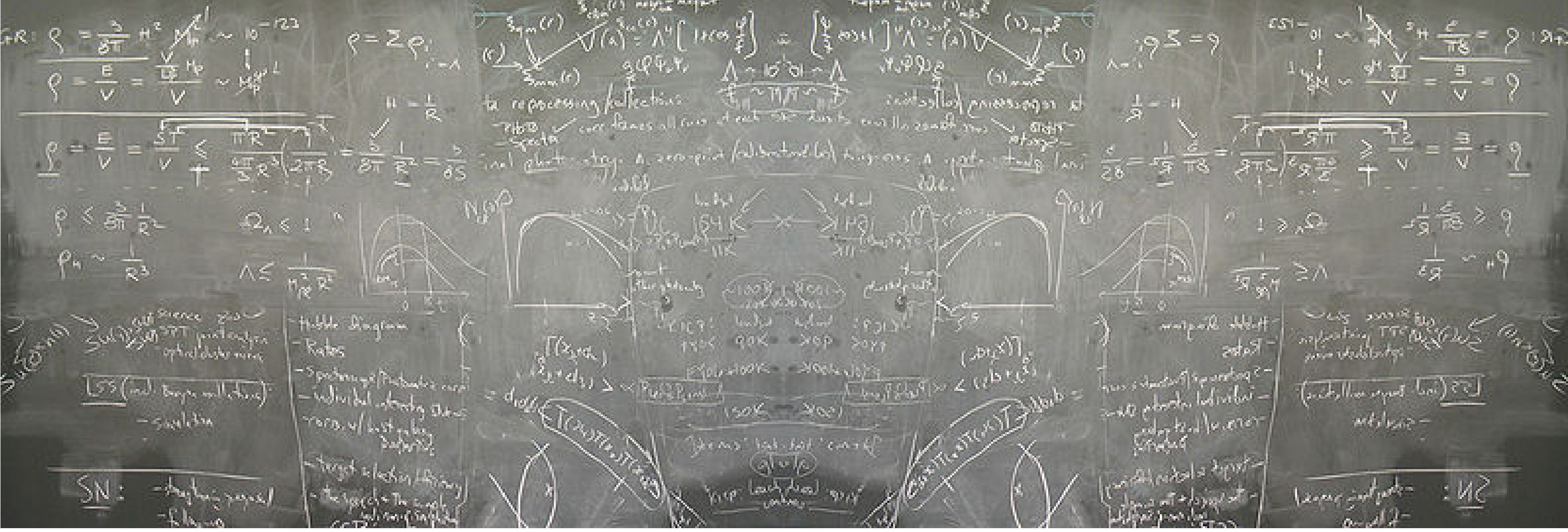


Anexo: Fórmulas matemáticas

Exponenciales complejas

$$\begin{aligned}
e^{\pm j\theta} &= \cos\theta \pm j\sin\theta \\
Ae^{\pm j\theta} &= A\cos\theta \pm jA\sin\theta \\
\cos\theta &= \frac{e^{j\theta}+e^{-j\theta}}{2} \\
\sin\theta &= \frac{e^{j\theta}-e^{-j\theta}}{2j} \\
\tan\theta &= -j\left[\frac{e^{j\theta}-e^{-j\theta}}{e^{j\theta}+e^{-j\theta}}\right] \\
1+e^{-j\theta} &= (e^{j\frac{\theta}{2}}+e^{-j\frac{\theta}{2}})e^{-j\frac{\theta}{2}} \\
&= 2\cos\left(\tfrac{\theta}{2}\right)e^{-j\frac{\theta}{2}}
\end{aligned}$$

Identidades trigonométricas de Pitagoras

$$\begin{aligned}
\sin^2\theta+\cos^2\theta &= 1 \\
1+\tan^2\theta &= \sec^2\theta \\
1+\cot^2 &= \csc^2
\end{aligned}$$

Productos notables

$$\begin{aligned}
(a\pm b)^2 &= a^2 \pm 2ab + b^2 \\
x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a} \\
(a\pm b)^3 &= a^3 \pm 3a^2b + 3ab^2 \pm b^3
\end{aligned}$$

Variables complejas

$$\begin{aligned}
z &= x+jy = r\underline{/\theta} = re^{j\theta} \\
&= r(\cos\theta + j\sin\theta) \\
r &= |z| = \sqrt{x^2+y^2} \\
\theta &= \tan^{-1}\frac{y}{x} \\
z* &= x-jy = r\underline{/-\theta}
\end{aligned}$$

$j=\sqrt{-1}$, $\dfrac{1}{j}=-j$, $j^2=-1$

Trigonometría

$$\begin{aligned}
\cos(2k\pi) &= 1 \\
\mathrm{sen}(2k\pi) &= 0 \\
\cos(k\pi) &= (-1)^k \\
\sin(k\pi) &= 0 \\
\cos(k\tfrac{\pi}{2}) &= \begin{cases} (-1)^{\frac{k}{2}} & si \quad k \quad par \\ 0 \quad si \quad k \quad impar & \end{cases} \\
\sin(k\tfrac{\pi}{2}) &= \begin{cases} (-1)^{\frac{k-1}{2}} & si \quad k \quad impar \\ 0 \quad si \quad k \quad par & \end{cases} \\
\sin(\alpha\pm 2k\pi) &= \sin(\alpha) \\
\sin(\alpha+\tfrac{\pi}{2}) &= \cos(\alpha) \\
\cos(\alpha\pm 2k\pi) &= \cos(\alpha) \\
\cos(\alpha-\tfrac{\pi}{2}) &= \sin(\alpha) \\
\tan(\alpha\pm k\pi) &= \cos(\alpha) \\
e^{\pm j2k\pi} &= 1 \\
e^{jk\pi} &= (-1)^k
\end{aligned}$$

Identidades de ángulos dobles

$$\begin{aligned}
\sin 2\theta &= 2\sin\theta\cos\theta \\
\cos 2\theta &= \cos^2\theta - \sin^2\theta \\
&= 2\cos^2\theta - 1 = 1 - 2\sin^2\theta \\
\sin^2\theta &= \frac{1-\cos 2\theta}{2} \\
\cos^2\theta &= \frac{1+\cos 2\theta}{2}
\end{aligned}$$

Multiples ángulos

$$\begin{aligned}\sin(n\theta) &= 2\sin((n-1)\theta)\cos\theta - \sin((n-2)\theta)\\ \cos(n\theta) &= 2\cos((n-1)\theta)\cos\theta - \cos((n-2)\theta)\end{aligned}$$

Identidades trigonométricas de sumas de ángulos

$$\begin{aligned}\sin\alpha+\sin\beta &= 2\sin(\tfrac{\alpha+\beta}{2})\cos(\tfrac{\alpha-\beta}{2})\\ \sin\alpha-\sin\beta &= 2\cos(\tfrac{\alpha+\beta}{2})\sin(\tfrac{\alpha-\beta}{2})\\ \cos\alpha+\cos\beta &= 2\cos(\tfrac{\alpha+\beta}{2})\cos(\tfrac{\alpha-\beta}{2})\\ \cos\alpha-\cos\beta &= -2\sin(\tfrac{\alpha+\beta}{2})\sin(\tfrac{\alpha-\beta}{2})\\ \sin(\alpha\pm\beta) &= \sin\alpha\cos\beta\pm\cos\alpha\sin\beta\\ \cos(\alpha\pm\beta) &= \cos\alpha\cos\beta\mp\sin\alpha\sin\beta\end{aligned}$$

$$A\cos\alpha+B\sin\alpha=\sqrt{A^2+B^2}\cos(\alpha-\tan^{-1}(\tfrac{B}{A}))$$

Identidades trigonométricas de productos a suma

$$\begin{aligned}2\sin\alpha\sin\beta &= \cos(\alpha-\beta)-\cos(\alpha+\beta)\\ 2\cos\alpha\cos\beta &= \cos(\alpha-\beta)+\cos(\alpha+\beta)\\ 2\sin\alpha\cos\beta &= \sin(\alpha+\beta)+\sin(\alpha-\beta)\end{aligned}$$

Relaciones trigonométricas hiperbólicas

$$\begin{aligned}\sinh\theta &= \frac{e^{\theta}-e^{-\theta}}{2}\\ \cosh\theta &= \frac{e^{\theta}+e^{-\theta}}{2}\\ \cosh^2\theta-\sinh^2\theta &= 1\end{aligned}$$

Fracciones parciales

Polos simples:

$$X(s)=\frac{N(s)}{(s-p_1)(s-p_2)\dots(s-p_m)}$$

$$X(s)=\frac{k_1}{s-p_1}+\frac{k_2}{s-p_2}+\dots+\frac{k_m}{s-p_m}$$

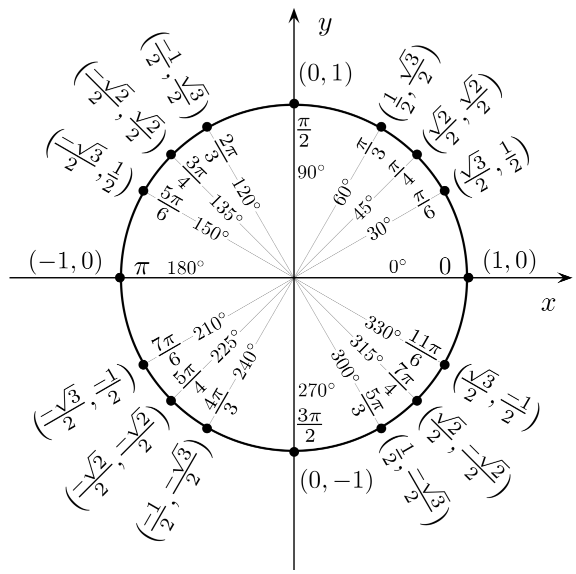
$$k_i=(s-p_i)X(s)|_{s=p_i}$$

Polos repetidos:

$$X(s)=\frac{k_r}{(s-p_1)^r}+\frac{k_{r-1}}{(s-p_1)^{r-1}}+\dots+\frac{k_2}{(s-p_1)^2}+\frac{k_1}{s-p_1}+X_1(s)$$

$$\begin{aligned}k_r &= (s-p_1)^rX(s)\Big|_{s=p_1}\\ k_{r-1} &= \frac{\mathrm{d}}{\mathrm{d}s}\left[(s-p_1)^rX(s)\right]\Big|_{s=p_1}\\ k_{r-2} &= \frac{1}{2!}\frac{\mathrm{d}^2}{\mathrm{d}s^2}\left[(s-p_1)^rX(s)\right]\Big|_{s=p_1}\\ &\vdots\\ k_{r-n} &= \frac{1}{n!}\frac{\mathrm{d}^n}{\mathrm{d}s^n}\left[(s-p_1)^rX(s)\right]\Big|_{s=p_1}\end{aligned}$$

Senos y cosenos de ángulos comunes



Integrales esenciales

$$
\begin{aligned}
\int a dt &= at + C \\
\int U\,dV &= UV - \int V\,dU \\
\int \cos(at)dt &= \tfrac{1}{a}\sin(at) \\
\int \sin(at)dt &= -\tfrac{1}{a}\cos(at) \\
\int \cos^2(at)dt &= \tfrac{t}{2} + \tfrac{\sin(2at)}{4a} \\
\int \sin^2(at)dt &= \tfrac{t}{2} - \tfrac{\sin(2at)}{4a} \\
\int t\cdot\cos(at)dt &= \tfrac{1}{a^2}\cos(at) + t\tfrac{1}{a}\sin(at) \\
\int t\cdot\sin(at)dt &= \tfrac{1}{a^2}\sin(at) - t\tfrac{1}{a}\cos(at) \\
\int t^a\cdot\sin(t)dt &= -t^a\cos(t) + n\int t^{a-1}\cdot\cos(t)dt \\
\int t^a\cdot\cos(t)dt &= t^a\sin(t) - n\int t^{a-1}\cdot\sin(t)dt \\
\int e^{at}dt &= \tfrac{1}{a}e^{at} \\
\int t\cdot e^{at}dt &= \frac{e^{at}}{a^2}(at-1) \\
\int e^{at}\cdot\sin(bt)dt &= \frac{e^{at}}{a^2+b^2}(a\sin(bt) - b\cos(bt)) \\
\int e^{at}\cdot\cos(bt)dt &= \frac{e^{at}}{a^2+b^2}(a\cos(bt) + b\sin(bt)) \\
\int t^n\,dt &= \tfrac{1}{n+1}t^{n+1} \quad \text{si } n \neq -1 \\
\int \frac{1}{t}\,dt &= \ln|t| \\
\int \frac{1}{at+b}\,dt &= \tfrac{1}{a}\ln|at+b| \quad \text{si } a \neq 0 \\
\int \frac{1}{a^2+(bt)^2}\,dt &= \tfrac{1}{ab}\tan^{-1}(\tfrac{bt}{a}) \\
\int \frac{1}{\sqrt{t^2 \pm a^2}}\,dt &= \ln|t + \sqrt{t^2 \pm a^2}|
\end{aligned}
$$

Series geométricas

$$
\begin{aligned}
\sum_{k=0}^{N-1} r^k &= \begin{cases} \dfrac{1-r^N}{1-r}, & si \quad r \neq 1 \\ N, & si \quad r = 1 \end{cases} \\
\sum_{k=a}^{N-1} r^k &= \begin{cases} \dfrac{r^a - r^N}{1-r}, & \text{para } r \neq 1 \\ N-a, & \text{para } r = 1 \end{cases}, \ a \text{ es constante.} \\
\sum_{k=0}^{N-1} kr^k &= \frac{r[1-(N+2)r^{N+1} + (N+1)r^{N+2}]}{(1-r)^2}, \quad \text{para } r \neq 1 \\
\sum_{k=1}^{N} k &= \frac{N(N+1)}{2} \\
\sum_{k=1}^{N} k^2 &= \frac{N(N+1)(2N+1)}{6} \\
\sum_{k=a}^{N} k^b &= \sum_{k=1}^{N} k^b - \sum_{k=1}^{a-1} k^b, \ b \text{ es constante} \\
\sum_{k=0}^{\infty} r^k &= \frac{1}{1-r}, \quad \text{para } |r| < 1 \\
\sum_{k=a}^{\infty} r^k &= \frac{r^a}{1-r}, \quad \text{para } |r| < 1, \ a \text{ es constante} \\
\sum_{k=0}^{\infty} kr^k &= \frac{r}{(1-r)^2}, \quad \text{para } |r| < 1 \\
\sum_{k=0}^{\infty} k^2r^k &= \frac{r^2+r}{(1+r)^3}, \quad \text{para } |r| < 1
\end{aligned}
$$

Integrales definidas

$$
\begin{aligned}
\int_0^\infty \frac{\sin(at)}{t}dt &= \frac{\pi}{2}, \ a > 0 \\
\int_0^\infty t^k\cdot e^{-at}dt &= \frac{k!}{a^{k+1}}, \ k \in \mathbb{Z}, \ a > 0 \\
\int_0^\infty e^{-at}\cdot\cos(bt)dt &= \frac{a}{a^2+b^2}, \ a > 0 \\
\int_0^\infty e^{-at}\cdot\sin(bt)dt &= \frac{b}{a^2+b^2}, \ a > 0
\end{aligned}
$$