

Local Action Diagrams

**Super awesome package for local action
diagrams.**

0.1

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Chapter 1

Introduction

A local action diagram $\Delta = (\Gamma, (G(v)), (X_a))$ consists of:

- a directed graph Γ ,
- a closed permutation group $G(v)$ for each $v \in V(\Gamma)$, and
- a set X_a for each $a \in A(\Gamma)$ such that each X_a is disjoint and X_a is an orbit of the action of $G(o(a))$ on $X_v := \bigsqcup_{a \in o^{-1}(v)} X_a$.

The definition of digraph used is different to that of the `Digraphs` package. For our purposes, a digraph Γ consists of:

- a vertex set V ,
- an arc set A ,
- a map $o : A \rightarrow V$ assigning each arc to an *origin* vertex, and
- a bijection $r : A \rightarrow A$ (denoted by $a \mapsto \bar{a}$) such that $r^2 = \text{id}$.

The bijection r defines a reverse arc for each arc of the graph. This is more specific than the definition in the `digraphs` package which does not require a reverse mapping.

The local action diagrams package provides a category for local action diagrams. It is built in the `IsDigraph` category from the `Digraphs` package.

Chapter 2

Creating Local Action Diagrams

2.1 Creating Local Action Diagrams

2.1.1 IsLocalActionDiagram (for IsDigraph)

▷ `IsLocalActionDiagram(lad)` (filter)

Returns: true if `lad` is of the category `IsLocalActionDiagram` and false otherwise.

Every local action diagram belongs to the `IsLocalActionDiagram` category. Every local action diagram is immutable.

2.1.2 Constructing From Data

▷ `LocalActionDiagramFromData(D, v_labels, e_labels, rev)` (operation)

▷ `LocalActionDiagramFromDataNC(D, v_labels, e_labels, rev)` (operation)

Returns: A local action diagram.

Constructs a local action diagram, checking that the arguments given are a valid local action diagram. The argument D is a digraph and rev must be a compatible involution on the edges of D . The argument v_labels is a list of vertex labels such that $v_labels[i]$ is the group labelling vertex i of D .

The argument e_labels is a list of edge labels. The edges of D are stored in lexicographical order and $edge_labels[i]$ is the set labelling edge i of D (when sorted in lexicographical order).

The NC variant of the operation does not check that the arguments given are a valid local action diagram.

2.1.3 LocalActionDiagramUniversalGroup (for IsPermGroup)

▷ `LocalActionDiagramUniversalGroup(arg)` (operation)

Constructs a local action diagram corresponding to the Burger-Moses group $U(F)$.

Chapter 3

Local Action Diagram Attributes and Operations

3.1 Local Action Diagram Attributes

3.1.1 LocalActionDiagramVertexLabels (for IsLocalActionDiagram)

▷ LocalActionDiagramVertexLabels(*lad*) (attribute)

Returns: A list of groups.

Returns the groups labelling the vertices of *lad*. Entry *i* of the list corresponds to the group labelling vertex *i* of the digraph.

3.1.2 LocalActionDiagramEdges (for IsLocalActionDiagram)

▷ LocalActionDiagramEdges(*lad*) (attribute)

Returns: A list of lists.

Returns a list of edges of *lad*. Each edge is stored as a list [*i*, *j*] where *i* is the origin vertex and *j* is the terminus vertex. The list is stored in lexicographical order.

3.1.3 LocalActionDiagramEdgeLabels (for IsLocalActionDiagram)

▷ LocalActionDiagramEdgeLabels(*lad*) (attribute)

Returns: A list of lists.

Returns the edge labels of *lad*. Entry *i* of the list corresponds to the label of edge *i* of the digraph.

3.1.4 LocalActionDiagramEdgeReversal (for IsLocalActionDiagram)

▷ LocalActionDiagramEdgeReversal(*lad*) (attribute)

Returns: A permutation.

Returns the reversal mapping of the local action digram *lad*.

3.2 Local Action Diagram Operations

3.2.1 LocalActionDiagramScopos (for IsLocalActionDiagram)

▷ LocalActionDiagramScopos(*lad*) (attribute)

Returns: A list of lists.

Returns a list of all scopos of *lad*. Each entry of the list is a list of edges in the scopo. This list will always contain the empty scopo [].

3.2.2 LocalActionDiagramGroupType (for IsLocalActionDiagram)

▷ LocalActionDiagramGroupType(*lad*) (attribute)

Returns: A string.

Returns the group type the local action diagram corresponds to. This is either "Fixed Vertex", "Edge Inversion", "Lineal", "Focal", or "General". Note that all "Horocyclic" local action diagrams have infinitely many vertices and so can never be the type returned by this function.

3.2.3 LocalActionDiagramIsDiscrete (for IsLocalActionDiagram)

▷ LocalActionDiagramIsDiscrete(*lad*) (attribute)

Returns: true or false.

Returns true if *lad* corresponds to a discrete group and false otherwise.

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