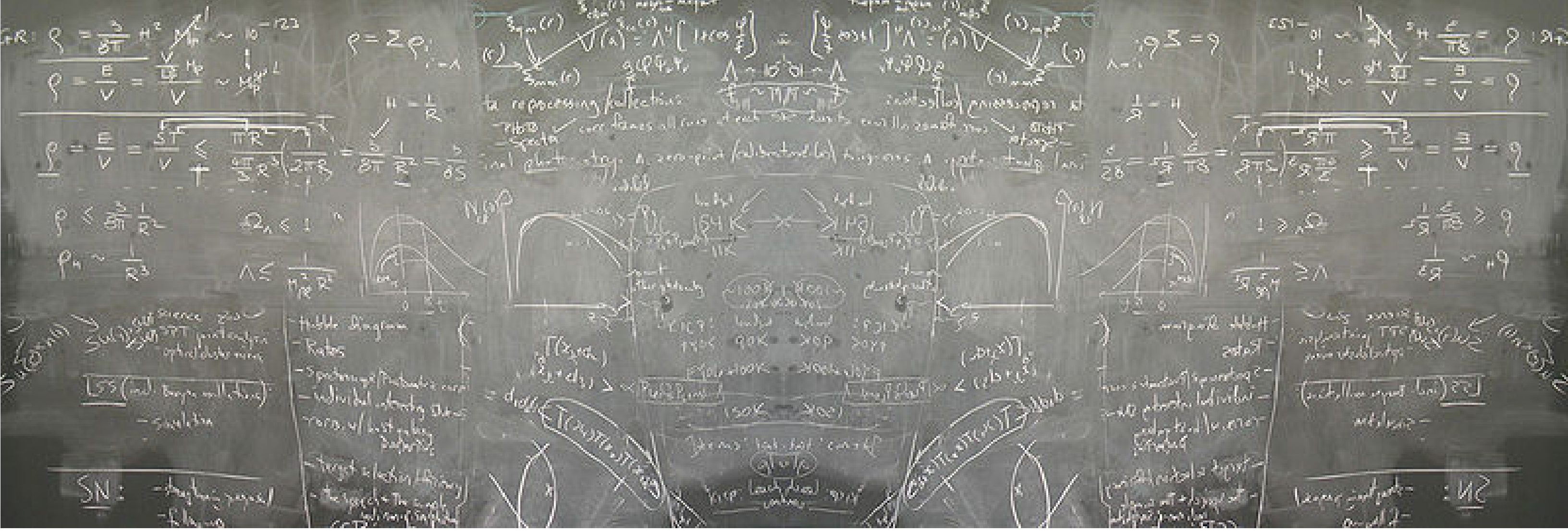


Anexo: Fórmulas matemáticas

Integrales esenciales

$$
\begin{aligned}
\int \cos(at)dt &= \tfrac{1}{a}\sin(at) \\
\int \sin(at)dt &= -\tfrac{1}{a}\cos(at) \\
\int t\cdot\cos(at)dt &= \tfrac{1}{a^2}\cos(at) + t\tfrac{1}{a}\sin(at) \\
\int t\cdot\sin(at)dt &= \tfrac{1}{a^2}sin(at) - t\tfrac{1}{a}cos(at) \\
\int e^t\cdot\sin(t)dt &= -\tfrac{1}{2}e^t\cos(t) + \tfrac{1}{2}e^t\sin(t) \\
\int x^n\,dx &= \tfrac{1}{n+1}x^{n+1} \;\; \text{si } n \neq -1 \\
\int \frac{1}{x}\,dx &= \ln|x| \\
\int \frac{1}{ax+b}\,dx &= \tfrac{1}{a}\ln|ax+b| \;\; \text{si } a \neq 0
\end{aligned}
$$

Exponenciales complejas

$$
\begin{aligned}
e^{\pm j\theta} &= \cos\theta \pm j\sin\theta \\
\cos\theta &= \tfrac{e^{j\theta}+e^{-j\theta}}{2} \\
\sin\theta &= \tfrac{e^{j\theta}-e^{-j\theta}}{2j}
\end{aligned}
$$

Identidades trigonométricas de Pitagoras

$$
\begin{aligned}
\sin^2\theta + \cos^2\theta &= 1 \\
1 + \tan^2\theta &= \sec^2\theta
\end{aligned}
$$

Series geométricas

$$
\begin{aligned}
\sum_{k=0}^{N-1} r^k &= \begin{cases} \dfrac{1-r^N}{1-r}, & si \quad r \neq 1 \\ N, & si \quad r = 1 \end{cases} \\
\sum_{k=a}^{N-1} r^k &= \begin{cases} \dfrac{r^a-r^N}{1-r}, & \text{para } r \neq 1 \\ N-a, & \text{para } r = 1 \end{cases}, \; a \text{ es constante.} \\
\sum_{k=1}^{n} k &= \frac{n(n+1)}{2} \\
\sum_{k=1}^{n} k^2 &= \frac{n(n+1)(2n+1)}{6} \\
\sum_{k=a}^{n} k^b &= \sum_{k=1}^{n} k^b - \sum_{k=1}^{a-1} k^b, \; b \text{ es constante} \\
\sum_{k=0}^{\infty} r^k &= \frac{1}{1-r}, \quad \text{para } |r| < 1 \\
\sum_{k=a}^{\infty} r^k &= \frac{r^a}{1-r}, \quad \text{para } |r| < 1, \; a \text{ es constante} \\
\sum_{k=0}^{\infty} kr^k &= \frac{r}{(1-r)^2}, \quad \text{para } |r| < 1 \\
\sum_{k=0}^{\infty} k^2r^k &= \frac{r^2+r}{(1+r)^3}, \quad \text{para } |r| < 1
\end{aligned}
$$

Productos notables

$$
\begin{aligned}
(a \pm b)^2 &= a^2 \pm 2ab + b^2 \\
x &= \frac{-b \pm \sqrt{b^2-4ac}}{2a} \\
(a \pm b)^3 &= a^3 \pm 3a^2b + 3ab^2 \pm b^3
\end{aligned}
$$

Trigonometría

$$\begin{aligned}
\cos(2k\pi) &= 1\\
\text{sen}(2k\pi) &= 0\\
\cos(k\pi) &= (-1)^k\\
\sin(k\pi) &= 0\\
\cos(k\tfrac{\pi}{2}) &= \begin{cases} (-1)^{\frac{k}{2}} & si \quad k \quad par \\ \\ 0 \quad si \quad k \quad impar \end{cases}\\
\sin(k\tfrac{\pi}{2}) &= \begin{cases} (-1)^{\frac{k-1}{2}} & si \quad k \quad impar \\ \\ \quad 0 \quad si \quad k \quad par \end{cases}\\
e^{j2k\pi} &= 1\\
e^{jk\pi} &= (-1)^k
\end{aligned}$$

Identidades de ángulos dobles

$$\begin{aligned}
\sin 2\theta &= 2\sin\theta\cos\theta\\
\cos 2\theta &= \cos^2\theta - \sin^2\theta\\
&= = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta\\
\sin^2\theta &= \frac{1-\cos 2\theta}{2}\\
\cos^2\theta &= \frac{1+\cos 2\theta}{2}
\end{aligned}$$

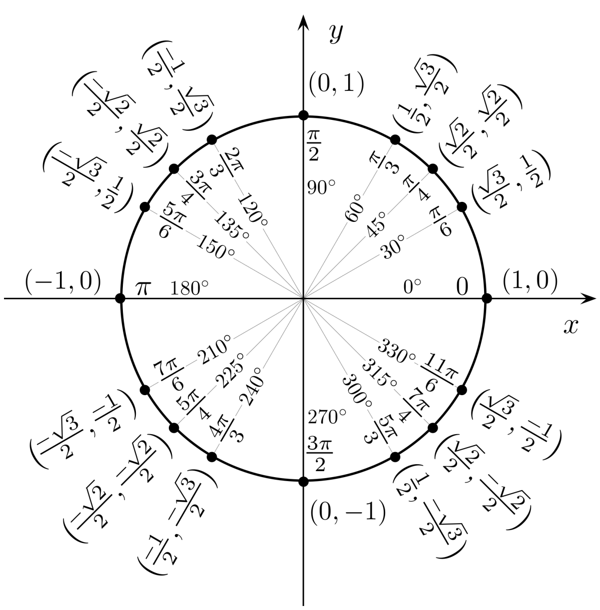
Identidades trigonométricas de sumas de ángulos

$$\begin{aligned}
\sin\alpha + \sin\beta &= 2\sin(\tfrac{\alpha+\beta}{2})\cos(\tfrac{\alpha-\beta}{2})\\
\sin\alpha - \sin\beta &= 2\cos(\tfrac{\alpha+\beta}{2})\sin(\tfrac{\alpha-\beta}{2})\\
\cos\alpha + \cos\beta &= 2\cos(\tfrac{\alpha+\beta}{2})\cos(\tfrac{\alpha-\beta}{2})\\
\cos\alpha - \cos\beta &= -2\sin(\tfrac{\alpha+\beta}{2})\sin(\tfrac{\alpha-\beta}{2})
\end{aligned}$$

Identidades trigonométricas de productos a suma

$$\begin{aligned}
\sin\alpha\sin\beta &= \tfrac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]\\
\cos\alpha\cos\beta &= \tfrac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]\\
\sin\alpha\cos\beta &= \tfrac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]
\end{aligned}$$

Senos y cosenos de ángulos comunes



Fracciones parciales

Polos simples:

$$X(s) = \frac{N(s)}{(s-p_1)(s-p_2)\ldots(s-p_m)}$$

$$X(s) = \frac{k_1}{s-p_1} + \frac{k_2}{s-p_2} + \ldots + \frac{k_m}{s-p_m}$$

$$k_i = (s-p_i)X(s)|_{s=p_i}$$

Laplace:

$$x(t) = (k_1e^{p_1t} + k_2e^{p_2t} + \ldots + k_me^{p_mt})u(t)$$

Polos repetidos:

$$X(s) = \frac{k_r}{(s-p_1)^r} + \frac{k_{r-1}}{(s-p_1)^{r-1}} + \ldots + \frac{k_2}{(s-p_1)^2} + \frac{k_1}{s-p_1} + X_1(s)$$

$$\begin{aligned}
k_r &= (s-p_1)^r X(s)\Big|_{s=p_1}\\
k_{r-1} &= \frac{\mathrm{d}}{\mathrm{d}s}\left[(s-p_1)^r X(s)\right]\Big|_{s=p_1}\\
k_{r-2} &= \frac{1}{2!}\frac{\mathrm{d}^2}{\mathrm{d}s^2}\left[(s-p_1)^r X(s)\right]\Big|_{s=p_1}\\
&\vdots\\
k_{r-n} &= \frac{1}{n!}\frac{\mathrm{d}^n}{\mathrm{d}s^n}\left[(s-p_1)^r X(s)\right]\Big|_{s=p_1}
\end{aligned}$$

Laplace:

$$x(t) = \Bigg(k_1e^{p_1t} + k_2te^{p_1t} + \frac{k_3}{2!}t^2e^{p_1t} + \ldots + \frac{k_m}{(m-1)!}t^{m-1}e^{p_1t}\Bigg)u(t) + x_1(t) \qquad (1)$$