1.1

$P(A,B,C,D,E,F) = P(A)P(B|A,F)P(C|B)P(D|C)P(E)P(F)$

1.2

a. $A \perp\!\!\!\perp B \Leftrightarrow A, B$ independent F

Because $P(B|A,F) \neq P(B) \Leftrightarrow$ Value of B depends on value of A

b. $A \perp\!\!\!\perp F \Leftrightarrow A, F$ independent T

Because $P(A,F) = P(A)P(F) \Leftrightarrow$ nodes A, F ($\Leftrightarrow$ corresponding r.v.) are not related to each other

c. $A \perp\!\!\!\perp C | \{B,E\} \Leftrightarrow A, C$ independent given B and E T

$= \underbrace{A \perp\!\!\!\perp C | B}_{T} \cup \underbrace{A \perp\!\!\!\perp C | E}_{T}$

T because T since E is indep on anything

once B fixed

$\Rightarrow A \perp\!\!\!\perp C$

Formally

$$A \perp\!\!\!\perp C | \{B,E\} \Leftrightarrow P(A,C|B,E) \overset{\text{Bayes}}{=} \frac{P(A,C,B,E)}{P(B,E)} = \frac{P(A)P(C|B)\cancel{P(B|A)P(E)}}{\cancel{P(B|A)P(E)}}$$

$$= P(A)P(C|B) \Leftrightarrow \text{Factorizes}$$

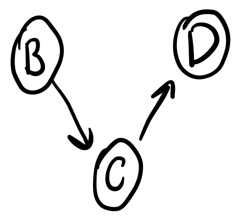
In poor words: Fixed the var in the middle, no one gives a s**t about the var on the other side of the fixed one

d. $F \perp\!\!\!\perp D | B \Leftrightarrow P(F,D|B) = \frac{P(F,D,B)}{P(B)} = \frac{P(F)\overbrace{P(D,B)}^{P(D|B)}}{P(B)}$

$$= P(F)P(D|B) \Rightarrow T$$

e. $B \perp\!\!\!\perp D \mid C \iff P(B,D \mid C) = \frac{P(B,D,C)}{P(C)} = \frac{P(D \mid C)\,\overbrace{P(C \mid B)\,P(B)}^{}}{P(C)}$ $\quad P(C \mid B)\,P(B) = P(B,C)$

$$= P(D \mid C)\,\underbrace{\frac{P(B,C)}{P(C)}}_{P(B \mid C)}$$



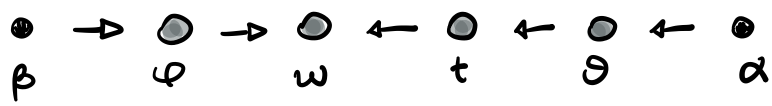
HEAD TO TAIL

$$= P(D \mid C)\,P(B \mid C)$$

$\Rightarrow$ Factorizes

$\Rightarrow$ T

2.1



GENERATIVE MODEL

$\varphi_k \sim \text{Dirichelet}_N(\beta)$ for $k = 1 \ldots T$

$\theta_i \sim \text{Dirichelet}_T(\alpha)$ for $i = 1 \ldots D$

for each word j, for $j = 1 \ldots W$,

$\quad t_{ij} \mid \theta_i \sim \text{Categorical}(\theta_i)$

$\quad w_{ij} \mid \varphi_{t_{ij}} \sim \text{Categorical}(\varphi_{t_{ij}})$