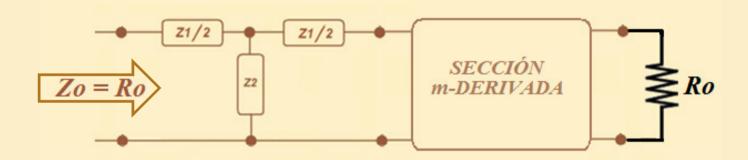


## **DEFINICIÓN:**

EL FILTRO M-DERIVADO, SE OBTIENE A PARTIR DEL FILTRO DE K-CONSTANTE Y SE EMPLEA EN CASCADA CON ESTOS.



#### **ESPECIFICACIONES:**

DEBE CUMPLIR CON LAS SIGUIENTES CONDICIONES :

- A. TENER IGUAL IMPEDANCIA CARACTERÍSTICA QUE EL FILTRO DE K-CONSTANTE.
- B. TENER IGUAL BANDA DE FRECUENCIA Y POR LO TANTO LAS MISMAS FRECUENCIAS DE CORTE QUE EL FILTRO DE K-CTE.

## A. IMPEDANCIAS CARACTERÍSTICAS IGUALES

#### PARTIMOS DE LA Zo DE UN CUADRIPOLO TIPO "T"

$$Z_{OT} = \sqrt{Z_{IN_{OC}} \cdot Z_{IN_{SH}}} = \sqrt{Z_{OUT_{OC}} \cdot Z_{OUT_{SH}}} =$$

$$Z_{OT} = \sqrt{Z_{1K} \cdot Z_{2K} + \frac{Z_{1K}^{2}}{4}} =$$

$$Z_{OT} = \sqrt{\frac{Z_{1K}}{2} \cdot \left(2 \cdot Z_{2K} + \frac{Z_{1K}}{2}\right)}$$
(1)

## INTRODUCIMOS UN COEFICIENTE REAL $0 < m \le 1$

$$Z_{OT} = \sqrt{\frac{m}{2}} \frac{Z_{1K}}{2} \cdot \left(2 \cdot Z_{2K} + \frac{Z_{1K}}{2}\right) \frac{1}{m}$$
 (2)

# DE LA ECUACIÓN (1) DEFINIMOS LA Zo DE UNA SECCIÓN M-DERIVADA :

$$Z_{OT} = \sqrt{\frac{Z_{1K}}{2}} \bullet \left(2 \bullet Z_{2K} + \frac{Z_{1K}}{2}\right) \tag{1}$$

$$Z_{OTm} = \sqrt{\frac{Z_{1Km}}{2} \cdot \left(2 \cdot Z_{2Km} + \frac{Z_{1Km}}{2}\right)} \tag{3}$$

#### DADO QUE Zot = Zotm , IGUALANDO (2) Y (3) :

$$Z_{OT} = \sqrt{\frac{Z_{1Km}}{2} \bullet \left(2 \bullet Z_{2Km} + \frac{Z_{1Km}}{2}\right)} = Z_{OTm} = \sqrt{\frac{mZ_{1K}}{2} \bullet \left(2 \bullet Z_{2K} + \frac{Z_{1K}}{2}\right) \frac{1}{m}}$$

$$\frac{Z_{1Km}}{2} \bullet \left(2 \bullet Z_{2Km} + \frac{Z_{1Km}}{2}\right) = \frac{mZ_{1K}}{2} \bullet \left(2 \bullet Z_{2K} + \frac{Z_{1K}}{2}\right) \frac{1}{m} \quad (4)$$

## DE LA ECUACIÓN (4) DESPEJANDO OBTENEMOS :

$$\frac{Z_{1Km}}{2} = \frac{m \cdot Z_{1K}}{2} \qquad \therefore$$

$$Z_{1Km} = m \bullet Z_{1K}$$

## DE LA ECUACIÓN (4) ADEMÁS:

$$\left(2 \bullet Z_{2Km} + \frac{Z_{1Km}}{2}\right) = \left(2 \bullet Z_{2K} + \frac{Z_{1K}}{2}\right) \frac{1}{m}$$

#### DESPEJANDO DE LA ÚLTIMA EXPRESIÓN :

$$Z_{2Km} = \frac{Z_{2K}}{m} + Z_{1K} \left( \frac{1 - m^2}{4m} \right)$$

## B. MISMA BANDA PASANTE

EN LOS FILTROS DE K-CTE TENEMOS :



$$|X_{\kappa}| = 0$$
  $\longrightarrow$  (DENTRO DE LA BANDA PASANTE)

EN LA SECCION DE K-CTE:

$$X_K = \sqrt{\frac{Z_{1K}}{4*Z_{2K}}}$$

Y EN LA m-DERIVADA SERÁ :

$$X_{Km} = \sqrt{\frac{Z_{1Km}}{4*Z_{2Km}}}$$

$$X_{Km} = \sqrt{\frac{m Z_{1K}}{4*\left(\frac{Z_{2K}}{m} + Z_{1K} \bullet \frac{1 - m^2}{4m}\right)}} = \sqrt{\frac{m Z_{1K}}{4 Z_{2K}} *\left(1 + \frac{Z_{1K}}{4 Z_{2K}} \bullet (1 - m^2)\right)}$$

$$X_{Km} = \sqrt{\frac{Z_{1K} \quad m^2}{4 \ Z_{2K} * \left(1 + \frac{Z_{1K}}{4 \ Z_{2K}} \bullet \left(1 - m^2\right)\right)}} \quad \text{Recordando } X_K^2 = \frac{Z_{1K}}{4 \ Z_{2K}}$$

$$X_{Km} = \sqrt{\frac{X_K^2 m^2}{\left(1 + X_K^2 \bullet \left(1 - m^2\right)\right)}}$$

Cuando  $X_K \to \infty$  $X_{Km} \rightarrow \text{valor finito Real}$ 

$$X_{Km} = \frac{X_K m}{\sqrt{1 + X_K^2 \bullet (1 - m^2)}}$$

$$\therefore \left| X_{Km} \right|_{X_K \to \infty} = \frac{m}{\sqrt{1 - m^2}}$$

## LA EXPRESIÓN DE XK PARA OBTENER XKm →∞ RECORDANDO QUE $X_K = j | X_K | ESTARA DADA POR$ :

$$X_{Km} = j \frac{\left| X_K \right| m}{\sqrt{1 - \left| X_K \right|^2 \bullet \left( 1 - m^2 \right)}} = j \frac{\left| X_K \right| m}{0} = \infty$$

$$|1-|X_K|^2 \bullet (1-m^2) = 0$$
 :  $|X_K|^2 \bullet (1-m^2) = 1$ 

$$\therefore |X_K|^2 \bullet (1-m^2) = 1$$

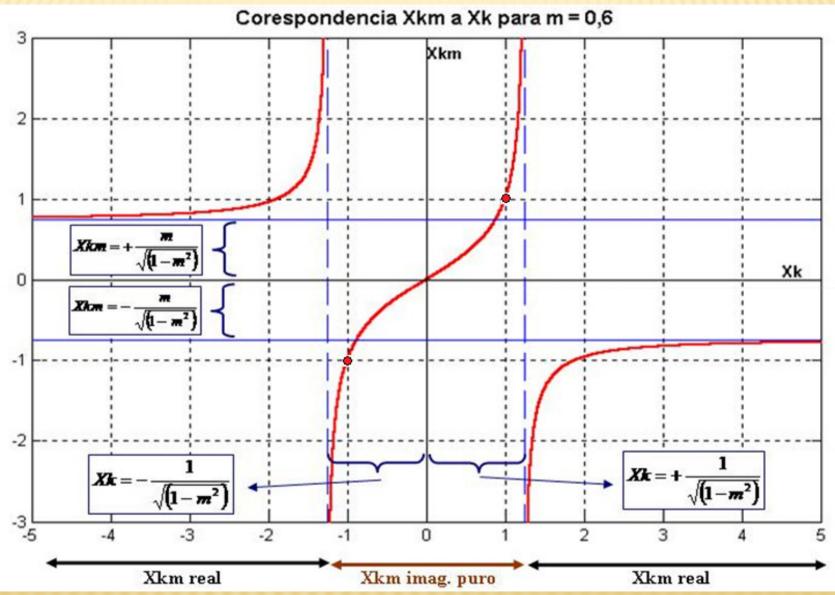
$$\left|X_K\right|^2 = \frac{1}{\left(1 - m^2\right)}$$

$$\therefore |X_K| = \frac{1}{\pm \sqrt{(1-m^2)}}$$

## TOMANDO m = 0,6 (VALOR MUY UTILIZADO) TENEMOS

$$|X_K| = \frac{1}{\pm \sqrt{(1-m^2)}} = \frac{1}{\pm \sqrt{(1-0.6^2)}} = \pm \frac{1}{0.8} = \pm 1.25$$

## CURVA DE CORRESPONDENCIA ENTRE XK Y XKm





## ETERMINACIÓN DE a y B EN FILTROS M-DERIVADOS

$$X_{K} = senh \frac{\gamma}{2} = j \ [X_{K}] = \sqrt{\frac{Z_{1K}}{4 * Z_{2K}}}$$

#### POR SIMILITUD CON LOS FILTROS DE K-CTE ESCRIBIMOS:

$$X_{Km} = j[X_{Km}] = \sqrt{\frac{Z_{1Km}}{4*Z_{2Km}}} = senh\frac{\alpha}{2}*cos\frac{\beta}{2} + jcosh\frac{\alpha}{2}*sen\frac{\beta}{2}$$

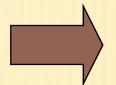
## EN LA ZONA EN QUE X<sub>Km</sub> ES IMAGINARIO PURO $|X_K| = \frac{1}{\pm \sqrt{(1-m^2)^2}}$ SUCEDE QUE:

$$senh\frac{\alpha}{2} * \cos\frac{\beta}{2} = 0$$

$$\beta = 0$$

$$\left| X_K \right| = \frac{1}{\pm \sqrt{1 - m^2}}$$

#### LA PARTE REAL DE senh (y/2) DEBE SER CERO:



$$senh\frac{\alpha}{2} * \cos\frac{\beta}{2} = 0$$

ESTO OCURRE SI :  $\alpha = 0$ 

$$\alpha = 0$$

$$\beta = \pm \Pi$$

PARA 
$$\alpha = 0 \rightarrow \cosh \alpha = 1$$

$$X_{Km} = \mathcal{N}[X_{Km}] = \mathcal{N}[\cos \frac{\alpha}{2} * \operatorname{sen} \frac{\beta}{2} \quad \therefore \quad \beta = 2 \operatorname{sen}^{-1}[X_{Km}]$$

$$\beta = 2 \operatorname{sen}^{-1} \left[ X_{Km} \right]$$

VÁLIDO PARA  $-1 \leq |X_{KM}| \leq +1$ 

PARA

$$\beta = \pm \Pi$$
  $\rightarrow sen \frac{\beta}{2} = \pm 1$ 

$$X_{Km} = \mathcal{N}[X_{Km}] = \mathcal{N}\cosh\frac{\alpha}{2} * sen\frac{\beta}{2} \quad \therefore \quad \alpha = 2 \cosh^{-1}[X_{Km}]$$

VÁLIDO PARA  $|X_{KM}| \geq \pm 1$ 

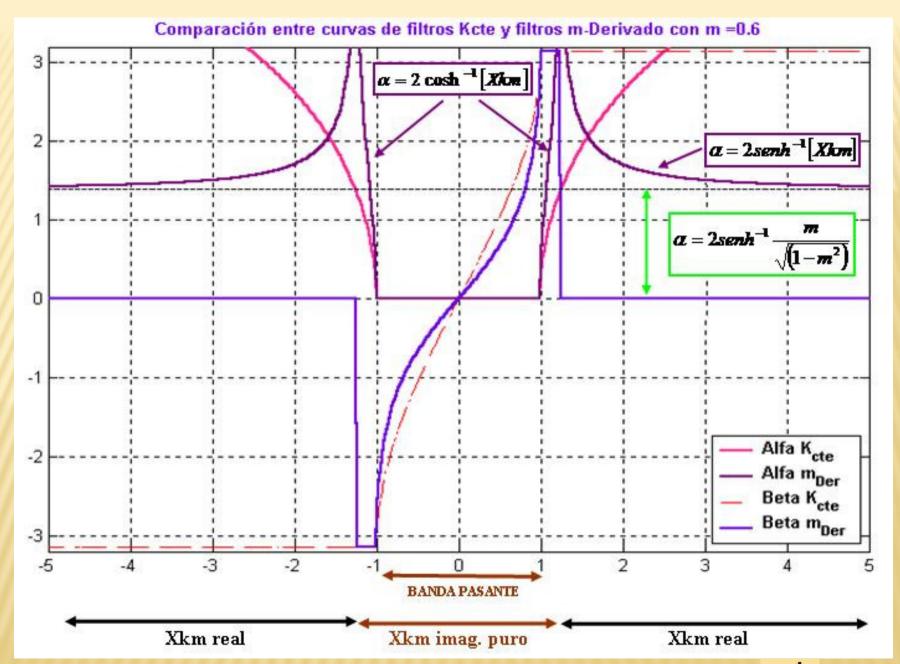
#### EN LA ZONA EN QUE XKM ES REAL, LA PARTE IMAGINARIA DE senh (y/2) DEBE SER CERO :

$$X_{Km} = j[X_{Km}] = \sqrt{\frac{Z_{1Km}}{4*Z_{2Km}}} = senh\frac{\alpha}{2}*cos\frac{\beta}{2} + jcosh\frac{\alpha}{2}*sen\frac{\beta}{2}$$

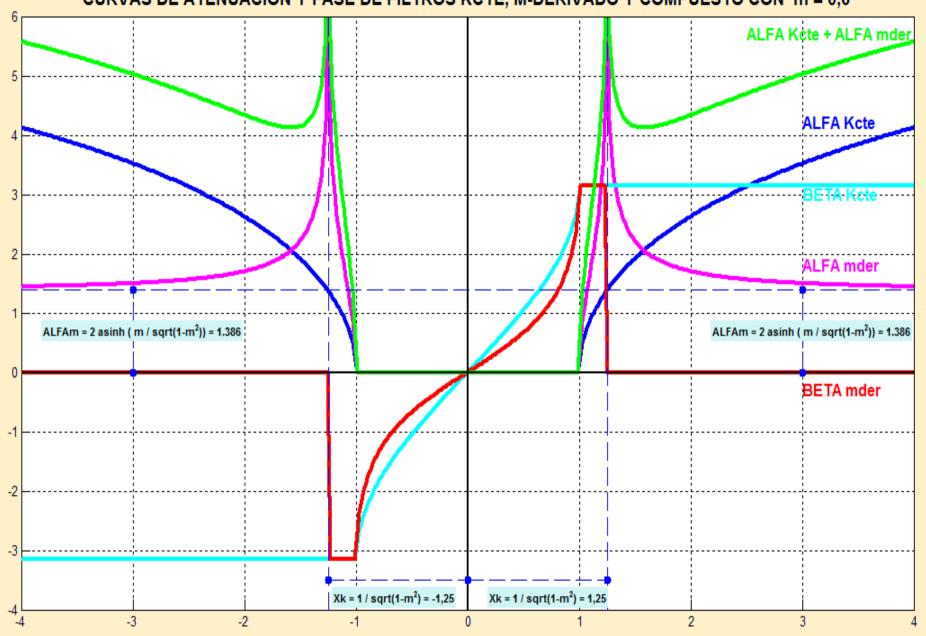
$$\cosh \frac{\alpha}{2} * sen \frac{\beta}{2} = 0$$
ESTO OCURRE CUANDO:  $\beta = 0$ 

#### ENTONCES EN ESTA ZONA:

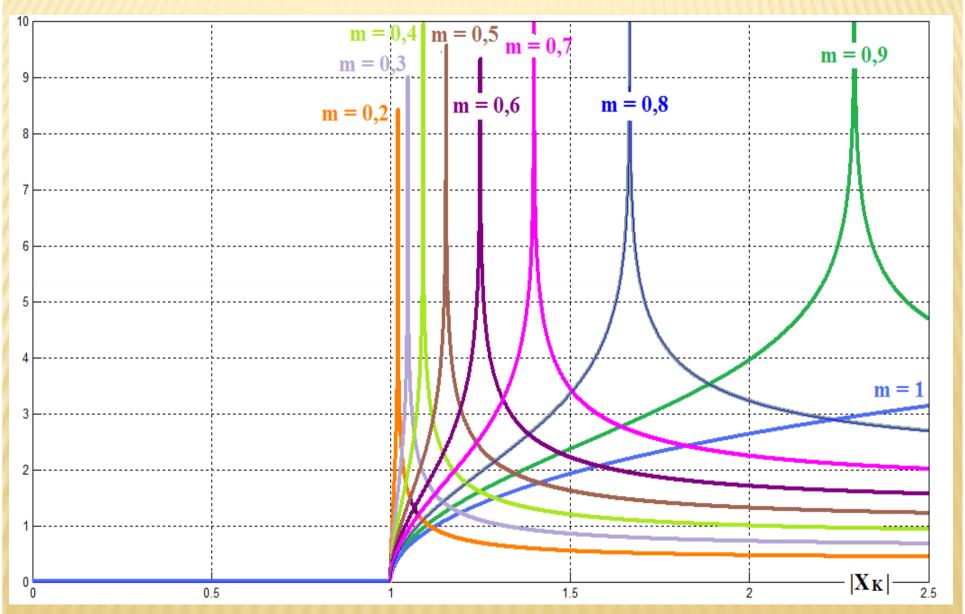
$$X_{Km} = j \left[ X_{Km} \right] = senh \frac{\alpha}{2} * cos \frac{\beta}{2} : \alpha = 2 senh^{-1} \left[ X_{Km} \right]$$



#### CURVAS DE ATENUACION Y FASE DE FILTROS KCTE, M-DERIVADO Y COMPUESTO CON m = 0,6



## CURVAS DE ATENUACIÓN DE FILTROS KCTE Y m-DERIVADOS

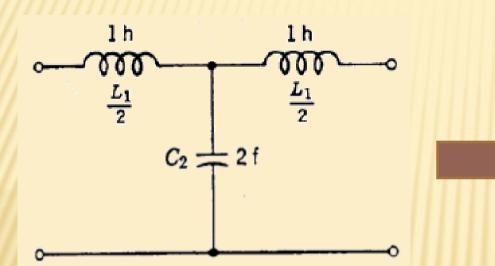


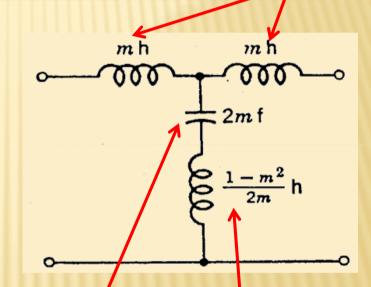
## DISEÑO DE FILTRO PASA-BAJOS m-DERIVADO

PARTIMOS DE UN FILTRO PASA BAJOS DE K<sub>KTE</sub> Y APLICAMOS

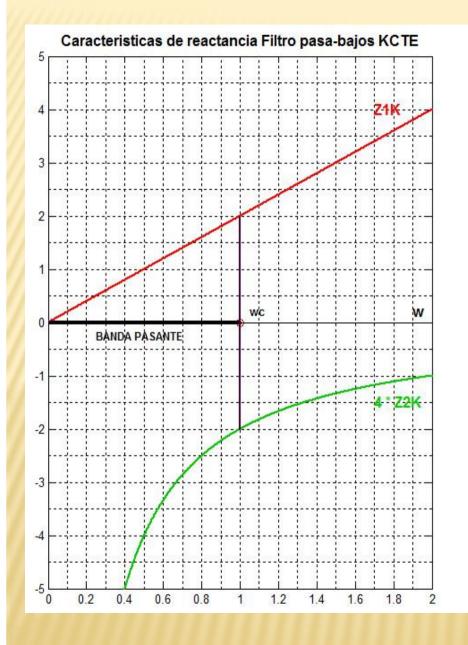
**NORMALIZACIÓN:** 

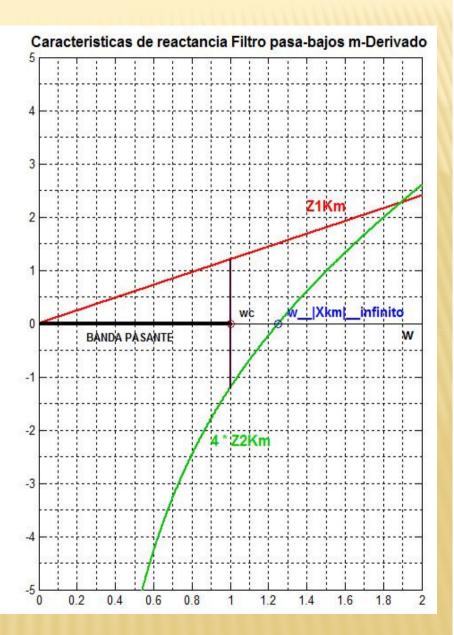
$$Z_{1Km} = m \bullet Z_{1K} = m \bullet pL = p mL$$





$$Z_{2Km} = \frac{Z_{2K}}{m} + Z_{1K} \left( \frac{1 - m^2}{4m} \right) = \frac{1}{P(mC)} + P\left( L \left( \frac{1 - m^2}{4m} \right) \right)$$



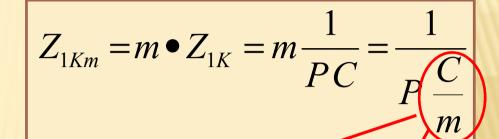


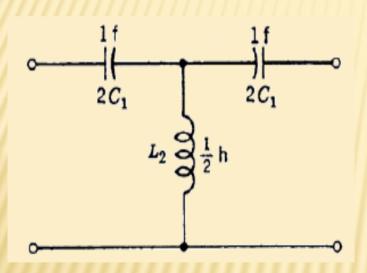


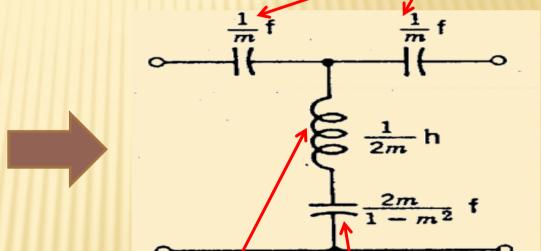
## DISEÑO DE FILTRO PASA-ALTOS m-DERIVADO

PARTIMOS DE UN FILTRO PASA ALTOS DE K<sub>KTE</sub> Y APLICAMOS

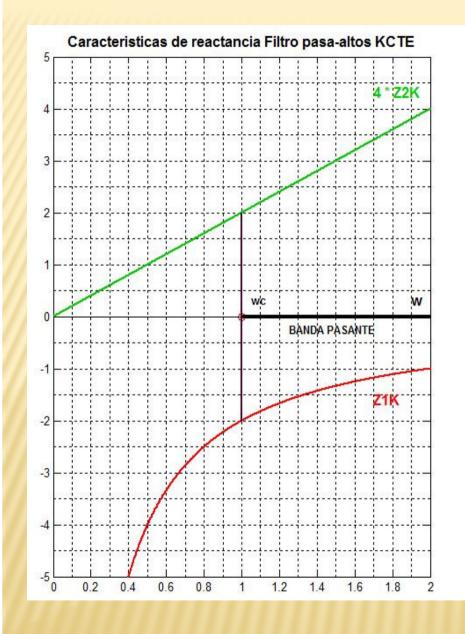
**NORMALIZACIÓN:** 

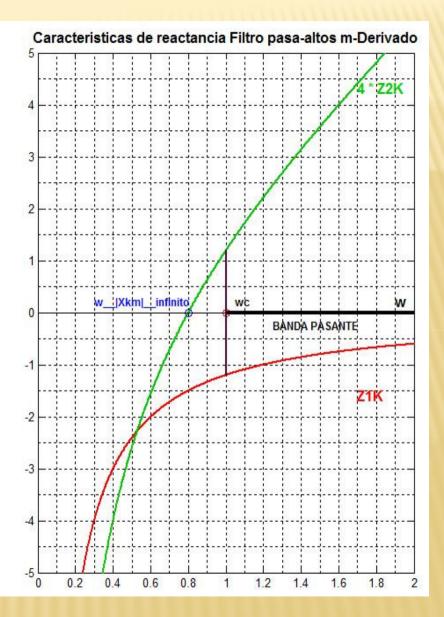






$$Z_{2Km} = \frac{Z_{2K}}{m} + Z_{1K} \left( \frac{1 - m^2}{4m} \right) = P \left( \frac{L}{m} \right) + \frac{1}{P \left( C \left( \frac{4m}{1 - m^2} \right) \right)}$$



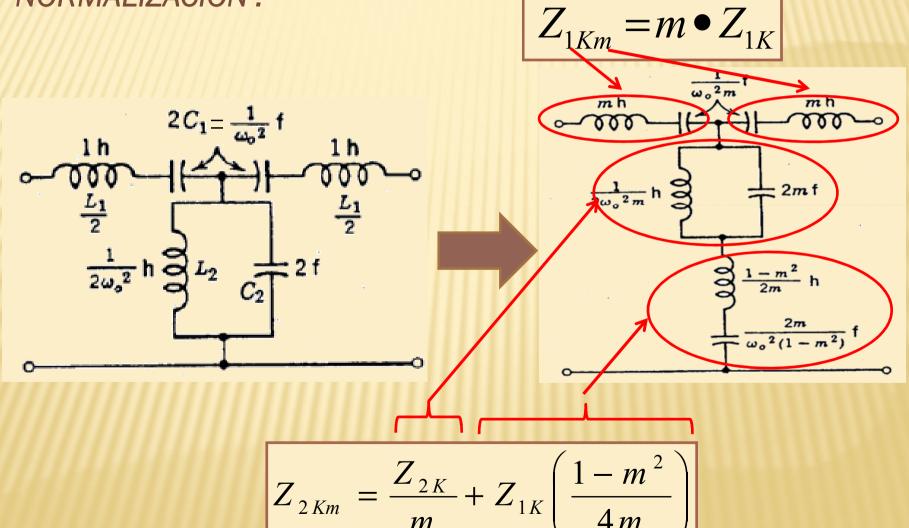


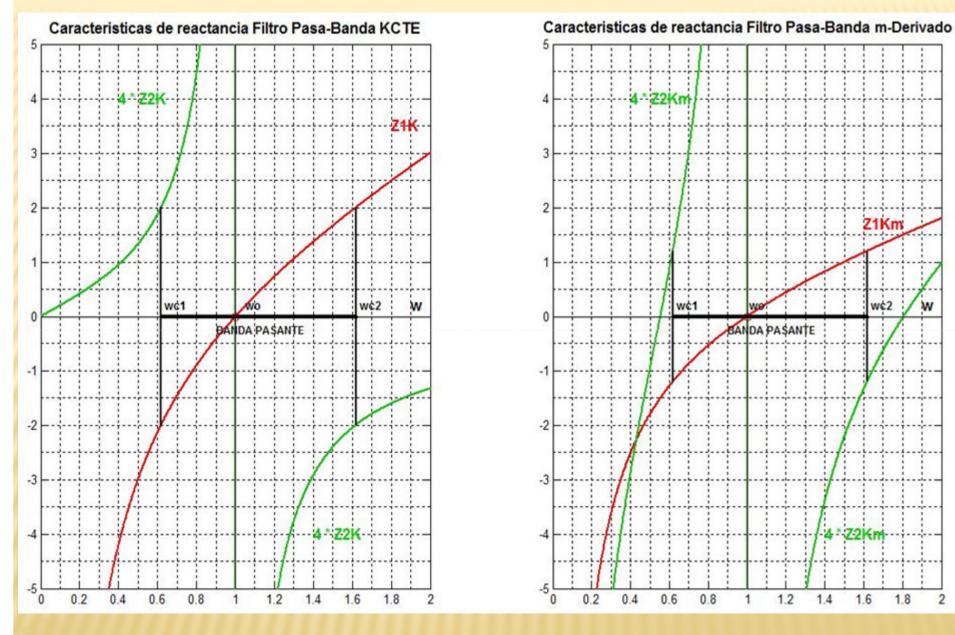


## DISEÑO DE FILTRO PASA-BANDA m-DERIVADO

PARTIMOS DE UN FILTRO PASA BANDA DE K<sub>KTF</sub> Y APLICAMOS





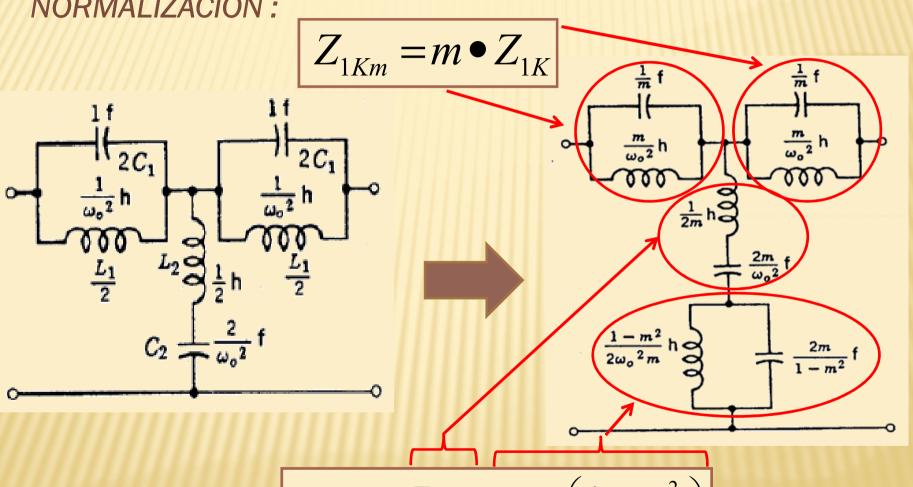




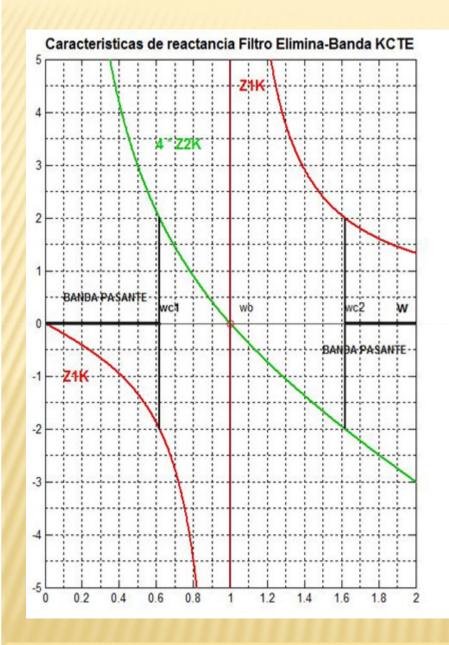
## DISEÑO DE FILTRO ELIMINA-BANDA m-DERIVADO

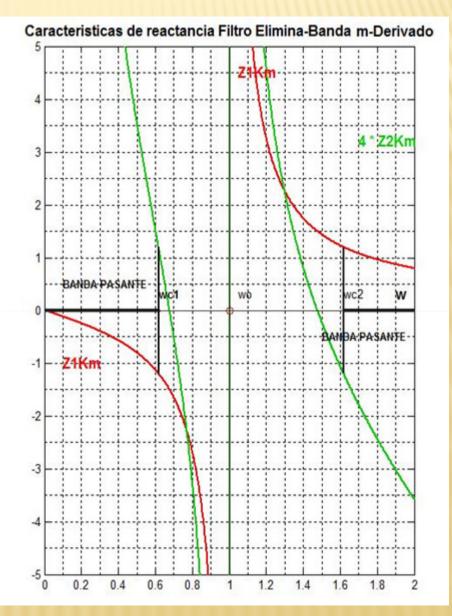
PARTIMOS DE UN FILTRO ELIMINA BANDA DE K<sub>KTF</sub> Y APLICAMOS





$$Z_{2Km} = \frac{Z_{2K}}{m} + Z_{1K} \left( \frac{1 - m^2}{4m} \right)$$



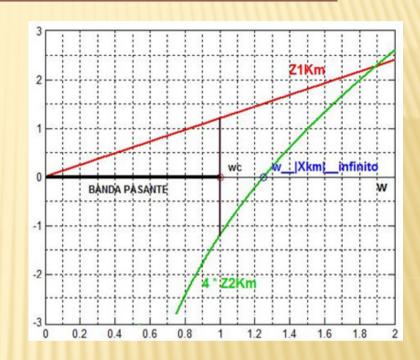




#### DETERMINACIÓN DEL VALOR DE m A PARTIR DE ωc Y ω∞

PARTIMOS DE LAS CURVAS DE REACTANCIA DE UN FILTRO PASA BAJOS DE K<sub>KTE</sub>.

DONDE  $4Z_{2KM} = 0$  SE PRODUCE LA ATENUACIÓN INFINITA ( $\alpha = \infty$ ) y CORREPONDE A  $X_{KM\infty}$ .



$$Z_{2Km} = \frac{Z_{2K}}{m} + Z_{1K} \left( \frac{1 - m^2}{4m} \right) = 0$$

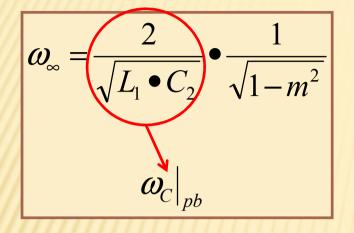


$$\left| \frac{Z_{1K}}{Z_{2K}} = -\frac{4}{1 - m^2} \right|$$

$$\frac{\omega_{\infty}L_1}{-\frac{1}{\omega_{\infty}C_2}} = -\frac{4}{1-m^2}$$

$$\omega_{\infty} = \frac{2}{\sqrt{L_1 \cdot C_2}} \cdot \frac{1}{\sqrt{1 - m^2}}$$

## DE LA ÚLTIMA EXPRESIÓN:





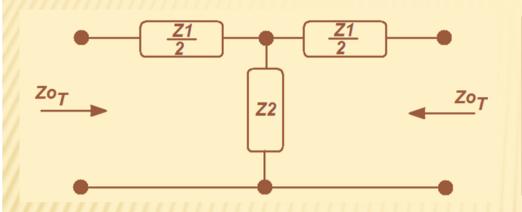
$$\omega_{\infty} = \omega_{C}|_{pb} \bullet \frac{1}{\sqrt{1 - m^{2}}}$$





$$m = \sqrt{1 - \left(\frac{\omega_C}{\omega_\infty}\right)^2} = \sqrt{1 - \left(\frac{f_C}{f_\infty}\right)^2}$$

## IMPEDANCIA CARACTERÍSTICA EN FILTROS TIPO "T" Y "Π" DE K<sub>CTE</sub>



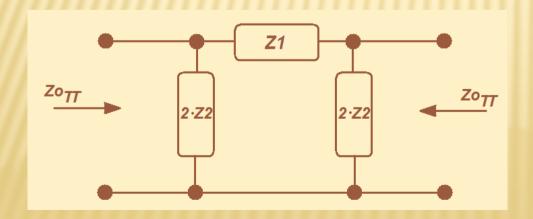
$$Zo_T = \sqrt{Z_1 \times Z_2} \times \sqrt{1 + \frac{Z_1}{4Z_2}}$$

$$Zo_T = Ro \times \sqrt{1 - |X_K|^2}$$

#### EN FILTROS DE K-CONSTANTE TENEMOS QUE:

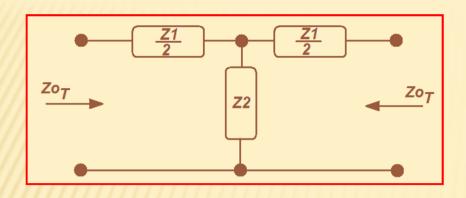
$$Ro = \sqrt{Z_1 \times Z_2}$$

$$-|X_K|^2 = \frac{Z_1}{4Z_2}$$

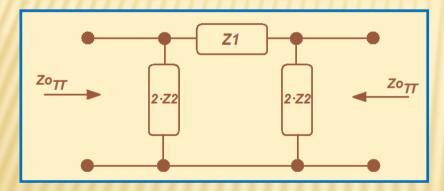


$$Zo_{\pi} = \frac{\sqrt{Z_1 \times Z_2}}{\sqrt{1 + \frac{Z_1}{4Z_2}}}$$

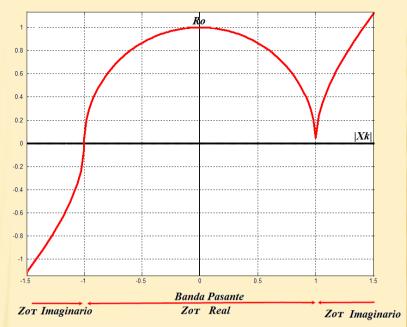
$$Zo_{\pi} = \frac{Ro}{\sqrt{1 - |X_K|^2}}$$

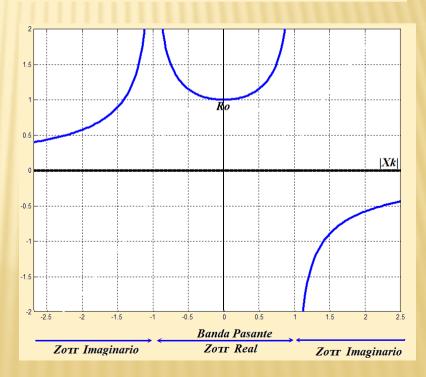


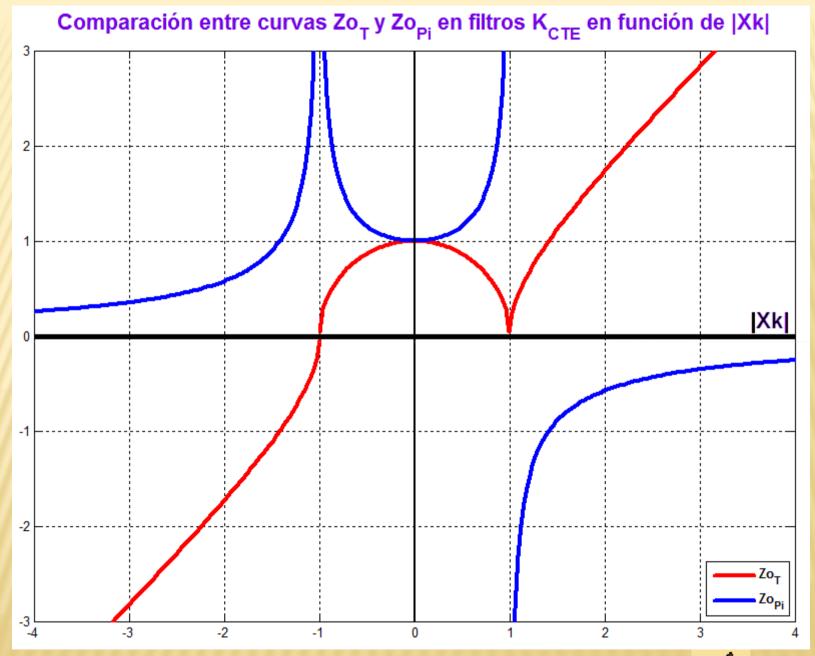
$$Zo_T = Ro \times \sqrt{1 - |X_K|^2}$$



$$Zo_{\pi} = \frac{Ro}{\sqrt{1-|X_K|^2}}$$







## IMPEDANCIA CARACTERÍSTICA DE UN FILTRO TIPO "Π" m-DERIVADO

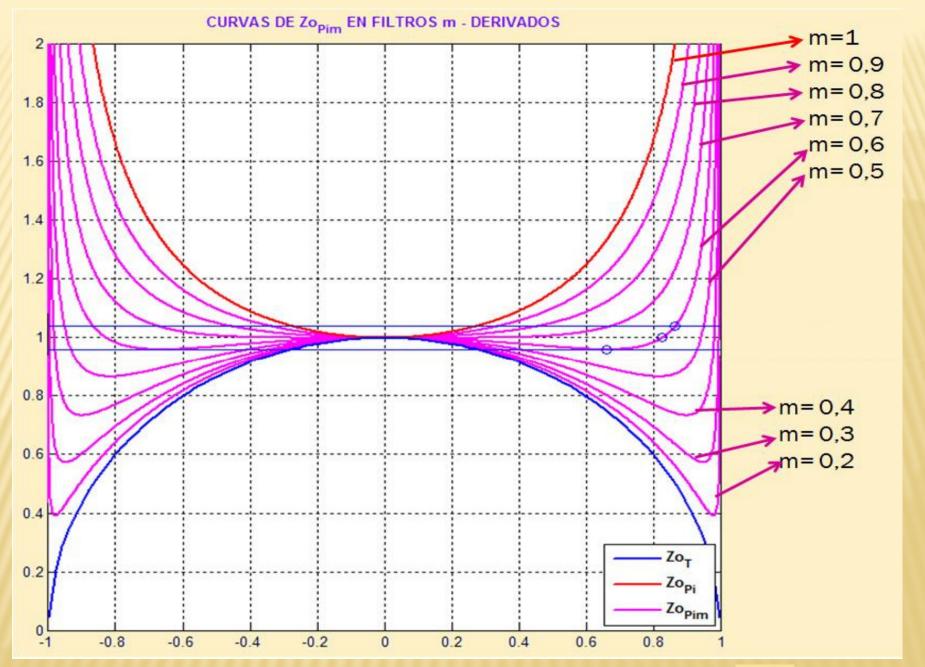
$$Z_{O\Pi m} = \frac{\sqrt{Z_{1Km} \times Z_{2Km}}}{\sqrt{1 + \frac{Z_{1Km}}{4Z_{2Km}}}}$$

$$Z_{1Km} = m \cdot Z_{1K}$$

$$Z_{2Km} = \frac{Z_{2K}}{m} + Z_{1K} \left( \frac{1 - m^2}{4m} \right)$$

#### REEMPLAZANDO Y OPERANDO NOS QUEDA:

$$Z_{O\Pi m} = \frac{Ro}{\sqrt{1 - \left|X_K\right|^2}} \bullet \left[1 - \left|X_K\right|^2 \bullet \left(1 - m^2\right)\right]$$





#### PARTIENDO DE LA EXPRESIÓN:

$$Z_{O\Pi m} = \frac{Ro}{\sqrt{1 - \left|X_K\right|^2}} \bullet \left[1 - \left|X_K\right|^2 \bullet \left(1 - m^2\right)\right]$$

#### DERIVANDO CON RESPECTO A |XK| E IGUALANDO A CERO

$$\frac{d(Z_{O\Pi m})}{d|X_K|} = 0$$



$$\left|X_{K}\right|_{\min} = \pm \sqrt{\frac{1-2m^{2}}{1-m^{2}}} \quad \begin{array}{c} VALOR\ DE\ |X_{K}| \\ PARA\ EL\ CUAL\ LA \\ Zorim\ ES\ MÍNIMA \end{array}$$

PARA |XK| = |XK| minimo

$$|Z_{O\Pi m}|_{\text{(minimo)}} = 2 \bullet Ro \bullet m \sqrt{1 - m^2}$$

PARA 
$$m = 0.6$$

$$Z_{O\Pi m}|_{\text{(minimo)}} = 0.96|_{m=0.6}$$

LA TOLERANCIA  $\mathcal{E}$  SE DEFINE COMO LA DIFERENCIA ENTRE LA IMPEDANCIA CARACTERÍSTICA DE UN FILTRO M DERIVADO EN CONFIGURACIÓN PI CUANDO  $|X_K| = 0$  Y LA MISMA IMPEDANCIA CUANDO  $|X_K| = |X_K|$  minimo DIVIDIDO POR EL VALOR DE Ro.

$$\varepsilon = \frac{Z_{O\Pi m}|_{\left(|X_K|=0\right)} - Z_{O\Pi m}|_{\left(|X_K|=|X_K|_{\min imo}\right)}}{Ro}$$

$$\varepsilon = \frac{Ro - 2 \cdot Ro \cdot m \sqrt{1 - m^2}}{Ro}$$

$$\varepsilon = 1 - 2 \bullet m \sqrt{1 - m^2}$$

PARA 
$$m = 0.6$$

$$|\varepsilon = 0.04|_{m=0.6}$$

## PARA OBTENER EL VALOR DE ZOMM (máximo) HACEMOS:

$$Z_{O\Pi m}|_{(\text{maximo})} = Ro + \varepsilon$$

$$Z_{O\Pi m}|_{\text{(maximo)}} = Ro + \frac{\left(Ro - 2 \bullet Ro \bullet m \bullet \sqrt{1 - m^2}\right)}{Ro}$$

$$Z_{O\Pi m}|_{\text{(maximo)}} = Ro \bullet \left[1 + \left(1 - 2 \bullet m \bullet \sqrt{1 - m^2}\right)\right]$$

PARA 
$$m = 0.6$$
  $Z_{O\Pi m}|_{(\text{maximo})} = 1.04|_{m=0.6}$ 

PARA OBTENER EL VALOR DE XK PARA CADA VALOR DE  $Z_{O\Pi m}$  CON m = 0.6, DESPEJAMOS DE LA SIGUIENTE EXPRESIÓN EL VALOR DE XK:

$$Z_{O\Pi m} = \frac{Ro}{\sqrt{1 - \left|X_K\right|^2}} \bullet \left[1 - \left|X_K\right|^2 \bullet \left(1 - m^2\right)\right]$$

$$Z_{O\Pi m} \bullet \sqrt{1 - |X_K|^2} = Ro \bullet \left[1 - |X_K|^2 \bullet (1 - m^2)\right] \quad pero \quad Ro = 1$$

$$Z_{O\Pi m}^2 \bullet \left(1 - |X_K|^2\right) = \left\{1 \bullet \left[1 - |X_K|^2 \bullet (1 - m^2)\right]^2\right\}$$

$$Z_{O\Pi m}^2 - Z_{O\Pi m}^2 |X_K|^2 = \left[1 - 2 \bullet |X_K|^2 \bullet (1 - m^2) + |X_K|^4 \bullet (1 - m^2)^2\right]$$

$$|X_K|^4 \bullet (1-m^2)^2 + |X_K|^2 \bullet [Z_{O\Pi m}^2 - 2 \bullet (1-m^2)] + (1-Z_{O\Pi m}^2) = 0$$

#### DE LA ÚLTIMA EXPRESIÓN:

$$|X_K|^4 \bullet (1-m^2)^2 + |X_K|^2 \bullet [Z_{O\Pi m}^2 - 2 \bullet (1-m^2)] + (1-Z_{O\Pi m}^2) = 0$$

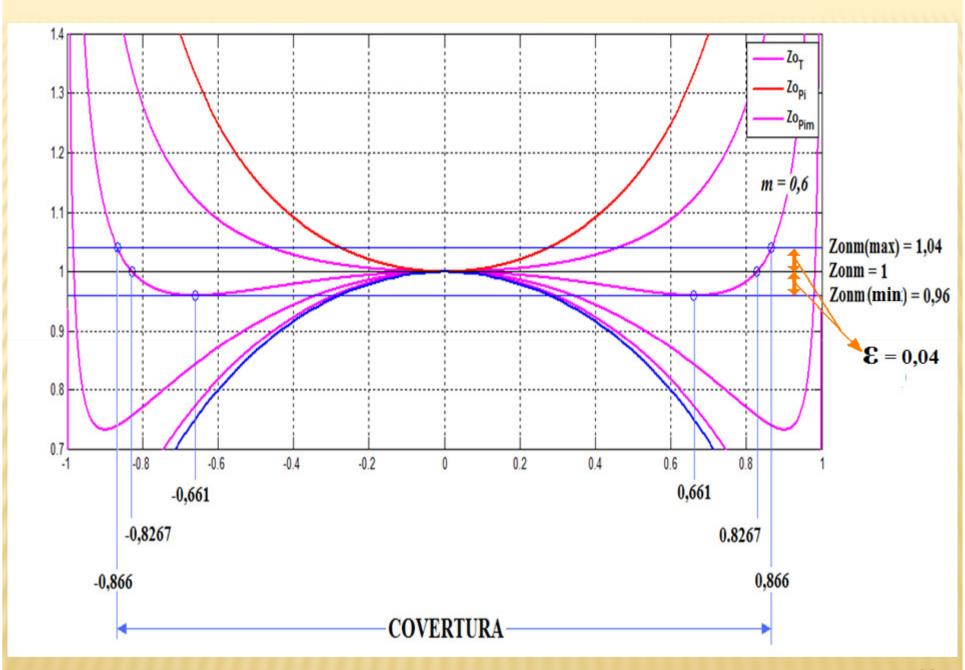
## OBTENEMOS LOS VALORES DE XK PARA CADA VALOR NOTABLE

DE ZOMM: 
$$|X_K|_{Z_{O\Pi m = 0.96}} = \pm 0.66143 \implies Z_{O\Pi m \text{(MINIMO)}}$$

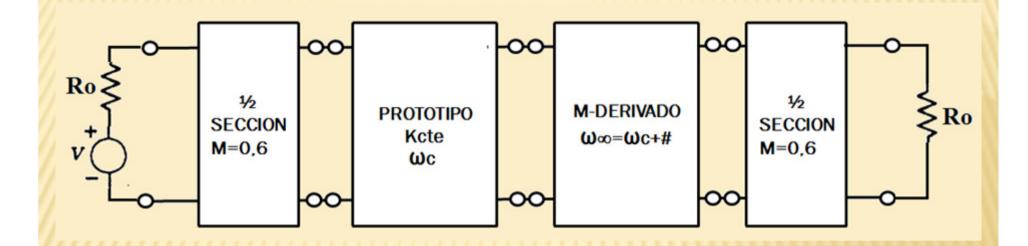
$$|X_K|_{Z_{O\Pi m=1}} = \pm 0.8267$$
  $\Rightarrow$   $Z_{O\Pi m(Ro)}$ 

$$|X_K|_{Z_{O\Pi m = 1,04}} = \pm 0,86602$$
  $\Rightarrow$   $Z_{O\Pi m \text{(MAXIMO)}}$ 

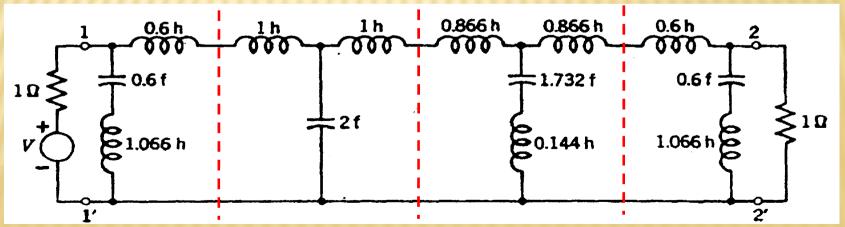
EL ÚLTIMO VALOR DEFINE LA COVERTURA = 86,6% PARA m = 0,6



#### FORMATO DE FILTRO COMPUESTO

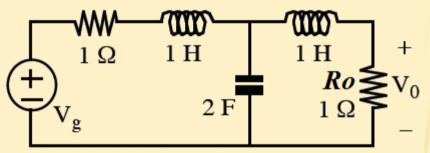


#### EJEMPLO FILTRO PASA BAJOS COMPUESTO NORMALIZADO Y $\omega \infty = 2$ [rps]



<u>EJEMPLO</u>: DISEÑE UN FILTRO PASA BAJOS COMPUESTO CON FRECUENCIA DE CORTE fc = 1[KHz],  $f\infty=1,05$  [KHz] y Zo=600 Ohms. EMPLEE SECCIONES DE TERMINACIÓN CON m=0,6.

#### CÁLCULO DE LA SECCIÓN DE KCTE:



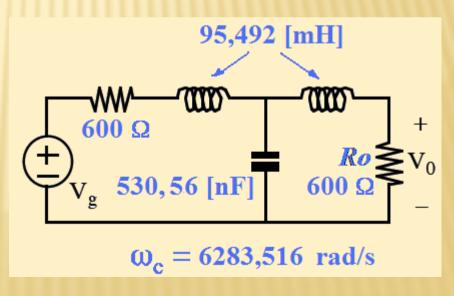
$$\omega_c = 1 \text{ rad/s}$$

$$\frac{L_1}{2} = \frac{\frac{L_{1N}}{2} \bullet b}{a} = \frac{1 \bullet 600}{6283,185} = 95,492 \text{ [mH]}$$

$$C_2 = \frac{C_{2N}}{a \bullet b} = \frac{2}{6283,185 \bullet 600} = 530,516 \text{ [nF]}$$

#### **COMPROBACION:**

$$\omega_C = \frac{2}{\sqrt{L_1 \cdot C_2}} = \frac{2}{\sqrt{0.095 \cdot 2 \cdot 530.51 \cdot 10^{-9}}} \cong 6283.185 \ [rps]$$



## CÁLCULO DE LA SECCIÓN m-DERIVADA PARA fc=1,05 [KHz]:

**RECORDANDO:** 

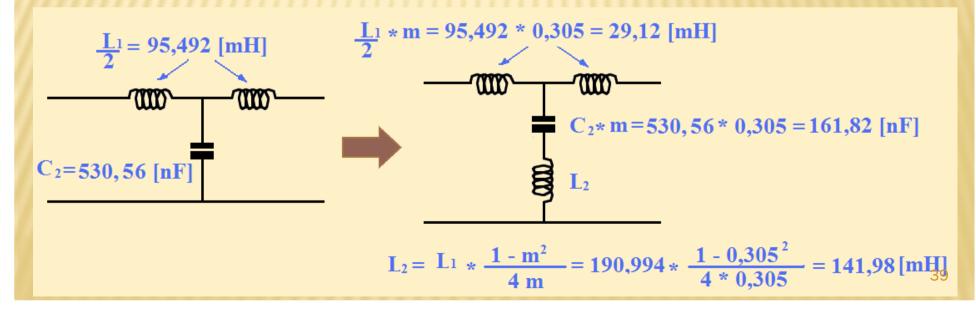
$$m = \sqrt{1 - \left(\frac{\omega_C}{\omega_\infty}\right)^2} = \sqrt{1 - \left(\frac{f_C}{f_\infty}\right)^2} = \sqrt{1 - \left(\frac{1000}{1050}\right)^2} = 0,305$$

Y LAS RELACIONES:

$$Z_{1Km} = m \bullet Z_{1K}$$

$$Z_{2Km} = \frac{Z_{2K}}{m} + Z_{1K} \left( \frac{1 - m^2}{4m} \right)$$

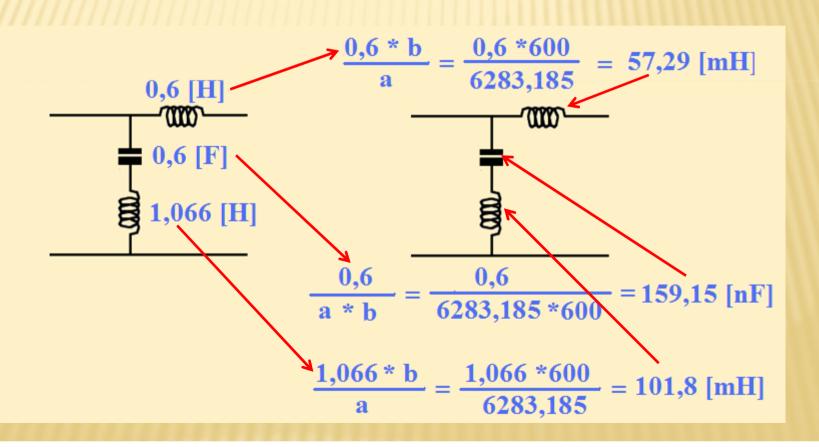
CÁLCULAMOS LA SECCIÓN m-DERIVADA PARA fc=1,05 [KHz] A PARTIR DE LA SECCIÓN DE KCTE



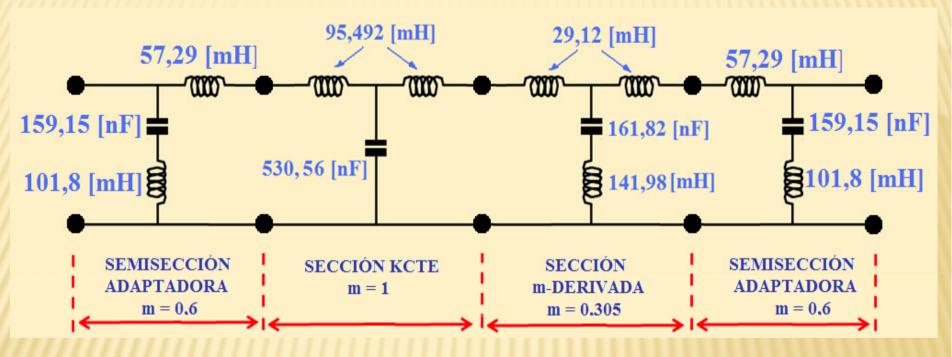
#### CÁLCULO DE LA SECCIÓN m-DERIVADA TIPO Π ADAPTADORA:

CÁLCULAMOS LA SEMI-SECCIÓN m-DERIVADA CON m=0,6 A PARTIR DE LA SEMI-SECCIÓN m-DERIVADA NORMALIZADA:

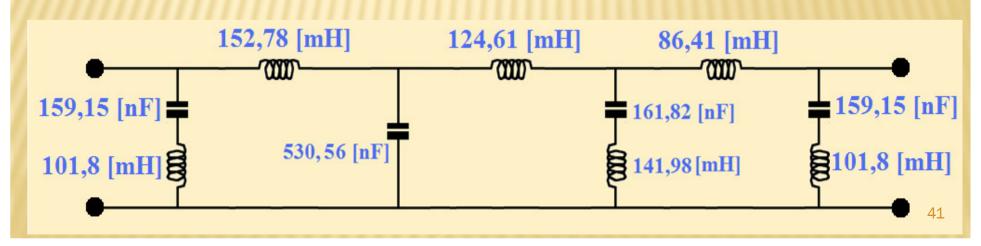
APLICAMOS CONCEPTO
DE NORMALIZACIÓN,
RECORDANDO:



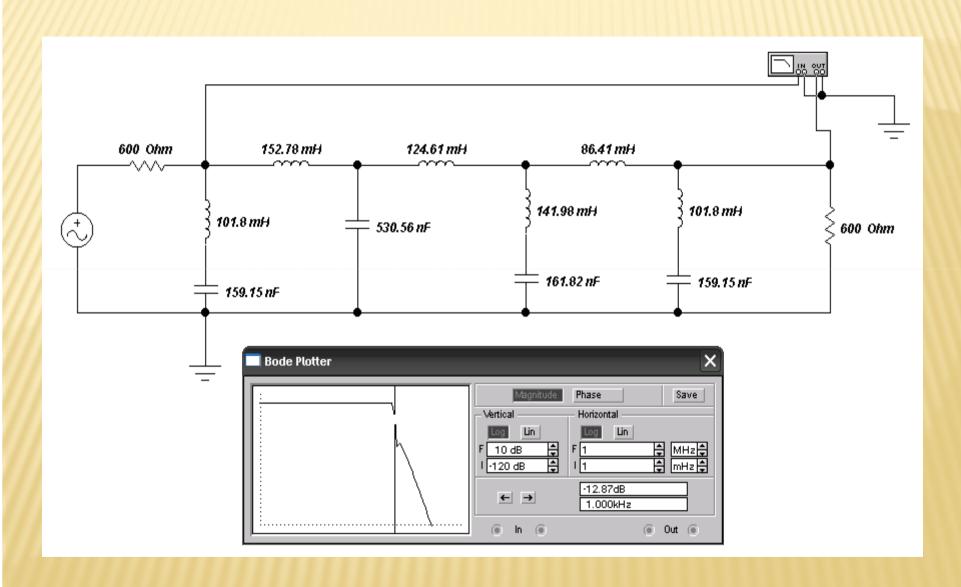
# FINALMENTE EL FILTRO COMPUESTO PROPUESTO ES TAL, COMO EL DE LA FIGURA:



#### POR ÚLTIMO:



#### CIRCUITO SIMULADO MEDIANTE EWB5 :



## CURVAS OBTENIDAS MEDIANTE EWB5 :

