1.3

$$x_1 \mid x_2 \sim \mathcal{N}\left(\mu_1 + \frac{\varrho}{\sigma_2^2}(x_2 - \mu_2),\ \sigma_1^2 - \frac{\varrho^2}{\sigma_2^2}\right)$$

$$x_2 \mid x_1 \sim \mathcal{N}\left(\mu_2 + \frac{\varrho}{\sigma_1^2}(x_1 - \mu_1),\ \sigma_2^2 - \frac{\varrho^2}{\sigma_1^2}\right)$$

} conditionals are both Gaussians

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$$p(x_1, x_2) = p(x_2 \mid x_1)\, \underbrace{p(x_1)}$$

Gaussian $x_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$

Bayes Th. $\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu \\ A\mu + b \end{pmatrix}, R^{-1}\right)$

$$\mu := \mu_1 \,, \quad Ax + b := \mu_2 - \frac{\varrho}{\sigma_1^2}\mu_1 + \frac{\varrho}{\sigma_1^2}x$$

$$\Rightarrow A\mu_1 + b = \mu_2$$

$$R^{-1} := \begin{pmatrix} \sigma_1^2 & \sigma_1^2 A \\ A\sigma_1^2 & \sigma_2^2 - \varrho/\sigma_1^2 + A\sigma_1^2 A \end{pmatrix} = \begin{pmatrix} \sigma_1^2 & \varrho \\ \varrho & \sigma_2^2 \end{pmatrix}$$

$\Rightarrow p(x_1, x_2)$ GAUSSIAN ※ EggrazieAlc···o