A Bayesian Approach to Modelling Graphical Vector Autoregressions

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Contents

1. VARs

VAR Model

Vector autoregressive (VAR) models are popular choice for studying the joint dynamics of multiple time series.

Consider x_t as a p-dimensional vector of time-series at time t, a VAR model is just a multivariate normal regression of x_t on its own-lags

$$x_t = c + \sum_{i=1}^{k} \Pi_i x_{t-i} + \epsilon_t \qquad (1)$$

where Π_i are $p \times p$ coefficient matrices determining the dynamics of the system, c is a deterministic vector of p components and $\epsilon_t \sim N_p(0, \Sigma)$.

Example of VAR(1)

A first order VAR model ($p = 1$) in two variables would be given by

$$\begin{pmatrix} x_{1,t} \\ x_{2,t} \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} + \begin{pmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{pmatrix} \begin{pmatrix} x_{1,t} \\ x_{2,t} \end{pmatrix} + \begin{pmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{pmatrix}$$

If, for example $\pi_{12} \neq 0$, this means that the history of x_2 helps explaining x_1.

Pros & Cons

- Main advantage: simplicity. VAR models require no special structure since the outcome variables are regressed on their own lagged variables.
- Main disadvantage: large number of regression coefficients, $K(Kp+m)$. The significant number of regression parameters is proportional to the number of lags, hence interpretation may be difficult and may not be good when fitted to small data.

Several tools have been proposed to aid in the interpretation of VAR models, most notably Impulse Response Functions (IRF) and Granger Causality tests (GC).

Granger Causality

The general idea behind Granger causality is that a variable X Granger causes Y if past values of X can help explain Y.

Of course, if Granger causality holds this does not guarantee that X causes Y. Nevertheless, if past values of X have explanatory power for current values of Y, it at least suggests that X improves Y's predictability.

In the context of a VAR model, testing for Granger causality simply implies to test the significance of the cross-coefficients (π_{12} and π_{21} in the VAR(1) example).

2. Graphical VARs

Graphical Model

Graphical models can be informally defined as statistical models represented in the form of a graph, where the nodes (vertices) represents the variables and the presence of an edge between a pair of vertices means that the variables are in some sense related.

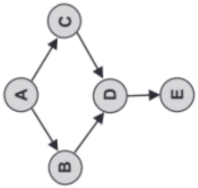
Some advantages of a graphical model are:

- represents graphically the logical implication of the relationships
- suitable representation of the causal relationships using directed edges
- clarity of interpretation when analyzing complex interactions.

DAG Model

In cross-sectional data, the graph usually represents the conditional independence structure of the system and (by symmetry) the edges are undirected. In time-series data, the time dimension of a process makes it more feasible to consider directed flow.

A Directed Acyclic Graph (DAG) is a collection $G = \{V, E\}$, where V is the set of vertices and E is the set of edges.



Connecting DAG and VAR

Given the VAR model in (1), there is a one-to-one relationship between the Π_i matrices and DAGs, that is

$$x_{a,t-i} \to x_{b,t} \iff \Pi_i(a,b) \neq 0$$

Where $x_{a,t-i} \to x_{b,t}$ means that $x_{a,t-i}$ "causes" somehow $x_{b,t}$

Hence, directed edges represent exactly the Granger causality relations.

Graphical VAR Model

3. Bayesian Model Assessment for Graphical VARs

4. Conclusions

Conclusions

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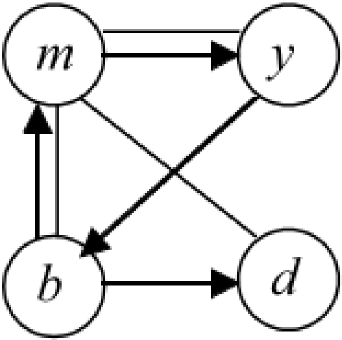

Bibliografy

Corander, J. & Villani, M. (2006), 'A Bayesian Approach to Modelling Graphical Vector Autoregressions', Journal of Time Series Analysis 27(1), 141–156.

Thank you!