UNIVERSITY OF PADUA
UNIVERSITA' DEGLI STUDI DI PADOVA

A Network of Thrones:
Shedding Light on Game of Thrones through Character Interaction Networks

Network Science, A.Y. 2023/24 – Final Project

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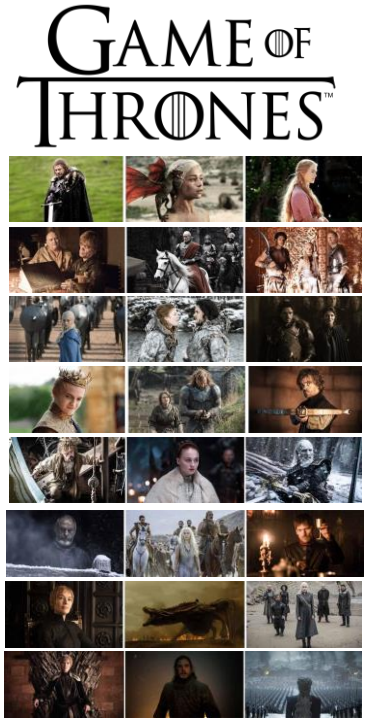
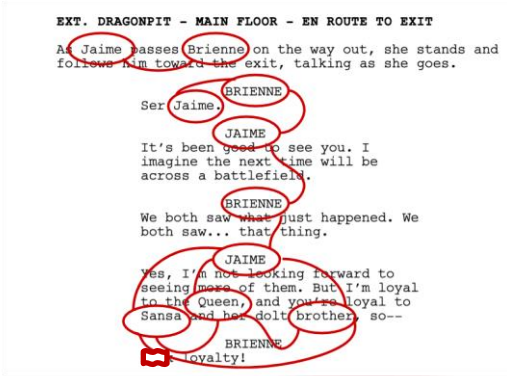
DIPARTIMENTO MATEMATICA

DIPARTIMENTO DI INGEGNERIA DELL'INFORMAZIONE

Introduction: Game of Thrones

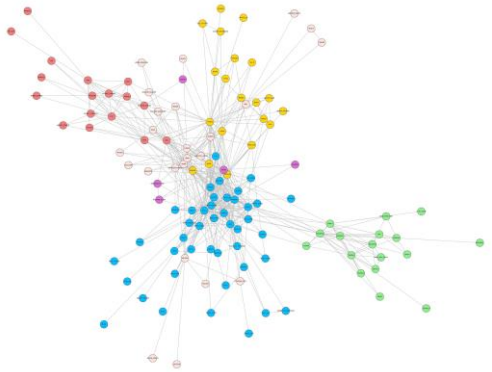
- **Game of Thrones**: popular fantasy drama TV Series.
- 8 Seasons, roughly 10 episodes each.
- Focuses on **character interactions**, political power and the quest for the Iron Throne → Interesting to investigate using Network Science!
- **Dataset**:
  - Scripts, stage directions and subtitles were processed into **undirected, weighted graphs** (one for each season).
  - The **weight** of an edge represents the **number of interactions.**
  - **Character A** and **Character B** are connected if:
    - Character A speaks directly after Character B
    - Character A speaks about Character B
    - Character C speaks about Character A & Character B
    - Character A & Character B are mentioned in the same stage direction
    - Character A & Character B appear in a scene together

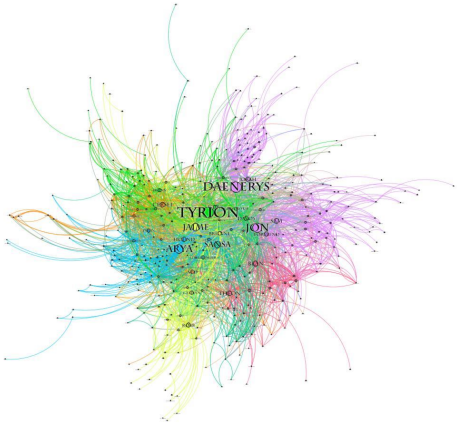




Approaching the Analysis

- Given the **8 Graphs** in the dataset, two approaches have been used:
  - **Analyzing each graph separately**:
    - **Prevents** the creation of a **large** and **messy** network, **facilitates** a more **in-depth** analysis of the story and the detection of communities.
    - Allows for **temporal** analysis through the seasons.
  - **Merge the graphs and analyze the overall GoT network**:
    - Overall overview of the whole story.





- Therefore:
  - **Season 1** is fully analyzed.
  - **Seasons 2-8** focus only on selected aspects, with focus on plot lines and character interactions.
  - The **Merged graph** is analyzed with focus on an overall overview of Game of Thrones, as well as on temporal analysis.

Season 1: Metrics

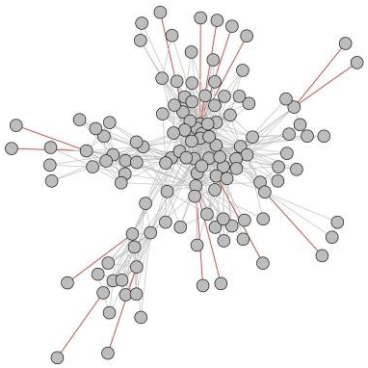


Fig 1. Bridges

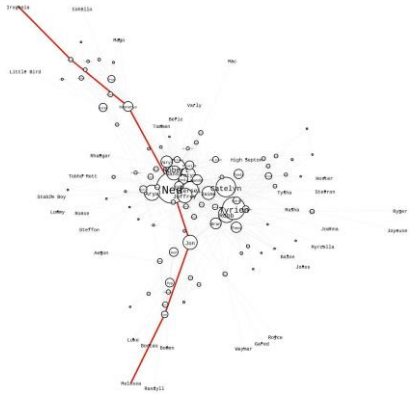


Fig 2. Diameter path

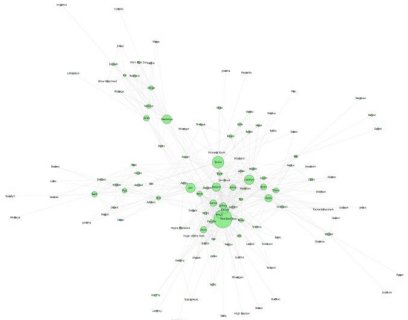
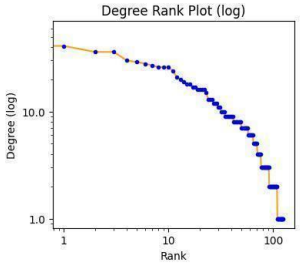
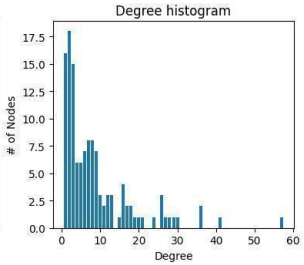


Fig 3. Season 1 (PR)

Description	Value
Nodes	126
Edges	549
Graph Density	0.697
Connected Graph	Yes
Number of Components	1
Small-World Network	Yes ($\lambda = 1.08$, $\gamma = 10.2$)
Diameter	6
Avg. Shortest Path	2.64
Avg. Degree	8.71, range [1, 57]
Most Freq. Degree	2
Number of Bridges	16
Deg. Assortativity Coeff.	-0.141
Global Clustering Coeff.	0.383
Average Clustering Coeff.	0.629

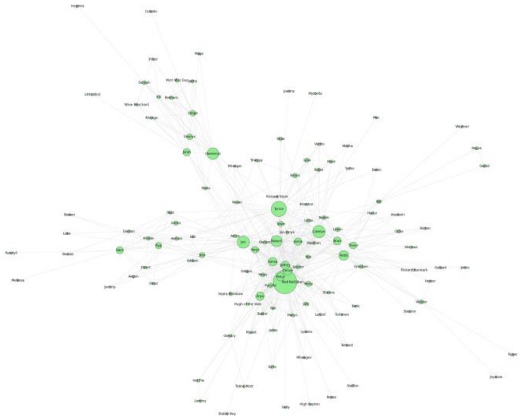
- **Graph density** is pretty high: not a lot of characters in S1, most of them interact.
- The network is **disassortative**: **high** degree nodes connect with **low** degree ones, and vice-versa. → **Feudal** society.
- **Clustering coefficients** indicate the presence of a significant number of triangles, and tendency of individual nodes to form well connected groups. Local clusters, though, are not well interconnected.
- **Bridges** involve **main** characters and **marginal** ones.
  - A lot of them involve Ned Stark and Tyrion Lannister.
  - Samuel Tarly's bridges with Melissa and Randyll highlight the fragility of his familial ties.
- **Diameter path** passes through the most central node (Ned).





Node	Degree	Avg. Deg. of Neighbours
Ned	57	13.2
Tyrion	41	14.7
Robert	36	17.3
Catelyn	36	17.0
Robb	30	17.7
Cersei	29	20.1
Arya	28	18.4
Joffrey	27	20.2
Petyr	26	18.9
Jon	26	18.7

Season 1: Centrality



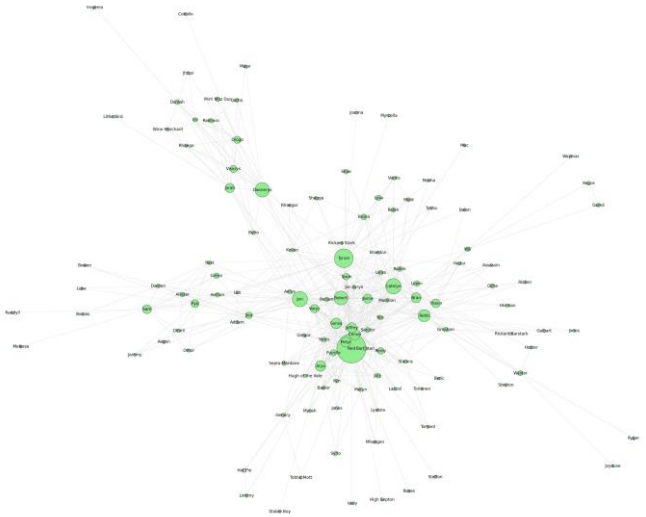
The most **central characters** are:

- **Ned**:
  - Top character for each centrality.
  - Hand of the King.
  - Events revolve around him and lead to his execution.
- **Tyrion:**
  - He is underestimated by most.
  - Though, his role as mediator and advisor is already clear, connecting disparate groups.
- **Robert:**
  - He is the king, therefore a central character.
  - Deeply trusts Ned.
- **Catelyn:**
  - Ned's wife. Begs Ned not to accept Robert's proposal to be his Hand of the King.

Measure	Character	Highest Centrality Score
Betweenness	Ned	0.303
	Tyrion	0.163
	Catelyn	0.118
	Robert	0.110
	Daenerys	0.101
Closeness	Ned	0.628
	Robert	0.553
	Catelyn	0.551
	Tyrion	0.543
	Jon	0.519
Eigenvector	Ned	0.315
	Robert	0.248
	Catelyn	0.239
	Tyrion	0.230
	Jon	0.229
Harmonic	Ned	0.011
	Tyrion	0.013
	Robert	0.013
	Catelyn	0.013
	Robb	0.014
Degree	Ned	0.456
	Tyrion	0.328
	Robert	0.288
	Catelyn	0.288
	Robb	0.240
Weighted Degree	Ned	1.000
	Tyrion	0.550
	Catelyn	0.453
	Robert	0.436
	Daenerys	0.415
PageRank	Ned	0.047
	Tyrion	0.034
	Catelyn	0.029
	Robert	0.028
	Robb	0.024

Season 1: Homophily

- **Assortativity**: -0.141
  - Network is **disassortative.**
  - Characters with high-weighted connections (frequent interactions) tend to connect with characters with low weighted connections.
  - Makes sense:
    - Prominent characters interact with less important ones and vice-versa, reflecting **hierchical and political dynamics**.
    - Very **marginal characters** appear on screen **because** they interact with more prominent ones.

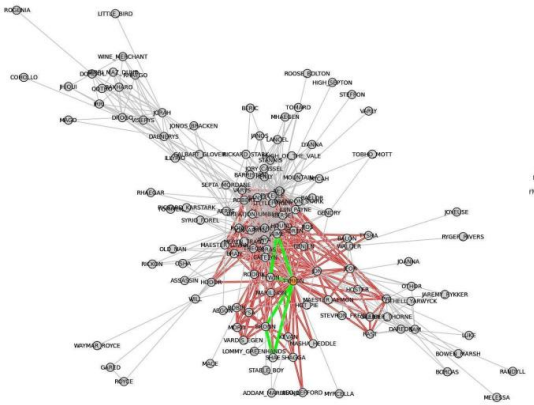


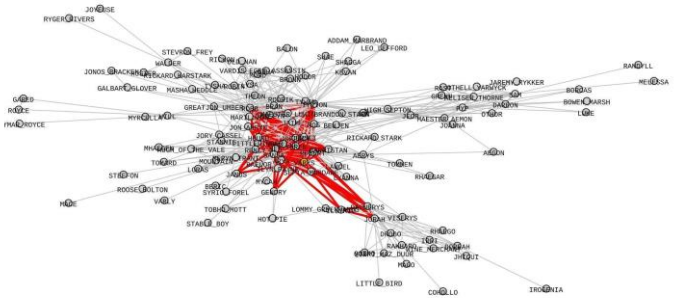
- **Jaccard Similarity**: results consistent with relationships and story.
  - Lysa Arryn is Robin Arryn's mother.
  - Vardis Egen served Jon Arryn – chosen to be the champion for house Arryn during Tyrion's trial by combat.
  - Quotho and Mirri Maz Duur are part of Daenerys storyline.
  - Grenn and Rast are both part of the Night's Watch.

First Node	Second Node	Jaccard Similarity Score
Lysa Arryn	Robin Arryn	0.75
Grenn	Rast	0.66
Mirri Maz Duur	Qotho	0.66
Robin Arryn	Vardis Egen	0.625
Lysa Arryn	Vardis Egen	0.625

Table 4. Top 5 characters for Jaccard Similarity Score

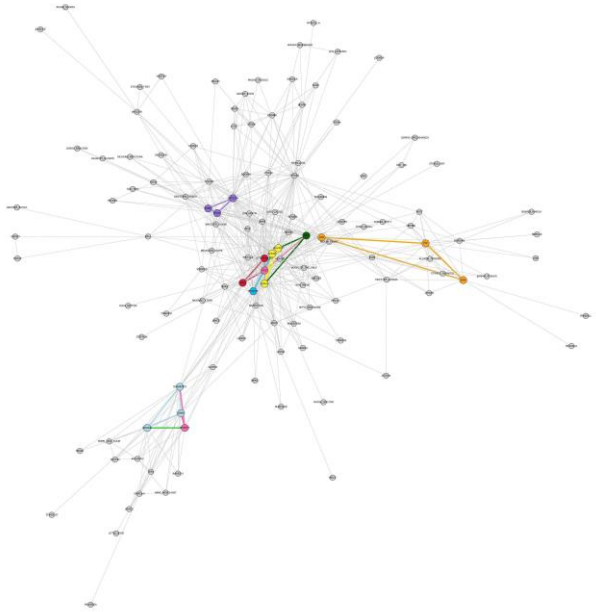
Season 1: Triangles



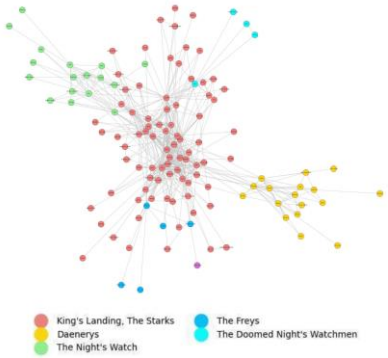


Triangle	Strength	Strongest Edge
(Ned, Robert, Cersei)	358	192 (Ned, Robert)
(Ned, Littlefinger, Varys)	276	107 (Ned, Littlefinger)
(Ned, Arya, Sansa)	192	90 (Ned, Arya)
(Arya, Joffrey, Sansa)	146	69 (Joffrey, Sansa)
(Cersei, Joffrey, Sansa)	159	69 (Joffrey, Sansa)
(Catelyn, Robb, Bran)	166	90 (Catelyn, Robb)
(Jon, Sam, Pyp)	193	121 (Jon, Sam)
(Daenerys, Drogo, Viserys)	184	91 (Daenerys, Drogo)
(Daenerys, Jorah, Viserys)	263	154 (Daenerys, Jorah)
(Daenerys, Jorah, Drogo)	265	154 (Daenerys, Jorah)

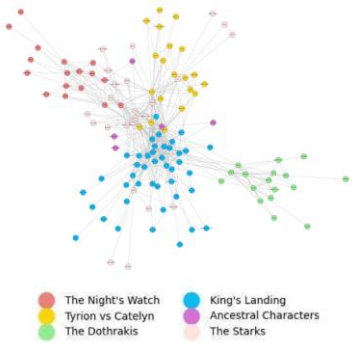
- **Characters** with **most** triangles:
  - Ned (280), Robert (185), Cersei (181), Catelyn (180), Tyrion (175), Joffrey (152), Robb (142), Littlefinger (139), Sansa (137), Arya (136).
  - All part of the Starks or the King's Landing community.
  - (Ned, Robert, Cersei) and (Daenerys, Jorah, Drogo) are important triangles.
- **Tyrion**'s triangles are **very loose**, capturing his independent spirit.
- **Varys** is a spy for the Lannisters: gathers information about Daenerys.



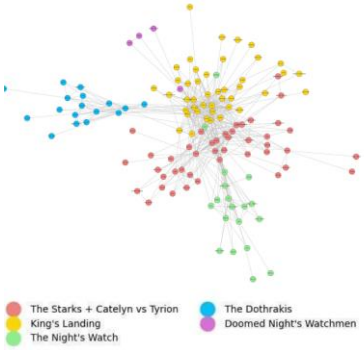
Season 1: Community Detection (1)



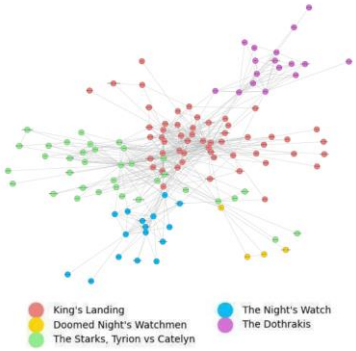
Girvan-Newman



Louvain

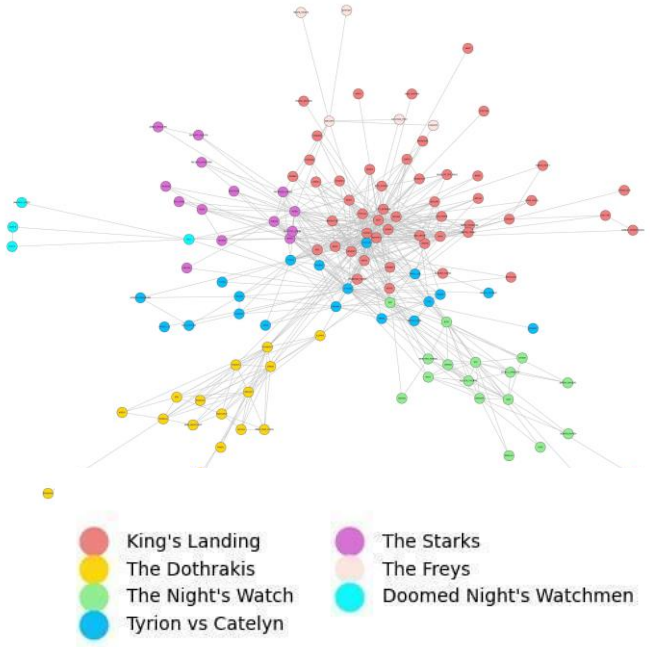


GMM



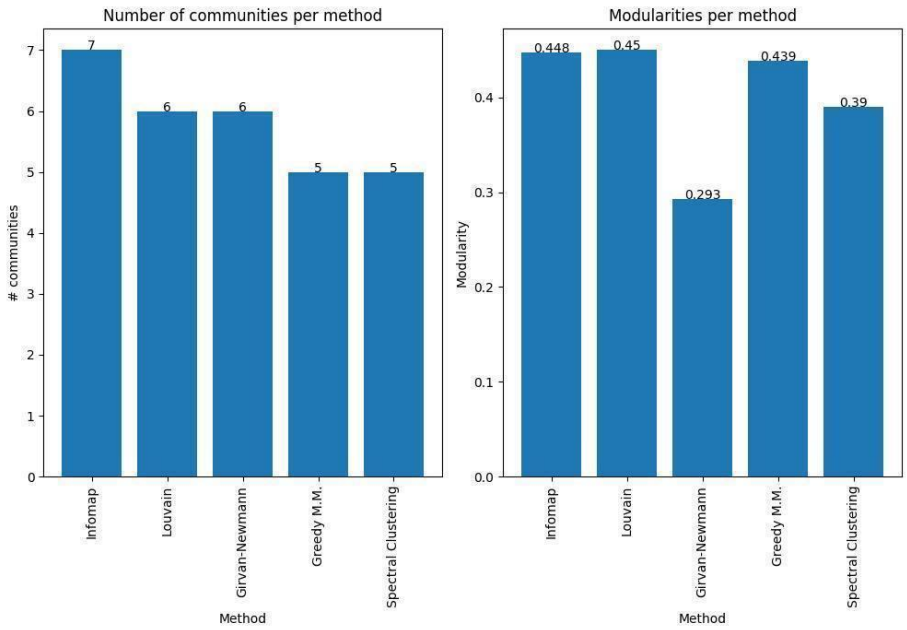
Spectral Clustering

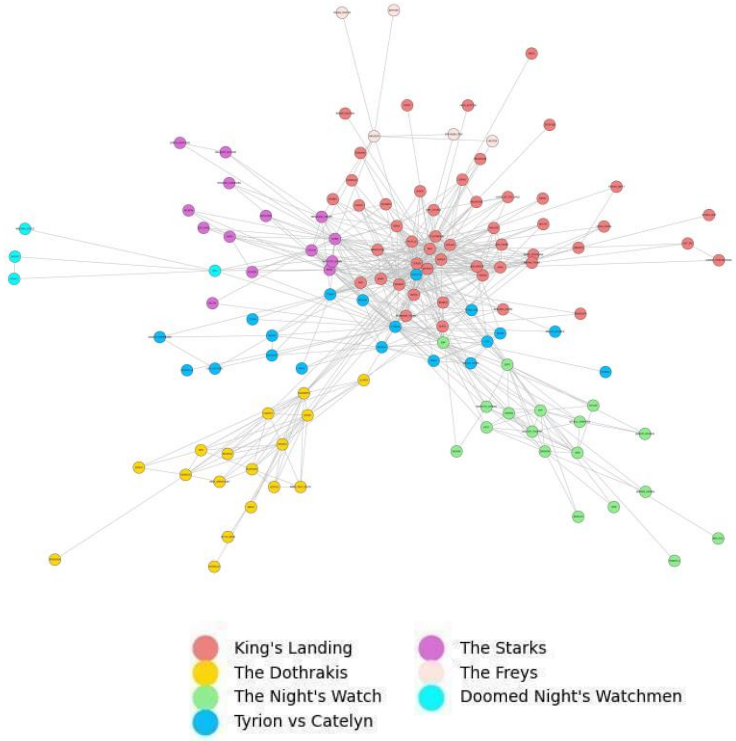
- **Infomap**: detects all communities correctly.
- **Girvan-Newman**:
  - less accurate, lower modularity.
  - King's Landing and The Starks are a single community.
- **Louvain**:
  - Highest modularity.
  - Small communities are clustered together.
  - But unexpectedly finds a community of ancestral characters!
    ('BRANDON_STARK', 'RHAEGAR', 'AEGON', 'RICKARD_STARK', 'AERYS')
- **Greedy Modularity Maximization**: less accurate, lower modularity.
  - The Starks and Tyrion vs Catelyn communities are a single one.
- **Spectral Clustering**: similar to GMM.



Infomap

Season 1: Community Detection (2)





Infomap's Clustering

- Overall, **Infomap** performed best on Season 1's graph.
  - Second highest modularity, after Louvain.
  - Correctly detected communities, even smaller, more nuanced ones.

Season 1: Robustness

- **Random Nodes Removal:**
  - **Shortest path increase** as more nodes are removed, indicating loss of robustness.
  - Diameter doesn't really fluctuate much.
  - Number of **components increases** as more nodes are removed (e.g. nodes_removed = 15).
  - **Centrality** measures **change.**

- **Centrality-based Node Removal:**
  - Attack the network by **removing most central** nodes according to **betweenness** centrality (serve as intermediaries).
  - Just with a few nodes, the network massively **fragmentates.**
  - Impact on average centrality measures:
    - **Betweenness increases**: remaining nodes take more intermediary roles.
    - **Closeness decreases**: reamaining nodes are further from each other.
    - **Eigenvector decreases**: influence of remaining nodes is lessened.
    - **Harmonic** massively **increases**: removal of key nodes affects network distances.
    - **Degree decreases**: remaining nodes are less connected.

Growing fragmentation shows that the network is not highly robust, especially in the case of a centrality-based attack.

Nodes Removed	Avg. Shortest Path	Diam.	Compon.
0 (Original)	2.644	6	1
5	2.620	6	2
10	2.688	6	1
15	2.950	7	7
20	2.595	6	4
25	2.731	6	2
30	2.765	6	6
35	2.518	5	2

Table 6. Changes in Network Metrics after random nodes removal

Nodes Remov.	Avg. Be-tween.	Avg. Close-ness	Avg. Eigen-vector	Avg. Har-monic	Avg. Degree
0 (Orig.)	0.013	0.389	0.059	0.018	0.069
5	0.013	0.386	0.060	0.386	0.060
10	0.014	0.383	0.061	0.383	0.061
15	0.016	0.314	0.061	0.314	0.061
20	0.014	0.375	0.064	0.375	0.064
25	0.017	0.371	0.067	0.371	0.067
30	0.016	0.337	0.063	0.337	0.063
35	0.016	0.399	0.071	0.399	0.071

Table 7. Changes in network centrality after random nodes removal

Nodes Removed	Avg. Shortest Path	Diam.	Compon.
0 (Original)	2.644	6	1
1	2.772	6	5
3	2.982	6	8
5	3.156	6	9

Table 8. Changes in Network Metrics after centrality-based nodes removal

Nodes Remov.	Avg. Be-tween.	Avg. Close-ness	Avg. Eigen-vector	Avg. Har-monic	Avg. Degree
0 (Orig.)	0.013	0.389	0.059	0.018	0.069
1	0.013	0.348	0.058	0.348	0.058
3	0.014	0.307	0.055	0.307	0.055
5	0.015	0.285	0.054	0.285	0.054

Table 9. Changes in network centrality after centrality-based nodes removal

Season 1: Link Prediction

Between Existing Nodes:

- **Preferential Attachment**
  - Make sense for the most part.
  - **Sansa – Tyrion**, and **Tyrion – Daenerys** will actually have strong connections in the following seasons!

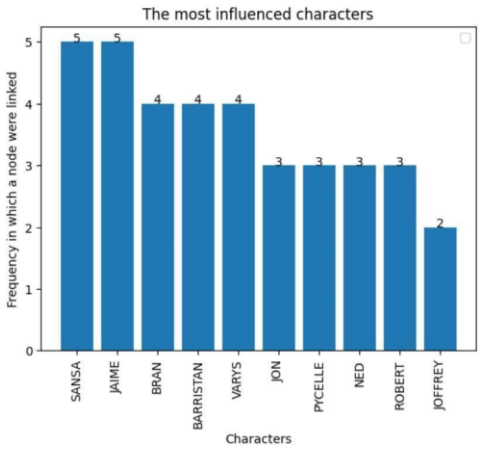
First node	Second node	Score
Sansa	Tyrion	1066
Littlefinger	Robb	780
Ned	Drogo	741
Tyrion	Daenerys	738
Renly	Tyrion	697

Between New Nodes:

- **Adding 1 Node with 1 Edge:**
  - **Barabasi-Albert** model.
  - New node is connected to Arya Stark with Preferential Attachment score of 29 (relatively low).
  - Realistic: Arya is adventurous and interacts with of minor characters.

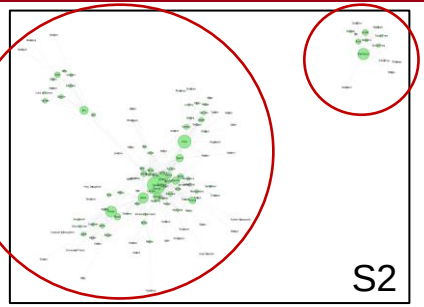
Node	Score
Ned	580
Robert	370
Catelyn	370
Arya	290
Littlefinger	270
Varys	220
Tywin	180
Pycelle	170
Maester Luwin	110
Vardis Egen	70

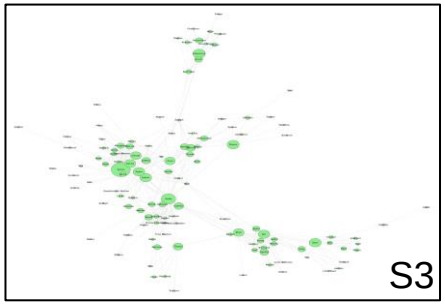
- **Adding 1 Node with 10 Edges:**
  - Central characters (e.g. Ned) tend to link with characters with high connectivity.
  - Pycelle and Luwin are maesters (not central), interact with characters with many edges.

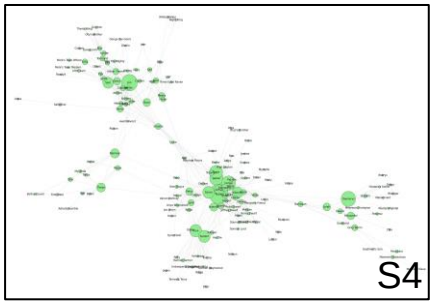


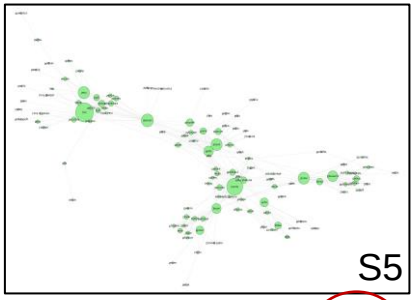
- **Adding 10 Nodes with x Edges** (with x being avg #edges per node):
  - Sansa, Jaime, Bran are central characters.

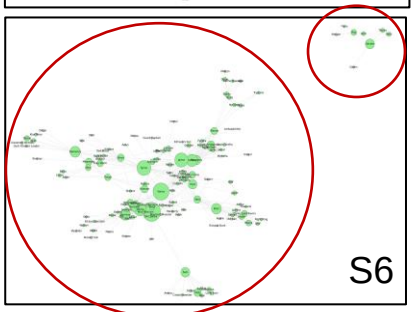
Seasons 2-8: General Info

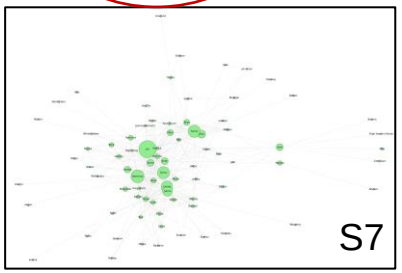


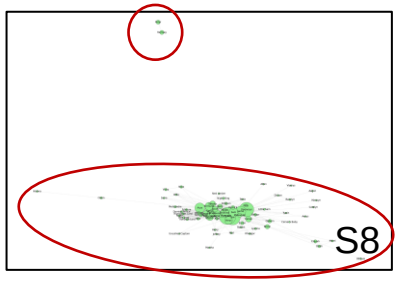












- **Most** networks are **connected**, composed of a **single component**
  - **Season 2** and **Season 6** have an isolated community (Daenerys in Essos and The Hound in the Riverlands, respectively).
  - Characters temporarily live **isolated** from the main hotspots of the conflicts, therefore rendering the graph **disconnected.**
  - **Season 8** is also disconnected, only because of Littlefinger and Ramsay, who are both dead.
- **Degree Assortativity Coefficient** is highest in season 6 (-0.08) and lowest in season 5 (-0.19)
- **Graph Density** is relatively **stable** through the seasons, but in **S7** and **S8 drastically increases.**
  - Plotlines are starting to **converge**, remaining characters start to interact way more.
  - The «White Walker's threat» and the «Last War» between Cersei and Daenerys' alliance are imminent.

Seasons 2-8: Community Detection (1)

Infomap (Season 5)

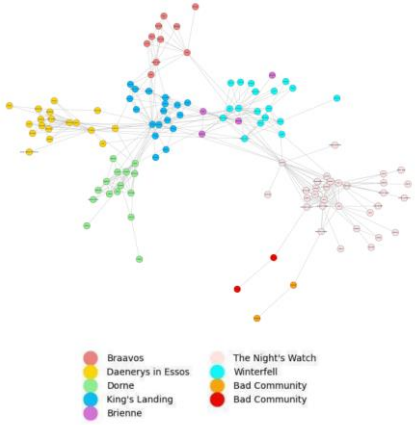


Figure 39. $\#communities = 9$, $modularity = 0.661$

Spectral Clustering (Season 6)

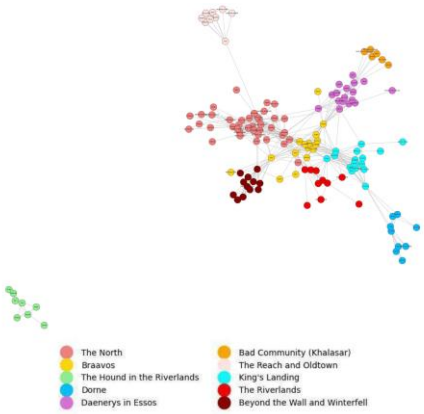


Figure 51. $\#communities = 10$, $modularity = 0.639$

Louvain (Season 6)

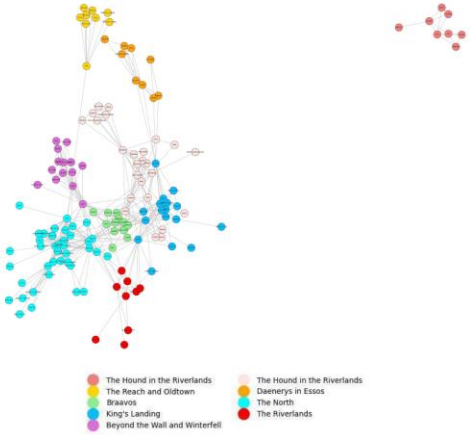


Figure 49. $\#communities = 9$, $modularity = 0.655$

- **Louvain**, **Infomap** and **Spectral Clustering** performed the best, considering each season – both in terms of modularity and accuracy in the detection.
- **Louvain** consistently scores the highest modularity in each season.
  - GoT communities are well structured communities with pretty high internal edge density.
- **Infomap** works well, but occasionally tends to «overcluster» and creates very small communities.
- **Spectral Clustering** also works fairly well (requires to specify the number of clusters)
  - Sometimes groups together large communities.
  - Can fail to recognize a small sub-community to be part of a bigger one (e.g. Khalasar).

Seasons 2-8: Community Detection (2)

Infomap (Season 7)

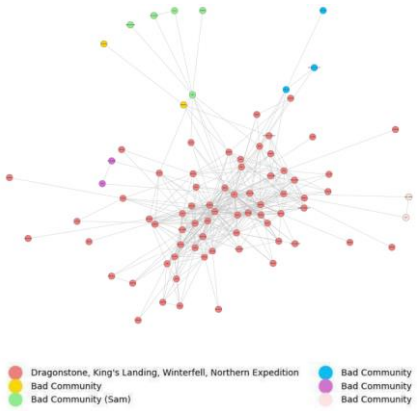


Figure 55. $\#communities = 6$, $modularity = 0.048$

Louvain (Season 7)

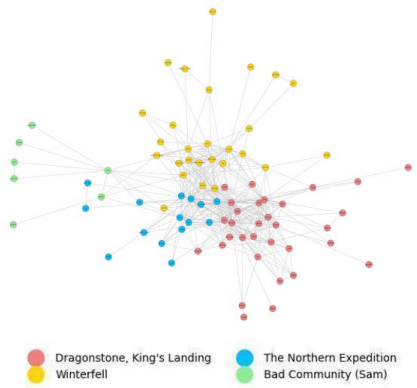


Figure 57. $\#communities = 4$, $modularity = 0.32$

Louvain (Season 8)

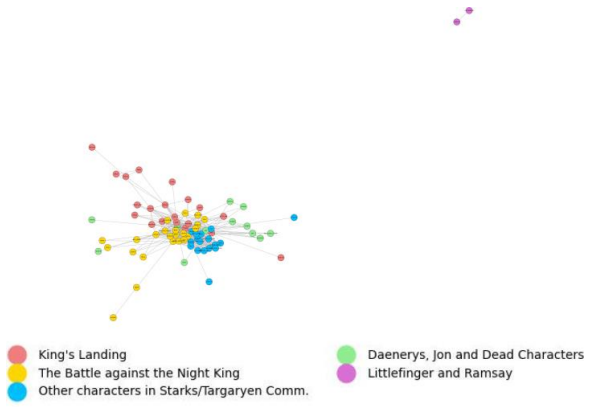


Figure 64. $\#communities = 6$, $modularity = 0.67$

- As plotlines come together (e.g. S7, S8), community detection becomes more **challenging**.
  - **Louvain** still **performs better.**
  - Other methods cluster together most of the communities.

- This intrinsically shows how the remaining characters **interact more** and more with others **outside** of their community. In particular, in Season 7-8:
  - Jon and Daenerys meet Cersei in King's Landing to discuss the impending threat (White Walkers).
  - The great majority of characters fight against the White Walkers during the Long Night, and the remaining also meet in King's Landing.

- Season 8 realistically only has 2 communities, but small communities appear from the analysis.

Spectral Clustering (Season 8)

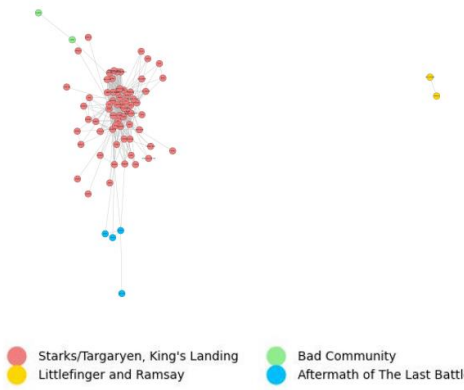
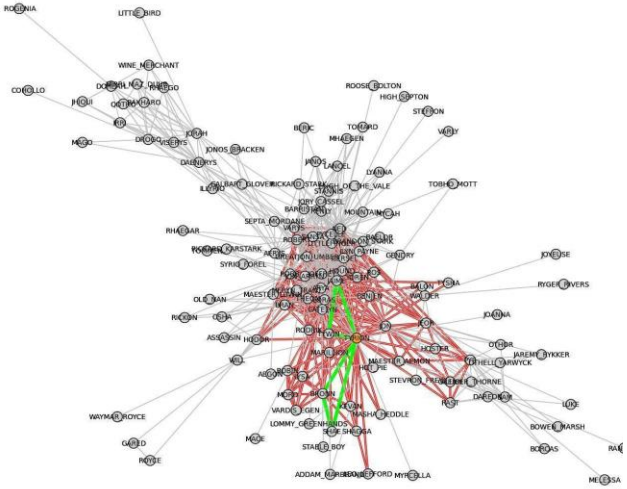
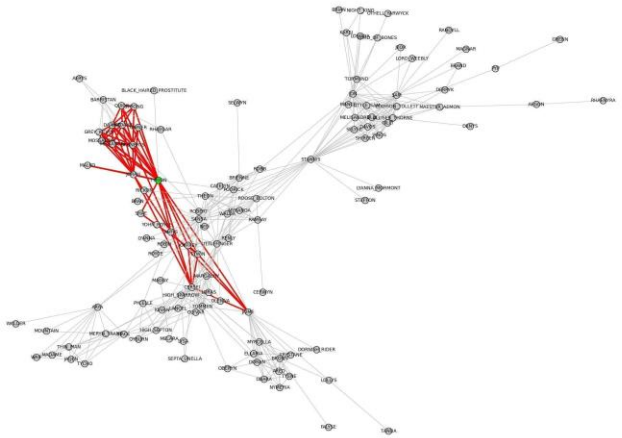


Figure 66. $\#communities = 6$, $modularity = 0.634$

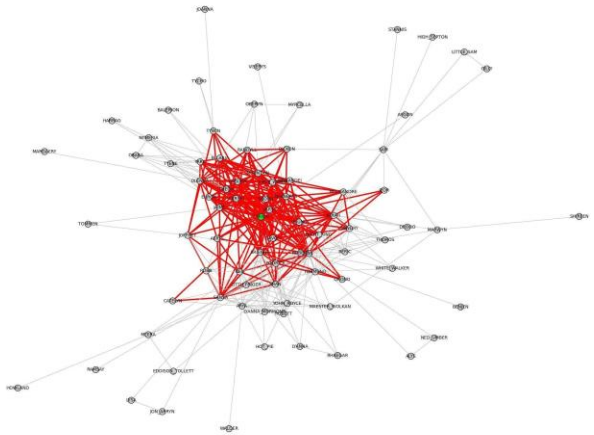
Seasons 2-8: Tyrion's Independence



Tyrion's triangles in Season 1



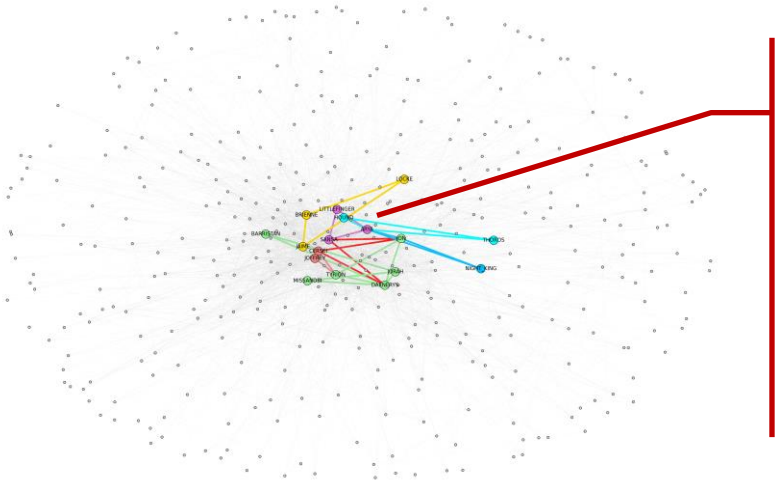
Tyrion's triangles in Season 5

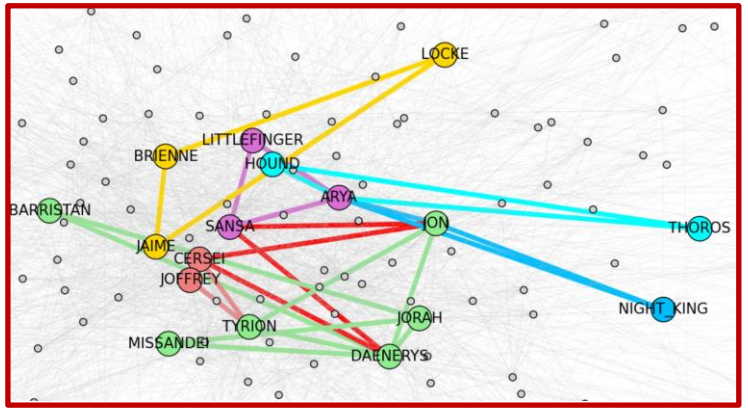


Tyrion's triangles in Season 7

- Analyzing the **triangles** involving a specific character can give **information** about the **story.**
- **Tyrion** has always been an **independent** character, interacting with people from **different communities**
  - Although always **strongly tied** with his family and King's Landing.
- **Season 5** is a **turning point** for his storyline:
  - In Season 4, Episode 2, The «Purple Wedding» happens and Joffrey is poisoned.
  - Tyrion is falsely accused, put on trial and convicted. Though, he escapes King's Landing.
  - His triangles in Season 5 indicate he has almost completely **cut ties** with his past life, and now serves Daenerys.
- In **Season 7**, storylines are **converging**: characters meet again.

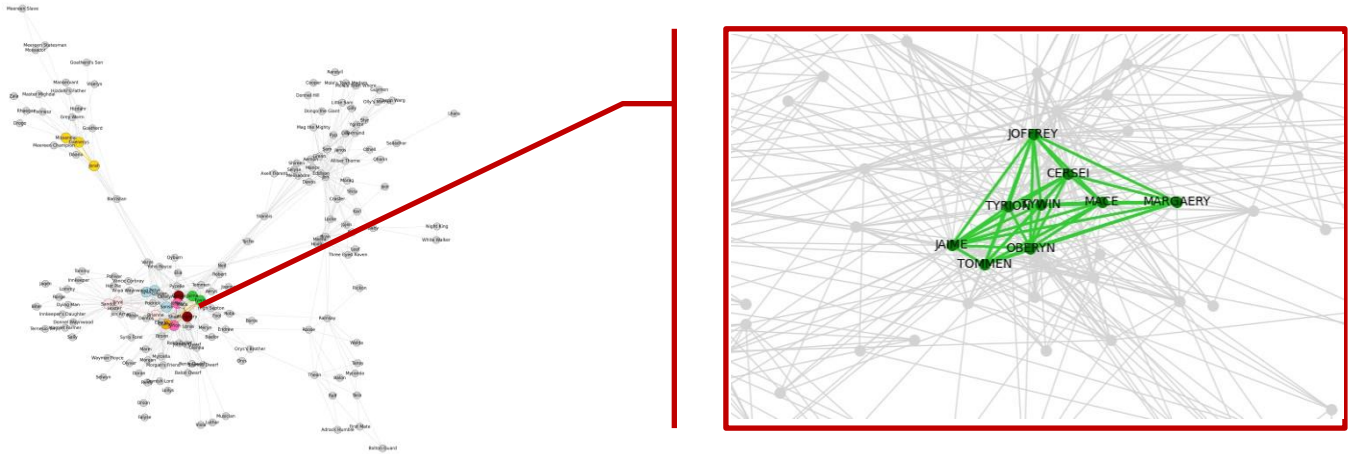
Seasons 2-8: Important Triangles





- (**Cersei, Tyrion**, Joffrey/Varys/Sansa) in S2, S3 shows how strongly Cersei hates Tyrion.
- (**Brienne, Jaime**, Locke) in S3 reflects how much Brienne and Jaime bond during their trek.
- (**Daenerys, Jorah**, Barristan), in S3, (**Daenerys, Jorah**, Missandei) in S4 reflect Daenerys' trust in Jorah.
- (**Arya, Hound**, Thoros) in S3, (**Arya, Hound**, Brienne) in S4 shows the bond Arya and The Hound develop.
- (**Daenerys, Tyrion**, Jon) in S7 highlights how Daenerys trusts Tyrion as her Hand of the Queen.
- (**Arya, Sansa**, Littlefinger) in S7 is a true testament of the strength of the bond between siblings.
- (**Daenerys, Jon**, Tyrion/Sansa/Cersei) in S8 describes the strong bittersweet end of the two's relationship.

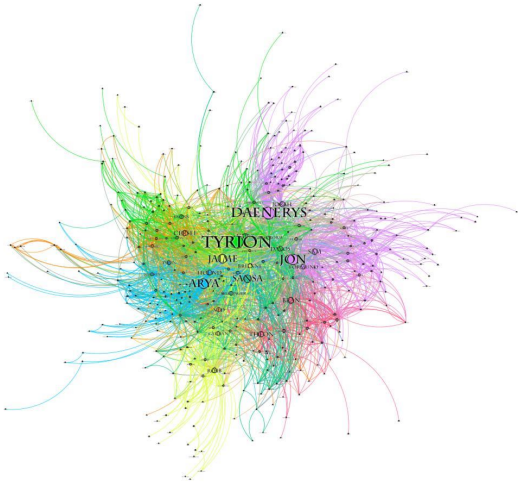
Seasons 2-8: From Triangles to Cliques

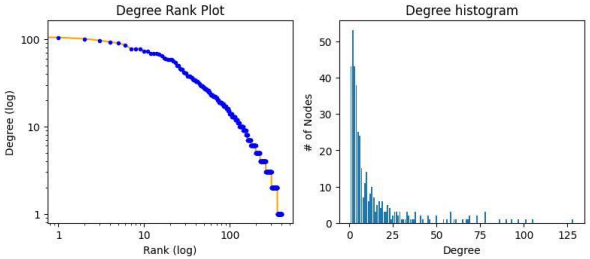


Let's consider **Season 4**:

- If we stray away from triangles only and consider **cliques**, we can find an interesting **9-clique**:
  - Composed of: Tyrion, Cersei, Jaime, Tywin, Joffrey, Tommen, Margaery, Mace and Oberyn.
  - After Joffrey's death, the power struggle between **Cersei** and **Margaery Tyrell** intensifies.
  - The tesions between the **Lannisters**, the **Tyrells** and the **Martells** are quite evident by this clique.

The Full Game of Thrones Network





Description	Value
Nodes	403
Edges	2634
Graph Density	0.03
Connected Graph	Yes
Number of Components	1
Small-World Network	Yes ($\lambda = 1.03$, $\gamma = 20.73$)
Diameter	6
Avg. Shortest Path	2.68
Avg. Degree	13.07 [1,128]
Most Freq. Degree	2
Number of Bridges	44
Deg. Assortativity Coeff.	-0.147
Global Clustering Coeff.	0.324
Average Clustering Coeff.	0.659

Table 40. Summary table for the complete GoT network

- **Small-world network**: despite being vast, characters are only a few steps away from each other. Storylines are interconnected.
- **Relatively sparse**, only a small fraction of all possible connections exist.
- The log-log plot of degree distribution follows a **power law**, typical for **scale-free networks** → Fewer nodes have very high degrees.
- The network is **disassortative**.
- **Bridges** composed of marginal characters interacting with a more central character. (e.g. Stable_Boy – Arya)
- **Clustering Coefficients** indicate significant local clustering, where local clusters aren't well interconnected → consistent with factionalism.

Full GoT Network: Centrality

- **Tyrion** is the most important character, anking first with respect to every centrality measure.
  - Connected to important characters, knows most people, and has the most connections across various parts of the network.
- **Jon** is very central, especially in Closeness and Degree centrality
  - He lacks in Eigenvector centrality, mostly dominated by the Lannisters. His alliance with Daenerys helps him to climb to third place.
- **Daenerys** is a crucial character – and the metrics tell a lot about her
  - Ranks second in Betweenness Centrality, meaning she ties many threads together.
  - She doesn't know as many people (fifth in degree centrality), though her ranking skyrockets from season 6.
  - Her long exile from Westeros impacts her Eigenvector Centrality (> Top 10).
- **Sansa** emerges as a capable ruler.
  - Her centrality measures ascend towards her being crowned Queen in the North, while **Cersei**'s progressively drop.
- **Arya** and **Jaime** can be compared
  - **Jaime** wins over Arya for Eigenvector and Closeness, **Arya** is far superior in Betweenness and Degree.
  - **Jaime** knows more important people.
  - **Arya** acts as a connector between various parts of the network.

Measure	Character	Highest Centrality Score
Betweenness	Tyrion	0.134
	Daenerys	0.114
	Arya	0.103
	Jon	0.097
	Theon	0.066
Closeness	Tyrion	0.579
	Jon	0.554
	Sansa	0.553
	Jaime	0.543
	Arya	0.542
Eigenvector	Tyrion	0.216
	Sansa	0.204
	Jon	0.182
	Arya	0.173
	Jaime	0.185
Harmonic	Tyrion	0.004
	Jon	0.004
	Sansa	0.004
	Arya	0.004
	Jaime	0.004
Degree	Tyrion	0.318
	Jon	0.261
	Sansa	0.251
	Arya	0.241
	Daenerys	0.231
Weighted Degree	Tyrion	1.000
	Daenerys	0.926
	Arya	0.705
	Jon	0.840
	Jaime	0.676
PageRank	Tyrion	0.022
	Daenerys	0.018
	Arya	0.018
	Jon	0.018
	Sansa	0.016

Table 41. Top characters with highest centrality scores

The Full GoT Network: Communities

Overall main communities:

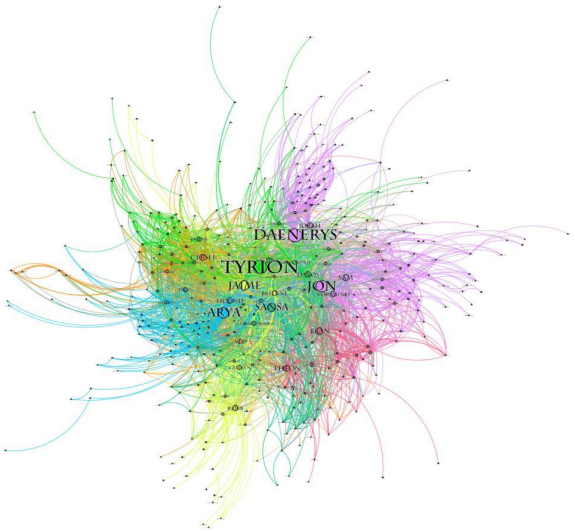
- **King's Landing**:
  - Very complex community, contains every Lannister, meaning that the conflict has always been **internal.**
  - **Tyrion**, **Cersei** and Jaime are the **main hubs.**
  - Cersei has the least connection outside King's Landing.

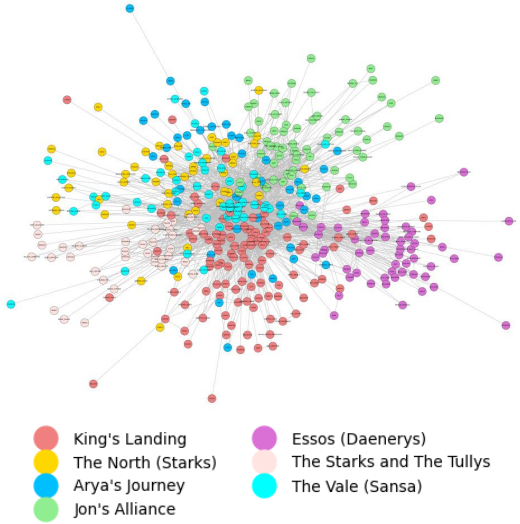
- **Jon's Alliance:**
  - Includes The Night's Watch, The Free Folk and House Baratheon.
  - Orbits around **Jon**, with Davos and Sam as **smaller hubs .**

- **Essos:**
  - **Daenerys** is the main **hub.**
  - Jorah, Missandei and Greyworm are her close advisors.

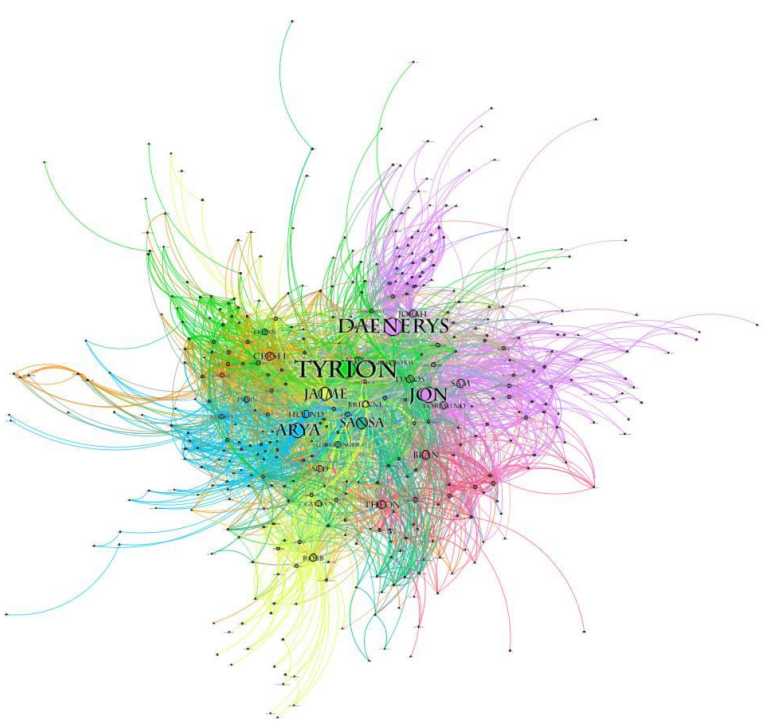
- **Arya's Journey**:
  - Mainly composed of minor characters met along the way. (e.g. The Hound, Gendry, The Brotherhood without Banners).

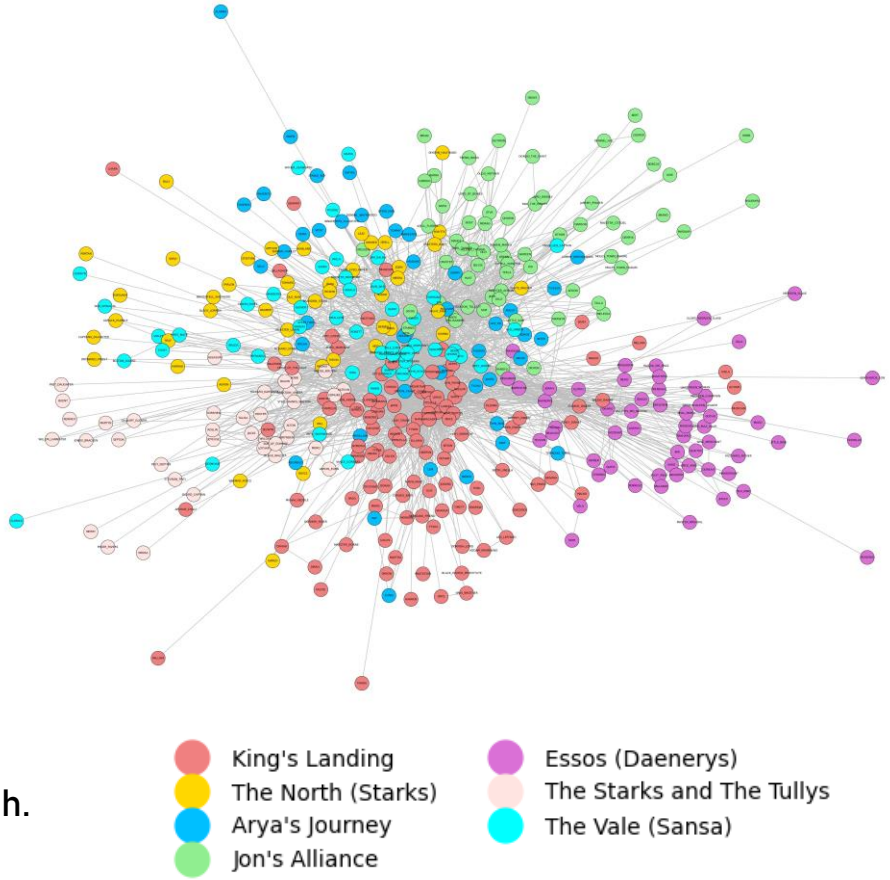
- **The Starks, The Tullys and Littlefinger:**
  - Composed various communities **tied to Winterfell.**
  - Littlefinger (Petyr Baelish) is central because of his political games aimed at gathering power.





The Full GoT Network: Communities





- Overall, **Louvain** performed best on the full GoT graph.
  - **Highest** modularity
  - **Correctly** detected communities
- Other methods don't perform as well, finding only 3-4 communities.

Temporal Evolution of Node Degree

- Having a graph **for each** season can be used as a **timestep**
  - A signal **di(t)** is computed, describing the **increment** in **degree** of a node with respect to time (i.e. cumulative sum of node degree).
- This can be used as a proxy to infer **characters' deaths**.
  - Expect an **active** character's degree to increase over time.
  - Events preventing the involvement of a character are expected to **influence** this metric.
- Indeed, a character's death **stops** – or **tapers off** – the growth of cumulative degree (fig. 1):
  - For some characters the effect is much more **noticeable** (e.g. Catelyl, Ygritte)
  - Central characters (e.g. Ned Stark) are **impacted less**, thanks to other people **talking** about them.
- Sudden changes in growth of cumulative node degree in **multiple characters** can be an indicator of a battle, or – more gruesomely – a wedding (fig. 2).

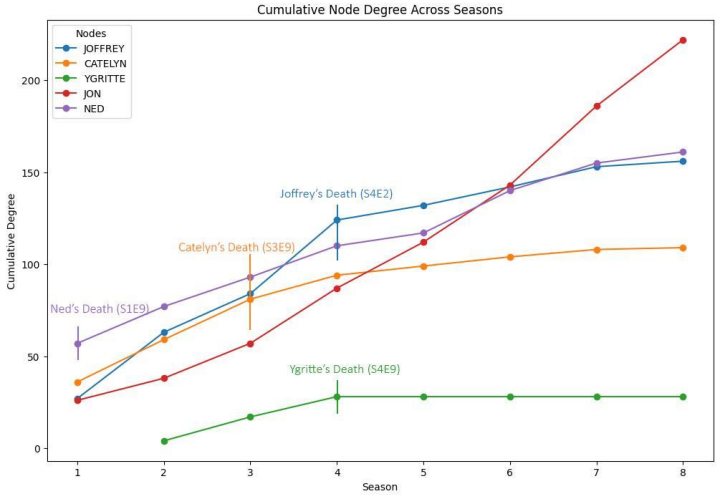


Figure 1. Death of characters along the series

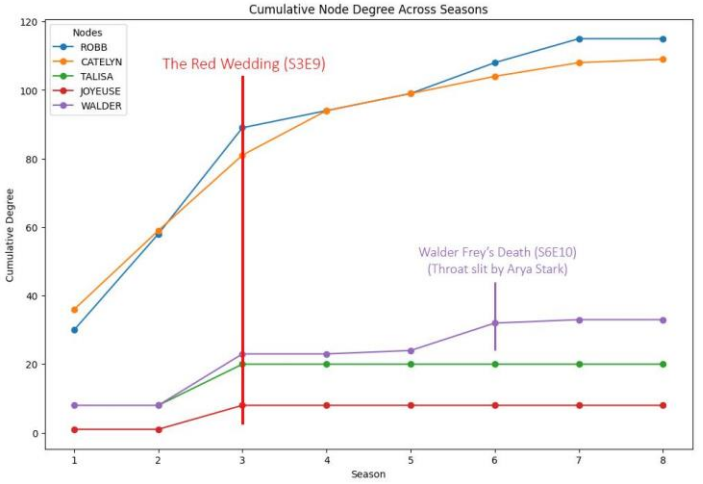


Figure 2. The effects of the «Red Wedding»

Thank You for Listening!