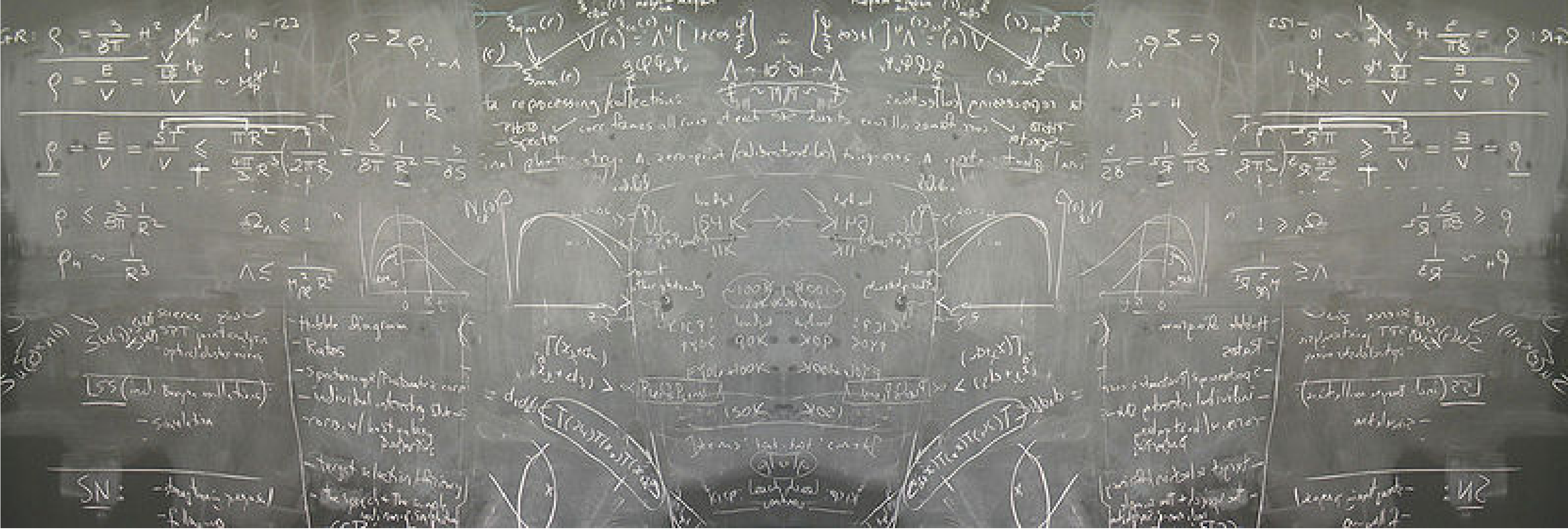

Anexo: Fórmulas matemáticas

Integrales esenciales

$$
\begin{aligned}
\int \cos(at)dt &= \tfrac{1}{a}\sin(at)\\
\int \sin(at)dt &= -\tfrac{1}{a}\cos(at)\\
\int t\cdot\cos(at)dt &= \tfrac{1}{a^2}\cos(at)+t\tfrac{1}{a}\sin(at)\\
\int t\cdot\sin(at)dt &= \tfrac{1}{a^2}sin(at)-t\tfrac{1}{a}cos(at)\\
\int e^t\cdot\sin(t)dt &= -\tfrac{1}{2}e^t\cos(t)+\tfrac{1}{2}e^t\sin(t)\\
\int x^n\,dx &= \tfrac{1}{n+1}x^{n+1}\ \text{ si } n\neq -1\\
\int \frac{1}{x}\,dx &= \ln|x|\\
\int \frac{1}{ax+b}\,dx &= \tfrac{1}{a}\ln|ax+b|\ \text{ si } a\neq 0
\end{aligned}
$$

Exponenciales complejas

$$
\begin{aligned}
e^{j\theta} &= \cos\theta + j\sin\theta\\
\cos\theta &= \tfrac{e^{j\theta}+e^{-j\theta}}{2}\\
\sin\theta &= \tfrac{e^{j\theta}-e^{-j\theta}}{2j}
\end{aligned}
$$

Identidades trigonométricas de Pitagoras

$$
\begin{aligned}
\sin^2\theta+\cos^2\theta &= 1\\
1+\tan^2\theta &= \sec^2\theta
\end{aligned}
$$

Series geométricas

$$
\begin{aligned}
\sum_{k=0}^{N-1} r^k &= \begin{cases}\dfrac{1-r^N}{1-r}, & si \quad r\neq 1\\ N, & si \quad r=1\end{cases}\\
\sum_{k=a}^{N-1} r^k &= \begin{cases}\dfrac{r^a-r^N}{1-r}, & \text{para } r\neq 1\\ N-a, & \text{para } r=1\end{cases},\ a \text{ es constante.}\\
\sum_{k=1}^{n} k &= \frac{n(n+1)}{2}\\
\sum_{k=1}^{n} k^2 &= \frac{n(n+1)(2n+1)}{6}\\
\sum_{k=a}^{n} k^b &= \sum_{k=1}^{n} k^b - \sum_{k=1}^{a-1} k^b,\ b \text{ es constante}\\
\sum_{k=0}^{\infty} r^k &= \frac{1}{1-r}, \quad \text{para } |r|<1\\
\sum_{k=a}^{\infty} r^k &= \frac{r^a}{1-r}, \quad \text{para } |r|<1,\ a \text{ es constante}\\
\sum_{k=0}^{\infty} kr^k &= \frac{r}{(1-r)^2}, \quad \text{para } |r|<1\\
\sum_{k=0}^{\infty} k^2r^k &= \frac{r^2+r}{(1+r)^3}, \quad \text{para } |r|<1
\end{aligned}
$$

Productos notables

$$
\begin{aligned}
(a\pm b)^2 &= a^2\pm 2ab+b^2\\
x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a}\\
(a\pm b)^3 &= a^3\pm 3a^2b+3ab^2\pm b^3
\end{aligned}
$$

Trigonometría

$$
\begin{aligned}
\cos(2k\pi) &= 1\\
\mathrm{sen}(2k\pi) &= 0\\
\cos(k\pi) &= (-1)^k\\
\sin(k\pi) &= 0\\
\cos(k\tfrac{\pi}{2}) &= \begin{cases} (-1)^{\frac{k}{2}} & si \quad k \quad par\\ 0 \quad si \quad k \quad impar & \end{cases}\\
\sin(k\tfrac{\pi}{2}) &= \begin{cases} (-1)^{\frac{k-1}{2}} & si \quad k \quad impar\\ 0 \quad si \quad k \quad par & \end{cases}\\
e^{j2k\pi} &= 1\\
e^{jk\pi} &= (-1)^k
\end{aligned}
$$

Identidades de ángulos dobles

$$
\begin{aligned}
\sin 2\theta &= 2\sin\theta\cos\theta\\
\cos 2\theta &= \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta\\
\sin^2\theta &= \frac{1-\cos 2\theta}{2}\\
\cos^2\theta &= \frac{1+\cos 2\theta}{2}
\end{aligned}
$$

Identidades trigonométricas de sumas de ángulos

$$
\begin{aligned}
\sin\alpha + \sin\beta &= 2\sin(\tfrac{\alpha+\beta}{2})\cos(\tfrac{\alpha-\beta}{2})\\
\sin\alpha - \sin\beta &= 2\cos(\tfrac{\alpha+\beta}{2})\sin(\tfrac{\alpha-\beta}{2})\\
\cos\alpha + \cos\beta &= 2\cos(\tfrac{\alpha+\beta}{2})\cos(\tfrac{\alpha-\beta}{2})\\
\cos\alpha - \cos\beta &= -2\sin(\tfrac{\alpha+\beta}{2})\sin(\tfrac{\alpha-\beta}{2})
\end{aligned}
$$

Identidades trigonométricas de productos a suma

$$
\begin{aligned}
\sin\alpha\sin\beta &= \tfrac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]\\
\cos\alpha\cos\beta &= \tfrac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]\\
\sin\alpha\cos\beta &= \tfrac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]
\end{aligned}
$$

Senos y cosenos de ángulos comunes