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LECTURE NOTES

SIGNAL DEFINITION

→ A signal @ it's most general sense is just a collection of observations.

→ The idea of an observation essentially answers two questions

(i) what → space of phenomena of interest

(ii) where → observational space

DEFN :

→ A signal is a pairing of an observational space & a space of phenomena

Examples

① temperature @ every point in the room

② temperature @ every point in the room over time.

③ an image

Of-course we are ~~@ this particular pt in~~ going to restrict our-selves to ~~phenomena over~~ observations over time.

→ "pairing" can be formalized to a map i.e a rule that tells us an observation element to assign to any point in time.

Defn: TIME SIGNAL

is a map $X : T \longrightarrow S$ → universe of possible observational values

Example:

→ room temp over time : $X : T \longrightarrow [\ \vdots\ ]$ → temperature @ point in the room; → matrix

→ video $X : T \longrightarrow [\ \ ]$ → image

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$x : T \longrightarrow$ Voltage (voltage over time)

→ Another restriction: We will restrict ourselves to scalar observations i.e observations that can be described by a single number.

TYPES OF SIGNALS

→ we can categorise signals into different classes depending on the kind of space T & S are

(i) Continuous time - continuous valued signals both T & S is cont.

$$x : \mathbb{R} \longmapsto \mathbb{R}$$

(ii) Discrete time - continuous values signals

$$\cancel{x : \mathbb{R} \longmapsto \mathbb{N}} \quad x : \mathbb{N} \longmapsto \mathbb{R}$$

(iii) Continous time - discrete valued signal (quanta)

$$x : \mathbb{R} \longmapsto \mathbb{N}$$

(iv) Discrete time - Discrete valued signal

$$x : \mathbb{N} \longmapsto \mathbb{N}$$

a special case Digital signal

$$x : \mathbb{N} \longmapsto \{0, 1\}$$

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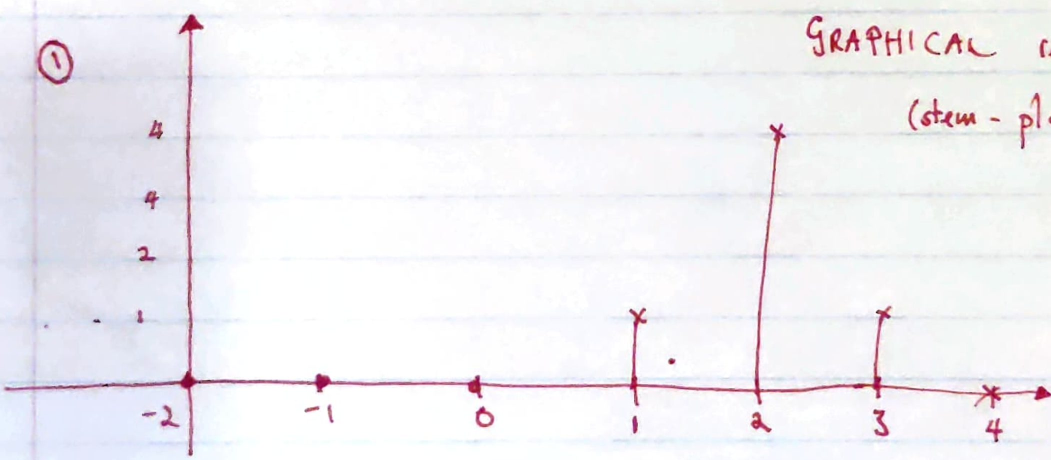
$$\begin{tikzcd}[column sep=huge, row sep=huge]
\overset{\text{II A}}{CT|CV} \arrow[r, bend left, "\text{sampling}"] \arrow[d, bend right, "\text{quantization}"'] & DT|CV \;\; \text{II B} \arrow[d, bend left, "\text{quantization}"] \\
CT|DT \arrow[r, bend right, "\text{sampling}"'] & \underset{\text{II B}}{DT|DT}
\end{tikzcd}$$

Representation of Discrete signals

1 GRAPHICAL representation (stem-plot)



2 Functional representation

$$x[n] = \begin{cases} 1 & \text{for } n = 1, 3 \\ 4 & \text{for } n = 2 \\ 0 & \text{else where} \end{cases}$$

3 Sequential Representation

$$x[n] = \{0, 0, 1, 4, 1, 1, 0, 0 \ldots\}$$

(4) Tabular

n	...	-2	-1	0	1	2	3	4	5	...
$x[n]$	..	0	0	0	1	4	1	0	0	..

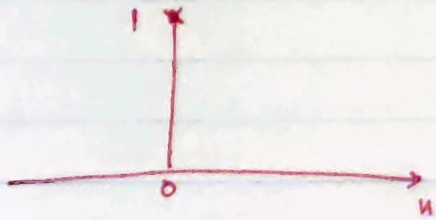
Tabular & Sequential data structures for computation.

SIGNAL MODELS

→ we are highlighting signals that we use as building blocks to represent any signal we want

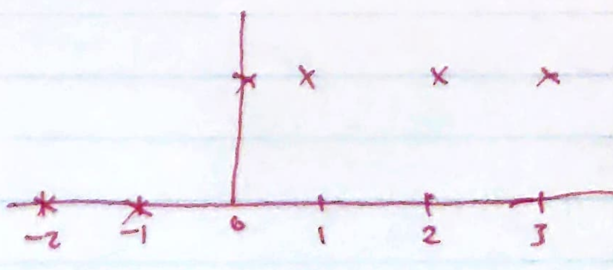
(1) Unit impulse function

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$



(2) Unit step function (switching function)

$$u(t) = \begin{cases} 1, & n \geq 0 \\ 0 & n < 0 \end{cases}$$

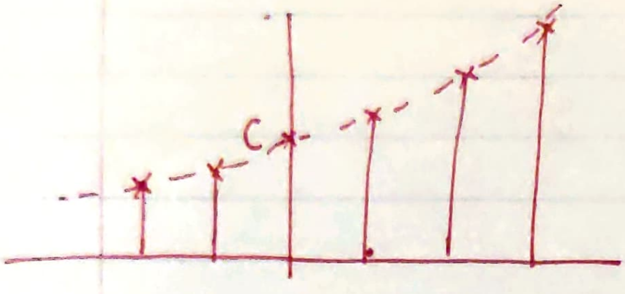


(3) Discrete exponential signal (Do sinusoids before this)

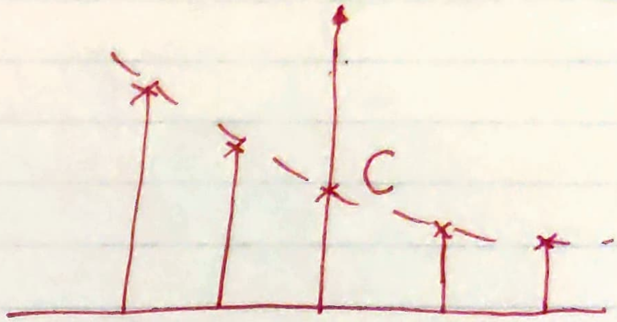
$$x[n] = Ca^n \qquad C, a \in \mathbb{C}$$

(4)

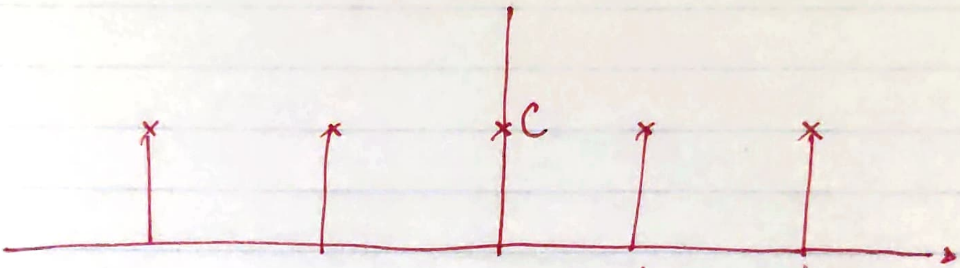
3a) assume C, a are both real.



$a > 1$

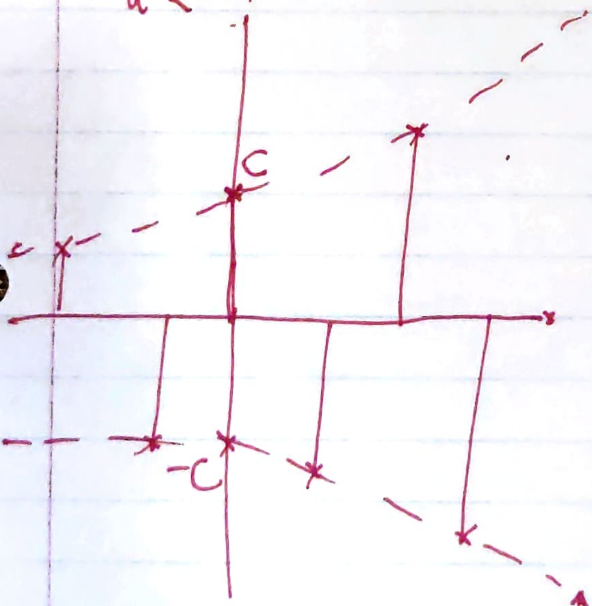


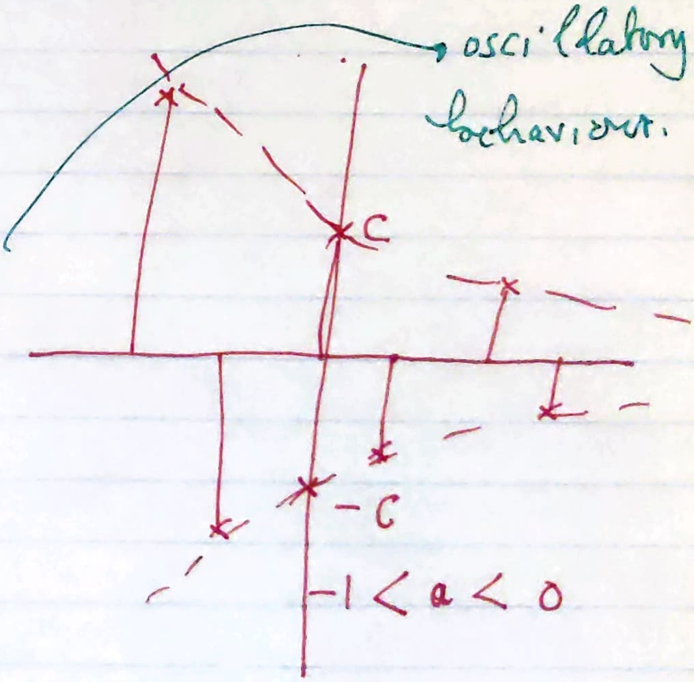
$0 < a < 1$



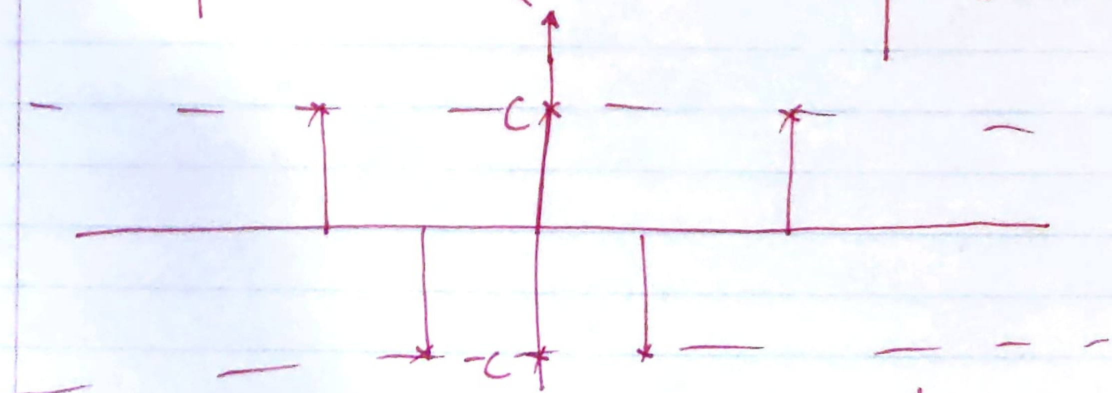
$a = 1$

$a < -1$





$-1 < a < 0$



$a = -1$

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3b) Assume C & a are complex (a has unit magnitude

$$C = Ae^{j\phi} = A\angle\phi$$

$$a = e^{j\Omega_0}$$

$$x[n] = Ca^n = Ae^{j\phi}e^{jn\Omega_0}$$
$$= Ae^{j(\phi+n\Omega_0)}$$
$$= A\cos(\phi+n\Omega_0) + jA\sin(\phi+n\Omega_0)$$

~~"seem periodic" but is it?~~

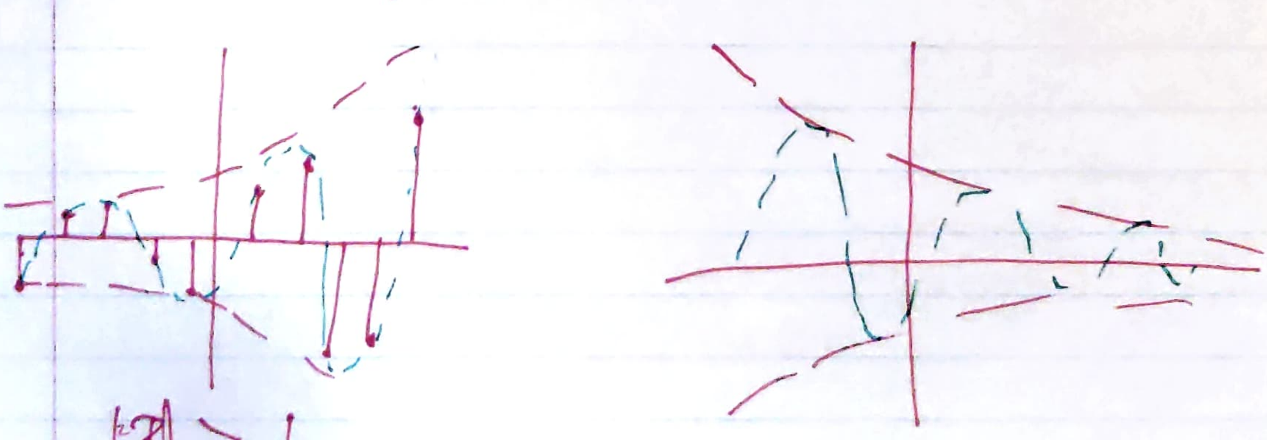
3c) C & a complex:

$$C = Ae^{j\phi} \quad \& \quad a = Be^{j\Omega_0}$$

$$\Rightarrow x[n] = Ca^n = Ae^{j\phi}\cdot B^n e^{nj\Omega_0}$$
$$= A\cdot B^n e^{j(\phi+n\Omega_0)}$$

$$\Rightarrow A\cdot B^n\cos(\phi+n\Omega_0) + jA\cdot B^n\sin(\phi+n\Omega_0)$$

Plot real part



$|B| > 1$ $\quad\quad$ $|B| < 1$

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(4) Sinusoidal signal

consider : $x[n] = \cos(n\Omega_0)$ → frequency

consider $\tilde{x}[n] = \cos(n(\Omega_0 + 2\pi)) = \cos(n\Omega_0 + n2\pi)$

$= \cos(n\Omega_0)\underbrace{\cos(n2\pi)}_{1} - \sin(n\Omega_0)\underbrace{\sin(n2\pi)}_{0}$

$\therefore \quad x[n] = \tilde{x}[n]$

we only need to consider frequencies $-\pi < \Omega_0 < \pi$

for $x[n]$ to be periodic $\exists$ an $N \in \mathbb{N}$ such that

$$x[n] = x[n+N]$$

$\Rightarrow \quad \cos(n\Omega_0) = \cos((n+N)\Omega_0) = \cos(n\Omega_0 + N\Omega_0)$

$= \cos(n\Omega_0)\cos(N\Omega_0) - \sin(n\Omega_0)\sin(N\Omega_0)$

$\Rightarrow$ we need $\cos(N\Omega_0) = 1 \Rightarrow N\Omega_0 = 2\pi k$

$\Downarrow$

$\sin(N\Omega_0) = 0 \Rightarrow N\Omega_0 = \pi m$

for periodicity we need $N\Omega_0 = 2\pi k$

$\Rightarrow \underbrace{\frac{\Omega_0}{2\pi}}_{\text{rational}} = \frac{k}{N}$

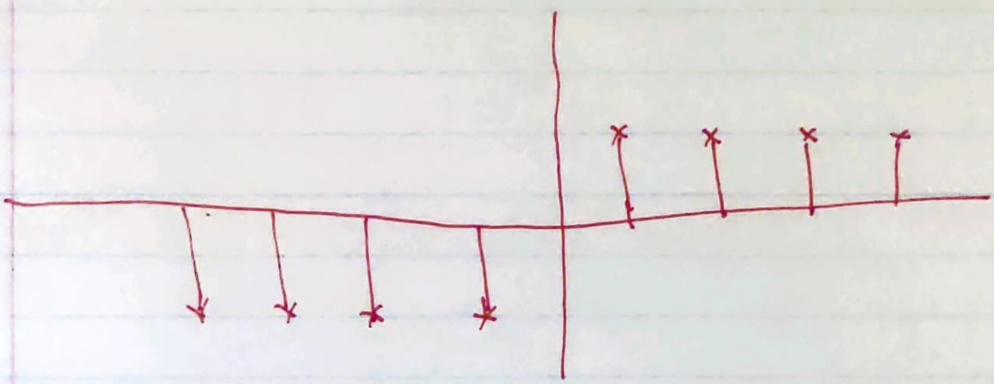
SIGNAL PROPERTIES

① Periodic $x[n]$ is periodic w̄ period N iff

$$x[n] = x[n+N]$$

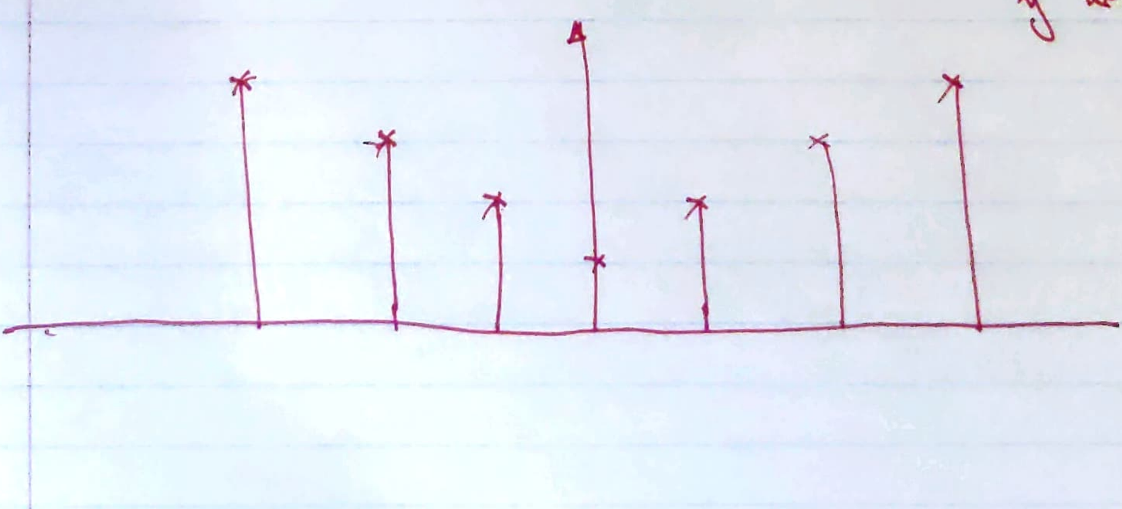
② Odd : $x_o[n]$ is said to be odd iff

$$x_o[n] = -x_o[-n]$$ ~~odd symmetry~~ (rotational symmetry)



③ Even : $x_e[n]$ is said to be even iff

$$x_e[n] = x_e[-n]$$ (symmetry about the y axis)



(9)

Any signal $x[n]$ can be expressed as the sum of an even & odd signal

PROOF,

consider these 2 signals :

$$x_e[n] = \frac{x[n] + x[-n]}{2} \quad ; \quad x_o[n] = \frac{x[n] - x[-n]}{2}$$

→ CLAIM : $x_e[n]$ is even !

$$x_e[-n] = \frac{x[-n] + x[-(-n)]}{2} \Rightarrow \frac{x[-n] + x[n]}{2} = x_e[n]$$

$\therefore$ $x_e[n]$ is even

→ CLAIM : $x_o[n]$ is odd

$$x_o[-n] = \frac{x[-n] - x[-(-n)]}{2} = \frac{x[-n] - x[n]}{2}$$

$$= -\frac{(x[n] - x[-n])}{2} = -x_o[n]$$

$\therefore$ $x_o[n]$ is odd

$$x_e[n] + x_o[n] = \frac{x[n] + x[-n]}{2} + \frac{x[n] - x[-n]}{2}$$

$$= x[n] \quad \text{Q.E.D}$$

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~~Power~~ Energy

if $x[n]$ is voltage then E is the ~~power~~ energy dissipated by a 1Ω - resistor

for signal $x[n]$ the energy is defined as

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

energy signal have a coherent idea of norm

* $x[n]$ is called an energy signal if $E < \infty$ (square summable)

* The unit step function is not an energy signal.
* any periodic signal
* Set of all energy signals is a Hilbert space with h_2 norm.

Power

$$P = \lim_{N \to \infty} \frac{1}{2N} \sum_{-N}^{N-1} |x[n]|^2$$

* non-decaying exponential signals are not power signals