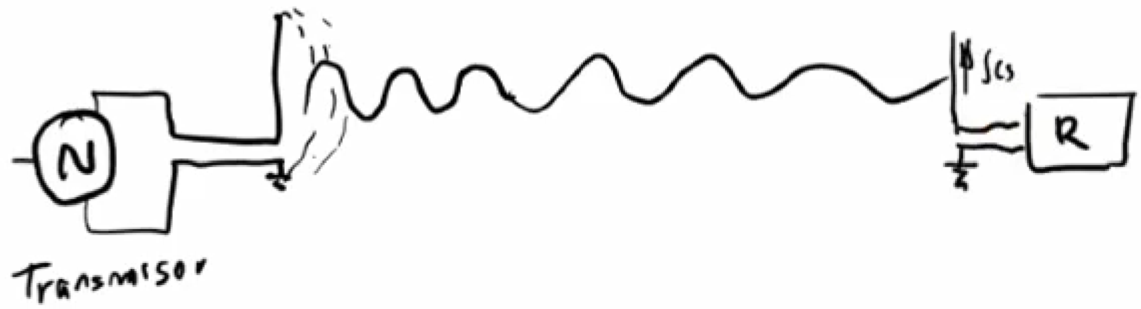
$$\oint_{Sup} \vec{D} \cdot \vec{dS} = -D_{n_1}\Delta S + D_r r d\phi \Delta x + D_{n_2}\Delta S = \frac{\rho_s}{\Delta X} \Delta S \Delta X$$

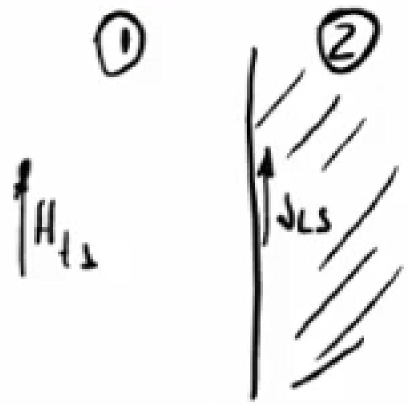
En Diel / Cond Perfecto

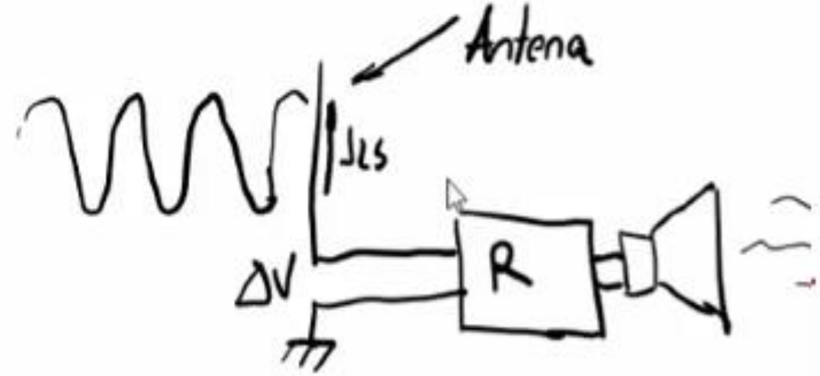
$$\boxed{D_{n1} = \rho_s} \qquad \boxed{D_{n2} = 0}$$

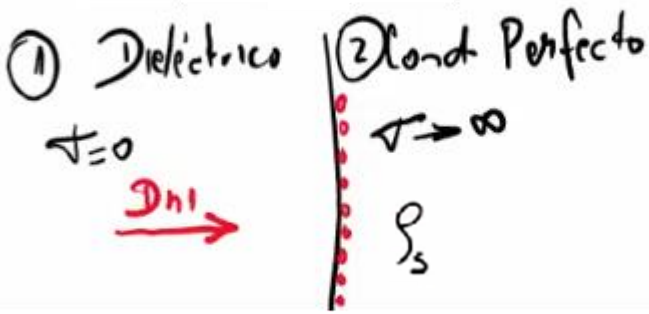
$$\rho_s = \frac{C}{m^2}$$

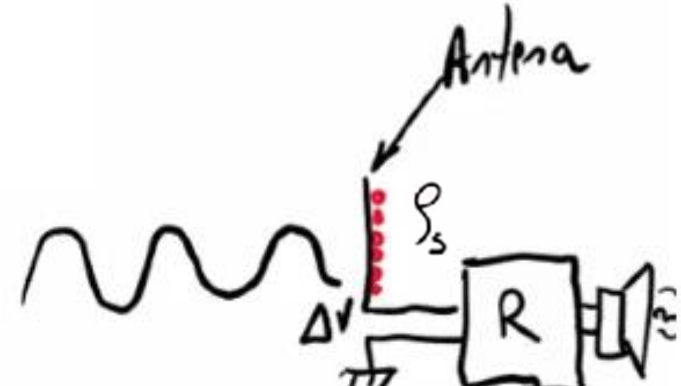
Densidad Superf.


N
R
Transmisor


1
2
H_{t1}
J_{LS}


Antena
J_{LS}
ΔV
R


1 Dieléctrico
2 Cond. Perfecto
$\sigma = 0$
$\sigma \to \infty$
D_{n1}
ρ_s


Antena
ρ_s
ΔV
R

Condiciones de Contorno

	DIEL/DIEL	DIEL/Cond Perf.	Ec Maxwell
TANGENC.	$H_{t1} = H_{t2}$	$H_{t1} = J_{ls}$ $H_{t2} = 0$	$\oint H\,dl = \int \left(J + \frac{\partial D}{\partial t}\right) ds$
	$E_{t1} = E_{t2}$	—	$\oint E\,dl = -\int \mu \frac{\partial H}{\partial t}\, ds$
Normales	$D_{n1} = D_{n2}$	$D_{n1} = \rho_s$ $D_{n2} = 0$	$\oint D\,ds = \rho$
	$B_{n1} = B_{n2}$	—	$\oint B\,ds = 0$