

Problem Set

1. Find the values of x and y if $\frac{b^x}{b^y} = b^9$ and $\frac{b^x \cdot b^2}{b^{3y}} = b^{13}$.

$$x - y = 9$$

$$x + 2 - 3y = 13$$

$$y + 9 + 2 - 3y = 13$$

$$y + 11 - 3y = 13$$

$$\begin{array}{r} -2y + 11 = 13 \\ -11 \quad -11 \end{array}$$

$$\frac{-2y}{-2} = \frac{2}{-2} \quad y = -1$$

$$x - (-1) = 9 \quad x + 1 = 9, \quad x = 8$$

$$x + 2 - 3(-1) = 13$$

$$x + 2 + 3 = 13$$

$$x + 5 = 13$$

$$x = 8$$

2. The speed of light is about 3.0×10^8 m/s. How long does it take sunlight to reach Jupiter, if its average distance to the sun is 7.8×10^8 kilometers? (Hint: watch out for the prefix!)

$$7.8 \times 10^8 \text{ km} \times 10^3 \frac{\text{m}}{\text{km}} = \frac{7.8 \times 10^{11}}{3.0 \times 10^8} = 2.6 \times 10^3$$

3. Rewrite each expression as a power of a product:

$$8a^3b^3 = (2 \cdot a \cdot b)^3$$

$$16x^2y^2 = (8xy)^2$$

$$64p^{18}q^{12} = (2 \cdot p^3 \cdot q^2)^6$$

$$81x^4y^8 = (3xy^2)^4$$

4. If $f(x) = 2\sqrt{x-3}$, what is the value of $f(19)$?

$$\begin{array}{l} f(19) = 2\sqrt{19-3} \\ 2\sqrt{16} = 8 \end{array}$$

$$f(19) = 8$$

5. Simplify:

$$\sqrt{27} = \sqrt{3 \cdot 3 \cdot 3} = 3\sqrt{3}$$

$$\sqrt{-16x^3} = \sqrt{-1 \cdot 4 \cdot 4 \cdot x \cdot x \cdot x}$$

$$\sqrt{-1} \sqrt{4} \sqrt{4} \sqrt{x} \sqrt{x} \sqrt{x}$$

$$4xi\sqrt{x}$$

$$32^{1.2} = 32^{6/5} = 32 \cdot 32^{1/5}$$

$$32 \sqrt[5]{32} = 32 \sqrt[5]{2^5}$$

$$32 \cdot 2 = \boxed{64}$$

Evaluate each expression. Remember, only four-function calculators are allowed:

6. $(-7)^0 = 1$

7. $4^0 = 1$

8. $5^{-4} = \frac{1}{5^4} = \frac{1}{625}$

9. $(-2)^{-5} = -1(2)^{-5}$

10. $\frac{2^{-4}}{3^2} = \frac{3^2}{2^4} = \frac{9}{16} = 194$

11. $\frac{-3^{-3}}{(-8)^{-2}} = \frac{(-1)(-8)^2}{3^3}$

$-1\frac{1}{32}$

12. A microscope magnifies an object 10^5 times. If the length of an object is 3.2×10^{-7} meters, what is its magnified length?

$100,000$
 $10^5 \cdot 10^{-7} = 10^{-2}$
 $3.2 \times 10^{-2} = 0.032$

13. The area of a rectangular computer chip is $112a^3b^2$ square microns. If its width is $8ab$ microns, what is its length?

$\frac{112a^3b^2}{8ab}$
 $14a^2b$

14. Coulomb's Law describes the electric force F (in Newtons) between two particles with charges q_1 and q_2 (in Coulombs) that are separated by a distance r (in meters):

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

Where $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2$ is a universal constant. What is the electric force between two particles of charges $q_1 = 3.2 \mu\text{C}$ (that's microCoulombs) and $q_2 = 6.1 \mu\text{C}$ that are separated by a distance of 12×10^{-8} meters? (Hint: convert the microCoulombs into Coulombs. Do you remember what the prefix micro means?)

$$F = \frac{1}{4\pi(8.85 \times 10^{-12})} \cdot \frac{(3.2 \times 10^{-6})(6.1 \times 10^{-6})}{(12 \times 10^{-8})^2}$$

$10^{-12} \cdot 35.4\pi \times 10^{-12}$
 $\frac{19.52 \times 10^{-12}}{144 \times 10^{-16}}$

Describe and correct the error in simplifying the expressions:

15. $2^4 \cdot 2^5 = 4^9$

2^9

$\frac{1}{35.4\pi} \cdot \frac{19.52}{144 \cdot 10^{-16}}$
 $\frac{1}{35.4\pi} \cdot \frac{19.52 \cdot 10^{16}}{144}$

16. $\frac{x^5 \cdot x^3}{x^4} = \frac{x^8}{x^4} = x^2$

x^4

$\frac{19.52 \times 10^{16}}{16,014.58}$
 0.0012×10^{16}

7. On a farm, about 9.46×10^{-1} pounds of potatoes were harvested from a 2.3×10^{-5} acre field. How many potatoes were harvested per acre?

$$\frac{9.46 \times 10^{-1}}{2.3 \times 10^{-5}}$$

$$\frac{0.946}{0.000023} = 41,130$$

18. Diffusion is the movement of molecules from one location to another. The time t (in seconds) it takes molecules to diffuse a distance of x centimeters is given by $t = \frac{x^2}{2D}$, where D is the diffusion coefficient. The diffusion coefficient for a drop of ink in water is about 2×10^{-5} square centimeters per second. How long will it take the ink to diffuse 3.4 micrometers (1 micrometer = 10^{-4} centimeter)?

$$2(2 \times 10^{-5})$$

$$3.4 \mu\text{m} \cdot \frac{10^{-4} \text{ cm}}{1 \mu\text{m}} = 3.4 \times 10^{-4} \text{ cm}$$

$$t = \frac{(3.4 \times 10^{-4})^2}{2(2 \times 10^{-5})} = \frac{11.56 \times 10^{-8}}{4 \times 10^{-5}} = \frac{11.56 \times 10^{-3}}{4}$$

$$2.89 \times 10^{-3}$$

19. If a cube has a side length of $6x$, write and simplify expressions for the surface area and volume of the cube.

20. Explain the steps used to solve the equation $3(2^{3x-4} - 5) = 27$

Step 1	$3 \cdot 2^{3x-4} - 15 = 27$	3×5
Step 2	$3 \cdot 2^{3x-4} = 42$	-15
Step 3	$2^{3x-4} = 14$	$\div 3$
Step 4	$\frac{2^{3x}}{2^4} = 14$	$x^{-a} = \frac{1}{x^a}$
Step 5	$2^{3x} = 224$	$2^4 \cdot 14$
Step 6	$8^x = 224$	2^3
Step 7	$x = 2$	$\div 8$

Use the rules of exponents to simplify. Negative exponents are usually considered unsimplified, so try to eliminate them.

$$21. 4x^8 \cdot 9x^3 = 36^{11}$$

$$22. \frac{4a^3 a^2}{a} =$$

$$23. 3x^{-3} \cdot x^2 = 3 \cdot x^{-1} = \frac{3}{x}$$

$$24. (3x^2 y)^3 \cdot (2x^2) = 54 \cdot 27x^2 \cdot 2x^5 \cdot x^7 \cdot 2y^3$$

$$(27x^5 y^3) \cdot (2x^2)$$

$$25. (4xy^2)^4 (x^3)^{-2} =$$

$$256x^4 y^8 \cdot x^{-6} = 256 \cdot x^4 \cdot x^{-6} \cdot y^8$$

$$= 256x^{-2} y^8$$

$$26. \frac{2x^2}{y^5} \cdot 3x^2 y^3$$

$$27. 18 \left(\frac{-4}{3x^5} \right)^3 =$$

$$28. \left(\frac{3^2}{x^3} \right)^{-2} \left(\frac{x^4}{2^2} \right)^2 = \frac{3^{-4}}{x^{-6}} \cdot \frac{x^8}{2^4}$$

$$29. \left(\frac{3x^2}{2y^5} \right)^{-3} = \frac{3^{-3} x^{-6}}{2^{-3} y^{-15}} = \frac{2^3 y^{15}}{3^3 x^6} = \frac{8 y^{15}}{27 x^6}$$

$$30. (6x^{-2})^3 = \frac{3^{-4} \cdot x^8}{x^{-6} \cdot 2^4}$$

$$(a \cdot b)^2 = a^2 b^2$$

$$6^3 x^{-6} = \frac{6^3}{x^6} = \frac{216}{x^6} \cdot \frac{3^{-4} \cdot x^{14}}{3^4 \cdot 2^4} = \frac{216 \cdot 3^{-4} \cdot x^{14}}{3^4 \cdot 2^4 \cdot x^6}$$

Distribute and Simplify:

$$31. 6x^2 y (3xy^2 + y^4 + \frac{1}{x}) = 18x^3 y^3 + 6x^2 y^5 + \frac{6x^2 y}{x}$$

$$18x^3 y^3 + 6x^2 y^5 + 6xy$$

$$32. (4x^2 + 3x)^2 = (4x^2 + 3x)(4x^2 + 3x) = 16x^4 + 12x^3 + 12x^3 + 9x^2$$

$$16x^4 + 24x^3 + 9x^2$$

Perform the following calculations. Express your answers in scientific notation.

13. $60(4.3 \times 10^3) =$

14. $(6.8 \times 10^{13})^2 =$

15. $(8.2 \times 10^4) \cdot (3.6 \times 10^{-5}) =$

16. $(5.4 \times 10^8) + (2.6 \times 10^7) =$

17. $\frac{6.2 \times 10^{13}}{2 \times 10^5} =$

Simplify the root:

18. $\sqrt{80} =$

19. $\sqrt{72} =$

20. $\sqrt[3]{16} =$

21. $\sqrt[3]{81} =$

22. $\sqrt{-25} =$

23. $\sqrt{-144} =$

24. $\sqrt{24x^2} =$

25. $\sqrt[3]{-8} =$

Evaluate each rational exponent without a calculator by first converting it into a radical:

26. $81^{1/2} =$

27. $125^{1/3} =$

28. $16^{3/4} =$

29. $32^{0.6} =$

30. If $8^{2x-3} = 4$, what must be the value of x ? Solve without a calculator

Selected Answers

SAT Answers: 4. A 6. D 14. A 20. 100 17. 19

Problem Set Answers: 2. $8x^5 - 12x^3 + 4x^2$ 3. $125x^8$ 4. $\frac{x^4}{3}$ 5. $\frac{3}{x^3}$ 6. $x^2 - \frac{2}{3}x + \frac{1}{9}$

7. 2.3×10^5 8. 5.838×10^4 9. 7.1×10^{-4} 10. 73000 11. 0.0000234 12. 576000000

13. 2.58×10^5 14. 4.624×10^{27} 15. 2.952 16. 5.66×10^8 17. 3.1×10^8

18. $4\sqrt{5}$ 19. $6\sqrt{2}$ 20. $2\sqrt[3]{2}$ 21. $3\sqrt[3]{3}$ 22. $5i$ 23. $12i$ 24. $2x\sqrt{6}$ 25. -2 26. 9 27. 5 28. 8 29. 8

SI Prefixes

10^{-9} ... 10^{-6} ... 10^{-3} ... 10^{-2} ... 10^{-1}
 nano micro milli centi deci
 n μ m c d

10^1 ... 10^2 ... 10^3 ... 10^6
 deca hecto kilo mega
 da h k M

Unit Symbols

Name	Symbol	Quantity
ampere	A	Electric current
coulomb	C	Electric charge
farad	F	Capacitance
hertz	Hz	Frequency
joule	J	Energy
kelvin	K	Temperature
kilogram	kg	Mass
meter	m	Distance
newton	N	Force
ohm	Ω	Resistance
pascal	Pa	Pressure
second	s	Time
tesla	T	Magnetic Field Strength
volt	V	Electric potential
watt	W	Power

Using SI Prefixes

Rewrite each quantity in standard units using scientific notation:

The distance to my house is 2.56 km = $2.56 \times 10^3 \text{ m}$

The charge of the particle is 583 μC = $583 \times 10^{-6} \text{ C}$

The total power used by this motor is 6.84 MW = $6.84 \times 10^6 \text{ W}$

The experiment lasted for 2.4 ns = $2.4 \times 10^{-9} \text{ s}$

The tip of the pen was 0.38 cm = $0.38 \times 10^{-2} \text{ m}$

The force on the brakes was 2040 kN = $2040 \times 10^3 \text{ N}$

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