Quantum Walks and Monte Carlo

Many real life problems require solving a partial differential equation (PDE), like the Boltzmann transport equation (BTE), or a system of PDEs. Often times these PDEs cannot be solved analytically, instead a numerical method is required and Monte Carlo is the go-to method of choice.

Monte Carlo methods rely on stochastic sampling and are implemented with simulations. The Galton board, in which The Price Is Right's Plinko game is based, has been thoroughly studied as a way to implement a universal statistical simulator.

Quantum computing provides an exponential speedup over classical computing at implementing the Galton board. Moreover, Carney and Varcoe[1] describe a method producing a shallower quantum circuit, which translates in less gate errors.

They model the board with $2n + 1$ working qubits and 1 control qubit, where n is number of layers of the quantum Galton board (QGB). To model each peg, 4 qubits are required, but qubits can be reused to minimize the circuit depth.

Figure 1 shows a 2 level balanced QGB. Level 1 has 1 peg, level 2 has 2 pegs, pegs in a level are separated with a CNOT. Each level begins resetting the control qubit (q_0) and applying a Hadamard gate. The Hadamard gate makes the QGB balanced because there is a 50/50 chance for the ball to go left or right. The ball is dropped on the middle working qubit, the peg of the first layer, so a Pauli-X is applied there. $n+1$ measurement gates are applied to alternating qubits.

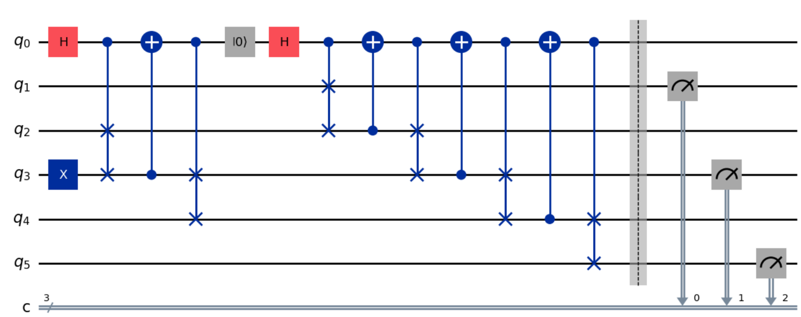

Figura 1: Two-level balanced QGB.

A biased QGB is realized replacing the Hadamard gate with a rotation around the X axis in the Bloch sphere. But in this case, each peg needs to reset the control qubit and apply the rotation. This biased QGB works as a universal statistical simulator, generating differen distributions by changing the rotation angle.

Referencias

[1] Carney, M and Varcoe, B. *Universal Statistical Simulator*. DOI:10.48550/arXiv.2202.01735, 2022

[2] Quantum AI. *Quantum walk*. Online. https://quantumai.google/cirq/experiments/quantum_walks. Accessed: Aug. 10, 2025.