Quantum tomography of structured light patterns from simple intensity measurements

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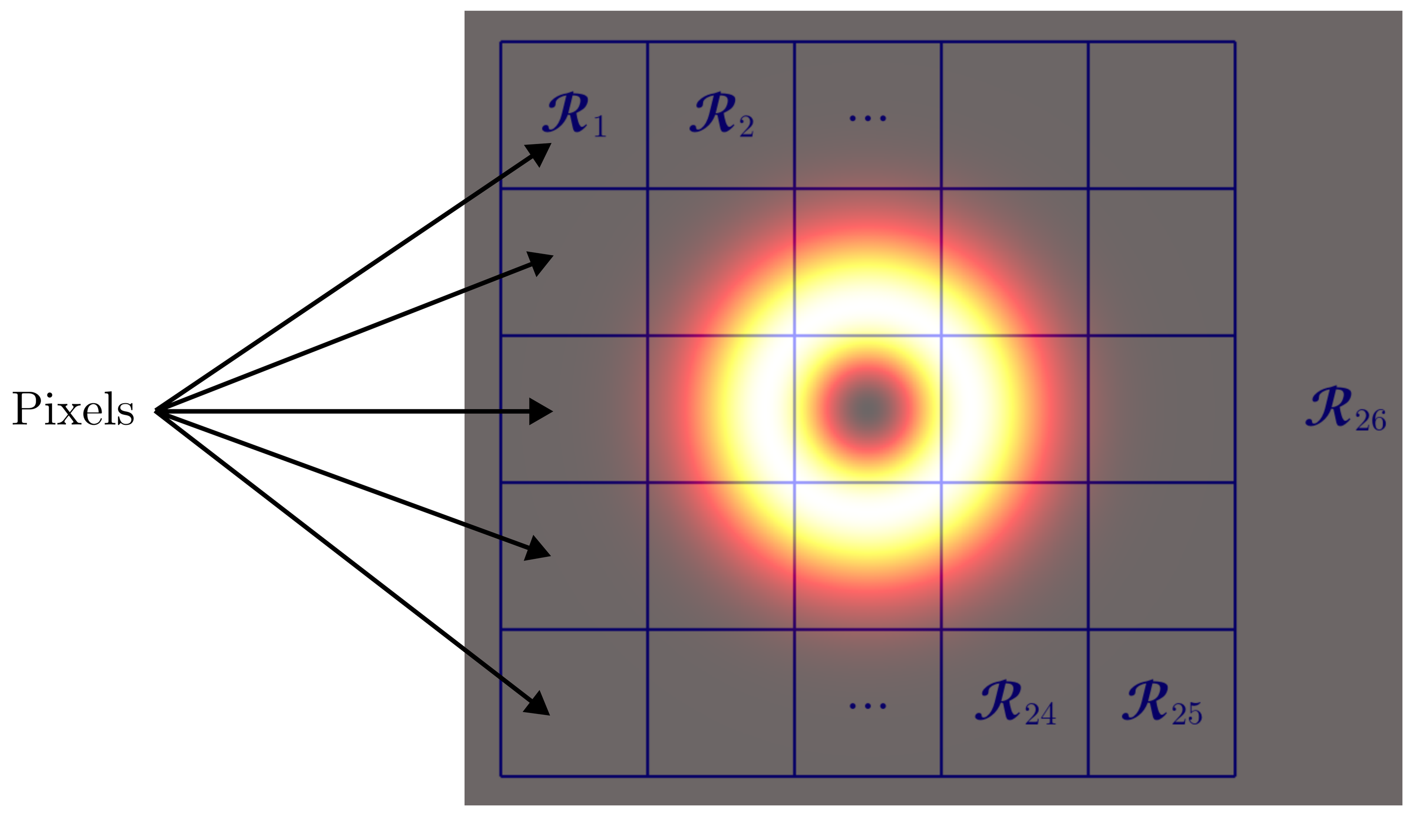
Optica Student Chapter

Introduction

- The transverse structure of light has many important applications in areas such as quantum cryptography and optical tweezing, so its characterization is extremely important.
- We want to produce a linear combination of modes and recover the coefficients that specify it.
- **We strive for simplicity**: only use the camera as a measuring device.
- We identify the problem as a standard Quantum State Tomography, which gives us access to the tools of this well-developed field.
- We experimentally show the power of our method in both the intense and photocount regimes, including even mixed states.

What does a camera measure?



A photocount will happen at a pixel with probability

$$P(\mathcal{R}) = \int_{\mathcal{R}} \Gamma(\mathbf{r},\mathbf{r})d\mathbf{r};$$

Hermite-Gaussian modes: $u_j = HG_{j,N-j}$

$$\Gamma(\mathbf{r},\mathbf{r}') = \frac{\langle E^-(\mathbf{r})E^+(\mathbf{r}')\rangle}{\int_{\mathbb{R}^2}\langle E^-(\mathbf{r})E^+(\mathbf{r})\rangle d\mathbf{r}} = \sum_{j,k=0}^{N} \rho_{jk} u_j^*(\mathbf{r}')u_k(\mathbf{r})$$

It is as if we have a quantum particle with position density matrix Γ and we measured

$$P(\mathcal{R}) = \langle F(\mathcal{R})\rangle; \quad F(\mathcal{R}) = \int_{\mathcal{R}} |\mathbf{r}\rangle\langle\mathbf{r}|\, d\mathbf{r}$$

It's is just quantum state tomography!

A Positive Operator Valued Measure (POVM) is a set of positive operators $\{F_m\}$ that satisfy $\sum_m F_m = I$. They specify the probability of outcomes in an experiment: $p(m) = \text{Tr}\ \rho F_m$. We can see that our set $\{F_m = F(\mathcal{R}_m)\}$ is a POVM! Consider the linear operator

$$T: \text{Her}(\mathcal{H}) \to \mathbb{R}^M; \quad T(\Omega) = (\text{Tr}\ \Omega F_1, \ldots, \text{Tr}\ \Omega F_M)$$

Space of modes of fixed order

Hermitian operator

POVM elements

Then, quantum state tomography reduces to the solution of the linear system

$$\boxed{T\rho = \mathbf{p}} \longrightarrow$$

Detection probability at each pixel

Experimentally determined in the intense regime

One needs a second image

We want that two different states produce two different experimetal outcomes, so we may be able to distinguish them. In other words, we want the transformation T to be injective. We know that it isn't: **Laguerre Gaussian modes with oposite l have the same intensity**.

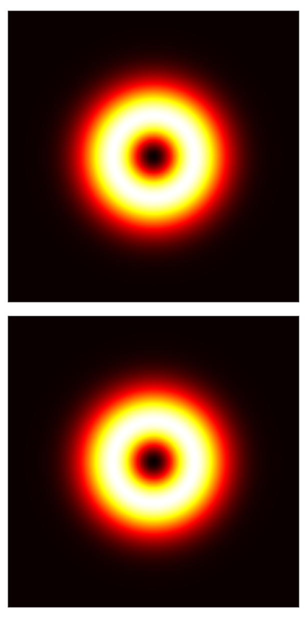
$$Y_{jk} = \frac{i}{\sqrt{2}}\left(|u_j\rangle\langle u_k| - |u_k\rangle\langle u_j|\right)$$

Hermite-Gaussian modes are real-valued

T is not injective

$$\text{Tr}\ F(\mathcal{R})Y_{jk} = \sqrt{2}\Im\int_{\mathcal{R}} u_j^*(\mathbf{r})u_k(\mathbf{r})d\mathbf{r} = 0 \Rightarrow T(Y_{jk}) = \mathbf{0}$$

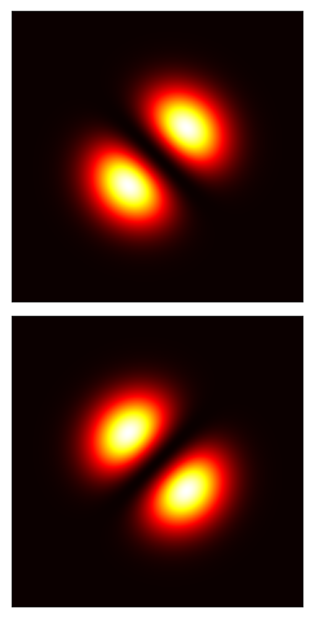
Solution: a mode converter implemented by a tilted cylindrical lens.



$$U_\theta = \sum_{m,n=0}^{\infty} e^{i(m-n)\theta}\,|HG_{mn}\rangle\langle HG_{mn}|$$

$$\longrightarrow$$

The angle can be adjusted experimentally. It is usually taken as $\pi/2$ but we will need other values.



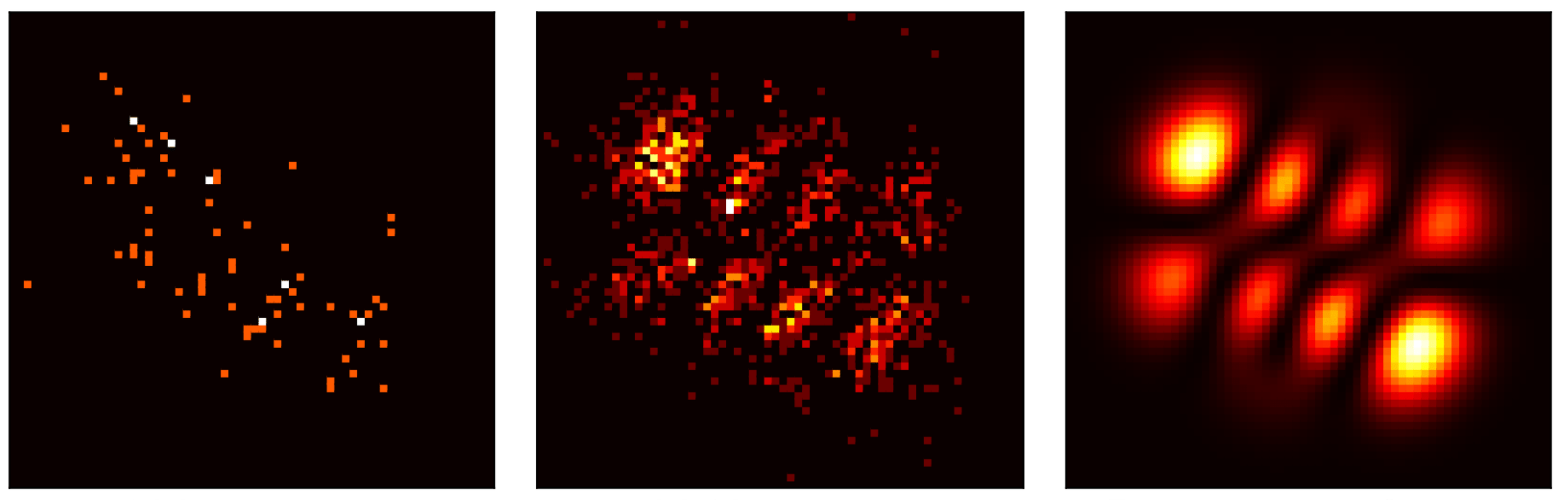
This generates a larger POVM $\{F_m/2, U_\theta^\dagger F_m U_\theta/2\}$ for which the transformation is shown to be injective.

Experimental results

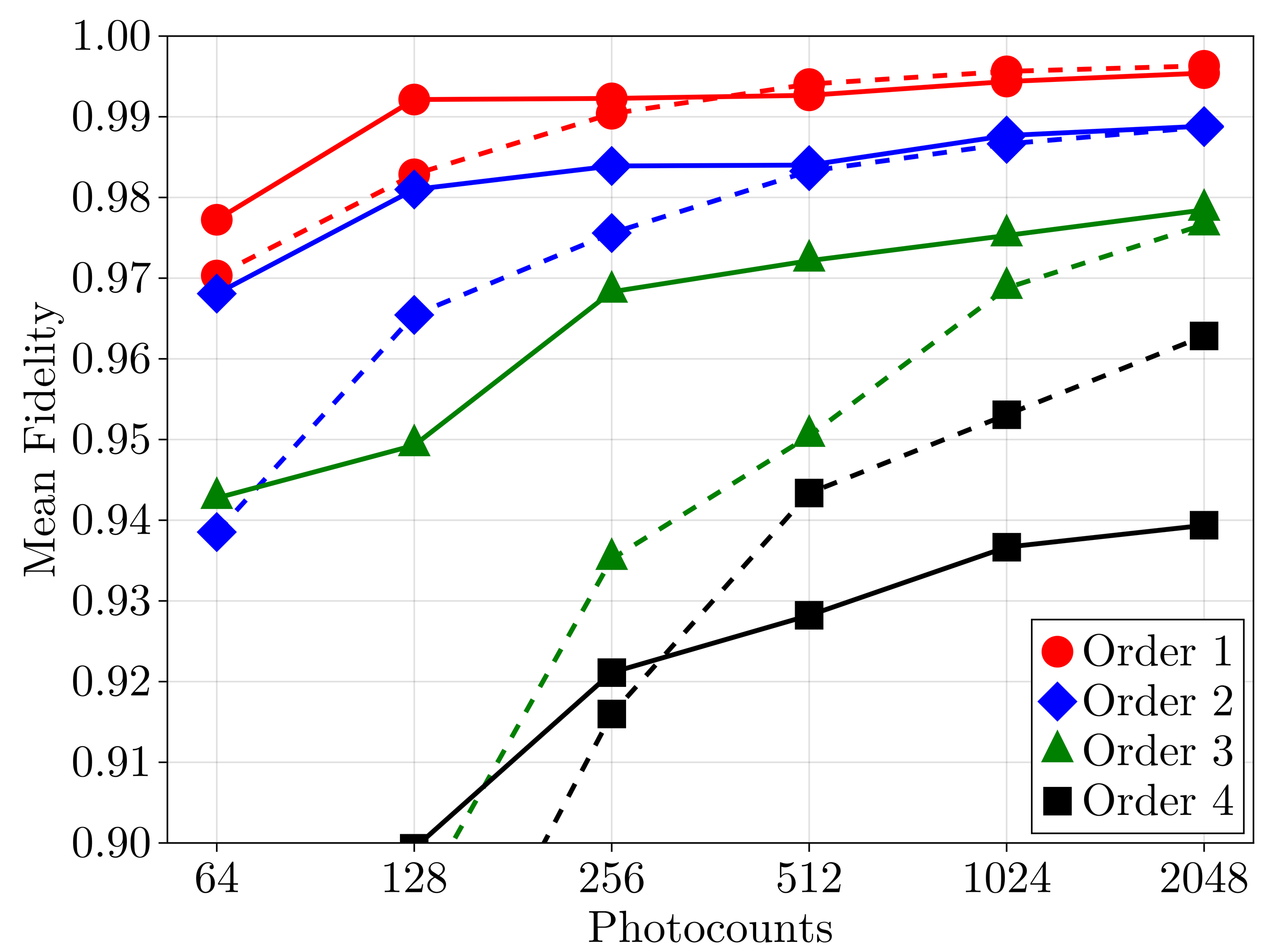
- Mixed states; intense regime; $\pi/6$ mode converter:

Method \Order	1	2	3	4	5
Linear Inversion	0.998	0.995	0.992	0.989	0.983
Machine Learning	0.995	0.988	0.985	0.979	0.968

- Pure states; photocount regime: $\pi/2$ mode converter



128 counts | 2048 counts | Intense simulation



Full lines: machine learning; **dashed lines**: Bayesian tomography

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