Soluciones 2do parcial 5R1 03/11/09.

1) El sistema con el valor de K ajustado es: (1)

$$G(s)H(s) = 30 \frac{s+1}{s^3+3s^2-4s-12}$$

El polinomio denominador factoreado es:

$$p(2) = 8+3.4-4.2-12=0 \therefore s_1=2$$

$$\begin{array}{l|l} s^3+3s^2-4s-12 & s-2 \\ \hline -s^3+2s^2 & = s^2+5s+6 \\ \quad 5s^2-4s & \\ \quad -5s^2+10s & \\ \qquad 6s-12 & \\ \qquad -6s+12 & \end{array}$$

$$s_{2-3} = \frac{-5\pm\sqrt{25-24}}{2} = \frac{-5\pm1}{2}$$

$$s_2=-2 \;;\; s_3=-3$$

$$G(s)H(s) = 30 \frac{s+1}{(s-2)(s+2)(s+3)}$$ En formato Bode:

$$G(s)H(s) = 30 \frac{s+1}{(-2)(-0{,}5s+1)\,2(0{,}5s+1)\,3(0{,}33s+1)}$$

$$G(s)H(s) = \frac{2{,}5}{(-1)} \frac{s+1}{(-0{,}5s+1)(0{,}5s+1)(0{,}33s+1)}$$

$$G(j\omega)H(j\omega) = \frac{2{,}5}{(-1)} \frac{1+j\omega}{(1-j0{,}5\omega)(1+j0{,}5\omega)(1+j0{,}33\omega)}$$

$$|G(j\omega)H(j\omega)|_{dB} = 20\log \frac{|2{,}5|}{|-1|} \frac{|1+j\omega|}{|1-j0{,}5\omega|\,|1+j0{,}5\omega|\,|1+j0{,}33\omega|}$$

$$|2,5|_{dB} = 20 \log 2,5 = 7,96$$

$$|1+j\omega|_{dB} = 20 \log \sqrt{1+\omega^2} = 10 \log(1+\omega^2)$$

$$\left|\frac{1}{(-1)}\right|_{dB} = 20 \log 1 - 20 \log 1 = 0$$

$$\left|\frac{1}{1-j0,5\omega}\right|_{dB} = 20 \log 1 - 20 \log \sqrt{1+0,25\omega^2} = -10 \log(1+0,25\omega^2)$$

$$\left|\frac{1}{1+j0,5\omega}\right|_{dB} = 20 \log 1 - 20 \log \sqrt{1+0,25\omega^2} = -10 \log(1+0,25\omega^2)$$

$$\left|\frac{1}{1+j0,33\omega}\right|_{dB} = 20 \log 1 - 20 \log \sqrt{1+0,11\omega^2} = -10 \log(1+0,11\omega^2)$$

$$\angle G(j\omega)H(j\omega) = \angle 2,5 + \angle 1+j\omega - \angle -1 - \angle 1-j0,5\omega - \angle 1+j0,5\omega - \angle 1+j0,33\omega$$

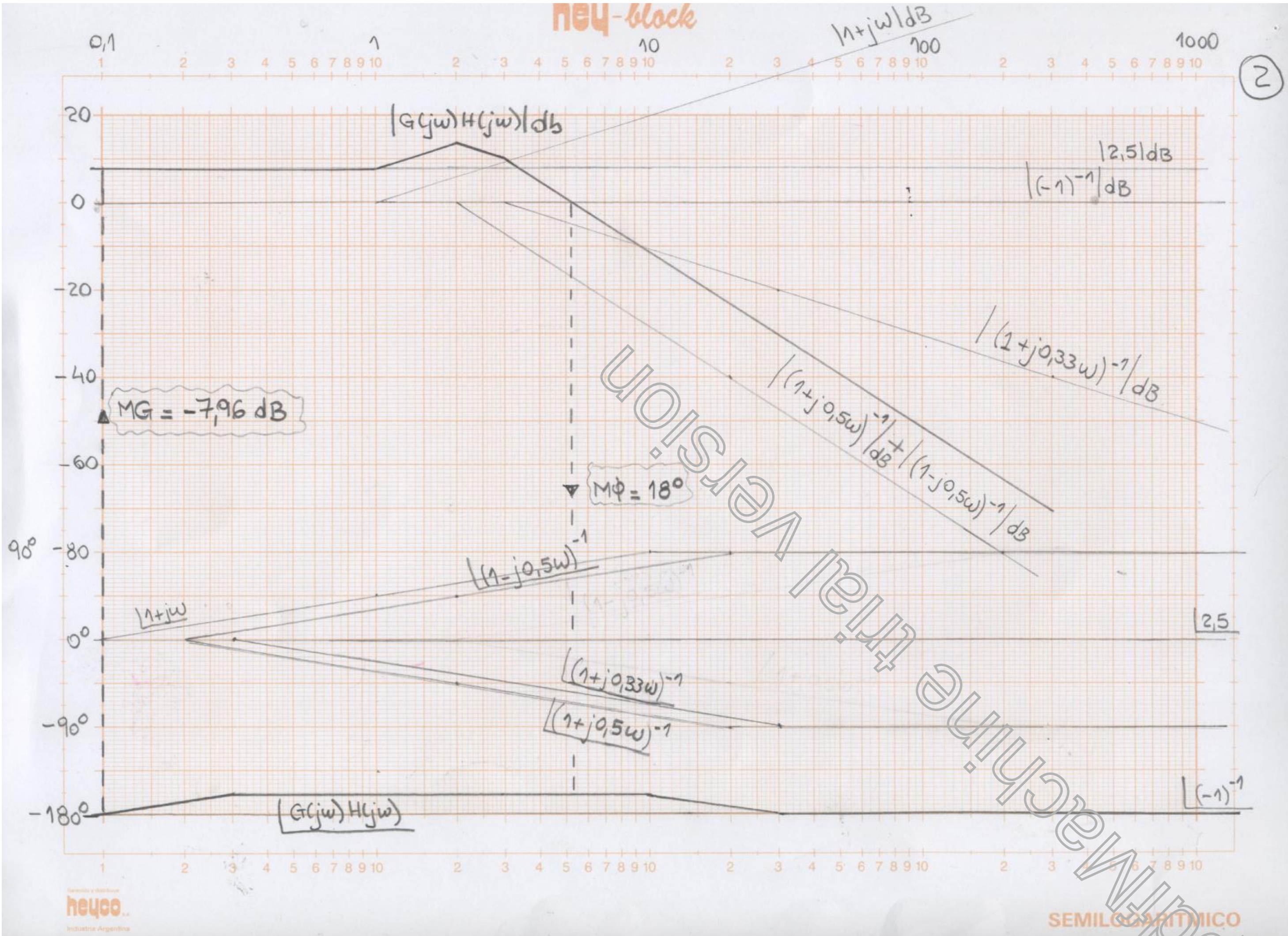
$$\angle 2,5 = \operatorname{arc\,tg} \frac{0}{2,5} = 0^\circ \qquad \angle 1+j\omega = \operatorname{arc\,tg} \omega$$

$$\angle \frac{1}{(-1)} = \angle 1 - \angle (-1) = 0^\circ - \operatorname{arc\,tg} \frac{0}{-1} = 0^\circ - 180^\circ = -180^\circ$$

$$\angle \frac{1}{1-j0,5\omega} = \angle 1 - \angle 1-j0,5\omega = -\operatorname{arc\,tg}(-0,5\omega) = \operatorname{arc\,tg} 0,5\omega$$

$$\angle \frac{1}{1+j0,5\omega} = \angle 1 - \angle 1+j0,5\omega = -\operatorname{arc\,tg} 0,5\omega$$

$$\angle \frac{1}{1+j0,33\omega} = \angle 1 - \angle 1+j0,33\omega = -\operatorname{arc\,tg} 0,33\omega$$



ney-block
0,1
1
10
100
1000
2
|G(jω)H(jω)|db
|1+jω|dB
|2,5|dB
$|(-1)^{-1}|$dB
20
0
-20
-40
-60
-80
MG = -7,96 dB
MΦ = 18°
$|(1+j0,33\omega)^{-1}|$dB
$|(1+j0,5\omega)^{-1}|$dB + $|(1-j0,5\omega)^{-1}|$dB
90°
0°
-90°
-180°
|1+jω
$\lfloor(1-j0,5\omega)^{-1}$
$\lfloor 2,5$
$\lfloor(1+j0,33\omega)^{-1}$
$\lfloor(1+j0,5\omega)^{-1}$
$\lfloor G(j\omega)H(j\omega)$
$\lfloor(-1)^{-1}$
heyco
SEMILOGARITMICO

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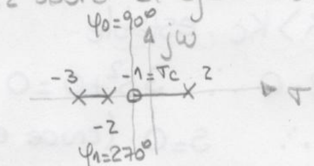
2) Veamos primero el lugar de raices sin la compensación en atraso (3)

$$G(s)H(s) = K \frac{s+1}{s^3+3s^2-4s-12} = K \frac{s+1}{(s-2)(s+2)(s+3)}$$

- Lugar de raiz sobre el eje real:



- Asintotas $p-z=3-1=2 \quad \therefore K=0,1$

$$\varphi_K = \frac{180°(2K+1)}{p-z} \;;\; \varphi_0 = \frac{180°}{2} = 90° \quad \varphi_1 = \frac{180°}{2}\cdot 3 = 270°$$

$$\sigma_c = \frac{\sum Re[p] - \sum Re[z]}{p-z} = \frac{-3\cancel{-2}\cancel{+2}-(-1)}{3-1} = \frac{-2}{2} = -1$$

- Punto de bifurcación:

$$K\frac{s+1}{s^3+3s^2-4s-12} + 1 = 0 \quad \therefore K = -\frac{s^3+3s^2-4s-12}{s+1}$$

$$\frac{\partial K}{\partial s} = -\frac{(3s^2+6s-4)(s+1)-(s^3+3s^2-4s-12)}{(s+1)^2} = 0$$

$$\frac{\partial K}{\partial s} = -\frac{3s^3+9s^2+2s-4-s^3-3s^2+4s+12}{(s+1)^2} = 0$$

$$2s^3+6s^2+6s+8=0 \quad \therefore \quad s^3+3s^2+3s+4=0.$$

$\boxed{s_1=-2,44=pb}$; $s_{2-3} = -0,28 \pm j1,25$ (complejas conjugadas no son PB)

$$K(pb) = 0,76.$$

- Criterio de Routh:

$$K\frac{s+1}{s^3+3s^2-4s-12} + 1 = 0$$

$$s^3+3s^2-4s-12+K(s+1) = 0$$

$$s^3+3s^2+(K-4)s+(K-12)=0$$

s^3	1	$K-4$
s^2	3	$K-12$
s	$2K$	
s^0	$K-12$	

$3(K-4)-(K-12)=3K-\cancel{12}-K+\cancel{12}=2K$

$K_c-12=0 \therefore K_c=12$

$K<K_c$ inestable con una raiz con Re[+]

$K>K_c$ estable

Punto de cruce: $3s^2+K_c-12=0 \therefore 3s^2+0=0$

$\therefore s=0$ (cruce eje imaginario)

Puntos auxiliares para trazado del LR

$K=1$	$s^3+3s^2-3s-11=0$	$s_1=1{,}86$	$s_{2-3}=-2{,}43\pm j0{,}3$
$K=5$	$s^3+3s^2+s-7=0$	$s_1=1{,}18$	$s_{2-3}=-2{,}09\pm j1{,}25$
$K=10$	$s^3+3s^2+6s-2=0$	$s_1=0{,}29$	$s_{2-3}=-1{,}65\pm j2{,}06$
$K=20$	$s^3+3s^2+16s+8=0$	$s_1=-0{,}55$	$s_{2-3}=-1{,}23\pm j3{,}63$ punto diseño
$K=30$	$s^3+3s^2+26s+18=0$	$s_1=-0{,}74$	$s_{2-3}=-1{,}13\pm j4{,}8$

Se asume el punto de diseño como L.R:

$$t_p=0{,}86\text{ seg}=\frac{\pi}{\omega_d} \therefore \omega_d=\frac{\pi}{0{,}86}=3{,}65\ \frac{\text{rad}}{\text{seg}}$$

$$t_{s5\%}=2{,}44\text{ seg}=\frac{3}{\sigma} \therefore \sigma=\frac{3}{2{,}44}=1{,}23\ \text{seg}^{-1}$$

punto de diseño: $\boxed{-1{,}23\pm j3{,}65}$

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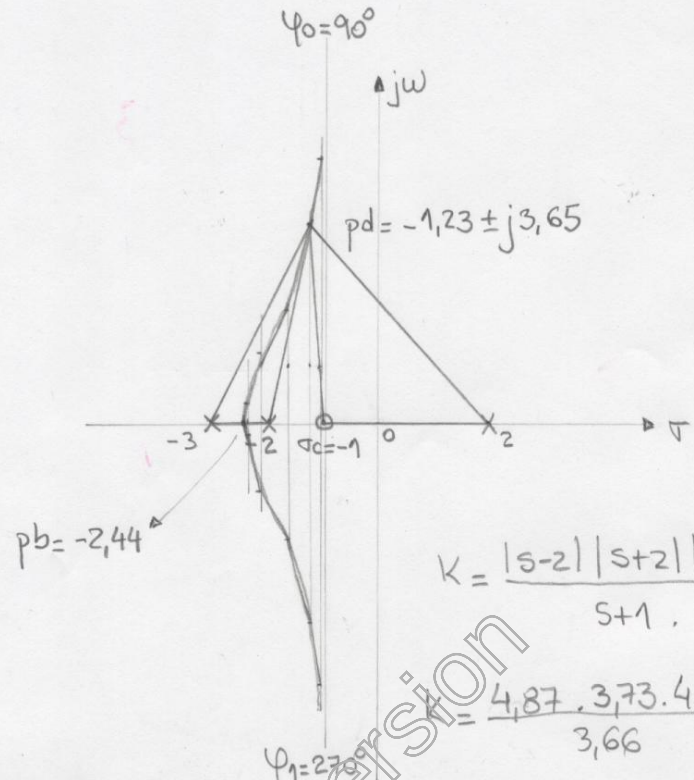

$$K = \frac{|s-2|\,|s+2|\,|s+3|}{s+1}.$$

$$K = \frac{4,87 \cdot 3,73 \cdot 4,06}{3,66} = 20,15$$

$$G(s)H(s) = 20,15 \,\frac{s+1}{s^3+3s^2-4s-12}$$

$$K_p = \lim_{s \to 0} 20,15 \,\frac{1}{(-12)} = -1,68$$

compensamos en atraso: $K_p = (-1,68)\,\dfrac{z_c}{p_c} = -3$

$\dfrac{z_c}{p_c} = 1,79.$ $\quad 0,62 > z_c > 0,12$

$z_c = 0,18 \;.\; \therefore\; p_c = \dfrac{0,18}{1,79} = 0,1.$

$$\boxed{G(s)H(s) = 20,15\,\frac{s+1}{s^3+3s^2-4s-12} \cdot \frac{s+0,18}{s+0,1}}$$