Second Assignment: Stock Market-discrete and continuous time models

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1 The "Thank you, Beyonce"'s

Consider the market $\mathcal{M} = (S^0, S^1, \Phi)$ whose prices evolve in discrete time following a four-periods trinomial additive model. Knowing that, $S_0^1 = 1$, the additive parameters are $u_1 = +1.8$, $m_1 = +0.2$ and $d_1 = -0.4$ in the first two periods while they are incremented of their 1% in the last two periods, and the term structure of spot prices is $\{B(0,1) = 0.82; B(0,2) = 0.75; B(0,3) = 0.72; B(0,4) = 0.75\}$, determine:

1. the set of martingale measures for S^1 assuming that it pays a dividend at the rate 0.03% at every node except the last one;
2. an arbitrage price of a Parisian down in, $H^d = 0.55$ (window up of 2 periods), put written on S^1 with maturity T=4, and strike K=3.2.

2 The Green Hedgers

Consider the foreign asset S^f, whose $\mathbb{P}$-dynamic is:

$$\frac{\mathrm{d}S_t^f}{S_t^f} = (r_t^f - \delta_t^f)\mathrm{d}t + \sigma_t^f(\mathrm{d}\hat{W}_t + \lambda_t^f\mathrm{d}t)$$

where r_t^f is the foreign risk-free rate, δ_t^f is the stochastic dividend rate payed by S^f and where $(\hat{W})_t$ is the $\mathbb{P}$-Brownian. Assuming that the interest rates are stochastic, evaluate:

1. the price and the hedging strategy in domestic currency of a compo call written on S^f, with strike price K^d, maturity T, assuring the absence of arbitrage opportunities between the two markets;
2. the price and the hedging strategy of a quanto put written on S^f, with strike price K^f, maturity T, assuring the absence of arbitrage opportunities between the two markets.

3 The Bondfathers

Given the continuous market $\mathcal{M} = (S^0, S, I, \Phi)$ characterized by the following $\mathbb{P}$-dynamics of the basic assets:

$$\begin{aligned}
\frac{\mathrm{d}S_t^0}{S_t^0} &= r_t\mathrm{d}t \\
\frac{\mathrm{d}S_t}{S_t} &= (r_t - a_t)\mathrm{d}t + \sigma_t^S(\lambda_t\mathrm{d}t + \mathrm{d}\widehat{W}_t) \\
\frac{\mathrm{d}I_t}{I_t} &= (r_t - b_t)\mathrm{d}t + \sigma_t^I(\lambda_t\mathrm{d}t + \mathrm{d}\widehat{W}_t)
\end{aligned}$$

where r_t is the risk-free rate, λ_t is the risk premium on the historical market and $(\widehat{W})_t$ is the $\mathbb{P}$-Brownian, compute:

1. the price and the hedging strategy of an exchange option whose underlying is I_t exchange for a quantity β of S_t^2 in the market $\mathcal{M}^{S^2}$ whose numeraire is S_t^2.

4 The martinguys

Let be $\mathcal{M}^d = (S^{0,d}, I^d, \Phi)$ the domestic market where the risk-free rate is $r^d = 0.01$ and the index I^f evolve (biennial) in discrete time following a multiplicative trinomial model. Assume that $I_0^f = 0.018$, the multiplicative parameters are $u = 1.032, m = 1.002$ and $d = 0.79$ and the exchange rate from domestic to foreign market is $X_0 = 0.77$ and evolve (annual) in discrete-time following a 2-period binomial model with parameters $u = 1.0056$ and $d = 0.077$.
Recover a domestic price of a compo put option written on I^f with variable strike price, i.e. $K^d = a \times 0.5$, where $a = 1.2$ in case of 2 up movements while $a = \sqrt{1.2}$ otherwise, and maturity after 2 years assuring the absence of arbitrage opportunities between the two economies in case $r^f = 0.76r^d$.

5 Group 5

Consider the market $\mathcal{M}^f = (S^{0,f}, S^{1,f}, \Phi)$ whose assets evolve in discrete time following a two-period multiplicative bino-trino model (1 period binomial-1 period trinomial). Knowing that $S_0^{0,f} = 1$, $S_0^{1,f} = 1.74$ and the exchange rate from the foreign to the domestic market is $X_0 = 0.37$, the risk-free rates are $r^d = r^f = 0$, it pays a rate of dividend only at maturity equal to 1% of the price and the multiplicative parameters are $u = 1.072, m = 1$ and $d = 0.85$ for the evolution of both $S^{1,f}$ and X_t, determine a future price (in domestic currency) of a contract written on the min of $S^{1,f}$ with maturity $T = 2$.

6 The Big Short

Given the continuous market $\mathcal{M} = (S^0, S^1, S^2, \Phi)$ whose $\mathbb{P}$-dynamics of the basic assets are:

$$\begin{aligned}\frac{\mathrm{d}S_t^0}{S_t^0} &= 0.03\mathrm{d}t \\ \frac{\mathrm{d}S_t^1}{S_t^1} &= (0.03 - 0.01)\mathrm{d}t + 0.1(\mathrm{d}\hat{W}_t + 0.5\mathrm{d}t) \\ \frac{\mathrm{d}S_t^2}{S_t^2} &= 0.03\mathrm{d}t + 0.5(\mathrm{d}\hat{W}_t + 0.5\mathrm{d}t)\end{aligned}$$

where $S_0^0 = 1$ and $S_0^1 = S_0^2 = 1.8$.
Recover the hedging strategy of a future written on $S_t^1 S_t^2$, with delivery price $K = 1.5$ and maturity $T = 3$, through the simple strategy $(\phi)_\Theta$ characterized by the following trading dates:

$$\Theta = \{t_0 = 0; t_1 = 1,5; t_2 = 2\}.$$

Recover a condition for the forwardation settlement of the future and for the incompleteness of the market.

7 Group 7

Consider the market $\mathcal{M} = (S^0, S^1, \Phi)$ whose prices evolve in discrete time following a two-periods trinomial additive model. Knowing that, $S_0^1 = 1$, the additive parameters are $u_1 = +1.7$, $m_1 = +0.3$ and $d_1 = +0.2$ in the first period while they are incremented of their 1% in the second period, and the term structure of spot prices is $\{B(0,1) = 0.83; B(0,2) = 0.65\}$, determine:

1. the set of martingale measures for S^1 assuming that it pays a dividend at the rate 0.3% at every node;
2. an arbitrage price of an asian knock in double barrier $H^d = 0.5, H^u = 1.67$, put written on S^1 with maturity T=2, and strike K=2.37.