Credit and Weather Derivatives assignment

1 Compute the zero coupon bond curve

Take the Euribor-swap curve in sheet 'swap curve" in the file "market data 2023.xlsx". Calibrate the Nelson-Siegel-Svensson model to the rates in column B in order to interpolated the swap rates each years from 1 year to 30 years. Then apply bootsptrapping technique to compute the price of zero risk free coupon bonds. Be care: the rates in column B with maturity less than 1 year are zero rates and we do not have to bootstrap them. Write a short paper with some graphs.

2 Compute default probabilities

Take the CDS spreads quoted in bps as in the sheet "CDS spread" in the file "market data 2023.xlsx". Compute the default & survival risk neutral probabilities boostrapping the CDS spread curve considering a time inhomogeneous Poisson process with piece constant intensity rate. In particular, consider the intensity rate to be constant between two consecutive quoted CDS spread. Write a short paper with some graphs.

3 Pricing a Multiname Credit Derivative

Suppose our trading desk ask you to price a multiname credit derivative whose payoff is defined as follow. You receive 1 Euro at maturity if no name of the underlying poll defaults until maturity, zero otherwise. The maturity is 5 years and the poll of name is constituted by ENI, Unicredit, Volkswagenk Allianz, Iberdrola. The fair value FV of that derivative is defined by:

$$FV = E^Q[D(0,5)\prod_{i=1}^{5} 1_{\tau_i > 5}] \tag{1}$$

Under the assumption of independence between interest rates and default times, we have

$$FV = P(0,5)E^Q[\prod_{i=1}^{5} 1_{\tau_i > 5}] \tag{2}$$

Compute the FV by Monte Carlo simulation using an intensity model with constant hazard rate for each name in the poll and the Gaussian copula (set $P(0,5) = 0.95$). Follow the step:

- fit the Gaussian copula the the time series of CDS spread in the file "CDS time series.xlsx". For each name, transform the CDS spread into the 5-years default probability using for each time t in the time series the formula:

  $$1 - e^{-CDS_i(t)/(1-REC_i)} \tag{3}$$

  where $i \in 1, \ldots, 5$ denotes a name in the poll. Set $REC_i = 0.4$ for each i (Remember: express the CDS spreads in bps).

- transform the time series of default probabilities obtained in the previous step into pseudo uniform number using the rank function (or the empirical CDF for example).

- fit the Gaussian copula to the time series of pseudo uniform number obtained in the previous steo

- using the correlation matrix C obtained in the previous fitting, sample multivariate normal random variable X with zero mean and correlation C. (hint: decompose the correlation matrix C with Cholesky)

- sample uniform random variable $U = \phi(X)$, where ϕ is the standard normal comulative function.

- simulate the default time of each name in the poll using the formula:

  $$\{\tau_i > 5\} = \{\int_0^5 \lambda_i dt > -\ln(1 - U_i)\} \tag{4}$$

  For each name in the poll, compute λ_i calibrating an intensity model with constant hazard rate to the last CDS spread in the time series (hint: use the credit triangle equality).

4 CDO

Watch the movie "The big short" and the documentary "Inside job".