# Transformers for Time Series Forecasting

Marco Zanotti

University Milano-Bicocca

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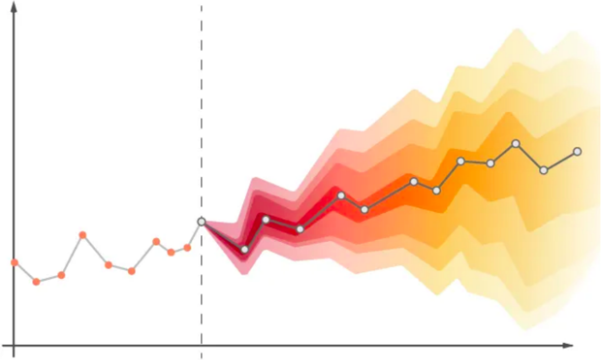
# 1. The TSF Problem

Time series forecasting (TSF) is the task of predicting future values of a given sequence based on previously observed values.



The TSF problem may be essentially identified by the following aspects:

- **Prediction objective**: point forecasting vs probabilistic forecasting
- **Forecast horizon**: short-term vs long-term forecasting
- **Input-Output dimension**: univariate vs multivariate forecasting
- **Forecasting task**: single-step vs multi-step forecasting

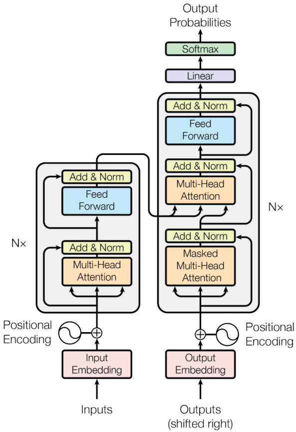
The TSF problem is usually faced with statistical models (ARIMA, ETS) or deep learning models (RNN, LSTM).

The main challenges of the TSF problem are:

- **uncertainty** increases as the forecast horizon increases
- difficulty in capturing **multiple complex patterns** over time
- difficulty in capturing **long-term dependencies** (critical for long-term forecasting)
- difficulty to handle **long input sequences**

# 2. Vanilla Transformer

- Based on Encoder-Decoder architecture
- Uses self-attention mechanism to access any part of the sequence history
- Positional encoding allows to account for element positions
- Residual connections and layer normalization help to stabilize the learning process
- Each encoder and decoder layer is composed of a self-attention layer and a feed-forward layer



# Can vanilla Transformers be used for TSF?

The TSF problem can be seen as a sequence learning problem such as machine translation.

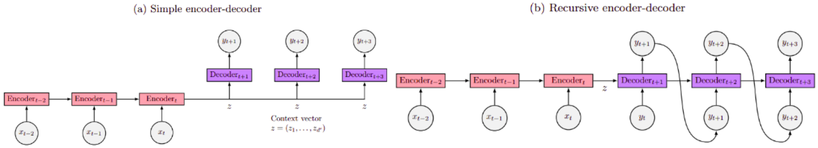
Main ingredients allowing to use vanilla Transformers for TSF:

- **Multi-head Self-attention** mechanism allows to access any part of the sequence history, capturing both short-term and long-term dependencies (but it is invariant to the order of elements in a sequence)
- **Positional encoding** allows to account for the sequence ordering
- **Masked self-attention** allows to avoid information leakage from future

# Can vanilla Transformers be used for TSF?

Just few changes are needed to adapt Transformers to TSF:

- **Remove the final activation** function (softmax) from the output layer and set the dimension of the linear layer equal to the forecasting horizon
- **Adapt the structure** to the desired forecasting task (single-step or multi-step)



# Problems with vanilla Transformers

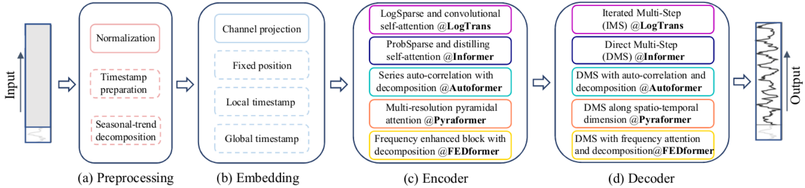
- **Locally agnostic**: the attention mechanism matches queries and keys without considering their local context being prone to temporal anomalies
- **Positional encoding**: only the order in which two elements occur is taken into account, but their temporal distance is not
- **Computational complexity**: given a sequence of length $L$, the time and memory burden is $O(L^2)$, making it difficult to learn patterns in long time series
- **Simple Architecture**: the architecture does not include any component of typical importance in TSF (e.g. autocorrelation, decomposition, recurrent layers, etc.)

# 3. TSF Transformers

# Classification

Transformers are **very appealing for long-term TSF** due to their ability to learn long-range dependencies.

Many solutions have been proposed to adapt Transformers to TSF, mainly in the direction to improve the encoding, adopt more efficient attention and expand the architecture.



# Informer

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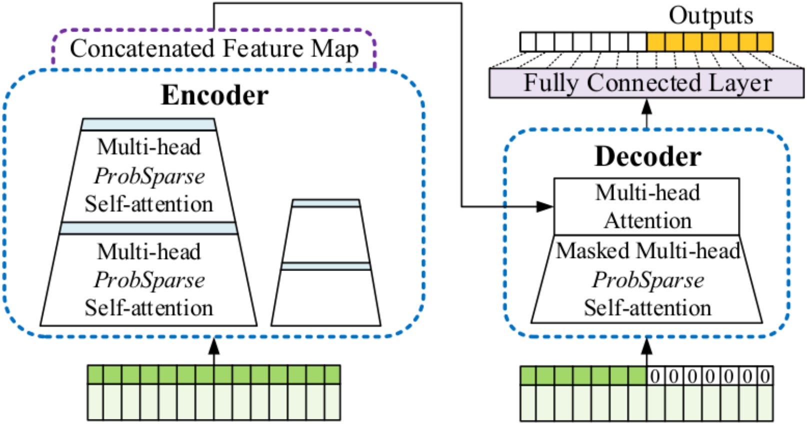
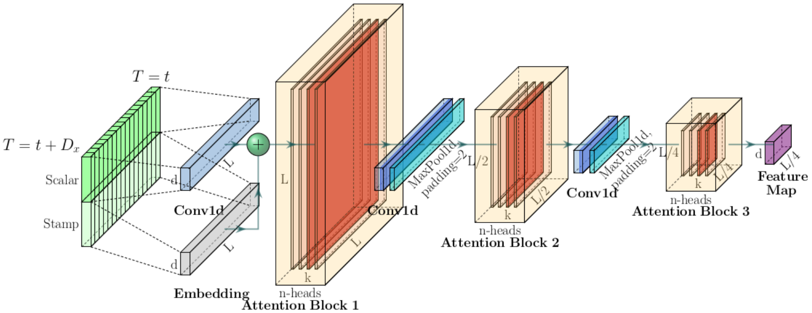
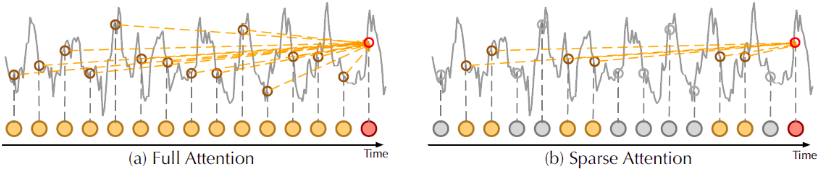
# Informer - Architecture



# Informer - Causal Convolution Layers



# Informer - ProbSparse Attention

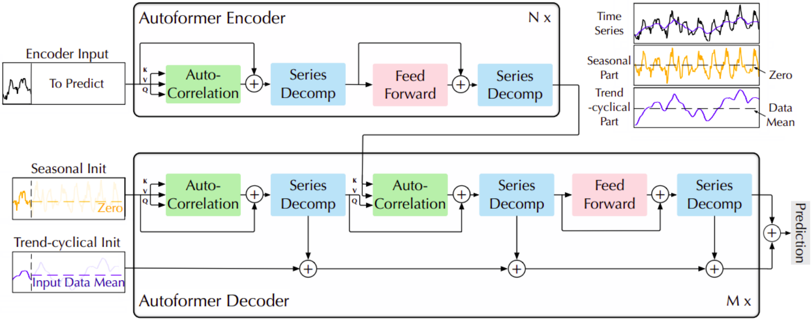


# Autoformer

**Autoformer** builds upon two traditional time series analysis methods:

- **Decomposition Layer**, which allows to decompose the time series into seasonality and trend-cycle components, enhancing the model's ability to capture these components accurately
- **Attention (Autocorrelation) Mechanism**, which replaces the standard self-attention used in the vanilla transformer with an autocorrelation mechanism, allowing to capture the temporal dependencies in the frequency domain

# Autoformer - Architecture



# Autoformer - Decomposition Layer

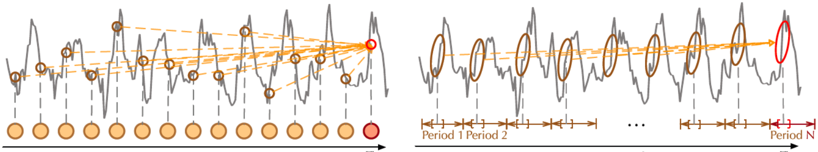
Autoformer incorporates **decomposition blocks** as an inner operation of the model.

Encoder and decoder use decomposition blocks to extract and aggregate the trend-cyclical and seasonal components from the series progressively, so to make raw data easier to predict.

For an input series $X_t$ with length $L$, the decomposition layer returns $X_{trend}$ and $X_{seasonal}$, both of length $L$. In practice, $X_{trend}$ is extracted using some form of moving-average and $X_{seasonal}$ is then obtained by difference.

# Autoformer - Attention Mechanism

Autoformer uses **autocorrelation within the self-attention** layer, extracting frequency-based dependencies from $(Q, K)$. The autocorrelation block measures the **time-delay similarity** and aggregates the top $n$ similar sub-series to reduce complexity.




(a) Full Attention (b) Auto-Correlation

In practice, autocorrelation of the queries and keys for all lags is calculated at once by Fast Fourier Transform, so to achieve $O(LlogL)$ time complexity (similar to Informer).

# 4. Conclusions

# Conclusions

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- While using various type of positional encoding can preserve some ordering information, it is still inevitable to have temporal information loss after applying self-attention.

# Bibliografy

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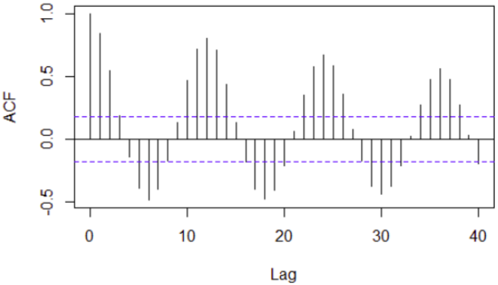
Thank you!

# Appendix

# Autocorrelation

In theory, given a time lag $k$, autocorrelation for a single discrete variable $Y$ is used to measure the "relationship" (pearson correlation) between the variable's current value at time $t$ to its past value at time $t-k$.

$$Autocorrelation(k) = Corr(Y_t, Y_{t-k})$$



# Time Series Decomposition

In time series analysis, decomposition is a method of breaking down a time series into three systematic components: trend-cycle, seasonal variation, and random fluctuations.