Capybara dreaming

Contents

1	First things first 1.1 Includes	2
2	Matemática 2.1 Algoritmo de euclides extendido 2.2 Máximo divisor comum 2.3 Mínimo múltiplo comum 2.4 Algoritmo de Pollard Rho 2.5 Transformada rápida de Fourrier 2.6 Matrizes 2.7 Fatoração em números primos 2.8 Modpow 2.9 Máximo e mínimo de funções 2.10Todos divisores de um número 2.11Crivo de Eratóstenes segmentado	3 3 3 5 6 6 7 7
3	Grafos 3.1 Grafos	11
4	Strings 4.1 Suffix array	15 15
5	Geometria 5.1 Linha de eventos radial	18 19
6	Estruturas de dados etc 6.1 Wavelet-tree	25 27 28

1 First things first

1.1 Includes

```
#include <bits/stdc++.h>
using namespace std;

#define ll long long
#define pb push_back
#define D(x) cout << #x " = " << (x) << endl

typedef vector<int> vi;
typedef vector<vi> vvi;

typedef pair<int, int> ii;
typedef vector<ii> vii;
```

2 Matemática

2.1 Algoritmo de euclides extendido

```
int xmdc(int a, int b, int &x, int &y)
{
    if (b == 0)
    {
        x = 1;
        y = 0;
        return a;
    }
    int x1, y1, mdc = xmdc(b, a % b, x1, y1);
    x = y1;
    y = x1 - (a / b) * y1;
    return mdc;
}
```

2.2 Máximo divisor comum

```
int mdc(int a, int b)
{
    int remainder;
    while (b != 0)
    {
        remainder = a % b;
        a = b;
        b = remainder;
    }
    return a;
}
```

2.3 Mínimo múltiplo comum

```
int mmc(int a, int b)
{
    int temp = mdc(a, b);
    return temp ? (a / temp * b) : 0;
}
```

2.4 Algoritmo de Pollard Rho

2.5 Transformada rápida de Fourrier

```
// Resolve:
// - De quantas maneiras conseguimos atingir Y com X tentativas
// - Dado X tentativas, conseguimos atingir Y?
// Complexidade:
// X * Ymax * Ymax(log Ymax)
```

```
// TEOREMA DA CONVOLUÇÃO:
// Podemos fazer a convolucão de 2 polinomios utilizando a FFT
// Reduzindo a complexidade de n^2 para n log n
// Definimos a convolucão como h[i] = sum(a[j] * b[j-i]) para todo j de 0 a i.
// Exemplo: h[5] = a[5] * b[0] + a[4] * b[1] + a[3] * b[2]...
// Segundo o teorema da convolução
// h(f \cdot g) = transformada inversa de (transformada <math>(f) * transformada (g))
// onde . é o operador de convolução.
// e * é o operador de multiplicação termo a termo.
#include <bits/stdc++.h>
using namespace std;
// primeira potência de 2 maior que o limite de H
#define MAX_DIST (262144 * 2)
typedef complex<double> cpx;
const double pi = acos(-1.0);
int p[MAX_DIST];
int maxDist;
// in:
           vector de entrada
           vector de saida
// out:
// n:
           Tamanho do input/output {DEVE SER DA ORDEM DE 2}
// type:
           1 = Transformada, -1 = Transformada inversa
void FFT(vector<cpx> &v, vector<cpx> &ans, int n, int type)
        assert(!(n & (n - 1)));
        int i, sz, o;
p[0] = 0;
        for (i = 1;
                    i < n; i++)
                 p[i] = (p[i >> 1] >> 1) | ((i \& 1) ? (n >> 1) : 0);
        for (i = 0; i < n; i++)
                 ans[i] = v[p[i]];
        for (sz = 1; sz < n; sz <<= 1)
        {
                 const cpx wn(cos(type * pi / sz), sin(type * pi / sz));
                 for (o = 0; o < n; o += (sz << 1))
                         cpx w = 1;
                         for (i = 0; i < sz; i++)
                                  const cpx u = ans[o + i], t = w * ans[o + sz + i];
                                  ans[o + i] = u + t;
ans[o + i + sz] = u - t;
                                  w ∗= wn:
                         }
                 }
        }
        if (type == -1)
                 for (i = 0; i < n; i++)
                         ans[i] /= n;
}
// Exemplo:
// Há um robo que pode disparar bolas em N distâncias diferentes.
// Queremos saber se ele alcanca uma distância M com 1 ou 2 tacadas.
// Resolucão:
// Podemos definir um vetor distances[MAX DIST],
// onde a distances[i] = 1 se ele pode tacar até a distancia i
// e distances[i] = 0 caso contrario
// Para ver se o robo acerta com 1 tacada, é trivial.
// Para ver se o robo acerta com 2 tacadas, podemos fazer a convolução de distances com distances.
// Ex: Acertar a Pode[10] é igual a: Pode[10] || Pode[9] * Pode[1] || Pode[8] * Pode[2]...
// Ou seja, H = FFTi(FFT(distances) ** 2);
// Complexidade:
// 2 * 200k * log(200k) = 8m
int main()
{
        int N, d;
        vector<cpx> distances, fftOut;
        while (cin >> N)
        {
                maxDist = 0;
                 distances = vector<cpx>(MAX_DIST);
                 fftOut = vector<cpx>(MAX_DIST);
```

```
// Distancia 0 é uma posicão de "possível"
        distances[0] = cpx(1, 0);
        for (int i = 0; i < N; i++)
        {
                cin >> d;
                if (d > maxDist)
                        maxDist = d;
                distances[d] = cpx(1, 0);
        }
        int shiftAmount;
        for (shiftAmount = 0; (maxDist >> shiftAmount) != 0; shiftAmount++)
       maxDist = 1 << (shiftAmount + 1);</pre>
        // fftOut <= transformada de distances
        FFT(distances, fftOut, maxDist, 1);
        // Multiplicacão termo a termo de f e g, no caso, f = g = fftOut
        // fftOut *= fftOut
        for (int i = 0; i < maxDist; i++)
                fftOut[i] = fftOut[i] * fftOut[i];
        // transformada inversa da multiplcacão termo a termo.
        FFT(fftOut, distances, maxDist, -1);
        cin >> N;
        int total = 0;
        for (int i = 0; i < N; i++)
                cin >> d;
                // Entra a distancia d
                // e verifica se a parte real da distância[d] é positiva
                // distância[d] guarda de quantas maneiras conseguimos atingir D
                if (distances[d].real() > 0.01)
                        total++;
        }
        cout << total << endl;</pre>
return 0;
```

2.6 Matrizes

}

```
#include <bits/stdc++.h>
using namespace std;
#define ll long long
typedef vector<ll> vl;
typedef vector<vl> vvl;
const int mod = 1000000;
// Retorna a matriz I_n
vvl matrixUnit(int n) {
      vvl res(n, vl(n));
for (int i = 0; i < n; i++)
    res[i][i] = 1;</pre>
      return res;
}
// Retorna a+b
vvl matrixAdd(const vvl &a, const vvl &b) {
      int n = a.size();
      int m = a[0].size();
     int m = a[0].3120,,
vvl res(n, vl(m));
for (int i = 0; i < n; i++)
    for (int j = 0; j < m; j++)
        res[i][j] = (a[i][j] + b[i][j]) % mod;</pre>
}
// Retorna a*b
vvl matrixMul(const vvl &a, const vvl &b) {
      int n = a.size();
      int m = a[0].size();
int k = b[0].size();
      vvl res(n, vl(k));
for (int i = 0; i < n; i++)</pre>
```

```
for (int j = 0; j < k; j++)
for (int p = 0; p < m; p++)
               res[i][j] = (res[i][j] + ((a[i][p] % mod) * (b[p][j] % mod) % mod)) % mod;
    return res;
}
// Retorna a matriz a^p
vvl matrixPow(const vvl &a, long long p) {
    if (p == 0)
        return matrixUnit(a.size());
    if (p & 1)
        return matrixMul(a, matrixPow(a, p - 1));
    return matrixPow(matrixMul(a, a), p / 2);
}
// Retorna sum^p_i=0 (a^i)
vvl matrixPowSum(const vvl &a, long long p) {
    long long n = a.size();
    if (p == 0)
        return vvl(n, vl(n));
    if (p % 2 == 0)
        return matrixMul(matrixPowSum(a, p / 2), matrixAdd(matrixUnit(n), matrixPow(a, p / 2)));
    return matrixAdd(a, matrixMul(matrixPowSum(a, p - 1), a));
}
int main() {
        long long n, l, k, i;
        while(scanf("%lld %lld %lld", &n, &l, &k) > 0) {
                 vvl matriz = vvl(2, vl(2));
                matriz[0][0] = 1;
                matriz[0][1] = k;
                matriz[1][0] = 1;
                matriz[1][1] = 0;
                matriz = matrixPow(matriz, n / 5);
                printf("%06lld\n", matriz[0][0]);
        }
}
```

2.7 Fatoração em números primos

```
vector<int> primeFactors(int n)
{
    vector<int> v:
    int sqrtn = sqrt(n);
    while (n \% 2 == 0)
        v.push_back(2);
        n = n / 2;
    for (int i = 3; i \le sqrtn; i = i + 2)
        while (n \% i == 0)
        {
            v.push_back(i);
            n = n / i;
    if (n > 2)
        v.push_back(n);
    return v;
}
```

2.8 Modpow

```
int modPow(int a, int b, int m)
{
    int res = 1;
    for (; b > 0; b >>= 1)
    {
        if (b & 1)
            res = (long long)res * a % m;
}
```

```
a = (long long)a * a % m;
}
return res;
}
```

2.9 Máximo e mínimo de funções

```
double gss(double a, double b, double (*f)(double), double e = 1e-6)
     double r = (sqrt(5) - 1) / 2; //=.618...=golden ratio-1
double x1 = b - r * (b - a), x2 = a + r * (b - a);
double f1 = f(x1), f2 = f(x2);
     while (b - a > e)
          if (f1 < f2)
          { //change to > to find maximum
               b = x2;
               x2 = x1;
               f2 = f1;
               x1 = b - r * (b - a);
               f1 = f(x1);
          else
               a = x1;
               x1 = x2;
               f1 = f2;
               x2 = a + r * (b - a);
               f2 = f(x2);
     return (b + a) / 2;
}
```

2.10 Todos divisores de um número

```
vector<int> divisores(int n)
{
    vector<int> div;
    sqrtn = sqrt(n);

    while(sqrtn * sqrtn < n)
        sqrtn++;

    for (i = 1; i < sqrtn; i++)
        if (!(n % i))
            div.push_back(i), div.push_back(n / i);

    if(sqrtn * sqrtn == n)
        div.push_back(sqrtn);
    return div;
}</pre>
```

2.11 Crivo de Eratóstenes segmentado

```
while (n--)
             scanf("%d %d", &a, &b);
             if (a > 100000 && b > 100000)
{ // (a > sqrt(N) && b > sqrt(N))
    for (i = a; i <= b; i++)</pre>
                          for (j = 0; j < primos.size(); j++)
    if (i % primos[j] == 0)</pre>
                                       goto ab;
                          printf("%d\n", i);
                   ab:;
}
            else if (a < 100001 && b < 100001)
{ // (a < sqrt(N) && b < sqrt(N))
for (i = a; i <= b; i++)
                          if (!nprimo[i])
                                printf("%d\n", i);
             }
else
             {
                    for (i = 0; i < primos.size(); i++)</pre>
                          if (primos[i] >= a)
                                break;
                   for (; i < primos.size(); i++)
    printf("%d\n", primos[i]);</pre>
                   for (; i <= b; i++)
                          for (j = 0; j < primos.size(); j++)
    if (i % primos[j] == 0)</pre>
         goto ac;

printf("%d\n", i);
ac:;
}

      }
}
```

3 Grafos

3.1 Grafos

```
#include <bits/stdc++.h>
using namespace std;
#define ll long long
#define pb push_back
typedef vector<int> vi;
struct Vertice
    int id, pai;
    11 dist;
    Vertice(int id, ll dist = 1, int pai = -1) : id(id), dist(dist), pai(pai) {}
    bool operator<(Vertice a) const</pre>
    {
        return a.dist < dist;</pre>
};
typedef vector<Vertice> vv;
typedef vector<vv> vvv;
struct Grafo
    vvv g;
    vi pais;
    int n;
    Grafo(int n) : n(n)
        g = vvv(n, vv());
        pais = vi(n);
    }
    void operator=(Grafo const &a)
        g = a.g;
        pais = a.pais;
n = a.n;
    }
    void addAresta(int a, int b, ll d = 0)
        g[a].pb(Vertice(b, d));
    void removeAresta(int a, int b)
        g[a].erase(remove_if(g[a].begin(), g[a].end(), [b](Vertice v) { return v.id == b; }));
    11 valAresta(int a, int b)
    {
        for (auto it : g[a])
    if (it.id == b)
                 return it.dist;
        return 0;
    }
    void modificaAresta(int a, int b, ll dif)
        for (auto &it : g[a])
             if (it.id == b)
             {
                 it.dist += dif;
                 break;
        g[a].erase(remove_if(g[a].begin(), g[a].end(), [b](Vertice v) { return v.dist == 0; }));
    }
    11 dijkstra(int s, int d)
        priority_queue<Vertice> fila;
        bool visitados[n];
        fill(visitados, visitados+n, 0);
        fill(pais.begin(), pais.end(), -1);
```

```
fila.push(Vertice(s, 0));
    auto top = fila.top();
    while (top.id != d)
        if (!visitados[top.id])
        {
            for (auto &it : g[top.id])
                 if (!visitados[it.id])
                     fila.push(Vertice(it.id, it.dist + top.dist, top.id));
            visitados[top.id] = 1;
            pais[top.id] = top.pai;
        }
        fila.pop();
        if (fila.empty())
            return -1;
        top = fila.top();
    }
    pais[top.id] = top.pai;
    return top.dist;
}
11 busca(int s, int d)
    queue<Vertice> fila;
    bool visitados[n];
    fill(visitados, visitados+n, 0);
    fill(pais.begin(), pais.end(), -1);
    fila.push(Vertice(s, 0));
    auto top = fila.front();
    while (top.id != d)
        if (!visitados[top.id])
            for (auto &it : g[top.id])
                 if (!visitados[it.id])
                     fila.push(Vertice(it.id, it.dist + 1, top.id));
            visitados[top.id] = 1;
            pais[top.id] = top.pai;
        }
        fila.pop();
        if (fila.empty())
            return -1;
        top = fila.front();
    }
    pais[top.id] = top.pai;
    return top.dist;
}
11 fluxo_maximo(int s, int d)
    int u, v;
    11 \text{ flow} = 0;
    Grafo g2 = *this;
    while (g2.busca(s, d) \ge 0)
    {
        11 path = 111 << 50;</pre>
        for (v = d; v != s; v = u)
            u = g2.pais[v];
            path = min(path, valAresta(u, v));
        }
        for (v = d; v != s; v = u)
            u = g2.pais[v];
            g2.modificaAresta(u, v, -path);
```

```
g2.modificaAresta(v, u, path);
               }
               flow += path;
          }
          return flow;
};
int main()
{
     Grafo g(20);
     g.addAresta(1, 2, 1);
g.addAresta(1, 3, 5);
     g.addAresta(2, 1, 6);
     g.addAresta(3, 2, 10);
     g.removeAresta(1, 2);
     for (auto it : g.g[1])
    cout << it.id << endl; // 3</pre>
     cout << g.dijkstra(1, 2) << endl; // 15</pre>
     cout << g.fluxo_maximo(1, 2) << endl; // 5</pre>
}
```

3.2 Matching máximo em grafo bipartido

```
const int MAXN1 = 50000, MAXN2 = 50000, MAXM = 150000;
int n1, n2, edges, last[MAXN1], prev[MAXM], head[MAXM], matching[MAXN2], dist[MAXN1], Q[MAXN1], used[MAXN1], vis
    [MAXN1];
void init(int _n1, int _n2)
{
    n1 = _n1;
n2 = _n2;
edges = 0;
    fill(last, last + n1, -1);
}
void addAresta(int u, int v)
{
    head[edges] = v;
    prev[edges] = last[u];
    last[u] = edges++;
}
void bfs()
    fill(dist, dist + n1, -1);
int sizeQ = 0;
    for (int u = 0; u < n1; ++u)
         if (!used[u])
             Q[sizeQ++] = u;
             dist[u] = 0;
    for (int i = 0; i < sizeQ; i++)
         int u1 = Q[i];
         for (int e = last[u1]; e \ge 0; e = prev[e])
         {
             int u2 = matching[head[e]];
             if (u2 >= 0 \&\& dist[u2] < 0)
                 dist[u2] = dist[u1] + 1;
                 Q[sizeQ++] = u2;
             }
        }
}
bool dfs(int u1)
    vis[u1] = true;
    for (int e = last[u1]; e >= 0; e = prev[e])
         int v = head[e];
```

```
int u2 = matching[v];
         if (u2 < 0 \mid | !vis[u2] \&\& dist[u2] == dist[u1] + 1 \&\& dfs(u2))
              matching[v] = u1;
              used[u1] = true;
              return true;
         }
     return false;
int maxMatching()
{
     fill(used, used + n1, false);
     fill(matching, matching + n2, -1);
for (int res = 0;;)
         bfs();
         fill(vis, vis + n1, false);
         int f = 0;
         for (int u = 0; u < n1; ++u)
if (!used[u] && dfs(u))
         if (!f)
              return res;
         res += f;
    }
}
```

3.3 Algoritmo húngaro

```
#define N 100
#define INF 100000000
int cost[N][N], n, max_match, lx[N], ly[N], xy[N], yx[N], slack[N], slackx[N], prev[N];
bool S[N], T[N];
void init_labels()
{
    memset(lx, 0, sizeof(lx));
    memset(ly, 0, sizeof(ly));
    for (int x = 0; x < n; x++)
for (int y = 0; y < n; y++)
             lx[x] = max(lx[x], cost[x][y]);
}
void update_labels()
    int x, y, delta = INF;
    for (y = 0; y < n; y++)
        if (!T[y])
            delta = min(delta, slack[y]);
    for (x = 0; x < n; x++)
        if (S[x])
            lx[x] -= delta;
    for (y = 0; \dot{y} < n; y++)
        if (T[y])
             ly[y] += delta;
    for (y = 0; y < n; y++)
        if (!T[y])
             slack[y] -= delta;
}
void add_to_tree(int x, int prevx)
    S[x] = true;
    prev[x] = prevx;
    for (int y = 0; y < n; y++)
	if (lx[x] + ly[y] - cost[x][y] < slack[y])
        {
             slack[y] = lx[x] + ly[y] - cost[x][y];
             slackx[y] = x;
}
void augment()
    if (max_match == n)
        return;
    int x, y, root, q[N], wr = 0, rd = 0;
```

```
memset(S, false, sizeof(S));
memset(T, false, sizeof(T));
memset(prev, -1, sizeof(prev));
     for (x = 0; x < n; x++)
          if (xy[x] == -1)
                q[wr++] = root = x;
               prev[x] = -2;
S[x] = true;
               break;
          }
     for (y = 0; y < n; y++)
          slack[y] = lx[root] + ly[y] - cost[root][y];
          slackx[y] = root;
     while (true)
          while (rd < wr)
               x = q[rd++];
               for (y = 0; y < n; y++)

if (cost[x][y] == lx[x] + ly[y] && !T[y])
                          if (yx[y] == -1)
                               break;
                          T[y] = true;
                          q[wr++] = yx[y];
add_to_tree(yx[y], x);
               if (y < n)
                     break;
          }
          if (y < n)
               break;
          update_labels();
          wr = rd = 0;
          for (y = 0; y < n; y++)
if (!T[y] \&\& slack[y] == 0)
                {
                     if (yx[y] == -1)
                          x = slackx[y];
                          break;
                     }
                     else
                          T[y] = true;
                          if (!S[yx[y]])
                                q[wr++] = yx[y];
                                add_to_tree(yx[y], slackx[y]);
                     }
               }
          if (y < n)
                break;
     }
     if (y < n)
          \max_{x \in \mathbb{R}} \operatorname{max_match}(x) + x; for (int cx = x, cy = y, ty; cx != -2; cx = \operatorname{prev}(x), cy = ty)
                ty = xy[cx];
               yx[cy] = cx;
xy[cx] = cy;
          augment();
int hungaro()
     int ret = 0;
```

}

{

```
max_match = 0;
memset(xy, -1, sizeof(xy));
memset(yx, -1, sizeof(yx));
init_labels();
augment();
for (int x = 0; x < n; x++)
    ret += cost[x][xy[x]];
return ret;
}</pre>
```

4 Strings

4.1 Suffix array

```
//Usage:
// Fill txt with the characters of the txting.
// Call SuffixSort(n), where n is the length of the txting stored in txt.
// That's it!
//Output:
// SA = The suffix array.
// Contains the n suffixes of txt sorted in lexicographical order.
// Each suffix is represented as a single integer (the SAition of txt where it starts).
// iSA = The inverse of the suffix array. iSA[i] = the index of the suffix txt[i..n)
     in the SA array. (In other words, SA[i] = k \iff iSA[k] = i)
     With this array, you can compare two suffixes in O(1): Suffix txt[i..n) is smaller than txt[j..n) if and only if iSA[i] < iSA[j]
const int MAX = 100010;
char txt[MAX];
                            //input
int iSA[MAX], SA[MAX]; //output
int cnt[MAX], next[MAX]; //internal
bool bh[MAX], b2h[MAX];
// Compares two suffixes according to their first characters
bool smaller_first_char(int a, int b)
    return txt[a] < txt[b];</pre>
}
void suffixSort(int n)
    for (int i = 0; i < n; ++i)
         SA[i] = i;
    sort(SA, SA + n, smaller_first_char);
    for (int i = 0; i < n; ++i)
         bh[i] = i == 0 \mid \mid txt[SA[i]] != txt[SA[i - 1]];
         b2h[i] = false;
    }
    for (int h = 1; h < n; h <<= 1)
         int buckets = 0;
         for (int i = 0, j; i < n; i = j)
         {
             j = i + 1;
             while (j < n \&\& !bh[j])
                 j++;
             next[i] = j;
             buckets++;
         }
         if (buckets == n)
             break:
         for (int i = 0; i < n; i = next[i])
             cnt[i] = 0;
             for (int j = i; j < next[i]; ++j)
    iSA[SA[j]] = i;</pre>
         cnt[iSA[n - h]]++;
         b2h[iSA[n - h]] = true;
         for (int i = 0; i < n; i = next[i])
             for (int j = i; j < next[i]; ++j)
                  int s = SA[j] - h;
                  if (s \ge 0)
                  {
                      int head = iSA[s];
                      iSA[s] = head + cnt[head]++;
                      b2h[iSA[s]] = true;
                  }
```

```
} for (int j = i; j < next[i]; ++j)
                int s = SA[j] - h;
                }
        } for (int i = 0; i < n; ++i)
            SA[iSA[i]] = i;
bh[i] |= b2h[i];
    } for (int i = 0; i < n; ++i)
        iSA[SA[i]] = i;
}
// End of suffix array algorithm
int lcp[MAX];

// lcp[i] = length of the longest common prefix of suffix SA[i] and suffix SA[i-1]

// lcp[0] = 0
// Begin of the O(n) longest common prefix algorithm
void getlcp(int n)
{
    for (int i = 0; i < n; ++i)
        iSA[SA[i]] = i;
    1cp[0] = 0;
    for (int i = 0, h = 0; i < n; ++i)
        if (iSA[i] > 0)
            int j = SA[iSA[i] - 1];
            while (i + h < n \&\& j + h < n \&\& txt[i + h] == txt[j + h])
            lcp[iSA[i]] = h;
            if (h > 0)
                h--;
        }
    }
}
```

5 Geometria

5.1 Linha de eventos radial

```
// - Radial sweep in Q2 quadrant in nlogn.
// - Sorts events using cross product to avoid dealing with
      numeric problems.
#include <bits/stdc++.h>
using namespace std;
struct Point {
        Point(int x = 0, int y = 0) : x(x), y(y) {}
        bool operator<(const Point& o) const {</pre>
                 // Order points in a quadrant by angle with origin:
                 // Uses anti-clockwise order by returning true when the
                 // cross product between the points is positive.
                 return (x*o.y - y*o.x) > 0;
        }
        bool operator<=(const Point& o) const {</pre>
        return (x*o.y - y*o.x) >= 0;
         */
        int x, y;
};
pair<int, int> solve(const vector<Point>& points) {
        map<Point, pair<int, int> > events;
        Point begin(0, 1);
        Point end(-1, 0);
        // Add events on the borders to guarantee that we consider them.
        events[begin];
        events[end];
        int superior = 0; // Number of points in Q1 quadrant.
                             // Number of points in origin.
// Number of current points in Q2 and Q4 quadrant better
        int same = 0;
        int active = 0;
                                              // than origin.
        int best_pos = points.size();
        int worst_pos = 0;
        for (const auto& p : points) {
                 if (p.x < 0 && p.y < 0) {}
else if (p.x > 0 && p.y > 0) superior++;
else if (p.x == 0 && p.y == 0) same++;
                 else if (p.x \le 0 \&\& p.y \ge 0) {
                          // assert(begin <= Point(p.x, p.y));</pre>
                          //assert(Point(p.x, p.y) <= end);</pre>
                          events[Point(p.x, p.y)].first++;
                 else if (p.x >= 0 \&\& p.y <= 0) {
                          //assert(begin <= Point(-p.x, -p.y));</pre>
                          //assert(Point(-p.x, -p.y) <= end);</pre>
                          active++;
                          events[Point(-p.x, -p.y)].second++;
                 else assert(false);
        for (const auto& e : events) {
                 int tie_best_pos = superior + active - e.second.second;
                 int tie_worst_pos = superior + active + e.second.first + same;
                 active += e.second.first - e.second.second;
                 best pos = min(best pos, tie best pos);
                 worst_pos = max(worst_pos, tie_worst_pos);
        }
        return make_pair(best_pos + 1, worst_pos + 1);
}
// Reads the set of points and centers them around Maria's product.
vector<Point> read() {
        int n, cx, cy;
        cin >> n >> cx >> cy;
        vector<Point> points(n - 1);
        for (Point& p : points) {
```

5.2 KD-Tree para pares mais próximos em O(log(n))

```
typedef pair<int, int> pii;
typedef vector<pii> vpii;
const int maxn = 100000;
int tx[maxn];
int ty[maxn];
bool divX[maxn];
bool cmpX(const pii &a, const pii &b)
         return a.first < b.first;</pre>
}
bool cmpY(const pii &a, const pii &b)
{
         return a.second < b.second;
}
void buildTree(int left, int right, pii points[])
{
         if (left >= right)
                  return;
         int mid = (left + right) >> 1;
         //sort(points + left, points + right + 1, divX ? cmpX : cmpY);
         int minx = INT MAX;
         int maxx = INT_MIN;
         int miny = INT_MAX;
int maxy = INT_MIN;
         for (int i = left; i < right; i++)</pre>
                  checkmin(minx, points[i].first);
                  checkmax(maxx, points[i].first);
                  checkmin(miny, points[i].second);
                  checkmax(maxy, points[i].second);
         divX[mid] = (maxx - minx) >= (maxy - miny);
nth_element(points + left, points + mid, points + right, divX[mid] ? cmpX : cmpY);
         tx[mid] = points[mid].first;
         ty[mid] = points[mid].second;
         if (left + 1 == right)
                  return;
         buildTree(left, mid, points);
buildTree(mid + 1, right, points);
}
long long closestDist;
int closestNode;
void findNearestNeighbour(int left, int right, int x, int y)
{
         if (left >= right)
         return;
int mid = (left + right) >> 1;
         int dx = x - tx[mid];
         int dy = y - ty[mid];
long long d = dx * (long long)dx + dy * (long long)dy;
         if (closestDist > d && d)
```

```
{
                 closestDist = d;
                 closestNode = mid:
         if (left + 1 == right)
                 return;
         int delta = divX[mid] ? dx : dy;
         long long delta2 = delta * (long long)delta;
         int l1 = left;
         int r1 = mid;
         int 12 = mid + 1;
         int r2 = right;
         if (delta > 0)
                 swap(11, 12), swap(r1, r2);
         findNearestNeighbour(l1, r1, x, y);
         if (delta2 < closestDist)</pre>
                 findNearestNeighbour(12, r2, x, y);
}
int findNearestNeighbour(int n, int x, int y)
{
         closestDist = LLONG_MAX;
         findNearestNeighbour(0, n, x, y);
         return closestNode;
}
int main()
{
        vpii p;
         p.push_back(make_pair(0, 2));
         p.push_back(make_pair(0, 3));
         p.push_back(make_pair(-1, 0));
         p.resize(unique(p.begin(), p.end()) - p.begin());
         int n = p.size();
        buildTree(1, 0, n - 1, &(vpii(p)[0]));
int res = findNearestNeighbour(n, 0, 0);
         cout << p[res].first << " " << p[res].second << endl;</pre>
         return 0;
}
```

5.3 Geometria (reduzido)

```
typedef pair < double > Ponto;
bool cw(Ponto a, Ponto b, Ponto c)
{
    return (b.first - a.first) * (c.second - a.second) - (b.second - a.second) * (c.first - a.first) < 0;
}
// Retorna o casco convexo do conjunto de pontos p
vector<Ponto> convexHull(vector<Ponto> p)
    int n = p.size();
    if (n <= 1)
        return p;
    int k = 0;
    sort(p.begin(), p.end());
    vector<Ponto> q(n * 2);
    for (int i = 0; i < n; q[k++] = p[i++])
        for (; k \ge 2 \& !cw(q[k - 2], q[k - 1], p[i]); --k)
    for (int i = n - 2, t = k; i >= 0; q[k++] = p[i--])
        for (; k > t \& !cw(q[k - 2], q[k - 1], p[i]); --k)
    q.resize(k - 1 - (q[0] == q[1]));
    return q;
}
//O dobro da área definida pelo triangulo de pontos pontos a, b e c (sem sinal).
double uArea2(Ponto a, Ponto b, Ponto c)
{
    return abs((b.first - a.first) * (c.second - a.second) - (b.second - a.second) * (c.first - a.first));
}
//O dobro da área definida pelo triangulo de pontos pontos a, b e c (com sinal).
double area2(Ponto a, Ponto b, Ponto c)
```

```
{
    return (b.first - a.first) * (c.second - a.second) - (b.second - a.second) * (c.first - a.first);
}
//Distância entre os pontos a e b
double dist(Ponto a, Ponto b)
{
    return hypot(a.first - b.first, a.second - b.second);
}
//Intersecão de semi-retas (p1 -> p2), (p3 -> p4)
bool segIntercept(Ponto p1, Ponto p2, Ponto p3, Ponto p4)
    return cw(p1, p2, p3) != cw(p1, p2, p4) & cw(p3, p4, p1) != cw(p3, p4, p2);
}
//Retorna a área do polígono p
double polygonArea(vector<Ponto> p)
    double s = 0.0;
    for (int i = 0; i < p.size(); i++)
        s += area2(Ponto(0, 0), p[i], p[(i + 1) % p.size()]);
    return fabs(s / 2.0);
}
//Retorna a área do polígono p definido pelos pontos p[i, f]
double polygonArea2(vector<Ponto> p, int i, int f)
    double s = 0.0;
    Ponto primeiro = p[i];
    for (; i != f; i++)
        s += area2(Ponto(0, 0), p[i], p[(i + 1)]);
    s += area2(Ponto(0, 0), p[i], primeiro);
    return fabs(s / 2.0);
}
//Retorna a menor largura do conjunto de pontos p
double raio(vector<Ponto> p)
    vector<Ponto> h = convexHull(p);
    int m = h.size();
    if (m == 1)
        return 0;
    if (m == 2)
        return 0;
    int k = 1;
    while (uArea2(h[m - 1], h[0], h[(k + 1) % m]) > uArea2(h[m - 1], h[0], h[k]))
        ++k:
    double res = 10000000;
    for (int i = 0, j = k; i \le k & j \le m; i++)
        res = min(res, dist(h[i], h[j]));
        while (j < m \& uArea2(h[i], h[(i + 1) % m], h[(j + 1) % m]) > uArea2(h[i], h[(i + 1) % m], h[j]))
        {
            res = min(res, dist(h[i], h[(j + 1) % m]));
            ++j;
        }
    return res;
}
//Retorna a maior largura do conjunto de pontos p
double diametro(vector<Ponto> p)
    vector<Ponto> h = convexHull(p);
    int m = h.size();
    if (m == 1)
        return 0;
    if (m == 2)
        return dist(h[0], h[1]);
    int k = 1;
    while (uArea2(h[m - 1], h[0], h[(k + 1) % m]) > uArea2(h[m - 1], h[0], h[k]))
        ++k;
    double res = 0;
    for (int i = 0, j = k; i \le k & j \le m; i++)
        res = max(res, dist(h[i], h[j]));
        while (j < m \& uArea2(h[i], h[(i + 1) % m], h[(j + 1) % m]) > uArea2(h[i], h[(i + 1) % m], h[j]))
            res = max(res, dist(h[i], h[(j + 1) % m]));
            ++j;
        }
```

```
}
return res;
```

5.4 Geometria (grande)

```
#include <bits/stdc++.h>
using namespace std;
const double EPS = 1e-10;
inline int cmp( double x, double y = 0, double tol = EPS ) {
    return (x \le y + tol)? (x + tol < y)? -1:0:1;
struct Point {
    double x, y;
    Point( double x = 0, double y = 0) : x(x), y(y) {}
    Point operator+( Point q ) const {
        return Point( x + q.x, y + q.y);
    Point operator-( Point q ) const {
        return Point( x - q.x, y - q.y );
    Point operator*( double t ) const {
        return Point( x * t, y * t );
    Point operator/( double t ) const {
        return Point( x / t, y / t );
    double operator*( Point q )const {
        return x * q.x + y * q.y;
    double operator^( Point q ) const {
        return x * q.y - y * q.x;
    }
    int cmp( Point q ) const {
        if ( int t = ::cmp( x, q.x ) )
                         return t;
        return ::cmp( y, q.y );
    }
    bool operator==( Point \underline{q} ) const {
        return cmp(q) == 0;
    bool operator!=( Point q ) const {
   return cmp( q ) != 0;
    bool operator<( Point q ) const {</pre>
        return cmp(q) < 0;
    static Point pivot;
};
Point Point::pivot;
typedef vector<Point> Polygon;
inline double abs( Point& p ) {
    return hypot( p.x, p.y );
}
inline double arg( Point& p ) {
    return atan2( p.y, p.x );
//Verifica o sinal do produto vetorial entre os vetores (p-r) e (q - r)
inline int ccw( Point& p, Point& q, Point& r ) {
   return cmp( ( p - r ) ^ ( q - r ) );
//calcula o angulo orientado entre os vetores (p-q) e (r - q)
inline double angle( Point& p, Point &q, Point& r ) {
    Point u = p - q, w = r - q;
```

```
return atan2( u ^ w, u * w );
}
//Decide se o ponto p esta sobre a reta que passa por p1p2.
bool pointoSobreReta( Point& p1, Point &p, Point& p2 ) {
     return ccw(p1, p2, p) == 0;
//Decide de p esta sobre o segmento p1p2
bool between( Point& p1, Point &p, Point& p2 ) {
     return ccw( p1, p2, p ) == 0.88 \text{ cmp}((p1 - p) * (p2 - p)) <= 0;
//Calcula a distancia do ponto p a reta que passa por p1p2
double retaDistance( Point& p1, Point& p2, Point &p ) {
    Point A = p1 - p, B = p2 - p1;
return fabs( A ^ B ) / sqrt( B * B );
}
//Calcula a distancia do ponto p ao segmento de reta que passa por p1p2
double segDistance( Point& p1, Point& p2, Point &p ) {
   Point A = p1 - p, B = p1 - p2, C = p2 - p;
     double a = A * A, b = B * B, c = C * C;
    if ( cmp( a, b + c ) >= 0 ) return sqrt( c );
if ( cmp( c, a + b ) >= 0 ) return sqrt( a );
return fabs( A ^ C ) / sqrt( b );
}
//Calcula a area orientada do poligono T.
double polygonArea( Polygon& T ) {
     double s = 0.0;
int n = T.size( );
     for ( int i = 0; i < n; i++ )
     {
         s += T[i] ^T[(i + 1) % n];
     return s / 2.0; //Retorna a area com sinal
//Classifica o ponto p em relacao ao poligono T dependendo se ele está
//na fronteira (-1) no exterior (0) ou no interior (1).
int inpoly( Point& p, Polygon& T ) {
     //-1 sobre, 0 fora, 1 dentro
     double a = 0.0;
     int n = T.size( );
for ( int i = 0; i < n; i++ )
         if ( between( T[i], p, T[( i + 1 ) % n] ) ) return -1;
a += angle( T[i], p, T[( i + 1 ) % n] );
     return cmp( a ) != 0;
//Ordenacao radial.
bool radialSort( Point p, Point q ) {
    Point P = p - Point::pivot, Q = q - Point::pivot; double R = P \land Q;
     if ( cmp( R ) ) return R > 0;
return cmp( P * P, Q * Q ) < 0;
//Determina o convex hull de T. ATENCAO. A lista de pontos T e destruida.
Polygon convexHull( vector<Point>& T ) {
     int j = 0, k, n = T.size();
Polygon U( n );
     Point::pivot = *min_element( T.begin( ), T.end( ) );
     sort( T.begin( ), T.end( ), radialSort );
     for ( k = n - 2; k \ge 0 && ccw(T[0], T[n - 1], T[k] ) == 0; <math>k--);
     reverse( ( k + 1 ) + T.begin( ), T.end( ) );
     for ( int i = 0; i < n; i++ )
     {
          // troque o >= por > para manter pontos colineares
         while (j > 1 & ccw(U[j - 1], U[j - 2], T[i]) >= 0) j--;
         U[j++] = T[i];
    U.resize( j );
    return U;
//Intersecão de semi-retas (p1 -> p2), (p3 -> p4)
bool segIntercept(Point p1, Point p2, Point p3, Point p4) {
         return ccw(p1, p2, p3) != ccw(p1, p2, p4) && ccw(p3, p4, p1) != ccw(p3, p4, p2);
}
```

6 Estruturas de dados etc

6.1 Wavelet-tree

```
#include <bits/stdc++.h>
using namespace std;
const int N = 10000;
struct KthSmallest
          struct Seg
          {
                    int 1, r, mid;
                    void set(int _l, int _r)
                              l = _l;
r = _r;
mid = l + r >> 1;
          } seg[N << 2];</pre>
          int b[25][N], left[25][N], sorted[N];
          void init(int *a, int n)
          {
                    for (int i = 0; i < n; i++)
                              b[0][i] = sorted[i] = a[i];
                    sort(sorted, sorted + n);
build(0, n, 0, 1);
          }
          void build(int 1, int r, int d, int idx)
                    seg[idx].set(1, r);
                    if (1 + 1 == r)
                              return;
                    int mid = seg[idx].mid;
                    int lsame = mid - 1;
                    for (int i = 1; i < r; i++)
         if (b[d][i] < sorted[mid])</pre>
                                        lsame--;
                    int lpos = 1, rpos = mid, same = 0;
                    for (int i = 1; i < r; ++i)
                    {
                              left[d][i] = (i != l ? left[d][i - 1] : 0);
                              if (b[d][i] < sorted[mid])</pre>
                              {
                                        left[d][i]++;
                                        b[d + 1][lpos++] = b[d][i];
                              else if (b[d][i] > sorted[mid])
b[d + 1][rpos++] = b[d][i];
                              else
                                        if (same < lsame)</pre>
                                        {
                                                   same++;
                                                   left[d][i]++;
                                                   b[d + 1][lpos++] = b[d][i];
                                        else
                                        {
                                                   b[d + 1][rpos++] = b[d][i];
                              }
                    }
                    build(1, mid, d + 1, idx << 1);
build(mid, r, d + 1, idx << 1 | 1);</pre>
          //Quando ordernarmos [l, r), qual é o k-ésimo termo? int kth(int l, int r, int k, int d = 0, int idx = 1) { // k : 1-origin!!!
                    if (1 + 1 == r)
                              return b[d][1];
```

```
int ltl = (l != seg[idx].l ? left[d][l - 1] : 0);
                  int tl = left[d][r - 1] - ltl;
                  if (tl >= k)
                           int newl = seg[idx].l + ltl;
                           int newr = seg[idx].l + ltl + tl;
                           return kth(newl, newr, k, d + 1, idx << 1);
                  }
                  else
                           int mid = seg[idx].mid;
                           int tr = r - l - tl;
int ltr = l - seg[idx].l - ltl;
                           int newl = mid + ltr;
                           int newr = mid + ltr + tr;
                           return kth(newl, newr, k - tl, d + 1, idx << 1 | 1);
                  }
         }
         //When sorting [1, r), what number will x come in?
         //If there are two or more x's, return the rank of the last one.
         //If there is no x, return the rank of the largest but less than x. //When there is no less than x, 0 is returned.
         int rank(int 1, int r, int x, int d = 0, int idx = 1)
                  if (seg[idx].l + 1 == seg[idx].r)
                           return 1 + 1 == r && sorted[1] <= x;
                  int ltl = (l != seg[idx].l ? left[d][l - 1] : 0);
                  int tl = left[d][r - 1] - ltl;
                  int mid = seg[idx].mid;
                  if (x < sorted[mid])</pre>
                  {
                           int newl = seg[idx].l + ltl;
                           int newr = seg[idx].l + ltl + tl;
                           return rank(newl, newr, x, d + 1, idx << 1);</pre>
                  élse
                           int tr = r - 1 - tl;
int ltr = l - seg[idx].l - ltl;
                           int newl = mid + ltr;
                           int newr = mid + ltr + tr;
                           return tl + rank(newl, newr, x, d + 1, idx << 1 | 1);
                  }
         }
         // Quantos x existem entre [1,r)
         int freq(int 1, int r, int x)
         {
                  return rank(1, r, x) - rank(1, r, x - 1);
} kth;
int main()
         int a[8] = \{6, 12, 5, 17, 10, 2, 7, 3\};
         kth.init(a, 8);
         cout << kth.kth(2, 7, 3) << endl; // 7
cout << kth.rank(2, 7, 7) << endl; // 3</pre>
```

6.2 Seg-tree

{

}

```
#include <algorithm>
using namespace std;
#define MAX 1000000 // 0 valor aqui tem que ser >= 2 * tamanho do maior n
#define INF 1 << 28
// Não necessáriamente é um int, pode ser uma segtree de struct etc;
int init[MAX], tree[MAX], lazy[MAX];
void build tree(int node, int a, int b)
```

```
{
         if (a > b)
                  réturn;
         // Se folha
         if (a == b)
                  tree[node] = init[a];
                  lazy[node] = 0;
                  return:
         }
         build_tree(node * 2, a, (a + b) / 2);
build_tree(node * 2 + 1, 1 + (a + b) / 2, b);
         tree[node] = tree[node * 2] + tree[node * 2 + 1];
         lazy[node] = 0;
}
void update_tree(int node, int a, int b, int i, int j, int value)
         // Se fora do intervalo - retorna
         if (a > b || a > j || b < i)
                  return;
         if (lazy[node] != 0)
         {
                  //Atualizacão atrasada.
                  tree[node] += lazy[node];
                  // Passa lazy para filhos
                  if (a != b)
                  {
                           lazy[node * 2] += lazy[node];
                           lazy[node * 2 + 1] += lazy[node];
                  }
                  //Reseta o nó
                  lazy[node] = 0;
         }
         // Se o nó atual cobre todo o intervalo
         if (a >= i \&\& b <= j)
         {
                  tree[node] += value;
                  if (a != b)
                  {
                           lazy[node * 2] += value;
lazy[node * 2 + 1] += value;
                  }
                  return;
         }
         // Se tem um pedaco em cada filho.
         // Atualiza os filhos.
         update_tree(node * 2, a, (a + b) / 2, i, j, value);
update_tree(1 + node * 2, 1 + (a + b) / 2, b, i, j, value);
         // Atualiza o pai.
         tree[node] = tree[node * 2] + tree[node * 2 + 1];
}
int query_tree(int node, int a, int b, int i, int j)
         // Se fora do intervalo
         if (a > b \mid | a > j \mid | b < i)
         {
                  //Aqui deverá ser retornado o elemento neutro para a operação desejada
                  return 0;
         }
         if (lazy[node] != 0)
                  //Atualizacão atrasada.
                  tree[node] += lazy[node];
                  //Se não folha, passa lazy pros filhos
                  if (a != b)
                  {
                           lazy[node * 2] += lazy[node];
                           lazy[node * 2 + 1] += lazy[node];
                  }
```

```
//Reseta o nó
                 lazy[node] = 0;
        }
        // Se o nó cobre o intervalo
        if (a >= i \&\& b <= j)
                 return tree[node];
        // Se o intervalo está um pedaco em cada filho.
        int q1 = query_tree(node * 2, a, (a + b) / 2, i, j);
int q2 = query_tree(1 + node * 2, 1 + (a + b) / 2, b, i, j);
        // Retorna a combinação dos intervalos.
        return q1 + q2;
}
/*
Uso:
Assumindo que "n" é o numero de termos que o segmento tem
Inicialize "init" com os valores iniciais:
        for(i = 0; i < n; scanf("%d", val), i++)
       init[i] = val;
E mande construir a arvore:
    build_tree(1, 0, n-1);
Para atualizar a arvore:
    update_tree(1, 0, n-1, inicio, fim, val);
    Onde inicio é a posicão inicial do segmento desejado e fim é a posicão final do mesmo
        e val é o quanto você quer alterar os valores desse seguimento
Para fazer queries
    query_tree(1, 0, n-1, inicio, fim);
        Onde inicio é a posicão inicial do segmento desejado e fim é a posicão final do mesmo
        o retorno terá o mesmo tipo que os dados guardados na arvore e será o resultado do segmento pesquisado
*/
```

6.3 Mergesort

```
typedef vector<int>::iterator vec_it;
void merge(vec_it left, vec_it left_end, vec_it right, vec_it right_end, vec_it numbers)
{
    while (left != left_end)
        if (*left < *right || right == right_end)</pre>
        {
            *numbers = *left;
            ++left;
        else
        {
            *numbers = *right;
            ++right;
        }
        ++numbers;
    }
    while (right != right_end)
        *numbers = *right;
        ++right;
        ++numbers;
    }
}
void merge_sort(vector<int> &numbers)
{
    if (numbers.size() <= 1)</pre>
        return;
    vector<int>::size_type middle = numbers.size() / 2;
    vector<int> left(numbers.begin(), numbers.begin() + middle);
    vector<int> right(numbers.begin() + middle, numbers.end());
    merge_sort(left);
    merge sort(right);
```

```
merge(left.begin(), left.end(), right.begin(), right.end(), numbers.begin());
}
```

6.4 Algoritmo de MO (queries offline)

```
#define N 311111
#define A 1111111
#define BLOCK 555 // ~sqrt(N)
int cnt[A], a[N], ans[N], answer = 0;
struct node
{
        int L, R, i;
} q[N];
bool cmp(node x, node y)
{
        if (x.L / BLOCK != y.L / BLOCK)
                 // different blocks, so sort by block.
                 return x.L / BLOCK < y.L / BLOCK;
         // same block, so sort by R value
         return x.R < y.R;
}
void add(int position)
         cnt[a[position]]++;
         if (cnt[a[position]] == 1)
                 answer++; // Verifica se é resposta aqui!!!
}
void remove(int position)
         cnt[a[position]]--;
         if (cnt[a[position]] == 0)
         {
                 answer--; // Verifica se é resposta aqui!!!
        }
}
int main()
{
        int m;
scanf("%d", &m);
for (int i = 0; i < m; i++)</pre>
                 scanf("%d%d", &q[i].L, &q[i].R);
                 q[i].L--;
                 q[i].R--;
                 q[i].i = i;
        }
         sort(q, q + m, cmp);
        int currentL = 0, currentR = 0;
for (int i = 0; i < m; i++)</pre>
                 int L = q[i].L, R = q[i].R;
                 while (currentL < L)</pre>
                 {
                          remove(currentL);
                          currentL++;
                 }
                 while (currentL > L)
                 {
                          add(currentL - 1);
                          currentL--;
                 while (currentR <= R)</pre>
```

6.5 Union-find

```
// Tamanho máximo de n
const int maxn = 200000;
int Rank[maxn], p[maxn], n;
void init(int _n)
{
     n = _n;
fill(Rank, Rank + n, 0);
for (int i = 0; i < n; i++)
   p[i] = i;
}
int find(int x)
{
     return x == p[x] ? x : (p[x] = find(p[x]));
}
void unir(int a, int b)
{
     a = find(a);
b = find(b);
      if (a == b)
           return;
      if (Rank[a] < Rank[b])</pre>
     swap(a, b);
if (Rank[a] == Rank[b])
++Rank[a];
     p[b] = a;
}
```