Segundo Parcial 5R1 01/11/11

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + \begin{bmatrix}1\\ 1\end{bmatrix}u(t) \;;\; y(t) = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}$$

1) Idem 5R2

2) " " con $K = 0{,}5$

3) " " $t_{s5\%} = 3$ seg.

4) " " $M\phi$, MG y K_V.

SOLUCIONES 5R1.

1)
$$\frac{Y(s)}{U(s)} = [C]\left(s[I]-[A]\right)^{-1}[B] + D$$

$$s[I]-[A] = \begin{bmatrix}s & 0\\ 0 & s\end{bmatrix} - \begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix} = \begin{bmatrix}s-1 & -1\\ 0 & s\end{bmatrix}$$

$$\left(s[I]-[A]\right)^{-1} = \frac{\operatorname{Adj}\left(s[I]-[A]\right)}{\left|s[I]-[A]\right|} = \frac{\begin{bmatrix}s & 0\\ 1 & s-1\end{bmatrix}^T}{s^2-s}.$$

$$\left(s[I]-[A]\right)^{-1} = \begin{bmatrix}\dfrac{s}{s^2-s} & \dfrac{1}{s^2-s}\\ \dfrac{0}{s^2-s} & \dfrac{s-1}{s^2-s}\end{bmatrix}$$

$$\begin{array}{c|cc} & \dfrac{s}{s^2-s} & \dfrac{1}{s^2-s}\\ & 0 & \dfrac{s-1}{s^2-s}\\ \hline 1\;0 & \dfrac{s}{s^2-s} & \dfrac{1}{s^2-s}\end{array}$$

$$[C]\left(s[I]-[A]\right)^{-1} = \begin{bmatrix}\dfrac{s}{s^2-s} & \dfrac{1}{s^2-s}\end{bmatrix}$$

	1
	1
$\frac{s}{s^2-s}$ $\quad 1$	$\frac{s+1}{s^2-s}$

$$\boxed{\frac{Y(s)}{U(s)} = \frac{s+1}{s(s-1)} = G(s)H(s)}$$

2) $$G(s)H(s) = \frac{Y(s)}{U(s)} = \frac{s+1}{s(s-1)}$$ con $K=0{,}5$ en cascada:

$$G(s)H(s) = 0{,}5 \; \frac{s+1}{s(s-1)} = 0{,}5 \; \frac{s+1}{s^2-s}.$$

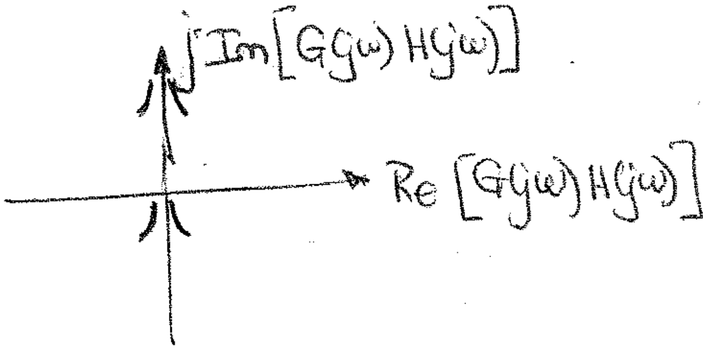
Análisis de BF: $$\lim_{s\to 0} G(s)H(s) = \lim_{s\to 0} 0{,}5 \; \frac{s+1}{s(s-1)}.$$

$$= \lim_{s\to 0} 0{,}5 \; \frac{1}{s(-1)} = \lim_{s\to 0} -\frac{0{,}5}{s}$$

con $s=j\omega$ $$\lim_{\omega\to 0} -\frac{0{,}5}{j\omega} = \lim_{\omega\to 0} j\,\frac{0{,}5}{\omega} \to j\infty.$$

Análisis de AT: $$\lim_{s\to\infty} 0{,}5 \; \frac{s+1}{s(s-1)} = \lim_{s\to\infty} 0{,}5 \; \frac{\overset{1}{\cancel{s}}}{\underset{1}{\cancel{s}}\cdot s}$$

$s=j\omega$ $$\lim_{\omega\to\infty} 0{,}5 \cdot \frac{1}{j\omega} \to -j0.$$



② SOLUCIONES 5R1.

$$G(s)H(s) = 0{,}5\,\frac{s+1}{s^2-s} \quad ; \quad G(j\omega)H(j\omega) = 0{,}5\,\frac{1+j\omega}{-\omega^2-j\omega}$$

$$G(j\omega)H(j\omega) = 0{,}5\,\frac{(1+j\omega)(-\omega^2+j\omega)}{\omega^4+\omega^2} = 0{,}5\,\frac{-\omega^2+j\omega-j\omega^3-\omega^2}{\omega^4+\omega^2}$$

$$G(j\omega)H(j\omega) = 0{,}5\,\frac{-2\omega^2+j(\omega-\omega^3)}{\omega^4+\omega^2}$$

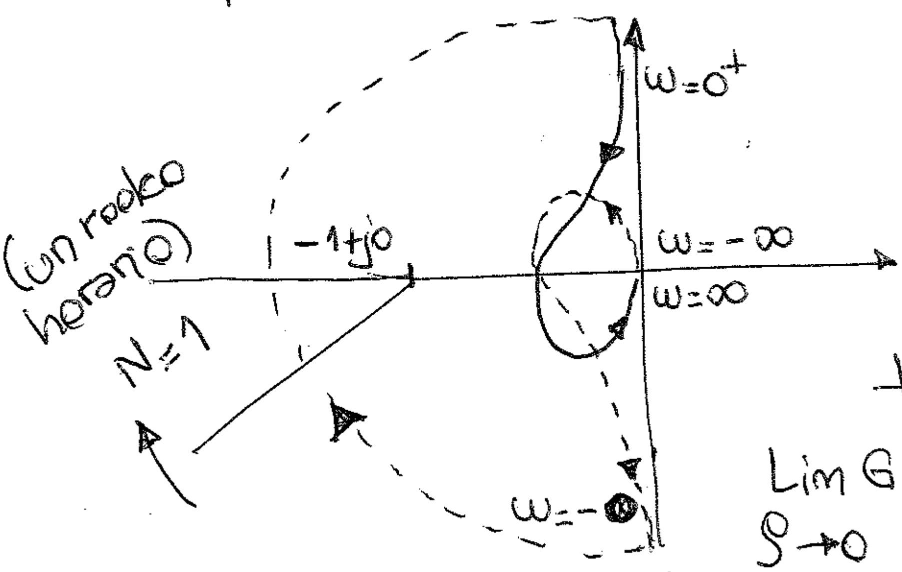
Como se observa hay corte al eje real: $\omega-\omega^3 = \omega(1-\omega^2)$

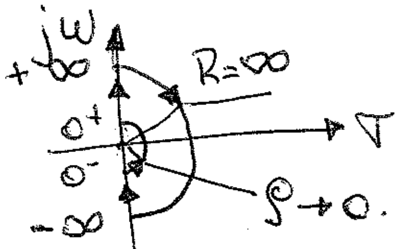
$\omega(1-\omega^2)=0$; $\omega=0$ y $1-\omega^2=0$ ∴

$$\omega = 1 \text{ rad/seg}.$$

$$G(j1)H(j1) = 0{,}5\,\frac{-2(1)^2}{1^4+1^2} = 0{,}5\,\frac{(-\cancel{2})^{1}}{\cancel{2}_{1}} = -0{,}5+j0.$$

La parte real es siempre negativa. Contorno:





Del análisis de BF tenemos (con $s=\rho e^{j\theta}$)

$$\lim_{\rho\to 0} G(\rho e^{j\theta})H(\rho e^{j\theta}) = \lim_{\rho\to 0} -\frac{0{,}5}{\rho e^{j\theta}} =$$

$$= \lim_{\rho\to 0}\frac{0{,}5}{\rho}\,\frac{e^{j\pi}}{e^{j\theta}} = \lim_{\rho\to 0}\frac{0{,}5}{\rho}\,e^{j(\pi-\theta)} \to \infty \;\underline{|\pi-\theta}$$

$P = 1$ (polo LA en +1) $N = Z - P$

$1 = Z - 1$ ∴ $\boxed{Z=2}$ $\boxed{\text{Inestable con dos polos con parte Re [+]}}$

3) Lugar de raices $G(s)H(s) = K \dfrac{s+1}{s(s-1)} = K \dfrac{s+1}{s^2-s}$

Asintotas: $\varphi_K = \dfrac{180^\circ (2K+1)}{p-z}$ $\quad K=0$.

$$\varphi_0 = \frac{180^\circ (2.0+1)}{2-1} = 180^\circ.$$

Punto trazado: $\sigma_C = \dfrac{\Sigma Re[p] - \Sigma Re[z]}{p-z} = \dfrac{0+1-(-1)}{2-1} = 2.$

Punto bifurcación: $\quad K \dfrac{s+1}{s^2-s} + 1 = 0.$

$$K = -\frac{s^2-s}{s+1} \; ; \; \frac{\partial K}{\partial s} = -\frac{(2s-1)(s+1)-(s^2-s)}{(s+1)^2} = 0.$$

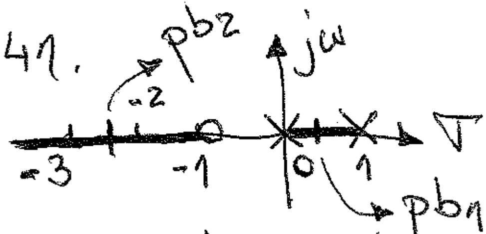
$$2s^2+2s-\cancel{s}-1-s^2+\cancel{s} = 0 = s^2+2s-1=0.$$

$$s_{1-2} = \frac{-2 \pm \sqrt{4-4(-1)}}{2} = \frac{-2 \pm \sqrt{8}}{2} \approx \frac{-2 \pm 2{,}83}{2}$$

$pb_1 = s_1$

$s_1 = 0{,}41 \; . \; s_2 = -2{,}41.$ $\quad pb_2 = s_2$

L.R. sobre eje real:



El LR es una circunsferencia de radio 1,41 con centro en $-1+j0$.

Criterio de Routh: $K \dfrac{s+1}{s^2-s} + 1 = 0 \approx \dfrac{K(s+1)+s^2-s}{s^2-s} = 0.$

$$Ks + K + s^2 - s = 0 \;\therefore\; s^2 + (K-1)s + K = 0.$$

s^2	1	K
s	K−1	
s^0	K	

$K_c - 1 = 0 \;\therefore\; K_c = 1$

$K < 1$ es inestable con 2 raices con parte $Re[+]$

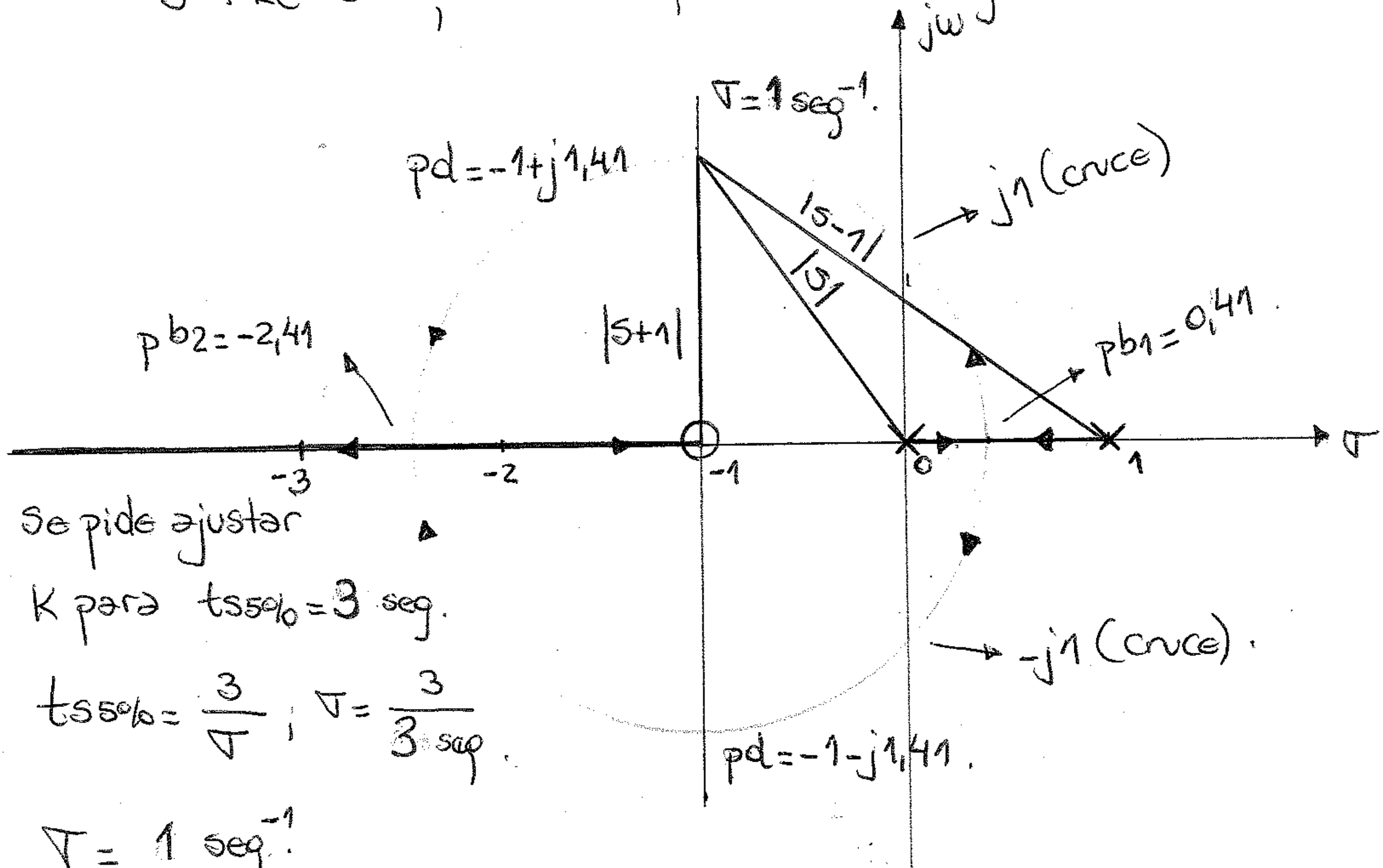
$K > 1$ estable.

Cortes eje $j\omega$:

(3)

SOLUCIONES SR1

$s^2 + K_c = 0$; $s^2+1=0$; $s = \sqrt{-1} = \pm j1$.



Se pide ajustar K para $ts_{5\%} = 3$ seg.

$$ts_{5\%} = \frac{3}{\sigma} \; ; \; \sigma = \frac{3}{3\ seg}.$$

$$\sigma = 1\ seg^{-1}$$

$$K = -\frac{s(s-1)}{s+1} \; ; \quad K = \frac{|s||s-1|}{|s+1|} = \frac{1{,}73 \cdot 2{,}45}{1{,}41} = \boxed{3 = K}$$

4).
$$G(s)H(s) = 3\,\frac{s+1}{s^2-s} = 3\,\frac{s+1}{s(s-1)}$$

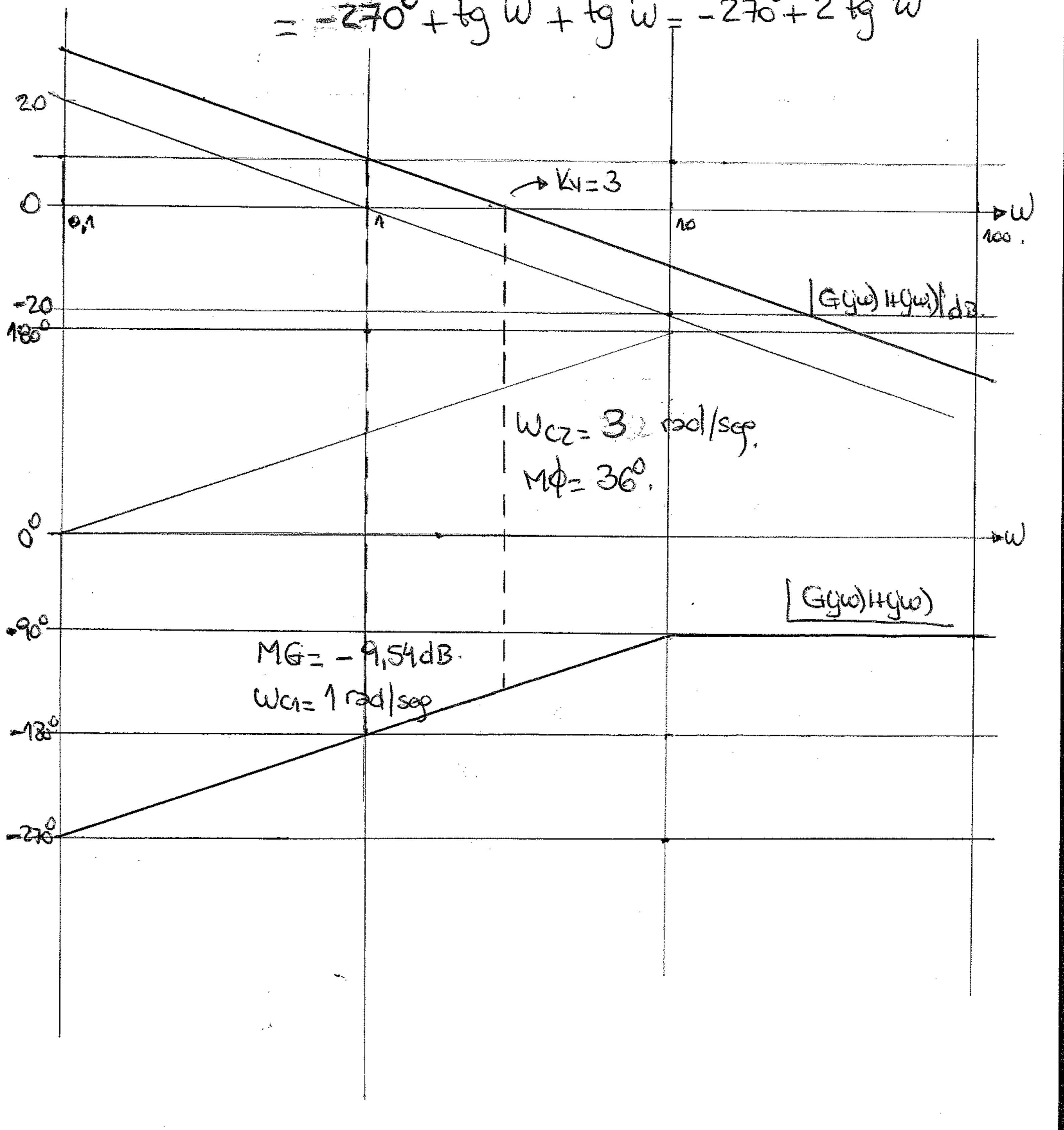
$$G(s)H(s) = 3\,\frac{s+1}{s\,(-1)\,(-s+1)} = \frac{3}{(-1)}\,\frac{s+1}{s(-s+1)}$$

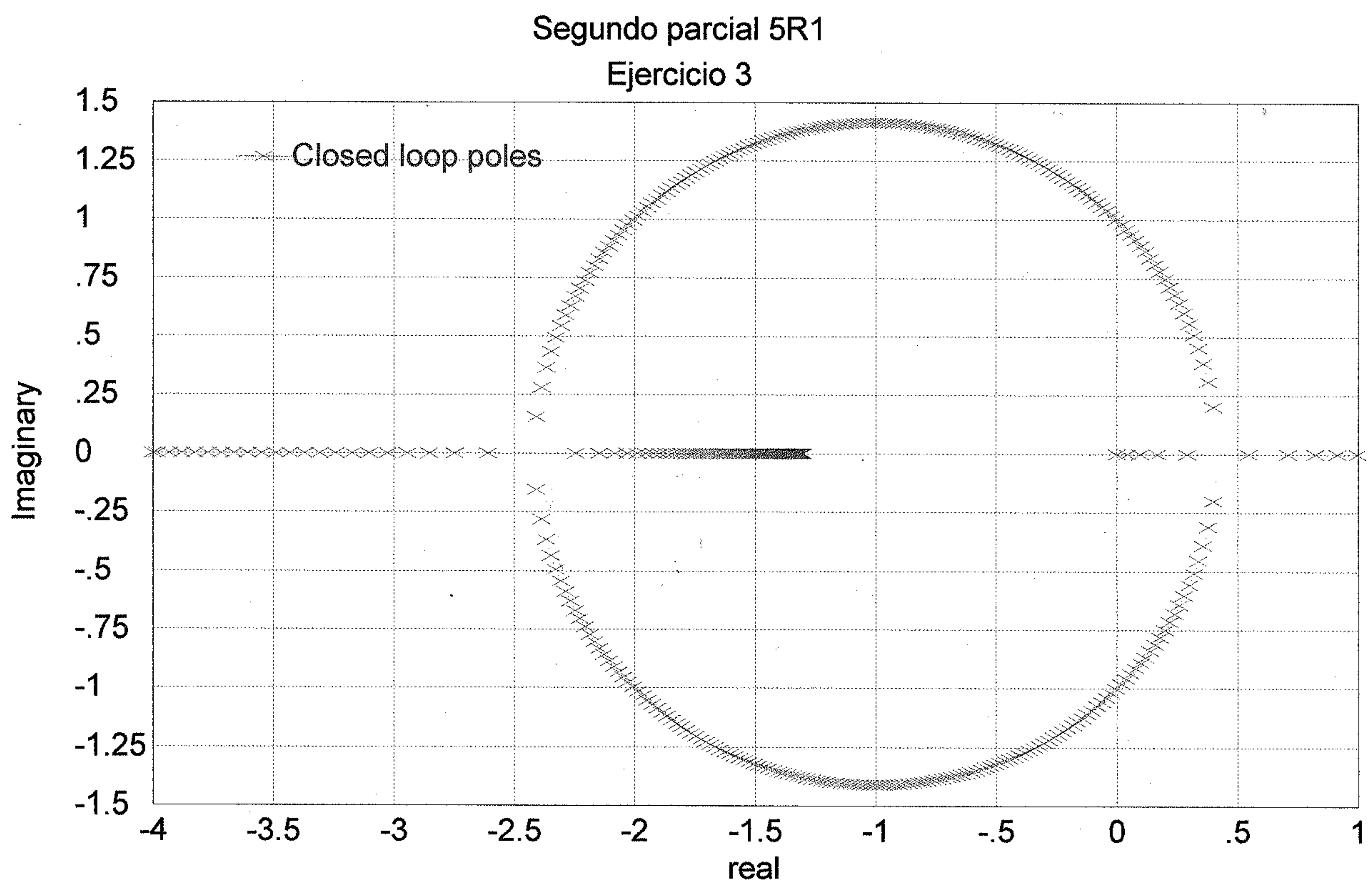
$$G(j\omega)H(j\omega) = \frac{3}{(-1)}\,\frac{1+j\omega}{(j\omega)(1-j\omega)}$$

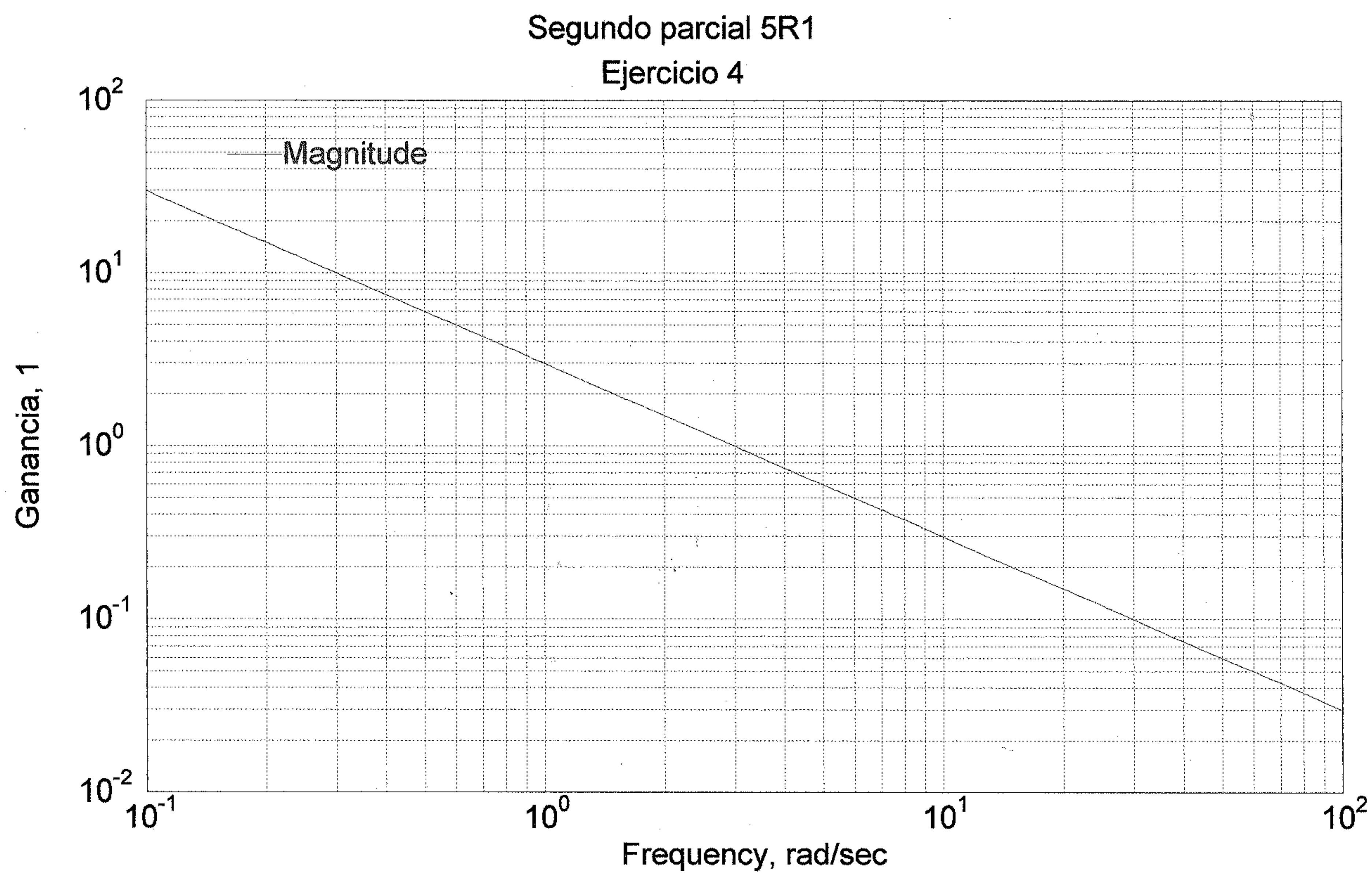
$$|G(j\omega)H(j\omega)| = \frac{3}{1}\,\frac{\cancel{\sqrt{1+\omega^2}}}{\omega\,\cancel{\sqrt{1+\omega^2}}} = \frac{3}{\omega}$$

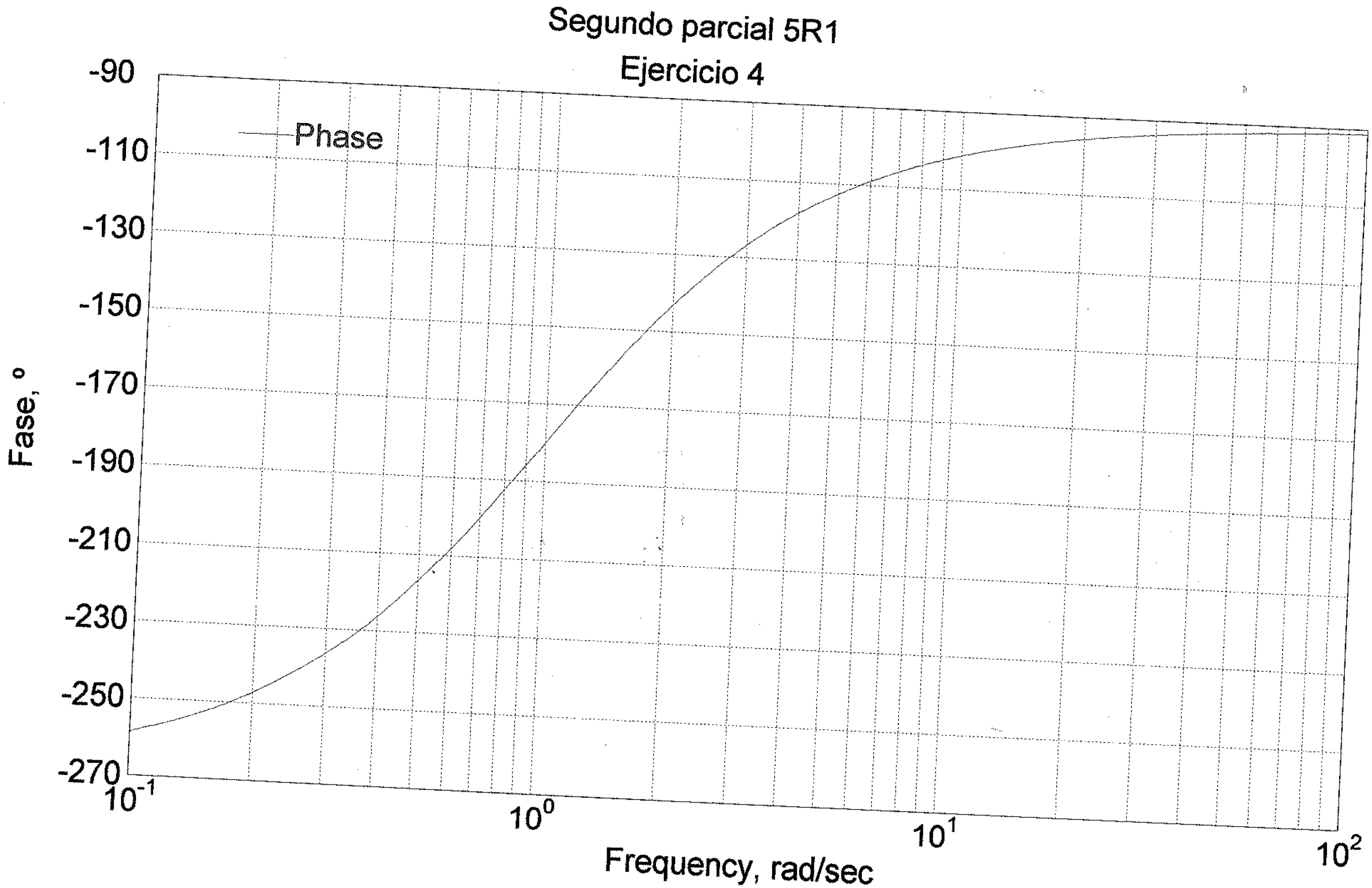
$$|G(j\omega)H(j\omega)|_{dB} = 20\log 3 - 20\log\omega$$
$$= 9{,}54 - 20\log\omega .$$

$$\underline{/G(j\omega)H(j\omega)} = 0^\circ + tg^{-1}\omega - 180^\circ - tg^{-1}(-\omega) - 90^\circ$$
$$= -270^\circ + tg^{-1}\omega + tg^{-1}\omega = -270^\circ + 2\,tg^{-1}\omega$$




Segundo parcial 5R1
Ejercicio 3
Closed loop poles
1.5
1.25
1
.75
.5
.25
0
-.25
-.5
-.75
-1
-1.25
-1.5
Imaginary
-4
-3.5
-3
-2.5
-2
-1.5
-1
-.5
0
.5
1
real


Segundo parcial 5R1
Ejercicio 4
Magnitude
Ganancia, 1
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency, rad/sec


Segundo parcial 5R1
Ejercicio 4
Phase
-90
-110
-130
-150
-170
-190
-210
-230
-250
-270
Fase, °
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency, rad/sec