

Quantum Theory of Radiation

Marcus Allen Denslow

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1.1 Transverse and Longitudinal Fields

In non-relativistic Quantum Mechanics, the static Electric field is represented by a scalar potential, magnetic fields by the vector potential, and the radiation field also through the vector potential. It will be convenient to keep this separation between the large static atomic Electric field and the radiation fields, however, the equations we have contain the four-vector A_μ with all the fields mixed. When we quantize the field, all E and B fields as well as electromagnetic waves will be made up of photons. It is useful to be able to separate the E fields due to fixed charges from the EM radiation from moving charges. This separation is not Lorentz invariant, but it is still useful. Enrico Fermi showed, in 1930, that A_\parallel together with A_0 give rise to Coulomb interactions between particles, whereas A_\perp gives rise to the EM radiation from moving charges. With this separation, we can maintain the form of our non-relativistic Hamiltonian.

$$\mathbf{H} = \sum_j \frac{1}{2m_j} \left(\vec{p} - \frac{e}{c} \vec{A}_\perp(\vec{x}_j) \right)^2 + \sum_{i>j} \frac{e_i e_j}{4\pi || \vec{x}_i - \vec{x}_j ||} + \mathbf{H}_{\text{rad}} \quad (1.1)$$

Where \mathbf{H}_{rad} is purely the Hamiltonian of the radiation (containing only \vec{A}_\perp) and \vec{A}_\perp is the part of the vector potential which satisfies $\nabla \cdot \vec{A}_\perp = 0$. Note that \vec{A}_\parallel and A_0 appear nowhere in the Hamiltonian. Instead, we have the Coulomb potential. This separation allows us to continue with our standard Hydrogen solution and just add radiation. We will not derive this result.

In a region in which there are no source terms,

$$j_\mu = 0 \quad (1.2)$$

we can make a gauge transformation which eliminates A_0 by choosing Λ such that

$$\frac{1}{c} \frac{\partial \Lambda}{\partial t} = A_0 \quad (1.3)$$