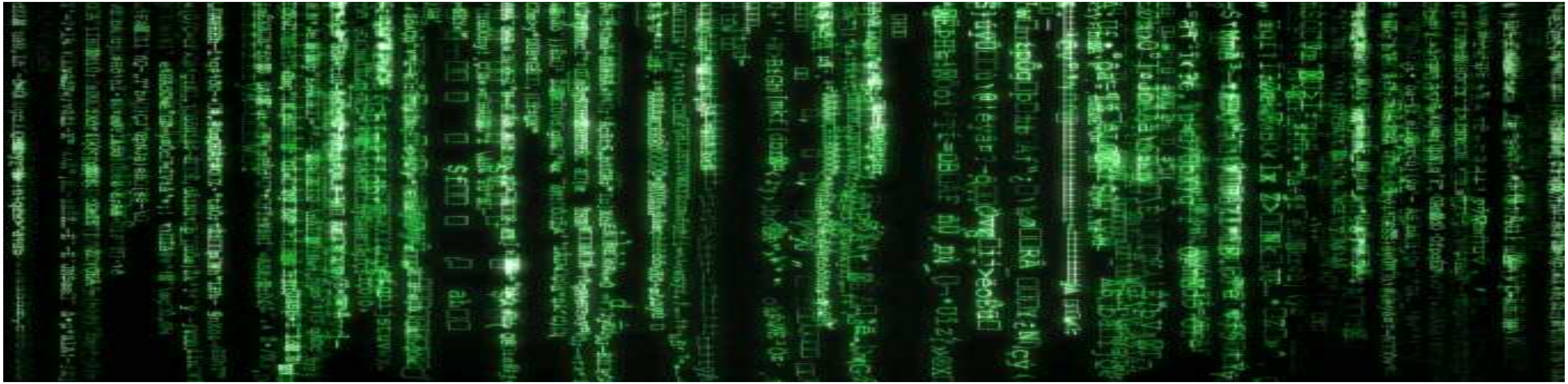


INF 110 Discovering Informatics

Probability

NAU
NORTHERN
ARIZONA
UNIVERSITY

Introduction

People use the term probability many times each day. For example, physician says that a patient has a 50-50 chance of surviving a certain operation. Another physician may say that she is 95% certain that a patient has a particular disease

Definition

If an event can occur in N mutually exclusive and equally likely ways, and if m of these possess a trait, E, the probability of the occurrence of E is read as

$$P(E) = m/N$$

Example: Coin Flips

- N = 2 ways a fair coin can appear
- m = 1 configuration where heads appears

P(Heads) = ½ = 0.5 = 50%

Example: More Dice (d20)

- N = 20 ways a fair d20 can appear
- m = 1 configuration where 20 appears

P(Twenty) = 1/20 = 0.05 = 5%

Random Selection

We can choose a random value (like a dice roll) with numpy:

```
np.random.choice(set, number=1)
```

Live Code Rolling Dice

Tasks: "Roll" a d20 in Python; Then roll a d20 100 times!

Learning Outcomes

- Using the choice method
- Understanding random sets

Definition

- Sample space: collection of unique, non-overlapping possible outcomes of a random circumstance.
- Simple event: one outcome in the sample space; a possible outcome of a random circumstance.
- Event: a collection of one or more simple events in the sample space; often written as
  A, B, C, and so on

Live Code Rolling Weighted Dice

Tasks: According to the World Health Organization (WHO) there are 105 male births for every 100 female births. The events in the sample space aren't equally likely. But we can treat them like they are if we change the sample space from 2 to 205.

P(M) = 0.512;   P(F) = 0.488

Learning Outcomes

- Using the choice method
- Understanding random sets

Definition

- Complement ==> sometimes, we want to know the probability that an event will not happen; an event opposite to the event of interest is called a complementary event.
- If A is an event, its complement is The probability of the complement is AC or !A
- Example: The complement of male event is the female

- *P*(A) + *P*(AC) = 1

Views of Probability

Subjective

- It is an estimate that reflects a person’s opinion, or best guess about whether an outcome will occur.
- This is a common view in medicine: probability forms the basis of a physician’s opinion about whether a patient has a specific disease. Estimate may change based on diagnostic procedures.

Views of Probability

Objective

- It is well known that the probability of flipping a fair coin and getting tails is 0.50.
- If a coin is flipped 10 times, is there a guarantee, that exactly 5 tails will be observed?
- If the coin is flipped 100 times? With 1000 flips?
- As the number of flips becomes larger, the proportion of coin flips that result in tails approaches 0.50

Live Code Flipping Coins

Tasks: Flip a coin 10x, 100x, 1000x and count the # of heads.
What happens as the number of flips grows larger?

Hint: To count the number of heads, use:

```
np.count_nonzero(rolls == "T")
```

Learning Outcomes

- Using the choice method
- Understanding random sets

Views of Probability

Objective

- Relative frequency
- Assuming that an experiment can be repeated many times and assuming that there are one or more outcomes that can result from each repetition. Then, the probability of a given outcome is the number of times that outcome occurs divided by the total number of repetitions.

Example Using Data

- P(Male and AB)?
- P(AB)?

Blood Group	Males	Females	Total
O A B AB	20 17 8 5	20 18 7 5	40 35 15 10
Total	50	50	100

Marginal Probabilities

Named because they appear on the “margins” of a probability table. It is probability of single outcome

Examples: P(Male), P(AB)

Conditional Probabilities

It is the probability of an event on condition that certain criteria is satisfied

Example: If a subject was selected randomly and found to be female what is the probability that she has a blood group O or P(O\F)

P(O\F) = 20/50 = 0.40

Joint Probability

It is the probability of occurrence of two or more events together

Example: Probability of being male & belong to blood group AB

$$P(M \text{ and } AB) = P(M \cap AB) = 5/100 = 0.05$$

$\cap$ = intersection

Probability Basics

- Probabilities ranges between 0 and 1
- If an outcome cannot occur, its probability is 0
- If an outcome is sure, it has a probability of 1
- The sum of probabilities of mutually exclusive outcomes is equal to 1

$$P(M) + P(F) = 1$$

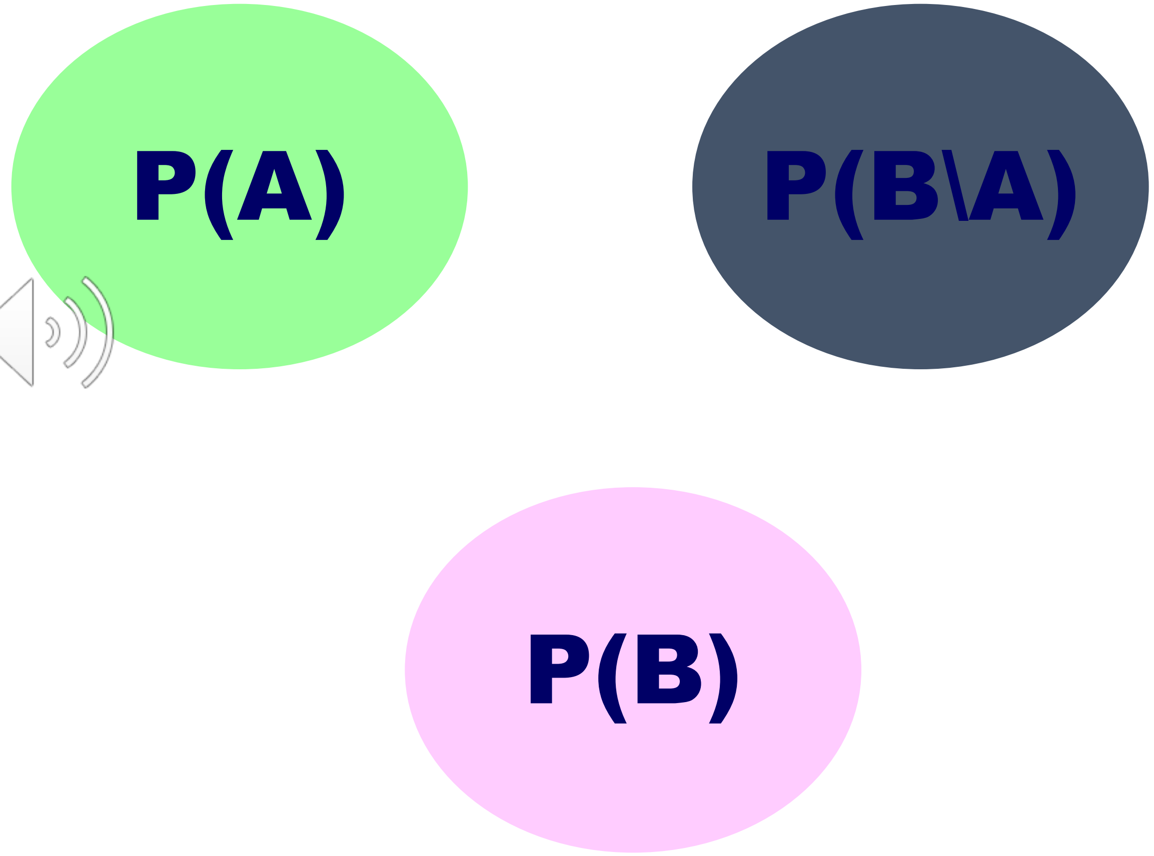
Multiplication Rule

Independence and multiplication rule

P(A and B) = P(A) P(B)

How do you know if two events are independent?

A and B are independent if
P(B\A) = P(B)



Example

Consider the joint probability of being male and having blood type O

P(O) = 40/100 = 0.40

P(O\M) = 20/50 = 0.40

The two events are independent! So we can calculate the probability of being male and having blood type O:

$P(O \cap M) = P(O) \times P(M) = (40/100) \times (50/100) = 0.20$

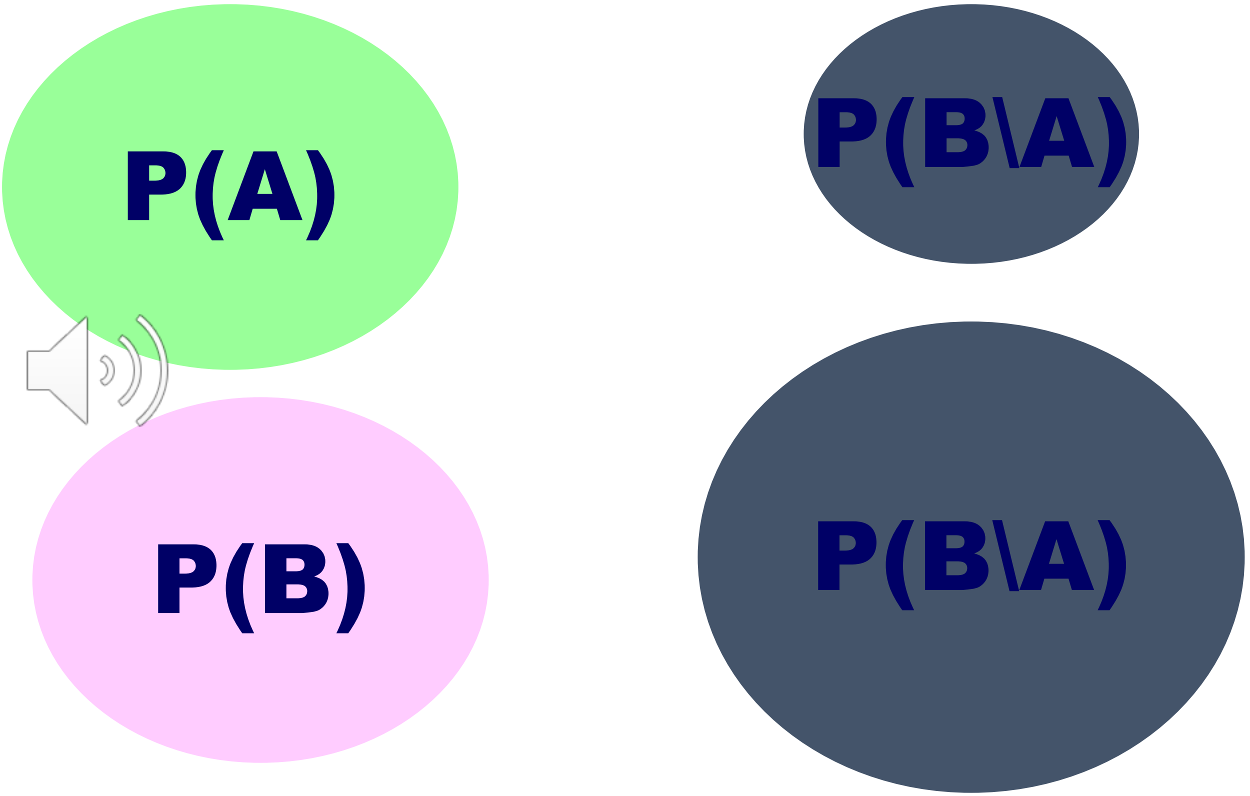
Multiplication Rule (Part 2)

Dependence and the modified multiplication rule

$P(A \text{ and } B) = P(A)\ P(B \backslash A)$

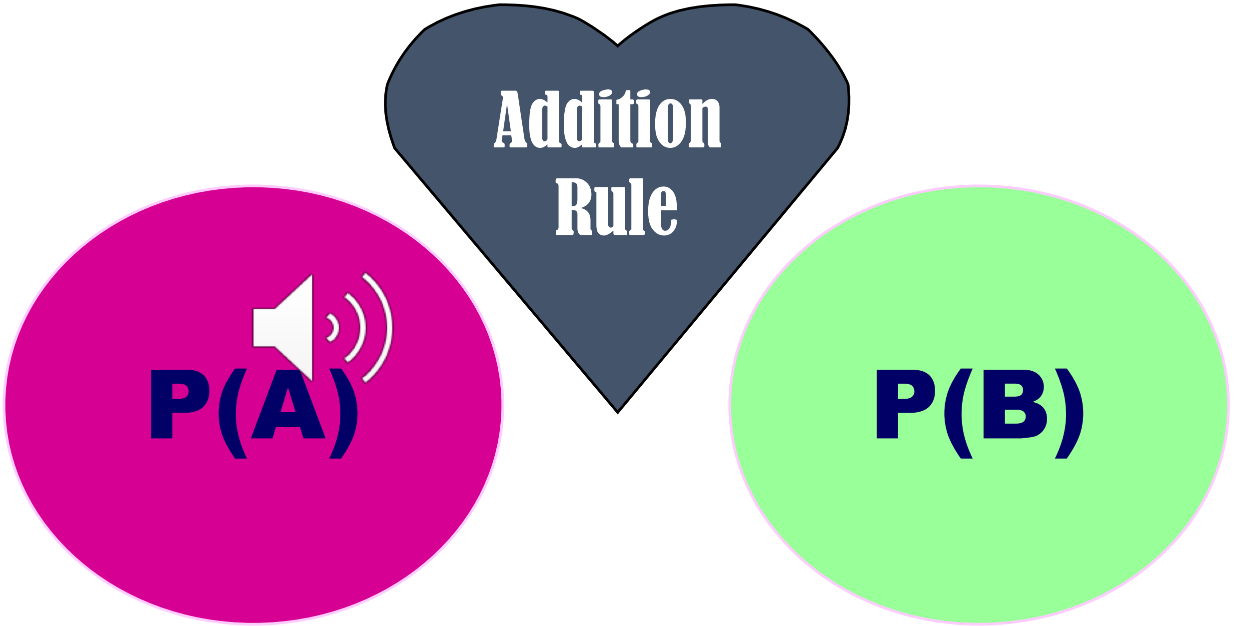
How do you know if two events are dependent?

A and B are dependent if
P(B\A) != P(B)



Rules of Probability: Addition

- A and B are mutually exclusive
- The occurrence of one event precludes the occurrence of the other



P(A OR B) = P(A U B) = P(A) + P(B)

Example

The probability of being either blood type O or blood type A:

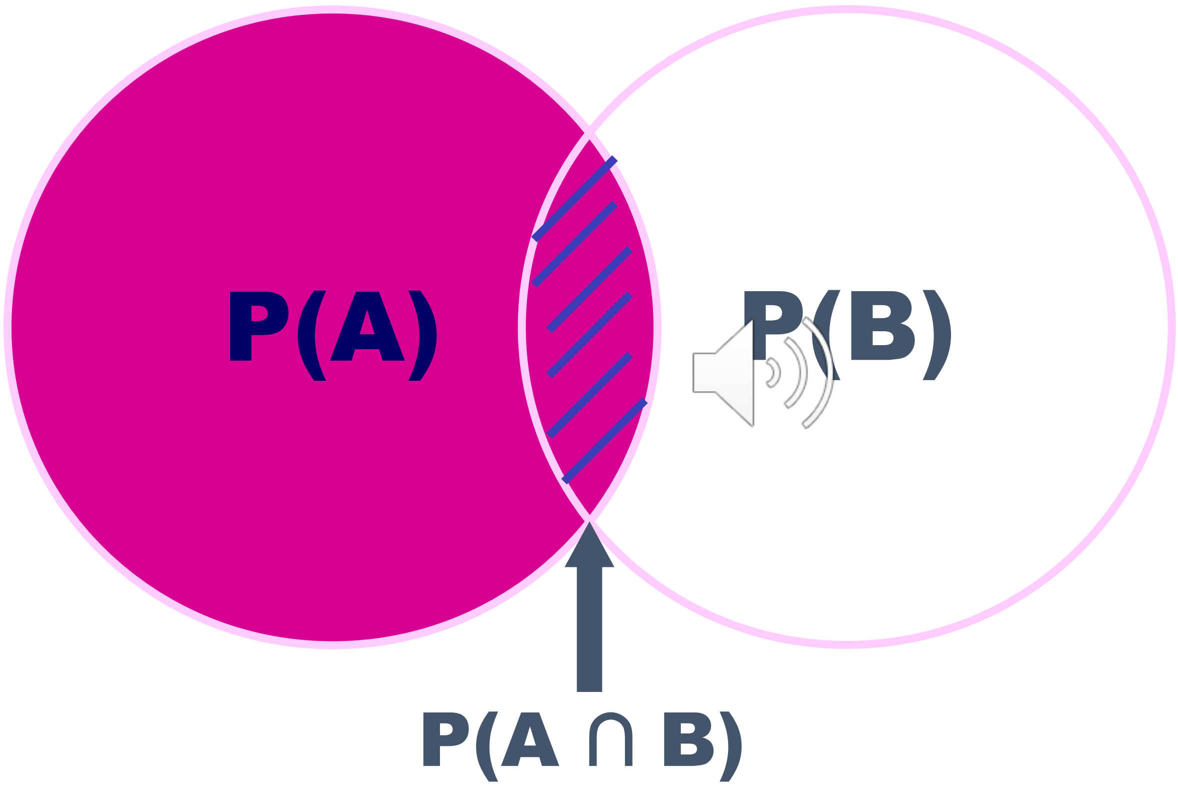
$$
\begin{aligned}
P(O \cup A) &= P(O) + P(A) \\
&= (40/100)+(35/100) \\
&= 0.75
\end{aligned}
$$

A and B are non mutually exclusive

(Can occur together)

Example: Male and smoker

Modified Addition Rule



$$P(A \text{ OR } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Example

Two events are not mutually exclusive (male gender and blood type O).

$$
\begin{aligned}
P(M \text{ OR } O) &= P(M)+P(O) - P(M \cap O) \\
&= 0.50 + 0.40 - 0.20 \\
&= 0.70
\end{aligned}
$$

end