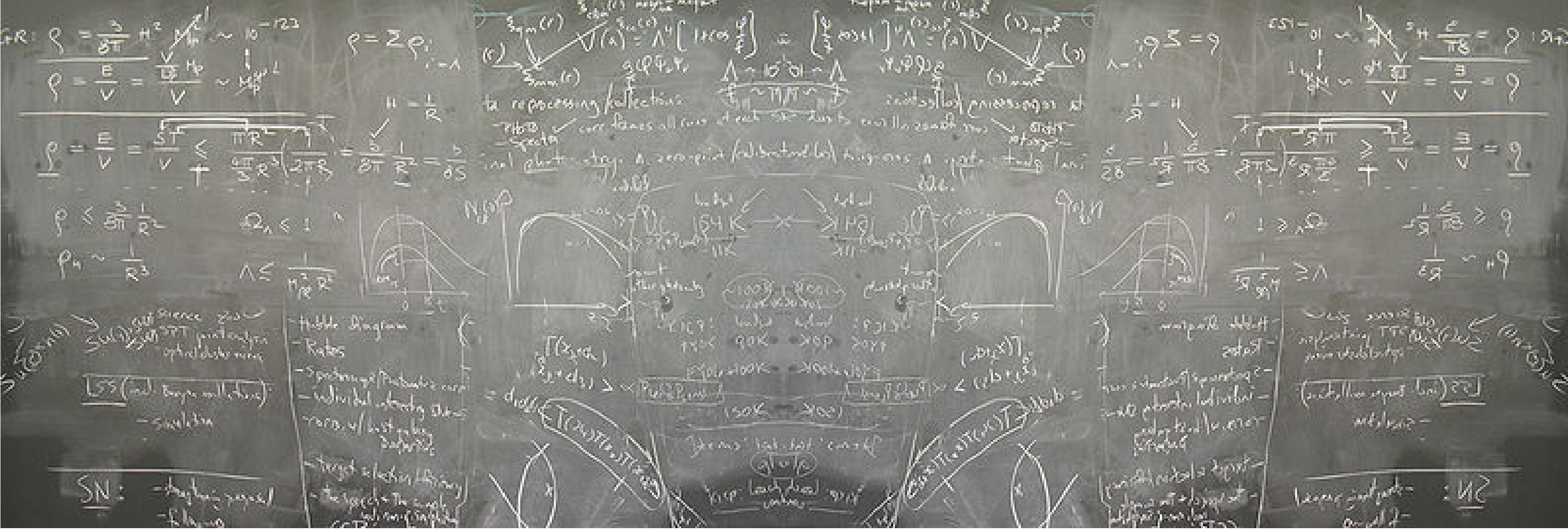


Anexo: Fórmulas matemáticas

Exponenciales complejas

$$
\begin{aligned}
e^{\pm j\theta} &= \cos\theta \pm j\sin\theta \\
Ae^{\pm j\theta} &= A\cos\theta \pm jA\sin\theta \\
\cos\theta &= \tfrac{e^{j\theta}+e^{-j\theta}}{2} \\
\sin\theta &= \tfrac{e^{j\theta}-e^{-j\theta}}{2j}
\end{aligned}
$$

Identidades trigonométricas de Pitagoras

$$
\begin{aligned}
\sin^2\theta + \cos^2\theta &= 1 \\
1+\tan^2\theta &= \sec^2\theta \\
1+\cot^2 &= \csc^2
\end{aligned}
$$

Productos notables

$$
\begin{aligned}
(a\pm b)^2 &= a^2 \pm 2ab + b^2 \\
x &= \frac{-b \pm \sqrt{b^2-4ac}}{2a} \\
(a\pm b)^3 &= a^3 \pm 3a^2b + 3ab^2 \pm b^3
\end{aligned}
$$

Variables complejas

$$
\begin{aligned}
z &= x + jy \\
&= r\underline{/\theta} = re^{j\theta} \\
&= r(\cos\theta + j\sin\theta) \\
r &= |z| = \sqrt{x^2+y^2} \\
\theta &= \tan^{-1}\frac{y}{x} \\
z* &= x - jy = r\underline{/-\theta}
\end{aligned}
$$

$$j = \sqrt{-1}, \quad \frac{1}{j} = -j, \quad j^2 = -1$$

Trigonometría

$$
\begin{aligned}
\cos(2k\pi) &= 1 \\
\mathrm{sen}(2k\pi) &= 0 \\
\cos(k\pi) &= (-1)^k \\
\sin(k\pi) &= 0 \\
\cos(k\tfrac{\pi}{2}) &= \begin{cases} (-1)^{\frac{k}{2}} & si \quad k \quad par \\ 0 \quad si \quad k \quad impar \end{cases} \\
\sin(k\tfrac{\pi}{2}) &= \begin{cases} (-1)^{\frac{k-1}{2}} & si \quad k \quad impar \\ 0 \quad si \quad k \quad par \end{cases} \\
\sin(\alpha \pm 2\pi) &= \sin(\alpha) \\
\cos(\alpha \pm 2\pi) &= \cos(\alpha) \\
\tan(\alpha \pm \pi) &= \cos(\alpha) \\
e^{\pm j2k\pi} &= 1 \\
e^{jk\pi} &= (-1)^k
\end{aligned}
$$

Identidades de ángulos dobles

$$
\begin{aligned}
\sin 2\theta &= 2\sin\theta\cos\theta \\
\cos 2\theta &= \cos^2\theta - \sin^2\theta \\
&= = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta \\
\sin^2\theta &= \frac{1-\cos 2\theta}{2} \\
\cos^2\theta &= \frac{1+\cos 2\theta}{2}
\end{aligned}
$$

Identidades trigonométricas de sumas de ángulos

$$
\begin{aligned}
\sin\alpha + \sin\beta &= 2\sin(\tfrac{\alpha+\beta}{2})\cos(\tfrac{\alpha-\beta}{2}) \\
\sin\alpha - \sin\beta &= 2\cos(\tfrac{\alpha+\beta}{2})\sin(\tfrac{\alpha-\beta}{2}) \\
\cos\alpha + \cos\beta &= 2\cos(\tfrac{\alpha+\beta}{2})\cos(\tfrac{\alpha-\beta}{2}) \\
\cos\alpha - \cos\beta &= -2\sin(\tfrac{\alpha+\beta}{2})\sin(\tfrac{\alpha-\beta}{2}) \\
\sin(\alpha \pm \beta) &= \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \\
\cos(\alpha \pm \beta) &= \cos\alpha\cos\beta \mp \sin\alpha\sin\beta
\end{aligned}
$$

Identidades trigonométricas de productos a suma

$$
\begin{aligned}
2\sin\alpha\sin\beta &= \cos(\alpha-\beta) - \cos(\alpha+\beta) \\
2\cos\alpha\cos\beta &= \cos(\alpha-\beta) + \cos(\alpha+\beta) \\
2\sin\alpha\cos\beta &= \sin(\alpha+\beta) + \sin(\alpha-\beta)
\end{aligned}
$$

Fracciones parciales

Polos simples:

$$X(s) = \frac{N(s)}{(s-p_1)(s-p_2)\ldots(s-p_m)}$$

$$X(s) = \frac{k_1}{s-p_1} + \frac{k_2}{s-p_2} + \ldots + \frac{k_m}{s-p_m}$$

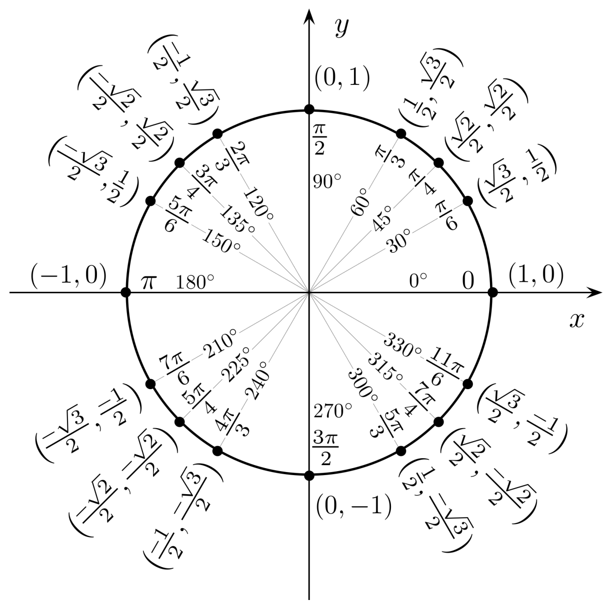
$$k_i = (s-p_i)X(s)|_{s=p_i}$$

Polos repetidos:

$$X(s) = \frac{k_r}{(s-p_1)^r} + \frac{k_{r-1}}{(s-p_1)^{r-1}} + \ldots + \frac{k_2}{(s-p_1)^2} + \frac{k_1}{s-p_1} + X_1(s)$$

$$
\begin{aligned}
k_r &= (s-p_1)^r X(s)\Big|_{s=p_1} \\
k_{r-1} &= \frac{\mathrm{d}}{\mathrm{d}s}\left[(s-p_1)^r X(s)\right]\Big|_{s=p_1} \\
k_{r-2} &= \frac{1}{2!}\frac{\mathrm{d}^2}{\mathrm{d}s^2}\left[(s-p_1)^r X(s)\right]\Big|_{s=p_1} \\
&\vdots \\
k_{r-n} &= \frac{1}{n!}\frac{\mathrm{d}^n}{\mathrm{d}s^n}\left[(s-p_1)^r X(s)\right]\Big|_{s=p_1}
\end{aligned}
$$

Senos y cosenos de ángulos comunes



Integrales esenciales

$$
\begin{aligned}
\int a dt &= at + C \\
\int U\, dV &= UV - \int V\, dU \\
\int \cos(at) dt &= \tfrac{1}{a}\sin(at) \\
\int \sin(at) dt &= -\tfrac{1}{a}\cos(at) \\
\int \cos^2(at) dt &= \tfrac{t}{2} + \tfrac{\sin(2at)}{4a} \\
\int \sin^2(at) dt &= \tfrac{t}{2} - \tfrac{\sin(2at)}{4a} \\
\int t \cdot \cos(at) dt &= \tfrac{1}{a^2}\cos(at) + t\tfrac{1}{a}\sin(at) \\
\int t \cdot \sin(at) dt &= \tfrac{1}{a^2} sin(at) - t\tfrac{1}{a} cos(at) \\
\int e^{at} dt &= \tfrac{1}{a}e^{at} \\
\int e^{at} \cdot \sin(bt) dt &= \tfrac{e^{at}}{a^2+b^2}(a\sin(bt) - b\cos(bt)) \\
\int e^{at} \cdot \cos(bt) dt &= \tfrac{e^{at}}{a^2+b^2}(a\cos(bt) + b\sin(bt)) \\
\int t^n\, dt &= \tfrac{1}{n+1}t^{n+1} \quad \text{si } n \neq -1 \\
\int \frac{1}{t}\, dt &= \ln|t| \\
\int \frac{1}{at+b}\, dt &= \tfrac{1}{a}\ln|at+b| \quad \text{si } a \neq 0
\end{aligned}
$$

Series geométricas

$$
\begin{aligned}
\sum_{k=0}^{N-1} r^k &= \begin{cases} \dfrac{1-r^N}{1-r}, & si \quad r \neq 1 \\ N, & si \quad r = 1 \end{cases} \\
\sum_{k=a}^{N-1} r^k &= \begin{cases} \dfrac{r^a - r^N}{1-r}, & \text{para } r \neq 1 \\ N-a, & \text{para } r = 1 \end{cases}, \ a \text{ es constante.} \\
\sum_{k=1}^{N} k &= \frac{N(N+1)}{2} \\
\sum_{k=1}^{N} k^2 &= \frac{N(N+1)(2N+1)}{6} \\
\sum_{k=a}^{N} k^b &= \sum_{k=1}^{N} k^b - \sum_{k=1}^{a-1} k^b, \ b \text{ es constante} \\
\sum_{k=0}^{\infty} r^k &= \frac{1}{1-r}, \quad \text{para } |r| < 1 \\
\sum_{k=a}^{\infty} r^k &= \frac{r^a}{1-r}, \quad \text{para } |r| < 1, \ a \text{ es constante} \\
\sum_{k=0}^{\infty} kr^k &= \frac{r}{(1-r)^2}, \quad \text{para } |r| < 1 \\
\sum_{k=0}^{\infty} k^2 r^k &= \frac{r^2 + r}{(1+r)^3}, \quad \text{para } |r| < 1
\end{aligned}
$$