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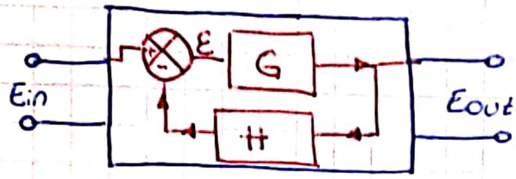
4R2

Resumen 2° Parcial Teoría.

Temas:

- Nyquist $G \times H$. ✓
- Routh - Herwitz. (con casos especiales). ✓
- Estabilización por Reducción de ganancia. ✓
- Margen de ganancia y margen de fase. ✓
- Cuadripolos:
  - Parámetros $(Z, Y, h, g, T_x, T_x^{-1})$. ✓
  - Relación entre parámetros ✓
    - → $Y = f(Z)$ ✓
    - → $T_x = f(Z)$. ✓
    - → $T_x^{-1} = f(Z)$. ✓
  - Conexiones (cascada, //, serie, ser-//, //-ser) ✓
  - Parámetros T_x en cuad. básicos. ✓
  - Transformación π a T y T a π. ✓
  - Cuadripolos cargados
    - → Zin en función de T_x.
    - → Zout " " " T_x
    - → Z iterativas (Z_k).
    - → Z imagenes (Z_{im}).
    - → Z característica (Z_0) (cuad. simétricos).
    - → Método alternativo cálculo Z_{im} y Z_0.
  - Funciones de propagación de tensiones y corrientes en función de T_x
    - → Cargados con Z general
    - → Cargados con Z_{k_2}
    - → cargados con Z_{im_2}
    - → cargados con Z_0
  - Adaptadores de impedancia.

Sistemas Realimentados.



$$F(P) = \frac{E_{out}(P)}{E_{in}(P)}$$

$$\varepsilon = E_{in}(P) - E_{out}(P) \cdot H(P)$$

$$E_{out}(P) = \varepsilon \cdot G(P) = E_{in}(P) \cdot G(P) - E_{out}(P) \cdot G(P) \cdot H(P)$$

$$E_{out}(P)\left[1 + G(P) \cdot H(P)\right] = E_{in}(P) \cdot G(P)$$

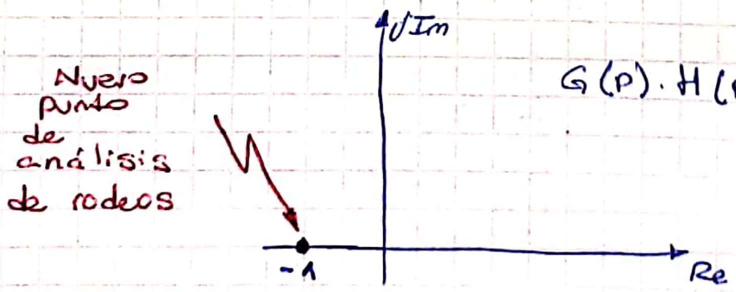
$$F(P) = \frac{E_{out}(P)}{E_{in}(P)} = \frac{G(P)}{G(P)H(P)+1} = \frac{\text{Zeros } F(P)}{\cancel{\text{Polos } F(P)}} = \frac{G(P)}{\frac{\cancel{\text{Zeros } G(P)H(P)+1}}{\text{Polos } G(P)H(P)+1}}$$

Los Polos de $F(P)$ son los Zeros de $G(P)H(P)+1$.

Analizar $G(P)H(P)+1 = 0$ es lo mismo que

$$G(P)H(P) = -1$$

Movemos punto de análisis en Nyquist y se cambia el criterio:



$N = Z - P$

$N = +\#$ → Sistema Inestable (Positivo ahora porque los ceros son los polos de $F(P)$).

$\left.\begin{array}{l} N = 0 \\ N = -\# \end{array}\right\}$ No se sabe

Pasos de Nyquist → No varían (sólo en paso 11 y en 12 el crit.)

1) $G(P)H(P)|_{P=0}$

2) $G(P)H(P)|_{P=\infty}$

3) $P \to jw$ $\left(G(P)H(P) \to G(jw)H(jw) \right)$

4) Separar en $Re + j\,Im$

5) $Re = 0$

6) $Im|_{Re=0}^{w}$

7) $Im = 0$

8) $Re|_{Im=0}^{w}$

9) Trazar Diag.

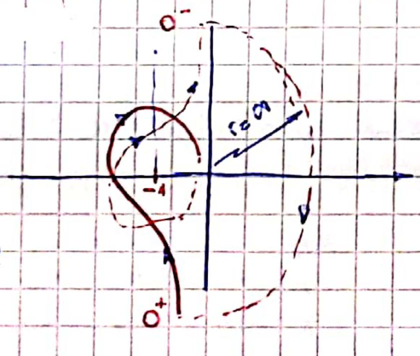
10) Cerrar p/ $w \to 0$.

11) No aplica (no importa que pasa en $w = \infty$)

12) Aplicar criterio.

Estabilización por Reducción de Ganancia. (No siempre es posible)

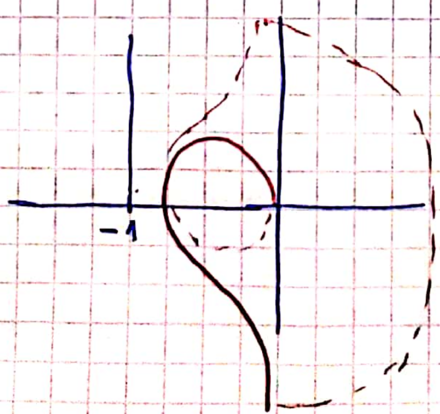
- Se realiza únicamente a $G(P)H(P)$



N = 2

La curva encierra al -1 pero la distancia no es infinita

Se busca que la gráfica quede a la derecha del -1.

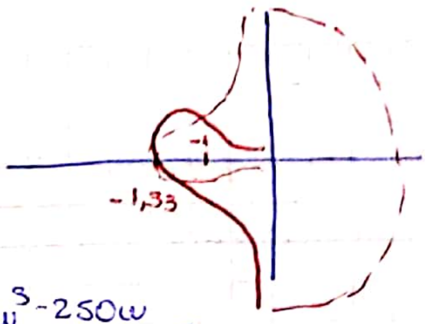


N = 0

De esta forma se estabiliza al sistema (Hacemos que el corte al eje Real sea mayor a -1)

Ejemplo

$$G(P)H(P) = \frac{10P+50}{P^3+3P^2+50} \longrightarrow$$



$$G(jw)H(jw) = \frac{-100w^2-10w^4}{9w^4+(5w-w^3)^2} + j\,\frac{20w^3-250w}{9w^4+(5w-w^3)^2}$$

7) $Im = 0$

$w = \pm 3{,}5355$

8) $Re\big|_{w=\pm 3{,}5355} = 10 \times (-0{,}1333) = -1{,}333$ → Se necesita que sea mayor a -1.

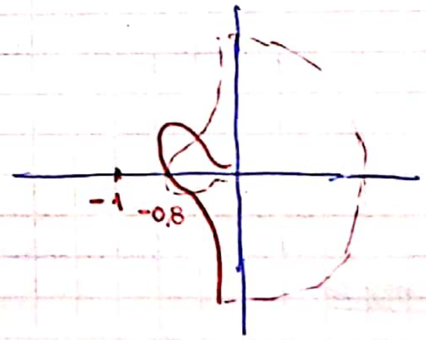
↓ Ganancia

↓ Si se disminuye a 6

$6 \times (-0{,}1333) = -0{,}8$ → No rodea a -1.

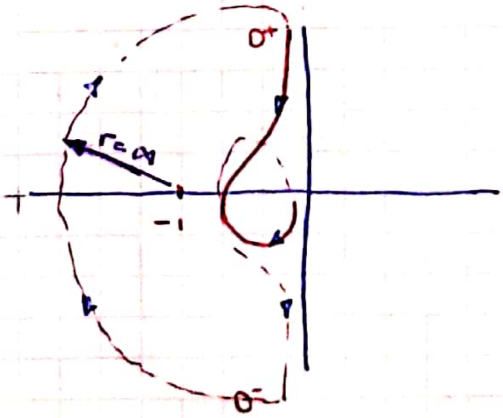
Nueva función de transferencia =

$$G(P)H(P) = \frac{6P+30}{P^3+3P^2+50} \longrightarrow$$



Tener en cuenta =

- No se podría estabilizar por este método una $G(P)H(P)$ con la sig. gráfica =



Hay una distancia infinita entre -1 y la curva.

Hay Sistemas Inestabilizables.

Algoritmo de Routh - Herwitz.

- Detecta raíces con parte real positiva.
- Produce tantos cambios de signo como raíces a parte $\mathbb{R}+$.
- Si se aplica al numerador de $G(P)\cdot H(P)+1$, se puede usar para determinar si el sistema es estable o no.
- Si se aplica a num. y den. de $G(P)\cdot H(P)+1$, podemos encontrar los N rodeos de Nyquist.

Ejemplo:

$$G(P)\cdot H(P) = \frac{10P+20}{P^4-2P^3+5P^2}$$

$\boxed{\text{Sumamos } 1 \text{ !!}}$

$$G(P)H(P)+1 = \frac{P^4-2P^3+5P^2+10P+20}{P^2(P^2-2P+5)}$$

R-H Num

P^4	1	5	20
P^3	-2	10	
P^2	$\frac{-2\cdot 5-1\cdot 10}{-2} = 10$	20	
P^1	14		
P^0	20		

Último elemento de filas pares iguales

Se analizan cambios de signo de esta columna

$Z = 2$

↳ Sistema Inestable

R-H Den (omitimos P^2 porque ya sabemos que la parte $\mathbb{R}$ no es +)

P^2	1	5
P^1	-2	
P^0	5	

$P = 2$

$N = Z - P = 0$ (No se sabría por Nyquist).

Casos especiales R-H =.

1°) Se anula un término de la columna principal. (Produciría un término infinito cuando se divida por él).

Se usan símbolos p/ expresar

- ε (número muy chico) → No importa el signo (≈ 0).
- # (número muy grande) → Importa el signo ($+\infty$ o $-\infty$).

También se puede multiplicar por un $P+\alpha$ aleatorio (otra opción).

Ejemplo:

$$2P^4 + 2P^3 + P^2 + P + 5.$$

P^4	2	1	5
P^3	2	1	
P^2	$\emptyset = \varepsilon$	5	
P^1	$-\#$		
P^0	5		

$$\# = \frac{\varepsilon \cdot 1 - 2 \cdot 5}{\varepsilon} = -\#$$

2 cambios de signo. = 2 r p $\mathbb{R}^+$

Otra opción =

$$\left(2P^4 + 2P^3 + P^2 + P + 5\right)(P+1) = 2P^5 + 4P^4 + 3P^3 + 2P^2 + 6P + 5.$$

P^5	2	3	6
P^4	4	2	5
P^3	2	3,5	
P^2	-5	5	
P^1	-5,5		
P^0	5		

2 cambios de signo = 2 r p $\mathbb{R}^+$.

2°) Se anula toda una fila → Se deriva la anterior y se colocan esos números

Ejemplo

$$P^8 + 6P^7 + 7P^6 - 6P^5 + 1016P^4 + 6144P^3 + 7168P^2 - 6144P - 8192$$

P^8	1	7	1016	7168	-8192
P^7	6	-6	6144	-6144	
P^6	8	-8	8192	-8192	
P^5	$\cancel{0}$ 48	$\cancel{0}$ -32	$\cancel{0}$ 16384		
P^4	-2,6	5461,3	-8192		
P^3	98272	-131072			
P^2	5457	-8192			
P^1	16432				
P^0	-8192				

$\longrightarrow 8P^6 - 8P^4 + 8192P^2 - 8192$

$$\frac{d}{dP} = 48P^5 - 32P^3 + 16384P$$

$\boxed{N^o \ r.p.R+ = 3}$

Se anula una fila completa cuando:

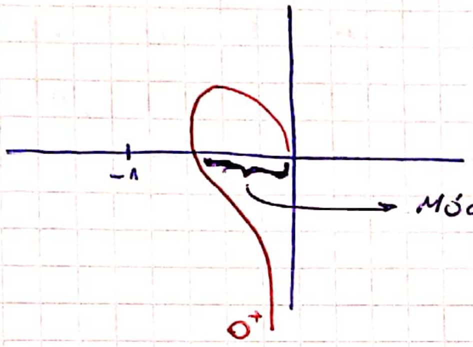
* 4 Raíces
  - $-\alpha \pm j\omega_0$
  - $+\alpha \pm j\omega_0$

Margen de Ganancia.

- Es la inversa del módulo, con fase $-180°$.

$$M.G = \frac{1}{|G(jw)\cdot H(jw)|}\Bigg|_{w \text{ donde la fase vale } -180°}$$

$$M.G\Big|_{dB} = 20\log(MG) = -20\log\left(G(jw)\cdot H(jw)\right)\Big|_{w \text{ donde fase } = -180°}$$



Si el M.G es mayor a 1 (en lineal) o positivo en dB el sistema es estable.

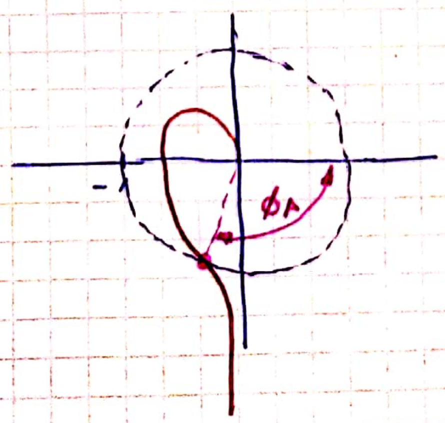
Margen de Fase.

- Valor de la fase cuando el módulo vale 0dB (1 en lineal).

$$|G(jw)\cdot H(jw) = 1| \longrightarrow 0dB.$$

$$M.F = 180° + \underbrace{\phi_A}$$
$\longrightarrow$ Ángulo negativo tomado desde 0° horario.

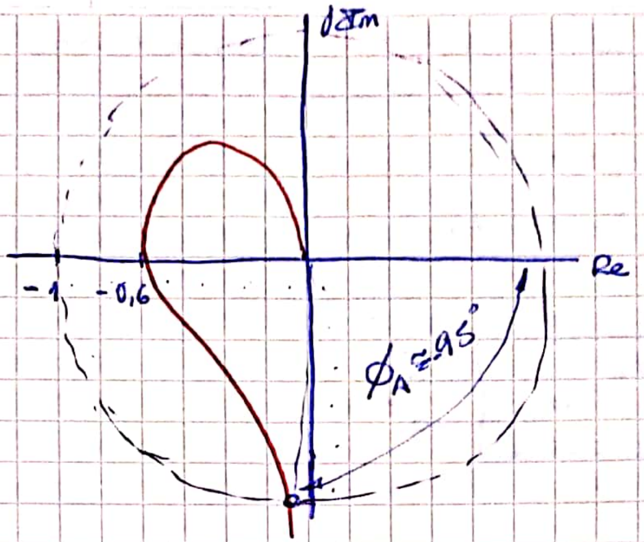
Ejemplo:



$MF > 0 \rightarrow$ Sistema estable.

(Si $MG_{dB} > 0$).

Ejemplo



$$MG = \frac{1}{0,6} = 1,67.$$

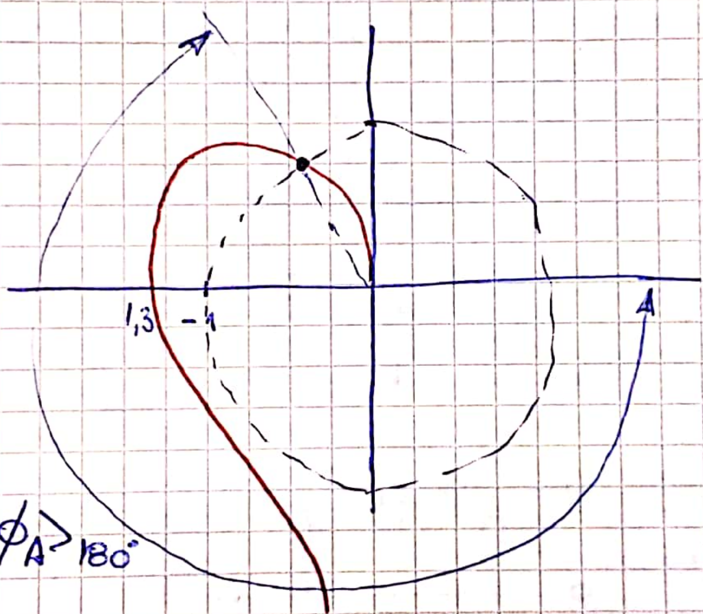
$$MG_{dB} = 4,44\,dB.$$

$$MF = 180° - 95° = 85°.$$

MG_{dB} y $MF > 0$

Sist. estable.

Ejemplo =



$$MG = \frac{1}{1,3} = 0,77.$$

$$MG_{dB} = -2,27\,dB.$$

ϕ MF = − ∢

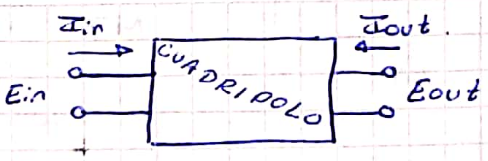
Sistema Inestable

Cuadripolos.

Cuadripolo = caja negra c/ 2 terminales de entrada y 2 de salida

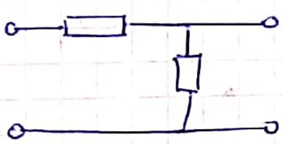
- No debe haber fuentes independientes dentro.
- La salida es nula si la alimentación es nula.

• Clasificación
  - → Pasivos ($E_{in} > E_{out}$ siempre)
  - → Activos (E_{in} puede ser menor a E_{out}) → Fuentes dependientes.



Configuraciones comunes:

• Conf. "L"



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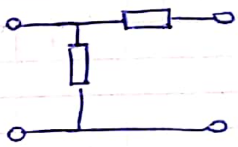
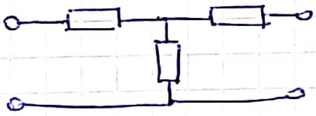
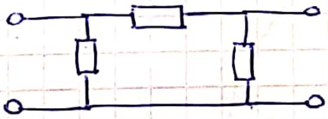


• Conf. "T"



• Conf. "π"



Parámetros → Relacionan las 4 variables de un cuadripolo de distintas maneras (E_{in}, E_{out}, I_{in}, I_{out}). Pueden ser impedancias, admitancias o adimensionales.

Par. Impedancia • [z]

$$\begin{cases} E_{in} = I_{in} \cdot Z_{11} + I_{out} \cdot Z_{12} \\ E_{out} = I_{in} \cdot Z_{21} + I_{out} \cdot Z_{22} \end{cases}$$

(Tensiones en función de corriente)

Par. Transconductancia • [g]

$$\begin{cases} I_{in} = E_{in} \cdot g_{11} + I_{out} \cdot g_{12} \\ E_{out} = E_{in} \cdot g_{21} + I_{out} \cdot g_{22} \end{cases}$$

→ (corriente entrada y tensión salida → FET)

Par. Admitancia • [Y]

$$\begin{cases} I_{in} = E_{in} \cdot Y_{11} - E_{out} \cdot Y_{12} \\ I_{out} = -E_{in} \cdot Y_{21} + E_{out} \cdot Y_{22} \end{cases}$$

(corrientes en función de tensiones)

Par. T_x Directa • [T_x]

$$\begin{cases} E_{in} = E_{out} \cdot A + I_{out} \cdot B \\ I_{in} = E_{out} \cdot C + I_{out} \cdot D \end{cases}$$

(Entrada en función de salida)

Parámetros híbridos • [h]

$$\begin{cases} E_{in} = I_{in} \cdot h_{11} + E_{out} \cdot h_{12} \\ I_{out} = I_{in} \cdot h_{21} + E_{out} \cdot h_{22} \end{cases}$$

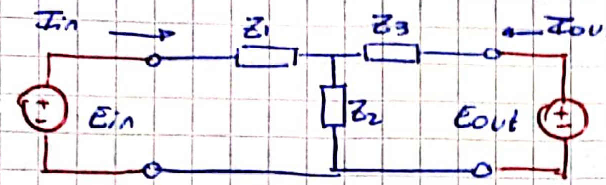
(Tensión entrada y corriente de salida) → $\boxed{\text{BJT}}$

Par. T_x Inversa • [T_x^{-1}]

$$\begin{cases} E_{out} = E_{in} \cdot E + I_{in} \cdot F \\ I_{out} = E_{in} \cdot G + I_{in} \cdot H \end{cases}$$

(Salida en función de entrada)

Parámetros Z de un cuadripolo "T"



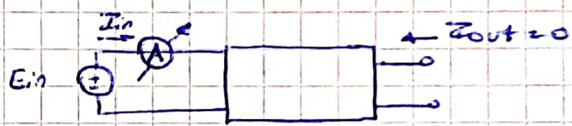
$$[Z]\begin{cases} E_{in} = I_{in} \cdot Z_{11} + I_{out} \cdot Z_{12} \\ E_{out} = I_{in} \cdot Z_{21} + I_{out} \cdot Z_{22} \end{cases} \longrightarrow \begin{cases} Z_{11} = Z_1 + Z_2 & [\Omega] \\ Z_{12} = Z_{21} = Z_2 & [\Omega] \\ Z_{22} = Z_2 + Z_3 & [\Omega] \end{cases}$$

(Ecuaciones método de las mallas)

En todo cuadripolo pasivo $Z_{12} = Z_{21}$.

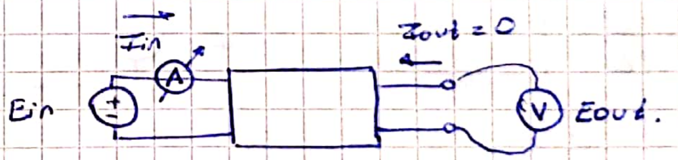
Cálculo o medición de cada parámetro: (p/ cualq. cuadripolo)

- $Z_{11} = \left.\dfrac{E_{in}}{I_{in}}\right|_{I_{out}=0}$

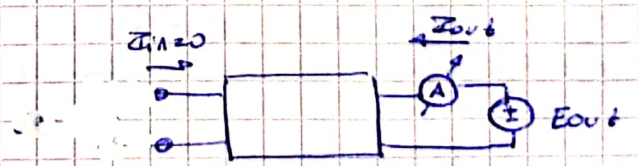


- $Z_{12} = \left.\dfrac{E_{in}}{I_{out}}\right|_{I_{in}=0}$

- $Z_{21} = \left.\dfrac{E_{out}}{I_{in}}\right|_{I_{out}=0}$

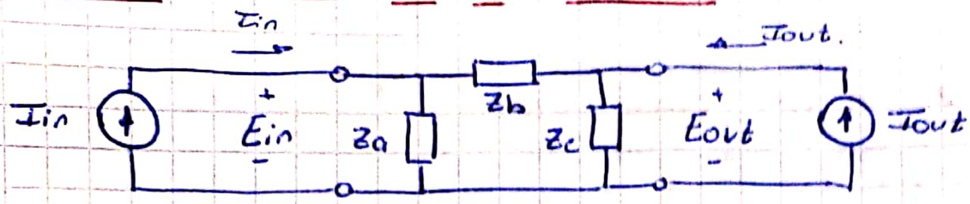


- $Z_{22} = \left.\dfrac{E_{out}}{I_{out}}\right|_{I_{in}=0}$



En este caso se usaron fuentes de tensión y se midieron corrientes, se pueden usar fuentes de corrientes y medir tensiones. Los valores a calcular / medir serán los mismos.

Parámetros Y de un cuadripolo "π"



$$Y \begin{cases} I_{in} = E_{in} \cdot Y_{11} - E_{out} \cdot Y_{12} \\ J_{out} = -E_{in} \cdot Y_{21} + E_{out} \cdot Y_{22} \end{cases}$$

(Ecuaciones método nodal)

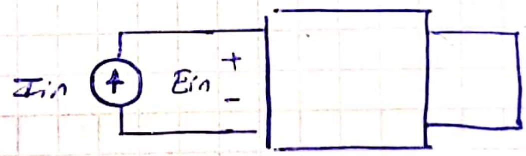
- $Y_{11} = \dfrac{1}{Z_a} + \dfrac{1}{Z_b}$ [℧] → Autoadmitancia nodo de entrada.

- $Y_{12} = Y_{21} = \dfrac{1}{Z_b}$ [℧] → Admitancia compartida.

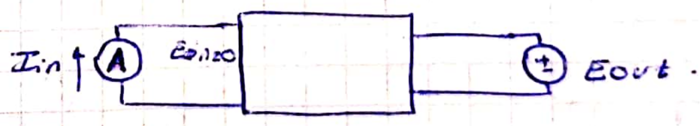
- $Y_{22} = \dfrac{1}{Z_b} + \dfrac{1}{Z_c}$ [℧] → Autoadmitancia nodo de salida.

Como medirlas o calcularlas: (General p/ cualquier cuadripolo).

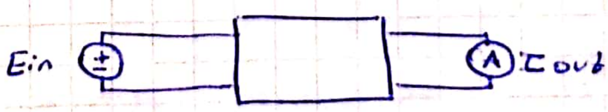
- $Y_{11} = \left.\dfrac{I_{in}}{E_{in}}\right|_{E_{out}=0}$



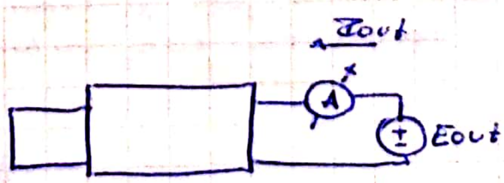
- $Y_{12} = \left.\dfrac{I_{in}}{E_{out}}\right|_{E_{in}=0}$



- $Y_{21} = \left.\dfrac{J_{out}}{E_{in}}\right|_{E_{out}=0}$



- $Y_{22} = \left.\dfrac{J_{out}}{E_{out}}\right|_{E_{in}=0}$



Relación entre parámetros

Los parámetros de un cuadripolo se pueden relacionar, así calculando solo 1 conjunto de los 6, mediante equivalencias calculamos los otros 5.

- Se Desarrollan
  - → Y en función de Z (a)
  - → T_x en función de Z. (b)
  - → T_x^{-1} en función de Z. (c)

(a) Parámetros Y en función de Z.

$$Y \begin{cases} I_{in} = E_{in} \cdot Y_{11} - E_{out} \cdot Y_{12} \\ I_{out} = -E_{in}\, Y_{21} + E_{out} \cdot Y_{22} \end{cases} \quad ; \quad Z \begin{cases} E_{in} = I_{in} \cdot Z_{11} + I_{out} \cdot Z_{12} \\ E_{out} = I_{in} \cdot Z_{21} + I_{out}\, Z_{22} \end{cases}$$

De las ecuaciones Z despejo I_{in} y I_{out}.

$$\begin{cases} I_{in} = E_{in} \cdot \dfrac{1}{Z_{11}} - I_{out} \cdot \dfrac{Z_{12}}{Z_{11}} \quad (1) \\ I_{out} = E_{out} \cdot \dfrac{1}{Z_{22}} - I_{in} \cdot \dfrac{Z_{21}}{Z_{22}} \quad (2) \end{cases}$$

Reemplazo cruzado (I_{in} en (2) y I_{out} en (1))

$$\begin{cases} I_{in} = E_{in} \cdot \dfrac{1}{Z_{11}} - E_{out} \cdot \dfrac{Z_{12}}{Z_{11} \cdot Z_{22}} + I_{in} \cdot \dfrac{Z_{21} \cdot Z_{12}}{Z_{11} \cdot Z_{22}} \\ I_{out} = E_{out} \cdot \dfrac{1}{Z_{22}} - E_{in} \cdot \dfrac{Z_{21}}{Z_{11}\, Z_{22}} + I_{out} \cdot \dfrac{Z_{21} \cdot Z_{12}}{Z_{11} \cdot Z_{22}} \end{cases}$$

$$\begin{cases} I_{in} \left(1 + \dfrac{Z_{21} \cdot Z_{12}}{Z_{11} \cdot Z_{22}}\right) = E_{in} \cdot \dfrac{1}{Z_{11}} - E_{out} \cdot \dfrac{Z_{12}}{Z_{11} \cdot Z_{22}} \\ I_{out} \left(1 + \dfrac{Z_{21} \cdot Z_{12}}{Z_{11} \cdot Z_{22}}\right) = -E_{in} \cdot \dfrac{Z_{21}}{Z_{11}\, Z_{22}} + E_{out} \cdot \dfrac{1}{Z_{22}} \end{cases}$$

$$1 + \frac{Z_{21} \cdot Z_{12}}{Z_{11}\, Z_{22}} = \frac{Z_{11}\, Z_{22} - Z_{21}\, Z_{12}}{Z_{11}\, Z_{22}} = \frac{\Delta Z}{Z_{11}\, Z_{22}}$$

$$\begin{cases} I_{in} \cdot \dfrac{\Delta Z}{Z_{11} Z_{22}} = E_{in} \cdot \dfrac{1}{Z_{11}} - E_{out} \cdot \dfrac{Z_{12}}{Z_{11}\, Z_{22}} \\ I_{out} \cdot \dfrac{\Delta Z}{Z_{11}\, Z_{22}} = -E_{in} \cdot \dfrac{Z_{21}}{Z_{11}\, Z_{22}} + E_{out} \cdot \dfrac{1}{Z_{22}} \end{cases}$$

$$\begin{cases} I_{in} = E_{in} \underbrace{\dfrac{Z_{22}}{\Delta Z}}_{Y_{11}} - E_{out} \cdot \underbrace{\dfrac{Z_{12}}{\Delta Z}}_{Y_{12}} \\ I_{out} = -E_{in} \underbrace{\dfrac{Z_{21}}{\Delta Z}}_{Y_{21}} + E_{out} \cdot \underbrace{\dfrac{Z_{11}}{\Delta Z}}_{Y_{22}} \end{cases} \quad ;$$

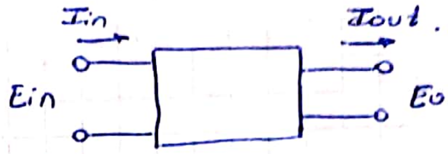
Equivalencia:

$$\boxed{[Y] = \frac{1}{\Delta Z} \begin{bmatrix} Z_{22} & Z_{12} \\ Z_{21} & Z_{11} \end{bmatrix}}$$

$$|T_x| = \begin{vmatrix} A & B \\ C & D \end{vmatrix} = A \cdot D - B \cdot C = 1 .$$ ⇒ Propiedad que se cumple siempre en cuad. pasivos

b) Parámetros Tx dir en función de Z.

$$[T_x] \begin{cases} E_{in} = E_{out} \cdot A + I_{out} \cdot B & \text{(I)} \\ I_{in} = E_{out} \cdot C + I_{out} \cdot D & \text{(II)} \end{cases}$$



⇒ Se cambia el sentido de Iout.

$$[Z] \begin{cases} E_{in} = I_{in} \cdot Z_{11} - I_{out} \cdot Z_{12} & (1) \\ -E_{out} = -I_{in} \cdot Z_{21} + I_{out} \cdot Z_{22} & (2) \end{cases}$$

Cambia signos de Iout y de Eout.

De (2) despejo Iin.

$$\boxed{I_{in} = E_{out} \cdot \frac{1}{Z_{21}} + I_{out} \cdot \frac{Z_{22}}{Z_{21}}}$$

⇒ Comparada con (II) ya está lista. $I_{in} = f(E_{out}, I_{out})$.

Reemplazo en (1).

$$E_{in} = E_{out} \cdot \frac{Z_{11}}{Z_{21}} + I_{out} \underbrace{\left(\frac{Z_{11} \cdot Z_{22}}{Z_{21}} - Z_{12} \right)}_{\frac{\Delta Z}{Z_{21}}}$$

$$\boxed{E_{in} = E_{out} \cdot \frac{Z_{11}}{Z_{21}} + I_{out} \cdot \frac{\Delta Z}{Z_{21}}}$$

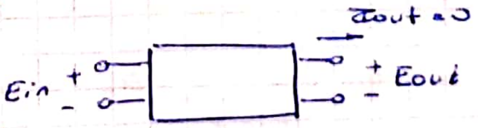
⇒ Comparada con (I) está lista. $E_{in} = f(E_{out}, I_{out})$.

$$\begin{cases} E_{in} = E_{out} \cdot \underbrace{\frac{Z_{11}}{Z_{21}}}_{A} + I_{out} \cdot \underbrace{\frac{\Delta Z}{Z_{21}}}_{B} \\ I_{in} = E_{out} \cdot \underbrace{\frac{1}{Z_{21}}}_{C} + I_{out} \cdot \underbrace{\frac{Z_{22}}{Z_{21}}}_{D} \end{cases}$$

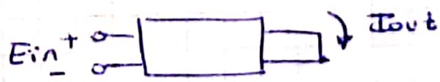
$$\boxed{[T_x] = \frac{1}{Z_{21}} \cdot \begin{bmatrix} Z_{11} & \Delta Z \\ 1 & Z_{22} \end{bmatrix}}$$

Medición / Cálculo Tx :

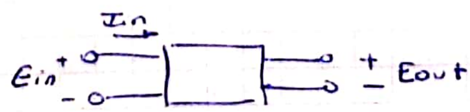
- $A = \frac{Z_{11}}{Z_{21}} = \left.\frac{E_{in}}{E_{out}}\right|_{I_{out}=0} = [-]$



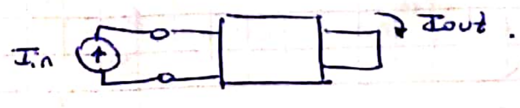
- $B = \frac{\Delta Z}{Z_{21}} = \left.\frac{E_{in}}{I_{out}}\right|_{E_{out}=0} = [\Omega]$



- $C = \frac{1}{Z_{21}} = \left.\frac{I_{in}}{E_{out}}\right|_{I_{out}=0} = [\mho]$



- $D = \frac{Z_{22}}{Z_{21}} = \left.\frac{I_{in}}{I_{out}}\right|_{E_{out}=0} = [-]$



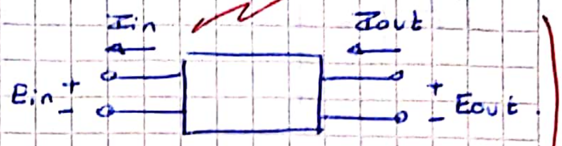
$$\left|T_x^{-1}\right| = \begin{vmatrix} \overset{(D)}{E} & \overset{(B)}{F} \\ \overset{(C)}{G} & \overset{(A)}{H} \end{vmatrix} = E\cdot H - F\cdot G = 1 \quad (D\cdot A - B\cdot C = 1) \Rightarrow$$

Propiedad que se cumple siempre en cuad. pasivos

(c) Parámetros T_x^{-1} en función de Z.

$$[T_x^{-1}] = \begin{cases} E_{out} = E_{in}\cdot E + I_{in}\cdot F & (I) \\ I_{out} = E_{in}\cdot G + I_{in}\cdot H & (II) \end{cases}$$

EFGH



Se invierte sentido de In

$$[Z] \begin{cases} -E_{in} = I_{in}\cdot Z_{11} - I_{out}\cdot Z_{12} & (1) \\ E_{out} = -I_{in}\cdot Z_{21} + I_{out}\cdot Z_{22} & (2) \end{cases}$$

Cambia signos de corrientes y de Ein

- De (1) despejo I_{out}.

$$\boxed{I_{out} = E_{in}\cdot \frac{1}{Z_{12}} + I_{in}\cdot \frac{Z_{11}}{Z_{12}}}$$

Comparable con (II).

- Reemplazando en (2):

$$E_{out} = -I_{in}\cdot Z_{21} + E_{in}\,\frac{Z_{22}}{Z_{12}} + I_{in}\cdot \frac{Z_{11}Z_{22}}{Z_{12}}.$$

$$E_{out} = E_{in}\cdot \frac{Z_{22}}{Z_{12}} + I_{in}\underbrace{\left(\frac{Z_{11}Z_{22}}{Z_{12}} - Z_{21}\right)}_{\frac{\Delta Z}{Z_{12}}}.$$

$$\boxed{E_{out} = E_{in}\cdot \frac{Z_{22}}{Z_{12}} + I_{in}\cdot \frac{\Delta Z}{Z_{12}}}$$

Comparable con (I).

$$\begin{cases} E_{out} = E_{in}\cdot \overset{E}{\frac{Z_{22}}{Z_{12}}} + I_{in}\cdot \overset{F}{\frac{\Delta Z}{Z_{12}}} \\ I_{out} = E_{in}\cdot \underset{G}{\frac{1}{Z_{12}}} + I_{in}\cdot \underset{H}{\frac{Z_{11}}{Z_{12}}} \end{cases}$$

Si el cuadripolo es pasivo $Z_{12} = Z_{21}$, entonces:

$$E = D = \frac{Z_{22}}{Z_{12}} = \frac{Z_{22}}{Z_{21}} \;;\; F = B = \frac{\Delta Z}{Z_{12}} = \frac{\Delta Z}{Z_{21}}.$$

$$G = C = \frac{1}{Z_{12}} = \frac{1}{Z_{21}} \;;\; H = A = \frac{Z_{11}}{Z_{12}} = \frac{Z_{11}}{Z_{21}}.$$

Demostración de que $|T_x| = 1$.

$$|T_x| = \begin{vmatrix} A & B \\ C & D \end{vmatrix} = \begin{vmatrix} \dfrac{z_{11}}{z_{21}} & \dfrac{\Delta z}{z_{21}} \\ \dfrac{1}{z_{21}} & \dfrac{z_{22}}{z_{21}} \end{vmatrix} = \frac{z_{11}}{z_{21}} \cdot \frac{z_{22}}{z_{21}} - \frac{\Delta z}{z_{21}} \cdot \frac{1}{z_{21}} =$$

$$= \frac{1}{(z_{21})^2} \left(z_{11} \cdot z_{22} - z_{11} \cdot z_{22} + z_{12} \cdot z_{21} \right) =$$

$$= \frac{1}{z_{21}^{2}} \left(z_{12} \cdot z_{21} \right)$$

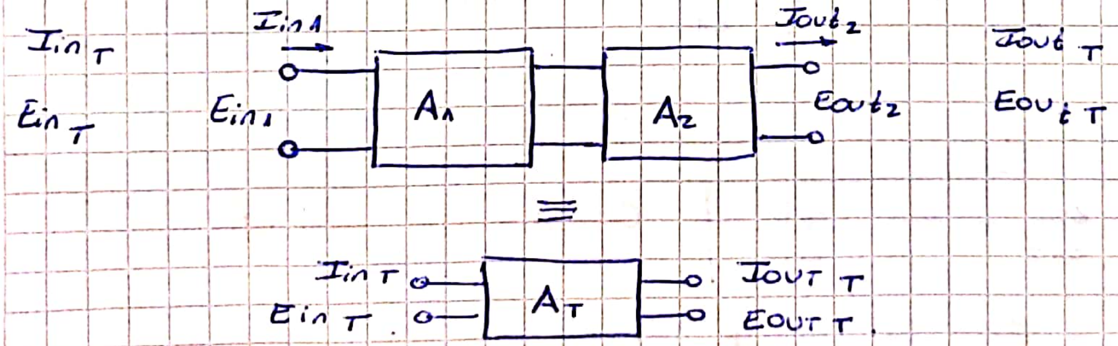
Si el cuad. es pasivo $z_{12} \cdot z_{21} = z_{21}^2$ porque $z_{12} = z_{21}$.

y $|T_x| = 1$.

Conexiones de cuadripolos.

Según la conexión, es conveniente usar distintos tipos de parámetros por facilidad para calcular el parámetro total de la conexión.

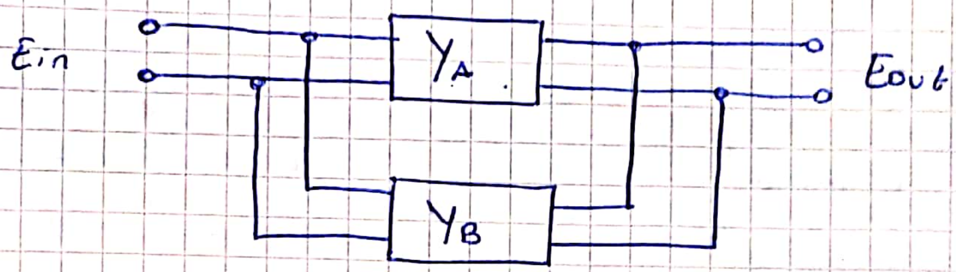
- Conexión cascada.



La manera más fácil es mediante los de transmisión directa.

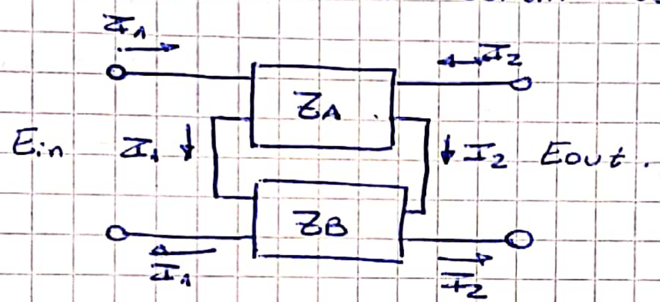
$$[T_{x_T}] = [T_{x_1}] \cdot [T_{x_2}] = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \times \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

- Conexión paralela → Se suman los parámetros "Y".



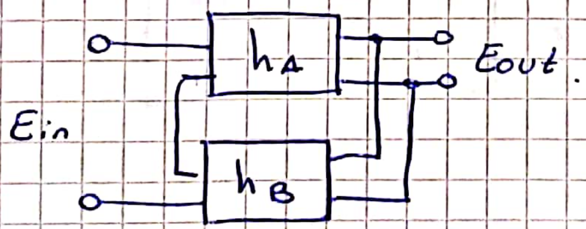
$$[Y_T] = [Y_A] + [Y_B]$$

- Conexión serie → Se suman los parámetros "Z"



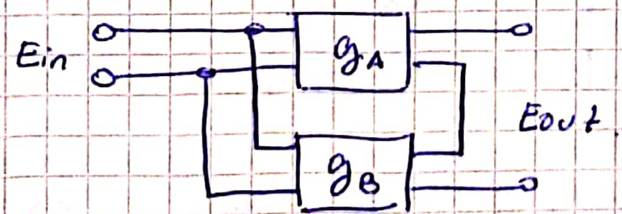
$$[Z_T] = [Z_A] + [Z_B]$$

- Serie - Paralelo → Se suman parámetros "h"



$$[h_T] = [h_A] + [h_B]$$

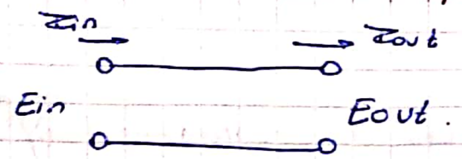
- Paralelo - Serie → Se suman parámetros "g"



$$[g_T] = [g_A] + [g_B]$$

Parámetros Tx cuadripolos básicos.

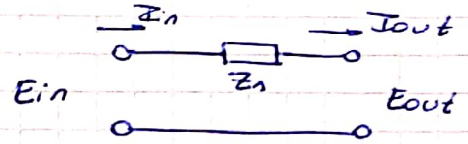
①



$$\begin{cases} E_{in} = E_{out}\cdot \overset{1}{A} + I_{out}\cdot \overset{0}{B} \\ I_{in} = E_{out}\cdot \overset{0}{C} + I_{out}\cdot \overset{1}{D}. \end{cases}$$

$$[T_x] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

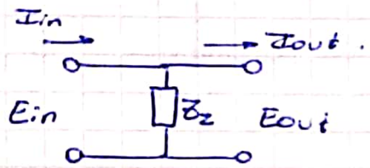
②



$$\begin{cases} E_{in} = E_{out}\cdot \overset{1}{A} + I_{out}\cdot \overset{Z_1}{B} \\ I_{in} = E_{out}\cdot \overset{0}{C} + I_{out}\cdot \overset{1}{D}. \end{cases}$$

$$[T_x] = \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix}$$

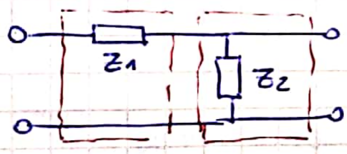
③



$$\begin{cases} E_{in} = E_{out}\cdot \overset{1}{A} + I_{out}\cdot \overset{0}{B} \\ I_{in} = E_{out}\cdot \underset{\frac{1}{Z_2}}{C} + I_{out}\cdot \overset{1}{D} \end{cases}$$

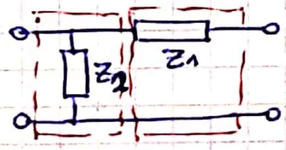
$$[T_x] = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_2} & 1 \end{bmatrix}$$

④



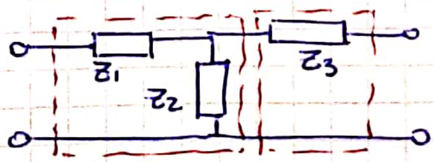
$$[T_x] = \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_2} & 1 \end{bmatrix} = \begin{bmatrix} 1 + \frac{Z_1}{Z_2} & Z_1 \\ 1/Z_2 & 1 \end{bmatrix}$$

⑤



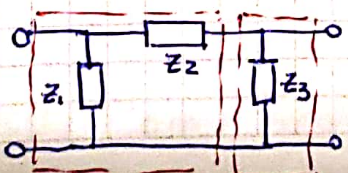
$$[T_x] = \begin{bmatrix} 1 & 0 \\ 1/Z_2 & 1 \end{bmatrix} \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & Z_1 \\ 1/Z_2 & 1 + \frac{Z_1}{Z_2} \end{bmatrix}$$

⑥



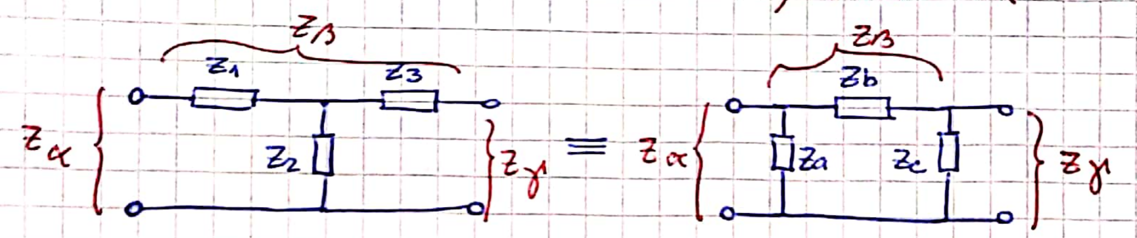
$$[T_x] = \begin{bmatrix} 1 + \frac{Z_1}{Z_2} & Z_1 \\ 1/Z_2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & Z_3 \\ 0 & 1 \end{bmatrix}$$

⑦



$$[T_x] = \begin{bmatrix} 1 + \frac{Z_2}{Z_1} & Z_2 \\ 1/Z_1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 1/Z_3 & 1 \end{bmatrix}$$

Transformación de cuad. π a T y viceversa (estrella - triángulo)



$$Z_\alpha = Z_1 + Z_2 = Z_a // (Z_b + Z_c) = \frac{Z_a \cdot Z_b + Z_a \cdot Z_c}{Z_a + Z_b + Z_c} \qquad (1)$$

$$Z_\beta = Z_1 + Z_3 = Z_b // (Z_a + Z_c) = \frac{Z_a \cdot Z_b + Z_b \cdot Z_c}{Z_a + Z_b + Z_c} \qquad (2)$$

$$Z_\gamma = Z_3 + Z_2 = Z_c // (Z_b + Z_a) = \frac{Z_a \cdot Z_c + Z_b \cdot Z_c}{Z_a + Z_b + Z_c} \qquad (3)$$

- Para $Z_1 \rightarrow (1) + (2) - (3)$:

$$Z_1 + \cancel{Z_2} + Z_1 + \cancel{Z_3} - \cancel{Z_2} - \cancel{Z_3} = \frac{Z_a Z_b + \cancel{Z_a Z_c} + Z_a Z_b + \cancel{Z_b Z_c} - \cancel{Z_a Z_c} - \cancel{Z_b Z_c}}{Z_a + Z_b + Z_c}$$

$$2 Z_1 = \frac{2 Z_a Z_b}{Z_a + Z_b + Z_c} \rightarrow \boxed{Z_1 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}} \quad (a)$$

- Para $Z_2 \rightarrow (1) + (3) - (2)$:

$$\cancel{Z_1} + Z_2 + \cancel{Z_3} + Z_2 - \cancel{Z_1} - \cancel{Z_3} = \frac{\cancel{Z_a Z_b} + Z_a Z_c + Z_a Z_c + \cancel{Z_b Z_c} - \cancel{Z_a Z_b} - \cancel{Z_b Z_c}}{Z_a + Z_b + Z_c}$$

$$\boxed{Z_2 = \frac{Z_a Z_c}{Z_a + Z_b + Z_c}} \quad (b)$$

- Para $Z_3 \rightarrow (2) + (3) - (1)$

$$\cancel{Z_1} + Z_3 + \cancel{Z_2} + Z_3 - \cancel{Z_1} - \cancel{Z_2} = \frac{\cancel{Z_a Z_b} + Z_b Z_c + \cancel{Z_a Z_c} + Z_b Z_c - \cancel{Z_a Z_b} - \cancel{Z_a Z_c}}{Z_a + Z_b + Z_c}$$

$$\boxed{Z_3 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c}} \quad (c)$$

(Hasta acá tenemos para pasar de π a T.)

Para pasar de T a π:

$$(a): Z_1 = \frac{Z_a \cdot Z_b}{Z_a + Z_b + Z_c} \; ; \; (b): Z_2 = \frac{Z_a Z_c}{Z_a + Z_b + Z_c} \; ; \; (c): Z_3 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c}.$$

$$\boxed{\frac{(a)}{(b)} = \frac{Z_1}{Z_2} = \frac{Z_b}{Z_c}} \quad \boxed{\frac{(a)}{(c)} = \frac{Z_1}{Z_3} = \frac{Z_a}{Z_c}} \quad \boxed{\frac{(b)}{(c)} = \frac{Z_2}{Z_3} = \frac{Z_a}{Z_b}}$$

(I) (II) (III).

Para $Z_a \rightarrow$ (a) ó (b)

$$(a): Z_1 = \frac{Z_a \cdot Z_b}{Z_a + Z_b + Z_c} = \frac{Z_a}{\underbrace{\frac{Z_a}{Z_b}}_{(III)} + 1 + \underbrace{\frac{Z_c}{Z_b}}_{(I)}} = \frac{Z_a}{\frac{Z_2}{Z_3} + 1 + \frac{Z_2}{Z_1}}$$

$$Z_a = Z_1 \left(1 + \frac{Z_2}{Z_3} + \frac{Z_2}{Z_1} \right) = Z_1 + \frac{Z_2 \cdot Z_1}{Z_3} + Z_2 = \frac{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}{Z_3}.$$

$$\boxed{Z_a = \frac{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}{Z_3}}$$

: Para $Z_b \rightarrow$ (a) ó (c)

$$(a): Z_1 = \frac{Z_a \cdot Z_b}{Z_a + Z_b + Z_c} = \frac{Z_b}{1 + \frac{Z_b}{Z_a} + \frac{Z_c}{Z_a}} = \frac{Z_b}{1 + \frac{Z_3}{Z_2} + \frac{Z_3}{Z_1}}$$

$$Z_b = Z_1 + \frac{Z_3 Z_1}{Z_2} + Z_3 = \frac{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}{Z_2} \rightarrow \boxed{Z_b = \frac{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}{Z_2}}$$

Para $Z_c \rightarrow$ (b) ó (c)

$$(b): Z_2 = \frac{Z_a Z_c}{Z_a + Z_b + Z_c} = \frac{Z_c}{1 + \frac{Z_b}{Z_a} + \frac{Z_c}{Z_a}} = \frac{Z_c}{1 + \frac{Z_3}{Z_2} + \frac{Z_3}{Z_1}}$$

$$Z_c = Z_2 + Z_3 + \frac{Z_2 Z_3}{Z_1}$$

$$\boxed{Z_c = \frac{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}{Z_1}}$$

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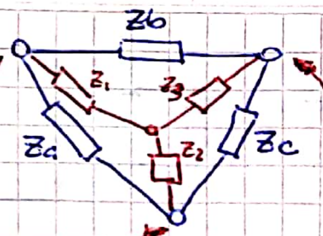
Regla memotécnica T a π:

Denominador = Suma de todos

$$Z_1 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}$$ → Los Z que tocan



$$Z_2 = \frac{Z_a Z_c}{Z_a + Z_b + Z_c}$$

$$Z_3 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c}.$$

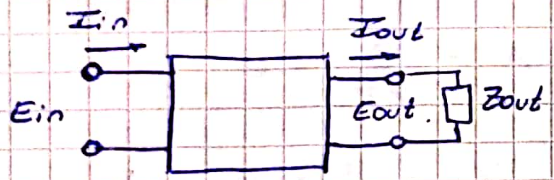
Numerador = Producto de Todos

$$Z_a = \frac{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}{Z_3}$$ → Opuesto a Za.

$$Z_b = \frac{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}{Z_2}$$ → Opuesto a Zb

$$Z_c = \frac{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}{Z_1}$$ → Opuesto a Zc.

Cuadripolos Cargados.



$$T_x \begin{cases} E_{in} = E_{out} \cdot A + I_{out} \cdot B \\ I_{in} = E_{out} \cdot C + I_{out} \cdot D. \end{cases}$$

Impedancia de entrada = .

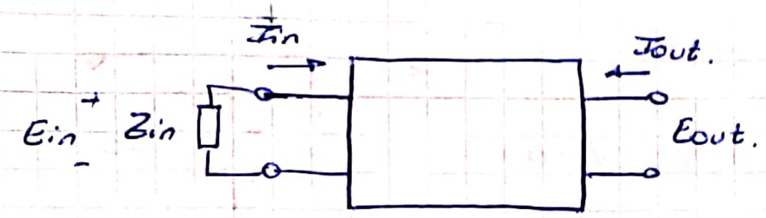
$$Z_{in} = \frac{E_{in}}{I_{in}} = \frac{E_{out} \cdot A + I_{out} \cdot B}{E_{out} \cdot C + I_{out} \cdot D}$$

$$E_{out} = I_{out} \cdot Z_{out}. \implies I_{out} = \frac{E_{out}}{Z_{out}}.$$

$$Z_{in} = \frac{\cancel{E_{out}} \cdot A + \frac{\cancel{E_{out}}}{Z_{out}} \cdot B}{\cancel{E_{out}} \cdot C + \frac{\cancel{E_{out}}}{Z_{out}} \cdot D}$$

$$\boxed{Z_{in} = \frac{A + B/Z_{out}}{C + D/Z_{out}}.}$$

Impedancia de salida.



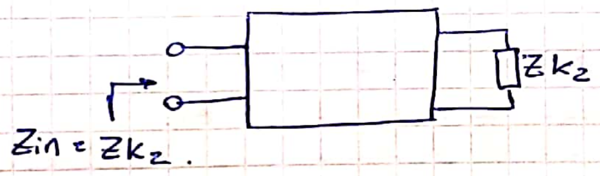
$$T_k^{-1}\begin{cases} E_{out} = E_{in}\cdot \overset{D}{E} + I_{in}\cdot \overset{B}{F}. \\ J_{out} = E_{in}\,\underset{C}{G} + I_{in}\cdot \underset{A}{H}. \end{cases}$$

$$Z_{out} = \frac{E_{out}}{J_{out}} = \frac{E_{in}\cdot D + I_{in}\cdot B}{E_{in}\cdot C + I_{in}\cdot A}\ ; \quad I_{in} = \frac{E_{in}}{Z_{in}}.$$

$$Z_{out} = \frac{E_{in}\,D + \frac{E_{in}}{Z_{in}}B}{E_{in}\cdot C + \frac{E_{in}}{Z_{in}}D} \longrightarrow \boxed{Z_{out} = \frac{D + B/Z_{in}}{C + D/Z_{in}}.}$$

Impedancia iterativa de salida (Z_{k_2}).

Impedancia que conectada en la salida, produce una Zin igual a esa carga.



$$Z_{in} = \frac{A + B/Z_{out}}{C + D/Z_{out}}\ ; \quad Z_{in} = Z_{out} = Z_{k_2}.$$

$$Z_{k_2} = \frac{A + B/Z_{k_2}}{C + D/Z_{k_2}} \longrightarrow Z_{k_2}\cdot C + D - A - \frac{B}{Z_{k_2}} = \varnothing.$$

$$Z_{k_2}^2\cdot C + (D-A)Z_{k_2} - B = \varnothing.$$

$$\underbrace{1}_{a} Z_{k_2}^2 + \underbrace{\left(\frac{D-A}{C}\right)}_{b}\cdot Z_{k_2} - \underbrace{\frac{B}{C}}_{c} = \varnothing.$$

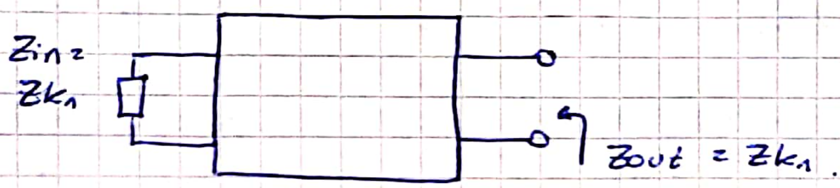
$$Z_{k_2} = \frac{-b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - c}$$

$$\boxed{Z_{k_2} = -\left(\frac{D-A}{2C}\right) \pm \sqrt{\left(\frac{D-A}{2C}\right)^2 + \frac{B}{C}}}$$

De los 2 resultados solo será válido aquel con $R>0$.

Impedancia iterativa de entrada (Zk_1).

Si pongo Zk_1 en la entrada, veo Zk_1 como $Zout$.



$$Zout = \frac{D + B/Zin}{C + A/Zin} \quad ; \quad Zout = Zin = Zk_1 .$$

$$Zk_1 = \frac{D + B/Zk_1}{C + A/Zk_1} .$$

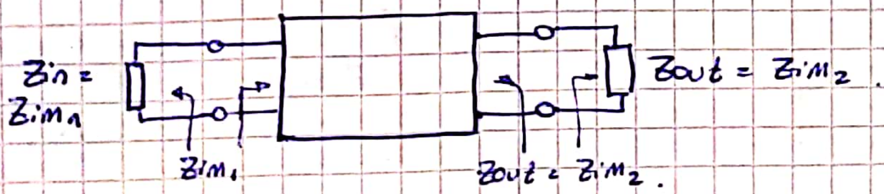
$$Zk_1 \cdot C + A - D - \frac{B}{Zk_1} = 0$$

$$Zk_1^2 \cdot C + (A-D)\, Zk_1 - B = \emptyset$$

$$Zk_1^2 + \left(\frac{A-D}{C}\right) Zk_1 - \frac{B}{C} = \emptyset$$

$$\boxed{Zk_1 = -\left(\frac{A-D}{C}\right) \pm \sqrt{\left(\frac{A-D}{C}\right)^2 + \frac{B}{C}}}$$

Impedancias imagen Zim_1 (entrada) y Zim_2 (salida)



Si tengo cargado el cuadripolo con Zim_1 en la entrada y con Zim_2 en la salida, tengo como Zin a Zim_1 mirando al cuadripolo de la entrada y a Zim_2 como $Zout$.

- Cargo con Zim_2 (en salida) $\longrightarrow$ Tengo $Zin = Zim_1$.
- Cargo con Zim_1 (en entrada) $\longrightarrow$ Tengo $Zout = Zim_2$.

} Son cruzadas (Alrevez de Zk_1 y Zk_2).

Z_{im_1}

$$Z_{in} = \frac{A + B/Z_{out}}{C + D/Z_{out}}$$

$$Z_{im_1} = \frac{A + B/Z_{im_2}}{C + D/Z_{im_2}}$$

Reemplazo cruzado

$$Z_{im_1} = \frac{A + B \cdot \dfrac{C + A/Z_{im_1}}{D + B/Z_{im_1}}}{C + D \cdot \dfrac{C + A/Z_{im_1}}{D + B/Z_{im_1}}}$$

$$Z_{im_1} = \frac{A \cdot D + AB/Z_{im_1} + BC + AB/Z_{im_1}}{CD + BC/Z_{im_1} + CD + AD/Z_{im_1}}$$

$$CD \cdot Z_{im_1} + \cancel{BC} + CD \cdot Z_{im_1} + \cancel{AD} + \ldots$$

$$- \cancel{AD} - AB/Z_{im_1} - \cancel{BC} - AB/Z_{im_1} = 0$$

$$2\, C \cdot D\, Z_{im_1} - 2 \frac{AB}{Z_{im_1}} = 0$$

$$Z_{im_1}^2\, CD - AB = 0$$

$$Z_{im_1}^2 = \frac{AB}{CD}$$

$$\boxed{Z_{im_1} = \sqrt{\frac{AB}{CD}}}$$

Z_{im_2}

$$Z_{out} = \frac{D + B/Z_{in}}{C + A/Z_{in}}$$

$$Z_{im_2} = \frac{D + B/Z_{im_1}}{C + A/Z_{im_1}}$$

$$Z_{im_2} = \frac{D + B \cdot \dfrac{C + D/Z_{im_2}}{A + B/Z_{im_2}}}{C + A \cdot \dfrac{C + D/Z_{im_2}}{A + B/Z_{im_2}}}$$

$$Z_{im_2} = \frac{DA + BD/Z_{im_2} + BC + BD/Z_{im_2}}{AC + BC/Z_{im_2} + AC + AD/Z_{im_2}}$$

$$AC\, Z_{im_2} + \cancel{BC} + AC\, Z_{im_2} + \cancel{AD} + \ldots$$

$$- \cancel{DA} - BD/Z_{im_2} - \cancel{BC} - BD/Z_{im_2} = \emptyset$$

$$2\, AC\, Z_{im_2} - 2 \frac{BD}{Z_{im_2}} = \emptyset$$

$$Z_{im_2}^2 \cdot AC - BD = \emptyset$$

$$Z_{im_2}^2 = \frac{BD}{AC}$$

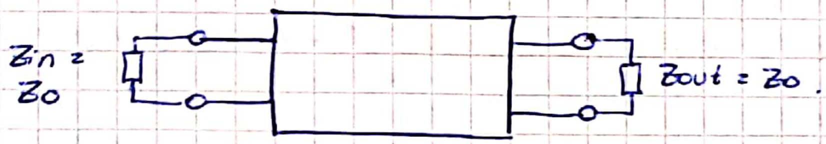
$$\boxed{Z_{im_2} = \sqrt{\frac{BD}{CA}}}$$

Regla memotécnica → ABCD. Intercambio D por A.

Cuadripolo cargado con Zo (impedancia característica)

Solo válido para cuadripolos simétricos (Es lo mismo conectarlo de una forma o alrevez)
→ Se puede intercambiar entrada con salida. Ej: cable coaxil.

$\hookrightarrow Z_{11} = Z_{22} \quad \Rightarrow \quad A = \frac{Z_{11}}{Z_{21}} = D = \frac{Z_{22}}{Z_{21}}.$



$$Z_{in} = Z_{out} = Z_o = \underbrace{\frac{A + B/Z_o}{C + D/Z_o}}_{\text{Fórmula } Z_{in}} = \underbrace{\frac{D + B/Z_o}{C + A/Z_o}}_{\text{Fórmula } Z_{out}}.$$

$$Z_o \cdot C + \cancel{D} - \cancel{A} - \frac{B}{Z_o} = 0 \qquad (D - A = 0 \text{ porque son iguales}).$$

$$Z_o^2 \cdot C = B$$

$$\boxed{Z_o = \sqrt{\frac{B}{C}}}$$

Si el cuadripolo es simétrico, además:

$$\boxed{Z_{k_1} = Z_{k_2} = Z_{im_1} = Z_{im_2} = Z_o.}$$

Resumen Cuadripolos cargados.

$$Z_{k_1} = -\left(\frac{A-D}{2C}\right) \pm \sqrt{\left(\frac{A-D}{2C}\right)^2 + \frac{B}{C}}$$

$$Z_{k_2} = -\left(\frac{D-A}{2C}\right) \pm \sqrt{\left(\frac{A-D}{2C}\right)^2 + \frac{B}{C}}.$$

$$Z_{im_1} = \sqrt{\frac{AB}{CD}}.$$

$$Z_{im_2} = \sqrt{\frac{BD}{AC}}$$

$Z_o = \sqrt{\frac{B}{C}} \quad \Rightarrow$ Sólo válido si es simétrico ; Y $\rightarrow Z_o = Z_{im_1} = Z_{im_2} = Z_{k_1} = Z_{k_2}$.

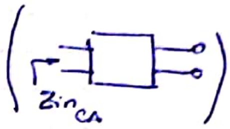
Método alternativo de cálculo de impedancias imágenes y característica (Zim y Zo).

* **Zim_1:**

$$T_x \begin{cases} E_{in} = E_{out} \cdot A + I_{out} \cdot B \\ I_{in} = E_{out} \cdot C + I_{out} \cdot D. \end{cases}$$

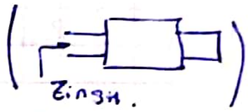
$$Z_{in} = \frac{E_{in}}{I_{in}} = \frac{E_{out} \cdot A + I_{out} \cdot B}{E_{out} \cdot C + I_{out} \cdot D}.$$

- Z_{in} a circ. abierto (O.C = open circuit) $\rightarrow I_{out} = 0$



$$Zin_{OC} = \frac{A}{C}.$$

- Z_{in} con salida a CC (SH = short circuit) $\rightarrow E_{out} = 0$



$$Zin_{SH} = \frac{B}{D}.$$

Z_{in} $\rightarrow$ Media geométrica Zin_{OC} y Zin_{SH}

$$Z_{in} = \sqrt{Zin_{OC} \times Zin_{SH}} = \sqrt{\frac{AB}{CD}} = Zim_1.$$

$$\boxed{Zim_1 = \sqrt{Zin_{OC} \times Zin_{SH}}}$$

* **Zim_2:**

$$T_x^{-1} \begin{cases} E_{out} = E_{in} \cdot D + I_{in} \cdot B \\ I_{out} = E_{in} \cdot C + I_{in} \cdot A. \end{cases}$$

$$Z_{out} = \frac{E_{out}}{I_{out}} = \frac{E_{in}\, D + I_{in}\, B}{E_{in}\, C + I_{in}\, A}.$$

- Z_{out} a C.A. ($I_{in} = 0$)

$$Zout_{OC} = \frac{D}{C}.$$

- Z_{out} a CC. ($E_{in} = 0$)

$$Zout_{SH} = \frac{B}{A}.$$

- Z_{out}:

$$Z_{out} = \sqrt{Zout_{OC} \times Zout_{SH}} = \sqrt{\frac{DB}{CA}} = Zim_2.$$

$$\boxed{Zim_2 = \sqrt{Zout_{OC} \times Zout_{SH}}}$$

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Zo (Cuadripolos simétricos) → A = D

$$\boxed{Zo = \sqrt{Zin_{OC} \times Zin_{SH}} = \sqrt{Zout_{OC} \times Zout_{SH}}}$$

* Función de Propagación de Tensión y corriente en cuadripolos cargados.

$$T_x \begin{cases} Ein = Eout \cdot A + Iout \cdot B \\ Iin = Eout \cdot C + Iout \cdot D. \end{cases}$$

$$Iout = \frac{Eout}{Zout}.$$

$$Eout = Iout \cdot Zout.$$

(1) Función de propagación de tensiones.

$$Ein = Eout \cdot A + Eout \frac{\cdot B}{Zout}.$$

$$\boxed{\frac{Ein}{Eout} = A + \frac{B}{Zout}.}$$

(2) Función de propagación de corrientes.

$$Iin = Iout \cdot C \cdot Zout + Iout \; D.$$

$$\boxed{\frac{Iin}{Iout} = D + C \cdot Zout}$$

Son las inversas de las F(P), pero con carga, me da la relación entrada salida, según lo que pongo, me da lo que saco. o cuanto tengo que poner para sacar x cosa.

Función de propagación con carga → Zk_2, Zim_2, Zo.

Desde acá sólo es válido para cuad pasivos $(AD - BC = 1)$.

Función de propagación de Tensión con Zk_2 como carga.

$$\left.\frac{E_{in}}{E_{out}}\right|_{Zk_2} = A + \frac{B}{-\left(\frac{D-A}{2C}\right) + \sqrt{\left(\frac{D-A}{2C}\right)^2 + \frac{B}{C}}} \qquad \left(A + \frac{B}{Zk_2} = \left.\frac{E_{in}}{E_{out}}\right|_{Zk_2} \right)$$

Solo tomamos la positiva.

Racionalizando (mult y div por $\left[-\left(\frac{D-A}{2C}\right) - \sqrt{\left(\frac{D-A}{2C}\right)^2 + \frac{B}{C}} \right]$

$$\left.\frac{E_{in}}{E_{out}}\right|_{Zk_2} = A + \frac{B \cdot \left[-\left(\frac{D-A}{2C}\right) - \sqrt{\left(\frac{D-A}{2C}\right)^2 + \frac{B}{C}} \right]}{\cancel{\left(\frac{D-A}{2C}\right)^2} - \cancel{\left(\frac{D-A}{2C}\right)^2} - \frac{B}{C}}$$

$\left((a+b)(a-b) = a^2 - b^2 \; \text{(Diferencia de cuadrados)} \right)$.

$$\left.\frac{E_{in}}{E_{out}}\right|_{Zk_2} = A + \frac{\cancel{B} \left[+ \left(\frac{D-A}{2C}\right) + \sqrt{\left(\frac{D-A}{2C}\right)^2 + \frac{B}{C}} \right]}{+\cancel{B}/C}$$

$$\left.\frac{E_{in}}{E_{out}}\right|_{Zk_2} = A + C \left[\frac{D-A}{2C} + \sqrt{\left(\frac{D-A}{2C}\right)^2 + \frac{B}{C}} \right.$$

$$\left.\frac{E_{in}}{E_{out}}\right|_{Zk_2} = A + \frac{D-A}{2} + \sqrt{\left(\frac{D-A}{2}\right)^2 + B \cdot C}.$$

$(BC = AD - 1 \ldots) \longrightarrow$ sale de que $|T_x| = 1$.

$$\left.\frac{E_{in}}{E_{out}}\right|_{Zk_2} = A + \frac{D}{2} - \frac{A}{2} + \sqrt{\frac{D^2}{4} - \frac{2AD}{4} + \frac{A^2}{4} + AD - 1}$$

$$\left.\frac{E_{in}}{E_{out}}\right|_{Zk_2} = \frac{A+D}{2} + \sqrt{\frac{D^2 + 2AD + A^2}{4} - 1} = \underbrace{\frac{A+D}{2}}_{\cosh\gamma} + \underbrace{\sqrt{\left(\frac{A+D}{2}\right) - 1}}_{\operatorname{senh}\gamma}.$$

$$\cosh\gamma = \mu \longrightarrow \operatorname{senh}\gamma = \sqrt{\mu^2 - 1}.$$

$$\boxed{\left.\frac{E_{in}}{E_{out}}\right|_{Zk_2} = \frac{A+D}{2} + \sqrt{\left(\frac{A+D}{2}\right)^2 - 1} = \cosh\gamma + \operatorname{senh}\gamma = e^{\gamma} = e^{\alpha} \cdot e^{dB}}$$

γ = Función de propagación. $= \alpha + j\beta$.

α = Función de atenuación

β = Función de fase.

$$\left.\frac{E_{in}}{E_{out}}\right|_{Zk_2} = e^{\gamma} = e^{\alpha}\cdot e^{j\beta}$$

$$\alpha = Ln\left(\left.\frac{E_{in}}{E_{out}}\right|_{Zk_2}\right) = \#\ [Neper]$$

De Neper a Lineal $\rightarrow e^{\#}$ [Lineal].

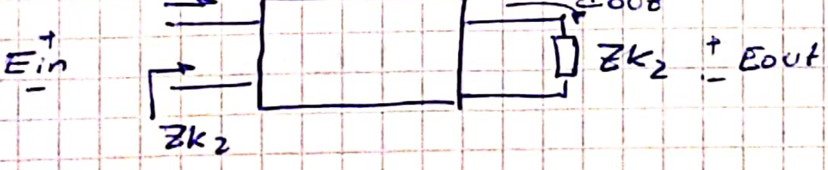
Función de propagación de corrientes cuad cargados con Zk_2

$$\left.\frac{I_{in}}{I_{out}}\right|_{Zk_2} = D + C\,Zk_2 = D + \cancel{C}\left[-\left(\frac{D-A}{2\cancel{C}}\right) + \sqrt{\left(\frac{D-A}{2\cancel{C}}\right)^2 + \frac{B\,C}{\cancel{C}}}\right]$$

$$\left.\frac{I_{in}}{I_{out}}\right|_{Zk_2} = D - \frac{D}{2} + \frac{A}{2} + \sqrt{\frac{D^2-2AD+A^2}{4} + AD - 1}.$$

$$\left.\frac{I_{in}}{I_{out}}\right|_{Zk_2} = \underbrace{\frac{A+D}{2}}_{\cosh\gamma} + \underbrace{\sqrt{\left(\frac{A+D}{2}\right)^2 - 1}}_{\operatorname{senh}\gamma} = e^{\gamma} = e^{\alpha}\cdot e^{j\beta} = \left.\frac{E_{in}}{E_{out}}\right|_{Zk_2}.$$

Si la carga es Zk_2, las funciones de propagación de corrientes y tensiones son iguales.



$$\frac{E_{in}}{I_{in}} = Zk_2 \qquad \frac{E_{out}}{I_{out}} = Zk_2.$$

$$\frac{E_{in}}{I_{in}} = \frac{E_{out}}{I_{out}} \rightarrow \boxed{\frac{E_{in}}{E_{out}} = \frac{I_{in}}{I_{out}}}$$

Son Iguales si cargamos c/ Zk_2

Función de propagación de tensión de cuad carg. c/ Zim_2.

$$\left.\frac{E_{in}}{E_{out}}\right|_{Zim_2} = A + \frac{B}{Zim_2} = A + \frac{B}{\sqrt{\frac{BD}{AC}}} = A + \frac{\sqrt{B^2}}{\sqrt{\frac{BD}{AC}}}$$

$$\left.\frac{E_{in}}{E_{out}}\right|_{Zim_2} = A + \sqrt{\frac{A\,BC}{D}} = A + \sqrt{\frac{A}{D}}\cdot\sqrt{BC} = A + \sqrt{\frac{A}{D}}\cdot\sqrt{AD-1}.$$

$$\left.\frac{E_{in}}{E_{out}}\right|_{Zim_2} = \frac{\sqrt{A}\cdot\sqrt{A}\;\sqrt{D}}{\sqrt{D}} + \sqrt{\frac{A}{D}}\sqrt{AD-1}. = \sqrt{\frac{A}{D}}\left(\underbrace{\sqrt{AD}}_{\underset{\cosh\theta}{\mu}} + \underbrace{\sqrt{AD-1}}_{\underset{\operatorname{senh}\theta}{\sqrt{\mu^2-1}}}\right)$$

$$\left.\frac{E_{in}}{E_{out}}\right|_{Zim_2} = \sqrt{\frac{\frac{\sqrt{A}\sqrt{B}}{\sqrt{C}\sqrt{D}}}{\frac{\sqrt{D}\sqrt{B}}{\sqrt{C}\sqrt{A}}}}\left(\sqrt{AD}+\sqrt{AD-1}\right) = \sqrt{\frac{Zim_1}{Zim_2}}\left(\sqrt{AD}+\sqrt{AD-1}\right)$$

$$\boxed{\left.\frac{E_{in}}{E_{out}}\right|_{Zim_2} = \sqrt{\frac{Zim_1}{Zim_2}}\left(\sqrt{AD}+\sqrt{AD-1}\right) = \sqrt{\frac{Zim_1}{Zim_2}}\left(\cosh\theta+\operatorname{senh}\theta\right) = \sqrt{\frac{Zim_1}{Zim_2}}\cdot e^{\theta} = e^{\gamma} = e^{\alpha}\cdot e^{j\beta}}$$

Función de propagación de corrientes de cuad. carg. c/ Zim_2.

$$\left.\frac{I_{in}}{I_{out}}\right|_{Zim_2} = C\cdot Zim_2 + D = D + C\cdot\sqrt{\frac{BD}{AC}} = D + \sqrt{\frac{BDC}{A}} = D + \sqrt{\frac{D}{A}}\cdot\sqrt{BC}.$$

$$\left.\frac{I_{in}}{I_{out}}\right|_{Zim_2} = D + \sqrt{\frac{D}{A}}\cdot\sqrt{AD-1} = \frac{\sqrt{D}\sqrt{D}\sqrt{A}}{\sqrt{A}} + \sqrt{\frac{D}{A}}\cdot\sqrt{AD-1}.$$

$$\left.\frac{I_{in}}{I_{out}}\right|_{Zim_2} = \sqrt{\frac{D}{A}}\left(\sqrt{DA}+\sqrt{DA-1}\right) = \sqrt{\frac{\frac{\sqrt{D}\sqrt{B}}{\sqrt{C}\sqrt{A}}}{\frac{\sqrt{A}\sqrt{B}}{\sqrt{C}\sqrt{D}}}}\left(\underbrace{\sqrt{DA}}_{\cosh\theta}+\underbrace{\sqrt{DA-1}}_{\operatorname{senh}\theta}\right)$$

$$\boxed{\left.\frac{I_{in}}{I_{out}}\right|_{Zim_2} = \sqrt{\frac{Zim_2}{Zim_1}}\left(\sqrt{DA}+\sqrt{DA-1}\right) = \sqrt{\frac{Zim_2}{Zim_1}}\left(\cosh\theta+\operatorname{senh}\theta\right) = \sqrt{\frac{Zim_2}{Zim_1}}\,e^{\theta} = e^{\gamma} = e^{\alpha}\cdot e^{j\beta}}$$

Con Zim_2 $\quad \dfrac{E_{in}}{E_{out}} \neq \dfrac{I_{in}}{I_{out}}$

Las Zim están cambiadas de orden.

* Sólo válido p/ cuad. simétrico:

Función de propagación de tensión de cuad cargados con Z_o.

$$\left.\frac{E_{in}}{E_{out}}\right|_{Z_o} = A + \frac{B}{Z_o} = A + \frac{B}{\sqrt{\frac{B}{C}}} = A + \sqrt{BC} = A + \sqrt{AD-1}$$

Como son cuad sim $\longrightarrow$ $A = D$.

$$\boxed{\left.\frac{E_{in}}{E_{out}}\right|_{Z_o} = A + \sqrt{A^2-1} = \cosh\gamma + \operatorname{senh}\gamma = e^{\gamma} = e^{\alpha} \cdot e^{d\beta}}$$

Función de propagación de corriente de cuad carg. con Z_o.

$$\left.\frac{I_{in}}{I_{out}}\right|_{Z_o} = D + C \cdot Z_o = D + C \cdot \sqrt{\frac{B}{C}} = D + \sqrt{BC} = D + \sqrt{AD-1}$$

$D = A$.

$$\boxed{\frac{I_{in}}{I_{out}} = A + \sqrt{A^2-1} = \cosh\gamma + \operatorname{senh}\gamma = e^{\gamma} = e^{\alpha} \cdot e^{d\beta}} \rightarrow \text{Igual a la de tensiones}$$

Resumen Funciones de propagación:

* Z_{k_2}
$$\left.\frac{E_{in}}{E_{out}}\right|_{Z_{k_2}} = \left.\frac{I_{in}}{I_{out}}\right|_{Z_{k_2}} = \frac{A+D}{2} + \sqrt{\left(\frac{A+D}{2}\right)^2 - 1} = \cosh\gamma + \operatorname{senh}\gamma = e^{\alpha} \cdot e^{d\beta}$$

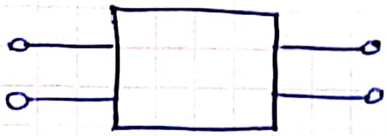
* Z_{im_2}
$$\left.\frac{E_{in}}{E_{out}}\right|_{Z_{im_2}} = \sqrt{\frac{Z_{im_1}}{Z_{im_2}}}\left(\sqrt{AD} + \sqrt{AD-1}\right) = \sqrt{\frac{Z_{im_1}}{Z_{im_2}}}\underbrace{\left(\cosh\theta + \operatorname{senh}\theta\right)}_{e^{\theta}} = e^{\gamma} = e^{\alpha} \cdot e^{d\beta}$$

$$\left.\frac{I_{in}}{I_{out}}\right|_{Z_{im_2}} = \sqrt{\frac{Z_{im_2}}{Z_{im_1}}}\left(\sqrt{AD} + \sqrt{AD-1}\right) = \sqrt{\frac{Z_{im_2}}{Z_{im_1}}}\underbrace{\left(\cosh\theta + \operatorname{senh}\theta\right)}_{e^{\theta}} = e^{\gamma} = e^{\alpha} e^{d\beta}$$

* Z_o
$$\left.\frac{E_{in}}{E_{out}}\right|_{Z_o} = \left.\frac{I_{in}}{I_{out}}\right|_{Z_o} = A + \sqrt{A^2-1} = \cosh\gamma + \operatorname{senh}\gamma = e^{\gamma} = e^{\alpha} \cdot e^{d\beta}$$

Cuadripolos Adaptadores de impedancia.

$Z_{IN} = Z_{im_1}$



$Z_{out} = Z_{im_2}$.

Se diseña el cuadripolo de tal forma que en la entrada vea Z_{im_1} (que será igual a R del generador por ejemplo), y que de la salida vea Z_{im_2} (que será igual a la carga).

$$Z_{im_1} = \sqrt{\frac{AB}{CD}} \quad ; \quad Z_{im_2} = \sqrt{\frac{BD}{AC}}.$$

$$\frac{Z_{im_1}}{Z_{im_2}} = \frac{\sqrt{\frac{AB}{CD}}}{\sqrt{\frac{BD}{AC}}} = \frac{A}{D} \Rightarrow \boxed{\sqrt{\frac{Z_{im_1}}{Z_{im_2}}} = \sqrt{\frac{A}{D}}.} \quad (1)$$

$$Z_{im_1} \times Z_{im_2} = \sqrt{\frac{AB}{CD}} \times \sqrt{\frac{BD}{AC}} = \frac{B}{C} \Rightarrow \boxed{\sqrt{Z_{im_1} Z_{im_2}} = \sqrt{\frac{B}{C}}.} \quad (2)$$

$$\left.\frac{E_{in}}{E_{out}}\right|_{Z_{im_2}} = \sqrt{\frac{Z_{im_1}}{Z_{im_2}}}\Big(\underbrace{\sqrt{AD}}_{\cosh\theta} + \underbrace{\sqrt{AD-1}}_{\operatorname{senh}\theta.}\Big) \quad \begin{cases} \boxed{\cosh\theta = \sqrt{AD}} & (A) \\ \boxed{\operatorname{senh}\theta = \sqrt{BC}} & (B). \end{cases}$$

$$(1) \times (A) \rightarrow \sqrt{\frac{Z_{im_1}}{Z_{im_2}}} \cdot \cosh\theta = \sqrt{\frac{A}{D}} \cdot \sqrt{AD} = A.$$

$$\boxed{A = \sqrt{\frac{Z_{im_1}}{Z_{im_2}}} \cdot \cosh\theta.}$$

$$(2) \times (B) \rightarrow \sqrt{Z_{im_1} \times Z_{im_2}} \cdot \operatorname{senh}\theta = \sqrt{\frac{B}{C}} \sqrt{BC} = B$$

$$\boxed{B = \sqrt{Z_{im_1} \times Z_{im_2}} = \operatorname{senh}\theta}$$

$$\frac{(B)}{(2)} \Rightarrow \frac{\operatorname{senh}\theta}{\sqrt{Z_{im_1} \cdot Z_{im_2}}} = \frac{\sqrt{BC}}{\sqrt{\frac{B}{C}}} = C \rightarrow \boxed{C = \frac{\operatorname{senh}\theta}{\sqrt{Z_{im_1} \times Z_{im_2}}}}$$

$$\frac{(A)}{(1)} \Rightarrow \frac{\cosh\theta}{\sqrt{\frac{Z_{im_1}}{Z_{im_2}}}} = \frac{\sqrt{AD}}{\sqrt{\frac{A}{D}}} \Rightarrow \boxed{D = \frac{\cosh\theta}{\sqrt{\frac{Z_{im_1}}{Z_{im_2}}}}}$$

Se debe diseñar un cuadripolo, el cual cumpla las ecuaciones encontradas de A, B, C y D. θ dependerá del modelo de cuadripolo a implementar.