Focused Beam

- Rigid baffle is valid to flat pistons

- Lenses or geometrically focused transducers

* Spherically focused transducer

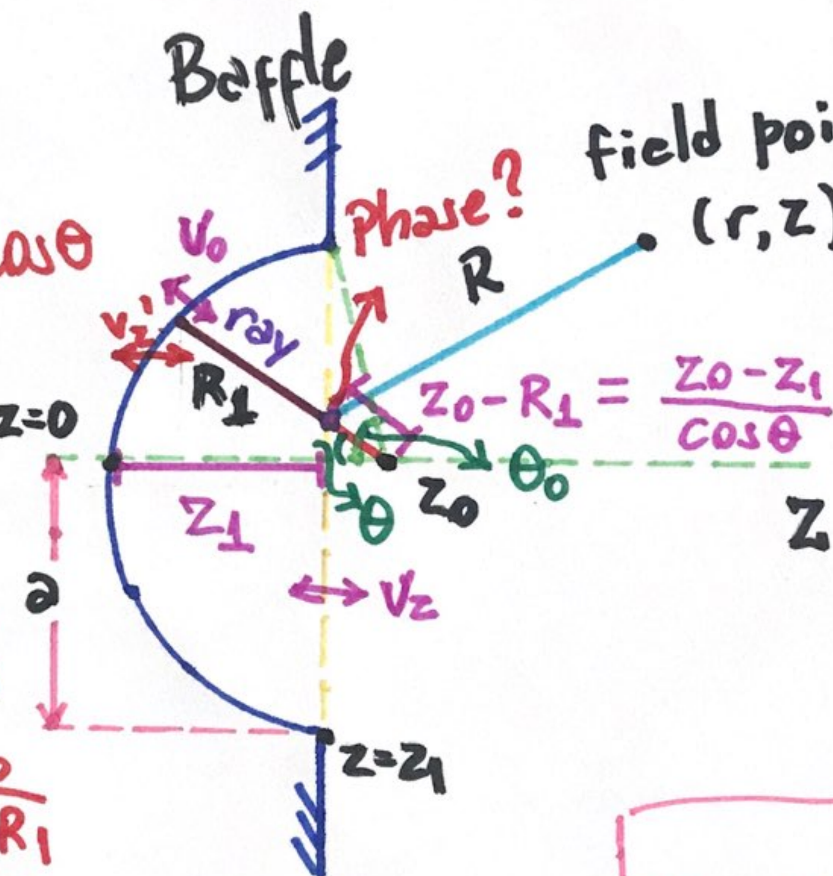
Spherically focused transducer

$$\frac{v_z'}{v_0} = \cos\theta$$

$$v_z' \;—\; v_z$$

$$z_0 - R_1 \;—\; z_0$$

$$v_z = v_z' \frac{z_0}{z_0 - R_1}$$



$$v_z = v_0 \cos\theta \frac{z_0}{z_0 - R_1}$$

$$v_z = v_0 \cos^2\theta \frac{z_0}{z_0 - z_1}$$

$$z_0 - z_1 \rightarrow z_0 \cos\theta_0$$

$$\boxed{v_z = v_0 \frac{\cos^2\theta}{\cos\theta_0} \cdot e^{ikR_1}}$$

Cont...

Rayleigh integral

$$p = -\frac{i\omega\rho_0 v_0}{2\pi} \int_0^a \int_0^{2\pi} \frac{e^{ik(R+R_1)} \cos^2\theta}{R \cos\theta_0} \, r' dr' d\varphi'$$

$$R = \sqrt{r'^2 + r^2 - 2rr'\cos(\varphi - \varphi') + (z - z_1)^2}$$

$z \gg r', r$

$$R \simeq z - z_1 + \frac{r'^2 + r^2 - 2rr'\cos(\varphi - \varphi')}{2z_1}$$

$$R_1 = z_0 - \sqrt{r'^2 + (z_0 - z_1)^2} \simeq z_1 - \frac{r'^2}{2z_0}$$

$z_0 \gg r', z_1$

Cont...

$$R_1 + R \simeq z + \frac{r'^2 + r^2 - 2rr'\cos(\varphi-\varphi')}{2z} - \frac{r'^2}{2z_0}$$

Paraxial approximation

$$\cos^2\theta \simeq 1, \quad \cos\theta_0 \simeq 1$$

$$p = -\frac{ik\rho_0 c_0 v_0}{2\pi} \underset{R\simeq}{\frac{1}{z}} \int_0^a \int_0^{2\pi} e^{\frac{ikr'^2}{2}\left(\frac{1}{z}-\frac{1}{z_0}\right)} e^{-\frac{ikrr'}{z}\cos(\varphi-\varphi')} r'dr'd\varphi'$$

$$\times\, e^{ik(z + r^2/2z)}$$

Cont...

$$p = \frac{-ik\rho_0 c_0 v_0}{\boxed{2\cancel{\pi}}} \frac{e^{ik(z+r^2/2z)}}{z} \int_0^a J_0\left(\frac{krr'}{z}\right) e^{ikr'^2/2\left(\frac{1}{z}-\frac{1}{z_0}\right)} r'dr'$$

At the geometrical focus $z = z_0$, $r = 0$

$$\boxed{p = -ik\rho_0 c_0 v_0 \frac{a^2}{2z_0} e^{ikz_0}}$$

At the focal plane $z = z_0$

jinc function

$$p = -2ik\rho_0 c_0 V_0 \frac{e^{ik(z+r^2/2z)}}{z_0} \frac{J_1(kra/z_0)}{kra/z_0}$$

Intensity (flux of energy) $I = \frac{\mathrm{Re}}{2}[p v_z^*]$

$$I = I_0 \left(\frac{ka^2}{2z_0}\right)^2 \mathrm{jinc}^2\left(\frac{kra}{2\pi z_0}\right)$$

Zero of the jinc-function

$$r_0(\text{zero}) = \frac{0{,}61\, z_0 \lambda}{a} = \frac{0.61\lambda}{\sin\theta_0}$$

Diameter at 3dB-points (FWHM)

$$d(3\text{dB}) = \frac{0.51\, z_0 \lambda}{a} = \frac{0.51\lambda}{\sin\theta_0} = 1.02\lambda F$$

$\frac{z}{2a}$

(F number)