ELEN3013: SIGNALS AND SYSTEMS IIB - Tutorial 5

1. Find the N-point DFT of the following sequences $x[n]$:
   (a) $x[n] = \delta[n]$.
   (b) $x[n] = u[n] - u[n-N]$

2. Consider two sequences $x[n]$ and $h[n]$ of length 4 given by

$$\begin{aligned} x[n] &= cos\left(\frac{\pi}{2}n\right) \quad n = 0,1,2,3 \\ h[n] &= \left(\frac{1}{2}\right)^n \quad n = 0,1,2,3 \end{aligned}$$

   Calculate $y[n]$ by DFT.

3. Consider the finite length complex exponential sequence

$$x[n] = \begin{cases} e^{j\Omega_0 n} & 0 \leq n \leq N-1 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find the Fourier transform $X(\Omega)$ of $x[n]$.
   (b) Find the N-point DFT $X[k]$ of $x[n]$.

4. Show that if $x[n]$ is real, then its DFT $X[k]$ satisfies the relation

$$X[N-k] = X^*[k]$$

   where $*$ is the complex conjugate.

5. Show that

$$x[n] = \text{IDTF}\{X[k]\} = \frac{1}{N}\left[\text{DFT}\{X^*[k]\}\right]^*$$

   where $*$ is the complex conjugate.

6. Consider the sequence

$$x[n] = \{1, 1, -1, -1, -1, 1, 1, -1\}.$$

   Determine the DFT $X[k]$ of $x[n]$ using the decimation-in-time FFT algorithm.

7. (a) Using the DFT, estimate the Fourier transform of the continuous time signal

$$x(t) = e^{-t}u(t)$$

   Assume the total recording time of $x(t)$ is $T_1 = 10s$ and the highest frequency of $x(t)$ is $\omega_M = 100rad/s$
   (b) Let $X[k]$ be the DFT of the sampled sequence of $x(t)$. Compare the values of $X[0]$, $X[1]$ and $X[10]$ with the values of $X(0)$, $X(\Delta\omega)$ and $X(10\Delta\omega)$.

8. Consider a continuous time signal $x(t)$ that has been prefiltered by a lowpass filter with a cut-off frequency of 10kHz. The spectrum of $x(t)$ is estimated by use of the N-point DFT. The desired frequency resolution is 0.1Hz. Determine the required value of N(assuming a power of 2) and the necessary data length T_1.