Calc 3 questions undertext

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Contents

Chapter 1

testing

1.1 questions

Oppgave

Evaluate the double integral

$$\iint_R x^2 y\, dA.$$

where R is the region bounded by $y = x^2$ and $y = 2x$.

Solution: First, find the intersection points of $y = x^2$ and $y = 2x$:

$$x^2 = 2x \implies x^2 - 2x = 0 \implies x(x-2) = 0.$$

so $x = 0$ and $x = 2$. The region R is described by $x^2 \leqslant 2x$ for $0 \leqslant x \leqslant 2$.

set up the integral:

$$\iint_R x^2 y\, dA = \int_0^2 \int_{x^2}^{2x} x^2 y\, dy\, dx.$$

Evaluate the inner integral:

$$\int_{x^2}^{2x} x^2 y\, dy = x^2 \left[\frac{y^2}{2}\right]_{x^2}^{2x} = x^2 \left(\frac{(2x)^2}{2} - \frac{(x^2)^2}{2}\right).$$

$$\boxed{= \frac{2(32)}{5} - \frac{128}{14} = \frac{64}{5} - \frac{64}{7} = \frac{448-320}{35} = \frac{128}{35}}.$$

Oppgave

Find the directional derivative of $f(x, y, z) = x^2yz^3$ at the point $P(2, 1, -1)$ in the direction of the vector $\mathbf{v} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$

Solution: First, compute the gradient of f:

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) = (2xyz^3, x^2z^3, 3x^2yz^2).$$

Evaluate at $P(2, 1, -1)$:

$$\nabla f(2, 1, -1) = (2 \cdot 2 \cdot 1 \cdot (-1)^3, 2^2 \cdot (-1)^3 3 \cdot 2^2 \cdot 1 \cdot (-1)^2).$$

$$= (2 \cdot 2 \cdot 1 \cdot (-1), 4 \cdot (-1), 3 \cdot 4 \cdot 1 * 1) = (-4, -4, 12).$$