Computational Physics: PS 3

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1 Discussion

My GitHub repo is `https://github.com/marcusHoskinsNYU/phys-ga2000`.

1.1 Problem 1

Testing the results of example 4.3 of Newman, we find that for the explicit matrix multiplication, the run time scales as N^3, as expected from Newman. Our actual results are seen in figure 1.

The results for something like the dot() method of matrix multiplication in python are seen in figure 2. All but the last N value have running times that are smaller than 10^{-19} seconds, and so we could not analyze the scaling behavior in N. Clearly the dot() method is much more efficient, and has some more complicated behavior in N.

1.2 Problem 2

Following the instruction given in exercise 10.2 of Newman, all four atoms numbers as functions of time are given in figure 3. And, as we can't quite see the profile of Pb209 and Tl209, these plots are given respectively by figures 4 and 5.

1.3 Problem 3

Following the instructions of Newman exercise 10.4, the desired plot is given in figure 6.

1.4 Problem 4

From the given problem, let's compute the mean and variance of y. To begin:

$$\mu_y = \mathbb{E}[y] = \frac{1}{N}\sum_{i=0}^{N}\mathbb{E}[x_i] = \frac{1}{N}\sum_{i=0}^{N}\int_0^\infty xe^{-x}x = \frac{N+1}{N},$$

where we have used the linearity of the expectation value. Notice that at large N, we expect the mean to go as 1. Then, for the variance, we first assume that each of the x_i are identically and

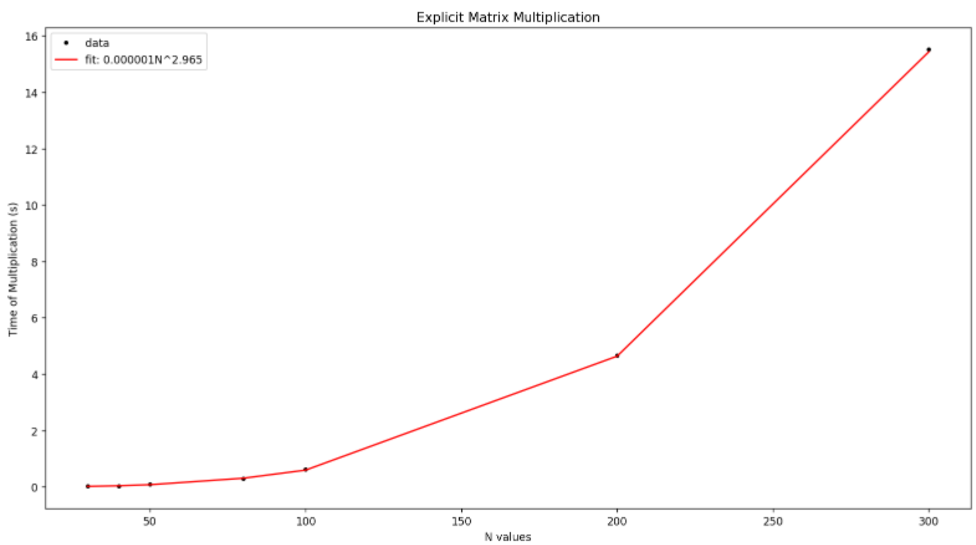

Figure 1: Run time vs N of explicit matrix multiplication of $N \times N$ matrices. Clearly this scales as N^3.

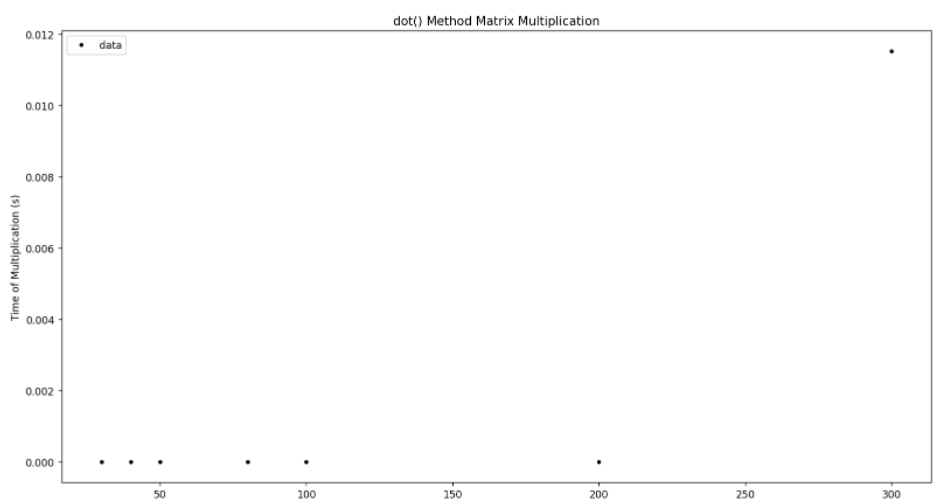

Figure 2: Run time vs N of dot() matrix multiplication. All but the last N value have running times that are smaller than 10^{-19} seconds, so this method was not amenable to a log power analysis.

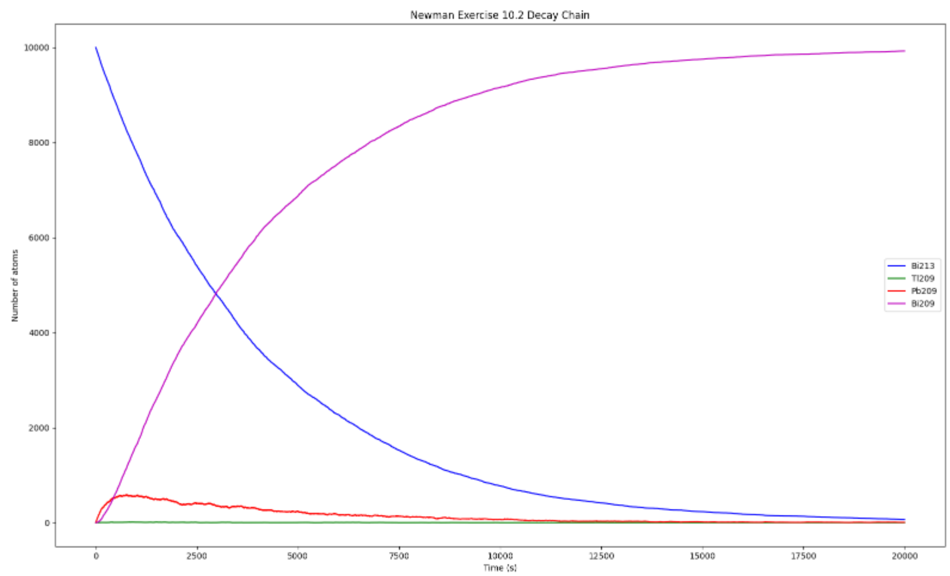

Figure 3: All four atom numbers as functions of time from Newman exercise 10.2. This describes the radioactive decay of Bi213 into Bi209 via Pb209 and Tl209.

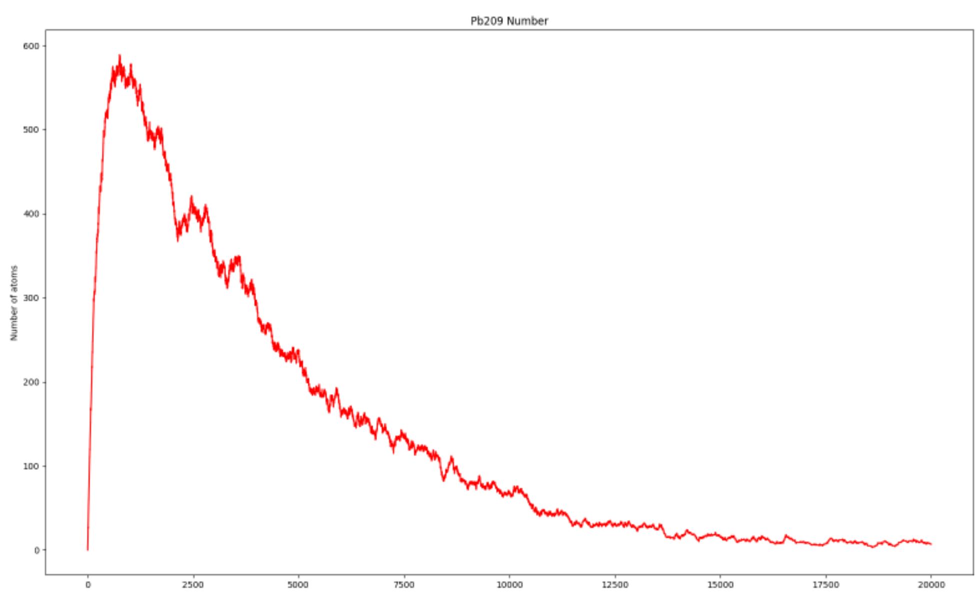

Figure 4: A zoomed in plot of 3, focusing just on Pb209.

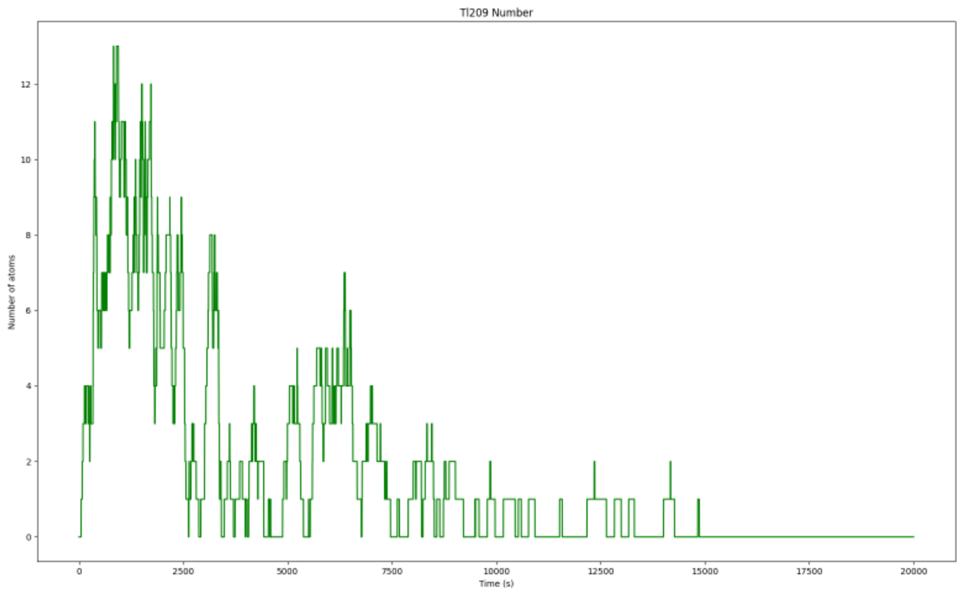

Figure 5: A zoomed in plot of 3, focusing just on Tl209.

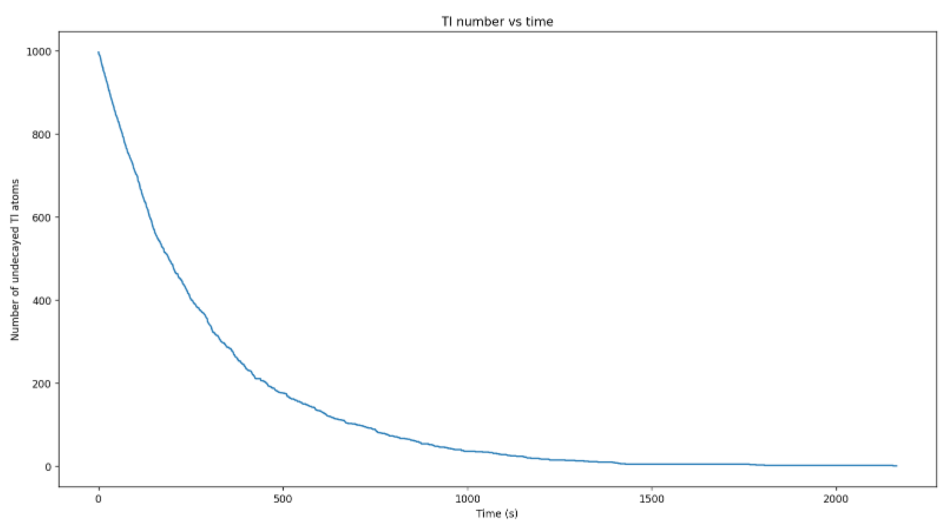

Figure 6: Number of Tl208 atoms as a function of time, computed using the more efficient transformation method.

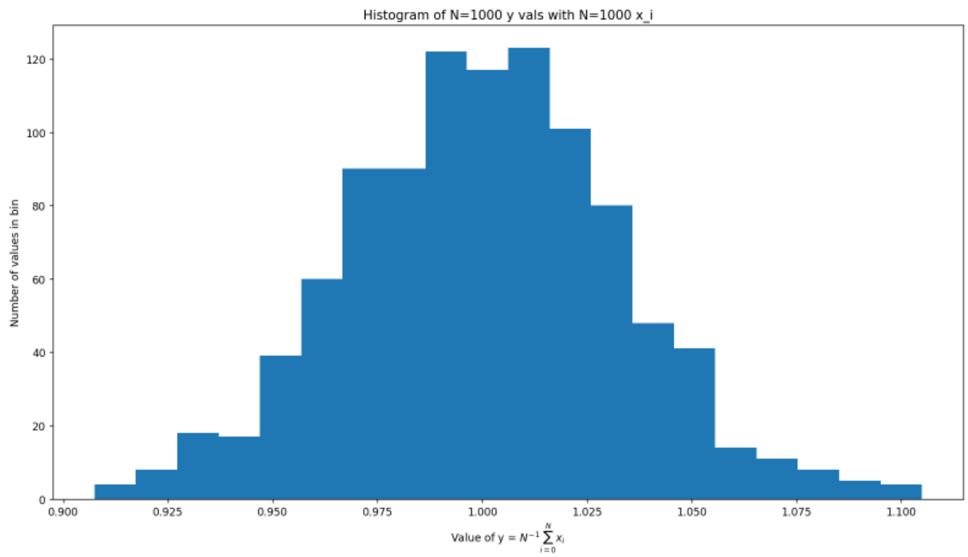

Figure 7: Visual proof of the Central Limit theorem. As N becomes large, the distribution of y tends towards a Gaussian.

independently distributed. Doing so, the variance then also becomes linear, with the caveat that when we pull out a constant, we must square it. Thus, for us:

$$\sigma_y^2 = \mathrm{var}(y) = \frac{1}{N^2}\sum_{i=0}^{N}\left(\mathbb{E}[x_i^2] - \mathbb{E}[x_i]^2\right).$$

Then, using the result from above for the mean:

$$\mathrm{var}(y) = \frac{1}{N^2}\sum_{i=0}^{N}\left(\int_0^\infty x^2 e^{-x}x - \left(\frac{N+1}{N}\right)^2\right) = \frac{1}{N^2}\sum_{i=0}^{N}\left(2 - \frac{N^2+2N+1}{N^2}\right) = \frac{N+1}{N^2}\left(1 - \frac{2}{N} - \frac{1}{N^2}\right).$$

In the large N limit, this variance goes as $\frac{1}{N}$.

As N becomes large (here $N = 1000$), the distribution of y tends towards a Gaussian, as seen in figure 7.

Then, the ways that the mean, variance, skewness, and kurtosis of the distribution change as functions of N are shown in figure 8. The formulas for skewness and kurtosis I used are:

$$\mathrm{skew}(y) = \mathbb{E}\left[\left(\frac{y-\mu_y}{\sigma_y}\right)^3\right] = \frac{1}{\sigma_y^3}\left(\mathbb{E}[y^3] - \mu_y\sigma_y^2 - \mu_y^3\right)$$

and

$$\mathrm{kurt}(y) = \mathbb{E}\left[\left(\frac{y-\mu_y}{\sigma_y}\right)^4\right] = \frac{1}{\sigma_y^4}\left(\mathbb{E}[y^4] + 6\mu_y^2\sigma_y^2 - 4\mu_y\mathbb{E}[y^3] + 3\mu_y^4\right),$$

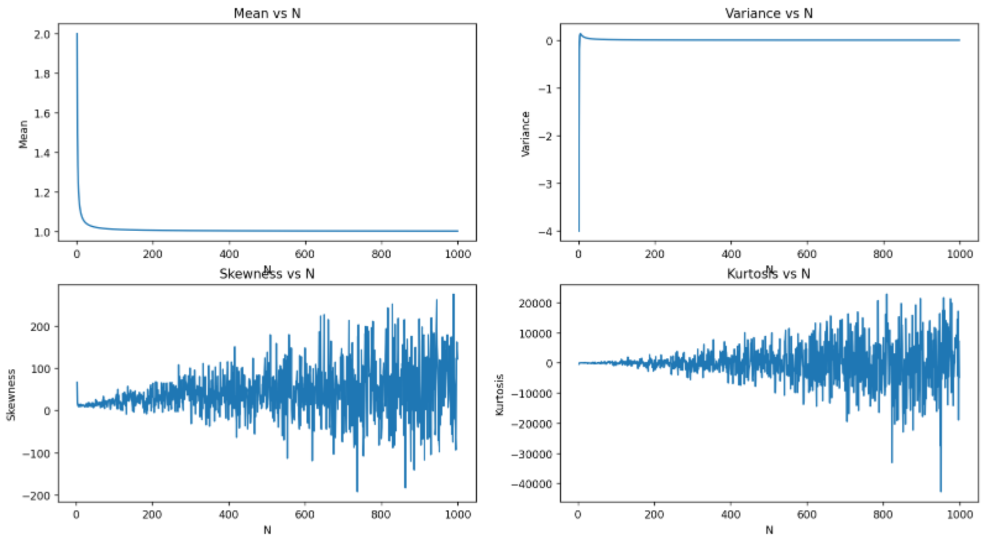

Figure 8: Plots of how the mean, variance, skewness, and kurtosis change as functions of N.

which are both easily derived from their respectively given definitions.

Using these formulas, the skewness reaches about 1% of its $N = 1$ value at $N = 45$ and the kurtosis never reaches 1% of its $N = 1$ value.