Computational Physics: PS 7

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1 Discussion

My GitHub repo is `https://github.com/marcusHoskinsNYU/phys-ga2000`. For images that are blurry here, please see the ps-8 folder for individual image files.

1.1 Problem 1

The desired plot of the logistic model (with optimized parameters) and answers versus age is seen in figure 1. For our logistic model, $\beta_0 = -5.62026237$ and $\beta_1 = 0.10956335$. The formal errors for these values are 1.057664 and 0.02084893, respectively. Moreover, the covariance matrix for β_0 and β_1 is seen in figure 2. These values seem to make sense when looking at the data in figure 1, as we know that $p(x = -\frac{\beta_0}{\beta_1}) = 0.5$, and for us $-\frac{\beta_0}{\beta_1} \approx 51.297$, which seems to be the age where the results begin to change from being mostly 0 to mostly 1.

1.2 Problem 2

1.2.1 Part (a)

The appropriate code is given in the repo. The plots from the application of this code to the two waveforms are seen in figures 3, 4, 5, and 6.

One can gain some insight about the nature of these two instruments from their Fourier coefficients. For the piano, we see that only one of the coefficients has a large value, with some higher coefficients having relatively small values. However, for the trumpet we see that several of the coefficients are excited by comparable amounts.

These observations point to the fact that a piano, when playing a particular note, largely excites a single frequency. On the other hand, when a trumpet plays a certain note, several other frequencies are excited, in addition to the one that is meant to be played.

1.2.2 Part (b)

Since both instruments are playing the same musical note, we will analyze the far easier piano spectrum. Zooming in on the largest peak, we see it occurs at a wavenumber of $k \approx 1190\text{m}^{-1}$. Then, as $k = \frac{2\pi f}{c_{sound}}$, where $c_{sound} = 343\text{m/s}$ is the speed of sound and f is the frequency, we find that the frequency of the note played by the piano and trumpet is $f \approx$

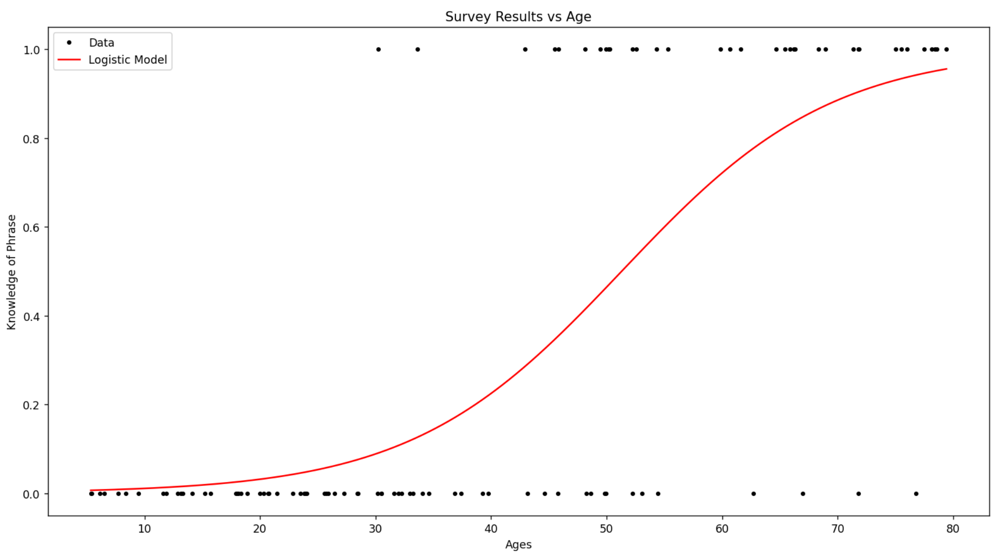

Figure 1: This is a plot of the logistic model, with $\beta_0 = -5.62026237$ and $\beta_1 = 0.10956335$, along with the answers of the survey, versus age.

```
[[ 1.1186532e+00 -2.1119451e-02]
 [-2.1119451e-02  4.3467787e-04]]
```

Figure 2: This is the covariance matrix for β_0 and β_1 given by scipy's minimize function.

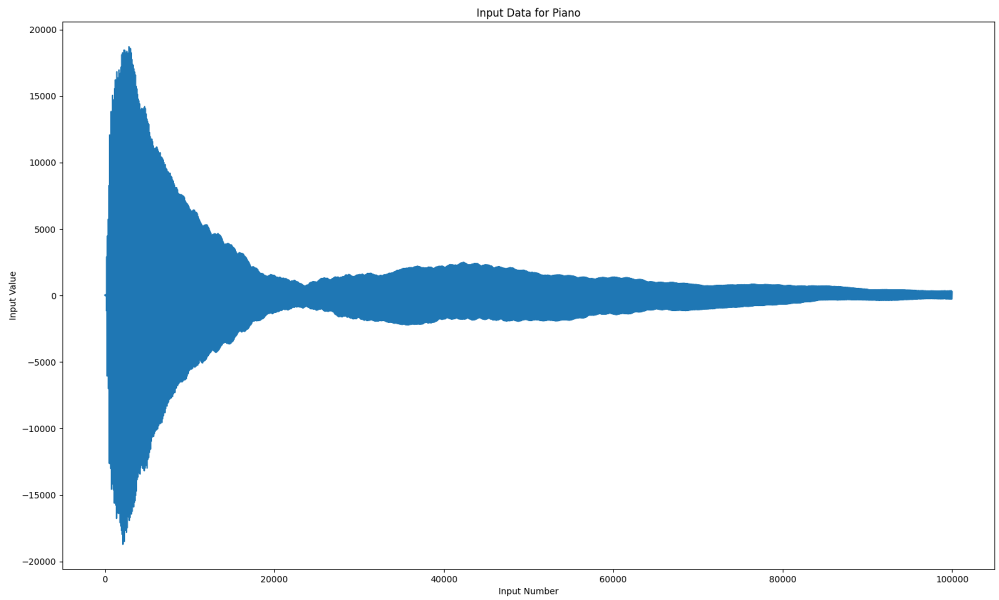

Figure 3: Input Waveform Data from Piano

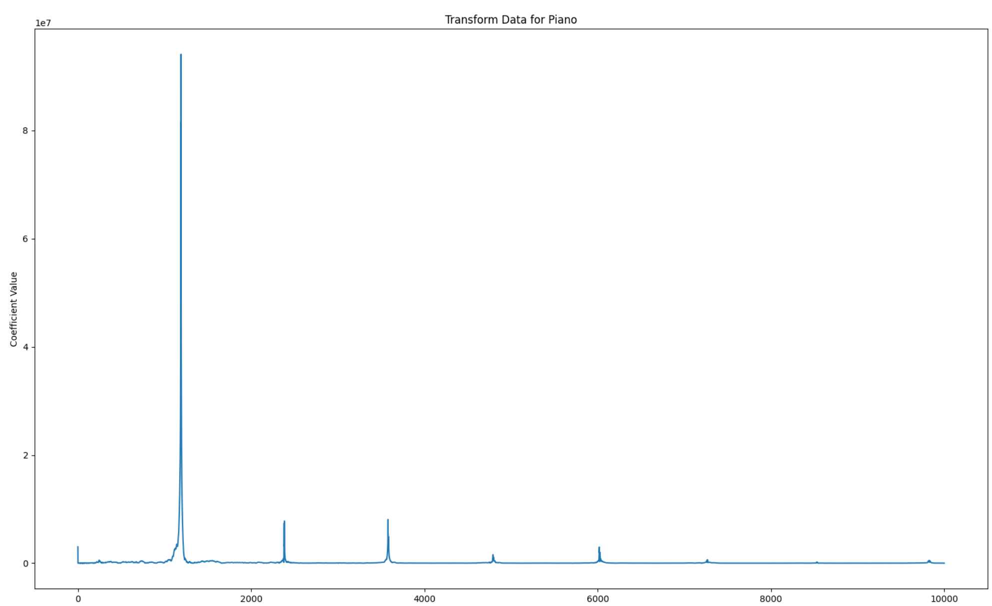

Figure 4: First 10000 coefficients from transform of piano waveform

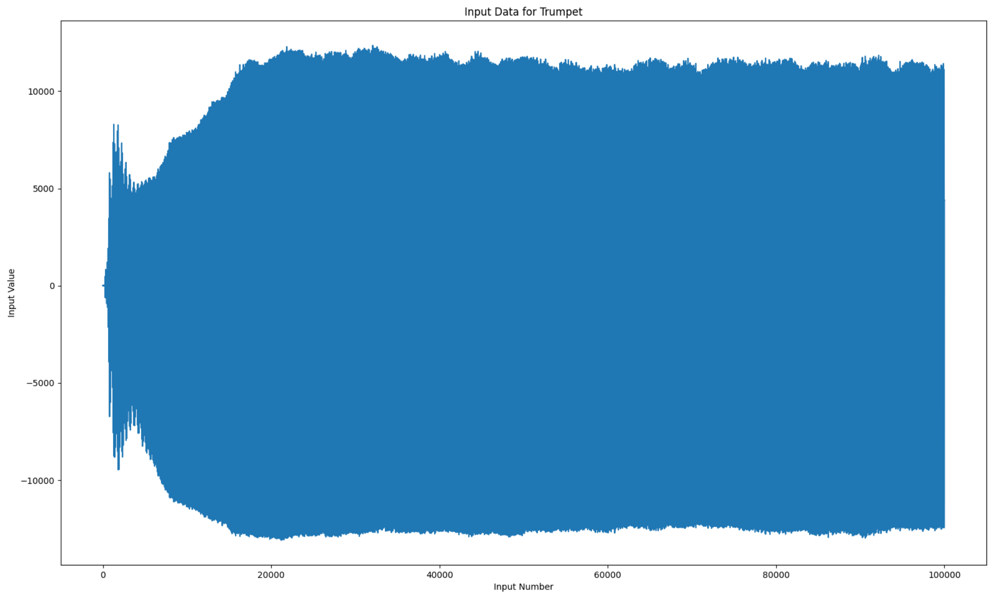

Figure 5: Input Waveform Data from Trumpet

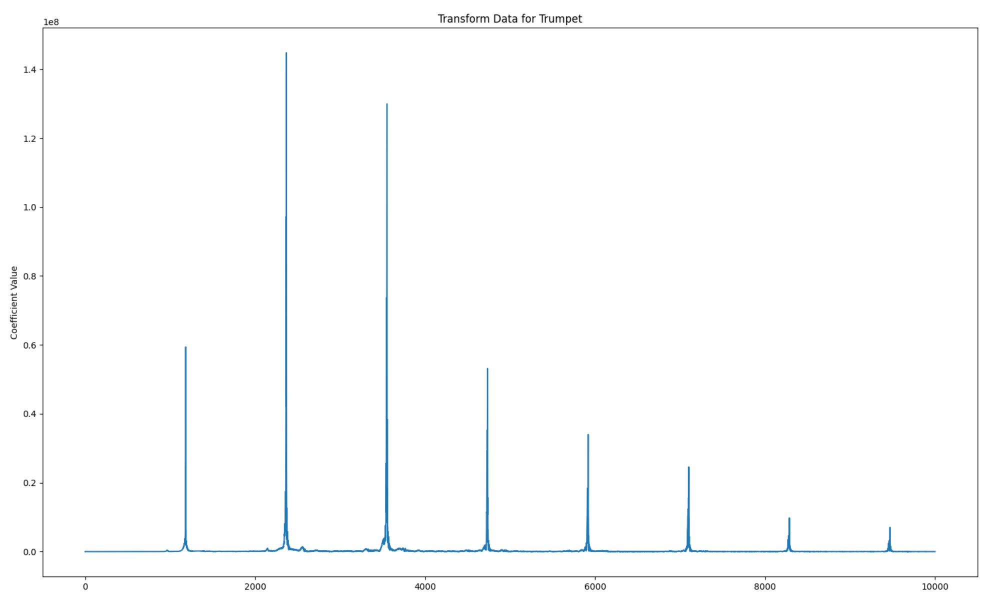

Figure 6: First 10000 coefficients from transform of trumpet waveform

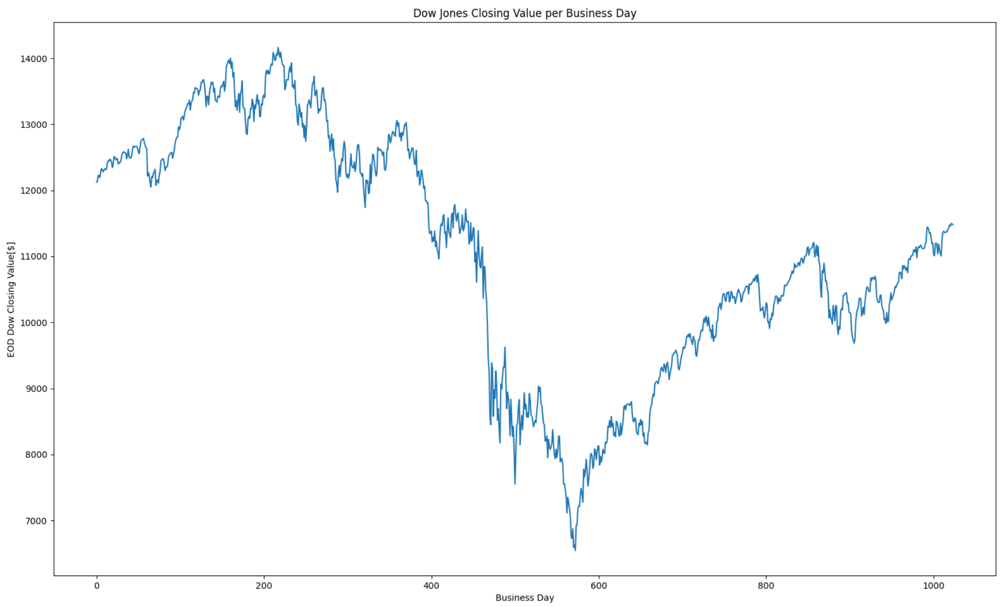

Figure 7: Daily EOD Dow Values between late 2006 and end of 2010.

1.3 Problem 3

1.3.1 Part (a)

The desired plot is seen in figure 7.

1.3.2 Part (b)

This calculation is done in the repo.

1.3.3 Part (c)

See the repo.

1.3.4 Part (d)

The desired plot is seen in figure 8. We see that much of the jaggedness and randomness from day to day has been smoothed out. This is because, when setting the higher Fourier coefficients to zero, we are not allowing sine waves with higher frequencies to contribute to our data, meaning we cannot have large short-time-scale variations. It's interesting to note that the Fourier expansion still captures the essence of the actual data.

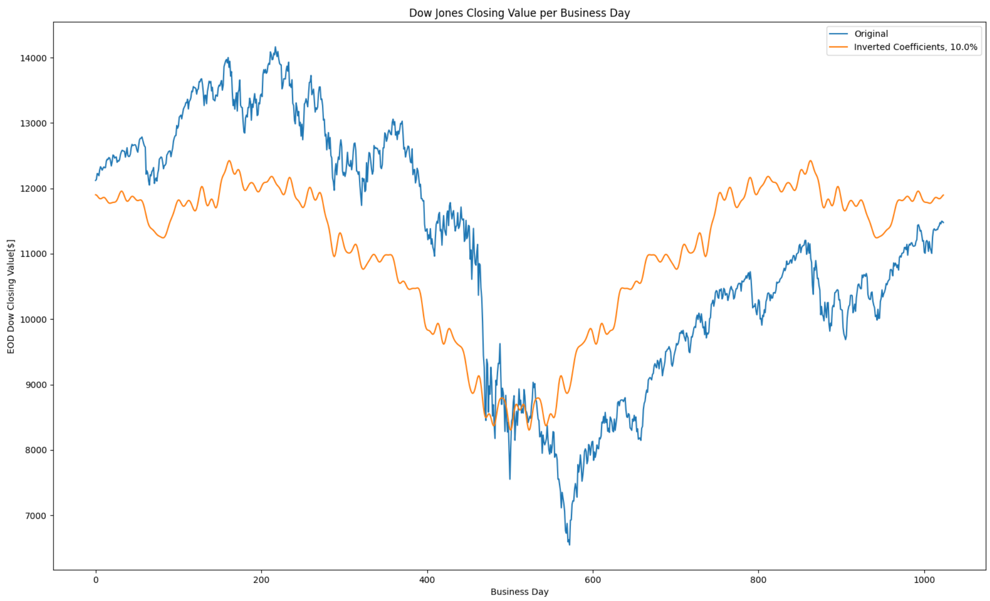

Figure 8: The first 10% of the Fourier coefficients used to construct the Dow Jones closing values.

1.3.5 Part (e)

The desired plot is seen in figure 9. Here we see even more than in part (d) that the jaggedness has been smoothed out. This is because we are now restricting to an even smaller set of relatively low frequencies, meaning the contributions to our Fourier expansion simply cannot have small time scale fluctuations. Here, too, it's interesting to note that the Fourier expansion still captures the essence of the actual data.

Figure 9: The first 2% of the Fourier coefficients used to construct the Dow Jones closing values.