Quantile Regression with Univariate Non-Response

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1. Introduction

Our work investigates the impact of **missing data in quantile regression**. Specifically, we consider **univariate non-response** for a covariate assuming **Missing Completely At Random** (MCAR) mechanism.

Main objectives:

- evaluate the impact of several strategies for missing values
- compare standard and bootstrap estimators when employing imputation methods

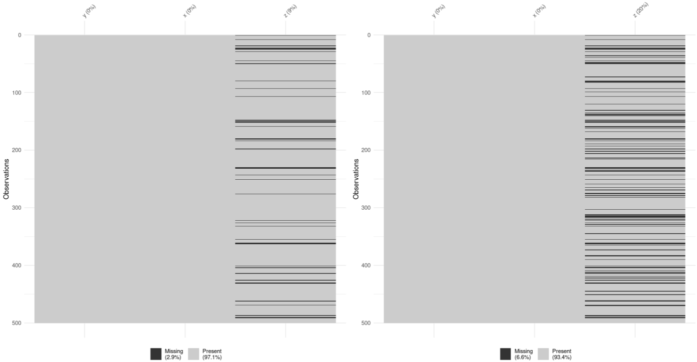
2. Data simulation

In this simulation, we assumed:

- $p = 2$ covariates (X, Z) with $X = (x_1, \dots, x_n) \sim U(3, 8)$ and $Z = (z_1, \dots, z_n) \sim U(-1, 5)$
- gaussian errors $\epsilon \sim N(0, 1)$, such that

$$y_i^{(j)} = 3x_i - 0.5z_i^{(j)} + \epsilon_i \quad \forall i = 1, \dots, 500$$

Z contains MCAR missing, with two scenarios



3. Model formulation

We considered the following model

$$Y_i = \beta_0 + \beta_1 x_i + \beta_2 z_i + \epsilon_i$$

The coefficients $\beta = (\beta_0, \beta_1, \beta_2)$ can be estimated by minimizing

$$\sum_{i=1}^{n} \rho_\tau (Y - (\beta_0 + \beta_1 x_i + \beta_2 z_i))$$

where $\tau \in (0,1)$ and $\rho_\tau(u) = (\tau - I(u < 0))u$.

The estimated model corresponds to

$$\hat{Q}_\tau(Y|x,z;\hat{\beta}) = \hat{\beta}_{0,\tau} + \hat{\beta}_{1,\tau} x + \hat{\beta}_{2,\tau} z$$

We compared several methods for handling missing data:

- complete-case analysis
- random imputation
- mean imputation
- median imputation

We estimated the quantile regression model considering the quantiles of order 0.25, 0.5 and 0.75.

	Variable Z: $\tau = 0.25$			Variable Z: $\tau = 0.5$			Variable Z: $\tau = 0.75$		
Method	**0%**	**10%**	**40%**	**0%**	**10%**	**40%**	**0%**	**10%**	**40%**
no-missing	-0.481			-0.429			-0.486		
	(0.036)			(0.037)			(0.04)		
deletion		-0.477	-0.476		-0.426	-0.451		-0.489	-0.455
		(0.037)	(0.043)		(0.038)	(0.053)		(0.042)	(0.059)
sample		-0.42	-0.343		-0.398	-0.289		-0.445	-0.21
		(0.038)	(0.046)		(0.039)	(0.047)		(0.043)	(0.05)
mean		-0.458	-0.449		-0.429	-0.452		-0.479	-0.476
		(0.038)	(0.045)		(0.039)	(0.051)		(0.042)	(0.058)
median		-0.458	-0.449		-0.429	-0.451		-0.475	-0.463
		(0.038)	(0.046)		(0.039)	(0.051)		(0.042)	(0.057)

Table 1. Estimates of regression coefficients related to Z and their standard errors (within the brackets) for each percentage of missing values and non-response method

Method	Variable X: $\tau = 0.25$ 0%	10%	40%	Variable X: $\tau = 0.5$ 0%	10%	40%	Variable X: $\tau = 0.75$ 0%	10%	40%
no-missing	3.012			3.02			2.938		
	(0.045)			(0.044)			(0.046)		
deletion		3.015	3.029		3.019	3.023		2.93	2.918
		(0.037)	(0.06)		(0.046)	(0.063)		(0.048)	(0.064)
sample		2.976	2.907		3.01	3.014		2.959	3.047
		(0.048)	(0.055)		(0.045)	(0.052)		(0.047)	(0.054)
mean		2.995	2.927		3.016	3.022		2.942	3.028
		(0.046)	(0.054)		(0.045)	(0.048)		(0.047)	(0.05)
median		2.995	2.929		3.016	3.027		2.946	3.012
		(0.046)	(0.054)		(0.045)	(0.048)		(0.047)	(0.049)

Table 2. Estimates of regression coefficients related to X and their standard errors (within the brackets) for each percentage of missing values and non-response method

4. Bootstrap Estimators

Let $D = (y, x, z)$ be the incomplete dataset.

For each repetition $b = 1, \ldots, 200$,

- we built a bootstrap sample D^* from D
- each value of D^*_{miss} was replaced by a single value following the chosen imputation method

Bootstrap estimates:

- regression coefficients

$$\hat{\beta}^*_\tau = \frac{1}{B}\sum_{b=1}^{B}\hat{\beta}^*_{\tau;b}$$

- standard errors

$$se^*(\hat{\beta}^*_\tau) = \sqrt{\frac{1}{B-1}\sum_{b=1}^{B}(\hat{\beta}^*_{\tau;b} - \hat{\beta}^*_\tau)(\hat{\beta}^*_{\tau;b} - \hat{\beta}^*_\tau)^T}$$

We finally compute the coverage for β_τ as

$$p(\hat{\beta}^*_\tau - z_{\alpha/2}se^*(\hat{\beta}^*_\tau) < \beta_\tau < \hat{\beta}^*_\tau + z_{\alpha/2}se^*(\hat{\beta}^*_\tau))$$

Method	Variable Z: $\tau = 0.25$ 10%	40%	Variable Z: $\tau = 0.5$ 10%	40%	Variable Z: $\tau = 0.75$ 10%	40%
sample	-0.428	-0.308	-0.408	-0.308	-0.45	-0.264
	(0.041 -0.955)	(0.042-0.965)	(0.036-0.945)	(0.038-0.945)	(0.042-0.955)	(0.057-0.96)
mean	-0.457	-0.453	-0.433	-0.457	-0.482	-0.48
	(0.04 -0.965)	(0.04-0.94)	(0.038-0.95)	(0.047-0.96)	(0.043-0.96)	(0.065-0.975)
median	-0.459	-0.45	-0.434	-0.455	-0.483	-0.473
	(0.039 -0.96)	(0.037-0.95)	(0.038-0.955)	(0.046-0.955)	(0.044-0.96)	(0.064-0.975)

Table 3. Bootstrap estimates of regression coefficients (standard errors - coverage) related to Z for each percentage of missing values and imputation method

Method	Variable X: $\tau = 0.25$ 10%	40%	Variable X: $\tau = 0.5$ 10%	40%	Variable X: $\tau = 0.75$ 10%	40%
sample	2.974	2.916	3.009	3.029	2.959	3.03
	(0.048 -0.98)	(0.05-0.95)	(0.033-0.95)	(0.04-0.955)	(0.049-0.95)	(0.05-0.955)
mean	2.979	2.933	3.011	3.027	2.963	3.014
	(0.05 -0.97)	(0.047-0.94)	(0.032-0.935)	(0.033-0.96)	(0.045-0.945)	(0.052-0.96)
median	2.98	2.93	3.011	3.03	2.961	3.021
	(0.049 -0.975)	(0.05-0.95)	(0.031-0.94)	(0.033-0.94)	(0.044-0.945)	(0.049-0.945)

Table 4. Bootstrap estimates of regression coefficients (standard errors - coverage) related to X for each percentage of missing values and imputation method

5. Conclusions

We investigated the impact of univariate non-response in quantile regression and we concluded that:

- the listwise deletion method yields estimates closely to those obtained with the complete dataset
- the other imputation methods provide less accurate estimates
- the sample imputation method exhibits really different estimates
- the analysis of bootstrap estimators indicates that some standard errors increased due to the additional variability introduced by the imputation method.

Bibliografy

Nicolini G., Marasini D., Montanari G.E., Pratesi M., Ranalli M.G., and Rocco E.: Metodi inferenziali in presenza di mancate risposte parziali. In: Metodi di stima in presenza di errori non campionari (2013). UNITEXT. Springer, Milano.

Little J.A., and Rubin D.: Statistical Analysis With Missing Data. 2nd Edition. Wiley Series in Probability and Statistics Book (2002)

Koenker, R.: Quantile Regression. Econometric Society Monographs. Cambridge University Press (2005)

Thank you!