65) Trazar diagrama polar de $F(P)$ y determinar estabilidad, aplicando criterio de Nyquist.

$$F(P) = \frac{1}{P^3 + 2P^2 + P + 3}$$

1) Origen diagrama.

$$F(P)\Big|_{P\to 0} = \frac{1}{3} = 0{,}\overline{333}$$

2) Fin diagrama.

$$F(P)\Big|_{P\to\infty} = |0| \underline{\lfloor -270^\circ}$$

3) $P \to jw$; $F(P) \to F(jw)$.

$$F(jw) = \frac{1}{(3-2w^2)+j(w-w^3)} \cdot \frac{(3-2w^2)+j(w^3-w)}{(3-2w^2)+j(w^3-w)}$$

4) $F(jw) = Re + j\,Im$.

$$F(jw) = \frac{3-2w^2}{(3-2w^2)^2+(w^3-w)^2} + j\frac{(w^3-w)}{(3-2w^2)^2+(w^3-w)^2}$$

5) $Re = \emptyset$.

$$3 - 2w^2 = 0$$

$$w = \sqrt{\frac{3}{2}} = \pm 1{,}2247$$

6) Corte eje Imag.

$$j\frac{(w^3-w)}{(3-2w^2)^2+(w^3-w)^2}\Bigg|_{w=+1{,}2247} = +j1{,}632$$

7) $Im = 0$.

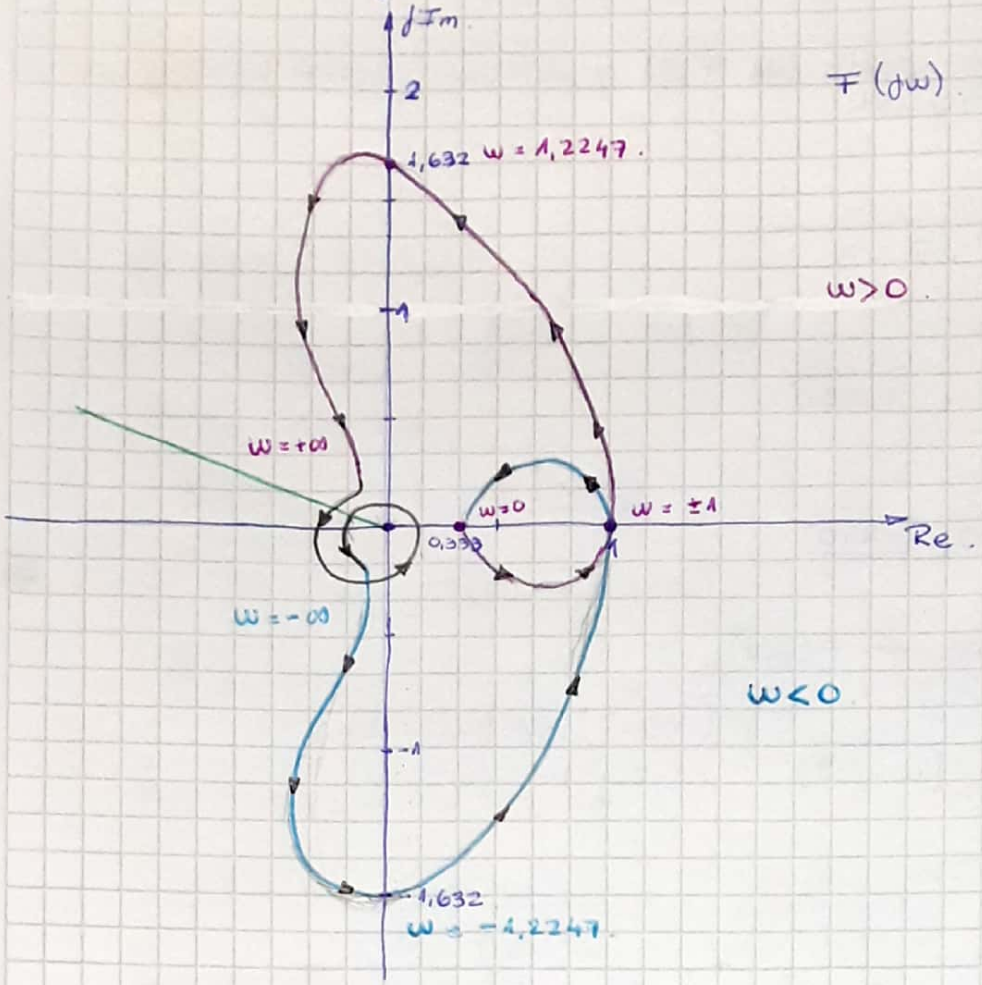
$$w^3 - w = 0$$

$$w = \pm 1$$

8) Corte eje Re.

$$\frac{3-2w^2}{(3-2w^2)^2+(w^3-w)^2}\Bigg|_{w=+1} = 1$$

9) Trazar Diagrama Polar.

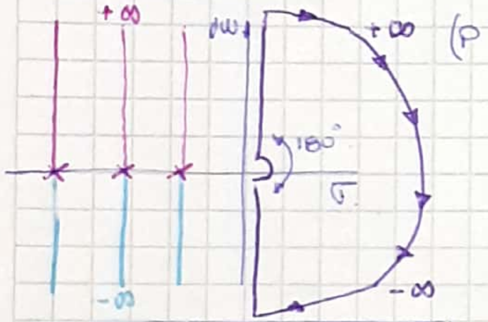


10) Cerrar diagrama p/ $P \to 0$ (No se aplica porque no hay polos en 0).

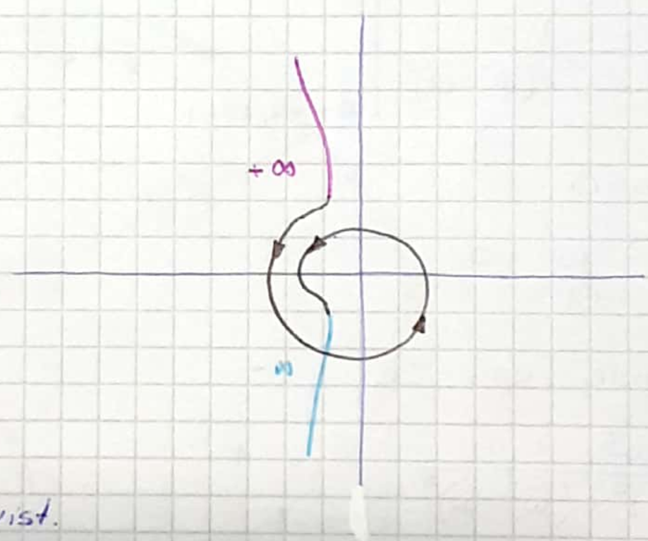
11) Cerrar p/ $P \to \infty$

$$F(P)\Big|_{P\to\infty} = \frac{1}{P^3}\Big|_{P\to\infty} = |0| \underline{\lfloor 3\theta}$$

Graficamente:



Zoom Origen



$\boxed{0^+ \to \infty^+ \to \infty^- \to 0^- \to 0^+}$

$\varphi = -3 \times 180° = -540°$

(Vuelta y media).

12) Aplicar criterio de Nyquist.

$\boxed{N = -2}$ ⟶ $\boxed{\text{Sistema Inestable}}$

66) Trazar diagrama polar de $F(P)$, determinar estabilidad aplicando Nyquist.

$$F(P) = \frac{10}{P(P+1)}$$

1) Origen diagrama.

$$F(P)\Big|_{P\to 0} = \frac{10}{P} = \infty \underline{|-90^\circ}$$

2) Fin diagrama.

$$F(P)\Big|_{P\to\infty} = 0 \underline{|-180^\circ}$$

3) $P \to j\omega$; $F(P) \to F(j\omega)$

$$F(j\omega) = \frac{10}{j\omega(1+j\omega)} = \frac{10}{-\omega^2 + j\omega} \cdot \frac{-\omega^2 - j\omega}{-\omega^2 - j\omega}$$

4) $F(j\omega) = Re + j\,Im.$

$$F(j\omega) = \frac{-10\omega^2}{\omega^4 + \omega^2} + j\,\frac{(-10\omega)}{\omega^4 + \omega^2} = \frac{-10}{\omega^2 + 1} + j\,\frac{-10}{\omega^3 + \omega}$$

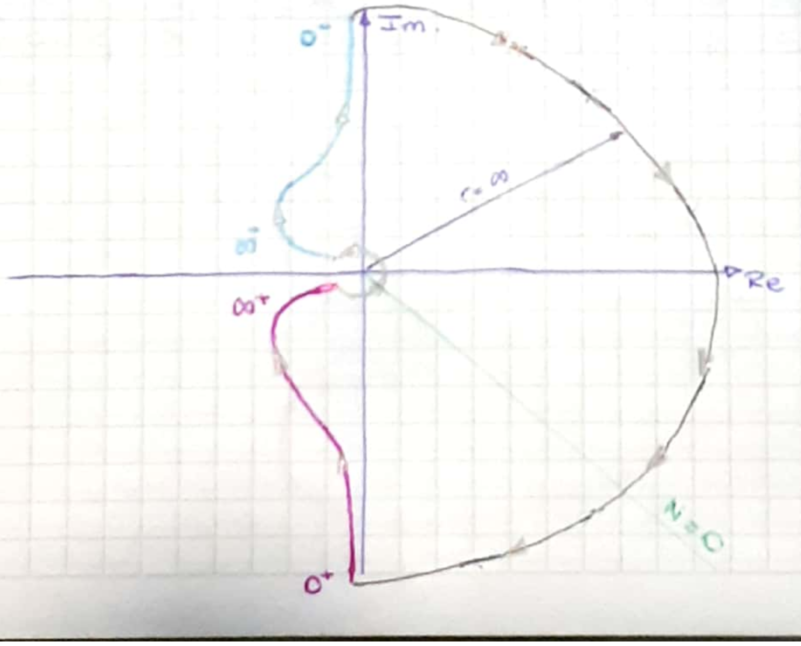
5) $Re = 0 \to$ No hay ω

6) Corte eje $Im \to$ No hay.

7) $Im = 0 \to$ No hay ω.

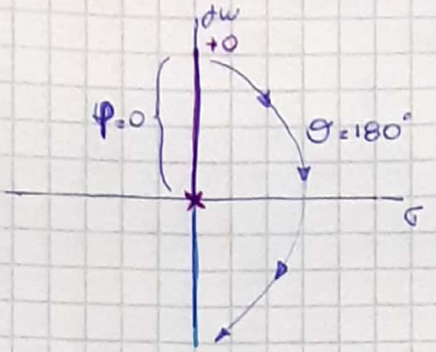
8) Corte eje $Re \to$ No hay.

9) Trazar Diagrama Polar.

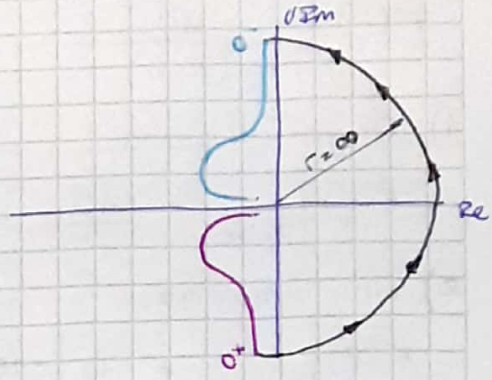


10) Cerrar Diagrama p/ $P \to 0$.

$$F(p)\Big|_{p\to 0} = \frac{10}{P} = \frac{10}{\gamma} \cdot e^{-j\theta} = |\infty| \cdot e^{-j\theta} = |\infty| \underline{|-\theta}$$



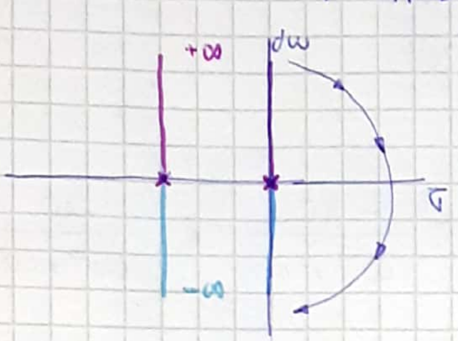
$\varphi = -180°$

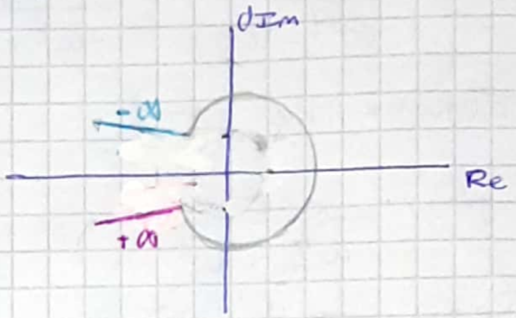


11) Cerrar p/ $P \to \infty$

$$F(p)\Big|_{p\to\infty} = \frac{10}{p^2}\Big|_{p\to\infty} = |0| \underline{|-2\theta}$$

Zoom origen =





$\varphi = -360°$ (1 vuelta).

12) Aplicar criterio de Nyquist.

N = 0.

No se determina por Nyquist la estabilidad.
Podemos decir que el sistema es estable, ya que al conocer los polos, verificamos que ninguno está en el semiplano $\sigma+$

67) Trazar diagrama polar, determinar estabilidad mediante Nyquist

$$F(p) = \frac{10}{(p+1)^2}$$

1) Origen diagrama

$$F(p)\Big|_{p\to 0} = 10.$$

2) Fin diagrama.

$$F(p)\Big|_{p\to\infty} = |0| \underline{|-180^\circ}$$

3) $p \to jw \quad \therefore \quad F(p) \to F(jw)$

$$F(jw) = \frac{10}{(1+jw)^2} = \frac{10}{(1-w^2)+j2w} \cdot \frac{(1-w^2)-j2w}{(1-w^2)-j2w}$$

4)

$$F(jw) = \frac{10(1-w^2)}{(1-w^2)^2+4w^2} + j\,\frac{(-20w)}{(1-w^2)^2+4w^2}$$

5) $Re = 0$

$$1 - w^2 = 0 \longrightarrow w = \pm 1.$$

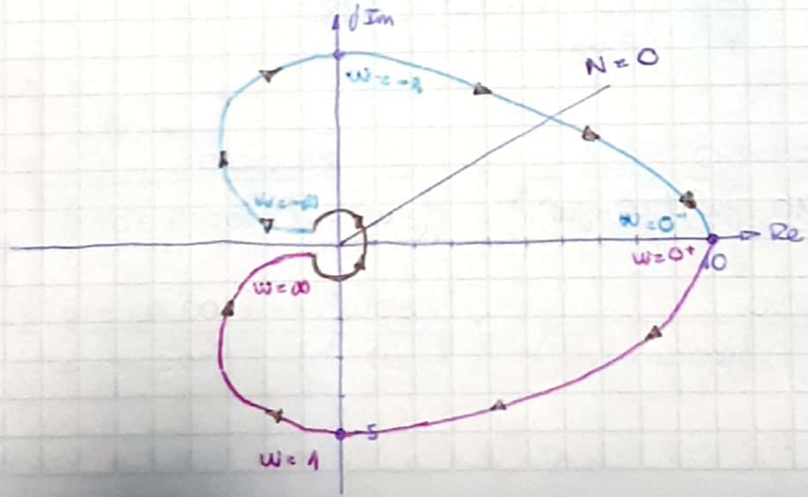
6) $$j\frac{(-20w)}{(1-w^2)^2+4w^2}\Bigg|_{w=1} = -j5$$

7) $Im = 0$

$$-20w = 0 \longrightarrow w = 0$$

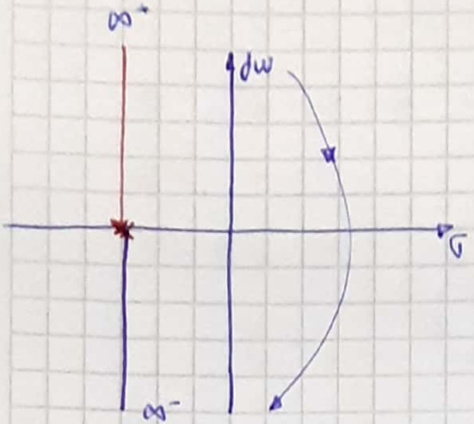
8) -

9)

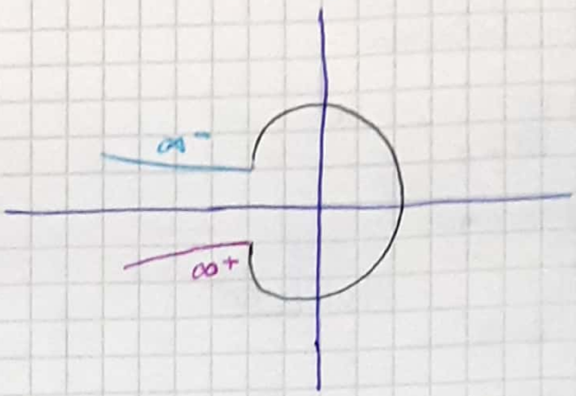


10) -

11) $F(P)\Big|_{P\to\infty} = \frac{1}{P^2}\Big|_{P\to\infty} = |0| \angle 2\emptyset$



Zoom origen



$\varphi = -2 \times 180^\circ = -360^\circ$.

12) Aplicando crit. Nyquist.

$0^+ \to \infty^+ \to \infty^- \to 0^- \to 0^+$

$\boxed{N = 0}$ Sistema sin determinar mediante Nyquist.
Sistema estable por no tener polos con $\sigma > 0$.

68) Diagrama Polar y aplicar Nyquist.

$$F(P) = \frac{50}{P\cdot(0{,}1P+1)(0{,}2P+1)} = \frac{50}{P\cdot 0{,}1\cdot 0{,}2\,(P+10)(P+\ )}$$

$$F(P) = \frac{2500}{P(P+10)(P+5)} = \frac{2500}{P(P^2+5P+10P+50)} = \frac{2500}{P(P^2+15P+50)}$$

1) $F(P)\Big|_{P\to 0} = |\infty| \angle -90^\circ$

2) $F(P)\Big|_{P\to\infty} = \frac{1}{P^3}\Big|_{P\to\infty} = |0| \angle -270^\circ$.

3) $$F(j\omega) = \frac{2500}{j\omega\left((j\omega)^2 + 15j\omega + 50\right)} = \frac{2500}{j\omega\left(-\omega^2 + 50 + j15\omega\right)}$$

$$F(j\omega) = \frac{2500}{(-15\omega^2) + j(50\omega - \omega^3)} \cdot \frac{(-15\omega^2) - j(50\omega - \omega^3)}{(-15\omega^2) - j(50\omega - \omega^3)}$$

4) $F(j\omega) = \dfrac{-375.00 \cdot \omega^2}{(15\,\omega^2)^2 + (50\,\omega - \omega^3)^2} + j\,\dfrac{(\omega^3 - 50\,\omega) \cdot 25.00}{(15\,\omega^2)^2 + (50\,\omega - \omega^3)^2}$

5) $Re = \emptyset \rightarrow -$

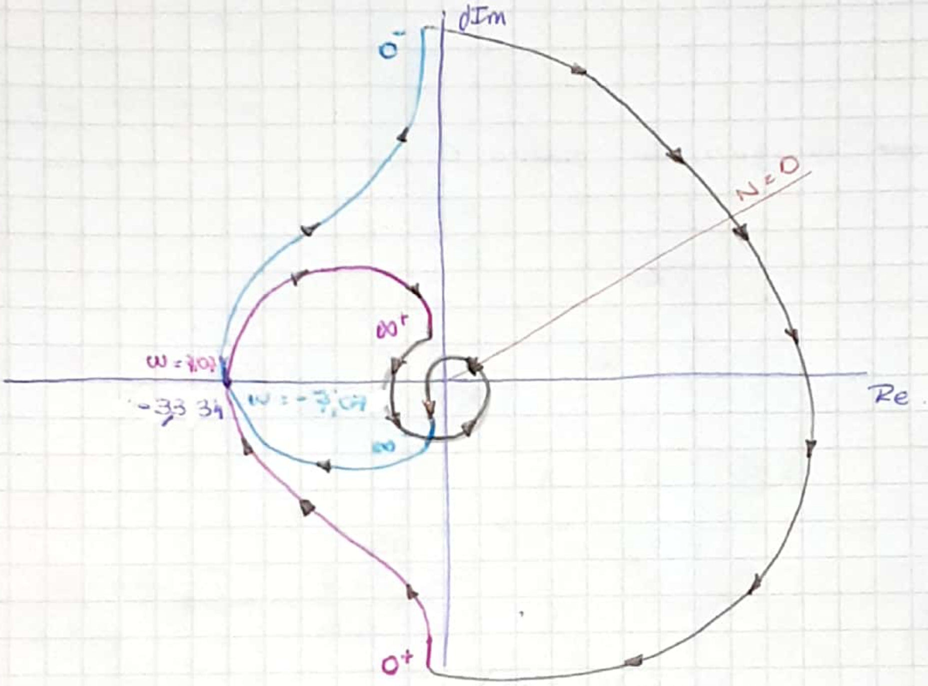
6) $-$

7) $Im = 0$

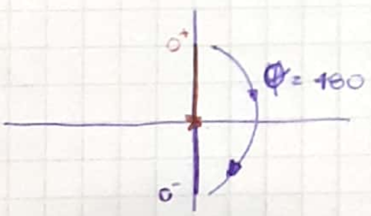
$$\omega^3 - 50\,\omega = 0$$
$$\omega^2 - 50 = 0 \longrightarrow \omega = \sqrt{50} = \pm 7{,}07$$

8) $$\left.\frac{-375.00 \cdot \omega^2}{(15\,\omega^2)^2 + (50\,\omega - \omega^3)^2}\right|_{\omega = 7{,}07} = -33{,}34$$

9)



10)

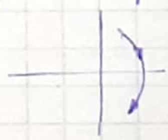


$\varphi = -1 \times 180^\circ = -180^\circ$

11) $\theta = 180^\circ$

$\varphi = -3 \times 180^\circ = -540^\circ$

(vuelta y media)



13) N = 0. No se determina por Nyquist.

Sistema estable por ubic. de polos.

69) Trazar Diag. Polar y determinar estabilidad mediante Nyquist.

$$F(P) = \frac{P+0,5}{P^2(P+10)}$$

1) $F(P)\Big|_{P\to 0} = |\infty| \underline{|-180°}$

2) $F(P)\Big|_{P\to\infty} = |0| \underline{|-180°}$

3) $P \to j\omega$; $F(P) \to F(j\omega)$

$$F(j\omega) = \frac{0,5+j\omega}{-\omega^2(10+j\omega)} = \frac{(0,5+j\omega)}{(-10\omega^2)-j\omega^3} \cdot \frac{(-10\omega^2)+j\omega^3}{(-10\omega^2)+j\omega^3}$$

4)
$$F(j\omega) = \frac{(-5\omega^2-\omega^4)}{100\omega^4+\omega^6} + j\frac{(0,5\omega^3-10\omega^3)}{100\omega^4+\omega^6}$$

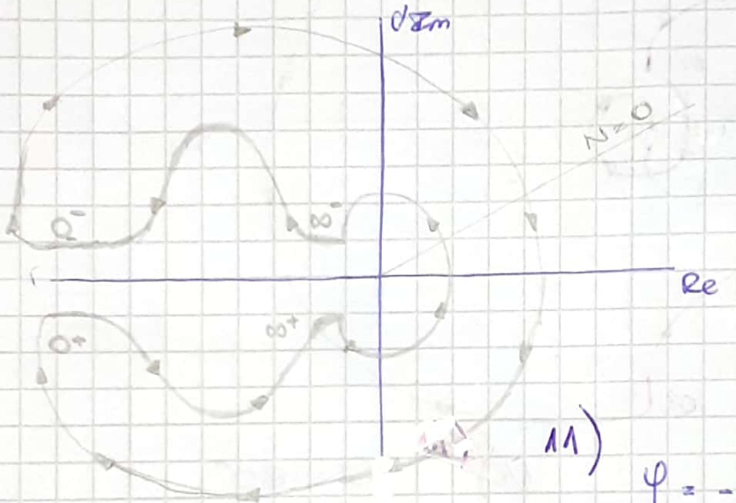
5) $-5-\omega^2 = 0 \to$ Ningún ω real.

6) –

7) $Im = 0 \longrightarrow$ Ningún ω salvo 0.

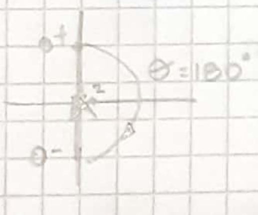
8) –

9)



10)

$\varphi = -360$



11)

$\varphi = -360°$

12) $N = 0$.

70) Trace diagrama polar de $F(P)$. Determinar estabilidad (Nyquist)

$$F(P) = \frac{10}{P^2+1}$$

1) $F(P)\Big|_{P=0}$: $\frac{10}{1} = 10$

2) $F(P)\Big|_{P\to\infty}$: $\frac{10}{P^2}\Big|_{P\to\infty} = |0| \underline{\lfloor -180^\circ}$

3) $P \to j\omega$

$$F(j\omega) = \frac{10}{(j\omega)^2+1} = \frac{10}{1-\omega^2}$$

4) $F(j\omega) = \frac{10}{1-\omega^2} + j \cdot 0$

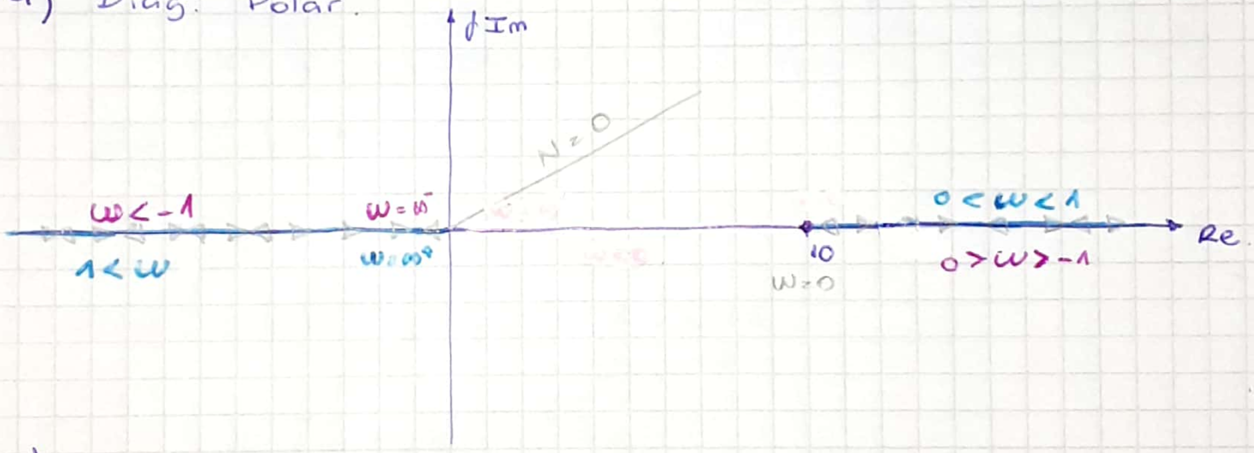
5) $Re = 0 \to$ No

6) -

7) -

8) -

9) Diag. Polar



10) -

11) -

12) $N = 0 \to$ No se especifica por Nyquist.

Sistema estable porque ∫ de polos es mayor a 0.

71) Trazar diag. polar y determinar estabilidad mediante Nyquist.

$$F(P) = \frac{P+2}{(P+1)(P^2+6{,}25)} = \frac{P+2}{P^3+P^2+6{,}25P+6{,}25}$$

1) $F(P)\Big|_{P\to 0} = \frac{2}{1 \times 6{,}25} = 0{,}32$

2) $F(P)\Big|_{P\to\infty} = \frac{1}{P^2}\Big|_{P\to\infty} = |0| \underline{|-180^\circ}$

3) $P \to jw$

$$F(jw) = \frac{2+jw}{(1+jw)(6{,}25-w^2)} = \frac{2+jw}{6{,}25-w^2+j(6{,}25w-w^3)}$$

$$F(jw) = \frac{2+jw}{(6{,}25-w^2)+j(6{,}25w-w^3)} \cdot \frac{(6{,}25-w^2)-j(6{,}25w-w^3)}{(6{,}25-w^2)-j(6{,}25w-w^3)}$$

4) $$F(jw) = \frac{12{,}5-2w^2+6{,}25w^2-w^4}{(6{,}25-w^2)^2+(6{,}25w-w^3)^2} + j\frac{6{,}25w-w^3-12{,}5w+2w^3}{(6{,}25-w^2)^2+(6{,}25w-w^3)^2}$$

$$F(jw) = \frac{12{,}5+4{,}25w^2-w^4}{(6{,}25-w^2)^2+(6{,}25w-w^3)^2} + j\frac{w^3-6{,}25w}{(6{,}25-w^2)^2+(6{,}25w-w^3)^2}$$

5) $12{,}5+4{,}25w^2-w^4 = 0$

$w^2 = x$

$w^4 = x^2$

$12{,}5+4{,}25x-x^2 = 0 \quad \begin{cases} x_1 = 6{,}25 \\ x_2 = -2 \end{cases}$

$w_{1,2} = \sqrt{6{,}25} = \pm 2{,}5$

$w_{3,4} = \sqrt{-2}$ = NO SIRVE

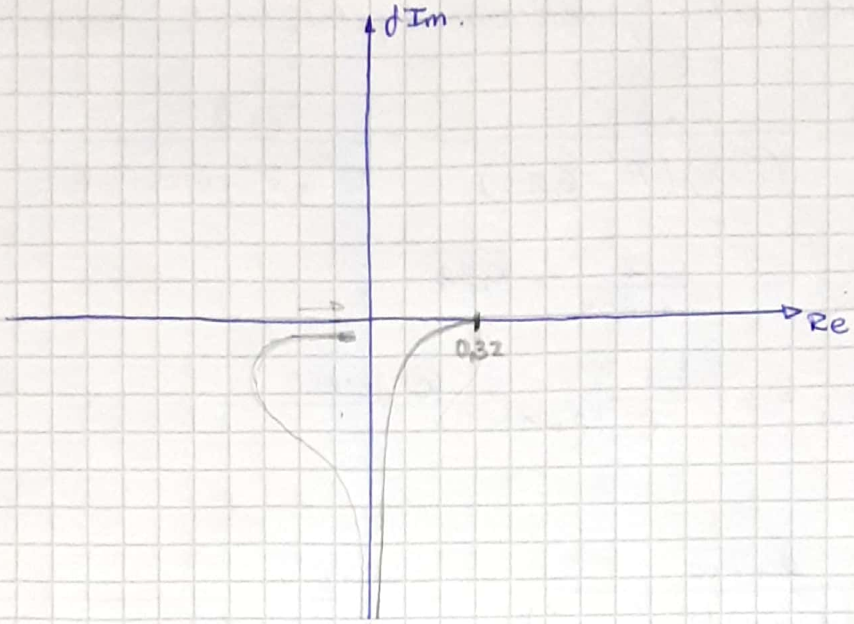
6) $$\frac{w^3-6{,}25w}{(6{,}25-w^2)^2+(6{,}25w-w^3)^2}\Bigg|_{w=2{,}5} = \frac{w(w^2-6{,}25)}{(6{,}25-w^2)^2+w^2(6{,}25-w^2)^2}\Bigg|_{w=2{,}5} =$$

$$= \frac{w(w^2-6{,}25)}{(w^2-6{,}25)^2[1+w^2]}\Bigg|_{w=2{,}5} = \frac{w}{(1+w^2)(w^2-6{,}25)}\Bigg|_{w=2{,}5} = +\infty$$

7) Im = 0 ⟹ Sólo en ω = 0

8) –

9)



Scanned by CamScanner

72) Trazar diag. polar y det. estabilidad empleando crit. Nyquist.

$$F(P) = \frac{20(P+4)}{P(P^2+5P+6)}$$

1) $F(P)\Big|_{P\to 0} = \frac{20 \times 4}{P}\Big|_{P\to 0} = |\infty| \underline{|-90°}$

2) $F(P)\Big|_{P\to\infty} = \frac{1}{P^2}\Big|_{P\to\infty} = |0| \underline{|-180°}$

3) $F(jw) = \frac{20(4+jw)}{jw(6-w^2+j5w)} = \frac{(80+j80w)}{(-5w^2)+j(6w-w^3)} \cdot \frac{(-5w^2)-j(6w-w^3)}{(-5w^2)-j(6w-w^3)}$

$$F(jw) = \frac{-400w^2+480w^2-80w^4}{25w^4+(6w-w^3)^2} + j\frac{-400w^3-480w+80w^3}{25w^4+(6w-w^3)^2}$$

4) $F(jw) = \frac{(80-80w^2)\cdot w^2}{25w^4+(6w-w^3)^2} + j\frac{w\cdot(-320w^2-480)}{25w^4+(6w-w^3)^2}$

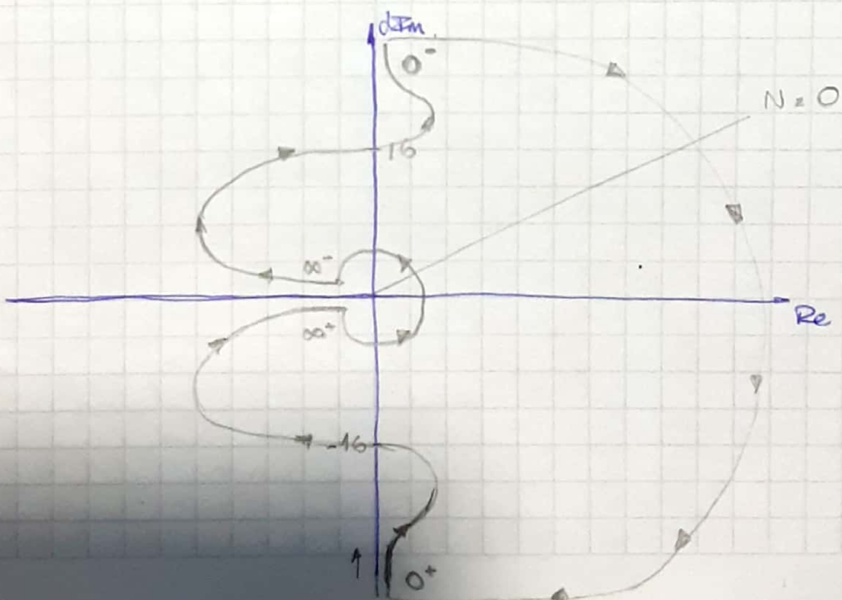
5) $80-80w^2=0 \longrightarrow w = \pm 1$.

6) $\frac{w(-320w^2-480)}{25w^4+(6w-w^3)^2}\Big|_{w=1} = -16$.

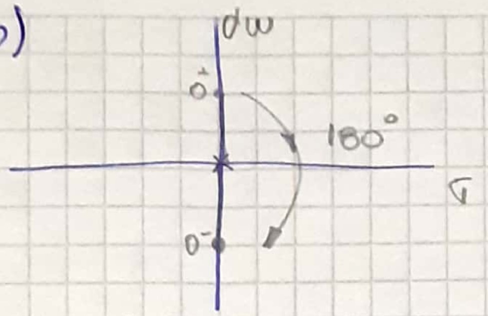
7) $-320w^2-480=0 \longrightarrow$ Ningún w.

8) –

9)

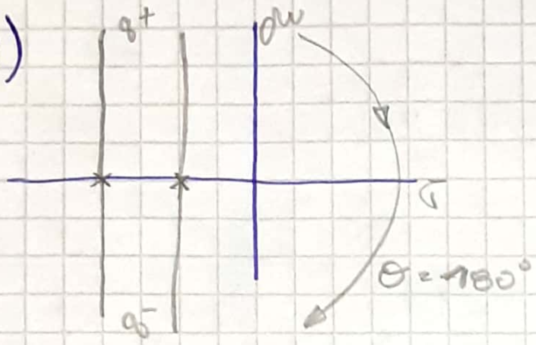


10)



$\varphi = -180°$

11)



$\varphi = -360°$

12) $N = 0$ (No se determina por Nyquist.)
Sist. estable (Polos con $\sigma \leq 0$).