

Calc 3 questions  
undertext

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# Chapter 1

## testing

### 1.1 questions

#### Oppgave

Evaluate the double integral

$$\iint_R x^2 y \, dA.$$

where  $R$  is the region bounded by  $y = x^2$  and  $y = 2x$ .

**Solution:** First, find the intersection points of  $y = x^2$  and  $y = 2x$  :

$$x^2 = 2x \implies x^2 - 2x = 0 \implies x(x - 2) = 0.$$

so  $x = 0$  and  $x = 2$ . The region  $R$  is described by  $x^2 \leq 2x$  for  $0 \leq x \leq 2$ .

set up the integral:

$$\iint_R x^2 y \, dA = \int_0^2 \int_{x^2}^{2x} x^2 y \, dy \, dx.$$

Evaluate the inner integral:

$$\int_{x^2}^{2x} x^2 y \, dy = x^2 \left[ \frac{y^2}{2} \right]_{x^2}^{2x} = x^2 \left( \frac{(2x)^2}{2} - \frac{(x^2)^2}{2} \right).$$

$$= \frac{2(32)}{5} - \frac{128}{14} = \frac{64}{5} - \frac{64}{7} = \frac{448 - 320}{35} = \frac{128}{35}.$$

#### Oppgave

Find the directional derivative of  $f(x, y, z) = x^2 y z^3$  at the point  $P(2, 1, -1)$  in the direction of the vector  $\mathbf{v} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$

**Solution:** First, compute the gradient of  $f$ :

$$\nabla f = \left( \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right) = (2xyz^3, x^2z^3, 3x^2yz^2).$$

Evaluate at  $P(2, 1, -1)$ :

$$\begin{aligned} \nabla f(2, 1, -1) &= (2 \cdot 2 \cdot 1 \cdot (-1)^3, 2^2 \cdot (-1)^3 \cdot 3, 3 \cdot 2^2 \cdot 1 \cdot (-1)^2) \\ &= (2 \cdot 2 \cdot 1 \cdot (-1), 4 \cdot (-1), 3 \cdot 4 \cdot 1 \cdot 1) = (-4, -4, 12). \end{aligned}$$

Normalize the direction vector  $\mathbf{v} = (1, 2, -2)$  :

$$|\mathbf{v}| = \sqrt{1^2 + 2^2 + (-2)^2} = \sqrt{1 + 4 + 4} = \sqrt{9} = 3.$$

$$\mathbf{u} = \frac{\mathbf{v}}{|\mathbf{v}|} = \left(\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}\right).$$

The direction derivative is:

$$\begin{aligned} D_{\mathbf{u}}f &= \nabla f \cdot \mathbf{u} = (-4, -4, 12) \cdot \left(\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}\right) \\ &= -\frac{4}{3} - \frac{8}{3} - \frac{24}{3} = -\frac{36}{3} = -12. \end{aligned}$$