Computational Physics: PS 6

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1 Discussion

My GitHub repo is `https://github.com/marcusHoskinsNYU/phys-ga2000`. For images that are blurry here, please see the ps-6 folder for individual image files.

1.1 Problem 1

1.1.1 Part (a)

The spectrum of a handful of galaxies is seen in figure 1. One feature to notice is that there are many spikes in the various spectra around the 3.82 x-value, which corresponds to a wavelength of $\lambda = 6606$ Angstroms, which is right around the emitted wavelength of light for an $n = 3$ to $n = 2$ Balmer series transition in the Hydrogen atom.

1.1.2 Part (b)

The code for this is in the repo.

1.1.3 Part (c)

The code for this is in the repo.

1.1.4 Part (d)

The first five eigenvectors of C are seen in figure 2. This is a density plot, and note that the majority of the entries of these 4001-entry vectors hover around 0. However, there are clear strips of entries that deviate from this.

1.1.5 Part (e)

Using the SVD method, we find the eigenvectors are in fact the same as those found in part (d) above, up to a factor of -1, which is perfectly allowed. This is seen by taking their difference and also by visual inspection. The columns that don't cancel when taking the difference in fact differ by a factor of -1, and hence their sum is 0. We can visually see that the two methods produce

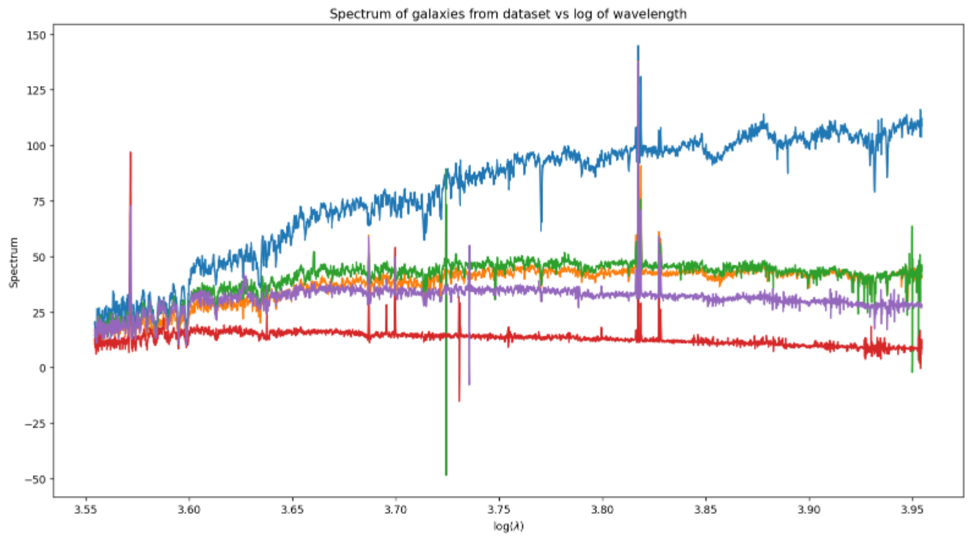

Figure 1: A handful of spectra from the galaxy data as a function of wavelength. The way to read this is that the y-values are the flux of light with the wavelength given by the corresponding x-value.

the same eigenvectors (again, up to a factor of -1) by comparing figures 3 and 4. Since we are physicists, this visual inspection of 6 entries per 4001-entry vector suffices as a proof.

On my machine, the computational cost of the direct diagonalization method of part (d) is around 6 seconds, whereas the cost for the SVD method here is about 30 seconds.

1.1.6 Part (f)

Using the hint as a springboard, one reason you may want to use SVD instead of constructing the covariance matrix and finding its eigenvectors is that the condition number of C is larger than that of R. That is, if we vary R slightly, C varies as the square of this change. So, effects are amplified in the covariance matrix approach, meaning it is more numerically unstable.

1.1.7 Part (g)

For the life of me I could not figure out how to reproduce the spectra properly. I believe where I'm at now is that I am getting the coefficients wrong. I don't know how to fix this. I even took the L with turning this in a day late, and still couldn't figure it out.

1.1.8 Part (h)

If I had done part (g) correctly, this part would be very easy. So, here is a photo of grilled cheese instead (c.f. figure 5).

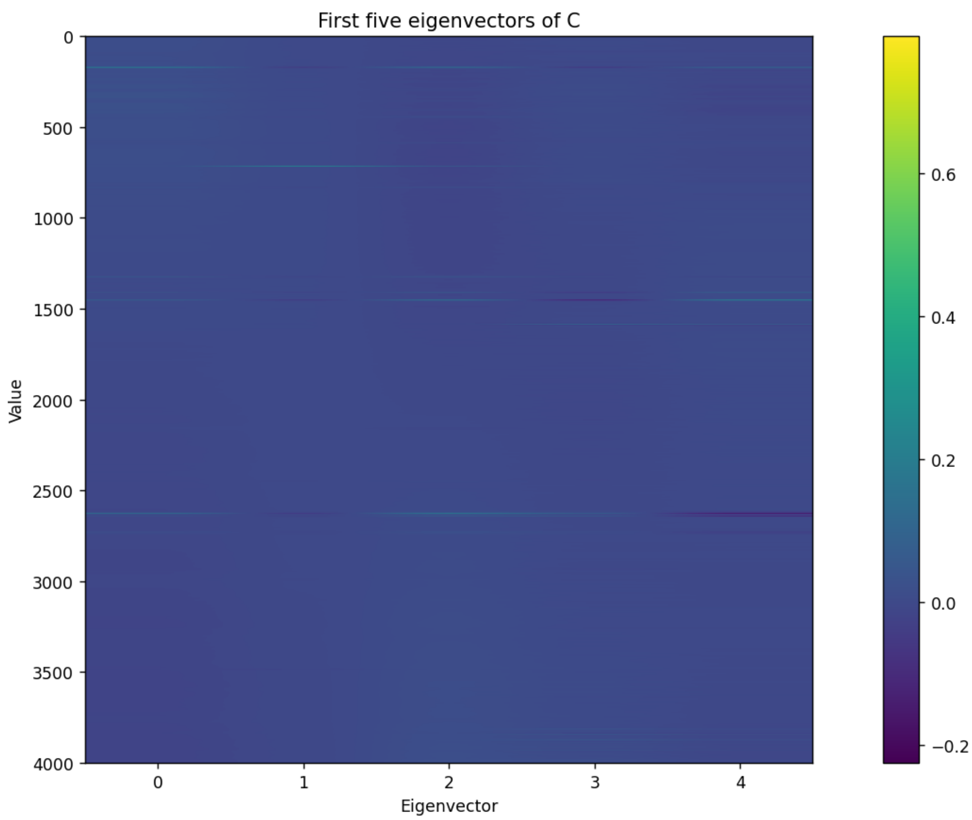

Figure 2: The first five eigenvectors of C. The aspect ratio here is 5:4001, so as to make the plot square. The majority of the entries of each vector hover around 0. However, there are clear strips that deviate from this.

```
[[ 0.01938475  0.00103864  0.00554674 ...  0.00105804 -0.00073563
  -0.00790685]
 [ 0.0198138   0.00121386  0.00635431 ...  0.00093607 -0.00033983
  -0.01581223]
 [ 0.01897847  0.00093465  0.00601822 ... -0.0012731   0.00118511
  -0.01581236]
 ...
 [-0.01683239 -0.00533577 -0.0212463  ... -0.00278504 -0.00106933
  -0.01581282]
 [-0.01672212 -0.00534546 -0.02117792 ...  0.00185148  0.00142532
  -0.01581212]
 [-0.01641501 -0.00545605 -0.0210479  ... -0.00108516 -0.00236447
  -0.00790622]]
```

Figure 3: Eigenvectors of the covariance matrix. The columns of the matrix are the actual eigenvectors. Notice how these are the same as those in figure 4, up to a factor of -1.

```
[[ 0.01938475  0.00103866 -0.00554674 ... -0.00104449  0.00073167
  -0.0079096 ]
 [ 0.0198138   0.00121387 -0.00635431 ... -0.00098387  0.00036038
  -0.01580746]
 [ 0.01897847  0.00093466 -0.00601822 ...  0.00126573 -0.00117832
  -0.01580941]
 ...
 [-0.01683239 -0.00533582  0.02124629 ...  0.00278856  0.00108613
 [-0.01672212 -0.00534551  0.02117791 ... -0.00187492 -0.00142488
  -0.01581124]
 [-0.01641501 -0.0054561   0.02104789 ...  0.00110322  0.00235153
  -0.00790463]]
```

Figure 4: Eigenvectors found from SVD. The columns of the matrix are the actual eigenvectors. Notice how these are the same as those in figure 3, up to a factor of -1.

Figure 5: Cheese (grilled).

1.1.9 Part (i)

Assuming I had done part (g) properly, this part would also be very easy. Please see figure 5 to clarify any confusions.