CUADRIPOLOS: PARÁMETROS

$$\mathbf{E}_{\mathrm{IN}} = \mathbf{I}_{\mathrm{IN}} * \mathbf{Z}_{11} + \mathbf{I}_{\mathrm{OUT}} * \mathbf{Z}_{12}$$

 $\mathbf{E}_{\text{OUT}} = \mathbf{I}_{\text{IN}} * \mathbf{Z}_{21} + \mathbf{I}_{\text{OUT}} * \mathbf{Z}_{22}$

PARÁMETROS	EXPRESA:	EN FUNCIÓN DE:
PARÁMETROS DE IMPEDANCIA "Z"	E _{IN} y E _{OUT}	$oldsymbol{I_{IN}} oldsymbol{y} oldsymbol{I_{OUT}}$
PARÁMETROS DE ADMITANCIA "Y"	I _{IN} y I _{OUT}	$oldsymbol{E_{IN}} oldsymbol{y} oldsymbol{E_{OUT}}$
PARÁMETROS HÍBRIDOS "h"	$oldsymbol{E_{IN}} oldsymbol{y} oldsymbol{I_{OUT}}$	$I_{IN} \ m{y} \ m{E}_{\mathrm{OUT}}$
PARÁMETRO S HÍBRIDO S "g"	I IN Y E OUT	E _{IN} y I _{OUT}
PARÁMETRO S DE TRANSMISIÓN DIRECTA "ABCD"	$oldsymbol{E_{IN}} oldsymbol{y} oldsymbol{I_{IN}}$	$oldsymbol{E}_{ ext{OUT}}$ $oldsymbol{y}$ $oldsymbol{I}_{ ext{OUT}}$
PARÁMETROS DE TRANSMISIÓN INVERSA "EFGH"	E out y I out	$oldsymbol{E_{IN}}$ $oldsymbol{y}$ $oldsymbol{I_{IN}}$

PARÁM	IETROS	Z	Y	ABCD	EFGH	h	g
Z	Z ₁₁		Υ ₂₂ / ΔΥ	A/C	h/G	$\Delta h/h_{22}$	1/g ₁₁
	Z ₁₂		$-\mathbf{Y_{12}}/\Delta\mathbf{Y}$	(AD-BC)/C	1/G	h_{12}/h_{22}	$-g_{12}/g_{11}$
	Z ₂₁		-Y ₂₁ / ΔY	1/C	(EH-GF)/G	$-h_{21}/h_{22}$	g_{21}/g_{11}
	Z ₂₂		Y ₁₁ / ΔΥ	D/C	E/G	$1/h_{22}$	$\Delta g/g_{11}$
	ΔZ	$Z_{11}Z_{22}\text{-}Z_{12}Z_{21}$	1 / ΔY	B / C	F/G	$\mathbf{h_{11}/h_{22}}$	g_{22}/g_{11}
	Y ₁₁	$Z_{22}/\Delta Z$		D/B	E/F	1/h ₁₁	$\Delta g/g_{22}$
	Y ₁₂	$-\mathbf{Z_{12}}/\Delta\mathbf{Z}$		-(AD-BC)/B	-1/F	$-h_{12}/h_{11}$	g_{12}/g_{22}
Y	Y ₂₁	$-\mathbf{Z}_{21}/\Delta\mathbf{Z}$		-1/B	-(EH-GF)/F	h_{21}/h_{11}	$-\mathbf{g_{21}}/\mathbf{g_{22}}$
1	\mathbf{Y}_{22}	$\mathbf{Z}_{11}/\Delta\mathbf{Z}$		A/B	H/F	$\Delta h/h_{11}$	$1/g_{22}$
	ΔY	1 / \D Z	$Y_{11}Y_{22} - Y_{12}Y_{21}$	C/B	G/F	h_{22}/h_{11}	g_{11}/g_{22}
_	A	Z_{11} / Z_{21}	$-Y_{22}/Y_{21}$		H/(H-GF)	$-\Delta h/h_{21}$	1/g ₂₁
A	В	$\Delta Z / Z_{21}$	$-1/Y_{21}$		F/(EH-GF)	$-h_{11}/h_{21}$	g_{22}/g_{21}
В	C	$1 / Z_{21}$	$-\Delta Y/Y_{21}$		G/(EH-GF)	$-h_{22}/h_{21}$	g_{11}/g_{21}
C	D	Z_{22} / Z_{21}	$-Y_{11}/Y_{21}$		E/(EH-GF)	$-1/h_{21}$	$\Delta g/g_{21}$
D	Δ_{ABCD}	Z_{12} / Z_{21}	Y_{12}/Y_{21}	(AD-BC)=1	1/(EH-GF)	$-h_{12}/h_{21}$	$-\mathbf{g_{12}}/\mathbf{g_{21}}$
\mathbf{E}	E	Z_{22} / Z_{12}	$-Y_{11}/Y_{12}$	D/(AD-BC)		1/h ₁₂	$-\Delta g/g_{12}$
\mathbf{F}	F	$\Delta Z / Z_{12}$	$-1/Y_{12}$	B/(AD-BC)		h_{11}/h_{12}	$-\mathbf{g_{22}/g_{12}}$
Ğ	G	$1 / Z_{12}$	-ΔY/Y ₁₂	C/(AD-BC)		h_{22}/h_{12}	$-\mathbf{g_{11}}/\mathbf{g_{12}}$
Н	H	Z_{11} / Z_{12}	$-\mathbf{Y}_{22}/\mathbf{Y}_{12}$	A/(AD-BC)		$\Delta h/h_{12}$	-1/g ₁₂
п	Δ_{EFGH}	Z_{12} / Z_{12}	Y_{21}/Y_{12}	1/(AD-BC)	(EH- FG)=1	$-h_{21}/h_{12}$	$-g_{21}/g_{12}$
Н	h ₁₁	$\Delta Z/Z_{22}$	1/Y ₁₁	B/D	F/E		$g_{22}/\Delta g$
	h ₁₂	Z_{12}/Z_{22}	$-Y_{12}/Y_{11}$	(AD-BC)/D	1/E		-g ₁₂ /∆g
	h ₂₁	$-Z21/Z_{22}$	Y_{21}/Y_{11}	-1/D	-(EH-GF)/E		-g ₂₁ /∆g
	h ₂₂	1/ Z ₂₂	$\Delta Y/Y_{11}$	C/D	G/E		g ₁₁ /∆g
	Δh	Z_{11}/Z_{22}	Y_{22}/Y_{11}	A/D	H/E	$\mathbf{h}_{11}\mathbf{h}_{22}$ - \mathbf{h}_{12} \mathbf{h}_{21}	1/∆g
G	g 11	1/ Z ₁₁	$\Delta Y/Y_{22}$	C/A	G/H	$h_{22}/\Delta h$	
	g 12	$-Z_{12}/Z_{11}$	Y_{12}/Y_{22}	-(AD-BC)/A	-1/H	$-h_{12}/\Delta h$	
	g ₂₁	Z_{21}/Z_{11}	$-Y_{21}/Y_{22}$	1/A	(EH-FG)/H	$-\mathbf{h}_{21}/\Delta\mathbf{h}$	
	g ₂₂	$\Delta Z/Z_{11}$	1/Y ₂₂	B/A	F/H	h ₁₁ /∆h	
	Δg	Z_{22}/Z_{11}	Y_{11}/Y_{22}	D/A	E/H	1/ ∆ h	g ₁₁ g ₂₂ -g ₁₂ g ₂₁

PARÁMETROS DE IMPEDANCIA Z

$$\begin{aligned} & Z_{11} = \frac{E_{IN}}{I_{IN}} \bigg|_{I_{OUT} = 0} & Z_{12} = \frac{E_{IN}}{I_{OUT}} \bigg|_{I_{IN} = 0} \\ & Z_{21} = \frac{E_{OUT}}{I_{IN}} \bigg|_{I_{OUT} = 0} & Z_{22} = \frac{E_{OUT}}{I_{OUT}} \bigg|_{I_{IN} = 0} \end{aligned}$$

PARÁMETROS DE ADMITANCIA Y

$$\begin{vmatrix} \mathbf{Y}_{11} = \frac{\mathbf{I}_{IN}}{\mathbf{E}_{IN}} \Big|_{\mathbf{E}_{OUT} = 0} & \mathbf{Y}_{12} = \frac{\mathbf{I}_{IN}}{\mathbf{E}_{OUT}} \Big|_{\mathbf{E}_{IN} = 0} \\ \mathbf{Y}_{21} = \frac{\mathbf{I}_{OUT}}{\mathbf{E}_{IN}} \Big|_{\mathbf{E}_{OUT} = 0} & \mathbf{Y}_{22} = \frac{\mathbf{I}_{OUT}}{\mathbf{E}_{OUT}} \Big|_{\mathbf{E}_{IN} = 0}$$

PARÁMETROS DE TRANSMISIÓN DIRECTA ABCD

$$A = \frac{Z_{11}}{Z_{21}} = \frac{E_{IN}}{E_{OUT}} \bigg|_{I_{OUT} = 0} \qquad B = \frac{\Delta z}{Z_{21}} = \frac{E_{IN}}{I_{OUT}} \bigg|_{E_{OUT} = 0}$$

$$C = \frac{1}{Z_{21}} = \frac{I_{IN}}{E_{OUT}} \bigg|_{I_{OUT} = 0} \qquad D = \frac{Z_{22}}{Z_{21}} = \frac{I_{IN}}{I_{OUT}} \bigg|_{E_{OUT} = 0}$$

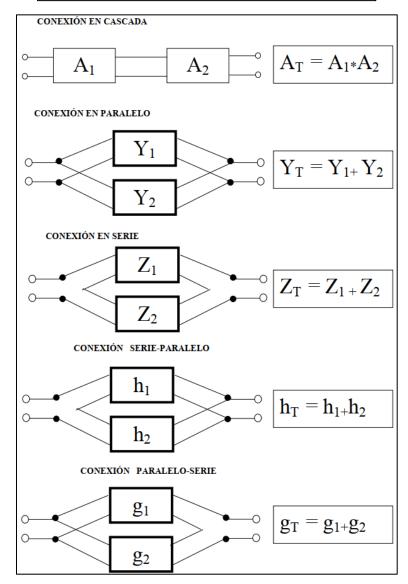
PARÁMETROS DE TRANSMISIÓN INVERSA EFGH

$$\begin{aligned} \mathbf{E} &= \frac{\mathbf{Z}_{22}}{\mathbf{Z}_{12}} = \frac{\mathbf{E}_{\text{OUT}}}{\mathbf{E}_{\text{IN}}} \bigg|_{\mathbf{I}_{\text{IN}} = 0} & \mathbf{F} &= \frac{\Delta_{Z}}{\mathbf{Z}_{12}} = \frac{\mathbf{E}_{\text{OUT}}}{\mathbf{I}_{\text{IN}}} \bigg|_{\mathbf{E}_{\text{IN}} = 0} \\ \mathbf{G} &= \frac{1}{\mathbf{Z}_{12}} = \frac{\mathbf{I}_{\text{OUT}}}{\mathbf{E}_{\text{IN}}} \bigg|_{\mathbf{I}_{\text{IN}} = 0} & \mathbf{H} &= \frac{\mathbf{Z}_{11}}{\mathbf{Z}_{12}} = \frac{\mathbf{I}_{\text{OUT}}}{\mathbf{I}_{\text{IN}}} \bigg|_{\mathbf{E}_{\text{IN}} = 0} \end{aligned}$$

De los parámetros de transmisión directa e inversa:

$$A = H = \frac{Z_{11}}{Z_{12}}$$
 $B = F = \frac{\Delta z}{Z_{12}}$ $C = G = \frac{1}{Z_{12}}$ $D = E = \frac{Z_{22}}{Z_{12}}$

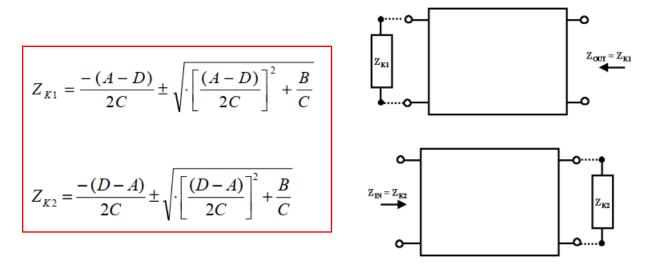
$$\Delta_{ABCD} = \Delta_{EFGH} = 1$$



CUADRIPOLOS CARGADOS

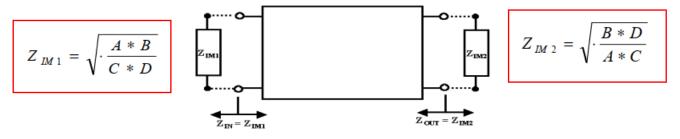
IMPEDANCIA ITERATIVA

Se carga solo uno de los extremos del cuadripolo. Cuando se carga un cuadripolo con su impedancia iterativa de entrada, el mismo valor se obtiene como impedancia de salida y si se carga con su impedancia iterativa de salida, el mismo valor se obtiene como impedancia de entrada.



IMPEDANCIA IMAGEN

Se cargan ambos extremos del cuadripolo simultáneamente.

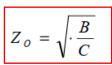


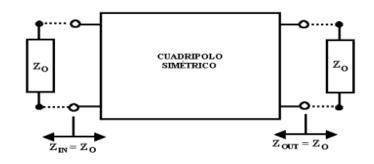
IMPEDANCIA CARACTERÍSTICA

Se produce en cuadripolos simétricos. Partiendo de la impedancia imagen podemos obtener la impedancia característica.

$$Z_o = \sqrt{\frac{A*B}{C*D}} = \sqrt{\frac{B*D}{A*C}}$$

Pero A = D pues Z_{11} = Z_{22} , por lo tanto:





Se puede verificar idéntico resultado a partir de las expresiones de impedancia iterativa en un cuadripolo simétrico donde A = D.

$$Z_{K1} = \frac{-(A-D)^{0}}{2C} \pm \sqrt{\left[\frac{(A-D)^{0}}{2C}\right]^{2} + \frac{B}{C}} = \sqrt{\frac{B}{C}} \qquad Z_{K2} = \frac{-(D-A)^{0}}{2C} \pm \sqrt{\left[\frac{(D-A)^{0}}{2C}\right]^{2} + \frac{B}{C}} = \sqrt{\frac{B}{C}}$$

FUNCIONES DE PROPAGACIÓN

Todas parten de:

$$\boxed{\frac{E_{IN}}{E_{OUT}} = A + \frac{B}{Z_{OUT}}} \boxed{\frac{I_{IN}}{I_{OUT}} = C * Z_{OUT} + D}$$

Para $Z_{OUT} = Z_{K2}$

$$\left|\frac{I_{IN}}{I_{OUT}}\right|_{Z_{K2}} = \left|\frac{E_{IN}}{E_{OUT}}\right|_{Z_{K2}} = \frac{(A+D)}{2} + \sqrt{\left[\frac{(A+D)}{2}\right]^2 - 1} = \cosh \ \gamma + senh \ \gamma = e^{\gamma} = e^{\alpha + j\beta} = e^{\alpha} \times e^{j\beta}$$

Para $Z_{OUT} = Z_{IM2}$

$$\left|\frac{E_{IN}}{E_{OUT}}\right|_{Z_{IM2}} = \sqrt{\frac{Z_{im1}}{Z_{im2}}} \times \left[\sqrt{A \times D} + \sqrt{(A \times D) - 1}\right] = \sqrt{\frac{Z_{im1}}{Z_{im2}}} \times \left(\cosh \theta + senh \theta\right) = e^{\gamma}$$

Para $Z_{OUT} = Z_{IM2}$

$$\left|\frac{I_{IN}}{I_{OUT}}\right|_{Z_{IM2}} = \sqrt{\frac{Z_{im2}}{Z_{im1}}} \times \left[\sqrt{A \times D} + \sqrt{(A \times D) - 1)}\right] = \sqrt{\frac{Z_{im2}}{Z_{im1}}} \times \left(\cosh \theta + senh \theta\right) = e^{\gamma}$$

Para $Z_{OUT} = Z_O$ ó para el caso de un cuadripolo simétrico, dado que A = D y $Z_{IM1} = Z_{IM2} = Z_O$, todos los casos se reducen a :

$$\frac{I_{IN}}{I_{OUT}} = \frac{E_{IN}}{E_{OUT}} = A + \sqrt{A^2 - 1} = (\cosh \gamma + senh \gamma) = e^{\gamma} = e^{\alpha + j\beta} = e^{\alpha} \times e^{j\beta}$$

γ es la constante de propagación.

 α es la constante de atenuación que se mide en [neper] .

 β es la constante de fase. cuadripolo es resistivo puro $\beta = 0$.

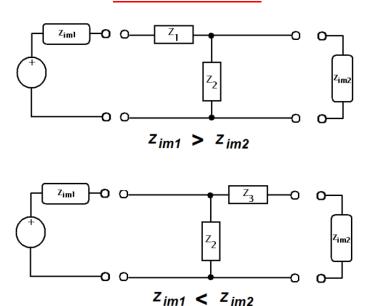
en caso de ser resistivo puro:

$$\frac{E_{IN}}{E_{OUT}} = veces$$
 $y = \ln\left[\frac{E_{IN}}{E_{OUT}}\right] = neper$ $y = 20 \log_{10}\left[\frac{E_{IN}}{E_{OUT}}\right] = dB$

CUADRIPOLOS ADAPTADORES DE IMPEDANCIA

$A = \sqrt{\frac{Zim1}{Zim2}} \bullet \cosh \theta$	$B = \sqrt{Zim1 \bullet Zim2} \bullet senh\theta$
$C = \frac{senh\theta}{\sqrt{Zim1 \bullet Zim2}}$	$D = \sqrt{\frac{Zim2}{Zim1}} \bullet \cosh \theta$

CUADRIPOLO TIPO "L"



Caso Zim1 > Zim2

$$A = \frac{Z_{11}}{Z_{21}} = \frac{Z1 + Z2}{Z2} = \sqrt{\frac{Zim1}{Zim2}} \cdot \cosh \theta \quad \therefore \quad Z1 = \left(\sqrt{\frac{Zim1}{Zim2}} \cdot \cosh \theta * Z2\right) - Z2 \quad [15]$$

$$B = \frac{\Delta_Z}{Z_{21}} = \frac{(Z1 + Z2) \bullet Z2 - Z2^2}{Z2} = Z1 = \sqrt{Zim1 \bullet Zim2} \bullet senh\theta$$

$$\therefore Z1 = \sqrt{Zim1 \bullet Zim2} \bullet senh\theta \qquad [16]$$

$$C = \frac{1}{Z_{21}} = \frac{1}{Z_2} = \frac{senh\theta}{\sqrt{Zim1 \bullet Zim2}} \quad \therefore \quad Z_2 = \frac{\sqrt{Zim1 \bullet Zim2}}{senh\theta}$$
[17]

$$D = \frac{Z_{22}}{Z_{21}} = \frac{Z2}{Z2} = 1 = \sqrt{\frac{Zim2}{Zim1}} \cdot \cosh \theta \quad \therefore \quad \theta = \cosh^{-1} \left(\sqrt{\frac{Zim1}{Zim2}} \right)$$
 [18]

$$\frac{E_{IN}}{E_{OUT}} = \sqrt{\frac{Zim1}{Zim2}} \bullet e^{\theta} \qquad [19]$$

Caso Zim1 > Zim2

$$A = \frac{Z_{11}}{Z_{21}} = \frac{Z^2}{Z^2} = 1 = \sqrt{\frac{Zim1}{Zim2}} \cdot \cosh\theta \quad \therefore \quad \theta = \cosh^{-1}\left(\sqrt{\frac{Zim2}{Zim1}}\right) \quad [19]$$

$$B = \frac{\Delta_Z}{Z_{21}} = \frac{Z_{11} \bullet Z_{22} - Z_{21}^2}{Z_{21}} = \frac{Z2 \bullet (Z2 + Z3) - Z2^2}{Z2} = Z3 = \sqrt{Zim1 \bullet Zim2} \bullet senh\theta$$

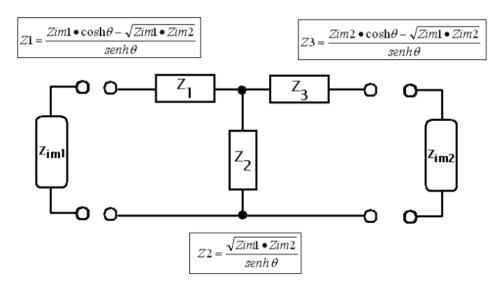
$$\therefore Z3 = \sqrt{Zim1 \bullet Zim2} \bullet senh\theta \qquad [20]$$

$$C = \frac{1}{Z_{21}} = \frac{1}{Z2} = \frac{senh\theta}{\sqrt{Zim1 \bullet Zim2}} \quad \therefore \quad Z2 = \frac{\sqrt{Zim1 \bullet Zim2}}{senh\theta}$$
 [21]

$$D = \frac{Z_{22}}{Z_{21}} = \frac{Z2 + Z3}{Z2} = \sqrt{\frac{Zim2}{Zim1}} \cdot \cosh \theta \quad \therefore \quad Z3 = \left(\sqrt{\frac{Zim2}{Zim1}} \cdot \cosh \theta * Z2\right) - Z2 \quad [22]$$

$$\frac{E_{IN}}{E_{OUT}} = \sqrt{\frac{Zim1}{Zim2}} \bullet e^{\theta} \qquad [23]$$

CUADRIPOLO TIPO "T"



$$\theta = \cosh^{-1}\left(\sqrt{\frac{Zim1}{Zim2}}\right) \ si \ Zim1 \rangle Zim2 \qquad y \qquad \theta = \cosh^{-1}\left(\sqrt{\frac{Zim2}{Zim1}}\right) \ si \ Zim1 \langle Zim2 \rangle Zim2$$

$$\frac{Ein}{Eout} = \sqrt{\frac{Zim1}{Zim2}} \bullet e^{\theta} \qquad \text{Re } cordemos \ ademas \ que \ \frac{Ein}{Eout} = \sqrt{\frac{Zim1}{Zim2}} \bullet e^{a+jb} = \sqrt{\frac{Zim1}{Zim2}} \bullet e^{a} \bullet e^{jb}$$

Si Zim1>Zim2 → Z3=0 o si Zim1<Zim2 → Z1=0 entonces se debe aumentar el valor de Ein/Eout y recalcular theta:

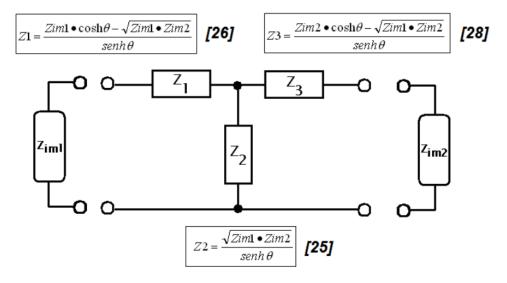
$$\theta = Ln \left(\left[\frac{Ein}{Eout} \right]_{RECALCULADO} \bullet \sqrt{\frac{Zim2}{Zim1}} \right) = a + jb$$

$$a = Ln \left(\left[\frac{Ein}{Eout} \right]_{RECALCULADO} \bullet \sqrt{\frac{Zim2}{Zim1}} \right) \qquad Func. Atenuación \ ALFA_{BASE IMAGEN} \qquad [29]$$

$$b = tg^{-1} \left[\frac{Im \left(\frac{Ein}{Eout} \right)_{RECALCULADO} \bullet \sqrt{\frac{Zim2}{Zim1}}}{Re \left(\frac{Ein}{Eout} \right)_{RECALCULADO} \bullet \sqrt{\frac{Zim2}{Zim1}}} \right] \qquad Fun. \ Fase \ BETA_{BASE IMAGEN} \qquad [30]$$

CUADRIPOLOS ATENUADORES

CUADRIPOLO SIMÉTRICO TIPO "T" RESISTIVO PURO



Al ser simétrico:

$$Z1 = Z3$$

$$Z_{11} = Z_{22}$$

A = D por lo que:

$$Zo = Zim1 = Zim2 = \sqrt{\frac{B}{C}}$$
 luego

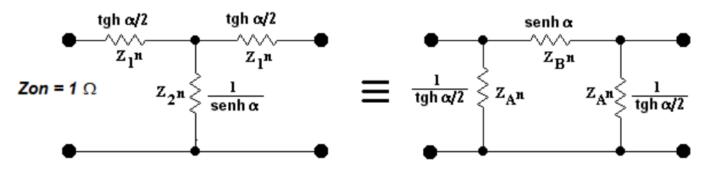
$$Z1 = Z3 = \frac{Zo \cdot \cosh \alpha - Zo}{senh\alpha}$$

$$Z2 = \frac{Zo}{senh\alpha}$$

Si se toma una carga Zon se deberá multiplicar Z1 y Z2 para desnormalizar. Con Zon = 1:

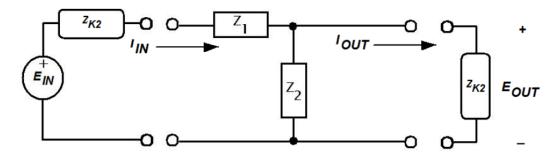
$$Z1n = Z3n = \frac{\cosh \alpha - 1}{\operatorname{senh}\alpha} = \operatorname{tgh}\frac{\alpha}{2}$$

$$Z2n = \frac{Zon}{senh\alpha} = \frac{1}{senh\alpha}$$



$$Z_{\#} = Z_{\#n} * Ro [\Omega]$$

CUADRIPOLO SIMÉTRICO TIPO "L" RESISTIVO PURO



Impedancia de carga y de generador = Zk2

Z1 y Z2 resistivas puras

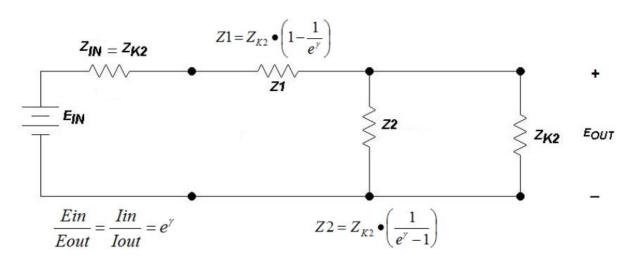
Función de propagación:

/

$$\frac{Ein}{Eout} = \frac{Iin}{Iout} = e^{\gamma}$$

$$Z1 = Z_{K2} \bullet \left(1 - \frac{1}{e^{\gamma}}\right)$$

$$Z2 = Z_{K2} \bullet \left(\frac{1}{e^{\gamma} - 1}\right)$$



<u>IDENTIDADES ALGEBRAICAS DE</u> UTILIDAD

$$senhX = \frac{e^X - e^{-X}}{2}$$

$$\cosh X = \frac{e^X + e^{-X}}{2}$$

$$tgX = \frac{senhX}{\cosh X} = \frac{e^X - e^{-X}}{e^X + e^{-X}}$$

$$\cosh^2 X + senh^2 X = 1$$

$$e^X = \cosh X + senhX$$

$$e^{2X} = \frac{1 + tgX}{1 - tgX}$$

$$senh\frac{X}{2} = \pm \sqrt{\frac{\cosh X - 1}{2}}$$

$$\cosh \frac{X}{2} = \pm \sqrt{\frac{\cosh X + 1}{2}}$$

$$tgh\frac{X}{2} = \frac{\cosh X - 1}{senhX}$$

$$Si \Rightarrow \cosh X = U$$

$$Si \Rightarrow \cosh X = U$$
 $senhX = \sqrt{U^2 - 1}$

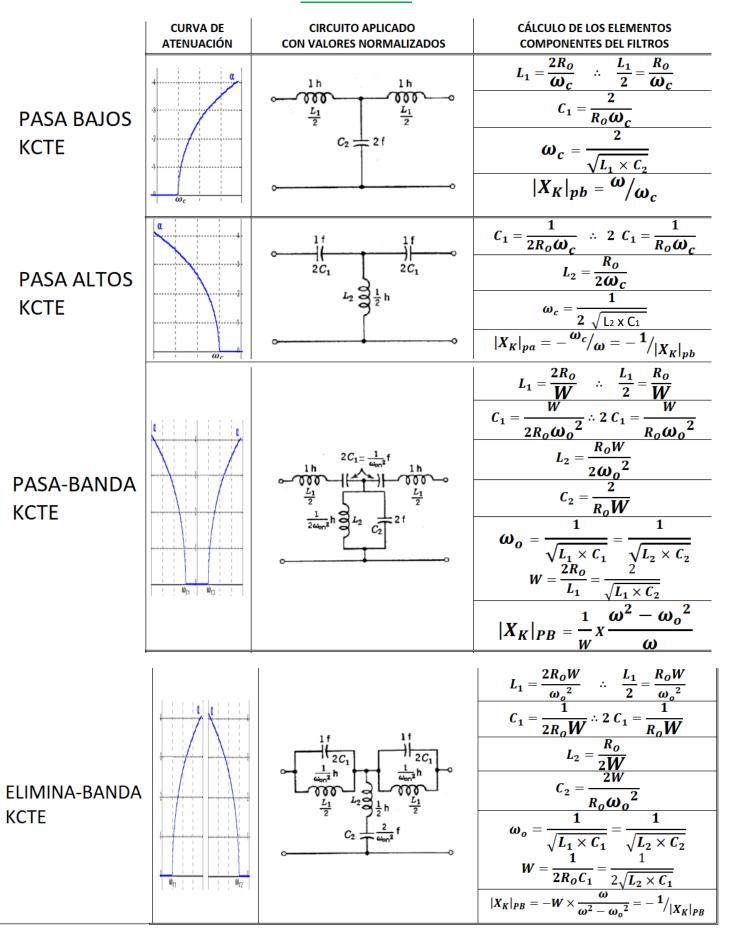
$$Si \Rightarrow senhX = U$$

$$Si \Rightarrow senhX = U$$
 $\cosh X = \sqrt{U^2 + 1}$

$$y = \ln\left(x + \sqrt{1 + x^2}\right) \iff y = \arg\sinh x$$

 $y = \ln\left(x + \sqrt{x^2 - 1}\right) \iff y = \arg\cosh x$
 $y = \frac{1}{2}\ln\left(\frac{1 + x}{1 - x}\right) \iff y = \arg\tanh x$

FILTROS K-CTE



 R_o = Impedancia de carga; ω_C = Pulsación de corte (pb y pa); ω_1 = Pulsación de corte inferior; ω_2 = Pulsación de corte superior; W = Ancho de banda = ω_2 - ω_1 y ω_0 = Pulsación de Resonancia.

$$AB = W = \omega_{c2} - \omega_{c1} \qquad \omega_0 = \sqrt{\omega_{c2} \cdot \omega_{c1}}$$