# Basic notation and properties

Notation:

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| X | A random variable |
| µ or µ(X) or EX | The mean of X (average value) |
| σ or σ(X) | The standard deviation = square-root of mean squared error = sqrt(µ((X - µ(X))2)) |
| σ2 or σ2(X) or V(X) | The variance of X = standard deviation squared |
| C(X, Y) | Covariance of X and Y = 1/2(µ(X - µ(X) µ(Y - µ(Y)) |
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Simple properties that follow from the definition:

1. E(aX + bY) = aE(X) + bE(Y)
2. C(X,Y) = E[(X−EX)(Y−EY)]=E[XY]−(EX)(EY)
3. C(X,X) = V(X)
4. C(X,Y) = 0 if X and Y are independent
5. V(X+Y) = V(X) + V(Y) + 2C(X, Y)
6. V(X) = E(X2) – E(X)2
7. V(aX) = a2V(X)

# Basic theorems

[Chebyshev's inequality](https://en.wikipedia.org/wiki/Chebyshev%27s_inequality): The probability that a sample from X is within k standard deviations is < 1/k2

**k % within k standard deviations of mean**

1 0%

2 75%

3 88.8889%

4 93.75%

5 96%

Vs Normal distribution of 66, 95, 99…

[Law of large numbers](https://en.wikipedia.org/wiki/Law_of_large_numbers): Let Xn = ΣX/n (average of n samples of X). Then lim n->ꝏ Xn = EX of X is finite variance.

Proof: V(Xn) = V(ΣX/n) = 1/n2V(ΣX) = 1/n2(n\*V(X)) = V(X)/n. Per Chebyshev’s inequality, the percent of values greater then any distance d will approach zero.