 TP1-1 Ej5. Mario Gustavo Salguero

Resolución Ej7. TP N°2.

La ecuación diferencia a resolver es la siguiente:

$$F(t) = m\ddot{x}(t) + f\dot{x}(t) + Kx(t)$$

Datos: $F(t) = 4\mu(t)$ [N] $\quad K = 2$ N/m

$m = 1$ Kg $\quad f = 0{,}2 \dfrac{N\,seg}{m}$

Por lo tanto $4\mu(t) = \ddot{x}(t) + 0{,}2\dot{x}(t) + 2x(t)$

Aplicando $\mathcal{L}$:

$$\frac{4}{s} = s^2X(s) - sX(0) - \dot{x}(0) + 0{,}2\left[sX(s) - x(0)\right] + 2X(s)$$

Como las condiciones iniciales son nulas

$$\frac{4}{s} = X(s)\left(s^2 + 0{,}2s + 2\right) \therefore X(s) = \frac{4}{s(s^2+0{,}2s+2)}$$

Se factorea el denominador

$$s_{1-2} = \frac{-0{,}2 \pm \sqrt{(0{,}2)^2 - 4.2}}{2} = -0{,}1 \pm j1{,}4107$$

$$X(s) = \frac{4}{s(s+0{,}1-j1{,}4107)(s+0{,}1+j1{,}4107)}$$

Por expansión en fracciones simples:

$$X(s) = \frac{A_0}{s} + \frac{A_1}{s+0{,}1-j1{,}4107} + \frac{\overline{A_1}}{s+0{,}1+j1{,}4107}$$

$$A_0 = \lim_{s \to 0} \frac{4}{s^2+0{,}2s+2} = 2$$

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$$A_1 = \lim_{s \to (-0{,}1+j1{,}4107)} \frac{4}{s(s+0{,}1+j1{,}4107)}$$

$$A_1 = \frac{\cancel{4}^{\,2}}{(-0{,}1+j1{,}4107)\,\cancel{2}j1{,}4107} = -j1{,}4177\,\frac{(-0{,}1-j1{,}4107)}{2{,}0001}$$

$$A_1 = -0{,}9999 + j0{,}0709 \quad \therefore \quad \bar{A}_1 = -0{,}9999 - j0{,}0709$$

$$X(s) = \frac{2}{s} + \frac{(-0{,}9999+j0{,}0709)}{s+0{,}1-j1{,}4107} + \frac{(-0{,}9999-j0{,}0709)}{s+0{,}1+j1{,}4107}$$

$$X(t) = \mathcal{L}^{-1}\left[X(s)\right]$$

$$X(t) = 2 + (-0{,}9999+j0{,}0709)\, e^{(-0{,}1+j1{,}4107)t} + \dots$$

$$\dots + (-0{,}9999-j0{,}0709)\, e^{(-0{,}1-j1{,}4107)t}$$

$$X(t) = 2 - 0{,}9999\, e^{-0{,}1t} \cdot e^{j1{,}4107t} + j0{,}0709\, e^{-0{,}1t} \cdot e^{j1{,}4107t} - \dots$$

$$\dots - 0{,}9999\, e^{-0{,}1t} \cdot e^{-j1{,}4107t} - j0{,}0709\, e^{-0{,}1t} \cdot e^{-j1{,}4107t}$$

$$X(t) = 2 - 0{,}9999\, e^{-0{,}1t}\left(e^{j1{,}4107t} + e^{-j1{,}4107t}\right) + \dots$$

$$\dots + j0{,}0709\, e^{-0{,}1t}\left(e^{j1{,}4107t} - e^{-j1{,}4107t}\right)$$

Según Euler $e^{j\theta} = \cos\theta + j\operatorname{sen}\theta$ y $e^{-j\theta} = \cos\theta - j\operatorname{sen}\theta$

$$\therefore \quad e^{j\theta} + e^{-j\theta} = 2\cos\theta \quad y$$

$$e^{j\theta} - e^{-j\theta} = j2\operatorname{sen}\theta$$

Con lo cual $X(t)$ será:

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$$x(t) = 2 - 0{,}9999\, e^{-0{,}1t} \left(2 \cos 1{,}4107t\right) + \dots$$

$$\dots + j\, 0{,}0709\, e^{-0{,}1t} \left(j\, 2 \operatorname{sen} 1{,}4107t\right)$$

$$x(t) = 2 - 2e^{-0{,}1t} \left(0{,}9999 \cos 1{,}4107t\right) - \dots$$

$$\dots - 2e^{-0{,}1t} \left(0{,}0709 \operatorname{sen} 1{,}4107t\right)$$

$$x(t) = 2 - 2e^{-0{,}1t} \left(0{,}9999 \cos 1{,}4107t + 0{,}0709 \operatorname{sen} 1{,}4107t\right)$$

Por trigonometría $A \cos\phi + B \operatorname{sen}\phi$

es igual a $\sqrt{A^2 + B^2} \cos\left(\phi - \operatorname{arctg}\frac{B}{A}\right)$

ó $\sqrt{A^2 + B^2} \operatorname{sen}\left(\phi + \operatorname{arctg}\frac{A}{B}\right)$

$$\therefore\; x(t) = 2 - 2e^{-0{,}1t} \left[1{,}0024 \cos(1{,}4107t - 4{,}06^\circ)\right]$$

$$x(t) = 2 - 2{,}0048\, e^{-0{,}1t} \cos(1{,}4107t - 4{,}06^\circ)$$