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Extended Kalman Filter Based Nonlinear Model Predictive Control

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Abstract

This paper formulates a nonlinear model predictive control algorithm based on successive linearization. The extended Kalman filter (EKF) technique is used to develop multi-step prediction of future states. The prediction is shown to be optimal under an affine approximation of the discrete state / measurement equations (obtained by integrating the nonlinear ODE model) made at each sampling time. Connections with previously available successive linearization based MPC techniques by Garcia (NLQDMC, 1984) and Gattu & Zafiriou (1992) are made. Potential benefits and shortcomings of the proposed technique are discussed using a bilinear control problem of paper machine.

1 Introduction

Model Predictive Control (MPC) has proven to be a powerful tool for dynamic optimization and control (see Garcia *et al.*, 1989 for review). Although initial versions of MPC (*e.g.*, Dynamic Matrix Control by Cutler & Ramaker, 1980) were heuristic in nature and limited in generality, many extensions, modifications and refinements have been proposed during the past decade. Motivated by the abundance of batch processes and continuous processes with wide operating ranges for which nonlinearity diminishes the effectiveness of linear MPC techniques, several researchers developed nonlinear MPC techniques that utilize nonlinear models in building prediction for the future output behavior. Proposed nonlinear MPC techniques range from a simple extension of linear MPC based on successive linearization of nonlinear models (NLQDMC by Garcia, 1984) to more elaborate and computationally intensive techniques involving discretization of the model followed by Nonlinear Programming (NLP) (see Biegler & Rawlings, 1991 for review). Another major refinement that has come along for MPC is in the method for estimating the disturbance effect on the future output. While most industrial MPC techniques have relied on a heuristic open-loop disturbance estimator, the late trend has been to use well-established state estimation techniques like the Kalman filtering in building prediction for the future output behavior (Ricker, 1990; Lee *et al.*, 1992a-b). This not only extends the applicability of MPC to open-loop unstable processes and inferential control problems, but also provides the *optimal* prediction in the presence of general stochastic state disturbances and measurement noise.

The objective of this article is to extend the recent state estimation based MPC approaches to processes with models described through nonlinear differential equations. This not only extends the NLQDMC technique to open-loop unstable processes, but also allows for more rigorous treatment of general stochastic disturbances leading to performance improvements and increased applicability (*e.g.*, to inferential control problems). We adopt the concept of the extended Kalman filtering (EKF, Kopp & Oxford, 1963) to develop multi-step prediction for the future output behavior. The prediction vector is linear with respect to the undecided manipulated input moves, yielding an optimization problem solvable by single Quadratic Programming (QP). While a more rigorous nonlinear treatment of filtering and prediction is possible in concept, significant theoretical and practical barriers weighed against often insignificant performance improvement do not justify such an approach at the present time. We make connections between our technique and previously published nonlinear QDMC techniques by Garcia (1984) and Gattu & Zafiriou (1992). Finally, we apply the proposed technique to a bilinear multi-variable control problem of a paper machine. The application demonstrates the effectiveness as well as some potential shortcomings of the technique.

2 Extended Kalman Filter Based NLMPC

2.1 Model

We assume that our model is expressed through the following nonlinear differential equation:

$$\dot{x} = f(x,u,d) \tag{1}$$
$$y = g(x,d) \tag{2}$$

x is the state vector, u the manipulated input vector, d the unmeasured disturbance vector and y the measured output vector. It is assumed that dynamics from u to y has relative degree of at least 1. For digital controller design purpose, u and d can be assumed to be constant between the sampling instants. Hence, we can write the discrete version of the model (1)-(2) as follows:

$$x_k = \bar{f}_{T_s}(x_{k-1}, u_{k-1}, d_{k-1}) \tag{3}$$
$$y_k = g(x_k, d_k) \tag{4}$$

where $\bar{f}_{T_s}(x_{k-1}, u_{k-1}, d_{k-1})$ denotes the terminal state vector resulting from integrating the ODE (1) for one sample interval (T_s) with the initial condition of x_{k-1} and constant inputs of $u = u_{k-1}$ and $d = d_{k-1}$. In general, $\bar{f}_{T_s}$ cannot be written in closed form.

For the purpose of state estimation, we need to express the unmeasured disturbance signal d as a stochastic process driven by white noise. Without loss of generality, we assume that d is generated through the following stochastic difference equation:

$$x_k^w = A^w x_{k-1}^w + B^w w_{k-1} \tag{5}$$
$$d_k = C^w x_k^w \tag{6}$$

w_k is discrete-time white noise with covariance R^w. It is also possible that the measurements of y_k are corrupted by measurement noise ν_k as follows:

$$\hat{y}_k = g(x_k, d_k) + \nu_k \tag{7}$$

We assume in this study that ν_k is white noise with covariance of R^ν. Combining (3)-(4) with (5)-(7), we obtain the following augmented model:

$$\begin{bmatrix} x_k \\ x_k^w \end{bmatrix} = \begin{bmatrix} \bar{f}_{T_s}(x_{k-1}, u_{k-1}, C^w x_{k-1}^w) \\ A^w x_{k-1}^w \end{bmatrix} + \begin{bmatrix} 0 \\ B^w \end{bmatrix} w_{k-1} \tag{8}$$
$$\hat{y}_k = g(x_k, C^w x_k^w) + \nu_k \tag{9}$$

2.2 State Estimation: Extended Kalman Filtering

A straightforward extension of the stochastic optimal linear filter ("Kalman filter") is the extended Kalman filter first proposed by Kopp & Oxford (1963). The technique uses an affine approximation of the nonlinear model obtained by linearization with respect to the previous state estimates and input values; this enables straightforward application of the linear filtering theory. For the nonlinear system (8)-(9), the EKF provides the state estimates in the following way:

Model Update:

$$\begin{bmatrix} x_{k|k-1} \\ x^w_{k|k-1} \end{bmatrix} = \begin{bmatrix} \tilde{f}_{T_s}(x_{k-1|k-1}, u_{k-1}, C^w x^w_{k-1|k-1}) \\ A^w x^w_{k-1|k-1} \end{bmatrix} \tag{10}$$

Measurement Correction:

$$\begin{bmatrix} x_{k|k} \\ x^w_{k|k} \end{bmatrix} = \begin{bmatrix} x_{k|k-1} \\ x^w_{k|k-1} \end{bmatrix} + L_k \left(\hat{y}_k - g(x_{k|k-1}, C^w x^w_{k|k-1})\right) \tag{11}$$

where

$$\begin{aligned} L_k &= \Sigma_{k|k-1}\Xi_k^T(\Xi_k\Sigma_{k|k-1}\Xi_k^T + R^\nu)^{-1} && (12)\\ \Sigma_{k|k-1} &= \Phi_{k-1}\Sigma_{k-1|k-1}\Phi_{k-1}^T + \Gamma^w R^w (\Gamma^w)^T && (13)\\ \Sigma_{k|k} &= (I - L_k\Xi_k)\Sigma_{k|k-1} && (14) \end{aligned}$$

and

$$\begin{aligned} \Phi_{k-1} &= \begin{bmatrix} A_{k-1} & B_{k-1}C^w \\ 0 & A^w \end{bmatrix}; \\ \Gamma^w &= \begin{bmatrix} 0 \\ B^w \end{bmatrix}; \quad \Xi_k = \begin{bmatrix} C_k & C_k^d \cdot C^w \end{bmatrix}; \\ A_{k-1} &= \exp\left(\tilde{A}_{k-1}T_s\right); \\ B_{k-1} &= \int_0^{T_s} \exp\left(\tilde{A}_{k-1}\tau\right) d\tau \cdot \tilde{B}_{k-1}; \\ \tilde{A}_{k-1} &= \left.\frac{\partial f(x,u,d)}{\partial x}\right|_{x=x_{k-1|k-1}, u=u_{k-1}, d=C^w x^w_{k-1|k-1}}; \\ \tilde{B}_{k-1} &= \left.\frac{\partial f(x,u,d)}{\partial d}\right|_{x=x_{k-1|k-1}, u=u_{k-1}, d=C^w x^w_{k-1|k-1}} \end{aligned} \tag{15}$$

Note that model update equation (10) requires nonlinear integration of ODE (1) with a known initial condition and constant input.

2.3 Prediction and Control

In order to implement a predictive control law, prediction for the future controlled output behavior must be developed based on the current state estimates. Since the underlying system is nonlinear, the future states are related to the current states and inputs in a complex nonlinear way. In order to keep the on-line computational requirement of the resulting algorithm from becoming exorbitant, we develop the the optimal multi-step estimates for the future states $x_{k+\ell|k}$ $(1 \le \ell \le p)$ under the first-order Taylor series approximation of $\tilde{f}_{T_s}(x_{k+\ell-1}, u_{k+\ell-1}, C^w x^w_{k+\ell-1})$ at known quantities (*i.e.*, $x_{k+\ell-1|k}, u_{k-1}$ and $C^w x^w_{k|k}$). This approach leads to the following prediction equation that is linear in the undecided input moves:

$$\begin{bmatrix} x_{k+\ell|k} \\ x^w_{k+\ell|k} \end{bmatrix} \approx \begin{bmatrix} \tilde{f}_{\ell\cdot T_s}(x_{k|k}, u_{k-1}, C^w(A^w)^i x^w_{k|k}|_{0\le i\le \ell-1}) \\ (A^w)^\ell x^w_{k|k} \end{bmatrix} + \begin{bmatrix} A_k^{\ell-1}B_k^u & \cdots & A_k B_k^u & B_k^u \\ & & 0 & \end{bmatrix} \begin{bmatrix} u_k - u_{k-1} \\ \vdots \\ u_{k+\ell-2} - u_{k-1} \\ u_{k+\ell-1} - u_{k-1} \end{bmatrix} \tag{16}$$

where

$$\begin{aligned} A_k &= \exp\left(\tilde{A}_k T_s\right) \\ B_k^u &= \int_0^{T_s} \exp\left(\tilde{A}_k\tau\right) d\tau \cdot \tilde{B}_k^u \\ \tilde{A}_k &= \left.\frac{\partial f(x,u,d)}{\partial x}\right|_{x=x_{k|k}, u=u_{k-1}, d=C^w x^w_{k|k}} \\ \tilde{B}_k^u &= \left.\frac{\partial f(x,u,d)}{\partial u}\right|_{x=x_{k|k}, u=u_{k-1}, d=C^w x^w_{k|k}} \end{aligned} \tag{17}$$

$\tilde{f}_{\ell\cdot T_s}(x_{k|k}, u_{k-1}, C^w(A^w)^i x^w_{k|k}|_{0\le i\le\ell-1})$ represents the terminal states resulting from integrating the nonlinear differential equation $\dot{x} = f(x,u,d)$ for ℓ sampling intervals with initial condition $x_{k|k}$, constant input $u = u_{k-1}$ and piece-wise constant input d taking the value of $C^w(A^w)^i x^w_{k|k}$ during the time interval $[k+i, k+i+1)$.

Assume that the control variable y^c is expressed as $y_k^c = Hx_k$. The following quadratic objective is commonly used to compute the undecided input moves $(\Delta u_k, \cdots, \Delta u_{k+m-1})$:

$$\min_{\Delta u_{k+i-1}, 1\le i\le m} \sum_{\ell=1}^{p} \|\Lambda^y[Hx_{k+\ell|k} - r_{k+\ell|k}]\|_2^2 + \sum_{i=1}^{m} \|\Lambda^u \Delta u_{k+i-1}\|_2^2 \tag{18}$$

$x_{k+\ell|k}$ is related to Δu_{k+i-1} linearly through the prediction equation (16). In the above, we provided the option of suppressing the last p-m input moves (*i.e.*, $\Delta u_{k+\ell} = 0$ for $m \le \ell \ge p-1$). $r_{k+\ell|k}$ is the future reference vector for $y^c_{k+\ell}$ available at time k. Λ^y and Λ^u are weighting matrices that are chosen as diagonal matrices in most cases. In general, in addition to the constraints imposed by the prediction equation, various constraints on the computed inputs and predicted outputs are added to the optimization and the optimization is solved via QP. The computed control moves are implemented in receding horizon fashion, that is, only the first move Δu_k is implemented and the whole optimization is repeated at the next sampling time.

3 Output Disturbance Model and Connections with NLQDMC

In the earlier versions of MPC (for examples, DMC, QDMC or IMC), it was customary to model unmeasured disturbance effects as a stochastic process added directly to each output (rather than as state disturbances). Such a disturbance model (referred to as "output disturbance model") can be treated in our framework by choosing the model of the following form:

$$\begin{aligned} \dot{x} &= f(x,u) && (19)\\ y &= g_1(x) + d && (20) \end{aligned}$$

(19)-(20) is certainly a special case of our general model (1)-(2) in which $f(x,u,d)$ has no dependence on d and $g(x,d) = g_1(x) + d$. Hence, all the techniques developed in the previous section apply straightforwardly.

Let us assume for this section that the system is open-loop *stable* (*i.e.*, all eigenvalues of A_k lie strictly inside the unit disk $\forall k$). Model the output disturbance d as a stochastic process $C^w(qI - A^w)^{-1}B^w$ driven by white noise w_k. The augmented discrete model corresponding to (19)-(20) is

$$\begin{aligned} \begin{bmatrix} x_k \\ x_k^w \end{bmatrix} &= \begin{bmatrix} \tilde{f}_{T_s}(x_{k-1}, u_{k-1}) \\ A^w x^w_{k-1} \end{bmatrix} + \begin{bmatrix} 0 \\ B^w \end{bmatrix} w_{k-1} && (21)\\ \hat{y}_k &= g_1(x_k) + C^w x_k^w + \nu_k && (22) \end{aligned}$$

Then, the optimal filter gain L_k for EKF reaches a steady state value as $k \to \infty$ and its steady state value $\bar{L}$ ($\triangleq \lim_{k\to\infty} L_k$) can be calculated by solving the following Algebraic Riccati Equation of reduced dimension:

$$\bar{L} = \begin{bmatrix} 0 \\ \bar{\Sigma}^w(C^w)^T(C^w\bar{\Sigma}^w(C^w)^T + R^\nu)^{-1} \end{bmatrix} \tag{23}$$

$$\bar{\Sigma}^w = A^w\bar{\Sigma}^w(A^w)^T + R^w \tag{24}$$
$$- \; A^w\bar{\Sigma}^w(C^w)^T\left(R^v + C^w\bar{\Sigma}^w(C^w)^T\right)^{-1}C^w\bar{\Sigma}^w(A^w)^T$$

For continuous processes, the above steady-state filter gain can be used for all practical purpose. For batch processes, it may be advantageous to employ the time-varying Kalman filter introduced earlier to correct for the initialization errors in x more efficiently.

In the case that $A^w = I, B^w = I, C^w = I$ (making d *integrated white noise* entering each output independently) and measurement noise is assumed negligible (*i.e.*, $R^v = 0$), $\bar{L} = I$ and the prediction resulting from the extended Kalman filter with the steady-state optimal gain $\bar{L}$ is identical to that used in NLQDMC by Garcia (1984). In NLQDMC, a constant bias corresponding to $\hat{y}_k - g(x_{k|0})$ (where $x_{k|0}$ is an open-loop estimate of x_k) is added to the open-loop output prediction $y_{k+\ell|0}$ to form prediction for $y_{k+\ell}$. The two approaches are equivalent.

While the use of the output disturbance model has proven useful in many applications, there are some potential hazards and limitations that one needs to be aware of. First, it is not applicable to unstable processes. Note that the steady state gain expression (23) is valid only for open-loop stable processes. For unstable processes, there is no steady state gain and the time-varying gain for EKF calculated from RDE of (13)–(14) initialized with a positive-definite covariance matrix $\Sigma_{0|0}$ must be used . Second, it is not applicable to processes with secondary measurements (where y differ from y^c) since the measurements of secondary outputs do not improve the prediction of controlled variables in the absence of a disturbance model correlating these outputs. Finally, although the estimates of the outputs are unbiased at steady state (because of the integrated white noise disturbance added to the output), the estimates for the states are clearly biased and this may adversely effect the quality of the linearized model.

3.1 NLQDMC for Unstable Processes and Connection with EKF based MPC

Gattu & Zafirious (1992) proposed a slightly different approach to handling unstable processes within the NLQDMC framework. In their approach, independent white noise is added to each state in addition to the integrated white noise disturbance added to each output:

$$\begin{bmatrix} x_k \\ x_k^w \end{bmatrix} = \begin{bmatrix} \bar{f}_{T_s}(x_{k-1}, u_{k-1}) \\ x_{k-1}^w \end{bmatrix} + \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} (w_1)_{k-1} \\ (w_2)_{k-1} \end{bmatrix} \tag{25}$$
$$\hat{y}_k = g_1(x_k) + x_k^w + \nu_k \tag{26}$$

where w_1 is white noise of scalar-times-identity covariance matrix R_1^w.

Although the technique of Gattu & Zafirious is best interpreted within the EKF framework as above, it differs from our EKF based technique in several aspects. First, instead of using the Kalman filter gain calculated explicitly via RDE (13)–(14), they suggest using

$$L_k = \begin{bmatrix} (\bar{L}_1)_k \\ I \end{bmatrix} \tag{27}$$

where $(\bar{L}_1)_k$ is the steady state Kalman filter gain matrix calculated via the formula

$$(\bar{L}_1)_k = \bar{\Sigma}^x C_k^T(C_k\bar{\Sigma}^x C_k^T + R^v)^{-1} \tag{28}$$
$$\bar{\Sigma}^x = A_{k-1}\bar{\Sigma}^x A_{k-1}^T + R_1^w \tag{29}$$
$$- \; A_{k-1}\bar{\Sigma}^x C_k^T\left(R^v + C_k\bar{\Sigma}^x C_k^T\right)^{-1}C_k\bar{\Sigma}^x A_{k-1}^T$$

Not only is the use of the steady state Kalman estimator gain for a time-varying linear model inappropriate, such an *ad hoc* construction of the gain matrix can lead to erroneous estimates and prediction. Second, in the prediction phase, instead of adding the measurement correction term $(\bar{L}_1)_k(y_k - y_{k|k-1})$ to the states and integrating the nonlinear ODE with the corrected states as initial condition, they add the correction term $\Phi_k(\bar{L}_1)_k(y_k - y_{k|k-1})$ uniformly to the predicted states for each time step of the prediction horizon after nonlinear integration.

Gattu & Zafiriou claim that, in addition to assuring that the Kalman filter gain is stabilising, the white noise disturbances to the states x_k also account for the state errors caused by model parameter uncertainty. However, errors caused by model parameter uncertainty are clearly correlated with the previous state and input values and therefore nonwhite. It is extremely difficult to account properly for the errors from such sources with white state excitation noise. Our experience has been that, in order to remove the *bias* in the states (more precisely, those in the measured combinations of states) in the presence of model parameter errors, one needs to add integrated white noise disturbances to the states. The number of integrated white noise disturbances added should coincide with the number of measurements so that the system remains detectable and estimates for the outputs (*i.e.*, linear combinations of the states that are measured) are unbiased.

4 Application to Paper Machine Headbox Control

4.1 Model

Ying *et al.* (1992) studied the control of composition and liquid level in a paper machine headbox, which they modeled as a bilinear, continuous-time system as follows:

$$\dot{x} = Ax + B_0u + \sum_{i=1}^{n_u} u_iB_ix + B_vv + B_dd \tag{30}$$
$$y = Cx \tag{31}$$

The state vector consists of the liquid level in the feed tank (H_1), the liquid level in the headbox (H_2), the consistency (percentage of pulp fibers in suspension) in the feed tank (N_1), and the consistency in the headbox (N_2). All states except H_1 are measured. The primary control objective is to regulate N_2 and H_2. Manipulated variables include the flowrate of stock entering the feed tank (G_p), and the flowrate of the recycled white water (G_w). Disturbances are the consistency of the stock entering the feed tank (N_p) which is measured and the consistency of the white water (N_w) which is unmeasured. Ying *et al.* derived the following constant parameter matrices:

$$A = \begin{bmatrix} -1.93 & 0 & 0 & 0 \\ 0.394 & -0.426 & 0 & 0 \\ 0 & 0 & -0.63 & 0 \\ 0.82 & -0.784 & 0.413 & -0.426 \end{bmatrix}; \tag{32}$$
$$B_0 = \begin{bmatrix} 1.274 & 1.274 \\ 0 & 0 \\ 1.34 & -0.65 \\ 0 & 0 \end{bmatrix}; \; B_1 = B_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -0.327 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$C = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}; \; B_v = \begin{bmatrix} 0 \\ 0 \\ 0.203 \\ 0 \end{bmatrix}; \; B_d = \begin{bmatrix} 0 \\ 0 \\ 0.406 \\ 0 \end{bmatrix}$$

All variables are zero at the nominal steady state. The process is open-loop stable.

We design a control system based on the known disturbance characteristics, assuming that N_w can be modeled as step-wise variations of random magnitude and duration (integrated white noise). Then, in (5)–(6), $A_w = 1$, $B_w = 1$, and $C_w = 1$. The Jacobian matrices required in Eqs (15) and (17) can be computed analytically:

$$\tilde{A}_{k-1} = A + ((u_1)_{k-1} + (u_2)_{k-1}) B_1 \quad (33)$$
$$\tilde{B}_{k-1} = \tilde{B}_k = B_d \quad (34)$$
$$\tilde{B}_k^u = B_0 + \left[\begin{array}{cc} B_1 x_{k|k} & B_2 x_{k|k} \end{array} \right] \quad (35)$$

The output is a linear, time-invariant function of the states, so $C_k = C$ and $C_k^d = 0$. The desired closed-loop response time is in the order of 1 minute, so we choose a sampling period of $T_s = 0.25$ minutes.

We note that, because the only nonlinearity of the model comes from the bilinear terms multiplying the state vector with the inputs, the system can be considered as a linear time-varying system and linearization at each time step provides a perfect model for the particular time step. Hence, we expect the EKF to provide good state estimates assuming the model parameters are accurate and covariance matrices are chosen properly.

4.2 Controller Design

The tuning parameters are chosen as follows:

$$\begin{array}{rcl} \Sigma_{0|0} &=& I, \;\; R^w = \alpha I, \;\; R^\nu = I \\ \Lambda^y &=& \text{diag}\{1,1,0\}, \;\; \Lambda^u = \lambda I, \;\; p = 5, \;\; m = 3 \end{array} \quad (36)$$

α and λ are used as tuning parameters to find a right balance between the closed-loop speed and robustness. The measured disturbance N_p and the setpoints for N_2 and H_2 are projected to remain constant at their current values throughout the prediction horizon.

4.3 Simulation Results for Ideal Case

Figure 1 shows the regulatory response when there is a bias in $x_{0|0}$. The tuning parameters used were $\alpha = 3$ and $\lambda = 0.2$. The initial state of the plant was $x_0 = \left[\begin{array}{cccc} -1.5794 & -1.6811 & 1.0311 & 2.1436 \end{array} \right]^T$ (chosen randomly), and that of the estimator was $x_{0|0} = 0$. The disturbances were held constant at $N_p = N_w = 0$. Within 15 minutes, the controller drives H_2 and N_2 to their setpoints of zero, and the state estimates converge to the true values. The closed-loop response time is about 3 minutes, and the outputs are well-damped (Fig. 1).

Figure 2(a) shows the servo response for a step change of -1 in the setpoint of H_2 from an initial steady state at $x_0 = 0$. The setpoint of N_2 is zero at all times, and there are no disturbances. The initial state estimates are unbiased, *i.e.*, $x_{0|0} = x_0$. Since the "plant" and "model" are the same set of ODEs, EKF provides perfect state estimates for all k. The response time is about 2 minutes with an overshoot of about 10%. The output decoupling is good – the setpoint change of 1.0 units in H_2 causes a maximum deviation of about 0.1 units in N_2. Oscillations are well-damped.

The shortcoming of linear MPC for the problem is illustrated in Figure 2(b), which shows the servo-response of the standard (linear) MPC. This MPC was designed using a linear model derived at the nominal steady-state. The state estimator is the usual DMC type, which assumes random-step disturbances at the outputs. The values of p, m and λ used in the EKF based NLMPC gave an unstable closed-loop response. The simulations shown in Figure 2(b) used $p = 20$, blocking of $m = [\; 3 \;\; 5 \;\; 12 \;]$ and $\lambda = 0.4$. The servo response is significantly worse than that of NLMPC. The setpoint change in H_2 causes large, poorly-damped oscillations in N_2. The usual tuning tricks cannot overcome this problem. For example, use of $m = 3, \lambda = 2$ reduces the magnitude of the oscillations in N_2, but the response is more sluggish and is still poorly damped (not shown). For setpoint changes of small magnitude, on the other hand, the MPC servo-response approaches that of NLMPC.

4.4 Simulation Results for Model Error Case

We next test the robustness of the various designs by introducing parametric plant/model mismatch. This is not a rigorous test of robustness, but gives one an idea of the sensitivity of each method to modeling errors. Each element of the matrices used to represent the plant was multiplied by 1+d, where d was from a normal distribution, $N(0, 0.1)$. Thus, the perturbations are 10 % of the nominal value on the average. The C matrix was not modified for an obvious reason, and the controller and estimator calculations used the same model parameters as before. The EKF based NLMPC degrades considerably (Figure 3(a)), exhibiting an offset of 0.3 units in H_2, and 0.6 units in N_2. The problem here is that N_w affects states 3 and 4 only. As the EKF converges to a steady-state condition, the first 2 rows of the gain matrix, L_k, go to zero. In other words, the original disturbance model stipulates that there should be no disturbances in the first 2 states, so they receive no feedback correction at steady-state. They are biased by model error, however. This causes the estimates of the remaining states to be biased too, as the EKF tries to eliminate the apparent error in the output prediction. In doing so, it only has one degree of freedom: the disturbance N_w. Thus, it is impossible for the EKF (formulated as discussed) to construct the states to track their measured values at steady-state.

However, this is by no means an intrinsic problem. Since we are measuring three of the four states, we should clearly be able to construct unbiased estimates for these states. We can accomplish this by adding two more integrated white noise disturbances into the measured states and making the number of state disturbances (with integrating characteristics) equal to the number of measurements. We put our new disturbances into the second and fourth states. Putting a disturbance into the third state would make it a linearly dependent disturbance (redundant with N_w). Putting another integrated white noise into the first state caused indetectability since the number of disturbances then exceeds the number of measurements. The covariance matrix R^w was chosen as a diagonal matrix and tuned for the best results. All other controller parameters were as in the simulations for the previous cases. Results (with model error) appear in Figure 3(b). There is less interaction and the process settles more quickly to the setpoint than for the previous simulation. Furthermore, at the final time we have $x_k = [-1.3597 \; -1.0000 \; 1.4679 \; 0.0000]^T$, and $x_{k|k} = [-1.5380 \; -1.0000 \; 1.4679 \; 0.0000]^T$, *i.e.*, bias has been eliminated from the three measured states. This is possible because the EKF now has three degrees of freedom (the three state disturbances). The unmeasured state is slightly biased, but this has no impact on the controller performance.

For the low-order paper machine process, the use of the "non-physical" state disturbances is easily tuned, but this might not be the case for poorly scaled, high-order processes, or those for which the outputs are a nonlinear function of the states. It is recommend in general to design these disturbances such that they enter the measured combinations of the states. This ensures that the estimates for the measured states are unbiased.

An example application to a bilinear paper machine control problem was shown to point out the advantages and potential pitfalls of the proposed technique. A more elaborate application of the technique to an inferential control problem arising in pulp digesters can be found in Lee & Datta (1992c).

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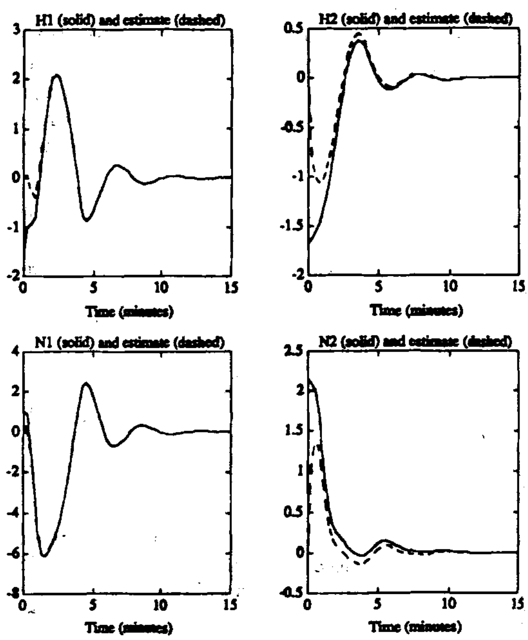

Figure 1: Responses of State Variables to the Intial State Biases Under EKF-based NLMPC

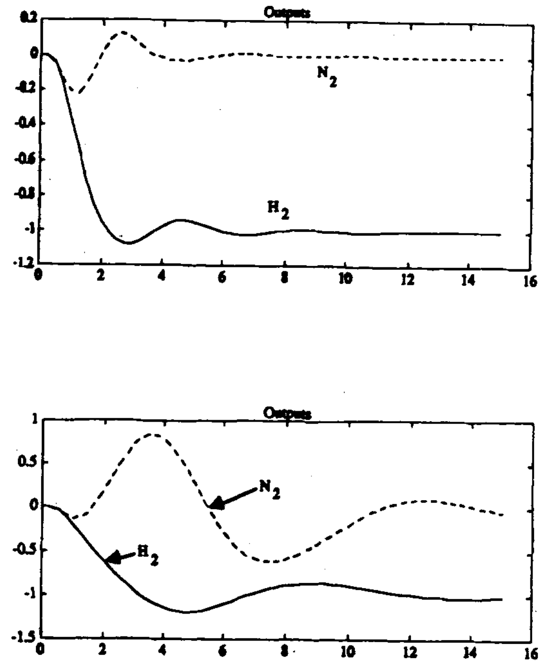

Figure 2: Responses of H_2 and N_2 to the Setpoint Change of -1 for H_2 Under (a) EKF-based NLMPC, (b) Linear MPC

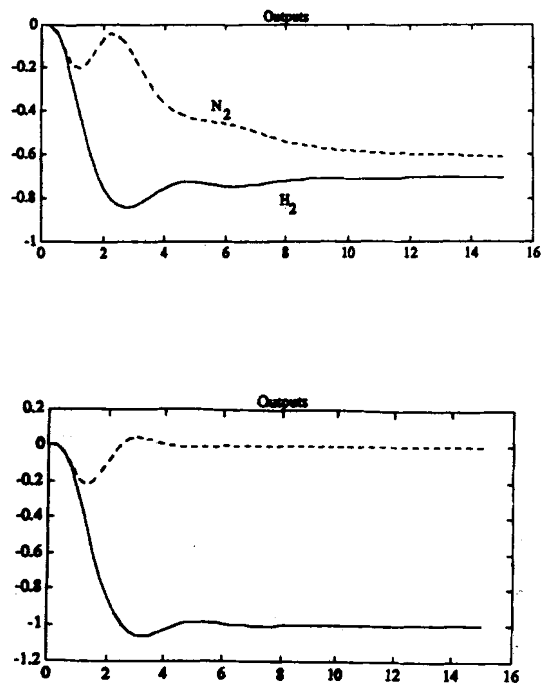

Figure 3: Responses of H_2 and N_2 to the Setpoint Change of -1 for H_2 Under EKF-based NLMPC with Model Error (a) One Degree-of-Freedom EKF, (b) Three Degree-of-Freedom EKF