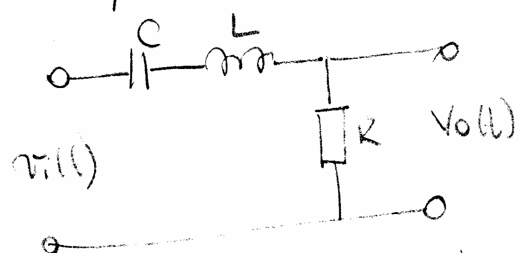
Primer Parcial 5R2 **22-06-2.010**

1) Resolver el siguiente sistema de ecuaciones

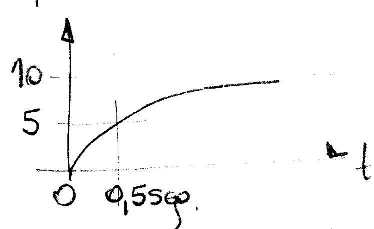
$$\begin{cases} x'(t) = x(t) - 4y(t) \\ y'(t) = -4x(t) + 2y(t) \end{cases} \qquad \begin{matrix} x(0) = 2 \\ y(0) = 4 \end{matrix}$$

2) Determinar la función de transferencia de la siguiente red

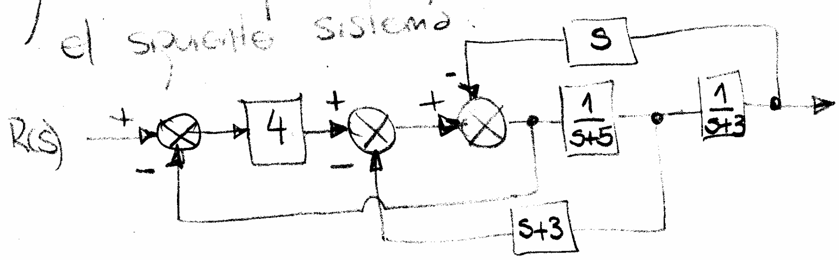
5R2



3) Obtener la función de transferencia de sistema que excitado con un escalón responde del siguiente modo.



4) Resolver por diagrama en bloques y Mason el siguiente sistema.



Soluciones **Primer Parcial 5R2** **22-06-2.010**

(1) Solución parciales 5R2 22/06/10

1)

$$\begin{cases} sX(s)-2 = X(s) - 4Y(s) \\ sY(s)-4 = -4X(s)+2Y(s) \end{cases} \qquad \begin{cases} (s-1)X(s)+4Y(s) = 2 \\ 4X(s)+(s-2)Y(s) = 4 \end{cases}$$

$$\Delta p(s) = \begin{vmatrix} s-1 & 4 \\ 4 & s-2 \end{vmatrix} = s^2-3s+2-16 = s^2-3s-14$$

$$\Delta sX(s) = \begin{vmatrix} 2 & 4 \\ 4 & s-2 \end{vmatrix} = 2s-4-16 = 2s-20$$

$$\Delta sY(s) = \begin{vmatrix} s-1 & 2 \\ 4 & 4 \end{vmatrix} = 4s-4-8 = 4s-12$$

$$X(s) = \frac{\Delta sX(s)}{\Delta p(s)} = \frac{2s-20}{s^2-3s-14}; \qquad s_{1-2} = \frac{3 \pm \sqrt{9-4(-14)}}{2} = \frac{3 \pm \sqrt{65}}{2}$$

$$s_1 = 5{,}53 \; ; \; s_2 = -2{,}53 \; ; \; X(s) = \frac{2s-20}{(s-5{,}53)\,(s+2{,}53)}$$

$$X(s) = \frac{A_1}{s-5{,}53} + \frac{A_2}{s+2{,}53}; \qquad A_1 = \lim_{s\to 5{,}53} \frac{2s-20}{s+2{,}53}$$

$$A_1 = \frac{2.\,5{,}53-20}{5{,}53+2{,}53} = -1{,}11; \qquad A_2 = \lim_{s\to -2{,}53} \frac{2s-20}{s-5{,}53}$$

$$A_2 = \frac{2(-2{,}53)-20}{-2{,}53-5{,}53} = 3{,}11$$

Soluciones **Primer Parcial 5R2** **22-06-2.010**

2

$$x(s) = \frac{-1{,}11}{s-5{,}53} + \frac{3{,}11}{s+2{,}53} \quad ; \quad x(t) = \mathcal{L}^{-1}\left[x(s)\right]$$

$$x(t) = -1{,}11\, e^{5{,}53t} + 3{,}11\, e^{-2{,}53t}$$

Condición inicial $x(0) = -1{,}11 \cdot 1 + 3{,}11 \cdot 1 = 2$

$$x'(t) = -6{,}14\, e^{5{,}53t} - 7{,}87\, e^{-2{,}53t}$$
$$-4x(t) = 4{,}44\, e^{5{,}53t} - 12{,}44\, e^{-2{,}53t}$$

$$Y(s) = \frac{\Delta Y(s)}{\Delta P(s)} = \frac{4s-12}{(s-5{,}53)(s+2{,}53)}$$

$$Y(s) = \frac{A_1}{s-5{,}53} + \frac{A_2}{s+2{,}53}$$

$$A_1 = \lim_{s \to 5{,}53} \frac{4s-12}{s+2{,}53} = \frac{4 \cdot 5{,}53 - 12}{5{,}53+2{,}53} = 1{,}26$$

$$A_2 = \lim_{s \to -2{,}53} \frac{4s-12}{s-5{,}53} = \frac{4(-2{,}53)-12}{-2{,}53-5{,}53} = 2{,}74$$

Condición inicial $y(t) = 1{,}26\, e^{5{,}53t} + 2{,}74\, e^{-2{,}53t}$

$$y(0) = 1{,}26 \cdot 1 + 2{,}74 \cdot 1 = 4$$

$$y'(t) = 6{,}97\, e^{5{,}53t} - 6{,}93\, e^{-2{,}53t}$$
$$-4y(t) = -5{,}04\, e^{5{,}53t} - 10{,}96\, e^{-2{,}53t}$$
$$2y(t) = 2{,}52\, e^{5{,}53t} + 5{,}48\, e^{-2{,}53t}$$

Soluciones **Primer Parcial 5R2** **22-06-2.010**

3)

$$\begin{cases} -6{,}14e^{5{,}53t} - 7{,}87e^{-2{,}53t} = -1{,}11e^{5{,}53t} + 3{,}11e^{-2{,}53t} - 5{,}04e^{5{,}53t} - 10{,}96e^{-2{,}53t} \\ 6{,}97e^{5{,}53t} - 6{,}93e^{-2{,}53t} = 4{,}44e^{5{,}53t} - 12{,}44e^{-2{,}53t} + 2{,}53e^{5{,}53t} + 5{,}48e^{-2{,}53t} \end{cases}$$

2)

$$\begin{cases} V_i(t) = \frac{1}{C}\int i(t)\,dt + L\frac{di(t)}{dt} + R\,i(t) \\ V_o(t) = R\,i(t) \end{cases}$$

$$\begin{cases} V_i(s) = \left(\frac{1}{sC} + Ls + R\right) I(s) \\ V_o(s) = R\,I(s) \end{cases}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{R}{\frac{1}{sC} + Ls + R} = \frac{R\,sC}{1 + sCLs + RsC}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{RC\,s}{LCs^2 + RCs + 1} = \frac{R\cancel{C}}{L\cancel{C}}\;\frac{s}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{R}{L}\;\frac{s}{s^2 + \frac{R}{L}s + \frac{1}{LC}}}$$

4

3)

$$y(t) = 10\left(1 - e^{-t/\tau}\right) \quad ; \quad 5 = 10\left(1 - e^{-0{,}5/\tau}\right)$$

$$0{,}5 = 1 - e^{-0{,}5/\tau} \quad ; \quad e^{-0{,}5/\tau} = 0{,}5$$

$$-\frac{0{,}5}{\tau} = \ln 0{,}5 \quad ; \quad \tau = -\frac{0{,}5}{\ln 0{,}5} = 0{,}72$$

$$y(t) = 10\left(1 - e^{-1{,}39\,t}\right) \quad ; \quad x(t) = \mu(t) \therefore X(s) = \frac{1}{s}$$

$$Y(s) = 10\left(\frac{1}{s} - \frac{1}{s+1{,}39}\right) = 10\left[\frac{\cancel{s}+1{,}39-\cancel{s}}{s(s+1{,}39)}\right]$$

$$Y(s) = \frac{13{,}9}{s(s+1{,}39)} \therefore \frac{Y(s)}{X(s)} = \frac{13{,}9}{\cancel{s}(s+1{,}39)} \cdot \cancel{s}$$

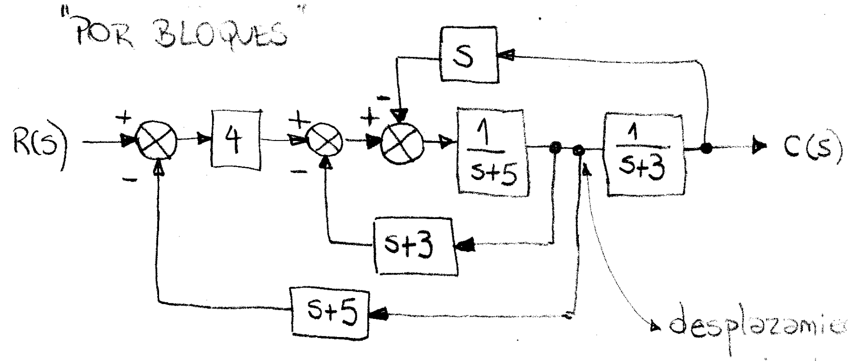
$$\frac{Y(s)}{X(s)} = \frac{13{,}9}{s+1{,}39}$$

Soluciones **Primer Parcial 5R2** **22-06-2.010**

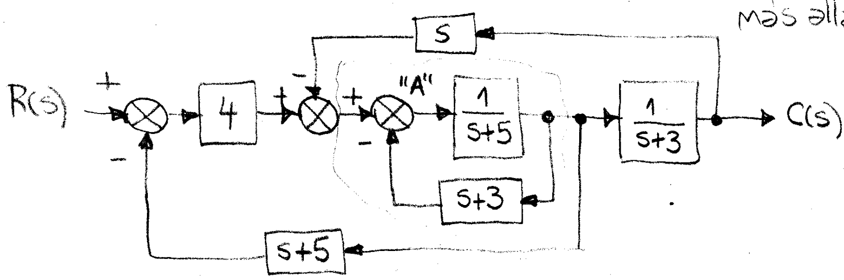
5

4)

"POR BLOQUES"

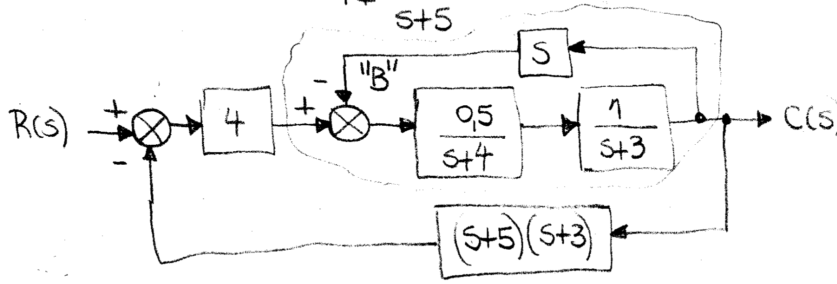


desplazamiento de un punto de toma más allá de un bloque.



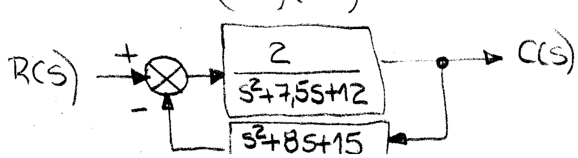
Lazo cerrado "A"

$$\frac{\frac{1}{S+5}}{1+\frac{S+3}{S+5}} = \frac{1}{S+5+S+3} = \frac{1}{2S+8} = \frac{0,5}{S+4}$$



Lazo cerrado "B"

$$\frac{\frac{0,5}{(S+4)(S+3)}}{1+\frac{0,5S}{(S+4)(S+3)}} = \frac{0,5}{S^2+7S+12+0,5S} = \frac{0,5}{S^2+7,5S+12}$$



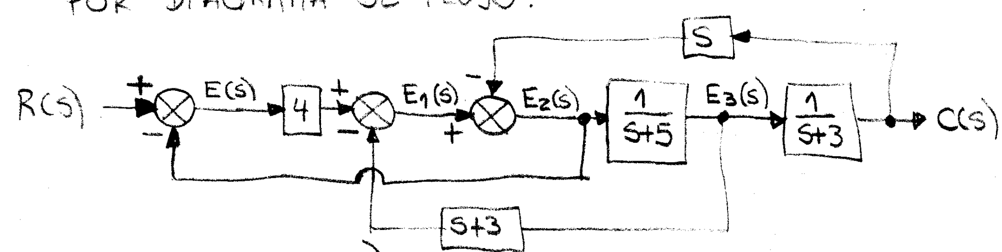
Soluciones **Primer Parcial 5R2** **22-06-2.010**

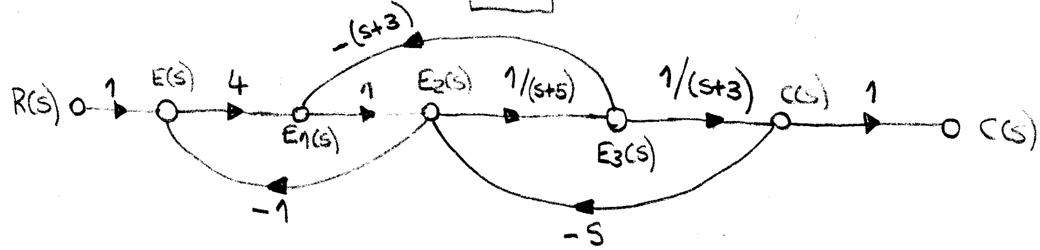
6)

$$\frac{C(s)}{R(s)} = \frac{\dfrac{2}{s^2+7{,}5s+12}}{1+2\,\dfrac{s^2+8s+15}{s^2+7{,}5s+12}} = \frac{2}{s^2+7{,}5s+12+2s^2+16s+30}$$

$$\frac{C(s)}{R(s)} = \frac{2}{3s^2+23{,}5s+42} = \frac{0{,}67}{s^2+7{,}83s+14} = \frac{C(s)}{R(s)}$$

"POR DIAGRAMA DE FLUJO.





Trayectoria directa. $P_1 = \dfrac{4}{(s+5)(s+3)} = \dfrac{4}{s^2+8s+15}$

Lazos cerrados $L_1 = -4$; $L_2 = -\dfrac{s+3}{s+5}$; $L_3 = -\dfrac{s}{s^2+8s+15}$

No hay lazos disjuntos.

$$\Delta = 1 - \sum L_a = 1 - \left[-4 - \frac{s+3}{s+5} - \frac{s}{s^2+8s+15}\right]$$

$$\Delta = 1+4+\frac{s+3}{s+5}+\frac{s}{s^2+8s+15} = 5+\frac{s+3}{s+5}+\frac{s}{s^2+8s+15}$$

$$\Delta = \frac{5(s^2+8s+15)+(s+3)^2+s}{s^2+8s+15} = \frac{5s^2+40s+75+s^2+6s+9+s}{s^2+8s+15}$$

$$\Delta = \frac{6s^2+47s+84}{s^2+8s+15} = 6\,\frac{s^2+7{,}83s+14}{s^2+8s+15}$$

Soluciones **Primer Parcial 5R2** **22-06-2.010**

7

$$\Delta_1 = 1$$

$$\frac{C(s)}{R(s)} = \frac{1}{\Delta} P_1 \Delta_1 = \frac{1}{6} \frac{\cancel{s^2+8s+15}}{s^2+7,83s+14} \cdot \frac{4}{s^2+8s+15} \cdot 1$$

$$\frac{C(s)}{R(s)} = \frac{0,67}{s^2+7,83s+14}$$