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NON LINEAR CONTROL
Collaborative Laboratory

097469 – NonLinear Control Course – A.Y. 2023/2024 – Semester I

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DIPARTIMENTO DI ELETTRONICA
INFORMAZIONE E BIOINGEGNERIA

NonLinear Control – Collaborative Laboratory

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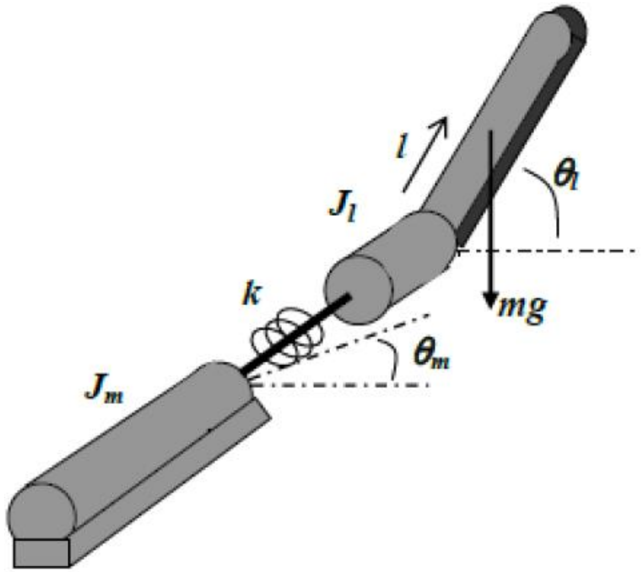
Introduction

Model Dynamical Equations :

$$J_l\ddot{\theta}_l + B_l\dot{\theta}_l + k(\theta_l + \theta_m) + mgl\cos(\theta_l) = 0$$
$$J_m\ddot{\theta}_m + B_m\dot{\theta}_m \ - k\ (\theta_l \ - \theta_m) = u$$



Parameters :

k	J_l	J_m	B_l	B_m	m	l
0.8 Nm/rad	4e-4 Nms²/rad	4e-4 Nms²/rad	0 Nms/rad	0 .015Nms/rad	0.3 kg	0.3 m

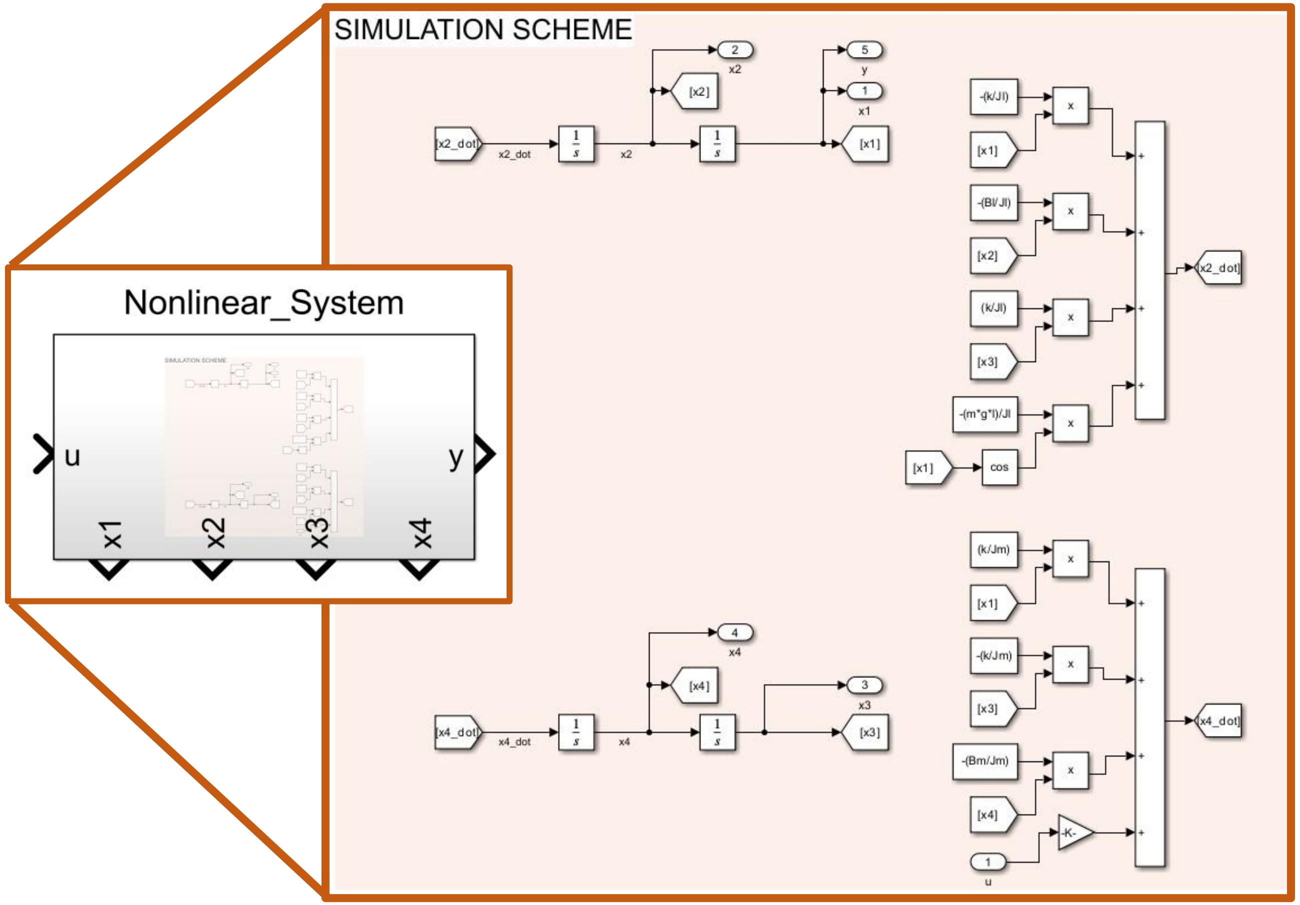
Request 1 : Give a state space representation of the system and provide a related simulation scheme

State-Space Representation:

$$J_l\ddot{\theta}_l + B_l\dot{\theta}_l + k(\theta_l - \theta_m) + mgl\,cos(\theta_l) = 0$$
$$J_m\ddot{\theta}_m + B_m\dot{\theta}_m - k\,(\theta_l - \theta_m) = u$$

$$\boldsymbol{x_1 = \theta_l\ ;\ x_2 = \dot{\theta}_l\ ;\ x_3 = \theta_m\ ;\ x_4 = \dot{\theta}_m}$$

$$\dot{\boldsymbol{x}}_{\boldsymbol{1}} = x_2$$
$$\dot{\boldsymbol{x}}_{\boldsymbol{2}} = -\frac{B_l}{J_l}x_2 - \frac{k}{J_l}x_1 + \frac{k}{J_l}x_3 - \frac{mgl}{J_l}\cos x_1$$
$$\dot{\boldsymbol{x}}_{\boldsymbol{3}} = x_4$$
$$\dot{\boldsymbol{x}}_{\boldsymbol{4}} = -\frac{B_m}{J_m}x_4 + \frac{k}{J_m}x_1 - \frac{k}{J_m}x_3 + \frac{1}{J_m}u$$
$$\boldsymbol{y} = x_1$$



Linear Tangent Approximation

Linearized Model around the Equilibrium

Request 2 : Give Linear Tangent Approximation of the System at the point corresponding to $\theta_l = \frac{\pi}{4}$ and propose a stabilizing feedback of this basis

Linear Tangent Approximation :

$$\dot{x} = A\,x + B\,u$$
$$y = C\,x$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{J_l} & -\frac{\beta}{J_l} & \frac{k}{J_l} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{J_m} & 0 & -\frac{k}{J_m} & -\frac{\beta_m}{J_m} \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_m} \end{bmatrix}$$

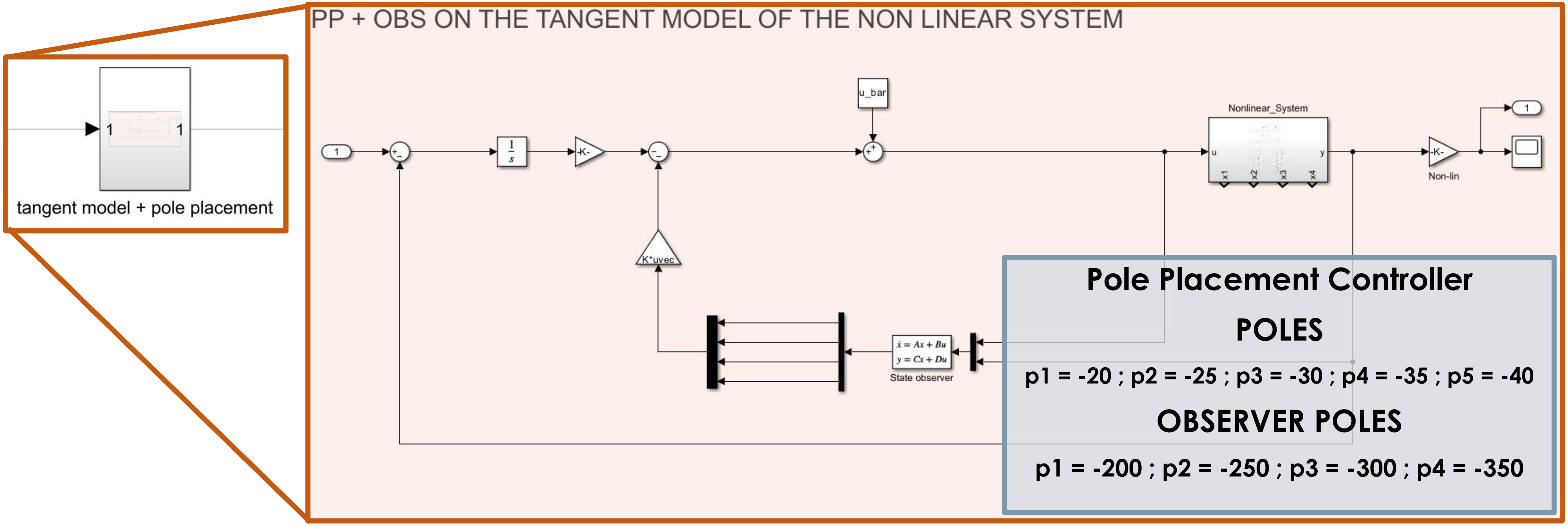
$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

$$D = 0$$

Linear Tangent Approximation

Stabilizing Control Law

As stabilizing control law we used a Pole Placement controller where we estimated the state through a Luenberger estimator and we enlarged the system with an integrator to track a reference on θ_l



Pole Placement Controller

POLES

p1 = -20 ; p2 = -25 ; p3 = -30 ; p4 = -35 ; p5 = -40

OBSERVER POLES

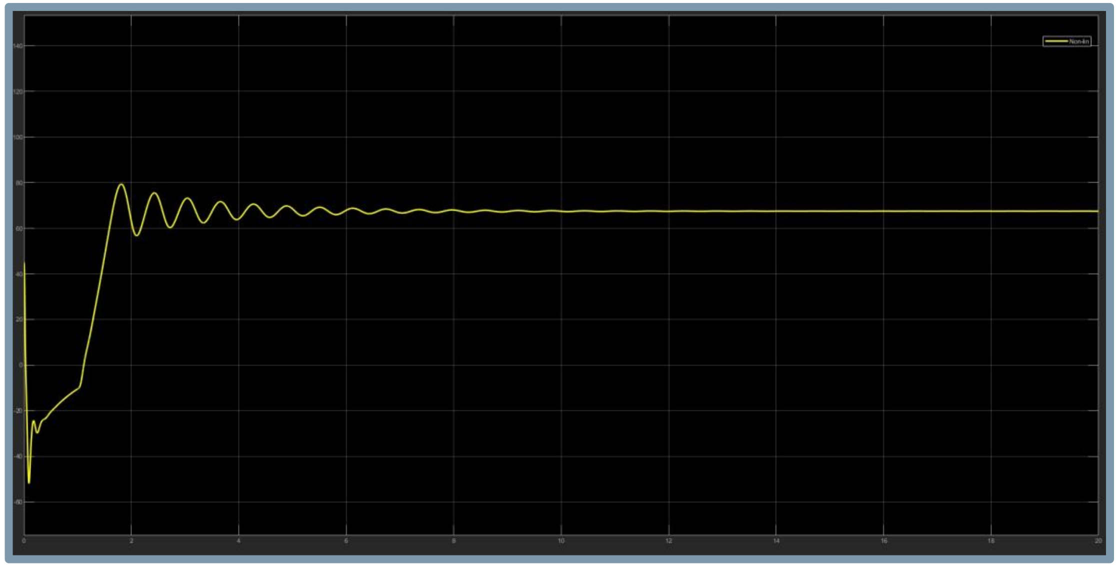
p1 = -200 ; p2 = -250 ; p3 = -300 ; p4 = -350

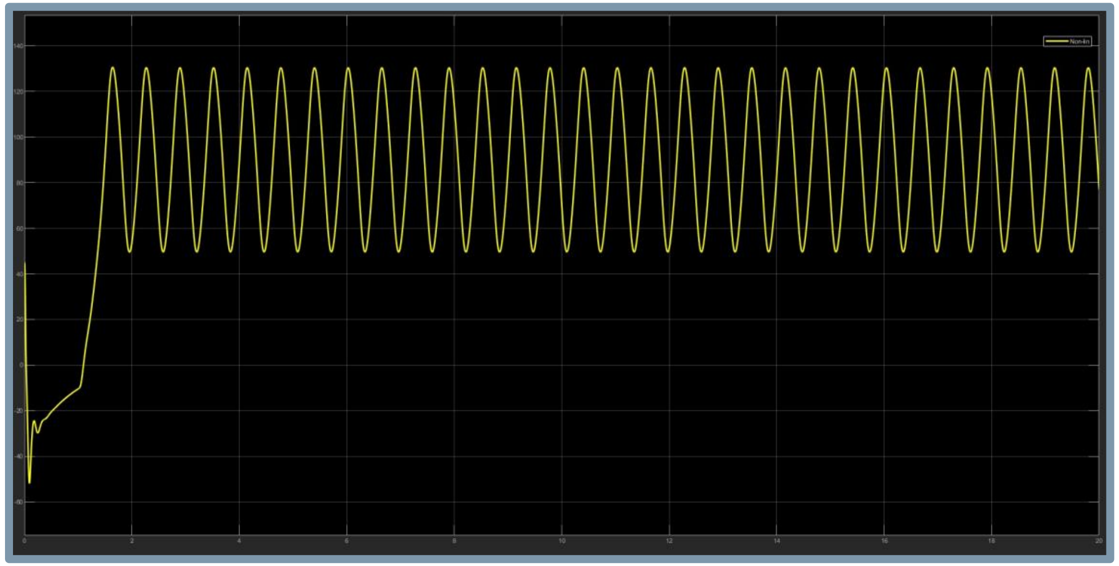
Linear Tangent Approximation

Simulation Performance

Request 3 : Implement the obtained control law in the simulation scheme and test its behavior

We run **two different simulations** to show the behaviour of this controller. In the first case (right image),we set a reference of **π/4 + π/8**, and as we can see the output tracks the reference after some initial small oscillations.





In the second case (left image) we used a reference of **π/2** and from the plot we can see that the tracking performances are not so good as in the other case. This is due to the fact that the tangent model is accurate only in a neighborhood of the equilibrium point.

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FeedBack Linearization

Fully Linearizability Condition

Request 4 : Check that the original nonlinear system is fully exactly I/O linearizable by diffeomorphism and state feedback, with $\frac{\pi}{4} - \theta_l$ as the output, and propose a stabilizing state feedback law on this basis

First step is to check if our NonLinear, affine in the input, TI system is Fully Linearizable (Input/State Linearizable). This is equal to check that our system S has relative degree r in x°, equal to the grade of the system, n. Then we can obtain a (locally) linear system via state feedback :

$$v := L_a^r c + u L_b L_a^{r-1} c \quad \Rightarrow \quad u = \frac{1}{L_b L_a^{r-1} c}(v - L_a^r c)$$

System S has relative degree r in x° if, in a neighborhood of x° we got :

$$L_b L_a^k c \equiv 0, \qquad k = 0, 1, 2, \ldots, r-2 \qquad \text{and} \qquad [L_b L_a^{r-1} c]_x \neq 0$$

Then, to build the feedback linearization block we need to compute 2 functions : $L_a^r c$ $\quad L_b L_a^{r-1} c$

In order to do so we define the Lie derivative operator along a vector field f as : $L f := \sum_{i=1}^{n} f_i(x) \frac{\delta}{\delta x_i}$

We have done all the other computations through a MatLab script

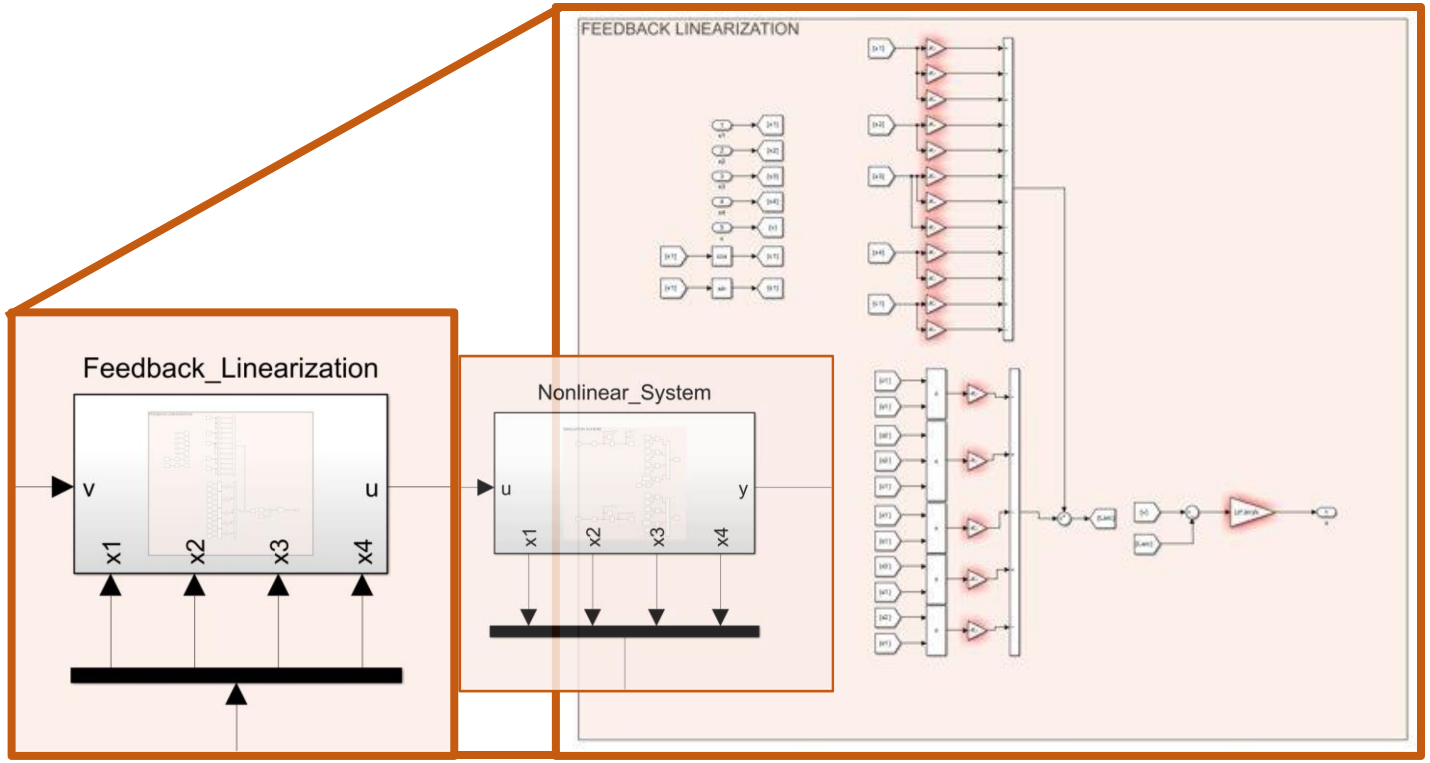
(results are on "symbolic_derivatives.m" file attached to the presentation)

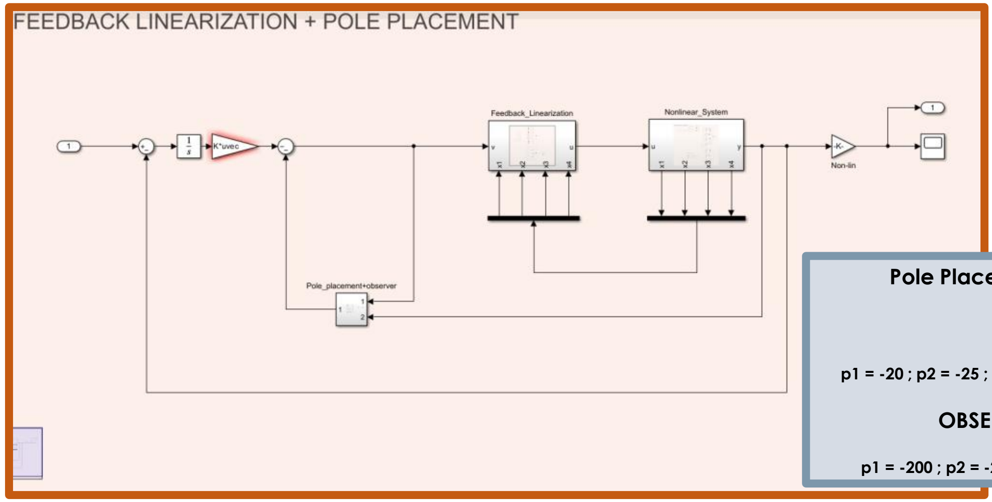
FeedBack Linearization

Model + Stabilizing State FeedBack Law

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad N = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C_{\text{fl}} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \qquad D_{\text{fl}} = 0$$





Pole Placement Controller

POLES

p1 = -20 ; p2 = -25 ; p3 = -30 ; p4 = -35 ; p5 = -40

OBSERVER POLES

p1 = -200 ; p2 = -250 ; p3 = -300 ; p4 = -350

As stabilizing control law we used a **pole placement approach** and we extended the system with an integrator in order to track a reference of θ_l

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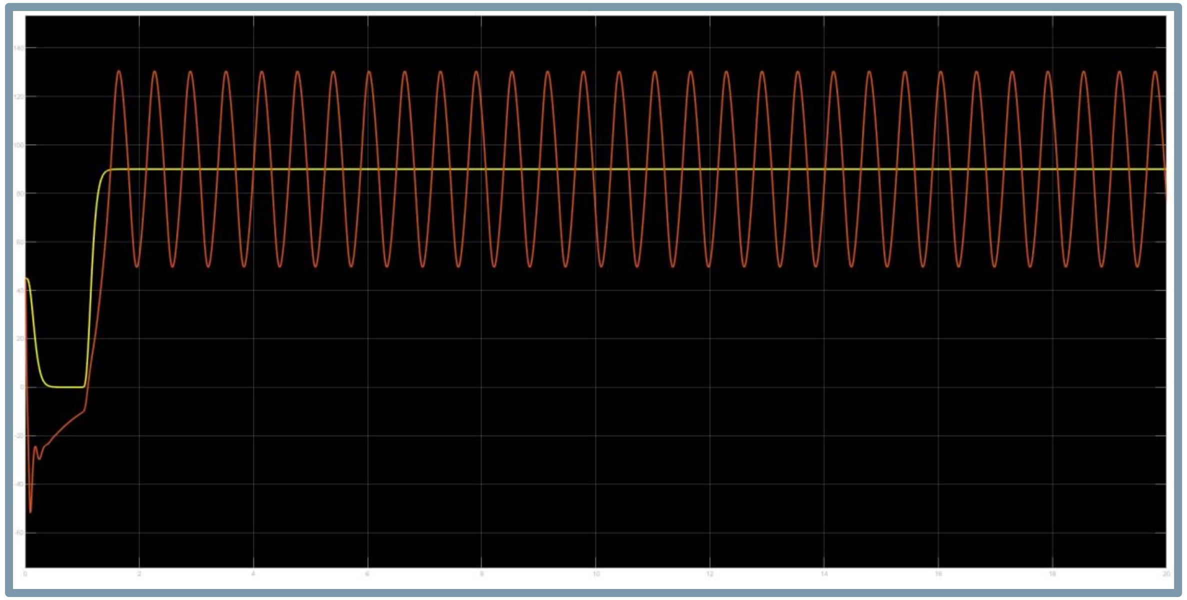
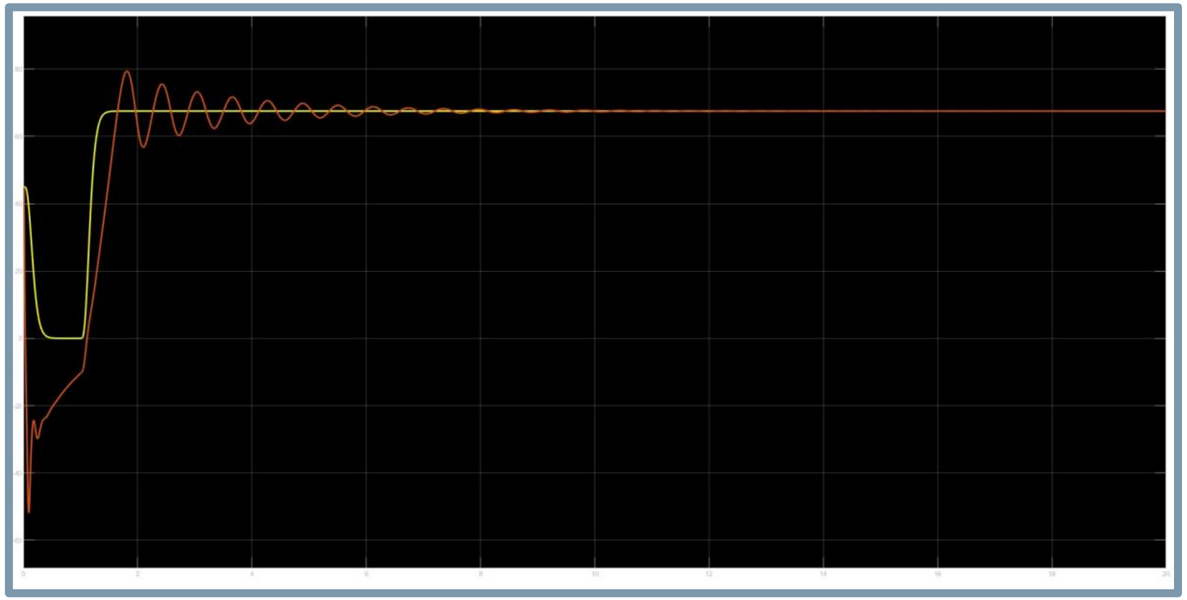
FeedBack Linearization

Simulation Performance

Request 5 : Verify the results in simulation and comment

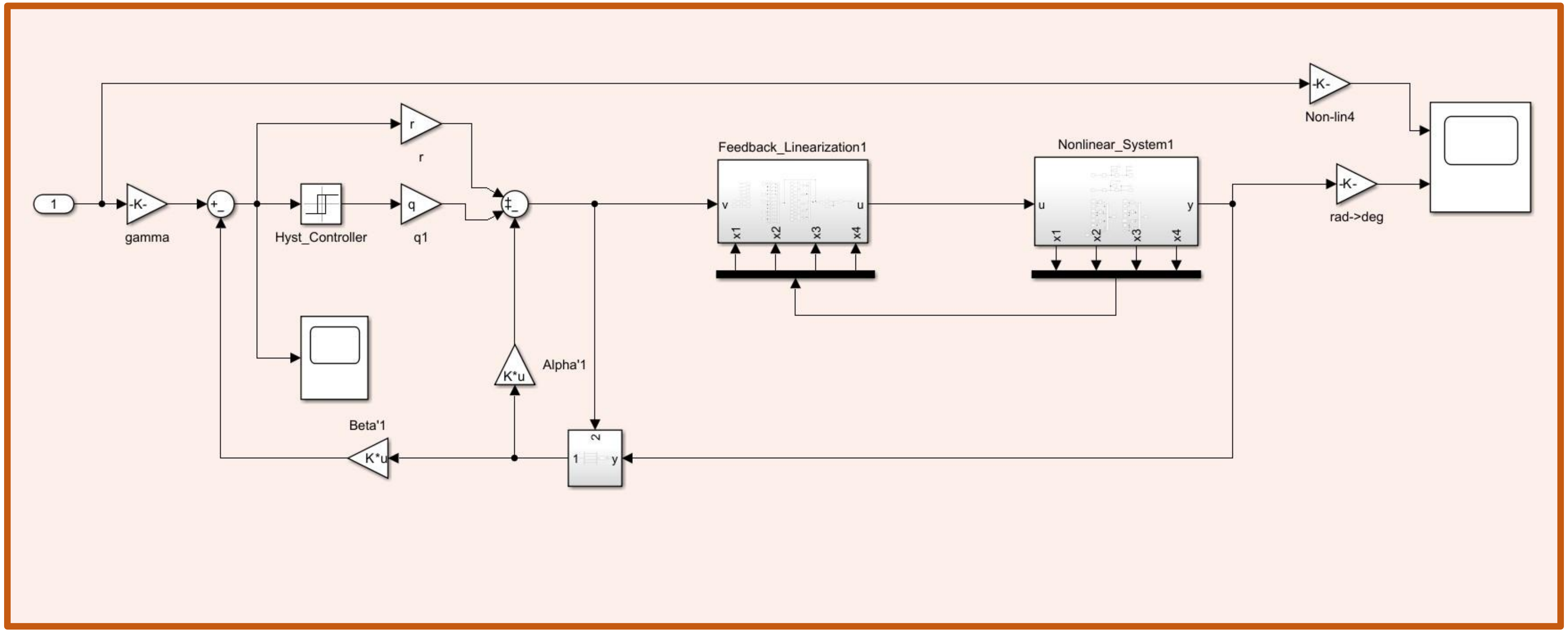
We run **two different simulations** with the same reference values we used for the linear tangent approximation. As we can see the feedback linearization performs better than the linear tangent approximation model tracking the reference way faster without oscillations.





Moreover we can also see that the feedback linearized system can track a reference far from the equilibrium point in a more performant way with respect to the Linear Tangent Approximation.

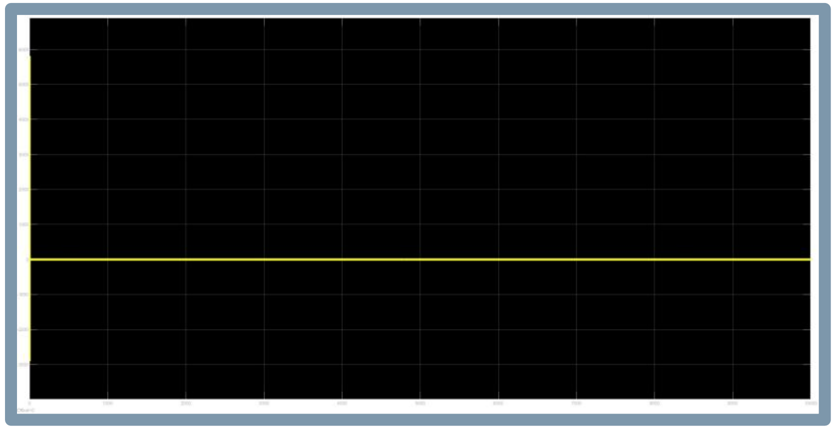
Request 6 : Propose a Variable Structure Controller for the system linearized via feedback linearization to make $y(t) = \frac{\pi}{4} - \theta_l$ track some (constant) reference signal y°

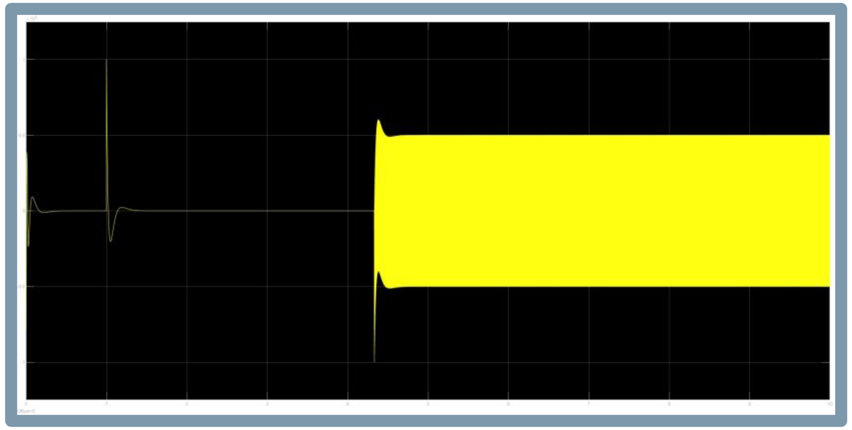


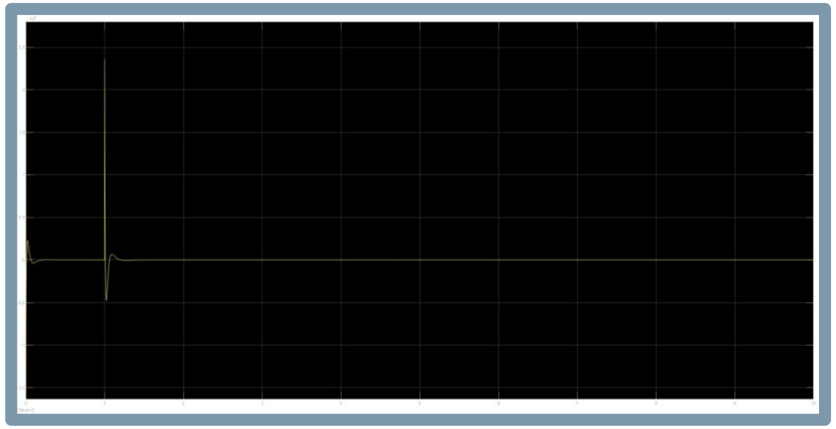
Variable Structure Control

Simulation Performance

Request 7 : Analyze the performance achieved by the variable structure controller devised in point 6 focusing on high frequency switching



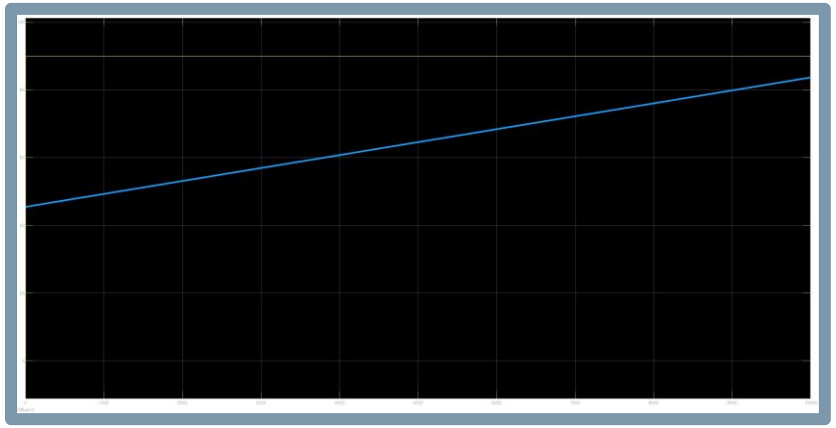


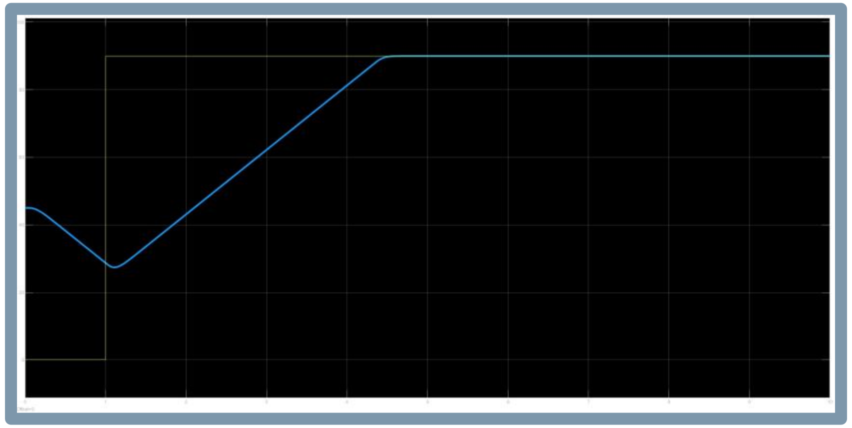


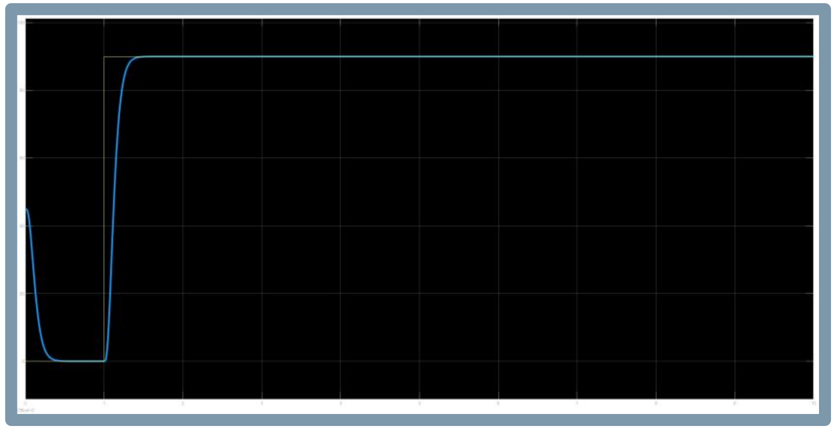
q = 1 ; r = 0

q = 5000 ; r = 0

q = 10 ; r = 100







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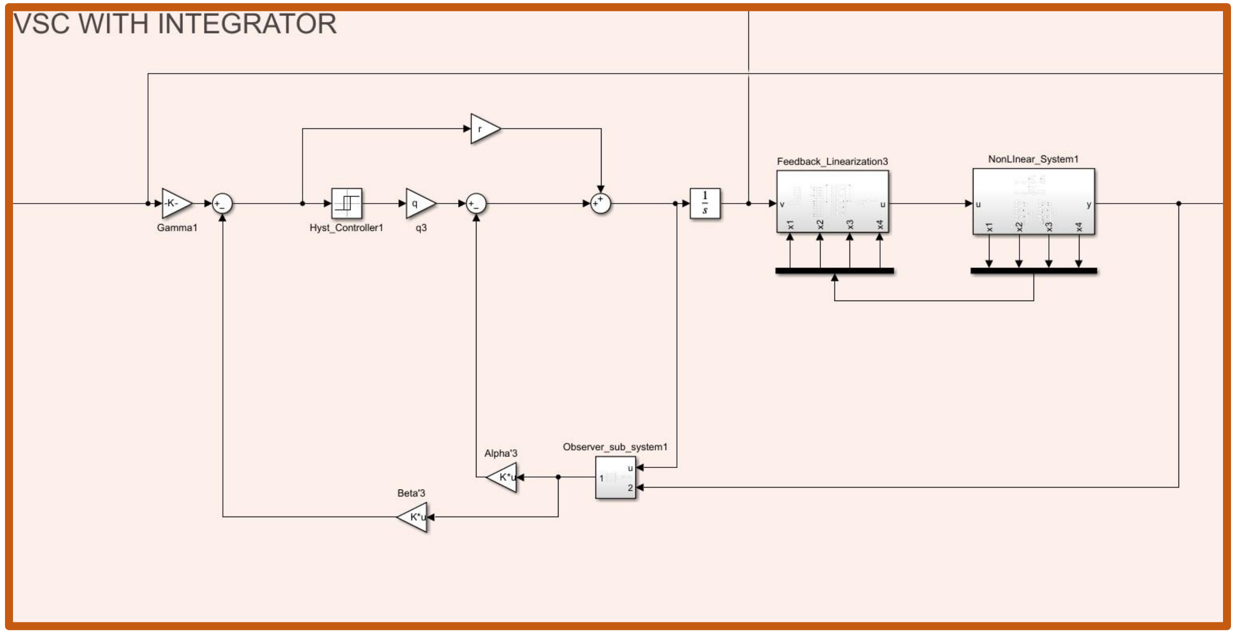
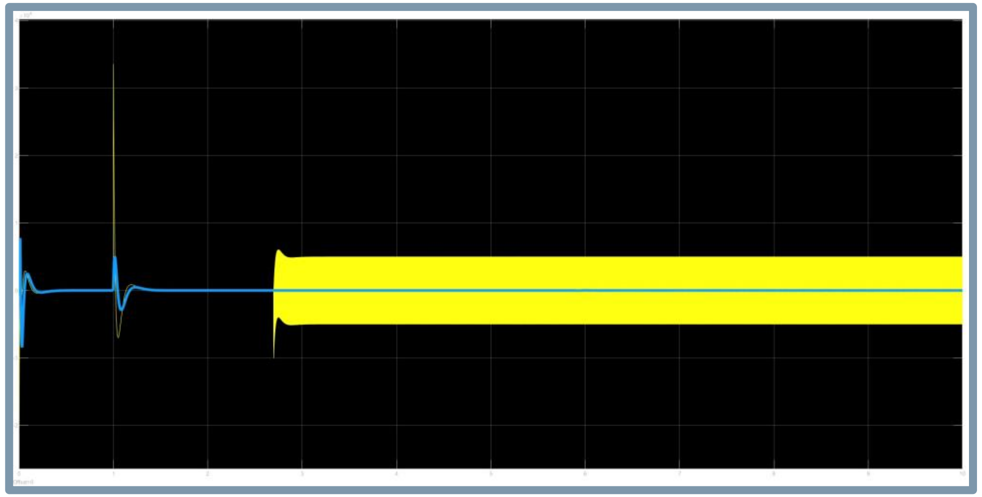
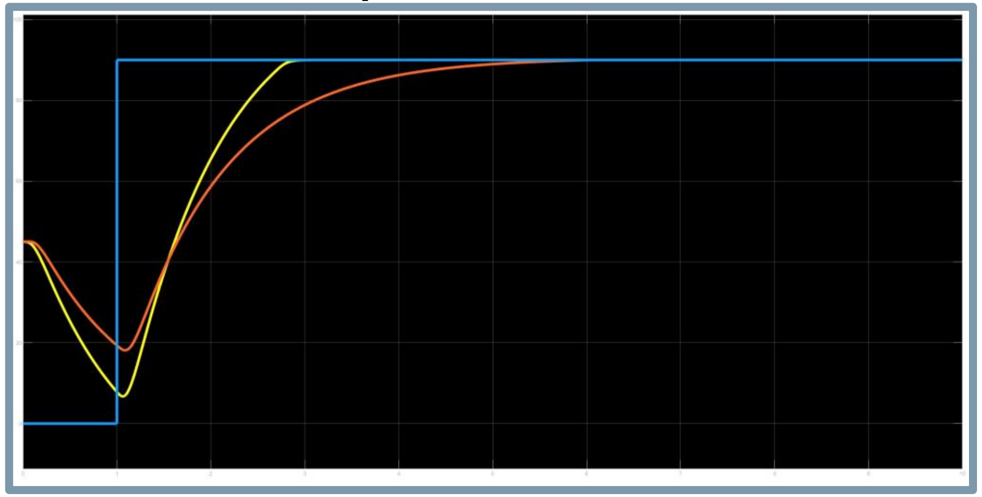
Variable Structure Control

High Frequency Switching countermeasure + Simulation performance





q = 5000 ; r = 1



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