

# Capybara dreaming

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# 1 First things first

## 1.1 Includes

```
#include <bits/stdc++.h>
using namespace std;

#define ll long long
#define pb push_back
#define D(x) cout << #x " = " << (x) << endl

typedef vector<int> vi;
typedef vector<vi> vvi;

typedef pair<int, int> ii;
typedef vector<ii> vii;
```

---

---

## 2 Matemática

### 2.1 Algoritmo de euclides extendido

```
int xmdc(int a, int b, int &x, int &y)
{
    if (b == 0)
    {
        x = 1;
        y = 0;
        return a;
    }
    int x1, y1, mdc = xmdc(b, a % b, x1, y1);
    x = y1;
    y = x1 - (a / b) * y1;
    return mdc;
}
```

---

### 2.2 Máximo divisor comum

```
int mdc(int a, int b)
{
    int remainder;
    while (b != 0)
    {
        remainder = a % b;
        a = b;
        b = remainder;
    }
    return a;
}
```

---

### 2.3 Mínimo múltiplo comum

```
int mmc(int a, int b)
{
    int temp = mdc(a, b);
    return temp ? (a / temp * b) : 0;
}
```

---

### 2.4 Algoritmo de Pollard Rho

```
int pollard(int number)
{
    x_fixed = 2, cycle_size = 2, x = 2, factor = 1;
    while (factor == 1)
    {
        for (int count = 1; count <= cycle_size && factor <= 1; count++)
        {
            x = (x * x + 1) % number;
            factor = mdc(x - x_fixed, number);
        }
        cycle_size *= 2;
        x_fixed = x;
    }
    return factor;
}
```

---

### 2.5 Transformada rápida de Fourier

```
// Resolve:
// - De quantas maneiras conseguimos atingir Y com X tentativas
// - Dado X tentativas, conseguimos atingir Y?
// Complexidade:
// X * Ymax * Ymax(log Ymax)
```

```

// TEOREMA DA CONVOLUÇÃO:
// Podemos fazer a convolução de 2 polinômios utilizando a FFT
// Reduzindo a complexidade de  $n^2$  para  $n \log n$ 
// Definimos a convolução como  $h[i] = \sum(a[j] * b[j-i])$  para todo  $j$  de 0 a  $i$ .
// Exemplo:  $h[5] = a[5] * b[0] + a[4] * b[1] + a[3] * b[2]...$ 
// Segundo o teorema da convolução
//  $h(f \cdot g) = \text{transformada inversa de } (\text{transformada}(f) * \text{transformada}(g))$ 
// onde  $\cdot$  é o operador de convolução.
// e  $*$  é o operador de multiplicação termo a termo.

#include <bits/stdc++.h>
using namespace std;

// primeira potência de 2 maior que o limite de H
#define MAX_DIST (262144 * 2)

typedef complex<double> cpx;
const double pi = acos(-1.0);

int p[MAX_DIST];
int maxDist;

// in:    vector de entrada
// out:   vector de saída
// n:     Tamanho do input/output {DEVE SER DA ORDEM DE 2}
// type:  1 = Transformada, -1 = Transformada inversa
void FFT(vector<cpx> &v, vector<cpx> &ans, int n, int type)
{
    assert(!(n & (n - 1)));
    int i, sz, o;
    p[0] = 0;
    for (i = 1; i < n; i++)
        p[i] = (p[i >> 1] >> 1) | ((i & 1) ? (n >> 1) : 0);
    for (i = 0; i < n; i++)
        ans[i] = v[p[i]];
    for (sz = 1; sz < n; sz <= 1)
    {
        const cpx wn(cos(type * pi / sz), sin(type * pi / sz));
        for (o = 0; o < n; o += (sz < 1))
        {
            cpx w = 1;
            for (i = 0; i < sz; i++)
            {
                const cpx u = ans[o + i], t = w * ans[o + sz + i];
                ans[o + i] = u + t;
                ans[o + i + sz] = u - t;
                w *= wn;
            }
        }
    }
    if (type == -1)
        for (i = 0; i < n; i++)
            ans[i] /= n;
}

// Exemplo:
// Há um robo que pode disparar bolas em N distâncias diferentes.
// Queremos saber se ele alcança uma distância M com 1 ou 2 tacadas.
// Resolução:
// Podemos definir um vetor distances[MAX_DIST],
// onde a distances[i] = 1 se ele pode tacar até a distancia i
// e distances[i] = 0 caso contrario
// Para ver se o robo acerta com 1 tacada, é trivial.
// Para ver se o robo acerta com 2 tacadas, podemos fazer a convolução de distances com distances.
// Ex: Acertar a Poda[10] é igual a: Poda[10] || Poda[9] * Poda[1] || Poda[8] * Poda[2]...
// Ou seja,  $H = \text{FFTi}(\text{FFT}(\text{distances}) ** 2)$ ;

// Complexidade:
//  $2 * 200k * \log(200k) = 8m$ 

int main()
{
    int N, d;
    vector<cpx> distances, fftOut;

    while (cin >> N)
    {
        maxDist = 0;
        distances = vector<cpx>(MAX_DIST);
        fftOut = vector<cpx>(MAX_DIST);
    }
}

```

---

```

    // Distancia 0 é uma posição de "possível"
    distances[0] = cpx(1, 0);

    for (int i = 0; i < N; i++)
    {
        cin >> d;
        if (d > maxDist)
            maxDist = d;
        distances[d] = cpx(1, 0);
    }

    int shiftAmount;
    for (shiftAmount = 0; (maxDist >> shiftAmount) != 0; shiftAmount++)
        ;

    maxDist = 1 << (shiftAmount + 1);

    // fftOut <= transformada de distances
    FFT(distances, fftOut, maxDist, 1);

    // Multiplicação termo a termo de f e g, no caso, f = g = fftOut
    // fftOut *= fftOut
    for (int i = 0; i < maxDist; i++)
        fftOut[i] = fftOut[i] * fftOut[i];

    // transformada inversa da multiplicação termo a termo.
    FFT(fftOut, distances, maxDist, -1);

    cin >> N;
    int total = 0;
    for (int i = 0; i < N; i++)
    {
        cin >> d;

        // Entra a distancia d
        // e verifica se a parte real da distância[d] é positiva
        // distância[d] guarda de quantas maneiras conseguimos atingir D
        if (distances[d].real() > 0.01)
            total++;
    }

    cout << total << endl;
}
return 0;
}

```

---

## 2.6 Matrizes

```

#include <bits/stdc++.h>
using namespace std;

#define ll long long

typedef vector<ll> vl;
typedef vector<vl> vvl;
const int mod = 1000000;

// Retorna a matriz I_n
vvl matrixUnit(int n) {
    vvl res(n, vl(n));
    for (int i = 0; i < n; i++)
        res[i][i] = 1;
    return res;
}

// Retorna a+b
vvl matrixAdd(const vvl &a, const vvl &b) {
    int n = a.size();
    int m = a[0].size();
    vvl res(n, vl(m));
    for (int i = 0; i < n; i++)
        for (int j = 0; j < m; j++)
            res[i][j] = (a[i][j] + b[i][j]) % mod;
    return res;
}

// Retorna a*b
vvl matrixMul(const vvl &a, const vvl &b) {
    int n = a.size();
    int m = a[0].size();
    int k = b[0].size();
    vvl res(n, vl(k));
    for (int i = 0; i < n; i++)

```

---

```

        for (int j = 0; j < k; j++)
            for (int p = 0; p < m; p++)
                res[i][j] = (res[i][j] + ((a[i][p] % mod) * (b[p][j] % mod) % mod)) % mod;
    return res;
}

// Retorna a matriz a^p
vvl matrixPow(const vvl &a, long long p) {
    if (p == 0)
        return matrixUnit(a.size());
    if (p & 1)
        return matrixMul(a, matrixPow(a, p - 1));
    return matrixPow(matrixMul(a, a), p / 2);
}

// Retorna sum^p_i=0 (a^i)
vvl matrixPowSum(const vvl &a, long long p) {
    long long n = a.size();
    if (p == 0)
        return vvl(n, vl(n));
    if (p % 2 == 0)
        return matrixMul(matrixPowSum(a, p / 2), matrixAdd(matrixUnit(n), matrixPow(a, p / 2)));
    return matrixAdd(a, matrixMul(matrixPowSum(a, p - 1), a));
}

int main() {
    long long n, l, k, i;
    while(scanf("%lld %lld %lld", &n, &l, &k) > 0) {
        vvl matriz = vvl(2, vl(2));

        matriz[0][0] = l;
        matriz[0][1] = k;
        matriz[1][0] = 1;
        matriz[1][1] = 0;

        matriz = matrixPow(matriz, n / 5);
        printf("%06lld\n", matriz[0][0]);
    }
}

```

---

## 2.7 Fatoração em números primos

```

vector<int> primeFactors(int n)
{
    vector<int> v;
    int sqrtn = sqrt(n);
    while (n % 2 == 0)
    {
        v.push_back(2);
        n = n / 2;
    }
    for (int i = 3; i <= sqrtn; i = i + 2)
    {
        while (n % i == 0)
        {
            v.push_back(i);
            n = n / i;
        }
    }
    if (n > 2)
        v.push_back(n);
    return v;
}

```

---

## 2.8 Modpow

```

int modPow(int a, int b, int m)
{
    int res = 1;
    for (; b > 0; b >>= 1)
    {
        if (b & 1)
            res = (long long)res * a % m;
    }
}

```

---

```

        a = (long long)a * a % m;
    }
    return res;
}

```

---

## 2.9 Máximo e mínimo de funções

```

double gss(double a, double b, double (*f)(double), double e = 1e-6)
{
    double r = (sqrt(5) - 1) / 2; //=.618...=golden ratio-1
    double x1 = b - r * (b - a), x2 = a + r * (b - a);
    double f1 = f(x1), f2 = f(x2);
    while (b - a > e)
    {
        if (f1 < f2)
        { //change to > to find maximum
            b = x2;
            x2 = x1;
            f2 = f1;
            x1 = b - r * (b - a);
            f1 = f(x1);
        }
        else
        {
            a = x1;
            x1 = x2;
            f1 = f2;
            x2 = a + r * (b - a);
            f2 = f(x2);
        }
    }
    return (b + a) / 2;
}

```

---

## 2.10 Todos divisores de um número

```

vector<int> divisores(int n)
{
    vector<int> div;
    sqrtn = sqrt(n);
    while(sqrtn * sqrtn < n)
        sqrtn++;
    for (i = 1; i < sqrtn; i++)
        if (!(n % i))
            div.push_back(i), div.push_back(n / i);
    if(sqrtn * sqrtn == n)
        div.push_back(sqrtn);
    return div;
}

```

---

## 2.11 Crivo de Eratóstenes segmentado

```

char nprimo[100001] = {0}; // tamanho = sqrt(maximo)
std::vector<int> primos;

int main()
{
    int n, a, b, i, j;
    nprimo[1] = 1;
    nprimo[0] = 1;
    for (i = 2; i < 320; i++) // i [2, sqrt(sqrt(maximo))]
        if (!nprimo[i])
            for (j = i * i; j < 100001; j += i) // j [i^2, sqrt(maximo)]
                nprimo[j] = 1;
    for (i = 2; i < 100001; i++)
        if (!nprimo[i])
            primos.push_back(i);
    scanf("%d", &n);
}

```

---

```

while (n--)
{
    scanf("%d %d", &a, &b);
    if (a > 100000 && b > 100000)
    { // (a > sqrt(N) && b > sqrt(N))
        for (i = a; i <= b; i++)
        {
            for (j = 0; j < primos.size(); j++)
                if (i % primos[j] == 0)
                    goto ab;

            printf("%d\n", i);
            ab;;
        }
    }
    else if (a < 100001 && b < 100001)
    { // (a < sqrt(N) && b < sqrt(N))
        for (i = a; i <= b; i++)
            if (!nprimo[i])
                printf("%d\n", i);
    }
    else
    {
        for (i = 0; i < primos.size(); i++)
            if (primos[i] >= a)
                break;

        for (; i < primos.size(); i++)
            printf("%d\n", primos[i]);

        for (; i <= b; i++)
        {
            for (j = 0; j < primos.size(); j++)
                if (i % primos[j] == 0)
                    goto ac;

            printf("%d\n", i);
            ac;;
        }
    }
}
}

```

---



---

## 3 Grafos

### 3.1 Grafos

```
#include <bits/stdc++.h>
using namespace std;

#define ll long long
#define pb push_back

typedef vector<int> vi;

struct Vertice
{
    int id, pai;
    ll dist;

    Vertice(int id, ll dist = 1, int pai = -1) : id(id), dist(dist), pai(pai) {}

    bool operator<(Vertice a) const
    {
        return a.dist < dist;
    }
};

typedef vector<Vertice> vv;
typedef vector<vv> vvv;

struct Grafo
{
    vvv g;
    vi pais;
    int n;

    Grafo(int n) : n(n)
    {
        g = vvv(n, vv());
        pais = vi(n);
    }

    void operator=(Grafo const &a)
    {
        g = a.g;
        pais = a.pais;
        n = a.n;
    }

    void addAresta(int a, int b, ll d = 0)
    {
        g[a].pb(Vertice(b, d));
    }

    void removeAresta(int a, int b)
    {
        g[a].erase(remove_if(g[a].begin(), g[a].end(), [b](Vertice v) { return v.id == b; }));
    }

    ll valAresta(int a, int b)
    {
        for (auto it : g[a])
            if (it.id == b)
                return it.dist;

        return 0;
    }

    void modificaAresta(int a, int b, ll dif)
    {
        for (auto &it : g[a])
            if (it.id == b)
            {
                it.dist += dif;
                break;
            }

        g[a].erase(remove_if(g[a].begin(), g[a].end(), [b](Vertice v) { return v.dist == 0; }));
    }

    ll dijkstra(int s, int d)
    {
        priority_queue<Vertice> fila;
        bool visitados[n];
        fill(visitados, visitados+n, 0);
        fill(pais.begin(), pais.end(), -1);
```

```

    fila.push(Vertice(s, 0));
    auto top = fila.top();
    while (top.id != d)
    {
        if (!visitados[top.id])
        {
            for (auto &it : g[top.id])
                if (!visitados[it.id])
                    fila.push(Vertice(it.id, it.dist + top.dist, top.id));

            visitados[top.id] = 1;
            pais[top.id] = top.pai;
        }
        fila.pop();
        if (fila.empty())
            return -1;
        top = fila.top();
    }
    pais[top.id] = top.pai;
    return top.dist;
}

ll busca(int s, int d)
{
    queue<Vertice> fila;
    bool visitados[n];
    fill(visitados, visitados+n, 0);
    fill(pais.begin(), pais.end(), -1);
    fila.push(Vertice(s, 0));
    auto top = fila.front();
    while (top.id != d)
    {
        if (!visitados[top.id])
        {
            for (auto &it : g[top.id])
                if (!visitados[it.id])
                    fila.push(Vertice(it.id, it.dist + 1, top.id));

            visitados[top.id] = 1;
            pais[top.id] = top.pai;
        }
        fila.pop();
        if (fila.empty())
            return -1;
        top = fila.front();
    }
    pais[top.id] = top.pai;
    return top.dist;
}

ll fluxo_maximo(int s, int d)
{
    int u, v;
    ll flow = 0;
    Grafo g2 = *this;
    while (g2.busca(s, d) >= 0)
    {
        ll path = 1ll << 50;
        for (v = d; v != s; v = u)
        {
            u = g2.pais[v];
            path = min(path, valAresta(u, v));
        }
        for (v = d; v != s; v = u)
        {
            u = g2.pais[v];
            g2.modificaAresta(u, v, -path);
        }
    }
}

```

```

        g2.modificaAresta(v, u, path);
    }
    flow += path;
}
return flow;
}
};

int main()
{
    Grafo g(20);

    g.addAresta(1, 2, 1);
    g.addAresta(1, 3, 5);
    g.addAresta(2, 1, 6);
    g.addAresta(3, 2, 10);

    g.removeAresta(1, 2);

    for (auto it : g.g[1])
        cout << it.id << endl; // 3

    cout << g.dijkstra(1, 2) << endl; // 15

    cout << g.fluxo_maximo(1, 2) << endl; // 5
}

```

## 3.2 Todas as pontes de um grafo

```

#include <bits/stdc++.h>
#define NIL -1
using namespace std;

// A class that represents an undirected graph
class Graph
{
    int V; // No. of vertices
    list<int> *adj; // A dynamic array of adjacency lists
    void bridgeUtil(int v, bool visited[], int disc[], int low[],
                    int parent[]);

public:
    Graph(int V); // Constructor
    void addEdge(int v, int w); // to add an edge to graph
    void bridge(); // prints all bridges
};

Graph::Graph(int V)
{
    this->V = V;
    adj = new list<int>[V];
}

void Graph::addEdge(int v, int w)
{
    adj[v].push_back(w);
    adj[w].push_back(v); // Note: the graph is undirected
}

// A recursive function that finds and prints bridges using
// DFS traversal
// u --> The vertex to be visited next
// visited[] --> keeps track of visited vertices
// disc[] --> Stores discovery times of visited vertices
// parent[] --> Stores parent vertices in DFS tree
void Graph::bridgeUtil(int u, bool visited[], int disc[],
                        int low[], int parent[])
{
    // A static variable is used for simplicity, we can
    // avoid use of static variable by passing a pointer.
    static int time = 0;

    // Mark the current node as visited
    visited[u] = true;

    // Initialize discovery time and low value
    disc[u] = low[u] = ++time;

    // Go through all vertices adjacent to this
    list<int>::iterator i;
    for (i = adj[u].begin(); i != adj[u].end(); ++i)

```

```

{
    int v = *i; // v is current adjacent of u
    // If v is not visited yet, then recur for it
    if (!visited[v])
    {
        parent[v] = u;
        bridgeUtil(v, visited, disc, low, parent);

        // Check if the subtree rooted with v has a
        // connection to one of the ancestors of u
        low[u] = min(low[u], low[v]);

        // If the lowest vertex reachable from subtree
        // under v is below u in DFS tree, then u-v
        // is a bridge
        if (low[v] > disc[u])
            cout << u << " " << v << endl;
    }

    // Update low value of u for parent function calls.
    else if (v != parent[u])
        low[u] = min(low[u], disc[v]);
}
}

// DFS based function to find all bridges. It uses recursive
// function bridgeUtil()
void Graph::bridge()
{
    // Mark all the vertices as not visited
    bool *visited = new bool[V];
    int *disc = new int[V];
    int *low = new int[V];
    int *parent = new int[V];

    // Initialize parent and visited arrays
    for (int i = 0; i < V; i++)
    {
        parent[i] = NIL;
        visited[i] = false;
    }

    // Call the recursive helper function to find Bridges
    // in DFS tree rooted with vertex 'i'
    for (int i = 0; i < V; i++)
        if (visited[i] == false)
            bridgeUtil(i, visited, disc, low, parent);
}

// Driver program to test above function
int main()
{
    // Create graphs given in above diagrams
    cout << "\nBridges in first graph \n";
    Graph g1(5);
    g1.addEdge(1, 0);
    g1.addEdge(0, 2);
    g1.addEdge(2, 1);
    g1.addEdge(0, 3);
    g1.addEdge(3, 4);
    g1.bridge();

    cout << "\nBridges in second graph \n";
    Graph g2(4);
    g2.addEdge(0, 1);
    g2.addEdge(1, 2);
    g2.addEdge(2, 3);
    g2.bridge();

    cout << "\nBridges in third graph \n";
    Graph g3(7);
    g3.addEdge(0, 1);
    g3.addEdge(1, 2);
    g3.addEdge(2, 0);
    g3.addEdge(1, 3);
    g3.addEdge(1, 4);
    g3.addEdge(1, 6);
    g3.addEdge(3, 5);
    g3.addEdge(4, 5);
    g3.bridge();

    return 0;
}

```

---

```
}
```

### 3.3 Matching máximo em grafo bipartido

```
const int MAXN1 = 50000, MAXN2 = 50000, MAXM = 150000;
int n1, n2, edges, last[MAXN1], prev[MAXM], head[MAXM], matching[MAXN2], dist[MAXN1], Q[MAXN1], used[MAXN1], vis
[MAXN1];
void init(int _n1, int _n2)
{
    n1 = _n1;
    n2 = _n2;
    edges = 0;
    fill(last, last + n1, -1);
}
void addAresta(int u, int v)
{
    head[edges] = v;
    prev[edges] = last[u];
    last[u] = edges++;
}
void bfs()
{
    fill(dist, dist + n1, -1);
    int sizeQ = 0;
    for (int u = 0; u < n1; ++u)
    {
        if (!used[u])
        {
            Q[sizeQ++] = u;
            dist[u] = 0;
        }
    }
    for (int i = 0; i < sizeQ; i++)
    {
        int u1 = Q[i];
        for (int e = last[u1]; e >= 0; e = prev[e])
        {
            int u2 = matching[head[e]];
            if (u2 >= 0 && dist[u2] < 0)
            {
                dist[u2] = dist[u1] + 1;
                Q[sizeQ++] = u2;
            }
        }
    }
}
bool dfs(int u1)
{
    vis[u1] = true;
    for (int e = last[u1]; e >= 0; e = prev[e])
    {
        int v = head[e];
        int u2 = matching[v];
        if (u2 < 0 || !vis[u2] && dist[u2] == dist[u1] + 1 && dfs(u2))
        {
            matching[v] = u1;
            used[u1] = true;
            return true;
        }
    }
    return false;
}
int maxMatching()
{
    fill(used, used + n1, false);
    fill(matching, matching + n2, -1);
    for (int res = 0;;)
    {
        bfs();
        fill(vis, vis + n1, false);
        int f = 0;
        for (int u = 0; u < n1; ++u)
            if (!used[u] && dfs(u))
                ++f;
        if (!f)
            return res;
    }
}
```

---

```

    }
    res += f;
}

```

---

### 3.4 Algoritmo húngaro

```

#define N 100
#define INF 1000000000

int cost[N][N], n, max_match, lx[N], ly[N], xy[N], yx[N], slack[N], slackx[N], prev[N];
bool S[N], T[N];

void init_labels()
{
    memset(lx, 0, sizeof(lx));
    memset(ly, 0, sizeof(ly));
    for (int x = 0; x < n; x++)
        for (int y = 0; y < n; y++)
            lx[x] = max(lx[x], cost[x][y]);
}

void update_labels()
{
    int x, y, delta = INF;
    for (y = 0; y < n; y++)
        if (!T[y])
            delta = min(delta, slack[y]);
    for (x = 0; x < n; x++)
        if (S[x])
            lx[x] -= delta;
    for (y = 0; y < n; y++)
        if (T[y])
            ly[y] += delta;
    for (y = 0; y < n; y++)
        if (!T[y])
            slack[y] -= delta;
}

void add_to_tree(int x, int prevx)
{
    S[x] = true;
    prev[x] = prevx;
    for (int y = 0; y < n; y++)
        if (lx[x] + ly[y] - cost[x][y] < slack[y])
        {
            slack[y] = lx[x] + ly[y] - cost[x][y];
            slackx[y] = x;
        }
}

void augment()
{
    if (max_match == n)
        return;
    int x, y, root, q[N], wr = 0, rd = 0;
    memset(S, false, sizeof(S));
    memset(T, false, sizeof(T));
    memset(prev, -1, sizeof(prev));

    for (x = 0; x < n; x++)
        if (xy[x] == -1)
        {
            q[wr++] = root = x;
            prev[x] = -2;
            S[x] = true;
            break;
        }

    for (y = 0; y < n; y++)
    {
        slack[y] = lx[root] + ly[y] - cost[root][y];
        slackx[y] = root;
    }

    while (true)
    {
        while (rd < wr)
        {
            x = q[rd++];

```

---

```

    for (y = 0; y < n; y++)
        if (cost[x][y] == lx[x] + ly[y] && !T[y])
        {
            if (yx[y] == -1)
                break;
            T[y] = true;
            q[wr++] = yx[y];
            add_to_tree(yx[y], x);
        }
    if (y < n)
        break;
}
if (y < n)
    break;
update_labels();
wr = rd = 0;
for (y = 0; y < n; y++)
    if (!T[y] && slack[y] == 0)
    {
        if (yx[y] == -1)
        {
            x = slackx[y];
            break;
        }
        else
        {
            T[y] = true;
            if (!S[yx[y]])
            {
                q[wr++] = yx[y];
                add_to_tree(yx[y], slackx[y]);
            }
        }
    }
    if (y < n)
        break;
}
if (y < n)
{
    max_match++;
    for (int cx = x, cy = y, ty; cx != -2; cx = prev[cx], cy = ty)
    {
        ty = xy[cx];
        yx[cy] = cx;
        xy[cx] = cy;
    }
    augment();
}
}

int hungaro()
{
    int ret = 0;
    max_match = 0;
    memset(xy, -1, sizeof(xy));
    memset(yx, -1, sizeof(yx));
    init_labels();
    augment();

    for (int x = 0; x < n; x++)
        ret += cost[x][xy[x]];

    return ret;
}

```

---

## 4 Strings

### 4.1 Suffix array

```
//Usage:
// Fill txt with the characters of the txtng.
// Call SuffixSort(n), where n is the length of the txtng stored in txt.
// That's it!

//Output:
// SA = The suffix array.
// Contains the n suffixes of txt sorted in lexicographical order.
// Each suffix is represented as a single integer (the SAition of txt where it starts).
// iSA = The inverse of the suffix array. iSA[i] = the index of the suffix txt[i..n)
// in the SA array. (In other words, SA[i] = k <=> iSA[k] = i)
// With this array, you can compare two suffixes in O(1): Suffix txt[i..n) is smaller
// than txt[j..n) if and only if iSA[i] < iSA[j]

const int MAX = 100010;
char txt[MAX]; //input
int iSA[MAX], SA[MAX]; //output
int cnt[MAX], next[MAX]; //internal
bool bh[MAX], b2h[MAX];

// Compares two suffixes according to their first characters
bool smaller_first_char(int a, int b)
{
    return txt[a] < txt[b];
}

void suffixSort(int n)
{
    for (int i = 0; i < n; ++i)
        SA[i] = i;
    sort(SA, SA + n, smaller_first_char);
    for (int i = 0; i < n; ++i)
    {
        bh[i] = i == 0 || txt[SA[i]] != txt[SA[i - 1]];
        b2h[i] = false;
    }
    for (int h = 1; h < n; h <= 1)
    {
        int buckets = 0;
        for (int i = 0, j; i < n; i = j)
        {
            j = i + 1;
            while (j < n && !bh[j])
                j++;
            next[i] = j;
            buckets++;
        }
        if (buckets == n)
            break;
        for (int i = 0; i < n; i = next[i])
        {
            cnt[i] = 0;
            for (int j = i; j < next[i]; ++j)
                iSA[SA[j]] = i;
        }
        cnt[iSA[n - h]]++;
        b2h[iSA[n - h]] = true;
        for (int i = 0; i < n; i = next[i])
        {
            for (int j = i; j < next[i]; ++j)
            {
                int s = SA[j] - h;
                if (s >= 0)
                {
                    int head = iSA[s];
                    iSA[s] = head + cnt[head]++;
                    b2h[iSA[s]] = true;
                }
            }
        }
    }
}
```



---

```

    }
    for (int j = i; j < next[i]; ++j)
    {
        int s = SA[j] - h;
        if (s >= 0 && b2h[iSA[s]])
            for (int k = iSA[s] + 1; !bh[k] && b2h[k]; k++)
                b2h[k] = false;
    }
    for (int i = 0; i < n; ++i)
    {
        SA[iSA[i]] = i;
        bh[i] |= b2h[i];
    }
    for (int i = 0; i < n; ++i)
    {
        iSA[SA[i]] = i;
    }
}
// End of suffix array algorithm

// Begin of the O(n) longest common prefix algorithm
int lcp[MAX];
// lcp[i] = length of the longest common prefix of suffix SA[i] and suffix SA[i-1]
// lcp[0] = 0
void getlcp(int n)
{
    for (int i = 0; i < n; ++i)
        iSA[SA[i]] = i;
    lcp[0] = 0;
    for (int i = 0, h = 0; i < n; ++i)
    {
        if (iSA[i] > 0)
        {
            int j = SA[iSA[i] - 1];
            while (i + h < n && j + h < n && txt[i + h] == txt[j + h])
                h++;
            lcp[iSA[i]] = h;
            if (h > 0)
                h--;
        }
    }
}
}

```

---

## 5 Geometria

### 5.1 Linha de eventos radial

```
// - Radial sweep in Q2 quadrant in nlogn.
// - Sorts events using cross product to avoid dealing with
//   numeric problems.
#include <bits/stdc++.h>
using namespace std;

struct Point {
    Point(int x = 0, int y = 0) : x(x), y(y) {}

    bool operator<(const Point& o) const {
        // Order points in a quadrant by angle with origin:
        // Uses anti-clockwise order by returning true when the
        // cross product between the points is positive.
        return (x*o.y - y*o.x) > 0;
    }

    /*
    bool operator<=(const Point& o) const {
        return (x*o.y - y*o.x) >= 0;
    }
    */
    int x, y;
};

pair<int, int> solve(const vector<Point>& points) {
    map<Point, pair<int, int> > events;

    Point begin(0, 1);
    Point end(-1, 0);

    // Add events on the borders to guarantee that we consider them.
    events[begin];
    events[end];

    int superior = 0; // Number of points in Q1 quadrant.
    int same = 0; // Number of points in origin.
    int active = 0; // Number of current points in Q2 and Q4 quadrant better
                    // than origin.

    int best_pos = points.size();
    int worst_pos = 0;

    for (const auto& p : points) {
        if (p.x < 0 && p.y < 0) {}
        else if (p.x > 0 && p.y > 0) superior++;
        else if (p.x == 0 && p.y == 0) same++;
        else if (p.x <= 0 && p.y >= 0) {
            // assert(begin <= Point(p.x, p.y));
            // assert(Point(p.x, p.y) <= end);
            events[Point(p.x, p.y)].first++;
        }
        else if (p.x >= 0 && p.y <= 0) {
            // assert(begin <= Point(-p.x, -p.y));
            // assert(Point(-p.x, -p.y) <= end);
            active++;
            events[Point(-p.x, -p.y)].second++;
        }
        else assert(false);
    }

    for (const auto& e : events) {
        int tie_best_pos = superior + active - e.second.second;
        int tie_worst_pos = superior + active + e.second.first + same;
        active += e.second.first - e.second.second;

        best_pos = min(best_pos, tie_best_pos);
        worst_pos = max(worst_pos, tie_worst_pos);
    }

    return make_pair(best_pos + 1, worst_pos + 1);
}

// Reads the set of points and centers them around Maria's product.
vector<Point> read() {
    int n, cx, cy;
    cin >> n >> cx >> cy;
    vector<Point> points(n - 1);
    for (Point& p : points) {
```

---

```

        cin >> p.x >> p.y;
        p.x -= cx;
        p.y -= cy;
    }
    return points;
}

int main() {
    auto input = read();
    auto solution = solve(input);

    for (auto& i : input)
        swap(i.x, i.y);

    assert(solution == solve(input));
    cout << solution.first << " " << solution.second << endl;
    return 0;
}

```

---

## 5.2 KD-Tree para pares mais próximos em $O(\log(n))$

```

typedef pair<int, int> pii;
typedef vector<pii> vpii;

const int maxn = 100000;
int tx[maxn];
int ty[maxn];
bool divX[maxn];

bool cmpX(const pii &a, const pii &b)
{
    return a.first < b.first;
}

bool cmpY(const pii &a, const pii &b)
{
    return a.second < b.second;
}

void buildTree(int left, int right, pii points[])
{
    if (left >= right)
        return;
    int mid = (left + right) >> 1;

    //sort(points + left, points + right + 1, divX ? cmpX : cmpY);
    int minx = INT_MAX;
    int maxx = INT_MIN;
    int miny = INT_MAX;
    int maxy = INT_MIN;
    for (int i = left; i < right; i++)
    {
        checkmin(minx, points[i].first);
        checkmax(maxx, points[i].first);
        checkmin(miny, points[i].second);
        checkmax(maxy, points[i].second);
    }
    divX[mid] = (maxx - minx) >= (maxy - miny);
    nth_element(points + left, points + mid, points + right, divX[mid] ? cmpX : cmpY);

    tx[mid] = points[mid].first;
    ty[mid] = points[mid].second;

    if (left + 1 == right)
        return;
    buildTree(left, mid, points);
    buildTree(mid + 1, right, points);
}

long long closestDist;
int closestNode;

void findNearestNeighbour(int left, int right, int x, int y)
{
    if (left >= right)
        return;
    int mid = (left + right) >> 1;
    int dx = x - tx[mid];
    int dy = y - ty[mid];
    long long d = dx * (long long)dx + dy * (long long)dy;
    if (closestDist > d && d)

```

---

```

    {
        closestDist = d;
        closestNode = mid;
    }
    if (left + 1 == right)
        return;

    int delta = divX[mid] ? dx : dy;
    long long delta2 = delta * (long long)delta;
    int l1 = left;
    int r1 = mid;
    int l2 = mid + 1;
    int r2 = right;
    if (delta > 0)
        swap(l1, l2), swap(r1, r2);

    findNearestNeighbour(l1, r1, x, y);
    if (delta2 < closestDist)
        findNearestNeighbour(l2, r2, x, y);
}

int findNearestNeighbour(int n, int x, int y)
{
    closestDist = LLONG_MAX;
    findNearestNeighbour(0, n, x, y);
    return closestNode;
}

int main()
{
    vpii p;
    p.push_back(make_pair(0, 2));
    p.push_back(make_pair(0, 3));
    p.push_back(make_pair(-1, 0));

    p.resize(unique(p.begin(), p.end()) - p.begin());

    int n = p.size();
    buildTree(1, 0, n - 1, &(vpil(p)[0]));
    int res = findNearestNeighbour(n, 0, 0);

    cout << p[res].first << " " << p[res].second << endl;

    return 0;
}

```

---

### 5.3 Geometria (reduzido)

```

typedef pair<double, double> Ponto;
bool cw(Ponto a, Ponto b, Ponto c)
{
    return (b.first - a.first) * (c.second - a.second) - (b.second - a.second) * (c.first - a.first) < 0;
}

// Retorna o casco convexo do conjunto de pontos p
vector<Ponto> convexHull(vector<Ponto> p)
{
    int n = p.size();
    if (n <= 1)
        return p;
    int k = 0;
    sort(p.begin(), p.end());
    vector<Ponto> q(n * 2);
    for (int i = 0; i < n; q[k++] = p[i++])
        for (; k >= 2 && !cw(q[k - 2], q[k - 1], p[i]); --k)
            ;
    for (int i = n - 2, t = k; i >= 0; q[k++] = p[i--])
        for (; k > t && !cw(q[k - 2], q[k - 1], p[i]); --k)
            ;
    q.resize(k - 1 - (q[0] == q[1]));
    return q;
}

//O dobro da área definida pelo triangulo de pontos pontos a, b e c (sem sinal).
double uArea2(Ponto a, Ponto b, Ponto c)
{
    return abs((b.first - a.first) * (c.second - a.second) - (b.second - a.second) * (c.first - a.first));
}

//O dobro da área definida pelo triangulo de pontos pontos a, b e c (com sinal).
double area2(Ponto a, Ponto b, Ponto c)

```

```

{
    return (b.first - a.first) * (c.second - a.second) - (b.second - a.second) * (c.first - a.first);
}

//Distância entre os pontos a e b
double dist(Ponto a, Ponto b)
{
    return hypot(a.first - b.first, a.second - b.second);
}

//Interseção de semi-retas (p1 -> p2), (p3 -> p4)
bool segIntercept(Ponto p1, Ponto p2, Ponto p3, Ponto p4)
{
    return cw(p1, p2, p3) != cw(p1, p2, p4) && cw(p3, p4, p1) != cw(p3, p4, p2);
}

//Retorna a área do polígono p
double polygonArea(vector<Ponto> p)
{
    double s = 0.0;
    for (int i = 0; i < p.size(); i++)
        s += area2(Ponto(0, 0), p[i], p[(i + 1) % p.size()]);
    return fabs(s / 2.0);
}

//Retorna a área do polígono p definido pelos pontos p[i, f]
double polygonArea2(vector<Ponto> p, int i, int f)
{
    double s = 0.0;
    Ponto primeiro = p[i];
    for (; i != f; i++)
        s += area2(Ponto(0, 0), p[i], p[(i + 1) % p.size()]);
    s += area2(Ponto(0, 0), p[i], primeiro);
    return fabs(s / 2.0);
}

//Retorna a menor largura do conjunto de pontos p
double raio(vector<Ponto> p)
{
    vector<Ponto> h = convexHull(p);
    int m = h.size();
    if (m == 1)
        return 0;
    if (m == 2)
        return 0;
    int k = 1;
    while (uArea2(h[m - 1], h[0], h[(k + 1) % m]) > uArea2(h[m - 1], h[0], h[k]))
        ++k;
    double res = 100000000;
    for (int i = 0, j = k; i <= k && j < m; i++)
    {
        res = min(res, dist(h[i], h[j]));
        while (j < m && uArea2(h[i], h[(i + 1) % m], h[(j + 1) % m]) > uArea2(h[i], h[(i + 1) % m], h[j]))
        {
            res = min(res, dist(h[i], h[(j + 1) % m]));
            ++j;
        }
    }
    return res;
}

//Retorna a maior largura do conjunto de pontos p
double diametro(vector<Ponto> p)
{
    vector<Ponto> h = convexHull(p);
    int m = h.size();
    if (m == 1)
        return 0;
    if (m == 2)
        return dist(h[0], h[1]);
    int k = 1;
    while (uArea2(h[m - 1], h[0], h[(k + 1) % m]) > uArea2(h[m - 1], h[0], h[k]))
        ++k;
    double res = 0;
    for (int i = 0, j = k; i <= k && j < m; i++)
    {
        res = max(res, dist(h[i], h[j]));
        while (j < m && uArea2(h[i], h[(i + 1) % m], h[(j + 1) % m]) > uArea2(h[i], h[(i + 1) % m], h[j]))
        {
            res = max(res, dist(h[i], h[(j + 1) % m]));
            ++j;
        }
    }
}

```

```

    }
    return res;
}

```

## 5.4 Geometria (grande)

```

#include <bits/stdc++.h>
using namespace std;
const double EPS = 1e-10;
inline int cmp( double x, double y = 0, double tol = EPS ) {
    return (x <= y + tol) ? (x + tol < y) ? -1 : 0 : 1;
}
struct Point {
    double x, y;
    Point( double x = 0, double y = 0 ) : x( x ), y( y ) {}
    Point operator+( Point q ) const {
        return Point( x + q.x, y + q.y );
    }
    Point operator-( Point q ) const {
        return Point( x - q.x, y - q.y );
    }
    Point operator*( double t ) const {
        return Point( x * t, y * t );
    }
    Point operator/( double t ) const {
        return Point( x / t, y / t );
    }
    double operator*( Point q )const {
        return x * q.x + y * q.y;
    }
    double operator^( Point q ) const {
        return x * q.y - y * q.x;
    }
    int cmp( Point q ) const {
        if ( int t = ::cmp( x, q.x ) )
            return t;
        return ::cmp( y, q.y );
    }
    bool operator==( Point q ) const {
        return cmp( q ) == 0;
    }
    bool operator!=( Point q ) const {
        return cmp( q ) != 0;
    }
    bool operator<( Point q ) const {
        return cmp( q ) < 0;
    }
    static Point pivot;
};
Point Point::pivot;
typedef vector<Point> Polygon;
inline double abs( Point& p ) {
    return hypot( p.x, p.y );
}
inline double arg( Point& p ) {
    return atan2( p.y, p.x );
}
//Verifica o sinal do produto vetorial entre os vetores (p-r) e (q - r)
inline int ccw( Point& p, Point& q, Point& r ) {
    return cmp( ( p - r ) ^ ( q - r ) );
}
//calcula o angulo orientado entre os vetores (p-q) e (r - q)
inline double angle( Point& p, Point &q, Point& r ) {
    Point u = p - q, w = r - q;

```

```

    return atan2( u ^ w, u * w );
}

//Decide se o ponto p esta sobre a reta que passa por p1p2.
bool pontoSobreReta( Point& p1, Point &p, Point& p2 ) {
    return ccw( p1, p2, p ) == 0;
}

//Decide de p esta sobre o segmento p1p2
bool between( Point& p1, Point &p, Point& p2 ) {
    return ccw( p1, p2, p ) == 0 && cmp( ( p1 - p ) * ( p2 - p ) ) <= 0;
}

//Calcula a distancia do ponto p a reta que passa por p1p2
double retaDistance( Point& p1, Point& p2, Point &p ) {
    Point A = p1 - p, B = p2 - p1;
    return fabs( A ^ B ) / sqrt( B * B );
}

//Calcula a distancia do ponto p ao segmento de reta que passa por p1p2
double segDistance( Point& p1, Point& p2, Point &p ) {
    Point A = p1 - p, B = p1 - p2, C = p2 - p;
    double a = A * A, b = B * B, c = C * C;
    if ( cmp( a, b + c ) >= 0 ) return sqrt( c );
    if ( cmp( c, a + b ) >= 0 ) return sqrt( a );
    return fabs( A ^ C ) / sqrt( b );
}

//Calcula a area orientada do poligono T.
double polygonArea( Polygon& T ) {
    double s = 0.0;
    int n = T.size( );
    for ( int i = 0; i < n; i++ )
    {
        s += T[i] ^ T[( i + 1 ) % n];
    }
    return s / 2.0; //Retorna a area com sinal
}

//Classifica o ponto p em relacao ao poligono T dependendo se ele está
//na fronteira (-1) no exterior (0) ou no interior (1).
int inpoly( Point& p, Polygon& T ) {
    // -1 sobre, 0 fora, 1 dentro
    double a = 0.0;
    int n = T.size( );
    for ( int i = 0; i < n; i++ )
    {
        if ( between( T[i], p, T[( i + 1 ) % n] ) ) return -1;
        a += angle( T[i], p, T[( i + 1 ) % n] );
    }
    return cmp( a ) != 0;
}

//Ordenacao radial.
bool radialSort( Point p, Point q ) {
    Point P = p - Point::pivot, Q = q - Point::pivot;
    double R = P ^ Q;
    if ( cmp( R ) ) return R > 0;
    return cmp( P * P, Q * Q ) < 0;
}

//Determina o convex hull de T. ATENCAO. A lista de pontos T e destruida.
Polygon convexHull( vector<Point>& T ) {
    int j = 0, k, n = T.size( );
    Polygon U( n );
    Point::pivot = *min_element( T.begin( ), T.end( ) );
    sort( T.begin( ), T.end( ), radialSort );

    for ( k = n - 2; k >= 0 && ccw( T[0], T[n - 1], T[k] ) == 0; k-- );
    reverse( ( k + 1 ) + T.begin( ), T.end( ) );

    for ( int i = 0; i < n; i++ )
    {
        // troque o >= por > para manter pontos colineares
        while ( j > 1 && ccw( U[j - 1], U[j - 2], T[i] ) >= 0 ) j--;
        U[j++] = T[i];
    }
    U.resize( j );
    return U;
}

//Interseção de semi-retas (p1 -> p2), (p3 -> p4)
bool segIntercept( Point p1, Point p2, Point p3, Point p4 ) {
    return ccw(p1, p2, p3) != ccw(p1, p2, p4) && ccw(p3, p4, p1) != ccw(p3, p4, p2);
}

```

---

---



## 6 Estruturas de dados etc

### 6.1 Wavelet-tree

```
#include <bits/stdc++.h>
using namespace std;
const int N = 10000;
struct KthSmallest
{
    struct Seg
    {
        int l, r, mid;
        void set(int _l, int _r)
        {
            l = _l;
            r = _r;
            mid = l + r >> 1;
        }
    } seg[N << 2];
    int b[25][N], left[25][N], sorted[N];
    void init(int *a, int n)
    {
        for (int i = 0; i < n; i++)
            b[0][i] = sorted[i] = a[i];
        sort(sorted, sorted + n);
        build(0, n, 0, 1);
    }
    void build(int l, int r, int d, int idx)
    {
        seg[idx].set(l, r);
        if (l + 1 == r)
            return;
        int mid = seg[idx].mid;
        int lsame = mid - 1;
        for (int i = l; i < r; i++)
            if (b[d][i] < sorted[mid])
                lsame--;
        int lpos = l, rpos = mid, same = 0;
        for (int i = l; i < r; ++i)
        {
            left[d][i] = (i != l ? left[d][i - 1] : 0);
            if (b[d][i] < sorted[mid])
            {
                left[d][i]++;
                b[d + 1][lpos++] = b[d][i];
            }
            else if (b[d][i] > sorted[mid])
                b[d + 1][rpos++] = b[d][i];
            else
            {
                if (same < lsame)
                {
                    same++;
                    left[d][i]++;
                    b[d + 1][lpos++] = b[d][i];
                }
                else
                {
                    b[d + 1][rpos++] = b[d][i];
                }
            }
        }
        build(l, mid, d + 1, idx << 1);
        build(mid, r, d + 1, idx << 1 | 1);
    }
}

//Quando ordenarmos [l, r), qual é o k-ésimo termo?
int kth(int l, int r, int k, int d = 0, int idx = 1)
{
    // k : 1-origin!!!
    if (l + 1 == r)
        return b[d][l];
}
```

```

int ltl = (l != seg[idx].l ? left[d][l - 1] : 0);
int tl = left[d][r - 1] - ltl;
if (tl >= k)
{
    int newl = seg[idx].l + ltl;
    int newr = seg[idx].l + ltl + tl;

    return kth(newl, newr, k, d + 1, idx << 1);
}
else
{
    int mid = seg[idx].mid;
    int tr = r - l - tl;
    int ltr = l - seg[idx].l - ltl;
    int newl = mid + ltr;
    int newr = mid + ltr + tr;

    return kth(newl, newr, k - tl, d + 1, idx << 1 | 1);
}
}

//When sorting [l, r), what number will x come in?
//If there are two or more x's, return the rank of the last one.
//If there is no x, return the rank of the largest but less than x.
//When there is no less than x, 0 is returned.
int rank(int l, int r, int x, int d = 0, int idx = 1)
{
    if (seg[idx].l + 1 == seg[idx].r)
        return l + 1 == r && sorted[l] <= x;

    int ltl = (l != seg[idx].l ? left[d][l - 1] : 0);
    int tl = left[d][r - 1] - ltl;
    int mid = seg[idx].mid;

    if (x < sorted[mid])
    {
        int newl = seg[idx].l + ltl;
        int newr = seg[idx].l + ltl + tl;

        return rank(newl, newr, x, d + 1, idx << 1);
    }
    else
    {
        int tr = r - l - tl;
        int ltr = l - seg[idx].l - ltl;
        int newl = mid + ltr;
        int newr = mid + ltr + tr;

        return tl + rank(newl, newr, x, d + 1, idx << 1 | 1);
    }
}

// Quantos x existem entre [l,r)
int freq(int l, int r, int x)
{
    return rank(l, r, x) - rank(l, r, x - 1);
}
} kth;

int main()
{
    int a[8] = {6, 12, 5, 17, 10, 2, 7, 3};
    kth.init(a, 8);

    cout << kth.kth(2, 7, 3) << endl; // 7
    cout << kth.rank(2, 7, 7) << endl; // 3
}

```

## 6.2 Seg-tree

```

#include <algorithm>
using namespace std;

#define MAX 1000000 // 0 valor aqui tem que ser >= 2 * tamanho do maior n
#define INF 1 << 28

// Não necessariamente é um int, pode ser uma segtree de struct etc;
int init[MAX], tree[MAX], lazy[MAX];

void build_tree(int node, int a, int b)

```

```

{
    if (a > b)
        return;
    // Se folha
    if (a == b)
    {
        tree[node] = init[a];
        lazy[node] = 0;
        return;
    }
    build_tree(node * 2, a, (a + b) / 2);
    build_tree(node * 2 + 1, 1 + (a + b) / 2, b);
    // Se nó
    tree[node] = tree[node * 2] + tree[node * 2 + 1];
    lazy[node] = 0;
}

void update_tree(int node, int a, int b, int i, int j, int value)
{
    // Se fora do intervalo - retorna
    if (a > b || a > j || b < i)
        return;
    if (lazy[node] != 0)
    {
        //Atualização atrasada.
        tree[node] += lazy[node];

        // Passa lazy para filhos
        if (a != b)
        {
            lazy[node * 2] += lazy[node];
            lazy[node * 2 + 1] += lazy[node];
        }

        //Reseta o nó
        lazy[node] = 0;
    }
    // Se o nó atual cobre todo o intervalo
    if (a >= i && b <= j)
    {
        tree[node] += value;
        if (a != b)
        {
            lazy[node * 2] += value;
            lazy[node * 2 + 1] += value;
        }
        return;
    }
    // Se tem um pedaco em cada filho.
    // Atualiza os filhos.
    update_tree(node * 2, a, (a + b) / 2, i, j, value);
    update_tree(1 + node * 2, 1 + (a + b) / 2, b, i, j, value);
    // Atualiza o pai.
    tree[node] = tree[node * 2] + tree[node * 2 + 1];
}

int query_tree(int node, int a, int b, int i, int j)
{
    // Se fora do intervalo
    if (a > b || a > j || b < i)
    {
        //Aqui deverá ser retornado o elemento neutro para a operação desejada
        return 0;
    }
    if (lazy[node] != 0)
    {
        //Atualização atrasada.
        tree[node] += lazy[node];

        //Se não folha, passa lazy pros filhos
        if (a != b)
        {
            lazy[node * 2] += lazy[node];
            lazy[node * 2 + 1] += lazy[node];
        }
    }
}

```

---

```

        //Reseta o nó
        lazy[node] = 0;
    }

    // Se o nó cobre o intervalo
    if (a >= i && b <= j)
        return tree[node];

    // Se o intervalo está um pedaco em cada filho.
    int q1 = query_tree(node * 2, a, (a + b) / 2, i, j);
    int q2 = query_tree(1 + node * 2, 1 + (a + b) / 2, b, i, j);

    // Retorna a combinação dos intervalos.
    return q1 + q2;
}

/*
Uso:
Assumindo que "n" é o numero de termos que o segmento tem
Inicialize "init" com os valores iniciais:
*   for(i = 0; i < n; scanf("%d", val), i++)
*   init[i] = val;
E mande construir a arvore:
*   build_tree(1, 0, n-1);
Para atualizar a arvore:
*   update_tree(1, 0, n-1, inicio, fim, val);
*   Onde inicio é a posição inicial do segmento desejado e fim é a posição final do mesmo
*   e val é o quanto você quer alterar os valores desse seguimento
Para fazer queries
*   query_tree(1, 0, n-1, inicio, fim);
*   Onde inicio é a posição inicial do segmento desejado e fim é a posição final do mesmo
*   o retorno terá o mesmo tipo que os dados guardados na arvore e será o resultado do segmento pesquisado
*/

```

---

## 6.3 Mergesort

```

typedef vector<int>::iterator vec_it;
void merge(vec_it left, vec_it left_end, vec_it right, vec_it right_end, vec_it numbers)
{
    while (left != left_end)
    {
        if (*left < *right || right == right_end)
        {
            *numbers = *left;
            ++left;
        }
        else
        {
            *numbers = *right;
            ++right;
        }
        ++numbers;
    }
    while (right != right_end)
    {
        *numbers = *right;
        ++right;
        ++numbers;
    }
}

void merge_sort(vector<int> &numbers)
{
    if (numbers.size() <= 1)
        return;

    vector<int>::size_type middle = numbers.size() / 2;
    vector<int> left(numbers.begin(), numbers.begin() + middle);
    vector<int> right(numbers.begin() + middle, numbers.end());

    merge_sort(left);
    merge_sort(right);
}

```

---

```

    merge(left.begin(), left.end(), right.begin(), right.end(), numbers.begin());
}

```

---

## 6.4 Algoritmo de MO (queries offline)

```

#define N 311111
#define A 1111111
#define BLOCK 555 // ~sqrt(N)

int cnt[A], a[N], ans[N], answer = 0;
struct node
{
    int L, R, i;
} q[N];
bool cmp(node x, node y)
{
    if (x.L / BLOCK != y.L / BLOCK)
    {
        // different blocks, so sort by block.
        return x.L / BLOCK < y.L / BLOCK;
    }
    // same block, so sort by R value
    return x.R < y.R;
}
void add(int position)
{
    cnt[a[position]]++;
    if (cnt[a[position]] == 1)
    {
        answer++; // Verifica se é resposta aqui!!!
    }
}
void remove(int position)
{
    cnt[a[position]]--;
    if (cnt[a[position]] == 0)
    {
        answer--; // Verifica se é resposta aqui!!!
    }
}
int main()
{
    int n;
    scanf("%d", &n);
    for (int i = 0; i < n; i++)
        scanf("%d", &a[i]);

    int m;
    scanf("%d", &m);
    for (int i = 0; i < m; i++)
    {
        scanf("%d%d", &q[i].L, &q[i].R);
        q[i].L--;
        q[i].R--;
        q[i].i = i;
    }

    sort(q, q + m, cmp);

    int currentL = 0, currentR = 0;
    for (int i = 0; i < m; i++)
    {
        int L = q[i].L, R = q[i].R;
        while (currentL < L)
        {
            remove(currentL);
            currentL++;
        }
        while (currentL > L)
        {
            add(currentL - 1);
            currentL--;
        }
        while (currentR <= R)
        {

```

---

```

        add(currentR);
        currentR++;
    }
    while (currentR > R + 1)
    {
        remove(currentR - 1);
        currentR--;
    }
    ans[q[i].i] = answer;
}
for (int i = 0; i < m; i++)
    printf("%d\n", ans[i]);
}

```

---

## 6.5 Union-find

```

// Tamanho máximo de n
const int maxn = 200000;
int Rank[maxn], p[maxn], n;

void init(int _n)
{
    n = _n;
    fill(Rank, Rank + n, 0);
    for (int i = 0; i < n; i++)
        p[i] = i;
}

int find(int x)
{
    return x == p[x] ? x : (p[x] = find(p[x]));
}

void unir(int a, int b)
{
    a = find(a);
    b = find(b);
    if (a == b)
        return;
    if (Rank[a] < Rank[b])
        swap(a, b);
    if (Rank[a] == Rank[b])
        ++Rank[a];
    p[b] = a;
}

```

---