[redacted]

Imports

In [ ]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns

import os

import statsmodels.api as sm
import scipy
from scipy.stats import multivariate_normal
import itertools

!pip install torch
import torch

!pip install ipca
import ipca
```

```
Requirement already satisfied: torch in c:\users\marcu\anaconda3\lib\site-pa
ckages (2.7.0)
Requirement already satisfied: filelock in c:\users\marcu\anaconda3\lib\site
-packages (from torch) (3.13.1)
Requirement already satisfied: typing-extensions>=4.10.0 in c:\users\marcu\a
naconda3\lib\site-packages (from torch) (4.11.0)
Requirement already satisfied: sympy>=1.13.3 in c:\users\marcu\anaconda3\lib
\site-packages (from torch) (1.14.0)
Requirement already satisfied: networkx in c:\users\marcu\anaconda3\lib\site
-packages (from torch) (3.3)
Requirement already satisfied: jinja2 in c:\users\marcu\anaconda3\lib\site-p
ackages (from torch) (3.1.4)
Requirement already satisfied: fsspec in c:\users\marcu\anaconda3\lib\site-p
ackages (from torch) (2024.6.1)
Requirement already satisfied: setuptools in c:\users\marcu\anaconda3\lib\si
te-packages (from torch) (75.1.0)
Requirement already satisfied: mpmath<1.4,>=1.1.0 in c:\users\marcu\anaconda
3\lib\site-packages (from sympy>=1.13.3->torch) (1.3.0)
Requirement already satisfied: MarkupSafe>=2.0 in c:\users\marcu\anaconda3\l
ib\site-packages (from jinja2->torch) (2.1.3)
Requirement already satisfied: ipca in c:\users\marcu\anaconda3\lib\site-pac
kages (0.6.7)
Requirement already satisfied: numpy in c:\users\marcu\anaconda3\lib\site-pa
ckages (from ipca) (1.26.4)
Requirement already satisfied: progressbar in c:\users\marcu\anaconda3\lib\s
ite-packages (from ipca) (2.5)
Requirement already satisfied: numba in c:\users\marcu\anaconda3\lib\site-pa
ckages (from ipca) (0.60.0)
Requirement already satisfied: scipy in c:\users\marcu\anaconda3\lib\site-pa
ckages (from ipca) (1.13.1)
Requirement already satisfied: joblib in c:\users\marcu\anaconda3\lib\site-p
ackages (from ipca) (1.4.2)
Requirement already satisfied: scikit-learn in c:\users\marcu\anaconda3\lib
\site-packages (from ipca) (1.5.1)
Requirement already satisfied: llvmlite<0.44,>=0.43.0dev0 in c:\users\marcu
\anaconda3\lib\site-packages (from numba->ipca) (0.43.0)
Requirement already satisfied: threadpoolctl>=3.1.0 in c:\users\marcu\anacon
da3\lib\site-packages (from scikit-learn->ipca) (3.5.0)
```

In [ ]:

```

```

In [ ]:

```
pd.reset_option('display.max_rows')
pd.reset_option('display.max_columns')
```

In [ ]:

```
# ! pip install numpy==1.22.4
```

In [ ]:

```
os.getcwd()
```

Out[ ]:

```
'C:\\Users\\marcu\\Downloads'
```

In [ ]:

```
#!pip uninstall tensorflow
#!pip install tensorflow==2.17.0
```

In [ ]:

```
#import tensorflow as tf
```

In [ ]:
```
#print("Num of GPUs Available:", len(tf.config.list_physical_devices('GPU'))
```

In [ ]:
```
#device = torch.device('cuda' if torch.cuda.is_available() else 'cpu')
```

In [ ]:
```
#tf.test.gpu_device_name()
```

In [ ]:
```
# Set the device to GPU if available
# device = torch.device('cuda' if torch.cuda.is_available() else 'cpu')
```

IPCA Features

- These are the features data frame created from the students.

In [ ]:
```
### NOTE: this data might only have investment grade bonds in it, we might r
```

In [ ]:
```
data = pd.read_csv('OLD_IPCA_Features.csv')

data.set_index(['Cusip', 'Date'], inplace=True)
data.sort_index(inplace=True)
```

In [ ]:
```
data
```

Out[ ]:

Cusip	Date	Index	Company	Industry	Excess_returns	Issue_date	Bo
001084AQ	**2012-12-01**	C0A0	AGCO CORP	Capital Goods	2.216	2012-06-01	0
	2013-01-01	C0A0	AGCO CORP	Capital Goods	-0.390	2012-06-01	0
	2013-02-01	C0A0	AGCO CORP	Capital Goods	0.319	2012-06-01	0
	2013-03-01	C0A0	AGCO CORP	Capital Goods	-2.567	2012-06-01	0
	2013-04-01	C0A0	AGCO CORP	Capital Goods	4.725	2012-06-01	0
...	**...**	...	...	...	...	...	
98978VAT	**2023-01-01**	C0A0	Zoetis Inc.	Healthcare	0.413	2020-06-01	2
	2023-02-01	C0A0	Zoetis Inc.	Healthcare	-1.077	2020-06-01	2
	2023-03-01	C0A0	Zoetis Inc.	Healthcare	1.050	2020-06-01	2
	2023-04-01	C0A0	Zoetis Inc.	Healthcare	0.904	2020-06-01	2
	2023-05-01	C0A0	Zoetis Inc.	Healthcare	-1.715	2020-06-01	2

228806 rows × 54 columns

In [ ]:

```
(data != 0).sum()
```

Out[ ]:
```
Index                        228806
Company                      228806
Industry                     228806
Excess_returns               228720
Issue_date                   228806
Bond_age                     228806
Face_value                   228806
Coupon                       228760
Duration                     228806
Spread                       228806
Rating                       228806
Distance_to_default          228806
Book_leverage                228750
Market_leverage              228262
Operating_leverage           228531
Book_to_price                228806
Earnings_to_price            228806
Marketcap                    228806
Debt                         228262
Debt_to_ebitda               228262
Spread_to_d2d                228806
Profitability                228802
Prof_change                   76701
Mom_6m_equity                228686
Mom_6m                       228711
Mom_6m_rating                228711
Mom_6m_spread                228666
Stock_vol                    228806
Turnover_vol                 228806
VaR                          228806
VIX_Beta                     228806
Mom_6m_industry              228790
Bond_vol                     226807
Bond_skew                    226807
Banking                          64
Basic Industry                12758
Telecommunications             2426
Energy                        23591
Consumer Non-Cyclical          3733
Leisure                         759
Technology & Electronics      25893
Healthcare                    30130
Consumer Goods                13574
Transportation                 9664
Consumer Cyclical              7027
Services                       8334
Financial Services             5876
Insurance                      1184
Automotive                     5115
Retail                        11425
Capital Goods                 20578
Utility                       27676
Media                         14211
Real Estate                    4788
dtype: int64
```

Bond Excess Returns Construction R_t

In [ ]:
```
returns = data[['Excess_returns']].copy()
returns_unstacked = returns.unstack(level='Cusip')
## how do we handle null returns in the panel coming from the unbalanced par
returns_unstacked
```

Out[ ]:

Cusip Date	001084AQ	00108WAD	00108WAF	00108WAH	00108WAJ	00108WAK
2010-01-01	NaN	NaN	NaN	NaN	NaN	NaN
2010-02-01	NaN	NaN	NaN	NaN	NaN	NaN
2010-03-01	NaN	NaN	NaN	NaN	NaN	NaN
2010-04-01	NaN	NaN	NaN	NaN	NaN	NaN
2010-05-01	NaN	NaN	NaN	NaN	NaN	NaN
...	...	...	...	...	...	...
2023-01-01	NaN	NaN	NaN	NaN	NaN	NaN
2023-02-01	NaN	NaN	NaN	NaN	NaN	NaN
2023-03-01	NaN	NaN	NaN	NaN	NaN	NaN
2023-04-01	NaN	NaN	NaN	NaN	NaN	NaN
2023-05-01	NaN	NaN	NaN	NaN	NaN	NaN

161 rows × 6704 columns

In [ ]:
```
returns
```

Out[ ]:

Cusip	Date	Excess_returns
001084AQ	**2012-12-01**	2.216
	2013-01-01	-0.390
	2013-02-01	0.319
	2013-03-01	-2.567
	2013-04-01	4.725
...	**...**	...
98978VAT	**2023-01-01**	0.413
	2023-02-01	-1.077
	2023-03-01	1.050
	2023-04-01	0.904
	2023-05-01	-1.715

228806 rows × 1 columns

Lagged Characteristics Construction X_{t-1}

In [ ]:

```
# characteristics
features = data.drop(columns=['Excess_returns', 'Issue_date', 'Index', 'Comp

features_lag = features.groupby('Cusip').shift().dropna()

features_lag
```

Out[ ]:

Cusip	Date	Bond_age	Face_value	Coupon	Duration	Spread	Ratin
001084AQ	2013-01-01	0.501370	280000000.0	5.875	7.037407	345.2777	11.
	2013-02-01	0.586301	280000000.0	5.875	6.971084	320.7451	11.
	2013-03-01	0.671233	280000000.0	5.875	6.903765	333.3739	11.
	2013-04-01	0.747945	280000000.0	5.875	6.810539	335.8488	11.
	2013-05-01	0.832877	280000000.0	5.875	6.690296	380.7369	11.
...	...	...	...	...	...	...	.
98978VAT	2023-01-01	2.501370	46700.0	3.000	16.500000	112.0000	9.
	2023-02-01	2.586301	46100.0	3.000	16.277000	113.0000	9.
	2023-03-01	2.671233	47100.0	3.000	16.586000	112.0000	9.
	2023-04-01	2.747945	45500.0	3.000	16.002000	120.0000	9.
	2023-05-01	2.832877	46700.0	3.000	16.359000	115.0000	9.

222102 rows × 49 columns

In [ ]:

```
features_lag.info()
```

```
<class 'pandas.core.frame.DataFrame'>
MultiIndex: 222102 entries, ('001084AQ', '2013-01-01') to ('98978VAT', '2023
-05-01')
Data columns (total 49 columns):
 #   Column                      Non-Null Count   Dtype  
---  ------                      --------------   -----  
 0   Bond_age                    222102 non-null  float64
 1   Face_value                  222102 non-null  float64
 2   Coupon                      222102 non-null  float64
 3   Duration                    222102 non-null  float64
 4   Spread                      222102 non-null  float64
 5   Rating                      222102 non-null  float64
 6   Distance_to_default         222102 non-null  float64
 7   Book_leverage               222102 non-null  float64
 8   Market_leverage             222102 non-null  float64
 9   Operating_leverage          222102 non-null  float64
 10  Book_to_price               222102 non-null  float64
 11  Earnings_to_price           222102 non-null  float64
 12  Marketcap                   222102 non-null  float64
 13  Debt                        222102 non-null  float64
 14  Debt_to_ebitda              222102 non-null  float64
 15  Spread_to_d2d               222102 non-null  float64
 16  Profitability               222102 non-null  float64
 17  Prof_change                 222102 non-null  float64
 18  Mom_6m_equity               222102 non-null  float64
 19  Mom_6m                      222102 non-null  float64
 20  Mom_6m_rating               222102 non-null  float64
 21  Mom_6m_spread               222102 non-null  float64
 22  Stock_vol                   222102 non-null  float64
 23  Turnover_vol                222102 non-null  float64
 24  VaR                         222102 non-null  float64
 25  VIX_Beta                    222102 non-null  float64
 26  Mom_6m_industry             222102 non-null  float64
 27  Bond_vol                    222102 non-null  float64
 28  Bond_skew                   222102 non-null  float64
 29  Banking                     222102 non-null  float64
 30  Basic Industry              222102 non-null  float64
 31  Telecommunications          222102 non-null  float64
 32  Energy                      222102 non-null  float64
 33  Consumer Non-Cyclical       222102 non-null  float64
 34  Leisure                     222102 non-null  float64
 35  Technology & Electronics    222102 non-null  float64
 36  Healthcare                  222102 non-null  float64
 37  Consumer Goods              222102 non-null  float64
 38  Transportation              222102 non-null  float64
 39  Consumer Cyclical           222102 non-null  float64
 40  Services                    222102 non-null  float64
 41  Financial Services          222102 non-null  float64
 42  Insurance                   222102 non-null  float64
 43  Automotive                  222102 non-null  float64
 44  Retail                      222102 non-null  float64
 45  Capital Goods               222102 non-null  float64
 46  Utility                     222102 non-null  float64
 47  Media                       222102 non-null  float64
 48  Real Estate                 222102 non-null  float64
```

```
dtypes: float64(49)
memory usage: 84.2+ MB
```

In [ ]:
```python
# Intersect characteristics index with excess returns index
returns = returns.loc[features_lag.index, :]
```

Dealing with Missing Values and Preprocessing

- The IPCA algorithm can deal with unbalanced panels by summing over only the observed i, t panel entries.
- In practice, we should "fill up" data $R_{i,t+1}$ and $X_{i,t}$ at any unobserved (i, t) entry with zeros, and then use the alternating least squares algorithm for the completed panel.
  - this is equivalent to summing over only the observed entries
  - in other implementations of IPCA, it is common to drop any null values, and that's what we'll follow

In [ ]:
```python
returns.isna().sum()
```

Out[ ]:
```
Excess_returns    0
dtype: int64
```

In [ ]:
```python
features_lag.isna().sum().sum()
```

Out[ ]:
```
0
```

In [ ]:
```python
# check for equal number of rows of returns and characteristics
assert (features_lag.shape[0] == returns.shape[0])
```

In [ ]:
```python
## Check their bond paper to see what preprocessing they do and how they har
```

Do we need to cross-sectionally rank standardize all of the characteristics, or cross-sectionally orthonormalize characteristics each period?

- See Kelly's Characteristics are Covariances and Modeling Corporate Bond Returns paper.
- "We cross-sectionally transform instruments period-by-period. In particular, we calculate stocks' ranks for each characteristic, then divide ranks by the number of non-missing observations and subtract 0.5. This maps characteristics into the [−0.5,+0.5] interval and focuses on their ordering as opposed to magnitude. We use this standardization for its insensitivity to outliers, and find in unreported robustness analyses that results are qualitatively unchanged with no characteristic transformation."

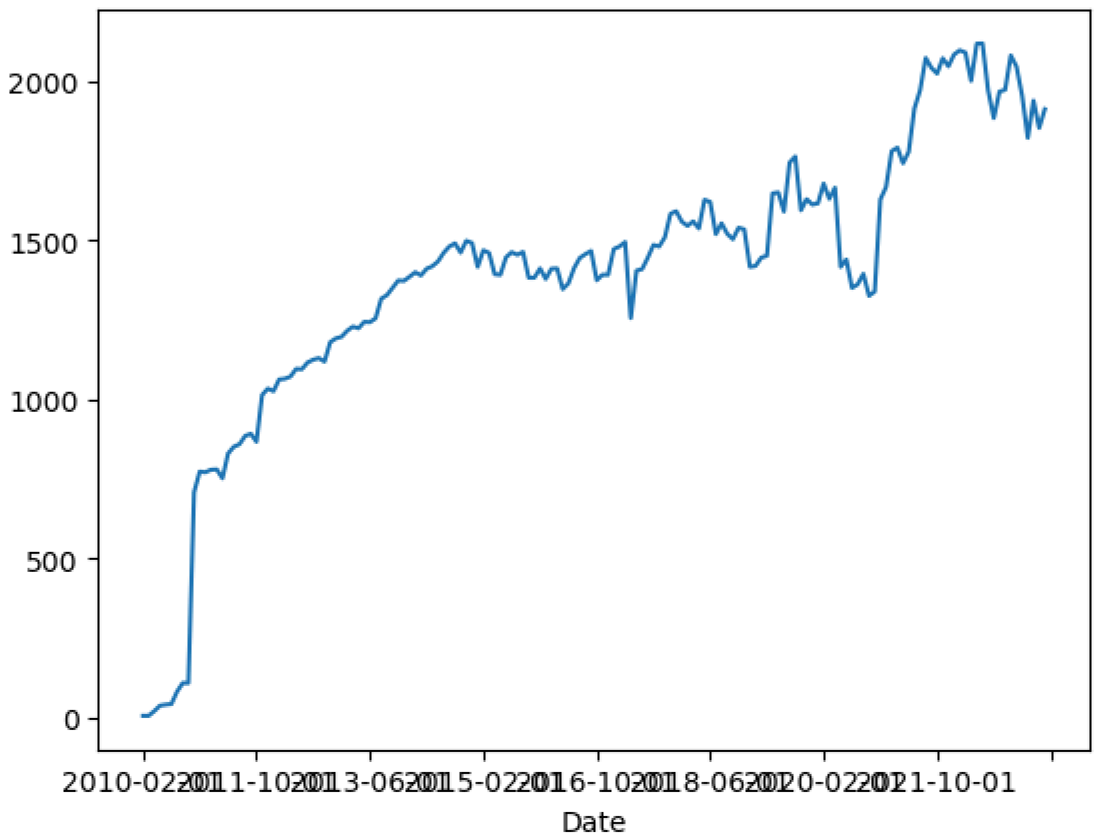
In [ ]:
```
num_bonds_ts = returns.groupby('Date')['Excess_returns'].apply(len)

print(num_bonds_ts.describe().T)

num_bonds_ts.plot()
```

```
count     160.000000
mean     1388.137500
std       456.052359
min         5.000000
25%      1243.750000
50%      1442.500000
75%      1618.000000
max      2120.000000
Name: Excess_returns, dtype: float64
```

Out[ ]: <Axes: xlabel='Date'>



In [ ]:
```
((features_lag.groupby('Date').get_group('2013-01-01').rank() /  num_bonds_t
```

Out[ ]:

	count	mean	std	min	25%	50%
Bond_age	1197.0	0.000418	0.283179	-0.483292	-0.258563	-0.000418
Face_value	1197.0	0.000418	0.287690	-0.442774	-0.249373	0.000418
Coupon	1197.0	0.000418	0.288777	-0.499165	-0.249373	0.000835
Duration	1197.0	0.000418	0.288796	-0.499165	-0.249373	0.000418
Spread	1197.0	0.000418	0.288796	-0.499165	-0.249373	0.000418
Rating	1197.0	0.000418	0.284193	-0.494570	-0.259816	0.008772
Distance_to_default	1197.0	0.000418	0.288780	-0.494570	-0.244361	0.001671
Book_leverage	1197.0	0.000418	0.288773	-0.497911	-0.245614	0.000418
Market_leverage	1197.0	0.000418	0.288773	-0.496241	-0.248956	0.004177
Operating_leverage	1197.0	0.000418	0.288773	-0.499165	-0.249373	-0.000418
Book_to_price	1197.0	0.000418	0.288773	-0.498329	-0.247285	-0.002089
Earnings_to_price	1197.0	0.000418	0.288773	-0.499165	-0.244779	-0.000418
Marketcap	1197.0	0.000418	0.288773	-0.497494	-0.248956	0.000000
Debt	1197.0	0.000418	0.288773	-0.496241	-0.250209	0.003342
Debt_to_ebitda	1197.0	0.000418	0.288773	-0.497911	-0.246867	0.000418
Spread_to_d2d	1197.0	0.000418	0.288796	-0.499165	-0.249373	0.000418
Profitability	1197.0	0.000418	0.288773	-0.483709	-0.249373	0.001253
Prof_change	1197.0	0.000418	0.094145	-0.498329	0.007937	0.007937
Mom_6m_equity	1197.0	0.000418	0.288773	-0.499165	-0.248120	-0.006683
Mom_6m	1197.0	0.000418	0.288796	-0.499165	-0.249373	0.000418
Mom_6m_rating	1197.0	0.000418	0.288796	-0.499165	-0.249373	0.000418
Mom_6m_spread	1197.0	0.000418	0.288796	-0.499165	-0.249373	0.000418
Stock_vol	1197.0	0.000418	0.288775	-0.497494	-0.250627	-0.000835
Turnover_vol	1197.0	0.000418	0.288781	-0.499165	-0.249373	0.000418
VaR	1197.0	0.000418	0.288796	-0.499165	-0.249373	0.000418
VIX_Beta	1197.0	0.000418	0.288796	-0.499165	-0.249373	0.000418
Mom_6m_industry	1197.0	0.000418	0.288796	-0.499165	-0.249373	0.000418
Bond_vol	1197.0	0.000418	0.051846	-0.499165	0.005848	0.005848
Bond_skew	1197.0	0.000418	0.051846	-0.005013	-0.005013	-0.005013
Banking	1197.0	0.000418	0.014452	0.000000	0.000000	0.000000
Basic Industry	1197.0	0.000418	0.104730	-0.022556	-0.022556	-0.022556
Telecommunications	1197.0	0.000418	0.068668	-0.009190	-0.009190	-0.009190
Energy	1197.0	0.000418	0.180815	-0.076859	-0.076859	-0.076859

	count	mean	std	min	25%	50%
Consumer Non-Cyclical	1197.0	0.000418	0.094124	-0.017962	-0.017962	-0.01796
Leisure	1197.0	0.000418	0.000000	0.000418	0.000418	0.00041
Technology & Electronics	1197.0	0.000418	0.128475	-0.035088	-0.035088	-0.03508
Healthcare	1197.0	0.000418	0.167479	-0.063910	-0.063910	-0.06391
Consumer Goods	1197.0	0.000418	0.104730	-0.022556	-0.022556	-0.02255
Transportation	1197.0	0.000418	0.062518	-0.007519	-0.007519	-0.00751
Consumer Cyclical	1197.0	0.000418	0.120466	-0.030493	-0.030493	-0.03049
Services	1197.0	0.000418	0.102898	-0.021721	-0.021721	-0.02172
Financial Services	1197.0	0.000418	0.080685	-0.012949	-0.012949	-0.01294
Insurance	1197.0	0.000418	0.028867	-0.001253	-0.001253	-0.00125
Automotive	1197.0	0.000418	0.072916	-0.010443	-0.010443	-0.01044
Retail	1197.0	0.000418	0.085433	-0.014620	-0.014620	-0.01462
Capital Goods	1197.0	0.000418	0.148546	-0.048454	-0.048454	-0.04845
Utility	1197.0	0.000418	0.167014	-0.063492	-0.063492	-0.06349
Media	1197.0	0.000418	0.125641	-0.033417	-0.033417	-0.03341
Real Estate	1197.0	0.000418	0.000000	0.000418	0.000418	0.00041

In [ ]:
```
features_lag.head()
```

Out[ ]:

		Bond_age	Face_value	Coupon	Duration	Spread	Rating
Cusip	**Date**						
001084AQ	**2013-01-01**	0.501370	280000000.0	5.875	7.037407	345.2777	11.0
	2013-02-01	0.586301	280000000.0	5.875	6.971084	320.7451	11.0
	2013-03-01	0.671233	280000000.0	5.875	6.903765	333.3739	11.0
	2013-04-01	0.747945	280000000.0	5.875	6.810539	335.8488	11.0
	2013-05-01	0.832877	280000000.0	5.875	6.690296	380.7369	11.0

5 rows × 49 columns

In [ ]:
```
features_lag = features_lag.groupby('Date', group_keys=False).apply(lambda g
```

In [ ]:
```
features_lag.groupby('Date').apply(np.min).describe(), features_lag.groupby(
```

Out[ ]:
```
(count    160.000000
 mean      -0.495840
 std        0.022718
 min       -0.499528
 25%       -0.499382
 50%       -0.499307
 75%       -0.499196
 max       -0.300000
 dtype: float64,
 count    160.0
 mean       0.5
 std        0.0
 min        0.5
 25%        0.5
 50%        0.5
 75%        0.5
 max        0.5
 dtype: float64)
```

In [ ]:
```
#### NOTE: Potential issues with student code, for example I don't see them
#### The following in this section is student code, meant for comparison wit
```

IPCA Package

- Docs

In [ ]:
```
#### IPCA Codebase Example
# data = statsmodels.datasets.grunfeld.load_pandas().data
# data.year = data.year.astype(np.int64)

# # Establish unique IDs to conform with package
# N = len(np.unique(data.firm))
# ID = dict(zip(np.unique(data.firm).tolist(),np.arange(1,N+1)))
# data.firm = data.firm.apply(lambda x: ID[x])

# # use multi-index for panel groups
# data = data.set_index(['firm', 'year'])
# y = data['invest']
# X = data.drop('invest', axis=1)

# # Call ipca
# from ipca import InstrumentedPCA
# regr = InstrumentedPCA(n_factors=1, intercept=False)
# regr = regr.fit(X=X, y=y)
# Gamma, Factors = regr.get_factors(label_ind=True)
```

In [ ]:
```
# Call ipca

regr = ipca.InstrumentedPCA(n_factors=4, intercept=False)
regr = regr.fit(X=features_lag, y=returns.squeeze())
```

```
[=================                                                       ]
23%
```

```
The panel dimensions are:
n_samples: 6492 , L: 49 , T: 160
```

```
[========================================================================] 1
00%
```

```
Step 1 - Aggregate Update: 4399.4042273758705
Step 2 - Aggregate Update: 3763.2806584552145
Step 3 - Aggregate Update: 673.6998577139684
Step 4 - Aggregate Update: 360.1630399205583
Step 5 - Aggregate Update: 92.85963422955149
Step 6 - Aggregate Update: 31.42185008377019
Step 7 - Aggregate Update: 17.54386837139532
Step 8 - Aggregate Update: 15.077989664669076
Step 9 - Aggregate Update: 14.529448632498315
Step 10 - Aggregate Update: 13.342160538792086
Step 11 - Aggregate Update: 37.3115826799718
Step 12 - Aggregate Update: 10.725221366637925
Step 13 - Aggregate Update: 9.635028420405433
Step 14 - Aggregate Update: 8.880389170085891
Step 15 - Aggregate Update: 8.400712544419093
Step 16 - Aggregate Update: 7.917857010259013
Step 17 - Aggregate Update: 7.469421062695801
Step 18 - Aggregate Update: 7.065610691629445
Step 19 - Aggregate Update: 6.705206204623664
Step 20 - Aggregate Update: 6.382913437340449
Step 21 - Aggregate Update: 6.092562194948641
Step 22 - Aggregate Update: 5.82837412517847
Step 23 - Aggregate Update: 5.585370845386024
Step 24 - Aggregate Update: 5.359421228707561
Step 25 - Aggregate Update: 5.147155083026519
Step 26 - Aggregate Update: 4.94584403461613
Step 27 - Aggregate Update: 4.753287479264145
Step 28 - Aggregate Update: 4.567718422029884
Step 29 - Aggregate Update: 4.387728827019146
Step 30 - Aggregate Update: 4.21221257218167
Step 31 - Aggregate Update: 4.040321324422621
Step 32 - Aggregate Update: 3.871430040198902
Step 33 - Aggregate Update: 3.705107918228805
Step 34 - Aggregate Update: 3.5410936405155553
Step 35 - Aggregate Update: 3.3792727866675705
Step 36 - Aggregate Update: 3.2196558583091246
Step 37 - Aggregate Update: 3.0623573396906068
Step 38 - Aggregate Update: 2.9075747286506157
Step 39 - Aggregate Update: 2.755568824589318
Step 40 - Aggregate Update: 2.6066434331214623
Step 41 - Aggregate Update: 2.4611277215503833
Step 42 - Aggregate Update: 2.3193590298847084
Step 43 - Aggregate Update: 2.1816677435026577
Step 44 - Aggregate Update: 2.048364763333211
Step 45 - Aggregate Update: 1.9197309214209781
Step 46 - Aggregate Update: 1.7960088371191887
Step 47 - Aggregate Update: 1.677397277774162
Step 48 - Aggregate Update: 1.564047840598505
Step 49 - Aggregate Update: 1.4560637997392405
Step 50 - Aggregate Update: 1.35350025672642
Step 51 - Aggregate Update: 1.2563663278251624
Step 52 - Aggregate Update: 1.1646283822751684
Step 53 - Aggregate Update: 1.0782135450112946
Step 54 - Aggregate Update: 0.9970149072214554
Step 55 - Aggregate Update: 0.9208957259284034
Step 56 - Aggregate Update: 0.8496948679407978
```

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Step 57 - Aggregate Update: 0.7832311248002952
Step 58 - Aggregate Update: 0.721308092741765
Step 59 - Aggregate Update: 0.663718467300896
Step 60 - Aggregate Update: 0.6102475771317586
Step 61 - Aggregate Update: 0.5606769161740885
Step 62 - Aggregate Update: 0.5147871469830534
Step 63 - Aggregate Update: 0.47236063910395387
Step 64 - Aggregate Update: 0.43318328100895087
Step 65 - Aggregate Update: 0.39704651159536297
Step 66 - Aggregate Update: 0.36374832381670785
Step 67 - Aggregate Update: 0.33309458978737894
Step 68 - Aggregate Update: 0.3048997151186086
Step 69 - Aggregate Update: 0.2789868363297501
Step 70 - Aggregate Update: 0.255188664878915
Step 71 - Aggregate Update: 0.23334694715231308
Step 72 - Aggregate Update: 0.2133132039721204
Step 73 - Aggregate Update: 0.19494797878587633
Step 74 - Aggregate Update: 0.17812071317894151
Step 75 - Aggregate Update: 0.16271008985086155
Step 76 - Aggregate Update: 0.14860256763495272
Step 77 - Aggregate Update: 0.13569308507339883
Step 78 - Aggregate Update: 0.12388418500722764
Step 79 - Aggregate Update: 0.11308547727230689
Step 80 - Aggregate Update: 0.10321339663369145
Step 81 - Aggregate Update: 0.09419103044319854
Step 82 - Aggregate Update: 0.085947201977433
Step 83 - Aggregate Update: 0.07841639891921659
Step 84 - Aggregate Update: 0.07153845909988377
Step 85 - Aggregate Update: 0.06525784701398152
Step 86 - Aggregate Update: 0.05952381356928527
Step 87 - Aggregate Update: 0.05428952946931531
Step 88 - Aggregate Update: 0.049512116891037294
Step 89 - Aggregate Update: 0.04515229886919769
Step 90 - Aggregate Update: 0.041174055560986744
Step 91 - Aggregate Update: 0.0375443434254521
Step 92 - Aggregate Update: 0.03423300922349881
Step 93 - Aggregate Update: 0.031212339439207426
Step 94 - Aggregate Update: 0.028457108200655057
Step 95 - Aggregate Update: 0.025944158160655206
Step 96 - Aggregate Update: 0.02365237041885848
Step 97 - Aggregate Update: 0.021562244792221463
Step 98 - Aggregate Update: 0.019656467762615648
Step 99 - Aggregate Update: 0.01791856871705022
Step 100 - Aggregate Update: 0.0163339930049915
Step 101 - Aggregate Update: 0.014889197162119672
Step 102 - Aggregate Update: 0.013572008020219073
Step 103 - Aggregate Update: 0.012371044734663883
Step 104 - Aggregate Update: 0.01127623062393468
Step 105 - Aggregate Update: 0.010278133671818068
Step 106 - Aggregate Update: 0.009368283358526241
Step 107 - Aggregate Update: 0.008538856272025441
Step 108 - Aggregate Update: 0.007782749128637079
Step 109 - Aggregate Update: 0.007093575371072802
Step 110 - Aggregate Update: 0.006465354702868353
Step 111 - Aggregate Update: 0.005892741052775818
Step 112 - Aggregate Update: 0.005370753574879927
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Step 113 - Aggregate Update: 0.004894984688235127
Step 114 - Aggregate Update: 0.004461370026092482
Step 115 - Aggregate Update: 0.004066118955051934
Step 116 - Aggregate Update: 0.003705842033468798
Step 117 - Aggregate Update: 0.0033774942785385065
Step 118 - Aggregate Update: 0.003078259053381771
Step 119 - Aggregate Update: 0.002805438499308366
Step 120 - Aggregate Update: 0.0025568638529875898
Step 121 - Aggregate Update: 0.002330284356503398
Step 122 - Aggregate Update: 0.002123765083069884
Step 123 - Aggregate Update: 0.0019355545596226875
Step 124 - Aggregate Update: 0.0017640114451751288
Step 125 - Aggregate Update: 0.0016076803412943264
Step 126 - Aggregate Update: 0.0014652011227553885
Step 127 - Aggregate Update: 0.0013353141389416123
Step 128 - Aggregate Update: 0.001216978047068551
Step 129 - Aggregate Update: 0.0011091253136328305
Step 130 - Aggregate Update: 0.0010107855205205851
Step 131 - Aggregate Update: 0.0009212179197675141
Step 132 - Aggregate Update: 0.0008395775580538611
Step 133 - Aggregate Update: 0.0007651353380424553
Step 134 - Aggregate Update: 0.0006972893393140112
Step 135 - Aggregate Update: 0.0006355306331897737
Step 136 - Aggregate Update: 0.0005791860899080348
Step 137 - Aggregate Update: 0.0005278414091662853
Step 138 - Aggregate Update: 0.00048104775942192646
Step 139 - Aggregate Update: 0.0004384111457511608
Step 140 - Aggregate Update: 0.0003995377628598362
Step 141 - Aggregate Update: 0.00036413545694813365
Step 142 - Aggregate Update: 0.0003318675176160468
Step 143 - Aggregate Update: 0.00030243015544328955
Step 144 - Aggregate Update: 0.0002756493836386653
Step 145 - Aggregate Update: 0.0002511867874090967
Step 146 - Aggregate Update: 0.0002289215362907271
Step 147 - Aggregate Update: 0.0002085734518004756
Step 148 - Aggregate Update: 0.00019018330704057007
Step 149 - Aggregate Update: 0.00017327970044789254
Step 150 - Aggregate Update: 0.00015794000637470162
Step 151 - Aggregate Update: 0.00014392011826203088
Step 152 - Aggregate Update: 0.00013114719422446797
Step 153 - Aggregate Update: 0.00011954885849263519
Step 154 - Aggregate Update: 0.00010893293165281648
Step 155 - Aggregate Update: 9.927987002811278e-05
Step 156 - Aggregate Update: 9.048648598763975e-05
Step 157 - Aggregate Update: 8.247428331742412e-05
Step 158 - Aggregate Update: 7.516247251260211e-05
Step 159 - Aggregate Update: 6.847442409707583e-05
Step 160 - Aggregate Update: 6.24086592324602e-05
Step 161 - Aggregate Update: 5.6880805914261146e-05
Step 162 - Aggregate Update: 5.1869686558347894e-05
Step 163 - Aggregate Update: 4.7234010253305314e-05
Step 164 - Aggregate Update: 4.306367418394075e-05
Step 165 - Aggregate Update: 3.923760141333332e-05
Step 166 - Aggregate Update: 3.5764254789683037e-05
Step 167 - Aggregate Update: 3.259628510932089e-05
Step 168 - Aggregate Update: 2.9699387141590705e-05
```

```
Step 169 - Aggregate Update: 2.704722555790795e-05
Step 170 - Aggregate Update: 2.468945103828446e-05
Step 171 - Aggregate Update: 2.249863928227569e-05
Step 172 - Aggregate Update: 2.0450898318813415e-05
Step 173 - Aggregate Update: 1.8710321910475614e-05
Step 174 - Aggregate Update: 1.7021456642396515e-05
Step 175 - Aggregate Update: 1.5494700164708775e-05
Step 176 - Aggregate Update: 1.4143965927360114e-05
Step 177 - Aggregate Update: 1.287049644815852e-05
Step 178 - Aggregate Update: 1.1725644981197547e-05
Step 179 - Aggregate Update: 1.0725248102971818e-05
Step 180 - Aggregate Update: 9.746984687808435e-06
-- Convergence Reached --
```

In [ ]:

```
Gamma, Factors = regr.get_factors(label_ind=True)

Gamma
```

Out[ ]:

	0	1	2	3
Bond_age	-0.000063	-0.008996	-0.012198	0.062035
Face_value	0.000027	0.017618	-0.124769	-0.087786
Coupon	0.000042	0.049795	0.020337	0.072009
Duration	0.001760	-0.059232	-0.102919	-0.647445
Spread	0.001079	0.101201	0.239167	-0.072481
Rating	0.000367	-0.061616	0.005257	-0.070395
Distance_to_default	-0.000265	0.082676	0.086799	0.050178
Book_leverage	-0.000023	0.061523	0.035238	0.007143
Market_leverage	0.000425	-0.068900	-0.005024	-0.008977
Operating_leverage	0.000005	0.023366	0.025892	-0.014023
Book_to_price	-0.000299	0.126884	0.052326	-0.042216
Earnings_to_price	0.000073	-0.026771	-0.036894	-0.029142
Marketcap	0.000002	0.100726	-0.001626	-0.068633
Debt	-0.000041	-0.044905	-0.036568	-0.017913
Debt_to_ebitda	-0.000145	0.000393	-0.008308	-0.003716
Spread_to_d2d	-0.000143	0.087792	0.137692	0.103796
Profitability	-0.000157	0.006848	-0.035389	0.021918
Prof_change	-0.000056	0.030014	0.018872	-0.006542
Mom_6m_equity	-0.000164	0.011181	0.007235	0.013111
Mom_6m	0.001797	-0.489719	0.135320	0.186605
Mom_6m_rating	0.000333	0.003835	0.187109	-0.023621
Mom_6m_spread	-0.002531	0.514517	-0.433039	-0.111717
Stock_vol	0.000250	-0.035200	0.011047	0.006491
Turnover_vol	0.000012	0.000957	0.013230	-0.045597
VaR	-0.000822	0.029378	-0.094180	0.031848
VIX_Beta	-0.000374	0.058878	0.016880	-0.134657
Mom_6m_industry	-0.000372	0.029407	-0.038034	-0.064507
Bond_vol	-0.000301	-0.107409	-0.014618	-0.209911
Bond_skew	0.000210	0.062644	0.035838	0.183789
Banking	0.223641	-0.082190	-0.215359	0.004710
Basic Industry	0.223153	0.091647	0.012870	-0.136994
Telecommunications	0.223687	0.347383	0.350684	0.259224
Energy	0.225010	-0.254839	0.010351	-0.169109

	0	1	2	3
Consumer Non-Cyclical	0.223346	0.018412	-0.074214	0.047004
Leisure	0.225069	-0.260928	0.212901	-0.272319
Technology & Electronics	0.223091	0.066418	-0.071503	-0.004932
Healthcare	0.223043	0.033788	-0.128856	0.059966
Consumer Goods	0.223349	-0.010046	-0.048163	0.082787
Transportation	0.223392	0.054023	-0.074105	-0.021125
Consumer Cyclical	0.223198	0.022944	0.005385	-0.004574
Services	0.223458	-0.025209	-0.109354	0.119067
Financial Services	0.223826	-0.115383	-0.074962	-0.059640
Insurance	0.223554	-0.130046	0.031409	0.212989
Automotive	0.223951	-0.069485	-0.011552	-0.045856
Retail	0.223451	0.026156	-0.039793	0.063102
Capital Goods	0.223253	-0.003035	-0.077895	0.070120
Utility	0.223247	-0.048438	-0.129769	0.157444
Media	0.223083	0.075353	-0.140132	-0.086265
Real Estate	0.224284	0.277033	0.560679	-0.269298

In [ ]:

```
Factors
```

Out[ ]:

	2010-02-01	2010-03-01	2010-04-01	2010-05-01	2010-06-01	2010-07-01	2010-08-01	2
0	1.412879	1.946483	-4.605450	10.121840	7.377150	6.771170	14.898849	1
1	0.498286	11.498442	-0.143923	-1.515086	-0.659622	0.204946	1.705582	-
2	2.351042	4.032427	1.478439	0.703931	-1.414143	1.389804	0.977772	-
3	0.225037	1.154040	5.329017	-1.624054	0.935380	0.602441	0.275163	(

4 rows × 160 columns

In [ ]:

```
ret_hat_package = pd.DataFrame(regr.predict(features_lag), index=features_la

ret_hat_package
```

Out[ ]:

Cusip	Date	Returns_forecast_package
001084AQ	2013-01-01	0.338043
	2013-02-01	1.905015
	2013-03-01	1.169758
	2013-04-01	1.002299
	2013-05-01	-0.383832
...	...	...
98978VAT	2023-01-01	1.360921
	2023-02-01	-1.302526
	2023-03-01	0.502499
	2023-04-01	0.396601
	2023-05-01	-0.340776

222102 rows × 1 columns

In [ ]:

```
ret_ret_hat_approx_ret_hat = pd.concat([ret_ret_hat_approx, ret_ret_hat.iloc

ret_ret_hat_approx_ret_hat.iloc[:, -3:].corr()
```

```
---------------------------------------------------------------------------
NameError                                 Traceback (most recent call last)
Cell In[143], line 1
----> 1 ret_ret_hat_approx_ret_hat = pd.concat([ret_ret_hat_approx, ret_ret_
hat.iloc[:, -1], ret_hat_package], axis=1)
      3 ret_ret_hat_approx_ret_hat.iloc[:, -3:].corr()

NameError: name 'ret_ret_hat_approx' is not defined
```

In [ ]:

```
regr.score(X=features_lag, y=returns.squeeze())
```

Out[ ]: 0.5072485752762694

Model Evaluation

- Consider the following measurements of statistical goodness-of-fit: total R^2, time-series R^2, cross-section R^2, and the relative pricing error.

Variety of R^2's

In [ ]:

```
import sklearn.metrics
```

```
sklearn.metrics.r2_score(ret_ret_hat['Excess_returns'], ret_ret_hat['Returns
```

```
---------------------------------------------------------------------------
NameError                                 Traceback (most recent call last)
Cell In[149], line 3
      1 import sklearn.metrics
----> 3 sklearn.metrics.r2_score(ret_ret_hat['Excess_returns'], ret_ret_hat
['Returns_forecast'])

NameError: name 'ret_ret_hat' is not defined
```

Total R^2

- $$\text{Total } R^2 = 1 - \frac{\sum_{i,t}(R_{i,t+1} - X_{i,t}^T \tilde{\mathrm{F}} \tilde{F}_{t+1})^2}{\sum_{i,t} R_{i,t+1}^2}$$
- Total R^2 describes how well the model describes the realized returns in a panel context.

In [ ]:
```
# total_R_2 = 1 - ((ret_ret_hat['Excess_returns'] - ret_ret_hat['Returns_for
```

In [ ]:
```
def total_R2(data):
    total_R2 = 1 - ((data['Excess_returns'] - data['Returns_forecast'])**2).

    return total_R2
```

Time-Series R^2

- Fix an asset i, compute the model's R_i^2 using time-series data, and then average those estimates across assets.
- $$\text{Time-Series } R^2 = \sum_i \frac{T_i}{\sum_i T_i} R_i^2,$$

  where

  $$R_i^2 = 1 - \frac{\sum_t (R_{i,t+1} - X_{i,t}^T \tilde{\mathrm{F}} \tilde{F}_{t+1})^2}{\sum_t R_{i,t+1}^2}$$
- Time-series R^2 summarizes pricing performance as the cross-sectional average of performance for each test asset i (weight each asset's performance by the number of time-series observations for that asset T_i).

In [ ]:
```
num_obs_per_bond = returns.groupby('Cusip')['Excess_returns'].apply(len)

percentage_obs_per_bond = num_obs_per_bond / num_obs_per_bond.sum()
```

```
percentage_obs_per_bond
```

Out[ ]:

```
Cusip
001084AQ    0.000261
00108WAD    0.000005
00108WAF    0.000005
00108WAH    0.000005
00108WAJ    0.000005
              ...
98978VAN    0.000221
98978VAP    0.000221
98978VAQ    0.000072
98978VAS    0.000131
98978VAT    0.000131
Name: Excess_returns, Length: 6492, dtype: float64
```

In [ ]:

```
# asset_R_2 = 1 - ((ret_ret_hat['Excess_returns'] - ret_ret_hat['Returns_for
# time_series_R_2 = (percentage_obs_per_bond * asset_R_2).sum()
```

In [ ]:

```
def time_series_R2(data, num_obs_per_asset):
    percentage_obs_per_asset = num_obs_per_asset / num_obs_per_asset.sum()
    asset_R2 = 1 - ((data['Excess_returns'] - data['Returns_forecast'])**2).
    time_series_R_2 = (percentage_obs_per_asset * asset_R2).sum()

    return time_series_R_2
```

Cross-Sectional R^2

- Fix a time t, compute the model's R_t^2 using cross-sectional data, and then average those estimates across time.
- $$\text{Cross-Section } R^2 = \frac{1}{T}\sum_t R_t^2,$$

  where

  $$R_t^2 = 1 - \frac{\sum_i (R_{i,t+1} - X_{i,t}^T \tilde{\Gamma} \tilde{F}_{t+1})^2}{\sum_i R_{i,t+1}^2}$$
- Cross-sectional R^2 quantifies the cross-sectional predictive performance as in a Fama-MacBeth context.

In [ ]:

```
# time_R_2 = 1 - ((ret_ret_hat['Excess_returns'] - ret_ret_hat['Returns_fore
# cross_section_R_2 = time_R_2.mean()
```

In [ ]:

```
def cross_sectional_R2(data):
    time_R2 = 1 - ((data['Excess_returns'] - data['Returns_forecast'])**2).g
    cross_section_R2 = time_R2.mean()
```

```python
    return cross_section_R2
```

Relative Pricing Error

- Fix an asset i, and compute the average error over time of the model, then square that average error, and sum across assets relative to average error squared.
- $$\frac{\sum_i \left[\frac{1}{T_i}\sum_t (R_{i,t+1} - X_{i,t}^T \tilde{\mathrm{F}} \tilde{F}_{t+1})\right]^2}{\sum_i \left[\frac{1}{T_i}\sum_t R_{i,t+1}\right]^2}$$
- A lower relative pricing number is better.

In [ ]:
```python
## TODO
```

Out-of-Sample Predictive Cross-Sectional R^2 Relative to a Benchmark

- This is for a fixed out-of-sample period $t+1$.
- $$R^2_{t+1} = 1 - \frac{\sum_i (R_{i,t+1} - X_{i,t}^T \tilde{\mathrm{F}} \tilde{F}_{t+1})^2}{\sum_i (R_{i,t+1} - \bar{R}_{i,t})^2},$$

  where the benchmark $\bar{R}_{i,t}$ is the in-sample (training sample) average return for the individual asset.
- We can average this out-of-sample predictive cross-sectional R^2 over the entire out-of-sample period and define that as the

$$\text{OOS Cross-Section } R^2 = \frac{1}{\#\{\text{ OOS set }\}} \sum_{t+1 \in \text{ OOS set}} R^2_{t+1}$$

In [ ]:
```python
def OOS_predictive_cross_sectional_R2_relative_to_benchmark(data):
    cross_section_R2 = 1 - ((data['Excess_returns'] - data['Returns_forecast

    return cross_section_R2
```

[redacted]

In-Sample Analysis

In [ ]:
```
%%time

regr = ipca.InstrumentedPCA(n_factors=4, intercept=False)
regr = regr.fit(X=features_lag, y=returns.squeeze())
# Gamma, Factors = regr.get_factors(label_ind=True)


ret_hat_package = pd.DataFrame(regr.predict(features_lag),
                               index=features_lag.index,
                               columns=['Returns_forecast'])

ret_ret_hat_package = pd.concat([returns, ret_hat_package], axis=1)

ret_ret_hat_package.head()
```

In [ ]:
```
in_sample_results = pd.DataFrame({'Total R^2': [total_R2(ret_ret_hat_package
                                  'Time-Series R^2': [time_series_R2(ret_ret
                                  'Cross-Sectional R^2': [cross_sectional_R2
                                  index=pd.MultiIndex.from_tuples([(False, 4

in_sample_results
```

In [ ]:
```
%%time

in_sample_results_list = []

for num_factors in range(1, 6):
    for intercept in [True, False]:
        print(f'***Num Factors: {num_factors}, Intercept: {intercept}***')
        regr = ipca.InstrumentedPCA(n_factors=num_factors, intercept=interce
        regr = regr.fit(X=features_lag, y=returns.squeeze(), quiet=True)

        ret_hat_package = pd.DataFrame(regr.predict(features_lag),
                                       index=features_lag.index,
                                       columns=['Returns_forecast'])

        ret_ret_hat_package = pd.concat([returns, ret_hat_package], axis=1)

        in_sample_results = pd.DataFrame({'Total R^2': [total_R2(ret_ret_hat
                                          'Time-Series R^2': [time_series_R2(r
                                          'Cross-Sectional R^2': [cross_sectio
                                          index=pd.MultiIndex.from_tuples([(ir
        in_sample_results_list.append(in_sample_results)

in_sample_results = pd.concat(in_sample_results_list, axis=0)

in_sample_results
```

In [ ]:
```
in_sample_results.sort_index().T
```

[redacted]

```
# This is formatted as code
```

Out-of-Sample Analysis

- We should pay careful attention to the unbalanced panel coming from bonds entering and exiting the sample.
- We also need to establish our training and testing window, and if we want a 1-period placeholder between the train and test period
  - i.e. train on t-1, trade on t, evaluate on t+1?, or train on t-1 and trade/evaluate on t?
  - note that we already have the lags built-in to the features!
- The minimum training window has influence over linear algebra errors in the IPCA package, i.e. if the window is too small, we'll get a LinAlg error (Matrix is singular to machine precision)

In [ ]:
```
## Students were doing rolling in-sample analysis, as opposed to rolling OOS
```

In [ ]:
```
idx = pd.IndexSlice
```

In [ ]:
```
## It appears "mean_factor=True" might be a bad decision, the R2s are very s
# but then this isn't tradeable (maybe we use F_hat_t for F_hat_t+1 as a pro

## What are appropriate OOS R2s for monthly data? As a first step, look at r

## What's happening on 2019-04-01?
```

In [ ]:
```
dates = features_lag.index.get_level_values('Date').drop_duplicates().sort_v
```

In [ ]:
```
len(features_lag.index.get_level_values('Date').drop_duplicates().sort_value
```

Out[ ]:
```
160
```

In [ ]:
```
features_lag_small = features_lag.iloc[:100]
returns_small = returns.loc[features_lag_small.index, :]
```

In [ ]:
```
# if not os.path.exists(os.path.join(data_path, 'Forecasts')):
#     os.makedirs(os.path.join(data_path, 'Forecasts'))

# if not os.path.exists(os.path.join(data_path, 'Results')):
#     os.makedirs(os.path.join(data_path, 'Results'))
```

```
In [ ]: %%time
## Takes about 2438mins <=> 40 hours to run

dates = features_lag.index.get_level_values('Date').drop_duplicates().sort_v

# out_of_sample_forecasts_dict = {}
# out_of_sample_results_dict = {}


for min_training_window in [60]:

    months = features_lag.index.get_level_values('Date').drop_duplicates().s

    for training_lag in [25]: #range(0, 26, 5):
        # print(f'***Training Window: {min_training_window}, Training Lag: {
        out_of_sample_forecasts = []
        out_of_sample_results = []

        ####################################################################

        for num_factors in range(1, 6):
            for intercept in [True, False]:
                for t in months:
                    # print(f'***Num Factors: {num_factors}, Intercept: {int

                    # (t-1)-measurable
                    # (t-training_lag)-measurable
                    X_train = features_lag.loc[idx[:, dates[max(0, dates.get
                    Y_train = returns.squeeze().loc[idx[:, dates[max(0, date
                    # X_train = features_lag.loc[idx[:, : dates[dates.get_lo
                    # Y_train = returns.squeeze().loc[idx[:, : dates[dates.g

                    try:
                        regr = ipca.InstrumentedPCA(n_factors=num_factors, i
                        regr = regr.fit(X=X_train, y=Y_train, quiet=True)
                        # Gamma, Factors = regr.get_factors(label_ind=True)


                        # (t)-measurable
                        X_test = features_lag.loc[idx[:, t], :]
                        Y_test = returns[returns.index.get_level_values('Dat

                        ####################################################
                        #### This is where I am dealing with the imbalanced
                        #### check how we're handling the imbalanced panel
                        ret_hat_package = pd.DataFrame(regr.predict(X_test,
                                                       index=X_test.index,
                                                       columns=['Returns_foreca

                        ret_hat_benchmark = Y_train.groupby('Cusip').mean().
                        ret_ret_hat_benchmark = pd.merge(Y_test.reset_index(

                        #### NEED TO SAVE THESE ####
                        ret_ret_hat_benchmark_package = pd.concat([ret_ret_h
                        ret_ret_hat_benchmark_package['Intercept'] = interce
```

```python
                    ret_ret_hat_benchmark_package['Num_Factors'] = num_f
                    ####################################################

                    oos_results = pd.DataFrame({'OOS R^2 Over Benchmark'
                                                'OOS R^2 Over Zero' : [t
                                                index=pd.MultiIndex.from
                except Exception as e:
                    print(f"An error occurred: {e} on month {t}")
                    print(f'***Num Factors: {num_factors}, Intercept: {i
                    print(f'***Training Window: {min_training_window}, T
                    ret_ret_hat_benchmark_package = pd.DataFrame()  # Em
                    oos_results = pd.DataFrame({'OOS R^2 Over Benchmark'
                                                'OOS R^2 Over Zero': [np
                                                index=pd.MultiIndex.from
#####################################################################
#####################################################################
                out_of_sample_forecasts.append(ret_ret_hat_benchmark_pac
                out_of_sample_results.append(oos_results)

        out_of_sample_forecasts = pd.concat(out_of_sample_forecasts, axis=0)
        #out_of_sample_forecasts.to_csv(os.path.join(data_path, 'Forecasts',

        out_of_sample_results = pd.concat(out_of_sample_results, axis=0)
        #out_of_sample_results.to_csv(os.path.join(data_path, 'Results', f'w

        # out_of_sample_forecasts_dict[min_training_window] = {training_lag:
        # out_of_sample_results_dict[min_training_window] = {training_lag: c
#####################################################################
#####################################################################


# ## Write forecasts to disk
# for window_size in out_of_sample_forecasts_dict.keys():
#     for lags in out_of_sample_forecasts_dict[window_size].keys():
#         out_of_sample_forecasts_dict[window_size][lags].to_csv(os.path.joi

# ## Write results to disk
# for window_size in out_of_sample_results_dict.keys():
#     for lags in out_of_sample_results_dict[window_size].keys():
#         out_of_sample_results_dict[window_size][lags].to_csv(os.path.join(
```

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The panel dimensions are:
n_samples: 2546 , L: 49 , T: 36
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The panel dimensions are:
n_samples: 2524 , L: 49 , T: 36
An error occurred: Matrix is singular to machine precision. on month 2019-10
-01
```

```
---------------------------------------------------------------------------
LinAlgError                               Traceback (most recent call last)
File <timed exec>:34

File ~\anaconda3\Lib\site-packages\ipca\ipca.py:221, in InstrumentedPCA.fit
(self, X, y, indices, PSF, Gamma, Factors, data_type, label_ind, **kwargs)
    220 # Run IPCA
--> 221 Gamma, Factors = self._fit_ipca(X=X, y=y, indices=indices, Q=Q,
    222                                 W=W, val_obs=val_obs, PSF=PSF,
    223                                 Gamma=Gamma, Factors=Factors,
    224                                 data_type=data_type, **kwargs)
    226 # Store estimates

File ~\anaconda3\Lib\site-packages\ipca\ipca.py:1013, in InstrumentedPCA._fi
t_ipca(self, X, y, indices, PSF, Q, W, val_obs, Gamma, Factors, quiet, data_
type, **kwargs)
   1011 while((iter <= self.max_iter) and (tol_current > self.iter_tol)):
-> 1013     Gamma_New, Factor_New = ALS_fit(Gamma_Old, *ALS_inputs,
   1014                                     PSF=PSF, **kwargs)
   1016     if self.PSFcase:

File ~\anaconda3\Lib\site-packages\ipca\ipca.py:1099, in InstrumentedPCA._AL
S_fit_portfolio(self, Gamma_Old, Q, W, val_obs, PSF, **kwargs)
   1098 # ALS Step 2
-> 1099 Gamma_New = _Gamma_fit_portfolio(F_New, Q, W, val_obs, PSF, L, K,
   1100                                  Ktilde, T)
   1102 # condition checks

File ~\anaconda3\Lib\site-packages\ipca\ipca.py:1520, in _Gamma_fit_portfoli
o(F_New, Q, W, val_obs, PSF, L, K, Ktilde, T)
   1515         Denom += _numba_kron(W[:, :, t],
   1516                              PSF[:, t].reshape((-1, 1))
   1517                              .dot(PSF[:, t].reshape((-1, 1)).T))\
   1518                              * val_obs[t]
-> 1520 Gamma_New = _numba_solve(Denom, Numer).reshape((L, Ktilde))
   1522 return Gamma_New

File ~\anaconda3\Lib\site-packages\numba\np\linalg.py:899, in _inv_err_handl
er()
    898 if r > 0:
--> 899     raise np.linalg.LinAlgError(
    900         "Matrix is singular to machine precision.")

LinAlgError: Matrix is singular to machine precision.

During handling of the above exception, another exception occurred:

AttributeError                            Traceback (most recent call last)
File <timed exec>:63

AttributeError: 'str' object has no attribute 'date'
```

In [ ]:

```
%%time
import numpy as np
import pandas as pd
from numpy.linalg import LinAlgError
```

```
dates = features_lag.index.get_level_values('Date').drop_duplicates().sort_v

for min_training_window in [60]:
    months = dates[min_training_window:]

    for training_lag in [25]:
        out_of_sample_forecasts = []
        out_of_sample_results = []

        for num_factors in range(1, 6):
            for intercept in [True, False]:
                for t in months:
                    try:
                        # Training data (t - training_lag measurable)
                        start_idx = max(0, dates.get_loc(t) - min_training_w
                        end_idx = dates.get_loc(t) - training_lag
                        if end_idx <= start_idx:
                            continue  # Not enough data

                        X_train = features_lag.loc[idx[:, dates[start_idx] :
                        Y_train = returns.squeeze().loc[idx[:, dates[start_i

                        # Drop constant columns and rows with NaNs
                        X_train = X_train.loc[:, X_train.std() > 1e-8].dropn
                        Y_train = Y_train.loc[X_train.index].dropna()

                        if X_train.empty or Y_train.empty:
                            continue  # Skip if no usable data

                        regr = ipca.InstrumentedPCA(n_factors=num_factors, i
                        regr = regr.fit(X=X_train, y=Y_train, quiet=True)

                        # Test data (t measurable)
                        X_test = features_lag.loc[idx[:, t], :]
                        Y_test = returns.loc[idx[:, t]]

                        ret_hat_package = pd.DataFrame(
                            regr.predict(X_test, mean_factor=True),
                            index=X_test.index,
                            columns=['Returns_forecast']
                        )

                        # Benchmark: cross-sectional mean
                        ret_hat_benchmark = Y_train.groupby('Cusip').mean().
                        ret_hat_benchmark.columns = ['Returns_benchmark_fore

                        ret_ret_hat_benchmark = pd.merge(
                            Y_test.reset_index(), ret_hat_benchmark.reset_in
                            on='Cusip', how='inner'
                        ).set_index(['Cusip', 'Date'])

                        ret_ret_hat_benchmark_package = pd.concat(
                            [ret_ret_hat_benchmark, ret_hat_package], axis=1
                        )
                        ret_ret_hat_benchmark_package['Intercept'] = interce
```

```python
                        ret_ret_hat_benchmark_package['Num_Factors'] = num_f

                        oos_results = pd.DataFrame({
                            'OOS R^2 Over Benchmark': [
                                OOS_predictive_cross_sectional_R2_relative_t
                            ],
                            'OOS R^2 Over Zero': [
                                total_R2(ret_ret_hat_benchmark_package)
                            ]
                        }, index=pd.MultiIndex.from_tuples(
                            [(intercept, num_factors, t)], names=['Intercept
                        ))

                    except LinAlgError as e:
                        print(f"LinAlgError on {t}: {e}")
                        continue
                    except Exception as e:
                        print(f"Error on {t}: {e}")
                        continue

                    out_of_sample_forecasts.append(ret_ret_hat_benchmark_pac
                    out_of_sample_results.append(oos_results)

        out_of_sample_forecasts = pd.concat(out_of_sample_forecasts, axis=0)
        out_of_sample_results = pd.concat(out_of_sample_results, axis=0)
```

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The panel dimensions are:
n_samples: 1656 , L: 49 , T: 36
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Error on 2015-02-01: '2015-02-01'
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The panel dimensions are:
n_samples: 2399 , L: 49 , T: 36
[========================================================================] 1
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Error on 2018-11-01: '2018-11-01'
The panel dimensions are:
n_samples: 2430 , L: 49 , T: 36
[========================================================================] 1
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Error on 2018-12-01: '2018-12-01'
The panel dimensions are:
n_samples: 2432 , L: 49 , T: 36
[========================================================================] 1
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Error on 2019-01-01: '2019-01-01'
The panel dimensions are:
n_samples: 2447 , L: 49 , T: 36
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88%
Error on 2019-02-01: '2019-02-01'
The panel dimensions are:
n_samples: 2470 , L: 49 , T: 36
[========================================================================] 1
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91%
Error on 2019-03-01: '2019-03-01'
The panel dimensions are:
n_samples: 2468 , L: 49 , T: 36
[========================================================================] 1
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63%
Error on 2019-04-01: '2019-04-01'
The panel dimensions are:
n_samples: 2482 , L: 49 , T: 36
[========================================================================] 1
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91%
Error on 2019-05-01: '2019-05-01'
The panel dimensions are:
n_samples: 2500 , L: 49 , T: 36
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[========================================================================] 1
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91%
Error on 2019-06-01: '2019-06-01'
The panel dimensions are:
n_samples: 2497 , L: 49 , T: 36
[========================================================================] 1
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Error on 2019-07-01: '2019-07-01'
The panel dimensions are:
n_samples: 2500 , L: 49 , T: 36
[==============================================                          ]
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Error on 2019-08-01: '2019-08-01'
The panel dimensions are:
n_samples: 2546 , L: 49 , T: 36
[========================================================================] 1
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Error on 2019-09-01: '2019-09-01'
The panel dimensions are:
n_samples: 2524 , L: 49 , T: 36
[==========================================                              ]
58%
LinAlgError on 2019-10-01: Matrix is singular to machine precision.
The panel dimensions are:
n_samples: 2546 , L: 49 , T: 36
[========================================================================] 1
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LinAlgError on 2019-11-01: Matrix is singular to machine precision.
The panel dimensions are:
n_samples: 2621 , L: 49 , T: 36
[====================================================================    ]
94%
Error on 2019-12-01: '2019-12-01'
The panel dimensions are:
n_samples: 2593 , L: 49 , T: 36
[========================================================================] 1
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[====================================================================    ]
94%
Error on 2020-01-01: '2020-01-01'
The panel dimensions are:
n_samples: 2619 , L: 49 , T: 36
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[========================================================================] 1
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[===============================================================         ]
88%
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Error on 2020-02-01: '2020-02-01'
The panel dimensions are:
n_samples: 2647 , L: 49 , T: 36
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[========================================================================] 1
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91%
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Error on 2020-03-01: '2020-03-01'
The panel dimensions are:
n_samples: 2622 , L: 49 , T: 36
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[========================================================================] 1
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80%
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Error on 2020-04-01: '2020-04-01'
The panel dimensions are:
n_samples: 2651 , L: 49 , T: 36
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[========================================================================] 1
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Error on 2020-05-01: '2020-05-01'
The panel dimensions are:
n_samples: 2693 , L: 49 , T: 36
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[==============================================================          ]
86%
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Error on 2020-06-01: '2020-06-01'
The panel dimensions are:
n_samples: 2693 , L: 49 , T: 36
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Error on 2020-07-01: '2020-07-01'
The panel dimensions are:
n_samples: 2741 , L: 49 , T: 36
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86%
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Error on 2020-08-01: '2020-08-01'
The panel dimensions are:
n_samples: 2765 , L: 49 , T: 36
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[========================================================================] 1
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83%
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Error on 2020-09-01: '2020-09-01'
The panel dimensions are:
n_samples: 2733 , L: 49 , T: 36
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[=========================================================================] 1
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Error on 2020-10-01: '2020-10-01'
The panel dimensions are:
n_samples: 2771 , L: 49 , T: 36
[==================================================================       ]
91%
Error on 2020-11-01: '2020-11-01'
The panel dimensions are:
n_samples: 2785 , L: 49 , T: 36
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Error on 2020-12-01: '2020-12-01'
The panel dimensions are:
n_samples: 2759 , L: 49 , T: 36
[=========================================================================] 1
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88%
Error on 2021-01-01: '2021-01-01'
The panel dimensions are:
n_samples: 2786 , L: 49 , T: 36
[=========================================================================] 1
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80%
Error on 2021-02-01: '2021-02-01'
The panel dimensions are:
n_samples: 2809 , L: 49 , T: 36
[=========================================================================] 1
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88%
Error on 2021-03-01: '2021-03-01'
The panel dimensions are:
n_samples: 2801 , L: 49 , T: 36
[=========================================================================] 1
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94%
Error on 2021-04-01: '2021-04-01'
The panel dimensions are:
n_samples: 2813 , L: 49 , T: 36
[=========================================================================] 1
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86%
Error on 2021-05-01: '2021-05-01'
The panel dimensions are:
n_samples: 2847 , L: 49 , T: 36
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69%
Error on 2021-06-01: '2021-06-01'
The panel dimensions are:
n_samples: 2845 , L: 49 , T: 36
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75%
Error on 2021-07-01: '2021-07-01'
The panel dimensions are:
n_samples: 2856 , L: 49 , T: 36
[========================================================================] 1
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83%
Error on 2021-08-01: '2021-08-01'
The panel dimensions are:
n_samples: 2930 , L: 49 , T: 36
[========================================================================] 1
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77%
Error on 2021-09-01: '2021-09-01'
The panel dimensions are:
n_samples: 2934 , L: 49 , T: 36
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94%
Error on 2021-10-01: '2021-10-01'
The panel dimensions are:
n_samples: 2947 , L: 49 , T: 36
[========================================================================] 1
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91%
Error on 2021-11-01: '2021-11-01'
The panel dimensions are:
n_samples: 2983 , L: 49 , T: 36
[========================================================================] 1
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75%
Error on 2021-12-01: '2021-12-01'
The panel dimensions are:
n_samples: 2974 , L: 49 , T: 36
[========================================================================] 1
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83%
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Error on 2022-01-01: '2022-01-01'
The panel dimensions are:
n_samples: 3009 , L: 49 , T: 36
[========================================================================] 1
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75%
Error on 2022-02-01: '2022-02-01'
The panel dimensions are:
n_samples: 3047 , L: 49 , T: 36
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77%
Error on 2022-03-01: '2022-03-01'
The panel dimensions are:
n_samples: 3021 , L: 49 , T: 36
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Error on 2022-04-01: '2022-04-01'
The panel dimensions are:
n_samples: 3056 , L: 49 , T: 36
[========================================================================] 1
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80%
Error on 2022-05-01: '2022-05-01'
The panel dimensions are:
n_samples: 3070 , L: 49 , T: 36
[========================================================================] 1
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75%
Error on 2022-06-01: '2022-06-01'
The panel dimensions are:
n_samples: 3064 , L: 49 , T: 36
[========================================================================] 1
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58%
Error on 2022-07-01: '2022-07-01'
The panel dimensions are:
n_samples: 3076 , L: 49 , T: 36
[========================================================================] 1
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80%
Error on 2022-08-01: '2022-08-01'
The panel dimensions are:
n_samples: 3067 , L: 49 , T: 36
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63%
Error on 2022-09-01: '2022-09-01'
The panel dimensions are:
n_samples: 3077 , L: 49 , T: 36
[========================================================================] 1
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61%
Error on 2022-10-01: '2022-10-01'
The panel dimensions are:
n_samples: 3084 , L: 49 , T: 36
[========================================================================] 1
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Error on 2022-11-01: '2022-11-01'
The panel dimensions are:
n_samples: 3077 , L: 49 , T: 36
[========================================================================] 1
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61%
Error on 2022-12-01: '2022-12-01'
The panel dimensions are:
n_samples: 3363 , L: 49 , T: 36
[========================================================================] 1
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69%
Error on 2023-01-01: '2023-01-01'
The panel dimensions are:
n_samples: 3458 , L: 49 , T: 36
[========================================================================] 1
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66%
Error on 2023-02-01: '2023-02-01'
The panel dimensions are:
n_samples: 3533 , L: 49 , T: 36
[========================================================================] 1
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[====================================================                    ]
72%
Error on 2023-03-01: '2023-03-01'
The panel dimensions are:
n_samples: 3512 , L: 49 , T: 36
[========================================================================] 1
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69%
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Error on 2023-04-01: '2023-04-01'
The panel dimensions are:
n_samples: 3560 , L: 49 , T: 36
[==========================================================================] 1
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Error on 2023-05-01: '2023-05-01'
The panel dimensions are:
n_samples: 1656 , L: 49 , T: 36
[==========================================================================] 1
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LinAlgError on 2015-02-01: Matrix is singular to machine precision.
The panel dimensions are:
n_samples: 1696 , L: 49 , T: 36
[==========================================================================] 1
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LinAlgError on 2015-03-01: Matrix is singular to machine precision.
The panel dimensions are:
n_samples: 1710 , L: 49 , T: 36
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LinAlgError on 2015-04-01: Matrix is singular to machine precision.
The panel dimensions are:
n_samples: 1738 , L: 49 , T: 36
[==========================================================================] 1
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LinAlgError on 2015-05-01: Matrix is singular to machine precision.
The panel dimensions are:
n_samples: 1777 , L: 49 , T: 36
[==========================================================================] 1
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LinAlgError on 2015-06-01: Matrix is singular to machine precision.
The panel dimensions are:
n_samples: 1794 , L: 49 , T: 36
[==========================================================================] 1
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LinAlgError on 2015-07-01: Matrix is singular to machine precision.
The panel dimensions are:
n_samples: 1823 , L: 49 , T: 36
[==========================================================================] 1
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LinAlgError on 2015-08-01: Matrix is singular to machine precision.
The panel dimensions are:
n_samples: 1872 , L: 49 , T: 36
[==========================================================================] 1
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LinAlgError on 2015-09-01: Matrix is singular to machine precision.
The panel dimensions are:
n_samples: 1895 , L: 49 , T: 36
[==========================================================================] 1
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LinAlgError on 2015-10-01: Matrix is singular to machine precision.
The panel dimensions are:
n_samples: 1914 , L: 49 , T: 36
```

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[===========================================================================] 1
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LinAlgError on 2015-11-01: Matrix is singular to machine precision.
The panel dimensions are:
n_samples: 1951 , L: 49 , T: 36
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[===========================================================================] 1
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LinAlgError on 2015-12-01: Matrix is singular to machine precision.
The panel dimensions are:
n_samples: 1957 , L: 49 , T: 36
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[===========================================================================] 1
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LinAlgError on 2016-01-01: Matrix is singular to machine precision.
The panel dimensions are:
n_samples: 1979 , L: 49 , T: 36
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[===========================================================================] 1
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LinAlgError on 2016-02-01: Matrix is singular to machine precision.
The panel dimensions are:
n_samples: 1987 , L: 49 , T: 36
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[===========================================================================] 1
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LinAlgError on 2016-03-01: Matrix is singular to machine precision.
The panel dimensions are:
n_samples: 1982 , L: 49 , T: 36
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[===========================================================================] 1
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LinAlgError on 2016-04-01: Matrix is singular to machine precision.
The panel dimensions are:
n_samples: 2008 , L: 49 , T: 36
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[===========================================================================] 1
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LinAlgError on 2016-05-01: Matrix is singular to machine precision.
The panel dimensions are:
n_samples: 2031 , L: 49 , T: 36
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[===========================================================================] 1
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LinAlgError on 2016-06-01: Matrix is singular to machine precision.
The panel dimensions are:
n_samples: 2041 , L: 49 , T: 36
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[===========================================================================] 1
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LinAlgError on 2016-07-01: Matrix is singular to machine precision.
The panel dimensions are:
n_samples: 2072 , L: 49 , T: 36
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[=========================================================                ]
77%
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Error on 2016-08-01: '2016-08-01'
The panel dimensions are:
n_samples: 2087 , L: 49 , T: 36
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[========================================================================] 1
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LinAlgError on 2016-09-01: Matrix is singular to machine precision.
The panel dimensions are:
n_samples: 2097 , L: 49 , T: 36
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LinAlgError on 2016-10-01: Matrix is singular to machine precision.
The panel dimensions are:
n_samples: 2106 , L: 49 , T: 36
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LinAlgError on 2016-11-01: Matrix is singular to machine precision.
The panel dimensions are:
n_samples: 2146 , L: 49 , T: 36
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97%
LinAlgError on 2016-12-01: Matrix is singular to machine precision.
The panel dimensions are:
n_samples: 2129 , L: 49 , T: 36
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LinAlgError on 2017-01-01: Matrix is singular to machine precision.
The panel dimensions are:
n_samples: 2146 , L: 49 , T: 36
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97%
LinAlgError on 2017-02-01: Matrix is singular to machine precision.
The panel dimensions are:
n_samples: 2164 , L: 49 , T: 36
[========================================================================] 1
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Error on 2017-03-01: '2017-03-01'
The panel dimensions are:
n_samples: 2169 , L: 49 , T: 36
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Error on 2017-04-01: '2017-04-01'
The panel dimensions are:
n_samples: 2176 , L: 49 , T: 36
[======================================================                  ]
75%
Error on 2017-05-01: '2017-05-01'
The panel dimensions are:
n_samples: 2199 , L: 49 , T: 36
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88%
Error on 2017-06-01: '2017-06-01'
The panel dimensions are:
n_samples: 2190 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2017-07-01: '2017-07-01'
The panel dimensions are:
n_samples: 2215 , L: 49 , T: 36
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Error on 2017-08-01: '2017-08-01'
The panel dimensions are:
n_samples: 2217 , L: 49 , T: 36
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Error on 2017-09-01: '2017-09-01'
The panel dimensions are:
n_samples: 2232 , L: 49 , T: 36
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Error on 2017-10-01: '2017-10-01'
The panel dimensions are:
n_samples: 2242 , L: 49 , T: 36
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Error on 2017-11-01: '2017-11-01'
The panel dimensions are:
n_samples: 2277 , L: 49 , T: 36
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Error on 2017-12-01: '2017-12-01'
The panel dimensions are:
n_samples: 2274 , L: 49 , T: 36
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Error on 2018-01-01: '2018-01-01'
The panel dimensions are:
n_samples: 2300 , L: 49 , T: 36
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Error on 2018-02-01: '2018-02-01'
The panel dimensions are:
n_samples: 2299 , L: 49 , T: 36
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Error on 2018-03-01: '2018-03-01'
The panel dimensions are:
n_samples: 2308 , L: 49 , T: 36
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Error on 2018-04-01: '2018-04-01'
The panel dimensions are:
n_samples: 2304 , L: 49 , T: 36
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Error on 2018-05-01: '2018-05-01'
The panel dimensions are:
n_samples: 2301 , L: 49 , T: 36
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Error on 2018-06-01: '2018-06-01'
The panel dimensions are:
n_samples: 2325 , L: 49 , T: 36
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Error on 2018-07-01: '2018-07-01'
The panel dimensions are:
n_samples: 2357 , L: 49 , T: 36
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Error on 2018-08-01: '2018-08-01'
The panel dimensions are:
n_samples: 2366 , L: 49 , T: 36
[====================================================                     ]
72%
Error on 2018-09-01: '2018-09-01'
The panel dimensions are:
n_samples: 2385 , L: 49 , T: 36
```

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Error on 2018-10-01: '2018-10-01'
The panel dimensions are:
n_samples: 2399 , L: 49 , T: 36
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Error on 2018-11-01: '2018-11-01'
The panel dimensions are:
n_samples: 2430 , L: 49 , T: 36
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Error on 2018-12-01: '2018-12-01'
The panel dimensions are:
n_samples: 2432 , L: 49 , T: 36
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Error on 2019-01-01: '2019-01-01'
The panel dimensions are:
n_samples: 2447 , L: 49 , T: 36
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Error on 2019-02-01: '2019-02-01'
The panel dimensions are:
n_samples: 2470 , L: 49 , T: 36
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Error on 2019-03-01: '2019-03-01'
The panel dimensions are:
n_samples: 2468 , L: 49 , T: 36
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Error on 2019-04-01: '2019-04-01'
The panel dimensions are:
n_samples: 2482 , L: 49 , T: 36
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Error on 2019-05-01: '2019-05-01'
The panel dimensions are:
n_samples: 2500 , L: 49 , T: 36
[========================================================================] 1
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91%
Error on 2019-06-01: '2019-06-01'
The panel dimensions are:
n_samples: 2497 , L: 49 , T: 36
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Error on 2019-07-01: '2019-07-01'
The panel dimensions are:
n_samples: 2500 , L: 49 , T: 36
[========================================================================] 1
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83%
Error on 2019-08-01: '2019-08-01'
The panel dimensions are:
n_samples: 2546 , L: 49 , T: 36
[========================================================================] 1
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Error on 2019-09-01: '2019-09-01'
The panel dimensions are:
n_samples: 2524 , L: 49 , T: 36
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Error on 2019-10-01: '2019-10-01'
The panel dimensions are:
n_samples: 2546 , L: 49 , T: 36
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75%
Error on 2019-11-01: '2019-11-01'
The panel dimensions are:
n_samples: 2621 , L: 49 , T: 36
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75%
Error on 2019-12-01: '2019-12-01'
The panel dimensions are:
n_samples: 2593 , L: 49 , T: 36
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91%
Error on 2020-01-01: '2020-01-01'
The panel dimensions are:
n_samples: 2619 , L: 49 , T: 36
[========================================================================] 1
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72%
Error on 2020-02-01: '2020-02-01'
The panel dimensions are:
n_samples: 2647 , L: 49 , T: 36
[========================================================================] 1
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Error on 2020-03-01: '2020-03-01'
The panel dimensions are:
n_samples: 2622 , L: 49 , T: 36
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Error on 2020-04-01: '2020-04-01'
The panel dimensions are:
n_samples: 2651 , L: 49 , T: 36
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86%
Error on 2020-05-01: '2020-05-01'
The panel dimensions are:
n_samples: 2693 , L: 49 , T: 36
[========================================================================] 1
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61%
Error on 2020-06-01: '2020-06-01'
The panel dimensions are:
n_samples: 2693 , L: 49 , T: 36
[========================================================================] 1
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72%
Error on 2020-07-01: '2020-07-01'
The panel dimensions are:
n_samples: 2741 , L: 49 , T: 36
[========================================================================] 1
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75%
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Error on 2020-08-01: '2020-08-01'
The panel dimensions are:
n_samples: 2765 , L: 49 , T: 36
[=========================================================================] 1
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66%
Error on 2020-09-01: '2020-09-01'
The panel dimensions are:
n_samples: 2733 , L: 49 , T: 36
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77%
Error on 2020-10-01: '2020-10-01'
The panel dimensions are:
n_samples: 2771 , L: 49 , T: 36
[=========================================================================] 1
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83%
Error on 2020-11-01: '2020-11-01'
The panel dimensions are:
n_samples: 2785 , L: 49 , T: 36
[=========================================================================] 1
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72%
Error on 2020-12-01: '2020-12-01'
The panel dimensions are:
n_samples: 2759 , L: 49 , T: 36
[=========================================================================] 1
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75%
Error on 2021-01-01: '2021-01-01'
The panel dimensions are:
n_samples: 2786 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2021-02-01: '2021-02-01'
The panel dimensions are:
n_samples: 2809 , L: 49 , T: 36
[=========================================================================] 1
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83%
Error on 2021-03-01: '2021-03-01'
The panel dimensions are:
n_samples: 2801 , L: 49 , T: 36
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[========================================================================] 1
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75%
Error on 2021-04-01: '2021-04-01'
The panel dimensions are:
n_samples: 2813 , L: 49 , T: 36
[========================================================================] 1
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88%
Error on 2021-05-01: '2021-05-01'
The panel dimensions are:
n_samples: 2847 , L: 49 , T: 36
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[========================================================                ]
77%
Error on 2021-06-01: '2021-06-01'
The panel dimensions are:
n_samples: 2845 , L: 49 , T: 36
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66%
Error on 2021-07-01: '2021-07-01'
The panel dimensions are:
n_samples: 2856 , L: 49 , T: 36
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77%
Error on 2021-08-01: '2021-08-01'
The panel dimensions are:
n_samples: 2930 , L: 49 , T: 36
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63%
Error on 2021-09-01: '2021-09-01'
The panel dimensions are:
n_samples: 2934 , L: 49 , T: 36
[========================================================================] 1
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69%
Error on 2021-10-01: '2021-10-01'
The panel dimensions are:
n_samples: 2947 , L: 49 , T: 36
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[======================================                                  ]
52%
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Error on 2021-11-01: '2021-11-01'
The panel dimensions are:
n_samples: 2983 , L: 49 , T: 36
[========================================================================] 1
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[==========================================                              ]
58%
Error on 2021-12-01: '2021-12-01'
The panel dimensions are:
n_samples: 2974 , L: 49 , T: 36
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66%
Error on 2022-01-01: '2022-01-01'
The panel dimensions are:
n_samples: 3009 , L: 49 , T: 36
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52%
Error on 2022-02-01: '2022-02-01'
The panel dimensions are:
n_samples: 3047 , L: 49 , T: 36
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69%
Error on 2022-03-01: '2022-03-01'
The panel dimensions are:
n_samples: 3021 , L: 49 , T: 36
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38%
Error on 2022-04-01: '2022-04-01'
The panel dimensions are:
n_samples: 3056 , L: 49 , T: 36
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72%
Error on 2022-05-01: '2022-05-01'
The panel dimensions are:
n_samples: 3070 , L: 49 , T: 36
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Error on 2022-06-01: '2022-06-01'
The panel dimensions are:
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Error on 2022-07-01: '2022-07-01'
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Error on 2022-08-01: '2022-08-01'
The panel dimensions are:
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Error on 2022-09-01: '2022-09-01'
The panel dimensions are:
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Error on 2022-10-01: '2022-10-01'
The panel dimensions are:
n_samples: 3084 , L: 49 , T: 36
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Error on 2022-11-01: '2022-11-01'
The panel dimensions are:
n_samples: 3077 , L: 49 , T: 36
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55%
Error on 2022-12-01: '2022-12-01'
The panel dimensions are:
n_samples: 3363 , L: 49 , T: 36
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Error on 2023-01-01: '2023-01-01'
The panel dimensions are:
n_samples: 3458 , L: 49 , T: 36
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Error on 2023-02-01: '2023-02-01'
The panel dimensions are:
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Error on 2023-03-01: '2023-03-01'
The panel dimensions are:
n_samples: 3512 , L: 49 , T: 36
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47%
Error on 2023-04-01: '2023-04-01'
The panel dimensions are:
n_samples: 3560 , L: 49 , T: 36
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Error on 2023-05-01: '2023-05-01'
The panel dimensions are:
n_samples: 1656 , L: 49 , T: 36
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Error on 2015-02-01: '2015-02-01'
The panel dimensions are:
n_samples: 1696 , L: 49 , T: 36
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Error on 2015-03-01: '2015-03-01'
The panel dimensions are:
n_samples: 1710 , L: 49 , T: 36
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Error on 2015-04-01: '2015-04-01'
The panel dimensions are:
n_samples: 1738 , L: 49 , T: 36
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Error on 2015-05-01: '2015-05-01'
The panel dimensions are:
n_samples: 1777 , L: 49 , T: 36
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Error on 2015-06-01: '2015-06-01'
The panel dimensions are:
n_samples: 1794 , L: 49 , T: 36
[========================================================================] 1
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Error on 2015-07-01: '2015-07-01'
The panel dimensions are:
n_samples: 1823 , L: 49 , T: 36
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Error on 2015-08-01: '2015-08-01'
The panel dimensions are:
n_samples: 1872 , L: 49 , T: 36
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Error on 2015-09-01: '2015-09-01'
The panel dimensions are:
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Error on 2015-10-01: '2015-10-01'
The panel dimensions are:
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Error on 2015-11-01: '2015-11-01'
The panel dimensions are:
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Error on 2015-12-01: '2015-12-01'
The panel dimensions are:
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Error on 2016-01-01: '2016-01-01'
The panel dimensions are:
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Error on 2016-02-01: '2016-02-01'
The panel dimensions are:
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94%
Error on 2016-03-01: '2016-03-01'
The panel dimensions are:
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Error on 2016-04-01: '2016-04-01'
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Error on 2016-05-01: '2016-05-01'
The panel dimensions are:
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Error on 2016-06-01: '2016-06-01'
The panel dimensions are:
n_samples: 2041 , L: 49 , T: 36
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Error on 2016-07-01: '2016-07-01'
The panel dimensions are:
n_samples: 2072 , L: 49 , T: 36
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Error on 2016-08-01: '2016-08-01'
The panel dimensions are:
n_samples: 2087 , L: 49 , T: 36
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Error on 2016-09-01: '2016-09-01'
The panel dimensions are:
n_samples: 2097 , L: 49 , T: 36
[===========================================================================] 1
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Error on 2016-10-01: '2016-10-01'
The panel dimensions are:
n_samples: 2106 , L: 49 , T: 36
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83%
Error on 2016-11-01: '2016-11-01'
The panel dimensions are:
n_samples: 2146 , L: 49 , T: 36
[===========================================================================] 1
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Error on 2016-12-01: '2016-12-01'
The panel dimensions are:
n_samples: 2129 , L: 49 , T: 36
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Error on 2017-01-01: '2017-01-01'
The panel dimensions are:
n_samples: 2146 , L: 49 , T: 36
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Error on 2017-02-01: '2017-02-01'
The panel dimensions are:
n_samples: 2164 , L: 49 , T: 36
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77%
Error on 2017-03-01: '2017-03-01'
The panel dimensions are:
n_samples: 2169 , L: 49 , T: 36
[=========================================================================] 1
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86%
Error on 2017-04-01: '2017-04-01'
The panel dimensions are:
n_samples: 2176 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2017-05-01: '2017-05-01'
The panel dimensions are:
n_samples: 2199 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2017-06-01: '2017-06-01'
The panel dimensions are:
n_samples: 2190 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2017-07-01: '2017-07-01'
The panel dimensions are:
n_samples: 2215 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2017-08-01: '2017-08-01'
The panel dimensions are:
n_samples: 2217 , L: 49 , T: 36
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Error on 2017-09-01: '2017-09-01'
The panel dimensions are:
n_samples: 2232 , L: 49 , T: 36
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Error on 2017-10-01: '2017-10-01'
The panel dimensions are:
n_samples: 2242 , L: 49 , T: 36
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Error on 2017-11-01: '2017-11-01'
The panel dimensions are:
n_samples: 2277 , L: 49 , T: 36
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Error on 2017-12-01: '2017-12-01'
The panel dimensions are:
n_samples: 2274 , L: 49 , T: 36
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97%
Error on 2018-01-01: '2018-01-01'
The panel dimensions are:
n_samples: 2300 , L: 49 , T: 36
[========================================================================] 1
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Error on 2018-02-01: '2018-02-01'
The panel dimensions are:
n_samples: 2299 , L: 49 , T: 36
[========================================================================] 1
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Error on 2018-03-01: '2018-03-01'
The panel dimensions are:
n_samples: 2308 , L: 49 , T: 36
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Error on 2018-04-01: '2018-04-01'
The panel dimensions are:
n_samples: 2304 , L: 49 , T: 36
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Error on 2018-05-01: '2018-05-01'
The panel dimensions are:
n_samples: 2301 , L: 49 , T: 36
[========================================================================] 1
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Error on 2018-06-01: '2018-06-01'
The panel dimensions are:
n_samples: 2325 , L: 49 , T: 36
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Error on 2018-07-01: '2018-07-01'
The panel dimensions are:
n_samples: 2357 , L: 49 , T: 36
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97%
Error on 2018-08-01: '2018-08-01'
The panel dimensions are:
n_samples: 2366 , L: 49 , T: 36
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[=========================================================================] 1
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91%
Error on 2018-09-01: '2018-09-01'
The panel dimensions are:
n_samples: 2385 , L: 49 , T: 36
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Error on 2018-10-01: '2018-10-01'
The panel dimensions are:
n_samples: 2399 , L: 49 , T: 36
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Error on 2018-11-01: '2018-11-01'
The panel dimensions are:
n_samples: 2430 , L: 49 , T: 36
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Error on 2018-12-01: '2018-12-01'
The panel dimensions are:
n_samples: 2432 , L: 49 , T: 36
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Error on 2019-01-01: '2019-01-01'
The panel dimensions are:
n_samples: 2447 , L: 49 , T: 36
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Error on 2019-02-01: '2019-02-01'
The panel dimensions are:
n_samples: 2470 , L: 49 , T: 36
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Error on 2019-03-01: '2019-03-01'
The panel dimensions are:
n_samples: 2468 , L: 49 , T: 36
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Error on 2019-04-01: '2019-04-01'
The panel dimensions are:
n_samples: 2482 , L: 49 , T: 36
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[===============================================================          ]
86%
Error on 2019-05-01: '2019-05-01'
The panel dimensions are:
n_samples: 2500 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2019-06-01: '2019-06-01'
The panel dimensions are:
n_samples: 2497 , L: 49 , T: 36
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72%
Error on 2019-07-01: '2019-07-01'
The panel dimensions are:
n_samples: 2500 , L: 49 , T: 36
[=========================================================================] 1
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47%
Error on 2019-08-01: '2019-08-01'
The panel dimensions are:
n_samples: 2546 , L: 49 , T: 36
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Error on 2019-09-01: '2019-09-01'
The panel dimensions are:
n_samples: 2524 , L: 49 , T: 36
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Error on 2019-10-01: '2019-10-01'
The panel dimensions are:
n_samples: 2546 , L: 49 , T: 36
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Error on 2019-11-01: '2019-11-01'
The panel dimensions are:
n_samples: 2621 , L: 49 , T: 36
[=========================================================================] 1
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61%
Error on 2019-12-01: '2019-12-01'
The panel dimensions are:
n_samples: 2593 , L: 49 , T: 36
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[=========================================================================] 1
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91%
Error on 2020-01-01: '2020-01-01'
The panel dimensions are:
n_samples: 2619 , L: 49 , T: 36
[=========================================================================] 1
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80%
Error on 2020-02-01: '2020-02-01'
The panel dimensions are:
n_samples: 2647 , L: 49 , T: 36
[=========================================================================] 1
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88%
Error on 2020-03-01: '2020-03-01'
The panel dimensions are:
n_samples: 2622 , L: 49 , T: 36
[=========================================================================] 1
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80%
Error on 2020-04-01: '2020-04-01'
The panel dimensions are:
n_samples: 2651 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2020-05-01: '2020-05-01'
The panel dimensions are:
n_samples: 2693 , L: 49 , T: 36
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Error on 2020-06-01: '2020-06-01'
The panel dimensions are:
n_samples: 2693 , L: 49 , T: 36
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Error on 2020-07-01: '2020-07-01'
The panel dimensions are:
n_samples: 2741 , L: 49 , T: 36
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Error on 2020-08-01: '2020-08-01'
The panel dimensions are:
n_samples: 2765 , L: 49 , T: 36
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86%
Error on 2020-09-01: '2020-09-01'
The panel dimensions are:
n_samples: 2733 , L: 49 , T: 36
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Error on 2020-10-01: '2020-10-01'
The panel dimensions are:
n_samples: 2771 , L: 49 , T: 36
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Error on 2020-11-01: '2020-11-01'
The panel dimensions are:
n_samples: 2785 , L: 49 , T: 36
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Error on 2020-12-01: '2020-12-01'
The panel dimensions are:
n_samples: 2759 , L: 49 , T: 36
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Error on 2021-01-01: '2021-01-01'
The panel dimensions are:
n_samples: 2786 , L: 49 , T: 36
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Error on 2021-02-01: '2021-02-01'
The panel dimensions are:
n_samples: 2809 , L: 49 , T: 36
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Error on 2021-03-01: '2021-03-01'
The panel dimensions are:
n_samples: 2801 , L: 49 , T: 36
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Error on 2021-04-01: '2021-04-01'
The panel dimensions are:
n_samples: 2813 , L: 49 , T: 36
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75%
Error on 2021-05-01: '2021-05-01'
The panel dimensions are:
n_samples: 2847 , L: 49 , T: 36
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Error on 2021-06-01: '2021-06-01'
The panel dimensions are:
n_samples: 2845 , L: 49 , T: 36
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Error on 2021-07-01: '2021-07-01'
The panel dimensions are:
n_samples: 2856 , L: 49 , T: 36
[========================================================================] 1
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Error on 2021-08-01: '2021-08-01'
The panel dimensions are:
n_samples: 2930 , L: 49 , T: 36
[========================================================================] 1
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Error on 2021-09-01: '2021-09-01'
The panel dimensions are:
n_samples: 2934 , L: 49 , T: 36
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Error on 2021-10-01: '2021-10-01'
The panel dimensions are:
n_samples: 2947 , L: 49 , T: 36
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Error on 2021-11-01: '2021-11-01'
The panel dimensions are:
n_samples: 2983 , L: 49 , T: 36
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80%
Error on 2021-12-01: '2021-12-01'
The panel dimensions are:
n_samples: 2974 , L: 49 , T: 36
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[=========================================================================] 1
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75%
Error on 2022-01-01: '2022-01-01'
The panel dimensions are:
n_samples: 3009 , L: 49 , T: 36
[=========================================================================] 1
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77%
Error on 2022-02-01: '2022-02-01'
The panel dimensions are:
n_samples: 3047 , L: 49 , T: 36
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77%
Error on 2022-03-01: '2022-03-01'
The panel dimensions are:
n_samples: 3021 , L: 49 , T: 36
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Error on 2022-04-01: '2022-04-01'
The panel dimensions are:
n_samples: 3056 , L: 49 , T: 36
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Error on 2022-05-01: '2022-05-01'
The panel dimensions are:
n_samples: 3070 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2022-06-01: '2022-06-01'
The panel dimensions are:
n_samples: 3064 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2022-07-01: '2022-07-01'
The panel dimensions are:
n_samples: 3076 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2022-08-01: '2022-08-01'
The panel dimensions are:
n_samples: 3067 , L: 49 , T: 36
[=========================================================================] 1
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58%
Error on 2022-09-01: '2022-09-01'
The panel dimensions are:
n_samples: 3077 , L: 49 , T: 36
[=========================================================================] 1
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91%
Error on 2022-10-01: '2022-10-01'
The panel dimensions are:
n_samples: 3084 , L: 49 , T: 36
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77%
Error on 2022-11-01: '2022-11-01'
The panel dimensions are:
n_samples: 3077 , L: 49 , T: 36
[=========================================================================] 1
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75%
Error on 2022-12-01: '2022-12-01'
The panel dimensions are:
n_samples: 3363 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2023-01-01: '2023-01-01'
The panel dimensions are:
n_samples: 3458 , L: 49 , T: 36
[=========================================================================] 1
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88%
Error on 2023-02-01: '2023-02-01'
The panel dimensions are:
n_samples: 3533 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2023-03-01: '2023-03-01'
The panel dimensions are:
n_samples: 3512 , L: 49 , T: 36
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[========================================================================] 1
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86%
Error on 2023-04-01: '2023-04-01'
The panel dimensions are:
n_samples: 3560 , L: 49 , T: 36
[========================================================================] 1
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Error on 2023-05-01: '2023-05-01'
The panel dimensions are:
n_samples: 1656 , L: 49 , T: 36
[========================================================================] 1
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Error on 2015-02-01: '2015-02-01'
The panel dimensions are:
n_samples: 1696 , L: 49 , T: 36
[========================================================================] 1
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Error on 2015-03-01: '2015-03-01'
The panel dimensions are:
n_samples: 1710 , L: 49 , T: 36
[========================================================================] 1
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Error on 2015-04-01: '2015-04-01'
The panel dimensions are:
n_samples: 1738 , L: 49 , T: 36
[========================================================================] 1
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Error on 2015-05-01: '2015-05-01'
The panel dimensions are:
n_samples: 1777 , L: 49 , T: 36
[========================================================================] 1
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Error on 2015-06-01: '2015-06-01'
The panel dimensions are:
n_samples: 1794 , L: 49 , T: 36
[========================================================================] 1
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Error on 2015-07-01: '2015-07-01'
The panel dimensions are:
n_samples: 1823 , L: 49 , T: 36
[========================================================================] 1
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Error on 2015-08-01: '2015-08-01'
The panel dimensions are:
n_samples: 1872 , L: 49 , T: 36
[========================================================================] 1
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Error on 2015-09-01: '2015-09-01'
The panel dimensions are:
n_samples: 1895 , L: 49 , T: 36
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Error on 2015-10-01: '2015-10-01'
The panel dimensions are:
n_samples: 1914 , L: 49 , T: 36
[==========================================================================] 1
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Error on 2015-11-01: '2015-11-01'
The panel dimensions are:
n_samples: 1951 , L: 49 , T: 36
[==========================================================================] 1
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Error on 2015-12-01: '2015-12-01'
The panel dimensions are:
n_samples: 1957 , L: 49 , T: 36
[===============================================================           ]
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Error on 2016-01-01: '2016-01-01'
The panel dimensions are:
n_samples: 1979 , L: 49 , T: 36
[==========================================================================] 1
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Error on 2016-02-01: '2016-02-01'
The panel dimensions are:
n_samples: 1987 , L: 49 , T: 36
[=========================================================                 ]
77%
Error on 2016-03-01: '2016-03-01'
The panel dimensions are:
n_samples: 1982 , L: 49 , T: 36
[==========================================================================] 1
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Error on 2016-04-01: '2016-04-01'
The panel dimensions are:
n_samples: 2008 , L: 49 , T: 36
[==========================================================================] 1
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Error on 2016-05-01: '2016-05-01'
The panel dimensions are:
n_samples: 2031 , L: 49 , T: 36
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Error on 2016-06-01: '2016-06-01'
The panel dimensions are:
n_samples: 2041 , L: 49 , T: 36
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Error on 2016-07-01: '2016-07-01'
The panel dimensions are:
n_samples: 2072 , L: 49 , T: 36
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Error on 2016-08-01: '2016-08-01'
The panel dimensions are:
n_samples: 2087 , L: 49 , T: 36
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Error on 2016-09-01: '2016-09-01'
The panel dimensions are:
n_samples: 2097 , L: 49 , T: 36
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Error on 2016-10-01: '2016-10-01'
The panel dimensions are:
n_samples: 2106 , L: 49 , T: 36
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Error on 2016-11-01: '2016-11-01'
The panel dimensions are:
n_samples: 2146 , L: 49 , T: 36
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Error on 2016-12-01: '2016-12-01'
The panel dimensions are:
n_samples: 2129 , L: 49 , T: 36
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Error on 2017-01-01: '2017-01-01'
The panel dimensions are:
n_samples: 2146 , L: 49 , T: 36
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Error on 2017-02-01: '2017-02-01'
The panel dimensions are:
n_samples: 2164 , L: 49 , T: 36
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Error on 2017-03-01: '2017-03-01'
The panel dimensions are:
n_samples: 2169 , L: 49 , T: 36
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Error on 2017-04-01: '2017-04-01'
The panel dimensions are:
n_samples: 2176 , L: 49 , T: 36
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83%
Error on 2017-05-01: '2017-05-01'
The panel dimensions are:
n_samples: 2199 , L: 49 , T: 36
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Error on 2017-06-01: '2017-06-01'
The panel dimensions are:
n_samples: 2190 , L: 49 , T: 36
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Error on 2017-07-01: '2017-07-01'
The panel dimensions are:
n_samples: 2215 , L: 49 , T: 36
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Error on 2017-08-01: '2017-08-01'
The panel dimensions are:
n_samples: 2217 , L: 49 , T: 36
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Error on 2017-09-01: '2017-09-01'
The panel dimensions are:
n_samples: 2232 , L: 49 , T: 36
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Error on 2017-10-01: '2017-10-01'
The panel dimensions are:
n_samples: 2242 , L: 49 , T: 36
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Error on 2017-11-01: '2017-11-01'
The panel dimensions are:
n_samples: 2277 , L: 49 , T: 36
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Error on 2017-12-01: '2017-12-01'
The panel dimensions are:
n_samples: 2274 , L: 49 , T: 36
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Error on 2018-01-01: '2018-01-01'
The panel dimensions are:
n_samples: 2300 , L: 49 , T: 36
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Error on 2018-02-01: '2018-02-01'
The panel dimensions are:
n_samples: 2299 , L: 49 , T: 36
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97%
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Error on 2018-03-01: '2018-03-01'
The panel dimensions are:
n_samples: 2308 , L: 49 , T: 36
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77%
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Error on 2018-04-01: '2018-04-01'
The panel dimensions are:
n_samples: 2304 , L: 49 , T: 36
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Error on 2018-05-01: '2018-05-01'
The panel dimensions are:
n_samples: 2301 , L: 49 , T: 36
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Error on 2018-06-01: '2018-06-01'
The panel dimensions are:
n_samples: 2325 , L: 49 , T: 36
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Error on 2018-07-01: '2018-07-01'
The panel dimensions are:
n_samples: 2357 , L: 49 , T: 36
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Error on 2018-08-01: '2018-08-01'
The panel dimensions are:
n_samples: 2366 , L: 49 , T: 36
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Error on 2018-09-01: '2018-09-01'
The panel dimensions are:
n_samples: 2385 , L: 49 , T: 36
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Error on 2018-10-01: '2018-10-01'
The panel dimensions are:
n_samples: 2399 , L: 49 , T: 36
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Error on 2018-11-01: '2018-11-01'
The panel dimensions are:
n_samples: 2430 , L: 49 , T: 36
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Error on 2018-12-01: '2018-12-01'
The panel dimensions are:
n_samples: 2432 , L: 49 , T: 36
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Error on 2019-01-01: '2019-01-01'
The panel dimensions are:
n_samples: 2447 , L: 49 , T: 36
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Error on 2019-02-01: '2019-02-01'
The panel dimensions are:
n_samples: 2470 , L: 49 , T: 36
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Error on 2019-03-01: '2019-03-01'
The panel dimensions are:
n_samples: 2468 , L: 49 , T: 36
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Error on 2019-04-01: '2019-04-01'
The panel dimensions are:
n_samples: 2482 , L: 49 , T: 36
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Error on 2019-05-01: '2019-05-01'
The panel dimensions are:
n_samples: 2500 , L: 49 , T: 36
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Error on 2019-06-01: '2019-06-01'
The panel dimensions are:
n_samples: 2497 , L: 49 , T: 36
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Error on 2019-07-01: '2019-07-01'
The panel dimensions are:
n_samples: 2500 , L: 49 , T: 36
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Error on 2019-08-01: '2019-08-01'
The panel dimensions are:
n_samples: 2546 , L: 49 , T: 36
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Error on 2019-09-01: '2019-09-01'
The panel dimensions are:
n_samples: 2524 , L: 49 , T: 36
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The panel dimensions are:
n_samples: 2546 , L: 49 , T: 36
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The panel dimensions are:
n_samples: 2621 , L: 49 , T: 36
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Error on 2019-12-01: '2019-12-01'
The panel dimensions are:
n_samples: 2593 , L: 49 , T: 36
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Error on 2020-01-01: '2020-01-01'
The panel dimensions are:
n_samples: 2619 , L: 49 , T: 36
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Error on 2020-02-01: '2020-02-01'
The panel dimensions are:
n_samples: 2647 , L: 49 , T: 36
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Error on 2020-03-01: '2020-03-01'
The panel dimensions are:
n_samples: 2622 , L: 49 , T: 36
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Error on 2020-04-01: '2020-04-01'
The panel dimensions are:
n_samples: 2651 , L: 49 , T: 36
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Error on 2020-05-01: '2020-05-01'
The panel dimensions are:
n_samples: 2693 , L: 49 , T: 36
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Error on 2020-06-01: '2020-06-01'
The panel dimensions are:
n_samples: 2693 , L: 49 , T: 36
[========================================================================] 1
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Error on 2020-07-01: '2020-07-01'
The panel dimensions are:
n_samples: 2741 , L: 49 , T: 36
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Error on 2020-08-01: '2020-08-01'
The panel dimensions are:
n_samples: 2765 , L: 49 , T: 36
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Error on 2020-09-01: '2020-09-01'
The panel dimensions are:
n_samples: 2733 , L: 49 , T: 36
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Error on 2020-10-01: '2020-10-01'
The panel dimensions are:
n_samples: 2771 , L: 49 , T: 36
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Error on 2020-11-01: '2020-11-01'
The panel dimensions are:
n_samples: 2785 , L: 49 , T: 36
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Error on 2020-12-01: '2020-12-01'
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The panel dimensions are:
n_samples: 2759 , L: 49 , T: 36
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Error on 2021-01-01: '2021-01-01'
The panel dimensions are:
n_samples: 2786 , L: 49 , T: 36
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Error on 2021-02-01: '2021-02-01'
The panel dimensions are:
n_samples: 2809 , L: 49 , T: 36
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Error on 2021-03-01: '2021-03-01'
The panel dimensions are:
n_samples: 2801 , L: 49 , T: 36
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Error on 2021-04-01: '2021-04-01'
The panel dimensions are:
n_samples: 2813 , L: 49 , T: 36
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Error on 2021-05-01: '2021-05-01'
The panel dimensions are:
n_samples: 2847 , L: 49 , T: 36
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Error on 2021-06-01: '2021-06-01'
The panel dimensions are:
n_samples: 2845 , L: 49 , T: 36
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72%
Error on 2021-07-01: '2021-07-01'
The panel dimensions are:
n_samples: 2856 , L: 49 , T: 36
[========================================================================] 1
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94%
Error on 2021-08-01: '2021-08-01'
The panel dimensions are:
n_samples: 2930 , L: 49 , T: 36
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Error on 2021-09-01: '2021-09-01'
The panel dimensions are:
n_samples: 2934 , L: 49 , T: 36
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Error on 2021-10-01: '2021-10-01'
The panel dimensions are:
n_samples: 2947 , L: 49 , T: 36
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Error on 2021-11-01: '2021-11-01'
The panel dimensions are:
n_samples: 2983 , L: 49 , T: 36
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52%
Error on 2021-12-01: '2021-12-01'
The panel dimensions are:
n_samples: 2974 , L: 49 , T: 36
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88%
Error on 2022-01-01: '2022-01-01'
The panel dimensions are:
n_samples: 3009 , L: 49 , T: 36
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Error on 2022-02-01: '2022-02-01'
The panel dimensions are:
n_samples: 3047 , L: 49 , T: 36
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75%
Error on 2022-03-01: '2022-03-01'
The panel dimensions are:
n_samples: 3021 , L: 49 , T: 36
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75%
Error on 2022-04-01: '2022-04-01'
The panel dimensions are:
n_samples: 3056 , L: 49 , T: 36
[========================================================================] 1
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88%
Error on 2022-05-01: '2022-05-01'
The panel dimensions are:
n_samples: 3070 , L: 49 , T: 36
[========================================================================] 1
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38%
Error on 2022-06-01: '2022-06-01'
The panel dimensions are:
n_samples: 3064 , L: 49 , T: 36
[========================================================================] 1
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77%
Error on 2022-07-01: '2022-07-01'
The panel dimensions are:
n_samples: 3076 , L: 49 , T: 36
[========================================================================] 1
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55%
Error on 2022-08-01: '2022-08-01'
The panel dimensions are:
n_samples: 3067 , L: 49 , T: 36
[========================================================================] 1
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80%
Error on 2022-09-01: '2022-09-01'
The panel dimensions are:
n_samples: 3077 , L: 49 , T: 36
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[=========================================================================] 1
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83%
Error on 2022-10-01: '2022-10-01'
The panel dimensions are:
n_samples: 3084 , L: 49 , T: 36
[=========================================================================] 1
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88%
Error on 2022-11-01: '2022-11-01'
The panel dimensions are:
n_samples: 3077 , L: 49 , T: 36
[=========================================================================] 1
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86%
Error on 2022-12-01: '2022-12-01'
The panel dimensions are:
n_samples: 3363 , L: 49 , T: 36
[=========================================================================] 1
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58%
Error on 2023-01-01: '2023-01-01'
The panel dimensions are:
n_samples: 3458 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2023-02-01: '2023-02-01'
The panel dimensions are:
n_samples: 3533 , L: 49 , T: 36
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72%
Error on 2023-03-01: '2023-03-01'
The panel dimensions are:
n_samples: 3512 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2023-04-01: '2023-04-01'
The panel dimensions are:
n_samples: 3560 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2023-05-01: '2023-05-01'
The panel dimensions are:
n_samples: 1656 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2015-02-01: '2015-02-01'
The panel dimensions are:
n_samples: 1696 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2015-03-01: '2015-03-01'
The panel dimensions are:
n_samples: 1710 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2015-04-01: '2015-04-01'
The panel dimensions are:
n_samples: 1738 , L: 49 , T: 36
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Error on 2015-05-01: '2015-05-01'
The panel dimensions are:
n_samples: 1777 , L: 49 , T: 36
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Error on 2015-06-01: '2015-06-01'
The panel dimensions are:
n_samples: 1794 , L: 49 , T: 36
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Error on 2015-07-01: '2015-07-01'
The panel dimensions are:
n_samples: 1823 , L: 49 , T: 36
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Error on 2015-08-01: '2015-08-01'
The panel dimensions are:
n_samples: 1872 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2015-09-01: '2015-09-01'
The panel dimensions are:
n_samples: 1895 , L: 49 , T: 36
[===================================================================      ]
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Error on 2015-10-01: '2015-10-01'
The panel dimensions are:
n_samples: 1914 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2015-11-01: '2015-11-01'
The panel dimensions are:
n_samples: 1951 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2015-12-01: '2015-12-01'
The panel dimensions are:
n_samples: 1957 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2016-01-01: '2016-01-01'
The panel dimensions are:
n_samples: 1979 , L: 49 , T: 36
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Error on 2016-02-01: '2016-02-01'
The panel dimensions are:
n_samples: 1987 , L: 49 , T: 36
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Error on 2016-03-01: '2016-03-01'
The panel dimensions are:
n_samples: 1982 , L: 49 , T: 36
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Error on 2016-04-01: '2016-04-01'
The panel dimensions are:
n_samples: 2008 , L: 49 , T: 36
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Error on 2016-05-01: '2016-05-01'
The panel dimensions are:
n_samples: 2031 , L: 49 , T: 36
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Error on 2016-06-01: '2016-06-01'
The panel dimensions are:
n_samples: 2041 , L: 49 , T: 36
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Error on 2016-07-01: '2016-07-01'
The panel dimensions are:
n_samples: 2072 , L: 49 , T: 36
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Error on 2016-08-01: '2016-08-01'
The panel dimensions are:
n_samples: 2087 , L: 49 , T: 36
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Error on 2016-09-01: '2016-09-01'
The panel dimensions are:
n_samples: 2097 , L: 49 , T: 36
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Error on 2016-10-01: '2016-10-01'
The panel dimensions are:
n_samples: 2106 , L: 49 , T: 36
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Error on 2016-11-01: '2016-11-01'
The panel dimensions are:
n_samples: 2146 , L: 49 , T: 36
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Error on 2016-12-01: '2016-12-01'
The panel dimensions are:
n_samples: 2129 , L: 49 , T: 36
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Error on 2017-01-01: '2017-01-01'
The panel dimensions are:
n_samples: 2146 , L: 49 , T: 36
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Error on 2017-02-01: '2017-02-01'
The panel dimensions are:
n_samples: 2164 , L: 49 , T: 36
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Error on 2017-03-01: '2017-03-01'
The panel dimensions are:
n_samples: 2169 , L: 49 , T: 36
[==============================================                           ]
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Error on 2017-04-01: '2017-04-01'
The panel dimensions are:
n_samples: 2176 , L: 49 , T: 36
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83%
Error on 2017-05-01: '2017-05-01'
The panel dimensions are:
n_samples: 2199 , L: 49 , T: 36
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Error on 2017-06-01: '2017-06-01'
The panel dimensions are:
n_samples: 2190 , L: 49 , T: 36
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Error on 2017-07-01: '2017-07-01'
The panel dimensions are:
n_samples: 2215 , L: 49 , T: 36
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Error on 2017-08-01: '2017-08-01'
The panel dimensions are:
n_samples: 2217 , L: 49 , T: 36
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Error on 2017-09-01: '2017-09-01'
The panel dimensions are:
n_samples: 2232 , L: 49 , T: 36
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Error on 2017-10-01: '2017-10-01'
The panel dimensions are:
n_samples: 2242 , L: 49 , T: 36
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Error on 2017-11-01: '2017-11-01'
The panel dimensions are:
n_samples: 2277 , L: 49 , T: 36
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Error on 2017-12-01: '2017-12-01'
The panel dimensions are:
n_samples: 2274 , L: 49 , T: 36
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Error on 2018-01-01: '2018-01-01'
The panel dimensions are:
n_samples: 2300 , L: 49 , T: 36
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Error on 2018-02-01: '2018-02-01'
The panel dimensions are:
n_samples: 2299 , L: 49 , T: 36
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[====================================================================    ]
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Error on 2018-03-01: '2018-03-01'
The panel dimensions are:
n_samples: 2308 , L: 49 , T: 36
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Error on 2018-04-01: '2018-04-01'
The panel dimensions are:
n_samples: 2304 , L: 49 , T: 36
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Error on 2018-05-01: '2018-05-01'
The panel dimensions are:
n_samples: 2301 , L: 49 , T: 36
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Error on 2018-06-01: '2018-06-01'
The panel dimensions are:
n_samples: 2325 , L: 49 , T: 36
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Error on 2018-07-01: '2018-07-01'
The panel dimensions are:
n_samples: 2357 , L: 49 , T: 36
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Error on 2018-08-01: '2018-08-01'
The panel dimensions are:
n_samples: 2366 , L: 49 , T: 36
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Error on 2018-09-01: '2018-09-01'
The panel dimensions are:
n_samples: 2385 , L: 49 , T: 36
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Error on 2018-10-01: '2018-10-01'
The panel dimensions are:
n_samples: 2399 , L: 49 , T: 36
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Error on 2018-11-01: '2018-11-01'
The panel dimensions are:
n_samples: 2430 , L: 49 , T: 36
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Error on 2018-12-01: '2018-12-01'
The panel dimensions are:
n_samples: 2432 , L: 49 , T: 36
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[====================================================================    ]
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Error on 2019-01-01: '2019-01-01'
The panel dimensions are:
n_samples: 2447 , L: 49 , T: 36
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86%
Error on 2019-02-01: '2019-02-01'
The panel dimensions are:
n_samples: 2470 , L: 49 , T: 36
[========================================================================] 1
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61%
Error on 2019-03-01: '2019-03-01'
The panel dimensions are:
n_samples: 2468 , L: 49 , T: 36
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88%
Error on 2019-04-01: '2019-04-01'
The panel dimensions are:
n_samples: 2482 , L: 49 , T: 36
[========================================================================] 1
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Error on 2019-05-01: '2019-05-01'
The panel dimensions are:
n_samples: 2500 , L: 49 , T: 36
[====================================================                    ]
72%
```

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Error on 2019-06-01: '2019-06-01'
The panel dimensions are:
n_samples: 2497 , L: 49 , T: 36
[========================================================================] 1
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[================================                                        ]
44%
Error on 2019-07-01: '2019-07-01'
The panel dimensions are:
n_samples: 2500 , L: 49 , T: 36
[========================================================================] 1
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[==========================================================              ]
80%
Error on 2019-08-01: '2019-08-01'
The panel dimensions are:
n_samples: 2546 , L: 49 , T: 36
[========================================================================] 1
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[========================================                                ]
55%
Error on 2019-09-01: '2019-09-01'
The panel dimensions are:
n_samples: 2524 , L: 49 , T: 36
[========================================================================] 1
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Error on 2019-10-01: '2019-10-01'
The panel dimensions are:
n_samples: 2546 , L: 49 , T: 36
[==========================================                              ]
58%
Error on 2019-11-01: '2019-11-01'
The panel dimensions are:
n_samples: 2621 , L: 49 , T: 36
[========================================================================] 1
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[======================================================================  ]
97%
Error on 2019-12-01: '2019-12-01'
The panel dimensions are:
n_samples: 2593 , L: 49 , T: 36
[========================================================================] 1
00%
[============================================                            ]
61%
Error on 2020-01-01: '2020-01-01'
The panel dimensions are:
n_samples: 2619 , L: 49 , T: 36
[========================================================================] 1
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[==================================================================      ]
91%
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Error on 2020-02-01: '2020-02-01'
The panel dimensions are:
n_samples: 2647 , L: 49 , T: 36
[=========================================================================] 1
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[================================================================         ]
88%
Error on 2020-03-01: '2020-03-01'
The panel dimensions are:
n_samples: 2622 , L: 49 , T: 36
[=========================================================================] 1
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88%
Error on 2020-04-01: '2020-04-01'
The panel dimensions are:
n_samples: 2651 , L: 49 , T: 36
[=========================================================================] 1
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63%
Error on 2020-05-01: '2020-05-01'
The panel dimensions are:
n_samples: 2693 , L: 49 , T: 36
[=========================================================================] 1
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[================================================================         ]
88%
Error on 2020-06-01: '2020-06-01'
The panel dimensions are:
n_samples: 2693 , L: 49 , T: 36
[=========================================================================] 1
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83%
Error on 2020-07-01: '2020-07-01'
The panel dimensions are:
n_samples: 2741 , L: 49 , T: 36
[=========================================================================] 1
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88%
Error on 2020-08-01: '2020-08-01'
The panel dimensions are:
n_samples: 2765 , L: 49 , T: 36
[=========================================================================] 1
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[==========================================================               ]
80%
Error on 2020-09-01: '2020-09-01'
The panel dimensions are:
n_samples: 2733 , L: 49 , T: 36
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[========================================================================] 1
00%
[====================================================                    ]
72%
Error on 2020-10-01: '2020-10-01'
The panel dimensions are:
n_samples: 2771 , L: 49 , T: 36
[========================================================================] 1
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69%
Error on 2020-11-01: '2020-11-01'
The panel dimensions are:
n_samples: 2785 , L: 49 , T: 36
[========================================================================] 1
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77%
Error on 2020-12-01: '2020-12-01'
The panel dimensions are:
n_samples: 2759 , L: 49 , T: 36
[========================================================================] 1
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72%
Error on 2021-01-01: '2021-01-01'
The panel dimensions are:
n_samples: 2786 , L: 49 , T: 36
[========================================================================] 1
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72%
Error on 2021-02-01: '2021-02-01'
The panel dimensions are:
n_samples: 2809 , L: 49 , T: 36
[========================================================================] 1
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[==================================================================      ]
91%
Error on 2021-03-01: '2021-03-01'
The panel dimensions are:
n_samples: 2801 , L: 49 , T: 36
[========================================================================] 1
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80%
Error on 2021-04-01: '2021-04-01'
The panel dimensions are:
n_samples: 2813 , L: 49 , T: 36
[========================================================================] 1
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69%
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Error on 2021-05-01: '2021-05-01'
The panel dimensions are:
n_samples: 2847 , L: 49 , T: 36
[========================================================================] 1
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86%
Error on 2021-06-01: '2021-06-01'
The panel dimensions are:
n_samples: 2845 , L: 49 , T: 36
[========================================================================] 1
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72%
Error on 2021-07-01: '2021-07-01'
The panel dimensions are:
n_samples: 2856 , L: 49 , T: 36
[========================================================================] 1
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75%
Error on 2021-08-01: '2021-08-01'
The panel dimensions are:
n_samples: 2930 , L: 49 , T: 36
[========================================================================] 1
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61%
Error on 2021-09-01: '2021-09-01'
The panel dimensions are:
n_samples: 2934 , L: 49 , T: 36
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Error on 2021-10-01: '2021-10-01'
The panel dimensions are:
n_samples: 2947 , L: 49 , T: 36
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66%
Error on 2021-11-01: '2021-11-01'
The panel dimensions are:
n_samples: 2983 , L: 49 , T: 36
[========================================================================] 1
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63%
Error on 2021-12-01: '2021-12-01'
The panel dimensions are:
n_samples: 2974 , L: 49 , T: 36
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[========================================================================] 1
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50%
Error on 2022-01-01: '2022-01-01'
The panel dimensions are:
n_samples: 3009 , L: 49 , T: 36
[========================================================================] 1
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61%
Error on 2022-02-01: '2022-02-01'
The panel dimensions are:
n_samples: 3047 , L: 49 , T: 36
[========================================================================] 1
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52%
Error on 2022-03-01: '2022-03-01'
The panel dimensions are:
n_samples: 3021 , L: 49 , T: 36
[========================================================================] 1
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61%
Error on 2022-04-01: '2022-04-01'
The panel dimensions are:
n_samples: 3056 , L: 49 , T: 36
[========================================================================] 1
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61%
Error on 2022-05-01: '2022-05-01'
The panel dimensions are:
n_samples: 3070 , L: 49 , T: 36
[========================================================================] 1
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69%
Error on 2022-06-01: '2022-06-01'
The panel dimensions are:
n_samples: 3064 , L: 49 , T: 36
[========================================================================] 1
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61%
Error on 2022-07-01: '2022-07-01'
The panel dimensions are:
n_samples: 3076 , L: 49 , T: 36
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[==================================          ]
47%
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Error on 2022-08-01: '2022-08-01'
The panel dimensions are:
n_samples: 3067 , L: 49 , T: 36
[========================================================================] 1
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66%
Error on 2022-09-01: '2022-09-01'
The panel dimensions are:
n_samples: 3077 , L: 49 , T: 36
[========================================================================] 1
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72%
Error on 2022-10-01: '2022-10-01'
The panel dimensions are:
n_samples: 3084 , L: 49 , T: 36
[========================================================================] 1
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Error on 2022-11-01: '2022-11-01'
The panel dimensions are:
n_samples: 3077 , L: 49 , T: 36
[========================================================================] 1
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66%
Error on 2022-12-01: '2022-12-01'
The panel dimensions are:
n_samples: 3363 , L: 49 , T: 36
[========================================================================] 1
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69%
Error on 2023-01-01: '2023-01-01'
The panel dimensions are:
n_samples: 3458 , L: 49 , T: 36
[========================================================================] 1
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66%
Error on 2023-02-01: '2023-02-01'
The panel dimensions are:
n_samples: 3533 , L: 49 , T: 36
[========================================================================] 1
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75%
Error on 2023-03-01: '2023-03-01'
The panel dimensions are:
n_samples: 3512 , L: 49 , T: 36
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[=========================================================================] 1
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[==============================================                           ]
63%
Error on 2023-04-01: '2023-04-01'
The panel dimensions are:
n_samples: 3560 , L: 49 , T: 36
[=========================================================================] 1
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[=========================================================================] 1
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Error on 2023-05-01: '2023-05-01'
The panel dimensions are:
n_samples: 1656 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2015-02-01: '2015-02-01'
The panel dimensions are:
n_samples: 1696 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2015-03-01: '2015-03-01'
The panel dimensions are:
n_samples: 1710 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2015-04-01: '2015-04-01'
The panel dimensions are:
n_samples: 1738 , L: 49 , T: 36
[=========================================================================] 1
00%
Error on 2015-05-01: '2015-05-01'
The panel dimensions are:
n_samples: 1777 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2015-06-01: '2015-06-01'
The panel dimensions are:
n_samples: 1794 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2015-07-01: '2015-07-01'
The panel dimensions are:
n_samples: 1823 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2015-08-01: '2015-08-01'
The panel dimensions are:
n_samples: 1872 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2015-09-01: '2015-09-01'
The panel dimensions are:
n_samples: 1895 , L: 49 , T: 36
[==========================================================================] 1
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Error on 2015-10-01: '2015-10-01'
The panel dimensions are:
n_samples: 1914 , L: 49 , T: 36
[==========================================================================] 1
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Error on 2015-11-01: '2015-11-01'
The panel dimensions are:
n_samples: 1951 , L: 49 , T: 36
[==========================================================================] 1
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Error on 2015-12-01: '2015-12-01'
The panel dimensions are:
n_samples: 1957 , L: 49 , T: 36
[==========================================================================] 1
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Error on 2016-01-01: '2016-01-01'
The panel dimensions are:
n_samples: 1979 , L: 49 , T: 36
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Error on 2016-02-01: '2016-02-01'
The panel dimensions are:
n_samples: 1987 , L: 49 , T: 36
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Error on 2016-03-01: '2016-03-01'
The panel dimensions are:
n_samples: 1982 , L: 49 , T: 36
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Error on 2016-04-01: '2016-04-01'
The panel dimensions are:
n_samples: 2008 , L: 49 , T: 36
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Error on 2016-05-01: '2016-05-01'
The panel dimensions are:
n_samples: 2031 , L: 49 , T: 36
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97%
Error on 2016-06-01: '2016-06-01'
The panel dimensions are:
n_samples: 2041 , L: 49 , T: 36
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[========================================================================] 1
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[========================================================================] 1
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Error on 2016-07-01: '2016-07-01'
The panel dimensions are:
n_samples: 2072 , L: 49 , T: 36
[========================================================================] 1
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Error on 2016-08-01: '2016-08-01'
The panel dimensions are:
n_samples: 2087 , L: 49 , T: 36
[========================================================================] 1
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Error on 2016-09-01: '2016-09-01'
The panel dimensions are:
n_samples: 2097 , L: 49 , T: 36
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Error on 2016-10-01: '2016-10-01'
The panel dimensions are:
n_samples: 2106 , L: 49 , T: 36
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Error on 2016-11-01: '2016-11-01'
The panel dimensions are:
n_samples: 2146 , L: 49 , T: 36
[========================================================================] 1
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72%
Error on 2016-12-01: '2016-12-01'
The panel dimensions are:
n_samples: 2129 , L: 49 , T: 36
[========================================================================] 1
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Error on 2017-01-01: '2017-01-01'
The panel dimensions are:
n_samples: 2146 , L: 49 , T: 36
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Error on 2017-02-01: '2017-02-01'
The panel dimensions are:
n_samples: 2164 , L: 49 , T: 36
[========================================================================] 1
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Error on 2017-03-01: '2017-03-01'
The panel dimensions are:
n_samples: 2169 , L: 49 , T: 36
[====================================================                     ]
72%
Error on 2017-04-01: '2017-04-01'
The panel dimensions are:
n_samples: 2176 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2017-05-01: '2017-05-01'
The panel dimensions are:
n_samples: 2199 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2017-06-01: '2017-06-01'
The panel dimensions are:
n_samples: 2190 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2017-07-01: '2017-07-01'
The panel dimensions are:
n_samples: 2215 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2017-08-01: '2017-08-01'
The panel dimensions are:
n_samples: 2217 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2017-09-01: '2017-09-01'
The panel dimensions are:
n_samples: 2232 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2017-10-01: '2017-10-01'
The panel dimensions are:
n_samples: 2242 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2017-11-01: '2017-11-01'
The panel dimensions are:
n_samples: 2277 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2017-12-01: '2017-12-01'
The panel dimensions are:
n_samples: 2274 , L: 49 , T: 36
[=========================================================================] 1
00%
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Error on 2018-01-01: '2018-01-01'
The panel dimensions are:
n_samples: 2300 , L: 49 , T: 36
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[========================================================================] 1
00%
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Error on 2018-02-01: '2018-02-01'
The panel dimensions are:
n_samples: 2299 , L: 49 , T: 36
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[========================================================================] 1
00%
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Error on 2018-03-01: '2018-03-01'
The panel dimensions are:
n_samples: 2308 , L: 49 , T: 36
```

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[========================================================================] 1
00%
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Error on 2018-04-01: '2018-04-01'
The panel dimensions are:
n_samples: 2304 , L: 49 , T: 36
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[========================================================================] 1
00%
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Error on 2018-05-01: '2018-05-01'
The panel dimensions are:
n_samples: 2301 , L: 49 , T: 36
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[========================================================================] 1
00%
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Error on 2018-06-01: '2018-06-01'
The panel dimensions are:
n_samples: 2325 , L: 49 , T: 36
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[========================================================                ]
77%
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Error on 2018-07-01: '2018-07-01'
The panel dimensions are:
n_samples: 2357 , L: 49 , T: 36
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[========================================================================] 1
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Error on 2018-08-01: '2018-08-01'
The panel dimensions are:
n_samples: 2366 , L: 49 , T: 36
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[==================================================================      ]
91%
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Error on 2018-09-01: '2018-09-01'
The panel dimensions are:
n_samples: 2385 , L: 49 , T: 36
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Error on 2018-10-01: '2018-10-01'
The panel dimensions are:
n_samples: 2399 , L: 49 , T: 36
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[======================================================                  ]
75%
Error on 2018-11-01: '2018-11-01'
The panel dimensions are:
n_samples: 2430 , L: 49 , T: 36
[========================================================================] 1
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Error on 2018-12-01: '2018-12-01'
The panel dimensions are:
n_samples: 2432 , L: 49 , T: 36
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Error on 2019-01-01: '2019-01-01'
The panel dimensions are:
n_samples: 2447 , L: 49 , T: 36
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97%
Error on 2019-02-01: '2019-02-01'
The panel dimensions are:
n_samples: 2470 , L: 49 , T: 36
[========================================================================] 1
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Error on 2019-03-01: '2019-03-01'
The panel dimensions are:
n_samples: 2468 , L: 49 , T: 36
[====================================================================    ]
94%
Error on 2019-04-01: '2019-04-01'
The panel dimensions are:
n_samples: 2482 , L: 49 , T: 36
[========================================================================] 1
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[========================================================================] 1
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Error on 2019-05-01: '2019-05-01'
The panel dimensions are:
n_samples: 2500 , L: 49 , T: 36
[========================================================================] 1
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Error on 2019-06-01: '2019-06-01'
The panel dimensions are:
n_samples: 2497 , L: 49 , T: 36
[========================================================================] 1
00%
Error on 2019-07-01: '2019-07-01'
The panel dimensions are:
n_samples: 2500 , L: 49 , T: 36
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[============================================================            ]
83%
Error on 2019-08-01: '2019-08-01'
The panel dimensions are:
n_samples: 2546 , L: 49 , T: 36
[========================================================================] 1
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91%
Error on 2019-09-01: '2019-09-01'
The panel dimensions are:
n_samples: 2524 , L: 49 , T: 36
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Error on 2019-10-01: '2019-10-01'
The panel dimensions are:
n_samples: 2546 , L: 49 , T: 36
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Error on 2019-11-01: '2019-11-01'
The panel dimensions are:
n_samples: 2621 , L: 49 , T: 36
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72%
Error on 2019-12-01: '2019-12-01'
The panel dimensions are:
n_samples: 2593 , L: 49 , T: 36
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Error on 2020-01-01: '2020-01-01'
The panel dimensions are:
n_samples: 2619 , L: 49 , T: 36
[========================================================================] 1
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Error on 2020-02-01: '2020-02-01'
The panel dimensions are:
n_samples: 2647 , L: 49 , T: 36
[====================================================================    ]
94%
Error on 2020-03-01: '2020-03-01'
The panel dimensions are:
n_samples: 2622 , L: 49 , T: 36
```

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[========================================================================] 1
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[========================================================================] 1
00%
Error on 2020-04-01: '2020-04-01'
The panel dimensions are:
n_samples: 2651 , L: 49 , T: 36
[========================================================================] 1
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Error on 2020-05-01: '2020-05-01'
The panel dimensions are:
n_samples: 2693 , L: 49 , T: 36
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Error on 2020-06-01: '2020-06-01'
The panel dimensions are:
n_samples: 2693 , L: 49 , T: 36
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Error on 2020-07-01: '2020-07-01'
The panel dimensions are:
n_samples: 2741 , L: 49 , T: 36
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Error on 2020-08-01: '2020-08-01'
The panel dimensions are:
n_samples: 2765 , L: 49 , T: 36
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75%
Error on 2020-09-01: '2020-09-01'
The panel dimensions are:
n_samples: 2733 , L: 49 , T: 36
[========================================================================] 1
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Error on 2020-10-01: '2020-10-01'
The panel dimensions are:
n_samples: 2771 , L: 49 , T: 36
[========================================================================] 1
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[==================================================================      ]
91%
Error on 2020-11-01: '2020-11-01'
The panel dimensions are:
n_samples: 2785 , L: 49 , T: 36
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[=========================================================================] 1
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[========================================================                 ]
77%
Error on 2020-12-01: '2020-12-01'
The panel dimensions are:
n_samples: 2759 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2021-01-01: '2021-01-01'
The panel dimensions are:
n_samples: 2786 , L: 49 , T: 36
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86%
Error on 2021-02-01: '2021-02-01'
The panel dimensions are:
n_samples: 2809 , L: 49 , T: 36
[=========================================================================] 1
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91%
Error on 2021-03-01: '2021-03-01'
The panel dimensions are:
n_samples: 2801 , L: 49 , T: 36
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Error on 2021-04-01: '2021-04-01'
The panel dimensions are:
n_samples: 2813 , L: 49 , T: 36
[=========================================================================] 1
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94%
Error on 2021-05-01: '2021-05-01'
The panel dimensions are:
n_samples: 2847 , L: 49 , T: 36
[=========================================================================] 1
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[==========================================                               ]
58%
Error on 2021-06-01: '2021-06-01'
The panel dimensions are:
n_samples: 2845 , L: 49 , T: 36
[=========================================================================] 1
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[================================================================         ]
88%
Error on 2021-07-01: '2021-07-01'
The panel dimensions are:
n_samples: 2856 , L: 49 , T: 36
```

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[========================================================================] 1
00%
[================================================================        ]
88%
Error on 2021-08-01: '2021-08-01'
The panel dimensions are:
n_samples: 2930 , L: 49 , T: 36
[========================================================================] 1
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[================================================                        ]
66%
Error on 2021-09-01: '2021-09-01'
The panel dimensions are:
n_samples: 2934 , L: 49 , T: 36
[========================================================================] 1
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72%
Error on 2021-10-01: '2021-10-01'
The panel dimensions are:
n_samples: 2947 , L: 49 , T: 36
[========================================================================] 1
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[==========================================================              ]
80%
Error on 2021-11-01: '2021-11-01'
The panel dimensions are:
n_samples: 2983 , L: 49 , T: 36
[========================================================================] 1
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[==========================================================              ]
80%
Error on 2021-12-01: '2021-12-01'
The panel dimensions are:
n_samples: 2974 , L: 49 , T: 36
[========================================================================] 1
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[================================================================        ]
88%
Error on 2022-01-01: '2022-01-01'
The panel dimensions are:
n_samples: 3009 , L: 49 , T: 36
[========================================================================] 1
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[==================================================================      ]
91%
Error on 2022-02-01: '2022-02-01'
The panel dimensions are:
n_samples: 3047 , L: 49 , T: 36
[========================================================================] 1
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[==================================================                      ]
69%
```

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Error on 2022-03-01: '2022-03-01'
The panel dimensions are:
n_samples: 3021 , L: 49 , T: 36
[========================================================================] 1
00%
[==========================================================              ]
80%
Error on 2022-04-01: '2022-04-01'
The panel dimensions are:
n_samples: 3056 , L: 49 , T: 36
[========================================================================] 1
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[========================================================                ]
77%
Error on 2022-05-01: '2022-05-01'
The panel dimensions are:
n_samples: 3070 , L: 49 , T: 36
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Error on 2022-06-01: '2022-06-01'
The panel dimensions are:
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Error on 2022-07-01: '2022-07-01'
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Error on 2022-08-01: '2022-08-01'
The panel dimensions are:
n_samples: 3067 , L: 49 , T: 36
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Error on 2022-09-01: '2022-09-01'
The panel dimensions are:
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Error on 2022-10-01: '2022-10-01'
The panel dimensions are:
n_samples: 3084 , L: 49 , T: 36
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Error on 2022-12-01: '2022-12-01'
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The panel dimensions are:
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The panel dimensions are:
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Error on 2015-09-01: '2015-09-01'
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Error on 2015-10-01: '2015-10-01'
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n_samples: 1914 , L: 49 , T: 36
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The panel dimensions are:
n_samples: 1951 , L: 49 , T: 36
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Error on 2016-01-01: '2016-01-01'
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n_samples: 1979 , L: 49 , T: 36
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Error on 2016-03-01: '2016-03-01'
The panel dimensions are:
n_samples: 1982 , L: 49 , T: 36
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Error on 2016-04-01: '2016-04-01'
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n_samples: 2097 , L: 49 , T: 36
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Error on 2016-10-01: '2016-10-01'
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Error on 2016-12-01: '2016-12-01'
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The panel dimensions are:
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Error on 2017-04-01: '2017-04-01'
The panel dimensions are:
n_samples: 2176 , L: 49 , T: 36
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Error on 2017-05-01: '2017-05-01'
The panel dimensions are:
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Error on 2017-06-01: '2017-06-01'
The panel dimensions are:
n_samples: 2190 , L: 49 , T: 36
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Error on 2017-07-01: '2017-07-01'
The panel dimensions are:
n_samples: 2215 , L: 49 , T: 36
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Error on 2017-08-01: '2017-08-01'
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n_samples: 2217 , L: 49 , T: 36
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Error on 2017-09-01: '2017-09-01'
The panel dimensions are:
n_samples: 2232 , L: 49 , T: 36
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Error on 2017-10-01: '2017-10-01'
The panel dimensions are:
n_samples: 2242 , L: 49 , T: 36
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Error on 2017-11-01: '2017-11-01'
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Error on 2017-12-01: '2017-12-01'
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Error on 2018-01-01: '2018-01-01'
The panel dimensions are:
n_samples: 2300 , L: 49 , T: 36
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Error on 2018-02-01: '2018-02-01'
The panel dimensions are:
n_samples: 2299 , L: 49 , T: 36
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Error on 2018-03-01: '2018-03-01'
The panel dimensions are:
n_samples: 2308 , L: 49 , T: 36
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Error on 2018-04-01: '2018-04-01'
The panel dimensions are:
n_samples: 2304 , L: 49 , T: 36
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Error on 2018-05-01: '2018-05-01'
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Error on 2018-06-01: '2018-06-01'
The panel dimensions are:
n_samples: 2325 , L: 49 , T: 36
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Error on 2018-07-01: '2018-07-01'
The panel dimensions are:
n_samples: 2357 , L: 49 , T: 36
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Error on 2018-08-01: '2018-08-01'
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n_samples: 2366 , L: 49 , T: 36
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Error on 2018-09-01: '2018-09-01'
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Error on 2018-10-01: '2018-10-01'
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n_samples: 2399 , L: 49 , T: 36
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Error on 2018-11-01: '2018-11-01'
The panel dimensions are:
n_samples: 2430 , L: 49 , T: 36
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Error on 2018-12-01: '2018-12-01'
The panel dimensions are:
n_samples: 2432 , L: 49 , T: 36
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Error on 2019-01-01: '2019-01-01'
The panel dimensions are:
n_samples: 2447 , L: 49 , T: 36
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Error on 2019-02-01: '2019-02-01'
The panel dimensions are:
n_samples: 2470 , L: 49 , T: 36
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Error on 2019-03-01: '2019-03-01'
The panel dimensions are:
n_samples: 2468 , L: 49 , T: 36
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Error on 2019-04-01: '2019-04-01'
The panel dimensions are:
n_samples: 2482 , L: 49 , T: 36
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Error on 2019-05-01: '2019-05-01'
The panel dimensions are:
n_samples: 2500 , L: 49 , T: 36
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Error on 2019-06-01: '2019-06-01'
The panel dimensions are:
n_samples: 2497 , L: 49 , T: 36
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Error on 2019-07-01: '2019-07-01'
The panel dimensions are:
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Error on 2019-08-01: '2019-08-01'
The panel dimensions are:
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Error on 2019-09-01: '2019-09-01'
The panel dimensions are:
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Error on 2019-10-01: '2019-10-01'
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Error on 2019-11-01: '2019-11-01'
The panel dimensions are:
n_samples: 2621 , L: 49 , T: 36
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Error on 2019-12-01: '2019-12-01'
The panel dimensions are:
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Error on 2020-01-01: '2020-01-01'
The panel dimensions are:
n_samples: 2619 , L: 49 , T: 36
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Error on 2020-02-01: '2020-02-01'
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n_samples: 2647 , L: 49 , T: 36
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Error on 2020-03-01: '2020-03-01'
The panel dimensions are:
n_samples: 2622 , L: 49 , T: 36
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Error on 2020-04-01: '2020-04-01'
The panel dimensions are:
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Error on 2020-05-01: '2020-05-01'
The panel dimensions are:
n_samples: 2693 , L: 49 , T: 36
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Error on 2020-06-01: '2020-06-01'
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n_samples: 2693 , L: 49 , T: 36
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Error on 2020-07-01: '2020-07-01'
The panel dimensions are:
n_samples: 2741 , L: 49 , T: 36
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Error on 2020-08-01: '2020-08-01'
The panel dimensions are:
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The panel dimensions are:
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Error on 2020-10-01: '2020-10-01'
The panel dimensions are:
n_samples: 2771 , L: 49 , T: 36
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Error on 2020-11-01: '2020-11-01'
The panel dimensions are:
n_samples: 2785 , L: 49 , T: 36
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Error on 2020-12-01: '2020-12-01'
The panel dimensions are:
n_samples: 2759 , L: 49 , T: 36
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Error on 2021-01-01: '2021-01-01'
The panel dimensions are:
n_samples: 2786 , L: 49 , T: 36
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Error on 2021-02-01: '2021-02-01'
The panel dimensions are:
n_samples: 2809 , L: 49 , T: 36
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Error on 2021-03-01: '2021-03-01'
The panel dimensions are:
n_samples: 2801 , L: 49 , T: 36
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Error on 2021-04-01: '2021-04-01'
The panel dimensions are:
n_samples: 2813 , L: 49 , T: 36
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Error on 2021-05-01: '2021-05-01'
The panel dimensions are:
n_samples: 2847 , L: 49 , T: 36
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Error on 2021-06-01: '2021-06-01'
The panel dimensions are:
n_samples: 2845 , L: 49 , T: 36
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Error on 2021-07-01: '2021-07-01'
The panel dimensions are:
n_samples: 2856 , L: 49 , T: 36
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Error on 2021-08-01: '2021-08-01'
The panel dimensions are:
n_samples: 2930 , L: 49 , T: 36
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Error on 2021-09-01: '2021-09-01'
The panel dimensions are:
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Error on 2021-10-01: '2021-10-01'
The panel dimensions are:
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Error on 2022-01-01: '2022-01-01'
The panel dimensions are:
n_samples: 3009 , L: 49 , T: 36
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Error on 2022-02-01: '2022-02-01'
The panel dimensions are:
n_samples: 3047 , L: 49 , T: 36
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Error on 2022-03-01: '2022-03-01'
The panel dimensions are:
n_samples: 3021 , L: 49 , T: 36
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Error on 2022-04-01: '2022-04-01'
The panel dimensions are:
n_samples: 3056 , L: 49 , T: 36
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Error on 2022-05-01: '2022-05-01'
The panel dimensions are:
n_samples: 3070 , L: 49 , T: 36
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Error on 2022-06-01: '2022-06-01'
The panel dimensions are:
n_samples: 3064 , L: 49 , T: 36
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Error on 2022-07-01: '2022-07-01'
The panel dimensions are:
n_samples: 3076 , L: 49 , T: 36
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Error on 2022-08-01: '2022-08-01'
The panel dimensions are:
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Error on 2022-09-01: '2022-09-01'
The panel dimensions are:
n_samples: 3077 , L: 49 , T: 36
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Error on 2022-10-01: '2022-10-01'
The panel dimensions are:
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Error on 2022-11-01: '2022-11-01'
The panel dimensions are:
n_samples: 3077 , L: 49 , T: 36
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Error on 2022-12-01: '2022-12-01'
The panel dimensions are:
n_samples: 3363 , L: 49 , T: 36
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Error on 2023-01-01: '2023-01-01'
The panel dimensions are:
n_samples: 3458 , L: 49 , T: 36
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Error on 2023-02-01: '2023-02-01'
The panel dimensions are:
n_samples: 3533 , L: 49 , T: 36
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Error on 2023-03-01: '2023-03-01'
The panel dimensions are:
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Error on 2023-04-01: '2023-04-01'
The panel dimensions are:
n_samples: 3560 , L: 49 , T: 36
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Error on 2023-05-01: '2023-05-01'
The panel dimensions are:
n_samples: 1656 , L: 49 , T: 36
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Error on 2015-02-01: '2015-02-01'
The panel dimensions are:
n_samples: 1696 , L: 49 , T: 36
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Error on 2015-03-01: '2015-03-01'
The panel dimensions are:
n_samples: 1710 , L: 49 , T: 36
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Error on 2015-04-01: '2015-04-01'
The panel dimensions are:
n_samples: 1738 , L: 49 , T: 36
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Error on 2015-05-01: '2015-05-01'
The panel dimensions are:
n_samples: 1777 , L: 49 , T: 36
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Error on 2015-06-01: '2015-06-01'
The panel dimensions are:
n_samples: 1794 , L: 49 , T: 36
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Error on 2015-07-01: '2015-07-01'
The panel dimensions are:
n_samples: 1823 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2015-08-01: '2015-08-01'
The panel dimensions are:
n_samples: 1872 , L: 49 , T: 36
[=========================================================================] 1
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Error on 2015-09-01: '2015-09-01'
The panel dimensions are:
n_samples: 1895 , L: 49 , T: 36
```

In [ ]:
```
out_of_sample_forecasts
```

In [ ]:
```
out_of_sample_results
```

In [ ]:
```
out_of_sample_results.to_csv('oos_results')
```

In [ ]:
```
out_of_sample_forecasts.to_csv('oos_forecasrts')
```

In [ ]:
```
out_of_sample_results[' OOS R^2 Over Benchmark'].plot(kind = 'line')
```

```
---------------------------------------------------------------------------
TypeError                                 Traceback (most recent call last)
Cell In[197], line 1
----> 1 out_of_sample_results[' OOS R^2 Over Benchmark'].plot(kind = 'line')

TypeError: list indices must be integers or slices, not str
```

In [ ]:
```
# %%time

# from joblib import Parallel, delayed


# # Function to compute forecasts and results for a specific parameter set a
# def compute_forecasts_and_results(t, num_factors, intercept, min_training_
#     try:
#         # (t-1)-measurable
#         X_train = features_lag.loc[idx[:, dates[max(0, dates.get_loc(t) -
#         Y_train = returns.squeeze().loc[idx[:, dates[max(0, dates.get_loc(

#         # Fit the model
#         regr = ipca.InstrumentedPCA(n_factors=num_factors, intercept=inter
#         regr = regr.fit(X=X_train, y=Y_train, quiet=True)

#         # Test set
#         X_test = features_lag.loc[idx[:, t], :]
#         Y_test = returns[returns.index.get_level_values('Date').get_loc(t)

#         # Predictions
#         ret_hat_package = pd.DataFrame(regr.predict(X_test, mean_factor=Tr
#                                        index=X_test.index,
#                                        columns=['Returns_forecast'])

#         # Benchmarking
#         ret_hat_benchmark = Y_train.groupby('Cusip').mean().to_frame().ren
#         ret_ret_hat_benchmark = pd.merge(Y_test.reset_index(), ret_hat_ben

#         # Combine results
#         ret_ret_hat_benchmark_package = pd.concat([ret_ret_hat_benchmark,

#         # Compute OOS R^2
#         oos_results = pd.DataFrame({'OOS R^2 Over Benchmark': [OOS_predict
#                                     'OOS R^2 Over Zero' : [total_R2(ret_re
#                                    index=pd.MultiIndex.from_tuples([(inter
#     except Exception as e:
#         print(f"Error for {t}: {e}")

#         # Initialize variables in case of failure
#         ret_ret_hat_benchmark_package = pd.DataFrame()  # Empty DataFrame
#         oos_results = pd.DataFrame({'OOS R^2 Over Benchmark': [np.nan],
```

```
#                                     'OOS R^2 Over Zero' : [np.nan]},
#                                    index=pd.MultiIndex.from_tuples([(inter

#     return ret_ret_hat_benchmark_package, oos_results


# # Function to parallelize over combinations of parameters
# def process_parameter_combination(min_training_window, training_lag, dates
#     months = dates[min_training_window:]
#     results = Parallel(n_jobs=-1, backend='loky', verbose=10)(delayed(comp
#         t, num_factors, intercept, min_training_window, training_lag, date
#     ) for num_factors in range(1, 6) for intercept in [True, False] for t

#     forecasts, oos_results = zip(*results)
#     return pd.concat(forecasts, axis=0), pd.concat(oos_results, axis=0)

# # Main processing loop
# dates = features_lag.index.get_level_values('Date').drop_duplicates().sort
# out_of_sample_forecasts_dict = {}
# out_of_sample_results_dict = {}

# # Parallelize outer loop
# for min_training_window in [60, 72, 96, 120]:
#     training_results = Parallel(n_jobs=-1, verbose=10)(delayed(process_par
#         min_training_window, training_lag, dates, features_lag, returns
#     ) for training_lag in [1, 5, 10, 15, 20, 25])

#     forecasts_results, oos_results = zip(*training_results)
#     out_of_sample_forecasts_dict[min_training_window] = dict(zip([1, 5, 10
#     out_of_sample_results_dict[min_training_window] = dict(zip([1, 5, 10,


# ## Write forecasts to disk

# for window_size in out_of_sample_forecasts_dict.keys():
#     for lags in out_of_sample_forecasts_dict[window_size].keys():
#         out_of_sample_forecasts_dict[window_size][lags].to_csv(os.path.joi
# ## Write results to disk

# for window_size in out_of_sample_results_dict.keys():
#     for lags in out_of_sample_results_dict[window_size].keys():
#         out_of_sample_results_dict[window_size][lags].to_csv(os.path.join(
```

In [ ]:
```
## check how we're handling the imbalanced panel
```

In [ ]:
```
# ## Write forecasts to disk

# for window_size in out_of_sample_forecasts_dict.keys():
#     for lags in out_of_sample_forecasts_dict[window_size].keys():
#         out_of_sample_forecasts_dict[window_size][lags].to_csv(os.path.joi

# ## Write results to disk

# for window_size in out_of_sample_results_dict.keys():
```

```
#     for lags in out_of_sample_results_dict[window_size].keys():
#         out_of_sample_results_dict[window_size][lags].to_csv(os.path.join(
```

In [ ]:
```
window_sizes_lags = sorted([(int(file.split('_')[2]), int(file.split('_')[-1

window_sizes_lags
```

Out[ ]:
```
[(60, 1),
 (60, 5),
 (60, 10),
 (60, 15),
 (60, 20),
 (60, 25),
 (72, 1),
 (72, 5),
 (72, 10),
 (72, 15),
 (72, 20),
 (72, 25),
 (96, 1),
 (96, 5),
 (96, 10),
 (96, 15),
 (96, 20),
 (96, 25),
 (120, 1),
 (120, 5),
 (120, 10),
 (120, 15),
 (120, 20),
 (120, 25)]
```

In [ ]:
```
df = pd.read_csv(os.path.join(data_path, 'Results', f'window_size_{120}_lags

pd.DataFrame(df.groupby(['Intercept', 'Num Factors'])[['OOS R^2 Over Benchma
                 columns=pd.MultiIndex.from_tuples([(120, 1)], names=['WindowSiz
```

Out[ ]:

	Intercept					
	Num Factors	1		2		
		OOS R^2 Over Benchmark	OOS R^2 Over Zero	OOS R^2 Over Benchmark	OOS R^2 Over Zero	OOS R^ Ove Benchma
WindowSize	**Lag**					
120	**1**	0.210187	0.034452	0.207826	0.030225	0.2014

In [ ]:
```
results = []

for window_size, lag in window_sizes_lags:
    df = pd.read_csv(os.path.join(data_path, 'Results', f'window_size_{windo
    df = pd.DataFrame(df.groupby(['Intercept', 'Num Factors'])[['OOS R^2 Ove
                      columns=pd.MultiIndex.from_tuples([(window_size, lag)]
```

```
    results.append(df)

results = pd.concat(results, axis=0).sort_index()

results.to_csv(os.path.join(data_path, 'Results', 'OOS_R2_Results.csv'))

results
```

Out[ ]:

	Intercept					
	Num Factors	1		2		
		OOS R^2 Over Benchmark	OOS R^2 Over Zero	OOS R^2 Over Benchmark	OOS R^2 Over Zero	
WindowSize	Lag					
60	1	0.135131	0.023710	-0.332571	-0.455403	-12
	5	0.169251	0.029483	-0.121142	-0.273896	-5
	10	0.214035	0.029909	0.154182	-0.044164	-1
	15	0.207982	0.036087	-0.803672	-1.119572	
	20	0.206377	0.032552	-947.630432	-1609.603913	
	25	0.178747	0.028828	-4951.223751	-22199.746858	
72	1	0.148688	0.028047	0.148753	0.027007	
	5	0.188223	0.031884	0.188066	0.030820	
	10	0.237629	0.033290	0.239386	0.034089	
	15	0.230367	0.038684	-0.541273	-0.825998	-14
	20	0.227608	0.032063	-0.165763	-0.374080	-9
	25	0.203136	0.025635	-5.856829	-6.502777	
96	1	0.177480	-0.001186	0.179803	-0.000576	
	5	0.215760	0.003632	0.217693	0.004984	
	10	0.258053	0.006202	0.262430	0.011311	
	15	0.238690	0.012601	0.242420	0.017167	
	20	0.272811	0.013370	0.276480	0.016368	
	25	0.269668	0.016042	0.275127	0.023694	
120	1	0.210187	0.034452	0.207826	0.030225	
	5	0.294753	0.038783	0.292265	0.034563	
	10	0.357450	0.045434	0.357669	0.044864	
	15	0.344508	0.050764	0.346717	0.052754	
	20	0.389365	0.048486	0.390775	0.048426	
	25	0.373115	0.044115	0.376465	0.046657	

In [ ]:

```
# results = []
# multiindex_labels = []

# for window_size in out_of_sample_results_dict.keys():
#     for lags in out_of_sample_results_dict[window_size].keys():
```

```
#         df = pd.DataFrame(
#             out_of_sample_results_dict[window_size][lags]
#             .sort_index()['OOS R^2 Over Benchmark', 'OOS R^2 Over Zero']
#             .unstack(level=['Intercept', 'Num Factors']).mean(),
#             columns=[(window_size, lags)]  # Store as a tuple for MultiInd
#         )
#         results.append(df)
#         multiindex_labels.append((window_size, lags))

# # Concatenate all DataFrames and set the columns to a MultiIndex
# results = pd.concat(results, axis=1)
# results.columns = pd.MultiIndex.from_tuples(multiindex_labels, names=['Wir
```

In [ ]:

```
pd.set_option('display.max_rows', None)
pd.set_option('display.max_columns', None)

results
```

Out[ ]:

	Intercept					
	Num Factors	**1**		**2**		
		OOS R^2 Over Benchmark	**OOS R^2 Over Zero**	**OOS R^2 Over Benchmark**	**OOS R^2 Over Zero**	
WindowSize	**Lag**					
60	**1**	0.135131	0.023710	-0.332571	-0.455403	-12
	5	0.169251	0.029483	-0.121142	-0.273896	-5
	10	0.214035	0.029909	0.154182	-0.044164	-1
	15	0.207982	0.036087	-0.803672	-1.119572	
	20	0.206377	0.032552	-947.630432	-1609.603913	
	25	0.178747	0.028828	-4951.223751	-22199.746858	
72	**1**	0.148688	0.028047	0.148753	0.027007	
	5	0.188223	0.031884	0.188066	0.030820	
	10	0.237629	0.033290	0.239386	0.034089	
	15	0.230367	0.038684	-0.541273	-0.825998	-14
	20	0.227608	0.032063	-0.165763	-0.374080	-9
	25	0.203136	0.025635	-5.856829	-6.502777	
96	**1**	0.177480	-0.001186	0.179803	-0.000576	
	5	0.215760	0.003632	0.217693	0.004984	
	10	0.258053	0.006202	0.262430	0.011311	
	15	0.238690	0.012601	0.242420	0.017167	
	20	0.272811	0.013370	0.276480	0.016368	
	25	0.269668	0.016042	0.275127	0.023694	
120	**1**	0.210187	0.034452	0.207826	0.030225	
	5	0.294753	0.038783	0.292265	0.034563	
	10	0.357450	0.045434	0.357669	0.044864	
	15	0.344508	0.050764	0.346717	0.052754	
	20	0.389365	0.048486	0.390775	0.048426	
	25	0.373115	0.044115	0.376465	0.046657	

In [ ]:

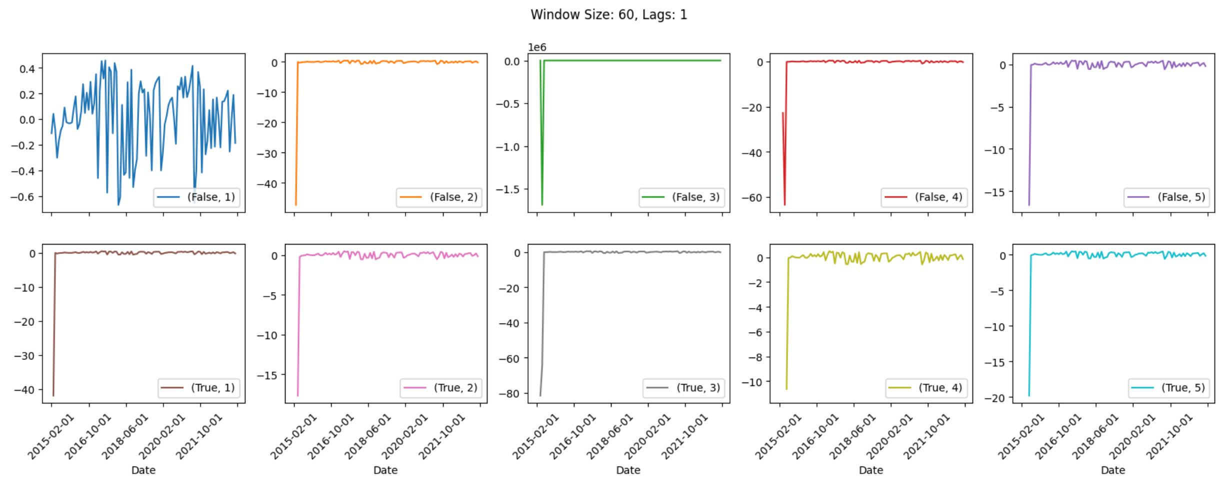
```
pd.reset_option('display.max_rows')
pd.reset_option('display.max_columns')
```

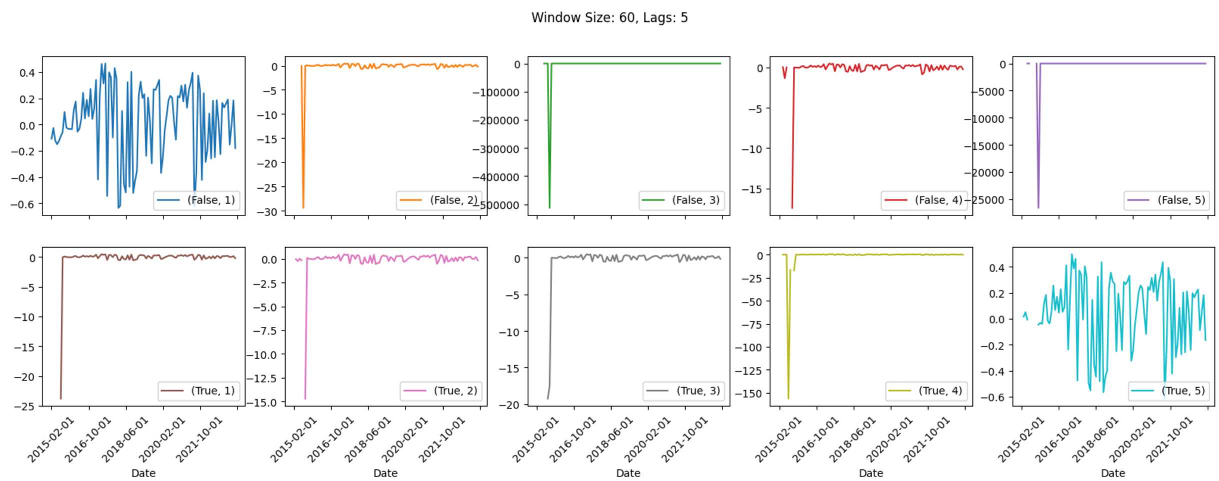
In [ ]:

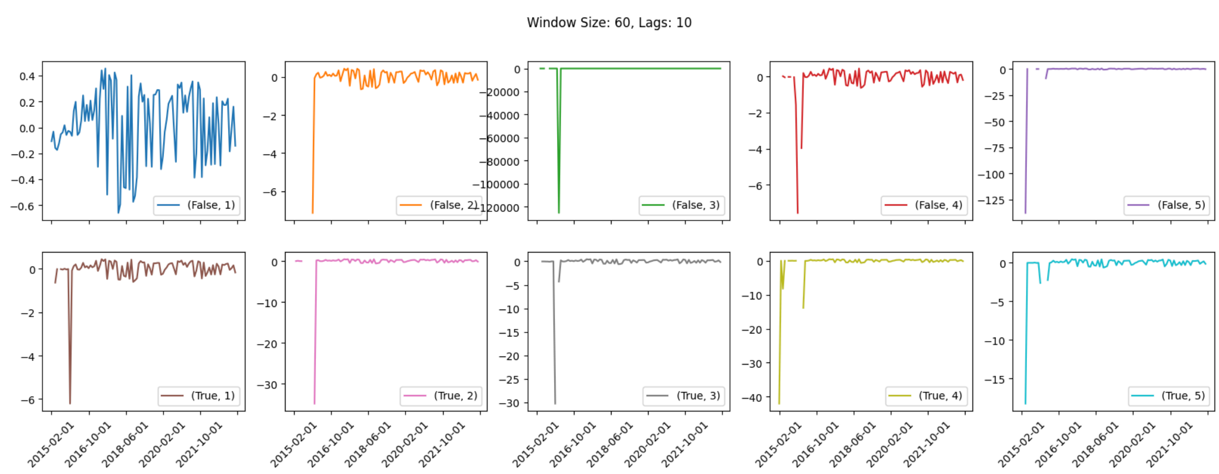
```
# df = pd.read_csv(os.path.join(data_path, 'Results', f'window_size_{60}_lag
```

```
# df.iloc[5:,:].plot()
```

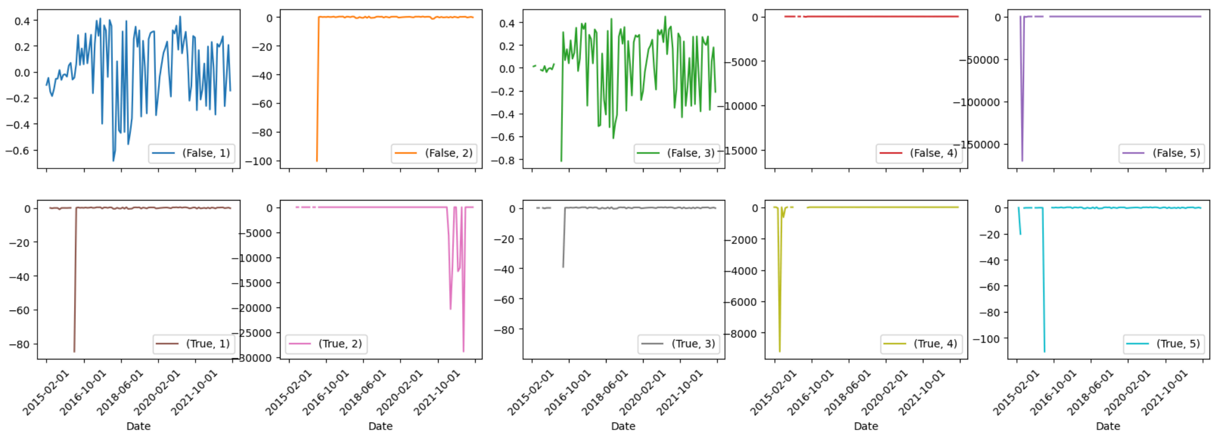
In [ ]:
```
for window_size, lag in window_sizes_lags:

    df = pd.read_csv(os.path.join(data_path, 'Results', f'window_size_{windo

    df.plot(subplots=True, figsize=(20, 6), layout=(2, 5), legend=True, rot=
    plt.suptitle(f"Window Size: {window_size}, Lags: {lag}")
    plt.show()
```



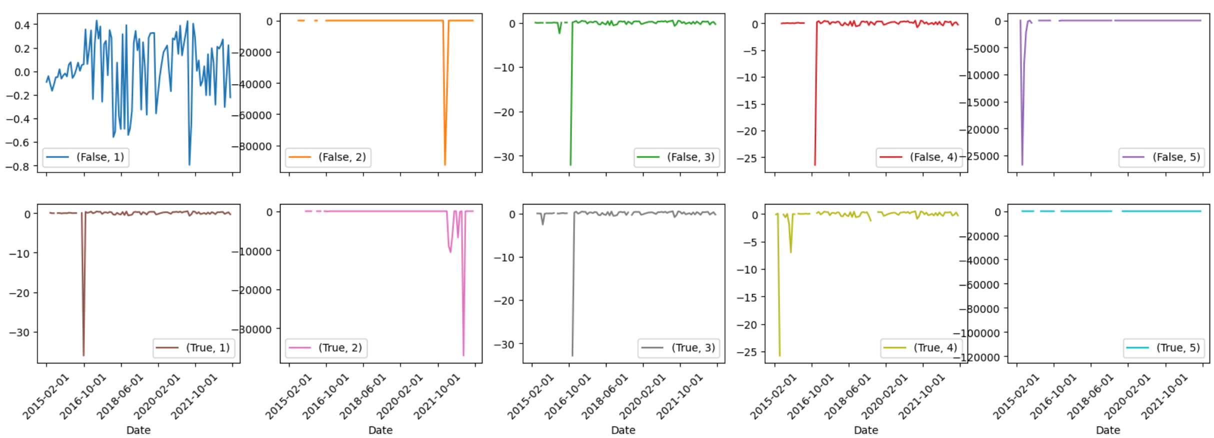




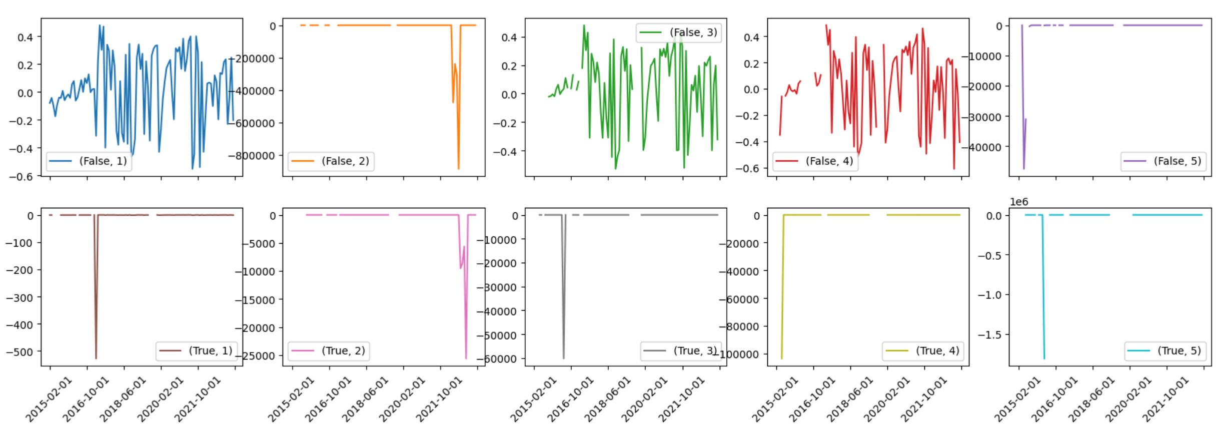
Window Size: 60, Lags: 15



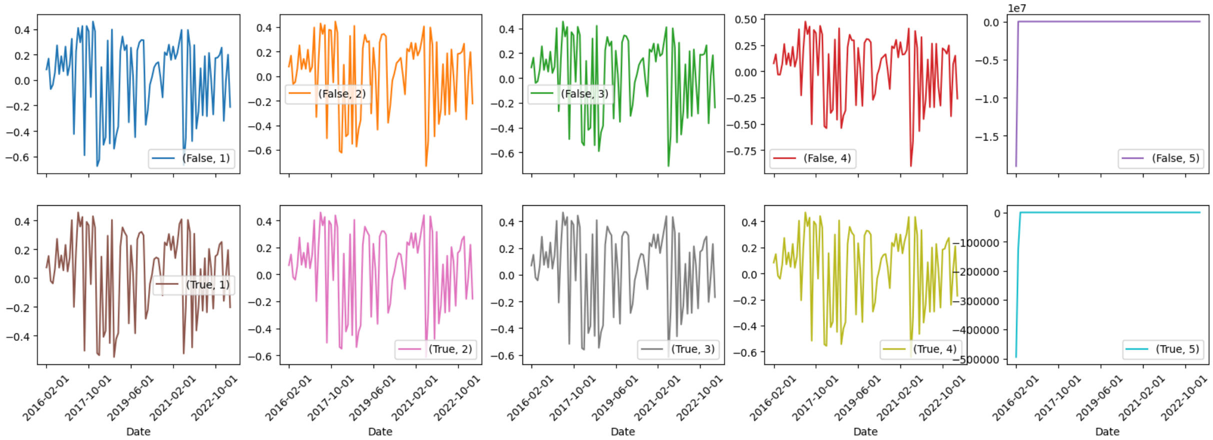
Window Size: 60, Lags: 20



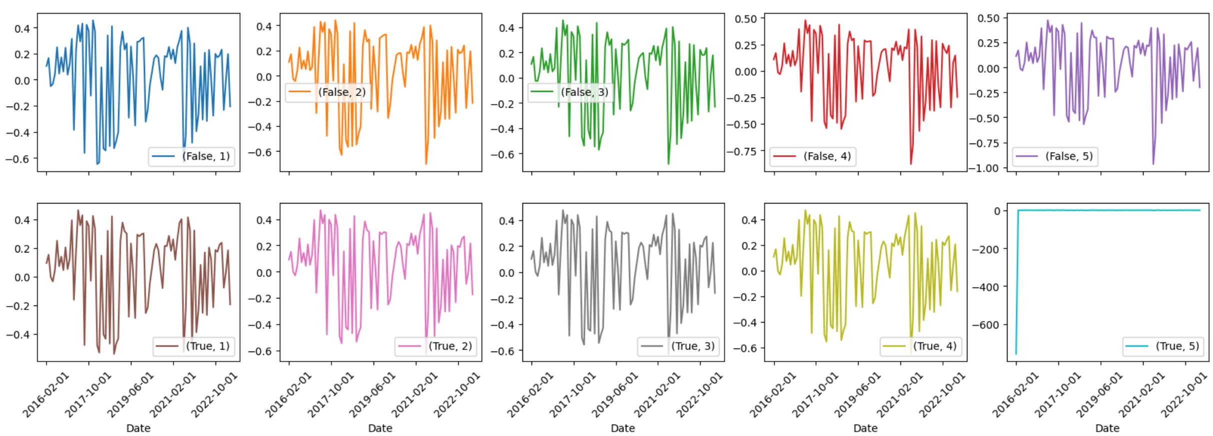
Window Size: 60, Lags: 25



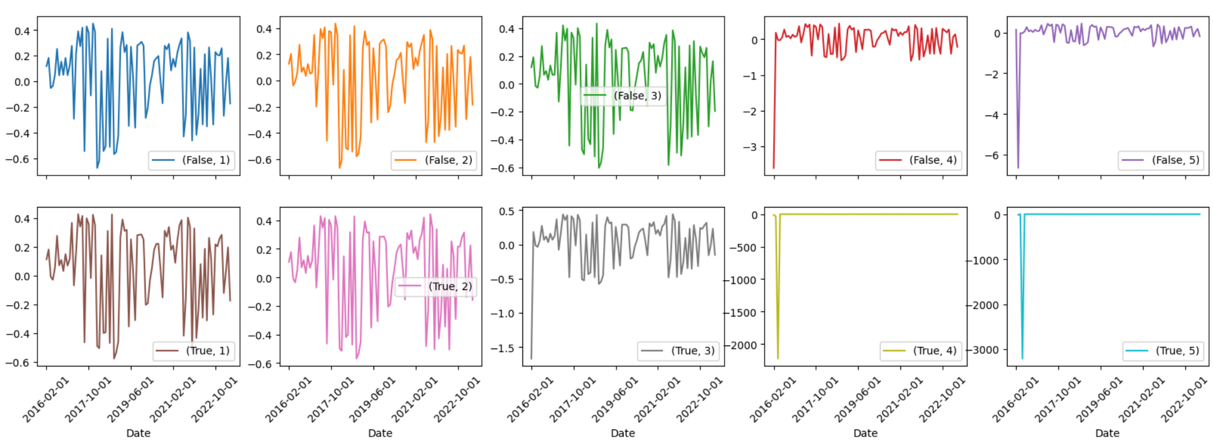
Window Size: 72, Lags: 1



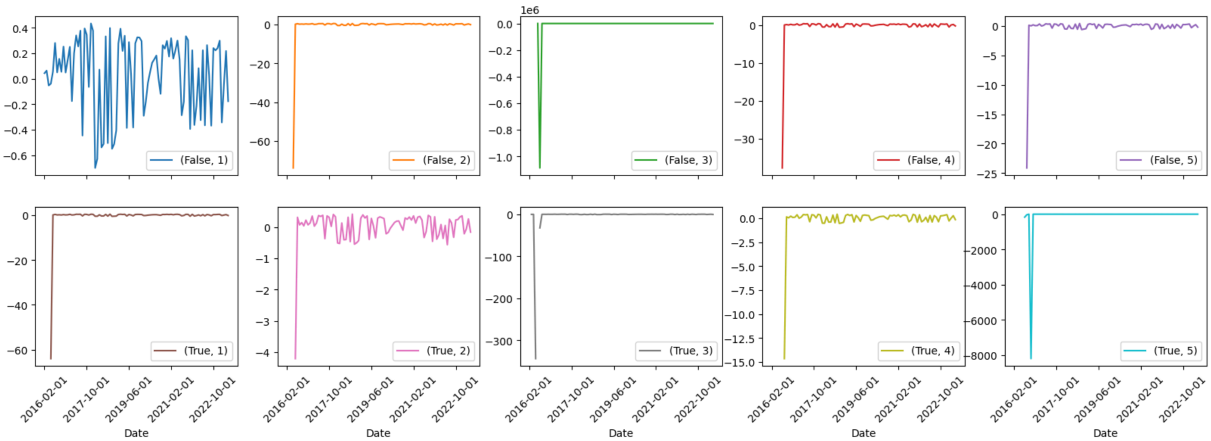
Window Size: 72, Lags: 5



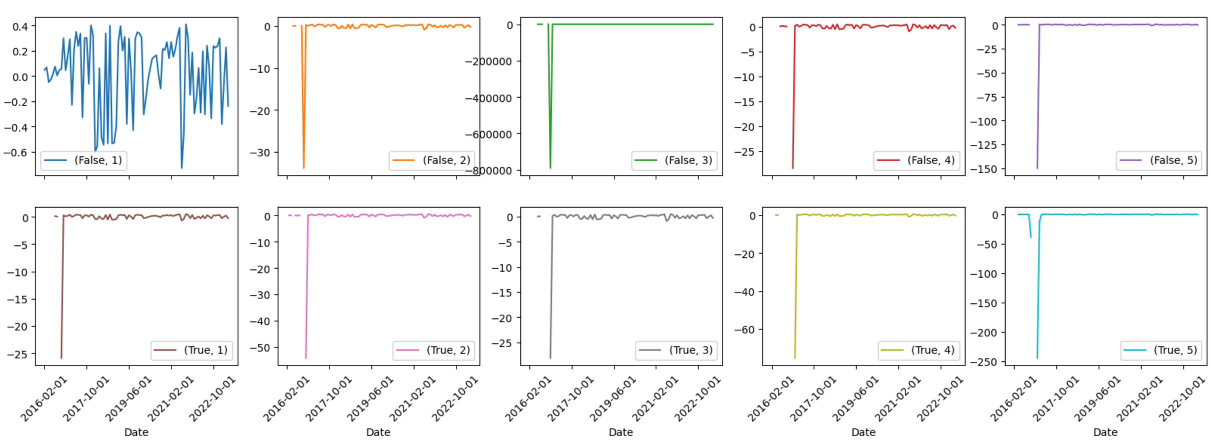
Window Size: 72, Lags: 10



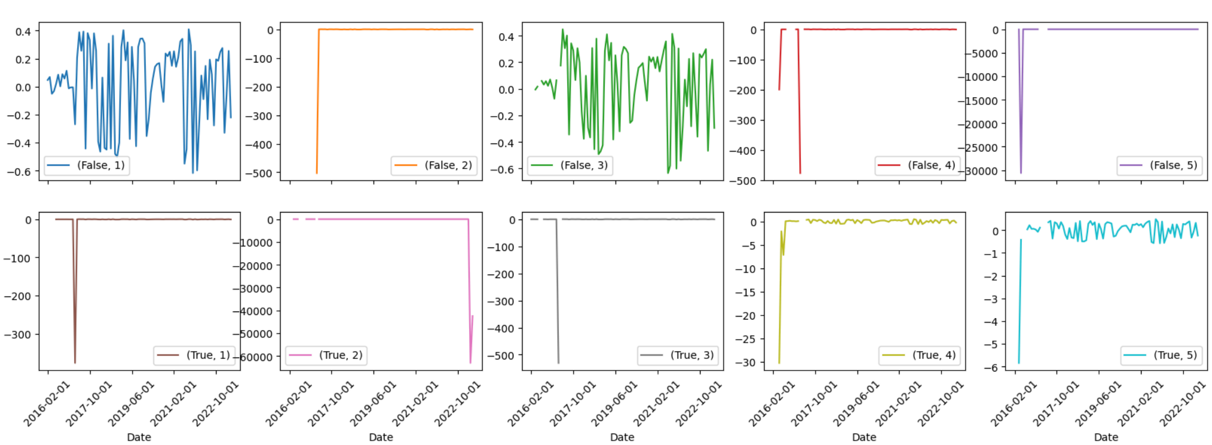
Window Size: 72, Lags: 15



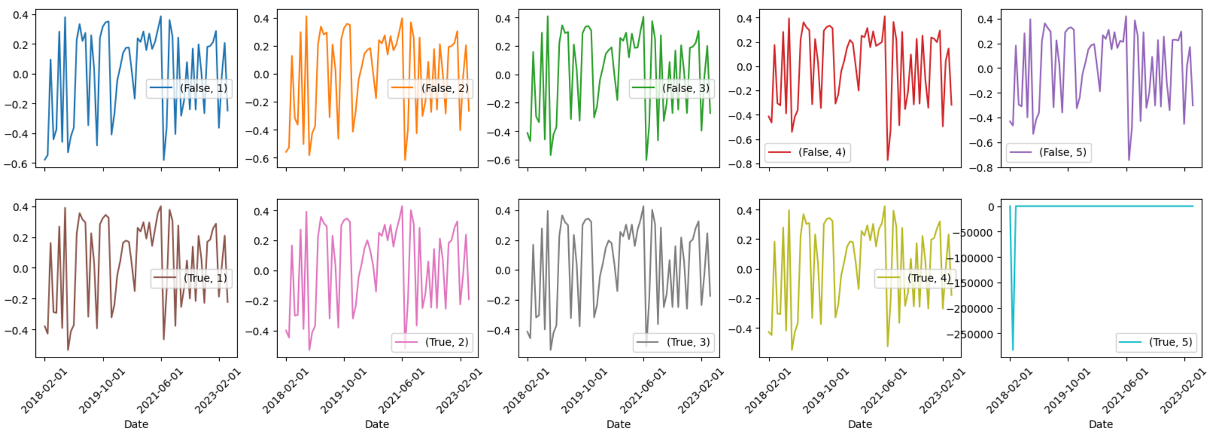
Window Size: 72, Lags: 20



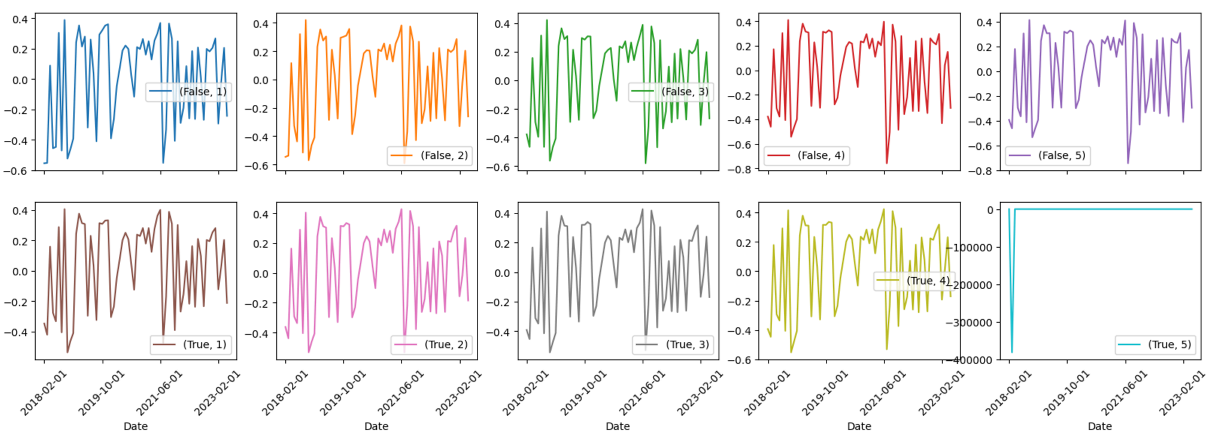
Window Size: 72, Lags: 25



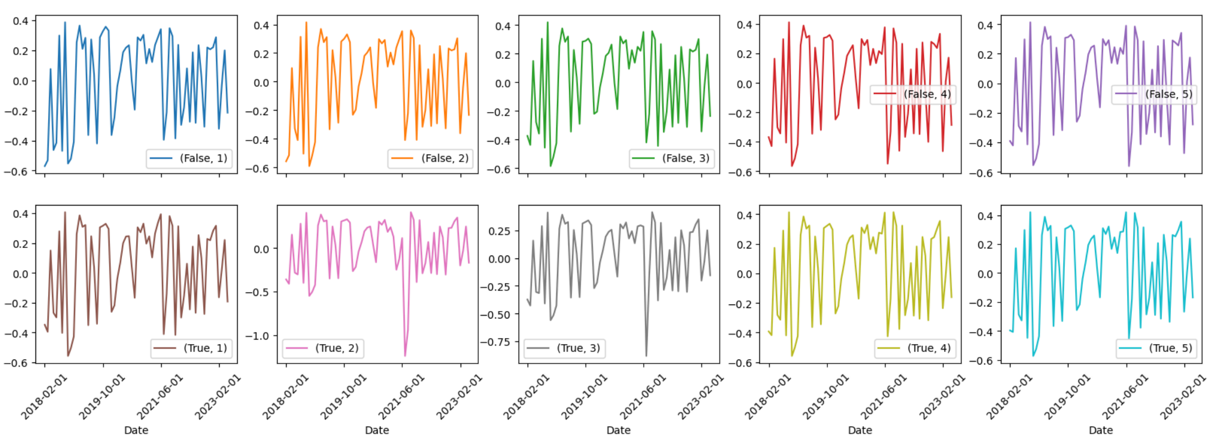
Window Size: 96, Lags: 1



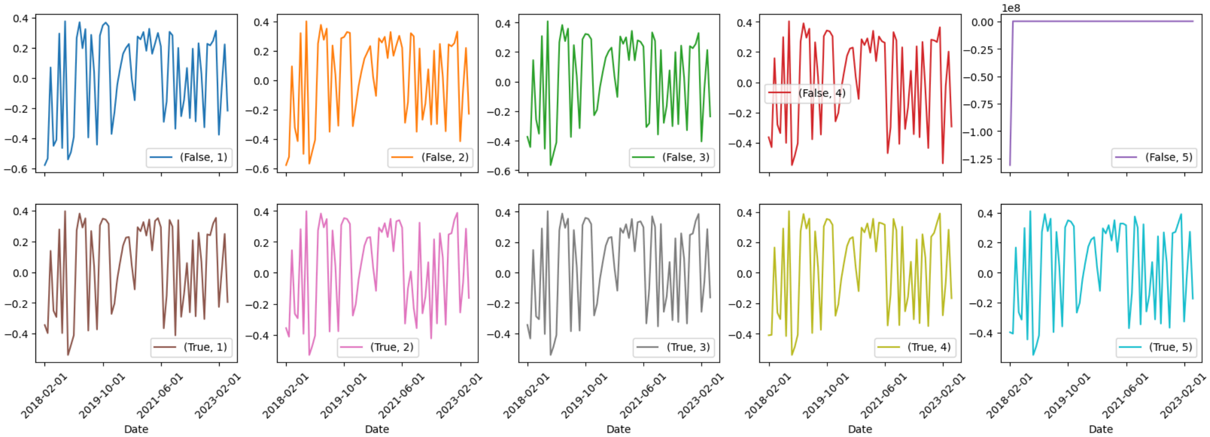
Window Size: 96, Lags: 5



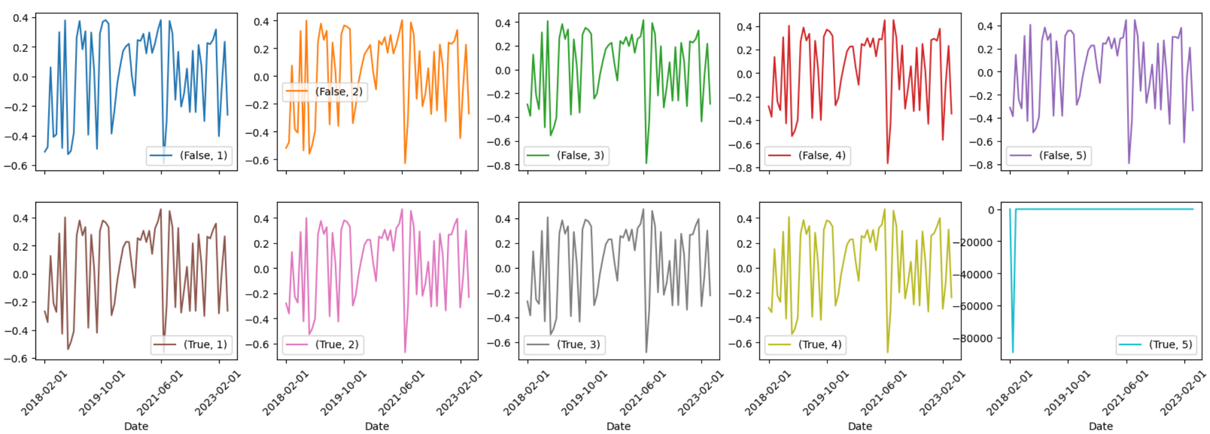
Window Size: 96, Lags: 10



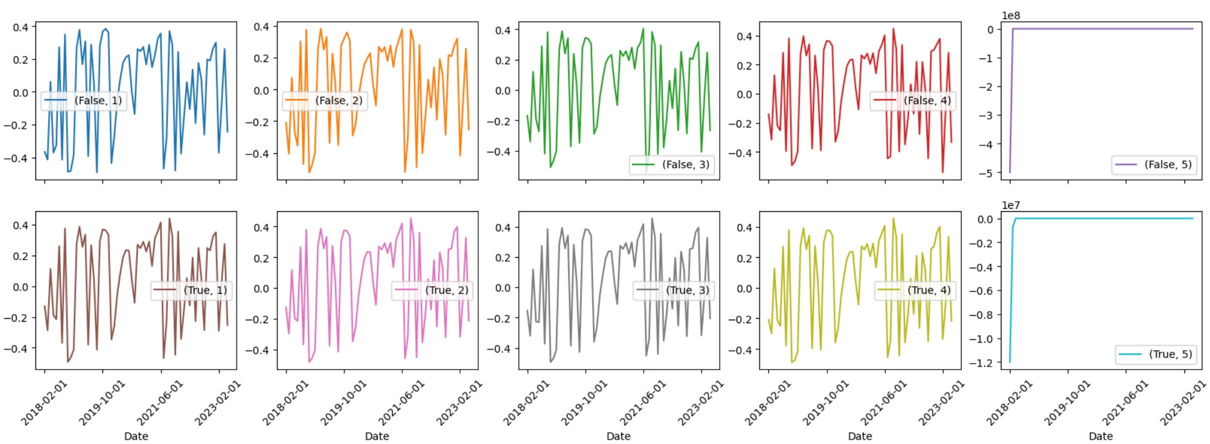
Window Size: 96, Lags: 15



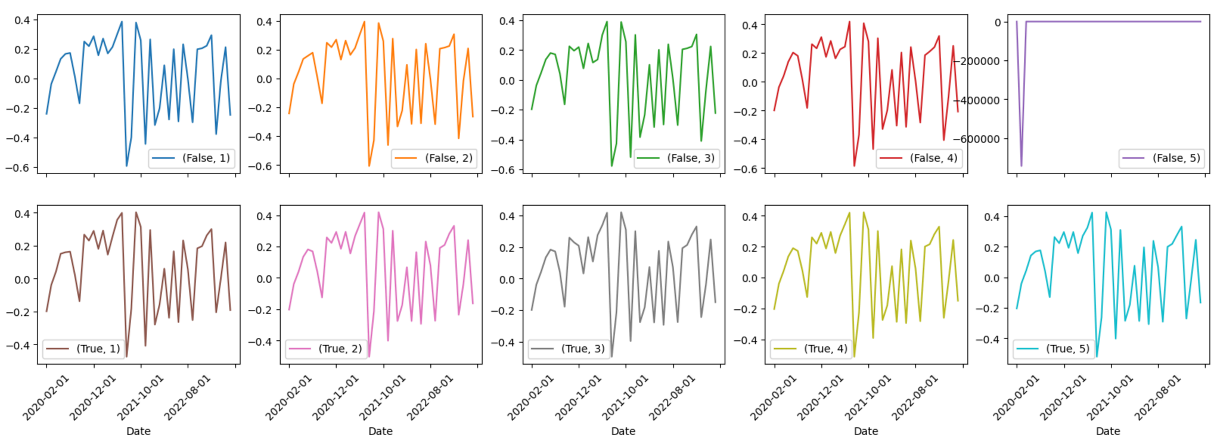
Window Size: 96, Lags: 20



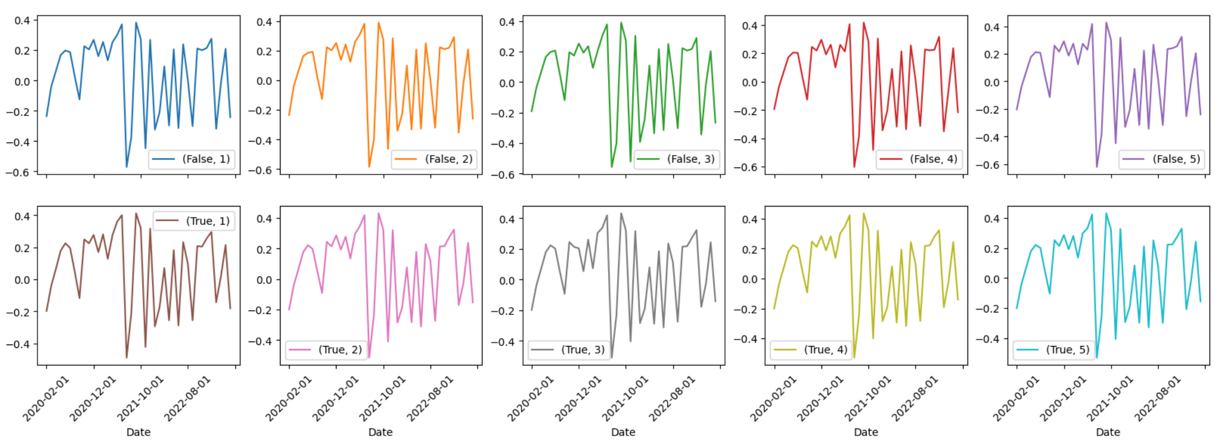
Window Size: 96, Lags: 25



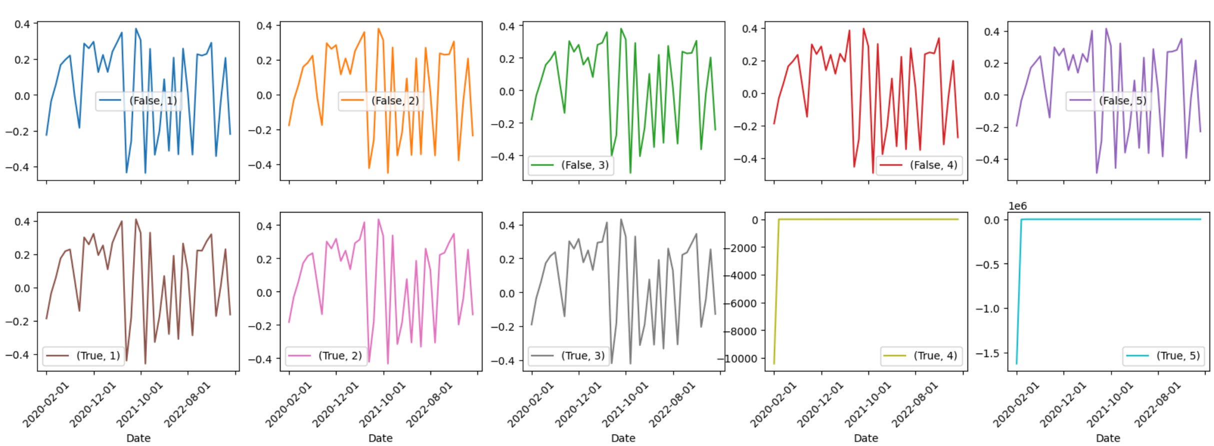
Window Size: 120, Lags: 1

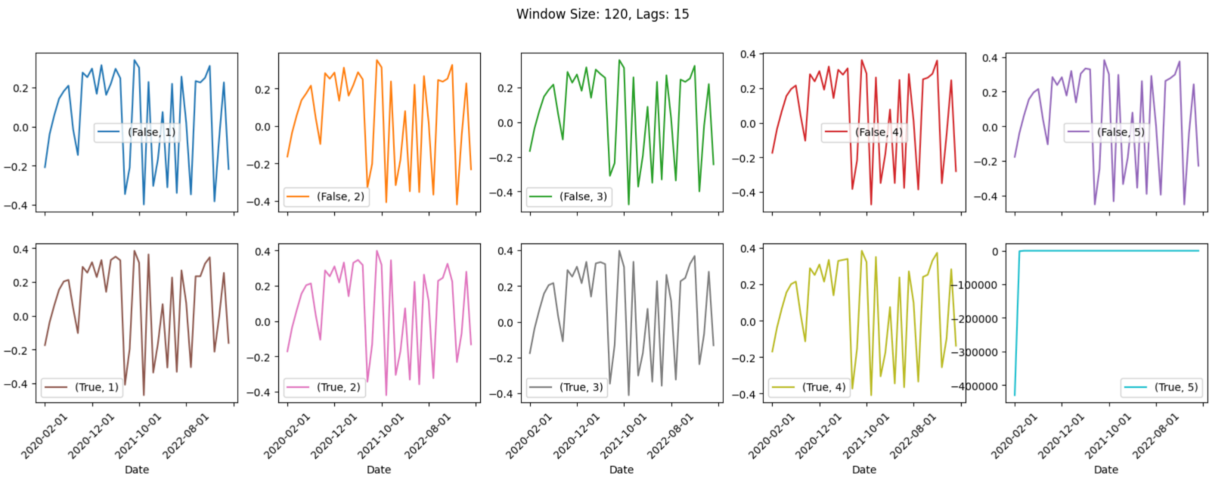


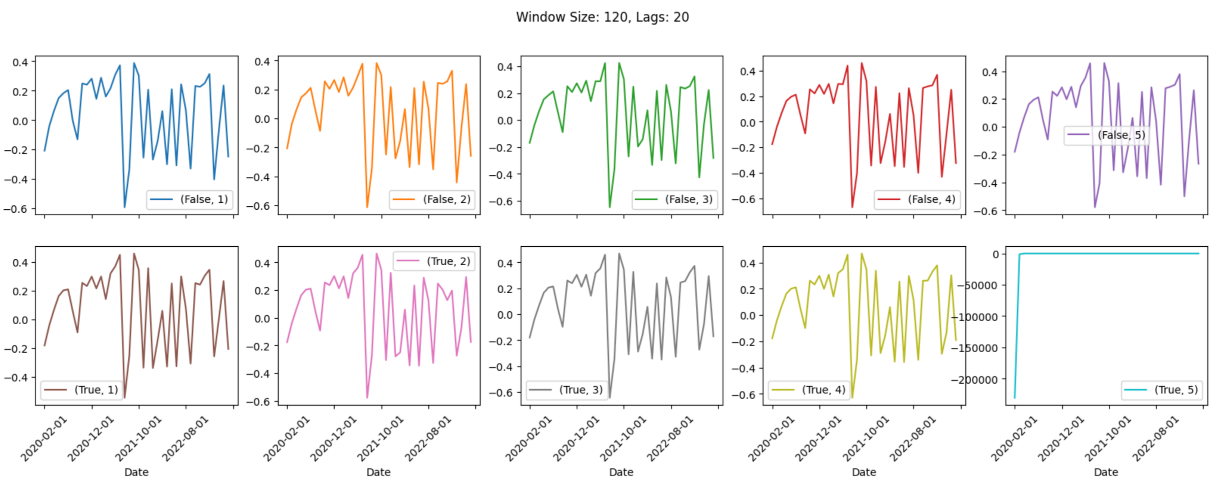
Window Size: 120, Lags: 5

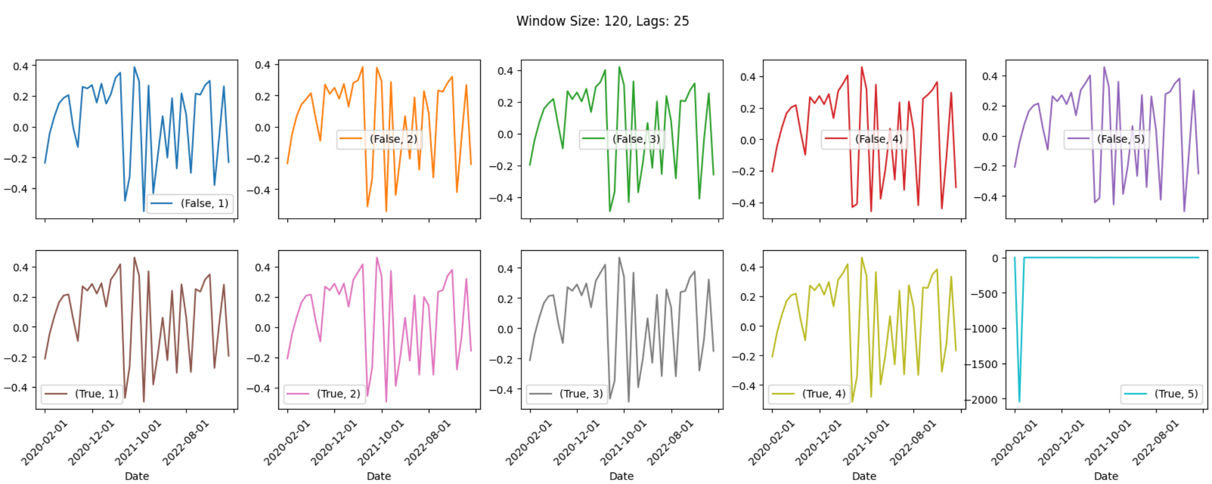


Window Size: 120, Lags: 10









In [ ]:

```
# for window_size in out_of_sample_results_dict.keys():
#     for lags in out_of_sample_results_dict[window_size].keys():

#         out_of_sample_results = out_of_sample_results_dict[window_size][la

#         for col in range(out_of_sample_results.sort_index()[['OOS R^2 Over
#             out_of_sample_results.sort_index()[['OOS R^2 Over Benchmark',
#             plt.title(out_of_sample_results.sort_index()[['OOS R^2 Over Be
#             plt.show()
```

In [ ]:

```
# Average the OOS R^2s over time
```

```
# out_of_sample_results.sort_index()['OOS R^2 Over Benchmark'].unstack(level
```

[redacted]

Portfolio Performance

In [ ]:
```
sorted(os.listdir(os.path.join(data_path, 'Forecasts')))
```

Out[ ]:
```
['window_size_120_lags_1.csv',
 'window_size_120_lags_10.csv',
 'window_size_120_lags_15.csv',
 'window_size_120_lags_20.csv',
 'window_size_120_lags_25.csv',
 'window_size_120_lags_5.csv',
 'window_size_60_lags_1.csv',
 'window_size_60_lags_10.csv',
 'window_size_60_lags_15.csv',
 'window_size_60_lags_20.csv',
 'window_size_60_lags_25.csv',
 'window_size_60_lags_5.csv',
 'window_size_72_lags_1.csv',
 'window_size_72_lags_10.csv',
 'window_size_72_lags_15.csv',
 'window_size_72_lags_20.csv',
 'window_size_72_lags_25.csv',
 'window_size_72_lags_5.csv',
 'window_size_96_lags_1.csv',
 'window_size_96_lags_10.csv',
 'window_size_96_lags_15.csv',
 'window_size_96_lags_20.csv',
 'window_size_96_lags_25.csv',
 'window_size_96_lags_5.csv']
```

In [ ]:
```
window_sizes_lags
```

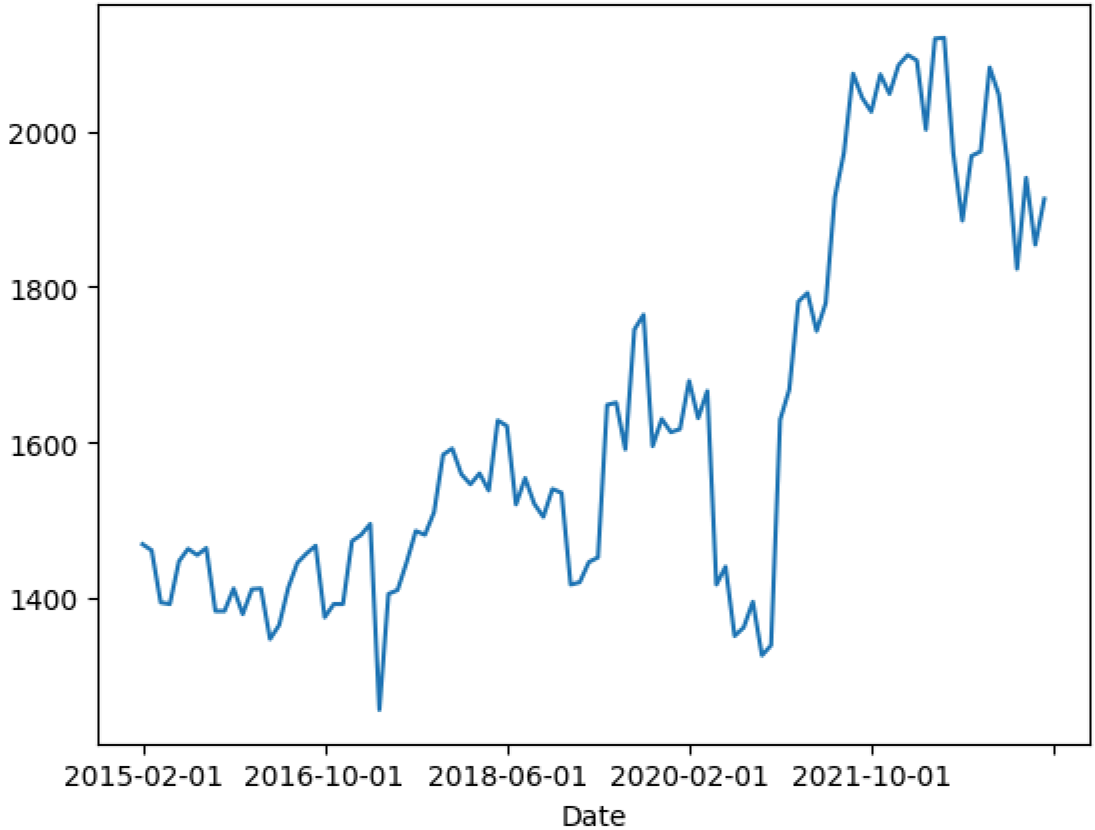
Out[ ]:
```
[(60, 1),
 (60, 5),
 (60, 10),
 (60, 15),
 (60, 20),
 (60, 25),
 (72, 1),
 (72, 5),
 (72, 10),
 (72, 15),
 (72, 20),
 (72, 25),
 (96, 1),
 (96, 5),
 (96, 10),
 (96, 15),
 (96, 20),
 (96, 25),
 (120, 1),
 (120, 5),
 (120, 10),
 (120, 15),
 (120, 20),
 (120, 25)]
```

Unbalanced Panel Analysis

In [ ]:
```
# how many bonds enter and exit a cross-section over time
```

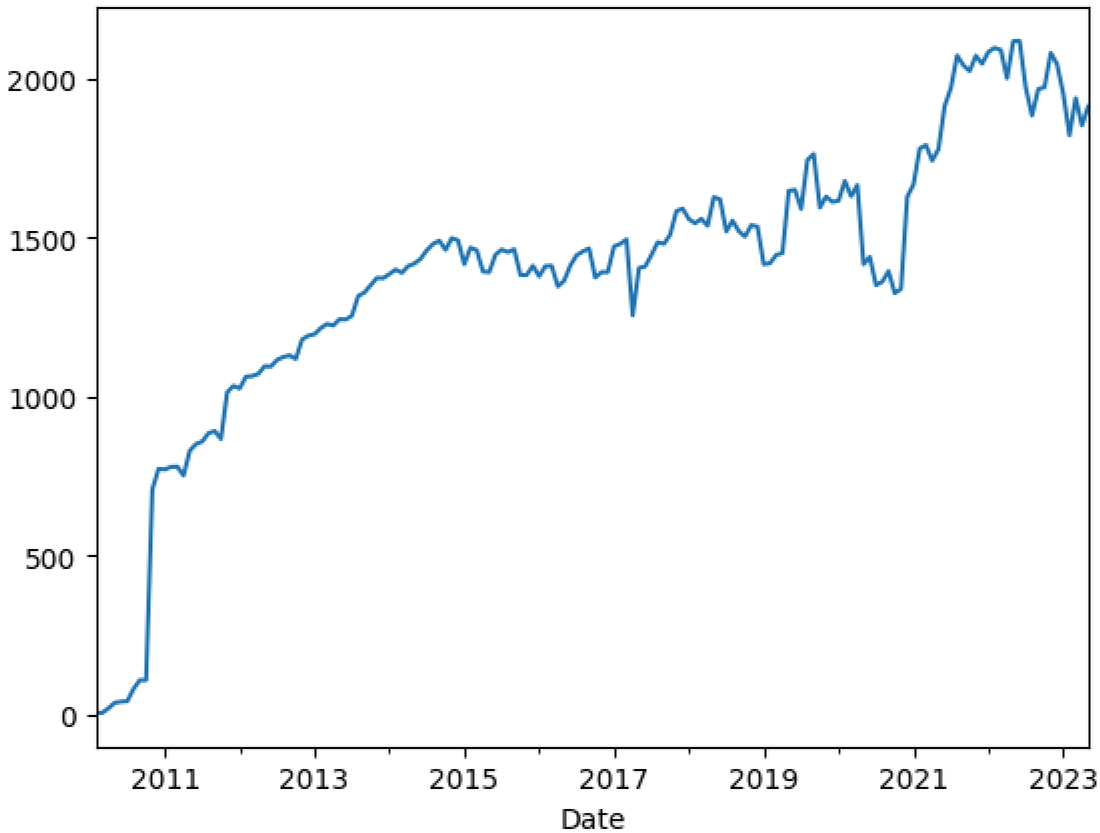
In [ ]:
```
### This looks roughly correct (slight difference between count and len base

# df.groupby('Date')[False].count().plot()
df.groupby('Date')[False].apply(len).plot()
```

```
<Axes: xlabel='Date'>
```



In [ ]:
```
num_bonds_ts = returns.groupby('Date')['Excess_returns'].apply(len)

print(num_bonds_ts.describe().T)

num_bonds_ts.plot()
```

```
count     160.000000
mean     1388.137500
std       456.052359
min         5.000000
25%      1243.750000
50%      1442.500000
75%      1618.000000
max      2120.000000
Name: Excess_returns, dtype: float64
<Axes: xlabel='Date'>
```




```
In [ ]: features_lag
```

Cusip	Date	Bond_age	Face_value	Coupon	Duration	Spread	Ratir
001084AQ	**2013-01-01**	-0.483292	-0.367168	0.121554	0.153300	0.485798	0.4920
	2013-02-01	-0.453947	-0.366776	0.143503	0.148849	0.486020	0.4913
	2013-03-01	-0.443043	-0.364117	0.143613	0.141172	0.491050	0.4918
	2013-04-01	-0.417484	-0.361111	0.154820	0.131536	0.493464	0.4926
	2013-05-01	-0.391077	-0.359325	0.160772	0.118167	0.497588	0.4923
...	**...**	...	...	...	...	...	
98978VAT	**2023-01-01**	-0.135806	-0.332992	-0.179028	0.447315	-0.096419	0.2156
	2023-02-01	-0.128634	-0.339002	-0.183763	0.445694	-0.052660	0.2188
	2023-03-01	-0.113660	-0.332216	-0.178866	0.439175	0.013918	0.2265
	2023-04-01	-0.101402	-0.339806	-0.188781	0.435275	-0.004585	0.2179
	2023-05-01	-0.097752	-0.325928	-0.178254	0.432044	-0.050706	0.2409

222102 rows × 49 columns

In [ ]:
```
# Each period, how many bonds get added to the cross-section, and how many b
```

Cross-Sectional Sorts by Forecasts into Deciles

- Take the order statistics of the return forecasts, and form equally weighted portfolios across deciles of these order statistics.

In [ ]:
```
# start with equal weighting -> average across buckets -> look at long-short

# compare results with Modeling Corporate Bond Returns by Kelly, et al.
# plot time-series graphs
# average over the sample
# sharpes and t-stats

## Excess Returns are in % format
```

In [ ]:
```
from IPython.display import display
```

```
high_minus_low_performance = pd.DataFrame(columns=['window_size', 'lag', 'nu

for window_size, lag in window_sizes_lags:


    df = pd.read_csv(os.path.join(data_path, 'Forecasts', f'window_size_{wir
    df.set_index(['Intercept', 'Num_Factors', 'Cusip', 'Date'], inplace=True
    df.sort_index(inplace=True)
    df = df[['Returns_forecast', 'Excess_returns']].unstack(['Intercept', 'N
    df = df.swaplevel(0,1,axis=1).swaplevel(1,2,axis=1).sort_index(axis=1)

    for num_factors in range(1, 6):
        for intercept in [False, True]:
            df.loc[:,idx[intercept, num_factors, 'decile']] = df.loc[:,idx[i
                                                                  .groupby('Da
                                                                  .apply(lambc

    df.sort_index(axis=1, inplace=True)
    ## Excess Returns are in % format

    # This is for a given lag period and a given window size
    for num_factors in range(1, 6):
        for intercept in [False, True]:

            port_eq_weight_decile_rets = df.loc[:,idx[intercept, num_factors
            # expected returns from long-short portfolio where we long the "
            port_eq_weight_decile_rets['high_minus_low'] = port_eq_weight_de

            port_stats = port_eq_weight_decile_rets.describe().T # none of t

            sharpe_annualized = (port_stats.loc['high_minus_low', 'mean'] / 
            t_stat = (port_stats.loc['high_minus_low', 'mean'] / port_stats.
            mean_annualized = port_stats.loc['high_minus_low', 'mean'] * 12 

            high_minus_low_performance = pd.concat([high_minus_low_performan
                                                    pd.DataFrame([window_siz
                                                    axis=0, ignore_index=Tru

high_minus_low_performance
```

	window_size	lag	num_factors	intercept	mean_annualized	sharpe_annua
0	60	1	1	False	0.302098	0.87
1	60	1	1	True	0.521045	1.49
2	60	1	2	False	0.239012	0.6
3	60	1	2	True	0.527101	1.50
4	60	1	3	False	0.245555	0.70
5	60	1	3	True	0.539449	1.54
6	60	1	4	False	0.44649	1.27
7	60	1	4	True	0.529272	1.51
8	60	1	5	False	0.583685	1.65
9	60	1	5	True	0.497862	1.40
10	60	5	1	False	0.282873	0.81
11	60	5	1	True	0.586032	1.65
12	60	5	2	False	0.219063	0.61
13	60	5	2	True	0.596877	1.70
14	60	5	3	False	0.289334	0.81
15	60	5	3	True	0.563908	1.58
16	60	5	4	False	0.357771	1.01
17	60	5	4	True	0.562652	1.60
18	60	5	5	False	0.525998	1.47
19	60	5	5	True	0.547365	1.5
20	60	10	1	False	0.321879	0.92
21	60	10	1	True	0.653399	1.85
22	60	10	2	False	0.366895	1.01
23	60	10	2	True	0.591699	1.66
24	60	10	3	False	0.404573	1.14
25	60	10	3	True	0.657499	1.8
26	60	10	4	False	0.446189	1.25
27	60	10	4	True	0.647286	1.83
28	60	10	5	False	0.508847	1.40
29	60	10	5	True	0.515261	1.44
30	60	15	1	False	0.356686	1.02
31	60	15	1	True	0.840304	2.38
32	60	15	2	False	0.300655	0.81

	window_size	lag	num_factors	intercept	mean_annualized	sharpe_annua
33	60	15	2	True	0.893372	2.48
34	60	15	3	False	0.507798	1.42
35	60	15	3	True	0.706242	1.96
36	60	15	4	False	0.63903	1.77
37	60	15	4	True	0.779858	2.18
38	60	15	5	False	0.63892	1.78
39	60	15	5	True	0.782578	2.19
40	60	20	1	False	0.358727	1.03
41	60	20	1	True	0.837043	2.35
42	60	20	2	False	0.292055	0.79
43	60	20	2	True	0.749644	2.05
44	60	20	3	False	0.523505	1.47
45	60	20	3	True	0.573319	1.60
46	60	20	4	False	0.607896	1.69
47	60	20	4	True	0.593697	1.64
48	60	20	5	False	0.600014	1.6
49	60	20	5	True	0.600336	1.62
50	60	25	1	False	0.308761	0.89
51	60	25	1	True	0.719577	1.97
52	60	25	2	False	0.487269	1.3
53	60	25	2	True	0.47871	1.28
54	60	25	3	False	0.512183	1.37
55	60	25	3	True	0.492054	1.3
56	60	25	4	False	0.538372	1.45
57	60	25	4	True	0.597708	1.60
58	60	25	5	False	0.527374	1.41
59	60	25	5	True	1.754848	4.5
60	72	1	1	False	0.502308	1.36
61	72	1	1	True	0.548026	1.4
62	72	1	2	False	0.288511	0.78
63	72	1	2	True	0.469313	1.27
64	72	1	3	False	0.487052	1.31
65	72	1	3	True	0.511342	1.38

	window_size	lag	num_factors	intercept	mean_annualized	sharpe_annua
66	72	1	4	False	0.571686	1.54
67	72	1	4	True	0.502137	1.35
68	72	1	5	False	0.601465	1.62
69	72	1	5	True	0.556216	1.50
70	72	5	1	False	0.460345	1.24
71	72	5	1	True	0.63689	1.72
72	72	5	2	False	0.28217	0.7
73	72	5	2	True	0.584608	1.58
74	72	5	3	False	0.401383	1.08
75	72	5	3	True	0.681737	1.84
76	72	5	4	False	0.525526	1.42
77	72	5	4	True	0.648486	1.75
78	72	5	5	False	0.568864	1.5
79	72	5	5	True	0.474119	1.2
80	72	10	1	False	0.471805	1.27
81	72	10	1	True	0.777382	2.1
82	72	10	2	False	0.578961	1.56
83	72	10	2	True	0.797364	2.15
84	72	10	3	False	0.649948	1.76
85	72	10	3	True	0.820734	2.22
86	72	10	4	False	0.782235	2.11
87	72	10	4	True	0.758646	2.05
88	72	10	5	False	0.729436	1.97
89	72	10	5	True	0.691612	1.86
90	72	15	1	False	0.514791	1.39
91	72	15	1	True	0.925277	2.46
92	72	15	2	False	0.568782	1.51
93	72	15	2	True	0.991172	2.63
94	72	15	3	False	0.768662	2.03
95	72	15	3	True	0.804348	2.15
96	72	15	4	False	0.883071	2.33
97	72	15	4	True	0.996197	2.61
98	72	15	5	False	0.832475	2.18

	window_size	lag	num_factors	intercept	mean_annualized	sharpe_annua
99	72	15	5	True	0.939214	2.47
100	72	20	1	False	0.476625	1.29
101	72	20	1	True	0.835622	2.1
102	72	20	2	False	0.428872	1.12
103	72	20	2	True	0.80292	2.12
104	72	20	3	False	0.550679	1.44
105	72	20	3	True	0.690621	1.80
106	72	20	4	False	0.634626	1.65
107	72	20	4	True	0.755399	1.97
108	72	20	5	False	0.752008	1.97
109	72	20	5	True	0.86175	2.29
110	72	25	1	False	0.452971	1.22
111	72	25	1	True	0.706253	1.87
112	72	25	2	False	0.37769	0.95
113	72	25	2	True	0.567238	1.48
114	72	25	3	False	0.459561	1.21
115	72	25	3	True	0.62438	1.66
116	72	25	4	False	0.565785	
117	72	25	4	True	0.630214	1.6
118	72	25	5	False	0.623097	1.62
119	72	25	5	True	0.562059	1.46
120	96	1	1	False	0.180136	0.41
121	96	1	1	True	0.401524	0.92
122	96	1	2	False	0.095254	0.2
123	96	1	2	True	0.367369	0.84
124	96	1	3	False	0.442888	1.02
125	96	1	3	True	0.375748	0.86
126	96	1	4	False	0.273491	0.63
127	96	1	4	True	0.395893	0.91
128	96	1	5	False	0.268361	0.61
129	96	1	5	True	0.443446	1.02
130	96	5	1	False	0.173708	0.40
131	96	5	1	True	0.525207	1.21

	window_size	lag	num_factors	intercept	mean_annualized	sharpe_annua
132	96	5	2	False	0.112867	0.26
133	96	5	2	True	0.504411	1.16
134	96	5	3	False	0.348729	0.80
135	96	5	3	True	0.526646	1.21
136	96	5	4	False	0.309875	0.71
137	96	5	4	True	0.511623	1.18
138	96	5	5	False	0.348338	0.80
139	96	5	5	True	0.527028	1.2
140	96	10	1	False	0.165403	0.38
141	96	10	1	True	0.663761	1.53
142	96	10	2	False	0.357267	0.82
143	96	10	2	True	0.559201	1.29
144	96	10	3	False	0.434831	1.00
145	96	10	3	True	0.572551	1.32
146	96	10	4	False	0.438126	1.0
147	96	10	4	True	0.584081	1.34
148	96	10	5	False	0.47017	1.0
149	96	10	5	True	0.577981	1.33
150	96	15	1	False	0.197154	0.45
151	96	15	1	True	0.786095	1.81
152	96	15	2	False	0.064223	0.14
153	96	15	2	True	0.718092	1.65
154	96	15	3	False	0.417671	0.96
155	96	15	3	True	0.68381	1.57
156	96	15	4	False	0.523012	1.20
157	96	15	4	True	0.747697	1.72
158	96	15	5	False	0.536179	1.23
159	96	15	5	True	0.734525	1.69
160	96	20	1	False	0.16565	0.38
161	96	20	1	True	0.73514	1.69
162	96	20	2	False	0.176946	0.4
163	96	20	2	True	0.630932	1.45
164	96	20	3	False	0.323982	0.74

	window_size	lag	num_factors	intercept	mean_annualized	sharpe_annua
165	96	20	3	True	0.612454	1.41
166	96	20	4	False	0.431603	0.99
167	96	20	4	True	0.610312	1.40
168	96	20	5	False	0.47615	1.09
169	96	20	5	True	0.632914	1.46
170	96	25	1	False	0.144626	[illegible]
171	96	25	1	True	0.603363	1.39
172	96	25	2	False	0.215162	0.49
173	96	25	2	True	0.549657	1.26
174	96	25	3	False	0.316502	0.73
175	96	25	3	True	0.517286	1.1
176	96	25	4	False	0.432606	0.99
177	96	25	4	True	0.523895	1.20
178	96	25	5	False	0.377824	0.87
179	96	25	5	True	0.495985	1.14
180	120	1	1	False	0.029385	0.05
181	120	1	1	True	0.170186	0.31
182	120	1	2	False	-0.084165	-0.15
183	120	1	2	True	0.130461	0.23
184	120	1	3	False	0.182927	0.33
185	120	1	3	True	0.175658	0.32
186	120	1	4	False	0.131186	0.23
187	120	1	4	True	0.213966	0.39
188	120	1	5	False	1.580359	2.88
189	120	1	5	True	0.210588	0.38
190	120	5	1	False	0.013184	0.0
191	120	5	1	True	0.378946	0.69
192	120	5	2	False	-0.056493	-0.10
193	120	5	2	True	0.320005	0.58
194	120	5	3	False	-0.049063	-0.08
195	120	5	3	True	0.257753	0.47
196	120	5	4	False	0.127437	0.23
197	120	5	4	True	0.336369	0.61

	window_size	lag	num_factors	intercept	mean_annualized	sharpe_annua
198	120	5	5	False	0.189659	0.34
199	120	5	5	True	0.350487	0
200	120	10	1	False	0.011844	0.02
201	120	10	1	True	0.527811	0.96
202	120	10	2	False	0.347866	0.63
203	120	10	2	True	0.62373	1.1
204	120	10	3	False	0.279384	0.51
205	120	10	3	True	0.399506	0.72
206	120	10	4	False	0.270043	0.49
207	120	10	4	True	0.417722	0.76
208	120	10	5	False	0.209412	0.38
209	120	10	5	True	1.395815	2.54
210	120	15	1	False	0.047934	0.08
211	120	15	1	True	0.82766	1.51
212	120	15	2	False	0.270741	0.49
213	120	15	2	True	0.80219	1.46
214	120	15	3	False	0.235154	0.42
215	120	15	3	True	0.603038	1.10
216	120	15	4	False	0.488765	0.89
217	120	15	4	True	0.597738	1.09
218	120	15	5	False	0.297179	0.54
219	120	15	5	True	1.697479	3.09
220	120	20	1	False	0.003426	0.00
221	120	20	1	True	0.621146	1.13
222	120	20	2	False	-0.105837	-0.19
223	120	20	2	True	0.484994	0.88
224	120	20	3	False	0.216502	0.39
225	120	20	3	True	0.409156	0.74
226	120	20	4	False	0.265756	0.48
227	120	20	4	True	0.432166	0.78
228	120	20	5	False	0.200954	0.36
229	120	20	5	True	2.088477	3.81
230	120	25	1	False	0.010691	0.01

	window_size	lag	num_factors	intercept	mean_annualized	sharpe_annua
231	120	25	1	True	0.441765	0.8
232	120	25	2	False	-0.224767	-0.41
233	120	25	2	True	0.456037	0.83
234	120	25	3	False	0.113506	0.20
235	120	25	3	True	0.431559	0.78
236	120	25	4	False	0.122172	0.22
237	120	25	4	True	0.423515	0.77
238	120	25	5	False	0.101811	0.18
239	120	25	5	True	1.906729	3.48

In [ ]:

```
pd.set_option('display.max_rows', None)
pd.set_option('display.max_columns', None)

high_minus_low_performance = high_minus_low_performance.set_index(['window_s
                                                        .sort_index().unstack
                                                        .swaplevel(0,2, axis=
```

		window_size				
		lag	1	5	10	
intercept	num_factors					
False	1	mean_annualized	0.302098	0.282873	0.321879	0.3566
		sharpe_annualized	0.872081	0.816583	0.929183	1.0296
		t_stat	3.858787	3.62039	4.08018	4.2582
	2	mean_annualized	0.239012	0.219063	0.366895	0.3006
		sharpe_annualized	0.68651	0.619604	1.015886	0.8141
		t_stat	2.782549	2.662802	4.821137	3.8215
	3	mean_annualized	0.245555	0.289334	0.404573	0.5077
		sharpe_annualized	0.701731	0.818359	1.144305	1.4212
		t_stat	2.631411	3.280107	3.805107	4.1559
	4	mean_annualized	0.44649	0.357771	0.446189	0.639
		sharpe_annualized	1.275951	1.011928	1.255424	1.7789
		t_stat	3.836153	3.435013	4.086548	4.9122
	5	mean_annualized	0.583685	0.525998	0.508847	0.638
		sharpe_annualized	1.650911	1.479978	1.408933	1.7882
		t_stat	4.74969	4.958605	4.522094	5.2731
True	1	mean_annualized	0.521045	0.586032	0.653399	0.8403
		sharpe_annualized	1.496588	1.657548	1.857693	2.3890
		t_stat	4.079739	5.231177	5.474282	6.1261
	2	mean_annualized	0.527101	0.596877	0.591699	0.8933
		sharpe_annualized	1.506318	1.705718	1.664837	2.4870
		t_stat	4.201353	5.310752	5.475806	6.696
	3	mean_annualized	0.539449	0.563908	0.657499	0.7062
		sharpe_annualized	1.549448	1.586644	1.86935	1.9660
		t_stat	4.237788	5.277266	5.902005	5.1401
	4	mean_annualized	0.529272	0.562652	0.647286	0.7798
		sharpe_annualized	1.512521	1.607912	1.830801	2.1826
		t_stat	3.989369	5.02547	5.130198	5.2524
	5	mean_annualized	0.497862	0.547365	0.515261	0.7825
		sharpe_annualized	1.408165	1.53197	1.449767	2.1902
		t_stat	4.036046	4.949295	4.315108	5.5009

In [ ]:
```
high_minus_low_performance#.T
```

	intercept				
	num_factors			1	
		mean_annualized	sharpe_annualized	t_stat	mea
window_size	lag				
60	1	0.302098	0.872081	3.858787	
	5	0.282873	0.816583	3.62039	
	10	0.321879	0.929183	4.08018	
	15	0.356686	1.029663	4.258282	
	20	0.358727	1.035555	4.274022	
	25	0.308761	0.891317	3.764746	
72	1	0.502308	1.360256	6.734437	
	5	0.460345	1.246621	6.205949	
	10	0.471805	1.277653	6.471617	
	15	0.514791	1.394061	6.633676	
	20	0.476625	1.290706	6.176397	
	25	0.452971	1.226652	5.845735	
96	1	0.180136	0.416006	2.679547	
	5	0.173708	0.401161	2.634742	
	10	0.165403	0.381981	2.596631	
	15	0.197154	0.455307	2.90578	
	20	0.16565	0.382551	2.453073	
	25	0.144626	0.334	2.140482	
120	1	0.029385	0.053649	0.474259	
	5	0.013184	0.02407	0.218112	
	10	0.011844	0.021624	0.197162	
	15	0.047934	0.087515	0.731489	
	20	0.003426	0.006254	0.054199	
	25	0.010691	0.019519	0.17347	

In [ ]:
```
window_size = 120
lag = [1,5,10,15,20,25]

intercept = True
num_factors = 3
```

```
high_minus_low_performance.loc[idx[window_size, lag], idx[intercept, num_fac
```

window_size	lag	mean_annualized	sharpe_annualized	t_stat
120	1	0.175658	0.320706	1.390613
	5	0.257753	0.470591	2.568523
	10	0.399506	0.729395	3.747632
	15	0.603038	1.100991	4.557753
	20	0.409156	0.747014	3.726411
	25	0.431559	0.787916	3.689114

In [ ]:
```
pd.reset_option('display.max_rows')
pd.reset_option('display.max_columns')
```

In [ ]:
```
from IPython.display import display

for window_size, lag in window_sizes_lags: #[(60,5)]:


    df = pd.read_csv(os.path.join(data_path, 'Forecasts', f'window_size_{wir
    df.set_index(['Intercept', 'Num_Factors', 'Cusip', 'Date'], inplace=True
    df.sort_index(inplace=True)
    df = df[['Returns_forecast', 'Excess_returns']].unstack(['Intercept', 'N
    df = df.swaplevel(0,1,axis=1).swaplevel(1,2,axis=1).sort_index(axis=1)

    for num_factors in range(1, 6):
        for intercept in [False, True]:
            df.loc[:,idx[intercept, num_factors, 'decile']] = df.loc[:,idx[i
                                                                  .groupby('Da
                                                                  .apply(lambc


    df.sort_index(axis=1, inplace=True)
    ## Excess Returns are in % format

    # This is for a given lag period and a given window size
    for num_factors in range(1, 6):
        for intercept in [False, True]:

            plt.figure(figsize=(16,8))

            port_eq_weight_decile_rets = df.loc[:,idx[intercept, num_factors
            # expected returns from long-short portfolio where we long the "
            port_eq_weight_decile_rets['high_minus_low'] = port_eq_weight_de
            port_eq_weight_decile_rets.plot(figsize=(16,8))
            plt.title(f'Monthly Portfolio Bucket Returns \n Window Size: {wi
            plt.show()


            port_stats = port_eq_weight_decile_rets.describe().T # none of t
```

```
            port_stats['sharpe_annualized'] = (port_stats['mean'] / port_sta
            port_stats['t_stat'] = (port_stats['mean'] / port_stats['std'])
            display(port_stats)
```

In [ ]:
```
# portstats.loc['ann Sharpe'] = portstats.loc['mean'] / portstats.loc['std']
# portstats.loc['t stat']     = portstats.loc['mean'] / portstats.loc['std']
# portstats.loc['% positive'] = 100 * (RP_longshort[['R_long','R_short','R',

# (1 + LEV*RP_longshort[['R','RC','R_short']]).cumprod().plot()
```

In [ ]:
```
## I will continue with disaggregating results based on IG, HY, and distance
# How are excess returns calculated?
```

Conditional Cross-Sectional Sorts by Forecasts into Deciles

- Stratify the Investable Universe by Bond Properties
- We will look at investment grade, high-yield, and bonds with near maturity (older, not the most recent issue) and far from maturity (newer, recently issued) (i.e. off / on the run)
- What about liquidity?
    - Bonds that have been issued most recently, high-quality (AAA), and "large size" (larger than 500 million)

In [ ]:
```
# ipca_df['Issue_date'] = ipca_df.groupby('Cusip')['Date'].transform('min')
# ipca_df['Bond_age'] = (ipca_df['Date'] - ipca_df['Issue_date']).dt.days /
# ipca_df['Bond_age'] = ipca_df['Bond_age'].apply(lambda x: min(x, 10))
```

In [ ]:
```
data
```

Out[ ]:

Cusip	Date	Index	Company	Industry	Excess_returns	Issue_date	Bo
001084AQ	2012-12-01	C0A0	AGCO CORP	Capital Goods	2.216	2012-06-01	0
	2013-01-01	C0A0	AGCO CORP	Capital Goods	-0.390	2012-06-01	0
	2013-02-01	C0A0	AGCO CORP	Capital Goods	0.319	2012-06-01	0
	2013-03-01	C0A0	AGCO CORP	Capital Goods	-2.567	2012-06-01	0
	2013-04-01	C0A0	AGCO CORP	Capital Goods	4.725	2012-06-01	0
...	...	...	...	...	...	...	
98978VAT	2023-01-01	C0A0	Zoetis Inc.	Healthcare	0.413	2020-06-01	2
	2023-02-01	C0A0	Zoetis Inc.	Healthcare	-1.077	2020-06-01	2
	2023-03-01	C0A0	Zoetis Inc.	Healthcare	1.050	2020-06-01	2
	2023-04-01	C0A0	Zoetis Inc.	Healthcare	0.904	2020-06-01	2
	2023-05-01	C0A0	Zoetis Inc.	Healthcare	-1.715	2020-06-01	2

228806 rows × 54 columns

In [ ]:

```
data.info()
```

```
<class 'pandas.core.frame.DataFrame'>
MultiIndex: 228806 entries, ('001084AQ', Timestamp('2012-12-01 00:00:00')) t
o ('98978VAT', Timestamp('2023-05-01 00:00:00'))
Data columns (total 54 columns):
 #   Column                      Non-Null Count   Dtype
---  ------                      --------------   -----
 0   Index                       228806 non-null  object
 1   Company                     228806 non-null  object
 2   Industry                    228806 non-null  object
 3   Excess_returns              228806 non-null  float64
 4   Issue_date                  228806 non-null  datetime64[ns]
 5   Bond_age                    228806 non-null  float64
 6   Face_value                  228806 non-null  float64
 7   Coupon                      228806 non-null  float64
 8   Duration                    228806 non-null  float64
 9   Spread                      228806 non-null  float64
 10  Rating                      228806 non-null  int64
 11  Distance_to_default         228806 non-null  float64
 12  Book_leverage               228806 non-null  float64
 13  Market_leverage             228806 non-null  float64
 14  Operating_leverage          228806 non-null  float64
 15  Book_to_price               228806 non-null  float64
 16  Earnings_to_price           228806 non-null  float64
 17  Marketcap                   228806 non-null  float64
 18  Debt                        228806 non-null  float64
 19  Debt_to_ebitda              228806 non-null  float64
 20  Spread_to_d2d               228806 non-null  float64
 21  Profitability               228806 non-null  float64
 22  Prof_change                 228806 non-null  float64
 23  Mom_6m_equity               228806 non-null  float64
 24  Mom_6m                      228806 non-null  float64
 25  Mom_6m_rating               228806 non-null  float64
 26  Mom_6m_spread               228806 non-null  float64
 27  Stock_vol                   228806 non-null  float64
 28  Turnover_vol                228806 non-null  float64
 29  VaR                         228806 non-null  float64
 30  VIX_Beta                    228806 non-null  float64
 31  Mom_6m_industry             228806 non-null  float64
 32  Bond_vol                    228806 non-null  float64
 33  Bond_skew                   228806 non-null  float64
 34  Banking                     228806 non-null  int64
 35  Basic Industry              228806 non-null  int64
 36  Telecommunications          228806 non-null  int64
 37  Energy                      228806 non-null  int64
 38  Consumer Non-Cyclical       228806 non-null  int64
 39  Leisure                     228806 non-null  int64
 40  Technology & Electronics    228806 non-null  int64
 41  Healthcare                  228806 non-null  int64
 42  Consumer Goods              228806 non-null  int64
 43  Transportation              228806 non-null  int64
 44  Consumer Cyclical           228806 non-null  int64
 45  Services                    228806 non-null  int64
 46  Financial Services          228806 non-null  int64
 47  Insurance                   228806 non-null  int64
 48  Automotive                  228806 non-null  int64
 49  Retail                      228806 non-null  int64
```

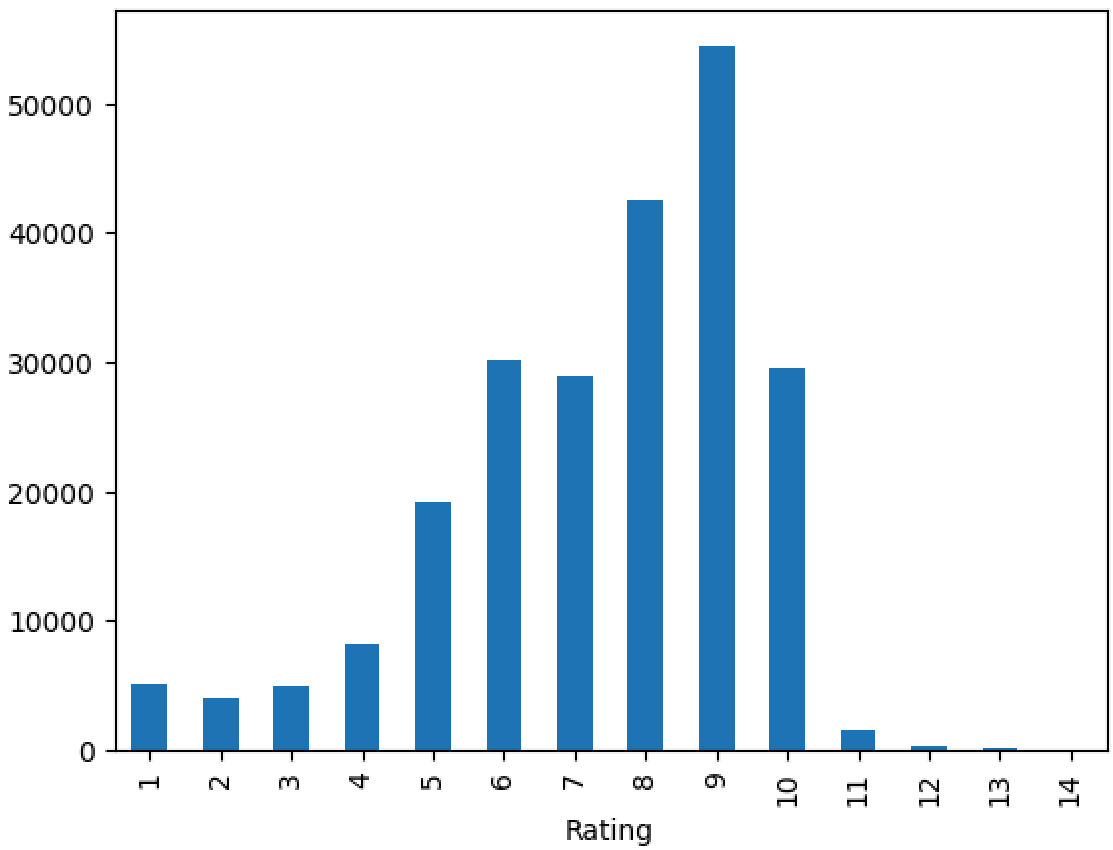
```
 50  Capital Goods              228806 non-null  int64
 51  Utility                    228806 non-null  int64
 52  Media                      228806 non-null  int64
 53  Real Estate                228806 non-null  int64
dtypes: datetime64[ns](1), float64(29), int64(21), object(3)
memory usage: 95.4+ MB
```

In [ ]:
```python
# we will want to stratify our results based on the following three characte
# C0A0 - US corporate index, H0A0 - US high yield index, ER00 - Euro index
# ICE BofA US Corporate Index (C0A0) tracks the performance of US dollar der
# The ICE BofA Merrill Lynch US High Yield Index (H0A0) tracks the performar


'''
Bond_age
Rating
Index
'''
```

Out[ ]: `'\nBond_age\nRating\nIndex\n'`

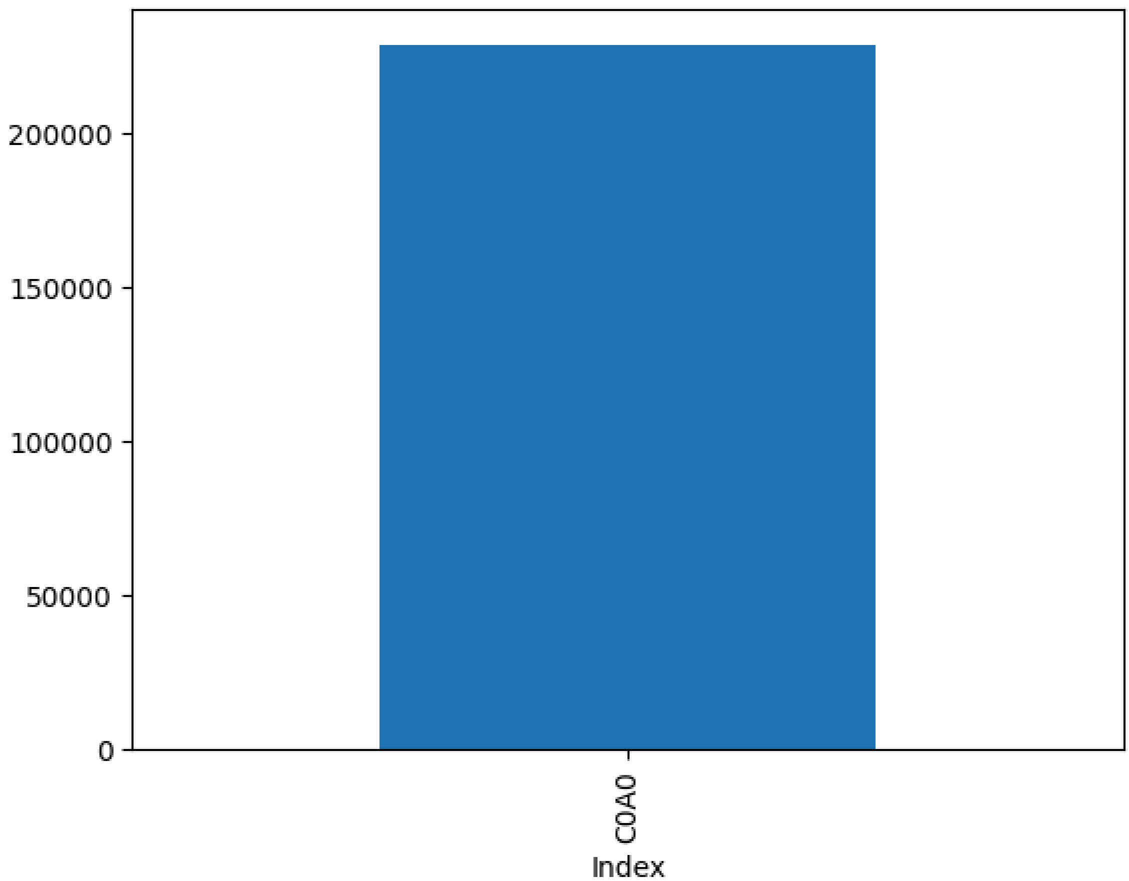
In [ ]:
```python
data['Rating'].value_counts().sort_index().plot(kind='bar')
```

Out[ ]: `<Axes: xlabel='Rating'>`



In [ ]:
```python
data['Index'].value_counts().plot(kind='bar')
```

Out[ ]: `<Axes: xlabel='Index'>`



In [ ]:
```
bond_characteristics = pd.DataFrame(index=data.index, columns=['Investment_G
bond_characteristics['Investment_Grade'] = np.where(data['Index']=='C0A0', 1
bond_characteristics
```

Out[ ]:

Cusip	Date	Investment_Grade	High_Yield
001084AQ	2012-12-01	1	NaN
	2013-01-01	1	NaN
	2013-02-01	1	NaN
	2013-03-01	1	NaN
	2013-04-01	1	NaN
...	...	...	...
98978VAT	2023-01-01	1	NaN
	2023-02-01	1	NaN
	2023-03-01	1	NaN
	2023-04-01	1	NaN
	2023-05-01	1	NaN

228806 rows × 2 columns

In [ ]:

```
# Date, CUSIP, IG Indicator, HY Yield Indicator, NearMaturity Indicator (off
# Inner merge this feature dataset (should these be features in the model?)
# Then filter based on desired properties of the bonds
```

In [ ]:

```
## Need to extract bond characteristics we can attach to the returns and for
```

In [ ]:

```
from IPython.display import display

high_minus_low_performance = pd.DataFrame(columns=['window_size', 'lag', 'nu

for window_size, lag in window_sizes_lags:

    df = pd.read_csv(os.path.join(data_path, 'Forecasts', f'window_size_{win
    df.set_index(['Intercept', 'Num_Factors', 'Cusip', 'Date'], inplace=True
    df.sort_index(inplace=True)
    df = df[['Returns_forecast', 'Excess_returns']].unstack(['Intercept', 'N
    df = df.swaplevel(0,1,axis=1).swaplevel(1,2,axis=1).sort_index(axis=1)

    for num_factors in range(1, 6):
        for intercept in [False, True]:
            df.loc[:,idx[intercept, num_factors, 'decile']] = df.loc[:,idx[i
                                                                  .groupby('Da
                                                                  .apply(lambd


    df.sort_index(axis=1, inplace=True)
    ## Excess Returns are in % format

    # This is for a given lag period and a given window size
    for num_factors in range(1, 6):
        for intercept in [False, True]:
```

```
            port_eq_weight_decile_rets = df.loc[:,idx[intercept, num_factors
            # expected returns from long-short portfolio where we long the "
            port_eq_weight_decile_rets['high_minus_low'] = port_eq_weight_de

            port_stats = port_eq_weight_decile_rets.describe().T # none of t

            sharpe_annualized = (port_stats.loc['high_minus_low', 'mean'] /
            t_stat = (port_stats.loc['high_minus_low', 'mean'] / port_stats.
            mean_annualized = port_stats.loc['high_minus_low', 'mean'] * 12

            high_minus_low_performance = pd.concat([high_minus_low_performan
                                                    pd.DataFrame([window_siz
                                                    axis=0, ignore_index=Tru

high_minus_low_performance
```

	window_size	lag	num_factors	intercept	mean_annualized	sharpe_annua
0	60	1	1	False	0.302098	0.87
1	60	1	1	True	0.521045	1.49
2	60	1	2	False	0.239012	0.6
3	60	1	2	True	0.527101	1.50
4	60	1	3	False	0.245555	0.70
5	60	1	3	True	0.539449	1.54
6	60	1	4	False	0.44649	1.27
7	60	1	4	True	0.529272	1.51
8	60	1	5	False	0.583685	1.65
9	60	1	5	True	0.497862	1.40
10	60	5	1	False	0.282873	0.81
11	60	5	1	True	0.586032	1.65
12	60	5	2	False	0.219063	0.61
13	60	5	2	True	0.596877	1.70
14	60	5	3	False	0.289334	0.81
15	60	5	3	True	0.563908	1.58
16	60	5	4	False	0.357771	1.01
17	60	5	4	True	0.562652	1.60
18	60	5	5	False	0.525998	1.47
19	60	5	5	True	0.547365	1.5
20	60	10	1	False	0.321879	0.92
21	60	10	1	True	0.653399	1.85
22	60	10	2	False	0.366895	1.01
23	60	10	2	True	0.591699	1.66
24	60	10	3	False	0.404573	1.14
25	60	10	3	True	0.657499	1.8
26	60	10	4	False	0.446189	1.25
27	60	10	4	True	0.647286	1.83
28	60	10	5	False	0.508847	1.40
29	60	10	5	True	0.515261	1.44
30	60	15	1	False	0.356686	1.02
31	60	15	1	True	0.840304	2.38
32	60	15	2	False	0.300655	0.81

	window_size	lag	num_factors	intercept	mean_annualized	sharpe_annua
33	60	15	2	True	0.893372	2.48
34	60	15	3	False	0.507798	1.42
35	60	15	3	True	0.706242	1.96
36	60	15	4	False	0.63903	1.77
37	60	15	4	True	0.779858	2.18
38	60	15	5	False	0.63892	1.78
39	60	15	5	True	0.782578	2.19
40	60	20	1	False	0.358727	1.03
41	60	20	1	True	0.837043	2.35
42	60	20	2	False	0.292055	0.79
43	60	20	2	True	0.749644	2.05
44	60	20	3	False	0.523505	1.47
45	60	20	3	True	0.573319	1.60
46	60	20	4	False	0.607896	1.69
47	60	20	4	True	0.593697	1.64
48	60	20	5	False	0.600014	1.6
49	60	20	5	True	0.600336	1.62
50	60	25	1	False	0.308761	0.89
51	60	25	1	True	0.719577	1.97
52	60	25	2	False	0.487269	1.3
53	60	25	2	True	0.47871	1.28
54	60	25	3	False	0.512183	1.37
55	60	25	3	True	0.492054	1.3
56	60	25	4	False	0.538372	1.45
57	60	25	4	True	0.597708	1.60
58	60	25	5	False	0.527374	1.41
59	60	25	5	True	1.754848	4.5
60	72	1	1	False	0.502308	1.36
61	72	1	1	True	0.548026	1.4
62	72	1	2	False	0.288511	0.78
63	72	1	2	True	0.469313	1.27
64	72	1	3	False	0.487052	1.31
65	72	1	3	True	0.511342	1.38

	window_size	lag	num_factors	intercept	mean_annualized	sharpe_annua
66	72	1	4	False	0.571686	1.54
67	72	1	4	True	0.502137	1.35
68	72	1	5	False	0.601465	1.62
69	72	1	5	True	0.556216	1.50
70	72	5	1	False	0.460345	1.24
71	72	5	1	True	0.63689	1.72
72	72	5	2	False	0.28217	0.7
73	72	5	2	True	0.584608	1.58
74	72	5	3	False	0.401383	1.08
75	72	5	3	True	0.681737	1.84
76	72	5	4	False	0.525526	1.42
77	72	5	4	True	0.648486	1.75
78	72	5	5	False	0.568864	1.5
79	72	5	5	True	0.474119	1.2
80	72	10	1	False	0.471805	1.27
81	72	10	1	True	0.777382	2.1
82	72	10	2	False	0.578961	1.56
83	72	10	2	True	0.797364	2.15
84	72	10	3	False	0.649948	1.76
85	72	10	3	True	0.820734	2.22
86	72	10	4	False	0.782235	2.11
87	72	10	4	True	0.758646	2.05
88	72	10	5	False	0.729436	1.97
89	72	10	5	True	0.691612	1.86
90	72	15	1	False	0.514791	1.39
91	72	15	1	True	0.925277	2.46
92	72	15	2	False	0.568782	1.51
93	72	15	2	True	0.991172	2.63
94	72	15	3	False	0.768662	2.03
95	72	15	3	True	0.804348	2.15
96	72	15	4	False	0.883071	2.33
97	72	15	4	True	0.996197	2.61
98	72	15	5	False	0.832475	2.18

	window_size	lag	num_factors	intercept	mean_annualized	sharpe_annua
99	72	15	5	True	0.939214	2.47
100	72	20	1	False	0.476625	1.29
101	72	20	1	True	0.835622	2.1
102	72	20	2	False	0.428872	1.12
103	72	20	2	True	0.80292	2.12
104	72	20	3	False	0.550679	1.44
105	72	20	3	True	0.690621	1.80
106	72	20	4	False	0.634626	1.65
107	72	20	4	True	0.755399	1.97
108	72	20	5	False	0.752008	1.97
109	72	20	5	True	0.86175	2.29
110	72	25	1	False	0.452971	1.22
111	72	25	1	True	0.706253	1.87
112	72	25	2	False	0.37769	0.95
113	72	25	2	True	0.567238	1.48
114	72	25	3	False	0.459561	1.21
115	72	25	3	True	0.62438	1.66
116	72	25	4	False	0.565785	
117	72	25	4	True	0.630214	1.6
118	72	25	5	False	0.623097	1.62
119	72	25	5	True	0.562059	1.46
120	96	1	1	False	0.180136	0.41
121	96	1	1	True	0.401524	0.92
122	96	1	2	False	0.095254	0.2
123	96	1	2	True	0.367369	0.84
124	96	1	3	False	0.442888	1.02
125	96	1	3	True	0.375748	0.86
126	96	1	4	False	0.273491	0.63
127	96	1	4	True	0.395893	0.91
128	96	1	5	False	0.268361	0.61
129	96	1	5	True	0.443446	1.02
130	96	5	1	False	0.173708	0.40
131	96	5	1	True	0.525207	1.21

	window_size	lag	num_factors	intercept	mean_annualized	sharpe_annua
132	96	5	2	False	0.112867	0.26
133	96	5	2	True	0.504411	1.16
134	96	5	3	False	0.348729	0.80
135	96	5	3	True	0.526646	1.21
136	96	5	4	False	0.309875	0.71
137	96	5	4	True	0.511623	1.18
138	96	5	5	False	0.348338	0.80
139	96	5	5	True	0.527028	1.2
140	96	10	1	False	0.165403	0.38
141	96	10	1	True	0.663761	1.53
142	96	10	2	False	0.357267	0.82
143	96	10	2	True	0.559201	1.29
144	96	10	3	False	0.434831	1.00
145	96	10	3	True	0.572551	1.32
146	96	10	4	False	0.438126	1.0
147	96	10	4	True	0.584081	1.34
148	96	10	5	False	0.47017	1.0
149	96	10	5	True	0.577981	1.33
150	96	15	1	False	0.197154	0.45
151	96	15	1	True	0.786095	1.81
152	96	15	2	False	0.064223	0.14
153	96	15	2	True	0.718092	1.65
154	96	15	3	False	0.417671	0.96
155	96	15	3	True	0.68381	1.57
156	96	15	4	False	0.523012	1.20
157	96	15	4	True	0.747697	1.72
158	96	15	5	False	0.536179	1.23
159	96	15	5	True	0.734525	1.69
160	96	20	1	False	0.16565	0.38
161	96	20	1	True	0.73514	1.69
162	96	20	2	False	0.176946	0.4
163	96	20	2	True	0.630932	1.45
164	96	20	3	False	0.323982	0.74

	window_size	lag	num_factors	intercept	mean_annualized	sharpe_annua
165	96	20	3	True	0.612454	1.41
166	96	20	4	False	0.431603	0.99
167	96	20	4	True	0.610312	1.40
168	96	20	5	False	0.47615	1.09
169	96	20	5	True	0.632914	1.46
170	96	25	1	False	0.144626	0
171	96	25	1	True	0.603363	1.39
172	96	25	2	False	0.215162	0.49
173	96	25	2	True	0.549657	1.26
174	96	25	3	False	0.316502	0.73
175	96	25	3	True	0.517286	1.1
176	96	25	4	False	0.432606	0.99
177	96	25	4	True	0.523895	1.20
178	96	25	5	False	0.377824	0.87
179	96	25	5	True	0.495985	1.14
180	120	1	1	False	0.029385	0.05
181	120	1	1	True	0.170186	0.31
182	120	1	2	False	-0.084165	-0.15
183	120	1	2	True	0.130461	0.23
184	120	1	3	False	0.182927	0.33
185	120	1	3	True	0.175658	0.32
186	120	1	4	False	0.131186	0.23
187	120	1	4	True	0.213966	0.39
188	120	1	5	False	1.580359	2.88
189	120	1	5	True	0.210588	0.38
190	120	5	1	False	0.013184	0.0
191	120	5	1	True	0.378946	0.69
192	120	5	2	False	-0.056493	-0.10
193	120	5	2	True	0.320005	0.58
194	120	5	3	False	-0.049063	-0.08
195	120	5	3	True	0.257753	0.47
196	120	5	4	False	0.127437	0.23
197	120	5	4	True	0.336369	0.61

	window_size	lag	num_factors	intercept	mean_annualized	sharpe_annua
198	120	5	5	False	0.189659	0.34
199	120	5	5	True	0.350487	0
200	120	10	1	False	0.011844	0.02
201	120	10	1	True	0.527811	0.96
202	120	10	2	False	0.347866	0.63
203	120	10	2	True	0.62373	1.1
204	120	10	3	False	0.279384	0.51
205	120	10	3	True	0.399506	0.72
206	120	10	4	False	0.270043	0.49
207	120	10	4	True	0.417722	0.76
208	120	10	5	False	0.209412	0.38
209	120	10	5	True	1.395815	2.54
210	120	15	1	False	0.047934	0.08
211	120	15	1	True	0.82766	1.51
212	120	15	2	False	0.270741	0.49
213	120	15	2	True	0.80219	1.46
214	120	15	3	False	0.235154	0.42
215	120	15	3	True	0.603038	1.10
216	120	15	4	False	0.488765	0.89
217	120	15	4	True	0.597738	1.09
218	120	15	5	False	0.297179	0.54
219	120	15	5	True	1.697479	3.09
220	120	20	1	False	0.003426	0.00
221	120	20	1	True	0.621146	1.13
222	120	20	2	False	-0.105837	-0.19
223	120	20	2	True	0.484994	0.88
224	120	20	3	False	0.216502	0.39
225	120	20	3	True	0.409156	0.74
226	120	20	4	False	0.265756	0.48
227	120	20	4	True	0.432166	0.78
228	120	20	5	False	0.200954	0.36
229	120	20	5	True	2.088477	3.81
230	120	25	1	False	0.010691	0.01

	window_size	lag	num_factors	intercept	mean_annualized	sharpe_annua
231	120	25	1	True	0.441765	0.8
232	120	25	2	False	-0.224767	-0.41
233	120	25	2	True	0.456037	0.83
234	120	25	3	False	0.113506	0.20
235	120	25	3	True	0.431559	0.78
236	120	25	4	False	0.122172	0.22
237	120	25	4	True	0.423515	0.77
238	120	25	5	False	0.101811	0.18
239	120	25	5	True	1.906729	3.48

In [ ]:

```
pd.set_option('display.max_rows', None)
pd.set_option('display.max_columns', None)

high_minus_low_performance = high_minus_low_performance.set_index(['window_s
                                                          .sort_index().unstack
                                                          .swaplevel(0,2, axis=
```

		window_size				
		lag	**1**	**5**	**10**	
intercept	**num_factors**					
False	**1**	**mean_annualized**	0.302098	0.282873	0.321879	0.3566
		sharpe_annualized	0.872081	0.816583	0.929183	1.0296
		t_stat	3.858787	3.62039	4.08018	4.2582
	2	**mean_annualized**	0.239012	0.219063	0.366895	0.3006
		sharpe_annualized	0.68651	0.619604	1.015886	0.8141
		t_stat	2.782549	2.662802	4.821137	3.8215
	3	**mean_annualized**	0.245555	0.289334	0.404573	0.5077
		sharpe_annualized	0.701731	0.818359	1.144305	1.4212
		t_stat	2.631411	3.280107	3.805107	4.1559
	4	**mean_annualized**	0.44649	0.357771	0.446189	0.639
		sharpe_annualized	1.275951	1.011928	1.255424	1.7789
		t_stat	3.836153	3.435013	4.086548	4.9122
	5	**mean_annualized**	0.583685	0.525998	0.508847	0.638
		sharpe_annualized	1.650911	1.479978	1.408933	1.7882
		t_stat	4.74969	4.958605	4.522094	5.2731
True	**1**	**mean_annualized**	0.521045	0.586032	0.653399	0.8403
		sharpe_annualized	1.496588	1.657548	1.857693	2.3890
		t_stat	4.079739	5.231177	5.474282	6.1261
	2	**mean_annualized**	0.527101	0.596877	0.591699	0.8933
		sharpe_annualized	1.506318	1.705718	1.664837	2.4870
		t_stat	4.201353	5.310752	5.475806	6.696
	3	**mean_annualized**	0.539449	0.563908	0.657499	0.7062
		sharpe_annualized	1.549448	1.586644	1.86935	1.9660
		t_stat	4.237788	5.277266	5.902005	5.1401
	4	**mean_annualized**	0.529272	0.562652	0.647286	0.7798
		sharpe_annualized	1.512521	1.607912	1.830801	2.1826
		t_stat	3.989369	5.02547	5.130198	5.2524
	5	**mean_annualized**	0.497862	0.547365	0.515261	0.7825
		sharpe_annualized	1.408165	1.53197	1.449767	2.1902
		t_stat	4.036046	4.949295	4.315108	5.5009

In [ ]:
```
high_minus_low_performance#.T
```

	intercept				
	num_factors			1	
		mean_annualized	sharpe_annualized	t_stat	mea
window_size	lag				
60	1	0.302098	0.872081	3.858787	
	5	0.282873	0.816583	3.62039	
	10	0.321879	0.929183	4.08018	
	15	0.356686	1.029663	4.258282	
	20	0.358727	1.035555	4.274022	
	25	0.308761	0.891317	3.764746	
72	1	0.502308	1.360256	6.734437	
	5	0.460345	1.246621	6.205949	
	10	0.471805	1.277653	6.471617	
	15	0.514791	1.394061	6.633676	
	20	0.476625	1.290706	6.176397	
	25	0.452971	1.226652	5.845735	
96	1	0.180136	0.416006	2.679547	
	5	0.173708	0.401161	2.634742	
	10	0.165403	0.381981	2.596631	
	15	0.197154	0.455307	2.90578	
	20	0.16565	0.382551	2.453073	
	25	0.144626	0.334	2.140482	
120	1	0.029385	0.053649	0.474259	
	5	0.013184	0.02407	0.218112	
	10	0.011844	0.021624	0.197162	
	15	0.047934	0.087515	0.731489	
	20	0.003426	0.006254	0.054199	
	25	0.010691	0.019519	0.17347	

In [ ]:
```
window_size = 120
lag = [1,5,10,15,20,25]

intercept = True
num_factors = 3
```

```
high_minus_low_performance.loc[idx[window_size, lag], idx[intercept, num_fac
```

window_size	lag	mean_annualized	sharpe_annualized	t_stat
120	**1**	0.175658	0.320706	1.390613
	5	0.257753	0.470591	2.568523
	10	0.399506	0.729395	3.747632
	15	0.603038	1.100991	4.557753
	20	0.409156	0.747014	3.726411
	25	0.431559	0.787916	3.689114

In [ ]:
```
pd.reset_option('display.max_rows')
pd.reset_option('display.max_columns')
```

[redacted]

Variants to the Model Formulation

- Using IPCA to Explan Delta-Hedged Option Returns

Regularization

Robustness

[redacted]

TODO

- ~~Implement IPCA with student dataset and match student results with the IPCA module~~
- Do some debugging & compare with
    - approximate solution when characteristics are cross-sectionally orthogonal
    - IPCA package
- Go back to raw files and reconstruct data at daily frequency

    - need to get comfortable with raw files and derived CaR files, as well as other processing steps
    - basic EDA to see that daily dataset is clean
- With daily data, we need to run IPCA
    - we might need to do more analysis and deal with serial correlation in the residuals (which will impact the risk model)
    - we can handle this by doing PCA on the residuals and using the PCs in the regression, or by using lags of certain variables in the regression
    - we're trying to apply this model for trading or market making

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- To what degree do we want to change the data, i.e. excess returns
    - how much can your return beat treasuries?
    - how much can your return beat the market (corporate bond index)?
    - how do you do the matching of the bonds?
        - duration of LQD ETF $\neq$ duration of corp bonds
        - use the OAS change in corp bond vs OAS change in ETF to do the matching

---

- Model the problem as linear neural networks for easy generalization to nonlinear deep learning.

▱

[redacted]

Inference

Bootstrapped Standard Errors

Background Notes

- Factor Models Notes: Different ways of interpreting and estimating the factors and the loadings
  - For a model where the factors are latent and need to be estimated, the primary methods of estimation (extraction) are **factor analysis** and **principal component analysis**.
- How to estimate factor exposure, risk premia, and discount factors
  - *“In a factor model, the total variance of an asset can be decomposed into a systematic risk component driven by covariances with the factors and a component that is idiosyncratic to the asset.*
  - There are many factor modeling strategies available that differ in their assumptions about whether or not factors and their exposures are assumed known, and whether the model uses a conditional or unconditional risk decomposition.
  - For a static factor model, if factors are known, we can estimate factor exposures via asset-by-asset time-series regressions…
  - For a conditional factor model, if factors are latent, but exposures are observable…we can estimate factors by cross-sectional regressions at each time point…This approach is most commonly used for individual stocks, for which their loadings can be proxied by firm characteristics. It is convenient for the cross-sectional regression to accommodate time-varying characteristics…
  - If neither factors nor loadings are known, we can resort to principal components analysis to extract latent factors and their loadings. Principal components analysis can identify factors and their loadings up to some unknown linear transformation…[and] extracts information about latent factors solely from realized return covariances…This decomposition yields a pair of estimates of factor innovations and exposures…Said differently, a rotation of factors and an inverse rotation of betas leaves model fits exactly unchanged. While allowing for latent factors and exposures can add great flexibility to a research project, this rotation indeterminacy makes it difficult to interpret the factors in a latent factor model…The principal components approach is also applicable if some but not all factors are observable.
  - A limitation of principal components analysis is that it only applies to static factor models. It also lacks the flexibility to incorporate other data beyond returns. To address both issues [one can] estimate the conditional factor model [in form of] instrumental principal components analysis…Given conditional betas, factors are estimated from cross-

section regressions of returns on betas [which] accommodates a potentially large number of characteristics…Conditional betas can be recovered from panel regressions of returns onto characteristics interacted with factors.

    - Deep learning [can be applied] to return factor models [through] a conditional autoencoder to explicitly account for the risk-return trade-off. The machine learning literature has long recognized the close connection between autoencoders and principal components analysis. However, [one can] introduce additional conditioning information into the autoencoder specification. The autoencoder allows betas to depend on stock characteristics in a more realistic, nonlinear way…[The figure below] illustrates the model's basic structure…On the left side of the network, factor loadings are a nonlinear function of covariates (e.g., firm characteristics), while the right side of the network models factors as portfolios of individual stock returns."

- Kent Daniel's Discussion of: Characteristics Are Covariances: A Unified Model of Risk and Return