Introduction to sample size and power calculations

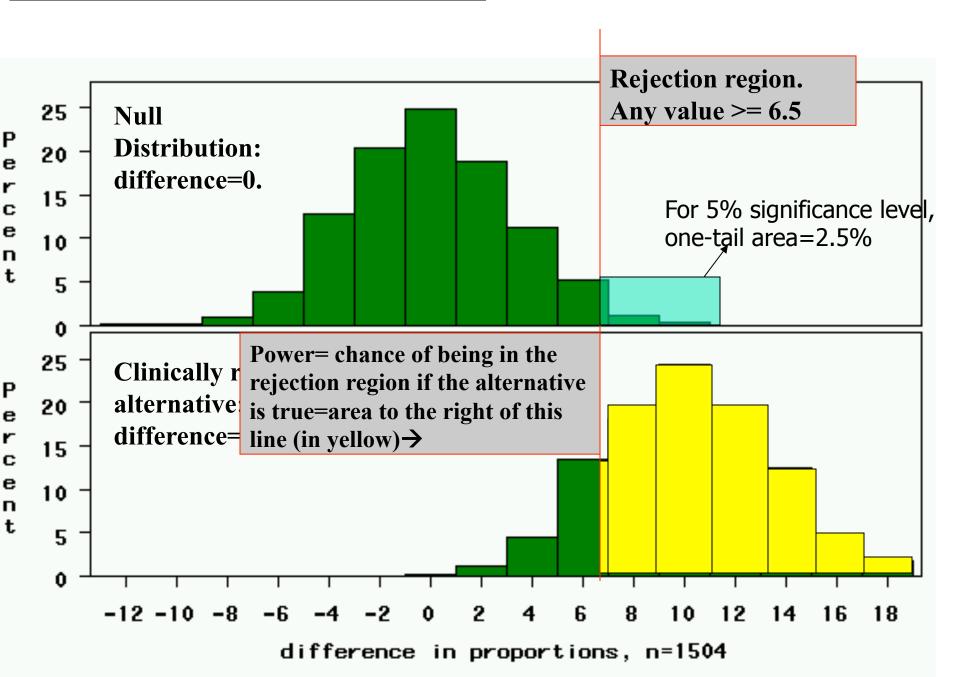
Power

- The probability of rejecting a false null hypothesis (H₀).
- The probability of obtaining a value of t that is large enough to reject H₀ when H₀ is actually false
- We always test the null hypothesis against an alternative/research hypothesis



Can we quantify how much power we have for given sample sizes?

study 1: 263 cases, 1241 controls



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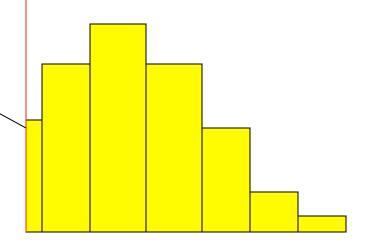
Rejection region. Any value >= 6.5

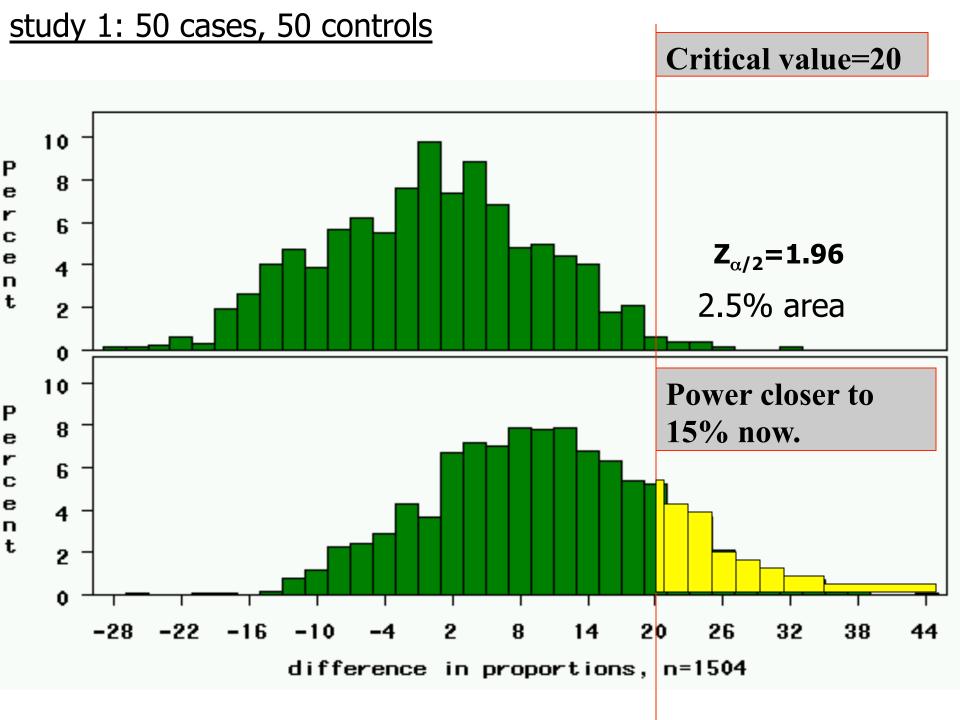
Power here:

$$P(Z > \frac{6.5-10}{3.3}) =$$

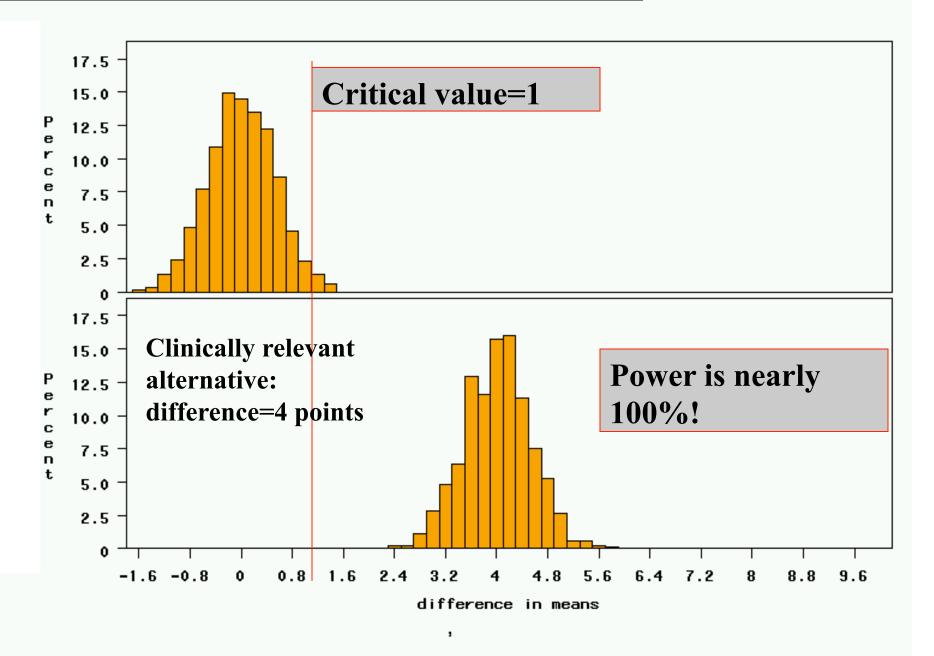
$$P(Z > -1.06) = 85\%$$

Power= chance of being in the rejection region if the alternative is true=area to the right of this line (in yellow)

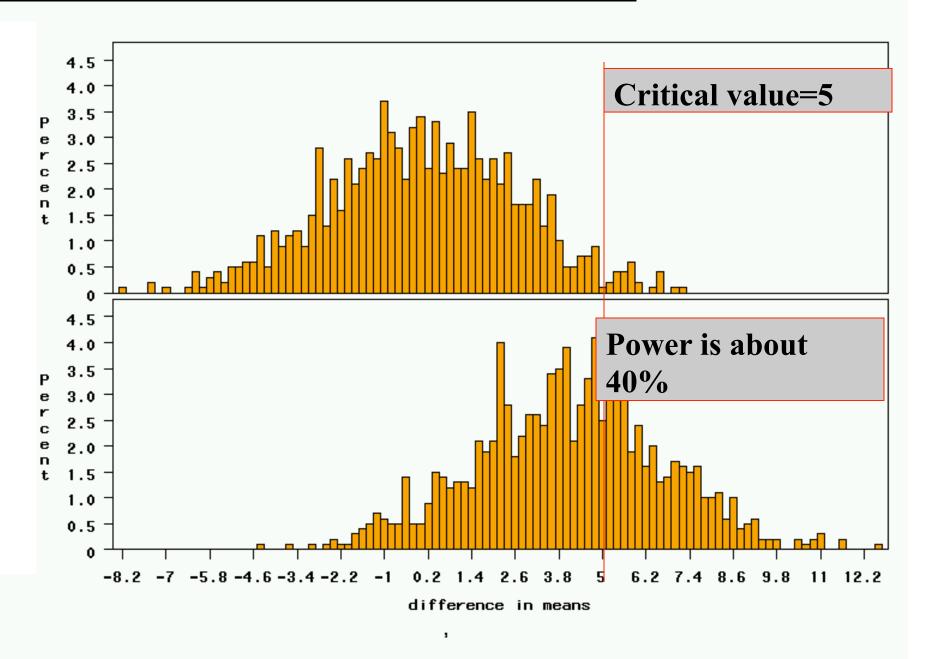




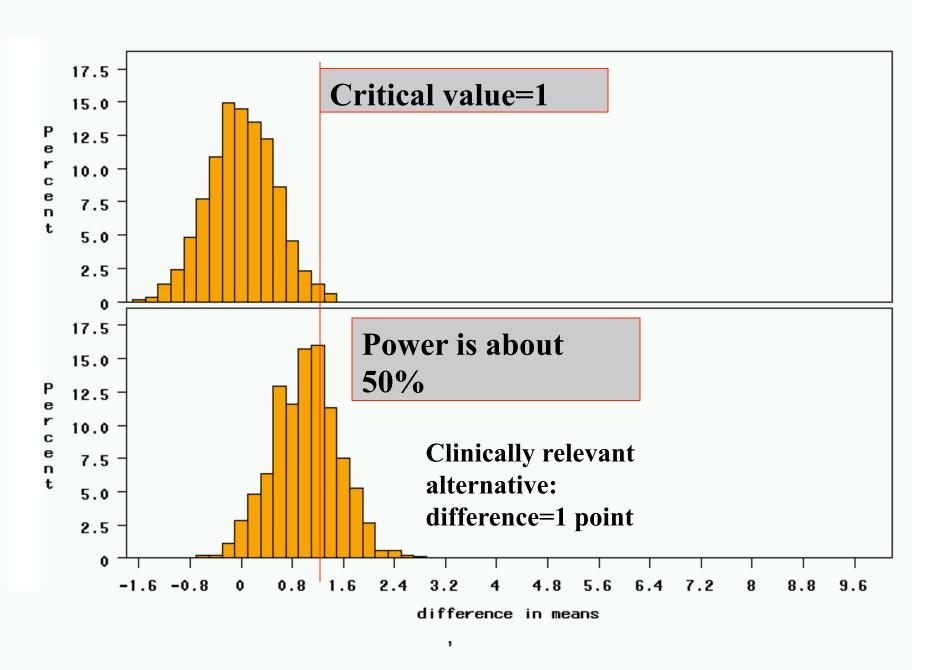
Study 2: 18 treated, 72 controls, STD DEV = 2



Study 2: 18 treated, 72 controls, STD DEV=10



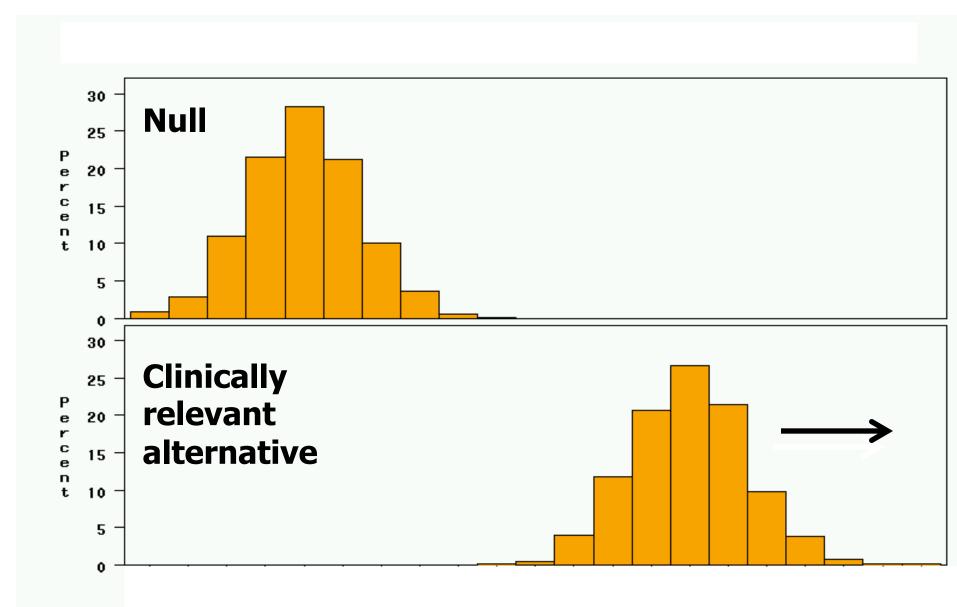
Study 2: 18 treated, 72 controls, effect size=1.0



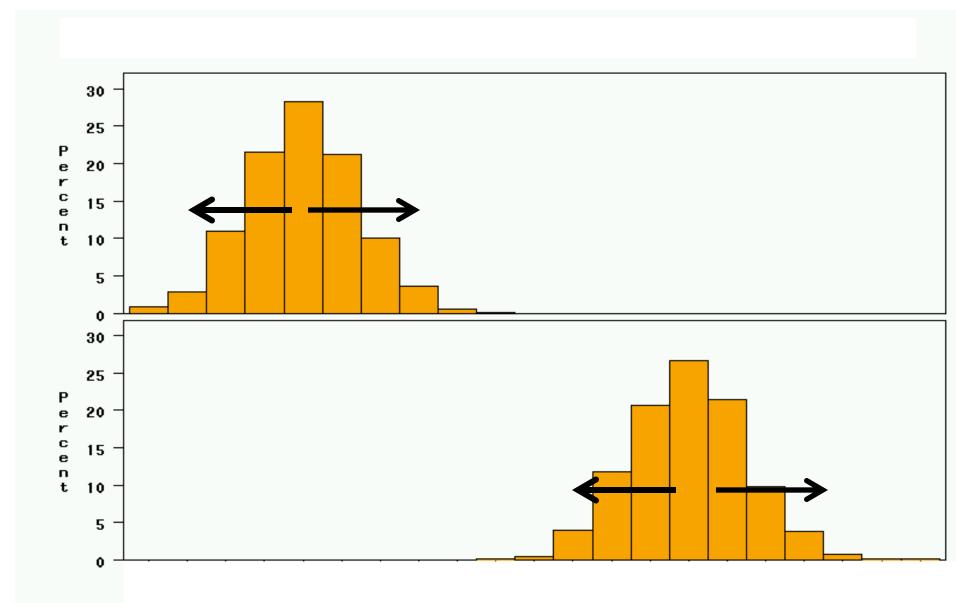
Factors Affecting Power

- 1. Size of the effect ↑
- 2. Standard deviation of the characteristic ↓
- 3. Bigger sample size ↑
- 4. Significance level desired ↓

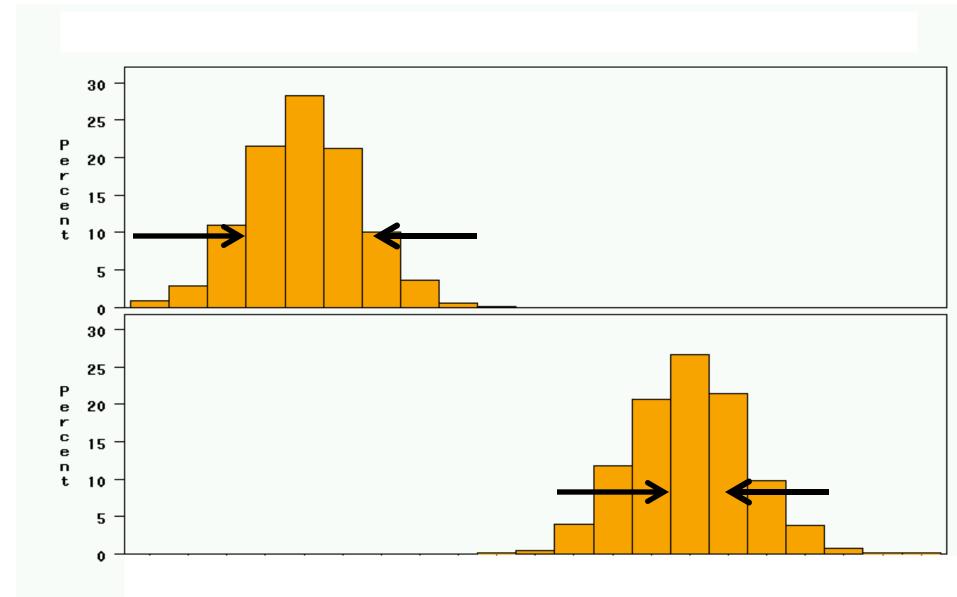
1. Bigger difference from the null mean



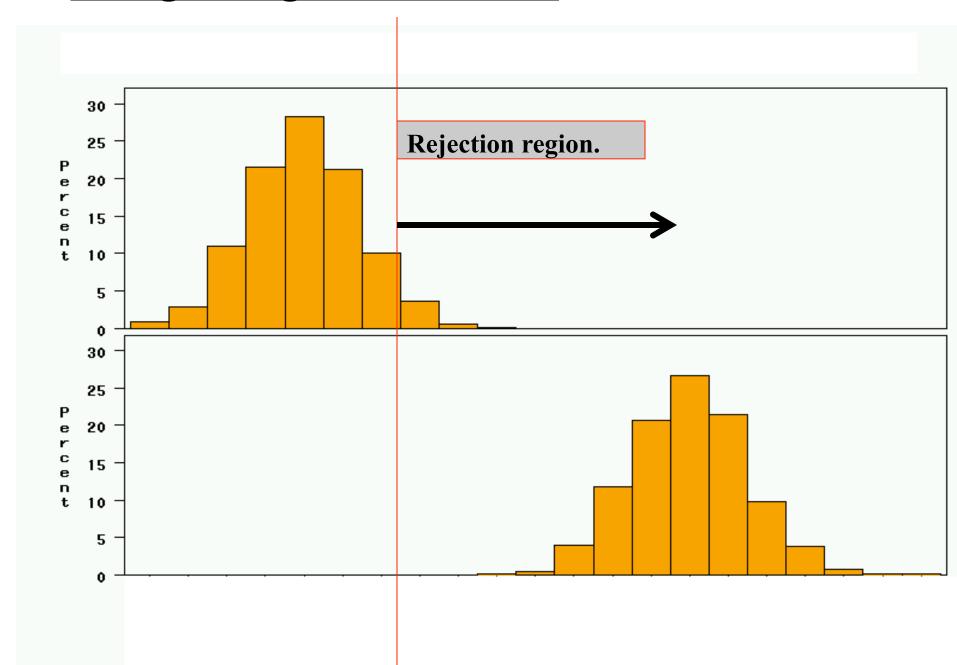
2. Bigger standard deviation



3. Bigger Sample Size



4. Higher significance level



Sample size calculations

 Based on these elements, you can write a formal mathematical equation that relates power, sample size, effect size, standard deviation, and significance level...

WE WILL DERIVE THESE FORMULAS FORMALLY SHORTLY

Simple formula for difference in proportions

Sample size in each group (assumes equal sized groups)

Represents the **desired power** (typically .84 for 80% power).

$$n = \frac{2(\bar{p})(1-\bar{p})(Z_{\beta} + Z_{\alpha/2})^{2}}{(p_{1}-p_{2})^{2}}$$

A measure of **variability** (similar to standard deviation)

Effect Size (the difference in proportions)

Represents the desired **level of statistical significance** (typically 1.96).



Example: How many people would you need to sample in each group to achieve power of 80%?

$$n = \frac{2\sigma^2 (Z_{\beta} + Z_{\alpha/2})^2}{(d^*)^2} = \frac{100(2)(.84 + 1.96)^2}{(3)^2} = 174$$

174/group; 348 altogether

In practice

- You need:
 - Alpha level (0.05)
 - Sought power (usually 80%)
 - Standard deviation
 - Estimate based on a pilot study or past literature
 - Effect size!
- https://clincalc.com/stats/samplesize.aspx

Effect size

- Difference between means
- For power, what is the smallest effect size of scientific interest?
 - Ex: Is a 1% decrease in anxiety meaningful? 10%?
 - Ex: Is a 40ms change in reaction time meaningful? 1ms?
- Subjective so be conservative

In class activity

- Calculate the sample size needed to answer
 - Does studying improve test performance?
 - Does playing on your phone while driving increase the number of accidents?
 - Does the treatment decrease anxiety?
- You need to come up with a reasonable standard deviation and effect size.