Test 22oct2024_statisticalmethods_CS

Question 1 – Bayes1 – 448797.1.1

A company has two machines, let's call these A and B. Machine A produces 60% of the items and machine B produces 40% of the items. Machine A has a 2% defect rate. The probability of an item being defect equals 0.1. We are given a defect item. The probability that this item was produced by machine A equals (round to 3 decimals)

0.120

Feedback at question level

P(D)=0.1, P(A)=0.6, P(D | A)=0.02, then
P(A | D) = 0.02*0.6/0.1=0.12 (Bayes)

Question 2 – birthday1 – 448803.1.1

In a group of 3 people, what is the probability that at least two people share the same birthday (assuming 365 possible birthdays and ignoring leap years)? Round to three decimals.

0.008

Feedback at question level

the requried probability is 1- P(all three different birthdays) = 1-(364/365)*(363/365) approx 0.008

Question 3 – CDF1 – 448802.1.1

Which of the following is a property of any cumulative distribution function (CDF)?

[✓ **A**] It is always non-decreasing.
B It can decrease at certain points.
C It is constant over the entire real line.
D Its value is always less than or equal to 0.
E It is always left-continuous.

Question 4 – ci−proportion – 449255.1.1

A study was conducted to estimate the proportion of people in a city who prefer public transportation. A random sample of 400 people shows that 240 prefer public transportation. What is an approximate 99% confidence interval for the proportion of people who prefer public transportation?

A (0.57, 0.63)

[✓ **B**] (0.54, 0.66)

C (0.51, 0.69)

D (0.59, 0.61)

Feedback at question level

CI equals bar(x) +- z_{0.005} sqrt(xbar(1-xbar)/n) where xbar=240/400=0.6, n=400 and z_{0.005}=2.58

Question 5 – condprob1 – 448804.1.0

Which statements on rules for computing probabilities are generally true?

A $P(A \cup B \mid C) = P(A \mid C) + P(B \mid C)$

B $P(A \cup B \mid C) = P(A \mid B \cup C)\, P(C)$

[✓ **C**] $P(A \cup B \cup C) = 1 - P(A^c \cap B^c \cap C^c)$

D $P(A \mid B) = P(B \mid A)\, P(B) / P(A)$

[✓ **E**] $P(A \cap B) = P(A \mid B)\, P(B)$

Question 6 – dice1 – 448794.1.0

Suppose you roll two fair six-sided dice. What is the probability that the sum of the dice is 7, given that exactly one of the dice shows a 3?

A 1

B 1/6

[✓ **C**] 1/5

D 1/3

Question 7 – expecbinomial – 448799.3.0

A random variable X follows the Binomial distribution with parameters n = 10 and p = 0.2. The expected value of X equals (round to 1 decimal)

2.0 2

Question 8 – expectationcontinuous1 – 448798.1.0

Let X be a random variable with probability density function
f_X(x) = 3 x^{-4} if x >1.
The expected value of X equals (round to 1 decimal)

1.5

Feedback at question level
\int_1^\infty x 3 x^{-4} d x = 3 \int_1^\infty x^{-3} d x = 3/2

Question 9 – hyp_b – 395879.1.1

Consider a testing problem with H_0: μ=21 versus H_1: μ<21. This test is

A right tailed (or right-sided).
[✓ **B**] left tailed (or left-sided).
C cross tailed (or cross-sided).
D center tailed (or center-sided).

Question 10 – hyp_c – 395880.1.0

A type I error occurs if

A a false null hypothesis is rejected
[✓ **B**] a true null hypothesis is rejected
C a false null hypothesis is not rejected
D a true null hypothesis is not rejected

Question 11 – lawoftotalprob1 – 448796.1.1

A company has two machines, let's call these A and B. Machine A produces 60% of the items and machine B produces 40% of the items. Machine A has a 2% defect rate and Machine B has a 5% defect rate. The probability that a randomly selected item is defect equals (round to 3 decimals)

0.032

Feedback at question level
A={produced by machine a}
D = {defect}
P(D | A) = 0.02, P(D | A^c)=0.05)
P(A)=0.6
dus 0.6*0.02 + 0.4* 0.05 = 0.032

Question 12 – lotus1 – 448801.1.0

Suppose X is a continuous random variable with probability density function $f_X(x) = 4x(1-x)$ if $x \in [0,1]$.
What is the $E[X^2]$?

A $(\int x \, 4x \, (1-x) \, d x)^2$

B $\int x \, (4x \, (1-x))^2 \, d x$

[✓ **C**] $\int x^2 \, 4x \, (1-x) \, d x$

D $\int (x \, 4x \, (1-x))^2 \, d x$

Question 13 – normaldistribution – 448805.1.0

If $X \sim N(3,4)$ and if Φ denotes the cumulative distribution function of the standard normal distribution, then $P(X>4)$ equals

[✓ **A**] $1 - \Phi(0.5)$

B $\Phi(0.5)$

C $1 - \Phi(0.25)$

D $\Phi(0.25)$

E $2 \, \Phi(0.5)$

F $2 \, \Phi(0.25)$

Question 14 – probunion – 448800.2.2

Consider three events A, B and C in a sample space with $P(A) = 0.4$, $P(B) = 0.5$ and $P(C) = 0.3$. If A, B and C are mutually independent, what is $P(A \cup B \cup C)$?

A 0.64

B 0.76

C 0.82

D 0.91

[✓ **E**] 0.79

F none of the given options

Feedback at question level

P(A cup B cup C)=1-P (complement event) = 1-P(A^c cap B^c cap C^c) = 1-0.6*0.5*0.7=0.79

Question 15 – pval – 449253.1.1

In a hypothesis test, the p-value obtained is 0.04. If the significance level (α) is set to 0.05, what is the correct conclusion?

A Fail to reject H_0

[✓ **B**] Reject H_0.

C Increase the sample size.

D Decrease the significance level.

E None of the above given answers.

Question 16 – spam_c – 395241.3.2

Suppose we developed some spam filter to identify email spam. It works as follows: if a message contains any of some flagged words, it is marked as spam. It is known that the probability of any message being spam is 20%, the probability that a flagged word is present in spam email is 0.9, whereas the probability that a flagged word is present in non-spam email is 0.05. An email message arrives. Denote W={message contains a flagged word}, S={message is spam}.

Indicate whether the below claims are true or false.

	True	Not true
The events W and S are independent.		X
The events W and S are disjoint.		X

Feedback at question level

S={spam}, W={flagged word}. Then P(S)=0.2, P(W|S)=0.9, P(W|not S)=0.05.
P(S)=0.2 not= 0.9=P(W|S), so W and S are not independent.
P(W and S) not= 0, W and S are clearly not disjoint.

Question 17 – sumrule1 – 448795.1.0

Consider two events A and B such that $P(A \cup B) = 0.8$, $P(A) = 0.5$ and $P(B) = 0.4$. What is $P(A \cap B)$)?

[✓ **A**] 0.1

B 0.3

C 0.4

D 0.5

Question 18 – testing_pval – 449254.1.2

Suppose X_1, X_2, ... X_n are independent random variables, each with the N(μ, σ^2)-distribution. A researcher is testing the hypothesis H_0: $\mu = 25$ *against* H_1: $\mu < 25$. The sample mean is 24, the sample size (n) is 36 and the population standard deviation is known to be 6. What is the p-value for this test?

In the options below, Φ denotes the cumulative distribution function of the standard Normal distribution and φ denotes the probability density function of the standard Normal distribution.

A $\Phi(-1/6)$

[✓ **B**] $\Phi(-1)$

C $\varphi(-1/6)$

D $\Phi(-6)$

E $1-\Phi(-1)$

F $1-\Phi(-1/6)$

Feedback at question level

t= sqrt(36) (24-25)/6

Question 19 – varsamplemean – 448806.1.1

If X_1, X_2, ..., X_n are independent and identically distributed random variables from a distribution with mean 10 and variance 5. The variance of the sample mean (which is defined as the average of X_1, X_2, ..., X_n) is given by

[✓ **A**] $5/n$

B $10/n$

C $5/\sqrt{n}$

D $25/n$

E $25/\sqrt{n}$

F $10/\sqrt{n}$

Question 20 – MSE – 449257.1.0

Suppose X_1, X_2,..., X_n is a random sample from the N(μ, 4)-distribution.

	True	Not true
The sample mean is an unbiased estimator for μ.	X	
The mean squared error for estimating μ equals $2/\sqrt{n}$.		X

Question 21 – mle – 449256.2.1

Suppose x_1, x_2,...,x_n are realizations of independent random variables, where each X_i (1≤ i ≤ n) has the Exponential distribution with parameter λ. Verify for the following statements whether they are true or false.

	True	Not true
The likelihood only depends on the sum of x_1, x_2,...,x_n	X	
The maximum likelihood estimate equals the sample mean.		X

Question 22 – probcontinuous – 449259.1.2

Suppose X is a random variable with probability denstiy function f(x)=x+1/2 for 0 ≤ x ≤ 1 and zero for other values of x. The probability of the event {X>1/3} equals (round to 2 decimals)

0.77 0.78

Feedback at question level

\int_{1/3}^1 (x+1/2) dx = [1/2x^2 +1/2 x]_{1/3}^1 =21/27\approx 0.7777

Question 23 – cdf_geometricdistribution – 449258.1.1

Suppose X has the geometric distribution with parameter 0.2. The value of the cumulative distribution function at 2.5 (i.e. $F_X(2.5)$) is given by (round to 2 decimals)

0.36

Feedback at question level

P(X\lge 2.5)=P(X=1)+P(X=2) = 0.2+0.2*0.8=0.36

Question 24 – compute type I error – 449262.2.1

Suppose X_1, X_2,..., X_9 is a random sample from the N(μ, 1)-distribution. We test the hypothesis H_0 : μ = 1 versus H_1 : μ > 1. We decide to reject H_0 if the sample mean is larger than 1.5. Denote the cumulative distribution function of the standard Normal distribution by Φ. The probability of a type I error is given by

A Φ(3/2)

B Φ(4/5)

C 1 - Φ(4.5)

[✓ **D**] 1 - Φ(3/2)

E Φ(-3/2)

F Φ(-4.5)

Feedback at question level

P_{\mu=1}(barX > 3/2)+P(Z >(3/2-1)/sqrt(1/9))=P(Z>3/2)=1-Phi(3/2)