5). Idem 3 con $V_C(t)$ determinar FT y comprobar.

$$V_C(t) = R i_L(t) + L \frac{d i_L(t)}{dt}$$

$$\text{con } i_L(t) = i(t) - i_C(t) = i(t) - C \frac{dV_C(t)}{dt}$$

$$V_C(t) = R i(t) - RC \frac{dV_C(t)}{dt} + L \frac{d i_L(t)}{dt}$$

$$\frac{d i_L(t)}{dt} = \frac{d i(t)}{dt} - C \frac{d^2 V_C(t)}{dt}$$

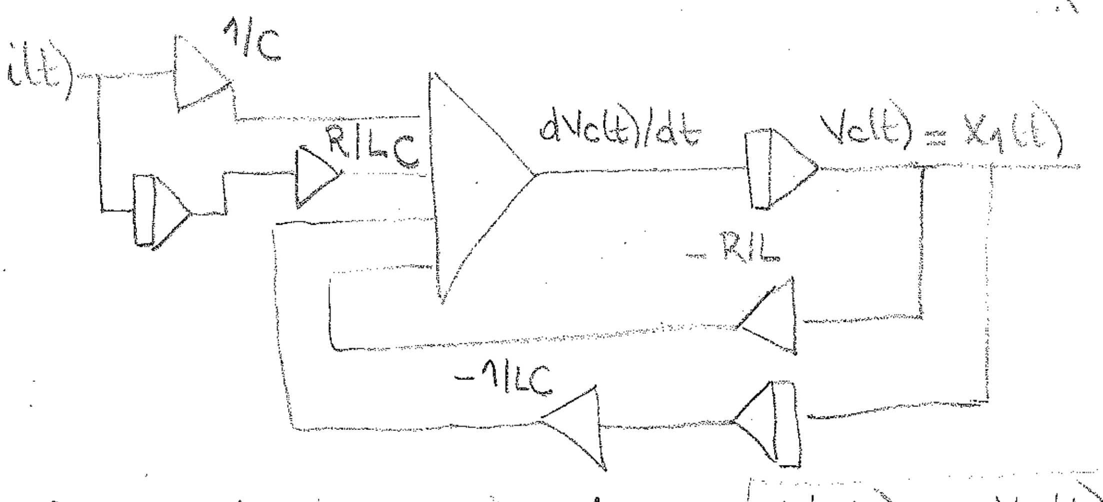
$$V_C(t) = R i(t) - RC \frac{dV_C(t)}{dt} + L \frac{d i(t)}{dt} - LC \frac{d^2 V_C(t)}{dt^2}$$

$$LC \frac{d^2 V_C(t)}{dt^2} = R i(t) + L \frac{d i(t)}{dt} - RC \frac{dV_C(t)}{dt} - V_C(t)$$

$$\frac{d^2 V_C(t)}{dt^2} = \frac{R}{LC} i(t) + \frac{1}{C} \frac{d i(t)}{dt} - \frac{R}{L} \frac{dV_C(t)}{dt} - \frac{1}{LC} V_C(t)$$

Así el diagrama de simulación analógica es

$$\frac{dV_C(t)}{dt} = \frac{R}{LC} \int i(t) dt + \frac{1}{C} i(t) - \frac{R}{L} V_C(t) - \frac{1}{LC} \int V_C(t) dt$$



Con $X_1(t) = V_C(t)$ $\therefore$ $\boxed{V_C(t) = X_1(t)}$

ecuación de salida

$$x_1(t) = \int \left[-\frac{R}{L} x_1(t) - \frac{1}{LC}\int x_1(t)\,dt + \frac{1}{C} i(t) + \frac{R}{LC}\int i(t)\,dt \right] dt.$$

$$\therefore \quad \dot{x}_1(t) = -\frac{R}{L} x_1(t) \underbrace{- \frac{1}{LC}\int x_1(t)\,dt}_{A} + \frac{1}{C} i(t) + \underbrace{\frac{R}{LC}\int i(t)\,dt}_{B}.$$

Los términos A y B no pueden formar parte de la ecuación de estado, en este caso no podemos elegir la variable de estado como salida del integrador solamente. Al no poder incluir en el segundo miembro derivadas / integrales, definiremos a $x_2(t)$ como

$$x_2(t) = -\frac{1}{LC}\int x_1(t)\,dt + \frac{R}{LC}\int i(t)\,dt$$

con lo que

$$\begin{cases} \dot{x}_1(t) = -\frac{R}{L} x_1(t) + x_2(t) + \frac{1}{C} i(t) \\ \dot{x}_2(t) = -\frac{1}{LC} x_1(t) + \frac{R}{LC} i(t) \end{cases}$$

ecuaciones de estado.

$$[\dot{x}(t)] = [A]\,[x(t)] + [B]\,[u(t)]$$

$$[V_C(t)] = [C]\,[x(t)] + [D]\,[u(t)].$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & 1 \\ -\frac{1}{LC} & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1/C \\ R/LC \end{bmatrix} [u(t)].$$

$$\left[V_c(t)\right] = \left[1 \quad 0\right] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + [0] \left[u(t)\right]$$

Función de transferencia:

$$V_c(t) = R\,i(t) - RC\frac{dV_c(t)}{dt} + L\frac{di(t)}{dt} - LC\frac{d^2V_c(t)}{dt^2}$$

$$LC\frac{d^2V_c(t)}{dt^2} + RC\frac{dV_c(t)}{dt} + V_c(t) = R\,i(t) + L\frac{di(t)}{dt}$$

Aplicando $\mathcal{L}$:

$$\left(LCs^2 + RCs + 1\right)V_c(s) = \left(R + sL\right) I(s)$$

$$\frac{V_c(s)}{I(s)} = \frac{L}{LC} \frac{s + R/L}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$\boxed{\frac{V_c(s)}{I(s)} = \frac{1}{C} \frac{s + R/L}{s^2 + \frac{R}{L}s + \frac{1}{LC}}}$$

$$\frac{V_c(s)}{I(s)} = [C]\left(s[I] - [A]\right)^{-1}[B] + D$$

$$s[I] - [A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -\frac{R}{L} & 1 \\ -\frac{1}{LC} & 0 \end{bmatrix} = \begin{bmatrix} s + \frac{R}{L} & -1 \\ \frac{1}{LC} & s \end{bmatrix}$$

$$\left| s[I]-[A] \right| = s^2+\frac{R}{L}s+\frac{1}{LC}$$

Adjunto:

$$\text{Adj}\left(s[I]-[A] \right) = \begin{bmatrix} s & -\frac{1}{LC} \\ 1 & s+\frac{R}{L} \end{bmatrix}^T = \begin{bmatrix} s & 1 \\ -\frac{1}{LC} & s+\frac{R}{L} \end{bmatrix}$$

$$\left(s[I]-[A] \right)^{-1} = \frac{\text{Adj}\left(s[I]-[A] \right)}{\left| s[I]-[A] \right|}$$

$$\left(s[I]-[A] \right)^{-1} = \begin{bmatrix} \dfrac{s}{s^2+\frac{R}{L}s+\frac{1}{LC}} & \dfrac{1}{s^2+\frac{R}{L}s+\frac{1}{LC}} \\ \dfrac{-1/LC}{s^2+\frac{R}{L}s+\frac{1}{LC}} & \dfrac{s+R/L}{s^2+\frac{R}{LC}s+\frac{1}{LC}} \end{bmatrix}$$

$$[C]\left(s[I]-[A] \right)^{-1} =$$

$$\begin{array}{c|cc} & \dfrac{s}{s^2+\frac{R}{L}s+\frac{1}{LC}} & \dfrac{1}{s^2+\frac{R}{L}s+\frac{1}{LC}} \\ & \dfrac{-1/LC}{s^2+\frac{R}{L}s+\frac{1}{LC}} & \dfrac{s+R/L}{s^2+\frac{R}{LC}s+\frac{1}{LC}} \\ \hline 1 \quad 0 & \dfrac{s}{s^2+\frac{R}{L}s+\frac{1}{LC}} & \dfrac{1}{s^2+\frac{R}{L}s+\frac{1}{LC}}. \end{array}$$

$$[C]\left(s[I]-[A]\right)^{-1}[B]=$$

$$\begin{bmatrix} \dfrac{s}{s^2+\frac{R}{L}s+\frac{1}{LC}} & \dfrac{1}{s^2+\frac{R}{L}s+\frac{1}{LC}} \end{bmatrix} \begin{bmatrix} \dfrac{1}{C} \\ \dfrac{R}{LC} \end{bmatrix} = \frac{\frac{1}{C}s+\frac{R}{LC}}{s^2+\frac{R}{L}s+\frac{1}{LC}}$$

como $D=0$

$$\frac{V_c(S)}{I(S)}=\frac{\frac{1}{C}s+\frac{R}{LC}}{S^2+\frac{R}{L}s+\frac{1}{LC}}=\frac{1}{C}\,\frac{s+\frac{R}{L}}{S^2+\frac{R}{L}s+\frac{1}{LC}}$$