

XVII J. A. Swieca School - Curitiba (March, 2023)

Nonlinear Optics

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Founded in 1535

2 millions of inhabitants

Optics in Recife

Nonlinear spectroscopy of
condensed matter systems

Random lasers

Nano optics: Optical
microcavities

High-harmonic generation

Ultrafast phenomena

Metrology of optical frequencies

Cold atoms: coherent effects

Quantum information and
entanglement

Laser dynamics



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DSc PUC-Rio 1975

PD Harvard 1976-1977



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PD Brown Univ. 1993



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DSc UFPE 2001

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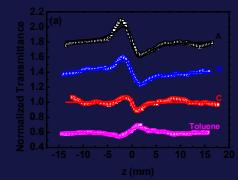
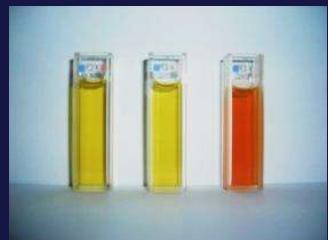
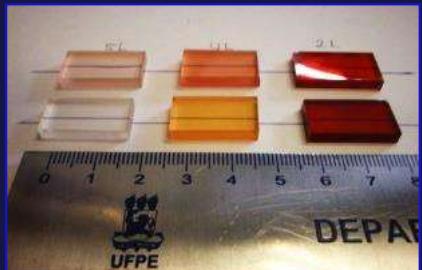
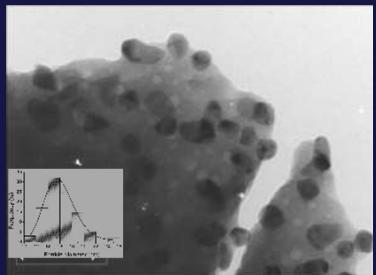
PD UFPE 2021-2022

Michelle O. Araujo

Univ. Côte d'Azur 2018

PD UFPE 2019-2022

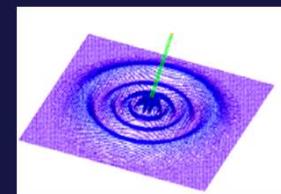
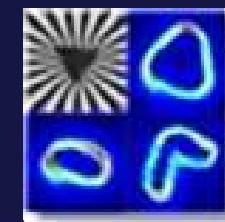
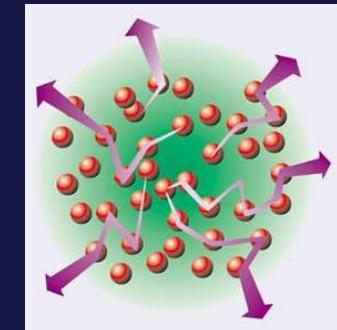
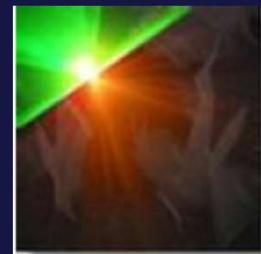
Plasmonics in random media



NLO LAB

Transition metal oxides
Organic materials
(nanoparticles and films)

Random lasers



NL transverse effects (phase
modulation, vortices, solitons...)

J. A. Swieca School 2023

1st. lecture

Introduction to nonlinear optics

2nd lecture

Nonlinear Photonics in Plasmonic media

3rd lecture

Stimulated Emission and Random Lasers

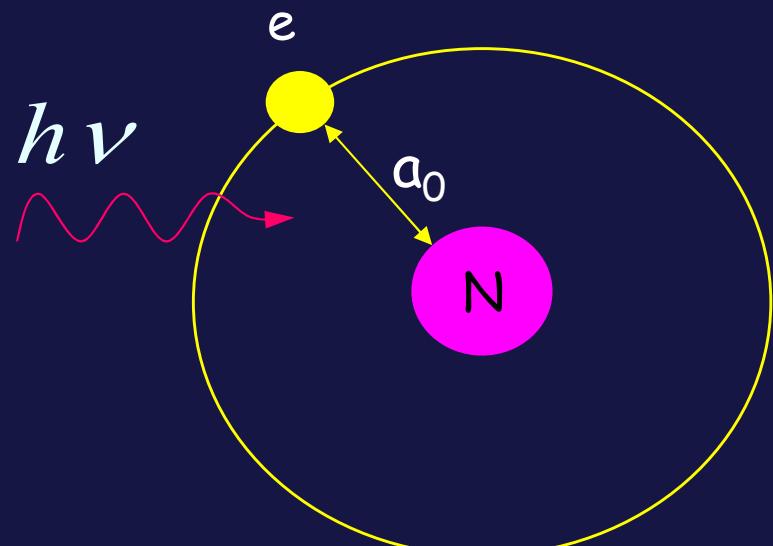
How does optical nonlinearity appear ?

The strength of the electric field of the light wave should be in the range of atomic fields

$$E_{at} = e / a_0^2$$

$$a_0 = \hbar^2 / me^2$$

$$E_{at} \approx 2 \times 10^{-7} \text{ esu} \quad 3 \times 10^8 \text{ V/cm}$$



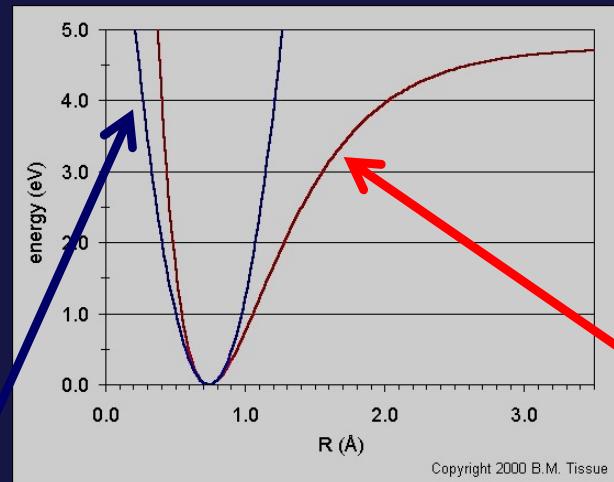
Atom as an oscillator

Electron potential energy

Small optical intensity

$$\omega_0 = \sqrt{\frac{K}{m_0}}$$

Harmonic oscillator



Large optical intensity

Anharmonic oscillator

$$U(x) = \frac{1}{2}m_0\omega_0^2x^2 + \frac{1}{3}m_0K_a x^3 + \frac{1}{4}m_0K_b x^4 + \dots$$

Restoring Force $F(x) = -\frac{dU}{dx} = -m_0\omega_0^2x - m_0K_a x^2 - m_0K_b x^3 - \dots$

Considering only the second order nonlinearity

$$m_0 \frac{d^2x}{dt^2} + m_0\gamma \frac{dx}{dt} + m_0\omega_0^2 x + \textcircled{m_0 K_a x^2} = -eE(t)$$

$$E(t) = E_0 \cos \omega t = \frac{1}{2} E_0 (e^{i\omega t} + e^{-i\omega t})$$

Tentative solution

$$x(t) = \frac{1}{2} (X_1 e^{i\omega t} + X_2 e^{i2\omega t} + \text{c. c.}) \quad X_1 \gg X_2$$

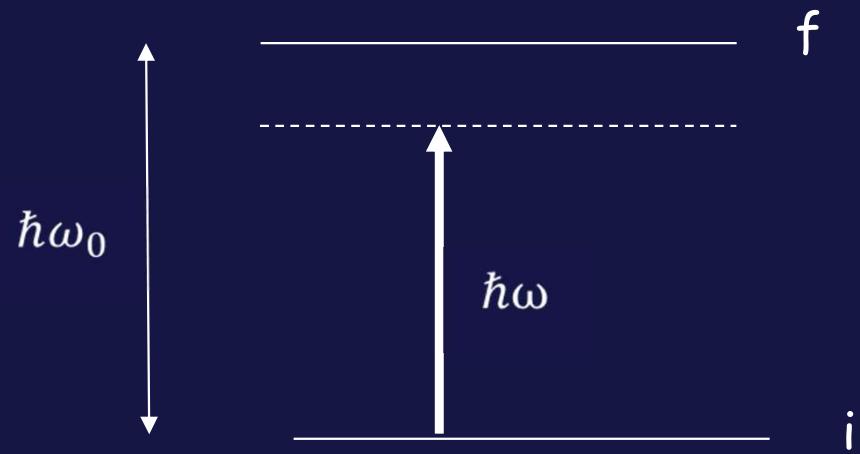
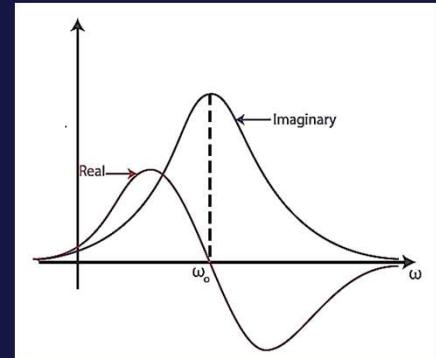
$$X_1 = - \frac{eE_0}{m_0} \frac{1}{(\omega_0^2 - \omega^2) + i\gamma\omega}$$

Induced polarization

$$P(\omega, t) = -Nex(\omega, t) = -\frac{Ne}{2} (X_1 e^{i\omega t} + \text{c. c.}) = \epsilon_0 \chi^{(1)}(\omega) E(t)$$

$$\chi^{(1)}(\omega) = \frac{N e^2}{m_0 \epsilon_0 [(\omega_0^2 - \omega^2) + i\gamma\omega]}$$

$$\chi^{(1)}(\omega, \omega) = \frac{Ne^2}{m\epsilon_0[(\omega_0^2 - \omega^2) + i\gamma\omega]}$$



Representation based on
the quantum description

$$n_0 \propto \operatorname{Re} \chi^{(1)}(\omega, \omega)$$

Linear refractive index

$$\alpha_0 \propto \operatorname{Im} \chi^{(1)}(\omega, \omega)$$

Linear absorption coefficient

By considering the next term in the perturbative solution

$$P(2\omega, t) = -Nex(2\omega, t) = -\frac{Ne}{2} (X_2 e^{i2\omega t} + \text{c. c.}) = \epsilon_0 \chi^{(2)}(2\omega) E^2(t)$$

$$\chi^{(2)}(2\omega) = \frac{Ne^3 K}{\epsilon_0 m^2 [(\omega_0^2 - \omega^2) + i\gamma\omega]^2 (\omega_0^2 - 4\omega^2 + i2\omega\gamma)}$$

Second Harmonic Generation

$$\begin{aligned} P^{(2)} &= \epsilon_0 \chi^{(2)}(2\omega) [E \cos \omega t]^2 \\ &= \frac{1}{2} \epsilon_0 \chi^{(2)}(2\omega) E^2 \{1 + \cos 2\omega t\} \end{aligned}$$

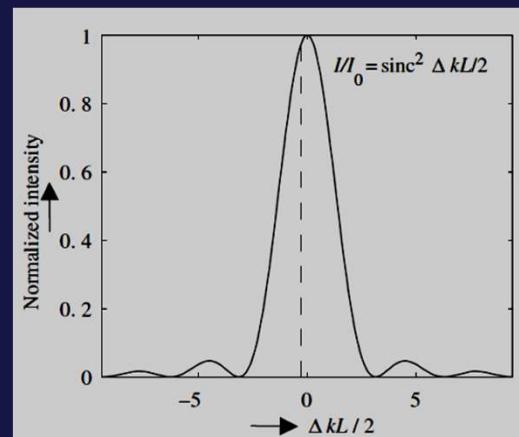
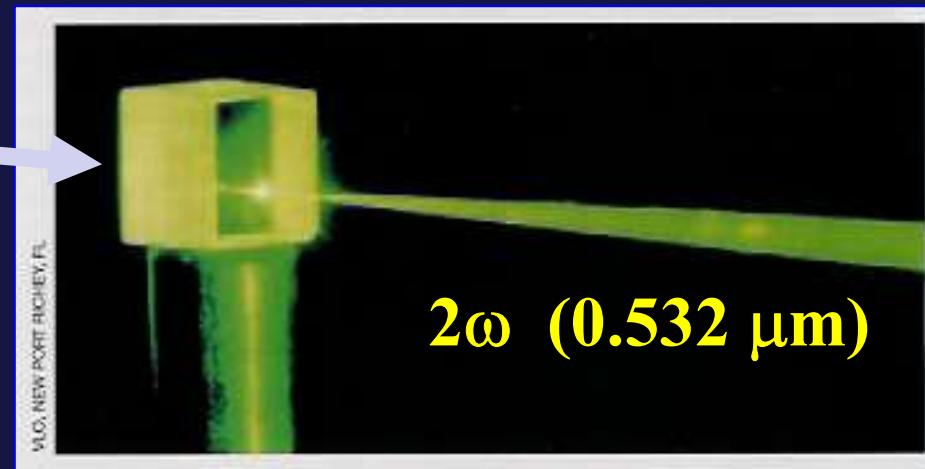
GENERATION OF OPTICAL HARMONICS*

P. A. Franken, A. E. Hill, C. W. Peters, and G. Weinreich

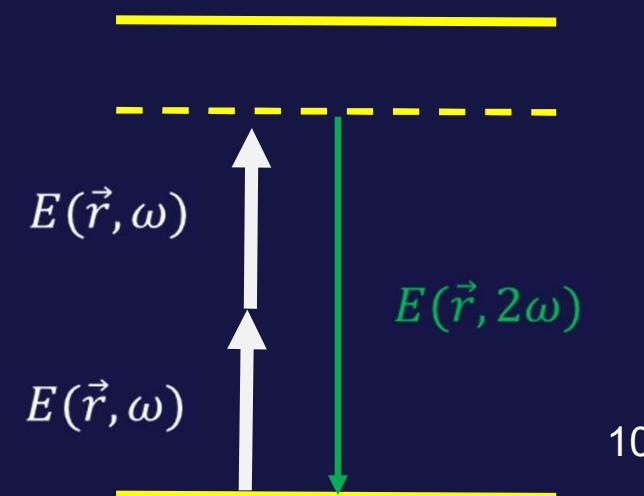
The Harrison M. Randall Laboratory of Physics, The University of Michigan, Ann Arbor, Michigan
(Received July 21, 1961)

Transparent crystal

Infra-red

 ω (1.064 μm)

Phase matching:
 $k(2\omega) = 2k(\omega)$

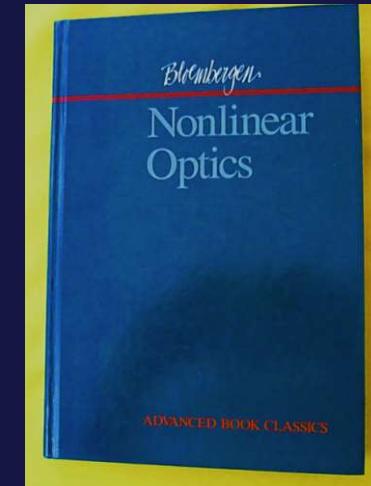


Classical and Quantum formalisms (1962-1964)



N. Bloembergen
(1920-2017)

1981
Physics Nobel Prize



Induced
polarization

Nonlinear susceptibilities

$$\bullet \vec{P} = \epsilon_0 \{ \vec{\chi}^{(1)} \cdot \vec{E} + \vec{\chi}^{(2)} : \vec{E} \vec{E} + \vec{\chi}^{(3)} : \vec{E} \vec{E} \vec{E} + \dots \}$$

\vec{P} induces changes in the speed of light in the medium and new frequencies may be generated

Meaning of each term

Polarization vector

$$\overrightarrow{P^{(1)}} = \epsilon_0 \overleftrightarrow{\chi^{(1)}} \cdot \overrightarrow{E}$$

$$\overrightarrow{P^{(2)}} = \epsilon_0 \overleftrightarrow{\chi^{(2)}} : \vec{E} \vec{E}$$

$$\overrightarrow{P^{(3)}} = \epsilon_0 \overleftrightarrow{\chi^{(3)}} : \vec{E} \vec{E} \vec{E}$$

Vector components

$$P_i^{(1)} = \epsilon_0 \chi_{ij}^{(1)} E_j$$

$$P_i^{(2)} = \epsilon_0 \chi_{ijk}^{(2)} E_j E_k$$

$$P_i^{(3)} = \epsilon_0 \chi_{ijkl}^{(3)} E_j E_k E_l$$

Analogously for the other terms...

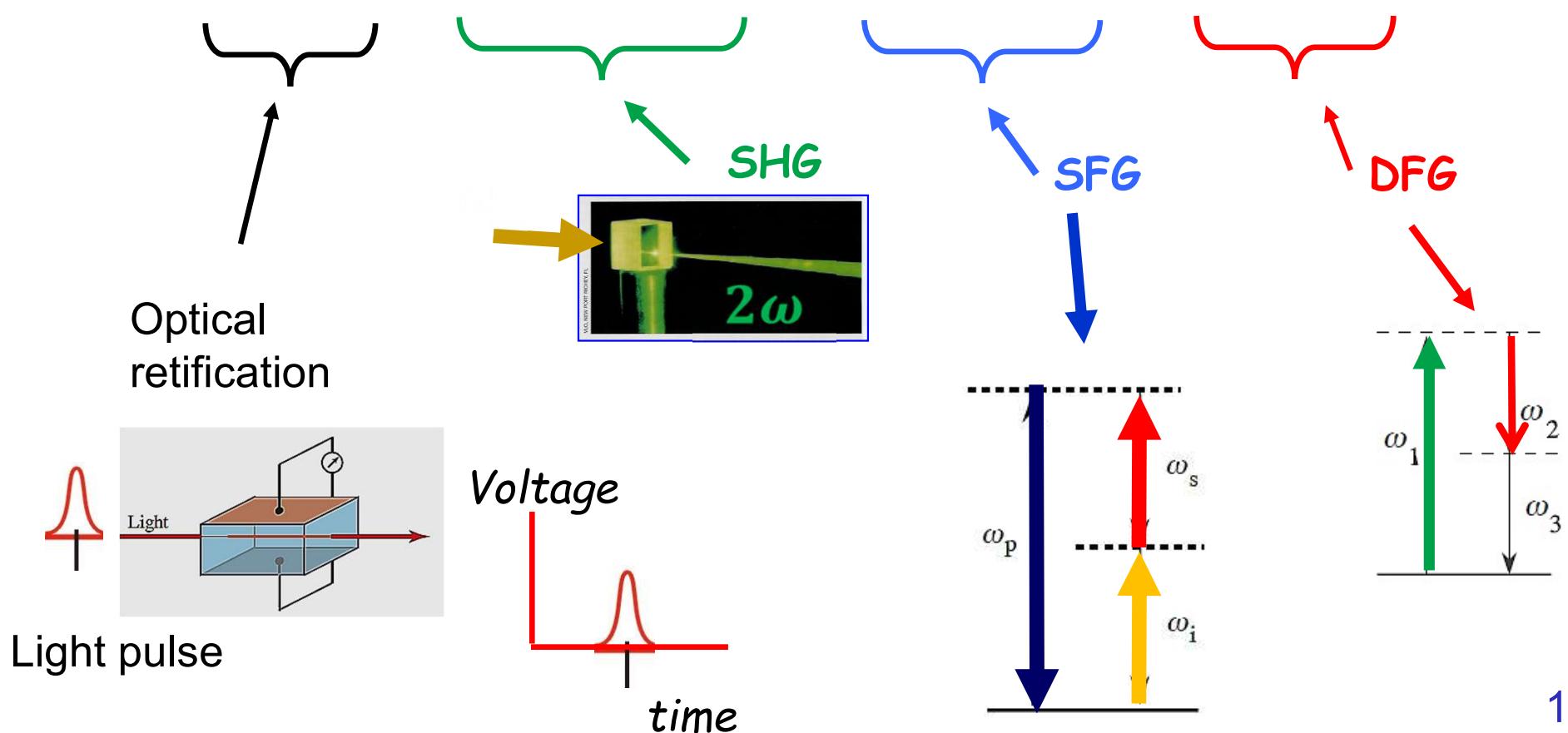
The linear “superposition principle” is not valid

$\chi^{(2)}$ 2nd. order polarization

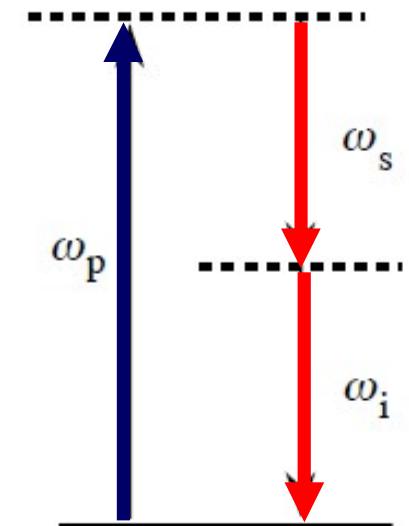
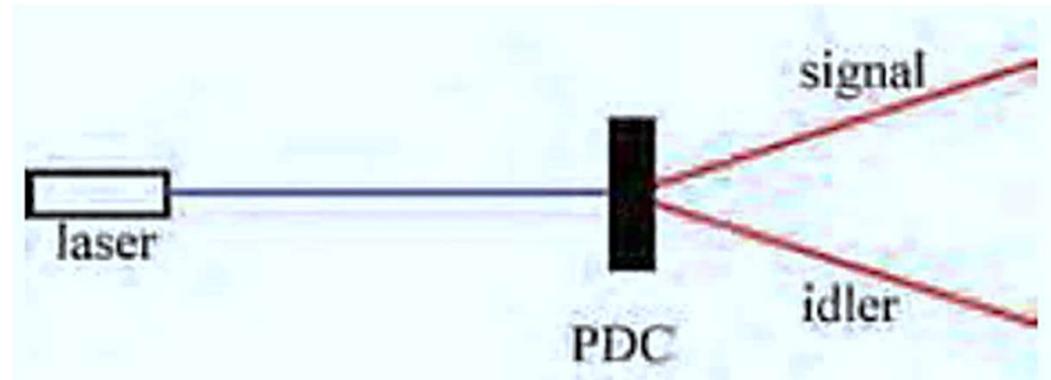
Three Wave- Mixing

$$P^{(2)} = \epsilon_0 \chi^{(2)} (E_1 \cos \omega_1 t + E_2 \cos \omega_2 t)^2 =$$

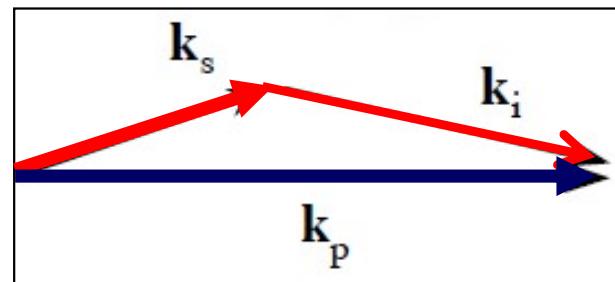
$$P_0^{(2)} + P_{2\omega_1}^{(2)} + P_{2\omega_2}^{(2)} + P_{\omega_1+\omega_2}^{(2)} + P_{\omega_1-\omega_2}^{(2)}$$



$\chi^{(2)}$ Parametric down-conversion



Phase - matching



Signal and idler photons are entangled in polarization, time, energy, position, transverse momentum, angular position and orbital angular momentum

Entanglement is important for fundamental tests of QM and quantum technologies.

Third order nonlinearity

Medium with inversion symmetry $\chi^{(n)} = 0$, n = 2,4,...

$$E(t) = E \cos \omega t$$

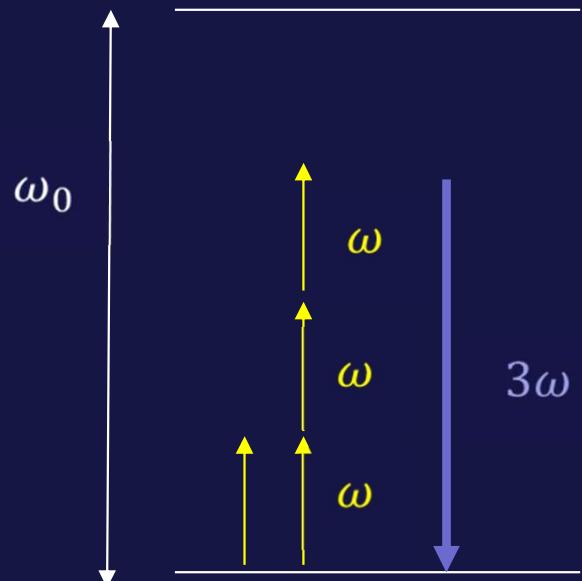
$$P^{(1)}(t) + P^{(3)}(t) + \dots = \epsilon_0 \left\{ \chi^{(1)} E(t) + \chi^{(3)} E^3(t) + \dots \right\}$$

$$\cos^3 \omega t = \frac{1}{4} \cos 3\omega t + \frac{3}{4} \cos \omega t$$

$$\begin{aligned} P(t) &= P^{(1)}(t) + P^{(3)}(t) \\ &= \epsilon_0 \chi^{(1)} E \cos \omega t + \frac{1}{4} \epsilon_0 \chi^{(3)} E^3 \cos 3\omega t + \frac{3}{4} \epsilon_0 \chi^{(3)} E^3 \cos \omega t \end{aligned}$$

$$P(t) = \epsilon_0 \left\{ \chi^{(1)} + \frac{3}{4} \chi^{(3)} E^2 \right\} E \cos \omega t + \frac{1}{4} \epsilon_0 \chi^{(3)} E^3 \cos 3\omega t$$

$$P(t) = \epsilon_0 \left\{ \chi^{(1)} + \frac{3}{4} \chi^{(3)} E^2 \right\} E \cos \omega t + \frac{1}{4} \epsilon_0 \chi^{(3)} E^3 \cos 3\omega t$$



This polarization generates the third harmonic

NL refractive index

$$n \propto \text{Re} \left\{ \chi^{(1)} + \frac{3}{4} \chi^{(3)} E^2 \right\}$$

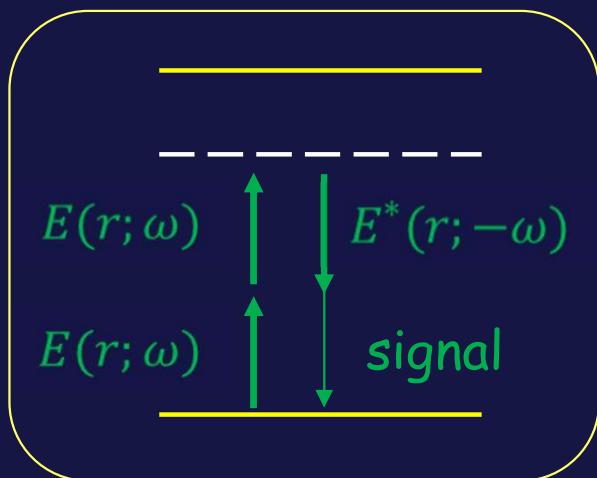
$$n = n_0 + n_2 I \quad n_2 = \frac{3 \text{ Re } \chi^{(3)}}{2n_0^2 \epsilon_0 c}$$

$$P^{(3)}(\omega) = \epsilon_0 \{ \chi^{(3)} (E e^{+i\omega t} + c.c.) \}^3$$

$$P^{(3)}(\omega) e^{-i\omega t} = \epsilon_0 \chi^{(3)}(\omega, \omega, \omega, -\omega) E(\omega) E(\omega) E^*(-\omega) e^{-i\omega t}$$

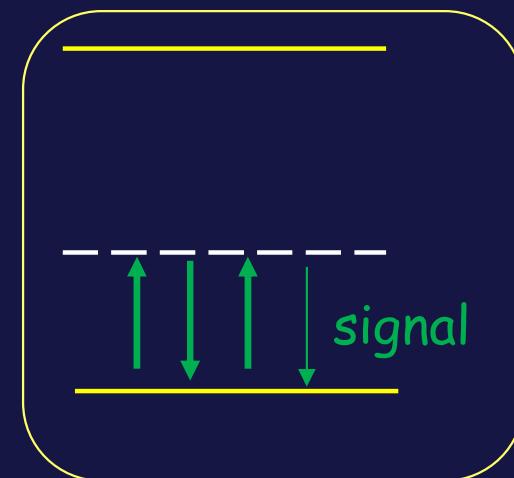
Degenerate wave-mixing

Transparent medium

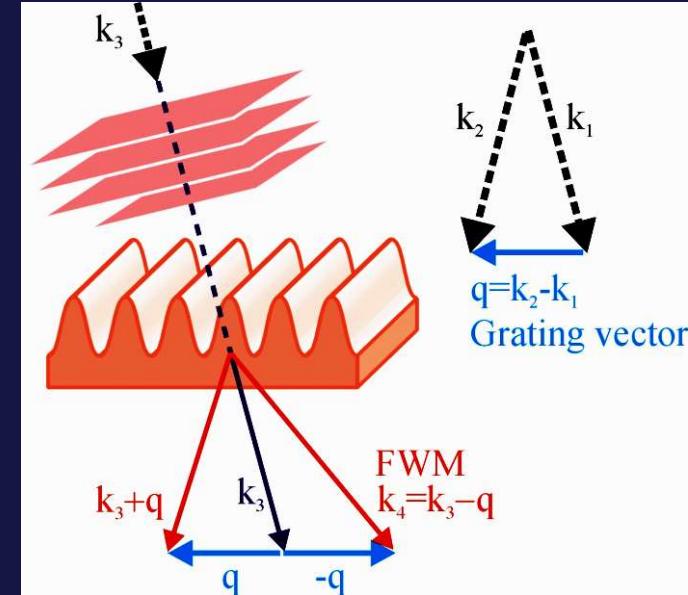
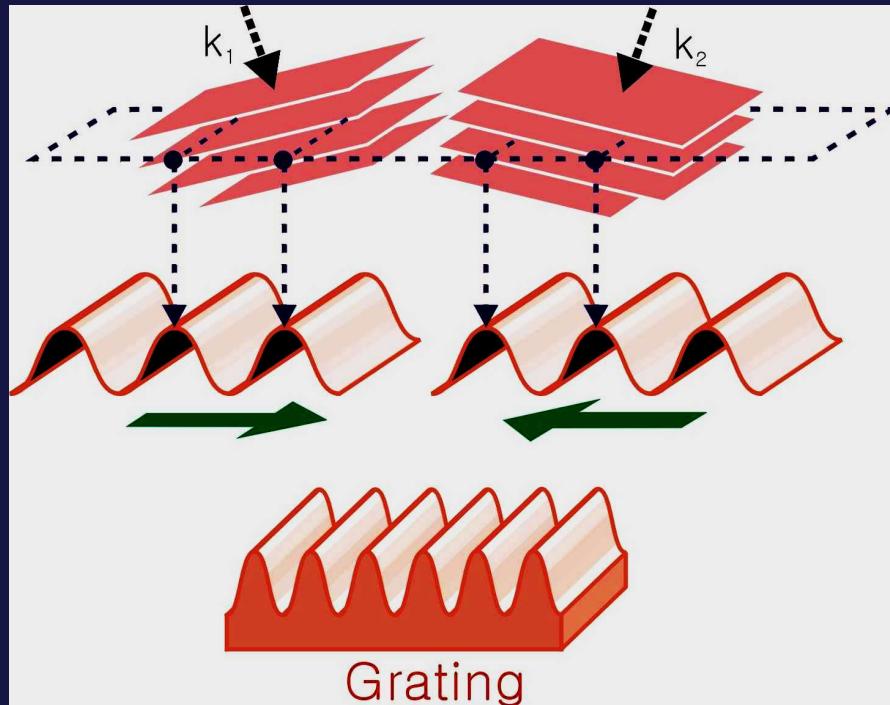


$$\chi^{(3)}(\omega, \omega, \omega, -\omega)$$

$$\chi^{(3)}(\omega, \omega, -\omega, \omega)$$

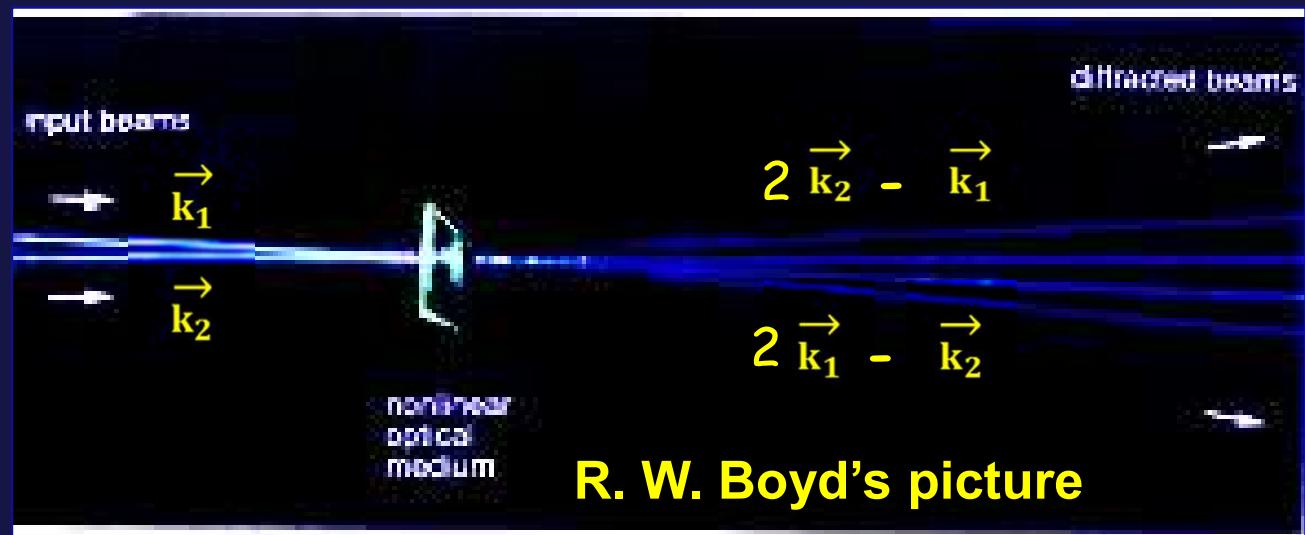


$\chi^{(3)}$: degenerate four-wave-mixing



$$P^{(3)} = \epsilon_0 \chi^{(3)} E^3$$

Only one optical frequency



R. W. Boyd's picture

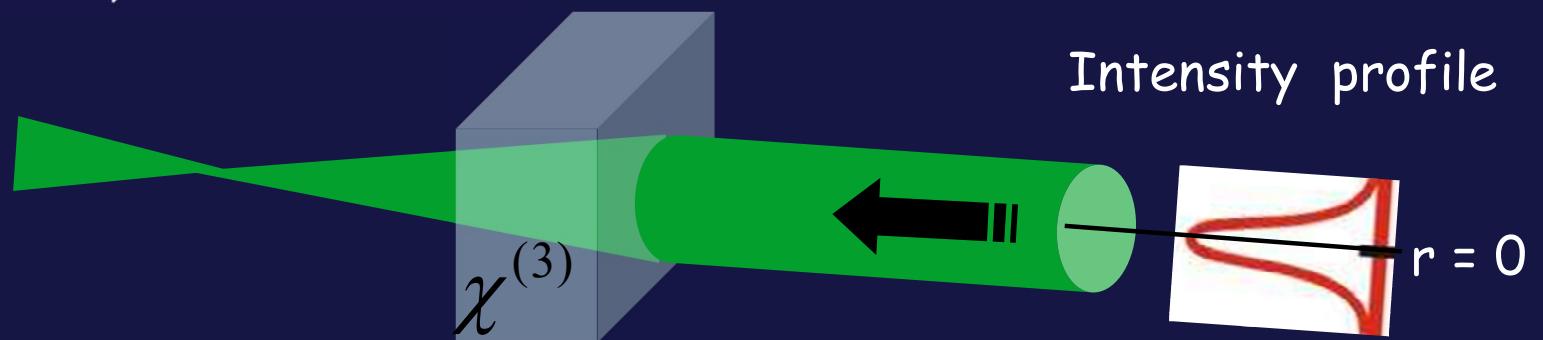
$\chi^{(3)}$: self - focusing

$$n_2 \propto \operatorname{Re} \chi^{(3)}$$

A laser beam with Gaussian intensity profile will induce a Gaussian refractive index profile inside the NL sample.

$$E(\omega) \exp\left(\frac{-r^2}{\Delta^2}\right) \exp(-i\omega t)$$

$$n_2 > 0$$



$$n = n_0 + n_2 I_0 \exp\{-r^2/w^2\}$$

Sample behaves as
a convergent lens

$$n_2 \propto Re \chi^{(3)}$$

Mechanism	n_2 (cm ² /W)	$\chi^{(3)}_{1111}$ (esu)	Response time (sec)
Electronic Polarization	10^{-16} - 10^{-13}	10^{-14} - 10^{-11}	10^{-15}
Molecular Orientation	10^{-14}	10^{-12}	10^{-12}
Electrostriction	10^{-14}	10^{-12}	10^{-9}
Saturated Absorption	10^{-10}	10^{-8}	10^{-8}
Thermal effects	10^{-6}	10^{-4}	10^{-3}

Electronic polarization

$$\Delta n (r = 0) = n_2 I_0 \quad I_0 = 1 \text{ GW/cm}^2$$

$$\Delta n = 10^{-7} \text{ to } 10^{-4}$$

$$P^{(3)}(t) = \epsilon_0 \chi^{(3)} E^3(t)$$

$$E(t) = E_1 e^{-i\omega_1 t} + E_2 e^{-i\omega_2 t} + E_3 e^{-i\omega_3 t}$$

$$\begin{array}{ccccccccc} \omega_1, & \omega_2, & \omega_3, & 3\omega_1, & 3\omega_2, & 3\omega_3 & , & (\omega_1 - \omega_2 + \omega_3), \\ (-\omega_1 + \omega_2 + \omega_3), & (2\omega_1 \pm \omega_2), & (2\omega_1 \pm \omega_3), & (2\omega_2 \pm \omega_1), & (2\omega_2 \pm \omega_3), & (2\omega_3 \pm \omega_1), \\ & & (2\omega_3 \pm \omega_2) \end{array}$$

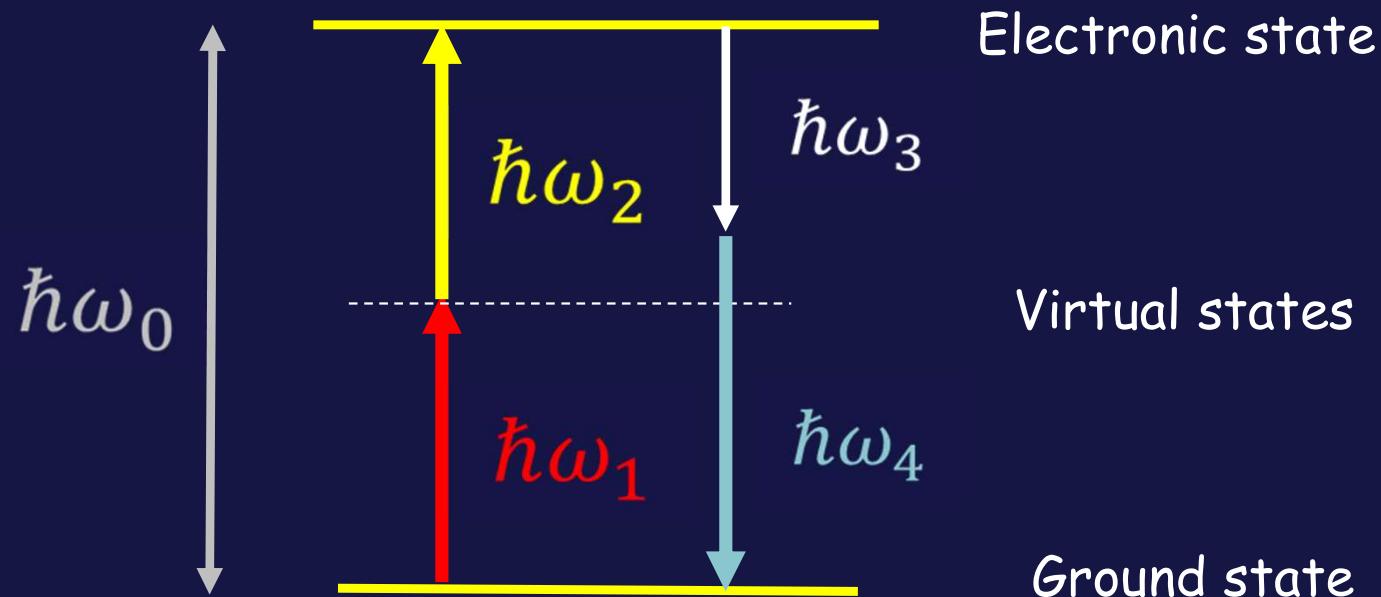
$$P^{(3)}(\omega_4 = 3\omega_1) = \epsilon_0 \chi^{(3)}(3\omega_1, \omega_1, \omega_1, \omega_1) E_1^3$$

$$P^{(3)}(\omega_4 = \omega_1 + \omega_2 - \omega_3) = 6\epsilon_0 \chi^{(3)}(\omega_4, \omega_1, \omega_2, -\omega_3) E_1 E_2 E_3^*$$

$$P^{(3)}(t) = \sum_n P^{(3)}(\omega_n) e^{i\omega_n t}$$

Two-photon resonant four wave-mixing

$$P(\omega_4 = \omega_1 + \omega_2 - \omega_3) = 6\epsilon_0\chi^{(3)}(\omega_4, \omega_1, \omega_2, -\omega_3) E_1 E_2 E_3^*$$

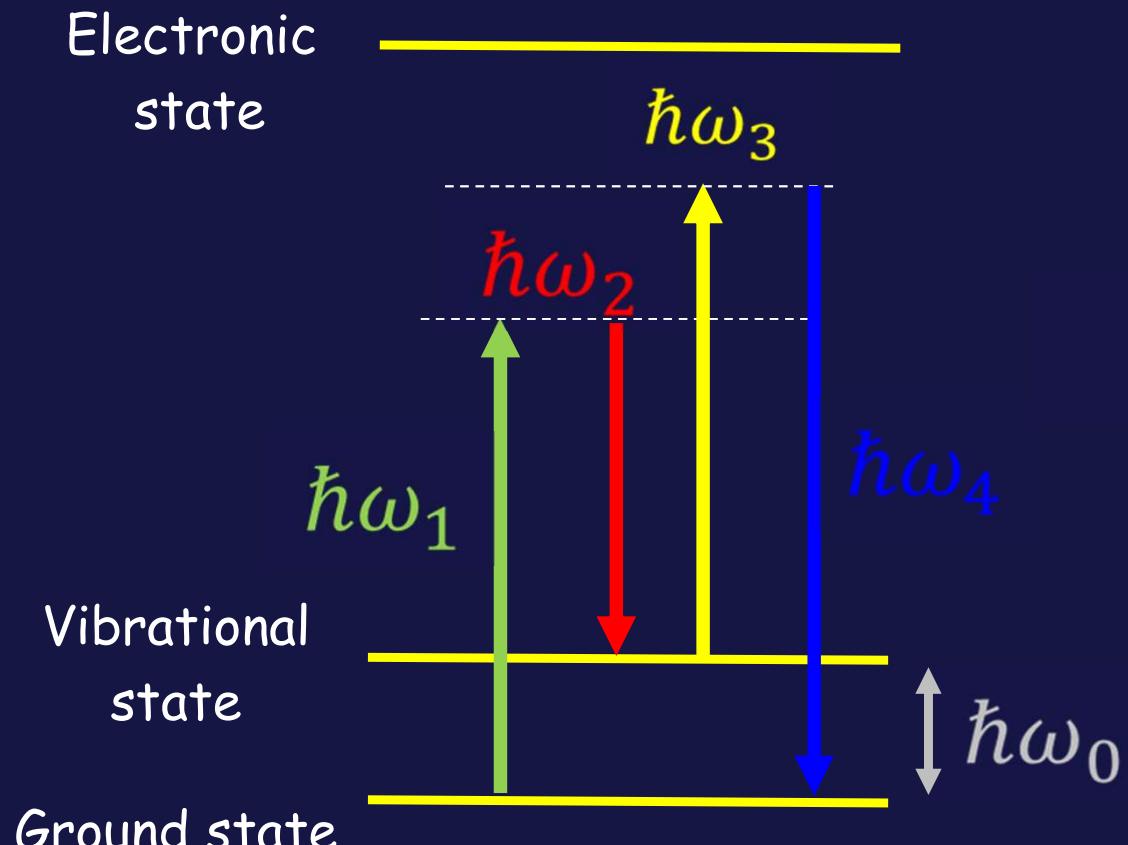


Excited state with same parity than the ground state

Four wave-mixing with Raman resonance

CARS

$$P(\omega_4 = \omega_1 - \omega_2 + \omega_3) = 6\epsilon_0\chi^{(3)}(\omega_4, \omega_1, -\omega_2, \omega_3) E_1 E_2^* E_3$$



$$\hbar\omega_1 - \hbar\omega_2 = \hbar\omega_0$$

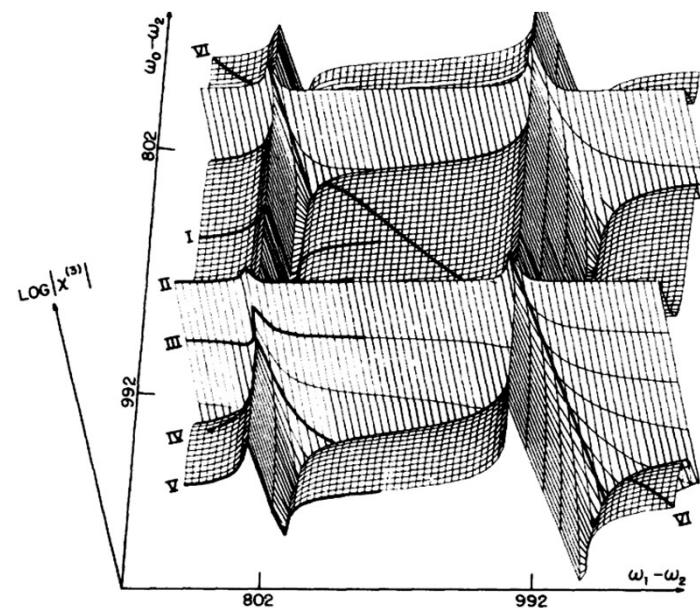
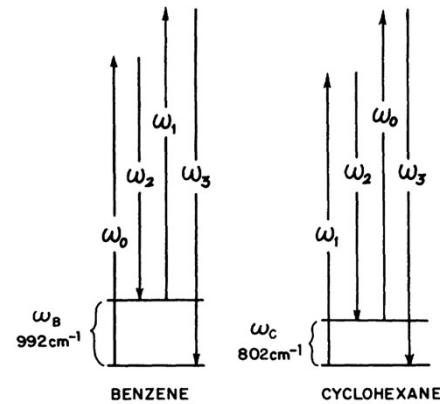
Interaction with
non-resonant background

$$P_{NL}^{(3)} = P_{NR}^{(3)} + P_{RES}^{(3)}$$

Y. R. Shen
The Principles of NLO
Wiley, 1984.

M. D. Levenson
Introduction to NL spectroscopy
Academic, 1982

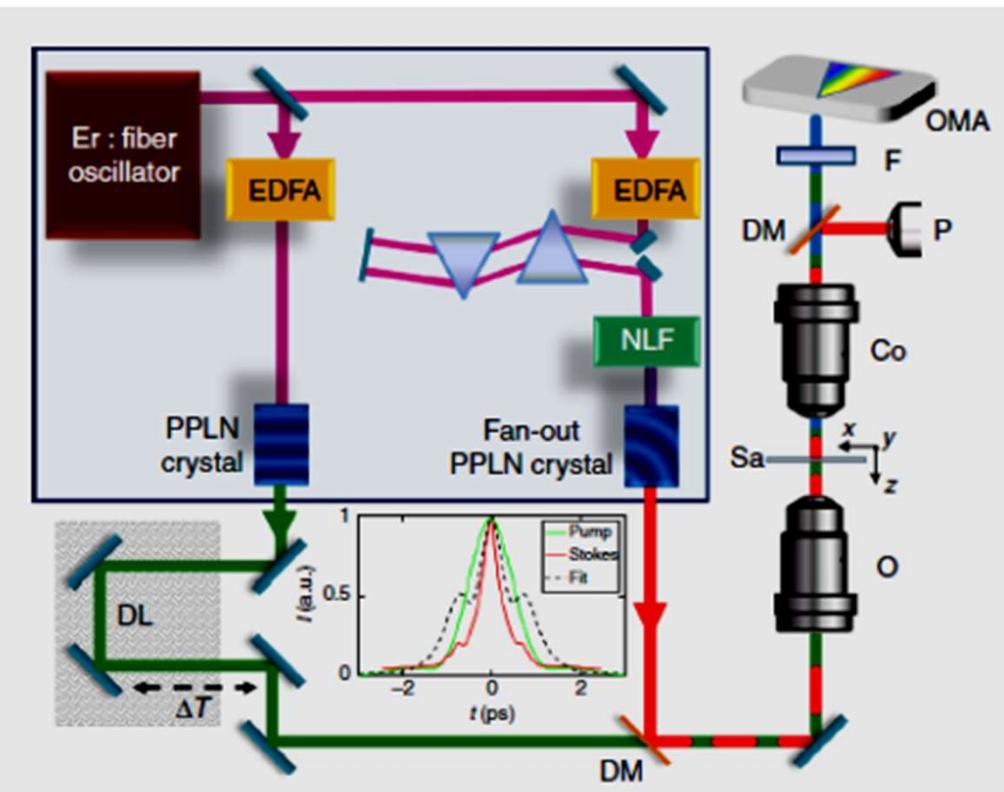
Lotem, Lynch, Bloembergen
Phys. Rev. A 14 (1976) 1748



A. Virga,*G. Cerullo*... et al
Coherent anti-Stokes Raman
spectroscopy of single and
multi-layer graphene

Nature Commun. 10 (2019) 3658

$$P_{NL}^{(3)} = P_{NR}^{(3)} + P_{RES}^{(3)}$$



Optical susceptibilities depend on the geometrical arrangements of atoms

Susceptibility tensors \Leftrightarrow structural symmetry

$$\chi_{ij}^{(1)} (\omega; \omega)$$

9 elements
Isotropic media: scalar

$$\chi_{ijk}^{(2)} (\omega; \omega_1, \omega_2)$$

27 elements
All elements are identically null
in systems with inversion symmetry

$$\chi_{ijkl}^{(3)} (\omega; \omega_1, \omega_2, \omega_3)$$

81 elements
In isotropic media: 21 nonzero
elements (only 3 are independent)

Higher order susceptibilities: more elements

Quantum approach

$$\langle P \rangle = N \text{Tr}(\rho \mu) \quad \frac{\partial \rho}{\partial t} = -\frac{i}{\hbar} [H, \rho]$$

$$H = H_o + H_{\text{int}} + H_{\text{random}}$$

$$H_{\text{int}} = -\vec{\mu} \cdot \vec{E} - \frac{e}{2} \vec{Q} \nabla \vec{E} + \vec{M} \cdot \vec{H} = \sum_i H_{\text{int}}(\omega_i)$$

Electric
dipole
coupling

Electric
quadrupole
coupling

Magnetic
dipole
coupling

$$H_{\text{int}} \propto E_i \exp(-i\omega_i t)$$

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar} [H_0 + H_{int}, \rho] + \left(\frac{\partial \rho}{\partial t} \right)_{relax}$$

$$\left(\frac{\partial \rho}{\partial t} \right)_{relax} = -\frac{i}{\hbar} [H_{random}, \rho] \quad \Rightarrow \quad \left(\frac{\partial \rho_{nm}}{\partial t} \right)_{relax} = -\Gamma_{nm} \rho_m$$

Diagonal elements \Leftrightarrow **atomic population**

Off-diagonal elements \Leftrightarrow **quantum coherences**

$$\rho = \rho^{(0)} + \rho^{(1)} + \rho^{(2)} + \dots \quad \rho^{(n)} = \sum_j \rho^{(n)}(\omega_j)$$

After solving system
of coupled equations

$$\rho^{(n)} \Rightarrow \langle P^{(n)} \rangle \Rightarrow \chi^{(n)}$$

Rabi frequency has to be smaller than the frequency detuning

When there is inversion symmetry:

$$\chi^{(j)} \equiv 0$$

j = even

$$P_L + P_{NL} = \epsilon_0 \sum_{N=0}^{\infty} \chi^{(2N+1)} E^{(2N+1)}$$

$$n_N \propto \text{Re } \chi^{(2N+1)}$$

Nonlinear refractive index

$$\alpha_N \propto \text{Im } \chi^{(2N+1)}$$

Nonlinear absorption coefficient

linear + nonlinear

$$n = n_0 + n_2 I + n_4 I^2 + n_6 I^3 + \dots$$

$$\alpha = \alpha_0 + \alpha_2 I + \alpha_4 I^2 + \alpha_6 I^3 + \dots$$

High-order nonlinearities (HON)

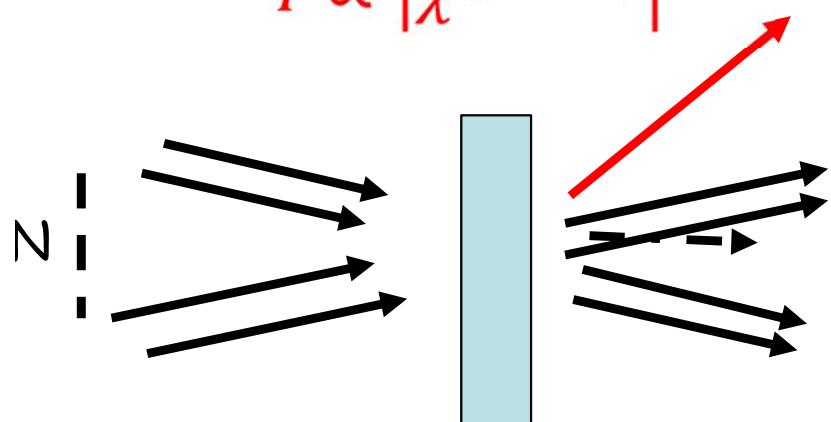
Normally HON are weaker than low-order effects

- Generation of high-order harmonics
- Multiphoton excitation processes
- Multi-wave mixing
- Stability of multidimension solitons

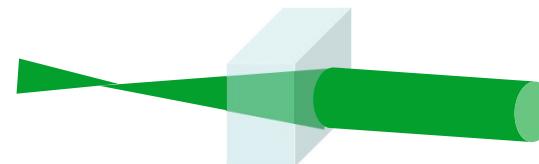
2nd lecture

Multi wave-mixing

$$I \propto |\chi^{(2N+1)}|^2$$

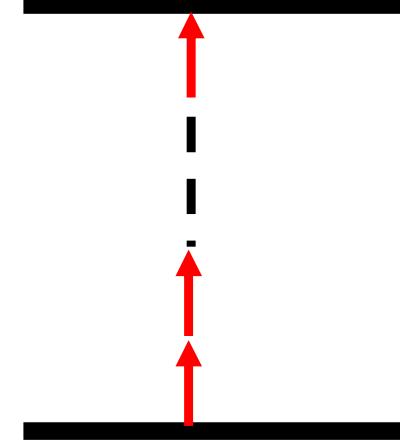


Self focusing



$$n_N \propto \operatorname{Re} \chi^{(2N+1)}$$

Multiphoton absorption



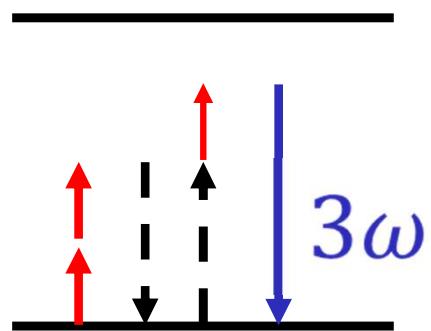
$$\alpha_N \propto \operatorname{Im} \chi^{(2N+1)}$$

Cascade processes

HON may be due to repeated low-order susceptibilities

Macroscopic cascading: involves propagation effects

$\omega + \omega$ creates 2ω , then $(2\omega + \omega)$ creates 3ω



$$\chi^{(2)} \cdot \chi^{(2)} \equiv \chi_{eff}^{(3)}$$

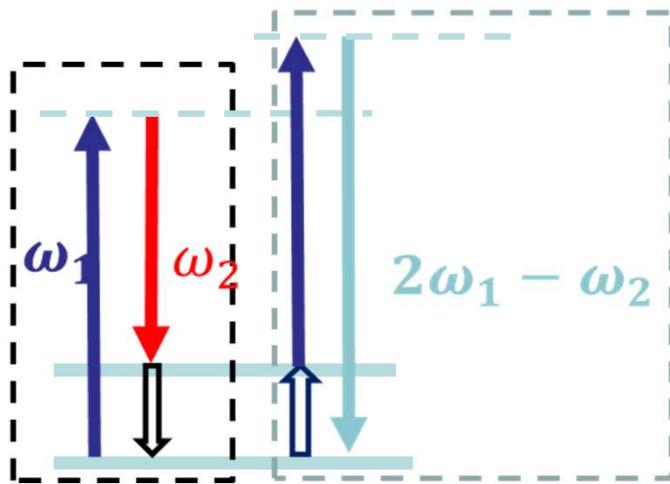
Microscopic cascading:

two neighbor atoms interact through local field effects
to create a HON process

Medium without inversion symmetry

$$\chi^{(2)} \neq 0$$

Infrared transition (polariton)



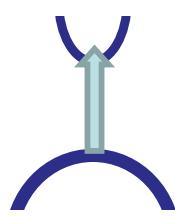
Yablonovitch, Flytzanis, Bloembergen
Phys. Rev. Lett. 29 (1972) 865

Local OR
non-local

$$E(\omega_1 - \omega_2) \propto \chi^{(2)} E(\omega_1) E^*(\omega_2)$$

$$E(2\omega_1 - \omega_2) \propto \chi^{(2)} E(\omega_1 - \omega_2) E(\omega_1) = \\ \propto \chi^{(2)} \{ \chi^{(2)} E(\omega_1) E^*(\omega_2) \} E(\omega_1)$$

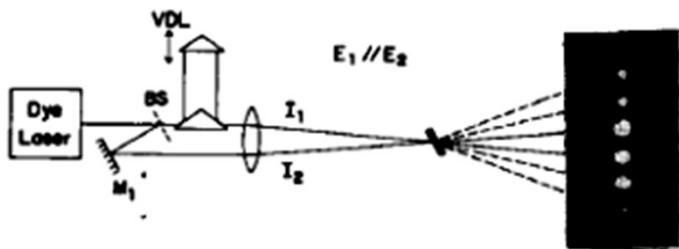
Effective susceptibility $\chi_{eff}^{(3)} = A \chi^{(2)} \cdot \chi^{(2)}$



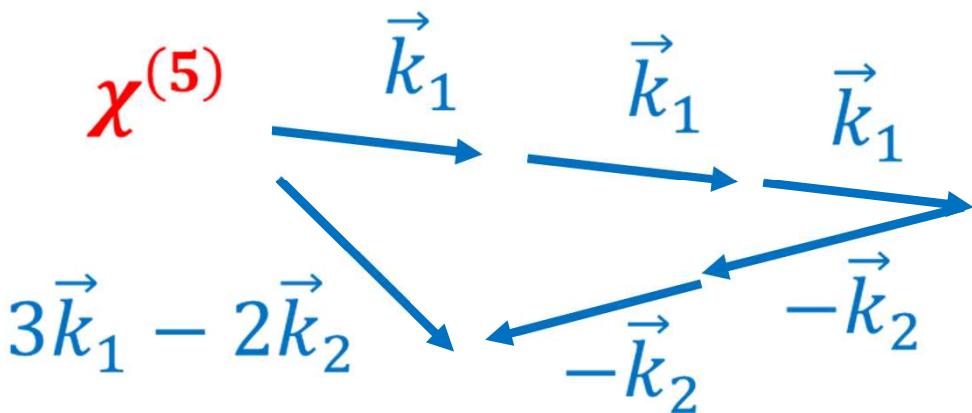
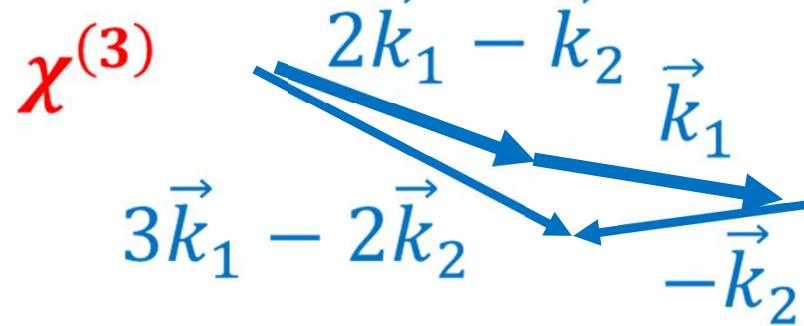
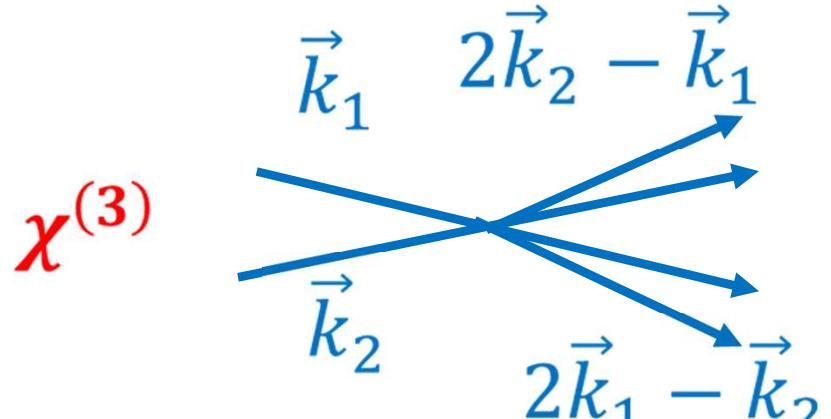
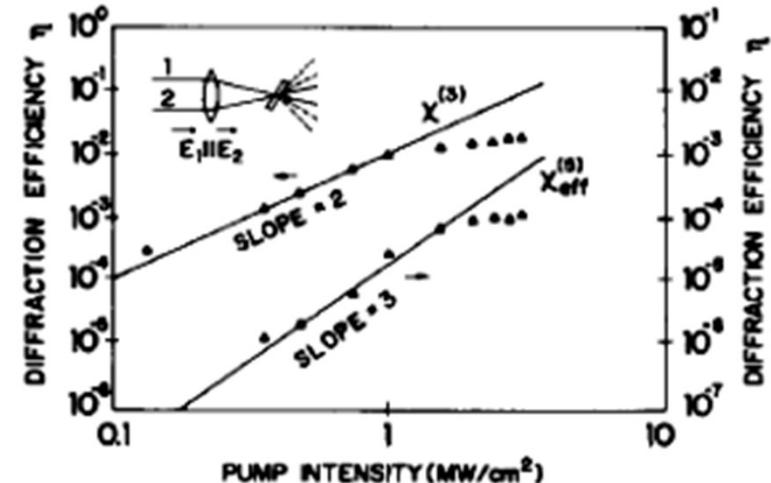
Glass with nanocrystals of $\text{CdS}_x\text{Se}_{1-x}$

Acioli et al. IEEE QE 26 (1990) 1277

$$\hbar\omega_{\text{laser}} \approx E_{\text{gap}}$$



Self-diffraction



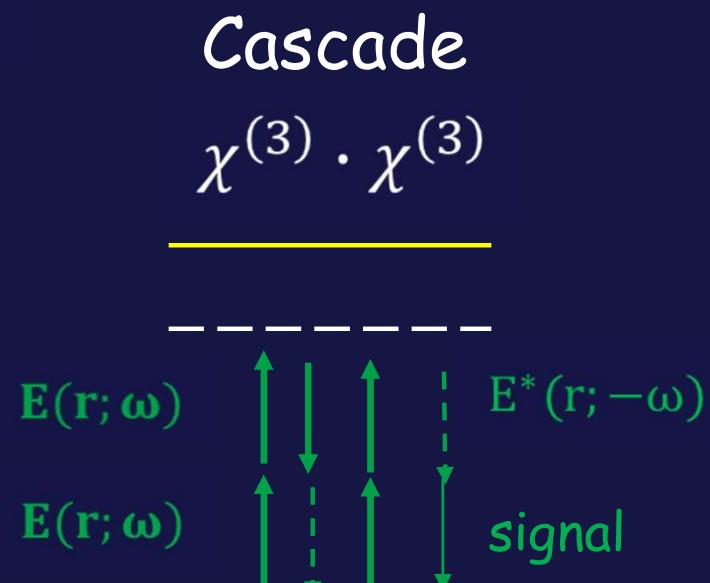
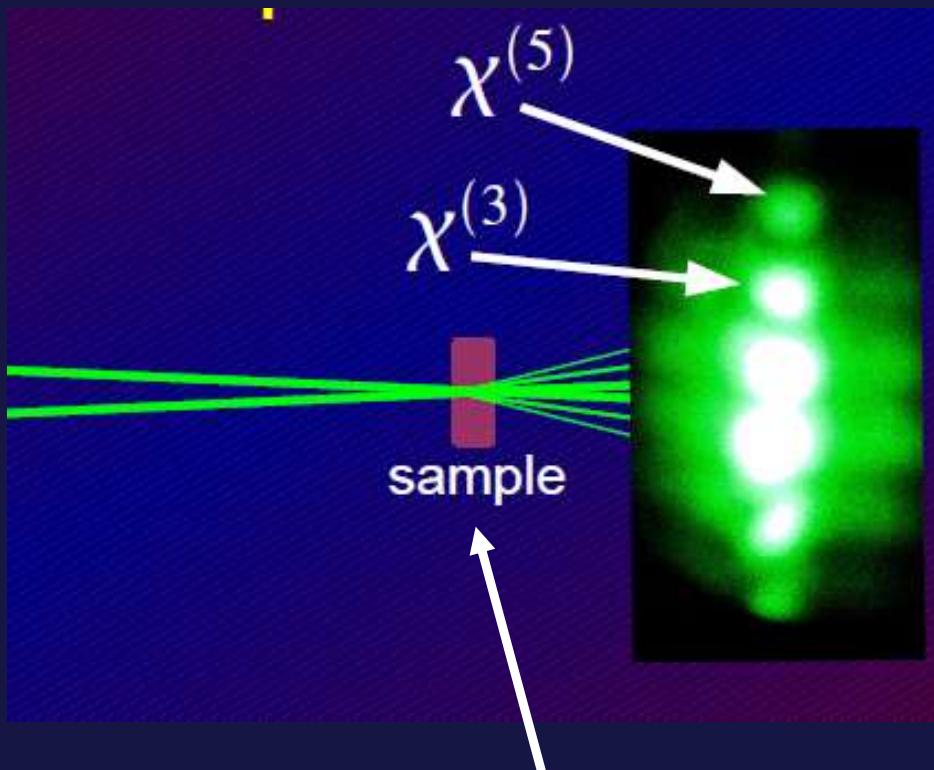
$$\chi^{(3)} \cdot \chi^{(3)} \equiv \chi^{(5)}$$

$$\boxed{\chi_{eff}^{(5)} = A\chi^{(5)} + B\chi^{(3)} \cdot \chi^{(3)}}$$

Observation of a Microscopic Cascaded Contribution to the Fifth-Order Nonlinear Susceptibility

Ksenia Dolgaleva,* Heedeuk Shin, and Robert W. Boyd

$$\chi_{eff}^{(5)} \propto A\chi^{(3)}:\chi^{(3)} + B\chi^{(5)}$$



There are other possibilities

Mixture of CS_2 and Fulerene (C_{60})

Microscopic cascading by local field effects

Third-order hyperpolarizability

$$\chi^{(3)} = N \gamma_{\text{at}}^{(3)} |L|^2 L^2.$$

Lorentz local field factor

$$L = \frac{\varepsilon^{(1)} + 2}{3}$$

Direct contribution from the fifth-order hyperpolarizability

$$\begin{aligned} \chi^{(5)} &= N \gamma_{\text{at}}^{(5)} |L|^4 L^2 \\ &+ \frac{24\pi}{10} N^2 (\gamma_{\text{at}}^{(3)})^2 |L|^4 L^3 + \frac{12\pi}{10} N^2 |\gamma_{\text{at}}^{(3)}|^2 |L|^6 L. \end{aligned}$$

Contributions by the third-order hyperpolarizability

Optical solitons

solutions of Maxwell's equation with NL terms

Wave with special shape such that propagates without change its shape.

When one soliton interacts with another soliton they do not change their shape but, the phases of the electric fields change.

Temporal solitons

Spatial solitons

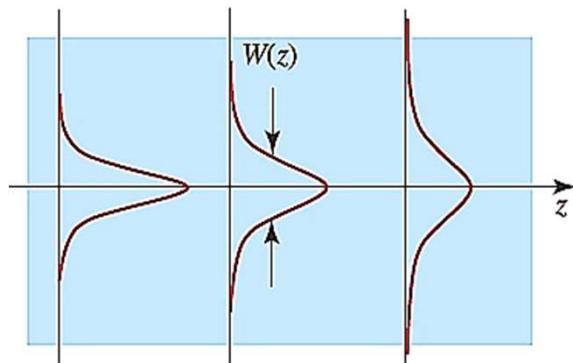
Temporal + spatial solitons ("light bullets")

Spatial Optical Solitons

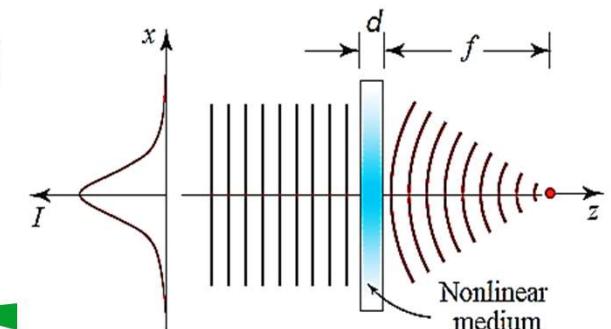
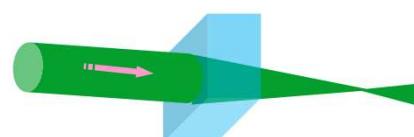
Solutions of Maxwell equations including nonlinear polarization

Example: NL Schrodinger equation

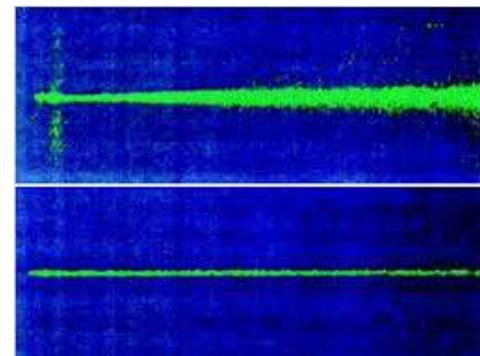
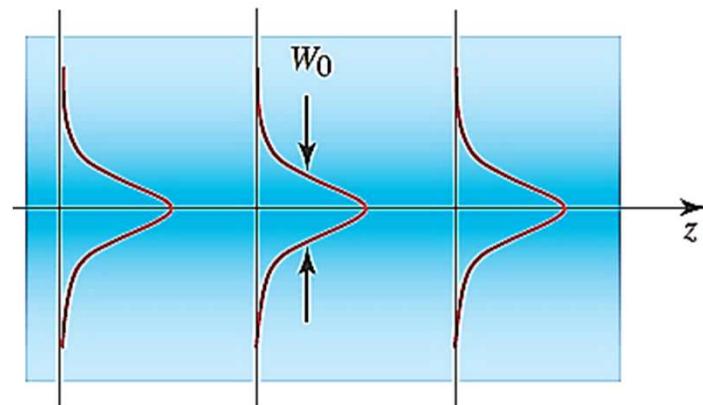
Diffraction



Self-focusing material



Bright spatial soliton



Small intensity
Large intensity
Soliton

Light propagation inside a NL medium

$$\nabla^2 \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \vec{P}_L}{\partial t^2} + \mu_0 \frac{\partial^2 \vec{P}_{NL}}{\partial t^2}$$

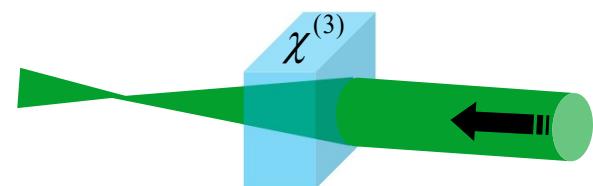
$$\vec{E}(\vec{r}, t) = \vec{A}(\vec{r}) \exp[i(kz - \omega t)]$$

third-order nonlinearity

$$i \frac{\partial \vec{A}}{\partial z} = -\frac{1}{2k} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \vec{A} - \frac{n_{2E}}{n_0} k |\vec{A}|^2 \vec{A}$$

diffraction

Self-focusing



Nonlinear Schrödinger Equation: (1+1)D

Scalar theory $E(x, z, t) = A_m a(x, z) e^{i(k_0 n z - \omega t)}$

Diffraction x self-focusing

Inside a waveguide

$$\frac{1}{2k_0 n_0} \frac{\partial^2 a}{\partial x^2} + i \frac{\partial a}{\partial z} + \frac{k_0 n_0 n_2}{2\eta_0} |a|^2 a = 0$$

$$\left| \frac{\partial^2 a(x, z)}{\partial z^2} \right| \ll \left| k_0 \cdot \frac{\partial a(x, z)}{\partial z} \right| \quad \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$a(x, z) = \text{sech} \left(\frac{x}{X_0} \right) \exp(iz/2L_d)$$

$$L_d = X_0^2 k_0 n_0$$

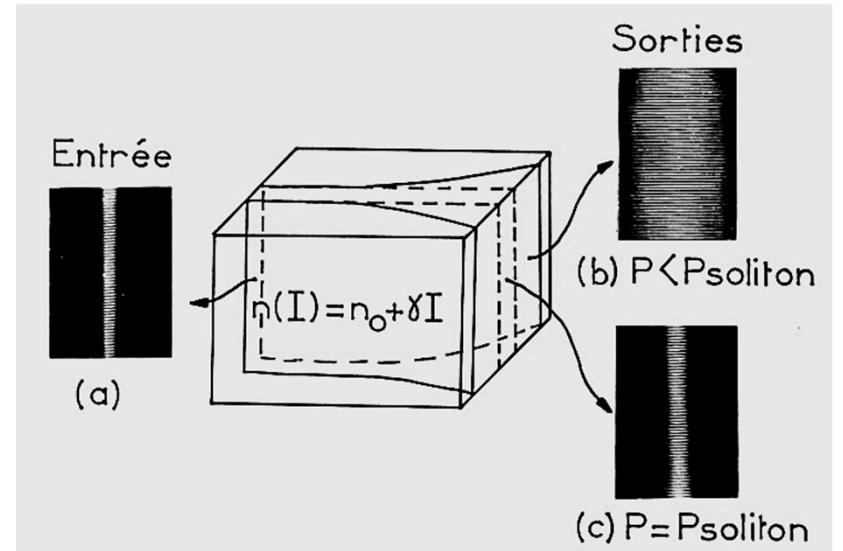
$$X_0 = \frac{1}{k_0 n_0} \sqrt{\frac{2\eta_0}{|n_2| |A_m|}}$$

(1+1)D spatial solitons in CS_2

$$n_2 > 0$$

Beam focused by cylindrical lenses

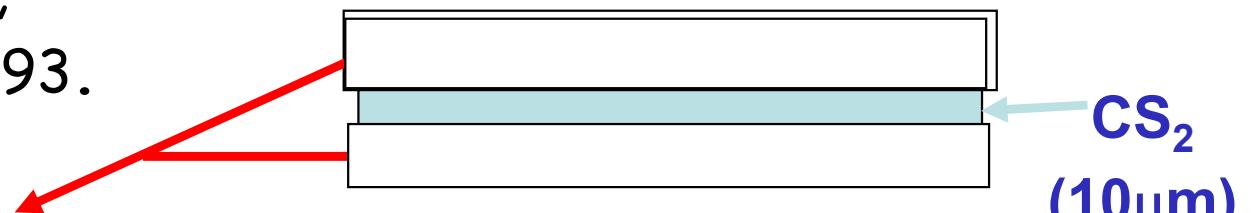
Barthélémy, Maneuf, Froehly,
Opt. Commun. 55 (1985) 201.



Planar waveguides

Maneuf, Desailly, Froehly,
Opt. Commun. 65 (1988) 193.

Quartz plates



Picosecond lasers - 532 nm

(2+1)D Soliton

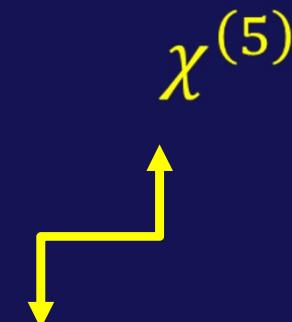
$$E(x, y, z, t) = C a(x, y, z, t) \exp[i(k_0 n z - \omega t)]$$

$$i \frac{\partial a}{\partial z} + \frac{1}{2k_0 n_0} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) a + \frac{k_0 n_0 n_2}{2\eta_0} |a|^2 a = 0$$

$n_2 > 0$
neglecting 2PA

(2+1)D soliton is unstable in a pure Kerr medium
Catastrophic self-focusing

High-order nonlinearity



$$i \frac{\partial a}{\partial z} + \frac{1}{2k_0 n_0} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) a + \frac{k_0 n_0 n_2}{2\eta_0} |a|^2 a + \left[\frac{k_0 n_0^2}{4\eta_0^2} n_4 - \frac{\alpha_3}{2} \left(\frac{n_0}{2\eta_0} \right)^2 \right] |a|^4 a = 0$$

Stable solution if $n_2 > 0$ and $n_4 < 0$

This problem was known
since the years 60s but
no homogeneous and isotropic
material was identified up until recently.

First demonstration of (2+1)D soliton propagating in a homogeneous medium with local electronic nonlinearity

PRL 110, 013901 (2013)

PHYSICAL REVIEW LETTERS

week ending
4 JANUARY 2013

Robust Two-Dimensional Spatial Solitons in Liquid Carbon Disulfide

Edilson L. Falcão-Filho* and Cid B. de Araújo

Departamento de Física, Universidade Federal de Pernambuco, 50670-901 Recife, Pernambuco, Brazil

Georges Boudebs, Hervé Leblond, and Vladimir Skarka

LUNAM Université, Université d'Angers, Laboratoire de Photonique d'Angers, EA 4464, 49045 Angers, France

Important: contributions of third
and fifth order of opposite signs

Carbon disulfide: CS_2

VOLUME 74, NUMBER 15

PHYSICAL REVIEW LETTERS

10 APRIL 1995

Fifth Order Optical Response of Liquid CS_2 Observed by Ultrafast Nonresonant Six-Wave Mixing

Keisuke Tominaga and Keitaro Yoshihara

Results in the femtosecond regime:

n_2 : Couris et al., Chem. Phys. Lett. 369, 318 (2003).

n_4 : Kong et al., J. Phys. B: At. Mol. Phys. 42, 065401 (2009).

$$n_2 = +3.1 \times 10^{-19} \text{ m}^2/\text{W}$$



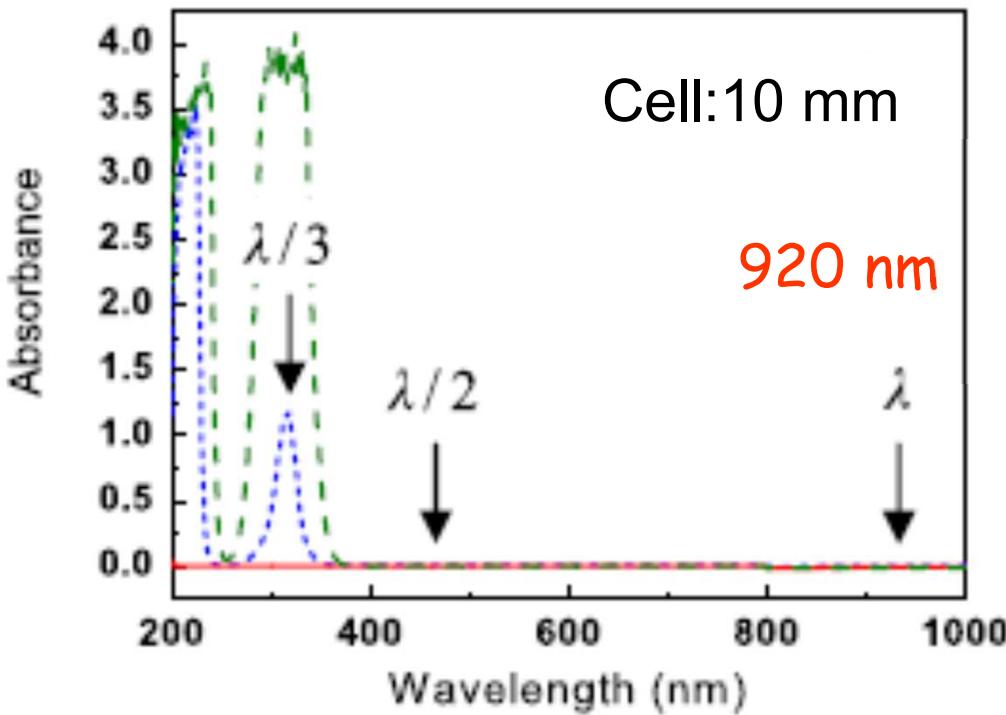
Focusing nonlinearity

$$n_4 = -2.0 \times 10^{-35} \text{ m}^4/\text{W}^2$$



Defocusing nonlinearity

Absorbance spectrum of CS_2 diluted in ethanol



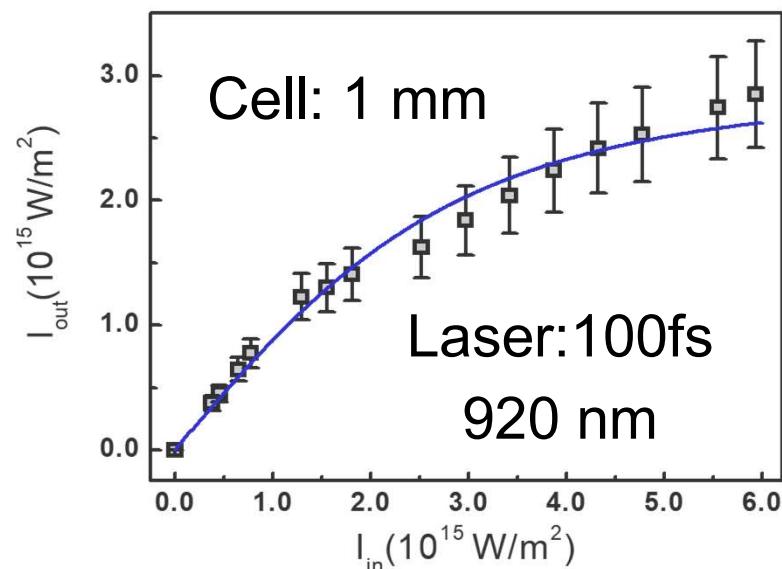
Three photon absorption
of pure CS_2

$$n_2 > 0 \quad n_4 < 0$$

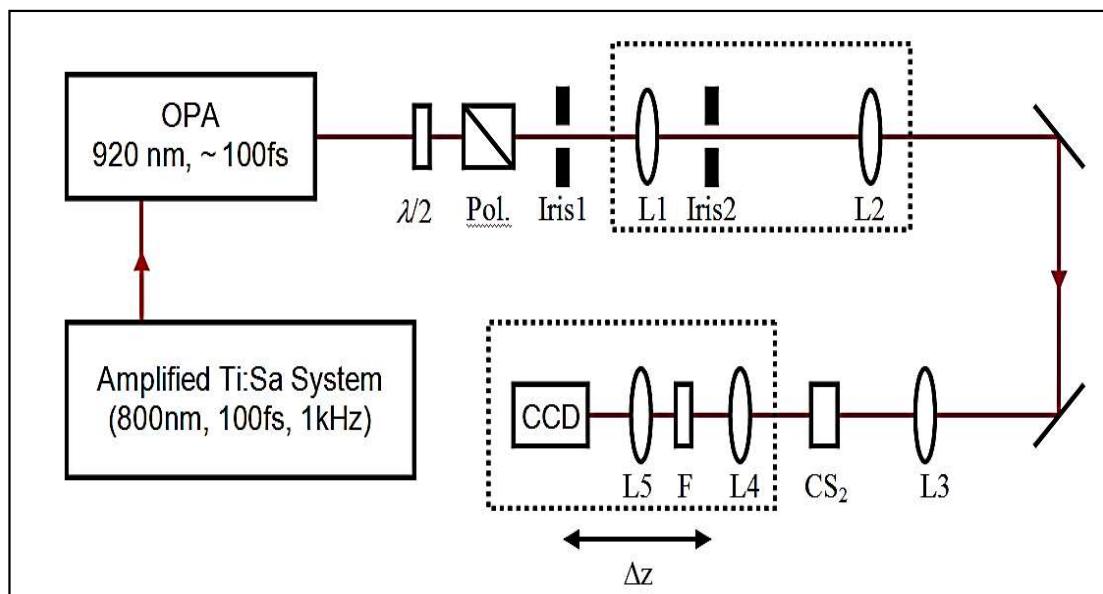
Optical limiting experiment

$$I_L = (1 - R)^2 \frac{I_0}{\sqrt{1 + 2I_0^2(1 - R)^2\alpha_4 L}}$$

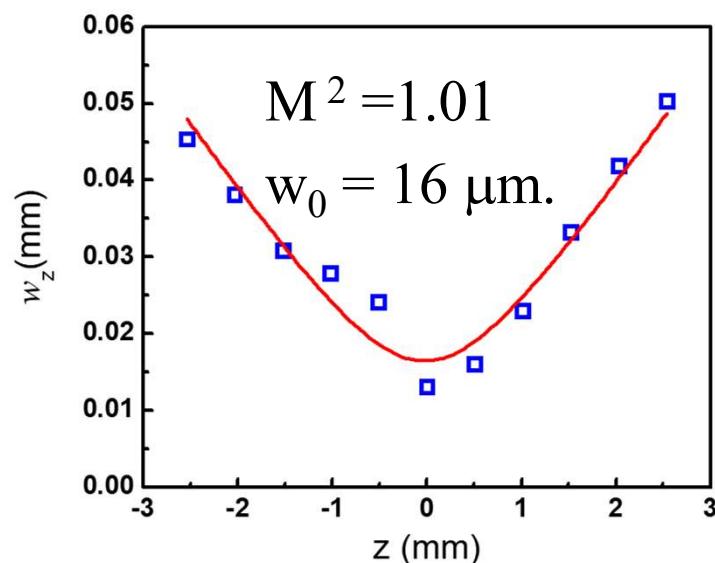
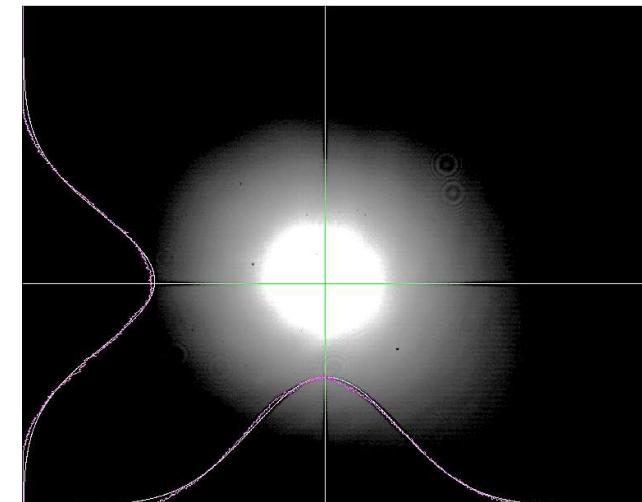
$$\alpha_4 = 5.8 \times 10^{-29} \text{ m}^3/\text{W}^2$$



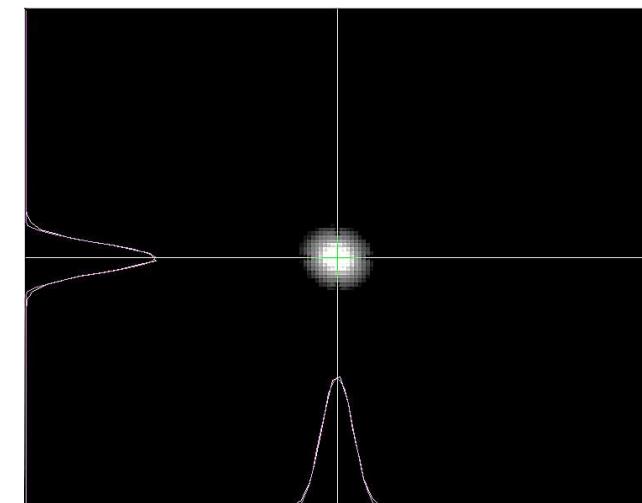
experimental setup



Beam waist on L3: $w = 2.0 \text{ mm}$

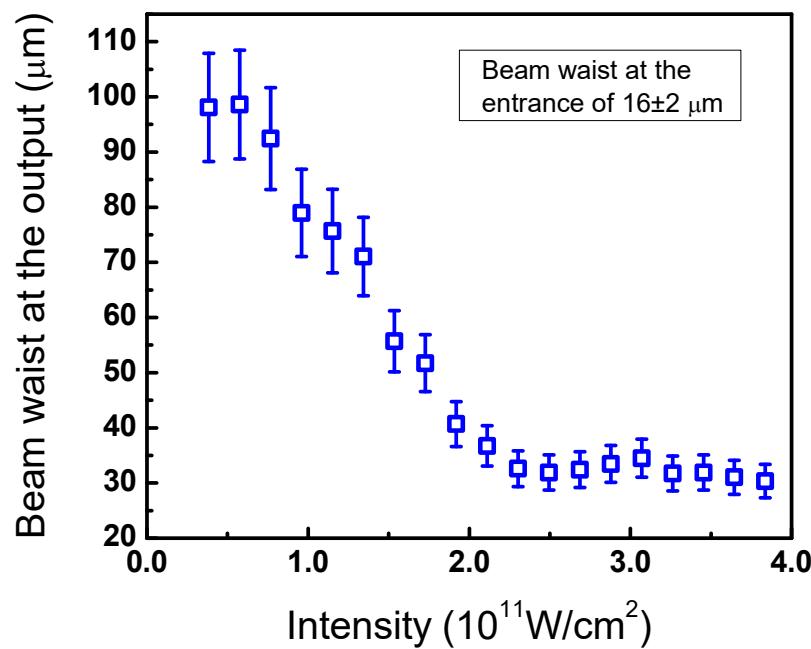


Focal region: $w_0 = 16 \mu\text{m}$

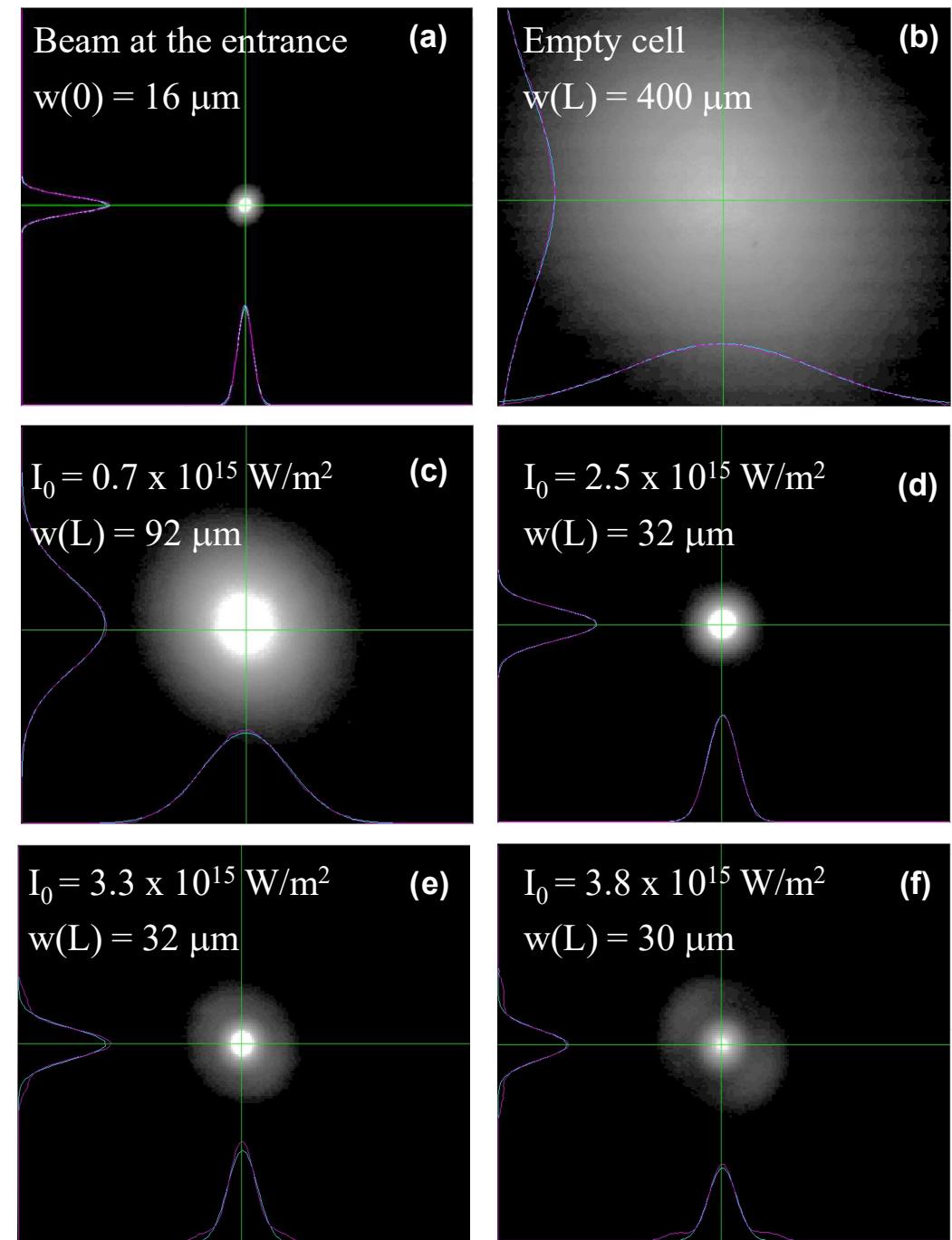


cell length: 1.0 cm

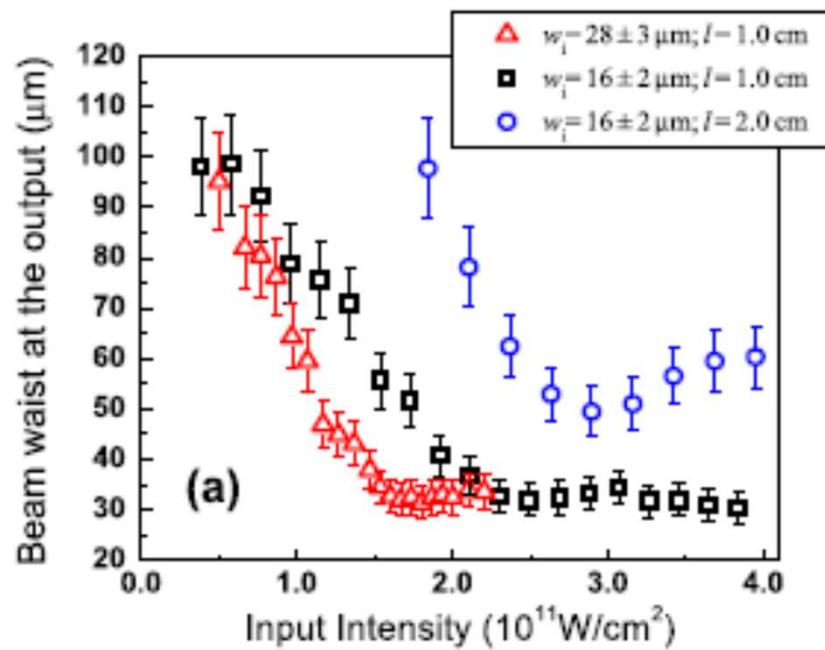
Beam focused on
the entrance face



1 cm $\rightarrow 10z_0$

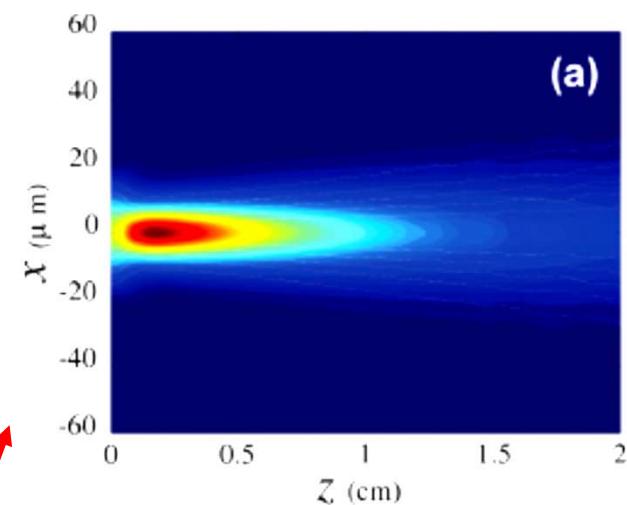


Beam waist versus laser intensity

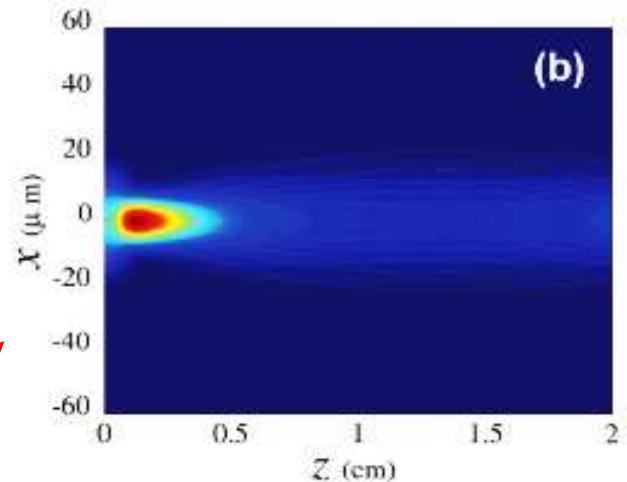


Cells: 1 and 2 mm

NLS equation



$$0.8 \times 10^{11} \text{ W/cm}^2$$



$$1.6 \times 10^{11} \text{ W/cm}^2$$

Summary - bright spatial solitons

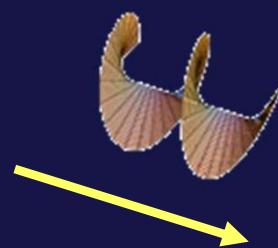
(2+1)D spatial soliton propagation over more than $10z_0$ in CS_2 due to simultaneous contribution of the third- and the fifth-order susceptibilities.

- Intensity clamping effect which corroborates the soliton stability

Computer simulations with the NLSE. Results in agreement with the experimental data.

Optical vortices

- Beams with phase singularity
- Zero field in the center of the vortex
- Helical wavefront
- Phase



$$\phi(t, z, \theta) = kz + \omega t + m\theta$$

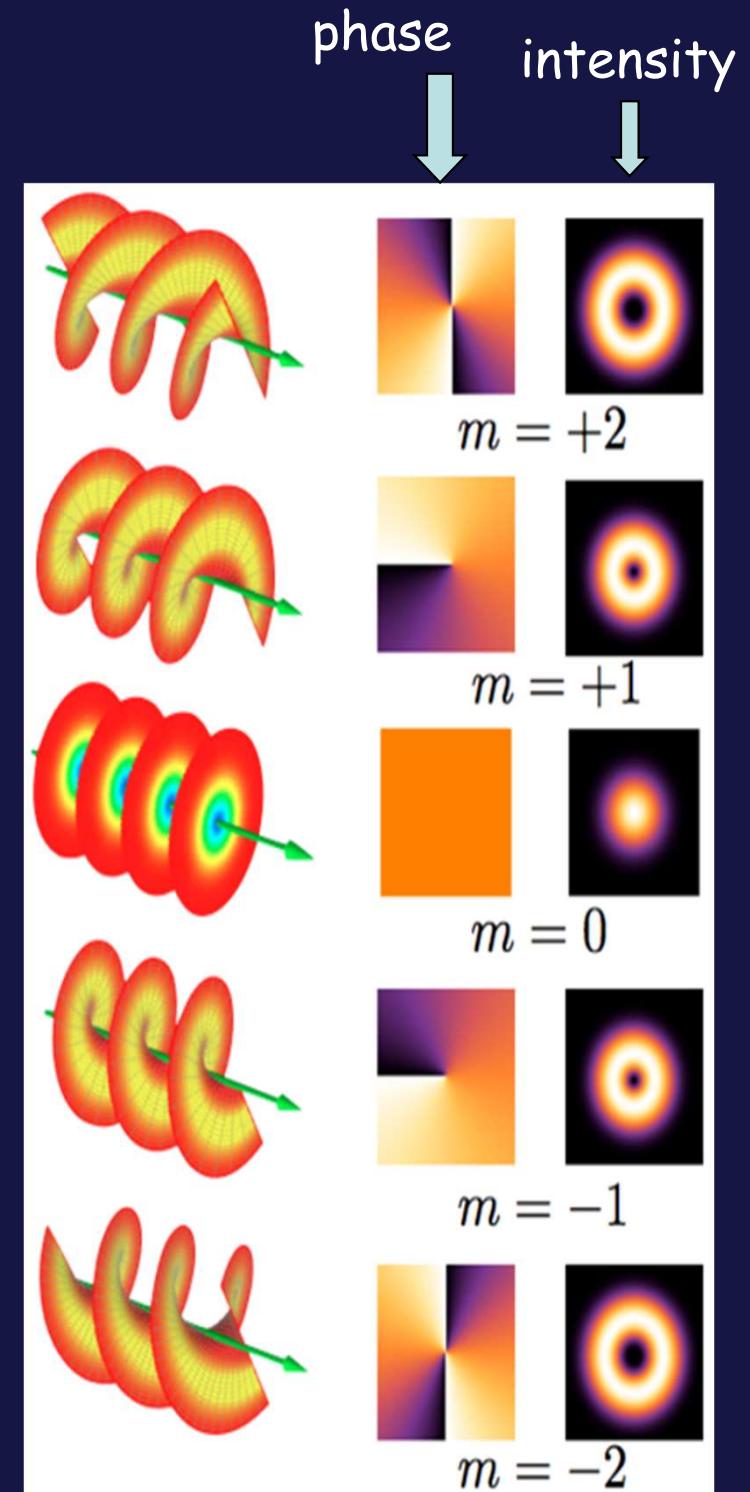
$m = 0$

Plane wave

$m \neq 0$

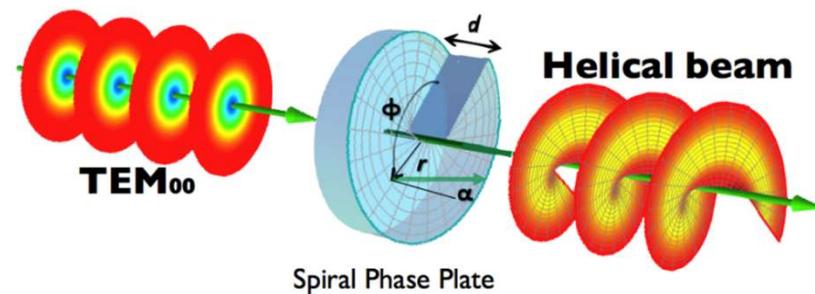
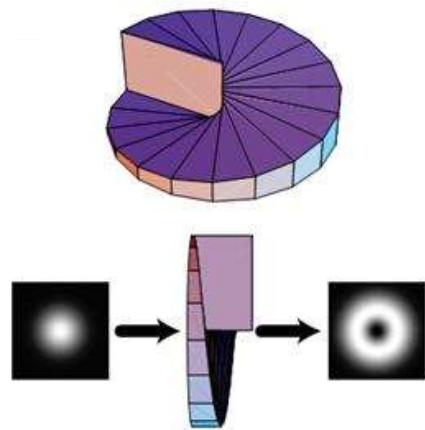
Wave with topological phase

m is the "topological charge"



Optical Vortex beam carries an orbital angular momentum of $m\hbar$ per photon

Vortex phase plate



Images
from the
internet

Spatial light
modulator



Optical Vortex Soliton - OVS

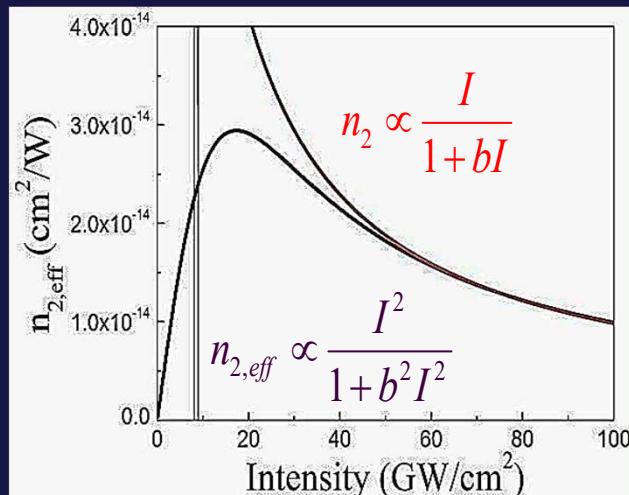
Stable propagation in a self-defocusing medium

Unstable propagation in self-focusing media

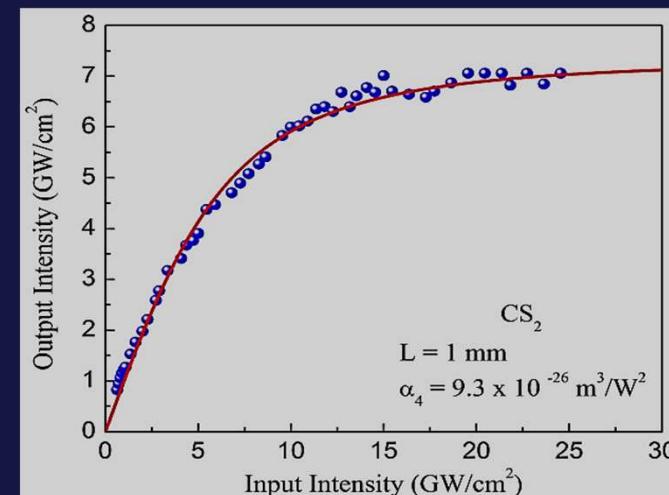
Conditions on the NLSE to observe stable OVS in a self-focusing medium?

Saturable NL refractive index and NL absorption

CS_2
532 nm
psec



Effective NL refractive index



NL transmittance

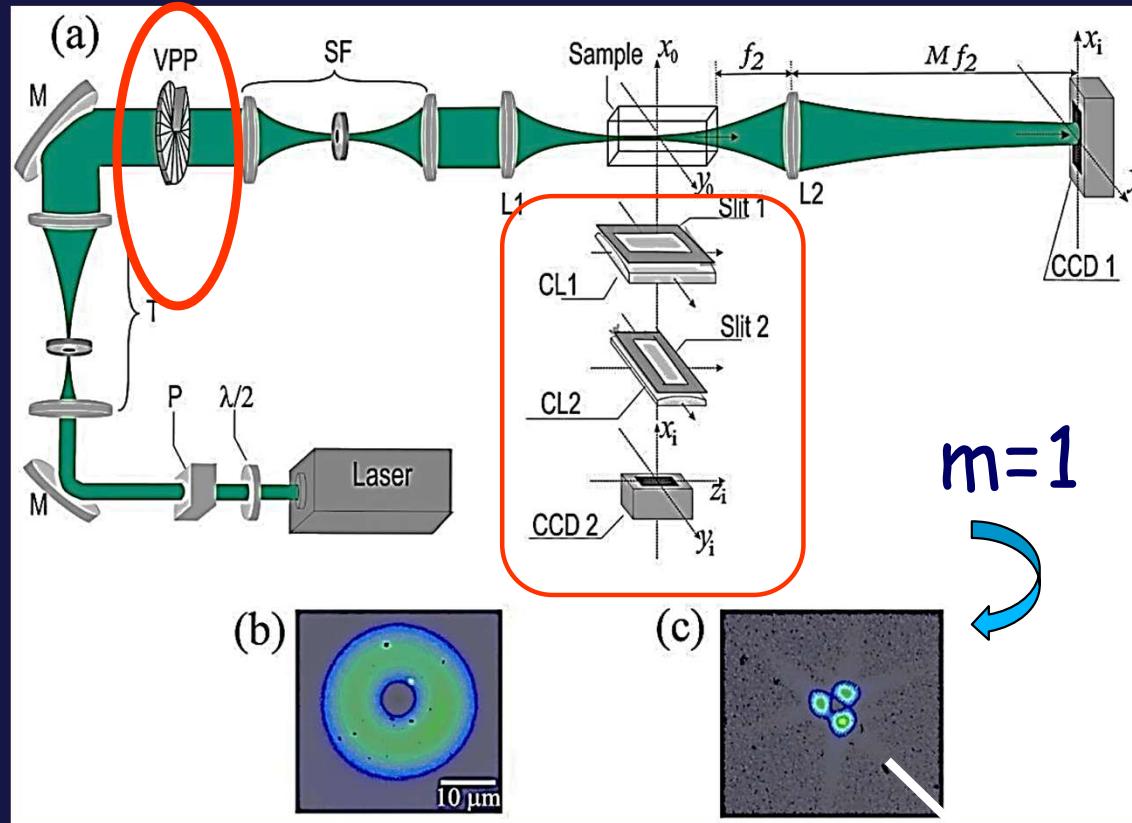
PHYSICAL REVIEW A 93, 013840 (2016)

Robust self-trapping of vortex beams in a saturable optical medium

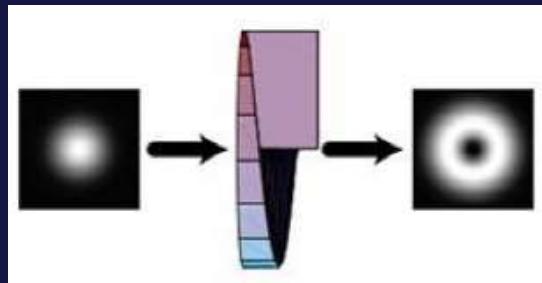
Albert S. Reyna,^{1,*} Georges Boudebs,² Boris A. Malomed,^{1,†} and Cid B. de Araújo¹

Propagation of OVS in CS_2

532 nm
80 ps
10 Hz



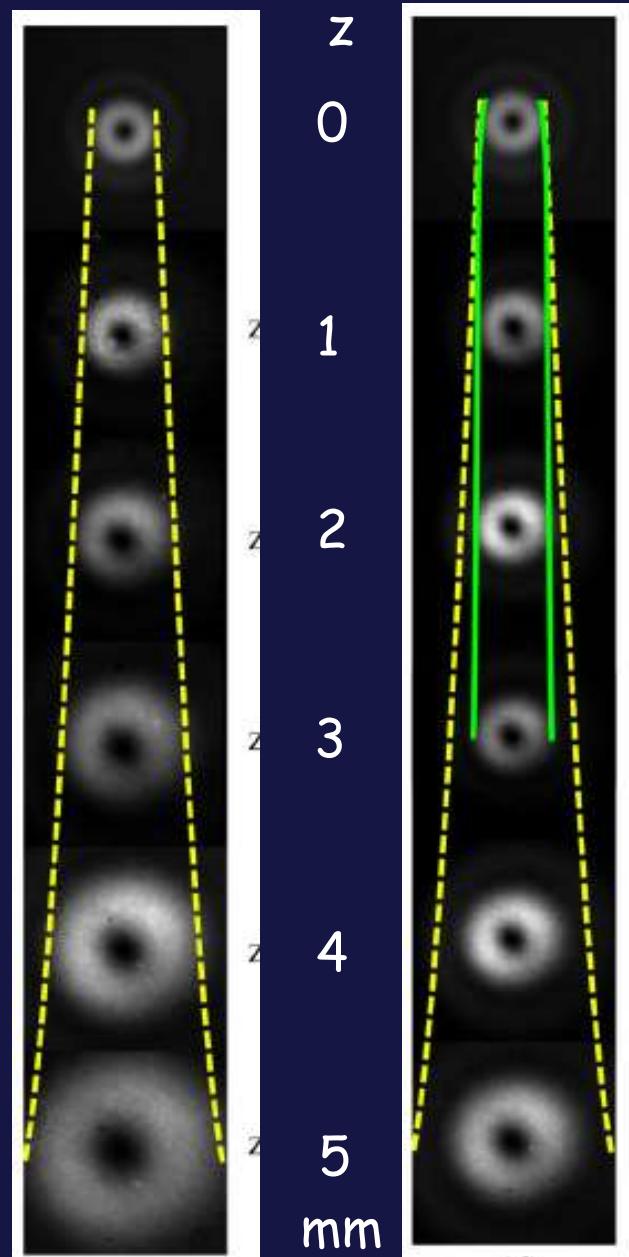
VPP- vortex phase plate



Optical Vortex beam
carries an orbital angular
momentum of $m\hbar$ per
photon

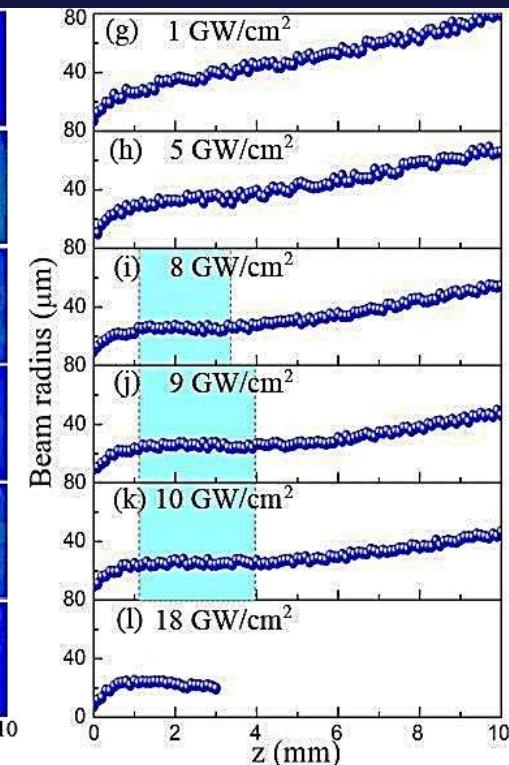
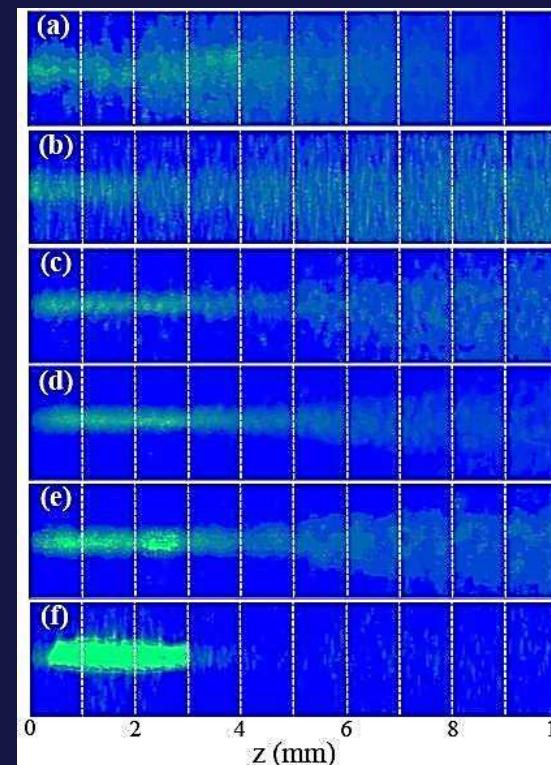
Hickmann et al.
PRL 105 (2010) 053904

Optical Vortex Solitons in CS_2



1.0 GW/ cm²

9.0 GW/ cm² homogenous medium with local nonlinearity

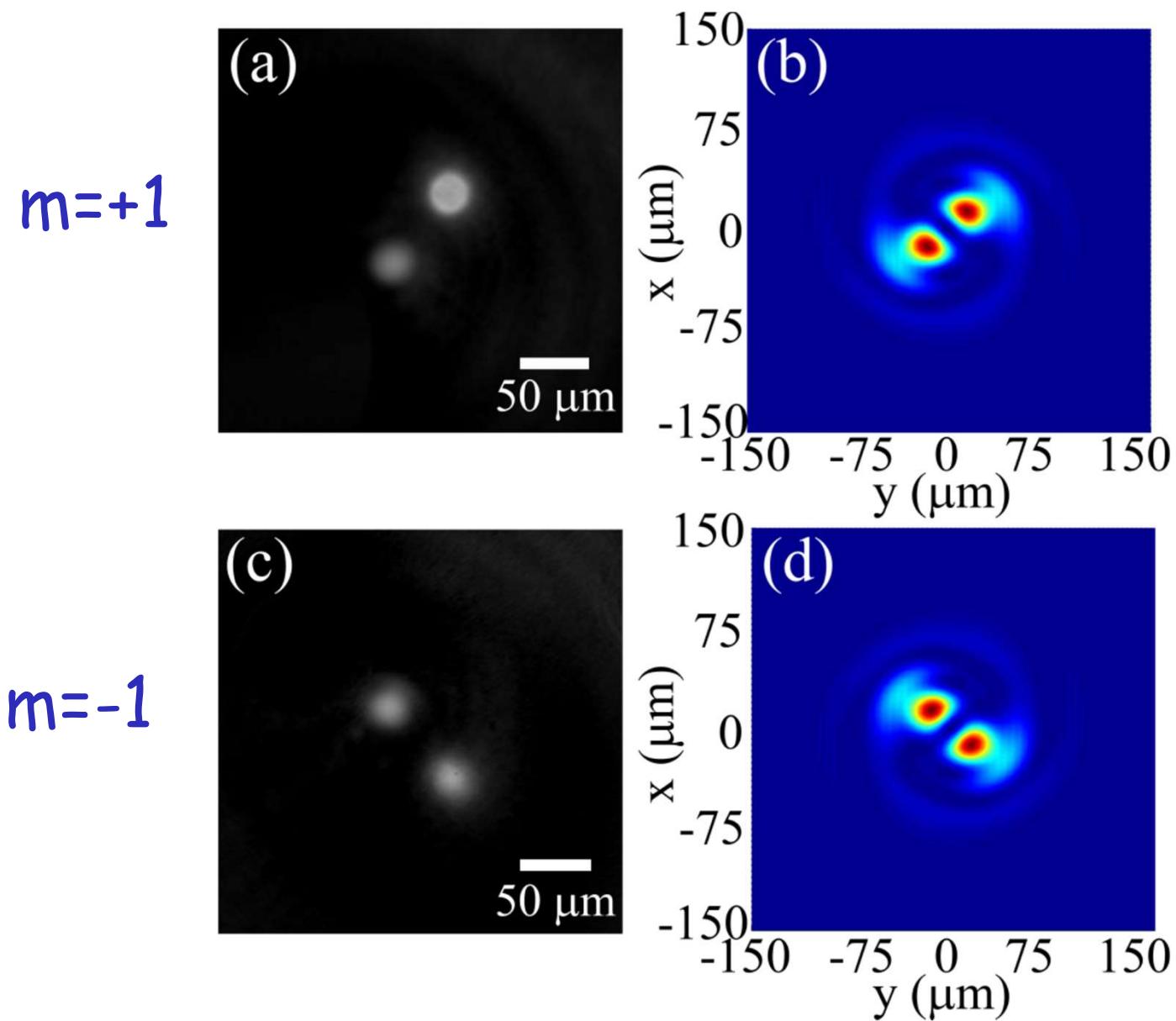


$$i \frac{\partial E}{\partial z} = -\frac{1}{2n_0 k} \nabla_{\perp} E - \left[\frac{k a I^2}{1 + b^2 I^2} + i \frac{\gamma I^2}{2} \right] E$$

First observation of an OVS in a

homogenous medium with local nonlinearity

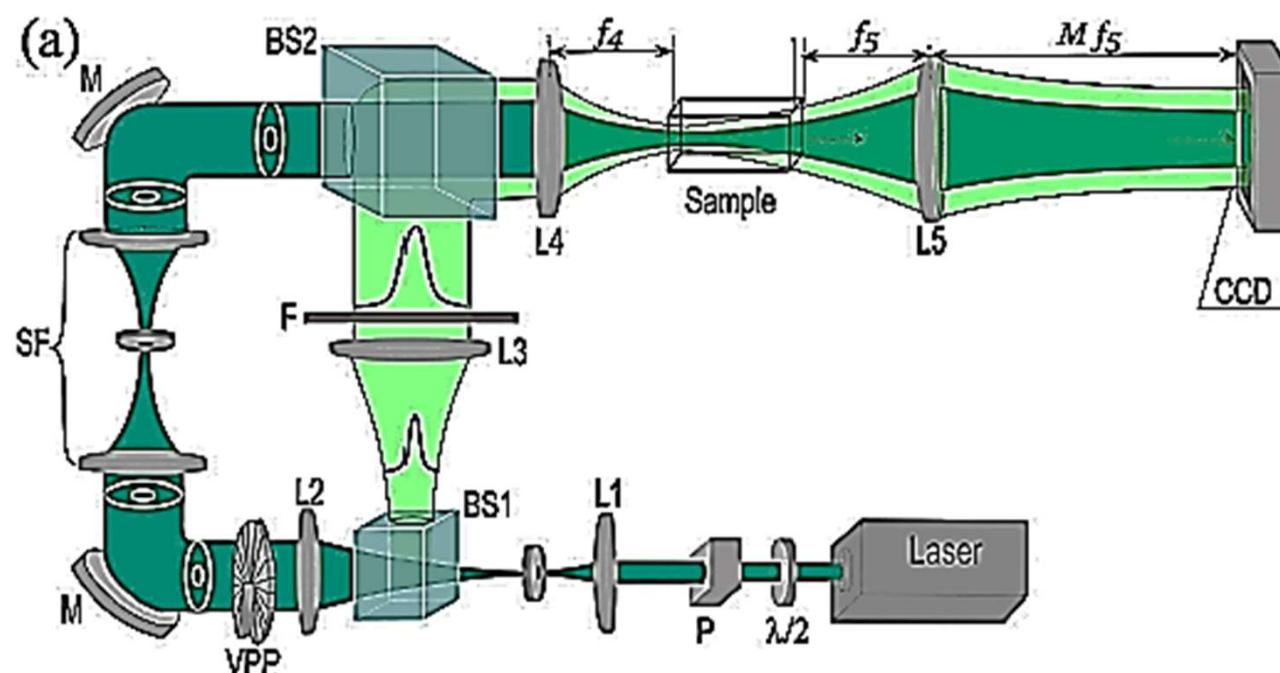
Image at the output of cell after fragmentation



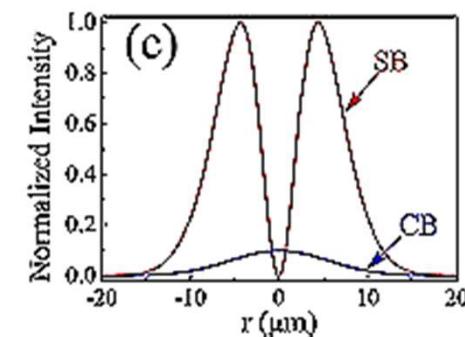
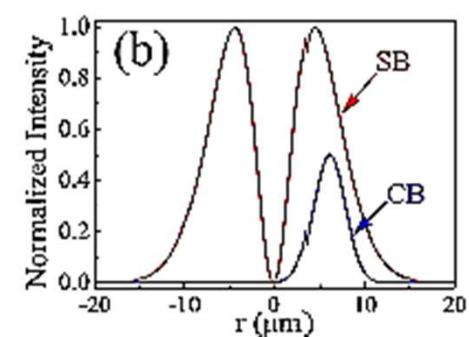
Experiment: 18 GW/cm^2 Simulation 15 GW/cm^2

Taming the emerging beams after the split of optical vortex solitons in a saturable medium

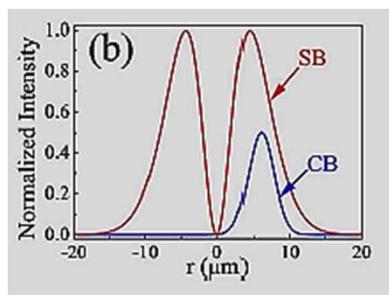
Albert S. Reyna* and Cid B. de Araújo



532 nm, 80 ps, 10Hz

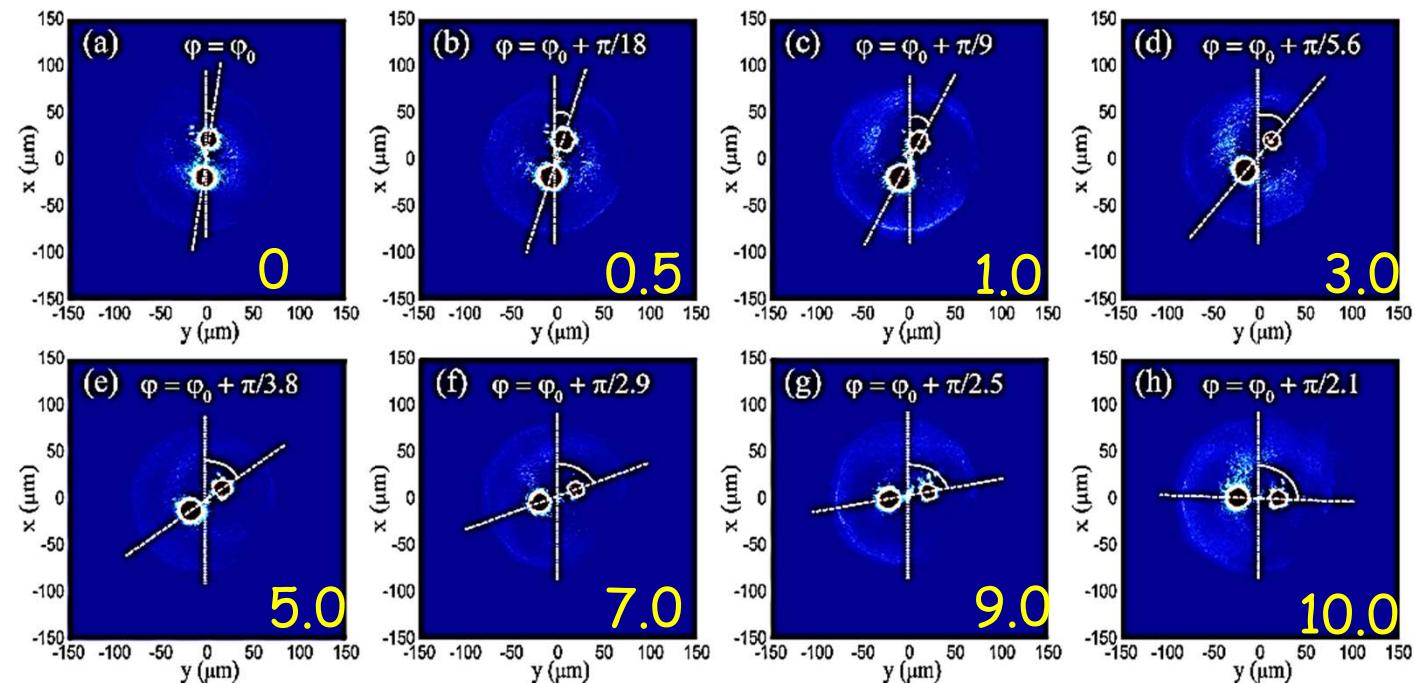


cell length: 10 mm

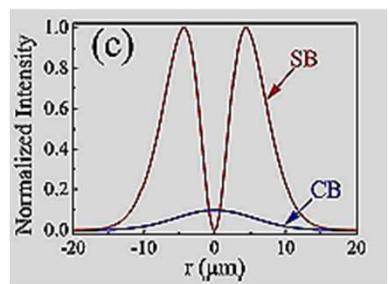


Vortex beam
18 GW/cm 2

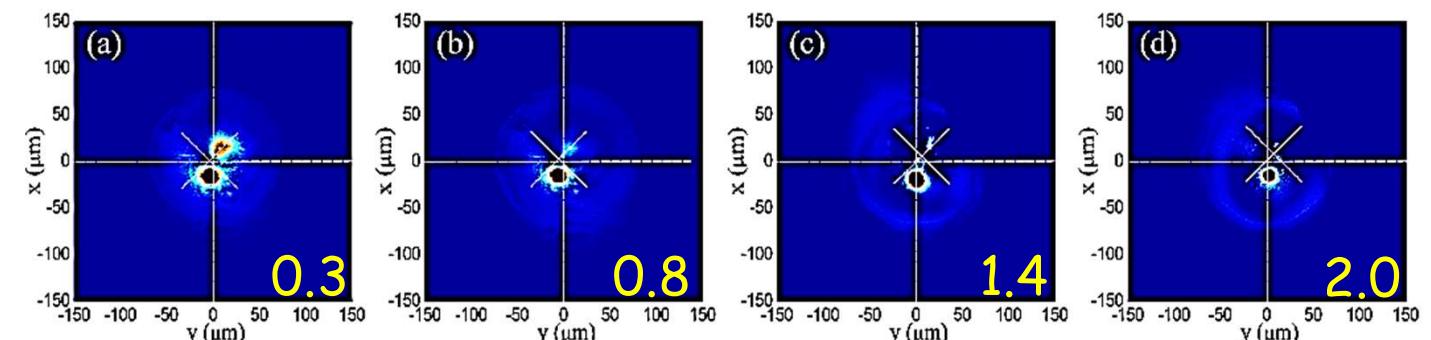
Control beam
Gaussian
GW/cm 2



Control beam less intense than the signal beam



Control beam
0.3-2.0 GW/cm 2



Control of energy transfer

Final comments

Exploitation of high-order electronic nonlinearities allows::

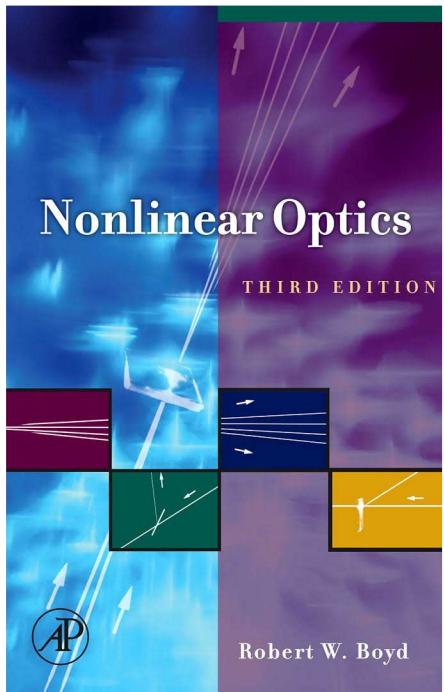
observation of (2+1)D bright spatial solitons in a homogeneous NL medium with electronic nonlinearity.

observation of stable propagation of optical vortex solitons (OVS) in a medium with saturable refractive index and presenting NL absorption.

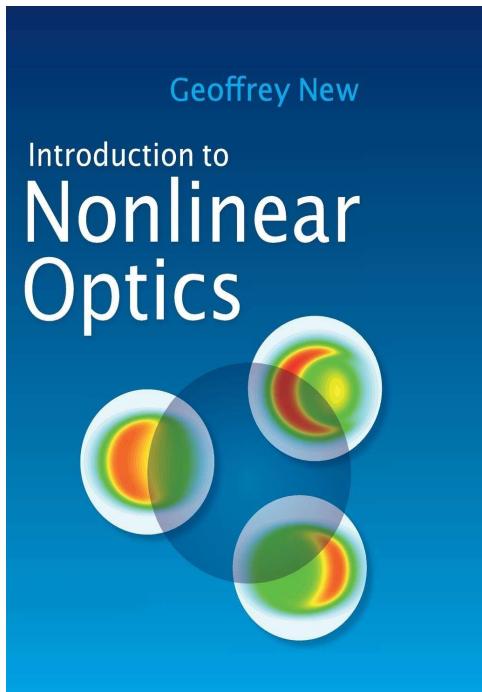
controlling the instability of an OVS by using a control beam with smaller intensity than the OVS.

Optical systems offer a simple ground for studies of fundamental effects in physics of nonlinear waves and different types of nonlinear phenomena

In the next two lectures I will present examples to illustrate these statements in Plasmonic systems and Random Lasers



R. W. Boyd



G. New

Christodoulides, Khoo, Salamo,
Stegeman, van Stryland
Nonlinear refraction and
absorption: mechanisms and
magnitudes,
Adv. Opt. Photon. **2** (2010) 60-200

Reyna, de Araújo
High-order optical nonlinearities
in plasmonic nanocomposites - a review
Adv. Opt. Photon. **9** (2017) 720-774

IOP Publishing

Rep. Prog. Phys. **79** (2016) 036401 (30pp)

Techniques for nonlinear optical characterization of materials: a review

Cid B de Araújo¹, Anderson S L Gomes¹ and Georges Boudebs²

Reports on Progress in Physics

[doi:10.1088/0034-4885/79/3/036401](https://doi.org/10.1088/0034-4885/79/3/036401)

2nd lecture - tomorrow

Nonlinear Photonics in Plasmonic media

Thank you for your attention

Our work has been supported by the Brazilian agencies

