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SOLUCION EJERCICIO 7 TP1-1: Dadas $R(s)=\frac{1}{s^2}$ y $G(s)=\frac{10}{s+1}$

obtendremos a $y(t)$ como $y(t)=\mathcal{L}^{-1}[Y(s)]=\mathcal{L}^{-1}[R(s)G(s)]$.

$Y(s)=\frac{10}{s^2(s+1)}$ para antitransformar expandimos en fracciones

simples: $Y(s)=\frac{A_0}{s^2}+\frac{B_0}{s}+\frac{A_1}{s+1}$.

$A_0=\lim_{s\to 0}\frac{10}{s+1}=10$; $B_0=\lim_{s\to 0}\frac{d}{ds}\left[\frac{10}{s+1}\right]=\lim_{s\to 0}-\frac{10}{(s+1)^2}=-10$

$A_1=\lim_{s\to -1}\frac{10}{s^2}=10$; con esto $Y(s)=\frac{10}{s^2}-\frac{10}{s}+\frac{10}{s+1}$.

$$y(t)=10t-10+10e^{-t}=\boxed{10(t-1+e^{-t})=y(t)} \quad a$$

Ahora en la segunda parte del ejercicio obtendremos $r(t)$ y $g(t)$:

$$r(t)=\mathcal{L}^{-1}[R(s)]=t \quad ; \quad g(t)=\mathcal{L}^{-1}[G(s)]=10e^{-t}.$$

Obtenemos ahora el producto de convolución:

$$y(t)=r(t)*g(t)=\int_0^t r(\tau)\,g(t-\tau)\,d\tau$$

Operamos el cambio de variable para $r(t)$ y $g(t)$:

$$r(\tau)=\tau \quad g(t-\tau)=10e^{-(t-\tau)}=10e^{\tau-t}$$

$$y(t)=\int_0^t \tau\cdot 10e^{\tau-t}\,d\tau=10e^{-t}\int_0^t \tau e^{\tau}\,d\tau.$$

A la integral la resolvemos por partes:

$$\int u\,dv=uv-\int v\,du.$$

$u = \tau \quad dv = e^{\tau} d\tau.$

$v = \int e^{\tau} d\tau = e^{\tau} \; ; \; du = d\tau.$

$$\int \tau e^{\tau} d\tau = \tau e^{\tau} - \int e^{\tau} d\tau = \tau e^{\tau} - e^{\tau} = e^{\tau}(\tau - 1).$$

$$\therefore \quad y(t) = 10\, e^{-t} \left[e^{\tau}(\tau-1) \Big|_0^t \right]$$

$$y(t) = 10\, e^{-t} \left[e^{t}(t-1) - (-1) \right] = 10\, e^{-t} \left[t e^{t} - e^{t} + 1 \right].$$

$$y(t) = 10t - 10 + 10\, e^{-t} = \boxed{10\left(t - 1 + e^{-t}\right) = y(t)} \quad a$$

Esto indica que $\mathcal{L}^{-1}\left[R(s)\,G(s)\right] = r(t) * g(t).$

o sea $\boxed{R(s)\,G(s) = \mathcal{L}\left[r(t) * g(t)\right]}.$