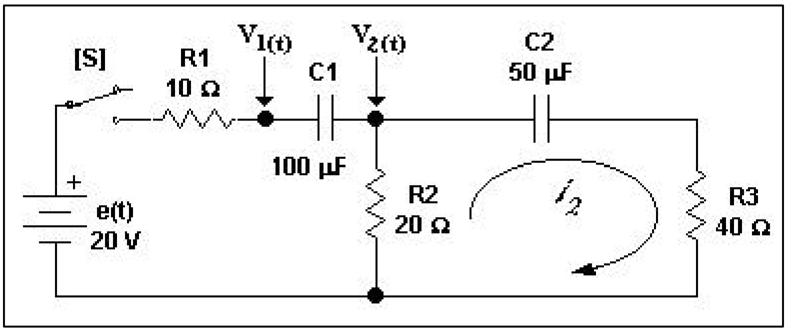
SOLUCIÓN PROBLEMA 20 :

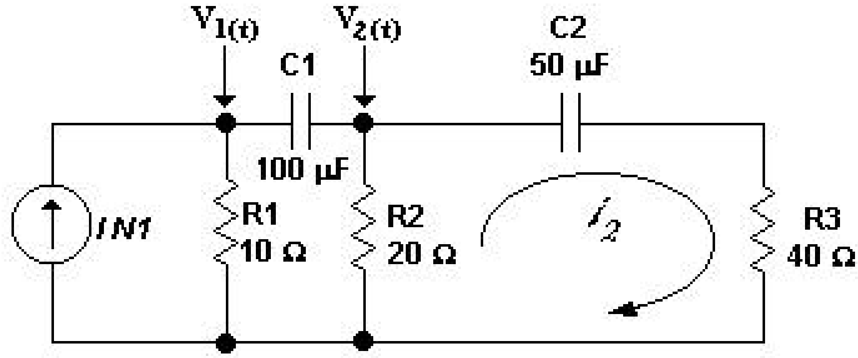
Mediante método nodal, determine las tensiones en los nudos $v_1(t)$ y $v_2(t)$, del siguiente circuito. Calcule también el valor de la corriente $i_2(t)$.



En primer lugar cambiamos la fuente de tensión con el resistor en serie R1, a fuente de corriente con el mismo resistor en paralelo:

Donde:

$$I_{N1} = \frac{20}{p} \bullet \frac{1}{R_1} = \frac{20}{P \bullet 10} = \frac{2}{P}$$



Escribimos las ecuaciones de nudos correspondientes :

$$\boxed{\begin{aligned} \frac{2}{P} &= V_1 Y_{11} - V_2 Y_{12} \\ 0 &= -V_1 Y_{21} + V_2 Y_{22} \end{aligned}}$$

Identificamos las admitancias :

$$Y_{11} = \frac{1}{R_1} + C_1 P = 0{,}1 + 0{,}0001P$$

$$Y_{12} = Y_{12} = C_1 P = 0{,}0001P$$

$$Y_{22} = C_1 P + \frac{1}{R_2} + \frac{1}{R_3 + \frac{1}{C_2 P}} = 0{,}0001P + 0{,}05 + \frac{1}{40 + \frac{20000}{P}}$$

$$Y_{22} = 0{,}0001P + 0{,}05 + \frac{P}{40P + 20000}$$

Calculamos los determinantes principal (Δ_P) y sustitutos (Δ_{S1} y Δ_{S2}) :

$$\Delta_P = \begin{vmatrix} Y_{11} & -Y_{12} \\ -Y_{21} & Y_{22} \end{vmatrix} = \begin{vmatrix} 0{,}1 + 0{,}0001P & -0{,}0001P \\ -0{,}0001P & 0{,}0001P + 0{,}05 + \dfrac{1}{40 + \dfrac{20000}{P}} \end{vmatrix}$$

$$\Delta_P = 0{,}00001P + 0{,}005 + \frac{0{,}1P}{40P + 20000} + \underset{1}{\cancel{10^{-8}}}P^2 + 0{,}000005P + \frac{0{,}0001P^2}{40P + 20000} - \cancel{10^{-8}}P^2$$

$$\Delta_P = 0{,}000015P + 0{,}005 + \frac{0{,}1P}{40P+20000} + \frac{0{,}0001P^2}{40P+20000} =$$

$$\Delta_P = \frac{6 \bullet 10^{-4}P^2 + 0{,}2P + 0{,}3P + 100 + 0{,}1P + 0{,}0001P^2}{40P+20000} = \frac{7 \bullet 10^{-4}P^2 + 0{,}6P + 100}{40P+20000} =$$

$$\Delta_P = \frac{7 \bullet 10^{-4}(P^2 + 857{,}1428571P + 142857{,}1429)}{40(P+500)} =$$

$$\Delta_P = 1{,}75 \bullet 10^{-5}\frac{(P+226{,}5409197) \bullet (P+630{,}6019375)}{P+500}$$

$$\Delta_{S1} = \begin{vmatrix} 2/P & -Y_{12} \\ 0 & Y_{22} \end{vmatrix} = \begin{vmatrix} 2/P & -0{,}0001P \\ 0 & 0{,}0001P + 0{,}05 + \dfrac{1}{40 + \dfrac{20000}{P}} \end{vmatrix}$$

$$\Delta_{S1} = \frac{2}{P} \bullet 0{,}0001P + \frac{2}{P} \bullet 0{,}05 + \frac{\frac{2}{P} \bullet P}{40P+20000} = 0{,}0002 + \frac{0{,}1}{P} + \frac{2}{40P+20000} =$$

$$\Delta_{S1} = \frac{0{,}008P + 4 + 0{,}4 + \dfrac{2000}{P} + 2}{40P+20000} = \frac{\frac{1}{P}\left(0{,}008P^2 + 6{,}4P + 2000\right)}{40(P+500)} =$$

$$\Delta_{S1} = \frac{\frac{0{,}008}{P}\left(P^2 + 800P + 250000\right)}{40(P+500)} = \frac{0{,}0002 \bullet \left(P^2 + 800P + 250000\right)}{P \bullet (P+500)}$$

$$\Delta_{S2} = \begin{vmatrix} Y_{11} & 2/P \\ Y_{21} & 0 \end{vmatrix} = \begin{vmatrix} 0{,}1 + 0{,}0001P & 2/P \\ -0{,}0001P & 0 \end{vmatrix} =$$

$$\Delta_{S2} = \frac{2}{P} \bullet 0{,}0001P = 0{,}0002$$

Así :

$$V_{1(P)} = \frac{\Delta_{S1}}{\Delta_P} = \frac{\dfrac{0{,}0002 \bullet \left(P^2 + 800P + 250000\right)}{P \bullet \cancel{(P+500)}}}{1{,}75 \bullet 10^{-5}\dfrac{(P+226{,}5409197) \bullet (P+630{,}6019375)}{\cancel{P+500}}} =$$

$$V_{1(P)} = \frac{11{,}42857143 \bullet \left(P^2 + 800P + 250000\right)}{P \bullet (P+226{,}5409197) \bullet (P+630{,}6019375)}$$

$$V_{2(P)} = \frac{\Delta_{S2}}{\Delta_P} = \frac{0{,}0002}{1{,}75 \bullet 10^{-5}\dfrac{(P+226{,}5409197) \bullet (P+630{,}6019375)}{P+500}} =$$

$$V_{2(P)} = \frac{11{,}42857143 \bullet (P+500)}{(P+226{,}5409197) \bullet (P+630{,}6019375)}$$

Aplicamos TVI y TVF a las tensiones transformadas $V_{1(P)}$ y $V_{2(P)}$.

$\underline{TVI\ =}$

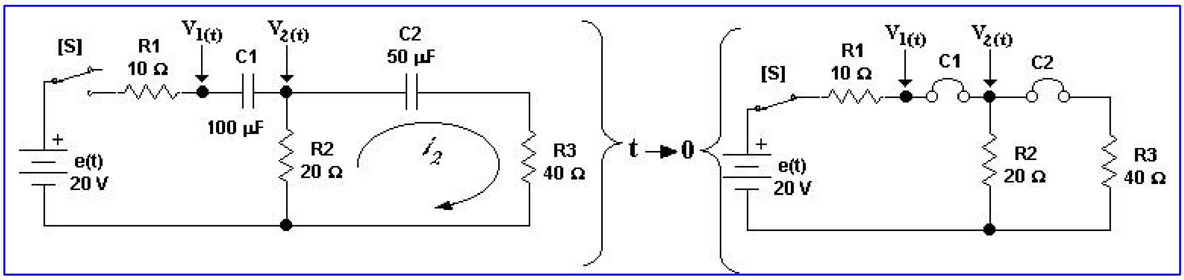
$$\lim|_{P\to\infty}\left(V_{1(P)}\right)\bullet P=\lim|_{P\to\infty}\left(\frac{11{,}42857143\bullet\left(P^2+800P+250000\right)}{P\bullet(P+226{,}5409197)\bullet(P+630{,}6019375)}\right)\bullet P=$$

$$\lim|_{P\to\infty}\left(V_{1(P)}\right)\bullet P=\lim|_{P\to\infty}\left(\frac{11{,}42857143\bullet P^3}{P^3}\right)=11{,}42857143[Volts]$$

$$\lim|_{P\to\infty}\left(V_{2(P)}\right)\bullet P=\lim|_{P\to\infty}\left(\frac{11{,}42857143\bullet(P+500)}{(P+226{,}5409197)\bullet(P+630{,}6019375)}\right)\bullet P=$$

$$\lim|_{P\to\infty}\left(V_{2(P)}\right)\bullet P=\lim|_{P\to\infty}\left(\frac{11{,}42857143\bullet P^2}{P^2}\right)\bullet P=11{,}42857143[Volts]$$

Observando el circuito para t→0 tendremos:



$$V_1=V_2=\frac{20}{R_1+(R_2\,//\,R_3)}\bullet(R_2\,//\,R_3)=\frac{20}{10+\frac{20\bullet 40}{20+40}}\bullet\frac{20\bullet 40}{20+40}=\frac{20\bullet 13{,}333}{10+13{,}333}=11{,}42857[Volts]$$

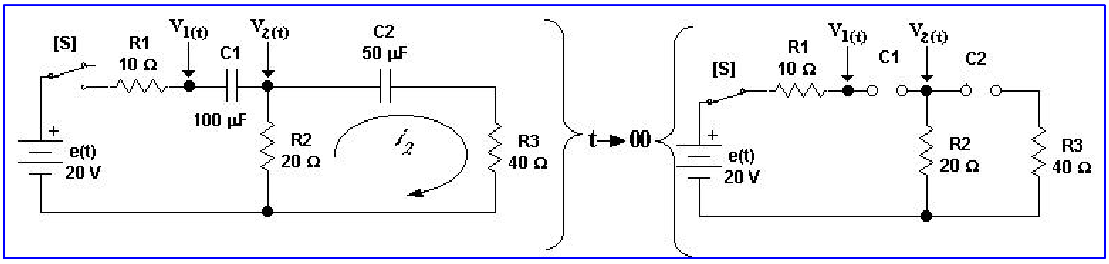
$\underline{TVF\ =}$

$$\lim|_{P\to 0}\left(V_{1(P)}\right)\bullet P=\lim|_{P\to 0}\left(\frac{11{,}42857143\bullet\left(P^2+800P+250000\right)}{\cancel{P}\bullet(P+226{,}5409197)\bullet(P+630{,}6019375)}\right)\bullet\cancel{P}=$$

$$\lim|_{P\to\infty}\left(V_{1(P)}\right)\bullet P=\lim|_{P\to\infty}\left(\frac{11{,}42857143\bullet 250000}{(226{,}5409197)\bullet(630{,}6019375)}\right)=20[Volts]$$

$$\lim|_{P\to\infty}\left(V_{2(P)}\right)\bullet P=\lim|_{P\to\infty}\left(\frac{11{,}42857143\bullet(P+500)}{(P+226{,}5409197)\bullet(P+630{,}6019375)}\right)\bullet P=0[Volts]$$

Observando el circuito para t→∞ tendremos:



Vemos que para t→∞ V_1 = 20 [Volts] y V_2 = 0 [Volts] .

Luego expandimos $V_{1(P)}$ y $V_{2(P)}$ en fracciones parciales simples para antitransformar:

$$V_{1(P)} = \frac{11{,}42857143 \bullet \left(P^2 + 800P + 250000\right)}{P \bullet (P + 226{,}5409197) \bullet (P + 630{,}6019375)} =$$

$$V_{1(P)} = \frac{20}{P} - \frac{14{,}99333126}{(P + 226{,}5409197)} + \frac{6{,}421902688}{(P + 630{,}6019375)}$$

$$V_{2(P)} = \frac{11{,}42857143 \bullet (P + 500)}{(P + 226{,}5409197) \bullet (P + 630{,}6019375)} =$$

$$V_{2(P)} = \frac{7{,}734590804}{(P + 226{,}5409197)} + \frac{3{,}69398626}{(P + 630{,}6019375)}$$

De esta manera :

$$v_{1(t))} = 20 - 14{,}99333126e^{-226{,}5409197\,t} + 6{,}421902688e^{-630{,}6019375\,t}$$

$$v_{2(t)} = 7{,}734590804e^{-226{,}5409197\,t} + 3{,}69398626e^{-630{,}6019375\,t}$$

Queda para el alumno aplicar TVI y TVF para comprobar los resultados.

Luego, para determinar el valor de la corriente $I_{2(P)}$, hacemos :

$$I_{2(P)} = \frac{V_{2(P)}}{R_3 + \frac{1}{C_2 P}} = \frac{11{,}42857143 \bullet (P + 500)}{(P + 226{,}5409197) \bullet (P + 630{,}6019375)} \bullet \frac{1}{40 + \frac{20000}{P}} =$$

$$I_{2(P)} = \frac{11{,}42857143 \bullet (P + 500)}{(P + 226{,}5409197) \bullet (P + 630{,}6019375)} \bullet \frac{1}{\frac{40}{P} \bullet (P + 500)} =$$

$$I_{2(P)} = \frac{0{,}285714285 \bullet P}{(P + 226{,}5409197) \bullet (P + 630{,}6019375)}$$

Aplicamos TVI y TVF y observamos que TVI = 0,285714285 [Amperes] y TVF = 0 [Amperes].
Del circuito observamos que para $t \to 0$:

$$i_{2(t)} = \frac{20}{R_1 + (R_2 // R_3)} \bullet (R_2 // R_3) \bullet \frac{1}{R_3} = \frac{20}{10 + \frac{20 \bullet 40}{20 + 40}} \bullet \frac{20 \bullet 40}{20 + 40} \bullet \frac{1}{40} = \frac{20 \bullet 13{,}333}{10 + 13{,}333} \bullet \frac{1}{40} = \frac{11{,}42857}{40} =$$

$$i_{2(t)} = 0{,}28571425\,[Amperes]$$

Mientras que para $t \to \infty$ dado que C_1 y C_2 son circuito abiertos **$i_{2(t)}$ = 0 [Amperes]**

Finalmente antitransformando $I_{2(P)}$ tendremos :

$$\boldsymbol{i_{2(t)} = -\,0{,}16018862\; e^{-\,226{,}5409197\,t} + 0{,}445902905\; e^{-\,630{,}6019375\,t}}$$

Aplicamos TVI y TVF y vemos que

$$\boldsymbol{TVI = -\,0{,}16018862\; e^{0} + 0{,}445902905\; e^{0} = 0{,}285714285\ [Amperes]}$$
$$\boldsymbol{TVF = -\,0{,}16018862\; e^{-\,¥} + 0{,}445902905\; e^{¥} = 0\ [Amperes]}$$