

RESUMEN DE CONSIGNAS CRITERIO DE NYQUIST

TIPO DE FUNCIÓN	FUNCIÓN DE TRANSFERENCIA TOTAL	FUNCIÓN DE TRANSFERENCIA DE LAZO ABIERTO
FUNCIÓN DE TRANSFERENCIA	$F(P)$	$G(P) \cdot H(P)$
DIAGRAMA EN BLOQUES DEL SISTEMA		
TRAZAR EL DIAGRAMA POLAR	EMPLEE "RECETA DE COCINA" DESDE EL PASO 1 HASTA EL PASO 11	EMPLEE "RECETA DE COCINA" DESDE EL PASO 1 HASTA EL PASO 10
LUGAR DE OBSERVACIÓN DE LOS RODEOS		
CONCLUSIONES SI: $N = Z - P$ $N = 0$	NO SE SABE POR CRITERIO DE NYQUIST. APLICAR OTRO MÉTODO	NO SE SABE POR CRITERIO DE NYQUIST. APLICAR OTRO MÉTODO
CONCLUSIONES SI: $N = Z - P$ $N = \text{Número positivo}$	NO SE SABE POR CRITERIO DE NYQUIST. APLICAR OTRO MÉTODO	INESTABLE
CONCLUSIONES SI: $N = Z - P$ $N = \text{Número negativo}$	INESTABLE	NO SE SABE POR CRITERIO DE NYQUIST. APLICAR OTRO MÉTODO

NYQUIST

Final Micaela 2020.

Función de Lazo Abierto $G(p) \cdot H(p)$ trae el Diagrama Polar y aplique criterio de Nyquist.

$$G \cdot H(p) = \frac{15(p-2)}{p^3 + 6p^2 + 10p}$$

$\neq p \in \{2, [0 - 3 + i - 3 - i], 15\}$

$$G \cdot H(p) = \frac{15p - 30}{p^3 + 6p^2 + 10p}$$

Paso 1: Origen del diagrama polar.

$$\left. F(p) \right|_{p \rightarrow 0}$$

$$F(p) \Big|_{p \rightarrow 0} = \frac{K_{TE}}{p^K} \Big|_{p \rightarrow 0} = \frac{-30}{p^K} = \infty$$

$$F(p) \Big|_{p \rightarrow 0} = \frac{K_{TE}}{(3e^{j0})^K} \Big|_{p \rightarrow 0} = \infty \angle -90^\circ \cdot K - 180^\circ = \infty \angle -90 \cdot 1 - 180^\circ$$

$= \infty \angle -270^\circ$

Paso 2: Punto final del diagrama polar.

$$p \rightarrow \infty$$

$$\lim_{p \rightarrow \infty} \frac{15p}{p^3} = 0 \angle -90^\circ \cdot K = 0 \angle -90^\circ \cdot 2 = 0 \angle -180^\circ$$

Paso 3: Cambio $P \rightarrow j\omega$

$$\begin{aligned}
 P^3 &= -j\omega^3 \\
 P^2 &= -\omega^2 \\
 P &= j\omega
 \end{aligned}
 \left. \right\} = \frac{15j\omega - 30}{-\omega^3 - 6\omega^2 + 10j\omega} = \frac{-30 + 15j\omega}{-\omega^2 - j(\omega^3 - 10\omega)}$$

$$= \frac{-30 + 15j\omega}{-\omega^2 - j(\omega^3 - 10\omega)} \cdot \frac{-6\omega^2 + j(\omega^3 - 10\omega)}{-6\omega^2 + j(\omega^3 - 10\omega)}$$

$$= \frac{180\omega^2 - j30\omega^3 + j300\omega - j90\omega^3 - 15\omega^4 + 150\omega^2}{36\omega^4 - j6\omega^5 + j60\omega^3 + j6\omega^5 + \omega^6 + j10\omega^4 + j60\omega^3 - j10\omega^4 + 10\omega^2}$$

$$= \frac{180\omega^2 + 150\omega^2 - 15\omega^4 + j(-120\omega^3 + 300\omega)}{36\omega^4 + \omega^6 + 100\omega^2 + j(-16\omega^4)}$$

$$= \frac{-15\omega^4 + 330\omega^2 + j(-120\omega^3 + 300\omega)}{\omega^6 - 16\omega^4 + 100\omega^2}$$

$$= \frac{-15\omega^4 + 330\omega^2}{\omega^6 + 16\omega^4 + 100\omega^2} + j \sqrt{\frac{-120\omega^3 + 300\omega}{\omega^6 + 16\omega^4 + 100\omega^2}}$$

Paso 4
 $\text{Re} + j\text{Im}$

Paso 5: Saco la raíz de la parte real.

$$-15\omega^4 + 330\omega^2 = 0$$

$$\omega^2(-15\omega^2 + 330) = 0$$

$$\omega_1 = \sqrt{22} \quad \text{Evaluo en la parte Img para carté}$$

$$\omega_1 = 4,690 \quad \text{sobre eje Img. (tomo siempre raíz positiva)}$$

Corte al eje imaginario:

$$6) \left. j \frac{-120 w^3 + 300w}{w^6 + 16w^4 + 100w^2} \right|_{\sqrt{22}} = -0,53300$$

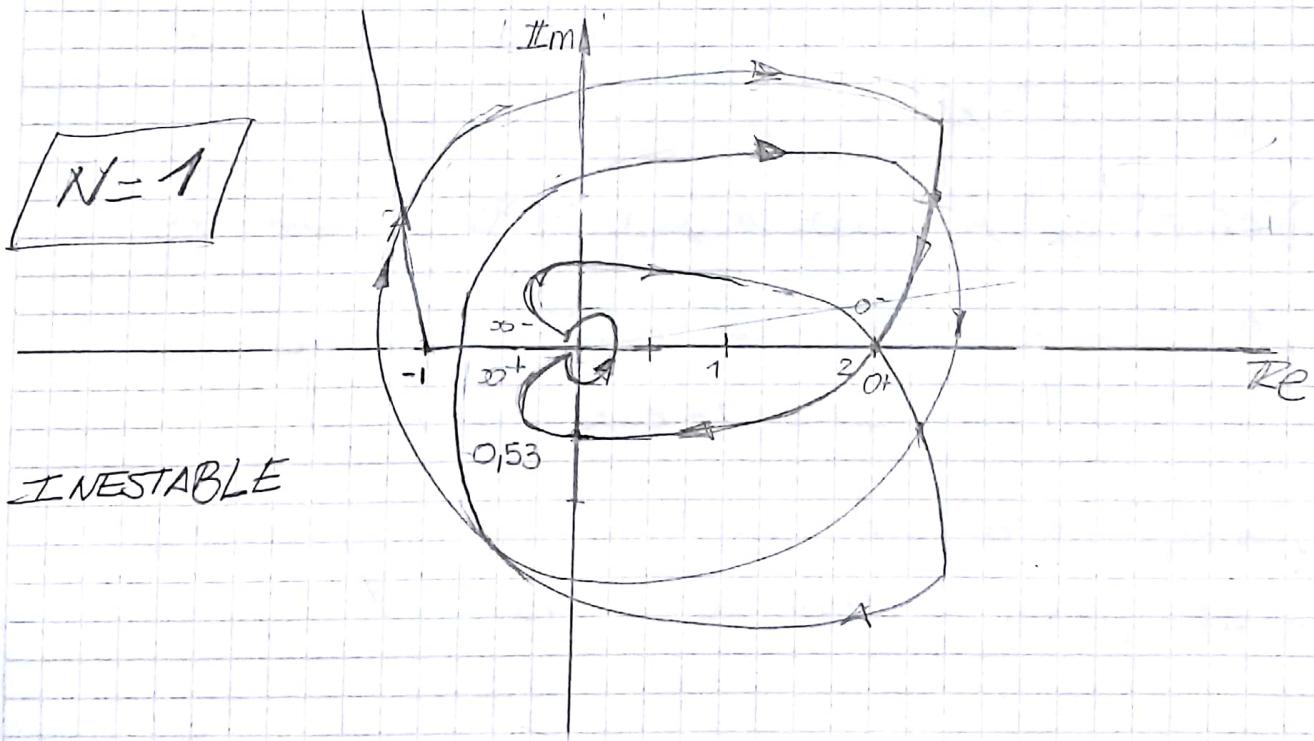
7) Obtengo Raíces de la parte imaginaria.

$$-120 w^3 + 300 w = 0$$

$$w(-120 w^2 + 300) = 0 \Rightarrow \frac{\sqrt{10}}{2} = 1,5811$$

8) Corte en el eje Real

$$\left. \frac{-15 w^4 + 330 w^2}{w^6 + 16w^4 + 100w^2} \right|_{\frac{\sqrt{10}}{2}} \Rightarrow w_1 = \frac{26}{5} = 5,2 \quad w_2 = 2,$$



Paso 90 Creree de $0^+ a 0^- \rightarrow 3 \times -180^\circ = -570^\circ$

2020

Final de Rojas.

Nyquist.

Idem al 05/12/2017
y 21/02/2018

$$G(P)H(P) = \frac{10P - 10}{P^3 + 4P^2 + 8P}$$

$$zpk([1], [0 -2+2j -2-2j], 10)$$

Paso 1: Inicio del Diagrama para $P \rightarrow 0$

$$P \rightarrow 0 \quad G(P)H(P) = \frac{-10}{P^n} = \infty \angle -90^\circ \times n = 180^\circ$$

$$G(P)H(P)|_{P \rightarrow 0} = -\frac{10}{P}|_{P \rightarrow 0} = 100 \angle -270^\circ$$

Paso 2: Final del Diagrama para $P \rightarrow \infty$

$$G(P)H(P)|_{P \rightarrow \infty} = \frac{10P}{P^3} = \frac{10}{P^2} = 100 \angle -90^\circ \times n = 100 \angle -180^\circ$$

Paso 3: Cambio P por jw

$$\begin{aligned} P^3 &= -jw^3 \\ P^2 &= -w^2 \\ P &= jw \end{aligned} \Rightarrow G(jw)H(jw) = \frac{10jw - 10}{-jw^3 + 4(-w^2) + 8jw} = \frac{10jw - 10}{-4w^2 + j(8w - w^3)}$$

$$= \frac{-10 + 10jw}{-4w^2 - j(w^3 - 8w)} \cdot \frac{-4w^2 + j(w^3 - 8w)}{-4w^2 + j(w^3 - 8w)}$$

$$= \frac{40w^2 - 10jw^3 + 30jw - 10jw^3 - 10w^4 + 80w^2}{16w^4 + w^6 - 8w^4 - 8w^4 + 64w^2}$$

$$= \frac{120w^2 - 10w^4 - j(50w^3 - 80w)}{w^6 + 64w^2} = \underbrace{\frac{120w^2 - 10w^4}{w^6 + 64w^2}}_{\text{Re}} + \underbrace{\frac{j(80w - 50w^3)}{w^6 + 64w^2}}_{\text{Im}}$$

Paso 6:

$$G(j\omega)H(j\omega) = \frac{120\omega^2 - 10\omega^4}{\omega^6 + 64\omega^2} + j \cdot \frac{(-50\omega^3 + 80\omega)}{\omega^6 + 64\omega^2}$$

Paso 5: $\text{Re}/\omega = 0$

$$\frac{120\omega^2 - 10\omega^4}{\omega^6 + 64\omega^2} = 0 \Rightarrow 120\omega^2 - 10\omega^4 = 0$$

Raíces = $\pm\sqrt{12} = 2\sqrt{3} = 3,464$

Paso 6: Corte al eje ~~Real~~ Imaginario

Evaluo la parte Imag en $\omega = 2\sqrt{3} = 3,464$

$$\left. \frac{-50\omega^3 + 80\omega}{\omega^6 + 64\omega^2} \right|_{\omega=2\sqrt{3}} = [-0,7216]$$

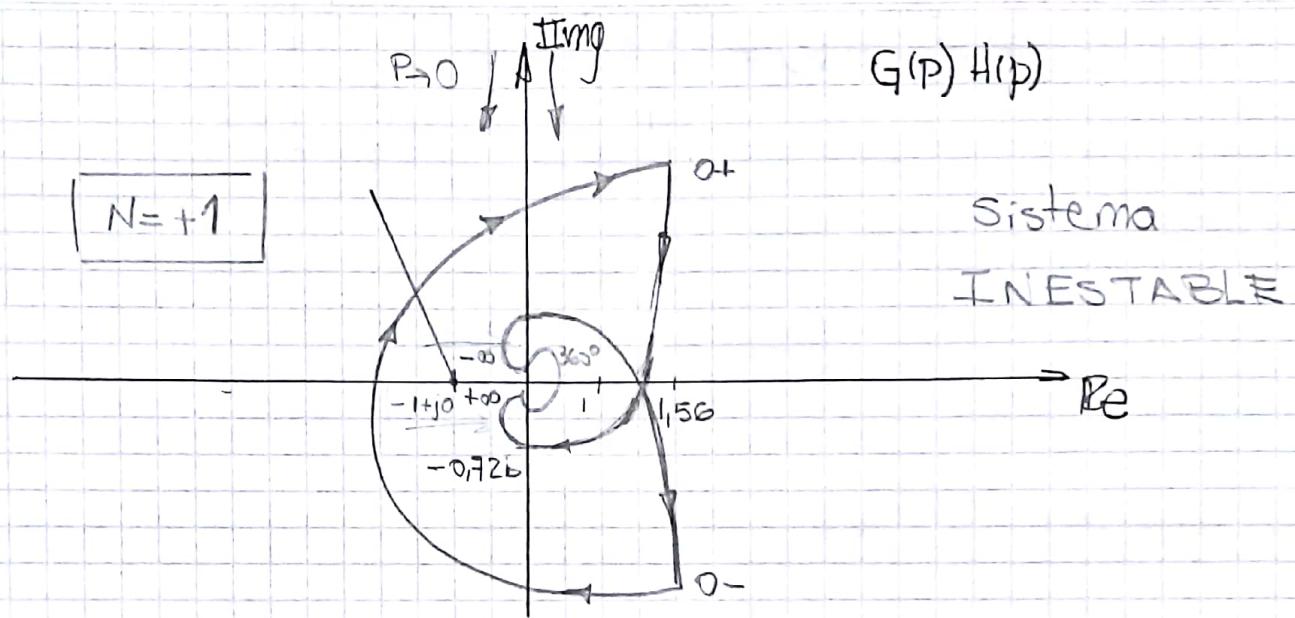
Paso 7: $\text{Imag}/\omega = 0$

$$-50\omega^3 + 80\omega = 0 \Rightarrow \text{Raíces: } \pm\frac{2\sqrt{10}}{5} \quad \omega = \frac{2\sqrt{10}}{5} = 1,2649$$

Paso 8: Corte al eje Real:

$$\left. \frac{120\omega^2 - 10\omega^4}{\omega^6 + 64\omega^2} \right|_{\omega=\frac{2\sqrt{10}}{5}} = 1,5625$$

Paso 9: Cantidad de rodeos al punto $-1+j0$



Paso 90:
Para cerrar el diagrama.

$$P \rightarrow \infty \quad \frac{Kt}{P^2} \Rightarrow 2 \times 180 = 360$$

$$P \rightarrow 0 \quad \frac{Kt}{P} \Rightarrow 1 \times 180 = 180$$

Final 08/05/2019

$$\boxed{G(P) H(P) = \frac{10 P + 50}{P(P^2 - P + 4)}} = \frac{10 P + 50}{P^3 - P^2 + 4P}$$

1) Inicio del Diagrama para $P \rightarrow 0$

$$\left| \frac{G(P) H(P)}{P \rightarrow 0} \right| = \frac{50}{P} = \infty \angle -90^\circ$$



2) Fin del Diagrama para $P \rightarrow \infty$

$$\left| \frac{G(P) H(P)}{P \rightarrow \infty} \right| = \frac{10P}{P^3} = \frac{10}{P^2} = 0 \angle -180^\circ$$



3) Cambio de $P \rightarrow j\omega$

$$P \rightarrow j\omega, P^2 \rightarrow -\omega^2 \quad G(j\omega) H(j\omega) = \frac{10 j\omega + 50}{j\omega(-\omega^2 - j\omega + 4)}$$

$$= \frac{50 + j10\omega}{-\omega^2 + j4\omega} = \frac{50 + j10\omega}{\omega^2 + j(4\omega - \omega^3)} \cdot \frac{\omega^2 - j(4\omega - \omega^3)}{\omega^2 - j(4\omega - \omega^3)}$$

$$= \frac{50\omega^2 - j200\omega + 50\omega^3 + j10\omega^3 + 40\omega^2 - 10\omega^4}{\omega^4 + (4\omega - \omega^3)^2}$$

$$= \frac{90\omega^2 + 50\omega^3 - 10\omega^4 + j(10\omega^3 - 200\omega)}{\omega^4 + \omega^6 - 8\omega^4 + 16\omega^2} \quad X$$

$$= \frac{90\omega^2 - 10\omega^4}{\omega^6 - 7\omega^4 + 16\omega^2} + j \frac{(60\omega^3 - 200\omega)}{\omega^6 - 7\omega^4 + 16\omega^2}$$

Re

Imag

Correcto

$$4) \operatorname{Re}/\omega = 0 \quad 90\omega^2 - 10\omega^4 = 0$$

$$10\omega^2 = 90 \Rightarrow \omega = \sqrt{9} = \boxed{3}$$

5) Corte al eje Imaginario

$$\frac{60\omega^3 - 200\omega}{\omega^6 - 7\omega^4 + 16\omega^2} \Big|_{\omega=3} = 3,33$$

6) $\operatorname{Im}/\omega = 0$

$$60\omega^3 - 200\omega = 0 \quad \text{Raíces} = \frac{\sqrt{30}}{3} \Rightarrow \omega = 1,825741$$

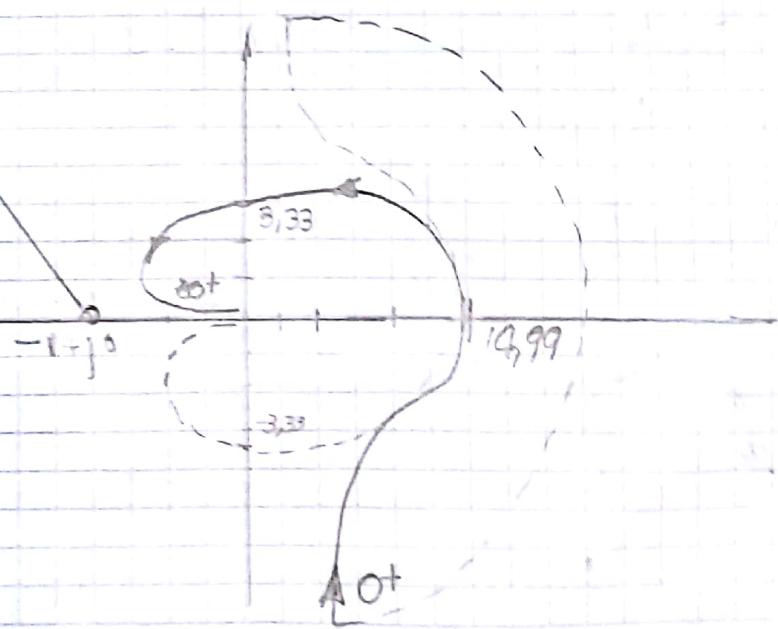
7) Corte al eje Real

$$\frac{90\omega^2 - 10\omega^4}{\omega^6 - 7\omega^4 + 16\omega^2} \Big|_{\omega=1,825} = 14,99$$

NO SE SABE

$N=0$

$N=0$



Final 21/02/2018 (idem al sept 2020 de Rgas)

$$G(p) + H(p) = \frac{10(p-1)}{p^3 + 4p^2 + 8p} = \frac{10p - 10}{p^3 + 4p^2 + 8p}$$

1) Inicio para $p \rightarrow 0$

$$\left. G(p) H(p) \right|_{p \rightarrow 0} = \frac{-10}{p} = \omega \angle -90^\circ - 180^\circ = \omega \angle -270^\circ$$

2) Final para $p \rightarrow \infty$

$$\left. G(p) H(p) \right|_{p \rightarrow \infty} = \frac{10p}{p^3} = \frac{10}{p^2} = 0 \angle -180^\circ$$

3) $p \rightarrow j\omega$ $p = j\omega$, $p^2 = -\omega^2$, $p^3 = -j\omega^3$

$$\begin{aligned} & \text{Resolução: } \frac{10j\omega - 10}{-j\omega^3 + 4(-\omega^2) + 8j\omega} = \frac{-10 + j10\omega}{-4\omega^2 + j(8\omega - \omega^3)} \cdot \frac{4\omega^2 - j(8\omega - \omega^3)}{4\omega^2 - j(8\omega - \omega^3)} \\ &= \frac{40\omega^2 + j80\omega - 10j\omega^3 - j40\omega^3 + 80\omega^2 - 10\omega^4}{16\omega^4 + (8\omega - \omega^3)^2} \\ &= \frac{120\omega^2 - 10\omega^4 + j(-50\omega^3 + 80\omega^2 + 80\omega)}{\omega^6 + 64\omega^2} \\ &= \underbrace{\frac{120\omega^2 - 10\omega^4}{\omega^6 + 64\omega^2}}_{\text{Re}} + j \underbrace{\frac{(-50\omega^3 + 80\omega^2 + 80\omega)}{\omega^6 + 64\omega^2}}_{\text{Imag}} \end{aligned}$$

4

Final ~~05/12/2017~~

$$G(P) H(P) = \frac{-10(P+10)}{P^3 + 4P^2 + 8P} = \frac{10P + 100}{P^3 + 4P^2 + 8P}$$

1) Inicio para $P \rightarrow 0$

$$\left| G(P) H(P) \right|_{P \rightarrow 0} = \frac{+100}{P} = \text{constante} \Rightarrow 90^\circ$$

2) Fin para $P \rightarrow \infty$

$$\left| G(H(P)) \right|_{P \rightarrow \infty} = \frac{10P}{P^3} = \frac{10}{P^2} = 0 \text{ } L-180^\circ$$

3) $P \rightarrow j\omega$; $P = j\omega$; $P^2 = -\omega^2$, $P^3 = -j\omega^3$

$$\begin{aligned} \frac{10j\omega + 100}{-j\omega^3 + 4(-\omega^2) + 8j\omega} &= \frac{100 + j10\omega}{-4\omega^2 + j(8\omega - \omega^3)} \cdot \frac{-4\omega^2 - j(8\omega - \omega^3)}{-4\omega^2 - j(8\omega - \omega^3)} \\ &= \frac{-400\omega^2 - j800\omega + j100\omega^3 - j40\omega^3 + 80\omega^2 - 10\omega^4}{\omega^6 + 64\omega^2} \\ &= \frac{-320\omega^2 - 10\omega^4 + j(60\omega^3 - 800\omega)}{\omega^6 + 64\omega^2} \end{aligned}$$

$$4) \text{Re}|_{\omega=0} -320\omega^2 - 10\omega^4 = 0 \Rightarrow \omega = j4\sqrt{2}$$

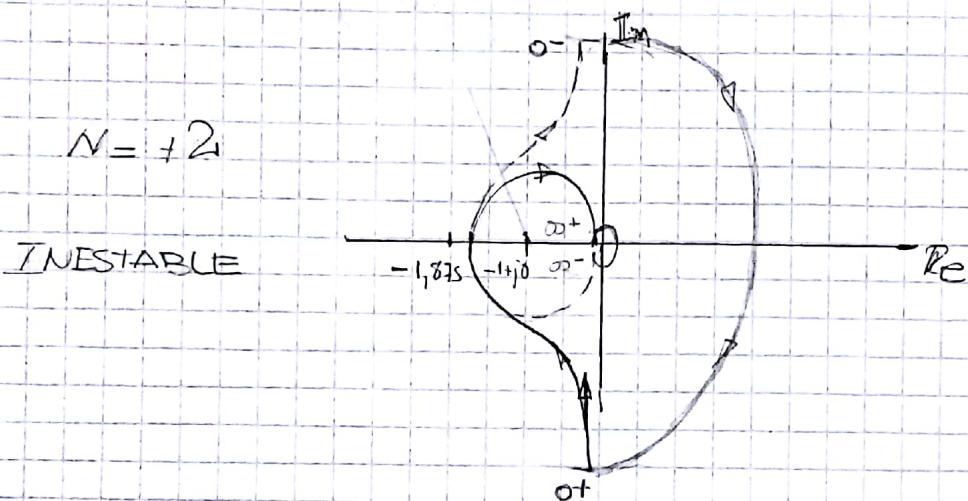
5) Si la frecuencia ω es un número real, por lo queNo hay corte al eje Imaginario.

$$6) \text{Im } / \omega = 0$$

$$60\omega^3 - 800\omega = 0 \quad \omega = \frac{2\sqrt[3]{30}}{3} = 3,6514$$

7) Corte al eje Real

$$\frac{-320\omega^3 - 10\omega^4}{\omega^6 + 64\omega^2} \Big|_{\omega = \frac{2\sqrt[3]{30}}{3}} = -\frac{15}{8} = -1,875$$



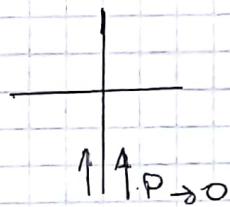
Final
30/05/2010

Nyquist.

$$G(P) \cdot H(P) = \frac{20(P+4)}{P^3 + 2P^2 + 4P} = \frac{20P + 80}{P^3 + 2P^2 + 4P}$$

1) Inicio para $P \rightarrow 0$

$$G(P) H(P) \Big|_{P \rightarrow 0} = \frac{80}{4P} = \infty \angle -90^\circ$$



2) Final para $P \rightarrow \infty$

$$G(P) H(P) \Big|_{P \rightarrow \infty} = -\frac{20P}{P^3} = \frac{20}{P^2} = 0 \angle -180^\circ \quad \begin{array}{c} \uparrow \\ P \rightarrow \infty \end{array}$$

3) $P \rightarrow jw$ $P = jw; P^2 = -w^2; P^3 = -jw^3$

$$\begin{aligned} G(jw) H(jw) &= \frac{20jw + 80}{-jw^3 + 2(-w^2) + 4jw} = \frac{80 + j20w}{-2w^2 + j(4w - w^3)} \cdot \frac{-2w^2 - j(4w - w^3)}{-2w^2 - j(4w - w^3)} \\ &= \frac{-20w^4 - 80w^2}{w^6 - 4w^4 + 16w^2} + j \frac{40w^3 - 320w}{w^6 - 4w^4 + 16w^2} \end{aligned}$$

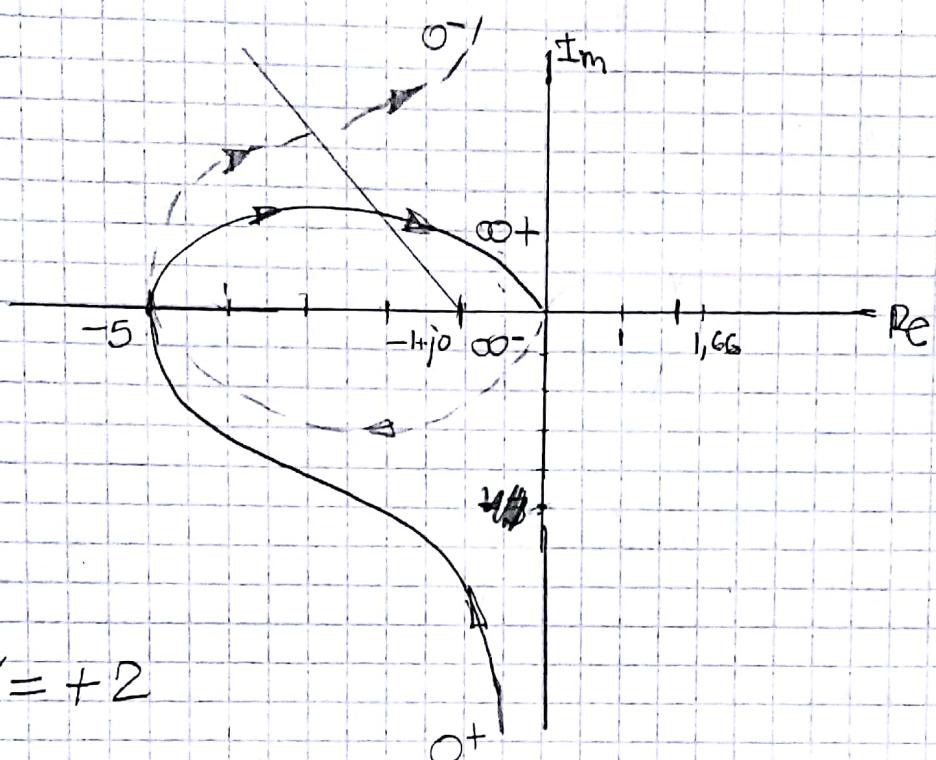
4) $\operatorname{Re} / \omega = 0$

$$-20w^4 - 80w^2 = 0 \Rightarrow w = 0 \quad (w \text{ es un imaginario} \Rightarrow \text{No hay})$$

5) $\operatorname{Im} / \omega = 0 \quad \text{Corte al eje Imaginario}$

$$40w^3 - 320w = 0 \Rightarrow w = 2\sqrt{2}$$

$$7) \text{Re} \Big|_{\omega=2\sqrt{3}} = \frac{-5}{3} = -1,66 \quad \underline{\text{Corte no eixo Real}}$$



$$N = +2$$

INESTABLE

Ejemplo
de Abad

Nyquist

2020

$$G(P)H(P) = \frac{10P+20}{P^2-3P} = \frac{10(P+2)}{P(P-3)}$$

1) Inicio para $P \rightarrow 0$: $\frac{20}{-3P} = -\infty \angle -90^\circ = \infty \angle -270^\circ$

2) Final para $P \rightarrow \infty$ $\frac{10P}{P^2} = \frac{10}{P} = 0 \angle -90^\circ$

3) $P \rightarrow j\omega$

$$G(j\omega)H(j\omega) = \frac{-50\omega^2}{\omega^4 + 9\omega^2} + j \frac{60\omega - 10\omega^3}{\omega^4 + 9\omega^2}$$

4) $\text{Re}/\omega = 0$

$$\frac{-50\omega^2}{\omega^4 + 9\omega^2} = 0 \quad -50\omega^2 = 0$$

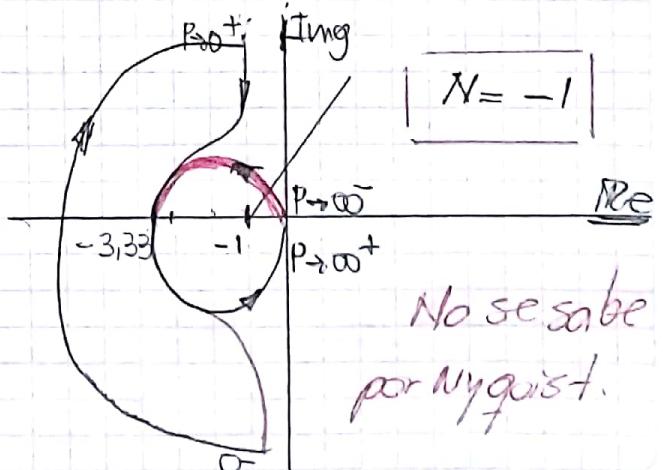
Como ω debe ser un número Real

5). Por lo tanto no hay corte al eje real, salvo para $\omega \rightarrow 0$ y $\omega \rightarrow \infty$

6) $\text{Imag}/\omega = 0$ $60\omega - 10\omega^3 = 0 \quad \therefore \omega = \pm \sqrt{\frac{60}{10}} = \pm 2,4494$

7) Corte al eje Real

$$\left. \frac{-50\omega^2}{\omega^4 + 9\omega^2} \right|_{\omega=2,4494} = -3,33$$



Ejemplos

$$G(P) H(P) = \frac{100}{P^3 + 5P^2 + 6P} \Rightarrow \text{Inestable}$$

$$G(P) H(P) = \frac{10P + 30}{P^2 - 4} \Rightarrow \text{Estable}$$

$$G(P) H(P) = \frac{50P + 150}{P^3 - P^2 + 11P - 51} \Rightarrow \text{Inestable}$$

$$G(P) H(P) = \frac{10P + 20}{P^3 + 3P^2} \Rightarrow \text{Estable.}$$

$$G(P) H(P) = \frac{20}{P^3 + 5P^2 + 6P} = \frac{20}{P(P+3)(P+2)} \Rightarrow \text{Estable}$$

$$G(P) H(P) = \frac{10P + 40}{P^3 - 6P^2 - 16P} \Rightarrow \text{Inestable} \quad N=1$$

(Aunque reduzco la

Ganancia, seguirá siendo
inestable)

$$G(P) H(P) = \frac{Kte}{P^3 + 2P^2 + 4P}$$

con $Kte = 10 \vee 8$

sistema. Inestable

con $Kte > 6 \Rightarrow \text{Estable}$

Si Reduzco Ganancia si es Estable)

Ejemplo:

$$G(p) \cdot H(p) = \frac{20}{p^3 + 4p^2 + 5p + 4} = \frac{20}{\underline{\quad}}$$

① Origen del diagrama

$$G(p) \cdot H(p) = \frac{20}{\lim_{p \rightarrow 0} H_2} = (5) \angle 0^\circ$$

② Determinar el final del diagrama

$$G(p) \cdot H(p) = \frac{20}{\lim_{p \rightarrow \infty}} \angle \frac{Kde}{p^3 (e^{j90})^3} = 0 \angle -90 \cdot 3 = -270^\circ$$

$$0 \angle -0 \cdot 3 =$$

③ $P \rightarrow j\omega =$

$$\frac{20}{(-j\omega^3) - 4 \cdot \omega^2 + j5\omega + 4} = \frac{20}{-4\omega^2 + 4 + j(\omega \cdot 5 - \omega^3)}$$

$$= \frac{20}{4 - 4\omega^2 + j(\omega \cdot 5 - \omega^3)} = \frac{(4 - 4\omega^2) - j(\omega \cdot 5 - \omega^3)}{(4 - 4\omega^2) - j(\omega \cdot 5 \cdot \omega^3)}$$

$$= \frac{80 - 80\omega^2 - j(100\omega - 20\omega^3)}{(4 - 4\omega^2)^2 + (\omega \cdot 5 - \omega^3)^2} = \frac{80 - 80\omega^2}{(4 - 4\omega^2)^2 + (5\omega - \omega^3)^2} - j \frac{(100\omega - 20\omega^3)}{(4 - 4\omega^2)^2 + (5\omega - \omega^3)^2}$$

④

$$\text{Real} = 0$$

$$80 - 80\omega^2 = 0 \quad \wedge \quad 80 = 80\omega^2$$

$$\sqrt{1} = \pm 1$$

$$\text{Im } \omega = \pm$$

$$-j \frac{(100 - 20)}{(4 - 4)(5 - 1)^2} = \frac{-80}{16} = \boxed{-5}$$

$$100\omega - 20\omega^3 = 0 \rightarrow \omega(100 - 20\omega^2) =$$

$$100 = 20\omega^2$$

$$\sqrt{\frac{100}{20}} = \omega = 2,23$$

$$\frac{80 - 80\omega^2}{((4 - 4\omega^2)^2 + (5\omega - \omega^3)^2} = \frac{80 - 80 \cdot (2,23)^2}{(4 - 4 \cdot (2,23)^2)^2 + (5 \cdot 2,23 - 2,23)^2}$$
$$= -4,25$$

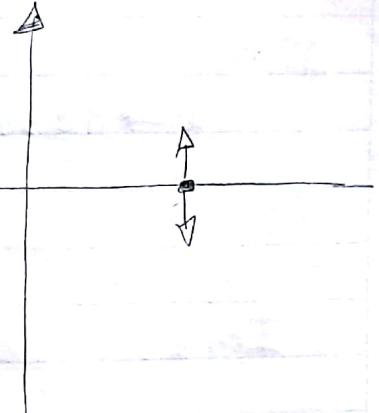
Análisis del Argomento:

$$\operatorname{tg}^{-1} \frac{Im}{Real} = \frac{(100\omega + 20\omega^3)}{80 - 80\omega^2} = -\omega \quad \operatorname{tg}^{-1} = -\#$$

$$F(p) = \frac{10}{p_0^3 + 2p_0^2 + 2p_0 + 40}$$

① Origem del Diagrama.

$$\lim_{p \rightarrow 0} \frac{10}{40} = 0,25 = 0,25 \angle 0^\circ$$



② Final del Diagrama.

$$\lim_{p \rightarrow \infty} \frac{10}{p^3 + 2p^2 + 2p + 40} = \frac{10}{p^3 (1 + \frac{2p}{p^3} + \frac{2p}{p^2} + \frac{40}{p^3})} = \frac{10}{(p \cdot e^{j90^\circ})^3} = 0, \angle -270^\circ$$

③ $p \rightarrow j\omega$



$$F(j\omega) = \frac{10}{(j\omega)^3 + 2(j\omega)^2 + 2j\omega + 40} = \frac{10}{-j\omega^3 - 2\omega^2 + j2\omega + 40} = \frac{10}{(40 - 2\omega^2) + j(2\omega - \omega^3)}$$

$$= \frac{10}{(40 - 2\omega^2) + j(2\omega - \omega^3)}$$

$$④ F(j\omega) = \text{Re} + \text{Im}$$

$$F(j\omega) = \frac{(10)}{(40 - 2\omega^2) + j(2\omega - \omega^3)} \cdot \frac{(40 - 2\omega^2) - j(2\omega - \omega^3)}{(40 - 2\omega^2) - j(2\omega - \omega^3)} =$$

$$= \frac{10(40 - 2\omega^2) - j10(2\omega - \omega^3)}{(40 - 2\omega^2)^2 + (2\omega - \omega^3)^2} = \frac{10(40 - 2\omega^2)}{(40 - 2\omega^2)^2 + (2\omega - \omega^3)^2} - \frac{j10(2\omega - \omega^3)}{(40 - 2\omega^2)^2 + (2\omega - \omega^3)^2}$$

⑤ $\text{Re} = 0$

$$\frac{10(40 - 2\omega^2)}{(40 - 2\omega^2)^2 + (2\omega - \omega^3)^2} = 0 \rightarrow 10(40 - 2\omega^2) = 0 \rightarrow 400 - 20\omega^2 = 0 \rightarrow \omega^2 = 20 \rightarrow \omega = \pm 2\sqrt{5}$$

(6)

$$\text{Im} \left|_{\substack{\omega \rightarrow \text{Re}=0}} \right. \frac{-10 \cdot (2 \cdot (2\sqrt{5}) - (2\sqrt{5})^3)}{(40 - 2(2\sqrt{5})^2)^2 + (2(2\sqrt{5}) - (2\sqrt{5})^3)^2}$$

$$\text{Im} \left|_{\substack{\omega \rightarrow \text{Re}=0}} \right. \frac{-10}{(2(2\sqrt{5}) - (2\sqrt{5})^3)} = 0,1242$$

Corte al eje

Im para $\omega = 2\sqrt{5} \text{ [rps]}$ (7) $\text{Im} = 0$ y despejar ω :

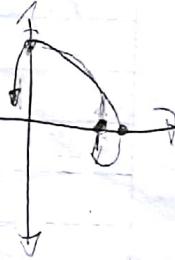
$$\frac{-10(2\omega - \omega^3)}{(40 - 2\omega^2)^2 + (2\omega - \omega^3)^2} = 0 \Rightarrow -10(2\omega - \omega^3) = 0$$

$$10\omega^3 - 20\omega = 0 \quad \begin{matrix} \sqrt[3]{2} = 1,4142 \\ \downarrow \quad \downarrow \\ 10 \quad 20 \end{matrix}$$

(8)

$$\text{Re} \left|_{\substack{\omega \rightarrow \text{Im}=0}} \right. \frac{10(40 - 2(\sqrt{2})^2)}{(40 - 2(\sqrt{2})^2)^2 + (2\sqrt{2} - (\sqrt{2})^3)^2} = \frac{10}{36} =$$

$$= 0,2778$$

↳ Corte al eje Re para $\omega = \sqrt{2} \text{ [rps]}$ 

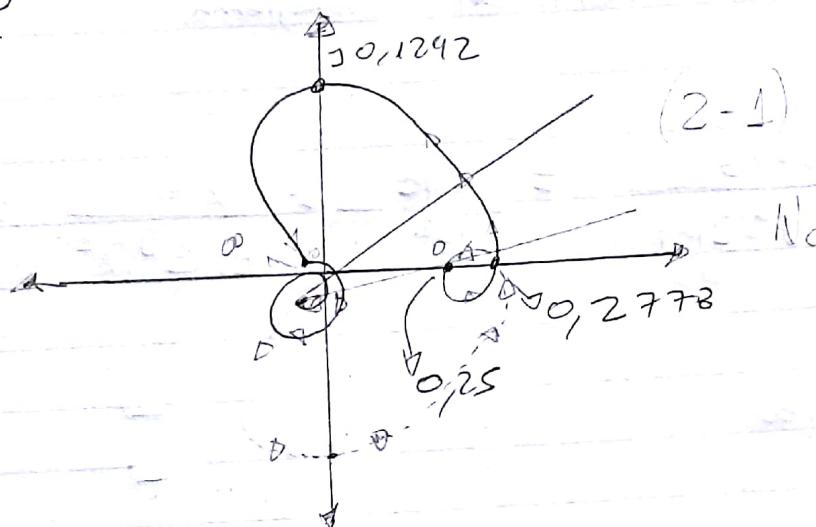
(9) Análisis del Argumento:

$$(\ell = \operatorname{tg}^{-1} \frac{\text{Im}}{\text{Re}} = \operatorname{tg}^{-1} \frac{-10(2\omega - \omega^3)}{10(40 - 2\omega^2)}) \stackrel{\omega > 0}{=} 2 \text{ I} + 4(-)$$

$$P/\omega \rightarrow 0 : \operatorname{tg}^{-1} = \frac{-}{+} \quad \begin{matrix} (4^\circ \text{ Cuadrante}) \\ (2^\circ \text{ Cuadrante}) \end{matrix}$$

$$P/\omega \rightarrow \infty : \operatorname{tg}^{-1} = \frac{-(2\omega - \omega^3)}{(40 - 2\omega^2)} = \frac{\pm \omega^3}{-2\omega^2} = \frac{\pm \omega^3}{2\omega^2} \quad \begin{matrix} 1^\circ \text{ Cuad} \\ 2^\circ \text{ Cuad} \end{matrix}$$

(9) Diagrama polar

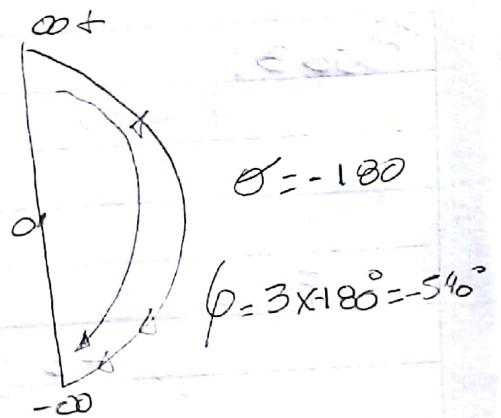


(2-1)

- (10) Cierre para $P \rightarrow 0$
Mismo punto que el 1: $0,25 < 0^\circ$

$$0 - 60^\circ = 300^\circ - 0^\circ$$

- (11) Cierre para $P \rightarrow \infty$
Mismo que la 2:
 $\lim_{P \rightarrow \infty} \frac{10}{P^3} = \frac{10}{(P \cdot e^{j390})^3} = (0 < -90) \cdot 3$

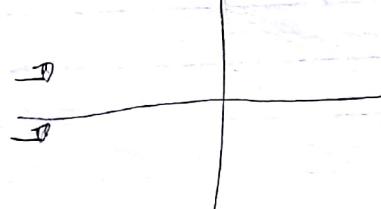


$$F(p) = \frac{10p+20}{p^4 - 2p^3 + 5p^2} = \frac{10p+20}{p^2(p^2 - 2p + 5)} =$$

Paso 1: Paso al dominio del Diagrama.

$$\lim_{p \rightarrow 0} F(p) = \frac{10p+20}{p^2(p^2 - 2p + 5)} = \frac{Kte}{p^2} = \frac{Vte}{(E \cdot e^{j\omega t})^2} = \frac{(Kt)}{E^2 \cdot e^{j2\omega t}}$$

$$= \infty \angle -180^\circ$$



Paso 2: Final de la grafica

$$\lim_{p \rightarrow \infty} F(p) = \frac{10p+20}{p^4 - 2p^3 + 5p^2} = \frac{Kte}{p^2} = \frac{Kt}{p^3 e^{j\omega t}} = 0 \angle -270^\circ$$

Paso 3:

$$p \rightarrow j\omega$$

$$\begin{aligned} j^1 &= j \\ j^2 &= -1 \\ j^3 &= -j \\ j^4 &= 1 \\ j^5 &= j \end{aligned}$$



$$F(j\omega) = \frac{10(j\omega) + 20}{(j\omega)^4 - 2 \cdot (j\omega)^3 + 5 \cdot (j\omega)^2} = \frac{10(j\omega) + 20}{\omega^4 - 5\omega^2 + j2\omega^3}$$

Paso 4: Separo $\text{Re} + j\text{Im}$.

$$= \frac{10(j\omega) + 20}{\omega^4 - 5\omega^2 + j2\omega^3} \cdot \frac{\omega^4 - 5\omega^2 - j2\omega^3}{\omega^4 - 5\omega^2 - j2\omega^3} =$$

$$= \frac{40\omega^4 - 100\omega^2}{(\omega^4 - 5\omega^2)^2 + (2\omega^3)^2} + j \frac{10\omega^5 - 90\omega^3}{(\omega^4 - 5\omega^2)^2 + (2\omega^3)^2}$$

Paso 5:

$$\text{Real} = 0$$

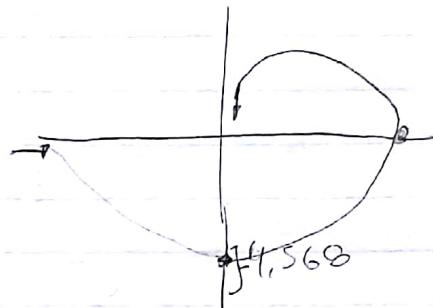
$$\frac{N^o}{\text{pde}} = \frac{\text{Grado } 4}{\text{Grado } 8}$$

$$\frac{40\omega^4 - 100\omega^2}{(\omega^4 - 5\omega^2)^2 + (2\omega^3)^2} = 0 \rightarrow 40\omega^4 - 100\omega^2 = 0 \quad \text{4 denominados}$$

$$40\omega^4 - 100\omega^2 = 0 \rightarrow \omega^2(40\omega^2 - 100) = 0$$

$$\omega = \sqrt{\frac{100}{40}} = \pm 1,5811$$

Paso 6: Lo inserto en Im. ω :



$$= \sqrt{\frac{10 \cdot (1,5811)^5 - 90 \cdot (1,5811)^3}{(1,5811)^4 - 5 \cdot (1,5811)^2 + (2 \cdot (1,5811)^3)^2}} = \frac{-256,92}{56,2407}$$

$$= \sqrt{\frac{(10\omega^5 - 90\omega^3)}{(\omega^4 - 5\omega^2) + 4\omega^6}} = \boxed{-4,568} \quad \begin{matrix} \text{Corte en el} \\ \text{eje Imaginario} \end{matrix}$$

Paso 7: $\text{Im} = 0$

$$\omega^3 (10\omega^5 - 90\omega^3) = 0$$

$$(10\omega^2 - 90) = 0 \rightarrow \omega = \sqrt{\frac{90}{10}} = \pm 3$$

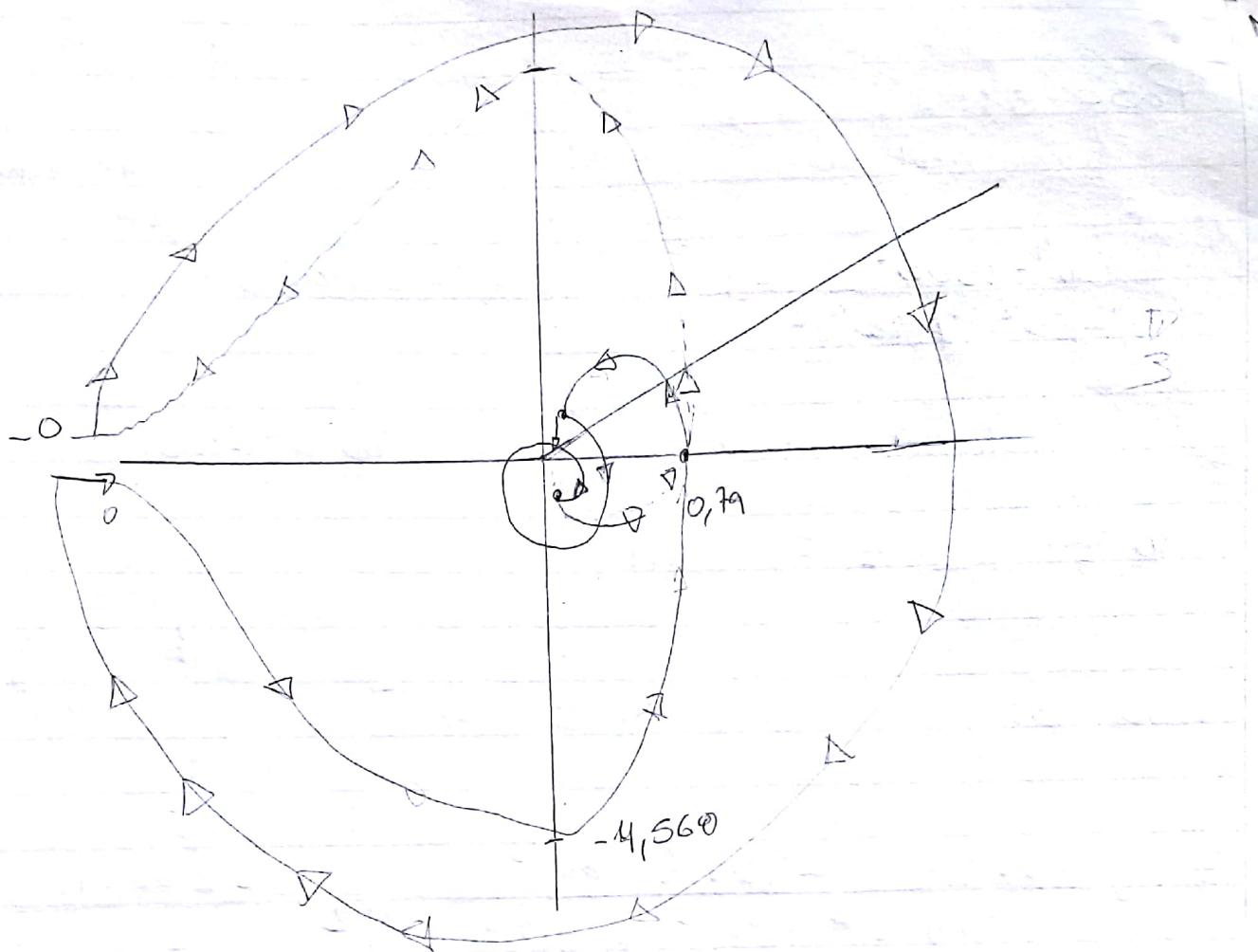
Paso 8: $\text{Real } \omega = 3$



$$\frac{40(3)^4 - 100(3)^2}{((3)^4 - 5 \cdot (3)^2)^2 + (2 \cdot 3^3)^2} = 0,79 \quad \begin{matrix} \text{Corte en el eje} \\ \text{Real} \end{matrix}$$

Paso 9: Análisis del Argumento:

$$\text{p } \omega \rightarrow 0 = \tan^{-1} \frac{\text{Im}}{\text{Real}} = \frac{10\omega^2 - 90}{40\omega^2 - 100} = + \quad \begin{matrix} 1^{\circ} \text{ Cuadrante} \\ 3^{\circ} \text{ Cuadrante} \end{matrix}$$



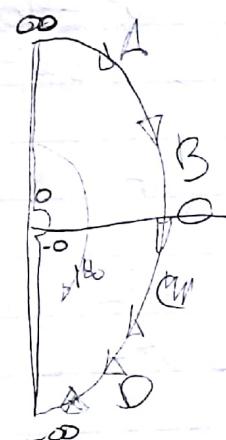
10) Crear para $p \rightarrow 0$

$0^+ \rightarrow 0^-$

$$\frac{F(p)}{p \rightarrow 0} = \frac{k_{de}}{p^2} = 100 / L 2 \cdot \theta$$

$$\theta = 180$$

$$2,760 = 360$$



$$\begin{aligned} F(p) &= \frac{10}{p^3} = \frac{10}{p \cdot (e^{j\alpha})^3} = 0 \angle 3 \cdot \theta - 180^\circ \\ &= 0 \angle -540^\circ \end{aligned}$$

$$(0^+) \rightarrow (\infty^+) \rightarrow (\infty^-) \rightarrow (-0)$$

Ejercicios de Nyquist.

Traza el diagrama polar y aplique criterio de Nyquist.

$$F(p) = \frac{10}{(p^4 + 6 \cdot p^3 + 8 \cdot p^2 + 10 \cdot p + 20)}$$

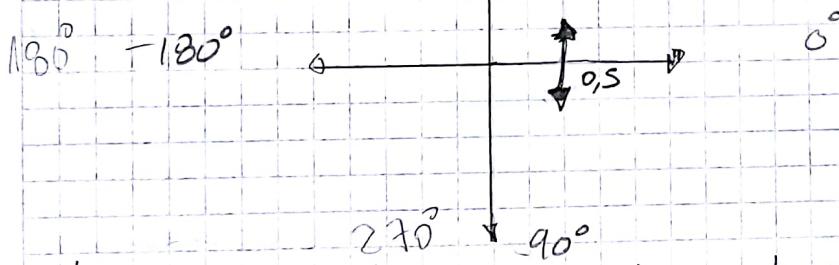
1º Paso: Determinar el Origen del diagrama polar:

$$\text{Evaluar } F(p)|_{p \rightarrow 0} \text{ y } G(p) \cdot H(p)|_{p \rightarrow 0}$$

$$F(p) = \frac{10}{20} = 0,5 \quad \text{Modulo } 0,5; \text{ angulo cero.}$$

-270° 90°
↑
Plano $F(p)$

Por qe no hay "P".

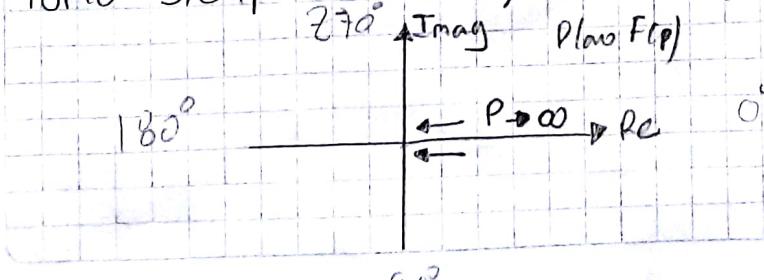


2º Paso: Determinar el final del diagrama Polar:

$$\text{Evaluar } F(p)|_{p \rightarrow \infty} \text{ y } G(p) \cdot H(p)|_{p \rightarrow \infty}$$

$$F(p)|_{p \rightarrow \infty} = \frac{10}{p^4} = \frac{10}{\infty} = 0 \quad \underline{-140^\circ} = |0| \cdot \underline{-360^\circ}$$

Tomo siempre el de mayor orden y la constante.



$$j^2 = -1 ; \quad j^0 = 1 ; \quad j^1 = j$$

Paso N°3: Cambiar en la función de transferencia
 $p \rightarrow j\omega$.

Es decir que $F(p) \rightarrow F(j\omega)$ & $G(p) \neq H(p) \rightarrow G(j\omega) \cdot H(j\omega)$

$$p^4 = j\omega^4 = j^2 \cdot j^2 \cdot \omega^4 = (-1) \cdot (-1) \cdot \omega^4 = 1 \omega^4$$

$$p^3 = j^2 \cdot j\omega^3 = -1 j\omega^3 = -j\omega^3$$

$$p^2 = j^2 \omega^2 = -\omega^2$$

$$p = j\omega = j\omega.$$

$$F(p) = \frac{10}{\omega^2 + 6 \cdot (-j\omega^3) + 8 \cdot (-\omega^2) + 10(j\omega) + 20}$$

$$F(p) = \frac{10}{\underbrace{\omega^4 - 6j\omega^3 - 8\omega^2}_{+} + \underbrace{10j\omega + 20}_{+}} = \frac{10}{(\omega^4 - 8\omega^2 + 20) + j(10j\omega - 6\omega^3)}$$

Paso 4: $F(j\omega) = R + jI$. Separa parte real y parte imaginaria.

$$F(p) = \frac{10}{(\omega^4 - 8\omega^2 + 20) + j(10\omega - 6\omega^3)} \cdot \frac{\omega^4 - 8\omega^2 + 20 - j(10\omega - 6\omega^3)}{\omega^4 - 8\omega^2 + 20 - j(10\omega - 6\omega^3)}$$

Resultados

$$F(p) = \frac{10 \cdot (\omega^4 - 8\omega^2 + 20)}{(\omega^4 - 8\omega^2 + 20)^2 + (10\omega - 6\omega^3)^2} + j \frac{10 \cdot (6\omega^3 - 10\omega)}{(\omega^4 - 8\omega^2 + 20)^2 + j(10\omega - 6\omega^3)^2}$$

Paso N°5:

Hacer la parte real de $F(j\omega) \cdot 0$ de $G(j\omega) * H(j\omega)$ igual a cero.

$$\text{Re } F(j\omega) = 0$$

$$\text{Re} = 0 = \frac{10(w^4 - 8w^2 + 20)}{(w^4 - 8w^2 + 20)^2 + (10w - 6w^3)^2} = 0$$

Calculo las raíces. No hay ningún valor real de w .

Recordatorio: Si hay un valor de w que me hace la parte real cero. Este valor de w lo evaluo en la parte imaginaria y eso va a ser el corte al eje imaginario.

Paso N°6: Corte imaginario:

$$\text{Im } |F(j\omega)| \mid w \rightarrow \text{Re} = 0.$$

No tenemos.

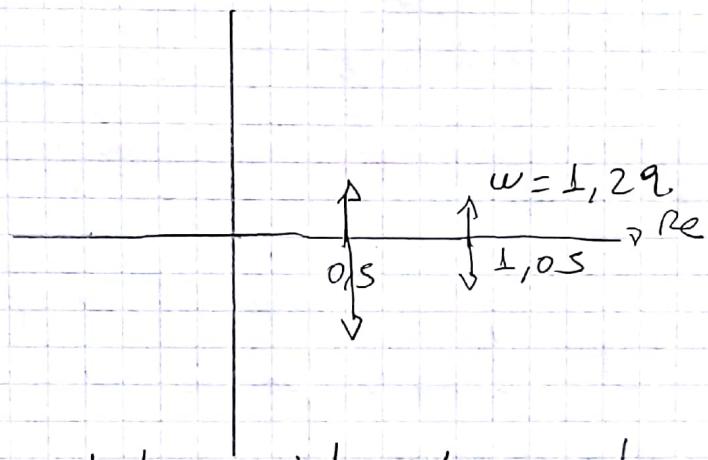
Paso N°7: Hacer la parte imaginaria de $F(j\omega) \cdot 0$ de $G(j\omega) * H(j\omega)$ igual a cero.

$$\text{Im } F(j\omega) = 0 = \text{Im } \frac{10(6w^3 - 10w)}{(w^4 - 8w^2 + 20)^2 + j(10w - 6w^3)^2} =$$

$$w = \sqrt{\frac{10}{6}} = \pm 1,290$$

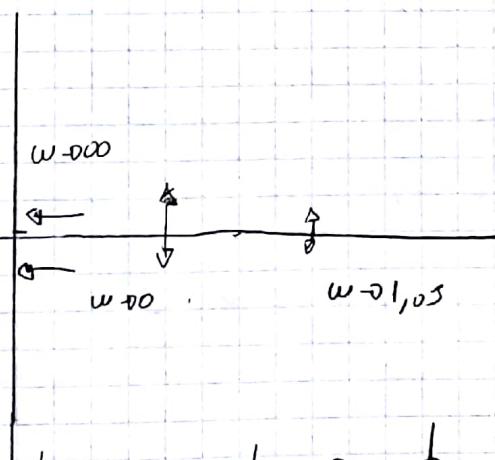
Paso 8: Determinar el corte al eje real, con " ω " obtenidos en Paso 7.

$$\operatorname{Re} \Big|_{\omega=+1,290} = \frac{10(\omega^4 - 8\omega^2 + z_0)}{(\omega^4 - 8\omega^2 + z_0)^2 + 10} = 1,0588.$$



Paso 9: Con los datos obtenidos en los pasos 1,2,6,y 8, Trazar el diagrama polar de las frecuencias positivas.

Luego Trazar las frecuencias negativas, Haciendo el espejo de la curva anterior sobre el eje real.



Si tengo dudas de como trazar la curva me fijo en el argumento.

a) $\text{Fase} = \operatorname{tg}^{-1} \frac{6\omega^3 - 10\omega}{\omega^4 - 8\omega^2 + z_0} = -\pi - \frac{10\omega}{z_0} = (-)$

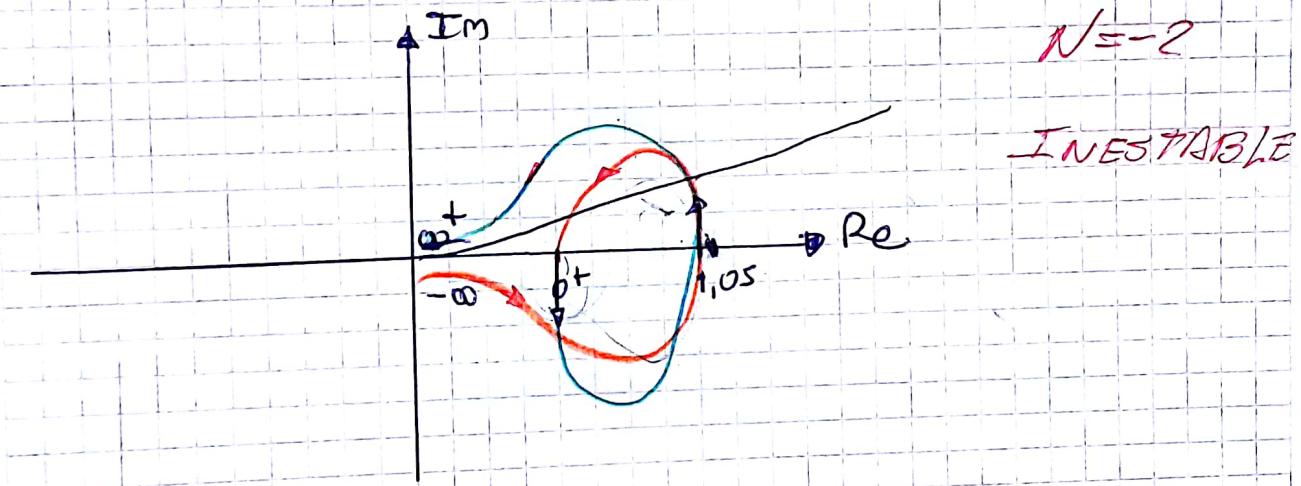
$$b) \text{ Fase} = \operatorname{tg}^{-1} \frac{6\omega^3 - 10\omega}{\omega^4 - 8\omega^2 + 20} = \frac{6 - 10}{1 - 8 + 20} = \frac{-4}{13}$$

$\omega = 1$

"Positiva fase paraje"

$$\text{Fase} = \operatorname{tg}^{-1} \frac{6\omega^3 - 10\omega}{\omega^4 - 8\omega^2 + 20} = \frac{6\omega^3}{\omega^4} = \frac{6}{\omega} = +$$

$\omega \rightarrow 0$



Paso N°10 = Cerrar diagrama para P>0

Dado que la función no tiene polos en el origen; este paso no se aplica.

$$D(s) = -10\omega^4 + 80\omega^2 + 40\omega + 7 (-40\omega^2 - 10\omega^3 + 8\omega)$$

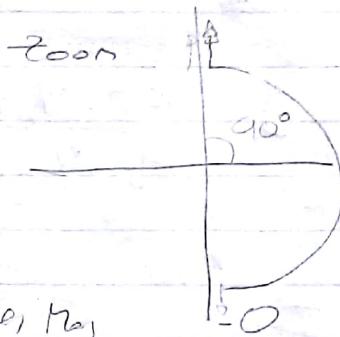
Ejemplo 2: Nyquist. $\omega \in [0, \infty)$

$$F(p) = \frac{5p+30}{p^5(3p^4+2p^3)} = \frac{5p+30}{p^3(p^2-3p+2)}$$

① Determinar el Círculo del diagrama

$$\lim_{p \rightarrow 0} = \frac{5}{p^3} = \frac{K(je)^{\infty}}{E^3(e^{390}))} = \infty \angle -270^\circ$$

$$= \infty \angle -3.0^\circ$$



② Fin del diagrama

Pregunto $\lim_{p \rightarrow \infty} = \frac{5p+30}{p^5(3p^4+2p^3)} = \frac{K(je)}{P^4} = 0 \angle 90^\circ$

$$= \frac{K(je)}{P(e^{390})} = 0 \angle 90^\circ$$

③ $p \rightarrow j\omega$

$$F(j\omega) = \frac{5(j\omega) + 30}{(j\omega^5 - 3\omega^4 - j2\omega^3)} =$$

④ Separar Real + Imag

$$F(j\omega) = \frac{5j\omega + 30}{3\omega^4 + j(\omega^5 - 2\omega^3)} = \frac{-3\omega^4 - j(\omega^5 - 2\omega^3)}{-3\omega^4 - j(\omega^5 - 2\omega^3)}$$

$$F(p) = \frac{5\omega^6 - 100\omega^4}{9\omega^8 + (\omega^5 - 2\omega^3)^2} + j \frac{(60\omega^3 - 45\omega^5)}{9\omega^8 + (\omega^5 - 2\omega^3)^2}$$

$$\textcircled{5} \quad F(j\omega) = \frac{5\omega^6 - 100\omega^4}{\omega^8} = 0 \rightarrow \omega^4(5\omega^2 - 100)$$

$$\omega = \sqrt{\frac{100}{5}} = \pm 4,97$$

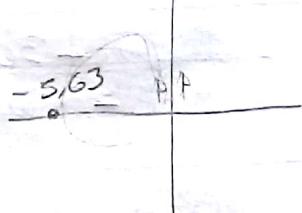
$$\textcircled{6} \quad \text{Im} / \left| \omega = \text{Re}\omega_0 = \right.$$

$$j \frac{60 \cdot (4,97)^3 - 45 \cdot (4,97)^5}{9 \cdot (4,97)^3 + ((-4,97)^5 - 2 \cdot (4,97)^3)^2} = -j 0,0186$$

$$\textcircled{7} \quad \text{Im} = 0 \quad 60\omega^3 - 45\omega^5 = 0 \rightarrow \omega^3 = \underbrace{(60 - 45\omega^2)}_{= 0}$$

$$\frac{60}{45} = \omega = 1,154$$

$$\text{Re}\omega = \left| \omega \text{Im} = 0 \right. \quad \frac{5\omega^6 - 100\omega^4}{9\omega^8 + (\omega^5 - 2\omega^3)^2} = -5,63$$



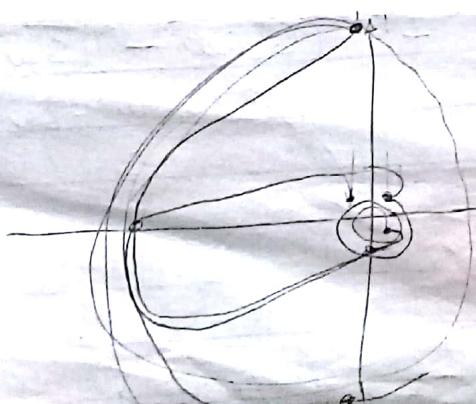
\textcircled{8} Análisis de Argumentos

$$P \rightarrow \omega = 0 \quad \operatorname{tg}^{-1} \frac{\text{Im}}{\text{Real}} = \frac{60 - 45\omega^2}{5\omega^2 - 100} = -20^\circ \text{ o Cuadrante II}$$

$$P \rightarrow \omega = \infty \quad \operatorname{tg}^{-1} \frac{\text{Im}}{\text{Real}} = \frac{60 - 45\omega^2}{5\omega^2 - 100} = \frac{-45\omega^2}{5\omega^2} = -\infty \text{ o Cuadrante IV}$$

Cierre para $P \rightarrow 0$

$$-3 \cdot 180 = -540^\circ$$



Cierre para $P \rightarrow \infty$

$$-4 \cdot 180 = -720^\circ$$