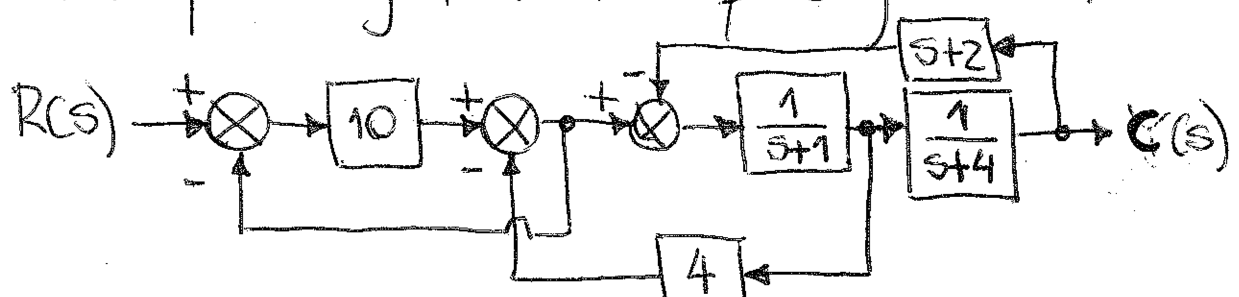
Primer parcial 5R1 10/06/14.

1) Resolver por diagrama en bloques y Mason:



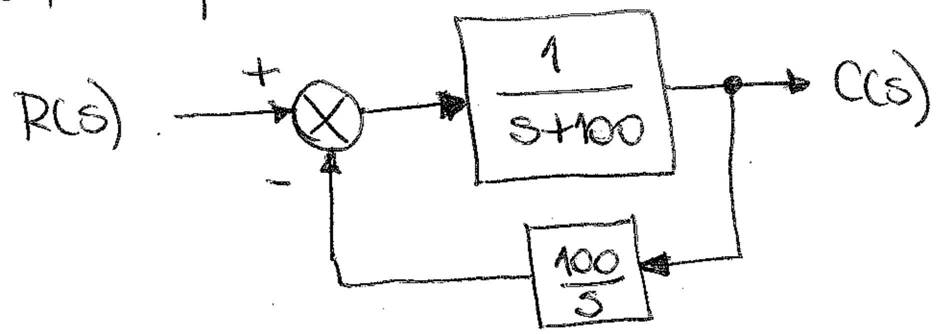
2) Obtener la respuesta temporal de un sistema mecánico compuesto por un resorte de $K = 4\frac{N}{m}$, un amortiguador de $f = 1\frac{N\,seg}{m}$ y masa $m = 1\,Kg$ cuando se excita con un impulso unitario.

3) Linealizar la expresión $z = x^2 \cdot y$ en la región:

$$9 \leq x \leq 11 \; ; \; 9 \leq y \leq 11$$

Calcular el error para $x = y = 9{,}5$.

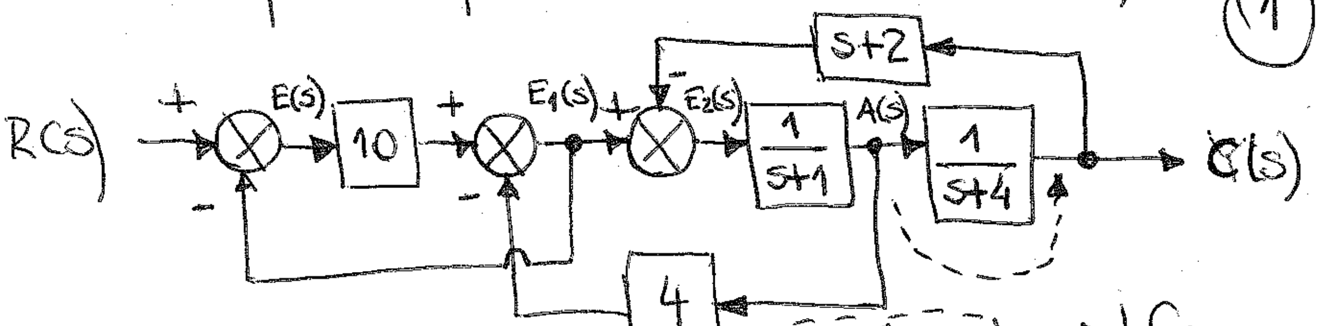
4) Determinar la forma temporal del error para una entrada rampa:

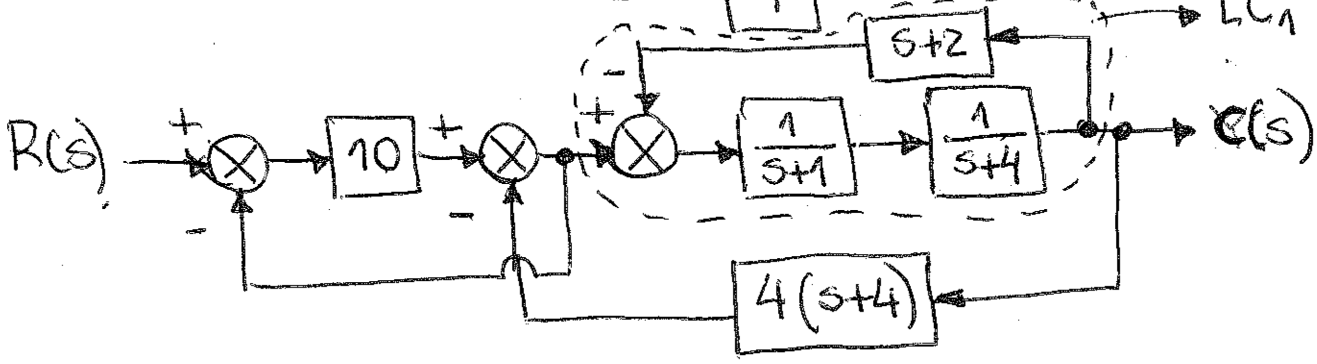


2,5 ptos c/u.

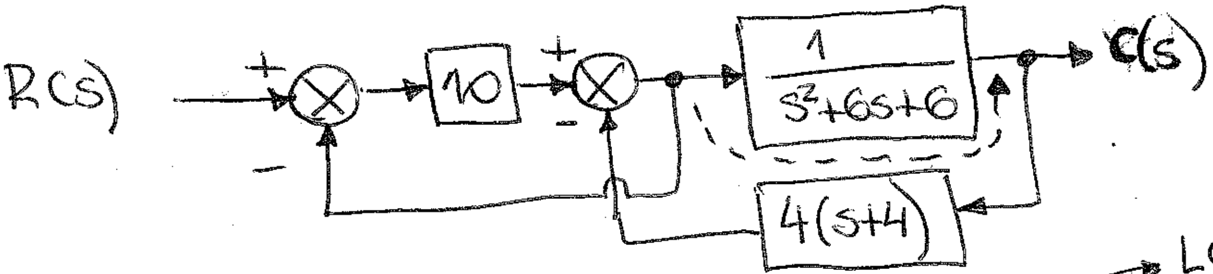
Soluciones primer parcial 5R1 10/06/14 (1)

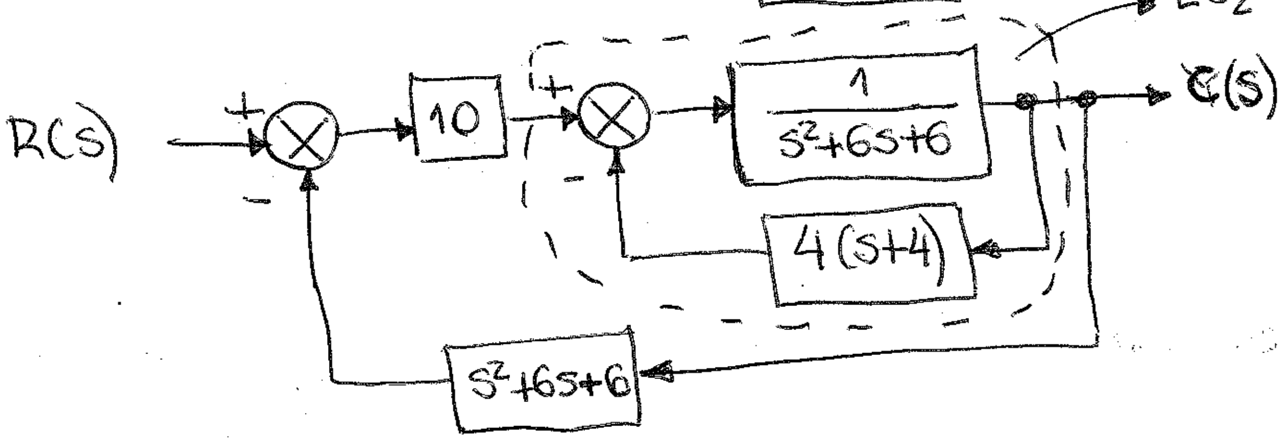
1)



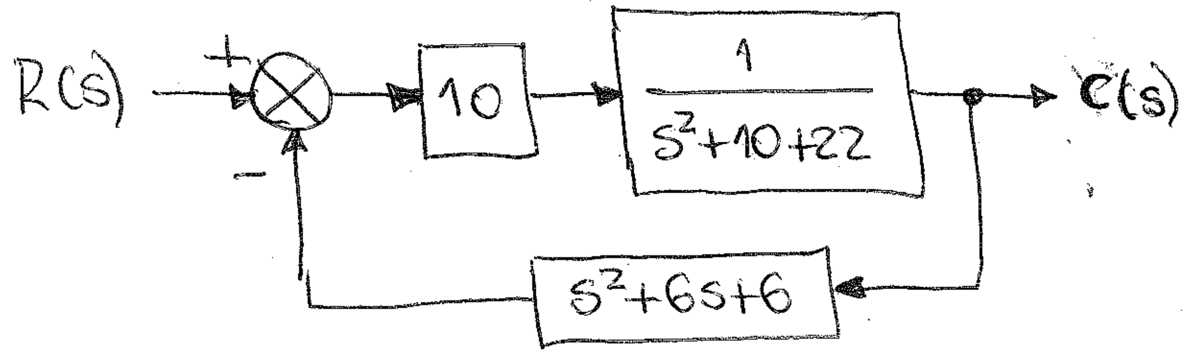


$$LC_1: \quad \frac{\frac{1}{s+1}\cdot\frac{1}{s+4}}{1+\frac{1}{s+1}\cdot\frac{1}{s+4}(s+2)} = \frac{1}{s^2+5s+4+s+2} = \frac{1}{s^2+6s+6}$$





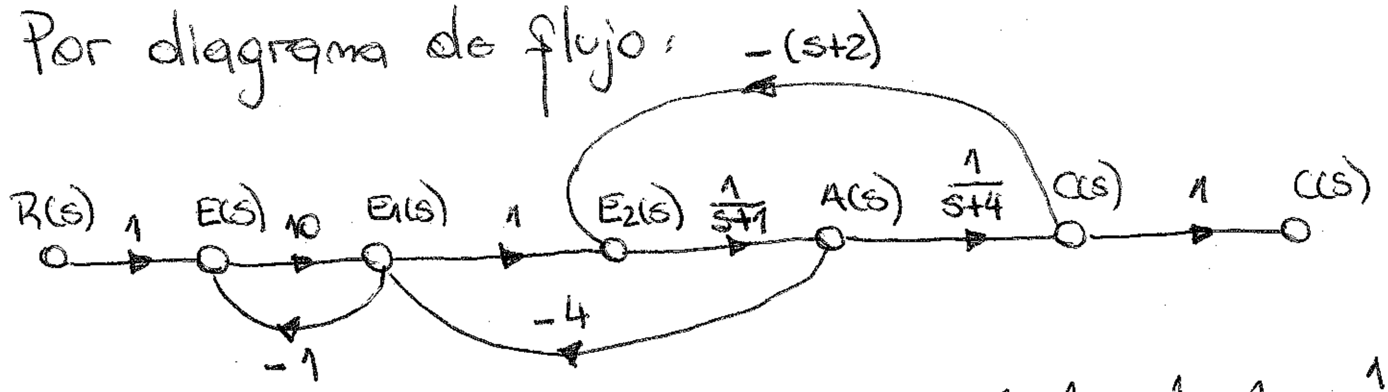
$$LC_2: \quad \frac{\frac{1}{s^2+6s+6}}{1+\frac{4(s+4)}{s^2+6s+6}} = \frac{1}{s^2+6s+6+4s+16} = \frac{1}{s^2+10s+22}$$



$$\frac{C(s)}{R(s)} = \frac{\dfrac{10}{s^2+10s+22}}{1+\dfrac{10}{s^2+10s+22}\cdot(s^2+6s+6)} = \frac{10}{s^2+10s+22+10s^2+60s+6\ldots}$$

$$\frac{C(s)}{R(s)} = \frac{10}{11s^2+70s+82} = \boxed{0{,}91\,\frac{1}{s^2+6{,}36s+7{,}45}}$$

Por diagrama de flujo:



Trayectoria directa: $P_1(s) = 1\cdot 10\cdot 1\cdot\frac{1}{s+1}\cdot\frac{1}{s+4}\cdot 1 = \frac{10}{s^2+5s+4}$

Lazos cerrados: $L_1 = 10\cdot(-1) = -10$

$$L_2 = 1\cdot\frac{1}{s+1}\cdot(-4) = -\frac{4}{s+1}\ ;\quad L_3 = \frac{1}{s+1}\cdot\frac{1}{s+4}\left[-(s+2)\right]$$

$$L_3 = -\frac{s+2}{(s+1)(s+4)}$$

L_1 y L_3 disjuntos.

P_1 no disjunto respecto lazos.

$$\Delta = 1-(L_1+L_2+L_3)+L_1\cdot L_3$$

$$\Delta = 1 - \left[-10 - \frac{4}{s+1} - \frac{s+2}{(s+1)(s+4)}\right] + (-10)\left[-\frac{s+2}{(s+1)(s+4)}\right]$$ 3

$$\Delta = 1 + 10 + \frac{4}{s+1} + \frac{s+2}{(s+1)(s+4)} + 10\,\frac{s+2}{(s+1)(s+4)}$$

$$\Delta = 11 + \frac{4}{s+1} + \frac{s+2}{(s+1)(s+4)} + \frac{10s+20}{(s+1)(s+4)}$$

$$\Delta = \frac{11\,(s^2+5s+4) + 4(s+4) + s+2 + 10s+20}{(s+1)(s+4)}$$

$$\Delta = \frac{11s^2+55s+44+4s+16+s+2+10s+20}{(s+1)(s+4)}$$

$$\Delta = \frac{11s^2+70s+82}{(s+1)(s+4)} \quad ; \quad \Delta_1 = 1$$

$$\frac{C(s)}{R(s)} = \frac{1}{\Delta}\,P_1\,\Delta_1 = \frac{\cancel{(s+1)(s+4)}}{11s^2+70s+82}\cdot\frac{10}{\cancel{(s+1)(s+4)}}$$

$$\frac{C(s)}{R(s)} = \frac{10}{11s^2+70s+82} = \boxed{\frac{0{,}91}{s^2+6{,}36s+7{,}45}}$$

2) $$F(t) = m\,\ddot{x}(t) + f\,\dot{x}(t) + k\,x(t)$$

$$F(s) = (m s^2 + f s + k)\; x(s)$$

$$\frac{X(s)}{F(s)} = \frac{1/m}{s^2 + f/m\, s + K/m}$$

$$\frac{X(s)}{F(s)} = \frac{1}{s^2 + s + 4} = \frac{1}{(s+0{,}5-j1{,}94)(s+0{,}5+j1{,}94)}$$

$$F(t) = \delta(t) \quad \therefore \quad F(s) = 1$$

$$X(s) = \frac{1}{(s+0{,}5-j1{,}94)(s+0{,}5+j1{,}94)} = \frac{A_1}{s+0{,}5-j1{,}94} + \frac{\overline{A_1}}{s+0{,}5+j1{,}94}$$

$$A_1 = \lim_{s \to -0{,}5+j1{,}94} \frac{1}{s+0{,}5+j1{,}94} = \frac{1}{-\cancel{0{,}5}+j1{,}94+\cancel{0{,}5}+j1{,}94}$$

$$A_1 = \frac{1}{j\,3{,}88} = -j\,\frac{1}{3{,}88}$$

$$X(s) = -j\,\frac{(1/3{,}88)}{s+0{,}5-j1{,}94} + j\,\frac{(1/3{,}88)}{s+0{,}5+j1{,}94}$$

$$x(t) = -j\,\frac{1}{3{,}88}\, e^{(-0{,}5+j1{,}94)t} + j\,\frac{1}{3{,}88}\, e^{(-0{,}5-j1{,}94)t}$$

$$x(t) = -j\,\frac{1}{3{,}88}\, e^{-0{,}5t} \cdot e^{j1{,}94t} + j\,\frac{1}{3{,}88}\, e^{-0{,}5t} \cdot e^{-j1{,}94t}$$

$$x(t) = 0{,}26\, e^{-0{,}5t}\left[-j\, e^{j1{,}94t} + j\, e^{-j1{,}94t}\right]$$

$$x(t) = -0{,}26\, e^{-0{,}5t}\left[j e^{j1{,}94t} - j e^{-j1{,}94t}\right]$$

$$x(t) = -0{,}26\, e^{-0{,}5t}\left[j\left(e^{j1{,}94t} - e^{-j1{,}94t}\right)\right]$$

$$\left.\begin{aligned} e^{j\theta} &= \cos\theta + j\,\text{sen}\,\theta \\ e^{-j\theta} &= \cos\theta - j\,\text{sen}\,\theta \end{aligned}\right\} \quad e^{j\theta} - e^{-j\theta} = 2j\,\text{sen}\,\theta$$

$$x(t) = -0{,}26\, e^{-0{,}5t}\left[j\left(2j\,\text{sen}\,1{,}94t\right)\right]$$

$$\boxed{x(t) = 0{,}52\, e^{-0{,}5t}\,\text{sen}\,1{,}94t}$$

3) Linealizar $z = x^2 \cdot y$ en $\begin{cases} 9 \leq x \leq 11 & \bar{x} = 10 \\ 9 \leq y \leq 11 & \bar{y} = 10 \end{cases}$

error para $x = y = 9{,}5$

$$z - \bar{z} = K_x(x - \bar{x}) + K_y(y - \bar{y})$$

$$\bar{z} = (10)^2 \cdot 10 = 1.000$$

$$K_x = \left.\frac{\partial z}{\partial x}\right|_{\substack{x=10\\y=10}} = \left. 2xy \right|_{\substack{x=10\\y=10}} = 200$$

$$K_y = \left.\frac{\partial z}{\partial y}\right|_{\substack{x=10\\y=10}} = \left. x^2 \right|_{\substack{x=10\\y=10}} = 100$$

$$z - 1.000 = 200(x-10) + 100(y-10)$$

$$z - \cancel{1.000} = 200x - 2.000 + 100y \cancel{- 1.000}$$

$$z_L = 200x + 100y - 2.000$$

$$z(9,5) = 857{,}38$$

$$z_L(9,5) = 850$$

$$\boxed{\varepsilon\% = \frac{z_L - z}{z} \cdot 100\% = -0{,}87\%}$$

4)

$$G(s) = \frac{1}{s+100} \; ; \quad H(s) = \frac{100}{s} \; ; \quad r(t) = t\,\mu(t)$$

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)H(s)} = \frac{1}{1+\frac{1}{s+100}\cdot\frac{100}{s}}$$

$$\frac{E(s)}{R(s)} = \frac{s(s+100)}{s^2+100s+100} \; ; \quad R(s) = \frac{1}{s^2}$$

$$E(s) = \frac{s+100}{s(s^2+100s+100)} = \frac{s+100}{s(s+1{,}01)(s+98{,}99)}$$

$$E(s) = \frac{A_0}{s} + \frac{A_1}{s+1{,}01} + \frac{A_2}{s+98{,}99}$$

$$A_0 = \lim_{s \to 0} \frac{s+100}{s^2+100s+100} = 1$$

$$A_1 = \lim_{s \to -1{,}01} \frac{s+100}{s(s+98{,}99)} = -1{,}0003.$$

$$A_2 = \lim_{s \to -98{,}99} \frac{s+100}{s(s+1{,}01)} = 0{,}0003.$$

$$E(s) = \frac{1}{s} - \frac{1{,}0003}{s+1{,}01} + \frac{0{,}0003}{s+98{,}99}.$$

$$\boxed{e(t) = 1 - 1{,}0003\, e^{-1{,}01t} + 0{,}0003\, e^{-98{,}99t}}$$

$$e_{ss} = \frac{1}{K_v} \; ; \quad K_v = \lim_{s \to 0} s\,G(s)H(s)$$

$$K_v = \lim_{s \to 0} \cancel{s}^{\,1} \cdot \frac{1}{s+100} \cdot \frac{100}{\cancel{s}_{\,1}} = 1.$$

$$\boxed{e_{ss} = 1.}$$