Engineering Ultrasound Waves

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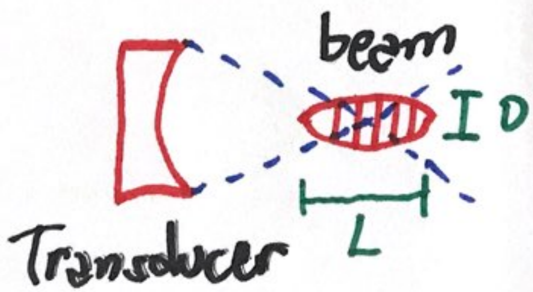
What do you want to do?

- Imaging
- Levitation
- Microparticle manipulation

Ultrasound imaging

- Pulse-echo (PE)
- Continuous wave (CW)

Focused ultrasound



Pulsed waves

$$\nabla^2 p + k^2 p = 0$$

(Works for a single frequency wave.)

$$p(\vec{r},t) = p(\vec{r})\, e^{-i\omega t}$$

Finding the wave equation

$$i\omega \Leftrightarrow \partial_t$$

$$-\frac{\omega^2}{c_0^2} = -K^2 \Leftrightarrow c_0^{-1} \partial_t^2$$

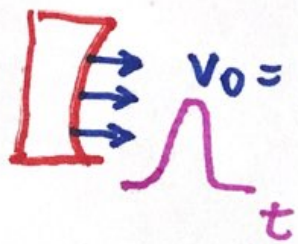
$$K^2 \Leftrightarrow -c_0^{-1} \partial_t^2$$

The wave equation

$$\nabla^2 p - \frac{1}{c_0^2}\partial_t^2 p = 0$$

$\oplus$ Boundary conditions

PE: velocity at the transducer surface is specified

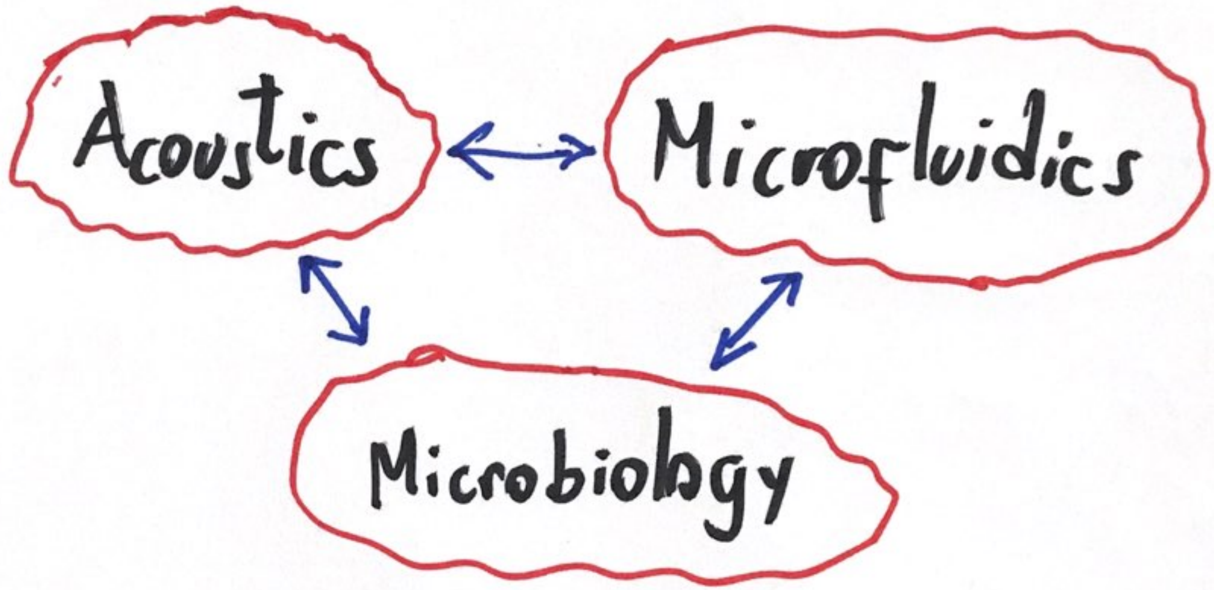


$v_0 = v_0(t)$ (a Gaussian pulse)

$$v_0 = A\, e^{-Bt^2}$$

Reading: Ch. 6 T. L. Szabo

Acoustofluidics

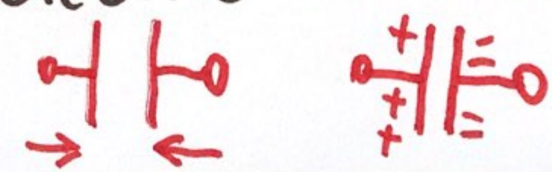


Produce Acoustic Landscapes of energy to manipulate microparticles.

Basic ingridients

- Submillimeter cavities

- Hard and soft materials
  - PDMS
  - Resin
  - Silicon
  - Glass, etc.
- Piezoelectric actuators



- Host liquid
- Microparticles (cells and microorganisms)

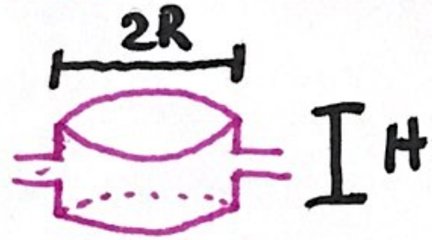
Lab-on-a-chip Tecnology

- Fabrication
  - 3D printing
  - Clean room
  - Owen, etc
- Tests
  - Microfluidics
  - Optics
  - Electronics & computers

Chip design

- Model equations
- Finite-element simulations
- Acoustic landscapes
- Forces, torques (maps)
- Villain: Acoustic streaming

Cylindrical Cavity



Lossless model

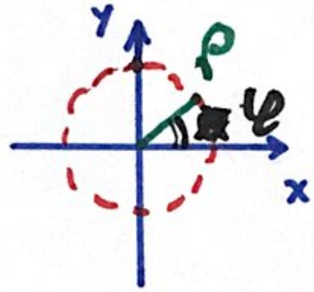
$$(\nabla^2 + k^2)p = 0$$

$$\vec{v} = \frac{\nabla p}{i\rho_0 c_0 k}$$

+ Boundary conditions
  - Hard walls $\}$ $v_n = 0$
    soft $\}$ $p = 0$
  - Actuation $v_b = v_0 e^{-i\omega t}$

Solution

- Cylindrical coordinates



$$\vec{r}(\varrho, \varphi, z)$$

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- Acoustic modes

$$p(\rho, \varphi, z) = p_0 J_n(k_\rho \rho) \cos(n\varphi + \varphi_0) \cos k_z z$$

$$k = \frac{\omega}{c_0} = \sqrt{k_\rho^2 + k_z^2}$$

Boudary conditions

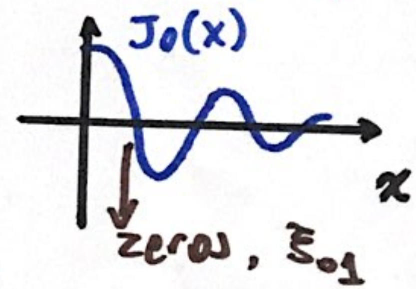
Hard : $v_z|_{z=-H/2} = v_z|_{z=H/2} = 0$

$v_\rho|_{\rho=R} = 0 \quad \Rightarrow \quad \begin{cases} \sin\left(\frac{k_z H}{2}\right) = 0 \\ J_n'(k_\rho R) = 0 \end{cases}$

Soft: $p(\rho = R) = 0$

$$\Downarrow$$

$$J_n(k_\rho R) = 0$$



Wavenumbers : $\begin{cases} k_z = \frac{m_z \pi}{2} \\ k_\rho = \frac{\eta_{nm}}{R} \text{ (hard)}; \ \frac{\xi_{nm}}{R} \text{ (soft)} \end{cases}$

Allowed frequencies:

$$f = \frac{k c_0}{2\pi} = \frac{c_0}{H}\sqrt{m_z^2 + \left(\frac{x_{nm} H}{2R}\right)^2}$$