2.1

$$\text{Gamma}(\alpha, \beta) \xrightarrow{\text{pdf}} \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$$

$$\text{Poisson}(\lambda) \xrightarrow{\text{pdf}} \frac{e^{-\lambda}\lambda^x}{x!}$$

Generative model: $\lambda \sim \text{Gamma}(\alpha, \beta)$

$x \sim \text{Poisson}(\lambda)$

Posterior:

$$p(\lambda | x) = \frac{p(x|\lambda)\, p(\lambda)}{p(x)}$$

$$p(x|\lambda)\, p(\lambda) = \underbrace{\prod_{i=1}^{n} \frac{e^{-\lambda}\lambda^{x_i}}{x_i!}}_{\{x_i\}_{i=1}^{n} \text{ n indep observations}} \frac{1}{\Gamma(\alpha)} \beta^\alpha \lambda^{\alpha-1} e^{-\beta\lambda}$$

$$= \frac{e^{-n\lambda} \lambda^{\sum_i x_i}}{\prod_i x_i!} \frac{1}{\Gamma(\alpha)} \beta^\alpha \lambda^{\alpha-1} e^{-\beta\lambda}$$

$$= \frac{1}{N} \beta^\alpha \underbrace{\lambda^{\alpha + \sum_i x_i - 1} e^{-(\beta+n)\lambda}}_{\text{Functional form of a Gamma}(\alpha + \sum_i x_i,\ \beta+n)}$$