Pressure from a circular piston

```
CC[ϕ_] := ϕ /. Complex [u_, v_] → Complex [u, -v];
```

Axial pressure

$$\text{pAxial} = -I\,\omega\,\rho_0\,v_0\,\text{Assuming}\left[r>0\,\&\&\,k>0\,\&\&\,a>0,\ \text{Integrate}\left[\frac{\text{Exp}\left[I\,k\sqrt{R^2+r^2}\right]}{\sqrt{R^2+r^2}}R,\ \{R,0,a\}\right]\right]$$

$$\frac{\left(e^{i k r}-e^{i k\sqrt{a^2+r^2}}\right)\omega\,v_0\,\rho_0}{k}$$

$$\text{pAxial} = \rho_0\,v_0\,c_0\left(e^{i k r}-e^{i k\sqrt{a^2+r^2}}\right)$$

$$\left(e^{i k r}-e^{i k\sqrt{a^2+r^2}}\right)c_0\,v_0\,\rho_0$$

$$\text{pAxial} = \rho_0\,v_0\,c_0\left(1-e^{i k\sqrt{a^2+r^2}-I k r}\right)e^{i k r}$$

$$e^{i k r}\left(1-e^{-i k r+i k\sqrt{a^2+r^2}}\right)c_0\,v_0\,\rho_0$$

$$\text{p2} = \text{CC}[\text{pAxial}]\ \text{pAxial}$$

$$\left(1-e^{i k r-i k\sqrt{a^2+r^2}}\right)\left(1-e^{-i k r+i k\sqrt{a^2+r^2}}\right)c_0^2\,v_0^2\,\rho_0^2$$

$$\text{p2} = e^{\frac{i}{2}\left(k r-k\sqrt{a^2+r^2}\right)}\left(e^{-\frac{i}{2}\left(k r-k\sqrt{a^2+r^2}\right)}-e^{\frac{i}{2}\left(k r-k\sqrt{a^2+r^2}\right)}\right) e^{-\frac{i}{2}\left(k r-k\sqrt{a^2+r^2}\right)}\left(e^{\frac{i}{2}\left(k r-k\sqrt{a^2+r^2}\right)}-e^{-\frac{i}{2}\left(k r-k\sqrt{a^2+r^2}\right)}\right)c_0^2\,v_0^2\,\rho_0^2$$

$$\left(e^{-\frac{1}{2}i\left(k r-k\sqrt{a^2+r^2}\right)}-e^{\frac{1}{2}i\left(k r-k\sqrt{a^2+r^2}\right)}\right)\left(-e^{-\frac{1}{2}i\left(k r-k\sqrt{a^2+r^2}\right)}+e^{\frac{1}{2}i\left(k r-k\sqrt{a^2+r^2}\right)}\right)c_0^2\,v_0^2\,\rho_0^2$$

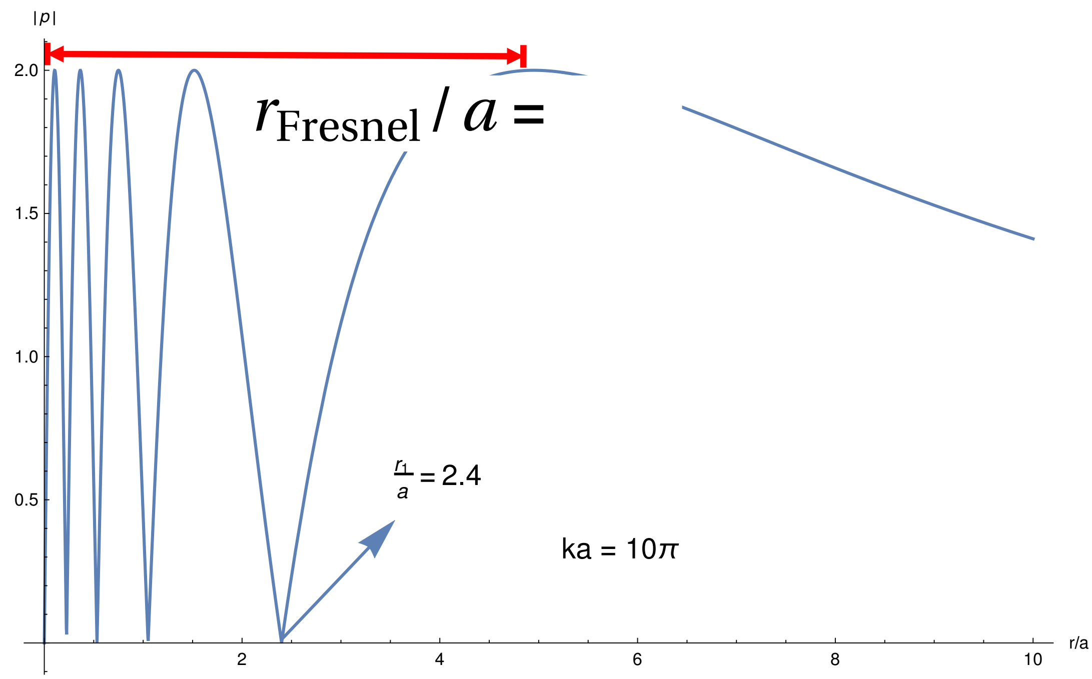
$$\text{pAxialAbs} = \sqrt{\text{ExpToTrig[p2]}}$$

$$2\sqrt{\left(\text{Sin}\left[\frac{k r}{2}-\frac{1}{2}k\sqrt{a^2+r^2}\right]^2 c_0^2\,v_0^2\,\rho_0^2\right)}$$

$$\text{pAxialAbs} = 2\,\text{Abs}\left[\text{Sin}\left[\frac{1}{2}k\sqrt{a^2+r^2}-\frac{k r}{2}\right]\right]c_0\,v_0\,\rho_0$$

$$2\,\text{Abs}\left[\text{Sin}\left[\frac{k r}{2}-\frac{1}{2}k\sqrt{a^2+r^2}\right]\right]c_0\,v_0\,\rho_0$$

$$\text{Plot}\left[\frac{\text{pAxialAbs}}{c_0\,v_0\,\rho_0}\ /.\ \{k\to 2\pi,\ r\to x\,a\}\ /.\ a\to 5,\ \{x,0,10\},\ \text{AxesLabel}\to\{\text{"r/a"},\ \text{Abs}[p]\}\right]$$



$$\mathbf{r_1} = \frac{(10\,\pi)^2 - (2\,\pi)^2}{4\,\pi\,2\,\pi}$$

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$$\frac{\mathbf{r_1}}{\mathbf{5.}}$$

2.4

$$\mathbf{pAxialAbs} = 2\,\mathrm{Abs}\left[\mathrm{Sin}\left[\frac{k\,r}{2}\left(1 + \frac{1}{2}\left(\frac{a}{r}\right)^2 - 1\right)\right]\right] c_0\, v_0\, \rho_0$$

$$2\,\mathrm{Abs}\left[\mathrm{Sin}\left[\frac{a^2\,k}{4\,r}\right]\right] c_0\, v_0\, \rho_0$$

Pressure zeros

$$\mathrm{Solve}\left[\frac{1}{2}k\,r_n\left(\sqrt{1 + \left(\frac{a}{r_n}\right)^2} - 1\right) == n\,\pi,\ r_n\right]$$

$$\left\{\left\{r_n \to \frac{a^2\,k^2 - 4\,n^2\,\pi^2}{4\,k\,n\,\pi}\right\}\right\}$$

Farfield region

$$\mathbf{R2} = (x - x0)^2 + (y - y0)^2 + z^2$$

Out[1]= $(x - x0)^2 + (y - y0)^2 + z^2$

In[2]:=
```
x = r Sin[θ] Cos[ϕ];
y = r Sin[θ] Sin[ϕ];
z = r Cos[θ];
x0 = rp Cos[ϕp];
y0 = rp Sin[ϕp];
```

In[8]:=
```
FullSimplify[R]
```

Out[8]= $r^2 + rp^2 - 2\, r\, rp \operatorname{Cos}[\phi - \phi p] \operatorname{Sin}[\theta]$

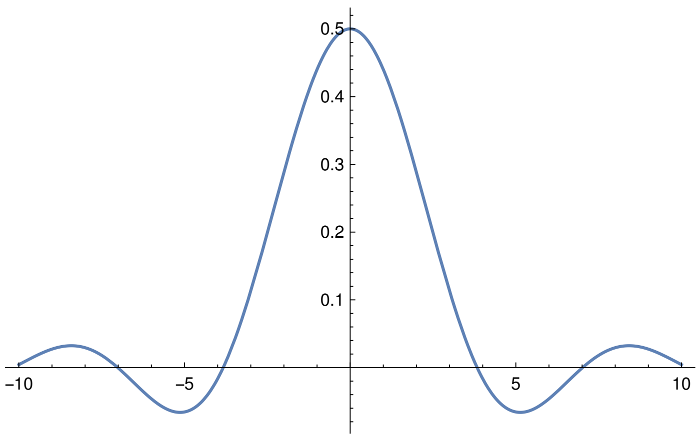
Zeros of the first-order Bessel

In[18]:=
```
BesselJZero[1, 1.]
```

Out[18]= 3.83171

In[21]:= $\text{Plot}\left[\frac{\text{BesselJ}[1, x]}{x}, \{x, -10, 10\}\right]$

Out[21]=



In[22]:= $\frac{3.831705970207512\text{`}}{2\,\pi}$

Out[22]= 0.609835

In[27]:=
```
DensityPlot[BesselJ[1, Sqrt[x1^2 + y1^2]], {x1, -10, 10}, {y1, -10, 10},
  PlotLegends → Automatic , PlotPoints→ 50, PlotRange → All]
```

Out[27]=