



Anexo: Fórmulas matemáticas

Integrales esenciales

$$\begin{aligned}
 \int \cos(at) dt &= \frac{1}{a} \sin(at) \\
 \int \sin(at) dt &= -\frac{1}{a} \cos(at) \\
 \int t \cdot \cos(at) dt &= \frac{1}{a^2} \cos(at) + t \frac{1}{a} \sin(at) \\
 \int t \cdot \sin(at) dt &= \frac{1}{a^2} \sin(at) - t \frac{1}{a} \cos(at) \\
 \int e^t \cdot \sin(t) dt &= -\frac{1}{2} e^t \cos(t) + \frac{1}{2} e^t \sin(t) \\
 \int x^n dx &= \frac{1}{n+1} x^{n+1} \quad \text{si } n \neq -1 \\
 \int \frac{1}{x} dx &= \ln |x| \\
 \int \frac{1}{ax+b} dx &= \frac{1}{a} \ln |ax+b| \quad \text{si } a \neq 0
 \end{aligned}$$

Exponenciales complejas

$$\begin{aligned}
 e^{j\theta} &= \cos \theta + j \sin \theta \\
 \cos \theta &= \frac{e^{j\theta} + e^{-j\theta}}{2} \\
 \sin \theta &= \frac{e^{j\theta} - e^{-j\theta}}{2j}
 \end{aligned}$$

Identidades trigonométricas de Pitágoras

$$\begin{aligned}
 \sin^2 \theta + \cos^2 \theta &= 1 \\
 1 + \tan^2 \theta &= \sec^2 \theta
 \end{aligned}$$

Series geométricas

$$\begin{aligned}
 \sum_{k=0}^{N-1} r^k &= \begin{cases} \frac{1-r^N}{1-r}, & \text{si } r \neq 1 \\ N, & \text{si } r = 1 \end{cases} \\
 \sum_{k=a}^{N-1} r^k &= \begin{cases} \frac{r^a - r^N}{1-r}, & \text{para } r \neq 1, a \text{ es constante.} \\ N-a, & \text{para } r = 1 \end{cases} \\
 \sum_{k=1}^n k &= \frac{n(n+1)}{2} \\
 \sum_{k=1}^n k^2 &= \frac{n(n+1)(2n+1)}{6} \\
 \sum_{k=a}^n k^b &= \sum_{k=1}^n k^b - \sum_{k=1}^{a-1} k^b, b \text{ es constante} \\
 \sum_{k=0}^{\infty} r^k &= \frac{1}{1-r}, \quad \text{para } |r| < 1 \\
 \sum_{k=a}^{\infty} r^k &= \frac{r^a}{1-r}, \quad \text{para } |r| < 1, a \text{ es constante} \\
 \sum_{k=0}^{\infty} k r^k &= \frac{r}{(1-r)^2}, \quad \text{para } |r| < 1 \\
 \sum_{k=0}^{\infty} k^2 r^k &= \frac{r^2 + r}{(1-r)^3}, \quad \text{para } |r| < 1
 \end{aligned}$$

Productos notables

$$\begin{aligned}
 (a \pm b)^2 &= a^2 \pm 2ab + b^2 \\
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 (a \pm b)^3 &= a^3 \pm 3a^2b + 3ab^2 \pm b^3
 \end{aligned}$$

Trigonometría

$$\begin{aligned}
 \cos(2k\pi) &= 1 \\
 \sin(2k\pi) &= 0 \\
 \cos(k\pi) &= (-1)^k \\
 \sin(k\pi) &= 0 \\
 \cos(k\frac{\pi}{2}) &= \begin{cases} (-1)^{\frac{k}{2}} & \text{si } k \text{ par} \\ 0 & \text{si } k \text{ impar} \end{cases} \\
 \sin(k\frac{\pi}{2}) &= \begin{cases} (-1)^{\frac{k-1}{2}} & \text{si } k \text{ impar} \\ 0 & \text{si } k \text{ par} \end{cases} \\
 e^{j2k\pi} &= 1 \\
 e^{jk\pi} &= (-1)^k
 \end{aligned}$$

Identidades de ángulos dobles

$$\begin{aligned}
 \sin 2\theta &= 2 \sin \theta \cos \theta \\
 \cos 2\theta &= \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta \\
 \sin^2 \theta &= \frac{1 - \cos 2\theta}{2} \\
 \cos^2 \theta &= \frac{1 + \cos 2\theta}{2}
 \end{aligned}$$

Identidades trigonométricas de sumas de ángulos

$$\begin{aligned}
 \sin \alpha + \sin \beta &= 2 \sin\left(\frac{\alpha+\beta}{2}\right) \cos\left(\frac{\alpha-\beta}{2}\right) \\
 \sin \alpha - \sin \beta &= 2 \cos\left(\frac{\alpha+\beta}{2}\right) \sin\left(\frac{\alpha-\beta}{2}\right) \\
 \cos \alpha + \cos \beta &= 2 \cos\left(\frac{\alpha+\beta}{2}\right) \cos\left(\frac{\alpha-\beta}{2}\right) \\
 \cos \alpha - \cos \beta &= -2 \sin\left(\frac{\alpha+\beta}{2}\right) \sin\left(\frac{\alpha-\beta}{2}\right)
 \end{aligned}$$

Identidades trigonométricas de productos a suma

$$\begin{aligned}
 \sin \alpha \sin \beta &= \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)] \\
 \cos \alpha \cos \beta &= \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)] \\
 \sin \alpha \cos \beta &= \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]
 \end{aligned}$$

Senos y cosenos de ángulos comunes

