Computational Physics: PS 5

Marcus Hoskins

October 8, 2024

1 Discussion

My GitHub repo is `https://github.com/marcusHoskinsNYU/phys-ga2000`. For images that are blurry here, please see the ps-5 folder for individual image files.

1.1 Problem 1

The desired functions are given in the github repo. Also, the analytic derivative of $f(x)$ is:

$$f'(x) = 1 - \tanh^2(2x).$$

The central difference, analytic, and jax autodiff results for the derivative of $f(x)$ are seen in figure 1

1.2 Problem 2

1.2.1 Part (a)

The desired code which plots the integrand of the gamma function is in the repo. The desired plots for $a = 2, 3, 4$ are seen in figure 2.

1.2.2 Part (b)

We wish to show that the maximum of the integrand of the gamma function, $f(x) = x^{a-1}e^{-x}$, falls at $x = a - 1$. Doing so:

$$\frac{df}{dx} = 0 = (a-1)x^{a-2}e^{-x} - x^{a-1}e^{-x},$$

which when solving for x gives $x = a - 1$, as expected.

1.2.3 Part (c)

We change variables to $z = \frac{x}{c+x}$. Then, if we set $z = \frac{1}{2}$ we trivially see that we must have $x = c$. So, if we want to put the maximum of f at $z = \frac{1}{2}$, and since we saw above that this maximum occurs at $x = a - 1$, we see that $c = a - 1$ puts the maximum at $z = \frac{1}{2}$.

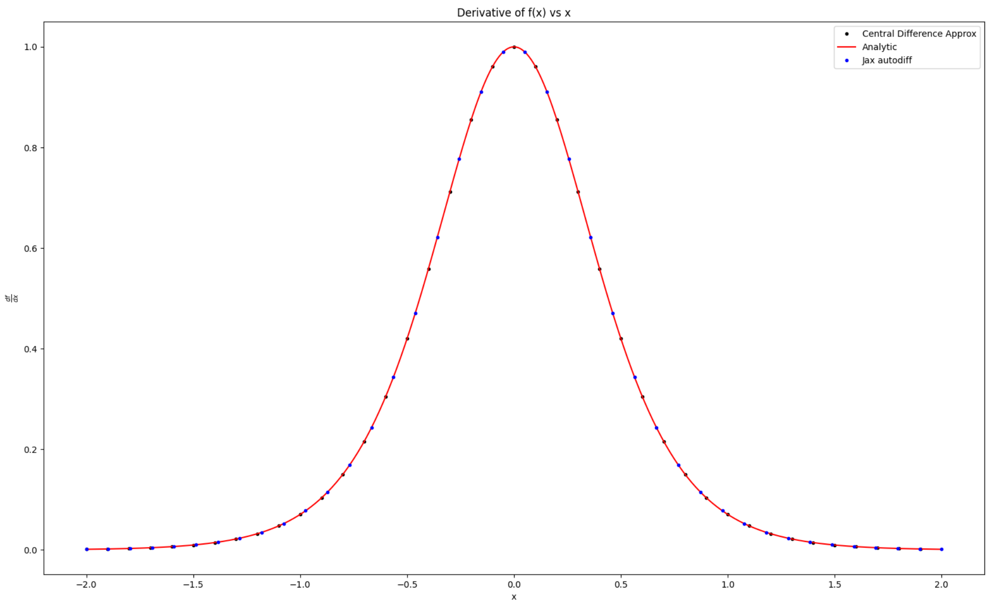

Figure 1: Central difference, analytic, and jax autodiff derivatives of the function $f(x) = 1 + \frac{1}{2}\tanh(2x)$.

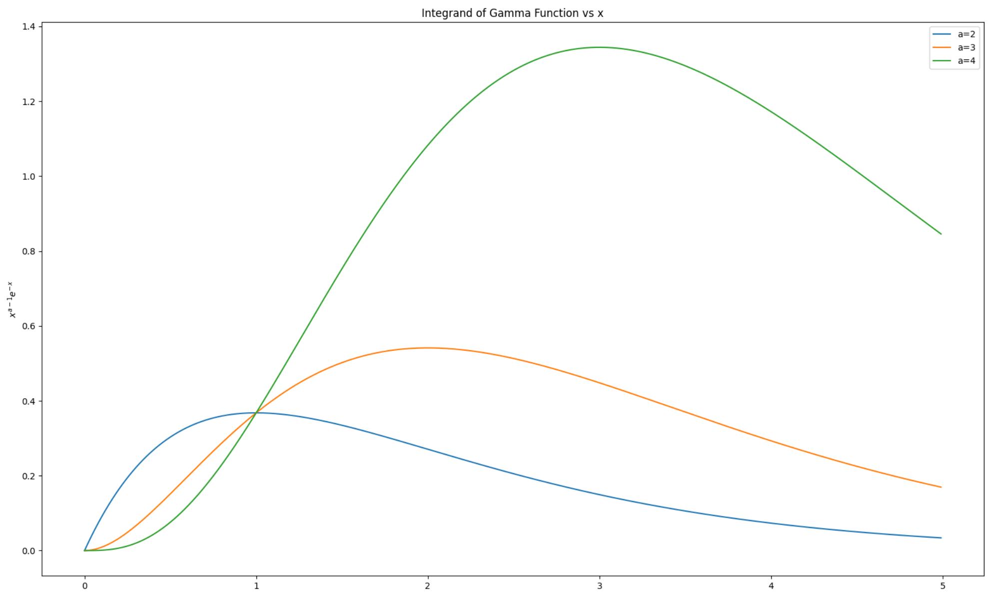

Figure 2: Plots of the integrand of the gamma function, $x^{a-1}e^{-x}$, for $a = 2, 3, 4$.

1.2.4 Part (d)

As described in the problem, the issue of $f(x)$ in its current form is that when x becomes large, x^{a-1} becomes large while e^{-x} becomes small. And, we know that large machine precision errors occur when we multiply a large number by a small number, which would occur in $f(x)$ at large x. So, we instead write $x^{a-1} = e^{(a-1)\ln(x)}$, meaning $f(x) = e^{(a-1)\ln(x)-x}$. Thus, for any x, $f(x)$ is the exponential of $(a-1)\ln(x) - x$, which will not give machine precision errors. The reason is that we are no longer multiplying large numbers by small numbers.

1.2.5 Part (e)

The desired function is given in the repo. We then use Gaussian quadrature integration with $n = 20$ sample points. Doing so, we find that $\Gamma(3/2) = 0.8862694302378592$, which is close to the expected value of $\frac{1}{2}\sqrt{\pi}$.

1.2.6 Part (f)

Using the same function as before, we find that $\Gamma(3) = 2.000002260422813$, $\Gamma(6) = 119.99997528617388$, and $\Gamma(10) = 362880.23320973915$, close to the expected values of $2! = 2$, $5! = 120$, and $9! = 362880$, respectively.

1.3 Problem 3

1.3.1 Part (a)

The plot of the given data is seen in figure 3.

1.3.2 Part (b)

Using the SVD technique to find the best third order polynomial fit of the form $a_0+a_1t+a_2t^2+a_3t^3$, we get the fit seen in figure 4. Here we have $a_0 = -0.07814319$, $a_1 = 1.16874352$, $a_2 = -0.22237109$, and $a_3 = -0.05547104$. Note here that the x-axis is the rescaled time, defined as $\frac{t-\mu_t}{\sigma_t}$, which must be used in order for the SVD technique to work.

1.3.3 Part (c)

The residuals of figure 4 are plotted in figure 5. We are given that all of the measurements have the same uncertainty of 2.0 in the signal units. Figure 5, however, has some periodic uncertainty which is not always 2.0. Thus, this is not a good explanation of the data.

1.3.4 Part (d)

In figure 6 we see a fit of the data with a polynomial of order 50. At the very least the polynomial has to be of order 15, as this is the number of times the data passes the x-axis. We found that an order 50 fit gave a better result than order 15, hinting that this data is actually some power series

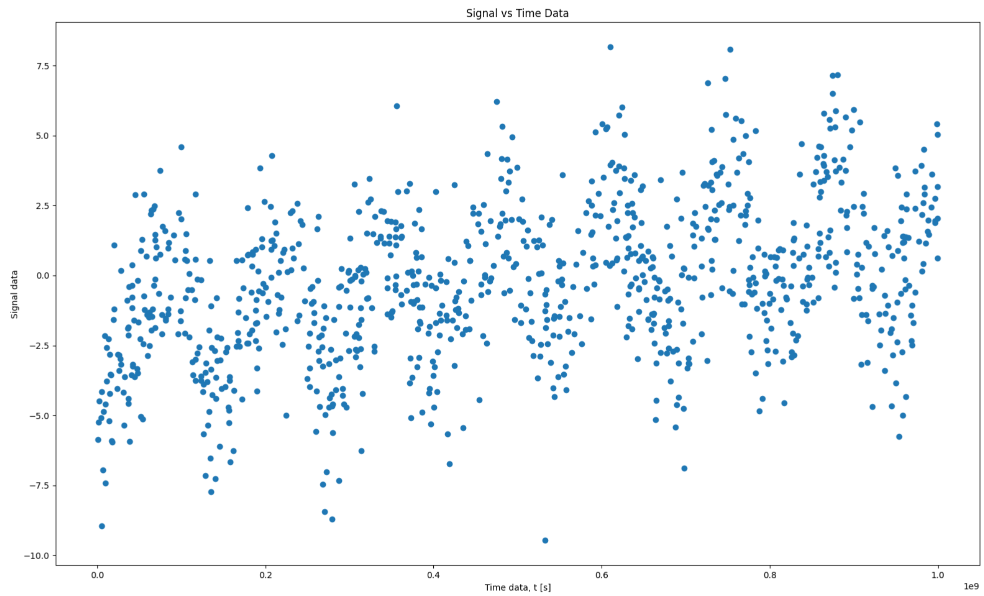

Figure 3: This is a plot of the provided signal data.

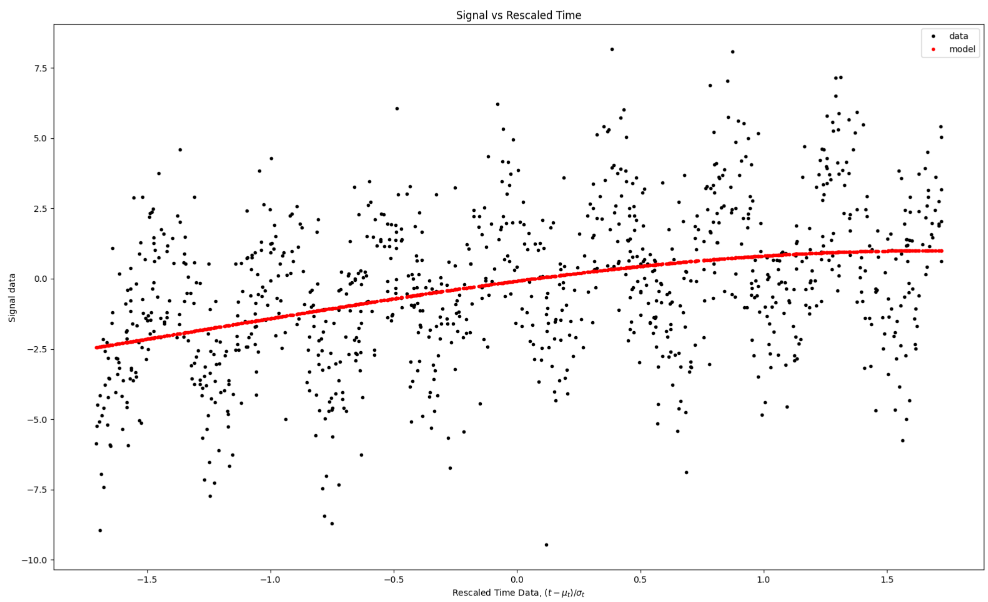

Figure 4: Third order polynomial fit of the data using the SVD technique. Here we have signal vs rescaled time.

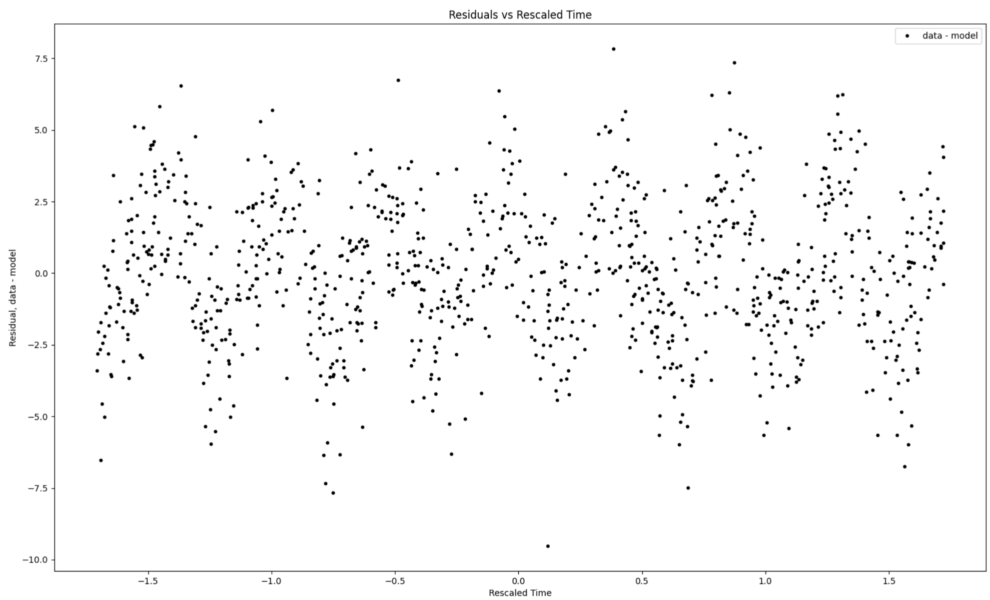

Figure 5: Residuals of figure 4.

in t that goes as a sum of sines and cosines. For some reason, an order 100 fit gave worse results than an order 50 one. One would imagine that if this data is in fact a sum of sines and cosines, the larger the order the better.

1.3.5 Part (e)

Finally fitting the data as a sum of sines and cosines, we see the results in figure 7. This seems to do a good job of explain the data, with model points lying close to the data points. In fact, the residuals are plotted in figure 8. We see that the values here are more constant than in figure 5, with the mean of the absolute value being 1.6767, fairly close to the true uncertainty of 2. Once again, the number of sines and cosines we have is 50, as this gives the best results. Each additional sine/cosine added has a different frequency, as in a Fourier series. From the fit, we see that the typical frequency (in units of the rescaled time variable) is about 2 Hz.

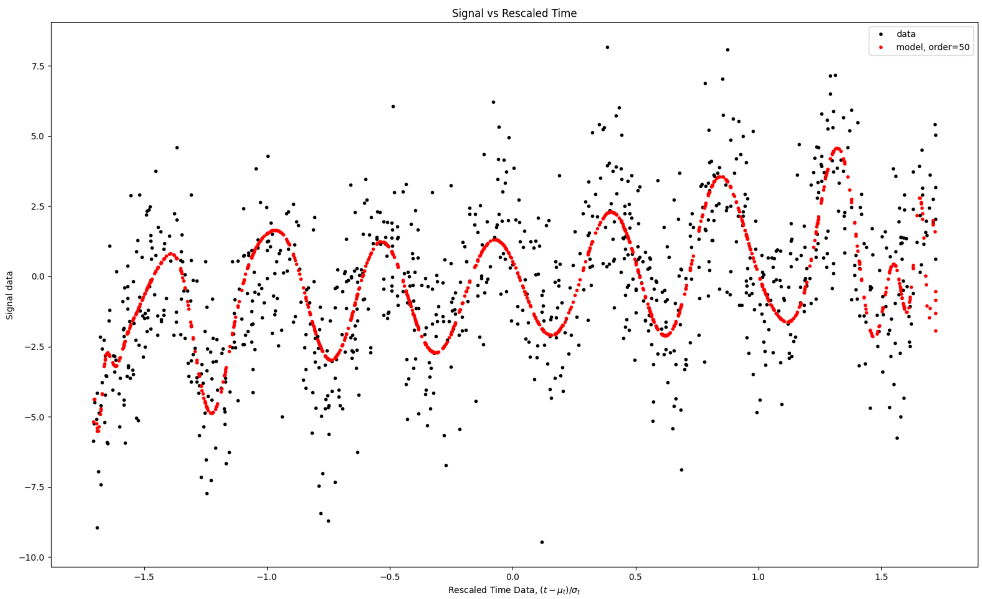

Figure 6: Order 50 polynomial fit of our signal data.

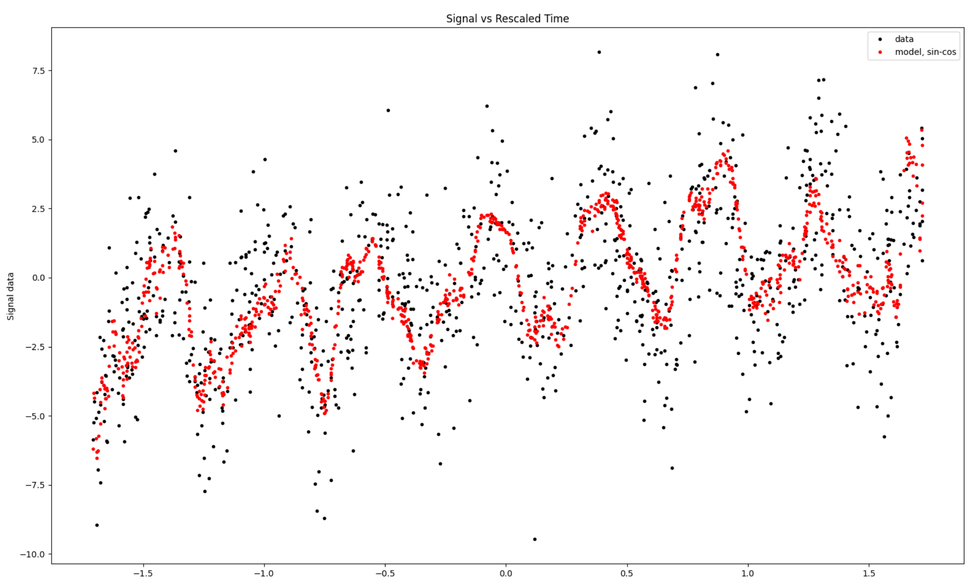

Figure 7: Fit of our signal data using a sum of sines and cosines, still using the SVD technique.

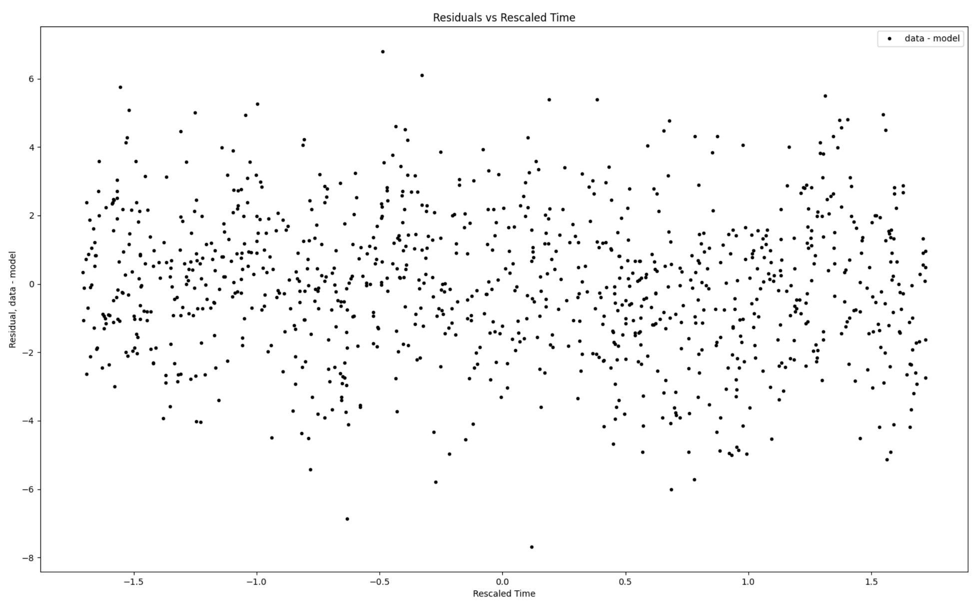

Figure 8: Residuals of figure 7