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Department of Mathematics

Title of the Thesis

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The work contained in this thesis is my own work unless otherwise stated.

Signed: STUDENT'S NAME Date: DATE

Abstract

ABSTRACT GOES HERE

Acknowledgements

ANY ACKNOWLEDGEMENTS GO HERE

1 Introduction

The introduction section goes here[1].

[1] Tip: write this section last.

2 Background

Background chapter.

2.1 My section

Section content goes here.

2.1.1 My subsection

A subsection.

My subsubsection

A subsubsection.

2.2 Figures

It is better to create figures in a vector-based format, such as PDF.

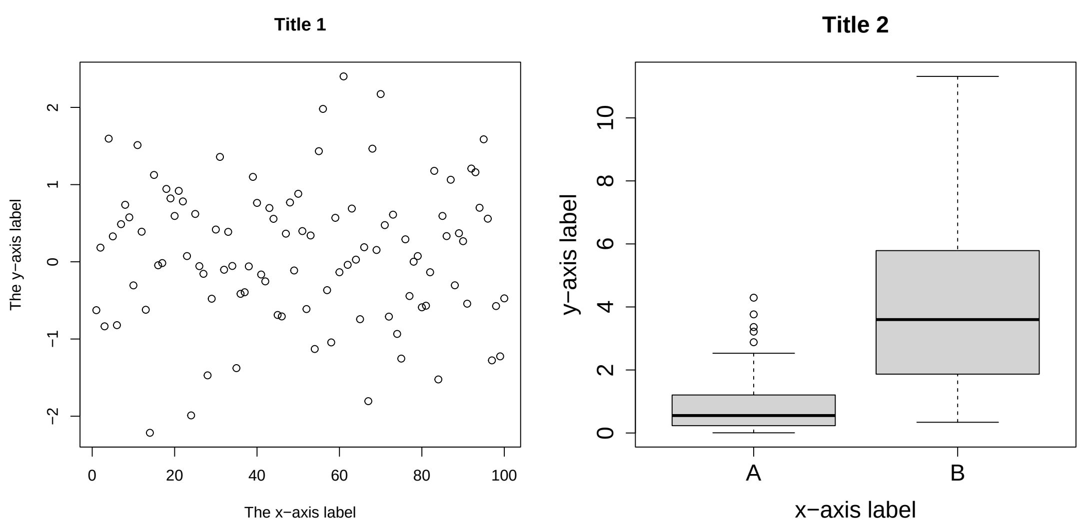

Figure 2.1: Remember to make fonts in figures large enough (compare the two figures.)

2.3 Tables

Here is an example of a table

z	P($Z < z$)
1.281	0.900
1.645	0.950
1.960	0.975
2.326	0.990
2.576	0.995

Table 2.1: Partial table showing values of z for P($Z < z$), where Z has a standard normal distribution.

2.4 Referencing sources, sections and items

A good book on the bootstrap is Efron and Tibshirani (1994), although the idea appeared in an earlier paper (Efron, 1979).

Note that to make the references appear, you will need to compile the bibtex, otherwise you may just see question marks where the references should be.

2.4.1 Referencing sections, results and equations

Theorem 2.5.2 is proved in Section 2.5; see Equation (2.2).

Referencing tables and figures

When labelling figures and tables, it is important that the label command `\label{LABELNAME}` comes **after** the caption command. See Table 2.1 and Figure 2.1 above.

2.4.2 Quoting sources

If you wish to quote a source, be sure to use quotation marks and cite the reference. The `\usequote` command is useful here:

"It was the best of times, it was the worst of times, it was the age of wisdom, it was the age of foolishness, it was the epoch of belief, it was the epoch of incredulity, it was the season of Light, it was the season of Darkness, it was the spring of hope, it was the winter of despair, we had everything before us, we had nothing before us, we were all going direct to Heaven, we were all going direct the other way - in short, the period was so far like the present period, that some of its noisiest authorities insisted on its being received, for good or for evil, in the superlative degree of comparison only." (Dickens, 1859)

2.5 Definitions, theorems and examples

The following environments are supported: Definition, Theorem, Proof, Proposition, Lemma, Remark, Example.

Definition 2.5.1. The **variance** of a random variable X is defined as

$$\mathrm{Var}(X) = \mathrm{E}[(X - \mathrm{E}[X])^2]. \tag{2.1}$$

Theorem 2.5.2. Given a random variable X, over all values $a \in \mathbb{R}$,

$$\min_{a\in\mathbb{R}} \mathrm{E}[(X - a)^2] = \mathrm{E}[(X - \mathrm{E}[X])^2]. \tag{2.2}$$

Proof. Starting with the left-hand side,

$$\begin{aligned}
\mathrm{E}[(X - a)^2] &= \mathrm{E}[(X - \mathrm{E}[X] + \mathrm{E}[X] - a)^2] \\
&= \mathrm{E}[(X - \mathrm{E}[X])^2] + 2\mathrm{E}[(X - \mathrm{E}[X])\,(\mathrm{E}[X] - a)] + \mathrm{E}[(\mathrm{E}[X] - a)^2] \\
&= \mathrm{E}[(X - \mathrm{E}[X])^2] + (\mathrm{E}[X] - a)^2 \\
&\geq \mathrm{E}[(X - \mathrm{E}[X])^2],
\end{aligned}$$

since $\mathrm{E}[X]$ is a real number and $(\mathrm{E}[X] - a)^2 \geq 0$, and the third line follows from linearity of expectation:

$$\mathrm{E}[(X - \mathrm{E}[X])\,(\mathrm{E}[X] - a)] = (\mathrm{E}[X] - a)\,\mathrm{E}[(X - \mathrm{E}[X])] = (\mathrm{E}[X] - a)\,(\mathrm{E}[X] - \mathrm{E}[X]) = 0,$$

since $\mathrm{E}[\mathrm{E}[X]] = \mathrm{E}[X]$, which proves the result. □

Remark 2.5.3. This theorem shows that that the minimum of the quantity $\mathrm{E}[(X-a)^2]$ is equal to $\mathrm{Var}(X)$. In some sense, this makes the variance a natural measure of dispersion if we are taking the metric to be the squared deviation of X.

Lemma 2.5.4 (Stein's Lemma)**.** Let $X \sim \mathrm{N}(\mu, \sigma^2)$, and let g be a differentiable function satisfying $\mathrm{E}[|g'(X)|] < \infty$. Then

$$\mathrm{E}[g(X)(X - \mu)] = \sigma^2 \mathrm{E}[g'(X)].$$

Proposition 2.5.5 (Popoviciu's inequality)**.** Suppose that the random variable X is known to only take values in the bounded range $[a, b]$. Then

$$\mathrm{Var}[X] \leq \frac{(b - a)^2}{4}.$$

Example 2.5.6. Suppose $X \sim \mathrm{Bern}(p)$, for some $p \in [0, 1]$. Then, since $X \in \{0, 1\}$, X is bounded between 0 and 1 and so $\mathrm{Var}[X] \leq \frac{1}{4}$.

3 Conclusion

Conclusion goes here.

References

Charles Dickens. *A Tale of Two Cities*. Chapman & Hall, 1859.

Bradley Efron. Bootstrap methods: Another look at the jackknife. *Annals of Statistics*, 7(1):1–26, 1979.

Bradley Efron and Robert J Tibshirani. *An introduction to the bootstrap*. CRC press, 1994.