65) Trazar diagrama polar de $F(P)$ y determinar estabilidad, aplicando criterio de Nyquist.

$$F(P) = \frac{1}{P^3 + 2P^2 + P + 3}$$

1) Origen diagrama.

$$F(P)\Big|_{P \to 0} = \frac{1}{3} = 0{,}3\widehat{33}$$

2) Fin diagrama.

$$F(P)\Big|_{P \to \infty} = |0| \underline{\lfloor -270^\circ}$$

3) $P \to jw$; $F(P) \to F(jw)$.

$$F(jw) = \frac{1}{(3-2w^2)+j(w-w^3)} \cdot \frac{(3-2w^2)+j(w^3-w)}{(3-2w^2)+j(w^3-w)}$$

4) $F(jw) = Re + j\,Im$.

$$F(jw) = \frac{3-2w^2}{(3-2w^2)^2+(w^3-w)^2} + j\,\frac{(w^3-w)}{(3-2w^2)^2+(w^3-w)^2}$$

5) $Re = \emptyset$.

$$3 - 2w^2 = 0$$

$$w = \sqrt{\frac{3}{2}} = \pm 1{,}2247$$

6) Corte eje Imag.

$$j\,\frac{(w^3-w)}{(3-2w^2)^2+(w^3-w)^2}\Bigg|_{w=+1{,}2247} = +j1{,}632$$

7) $Im = 0$.

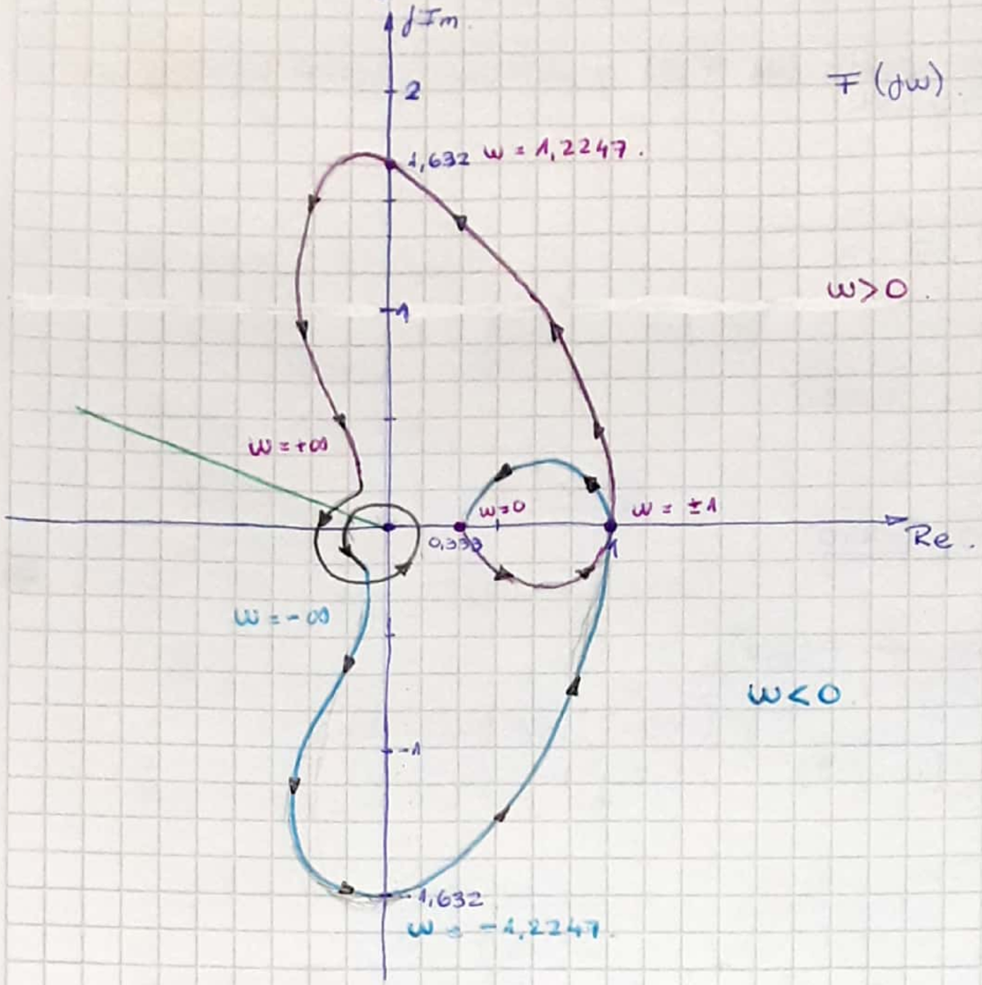
$$w^3 - w = 0$$

$$w = \pm 1$$

8) Corte eje Re.

$$\frac{3-2w^2}{(3-2w^2)^2+(w^3-w)^2}\Bigg|_{w=+1} = 1$$

9) Trazar Diagrama Polar.

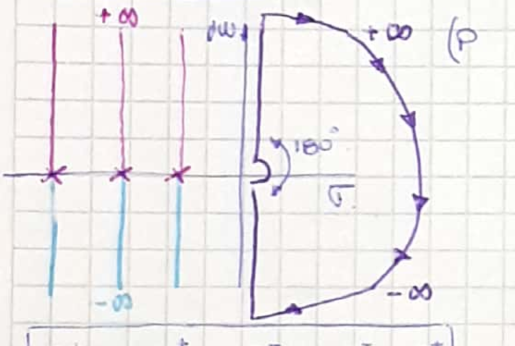


10) Cerrar diagrama p/ $P \to 0$ (No se aplica porque no hay polos en 0).

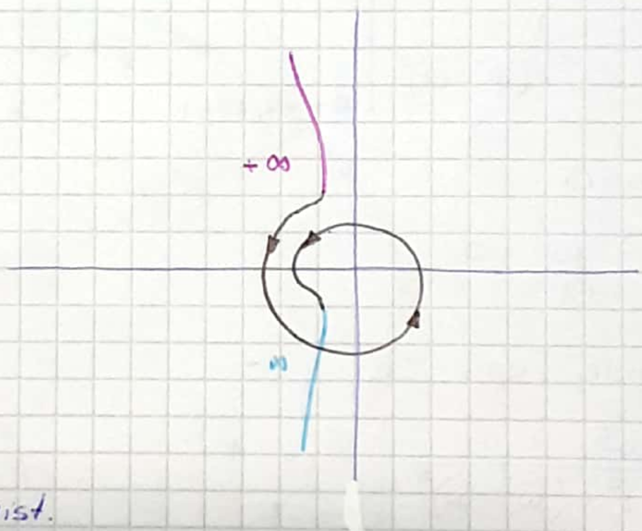
11) Cerrar p/ $P \to \infty$

$$F(P)\Big|_{P\to\infty} = \frac{1}{P^3}\Big|_{P\to\infty} = |0| \underline{\lfloor 3\theta}$$

Graficamente:



Zoom Origen



$\boxed{0^+ \to \infty^+ \to \infty^- \to 0^- \to 0^+}$

$\varphi = -3 \times 180° = -540°$

(Vuelta y media).

12) Aplicar criterio de Nyquist.

$\boxed{N = -2} \longrightarrow$ **Sistema Inestable**

66) Trazar diagrama polar de $F(P)$, determinar estabilidad aplicando Nyquist.

$$F(P) = \frac{10}{P\,(P+1)}$$

1) Origen diagrama.

$$F(P)\Big|_{P\to 0} = \frac{10}{P} = \infty\,\underline{|-90^\circ}$$

2) Fin diagrama.

$$F(P)\Big|_{P\to \infty} = 0\,\underline{|-180^\circ}$$

3) $P \to j\omega$; $F(P) \to F(j\omega)$

$$F(j\omega) = \frac{10}{j\omega\,(1+j\omega)} = \frac{10}{-\omega^2 + j\omega} \cdot \frac{-\omega^2 - j\omega}{-\omega^2 - j\omega}$$

4) $F(j\omega) = Re + j\,Im$.

$$F(j\omega) = \frac{-10\,\omega^2}{\omega^4 + \omega^2} + j\,\frac{(-10\,\omega)}{\omega^4 + \omega^2} = \frac{-10}{\omega^2 + 1} + j\,\frac{-10}{\omega^3 + \omega}$$

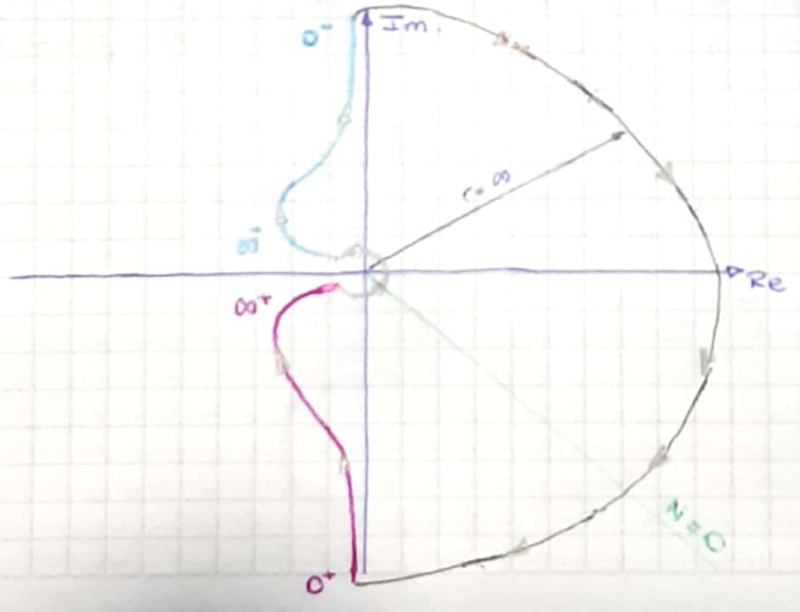
5) $Re = 0 \to$ No hay ω

6) Corte eje $Im \to$ No hay.

7) $Im = 0 \to$ No hay ω.

8) Corte eje $Re \to$ No hay.

9) Trazar Diagrama Polar.



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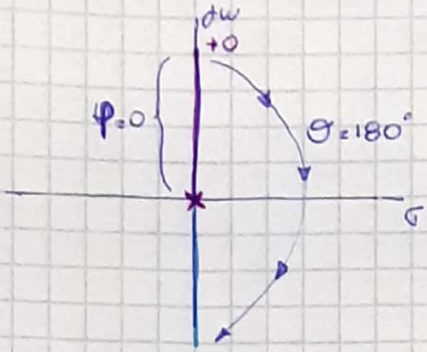
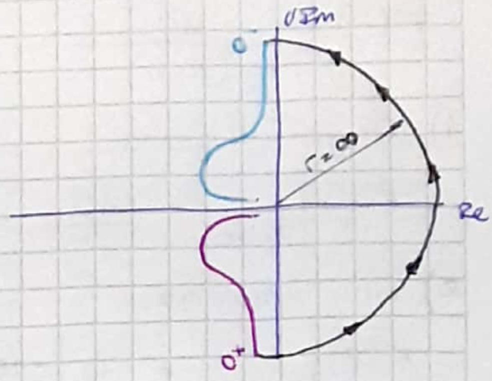
10) Cerrar Diagrama p/ $P \to 0$.

$$F(p)\Big|_{p\to 0} = \frac{10}{p} = \frac{10}{\gamma} e^{-j\theta} = |\infty| \cdot e^{-j\theta} = |\infty| \underline{|-\theta}$$

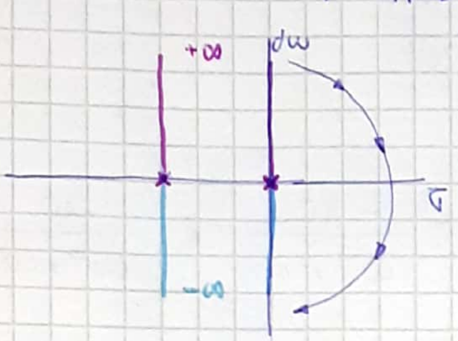


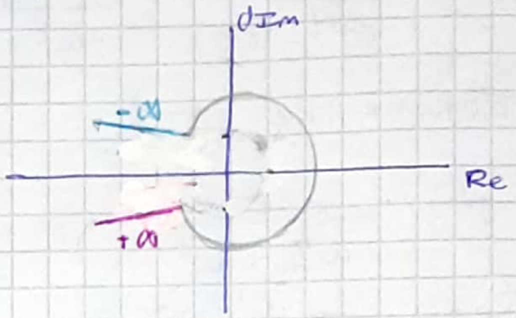
$\varphi = -180°$



11) Cerrar p/ $P \to \infty$

$$F(p)\Big|_{p\to\infty} = \frac{10}{p^2}\Big|_{p\to\infty} = |0| \underline{|-2\theta}$$ Zoom origen =





$\varphi = -360°$ (1 vuelta).

12) Aplicar criterio de Nyquist.

$N = 0$.

No se determina por Nyquist la estabilidad.
Podemos decir que el sistema es estable, ya que al conocer los polos, verificamos que ninguno esté en el semiplano $\sigma+$

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67) Trazar diagrama polar, determinar estabilidad mediante Nyquist

$$F(p) = \frac{10}{(p+1)^2}$$

1) Origen diagrama

$$F(p)\Big|_{p\to 0} = 10.$$

2) Fin diagrama.

$$F(p)\Big|_{p\to\infty} = |0| \underline{\lfloor -180^\circ}$$

3) $p \to j\omega \quad \therefore \quad F(p) \to F(j\omega)$

$$F(j\omega) = \frac{10}{(1+j\omega)^2} = \frac{10}{(1-\omega^2)+j2\omega} \cdot \frac{(1-\omega^2)-j2\omega}{(1-\omega^2)-j2\omega}$$

4)

$$F(j\omega) = \frac{10(1-\omega^2)}{(1-\omega^2)^2+4\omega^2} + j\frac{(-20\omega)}{(1-\omega^2)^2+4\omega^2}$$

5) $Re = 0$

$$1-\omega^2 = 0 \longrightarrow \omega = \pm 1.$$

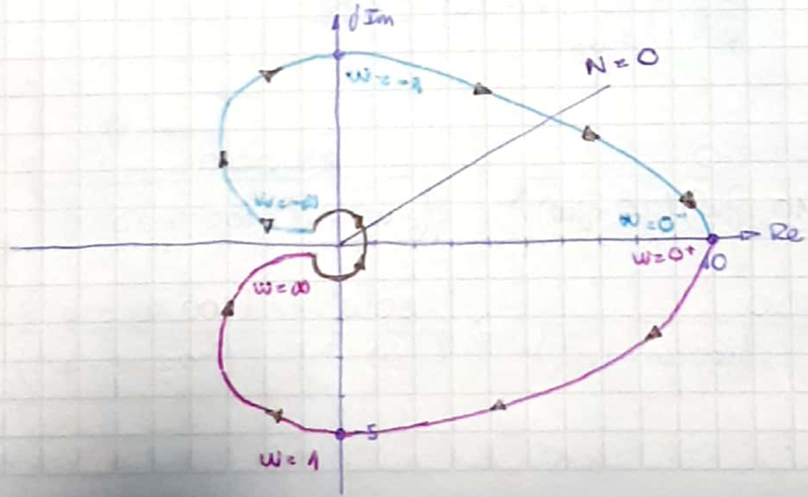
6) $$j\frac{(-20\omega)}{(1-\omega^2)^2+4\omega^2}\Bigg|_{\omega=1} = -j5$$

7) $Im = 0$

$$-20\omega = 0 \longrightarrow \omega = 0.$$

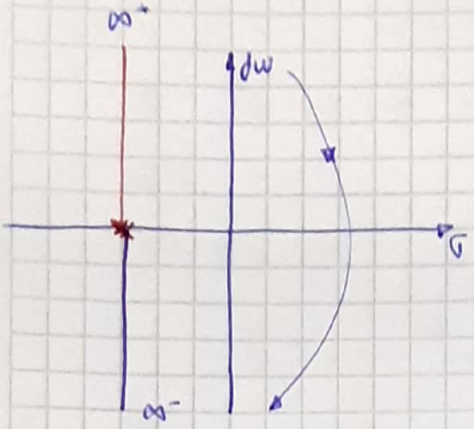
8) –

9)

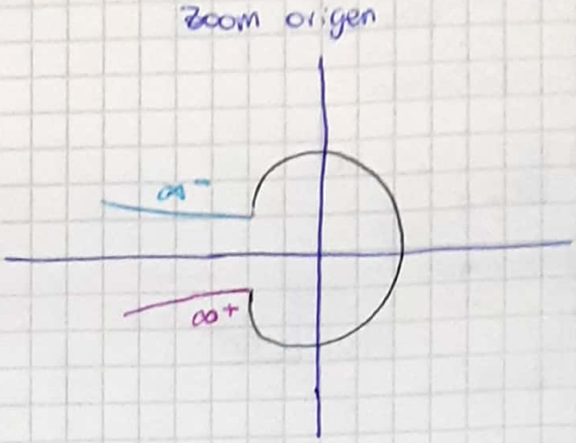


10) -

11) $F(P)\Big|_{P\to\infty} = \frac{1}{P^2}\Big|_{P\to\infty} = |0| \underline{|2\varnothing}$



$\varphi = -2 \times 180^\circ = -360^\circ$.



12) Aplicando crit. Nyquist.

$0^+ \to \infty^+ \to \infty^- \to 0^- \to 0^+$

$\boxed{N = 0}$ Sistema sin determinar mediante Nyquist.
Sistema estable por no tener polos con $\sigma > 0$.

68) Diagrama Polar y aplicar Nyquist.

$$F(P) = \frac{50}{P \cdot (0{,}1P+1)(0{,}2P+1)} = \frac{50}{P \cdot 0{,}1 \cdot 0{,}2\,(P+10)(P+\quad)}$$

$$F(P) = \frac{2500}{P(P+10)(P+5)} = \frac{2500}{P(P^2+5P+10P+50)} = \frac{2500}{P(P^2+15P+50)}$$

1) $F(P)\Big|_{P\to 0} = |\infty| \underline{|-90^\circ}$

2) $F(P)\Big|_{P\to\infty} = \frac{1}{P^3}\Big|_{P\to\infty} = |0| \underline{|-270^\circ}$

3) $$F(j\omega) = \frac{2500}{j\omega((j\omega)^2+15j\omega+50)} = \frac{2500}{j\omega(-\omega^2+50+j15\omega)}$$

$$F(j\omega) = \frac{2500}{(-15\omega^2)+j(50\omega-\omega^3)} \cdot \frac{(-15\omega^2)-j(50\omega-\omega^3)}{(-15\omega^2)-j(50\omega-\omega^3)}$$

4) $F(j\omega) = \dfrac{-375.00 \cdot \omega^2}{(15\omega^2)^2 + (50\omega - \omega^3)^2} + j \dfrac{(\omega^3 - 50\omega) \cdot 25.00}{(15\omega^2)^2 + (50\omega - \omega^3)^2}$

5) $Re = \emptyset \rightarrow -$

6) $-$

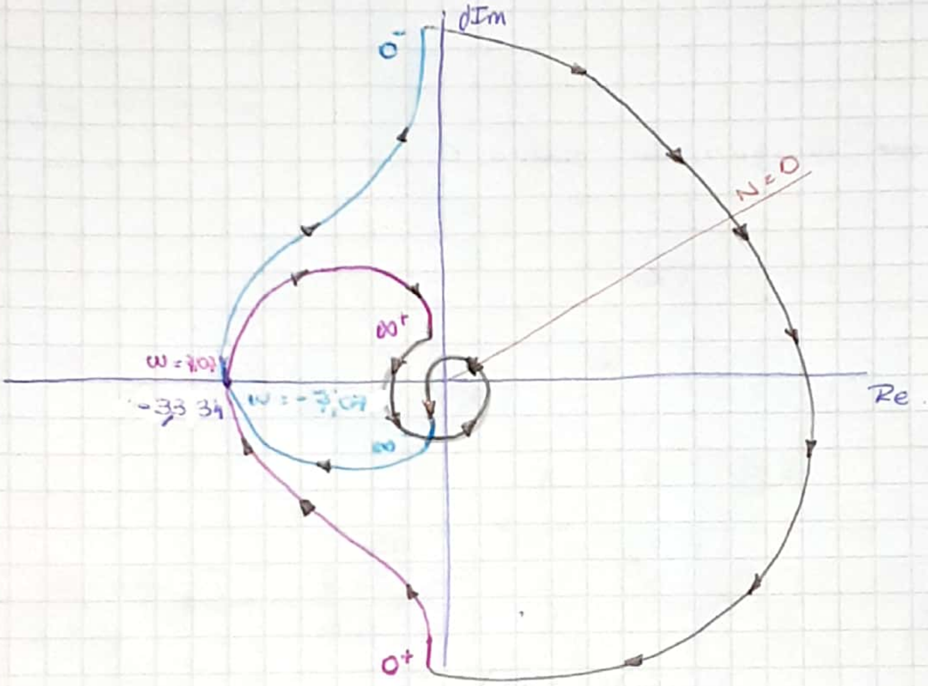
7) $Im = 0$

$\omega^3 - 50\omega = 0$

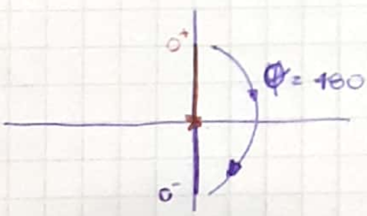
$\omega^2 - 50 = 0 \longrightarrow \omega = \sqrt{50} = \pm 7{,}07$

8) $\left.\dfrac{-375.00 \cdot \omega^2}{(15\omega^2)^2 + (50\omega - \omega^3)^2}\right|_{\omega = 7{,}07} = -33{,}34$

9)



10)

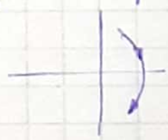


$\varphi = -1 \times 180° = -180°$

11) $\theta = 180°$

$\varphi = -3 \times 180° = -540°$

(vuelta y media)



13) N = 0. No se determina por Nyquist.

Sistema estable por ubic. de polos.

69) Trazar Diag. Polar y determinar estabilidad mediante Nyquist.

$$F(p) = \frac{p+0{,}5}{p^2(p+10)}$$

1) $F(p)\big|_{p\to 0} = |\infty| \,\underline{|-180^\circ}$

2) $F(p)\big|_{p\to\infty} = |0| \,\underline{|-180^\circ}$

3) $p \to j\omega \; ; \; F(p) \to F(j\omega)$

$$F(j\omega) = \frac{0{,}5+j\omega}{-\omega^2(10+j\omega)} = \frac{(0{,}5+j\omega)}{(-10\omega^2)-j\omega^3} \cdot \frac{(-10\omega^2)+j\omega^3}{(-10\omega^2)+j\omega^3}$$

4)
$$F(j\omega) = \frac{(-5\omega^2-\omega^4)}{100\omega^4+\omega^6} + j\,\frac{(0{,}5\omega^3-10\omega^3)}{100\omega^4+\omega^6}$$

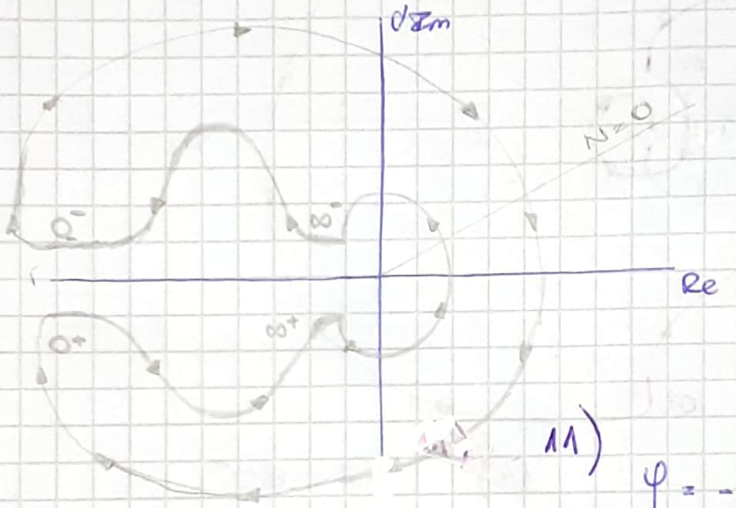
5) $-5-\omega^2 = 0 \rightarrow$ Ningún ω. real.

6) –

7) $Im = 0 \longrightarrow$ Ningún ω salvo 0.

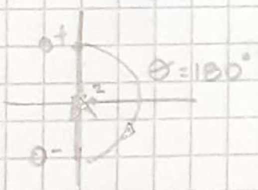
8) –

9)



10)

$\varphi = -360$



11) $\varphi = -360^\circ$.

12) $N = 0$.

70) Trace diagrama polar de $F(P)$ Determinar estabilidad. (Nyquist)

$$F(P) = \frac{10}{P^2+1}$$

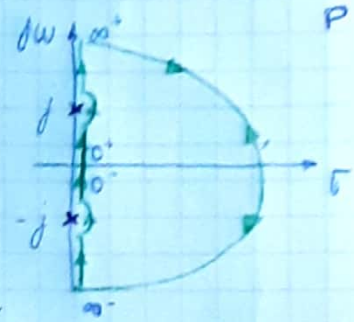
1) $F(P)\Big|_{P=0} \quad \frac{10}{1} = 10$

2) $F(P)\Big|_{P\to\infty} \quad \frac{10}{P^2}\Big|_{P\to\infty} = |0| \underline{|-180^\circ}$

3) $P \to j\omega$

$$F(j\omega) = \frac{10}{(j\omega)^2+1} = \frac{10}{1-\omega^2}$$

4) $F(j\omega) = \frac{10}{1-\omega^2} + j\,0$



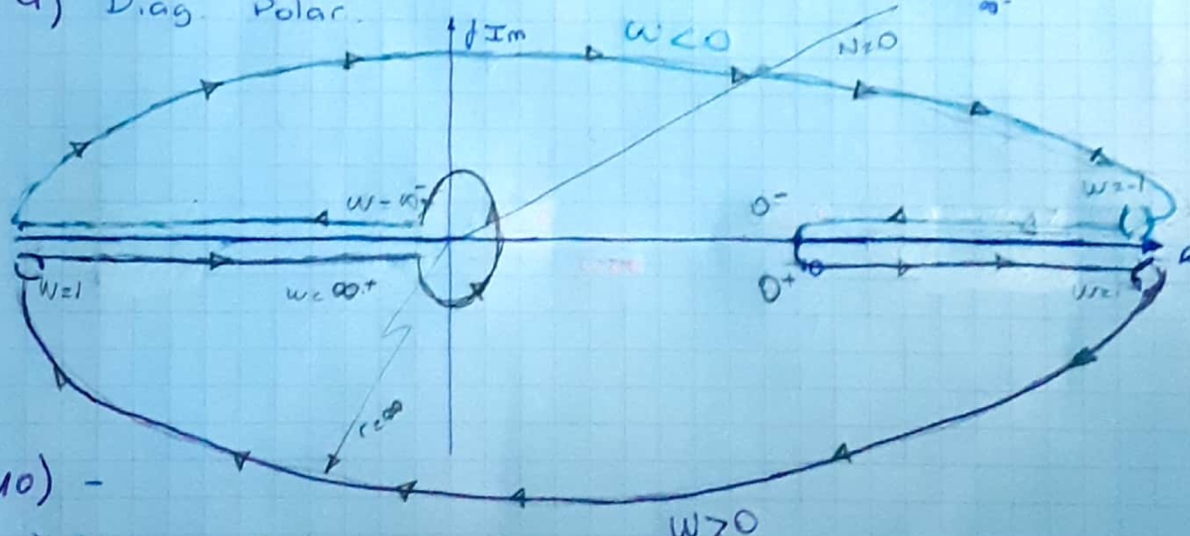
5) $Re = 0 \to$ No

6) -

7) -

8) -

9) Diag. Polar.



10) -

11) $\varphi = -360^\circ$

12) $N = 0 \to$ No se especifica por Nyquist.

Sistema estable porque σ de polos no es mayor a 0.

71) Trazar diag. polar y det. estab. mediante Nyquist.

$$F(P) = \frac{P+2}{(P+1)(P^2+6,25)} = \frac{P+2}{P^3+P^2+6,25P+6,25}$$

1) $F(P)\Big|_{P\to 0} = \frac{2}{6,25} = 0,32$

2) $F(P)\Big|_{P\to\infty} = \frac{k}{P^2}\Big|_{P\to M} = |0| \underline{\big|-180^\circ}$

3) $P \to j\omega$

$$F(j\omega) = \frac{2+j\omega}{(1+j\omega)(6,25-\omega^2)} = \frac{2+j\omega}{6,25-\omega^2+j(6,25\omega-\omega^3)}$$

$$F(j\omega) = \frac{(2+j\omega)}{(6,25-\omega^2)+j(6,25\omega-\omega^3)} \cdot \frac{(6,25-\omega^2)-j(6,25\omega-\omega^3)}{(6,25-\omega^2)-j(6,25\omega-\omega^3)}$$

4) $$F(j\omega) = \frac{12,5-2\omega^2+6,25\omega^2-\omega^4}{(6,25-\omega^2)^2+(6,25\omega-\omega^3)^2} + j\frac{6,25\omega-\omega^3-12,5\omega+2\omega^3}{(6,25-\omega^2)^2+(6,25\omega-\omega^3)^2}$$

$$F(j\omega) = \frac{12,5+4,25\omega^2-\omega^4}{(6,25-\omega^2)^2+\omega^2(6,25-\omega^2)^2} + j\frac{\omega^3-6,25\omega}{(6,25-\omega^2)^2+\omega^2(6,25-\omega^2)^2}$$

$$F(j\omega) = \frac{-(6,25-\omega^2)(\omega^2+2)}{(1+\omega^2)(6,25-\omega^2)^2} + j\frac{-\omega(6,25-\omega^2)}{(1+\omega^2)(-6,25-\omega^2)^2}$$

$$F(j\omega) = \frac{\omega^2+2}{(1+\omega^2)(-\omega^2+6,25)} + j\frac{(-\omega)}{(1+\omega^2)(-\omega^2+6,25)}$$

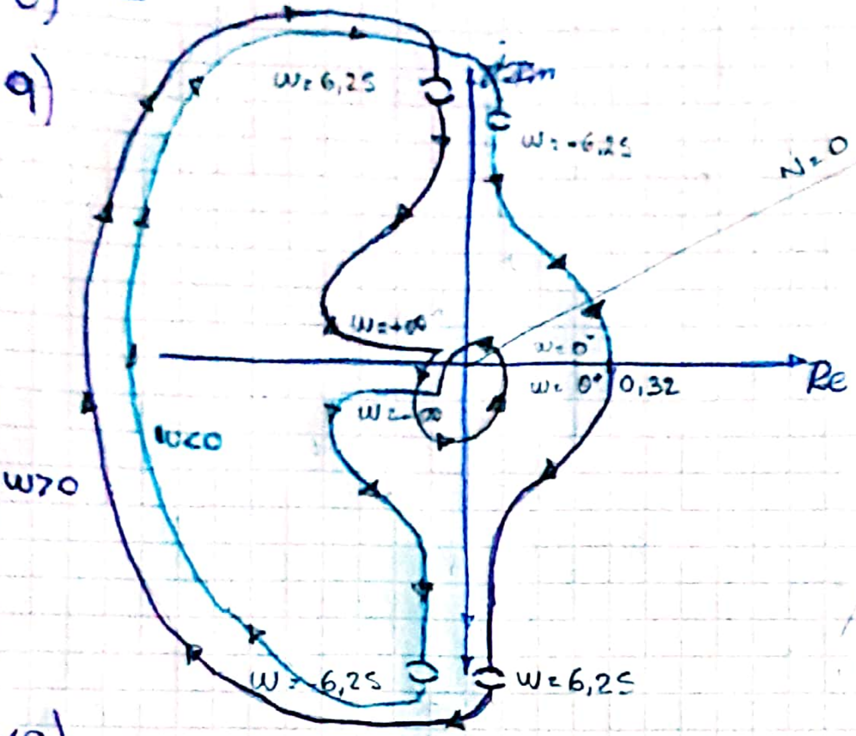
5) -

6) -

7) -

8) -

9)



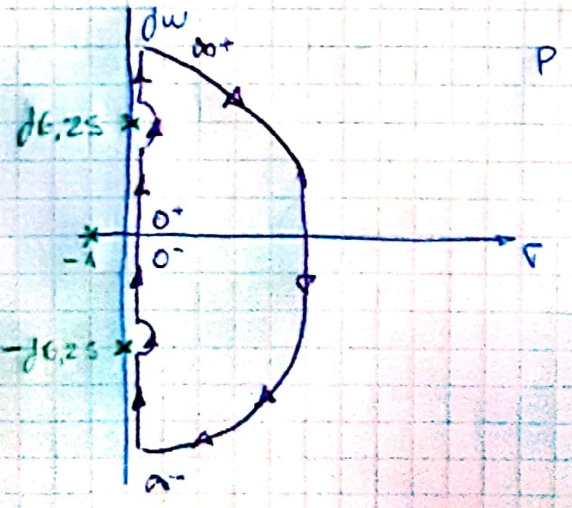
10) -

11) $\varphi = -360°$.

12) $N=0$

No se determina por Nyquist.

(Sistema estable al conocer polos).



72) Trazar diag. polar y det. estabilidad empleando crit. Nyquist.

$$F(P) = \frac{20\,(P+4)}{P\,(P^2+5P+6)}$$

1) $F(P)\Big|_{P\to 0} = \dfrac{20\times 4}{P}\Big|_{P\to 0} = |\infty| \angle -90°$

2) $F(P)\Big|_{P\to\infty} = \dfrac{1}{P^2}\Big|_{P\to\infty} = |0| \angle -180°$

3) $F(j\omega) = \dfrac{20\,(4+j\omega)}{j\omega\,(6-\omega^2+j5\omega)} = \dfrac{(80+j20\omega)}{(-5\omega^2)+j\,(6\omega-\omega^3)} \cdot \dfrac{(-5\omega^2)-j\,(6\omega-\omega^3)}{(-5\omega^2)-j\,(6\omega-\omega^3)}$

$$F(j\omega) = \frac{-400\,\omega^2+120\,\omega^2-20\,\omega^4}{25\omega^4+(6\omega-\omega^3)^2} + j\,\frac{-100\,\omega^3-480\,\omega+80\,\omega^3}{25\omega^4+(6\omega-\omega^3)^2}$$

4) $F(j\omega) = \dfrac{-(280+20\,\omega^2)\cdot\omega^2}{25\omega^4+(6\omega-\omega^3)^2} + j\,\dfrac{-\omega\cdot(20\,\omega^2+480)}{25\omega^4+(6\omega-\omega^3)^2}$

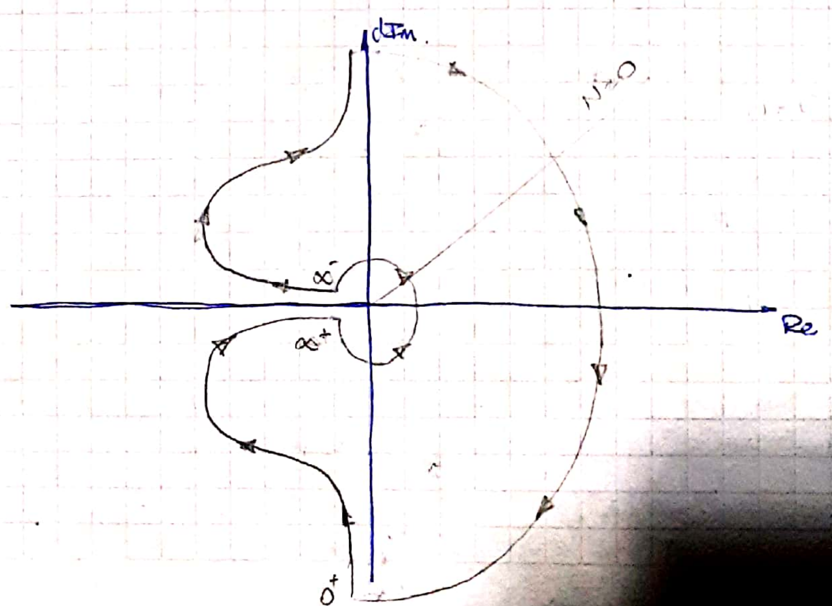
5) $280+20\omega^2=0 \longrightarrow$ Ningún ω

6) -

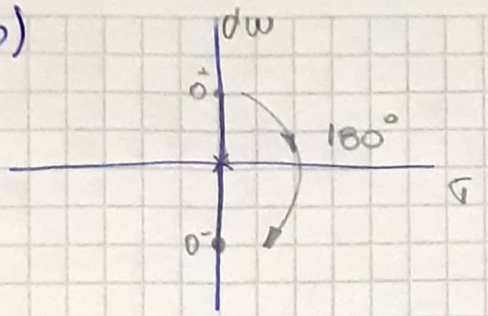
7) $20\omega^2+480=0 \longrightarrow$ Ningún ω.

8) -

9)

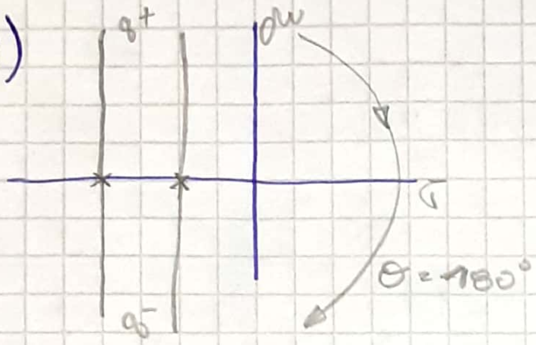


10)



$\varphi = -180°$

11)



$\varphi = -360°$

12) N = 0. (No se determina por Nyquist.)
Sist. estable. (Polos con $\sigma \leq 0$).

73) $G(p)\,H(p) = \dfrac{10 \cdot (p+5)}{p(p^2+3p+5)}$, Trazar diag. polar y determinar estabilidad mediante Nyquist.

1) $G(p)H(p)\Big|_{p\to 0} = |\infty| \underline{\lfloor -90^\circ}$

2) $G(p)H(p)\Big|_{p\to\infty} = |0| \underline{\lfloor -180^\circ}$

3) $p \to j\omega$

$$G(j\omega)\cdot H(j\omega) = \frac{10\cdot(5+j\omega)}{j\omega(-\omega^2+3j\omega+5)}$$

$$G(j\omega)\cdot H(j\omega) = \frac{50 + j10\omega}{-3\omega^2+j(5\omega-\omega^3)} \cdot \frac{(-3\omega^2 - j(5\omega-\omega^3))}{(-3\omega^2 - j(5\omega-\omega^3))}$$

4)
$$G(j\omega)\cdot H(j\omega) = \frac{-150\omega^2+50\omega^2-10\omega^4}{9\omega^4+(5\omega-\omega^3)^2} + j\,\frac{-30\omega^3-250\omega+50\omega^3}{9\omega^4+(5\omega-\omega^3)^2}$$

$$G(j\omega)\,H(j\omega) = \frac{-100\omega^2-10\omega^4}{25\omega^2-\omega^4+\omega^6} + j\,\frac{20\omega^3-250\omega}{25\omega^2-\omega^4+\omega^6}$$

5) $Re = 0 \Rightarrow$ Ningún ω real.

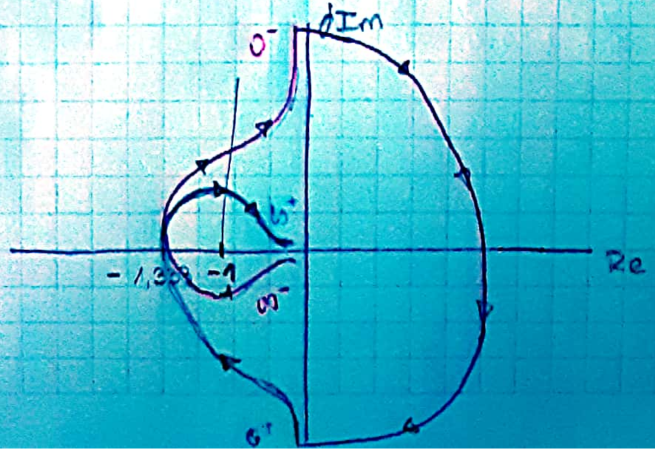
6) –

7) $Im = 0 \Rightarrow 20\omega - 250 = 0 \Rightarrow \omega = \pm 3{,}5355$

8) $Re\Big|_{\omega = +3{,}5355} = -1{,}333.$

9)

10) $\varphi = -180^\circ$



11) No aplica

12) N = 2

Sistema INESTABLE.

74) Trazar diagrama polar y aplicar criterio de Nyquist

$$G(p)\,H(p) = \frac{25}{p^3+3p^2+4p+1}$$

1) $G(p)\,H(p)\Big|_{p\to 0} = 25$

2) $G(p)\,H(p)\Big|_{p\to\infty} = \frac{k}{p^3} = |0| \angle -270^\circ$

3) $p \to j\omega$

$$G(j\omega)\,H(j\omega) = \frac{25}{-j\omega^3 - 3\omega^2 + 4j\omega + 1} = \frac{25}{(1-3\omega^2)+j(4\omega-\omega^3)} \cdot \frac{(1-3\omega^2)-j(4\omega-\omega^3)}{(1-3\omega^2)-j(4\omega-\omega^3)}$$

4) $$G(j\omega)\,H(j\omega) = \frac{25(1-3\omega^2)}{(1-3\omega^2)^2+(4\omega-\omega^3)^2} + j\,\frac{-25(4\omega-\omega^3)}{(1-3\omega^2)^2+(4\omega-\omega^3)^2}$$

5) $Re = 0$

$1 - 3\omega^2 = 0 \Rightarrow \omega = \pm\sqrt{\frac{1}{3}} = \pm 0,577$

6) $$Im\Big|_{\omega_{Re=0}} = \frac{-25(4\omega-\omega^3)}{(1-3\omega^2)^2+(4\omega-\omega^3)^2}\Bigg|_{\omega=0,577} = \frac{-52,877}{4,477} = -11,81$$

7) $Im = 0$

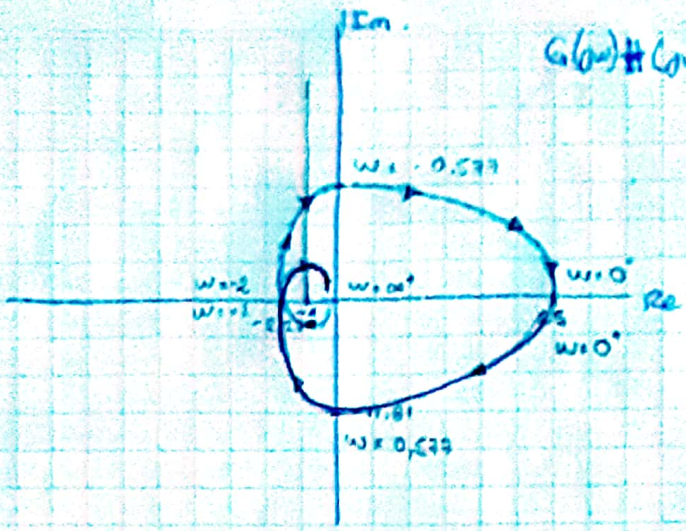
$4\omega - \omega^3 = 0$

$4 - \omega^2 = 0$

$\omega = \pm 2$

8) $$Re\Big|_{\omega_{Im=0}} = \frac{25(1-3\omega^2)}{(1-3\omega^2)^2+(4\omega-\omega^3)^2} = -2,27$$

9)



10) -

11) -

12) $N = 2$
Sistema Inestable

75) Criterio de Routh-Hourwitz para

$$G(P)\cdot H(P) = \frac{25}{P^3+3P^2+4P+1}, \quad (\text{Ejercicio } 74)$$

$$G(P)\cdot H(P)+1 = \frac{25}{P^3+3P^2+4P+1} + 1 = \frac{P^3+3P^2+4P+26}{P^3+3P^2+4P+1}$$

• R-H Numerador

P^3	1	4
P^2	3	26
P^1	-4,67	
P^0	26	

• 2 cambios de signo = 2 raíces a parte real positiva. → Sistema Inestable.

$Z = 2$

• R-H. Denominador

P^3	1	4
P^2	3	1
P^1	3,67	
P^0	1	

• Sin cambios de signo.

$P = 0$

Similitud c/ Nyquist.

$$N = Z - P = 2 - 0$$

$$\boxed{N = 2}$$

76) Trazar Diag. polar y determ. estabilidad por crit. de Nyquist.

$$G(p)\,H(p) = \frac{5}{p^4 + 4p^3 + 10p^2 + p}$$

1) $G(p)H(p)\Big|_{p\to 0} = \frac{k}{p}\Big|_{p\to 0} = |\infty| \underline{|-90^\circ}$

2) $G(p)\,H(p)\Big|_{p\to\infty} = \frac{k}{p^4}\Big|_{p\to\infty} = |0| \underline{|-360^\circ}$

3) $p \to j\omega$

$$G(j\omega)H(j\omega) = \frac{5}{\omega^4 - j\,4\omega^3 - 10\omega^2 + j\omega} = \frac{5}{(\omega^4 - 10\omega^2) + j(\omega - 4\omega^3)}$$

4) $$G(j\omega)H(j\omega) = \frac{5\cdot(\omega^4 - 10\omega^2)}{(\omega^4 - 10\omega^2)^2 + (\omega - 4\omega^3)^2} + j\,\frac{-5\,(\omega - 4\omega^3)}{(\omega^4 - 10\omega^2)^2 + (\omega - 4\omega^3)^2}$$

5) $Re = 0$

$$\omega^2 - 10 = 0 \Rightarrow \omega = \pm 3{,}16$$

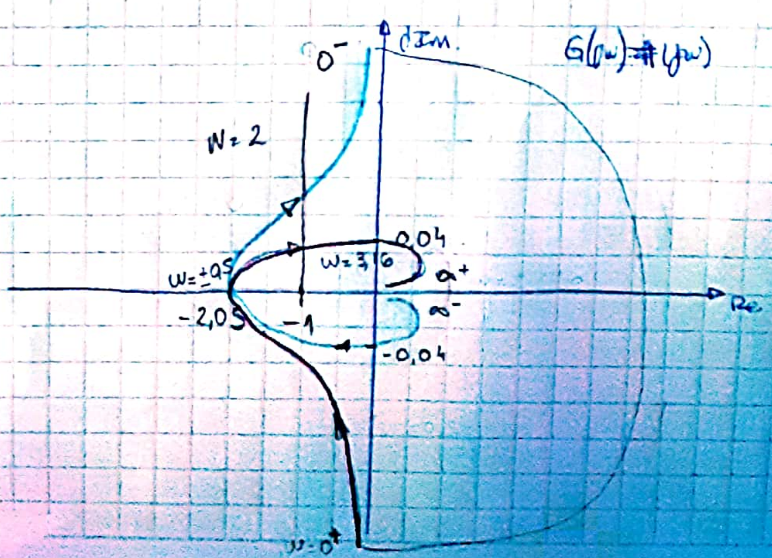
6) $$Im\Big|_{p/\omega = 3{,}16} = \frac{-5\,(\omega - 4\omega^3)}{(\omega^4 - 10\omega^2)^2 + (\omega - 4\omega^3)^2} = 0{,}04$$

7) $Im = 0$

$$1 - 4\omega^2 = 0 \Rightarrow \omega = \pm 0{,}5$$

8) $$Re\Big|_{\omega = 0{,}5} = \frac{5\,(\omega^4 - 10\omega^2)}{(\omega^4 - 10\omega^2)^2 + (\omega - 4\omega^3)^2}\Bigg|_{\omega = 0{,}5} = -2{,}05$$

9)



10) $\varphi = -180^\circ$

11) -

12) $N = 2$

Sistema Inestable.

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77) Criterio de R-H a

$$G(p)\,H(p) = \frac{5}{p(p^3+4p^2+10p+1)} = \frac{5}{p^4+4p^3+10p^2+p}$$

$$G(p)\,H(p)+1 = \frac{p^4+4p^3+10p^2+p+5}{p(p^3+4p^2+10p+1)}$$

· R-H Num

p^4	1	10	5
p^3	4	1	0
p^2	9,75	5	
p^1	-1,05		
p^0	5		

2 cambios de signo

$Z = 2$

Sistema Inestable

· R-H Den

p^3	1	10
p^2	4	1
p^1	9,75	
p^0	1	

$P = 0$

$N = Z - P$

$\boxed{N = 2}$

78) Trazar diag. polar y aplicar crit. de Nyquist.

$$G(p).H(p) = \frac{15}{p^3+4p^2+2p+1}$$

1) $G(p)\,H(p)\Big|_{p\to 0} = 15$

2) $G(p)\,H(p)\Big|_{p\to\infty} = \frac{k}{p^3}\Big|_{p\to\infty} = |0|\;\underline{|-270°}$

3) $p \to j\omega$

$$G(j\omega)\,H(j\omega) = \frac{15}{-j\omega^3-4\omega^2+2j\omega+1} = \frac{15}{(1-4\omega^2)+j(2\omega-\omega^3)}\cdot\frac{(1-4\omega^2)-j(2\omega-\omega^3)}{(1-4\omega^2)-j(2\omega-\omega^3)}$$

4) $$G(j\omega)\,H(j\omega) = \frac{15\cdot(1-4\omega^2)}{(1-4\omega^2)^2+(2\omega-\omega^3)^2} + j\,\frac{-15\,(2\omega-\omega^3)}{(1-4\omega^2)^2+(2\omega-\omega^3)^2}$$

5) $Re = \emptyset$

$1 - 4w^2 = 0 \Rightarrow w = \pm 0,5.$

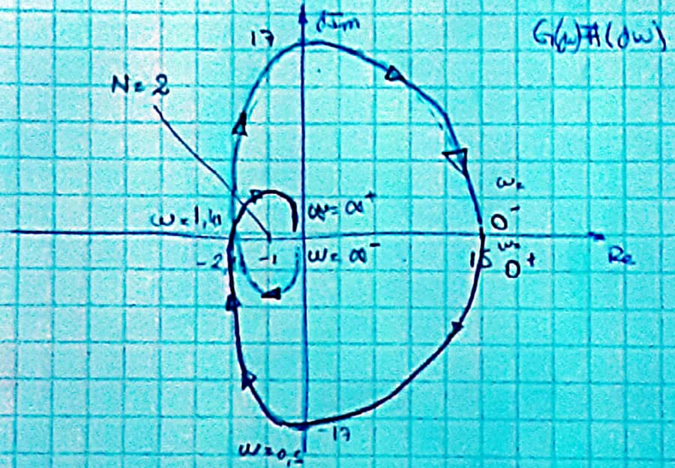
6) $$Im\Big|_{w=0,5} = \frac{-15 \cdot (2w - w^3)}{(1-4w^2)^2 + (2w-w^3)^2}\Bigg|_{w=0,5} = -17,14.$$

7) $Im = \emptyset$

$2 - w^2 = 0 \Rightarrow w = \pm\sqrt{2} = \pm 1,41.$

8) $$Re\Big|_{w=1,41} = \frac{15 \cdot (1-4w^2)}{(1-4w^2)^2 + (2w-w^3)^2} = -2,14.$$

9)



10) -

11) -

12) N = 2

Sistema Inestable.

79) R-H. de G(P) H(P) anterior

$$G(P)\cdot H(P) = \frac{15}{P^3+4P^2+2P+1}$$

$$G(P)\,H(P)+1 = \frac{P^3+4P^2+2P+16}{P^3+4P^2+2P+1}$$

• R-H Num

P^3	1	2
P^2	4	16
P^1	-2	
P^0	16	

2 cambios de signo: Z = 2.
Sistema inestable

• R-H Den

P^3	1	2
P^2	4	1
P^1	1,75	
P^0	1	

P = 0

N = Z - P

$\boxed{N = 2}$

80) Trazar diag. polar de G(P)·H(P) y aplicar crit. de Nyquist.

$$G(P)\cdot H(P) = \frac{20}{P^3+3P^2+4P+1}$$

1) $G(P)\cdot H(P)\Big|_{P\to 0} : 20$

2) $G(P)\cdot H(P)\Big|_{P\to\infty} : \frac{k}{P^3}\Big|_{P\to\infty} = |0|\ \underline{|-270^\circ}$

3) $P \to j\omega$

$$G(j\omega)\,H(j\omega) = \frac{20}{-j\omega^3-\omega^2+j4\omega+1} = \frac{20}{(1-\omega^2)+j(4\omega-\omega^3)}\cdot\frac{(1-\omega^2)-j(4\omega-\omega^3)}{(1-\omega^2)-j(4\omega-\omega^3)}$$

4) $$G(j\omega)\,H(j\omega) = \underbrace{\frac{20(1-\omega^2)}{(1-\omega^2)^2+(4\omega-\omega^3)^2}}_{Re} + j\underbrace{\frac{-20(4\omega-\omega^3)}{(1-\omega^2)^2+(4\omega-\omega^3)^2}}_{Im}$$

5) $Re = 0$

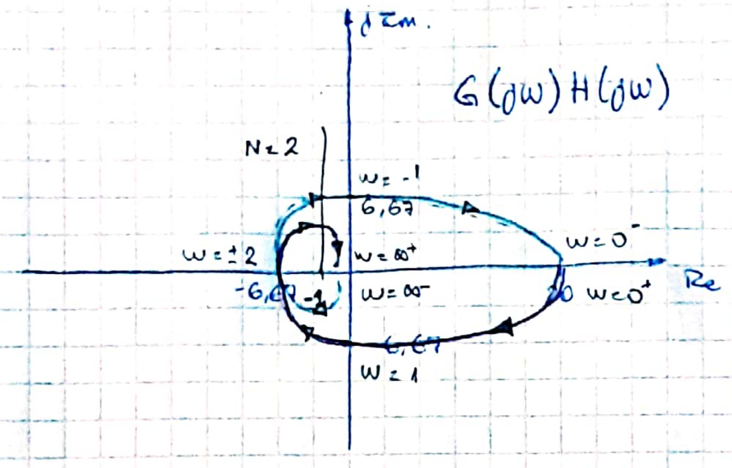
$1 - w^2 = 0 \Rightarrow w = \pm 1$

6)
$$Im\Big|_{w=1} = \frac{-20\,(4w - w^3)}{(1-w^2)^2 + (4w - w^3)^2} = -6{,}67$$

7) $Im = 0$: $4w - w^3 = 0 \Rightarrow w = \pm 2$

8)
$$Re\Big|_{w=2} = \frac{20\,(1-w^2)}{(1-w^2)^2 + (4w - w^3)}\Bigg|_{w=2} = -6{,}67$$

9)



10) -

11) -

12) $N = 2$
Sistema Inestable.

81) Criterio R-H a $G(P)H(P)$ de ej. 80.

$$G(P)\,H(P) = \frac{20}{P^3 + 3P^2 + 4P + 1}$$

$$G(P)\,H(P) + 1 = \frac{P^3 + 3P^2 + 4P + 21}{P^3 + 3P^2 + 4P + 1}$$

- R-H Num

P^3	1	4
P^2	3	21
P^1	-3	
P^0	21	

2 cambios de signo :

$Z = 2$

Sistema Inestable

- R-H Den

P^3	1	4
P^2	3	1
P^1	3,67	
P^0	1	

$P = 0$

$N = Z - P$.

$\boxed{N = 2}$

82) Routh - Hourwitz

$$5P^4 + 6P^3 + 4P^2 + 2P + 3 = 0$$

P^4	5	4	3
P^3	6	2	0
P^2	2,33	3	
P^1	-5,72		
P^0	3		

Cambios de signo = 2

El polinomio tiene 2 raices a parte real positiva.

83) Routh - Hourwitz

$$25P^5 + 105P^4 + 120P^3 + 120P^2 + 20P + 1 = 0$$

P^5	25	120	20
P^4	105	120	1
P^3	91,43	19,76	0
P^2	97,7	1	
P^1	18,51		
P^0	1		

No hay raices a parte real positiva.

84) R - H

$$2P^6 + 2P^5 + 3P^4 + 2P^3 + 4P^2 + 3P + 2 = 0$$

P^6	2	3	4	2
P^5	2	2	3	
P^4	1	1	2	
P^3	$+\varepsilon$ $\cancel{0}$	-1		
P^2	$+\#$	2		
P^1	-1			
P^0	2			

$$\# = \frac{\varepsilon \cdot 1 + (1)(1)}{\varepsilon} \cong +\#$$

2 cambios de signo = 2 raices a parte real positiva

85) R - H

$P^4 + 4P^3 + 3P^2 + P + 1$.

P^4	1	3	1
P^3	4	1	
P^2	2,75	1	
P^1	-0,45		
P^0	1		

2 cambios de signo =

• 2 raíces a parte real positiva.

86) R - H

$P^3 + 2P^2 + P + 2$

P^3	1	1
P^2	2	2
P^1	0	
P^1	4	
P^0	2	

$2P^2 + 2$

→ Se anula toda la fila → $\frac{d}{dP} = 4P$.

0 cambios de signos = 0 raíces a parte real +.

87) R - H

$P^4 + 3P^3 + 6P^2 + 12P + 8$.

P^4	1	6	8
P^3	3	12	
P^2	2	8	
P^1	0		
P^1	4		
P^0	8		

$2P^2 + 8$

→ Se anula la fila $\frac{d}{dP} = 4P$

Sin cambios de signo = 0 raíces a parte real +

88) R-H.

$$P^6 + 5P^5 + 11P^4 + 25P^3 + 36P^2 + 30P + 36$$

P^6	1	11	36	36
P^5	5	25	30	
P^4	6	30	36	
P^3	0	0		
P^3	24	60		
P^2	15	36		
P^1	2,4			
P^0	36			

$\rightarrow 6P^4 + 30P^2 + 36$

$\Rightarrow$ Se anula la fila, $\frac{d}{dP} = 24P^3 + 60P$

Sin cambios de signo = No hay raices a parte real +.

89) Aplicar criterio de Nyquist y criterio de R-H a:

$$G(P)\cdot H(P) = \frac{10P + 10}{P^5 - P^4 - P^3} = \frac{10(P+1)}{P^3(P^2 - P - 1)}$$

- Nyquist.

1) $G(P)H(P)\Big|_{P\to 0} = \frac{-k}{P^3}\Big|_{P\to 0} = |\infty| \angle -270° + 180° = |\infty| \angle -90°$

2) $G(P)\cdot H(P)\Big|_{P\to\infty} = \frac{K}{P^4}\Big|_{P\to\infty} = |0| \angle -360°$

3) $P \to j\omega$.

$$G(j\omega)H(j\omega) = \frac{10 + j10\omega}{j\omega^5 - \omega^4 + j\omega^3} = \frac{10 + j10\omega}{-\omega^4 + j(\omega^3 + \omega^5)} \cdot \frac{-\omega^4 - j(\omega^3 + \omega^5)}{-\omega^4 - j(\omega^3 + \omega^5)}$$

$$G(j\omega)H(j\omega) = \frac{-10\omega^4 + 10\omega^4 + 10\omega^6}{\omega^8 + (\omega^3 + \omega^5)^2} + j\frac{-10\omega^5 - 10\omega^3 - 10\omega^5}{\omega^8 + (\omega^3 + \omega^5)^2}$$

4) $$G(j\omega)\cdot H(j\omega) = \frac{10\omega^6}{\omega^8 + (\omega^3 + \omega^5)^2} + j\frac{-20\omega^5 - 10\omega^3}{\omega^8 + (\omega^3 + \omega^5)^2}$$

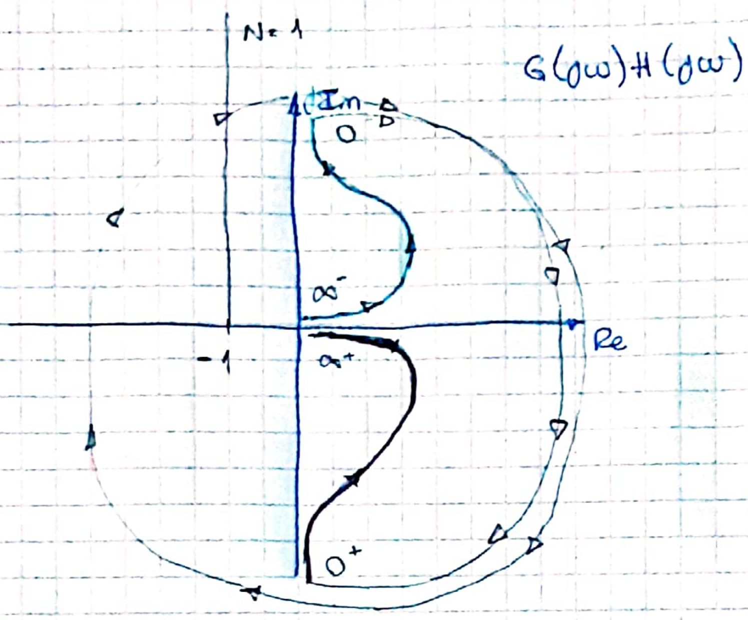
5) Re = 0 ⟶ -

6) -

7) Im = 0 ⟶ -

8) -

9)



10) $\varphi = -540°$

11) -

12) N = 1 ⟹ Sistema Inestable

- R-H

$$G(p)H(p)+1 = \frac{10P+10+P^5-P^4-P^3}{P^5-P^4-P^3} = \frac{P^5-P^4-P^3+0 \cdot P^2+10P+10}{P^3(P^2-P-1)}$$

R-H Num

p^5	1	-1	10
p^4	-1	0	+10
p^3	-1	20	
p^2	-20	+10	
p^1	19,5		
p^0	+10		

2 cambios de signo.

Z = 2

⟶ Sistema Inestable

R-H Den

p^2	1	-1
p^1	-1	
p^0	-1	

1 cambio.

P = 1

$$\boxed{N = Z - P = 1}$$

90) Aplicar criterios de Nyquist y R-H.

$$G(P)H(P) = \frac{5(P+2)}{P(P+1)(P-5)} = \frac{5P+10}{P(P^2-4P-5)} = \frac{5P+10}{P^3-4P^2-5P}$$

• Nyquist.

1) $G(P)H(P)\Big|_{P\to 0} = \frac{-k}{P}\Big|_{P\to 0} = |\infty| \underline{|-90-180^\circ} = |\infty| \underline{|-270^\circ}$

2) $G(P)H(P)\Big|_{P\to\infty} = \frac{k}{P^2}\Big|_{P\to\infty} = |0| \underline{|-180^\circ}$

3) $P \to jw$

$$G(jw)H(jw) = \frac{10 + j5w}{-jw^3 + 4w^2 - j5w} = \frac{10 + j5w}{4w^2 + j(-w^3 - 5w)} \cdot \frac{4w^2 - j(-w^3 - 5w}{4w^2 - j(-w^3 - 5w}$$

4) $$G(jw)H(jw) = \frac{40w^2 - 5w^4 - 25w^2}{16w^4 + (-w^3 - 5w)^2} + j\frac{20w^3 + 10w^3 + 50w}{16w^4 + (-w^3 - 5w)^2}$$

$$G(jw)H(jw) = \frac{15w^2 - 5w^4}{16w^4 + (w^3 + 5w)^2} + j\frac{30w^3 + 50w}{16w^4 + (w^3 + 5w)^2}$$

5) $Re = 0$.

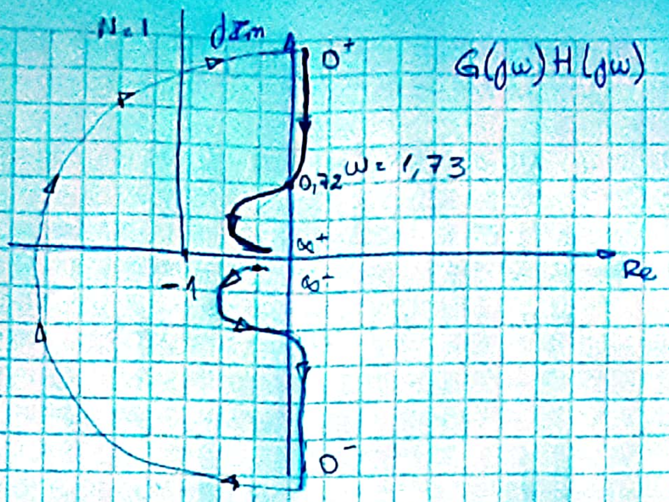
$15 - 5w^2 = 0 \Rightarrow w = \sqrt{3} = \pm 1{,}73$

6) $Im\Big|_{w=1,73} = \frac{30w^3 + 50w}{16w^4 + (w^3 + 5w)^2}\Big|_{w=1,73} = 0{,}72$.

7) $Im = 0$ -

8) -

9)



10) $\varphi = -180°$

11) -

12) N = 1

Sist. Inestable.

• R-H

$$G(P)H(P)+1 = \frac{5P+10+P^3-4P^2-5P}{P^3-4P^2-5P}$$

$$G(P)H(P)+1 = \frac{P^3-4P^2+0\cdot P+10}{P(P^2-4P-5)}$$

• R-H Num

P^3	1	0
P^2	-4	10
P^1	2,5	
P^0	10	

2 cambios de signo

Z = 2

↳ Sistema Inestable.

• R-H Den

P^2	1	-5
P^1	-4	
P^0	-5	

1 cambio de signo

P = 1

$$N = Z - P = 2 - 1$$

$$\boxed{N = 1}$$

91) Aplicar criterios de Nyquist y R-H.

$$G(P)\ H(P) = \frac{10P-10}{P^3+4P^2+8P}$$

• Nyquist

1) $G(P)H(P)\Big|_{P\to 0} = \frac{-k}{P}\Big|_{P\to 0} = |\infty| \underline{\lfloor -90-180^\circ} = |\infty| \underline{\lfloor -270^\circ}$

2) $G(P)H(P)\Big|_{P\to\infty} = \frac{k}{P^2}\Big|_{P\to\infty} = |0| \underline{\lfloor -180^\circ}$

3) $P \to j\omega$

$$G(j\omega).H(j\omega) = \frac{-10+j10\omega}{-j\omega^3-4\omega^2+j8\omega} = \frac{-10+j10\omega}{-4\omega^2+j(8\omega-\omega^3)} \cdot \frac{-4\omega^2-j(8\omega-\omega^3)}{-4\omega^2-j(8\omega-\omega^3)}$$

4)

$$G(j\omega)H(j\omega) = \frac{40\omega^2+80\omega^2-10\omega^4}{16\omega^4+(8\omega-\omega^3)^2} + j\frac{-40\omega^3+80\omega-10\omega^3}{16\omega^4+(8\omega-\omega^3)^2}$$

$$G(j\omega).H(j\omega) = \frac{120\omega^2-10\omega^4}{16\omega^4+(8\omega-\omega^3)^2} + j\frac{-50\omega^3+80\omega}{16\omega^4+(8\omega-\omega^3)^2}$$

5) $Re = 0$

$120-10\omega^2 = 0 \quad \Rightarrow \omega = \pm\sqrt{12} = \pm 3,46$

6) $Im\Big|_{\omega=3,46} = \frac{80\omega-50\omega^3}{16\omega^4+(8\omega-\omega^3)^2}\Big|_{\omega=3,46} = -0,72$

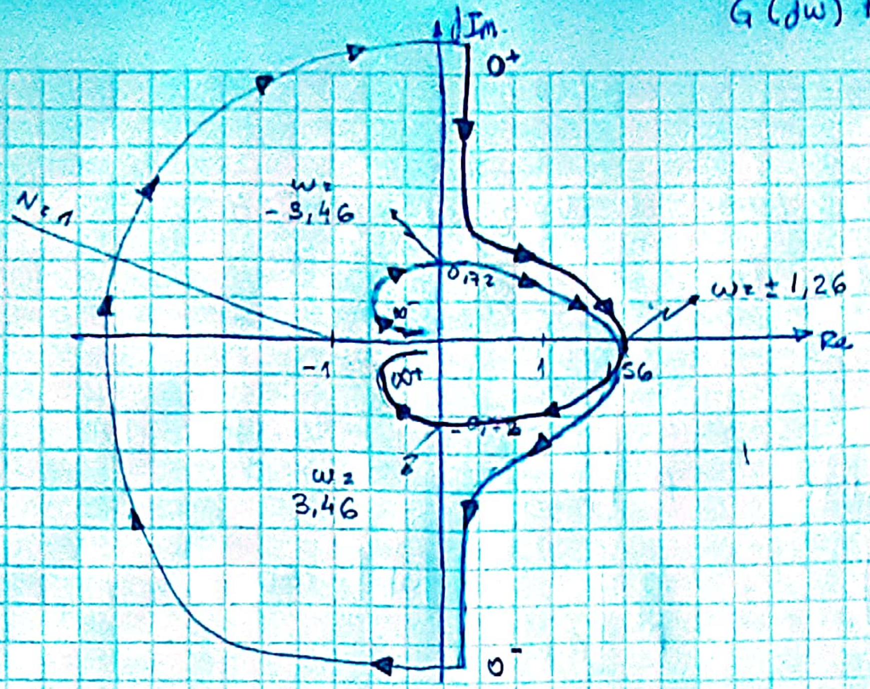
7) $Im = 0$

$-50\omega^2+80 = 0 \quad \Rightarrow \omega = \pm\sqrt{\frac{8}{5}} = \pm 1,26$

8) $Re\Big|_{\omega=1,26} = \frac{120\omega^2-10\omega^4}{16\omega^4+(8\omega-\omega^3)^2}\Big|_{\omega=1,26} = 1,56$

$G(j\omega)\,H(j\omega)$

9)



10) $\varphi = -180°$.

11) –

12) N = 1 ⇒ Sistema Inestable.

• R-H:

$$G(p)\,H(p) + 1 = \frac{10p - 10 + p^3 + 4p^2 + 8p}{p(p^2 + 4p + 8)} = \frac{p^3 + 4p^2 + 18p - 10}{p(p^2 + 4p + 8)}$$

• R-H Num

p^3	1	18
p^2	4	-10
p^1	20,5	
p^0	-10	

1 cambio de signo = 1 raíz a parte $\mathbb{R}$ +.

↳ Sistema Inestable

Z = 1.

• R-H Den

p^2	1	8
p^1	4	
p^0	8	

0 cambios

P = 0

$$N = Z - P = 1 - 0$$

$$\boxed{N = 1}$$

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