En la exposición siguiente se presentan las transformadas de Laplace de funciones, así como teoremas sobre la transformada de Laplace, útiles en el estudio de sistemas lineales de control.

Tabla 1-1 Pares de transformadas de Laplace

	$f(t)$	$F(s)$
1	Impulso unitario $\delta(t)$	1
2	Escalón unitario $1(t)$	$\frac{1}{s}$
3	t	$\frac{1}{s^2}$
4	$\frac{t^{n-1}}{(n-1)!} \quad (n = 1,2,3, \ldots)$	$\frac{1}{s^n}$
5	$t^n \quad (n = 1,2,3, \ldots)$	$\frac{n!}{s^{n+1}}$
6	e^{-at}	$\frac{1}{s+a}$
7	te^{-at}	$\frac{1}{(s+a)^2}$
8	$\frac{1}{(n-1)!} t^{n-1} e^{-at} \quad (n = 1,2,3, \ldots)$	$\frac{1}{(s+a)^n}$
9	$t^n e^{-at} \quad (n = 1,2,3, \ldots)$	$\frac{n!}{(s+a)^{n+1}}$
10	sen ωt	$\frac{\omega}{s^2+\omega^2}$
11	cos ωt	$\frac{s}{s^2+\omega^2}$
12	sen h ωt	$\frac{\omega}{s^2-\omega^2}$
13	cosh ωt	$\frac{s}{s^2-\omega^2}$
14	$\frac{1}{a}(1 - e^{-at})$	$\frac{1}{s(s+a)}$
15	$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(s+a)(s+b)}$
16	$\frac{1}{b-a}(be^{-bt} - ae^{-at})$	$\frac{s}{(s+a)(s+b)}$
17	$\frac{1}{ab}\left[1 + \frac{1}{a-b}(be^{-at} - ae^{-bt})\right]$	$\frac{1}{s(s+a)(s+b)}$

Tabla 1-1 Continuación

18	$\frac{1}{a^2}(1 - e^{-at} - ate^{-at})$	$\frac{1}{s(s+a)^2}$
19	$\frac{1}{a^2}(at - 1 + e^{-at})$	$\frac{1}{s^2(s+a)}$
20	$e^{-at}\,\text{sen}\,\omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
21	$e^{-at}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
22	$\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\,\text{sen}\,\omega_n\sqrt{1-\zeta^2}\,t$	$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$
23	$-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\,\text{sen}(\omega_n\sqrt{1-\zeta^2}\,t-\phi)$ $\phi = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$	$\frac{s}{s^2+2\zeta\omega_n s+\omega_n^2}$
24	$1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\,\text{sen}(\omega_n\sqrt{1-\zeta^2}\,t+\phi)$ $\phi = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$	$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$
25	$1-\cos\omega t$	$\frac{\omega^2}{s(s^2+\omega^2)}$
26	$\omega t - \text{sen}\,\omega t$	$\frac{\omega^3}{s^2(s^2+\omega^2)}$
27	$\text{sen}\,\omega t - \omega t\cos\omega t$	$\frac{2\omega^3}{(s^2+\omega^2)^2}$
28	$\frac{1}{2\omega}t\,\text{sen}\,\omega t$	$\frac{s}{(s^2+\omega^2)^2}$
29	$t\cos\omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
30	$\frac{1}{\omega_2^2-\omega_1^2}(\cos\omega_1 t-\cos\omega_2 t)\qquad(\omega_1^2\neq\omega_2^2)$	$\frac{s}{(s^2+\omega_1^2)(s^2+\omega_2^2)}$
31	$\frac{1}{2\omega}(\text{sen}\,\omega t+\omega t\cos\omega t)$	$\frac{s^2}{(s^2+\omega^2)^2}$

Traslación de una función. Se requiere obtener la transformada de Laplace de una función trasladada $f(t-\alpha)1(t-\alpha)$, donde $\alpha \geq 0$. Esta función es cero para $t < \alpha$. Las funciones $f(t)1(t)$ y $f(t-\alpha)1(t-\alpha)$ aparecen en la figura 1-8.

Tabla 1-2 Propiedades de las transformadas de Laplace

1	$\mathscr{L}[Af(t)] = AF(s)$
2	$\mathscr{L}[f_1(t) \pm f_2(t)] = F_1(s) \pm F_2(s)$
3	$\mathscr{L}_{\pm}\left[\dfrac{d}{dt}f(t)\right] = sF(s) - f(0\pm)$
4	$\mathscr{L}_{\pm}\left[\dfrac{d^2}{dt^2}f(t)\right] = s^2F(s) - sf(0\pm) - \dot{f}(0\pm)$
5	$\mathscr{L}_{\pm}\left[\dfrac{d^n}{dt^n}f(t)\right] = s^nF(s) - \sum_{k=1}^{n} s^{n-k}\overset{(k-1)}{f}(0\pm)$ donde $\overset{(k-1)}{f}(t) = \dfrac{d^{k-1}}{dt^{k-1}}f(t)$
6	$\mathscr{L}_{\pm}\left[\int f(t)\,dt\right] = \dfrac{F(s)}{s} + \dfrac{\left[\int f(t)\,dt\right]_{t=0\pm}}{s}$
7	$\mathscr{L}_{\pm}\left[\iint f(t)\,dt\,dt\right] = \dfrac{F(s)}{s^2} + \dfrac{\left[\int f(t)\,dt\right]_{t=0\pm}}{s^2} + \dfrac{\left[\iint f(t)\,dt\,dt\right]_{t=0\pm}}{s}$
8	$\mathscr{L}_{\pm}\left[\int \cdots \int f(t)(dt)^n\right] = \dfrac{F(s)}{s^n} + \sum_{k=1}^{n} \dfrac{1}{s^{n-k+1}}\left[\int \cdots \int f(t)(dt)^k\right]_{t=0\pm}$
9	$\mathscr{L}\left[\int_0^t f(t)\,dt\right] = \dfrac{F(s)}{s}$
10	$\int_0^\infty f(t)\,dt = \lim_{s\to 0} F(s) \qquad \text{si } \int_0^\infty f(t)\,dt \text{ existe}$
11	$\mathscr{L}[e^{-at}f(t)] = F(s+a)$
12	$\mathscr{L}[f(t-\alpha)1(t-\alpha)] = e^{-\alpha s}F(s) \qquad \alpha \geq 0$
13	$\mathscr{L}[tf(t)] = -\dfrac{dF(s)}{ds}$
14	$\mathscr{L}[t^2f(t)] = \dfrac{d^2}{ds^2}F(s)$
15	$\mathscr{L}[t^nf(t)] = (-1)^n\dfrac{d^n}{ds^n}F(s) \qquad n = 1,2,3,\ldots$
16	$\mathscr{L}\left[\dfrac{1}{t}f(t)\right] = \int_s^\infty F(s)\,ds$
17	$\mathscr{L}\left[f\left(\dfrac{t}{a}\right)\right] = aF(as)$