SEGUNDO PARCIAL 5R2 01/11/11

La siguiente representación en el espacio de estado muestra la F.T.L.A. de un sistema que se desea ajustar:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t) \quad ; \quad y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

Se pide:

1) Encontrar la F. de T. $\frac{Y(s)}{U(s)} = G(s)H(s)$.

2) Agregando un amplificador en cascada se pretende corregir al sistema; con $K = 0{,}4$ determinar mediante Nyquist la estabilidad del sistema.

3) Ajustar K para que $t_{s5\%}$ sea 1,5 seg.

4) Con la ganancia anterior determinar mediante Bode $M\phi$, MG y Kp.

SOLUCIONES 5R2.

$$\frac{Y(s)}{U(s)} = [C]\left(s[I]-[A]\right)^{-1}[B] + D \quad , \quad s[I]-[A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$s[I]-[A] = \begin{bmatrix} s & -1 \\ -1 & s \end{bmatrix} \; ; \; \left(s[I]-[A]\right)^{-1} = \frac{Adj\left(s[I]-[A]\right)}{\left|s[I]-[A]\right|} =$$

$$= \frac{\begin{bmatrix} s & 1 \\ 1 & s \end{bmatrix}^T}{s^2-1} = \frac{\begin{bmatrix} s & 1 \\ 1 & s \end{bmatrix}}{s^2-1} = \begin{bmatrix} \frac{s}{s^2-1} & \frac{1}{s^2-1} \\ \frac{1}{s^2-1} & \frac{s}{s^2-1} \end{bmatrix}$$

$$\begin{array}{c|cc} & \frac{s}{s^2-1} & \frac{1}{s^2-1} \\ & \frac{1}{s^2-1} & \frac{s}{s^2-1} \\ \hline 1 \quad 0 & \frac{s}{s^2-1} & \frac{1}{s^2-1} \end{array} = [C]\left(s[I]-[A]\right)^{-1}$$

	1
	2
$\frac{s}{s^2-1}$ $\frac{1}{s^2-1}$	$\boxed{\frac{s+2}{s^2-1} = \frac{Y(s)}{U(s)} = G(s)\,H(s)}$

2) Con $K = 0{,}4$ $\quad G(s)H(s) = 0{,}4\,\frac{s+2}{(s+1)(s-1)}$

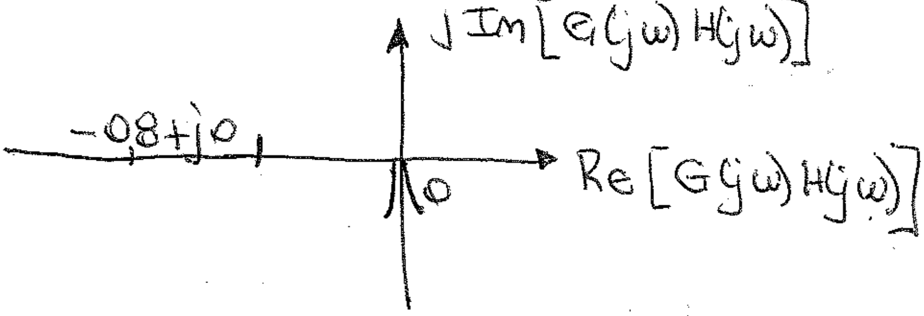
Análisis de B.F.

$$\lim_{s\to 0} 0{,}4\,\frac{s+2}{(s+1)(s-1)} = 0{,}4\,\frac{2}{(-1)} = -0{,}8 + j0$$

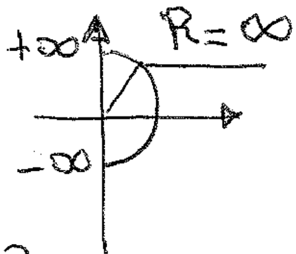
Análisis de A.F.

$$\lim_{s\to\infty} 0{,}4\,\frac{s+2}{(s+1)(s-1)} = \lim_{s\to\infty} 0{,}4\,\frac{\cancel{s}}{\cancel{s}\cdot s} = \lim_{s\to\infty}\frac{0{,}4}{s}$$

con $s = j\omega$ $\quad \lim_{\omega\to\infty}\frac{0{,}4}{j\omega} \rightarrow -j0$



Contorno:

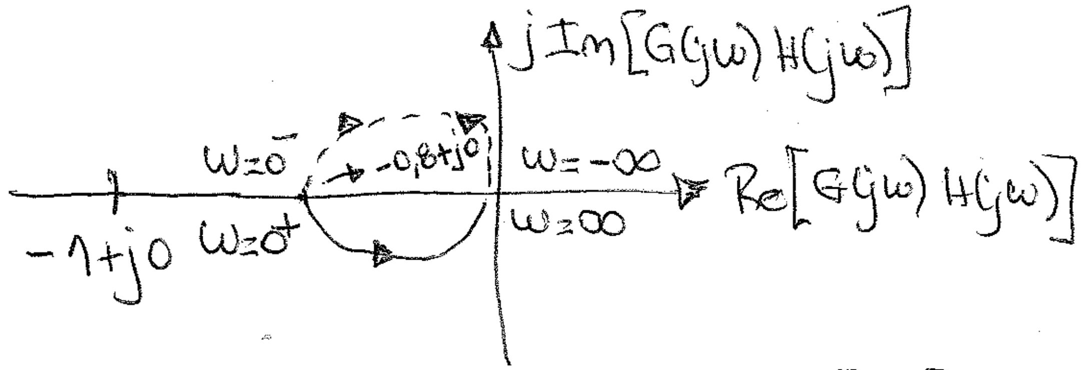


Corte a los ejes: $G(s)H(s) = 0{,}4\,\frac{s+2}{s^2-1}$

$$G(j\omega)H(j\omega) = 0{,}4\,\frac{2+j\omega}{-\omega^2-1} = 0{,}4\,(-1)\,\frac{2+j\omega}{\omega^2+1}$$

$$G(j\omega)H(j\omega) = 0{,}4\,\frac{-2-j\omega}{\omega^2+1}.$$

No hay cortes a los ejes para valores finitos de ω, partes real e imaginaria siempre negativas.



No hay rodeos $N=0$; $N=Z-P=0$

$P=1$ (polo de LA en $+1$) $\therefore Z-1=0$

$\therefore \boxed{Z=1}$ inestable con una raiz con parte Re [+].

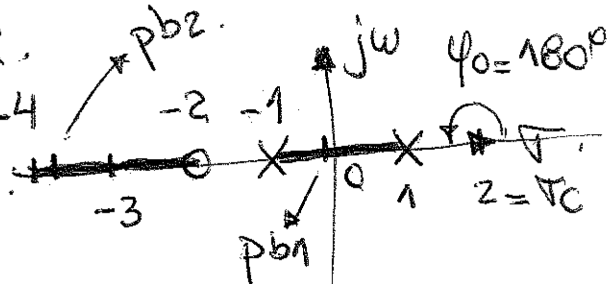
3) Lugar de raices: $G(s)H(s) = K \dfrac{s+2}{(s+1)(s-1)} = K \dfrac{s+2}{s^2-1}$.

Asintotas: $\varphi_K = \dfrac{180^\circ}{p-z}(2K+1)$ con $K=0$.

$\varphi_0 = \dfrac{180^\circ}{2-1}(2.0+1) = 180^\circ$; $\sigma_c = \dfrac{\Sigma Re[p] - \Sigma Re[z]}{p-z}$

$\sigma_c = \dfrac{\cancel{-1}+\cancel{1}-(-2)}{2-1} = 2$.

L.R. sobre eje real:



Puntos bifurcación: $K \dfrac{s+2}{s^2-1} + 1 = 0$; $K = -\dfrac{s^2-1}{s+2}$

$$\frac{\partial K}{\partial s} = -\frac{2s(s+2)-(s^2-1)}{(s+2)^2} = \frac{2s^2+4s-s^2+1}{(s+2)^2} = 0.$$

$s^2+4s+1=0$. $s_{1-2} = \dfrac{-4\pm\sqrt{16-4}}{2}$

$s_{1-2} = \dfrac{-4\pm\sqrt{12}}{2} = \dfrac{-4\pm 3{,}46}{2}$ $\begin{cases} s_1 = -0{,}27 = pb_1 \\ s_2 = -3{,}73 = pb_2 \end{cases}$

Criterio de Routh: $K\dfrac{s+2}{s^2-1}+1=0$.

$$\frac{K(s+2)+s^2-1}{s^2-1}=0 \;; \quad Ks+2K+s^2-1=0.$$

$$s^2+Ks+(2K-1)=0.$$

s^2	1	$2K-1$
s	K	
s^0	$2K-1$.	

$2K_c-1=0$.

$\therefore \boxed{K_c=0{,}5}$

$K<0{,}5$ inestable. con una raiz con parte Re[+].

$K>0{,}5$ ESTABLE

CORTE EJE $j\omega$: $s^2+(2K_c-1)=0$.

$$s^2+(2\cdot 0{,}5-1)=0$$

$$s^2+0=0.$$

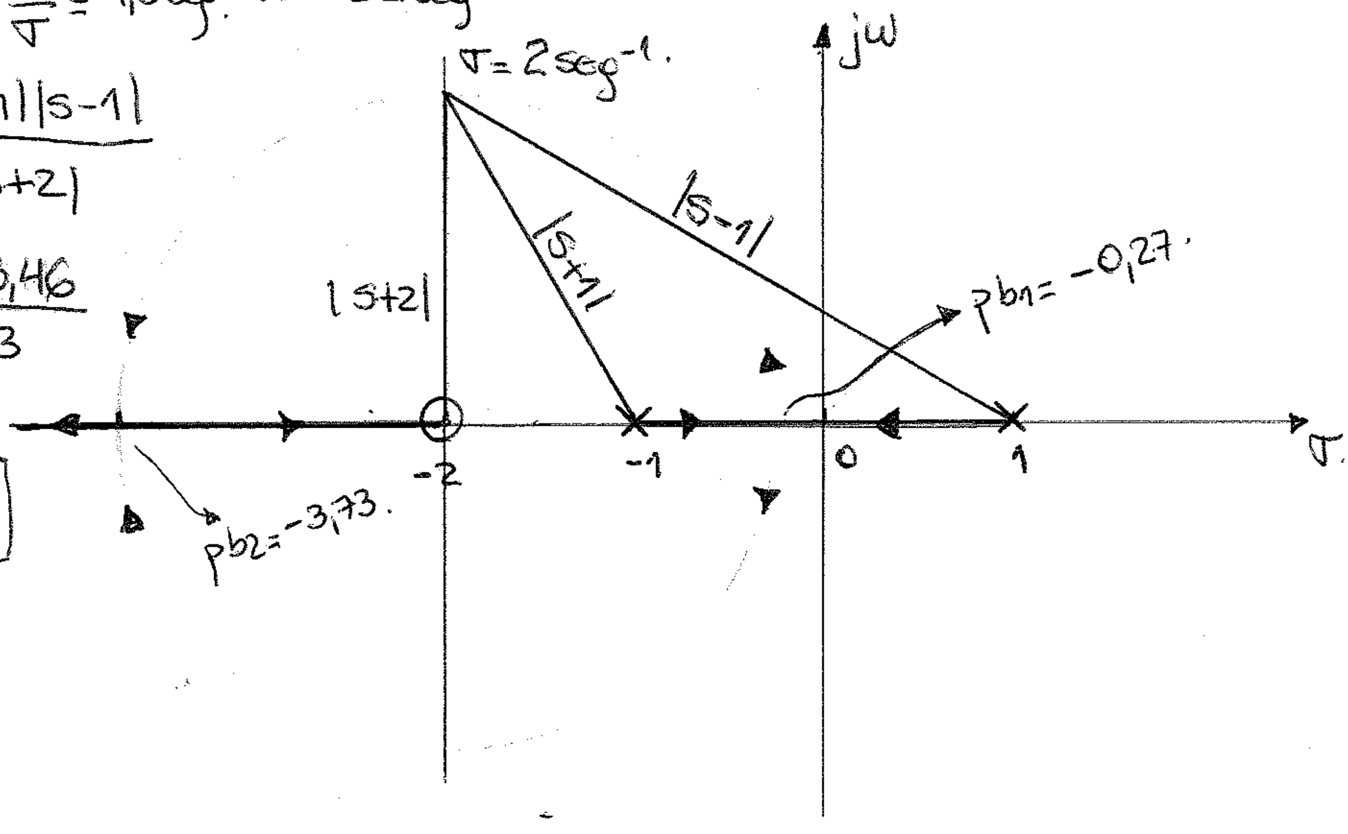
$\therefore \boxed{s=\pm\sqrt{0}=0.}$

$t_{s5\%}=\dfrac{3}{\sigma}=1{,}5\,seg. \therefore \sigma=2\,seg^{-1}$

$K=\dfrac{|s+1||s-1|}{|s+2|}$

$K=\dfrac{2\cdot 3{,}46}{1{,}73}$

$\boxed{K=4}$



4) $M\phi$, MG y Kp. $G(s)H(s) = 4\,\dfrac{s+2}{s^2-1}$.

En formato Bode: $G(s)H(s) = 4\,\dfrac{s+2}{(s+1)(s-1)} = 4\,\dfrac{2(0{,}5s+1)}{(s+1)(-1)(-s+1)}$

$$G(s)H(s) = \frac{8\,(0{,}5s+1)}{(-1)\,(s+1)(-s+1)}\;; \quad G(j\omega)H(j\omega) = \frac{8\,(1+j0{,}5\omega)}{(-1)(1+j\omega)(1-j\omega)}.$$

$$|G(j\omega)H(j\omega)| = \frac{|8|\,|1+j0{,}5\omega|}{|-1|\,|1+j\omega|\,|1-j\omega|} = \frac{8\,|1+j0{,}5\omega|}{1\,|1+j\omega|\,|1-j\omega|}.$$

$$|G(j\omega)H(j\omega)|_{dB} = 20\log 8 + 20\log\sqrt{1+0{,}25\omega^2} - \underbrace{20\log 1}_{=0} - 20\log\sqrt{1+\omega^2} - 20\log\sqrt{1+\omega^2}$$

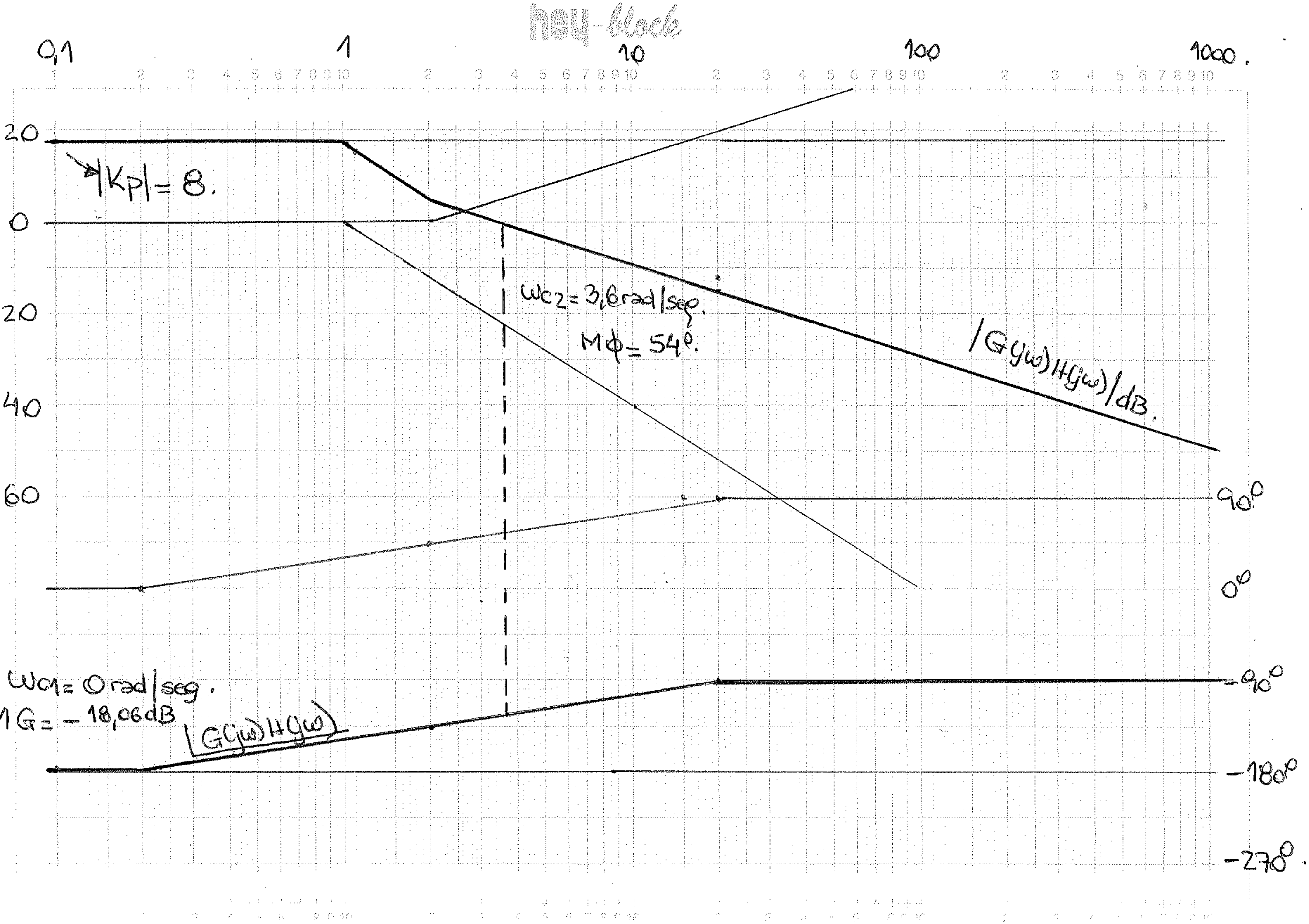
$$|G(j\omega)H(j\omega)|_{dB} = 18{,}06 + 10\log(1+0{,}25\omega^2) - 20\log(1+\omega^2).$$

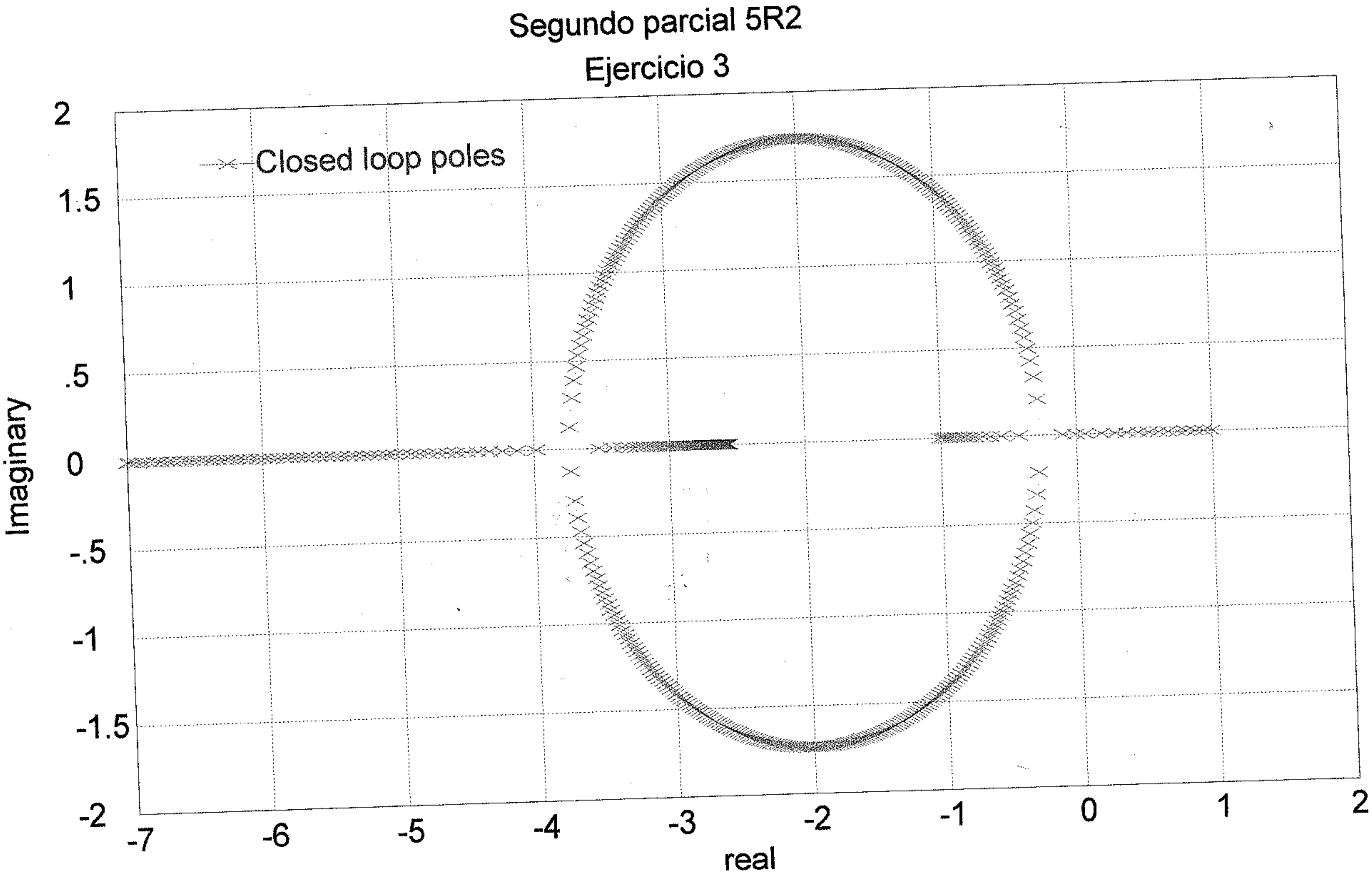
$$\underline{/G(j\omega)H(j\omega)} = \underbrace{tg^{-1}\frac{0}{8}}_{=0^\circ} + tg^{-1}0{,}5\omega - tg^{-1}\frac{0}{(-1)} - tg^{-1}\omega - tg^{-1}(-\omega)$$

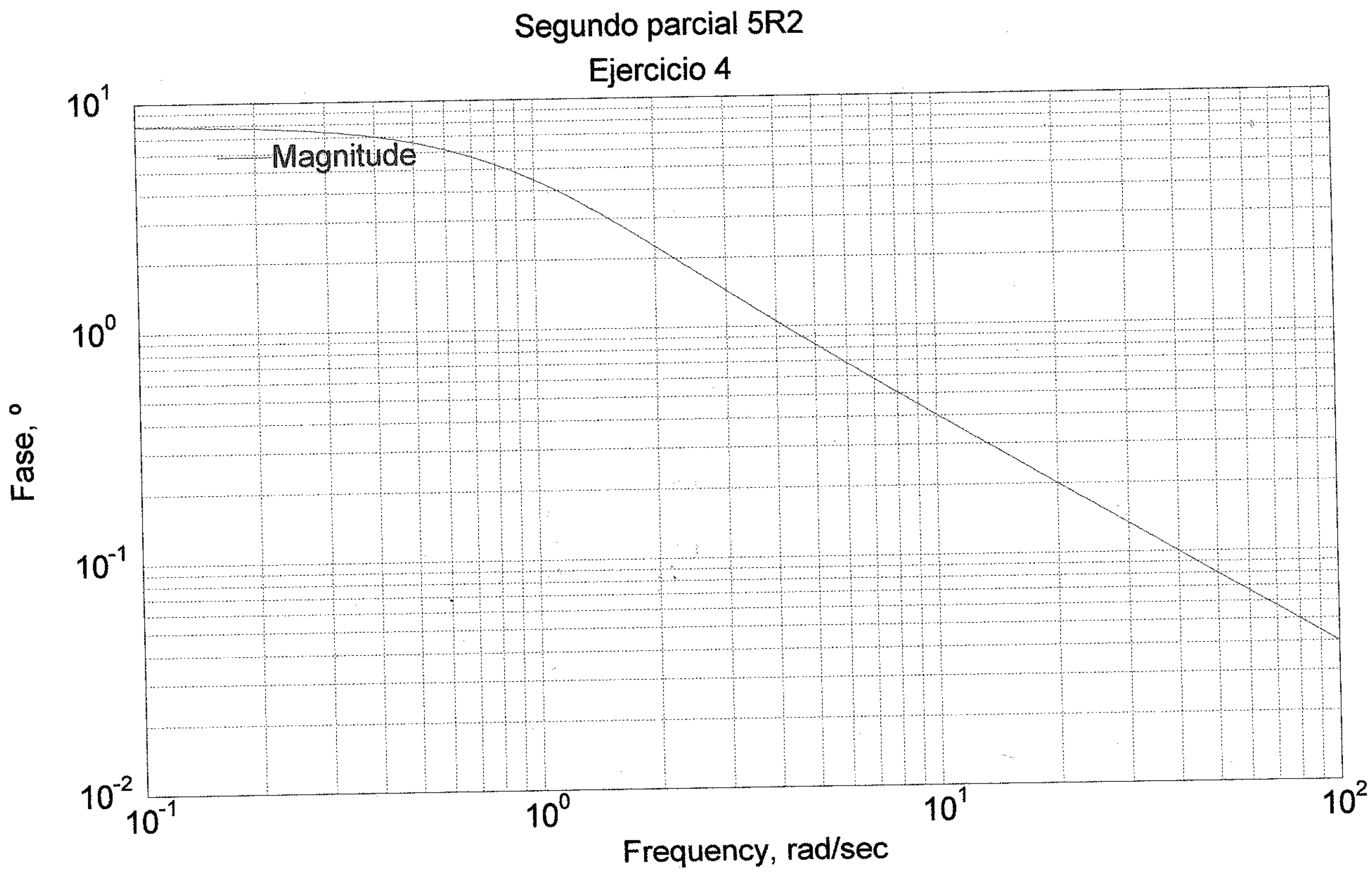
$$\underline{/G(j\omega)H(j\omega)} = tg^{-1}0{,}5\omega - 180^\circ \cancel{- tg^{-1}\omega} \cancel{+ tg^{-1}\omega}$$

$$\underline{/G(j\omega)H(j\omega)} = tg^{-1}0{,}5\omega - 180^\circ.$$

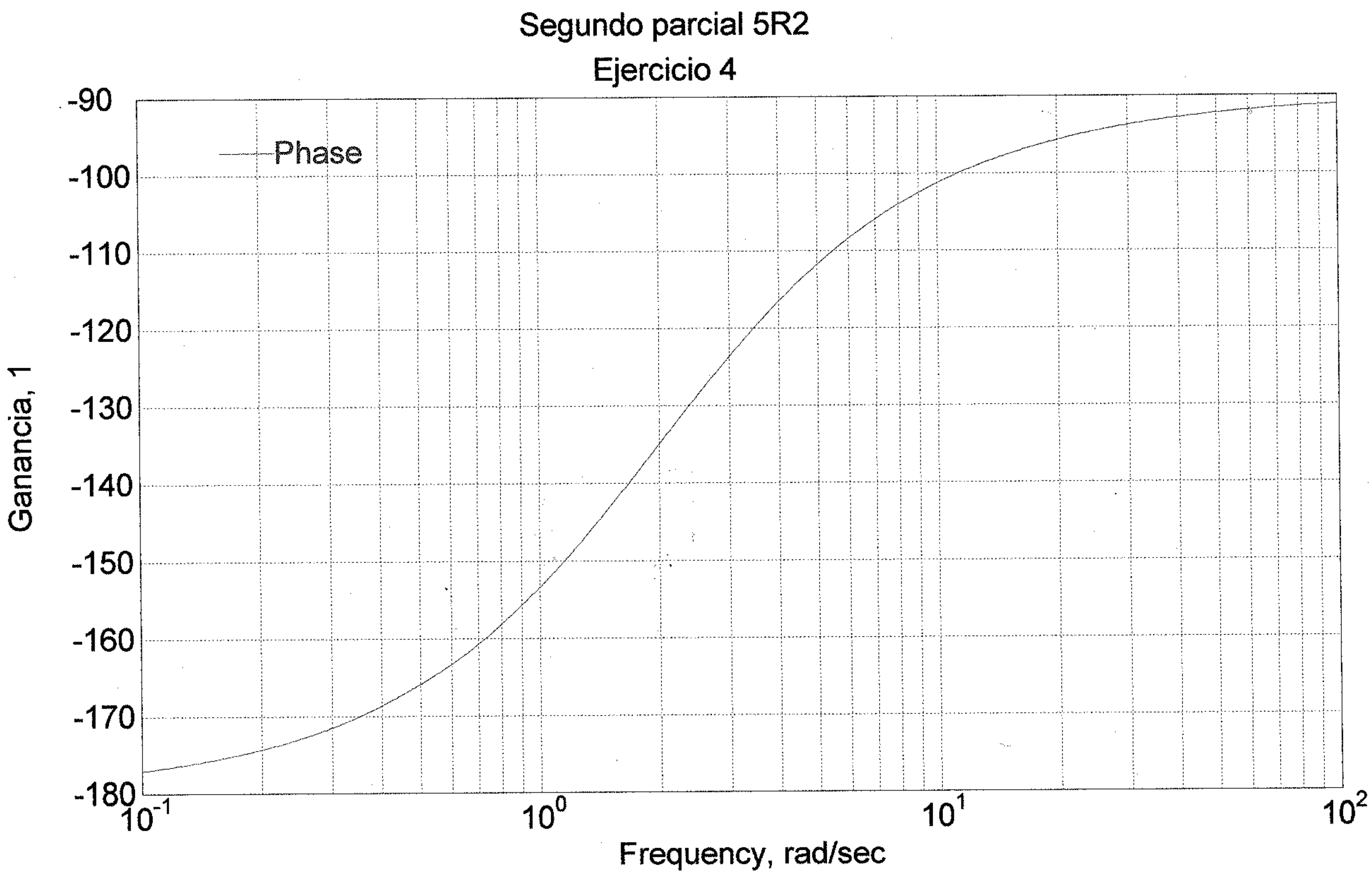
$$|K_p| = 10^{(18{,}06/20)} = 8.$$


hey-block
0,1
1
10
100
1000.
20
0
20
40
60
|Kp| = 8.
ωc2 = 3,6 rad/seg.
Mϕ = 54°.
|G(jω)H(jω)|dB.
90°
0°
-90°
-180°
-270°.
ωc1 = 0 rad/seg.
MG = -18,06 dB
∠G(jω)H(jω)


Segundo parcial 5R2
Ejercicio 3
Closed loop poles
Imaginary
real
2
1.5
1
.5
0
-.5
-1
-1.5
-2
-7
-6
-5
-4
-3
-2
-1
0
1
2



Segundo parcial 5R2
Ejercicio 4
Magnitude
Fase, º
Frequency, rad/sec
10^1
10^0
10^-1
10^-2
10^-1
10^0
10^1
10^2


Segundo parcial 5R2
Ejercicio 4
Phase
-90
-100
-110
-120
-130
-140
-150
-160
-170
-180
Ganancia, 1
10^-1
10^0
10^1
10^2
Frequency, rad/sec