Primer Parcial 5R1 **22-06-2.010**

1) Obtener la respuesta temporal de un sistema mecánico ideal compuesto por un resorte de $K = 2\,\frac{N}{m}$ y masa 1kg cuando se excita con un impulso unitario.

2) Obtener la velocidad en régimen de un motor de CC en RPM si se excita con 110 V y tiene las siguientes constantes:

$L_f = 10\,Hy$ $R_f = 100\,\Omega$ $J = 1\,kgm^2$ $B = 0{,}5\,\frac{Nm\,seg}{rad}$

$K_i = 90\,\frac{Nm}{A}$

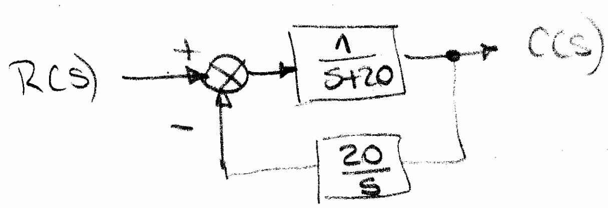
3) Linealizar la expresión $z = x\sqrt{y}$ en la región

$10 \leq x \leq 12$
$100 \leq y \leq 120$

Calcular error para $x = 10$ e $y = 100$.

4) Determinar la forma temporal del error del siguiente sistema cuando se excita con una entrada rampa.

5R1



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Soluciones parcial 1er 5R1. 22/06/10.

(1)

$F(t) = m\ddot{x}(t) + f\dot{x}(t) + Kx(t)$ al ser ideal $f = 0$.

$F(t) = m\ddot{x}(t) + Kx(t)$; $F(t) = \delta(t)$ $\therefore$ $F(s) = 1$.

$$F(s) = (ms^2 + K)\,X(s) \;\therefore\; \frac{X(s)}{F(s)} = \frac{1}{ms^2 + K} = \frac{1/m}{s^2 + K/m}$$

con $F(s) = 1$. excitación impulso unitario tenemos:

$$X(s) = \frac{1/m}{s^2 + K/m} \;;\; x(t) = \mathcal{L}^{-1}\left[X(s)\right]$$

Los polos son $s^2 + \frac{K}{m} = 0$; $s^2 = -\frac{K}{m}$; $s_{1-2} = \pm j\sqrt{\frac{K}{m}}$

$$X(s) = \frac{1/m}{\left(s - j\sqrt{\frac{K}{m}}\right)\left(s + j\sqrt{\frac{K}{m}}\right)} = \frac{A_1}{s - j\sqrt{\frac{K}{m}}} + \frac{\bar{A}_1}{s + j\sqrt{\frac{K}{m}}}$$

$$A_1 = \lim_{s \to j\sqrt{\frac{K}{m}}} \frac{1/m}{s + j\sqrt{\frac{K}{m}}} = \frac{1/m}{j2\sqrt{\frac{K}{m}}} = -j\,\frac{1}{2}\cdot\frac{1}{m}\sqrt{\frac{m}{K}}$$

$$A_1 = -j\,\frac{1}{2}\sqrt{\frac{\cancel{m}^{1}}{m^{\cancel{2}}\cdot K}} = -j\,\frac{1}{2}\cdot\frac{1}{\sqrt{Km}} \;;\; \bar{A}_1 = j\,\frac{1}{2}\cdot\frac{1}{\sqrt{Km}}$$

$$X(s) = \frac{1}{2}\,\frac{1}{\sqrt{Km}}\left[\frac{-j}{s - j\sqrt{\frac{K}{m}}} + \frac{j}{s + j\sqrt{\frac{K}{m}}}\right]$$

$$x(t) = \frac{1}{2}\,\frac{1}{\sqrt{Km}}\left[-j\,e^{j\sqrt{\frac{K}{m}}t} + j\,e^{-j\sqrt{\frac{K}{m}}t}\right]$$

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(2) $e^{j\theta} = \cos\theta + j\,\text{sen}\,\theta$ $e^{j\theta} - e^{-j\theta} = 2j\,\text{sen}\,\theta$

$e^{-j\theta} = \cos\theta - j\,\text{sen}\,\theta$ $j e^{j\theta} - j e^{-j\theta} = -2\,\text{sen}\,\theta$

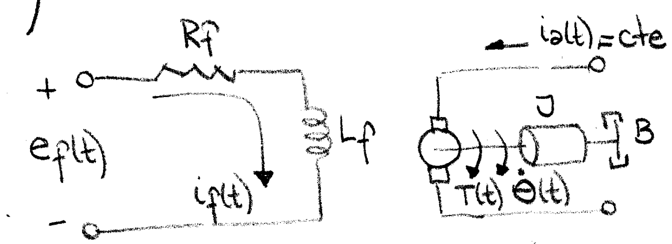
$-j e^{j\theta} + j e^{-j\theta} = 2\,\text{sen}\,\theta$

$$x(t) = \frac{1}{\cancel{2}}\,\frac{1}{\sqrt{km}}\left[\cancel{2}\,\text{sen}\sqrt{\frac{k}{m}}\,t\right] = \boxed{\frac{1}{\sqrt{km}}\,\text{sen}\sqrt{\frac{k}{m}}\,t = x(t)}$$

con $K = 2\,\frac{N}{m}$; $m = 1\,kg$.

$$x(t) = \frac{1}{\sqrt{2}}\,\text{sen}\,\sqrt{2}\cdot t = \boxed{0{,}71\,\text{sen}\,1{,}41t = x(t)}$$

2) Motor CC controlado por campo:



$$e_f(t) = R_f\,i_f(t) + L_f\,\frac{d\,i_f(t)}{dt}$$

$$T(t) = K_1\,\Phi(t)\,i_a(t) \quad ; \quad \Phi(t) = K_2\,i_f(t) \quad i_a(t) = K_3$$

$$T(t) = K_1 K_2\,i_f(t)\,K_3 = K_i\,i_f(t)$$

$$T(t) = J\,\ddot{\theta}(t) + B\,\dot{\theta}(t)$$

$$E_f(s) = (R_f + sL_f)\,I_f(s)$$

$$\frac{I_f(s)}{E_f(s)} = \frac{1/L_f}{s + R_f/L_f}$$

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3.

$$\frac{T(s)}{I_f(s)} = K_i \quad ; \quad T(s) = \left(s^2 J + s B\right)\Theta(s).$$

$$\dot{\Theta}(s) = \mathcal{L}\left[\dot{\Theta}(t)\right] \qquad \dot{\Theta}(s) = s\,\Theta(s)$$

$$T(s) = \left(J s + B\right)\dot{\Theta}(s) \quad ; \quad \dot{\Theta}(s) = \frac{1/J}{s + B/J}$$

$$E_f(s) \longrightarrow \boxed{\frac{1/L_f}{s + R_f/L_f}} \longrightarrow \boxed{K_i} \longrightarrow \boxed{\frac{1/J}{s + B/J}} \longrightarrow \dot{\Theta}(s)$$

$$\boxed{\frac{\dot{\Theta}(s)}{E_f(s)} = \frac{K_i}{J L_f}\;\frac{1}{s^2 + \left(\frac{R_f}{L_f} + \frac{B}{J}\right)s + \frac{B R_f}{J L_f}}}$$

Las unidades son todas MKS:

$$\frac{K_i}{J L_f} = \frac{90}{1 . 10} = 9 \qquad \frac{R_f}{L_f} + \frac{B}{J} = \frac{100}{10} + \frac{0,5}{1} = 10,5.$$

$$\frac{B R_f}{J L_f} = \frac{0,5 . 10\cancel{0}}{1 . 1\cancel{0}} = 5. \qquad \frac{\dot{\Theta}(s)}{E_f(s)} = 9\;\frac{1}{s^2 + 10,5 s + 5}.$$

Con $e_f(t) = 110\,\mu(t)\,[V]$ $\quad E_f(s) = \frac{110}{s}.$

$$\dot{\Theta}(s) = 990\;\frac{1}{s\left(s^2 + 10,5 s + 5\right)}$$

$$\dot{\Theta}(\infty) = \lim_{t \to \infty} \Theta(t) = \lim_{s \to 0} s\,\dot{\Theta}(s) = \frac{990}{5} = 198\;\frac{rad}{seg}$$

$$\dot{\Theta}(\infty) = 198\;\frac{rad}{\cancel{seg}} \cdot \frac{60\,\cancel{seg}}{1\,min} \cdot \frac{1\,rev}{2\pi\,rad} = 1890,76\ RPM.$$

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1) 3) $z = x\sqrt{y}$

elegimos $\bar{x} = 11$; $\bar{y} = 110$.

$$\bar{z} = \bar{x}\sqrt{\bar{y}} = 115{,}37$$

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=11\\y=110}} = \left.\sqrt{y}\right|_{\substack{x=11\\y=110}} = 10{,}49 = K_1$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=11\\y=110}} = \left.x \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{y}}\right|_{\substack{x=11\\y=110}} = \left.\frac{x}{2\sqrt{y}}\right|_{\substack{x=11\\y=110}} = 0{,}52 = K_2$$

$$z - \bar{z} = K_1(x - \bar{x}) + K_2(y - \bar{y})$$

$$z - 115{,}37 = 10{,}49(x - 11) + 0{,}52(y - 110)$$

$$z = 10{,}49x - \cancel{115{,}39} + 0{,}52y - 57{,}2 + \cancel{115{,}37}$$

$$\boxed{z = 10{,}49x + 0{,}52y - 57{,}2}$$ función linealizada.

$$z(10,100) = 104{,}9 + 52 - 57{,}2 = 99{,}7$$

$$z = 10 \cdot \sqrt{100} = 10 \cdot 10 = 100$$

$$\varepsilon\% = \frac{99{,}7 - 100}{100} \cdot 100\% = -0{,}3\%$$

4)

$$\frac{E(s)}{R(s)} = \frac{1}{1 + G(s)H(s)}$$

$$1 + G(s)H(s) = \frac{20}{s(s+20)} + 1 = \frac{20 + s^2 + 20s}{s(s+20)} = \frac{s^2 + 20s + 20}{s(s+20)}$$

$$\frac{E(s)}{R(s)} = \frac{s(s+20)}{s^2 + 20s + 20}$$

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(5) $R(s) = \frac{1}{s^2}$ dado que $r(t) = t\,\mu(t)$.

$$E(s) = \frac{1}{s^2}\,\frac{s(s+20)}{s^2+20s+20} = \frac{s+20}{s(s^2+20s+20)}$$

$$s_{1-2} = \frac{-20 \pm \sqrt{(20)^2 - 4.20}}{2} = \frac{-20 \pm \sqrt{320}}{2} = \frac{-20 \pm 17{,}89}{2}$$

$$s_1 = -1{,}06 \qquad s_2 = -18{,}94$$

$$E(s) = \frac{s+20}{s(s+1{,}06)(s+18{,}94)} = \frac{A_0}{s} + \frac{A_1}{s+1{,}06} + \frac{A_2}{s+18{,}94}$$

$$A_0 = \lim_{s \to 0} \frac{s+20}{s^2+20s+20} = 1$$

$$A_1 = \lim_{s \to -1{,}06} \frac{s+20}{s(s+18{,}94)} = -0{,}9993$$

$$A_2 = \lim_{s \to -18{,}94} \frac{s+20}{s(s+1{,}06)} = 0{,}0031$$

$$\boxed{e(t) = 1 - 0{,}999\,e^{-1{,}06t} + 0{,}003\,e^{-18{,}94t}}$$