Econometrics for Financial Markets

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1 Report on the Analysis

1.1 Univariate Volatility Analysis

The analysis of the financial asset expressed in logarithmic form ("lp") reveals time-varying volatility patterns that are common in financial markets.

The analysis begins with the unconditional statistical properties[1], where we observe the typical features of financial log-returns:

- A mean near zero (0.006891) with high variability (standard deviation of 0.222634), reflecting the stochasticity of asset price movements[1].

- **Stationarity** is confirmed by the Augmented Dickey-Fuller test (statistic $= -81.24$, p-value $= 0$), validating the application of GARCH methodology.

- **Normality** is strongly rejected by the Jarque-Bera test (statistic $= 2973.72$, p-value $= 0$), revealing fat tails in the returns distribution[2]. This non-normal behavior is aligned with the extreme market movements occurring in financial markets.

The Engle's ARCH test provides evidence of volatility clustering (conditional heteroscedasticity), with a statistic of 292.764 (p-value $= 1.55 \times 10^{55}$), indicating that large price movements tend to be followed by large movements in both directions[3]. The analysis also used a non-parametric volatility tool, the Exponentially Weighted Moving Average (EWMA)[4].

Further, the research reveals that the **GJR-GARCH(1,1)** specification with Student's t-distribution to model the residuals distribution provides the best characterization of volatility dynamics (Akaike Information Criterion $= 58742.89$). The GJR-GARCH(1,1) allows capturing asymmetric effects of positive and negative shocks on volatility through the γ parameter.

The parameters estimated with the Maximum Likelihood Estimation (MLE) offer an interpretation of the asset behavior:

- The sum of the parameters ($\alpha + \beta + 0.5\gamma \approx 0.9095$) is less than 1, ensuring the stationarity of the model.

- The parameter α represents the immediate reaction to shocks on the market and is equal to 0.1103.

- β represents how much of the past volatility persists in current volatility, and it is equal to 0.8162, showing high persistence[5].

- The negative leverage effect ($\gamma = -0.034$, p-value < 0.05) captures the asymmetric response of volatility to positive versus negative returns.

Since γ is negative, positive shocks tend to increase next-period volatility more than a negative return of equal size, which is quite unusual for the stock market, while this behavior has been observed for certain other asset classes such as safe-haven assets like gold (ResearchGate). This asymmetry is shown in the impulse response functions[6].

For risk management purposes, the VaR and ES were computed using two different methods: the parametric and the empirical[7].

- The parametric 1-day Value-at-Risk is -0.303440 at the 10% level and Expected Shortfall is -0.449485, providing quantitative measures for capital allocation and risk limits[8].

- The empirical 1-day Value-at-Risk is -0.296903 at the 10% level and Expected Shortfall is -0.3839.

Regarding distribution fitting, the Generalized Normal distribution emerges as optimal for modeling standardized residuals (AIC $= 18462.54$, KS p-value $= 0.361$)[9], while the Generalized Extreme Value distribution models accurately the left tail (AIC: 628.47 vs alternatives > 728), offering capability for capturing extreme loss scenarios[10].

1.2 Multivariate Volatility Analysis: Interconnected Market Dynamics

The multivariate analysis examines the interconnected volatility paths of three financial assets[11] between 2012 and 2021, revealing interdependence patterns and time-varying correlation, which tends to converge in periods of high market volatility and diminish during lower volatility regimes.

The methodological approach begins with univariate modeling[12] of the time-series, revealing that the assets require distinct specifications[13]: EGARCH for Prices 1 and 2, and GJR-GARCH for Price 3 [14][15][16].

- **Price 1** represents a case where EGARCH provides efficient asymmetric volatility capture with optimal model fit (AIC: 10174.3).

- **Price 2** exhibits extreme characteristics that demand EGARCH exponential formulation to manage fat-tailed behavior (kurtosis $= 14.94$, JB p-value $= 0$) with strong positive skewness (0.716908).

- **Price 3** demonstrates classic financial asset behavior where GJR-GARCH's explicit leverage parameter γ captures the volatility asymmetry.

The parameters for each model were estimated using the Quasi-Maximum Likelihood (QML) method. The universal selection of Student's t-distribution among all models confirms the necessity of heavy-tailed error distributions to explain the significant excess kurtosis present in all three return series. All models successfully eliminated residual ARCH effects in squared returns.

However, the Ljung-Box test on residuals (≈ 0.00) showed significant autocorrelation in the standardized errors, meaning the GARCH models removed volatility clustering but left unsolved predictable patterns in the return series. This led to the improvement of the models through the adoption of **ARMA-GARCH** models, in particular AR(1)-EGARCH(1,1)-t for Price 1 and Price 2, and AR(1)-GJR(1,1)-t for Price 3.

With the ARMA implementation, the mean equations were added. The residual analysis of this new model showed the elimination of the autocorrelation for Price 1 (LB p: $0.00 \to 0.494$) and improvement for Price 2 (LB p: $0.047 \to 0.088$), while Price 3 still exhibits slight autocorrelation (LB $p = 0.011$).

The **CCC (Constant Conditional Correlation)** matrix shows positive correlations: Price1-Price3 (0.374) strongest, Price1-Price2 (0.319) moderate, and Price2-Price3 (0.257). All correlations are statistically significant but constrained to be constant over time, ignoring the dynamic nature of financial market interdependencies. Hence, the **DCC (Dynamic Conditional Correlation)** model is implemented.

The DCC parameter estimates[17] reveal:

- Stationarity in correlation dynamics ($\zeta + \xi = 0.996282 < 1$).

- High persistence of the correlation structures, confirming long memory with dislocations taking a long time to normalize (half-life of 186 periods).

- Minimal responsiveness of correlation to new shocks ($\zeta = 0.007145$), representing slow adjustments of the market.

The statistical superiority of DCC over CCC is clear and economically significant[18]. The likelihood ratio test statistic of 40.0214 (p-value $= 0.00$) provides evidence against the constant correlation hypothesis. The dynamic correlation patterns[19] reveal meaningful economic relationships that evolve over time[20], reflecting changes in the perceptions of risk factors[21]. This underscores that diversification benefits are time-varying rather than static.

1.3 Vector Autoregression Analysis of Oil Price-Growth Rate-Inflation

The VAR analysis provides evidence of dynamic interrelationships among oil prices, inflation, and GDP growth[22], offering insights into how macro-economical data and the energy sector are linked in small open economies.

The results prove statistically significant and economically meaningful relationships[23]. Stationarity testing[24] confirms that all three variables exhibit strong stationarity properties (ADF p-values $= 0.000$), enabling the use of VAR modeling without transformation. The optimal lag length selection

process demonstrates support for lag 1, with AIC and BIC criteria.

The Johansen test reveals three cointegrating relationships at the 95% confidence level[25], indicating strong long-run equilibrium among the variables. The analysis follows a well-defined transmission channel: Oil Price Shocks → Inflation → GDP Growth.

Granger causality tests[26] reveal bidirectional relationships:

- Inflation significantly causes oil prices to increase ($p = 0.0001$).

- Oil prices cause GDP growth ($p = 0.0160$).

- Inflation causes GDP growth ($p = 0.0006$).

This causal structure reflects the macroeconomic reality where energy costs influence production costs and consequently affect overall price levels and economic activity[27].

VAR coefficient estimates[28] quantify these relationships, showing high persistence between all variables (0.46 for oil prices, 0.551 for inflation, 0.521 for GDP growth). More importantly, the cross-variable impacts show economic significance: oil prices have a 0.327 impact on inflation, while past GDP growth positively affects current GDP growth (0.521). The economic magnitude is relevant: a one standard deviation oil price shock (0.573) generates a 0.187 inflation impact, representing 31% of inflation's standard deviation.

The dynamic analysis through impulse response functions[29] reveals persistent shock propagation with oil price effects having peaks at 6–8 months and lasting more than 24 months. Forecast error variance decomposition[30] exhibits cross-variable influences, with oil shocks explaining 30–40% of inflation variance.

The model shows strong out-of-sample forecasting performance[31] with RMSE values of 0.2972 for oil prices and 0.2734 for inflation[32]. Model validation[33] confirms robust statistical properties (Ljung-Box p-values > 0.72, ARCH effects p-values > 0.47)[34].

From a policy perspective, the findings lead to crucial implications:

1. Monetary authorities should monitor oil prices given the strong (0.327) impact coefficient.

2. A 10% oil price increase generates a 0.033 inflation increase.

3. Oil price stabilization contributes to macroeconomic stability.

The choice to proceed with VAR in levels despite cointegration allows for the interpretation of short-run dynamics. The economic interpretation of cointegrated relationships suggests long-run interdependence, where no variable can permanently deviate from the others without eventual correction.

A   Methodological Foundations and Statistical Analysis

A.1   Analysis 1: Univariate Volatility Modeling

A.1.1   Theoretical Framework

The analysis employs GARCH-family models following Bollerslev (1986) and Engle (1982). The base GARCH(1,1) specification:

$$r_t = \mu + \varepsilon_t$$
$$\varepsilon_t = \sigma_t z_t$$
$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

Asymmetric extensions include GJR-GARCH for leverage effects:

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \gamma I_{t-1} \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

where $I_{t-1} = 1$ if $\varepsilon_{t-1} < 0$, and 0 otherwise.

A.1.2   Methodological Procedure

1. **Data Preparation**: Logarithmic returns computation with percentage scaling

2. **Tests**:

   - Stationarity testing (ADF test)

   - Normality test (Jarque-Bera test)

   - ARCH effects detection (Engle's test)

3. **Model Estimation**:

   - GARCH, EGARCH, GJR-GARCH, APARCH.

   - Student's t-distribution for residuals

   - Maximum likelihood estimation

4. **Model Selection**: Akaike Information Criterion (AIC) comparison.

5. **Diagnostic Testing**:

   - Standardized residual analysis

   - Ljung-Box tests for autocorrelation

   - ARCH-LM tests for remaining volatility clustering

6. **Risk Measurement**:

- Parametric VaR/ES using fitted distributions

- Empirical VaR/ES from historical quantiles

- Conditional volatility scaling

7. **Additional Analyses**:

    - Impulse response functions

    - Multi-period volatility forecasting

    - Tail distribution fitting

A.2 Analysis 2: Multivariate CCC-DCC GARCH Modeling

A.2.1 Theoretical Framework

Constant Conditional Correlation (CCC) Model The CCC model (Bollerslev, 1990) provides a multivariate volatility framework with time-varying volatilities but constant correlations matrix:

$$r_t = \mu_t + \varepsilon_t$$
$$\varepsilon_t = H_t^{1/2} z_t$$
$$H_t = D_t R D_t$$
$$D_t = \mathrm{diag}(\sigma_{1,t}, \sigma_{2,t}, \ldots, \sigma_{N,t})$$

Components:

- $D_t = \mathrm{diag}(\sigma_{1,t}, \sigma_{2,t}, \ldots, \sigma_{N,t})$: Univariate GARCH volatilities of each asset

- R: Unconditional correlation matrix between the assets

- $\sigma_{i,t}$: Conditional volatility for asset i at time t

- z_t: Standardized residuals vector $\sim i.i.d.(0, I)$

CCC Estimation:

1. Estimation of univariate GARCH models for each asset to obtain D_t

2. Find constant correlation matrix R from the standardized residuals

3. Construct conditional covariance matrix $H_t = D_t R D_t$

Dynamic Conditional Correlation (DCC) Model The DCC model (Engle, 2002) improves the CCC model by considering time-varying correlations between the variiables:

$$H_t = D_t R_t D_t$$
$$Q_t = (1 - \zeta - \xi)\bar{R} + \zeta(\varepsilon_{t-1}\varepsilon'_{t-1}) + \xi Q_{t-1}$$
$$R_t = \mathrm{diag}(Q_t)^{-1/2} Q_t \mathrm{diag}(Q_t)^{-1/2}$$

Key Components:

- $D_t = \text{diag}(\sigma_{1,t}, \sigma_{2,t}, \ldots, \sigma_{N,t})$: Univariate GARCH volatilities

- R_t: Time-dependent correlation matrix

- $\bar{R}$: Unconditional correlation matrix

- Q_t: dynamic correlation matrix, from which is built R_t

- ζ: Responsiveness of correlation to new shocks

- ξ: Persistence parameter (memory of past correlations)

- Constraints: $\zeta > 0$, $\xi > 0$, $\zeta + \xi < 1$

Two-Stage Estimation:

1. Estimate univariate GARCH models for each asset to obtain D_t, univariate volatilities

2. Estimate DCC parameters (ζ, ξ) using standardized residuals through Maximum/Quasimaximum Likelyhood Estimation tecniques

A.2.2 Methodological Procedure

1. **Data Preparation and Preliminary Analysis**:

   - Loading the price series from Excel Sheet 2

   - Computation of log-returns with percentage scaling ($\times 100$) for numerical stability

   - Cointegration testing using Johansen test to assess long-run relationships

2. **Preliminary Tests**:

   - Stationarity testing (ADF test) on return series

   - Normality test (Jarque-Bera test)

   - ARCH effects detection (Engle's ARCH-LM test)

3. **Univariate GARCH Modelling**:

   - Candidate specifications: GARCH(1,1)-t, GJR-GARCH(1,1)-t, EGARCH(1,1)-t

   - Maximum likelihood estimation with Student's t-distribution for residuals

   - Model selection via Akaike Information Criterion (AIC)

   - Diagnostic testing on standardized residuals

4. **ARMA-GARCH Implementation**:

   - AR(1) mean specification implemented to eliminate/reduce residual autocorrelation

   - Choice based on Ljung-Box test results showing significant autocorrelation

- Specifications: AR(1)-GARCH(1,1)-t, AR(1)-GJR-GARCH(1,1)-t, AR(1)-EGARCH(1,1)-t

- Final model selection based on AIC improvement over non-ARMA specifications

5. **Constant Conditional Correlation (CCC) Estimation**:

   - Computation of constant correlation matrix from aligned standardized residuals

   - Benchmark model for DCC comparison

6. **Standardized Residuals Preparation for DCC**:

   - Extraction of standardized residuals from selected ARMA-GARCH models

   - Ljung-Box and AIC test to confirm the reduction of autocorrelation

7. **DCC Model Estimation**:

   - Computation of unconditional correlation matrix $\bar{R} = \frac{1}{T} \sum \varepsilon_t \varepsilon_t'$

   - DCC log-likelihood optimization with constraints: $\zeta > 0$, $\xi > 0$, $\zeta + \xi < 1$

   - Multiple optimization attempts with QuasiMaximum Likelyhood

   - Numerical regularization ($\epsilon_{reg} = 1 \times 10^{-8}$) for positive definiteness

8. **Dynamic Correlation Computation**:

   - Time-varying correlation matrices: $R_t = \text{diag}(Q_t)^{-1/2} Q_t \text{diag}(Q_t)^{-1/2}$

   - Positive definiteness verification at each time period

   - Computation of average DCC correlation matrix

9. **Model Comparison**:

   - Likelihood ratio tests between CCC and DCC specifications

   - Information criteria comparison: Akaike (AIC) and Bayesian (BIC)

   - Testing of DCC improvement

10. **Validation**:

    - Standardized residual analysis for each ARMA-GARCH model

    - Ljung-Box tests for autocorrelation in levels and squares

    - ARCH-LM tests for remaining conditional heteroscedasticity

    - Jarque-Bera tests

A.3 Analysis 3: Vector Autoregression Framework and Estimation Methodology

A.3.1 Theoretical Framework

The Vector Autoregression (VAR) methodology , Sims (1980), provides a flexible multivariate framework to capture dynamic interrelationships among multiple time series without imposing strong restrictions.

The general VAR(p) model specification for our three-variable system:

$$
\begin{bmatrix} \text{oil\_price}_t \\ \text{inflation}_t \\ \text{gdp\_growth}_t \end{bmatrix} = c + \sum_{i=1}^{p} A_i \begin{bmatrix} \text{oil\_price}_{t-i} \\ \text{inflation}_{t-i} \\ \text{gdp\_growth}_{t-i} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{bmatrix}
$$

where c is a 3×1 vector of constants, A_i are 3×3 coefficient matrices, and $\varepsilon_t \sim N(0, \Sigma)$ is the vector of shocks with covariance matrix Σ.

A.3.2 Methodological Procedure

1. **Data Preparation and Quality Control**:

   - Loading monthly data from Excel Sheet 3

   - Variable standardization

   - Temporal alignment and missing value analysis

   - Monthly frequency conversion of data

2. **Data Analysis**:

   - Time series visualization of all three variables

   - Correlation matrix

   - Descriptive statisticsof the time series

3. **Granger Causality Test**:

   - Bivariate VAR tests across lags 1-8 to test cointegration pairwise

   - Minimum p-value approach to identify Granger causality

   - F-test statistic calculation with significance validation(How much adding a lagged variable improves the predictability of another variable)

   - Causality matrix construction to visualize the results

4. **Stationarity Test and Data Transformation**:

   - Augmented Dickey-Fuller test(ADF p ¡ 0.05)

- KPSS test(KPSS p ¿ 0.05)

- Data-driven differencing decision based on joint significance

5. **Lag Order Selection**:

- Multi-criteria comparison: Akaike (AIC), Bayesian (BIC)

- Maximum lag search up to 12 periods

- Final lag selection based on the previous criteria(Lag=1)

6. **VAR Model Estimation**:

- Maximum likelihood estimation with OrdinaryLeastSquares

- Train-test split (12-month of the sample for validation)

- Coefficient significance testing p values from the standard errors

- AIC and BIC criteria to choose the more appropriate lag and model specification

7. **Residual Diagnostics**:

- Durbin-Watson test for autocorrelation

- Ljung-Box test for residual correlation

- ARCH Engle's test for conditional heteroscedasticity

- Residual distribution and correlation analysis

8. **Cointegration Analysis**:

- Johansen trace test found strong cointegration(COMPLETE SYSTEM INTERDEPENDENCE)

- Maximum eigenvalue test for cointegration rank determination, evidence against the no cointegration hypotesis

- Critical value comparison at 95% confidence level

9. **Forecast Evaluation**:

- 12-month forecasting

- Root Mean Squared Error (RMSE) for forecast accuracy understanding

- Forecast performance comparison between variables using RMSE

10. **Plots and interpretation:**

- Impulse Response Functions

- Forecast Error Variance Decomposition, to understand interdependencies of variiances

- Structural break analysis, economic relationship stable in different regimes

- Rolling correlation and volatility analysis, to capture time-varying volatility patterns

11. **Model Validation**:

- Diagnostic check

- Out-of-sample performance assessment

- Economic consistency confirmed

B   Tables and Graphs

B.1   Univariate Analysis
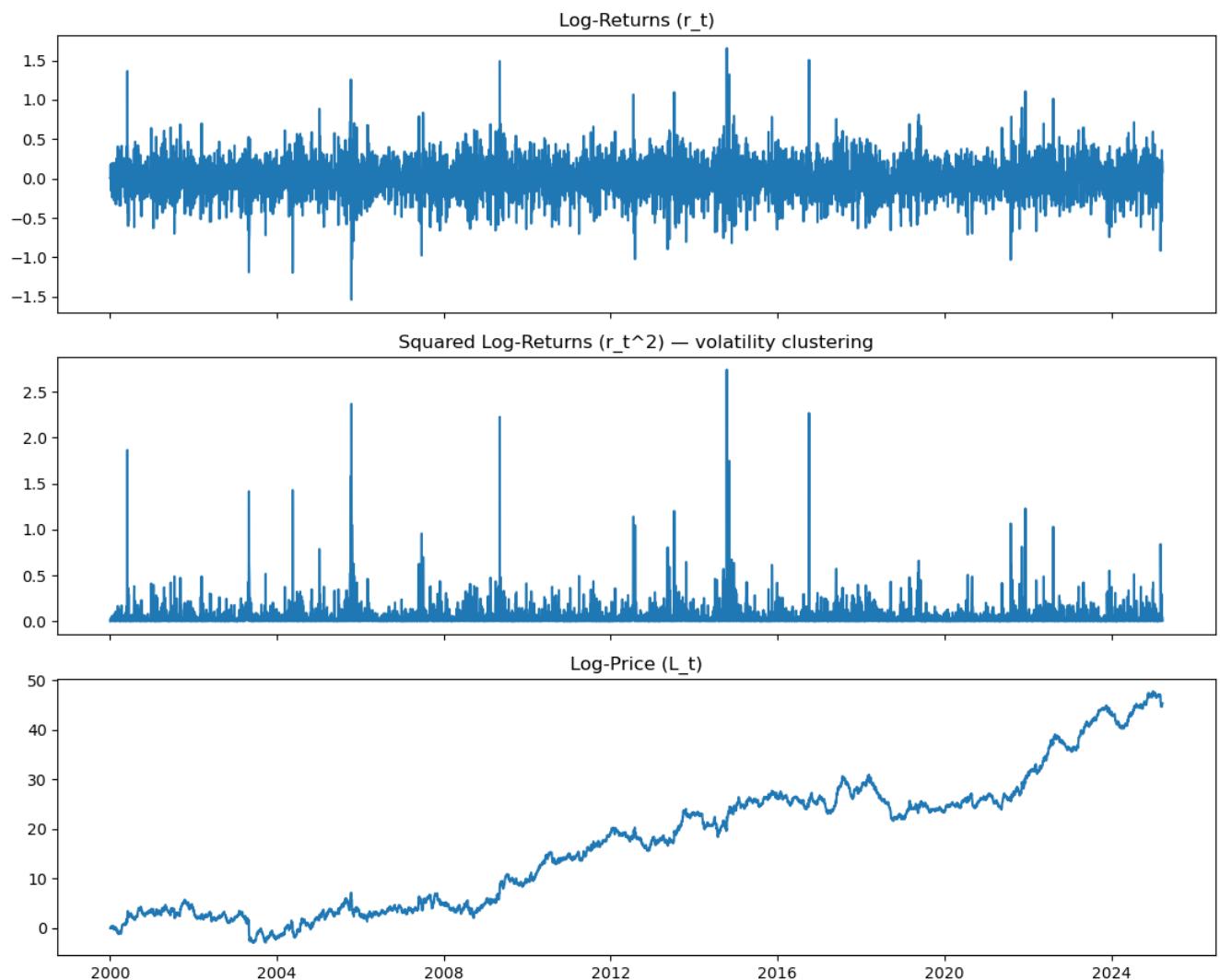

Figure 1: Log-returns time series
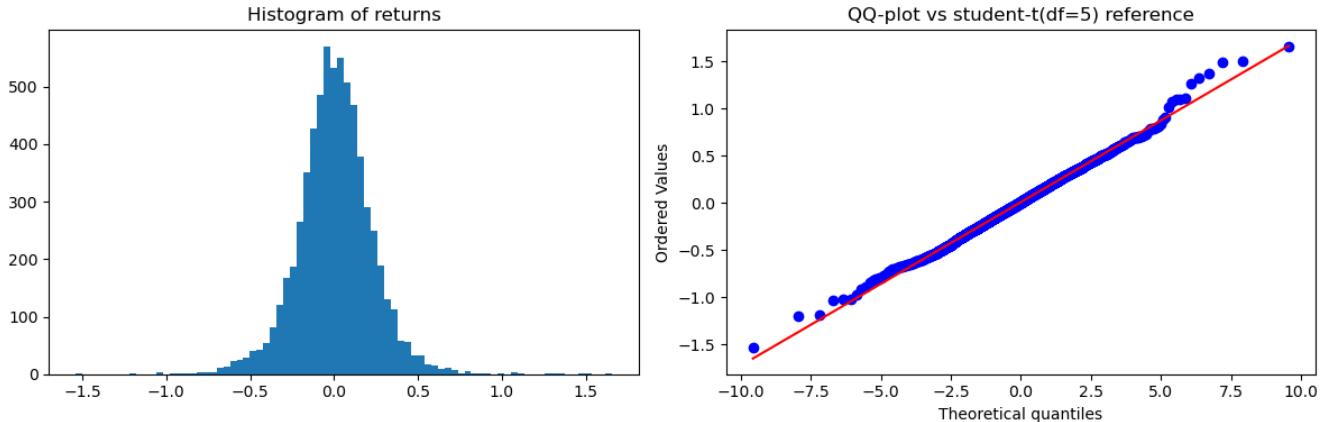
Figure 2: Histogram of returns and QQ plot(Student-t v=5)

Table 1: Descriptive Statistics of Returns

Statistic	Value
Count	6580.000000
Mean	0.006891
Std	0.222634
Min	-1.539114
25%	-0.120078
50%	0.005642
75%	0.135364
Max	1.655663
Skewness	0.107456
Kurtosis (Pearson)	6.289780
Jarque-Bera Statistic	2973.717662
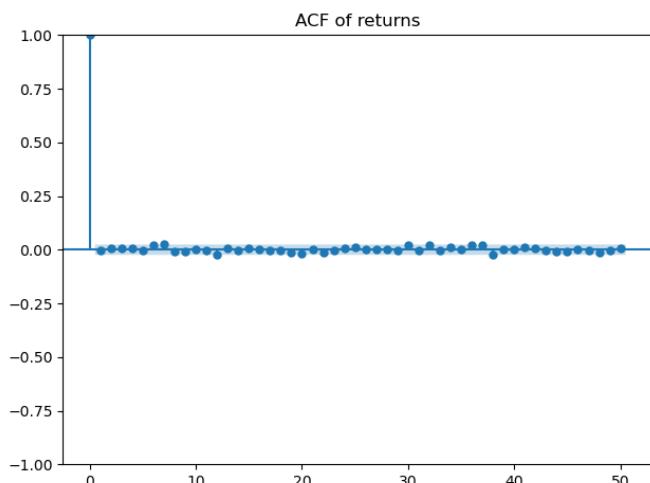
Figure 3: ACF of returns
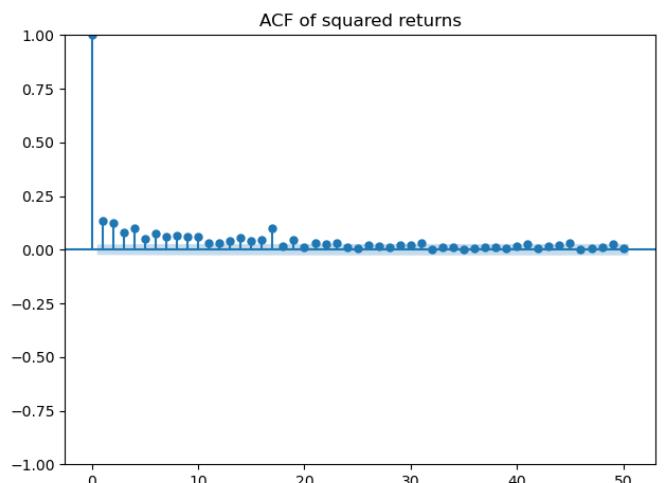
Figure 4: ACF of squared returns
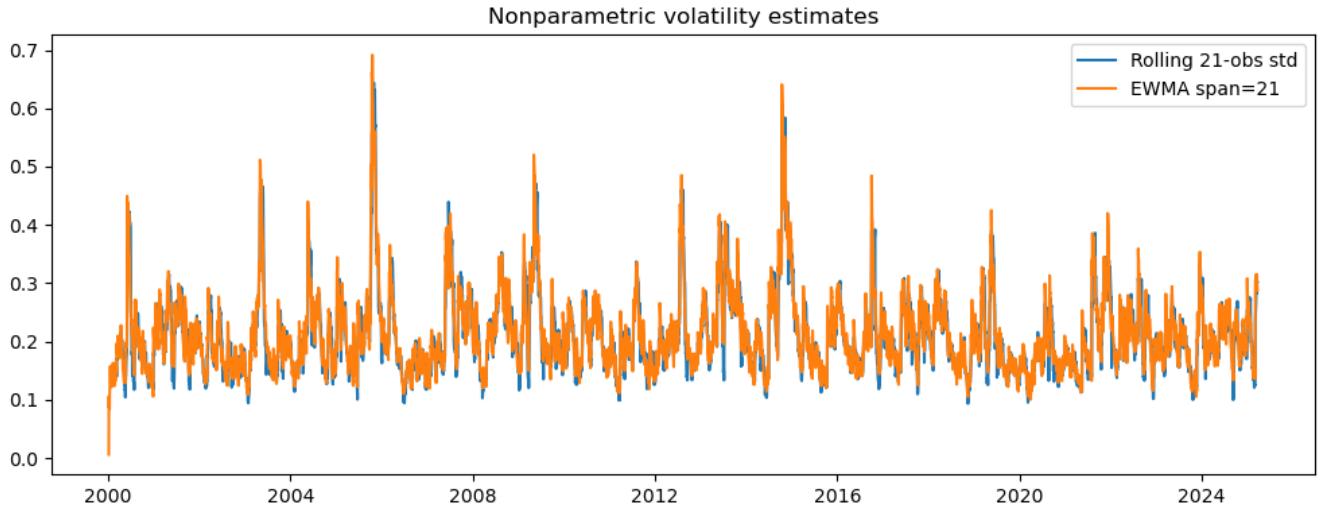
Figure 5: Nonparametric volatility estimates-EWMA

Table 2: GARCH Model Comparison and Statistical Significance

Model	AIC	α	β	γ
GARCH(1,1)	58745.15	0.0940***	0.8131***	-
EGARCH(1,1)	58762.04	0.1942***	0.9154***	0.0247**
GJR-GARCH(1,1)	**58742.89**	0.1103***	0.8162***	-0.0340**
APARCH(1,1)	58744.89	0.0921***	0.8160***	-0.0915**

Note: ***, ** denote significance at 1% and 5% levels respectively
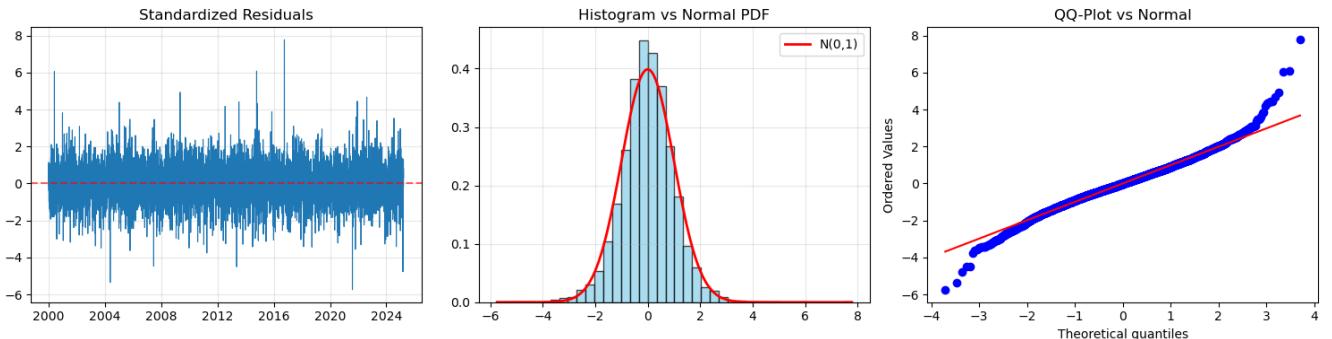
Figure 6: Residuals Diagnostic for the GJR-GARCH model
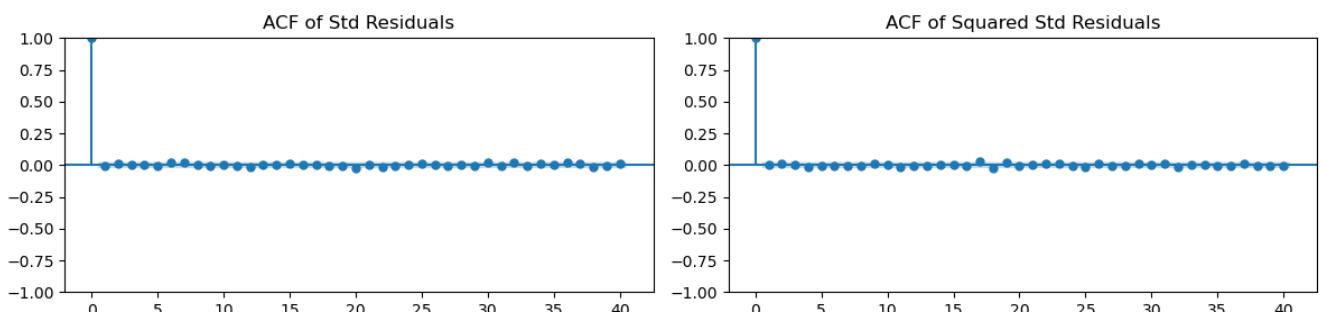
Figure 7: ACF of Standard Residuals and Standard Residuals squared
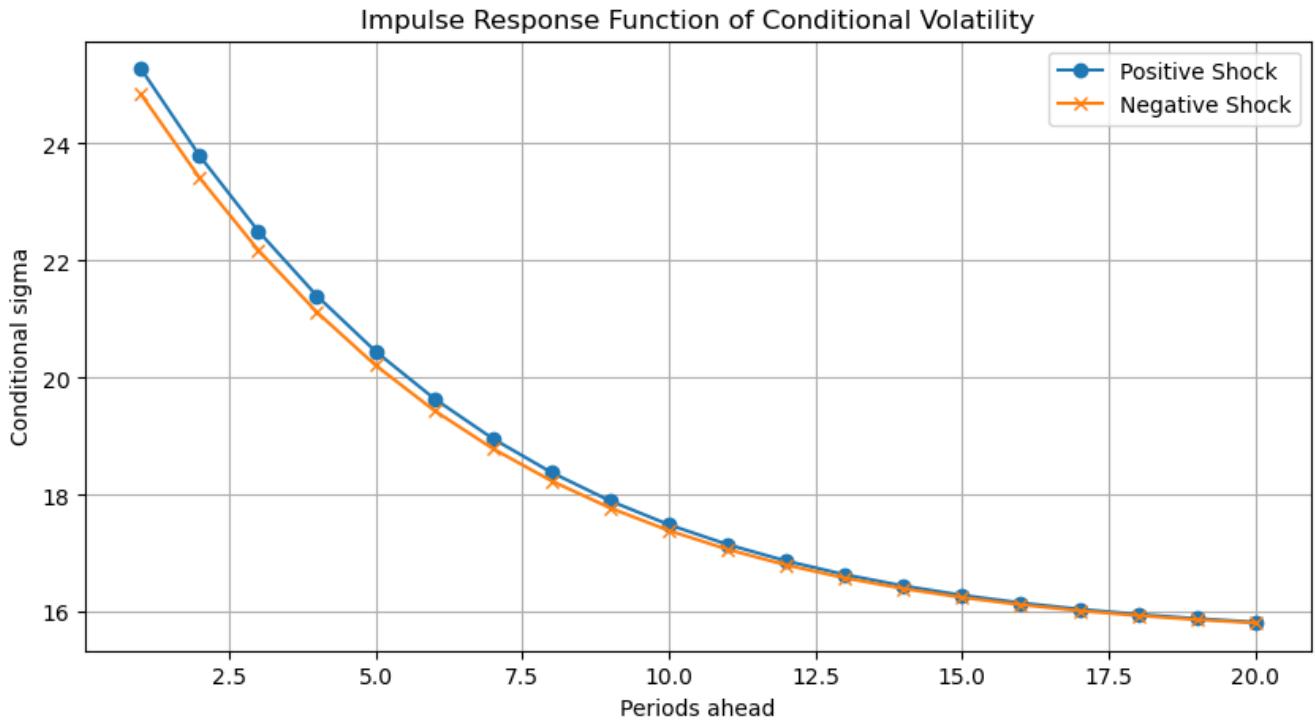
Figure 8: Impulse Response Function of conditional volatility showing asymmetric responses to positive and negative shocks

Table 3: Distribution Fitting Comparison for Standardized Residuals

Distribution	AIC	BIC	KS p-value	Rank
Generalized Normal	**18462.54**	**18482.91**	**0.3619**	1
Skew Normal	18686.97	18707.34	1.30e-05	2
Laplace	18690.05	18703.64	1.85e-08	3
Normal	18695.66	18709.24	3.38e-05	4
Generalized Extreme	19445.58	19465.96	7.34e-24	5

Note: Lower AIC/BIC values indicate better fit. KS p-value ¿ 0.05 suggests the distribution adequately fits the data.
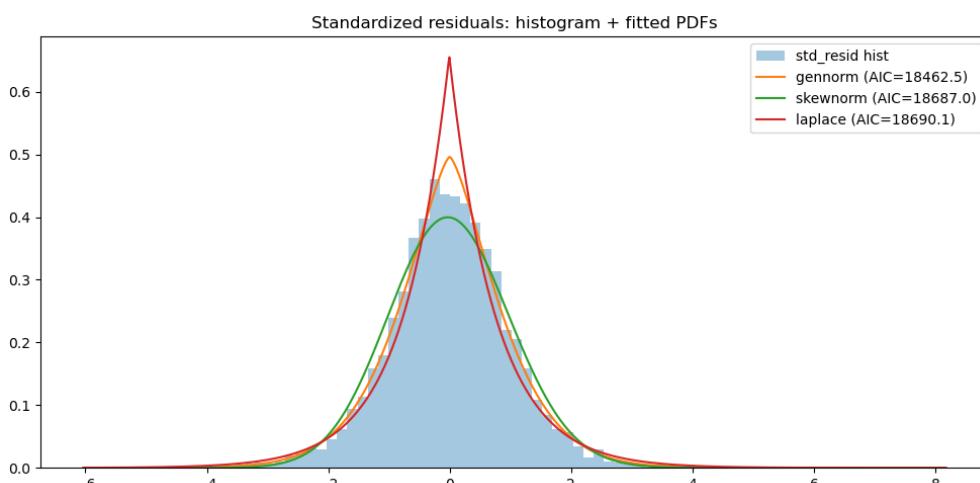
Figure 9: Standardized Residuals vs the Distribution functions tested

Table 4: Residual Diagnostics and Model Adequacy

Test	Statistical Interpretation
Standardized Residuals	Mean = -0.0009, Std = 1.0014
Jarque-Bera Test	p = 0.0000: Residuals strongly non-normal
Ljung-Box Residuals (lag 10)	p = 0.5893: No significant autocorrelation
Ljung-Box Squared Residuals (lag 10)	p = 0.9523: No remaining ARCH effects
Engle ARCH Test (12 lags)	p = 0.9185: No conditional heteroscedasticity in standardized residuals
Distribution Comparison	Generalized Normal optimal (AIC: 18462.54)
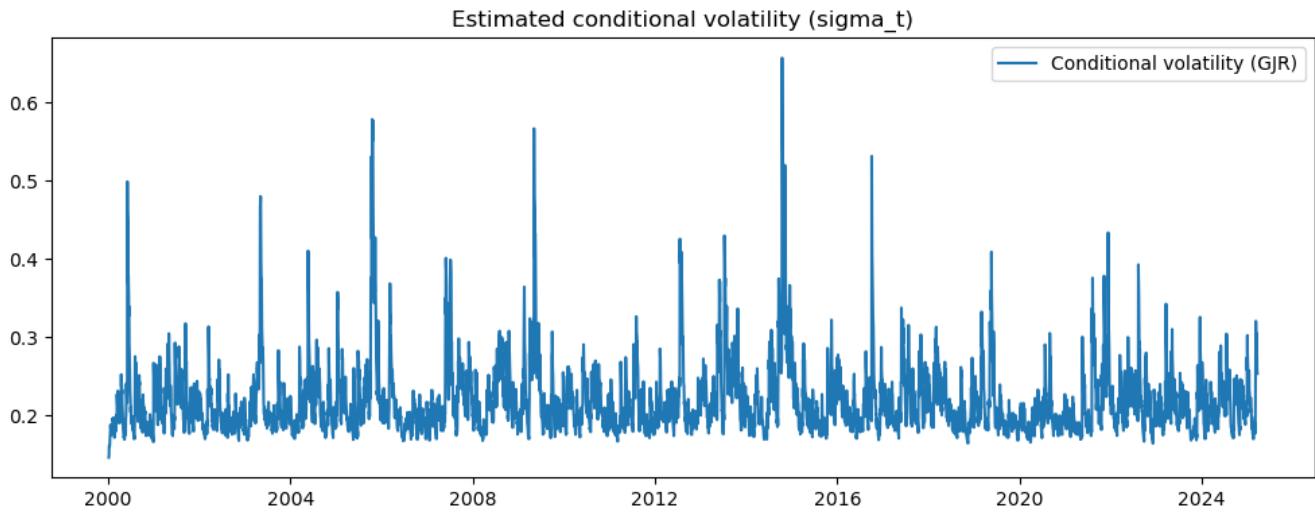
Figure 10: Estimated Conditional Volatility using GJR-GARCH

Table 5: Risk Measures and Distributional Analysis

| Metric	Value	Statistical Interpretation
1-day Parametric VaR (10%)	-0.303440	Using Generalized Normal distribution
1-day Parametric ES (10%)	-0.449485	Expected Shortfall from Generalized Normal distribution
1-day Empirical VaR (10%)	-0.296903	Based on empirical quantile of standardized residuals
Empirical Global ES (10%)	-0.383939	Average realized losses below empirical VaR
Best Distribution	Generalized Normal	Optimal fit (AIC: 18462.54, KS p-value: 0.3619)
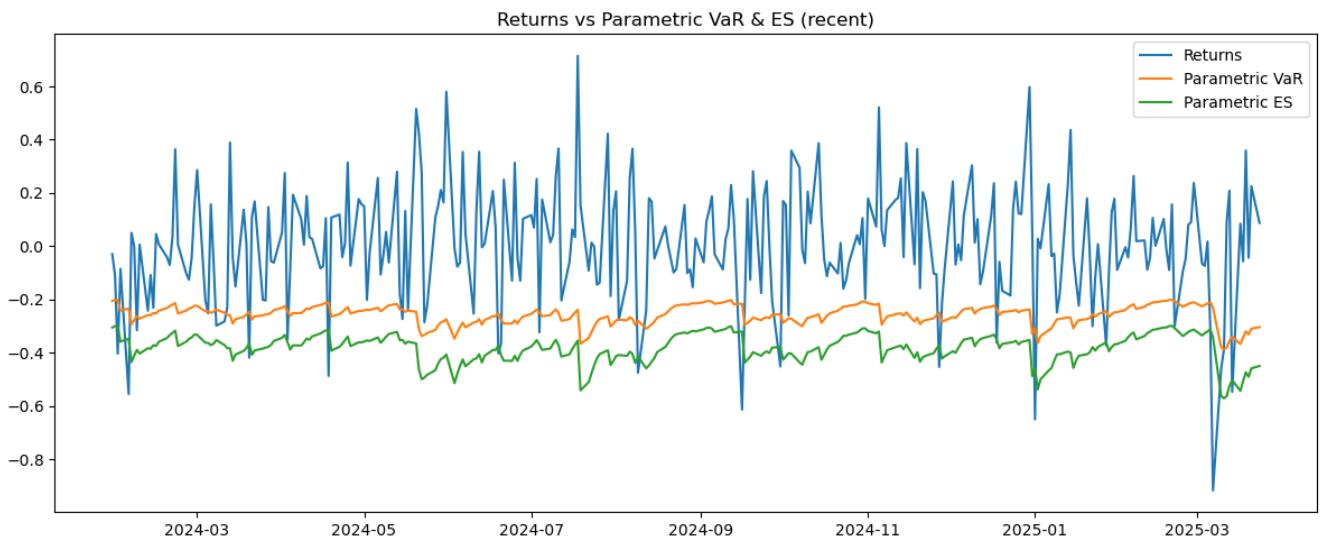
Figure 11: Returns compared to Parametric VaR and ES
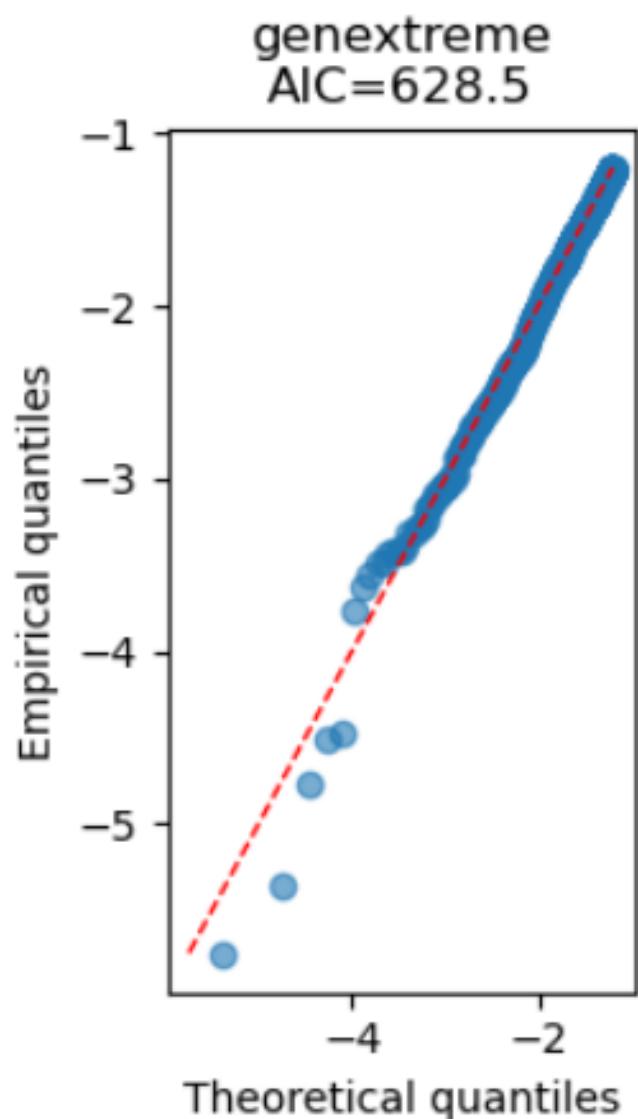
Figure 12: Generalized Extreme distribution to model the left tail of the distribution
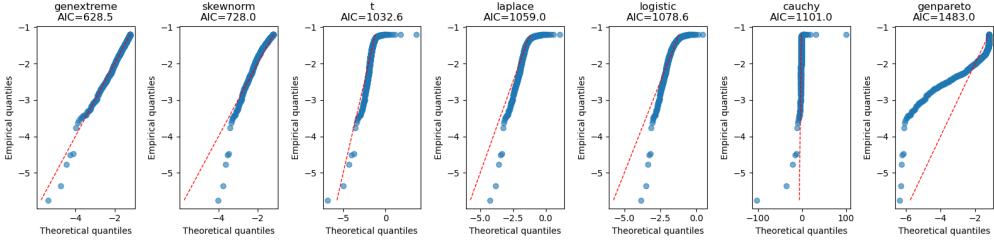
Figure 13: Some distributions to approximate the left tail distribution, from which was chosen the generalized extreme

B.2 Multivariate Analysis

Table 6: Distributional Statistics for All Series

| Series	Skewness	Kurtosis	JB Statistic	Statistical Interpretation
logret_Price1	0.095039	6.067290	885.474474	$p = 5.27 \times 10^{-193}$: Strongly non-normal
logret_Price2	0.716908	14.937035	13570.274902	$p = 0.000000$: Extreme non-normality
logret_Price3	0.025934	6.532402	1170.380647	$p = 7.16 \times 10^{-255}$: Strongly non-normal
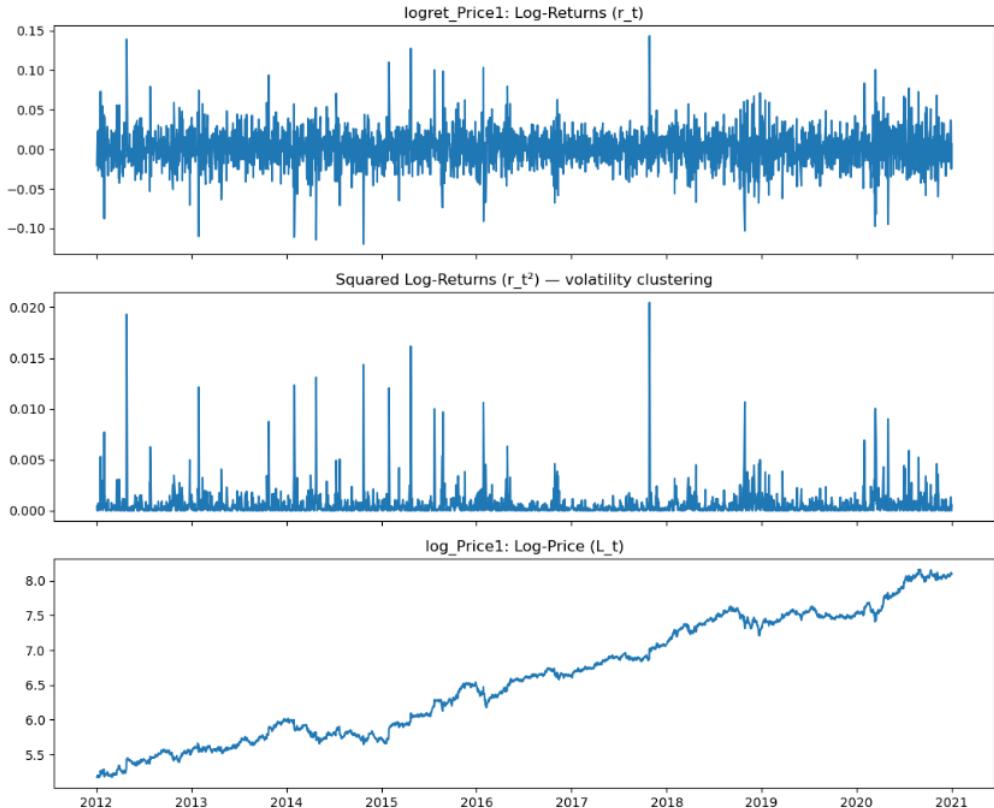
Figure 14: log return Price1
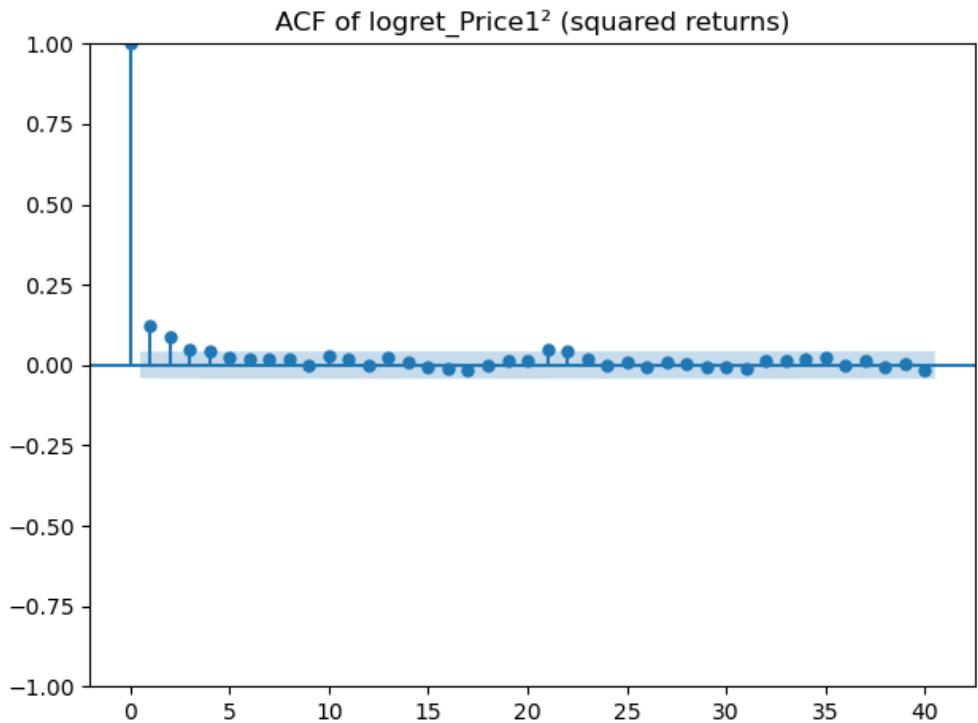
Figure 15: ACF log return Price1 squared
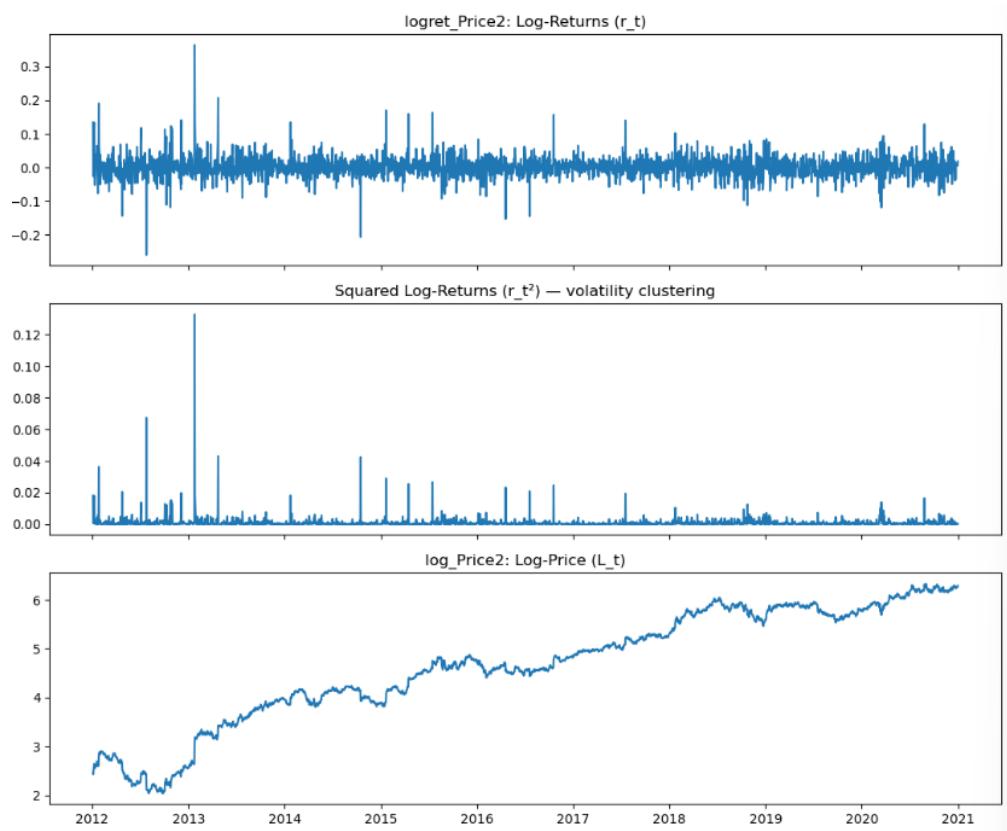

Figure 16: log return Price2
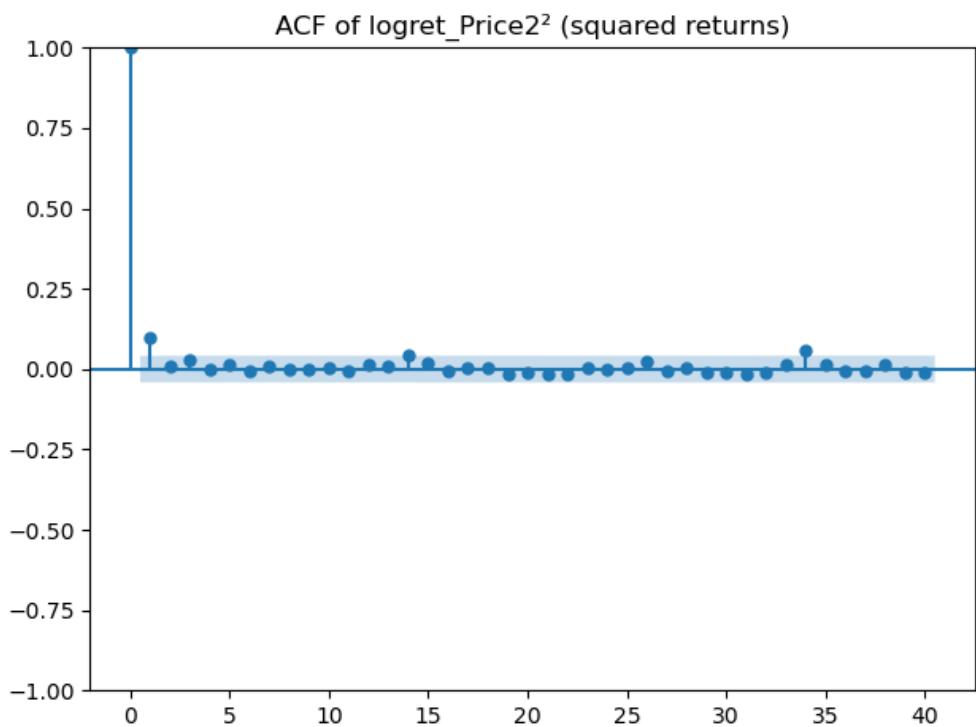
Figure 17: ACF log return Price2 squared
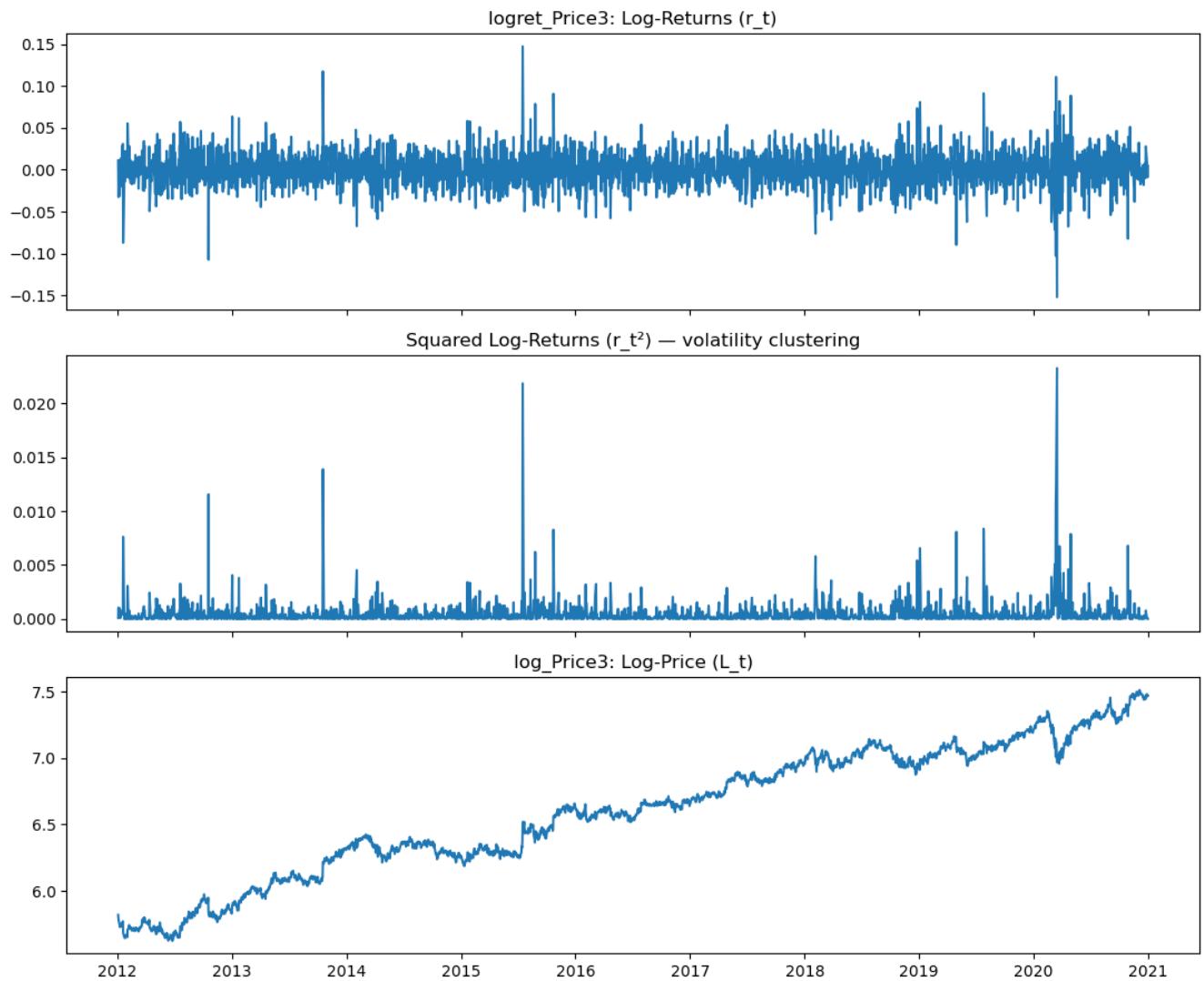
Figure 18: log return Price3
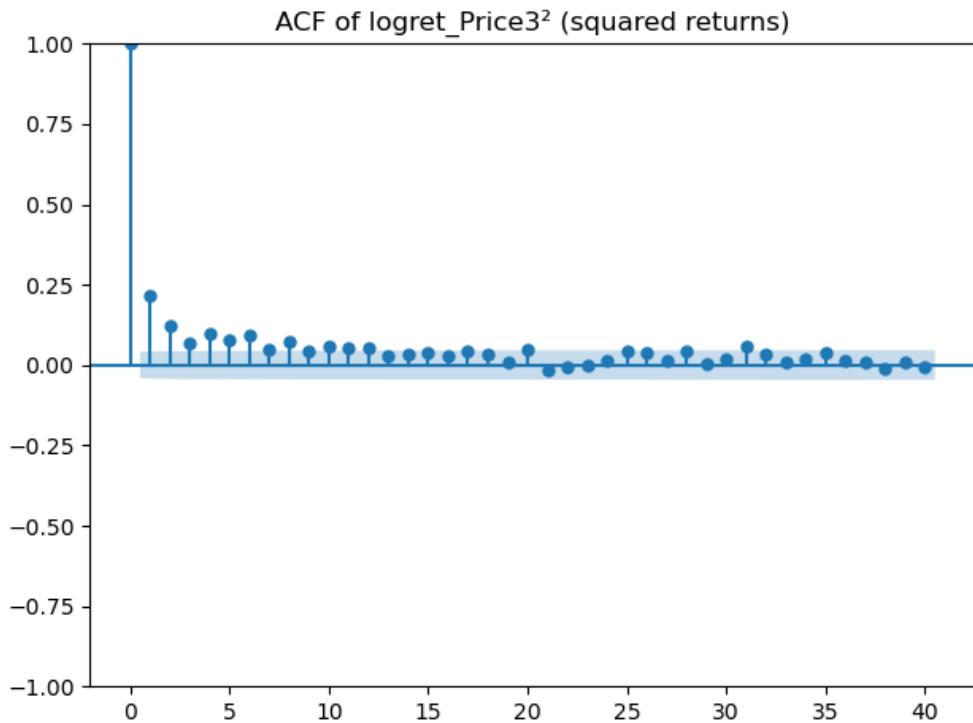
Figure 19: ACF log return Price3 squared

Table 7: Univariate GARCH Model Comparison and Selection

Series	Model	AIC	BIC	Log-Likelihood
logret_Price1				
	EGARCH11-t	**10174.34**	10208.69	-5081.17
	GJR11-t	10179.99	10214.34	-5084.00
	GARCH11-t	10181.83	10210.46	-5085.92
logret_Price2				
	EGARCH11-t	**11285.15**	11319.50	-5636.57
	GJR11-t	11304.72	11339.07	-5646.36
	GARCH11-t	11313.57	11342.20	-5651.79
logret_Price3				
	GJR11-t	**9761.95**	9796.30	-4874.98
	EGARCH11-t	9763.66	9798.01	-4875.83
	GARCH11-t	9769.99	9798.61	-4879.99

Note: Model selected based on lowest AIC criterion

Table 8: Model Selection Summary

Series	Selected Model	AIC	Economic Interpretation
logret_Price1	EGARCH11-t	10174.34	Logarithmic volatility with asymmetric response to news
logret_Price2	EGARCH11-t	11285.15	Exponential volatility specification for high kurtosis series
logret_Price3	GJR11-t	9761.95	Explicit leverage effects through negative return indicator

Table 9: Diagnostics on Standardized Residuals

Series	Model	Mean	Std	Skewness	Kurtosis	LB10 p
logret_Price1	EGARCH11-t	0.005862	1.010900	0.216443	6.252723	0.000000
logret_Price2	EGARCH11-t	0.021166	1.034695	0.660963	15.326964	0.046663
logret_Price3	GJR11-t	0.003133	1.008019	0.113378	5.919438	0.000000

Note: All models show excellent standardization (mean0, std1) and no remaining ARCH effects (p ¿ 0.82). Significant autocorrelation persists (LB10 p less than 0.05). All series exhibit heavy tails.
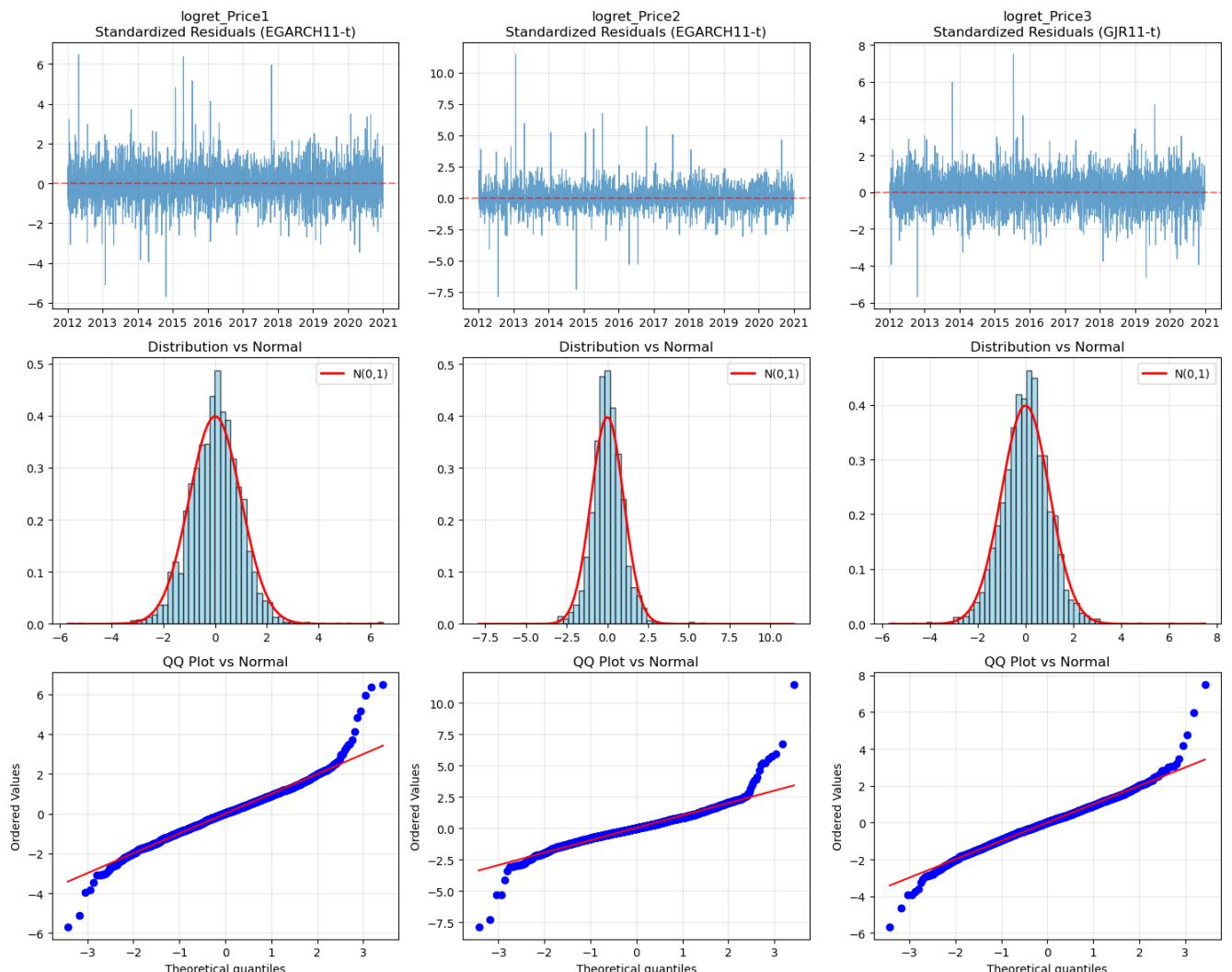

Figure 20: Standardized Residuals Diagnostic

Table 10: Standardized Residuals Diagnostics with Autocorrelation Assessment

Series	Model	Mean	Std	LB10 p	Autocorrelation
logret_Price1	AR1-EGARCH11-t	0.005377	1.010679	0.493829	Removed
logret_Price2	AR1-EGARCH11-t	0.019708	1.031549	0.087625	Mostly Removed
logret_Price3	AR1-GJR11-t	0.000942	1.011410	0.010617	Improved

Note: LB10 p = Ljung-Box test p-value at lag 10. Autocorrelation assessment: p ¿ 0.10 = Removed, 0.05 less than p 0.10 = Mostly Removed, p 0.05 = Improved.

Table 11: Dramatic Autocorrelation Reduction with ARMA-GARCH

Series	Model	LB10 without ARMA	LB10 with ARMA
logret_Price1	AR1-EGARCH11-t	0.000000	0.493829
logret_Price2	AR1-EGARCH11-t	0.046663	0.087625
logret_Price3	AR1-GJR11-t	0.000000	0.010617

Table 12: Optimal Univariate Model Selection for Multivariate Analysis

Asset	Selected Model	AIC
Price1	AR(1)-EGARCH(1,1)-t	10015.40
Price2	AR(1)-EGARCH(1,1)-t	11225.34
Price3	AR(1)-GJR-GARCH(1,1)-t	9582.43

COMPREHENSIVE DIAGNOSTICS: OPTIMIZED ARMA-GARCH STANDARDIZED RESIDUALS
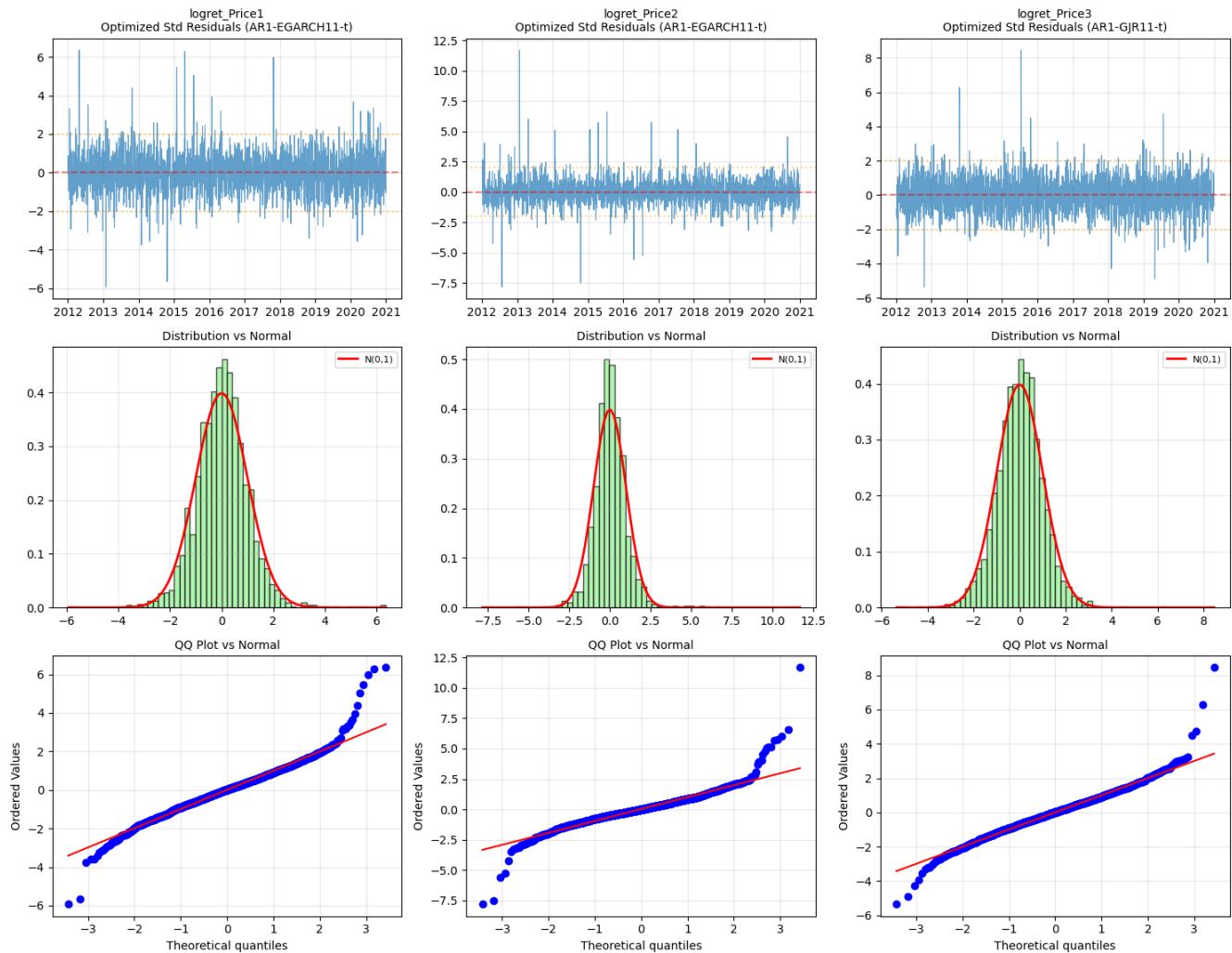


Figure 21: Optimized ARMA-GARCH Standardized Residuals Analysis
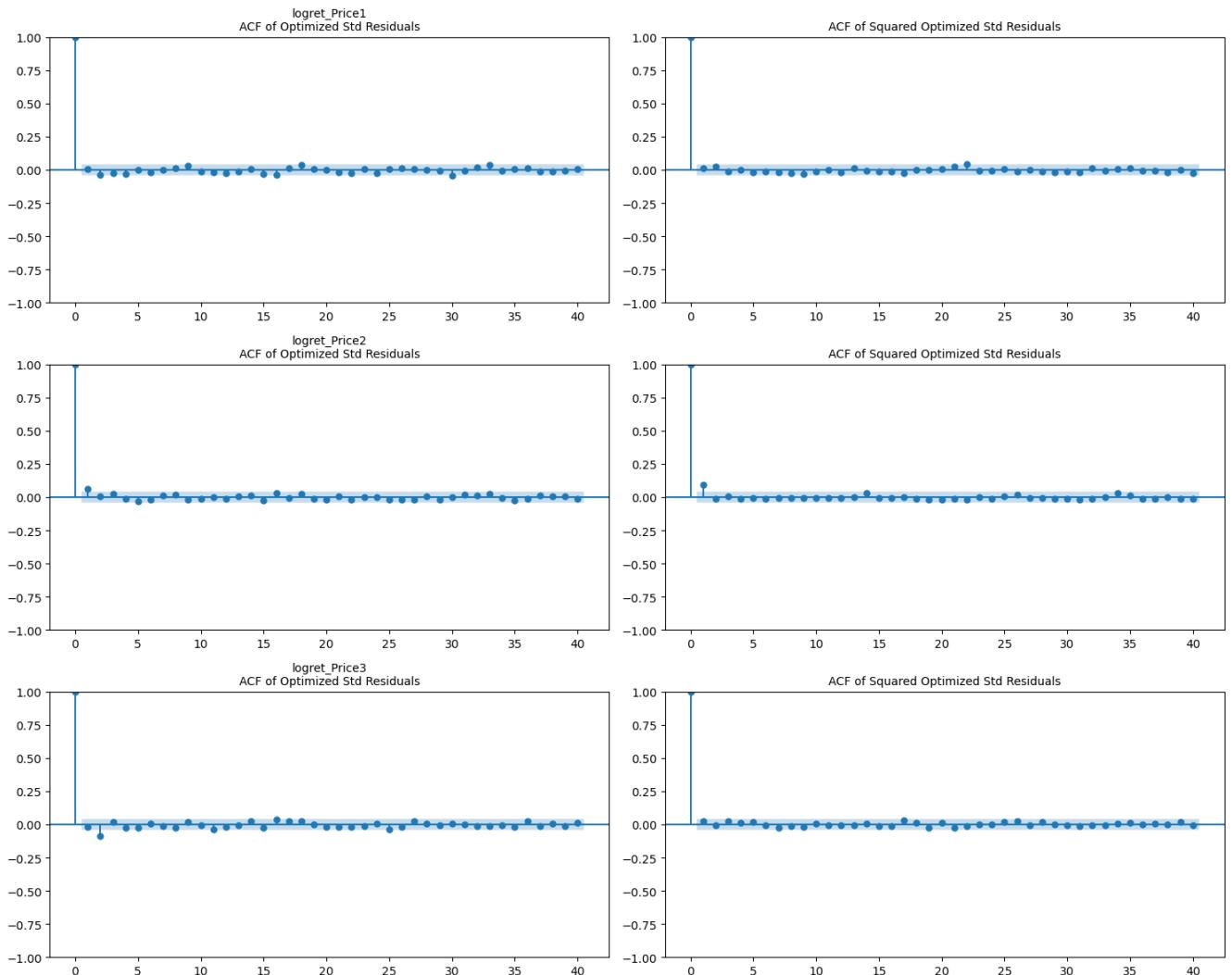
Figure 22: ACF of the Residuals in ARMA-GARCH models

CONSTANT CONDITIONAL CORRELATION (CCC) MATRIX
(Using ARMA-GARCH Standardized Residuals)
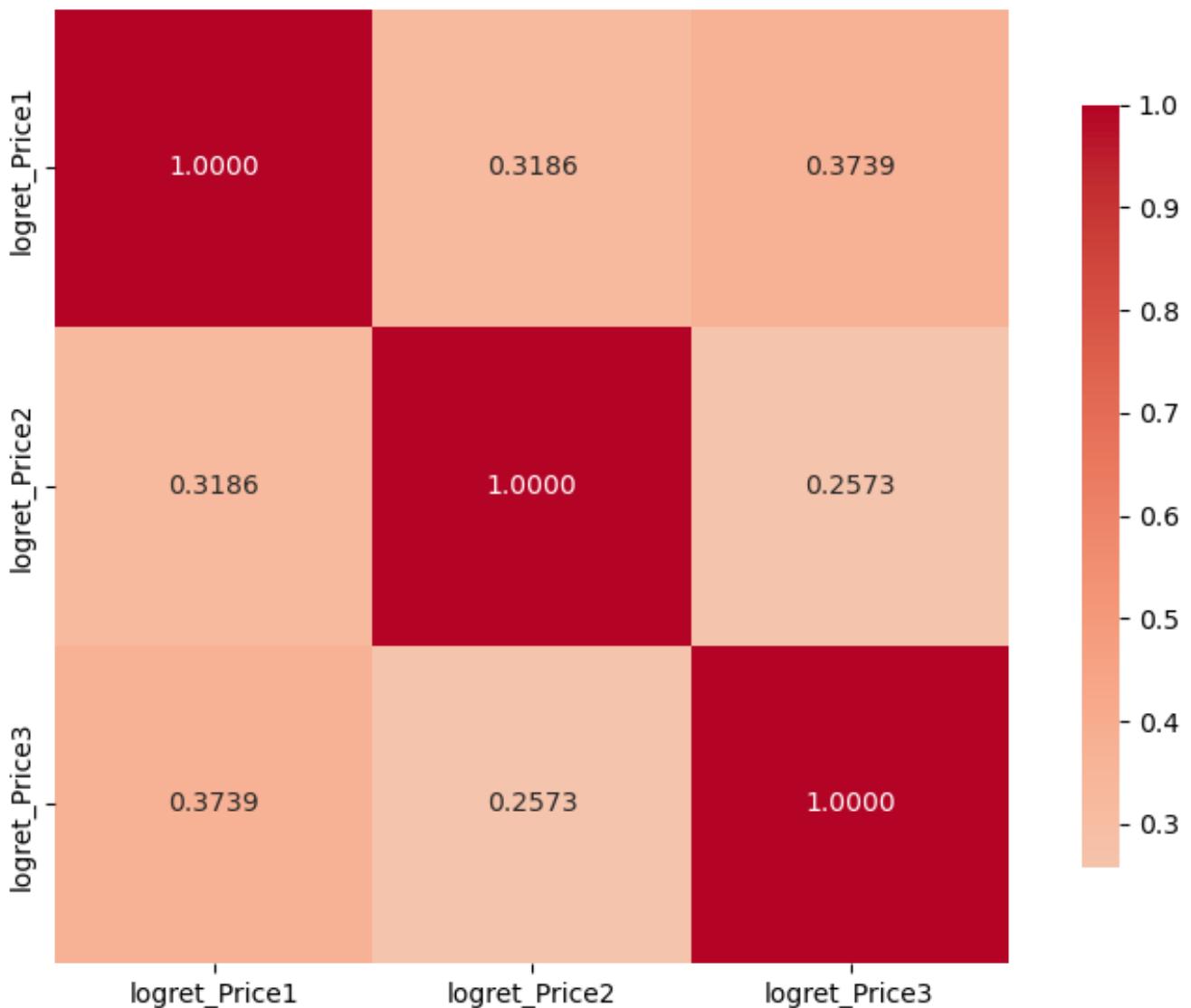
Figure 23: Constant Conditional Correlation Matrix

Table 13: DCC-GARCH Parameter Estimates

Parameter	Estimate	Statistical Meaning	Interpretation
ζ	0.007145	News Impact	0.71% of correlation shocks persist to next period
ξ	0.989137	Persistence	98.91% of correlation structure carries forward
$\zeta + \xi$	0.996282	Stationarity	Ensures covariance stationarity of DCC process
Half-life	186 periods	-	Time for correlation shocks to decay by half

Table 14: Statistical Model Comparison: CCC vs DCC

Criterion	CCC Model	DCC Model	Statistical Interpretation
Log-Likelihood	-3196.46	-3176.45	Higher likelihood indicates better fit
AIC	6398.93	6356.91	Lower AIC supports DCC specification
BIC	6416.10	6368.35	Lower BIC confirms DCC superiority
Likelihood Ratio	-	40.0214	Significant improvement (p less than 0.000001)

Table 15: Correlation Dynamics: Statistical Properties

| Asset Pair	CCC	DCC Mean	Std. Dev.	Range	Variation Coefficient	Half-life
Price1-Price2	0.3186	0.3145	0.0848	0.3661	0.2695	186
Price1-Price3	0.3739	0.3725	0.0659	0.3461	0.1770	186
Price2-Price3	0.2573	0.2609	0.0613	0.3211	0.2350	186
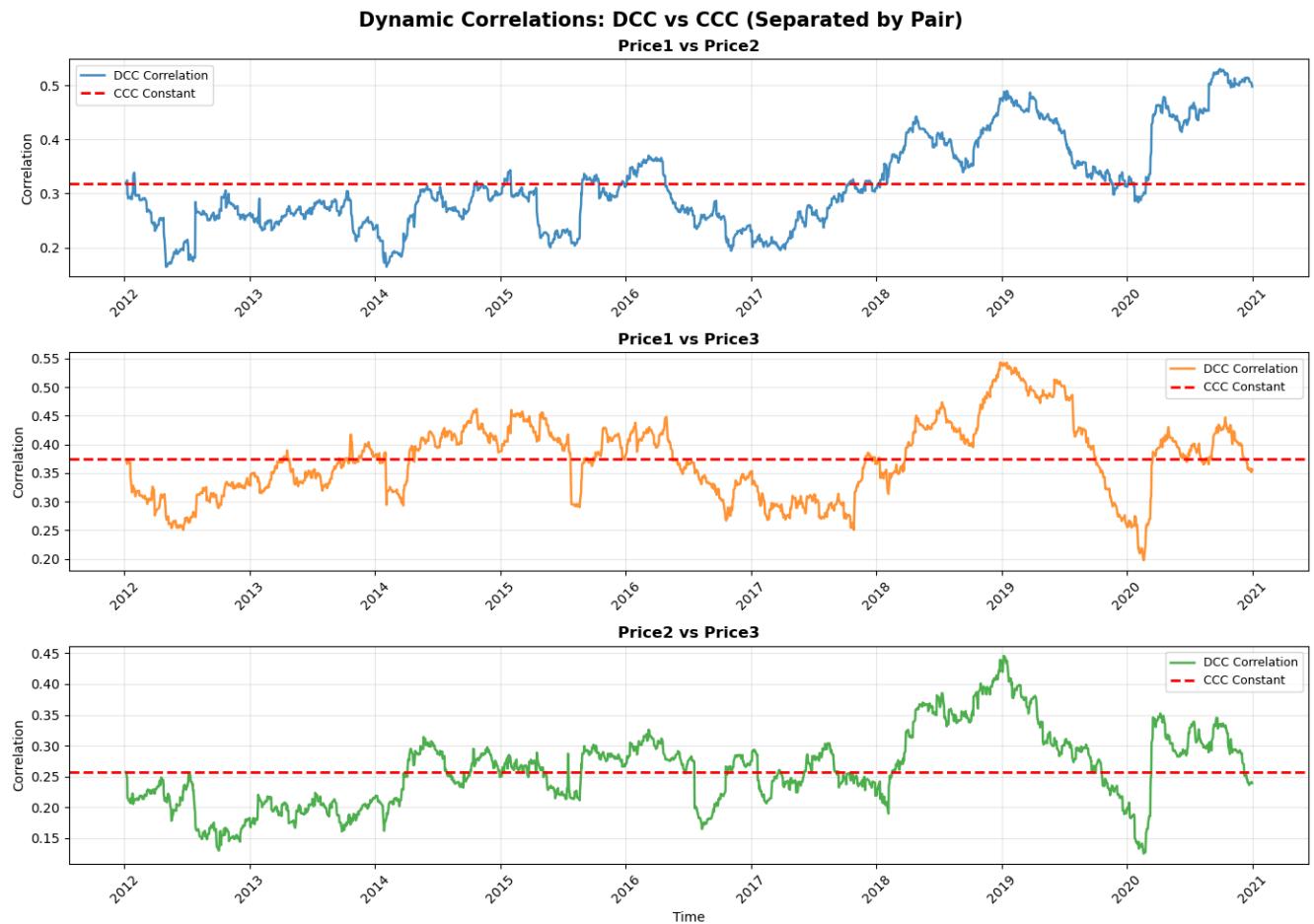
Figure 24: CCC vs DCC across time
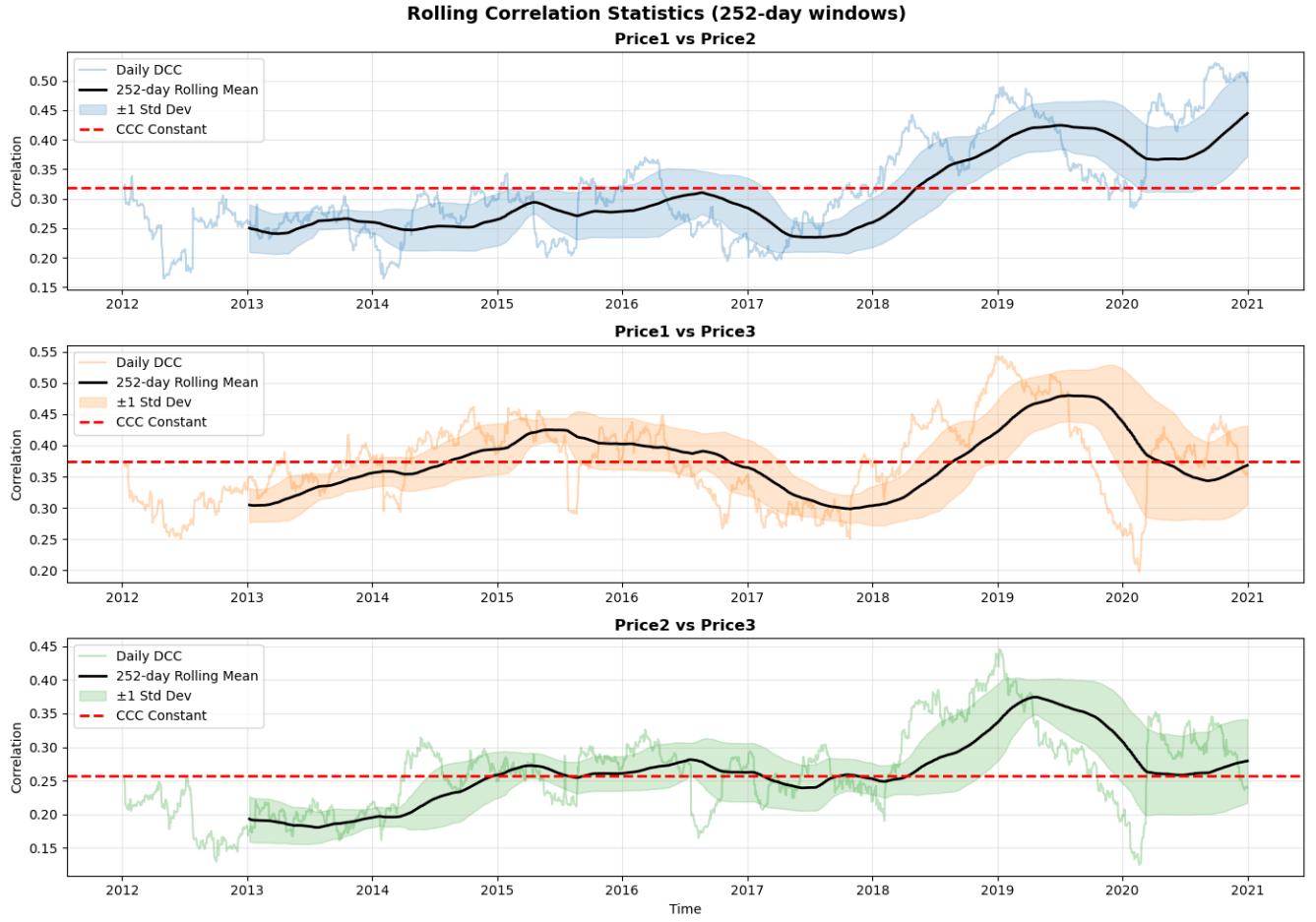
Figure 25: Rolling correlation statistics

B.3   VAR Model Analysis

Table 16: Descriptive Statistics and Distributional Properties

Variable	Mean	Std. Dev.	Min	Max	Skewness	Kurtosis
Oil Price	-0.0233	0.5731	-1.9831	1.6035	0.197	0.145
Inflation	-0.0095	0.6001	-1.6722	2.0177	0.422	0.481
GDP Growth	-0.0000	0.5330	-1.4505	1.7034	-0.158	-0.135

Table 17: Correlation Matrix

	Oil Price	Inflation	GDP Growth
Oil Price	1.000	0.643	-0.041
Inflation	0.643	1.000	-0.169
GDP Growth	-0.041	-0.169	1.000

Table 18: Stationarity Test Results and Modeling Decision

Variable	ADF p-value	KPSS p-value	Decision
Oil Price	0.0000	0.1000	Stationary
Inflation	0.0000	0.1000	Stationary
GDP Growth	0.0000	0.1000	Stationary

Table 19: Granger Causality Matrix

Predictors/Response	Oil Price	Inflation	GDP Growth
Oil Price	-	0.0001	0.1181
Inflation	0.0000	-	0.4206
GDP Growth	0.0160	0.0006	-

rows=Predictor; columns = Response

Table 20: VAR(1) Model Coefficient Estimates and Statistical Significance

	Oil Price	Inflation	GDP Growth
Constant	-0.0113	0.0032	0.0049
L1.Oil Price	0.4609**	0.3267**	0.0215
L1.Inflation	0.2195**	0.5507**	-0.1202
L1.GDP Growth	0.0879	-0.0621	0.5211**

Table 21: Residual Diagnostics and Model Adequacy Tests

Diagnostic Test	Results
Ljung-Box Test	All p-values ¿ 0.56
ARCH Effects Test	Oil: 0.9785, Inflation: 0.7857, GDP: 0.4761 (No ARCH)
Information Criteria	AIC: -5.337, BIC: -5.214, Log-Likelihood: -602.696
Residual Correlation	Oil-Inflation: 0.370, Oil-GDP: 0.005, Inflation-GDP: -0.008

Table 22: Forecast Accuracy and Out-of-Sample Performance

Variable	RMSE	Interpretation
Oil Price	0.2972	Good forecasting accuracy
Inflation	0.2734	predictive performance
GDP Growth	0.7544	Uncertainty - typical for growth forecasts

12-month out-of-sample forecast evaluation

Table 23: Cointegration Analysis: Johansen Test Results

Test Statistic	Value
Trace Statistic Critical Value (95%)	29.796
Number of Cointegrating Relationships	3
Cointegration Rank	Full Rank

Strong evidence of long-run equilibrium relationships among all three variables

Table 24: Comprehensive Model Validation Summary

Validation Check	Status and Interpretation
Stationarity	All variables strongly stationary
Residual Autocorrelation	No significant autocorrelation
ARCH Effects	No conditional heteroscedasticity
Cointegration	Strong evidence of long-run relationships
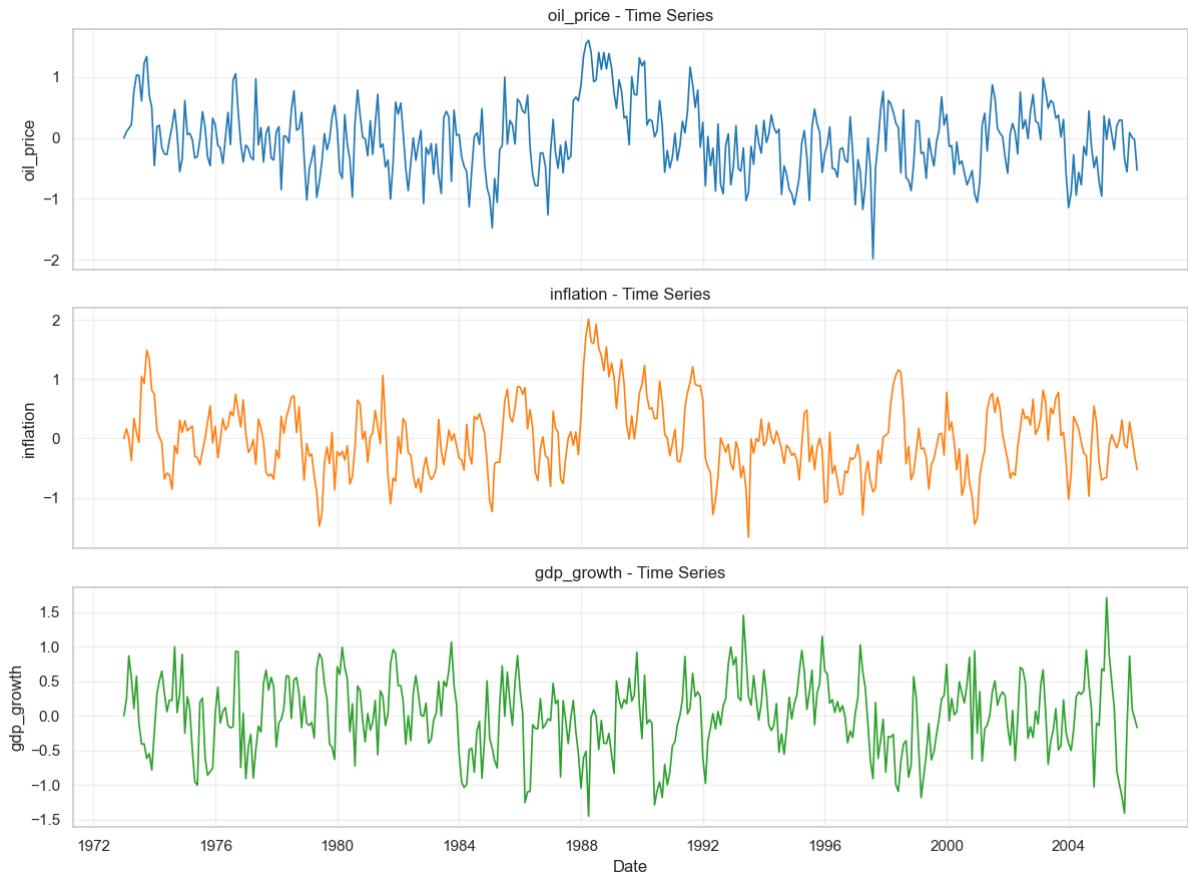
Figure 26: Time Series Evolution of Oil Prices, Inflation, and GDP Growth (1973-2006)
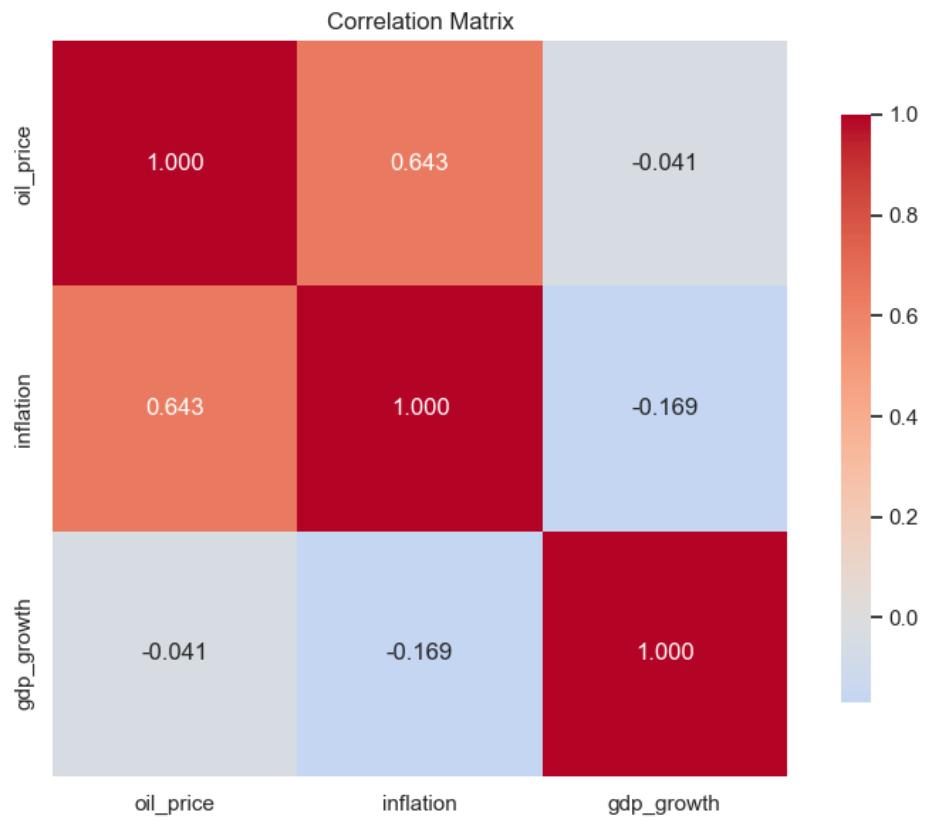
Figure 27: Correlation Heatmap Visualizing Intervariable Relationships
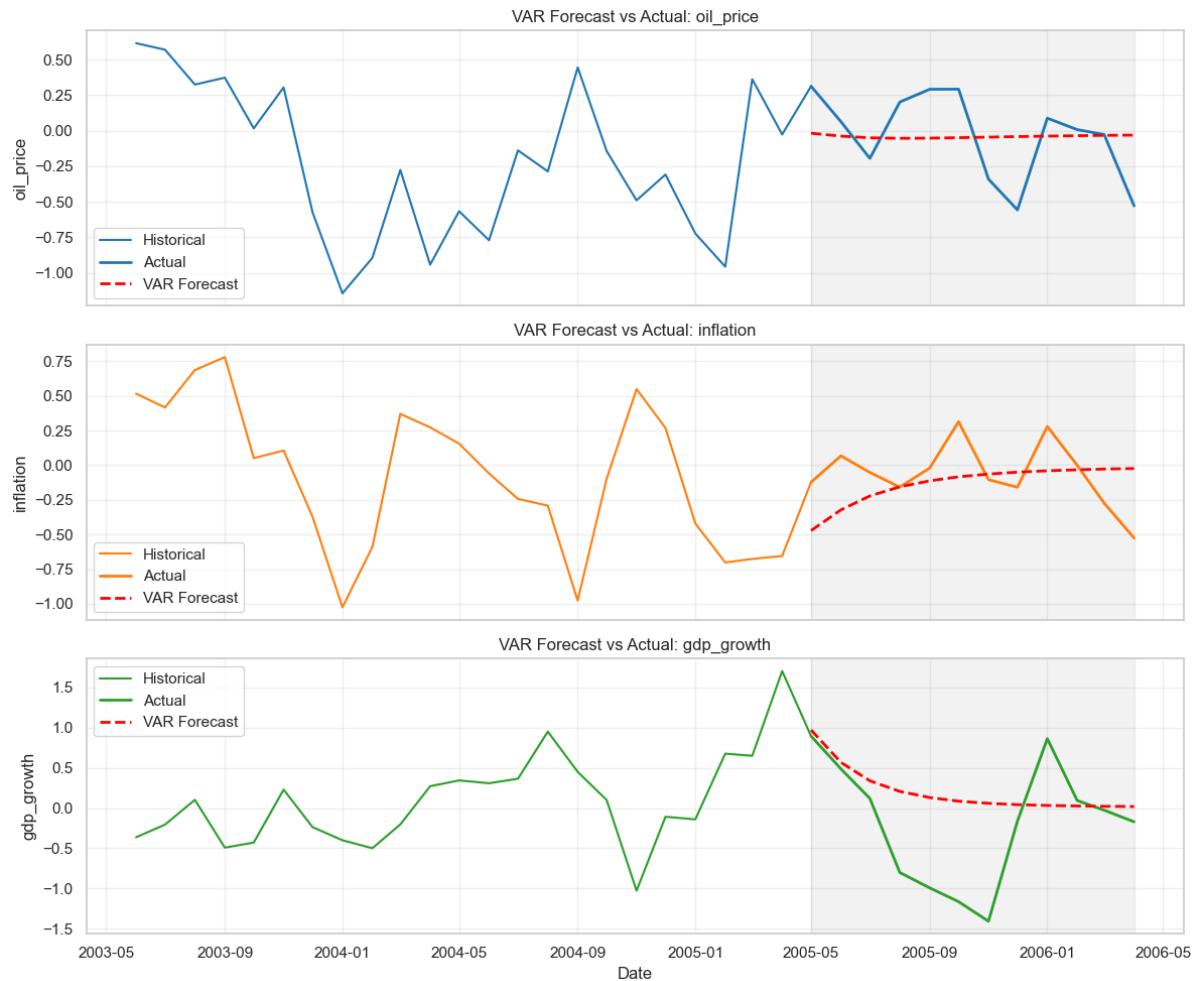
Figure 28: VAR Model Forecasts vs Actual Values (12-Month Out-of-Sample Horizon)
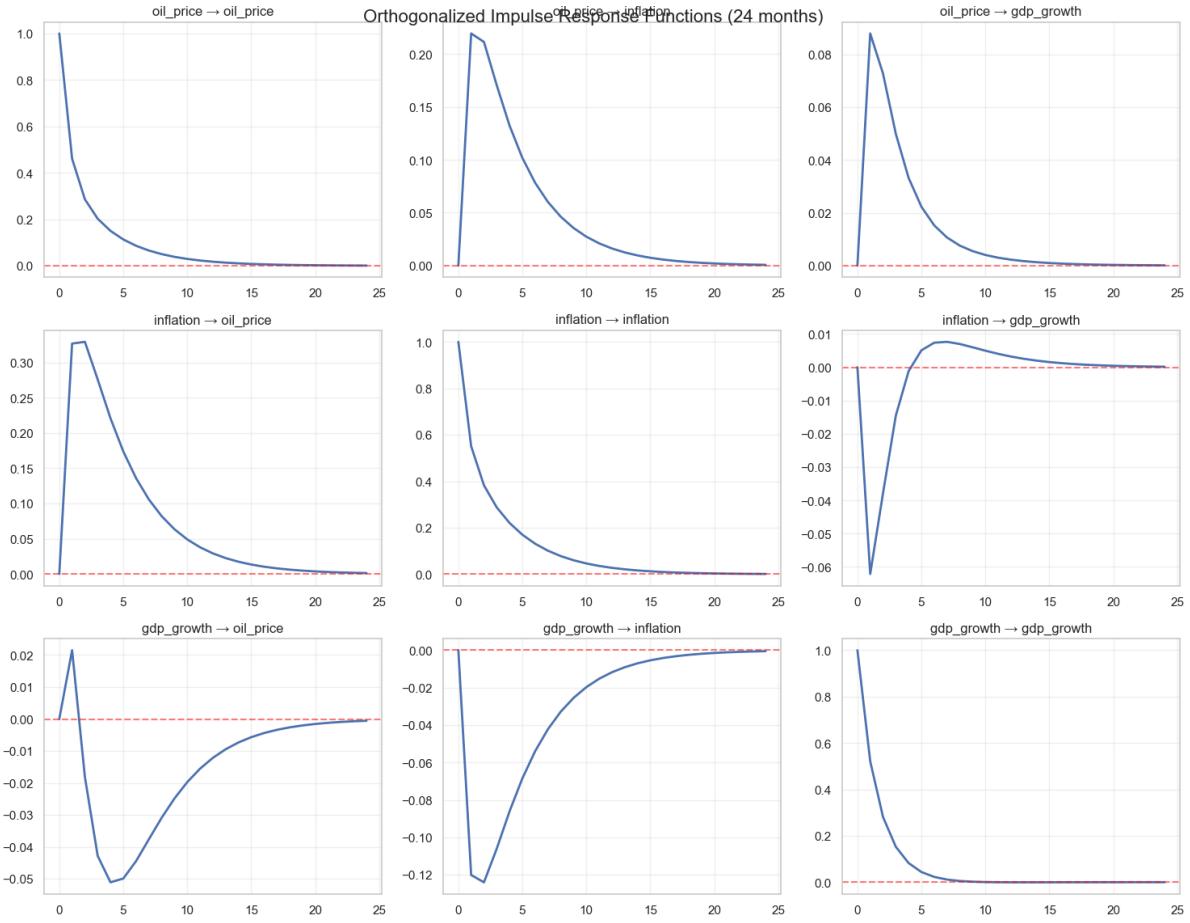
Figure 29: Impulse Response Functions (24-Month Horizon)
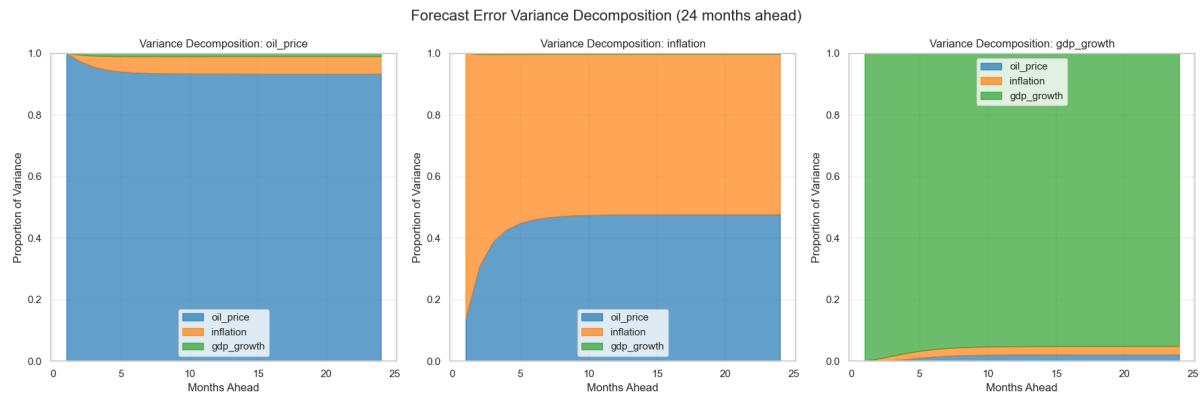
Figure 30: Forecast Error Variance Decomposition by Shock Source

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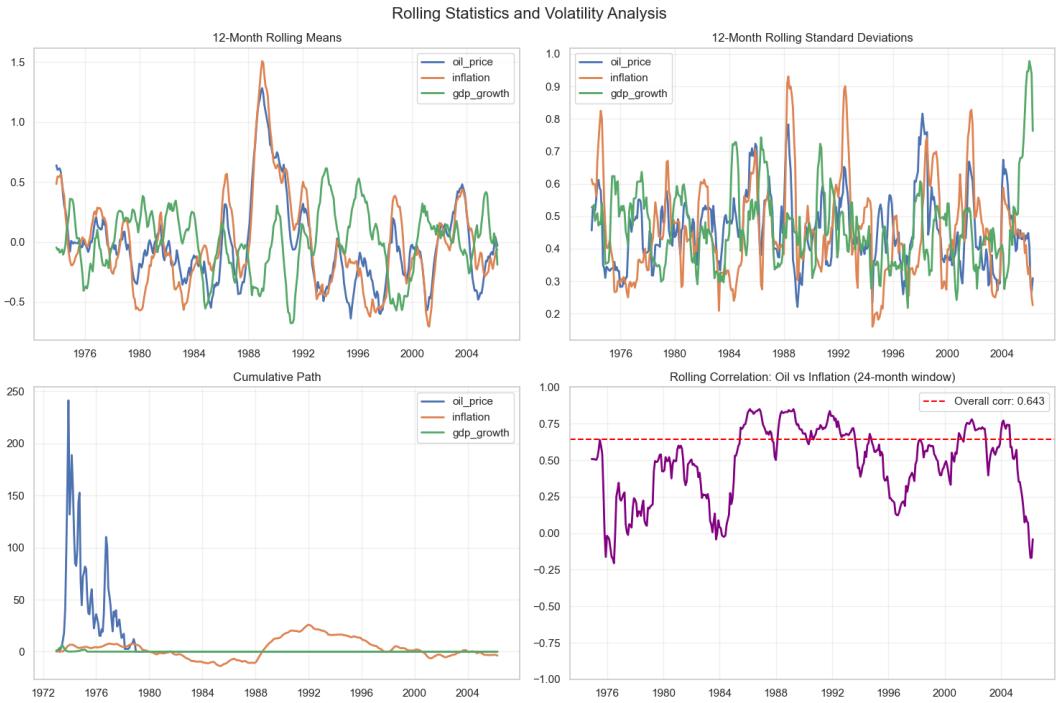
Figure 31: Rolling Statistics: Evolving Means and Volatilities (12-Month Window)