$$\frac{n-1}{\sigma^2} S^2 \sim \chi^2_{n-1}$$

$$E[S^2] = \sigma^2$$

$$VAR\left[\frac{n-1}{\sigma^2} S^2\right] = 2(n-1)$$

$$E\left[\left(\frac{n-1}{\sigma^2} S^2 - E\left[\frac{n-1}{\sigma^2} S^2\right]\right)^2\right] = E\left[\left(\frac{n-1}{\sigma^2} S^2 - (n-1)\right)^2\right] = E\left[\frac{(n-1)^2}{\sigma^4}\left(S^2 - \sigma^2\right)^2\right] =$$

$$= \frac{(n-1)^2}{\sigma^4} E\left[\left(S^2 - E[S^2]\right)^2\right] = \frac{(n-1)^2}{\sigma^4} VAR(S^2)$$

$$\Rightarrow \boxed{VAR(S^2) = \frac{2(n-1)\sigma^4}{(n-1)^2}}$$