

Capbara dreaming

Contents

1	First things first	2
1.1	Includes	2
2	Matemática	3
2.1	Algoritmo de euclides estendido	3
2.2	Máximo divisor comum	3
2.3	Mínimo múltiplo comum	3
2.4	Algoritmo de Pollard Rho	3
2.5	Transformada rápida de Fourier	3
2.6	Matrizes	5
2.7	Quantidade de fatores primos de um número	6
2.8	Fatoração em números primos	7
2.9	Modpow	7
2.10	Quantidade de coprimos.	7
2.11	Máximo e mínimo de funções (Busca ternária)	8
2.12	Todos divisores de um número	8
2.13	Crivo de Eratóstenes segmentado	9
3	Grafos	10
3.1	Grafos	10
3.2	Todas as pontes de um grafo	12
3.3	Isomorfismo de árvores	14
3.4	Matching máximo em grafo bipartido	15
3.5	Algoritmo húngaro	16
3.6	Isomorfismo de árvores para a biblioteca de grafos.	18
3.7	Fluxo máximo de custo mínimo	20
4	Strings	22
4.1	Suffix array	22
4.2	Algoritmo Z	23
4.3	Strings em Árvore	24
4.4	Split strings	25
5	Geometria	27
5.1	Linha de eventos radial	27
5.2	KD-Tree para pares mais próximos em $O(\log(n))$	28
5.3	Geometria (reduzido)	31
5.4	Geometria (grande)	32
5.5	Geometria (Marcelo)	34
6	Estruturas de dados etc	38
6.1	Wavelet-tree	38
6.2	Seg-tree 2D	39
6.3	Seg-tree	42
6.4	Mergesort	43
6.5	Algoritmo de MO (queries offline)	44
6.6	Union-find	45
6.7	Árvore para achar o K-ésimo termo	45
6.8	Índice de inversão ($n \log n$)	46
7	Python	48
7.1	Matrizes	48
7.2	Modpow	48
7.3	Busca binária	49

1 First things first

1.1 Includes

```
#include <bits/stdc++.h>
using namespace std;

#define ll long long
#define pb push_back
#define D(x) cout << #x " = " << (x) << endl

typedef vector<int> vi;
typedef vector<vi> vvi;

typedef pair<int, int> ii;
typedef vector<ii> vii;
```

2 Matemática

2.1 Algoritmo de euclides extendido

```
int xmdc(int a, int b, int &x, int &y)
{
    if (b == 0)
    {
        x = 1;
        y = 0;
        return a;
    }
    int x1, y1, mdc = xmdc(b, a % b, x1, y1);
    x = y1;
    y = x1 - (a / b) * y1;
    return mdc;
}
```

2.2 Máximo divisor comum

```
int mdc(int a, int b)
{
    int remainder;
    while (b != 0)
    {
        remainder = a % b;
        a = b;
        b = remainder;
    }
    return a;
}
```

2.3 Mínimo múltiplo comum

```
int mmc(int a, int b)
{
    int temp = mdc(a, b);
    return temp ? (a / temp * b) : 0;
}
```

2.4 Algoritmo de Pollard Rho

```
int pollard(int number)
{
    x_fixed = 2, cycle_size = 2, x = 2, factor = 1;
    while (factor == 1)
    {
        for (int count = 1; count <= cycle_size && factor <= 1; count++)
        {
            x = (x * x + 1) % number;
            factor = mdc(x - x_fixed, number);
        }
        cycle_size *= 2;
        x_fixed = x;
    }
    return factor;
}
```

2.5 Transformada rápida de Fourier

```
// Resolve:
// - De quantas maneiras conseguimos atingir Y com X tentativas
// - Dado X tentativas, conseguimos atingir Y?
// Complexidade:
// X * Ymax * Ymax(log Ymax)
```

```

// TEOREMA DA CONVOLUÇÃO:
// Podemos fazer a convolução de 2 polinômios utilizando a FFT
// Reduzindo a complexidade de  $n^2$  para  $n \log n$ 
// Definimos a convolução como  $h[i] = \sum(a[j] * b[j-i])$  para todo  $j$  de 0 a  $i$ .
// Exemplo:  $h[5] = a[5] * b[0] + a[4] * b[1] + a[3] * b[2]...$ 
// Segundo o teorema da convolução
//  $h(f * g)$  = transformada inversa de (transformada (f) * transformada (g))
// onde  $*$  é o operador de convolução.
// e  $*$  é o operador de multiplicação termo a termo.

#include <bits/stdc++.h>
using namespace std;

// primeira potência de 2 maior que o limite de H
const int MAX_DIST = 1 << 19;
typedef complex<double> cpx;
const double pi = acos(-1.0);
int maxDist;
void fft(vector<cpx> &a, bool invert)
{
    int n = (int)a.size();
    for (int i = 1, j = 0; i < n; ++i)
    {
        int bit = n >> 1;
        for (; j >= bit; bit >>= 1)
            j -= bit;
        j += bit;
        if (i < j)
            swap(a[i], a[j]);
    }
    for (int len = 2; len <= n; len <= 1)
    {
        double ang = 2 * pi / len * (invert ? -1 : 1);
        cpx wlen(cos(ang), sin(ang));
        for (int i = 0; i < n; i += len)
        {
            cpx w(1);
            for (int j = 0; j < len / 2; ++j)
            {
                cpx u = a[i + j], v = a[i + j + len / 2] * w;
                a[i + j] = u + v;
                a[i + j + len / 2] = u - v;
                w *= wlen;
            }
        }
    }
    if (invert)
        for (int i = 0; i < n; ++i)
            a[i] /= n;
}

// Exemplo:
// Há um robo que pode disparar bolas em N distâncias diferentes.
// Queremos saber se ele alcanca uma distância M com 1 ou 2 tacadas.
// Resolução:
// Podemos definir um vetor distances[MAX_DIST],
// onde a distances[i] = 1 se ele pode tacar até a distancia i
// e distances[i] = 0 caso contrario
// Para ver se o robo acerta com 1 tacada, é trivial.
// Para ver se o robo acerta com 2 tacadas, podemos fazer a convolução de distances com distances.
// Ex: Acertar a Pode[10] é igual a: Pode[10] || Pode[9] * Pode[1] || Pode[8] * Pode[2]...
// Ou seja,  $H = FFTi(FFT(distances) ** 2)$ ;

// Complexidade:
//  $2 * 200k * \log(200k) = 8m$ 

int main()
{
    ios::sync_with_stdio(false);
    cin.tie(0);
    int N, d;
    vector<cpx> distances;
    while (cin >> N)
    {
        maxDist = 0;
    }
}

```

```

distances = vector<cpx>(MAX_DIST);
// Distancia 0 é uma posição de "possível"
distances[0] = cpx(1, 0);
for (int i = 0; i < N; i++)
{
    cin >> d;
    if (d > maxDist)
        maxDist = d;
    distances[d] = cpx(1, 0);
}
int shiftAmount;
for (shiftAmount = 0; (maxDist >> shiftAmount) != 0; shiftAmount++)
{
    maxDist = 1 << (shiftAmount + 1);
    distances.resize(maxDist);
    fft(distances, false);
    // Multiplicação termo a termo de f e g, no caso, f = g = fftOut
    // fftOut *= fftOut
    for (int i = 0; i < maxDist; i++)
        distances[i] = distances[i] * distances[i];
    // transformada inversa da multiplicação termo a termo.
    fft(distances, true);
    cin >> N;
    int total = 0;
    for (int i = 0; i < N; i++)
    {
        cin >> d;
        // Entra a distancia d
        // e verifica se a parte real da distância[d] é positiva
        // distância[d] guarda de quantas maneiras conseguimos atingir D
        if (distances[d].real() > 0.01)
            total++;
    }
    cout << total << endl;
}
}

```

2.6 Matrizes

```

#include <bits/stdc++.h>
using namespace std;

#define ll long long

typedef vector<ll> vl;
typedef vector<vl> vvl;
const int mod = 1000000;

// Retorna a matriz I_n
vvl matrixUnit(int n)
{
    vvl res(n, vl(n));
    for (int i = 0; i < n; i++)
        res[i][i] = 1;
    return res;
}

// Retorna a+b
vvl matrixAdd(const vvl &a, const vvl &b)
{
    int n = a.size();
    int m = a[0].size();
    vvl res(n, vl(m));
    for (int i = 0; i < n; i++)
        for (int j = 0; j < m; j++)
            res[i][j] = (a[i][j] + b[i][j]) % mod;
    return res;
}

// Retorna a*b
vvl matrixMul(const vvl &a, const vvl &b)

```

```

{
    int n = a.size();
    int m = a[0].size();
    int k = b[0].size();
    vvl res(n, vl(k));
    for (int i = 0; i < n; i++)
        for (int j = 0; j < k; j++)
            for (int p = 0; p < m; p++)
                res[i][j] = (res[i][j] + ((a[i][p] % mod) * (b[p][j] % mod)) % mod) % mod;
    return res;
}

// Retorna a matriz a^p
vvl matrixPow(const vvl &a, long long p)
{
    if (p == 0)
        return matrixUnit(a.size());
    if (p & 1)
        return matrixMul(a, matrixPow(a, p - 1));
    return matrixPow(matrixMul(a, a), p / 2);
}

// Retorna sum^p_i=0 (a^i)
vvl matrixPowSum(const vvl &a, long long p)
{
    long long n = a.size();
    if (p == 0)
        return vvl(n, vl(n));
    if (p % 2 == 0)
        return matrixMul(matrixPowSum(a, p / 2), matrixAdd(matrixUnit(n), matrixPow(a, p / 2)));
    return matrixAdd(a, matrixMul(matrixPowSum(a, p - 1), a));
}

int main()
{
    long long n, l, k, i;
    while (scanf("%lld %lld %lld", &n, &l, &k) > 0)
    {
        vvl matriz = vvl(2, vl(2));
        matriz[0][0] = l;
        matriz[0][1] = k;
        matriz[1][0] = 1;
        matriz[1][1] = 0;
        matriz = matrixPow(matriz, n / 5);
        printf("%06lld\n", matriz[0][0]);
    }
}

```

2.7 Quantidade de fatores primos de um número

```

#define pb push_back
typedef vector<int> vi;
int main()
{
    long long i, j, n, qtd = 0, resp = 0;
    bool crivo[LIM] = {0};
    vi primos;
    for (i = 2; i < LIM; i++)
        if (!crivo[i])
        {
            primos.pb(i);
            for (j = i + i; j < LIM; j += i)
                crivo[j] = 1;
        }
    scanf("%lld", &n);
    for (auto it : primos)
        if (n % it == 0)
        {
            qtd++;
            n /= it;
            while (n % it == 0)

```

```

        n /= it;
    }
    else if (it > n)
        break;
    if (n != 1)
        qtd++;
    printf("%d\n", qtd);
}

```

2.8 Fatoração em números primos

```

vector<int> primeFactors(int n)
{
    vector<int> v;
    int sqrt = sqrt(n);
    while (n % 2 == 0)
    {
        v.push_back(2);
        n = n / 2;
    }
    for (int i = 3; i <= sqrt; i = i + 2)
    {
        while (n % i == 0)
        {
            v.push_back(i);
            n = n / i;
        }
    }
    if (n > 2)
        v.push_back(n);
    return v;
}

```

2.9 Modpow

```

int modPow(int a, int b, int m)
{
    int res = 1;
    for (; b > 0; b >>= 1)
    {
        if (b & 1)
            res = (long long)res * a % m;
        a = (long long)a * a % m;
    }
    return res;
}

```

2.10 Quantidade de coprimos.

```

#define pb push_back
typedef vector<int> vi;
bool crivo[100000] = {0};
vi primos;

int main()
{
    int i, j, n, resp;
    for(i = 2; i < 100000; i++)
        if(!crivo[i])
        {
            primos.pb(i);
            for(j = i + i; j < 100000; j += i)
                crivo[j] = 1;
        }
    while(scanf("%d", &n) > 0)
    {
        resp = n;
    }
}

```

```

for(auto &it : primos)
{
    if(it * it > n)
    {
        if(n != 1)
            resp -= resp / n;

        break;
    }
    if(n % it == 0)
    {
        resp -= resp / it;
        while(n % it == 0)
            n /= it;
    }
}
printf("%d\n", resp / 2);
}
}

```

2.11 Máximo e mínimo de funções (Busca ternaria)

```

double gss(double a, double b, double (*f)(double), double e = 1e-6)
{
    double r = (sqrt(5) - 1) / 2; //=.618...=golden ratio-1
    double x1 = b - r * (b - a), x2 = a + r * (b - a);
    double f1 = f(x1), f2 = f(x2);
    // while(fabs(f1 - f2) > e)
    while (b - a > e)
    {
        //change to > to find maximum
        if (f1 < f2)
        {
            b = x2;
            x2 = x1;
            f2 = f1;
            x1 = b - r * (b - a);
            f1 = f(x1);
        }
        else
        {
            a = x1;
            x1 = x2;
            f1 = f2;
            x2 = a + r * (b - a);
            f2 = f(x2);
        }
    }
    // return f1;
    return (b + a) / 2;
}

```

2.12 Todos divisores de um número

```

vector<int> divisores(int n)
{
    vector<int> divis;
    int sqrtn = sqrt(n);
    while(sqrtn * sqrtn < n)
        sqrtn++;
    for (i = 1; i < sqrtn; i++)
        if (!(n % i))
            divis.push_back(i), divis.push_back(n / i);
    if(sqrtn * sqrtn == n)
        divis.push_back(sqrtn);
    return divis;
}

```

2.13 Crivo de Eratóstenes segmentado

```
char nprimo[100001] = {0}; // tamanho = sqrt(maximo)
std::vector<int> primos;

int main()
{
    int n, a, b, i, j;

    nprimo[1] = 1;
    nprimo[0] = 1;

    for (i = 2; i < 320; i++) // i [2, sqrt(sqrt(maximo))]
        if (!nprimo[i])
            for (j = i * i; j < 100001; j += i) // j [i^2, sqrt(maximo)]
                nprimo[j] = 1;

    for (i = 2; i < 100001; i++)
        if (!nprimo[i])
            primos.push_back(i);

    scanf("%d", &n);
    while (n--)
    {
        scanf("%d %d", &a, &b);
        if (a > 100000 && b > 100000)
        {
            // (a > sqrt(N) && b > sqrt(N))
            for (i = a; i <= b; i++)
            {
                for (j = 0; j < primos.size(); j++)
                    if (i % primos[j] == 0)
                        goto ab;

                printf("%d\n", i);
            }
        }
        else if (a < 100001 && b < 100001)
        {
            // (a < sqrt(N) && b < sqrt(N))
            for (i = a; i <= b; i++)
                if (!nprimo[i])
                    printf("%d\n", i);
        }
        else
        {
            for (i = 0; i < primos.size(); i++)
                if (primos[i] >= a)
                    break;

            for (; i < primos.size(); i++)
                printf("%d\n", primos[i]);

            for (; i <= b; i++)
            {
                for (j = 0; j < primos.size(); j++)
                    if (i % primos[j] == 0)
                        goto ac;

                printf("%d\n", i);
            }
        }
    }
}
```

3 Grafos

3.1 Grafos

```
#include <bits/stdc++.h>
using namespace std;

#define ll long long
#define pb push_back

typedef vector<int> vi;

struct Vertice
{
    int id, pai;
    ll dist;

    Vertice(int id, ll dist = 1, int pai = -1) : id(id), dist(dist), pai(pai) {}

    bool operator<(Vertice a) const
    {
        return a.dist < dist;
    }
};

typedef vector<Vertice> vv;
typedef vector<vv> vvv;

struct Grafo
{
    vvv g;
    vi pais;
    int n;

    Grafo(int n) : n(n)
    {
        g = vvv(n, vv());
        pais = vi(n);
    }

    void operator=(Grafo const &a)
    {
        g = a.g;
        pais = a.pais;
        n = a.n;
    }

    void addAresta(int a, int b, ll d = 0)
    {
        g[a].pb(Vertice(b, d));
    }

    void removeAresta(int a, int b)
    {
        g[a].erase(remove_if(g[a].begin(), g[a].end(), [b](Vertice v)
        {
            return v.id == b;
        })));
    }

    ll valAresta(int a, int b)
    {
        for (auto it : g[a])
            if (it.id == b)
                return it.dist;

        return 0;
    }

    void modificaAresta(int a, int b, ll dif)
    {
        for (auto it = g[a].begin(); it != g[a].end(); it++)
        {
            if (it->id == b)
            {
                it->dist += dif;
                if (it->dist == 0)
                    g[a].erase(it);
                break;
            }
        }
    }

    ll dijkstra(int s, int d)
    {

```

```

priority_queue<Vertice> fila;
bool visitados[n];
fill(visitados, visitados + n, 0);
fill(pais.begin(), pais.end(), -1);
fila.push(Vertice(s, 0));
auto top = fila.top();
while (top.id != d)
{
    if (!visitados[top.id])
    {
        for (auto &it : g[top.id])
            if (!visitados[it.id])
                fila.push(Vertice(it.id, it.dist + top.dist, top.id));

        visitados[top.id] = 1;
        pais[top.id] = top.pai;
    }
    fila.pop();
    if (fila.empty())
        return -1;
    top = fila.top();
}
pais[top.id] = top.pai;
return top.dist;
}

ll busca(int s, int d)
{
    queue<Vertice> fila;
    bool visitados[n];
    fill(visitados, visitados + n, 0);
    fill(pais.begin(), pais.end(), -1);
    fila.push(Vertice(s, 0));
    auto top = fila.front();
    while (top.id != d)
    {
        if (!visitados[top.id])
        {
            for (auto &it : g[top.id])
                if (!visitados[it.id])
                    fila.push(Vertice(it.id, it.dist + 1, top.id));

            visitados[top.id] = 1;
            pais[top.id] = top.pai;
        }
        fila.pop();
        if (fila.empty())
            return -1;
        top = fila.front();
    }
    pais[top.id] = top.pai;
    return top.dist;
}

ll fluxo_maximo(int s, int d)
{
    int u, v;
    ll flow = 0;
    Grafo g2 = *this;
    while (g2.busca(s, d) >= 0)
    {
        ll path = 1ll << 50;
        for (v = d; v != s; v = u)
        {
            u = g2.pais[v];
            path = min(path, valAresta(u, v));
        }
    }
}

```

```

        for (v = d; v != s; v = u)
        {
            u = g2.pais[v];
            g2.modificaAresta(u, v, -path);
            g2.modificaAresta(v, u, path);
        }
        flow += path;
    }
    return flow;
};

int main()
{
    Grafo g(20);

    g.addAresta(1, 2, 1);
    g.addAresta(1, 3, 5);
    g.addAresta(2, 1, 6);
    g.addAresta(3, 2, 10);

    g.removeAresta(1, 2);

    for (auto it : g.g[1])
        cout << it.id << endl; // 3
    cout << g.dijkstra(1, 2) << endl; // 15
    cout << g.fluxo_maximo(1, 2) << endl; // 5
}

```

3.2 Todas as pontes de um grafo

```

#include <bits/stdc++.h>
#define NIL -1
using namespace std;

// A class that represents an undirected graph
class Graph
{
    int V; // No. of vertices
    list<int> *adj; // A dynamic array of adjacency lists
    void bridgeUtil(int v, bool visited[], int disc[], int low[],
                    int parent[]);

public:
    Graph(int V); // Constructor
    void addEdge(int v, int w); // to add an edge to graph
    void bridge(); // prints all bridges
};

Graph::Graph(int V)
{
    this->V = V;
    adj = new list<int>[V];
}

void Graph::addEdge(int v, int w)
{
    adj[v].push_back(w);
    adj[w].push_back(v); // Note: the graph is undirected
}

// A recursive function that finds and prints bridges using
// DFS traversal
// u --> The vertex to be visited next
// visited[] --> keeps track of visited vertices
// disc[] --> Stores discovery times of visited vertices
// parent[] --> Stores parent vertices in DFS tree
void Graph::bridgeUtil(int u, bool visited[], int disc[],
                        int low[], int parent[])
{
    // A static variable is used for simplicity, we can
    // avoid use of static variable by passing a pointer.
    static int time = 0;

    // Mark the current node as visited
    visited[u] = true;

    // Initialize discovery time and low value
    disc[u] = low[u] = ++time;
}

```

```

// Go through all vertices adjacent to this
list<int>::iterator i;
for (i = adj[u].begin(); i != adj[u].end(); ++i)
{
    int v = *i; // v is current adjacent of u

    // If v is not visited yet, then recur for it
    if (!visited[v])
    {
        parent[v] = u;
        bridgeUtil(v, visited, disc, low, parent);

        // Check if the subtree rooted with v has a
        // connection to one of the ancestors of u
        low[u] = min(low[u], low[v]);

        // If the lowest vertex reachable from subtree
        // under v is below u in DFS tree, then u-v
        // is a bridge
        if (low[v] > disc[u])
            cout << u << " " << v << endl;
    }

    // Update low value of u for parent function calls.
    else if (v != parent[u])
        low[u] = min(low[u], disc[v]);
}
}

// DFS based function to find all bridges. It uses recursive
// function bridgeUtil()
void Graph::bridge()
{
    // Mark all the vertices as not visited
    bool *visited = new bool[V];
    int *disc = new int[V];
    int *low = new int[V];
    int *parent = new int[V];

    // Initialize parent and visited arrays
    for (int i = 0; i < V; i++)
    {
        parent[i] = NIL;
        visited[i] = false;
    }

    // Call the recursive helper function to find Bridges
    // in DFS tree rooted with vertex 'i'
    for (int i = 0; i < V; i++)
        if (visited[i] == false)
            bridgeUtil(i, visited, disc, low, parent);
}

// Driver program to test above function
int main()
{
    // Create graphs given in above diagrams
    cout << "\nBridges in first graph \n";
    Graph g1(5);
    g1.addEdge(1, 0);
    g1.addEdge(0, 2);
    g1.addEdge(2, 1);
    g1.addEdge(0, 3);
    g1.addEdge(3, 4);
    g1.bridge();

    cout << "\nBridges in second graph \n";
    Graph g2(4);
    g2.addEdge(0, 1);
    g2.addEdge(1, 2);
    g2.addEdge(2, 3);
    g2.bridge();

    cout << "\nBridges in third graph \n";
    Graph g3(7);
    g3.addEdge(0, 1);
    g3.addEdge(1, 2);
    g3.addEdge(2, 0);
    g3.addEdge(1, 3);
    g3.addEdge(1, 4);
    g3.addEdge(1, 6);
}

```

```

    g3.addEdge(3, 5);
    g3.addEdge(4, 5);
    g3.bridge();

    return 0;
}

```

3.3 Isomorfismo de árvores

```

#include <bits/stdc++.h>
#define pb push_back
using namespace std;
typedef vector<int> vi;
typedef vector<vi> vvi;
int main()
{
    int n, i, a, b, count, atual;
    while (scanf("%d", &n) > 0)
    {
        map<multiset<int>, int> mapa;
        vvi grafo_esq(n), grafo_dir(n);
        vi valores_esq(n), valores_dir(n), pais_esq(n, -1), pais_dir(n, -1);
        set<int> centros_esq, centros_dir;

        for (i = 0; i < n - 1; i++)
            scanf("%d %d", &a, &b), grafo_esq[a - 1].pb(b - 1), grafo_esq[b - 1].pb(a - 1),
            centros_esq.insert(i);
        centros_esq.insert(i);

        for (i = 0; i < n - 1; i++)
            scanf("%d %d", &a, &b), grafo_dir[a - 1].pb(b - 1), grafo_dir[b - 1].pb(a - 1),
            centros_dir.insert(i);
        centros_dir.insert(i);

        atual = count = 0;
        while (centros_esq.size() > 2)
        {
            vi a_remover;
            for (auto &linha : centros_esq)
            {
                int count = 0, pai;
                for (auto &it : grafo_esq[linha])
                    if (centros_esq.count(it))
                        count++, pai = it;

                if (count == 1)
                {
                    pais_esq[linha] = pai;
                    a_remover.pb(linha);
                }
            }
            for (auto &it : a_remover)
            {
                multiset<int> valores;
                for (auto &it2 : grafo_esq[it])
                    if (pais_esq[it2] == it)
                        valores.insert(valores_esq[it2]);

                if (mapa.count(valores))
                    valores_esq[it] = mapa[valores];
                else
                    valores_esq[it] = mapa[valores] = atual++;
                centros_esq.erase(it);
            }
        }
        for (auto &it : centros_esq)
        {
            multiset<int> valores;
            for (auto &it2 : grafo_esq[it])
                if (pais_esq[it2] == it)
                    valores.insert(valores_esq[it2]);

```

```

        if (mapa.count(valores))
            valores_esq[it] = mapa[valores];
        else
            valores_esq[it] = mapa[valores] = atual++;
    }
    while (centros_dir.size() > 2)
    {
        vi a_remove;
        for (auto &linha : centros_dir)
        {
            int count = 0, pai;
            for (auto &it : grafo_dir[linha])
                if (centros_dir.count(it))
                    count++, pai = it;

            if (count == 1)
            {
                pais_dir[linha] = pai;
                a_remove.pb(linha);
            }
        }
        for (auto &it : a_remove)
        {
            multiset<int> valores;
            for (auto &it2 : grafo_dir[it])
                if (pais_dir[it2] == it)
                    valores.insert(valores_dir[it2]);

            if (mapa.count(valores))
                valores_dir[it] = mapa[valores];
            else
                valores_dir[it] = mapa[valores] = atual++;

            centros_dir.erase(it);
        }
    }
    for (auto &it : centros_dir)
    {
        multiset<int> valores;
        for (auto &it2 : grafo_dir[it])
            if (pais_dir[it2] == it)
                valores.insert(valores_dir[it2]);

        if (mapa.count(valores))
            valores_dir[it] = mapa[valores];
        else
            valores_dir[it] = mapa[valores] = atual++;
    }

    sort(valores_dir.begin(), valores_dir.end());
    sort(valores_esq.begin(), valores_esq.end());

    for(i = 0; i < valores_dir.size(); i++)
        if(valores_esq[i] != valores_dir[i])
        {
            puts("N");
            goto proximo;
        }

    puts("S");

proximo:
    ;
}
}

```

3.4 Matching máximo em grafo bipartido

```

const int MAXN1 = 50000, MAXN2 = 50000, MAXM = 150000;
int n1, n2, edges, last[MAXN1], prev[MAXM], head[MAXM], matching[MAXN2], dist[MAXN1], Q[MAXN1], used[MAXN1], vis
    [MAXN1];

void init(int _n1, int _n2)
{
    n1 = _n1;
    n2 = _n2;

```

```

    edges = 0;
    fill(last, last + n1, -1);
}

void addAresta(int u, int v)
{
    head[edges] = v;
    prev[edges] = last[u];
    last[u] = edges++;
}

void bfs()
{
    fill(dist, dist + n1, -1);
    int sizeQ = 0;
    for (int u = 0; u < n1; ++u)
    {
        if (!used[u])
        {
            Q[sizeQ++] = u;
            dist[u] = 0;
        }
    }
    for (int i = 0; i < sizeQ; i++)
    {
        int u1 = Q[i];
        for (int e = last[u1]; e >= 0; e = prev[e])
        {
            int u2 = matching[head[e]];
            if (u2 >= 0 && dist[u2] < 0)
            {
                dist[u2] = dist[u1] + 1;
                Q[sizeQ++] = u2;
            }
        }
    }
}

bool dfs(int u1)
{
    vis[u1] = true;
    for (int e = last[u1]; e >= 0; e = prev[e])
    {
        int v = head[e];
        int u2 = matching[v];
        if (u2 < 0 || !vis[u2] && dist[u2] == dist[u1] + 1 && dfs(u2))
        {
            matching[v] = u1;
            used[u1] = true;
            return true;
        }
    }
    return false;
}

int maxMatching()
{
    fill(used, used + n1, false);
    fill(matching, matching + n2, -1);
    for (int res = 0;;)
    {
        bfs();
        fill(vis, vis + n1, false);
        int f = 0;
        for (int u = 0; u < n1; ++u)
            if (!used[u] && dfs(u))
                ++f;
        if (!f)
            return res;
        res += f;
    }
}

```

3.5 Algoritmo húngaro

```

#define N 100
#define INF 100000000

int cost[N][N], n, max_match, lx[N], ly[N], xy[N], yx[N], slack[N], slackx[N], prev[N];
bool S[N], T[N];

```



```

void init_labels()
{
    memset(lx, 0, sizeof(lx));
    memset(ly, 0, sizeof(ly));
    for (int x = 0; x < n; x++)
        for (int y = 0; y < n; y++)
            lx[x] = max(lx[x], cost[x][y]);
}

void update_labels()
{
    int x, y, delta = INF;
    for (y = 0; y < n; y++)
        if (!T[y])
            delta = min(delta, slack[y]);
    for (x = 0; x < n; x++)
        if (S[x])
            lx[x] -= delta;
    for (y = 0; y < n; y++)
        if (T[y])
            ly[y] += delta;
    for (y = 0; y < n; y++)
        if (!T[y])
            slack[y] -= delta;
}

void add_to_tree(int x, int prevx)
{
    S[x] = true;
    prev[x] = prevx;
    for (int y = 0; y < n; y++)
        if (lx[x] + ly[y] - cost[x][y] < slack[y])
        {
            slack[y] = lx[x] + ly[y] - cost[x][y];
            slackx[y] = x;
        }
}

void augment()
{
    if (max_match == n)
        return;
    int x, y, root, q[N], wr = 0, rd = 0;
    memset(S, false, sizeof(S));
    memset(T, false, sizeof(T));
    memset(prev, -1, sizeof(prev));

    for (x = 0; x < n; x++)
        if (xy[x] == -1)
        {
            q[wr++] = root = x;
            prev[x] = -2;
            S[x] = true;
            break;
        }

    for (y = 0; y < n; y++)
    {
        slack[y] = lx[root] + ly[y] - cost[root][y];
        slackx[y] = root;
    }

    while (true)
    {
        while (rd < wr)
        {
            x = q[rd++];
            for (y = 0; y < n; y++)
                if (cost[x][y] == lx[x] + ly[y] && !T[y])
                {
                    if (yx[y] == -1)
                        break;
                    T[y] = true;
                    q[wr++] = yx[y];
                    add_to_tree(yx[y], x);
                }
            if (y < n)
                break;
        }
    }
}

```

```

    if (y < n)
        break;
    update_labels();
    wr = rd = 0;
    for (y = 0; y < n; y++)
        if (!T[y] && slack[y] == 0)
        {
            if (yx[y] == -1)
            {
                x = slackx[y];
                break;
            }
            else
            {
                T[y] = true;
                if (!S[yx[y]])
                {
                    q[wr++] = yx[y];
                    add_to_tree(yx[y], slackx[y]);
                }
            }
        }
    if (y < n)
        break;
}
if (y < n)
{
    max_match++;
    for (int cx = x, cy = y, ty; cx != -2; cx = prev[cx], cy = ty)
    {
        ty = xy[cx];
        yx[cy] = cx;
        xy[cx] = cy;
    }
    augment();
}
}
int hungaro()
{
    int ret = 0;
    max_match = 0;
    memset(xy, -1, sizeof(xy));
    memset(yx, -1, sizeof(yx));
    init_labels();
    augment();

    for (int x = 0; x < n; x++)
        ret += cost[x][xy[x]];

    return ret;
}

```

3.6 Isomorfismo de árvores para a biblioteca de grafos.

```

bool isomorfismo(Grafo &grafo)
{
    if (this->n != grafo.n)
        return false;

    map<multiset<int>, int> mapa;
    vi valores_esq(n), valores_dir(n), pais_esq(n, -1), pais_dir(n, -1);
    set<int> centros_esq, centros_dir;
    int atual = 0, count = 0, i;
    for (i = 0; i < n; i++)
        centros_esq.insert(i), centros_dir.insert(i);

    while (centros_esq.size() > 2)
    {
        vi a_remove;
        for (auto &linha : centros_esq)
        {
            int count = 0, pai;
            for (auto &it : this->g[linha])
                if (centros_esq.count(it.id))

```

```

        count++, pai = it.id;
    if (count == 1)
    {
        pais_esq[linha] = pai;
        a_remover.pb(linha);
    }
}
for (auto &it : a_remover)
{
    multiset<int> valores;
    for (auto &it2 : this->g[it])
        if (pais_esq[it2.id] == it)
            valores.insert(valores_esq[it2.id]);

    if (mapa.count(valores))
        valores_esq[it] = mapa[valores];
    else
        valores_esq[it] = mapa[valores] = atual++;
    centros_esq.erase(it);
}
}
for (auto &it : centros_esq)
{
    multiset<int> valores;
    for (auto &it2 : this->g[it])
        if (pais_esq[it2.id] == it)
            valores.insert(valores_esq[it2.id]);

    if (mapa.count(valores))
        valores_esq[it] = mapa[valores];
    else
        valores_esq[it] = mapa[valores] = atual++;
}
while (centros_dir.size() > 2)
{
    vi a_remover;
    for (auto &linha : centros_dir)
    {
        int count = 0, pai;
        for (auto &it : grafo.g[linha])
            if (centros_dir.count(it.id))
                count++, pai = it.id;

        if (count == 1)
        {
            pais_dir[linha] = pai;
            a_remover.pb(linha);
        }
    }
    for (auto &it : a_remover)
    {
        multiset<int> valores;
        for (auto &it2 : grafo.g[it])
            if (pais_dir[it2.id] == it)
                valores.insert(valores_dir[it2.id]);

        if (mapa.count(valores))
            valores_dir[it] = mapa[valores];
        else
            valores_dir[it] = mapa[valores] = atual++;
        centros_dir.erase(it);
    }
}
for (auto &it : centros_dir)
{
    multiset<int> valores;
    for (auto &it2 : grafo.g[it])
        if (pais_dir[it2.id] == it)
            valores.insert(valores_dir[it2.id]);

    if (mapa.count(valores))
        valores_dir[it] = mapa[valores];
    else

```

```

        valores_dir[it] = mapa[valores] = atual++;
    }
    sort(valores_dir.begin(), valores_dir.end());
    sort(valores_esq.begin(), valores_esq.end());
    for (i = 0; i < valores_dir.size(); i++)
        if (valores_esq[i] != valores_dir[i])
            return false;
    return true;
}

int main()
{
    int n, a, i, j, resp = 0;
    scanf("%d");
    vector<Grafo> arvores;
    while (scanf("%d", &n) > 0)
    {
        Grafo g(n);
        for (int i = 1; i < n; i++)
            scanf("%d", &a),
                g.addAresta(i, a - 1),
                g.addAresta(a - 1, i);

        arvores.push_back(g);
    }
    for(i = 0; i < arvores.size(); i++)
    {
        for(j = i + 1; j < arvores.size(); j++)
        {
            if(arvores[i].isomorfismo(arvores[j]))
                break;
        }
        if(j == arvores.size())
            resp++;
    }
    printf("%d\n", resp);
}

```

3.7 Fluxo maximo de custo minimo

```

int INFINITO = 1 << 30, MAXM = 10000, MAXN = 1000;
struct Node
{
    int x, y, cap, cost;
    int next;
} edge[MAXM];
class MinCost
{
public:
    int e, head[MAXN], dis[MAXN], pre[MAXN], record[MAXN], inq[MAXN];
    void Addedge(int x, int y, int cap, int cost)
    {
        edge[e].x = x, edge[e].y = y, edge[e].cap = cap, edge[e].cost = cost;
        edge[e].next = head[x], head[x] = e++;
        edge[e].x = y, edge[e].y = x, edge[e].cap = 0, edge[e].cost = -cost;
        edge[e].next = head[y], head[y] = e++;
    }
    int mincost(int s, int t)
    {
        int mncost = 0, flow, totflow = 0;
        int i, x, y;
        while (1)
        {
            memset(dis, 63, sizeof(dis));
            int oo = dis[0];
            dis[s] = 0;
            deque<int> Q;
            Q.push_front(s);
            while (!Q.empty())
            {
                x = Q.front(), Q.pop_front();

```

```

    inq[x] = 0;
    for (i = head[x]; i != -1; i = edge[i].next)
    {
        y = edge[i].y;
        if (edge[i].cap > 0 && dis[y] > dis[x] + edge[i].cost)
        {
            dis[y] = dis[x] + edge[i].cost;
            pre[y] = x, record[y] = i;
            if (inq[y] == 0)
            {
                inq[y] = 1;
                if (Q.size() && dis[Q.front()] > dis[y])
                    Q.push_front(y);
                else
                    Q.push_back(y);
            }
        }
    }
}
if (dis[t] == INFINITO)
    break;
flow = INFINITO;
for (x = t; x != s; x = pre[x])
{
    int ri = record[x];
    flow = min(flow, edge[ri].cap);
}
for (x = t; x != s; x = pre[x])
{
    int ri = record[x];
    edge[ri].cap -= flow;
    edge[ri ^ 1].cap += flow;
    edge[ri ^ 1].cost = -edge[ri].cost;
}
totflow += flow;
mncost += dis[t] * flow;
}
return mncost;
}
void init(int n)
{
    e = 0;
    for (int i = 0; i <= n; i++)
        head[i] = -1;
}
} g;

// Uso:
// Iniciar um grafo com 200 vértices:
// g.init(200);
// Adicionar aresta entre 1 e 2 com custo 10 e capacidade 5
// g.Addedge(1, 2, 10, 5);
// Achar o fluxo de custo mínimo entre 1 e 2
// g.mincost(1, 2);

```

4 Strings

4.1 Suffix array

```
//Usage:
// Fill txt with the characters of the txtng.
// Call SuffixSort(n), where n is the length of the txtng stored in txt.
// That's it!

//Output:
// SA = The suffix array.
// Contains the n suffixes of txt sorted in lexicographical order.
// Each suffix is represented as a single integer (the SAition of txt where it starts).
// iSA = The inverse of the suffix array. iSA[i] = the index of the suffix txt[i..n)
// in the SA array. (In other words, SA[i] = k <=> iSA[k] = i)
// With this array, you can compare two suffixes in O(1): Suffix txt[i..n) is smaller
// than txt[j..n) if and only if iSA[i] < iSA[j]

const int MAX = 100010;
char txt[MAX]; //input
int iSA[MAX], SA[MAX]; //output
int cnt[MAX], prox[MAX]; //internal
bool bh[MAX], b2h[MAX];

// Compares two suffixes according to their first characters
bool smaller_first_char(int a, int b)
{
    return txt[a] < txt[b];
}

void suffixSort(int n)
{
    for (int i = 0; i < n; ++i)
        SA[i] = i;
    sort(SA, SA + n, smaller_first_char);
    for (int i = 0; i < n; ++i)
    {
        bh[i] = i == 0 || txt[SA[i]] != txt[SA[i - 1]];
        b2h[i] = false;
    }
    for (int h = 1; h < n; h <= 1)
    {
        int buckets = 0;
        for (int i = 0, j; i < n; i = j)
        {
            j = i + 1;
            while (j < n && !bh[j])
                j++;
            prox[i] = j;
            buckets++;
        }
        if (buckets == n)
            break;
        for (int i = 0; i < n; i = prox[i])
        {
            cnt[i] = 0;
            for (int j = i; j < prox[i]; ++j)
                iSA[SA[j]] = i;
        }
        cnt[iSA[n - h]]++;
        b2h[iSA[n - h]] = true;
        for (int i = 0; i < n; i = prox[i])
        {
            for (int j = i; j < prox[i]; ++j)
            {
                int s = SA[j] - h;
                if (s >= 0)
                {
                    int head = iSA[s];
                    iSA[s] = head + cnt[head]++;
                    b2h[iSA[s]] = true;
                }
            }
        }
    }
}
```

```

    }
    for (int j = i; j < prox[i]; ++j)
    {
        int s = SA[j] - h;
        if (s >= 0 && b2h[iSA[s]])
            for (int k = iSA[s] + 1; !bh[k] && b2h[k]; k++)
                b2h[k] = false;
    }
    for (int i = 0; i < n; ++i)
    {
        SA[iSA[i]] = i;
        bh[i] |= b2h[i];
    }
}
for (int i = 0; i < n; ++i)
{
    iSA[SA[i]] = i;
}
}
// End of suffix array algorithm

// Begin of the O(n) longest common prefix algorithm
int lcp[MAX];
// lcp[i] = length of the longest common prefix of suffix SA[i] and suffix SA[i-1]
// lcp[0] = 0
void getlcp(int n)
{
    for (int i = 0; i < n; ++i)
        iSA[SA[i]] = i;
    lcp[0] = 0;
    for (int i = 0, h = 0; i < n; ++i)
    {
        if (iSA[i] > 0)
        {
            int j = SA[iSA[i] - 1];
            while (i + h < n && j + h < n && txt[i + h] == txt[j + h])
                h++;
            lcp[iSA[i]] = h;
            if (h > 0)
                h--;
        }
    }
}
}

```

4.2 Algoritmo Z

```

// Algoritmo Z:
// Entrada -> uma string S
// Saída -> Um vetor onde p[k] = i implica que a substring
// que começa em S[k] e termina em S[k + i - 1] é um prefixo de S

#include <bits/stdc++.h>
using namespace std;
void getZarr(string str, int Z[]);
// prints all occurrences of pattern in text using Z algo
void search(string text, string pattern)
{
    // Create concatenated string "P$T"
    string concat = pattern + "$" + text;
    int l = concat.length();

    // Construct Z array
    int Z[l];
    getZarr(concat, Z);

    // now looping through Z array for matching condition
    for (int i = 0; i < l; ++i)
    {
        // if Z[i] (matched region) is equal to pattern
        // length we got the pattern
        if (Z[i] == pattern.length())
            cout << "Pattern found at index "

```

```

        << i - pattern.length() - 1 << endl;
    }
}

// Fills Z array for given string str[]
void getZarr(string str, int Z[])
{
    int n = str.length();
    int L, R, k;

    // [L,R] make a window which matches with prefix of s
    L = R = 0;
    for (int i = 1; i < n; ++i)
    {
        // if i>R nothing matches so we will calculate.
        // Z[i] using naive way.
        if (i > R)
        {
            L = R = i;

            // R-L = 0 in starting, so it will start
            // checking from 0'th index. For example,
            // for "ababab" and i = 1, the value of R
            // remains 0 and Z[i] becomes 0. For string
            // "aaaaaa" and i = 1, Z[i] and R become 5
            while (R < n && str[R - L] == str[R])
                R++;
            Z[i] = R - L;
            R--;
        }
        else
        {
            // k = i-L so k corresponds to number which
            // matches in [L,R] interval.
            k = i - L;

            // if Z[k] is less than remaining interval
            // then Z[i] will be equal to Z[k].
            // For example, str = "ababab", i = 3, R = 5
            // and L = 2
            if (Z[k] < R - i + 1)
                Z[i] = Z[k];

            // For example str = "aaaaaa" and i = 2, R is 5,
            // L is 0
            else
            {
                // else start from R and check manually
                L = i;
                while (R < n && str[R - L] == str[R])
                    R++;
                Z[i] = R - L;
                R--;
            }
        }
    }
}

// Driver program
int main()
{
    string text = "GEEKS FOR GEEKS";
    string pattern = "GEEK";
    search(text, pattern);
    return 0;
}

```

4.3 Strings em Arvore

```

#include <bits/stdc++.h>
using namespace std;

struct No
{
    char c;
    map<char, No> filhos;
};

void imprime(No &atual)
{
    cout << atual.c << ": ";
}

```



```

    for(auto it : atual.filhos)
        cout << it.first << " ";
    cout << endl;
    for(auto it : atual.filhos)
        imprime(it.second);
}

int main()
{
    int n, i;
    string pal;

    // Lê n palavras e monta a arvore de todas as palavras, adicionando $ a cada fim de palavra
    while(cin >> n)
    {
        No inicio, *atual;
        inicio.c = '-';
        for(i = 0; i < n; i++)
        {
            atual = &inicio;
            cin >> pal;
            for(auto &it : pal)
            {
                if(atual->filhos.count(it))
                    atual = &(atual->filhos[it]);
                else
                {
                    atual->filhos[it] = No();
                    atual = &(atual->filhos[it]);
                    atual->c = it;
                }
            }
            atual->filhos['$'] = No();
            atual->filhos['$'].c = '$';
        }
        imprime(inicio);
    }
}

```

4.4 Split strings

```

vector<string> sSplit(string s, string div)
{
    vector<string> parseado;
    string aux = "";
    string aux2 = "";
    int i, j;
    for(i = 0, j = 0; i < s.length(); ++i)
    {
        if(s[i] != div[j])
        {
            aux += s[i];
        }
        else
        {
            aux2 = "";
            for(; j < div.length() && i < s.length() && s[i] == div[j]; ++i, ++j)
                aux2 += s[i];

            if(j == div.length())
            {
                parseado.push_back(aux);
                aux = "";
            }
            else
            {
                aux += aux2;
            }

            --i;
            j = 0;
        }
    }
    parseado.push_back(aux);
}

```

```
    return parseado;  
}
```

5 Geometria

5.1 Linha de eventos radial

```
// - Radial sweep in Q2 quadrant in nlogn.
// - Sorts events using cross product to avoid dealing with
//   numeric problems.
#include <bits/stdc++.h>
using namespace std;

struct Point
{
    Point(int x = 0, int y = 0) : x(x), y(y) {}

    bool operator<(const Point &o) const
    {
        // Order points in a quadrant by angle with origin:
        // Uses anti-clockwise order by returning true when the
        // cross product between the points is positive.
        return (x * o.y - y * o.x) > 0;
    }

    /*
    bool operator<=(const Point& o) const {
        return (x*o.y - y*o.x) >= 0;
    }
    */
    int x, y;
};

pair<int, int> solve(const vector<Point> &points)
{
    map<Point, pair<int, int> > events;

    Point begin(0, 1);
    Point end(-1, 0);

    // Add events on the borders to guarantee that we consider them.
    events[begin];
    events[end];

    int superior = 0; // Number of points in Q1 quadrant.
    int same = 0;    // Number of points in origin.
    int active = 0;  // Number of current points in Q2 and Q4 quadrant better
    // than origin.

    int best_pos = points.size();
    int worst_pos = 0;

    for (const auto &p : points)
    {
        if (p.x < 0 && p.y < 0) {}
        else if (p.x > 0 && p.y > 0) superior++;
        else if (p.x == 0 && p.y == 0) same++;
        else if (p.x <= 0 && p.y >= 0)
        {
            // assert(begin <= Point(p.x, p.y));
            //assert(Point(p.x, p.y) <= end);
            events[Point(p.x, p.y)].first++;
        }
        else if (p.x >= 0 && p.y <= 0)
        {
            //assert(begin <= Point(-p.x, -p.y));
            //assert(Point(-p.x, -p.y) <= end);
            active++;
            events[Point(-p.x, -p.y)].second++;
        }
        else assert(false);
    }

    for (const auto &e : events)
    {
        int tie_best_pos = superior + active - e.second.second;
        int tie_worst_pos = superior + active + e.second.first + same;
        active += e.second.first - e.second.second;

        best_pos = min(best_pos, tie_best_pos);
        worst_pos = max(worst_pos, tie_worst_pos);
    }

    return make_pair(best_pos + 1, worst_pos + 1);
}
```

```

// Reads the set of points and centers them around Maria's product.
vector<Point> read()
{
    int n, cx, cy;
    cin >> n >> cx >> cy;
    vector<Point> points(n - 1);
    for (Point &p : points)
    {
        cin >> p.x >> p.y;
        p.x -= cx;
        p.y -= cy;
    }
    return points;
}

int main()
{
    auto input = read();
    auto solution = solve(input);

    for (auto &i : input)
        swap(i.x, i.y);

    assert(solution == solve(input));

    cout << solution.first << " " << solution.second << endl;
    return 0;
}

```

5.2 KD-Tree para pares mais próximos em $O(\log(n))$

```

#include <bits/stdc++.h>
using namespace std;
typedef long long ntype;
const ntype sentry = numeric_limits<ntype>::max();

// point structure for 2D-tree, can be extended to 3D
struct point
{
    ntype x, y;
    point(ntype xx = 0, ntype yy = 0) : x(xx), y(yy) {}
};

bool operator==(const point &a, const point &b)
{
    return a.x == b.x && a.y == b.y;
}

// sorts points on x-coordinate
bool on_x(const point &a, const point &b)
{
    return a.x < b.x;
}

// sorts points on y-coordinate
bool on_y(const point &a, const point &b)
{
    return a.y < b.y;
}

// squared distance between points
ntype pdist2(const point &a, const point &b)
{
    ntype dx = a.x - b.x, dy = a.y - b.y;
    return dx * dx + dy * dy;
}

// bounding box for a set of points
struct bbox
{
    ntype x0, x1, y0, y1;

    bbox() : x0(sentry), x1(-sentry), y0(sentry), y1(-sentry) {}

    // computes bounding box from a bunch of points
    void compute(const vector<point> &v)
    {
        for (int i = 0; i < v.size(); ++i)
        {
            x0 = min(x0, v[i].x);

```

```

        x1 = max(x1, v[i].x);
        y0 = min(y0, v[i].y);
        y1 = max(y1, v[i].y);
    }
}

// squared distance between a point and this bbox, 0 if inside
ntype distance(const point &p)
{
    if (p.x < x0)
    {
        if (p.y < y0)    return pdist2(point(x0, y0), p);
        else if (p.y > y1) return pdist2(point(x0, y1), p);
        else            return pdist2(point(x0, p.y), p);
    }
    else if (p.x > x1)
    {
        if (p.y < y0)    return pdist2(point(x1, y0), p);
        else if (p.y > y1) return pdist2(point(x1, y1), p);
        else            return pdist2(point(x1, p.y), p);
    }
    else
    {
        if (p.y < y0)    return pdist2(point(p.x, y0), p);
        else if (p.y > y1) return pdist2(point(p.x, y1), p);
        else            return 0;
    }
}

};

// stores a single node of the kd-tree, either internal or leaf
struct kdnode
{
    bool leaf;        // true if this is a leaf node (has one point)
    point pt;         // the single point of this is a leaf
    bbox bound;       // bounding box for set of points in children

    kdnode *first, *second; // two children of this kd-node
    kdnode() : leaf(false), first(0), second(0) {}
    ~kdnode()
    {
        if (first) delete first;
        if (second) delete second;
    }

    // intersect a point with this node (returns squared distance)
    ntype intersect(const point &p)
    {
        return bound.distance(p);
    }

    // recursively builds a kd-tree from a given cloud of points
    void construct(vector<point> &vp)
    {
        // compute bounding box for points at this node
        bound.compute(vp);

        // if we're down to one point, then we're a leaf node
        if (vp.size() == 1)
        {
            leaf = true;
            pt = vp[0];
        }
        else
        {
            // split on x if the bbox is wider than high (not best heuristic...)
            if (bound.x1 - bound.x0 >= bound.y1 - bound.y0)
                sort(vp.begin(), vp.end(), on_x);
            // otherwise split on y-coordinate
            else
                sort(vp.begin(), vp.end(), on_y);

            // divide by taking half the array for each child
            // (not best performance if many duplicates in the middle)
            int half = vp.size() / 2;
            vector<point> vl(vp.begin(), vp.begin() + half);
            vector<point> vr(vp.begin() + half, vp.end());
            first = new kdnode();
            first->construct(vl);
            second = new kdnode();
            second->construct(vr);
        }
    }
};

```

```

    }
};

// simple kd-tree class to hold the tree and handle queries
struct kdtree
{
    kdnode *root;
    // constructs a kd-tree from a points (copied here, as it sorts them)
    kdtree(const vector<point> &vp)
    {
        vector<point> v(vp.begin(), vp.end());
        root = new kdnode();
        root->construct(v);
    }
    ~kdtree()
    {
        delete root;
    }

    // recursive search method returns squared distance to nearest point
    ntype search(kdnode *node, const point &p)
    {
        if (node->leaf)
        {
            // commented special case tells a point not to find itself
            // if (p == node->pt) return sentry;
            // else
            return pdist2(p, node->pt);
        }

        ntype bfirst = node->first->intersect(p);
        ntype bsecond = node->second->intersect(p);

        // choose the side with the closest bounding box to search first
        // (note that the other side is also searched if needed)
        if (bfirst < bsecond)
        {
            ntype best = search(node->first, p);
            if (bsecond < best)
                best = min(best, search(node->second, p));
            return best;
        }
        else
        {
            ntype best = search(node->second, p);
            if (bfirst < best)
                best = min(best, search(node->first, p));
            return best;
        }
    }

    // squared distance to the nearest
    ntype nearest(const point &p)
    {
        return search(root, p);
    }
};

int main()
{
    int n;
    while(scanf("%d", &n) && n)
    {
        vector<point> p(n);
        for(auto &it : p)
            scanf("%d %d", &it.x, &it.y);
        p.resize(unique(p.begin(), p.end()) - p.begin());
        kdtree tree(p);
        cout << tree.nearest(point(1000, 1000)) << endl;
    }
    return 0;
}

```

5.3 Geometria (reduzido)

```
typedef pair<double, double> Ponto;

bool cw(Ponto a, Ponto b, Ponto c)
{
    return (b.first - a.first) * (c.second - a.second) - (b.second - a.second) * (c.first - a.first) < 0;
}

// Retorna o casco convexo do conjunto de pontos p
vector<Ponto> convexHull(vector<Ponto> p)
{
    int n = p.size();
    if (n <= 1)
        return p;
    int k = 0;
    sort(p.begin(), p.end());
    vector<Ponto> q(n * 2);
    for (int i = 0; i < n; q[k++] = p[i++])
        for (; k >= 2 && !cw(q[k - 2], q[k - 1], p[i]); --k)
            ;
    for (int i = n - 2, t = k; i >= 0; q[k++] = p[i--])
        for (; k > t && !cw(q[k - 2], q[k - 1], p[i]); --k)
            ;
    q.resize(k - 1 - (q[0] == q[1]));
    return q;
}

//O dobro da área definida pelo triângulo de pontos a, b e c (sem sinal).
double uArea2(Ponto a, Ponto b, Ponto c)
{
    return abs((b.first - a.first) * (c.second - a.second) - (b.second - a.second) * (c.first - a.first));
}

//O dobro da área definida pelo triângulo de pontos a, b e c (com sinal).
double area2(Ponto a, Ponto b, Ponto c)
{
    return (b.first - a.first) * (c.second - a.second) - (b.second - a.second) * (c.first - a.first);
}

//Distância entre os pontos a e b
double dist(Ponto a, Ponto b)
{
    return hypot(a.first - b.first, a.second - b.second);
}

//Interseção de semi-retas (p1 -> p2), (p3 -> p4)
bool segIntercept(Ponto p1, Ponto p2, Ponto p3, Ponto p4)
{
    return cw(p1, p2, p3) != cw(p1, p2, p4) && cw(p3, p4, p1) != cw(p3, p4, p2);
}

//Retorna a área do polígono p
double polygonArea(vector<Ponto> p)
{
    double s = 0.0;
    for (int i = 0; i < p.size(); i++)
        s += area2(Ponto(0, 0), p[i], p[(i + 1) % p.size()]);
    return fabs(s / 2.0);
}

//Retorna a área do polígono p definido pelos pontos p[i, f]
double polygonArea2(vector<Ponto> p, int i, int f)
{
    double s = 0.0;
    Ponto primeiro = p[i];
    for (; i != f; i++)
        s += area2(Ponto(0, 0), p[i], p[(i + 1) % p.size()]);
    s += area2(Ponto(0, 0), p[i], primeiro);
    return fabs(s / 2.0);
}

//Retorna a menor largura do conjunto de pontos p
double raio(vector<Ponto> p)
{
    vector<Ponto> h = convexHull(p);
    int m = h.size();
    if (m == 1)
        return 0;
    if (m == 2)
        return 0;
    int k = 1;
```

```

while (uArea2(h[m - 1], h[0], h[(k + 1) % m]) > uArea2(h[m - 1], h[0], h[k]))
    ++k;
double res = 10000000;
for (int i = 0, j = k; i <= k && j < m; i++)
{
    res = min(res, dist(h[i], h[j]));
    while (j < m && uArea2(h[i], h[(i + 1) % m], h[(j + 1) % m]) > uArea2(h[i], h[(i + 1) % m], h[j]))
    {
        res = min(res, dist(h[i], h[(j + 1) % m]));
        ++j;
    }
}
return res;
}

//Retorna a maior largura do conjunto de pontos p
double diametro(vector<Ponto> p)
{
    vector<Ponto> h = convexHull(p);
    int m = h.size();
    if (m == 1)
        return 0;
    if (m == 2)
        return dist(h[0], h[1]);
    int k = 1;
    while (uArea2(h[m - 1], h[0], h[(k + 1) % m]) > uArea2(h[m - 1], h[0], h[k]))
        ++k;
    double res = 0;
    for (int i = 0, j = k; i <= k && j < m; i++)
    {
        res = max(res, dist(h[i], h[j]));
        while (j < m && uArea2(h[i], h[(i + 1) % m], h[(j + 1) % m]) > uArea2(h[i], h[(i + 1) % m], h[j]))
        {
            res = max(res, dist(h[i], h[(j + 1) % m]));
            ++j;
        }
    }
    return res;
}

```

5.4 Geometria (grande)

```

#include <bits/stdc++.h>
using namespace std;
const double EPS = 1e-10;
inline int cmp( double x, double y = 0, double tol = EPS ) {
    return (x <= y + tol) ? (x + tol < y) ? -1 : 0 : 1;
}
struct Point {
    double x, y;
    Point( double x = 0, double y = 0 ) : x( x ), y( y ) {}
    Point operator+( Point q ) const {
        return Point( x + q.x, y + q.y );
    }
    Point operator-( Point q ) const {
        return Point( x - q.x, y - q.y );
    }
    Point operator*( double t ) const {
        return Point( x * t, y * t );
    }
    Point operator/( double t ) const {
        return Point( x / t, y / t );
    }
    double operator*( Point q )const {
        return x * q.x + y * q.y;
    }
    double operator^( Point q ) const {
        return x * q.y - y * q.x;
    }
    int cmp( Point q ) const {
        if ( int t = ::cmp( x, q.x ) )

```



```

        return t;
    return ::cmp( y, q.y );
}

bool operator==( Point q ) const {
    return cmp( q ) == 0;
}

bool operator!=( Point q ) const {
    return cmp( q ) != 0;
}

bool operator<( Point q ) const {
    return cmp( q ) < 0;
}

static Point pivot;
};

Point Point::pivot;

typedef vector<Point> Polygon;

inline double abs( Point& p ) {
    return hypot( p.x, p.y );
}

inline double arg( Point& p ) {
    return atan2( p.y, p.x );
}

//Verifica o sinal do produto vetorial entre os vetores (p-r) e (q - r)
inline int ccw( Point& p, Point& q, Point& r ) {
    return cmp( ( p - r ) ^ ( q - r ) );
}

//calcula o angulo orientado entre os vetores (p-q) e (r - q)
inline double angle( Point& p, Point &q, Point& r ) {
    Point u = p - q, w = r - q;
    return atan2( u ^ w, u * w );
}

//Decide se o ponto p esta sobre a reta que passa por p1p2.
bool pontoSobreReta( Point& p1, Point &p, Point& p2 ) {
    return ccw( p1, p2, p ) == 0;
}

//Decide de p esta sobre o segmento p1p2
bool between( Point& p1, Point &p, Point& p2 ) {
    return ccw( p1, p2, p ) == 0 && cmp( ( p1 - p ) * ( p2 - p ) ) <= 0;
}

//Calcula a distancia do ponto p a reta que passa por p1p2
double retaDistance( Point& p1, Point& p2, Point &p ) {
    Point A = p1 - p, B = p2 - p1;
    return fabs( A ^ B ) / sqrt( B * B );
}

//Calcula a distancia do ponto p ao segmento de reta que passa por p1p2
double segDistance( Point& p1, Point& p2, Point &p ) {
    Point A = p1 - p, B = p1 - p2, C = p2 - p;
    double a = A * A, b = B * B, c = C * C;
    if ( cmp( a, b + c ) >= 0 ) return sqrt( c );
    if ( cmp( c, a + b ) >= 0 ) return sqrt( a );
    return fabs( A ^ C ) / sqrt( b );
}

//Calcula a area orientada do poligono T.
double polygonArea( Polygon& T ) {
    double s = 0.0;
    int n = T.size( );
    for ( int i = 0; i < n; i++ )
    {
        s += T[i] ^ T[( i + 1 ) % n];
    }
    return s / 2.0; //Retorna a area com sinal
}

//Classifica o ponto p em relacao ao poligono T dependendo se ele está
//na fronteira (-1) no exterior (0) ou no interior (1).
int inpoly( Point& p, Polygon& T ) {
    // -1 sobre, 0 fora, 1 dentro
    double a = 0.0;
    int n = T.size( );
    for ( int i = 0; i < n; i++ )
    {

```

```

        if ( between( T[i], p, T[( i + 1 ) % n] ) ) return -1;
        a += angle( T[i], p, T[( i + 1 ) % n] );
    }
    return cmp( a ) != 0;
}
//Ordenacao radial.
bool radialSort( Point p, Point q ) {
    Point P = p - Point::pivot, Q = q - Point::pivot;
    double R = P ^ Q;
    if ( cmp( R ) ) return R > 0;
    return cmp( P * P, Q * Q ) < 0;
}
//Determina o convex hull de T. ATENCAO. A lista de pontos T e destruida.
Polygon convexHull( vector<Point>& T ) {
    int j = 0, k, n = T.size( );
    Polygon U( n );
    Point::pivot = *min_element( T.begin( ), T.end( ) );
    sort( T.begin( ), T.end( ), radialSort );

    for ( k = n - 2; k >= 0 && ccw( T[0], T[n - 1], T[k] ) == 0; k-- );
    reverse( ( k + 1 ) + T.begin( ), T.end( ) );

    for ( int i = 0; i < n; i++ )
    {
        // troque o >= por > para manter pontos colineares
        while ( j > 1 && ccw( U[j - 1], U[j - 2], T[i] ) >= 0 ) j--;
        U[j++] = T[i];
    }
    U.resize( j );
    return U;
}
//Interseção de semi-retas (p1 -> p2), (p3 -> p4)
bool segIntercept(Point p1, Point p2, Point p3, Point p4) {
    return ccw(p1, p2, p3) != ccw(p1, p2, p4) && ccw(p3, p4, p1) != ccw(p3, p4, p2);
}

```

5.5 Geometria (Marcelo)

```

#include <bits/stdc++.h>
using namespace std;

#define D(x) cout << #x " = " << x << endl
//INT_MAX, UINT_MAX, LLONG_MAX, ULLONG_MAX, INFINITY
// Mudar para double dependendo do exercicio
typedef double Double;

static const Double EPS = 1e-10;
// Compara doubles
int cmp(Double x, Double y = 0.0, Double tol = EPS)
{
    return (x <= y + tol) ? (x + tol <= y) ? -1 : 0 : 1;
}

struct Vec3
{
    Double x, y, z;

    Vec3() : x(0.0), y(0.0), z(0.0) {}
    Vec3(Double x, Double y, Double z) : x(x), y(y), z(z) {}
    Vec3(const Vec3 &u, const Vec3 &v) : x(v.x - u.x), y(v.y - u.y), z(v.z - u.z) {}
    Vec3(const Vec3 &v) : x(v.x), y(v.y), z(v.z) {}

    void operator=(const Vec3 &v)
    {
        x = v.x;
        y = v.y;
        z = v.z;
    }

    //----- OPERADORES VETORIAIS
    Vec3 operator+(const Vec3 &v) const
    {
        return Vec3(x + v.x, y + v.y, z + v.z);
    }

    Vec3 operator-(const Vec3 &v) const
    {
        return Vec3(x - v.x, y - v.y, z - v.z);
    }
}

```

```

// Produto escalar
double operator*(const Vec3 &v) const
{
    return x * v.x + y * v.y + z * v.z;
}

// Produto vetorial
Vec3 operator^(const Vec3 &v) const
{
    return Vec3(y * v.z - z * v.y, z * v.x - x * v.z, x * v.y - y * v.x);
}

//-----
//----- OPERADORES COM ESCALARES
Vec3 operator+(const Double c) const
{
    return Vec3(x + c, y + c, z + c);
}

Vec3 operator-(const Double c) const
{
    return Vec3(x - c, y - c, z - c);
}

Vec3 operator*(const Double c) const
{
    return Vec3(x * c, y * c, z * c);
}

Vec3 operator/(const Double c) const
{
    assert(::cmp(c) != 0);
    return Vec3(x / c, y / c, z / c);
}

//-----
//----- NORMAS
// Retorna a norma
Double norma() const
{
    return sqrt(x * x + y * y + z * z);
}

// Retorna a norma ao quadrado
Double norma2() const
{
    return x * x + y * y + z * z;
}

// Retorna uma copia normalizada do vetor
Vec3 normalizado() const
{
    return *this / this->norma();
}

//-----
//----- COMPARADORES
//***** CUIDADO - ADAPTAR AO PROBLEMA *****
bool operator==(const Vec3 &v) const
{
    return !cmp(x, v.x) && !cmp(y, v.y) && !cmp(z, v.z);
}

bool operator!=(const Vec3 &v) const
{
    return !(*this == v);
}

// Critehrios de comparacao
bool operator<(const Vec3 &v) const
{
    return cmpMenorXYZ(v);
}

// Compara componentes na ordem x, y, z.
// Retorna true assim que encontrar a primeira menor
bool cmpMenorXYZ(const Vec3 &v) const
{
    int aux = cmp(x, v.x);
    if(aux < 0)
        return true;
    else if(aux == 0)
    {

```

```

        aux = cmp(y, v.y);
        if(aux < 0 || aux == 0 && cmp(z, v.z) < 0)
            return true;
    }
    return false;
}
//-----
//----- DEBUG
friend ostream &operator<<(ostream &os, const Vec3 &v)
{
    return os << "(" << v.x << ", " << v.y << ", " << v.z << ") ";
}
//-----
};

//----- AUXILIARES
// Retorna se os pontos a, b, c estao em sentido horario
bool cw(Vec3 a, Vec3 b, Vec3 c)
{
    return cmp((b.x - a.x) * (c.y - a.y) - (b.y - a.y) * (c.x - a.x)) < 0;
}

//O dobro da area definida pelo triangulo de pontos pontos a, b e c (com sinal).
Double area2(Vec3 a, Vec3 b, Vec3 c)
{
    return (b.x - a.x) * (c.y - a.y) - (b.y - a.y) * (c.x - a.x);
}

//Retorna a area do poligono p
Double areaPoligono(vector<Vec3> &p)
{
    Double s = 0.0;
    for (unsigned int i = 0; i < p.size(); i++)
        s += area2(Vec3(), p[i], p[(i + 1) % p.size()]);
    return fabs(s / 2.0);
}

// Retorna a projecao de u em v
Vec3 projecao(const Vec3 &u, const Vec3 &v)
{
    assert(v.x || v.y || v.z);
    return v * (u * v / v.norma2());
}

// (u ^ v) * w = [[ux, uy, uz], [vx, vy, vz], [wx, wy, wz]]
Double produtoMisto(const Vec3 &u, const Vec3 &v, const Vec3 &w)
{
    return (u ^ v) * w;
}
//-----
//----- DISTÂNCIAS
// Retorna a distância do ponto p ao segmento ab
Double distPontoSegmento(const Vec3 &p, const Vec3 &a, const Vec3 &b)
{
    Vec3 u = b - a, v = p - a;
    Double t = (u * v) / (u * u);
    if(cmp(t) == -1) t = 0.0;
    if(cmp(t, 1.0) == 1) t = 1.0;
    return (p - Vec3(a + u * t)).norma();
}

// Retorna a menor distância entre um ponto qualquer de a1b1 com um ponto qualquer de a2b2
Double distSegmentoSegmento(const Vec3 &a1, const Vec3 &b1, const Vec3 &a2, const Vec3 &b2)
{
    Vec3 u = b1 - a1, v = b2 - a2, w = a1 - a2;
    Double a = u * u, b = u * v, c = v * v, d = u * w, e = v * w, den = a * c - b * b, t1, t2;

    if(cmp(den) == 0)
    {
        t1 = 0;
        t2 = d / b;
    }
    else
    {
        t1 = (b * e - c * d) / den;
        t2 = (a * e - b * d) / den;
    }

    if(0 <= t1 && t1 <= 1 && 0 <= t2 && t2 <= 1)
    {

```

```

    Vec3 p = a1 + u * t1, q = a2 + v * t2;
    return (p - q).norma();
}
else
{
    Double option1 = min(distPontoSegmento(a2, a1, b1), distPontoSegmento(b2, a1, b1));
    Double option2 = min(distPontoSegmento(a1, a2, b2), distPontoSegmento(b1, a2, b2));
    return min(option1, option2);
}
}

// Retorna a menor distância entre o ponto p ao triângulo t1t2t3
Double distPontoTriangulo(const Vec3 &p, const Vec3 &t1, const Vec3 &t2, const Vec3 &t3)
{
    Vec3 u = t2 - t1, v = t3 - t1, n = u ^ v;
    // Se falhar os pontos do triângulo são colineares
    assert(cmp(n * n) != 0);
    Double s = -(n * (p - t1)) / (n * n);
    // q é o ponto do plano do triângulo mais próximo de p
    Vec3 q = p + n * s;
    // Verificando se q está dentro do triângulo
    Vec3 w = q - t1;
    Vec3 nv = n ^ v;
    Vec3 nu = n ^ u;
    Double a2 = (w * nv) / (u * nv);
    Double a3 = (w * nu) / (v * nu);
    Double a1 = 1 - a2 - a3;
    // Temos as coordenadas baricêntricas de q. q == a1*t1 + a2*t2 + a3*t3.
    if (0 <= a1 && a1 <= 1 && 0 <= a2 && a2 <= 1 && 0 <= a3 && a3 <= 1)
    {
        // O ponto está dentro do triângulo ou em sua borda.
        // Basta retornar a distância de p a q
        return (p - q).norma();
    }
    else
    {
        // O ponto mais próximo está no plano do triângulo.
        Double ans = distPontoSegmento(p, t1, t2);
        ans = min(ans, distPontoSegmento(p, t2, t3));
        ans = min(ans, distPontoSegmento(p, t3, t1));
        return ans;
    }
}

//----- CASCO CONVEXO
// Retorna o casco convexo do conjunto de pontos p em sentido ANTI-HORARIO
vector<Vec3> convexHull(vector<Vec3> &p)
{
    int n = p.size();
    if (n <= 1)
        return p;
    int k = 0;
    // CUIDADO COM O OPERADOR <
    sort(p.begin(), p.end());
    vector<Vec3> q(n * 2);
    for (int i = 0; i < n; q[k++] = p[i++])
        for (; k >= 2 && cw(q[k - 2], q[k - 1], p[i]); --k)
            ;
    for (int i = n - 2, t = k; i >= 0; q[k++] = p[i--])
        for (; k > t && cw(q[k - 2], q[k - 1], p[i]); --k)
            ;
    q.resize(k - 1 - (q[0] == q[1]));
    return q;
}

//-----
int main()
{
    Vec3 u(1, 0, 0), v(2, 0, 0), p(1, 0, 1), q(0, 0, 0);
    D(distPontoTriangulo(q, u, v, p));
}

```

6 Estruturas de dados etc

6.1 Wavelet-tree

```
#include <bits/stdc++.h>
using namespace std;
const int N = 10000;
struct KthSmallest
{
    struct Seg
    {
        int l, r, mid;
        void set(int _l, int _r)
        {
            l = _l;
            r = _r;
            mid = l + r >> 1;
        }
    } seg[N << 2];
    int b[25][N], left[25][N], sorted[N];
    void init(int *a, int n)
    {
        for (int i = 0; i < n; i++)
            b[0][i] = sorted[i] = a[i];
        sort(sorted, sorted + n);
        build(0, n, 0, 1);
    }
    void build(int l, int r, int d, int idx)
    {
        seg[idx].set(l, r);
        if (l + 1 == r)
            return;
        int mid = seg[idx].mid;
        int lsame = mid - 1;
        for (int i = l; i < r; i++)
            if (b[d][i] < sorted[mid])
                lsame--;
        int lpos = l, rpos = mid, same = 0;
        for (int i = l; i < r; ++i)
        {
            left[d][i] = (i != l ? left[d][i - 1] : 0);
            if (b[d][i] < sorted[mid])
            {
                left[d][i]++;
                b[d + 1][lpos++] = b[d][i];
            }
            else if (b[d][i] > sorted[mid])
                b[d + 1][rpos++] = b[d][i];
            else
            {
                if (same < lsame)
                {
                    same++;
                    left[d][i]++;
                    b[d + 1][lpos++] = b[d][i];
                }
                else
                {
                    b[d + 1][rpos++] = b[d][i];
                }
            }
        }
        build(l, mid, d + 1, idx << 1);
        build(mid, r, d + 1, idx << 1 | 1);
    }
}

//Quando ordenarmos [l, r), qual é o k-ésimo termo?
int kth(int l, int r, int k, int d = 0, int idx = 1)
{
    // k : 1-origin!!!
    if (l + 1 == r)
```

```

        return b[d][l];
int ltl = (l != seg[idx].l ? left[d][l - 1] : 0);
int tl = left[d][r - 1] - ltl;
if (tl >= k)
{
    int newl = seg[idx].l + ltl;
    int newr = seg[idx].l + ltl + tl;
    return kth(newl, newr, k, d + 1, idx << 1);
}
else
{
    int mid = seg[idx].mid;
    int tr = r - l - tl;
    int ltr = l - seg[idx].l - ltl;
    int newl = mid + ltr;
    int newr = mid + ltr + tr;
    return kth(newl, newr, k - tl, d + 1, idx << 1 | 1);
}
}

//When sorting [l, r), what number will x come in?
//If there are two or more x's, return the rank of the last one.
//If there is no x, return the rank of the largest but less than x.
//When there is no less than x, 0 is returned.
int rank(int l, int r, int x, int d = 0, int idx = 1)
{
    if (seg[idx].l + 1 == seg[idx].r)
        return l + 1 == r && sorted[l] <= x;

    int ltl = (l != seg[idx].l ? left[d][l - 1] : 0);
    int tl = left[d][r - 1] - ltl;
    int mid = seg[idx].mid;

    if (x < sorted[mid])
    {
        int newl = seg[idx].l + ltl;
        int newr = seg[idx].l + ltl + tl;
        return rank(newl, newr, x, d + 1, idx << 1);
    }
    else
    {
        int tr = r - l - tl;
        int ltr = l - seg[idx].l - ltl;
        int newl = mid + ltr;
        int newr = mid + ltr + tr;
        return tl + rank(newl, newr, x, d + 1, idx << 1 | 1);
    }
}

// Quantos x existem entre [l,r)
int freq(int l, int r, int x)
{
    return rank(l, r, x) - rank(l, r, x - 1);
}
} kth;

int main()
{
    int a[8] = {6, 12, 5, 17, 10, 2, 7, 3};
    kth.init(a, 8);
    cout << kth.kth(2, 7, 3) << endl; // 7
    cout << kth.rank(2, 7, 7) << endl; // 3
}

```

6.2 Seg-tree 2D

```

#include <bits/stdc++.h>
using namespace std;
#define Max 501
#define INF (1 << 30)
int P[Max][Max]; // container for 2D grid
/* 2D Segment Tree node */
struct Point

```

```

{
    int x, y, mx;
    Point() {}
    Point(int x, int y, int mx) : x(x), y(y), mx(mx) {}

    bool operator<(const Point &other) const
    {
        return mx < other.mx;
    }
};

struct Segtree2d
{
    // I didn't calculate the exact size needed in terms of 2D container size.
    // If anyone, please edit the answer.
    // It's just a safe size to store nodes for MAX * MAX 2D grids which won't cause stack overflow :)
    Point T[500000]; // TODO: calculate the accurate space needed

    int n, m;

    // initialize and construct segment tree
    void init(int n, int m)
    {
        this->n = n;
        this->m = m;
        build(1, 1, 1, n, m);
    }

    // build a 2D segment tree from data [ (a1, b1), (a2, b2) ]
    // Time: O(n logn)
    Point build(int node, int a1, int b1, int a2, int b2)
    {
        // out of range
        if (a1 > a2 or b1 > b2)
            return def();

        // if it is only a single index, assign value to node
        if (a1 == a2 and b1 == b2)
            return T[node] = Point(a1, b1, P[a1][b1]);

        // split the tree into four segments
        T[node] = def();
        T[node] = maxNode(T[node], build(4 * node - 2, a1, b1, (a1 + a2) / 2, (b1 + b2) / 2));
        T[node] = maxNode(T[node], build(4 * node - 1, (a1 + a2) / 2 + 1, b1, a2, (b1 + b2) / 2));
        T[node] = maxNode(T[node], build(4 * node + 0, a1, (b1 + b2) / 2 + 1, (a1 + a2) / 2, b2));
        T[node] = maxNode(T[node], build(4 * node + 1, (a1 + a2) / 2 + 1, (b1 + b2) / 2 + 1, a2, b2));
        return T[node];
    }

    // helper function for query(int, int, int, int);
    Point query(int node, int a1, int b1, int a2, int b2, int x1, int y1, int x2, int y2)
    {
        // if we out of range, return dummy
        if (x1 > a2 or y1 > b2 or x2 < a1 or y2 < b1 or a1 > a2 or b1 > b2)
            return def();

        // if it is within range, return the node
        if (x1 <= a1 and y1 <= b1 and a2 <= x2 and b2 <= y2)
            return T[node];

        // split into four segments
        Point mx = def();
        mx = maxNode(mx, query(4 * node - 2, a1, b1, (a1 + a2) / 2, (b1 + b2) / 2, x1, y1, x2, y2));
        mx = maxNode(mx, query(4 * node - 1, (a1 + a2) / 2 + 1, b1, a2, (b1 + b2) / 2, x1, y1, x2, y2));
        mx = maxNode(mx, query(4 * node + 0, a1, (b1 + b2) / 2 + 1, (a1 + a2) / 2, b2, x1, y1, x2, y2));
        mx = maxNode(mx, query(4 * node + 1, (a1 + a2) / 2 + 1, (b1 + b2) / 2 + 1, a2, b2, x1, y1, x2, y2));

        // return the maximum value
        return mx;
    }

    // query from range [ (x1, y1), (x2, y2) ]
    // Time: O(logn)
    Point query(int x1, int y1, int x2, int y2)
    {
        return query(1, 1, 1, n, m, x1, y1, x2, y2);
    }

    // helper function for update(int, int, int);
    Point update(int node, int a1, int b1, int a2, int b2, int x, int y, int value)
    {
        if (a1 > a2 or b1 > b2)
            return def();

```



```

    if (x > a2 or y > b2 or x < a1 or y < b1)
        return T[node];
    if (x == a1 and y == b1 and x == a2 and y == b2)
        return T[node] = Point(x, y, value);

    Point mx = def();
    mx = maxNode(mx, update(4 * node - 2, a1, b1, (a1 + a2) / 2, (b1 + b2) / 2, x, y, value));
    mx = maxNode(mx, update(4 * node - 1, (a1 + a2) / 2 + 1, b1, a2, (b1 + b2) / 2, x, y, value));
    mx = maxNode(mx, update(4 * node + 0, a1, (b1 + b2) / 2 + 1, (a1 + a2) / 2, b2, x, y, value));
    mx = maxNode(mx, update(4 * node + 1, (a1 + a2) / 2 + 1, (b1 + b2) / 2 + 1, a2, b2, x, y, value));
    return T[node] = mx;
}

// update the value of (x, y) index to 'value'
// Time: O(logn)
Point update(int x, int y, int value)
{
    return update(1, 1, 1, n, m, x, y, value);
}

// utility functions; these functions are virtual because they will be overridden in child class
virtual Point maxNode(Point a, Point b)
{
    return max(a, b);
}

// dummy node
virtual Point def()
{
    return Point(0, 0, -INF);
}
};

/* 2D Segment Tree for range minimum query; a override of Segtree2d class */
struct Segtree2dMin : Segtree2d
{
    // overload maxNode() function to return minimum value
    Point maxNode(Point a, Point b)
    {
        return min(a, b);
    }

    Point def()
    {
        return Point(0, 0, INF);
    }
};

// initialize class objects
Segtree2d Tmax;
Segtree2dMin Tmin;

/* Drier program */
int main(void)
{
    int n, m;
    // input
    scanf("%d %d", &n, &m);
    for (int i = 1; i <= n; i++)
        for (int j = 1; j <= m; j++)
            scanf("%d", &P[i][j]);

    // initialize
    Tmax.init(n, m);
    Tmin.init(n, m);

    // query
    int x1, y1, x2, y2;
    scanf("%d %d %d %d", &x1, &y1, &x2, &y2);

    Tmax.query(x1, y1, x2, y2).mx;
    Tmin.query(x1, y1, x2, y2).mx;

    // update
    int x, y, v;
    scanf("%d %d %d", &x, &y, &v);
    Tmax.update(x, y, v);
    Tmin.update(x, y, v);

    return 0;
}

```

6.3 Seg-tree

```
#include <algorithm>
using namespace std;
#define MAX 1000000 // 0 valor aqui tem que ser >= 2 * tamanho do maior n
#define INF 1 << 28
// Não necessariamente é um int, pode ser uma segtree de struct etc;
int init[MAX], tree[MAX], lazy[MAX];
void build_tree(int node, int a, int b)
{
    if (a > b)
        return;
    // Se folha
    if (a == b)
    {
        tree[node] = init[a];
        lazy[node] = 0;
        return;
    }
    build_tree(node * 2, a, (a + b) / 2);
    build_tree(node * 2 + 1, 1 + (a + b) / 2, b);
    // Se nó
    tree[node] = tree[node * 2] + tree[node * 2 + 1];
    lazy[node] = 0;
}
void update_tree(int node, int a, int b, int i, int j, int value)
{
    // Se fora do intervalo - retorna
    if (a > b || a > j || b < i)
        return;
    if (lazy[node] != 0)
    {
        //Atualização atrasada.
        tree[node] += lazy[node];
        // Passa lazy para filhos
        if (a != b)
        {
            lazy[node * 2] += lazy[node];
            lazy[node * 2 + 1] += lazy[node];
        }
        //Reseta o nó
        lazy[node] = 0;
    }
    // Se o nó atual cobre todo o intervalo
    if (a >= i && b <= j)
    {
        tree[node] += value;
        if (a != b)
        {
            lazy[node * 2] += value;
            lazy[node * 2 + 1] += value;
        }
        return;
    }
    // Se tem um pedaco em cada filho.
    // Atualiza os filhos.
    update_tree(node * 2, a, (a + b) / 2, i, j, value);
    update_tree(1 + node * 2, 1 + (a + b) / 2, b, i, j, value);
    // Atualiza o pai.
    tree[node] = tree[node * 2] + tree[node * 2 + 1];
}
int query_tree(int node, int a, int b, int i, int j)
{
    // Se fora do intervalo
    if (a > b || a > j || b < i)
    {
        //Aqui deverá ser retornado o elemento neutro para a operação desejada
    }
}
```

```

    return 0;
}
if (lazy[node] != 0)
{
    //Atualização atrasada.
    tree[node] += lazy[node];

    //Se não folha, passa lazy pros filhos
    if (a != b)
    {
        lazy[node * 2] += lazy[node];
        lazy[node * 2 + 1] += lazy[node];
    }

    //Reseta o nó
    lazy[node] = 0;
}

// Se o nó cobre o intervalo
if (a >= i && b <= j)
    return tree[node];

// Se o intervalo está um pedaco em cada filho.
int q1 = query_tree(node * 2, a, (a + b) / 2, i, j);
int q2 = query_tree(1 + node * 2, 1 + (a + b) / 2, b, i, j);

// Retorna a combinação dos intervalos.
return q1 + q2;
}

```

```

/*
Uso:

Assumindo que "n" é o numero de termos que o segmento tem
Inialize "init" com os valores iniciais:
*   for(i = 0; i < n; scanf("%d", val), i++)
*   init[i] = val;

E mande construir a arvore:
*   build_tree(1, 0, n-1);

Para atualizar a arvore:
*   update_tree(1, 0, n-1, inicio, fim, val);
*   Onde inicio é a posição inicial do segmento desejado e fim é a posição final do mesmo
*   e val é o quanto você quer alterar os valores desse seguimento

Para fazer queries
*   query_tree(1, 0, n-1, inicio, fim);
*   Onde inicio é a posição inicial do segmento desejado e fim é a posição final do mesmo
*   o retorno terá o mesmo tipo que os dados guardados na arvore e será o resultado do segmento pesquisado
*/

```

6.4 Mergesort

```

typedef vector<int>::iterator vec_it;
void merge(vec_it left, vec_it left_end, vec_it right, vec_it right_end, vec_it numbers)
{
    while (left != left_end)
    {
        if (*left < *right || right == right_end)
        {
            *numbers = *left;
            ++left;
        }
        else
        {
            *numbers = *right;
            ++right;
        }
        ++numbers;
    }
    while (right != right_end)
    {
        *numbers = *right;
        ++right;
    }
}

```

```

        ++numbers;
    }
}

void merge_sort(vector<int> &numbers)
{
    if (numbers.size() <= 1)
        return;

    vector<int>::size_type middle = numbers.size() / 2;
    vector<int> left(numbers.begin(), numbers.begin() + middle);
    vector<int> right(numbers.begin() + middle, numbers.end());

    merge_sort(left);
    merge_sort(right);

    merge(left.begin(), left.end(), right.begin(), right.end(), numbers.begin());
}

```

6.5 Algoritmo de MO (queries offline)

```

#define N 311111
#define A 1111111
#define BLOCK 555 // ~sqrt(N)

int cnt[A], a[N], ans[N], answer = 0;

struct node
{
    int L, R, i;
} q[N];

bool cmp(node x, node y)
{
    if (x.L / BLOCK != y.L / BLOCK)
    {
        // different blocks, so sort by block.
        return x.L / BLOCK < y.L / BLOCK;
    }
    // same block, so sort by R value
    return x.R < y.R;
}

void add(int position)
{
    cnt[a[position]]++;
    if (cnt[a[position]] == 1)
    {
        answer++; // Verifica se é resposta aqui!!!
    }
}

void remove(int position)
{
    cnt[a[position]]--;
    if (cnt[a[position]] == 0)
    {
        answer--; // Verifica se é resposta aqui!!!
    }
}

int main()
{
    int n;
    scanf("%d", &n);
    for (int i = 0; i < n; i++)
        scanf("%d", &a[i]);

    int m;
    scanf("%d", &m);
    for (int i = 0; i < m; i++)
    {
        scanf("%d%d", &q[i].L, &q[i].R);
        q[i].L--;
        q[i].R--;
        q[i].i = i;
    }

    sort(q, q + m, cmp);

    int currentL = 0, currentR = 0;
    for (int i = 0; i < m; i++)
    {

```

```

    int L = q[i].L, R = q[i].R;
    while (currentL < L)
    {
        remove(currentL);
        currentL++;
    }
    while (currentL > L)
    {
        add(currentL - 1);
        currentL--;
    }
    while (currentR <= R)
    {
        add(currentR);
        currentR++;
    }
    while (currentR > R + 1)
    {
        remove(currentR - 1);
        currentR--;
    }
    ans[q[i].i] = answer;
}
for (int i = 0; i < m; i++)
    printf("%d\n", ans[i]);
}

```

6.6 Union-find

```

// Tamanho máximo de n
const int maxn = 200000;
int Rank[maxn], p[maxn], n;

void init(int _n)
{
    n = _n;
    fill(Rank, Rank + n, 0);
    for (int i = 0; i < n; i++)
        p[i] = i;
}

int find(int x)
{
    return x == p[x] ? x : (p[x] = find(p[x]));
}

void unir(int a, int b)
{
    a = find(a);
    b = find(b);
    if (a == b)
        return;
    if (Rank[a] < Rank[b])
        swap(a, b);
    if (Rank[a] == Rank[b])
        ++Rank[a];
    p[b] = a;
}

```

6.7 Arvore para achar o K-esimo termo

```

#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>

using namespace std;
using namespace __gnu_pbds;

#define D(x) //cout << #x << " = " << x << endl

typedef pair<int, int> Ponto;
typedef pair<Ponto, int> Flor;

typedef tree <
pair<int, int>,
null_type,
less<pair<int, int>>,
rb_tree_tag,

```

```

    tree_order_statistics_node_update >
    ordered_set;
int main()
{
    vector<Flor> flores;
    vector<Ponto> cercas;

    int p, v, aux;
    long long resp;

    scanf("%d %d", &p, &v);

    ordered_set eventos;

    flores = vector<Flor>(p);
    cercas = vector<Ponto>(v);

    for(int i = 0; i < p; i++)
        scanf("%d %d", &flores[i].first.first, &flores[i].first.second),
            flores[i].second = i + 1;

    for(auto &it : cercas)
        scanf("%d %d", &it.first, &it.second);

    sort(flores.begin(), flores.end());
    sort(cercas.begin(), cercas.end());

    auto flor = flores.begin();
    auto cerca = cercas.begin();

    resp = 0;
    aux = 1;

    while(flor != flores.end())
    {
        if(cerca == cercas.end())
        {
            resp += flor->second;
            ++flor;
        }
        else if(flor->first < *cerca)
        {
            auto lb = eventos.lower_bound({flor->first.second, 5000000});
            auto order = eventos.order_of_key(*lb);

            if(lb == eventos.end() || order % 2 == 0)
                resp += flor->second;

            ++flor;
        }
        else
        {
            eventos.insert({cerca->second, aux++});
            cerca++;
        }
    }

    printf("%lld\n", resp);
}

```

6.8 Índice de inversao ($n \log n$)

```

#include <bits/stdc++.h>
using namespace std;

int _mergeSort(int arr[], int temp[], int left, int right);
int merge(int arr[], int temp[], int left, int mid, int right);

int mergeSort(int arr[], int array_size)
{
    int *temp = (int *)malloc(sizeof(int)*array_size);
    return _mergeSort(arr, temp, 0, array_size - 1);
}

/* An auxiliary recursive function that sorts the input array and
   returns the number of inversions in the array. */
int _mergeSort(int arr[], int temp[], int left, int right)
{
    int mid, inv_count = 0;
    if (right > left)
    {

```

```

    /* Divide the array into two parts and call _mergeSortAndCountInv()
       for each of the parts */
    mid = (right + left)/2;

    /* Inversion count will be sum of inversions in left-part, right-part
       and number of inversions in merging */
    inv_count = _mergeSort(arr, temp, left, mid);
    inv_count += _mergeSort(arr, temp, mid+1, right);

    /*Merge the two parts*/
    inv_count += merge(arr, temp, left, mid+1, right);
}
return inv_count;
}

/* This funt merges two sorted arrays and returns inversion count in
   the arrays.*/
int merge(int arr[], int temp[], int left, int mid, int right)
{
    int i, j, k;
    int inv_count = 0;
    i = left; /* i is index for left subarray*/
    j = mid; /* j is index for right subarray*/
    k = left; /* k is index for resultant merged subarray*/
    while ((i <= mid - 1) && (j <= right))
    {
        if (arr[i] <= arr[j])
        {
            temp[k++] = arr[i++];
        }
        else
        {
            temp[k++] = arr[j++];

            /*this is tricky -- see above explanation/diagram for merge()*/
            inv_count = inv_count + (mid - i);
        }
    }

    /* Copy the remaining elements of left subarray
       (if there are any) to temp*/
    while (i <= mid - 1)
        temp[k++] = arr[i++];

    /* Copy the remaining elements of right subarray
       (if there are any) to temp*/
    while (j <= right)
        temp[k++] = arr[j++];

    /*Copy back the merged elements to original array*/
    for (i=left; i <= right; i++)
        arr[i] = temp[i];

    return inv_count;
}

int main() {
    ios::sync_with_stdio(0);
    cin.tie(0);

    int n, i, entrada[100000];
    while(cin >> n && n) {
        for(i = 0; i < n; i++)
            cin >> entrada[i];

        cout << (mergeSort(entrada, n) % 2 ? "Marcelo" : "Carlos") << "\n";
    }

    return 0;
}

```

7 Python

7.1 Matrices

```
def matrixUnit(n):
    return [[1 if i == j else 0 for j in range(n)] for i in range(n)]

# Retorna a+b
def matrixAdd(a, b):
    return [[a[i][j] + b[i][j] for i in range(len(a[0]))] for j in range(len(a))]

# Retorna a*b
def matrixMul(a, b):
    n = len(a)
    m = len(a[0])
    k = len(b[0])

    res = [[0 for j in range(k)] for i in range(n)]

    for i in range(n):
        for j in range(k):
            for p in range(m):
                res[i][j] = res[i][j] + a[i][p] * b[p][j]

    return res

# Retorna a matriz a^p
def matrixPow(a, p):
    if (p == 0):
        return matrixUnit(len(a))

    if (p % 2 == 1):
        return matrixMul(a, matrixPow(a, p - 1))

    return matrixPow(matrixMul(a, a), p >> 1)

# Retorna sum_{i=0}^p (a^i)
def matrixPowSum(a, p):
    n = len(a)

    if (p == 0):
        return [[0 for j in range(n)] for i in range(n)]

    if (p % 2 == 0):
        return matrixMul(matrixPowSum(a, p >> 1), matrixAdd(matrixUnit(n), matrixPow(a, p >> 1)))

    return matrixAdd(a, matrixMul(matrixPowSum(a, p - 1), a))

def main():
    x, mf = map(int, input().split(' '))

    while x != 0 and mf != 0:
        matriz = [[0 for x in range(2)] for y in range(2)]

        matriz[0][0] = 1
        matriz[0][1] = 1
        matriz[1][0] = 1
        matriz[1][1] = 0

        matriz = matrixPow(matriz, mf + 1)

        print(x * matriz[0][0])

        x, mf = map(int, input().split(' '))

main()
```

7.2 Modpow

```
def modPow(a, b, m):
    res = 1

    while(b > 0):
        if (b & 1 == 1):
            res = res * a % m
            a = a * a % m
            b >>= 1

    return res

n = int(input())
v = modPow(2, n, 86400) - 1
```



```
print("%02d:%02d:%02d" % (v / 3600, (v / 60) % 60, v % 60))
```

7.3 Busca binaria

```
import bisect
fib = [1, 2]
for i in range(2, 1000):
    fib.append(fib[i-1] + fib[i - 2])
a, b = map(int, input().split(' '))
while a != 0 or b != 0:
    lb = bisect.bisect_left(fib, a)
    up = bisect.bisect_right(fib, b)
    print(up - lb)
    a, b = map(int, input().split(' '))
```
