1.

FIR FILTER DESIGN NOTES (WINDOWING METHOD)

~~→ start with a desired frequency response~~

~~$H_d(\omega)$ ↳ continuous time~~ ~~$H_d(\Omega)$ ↳ discrete time~~

~~→ our desired filter impulse response~~

$$\cancel{h_d[n] = \sum_{n=-\infty}^{\infty}}$$

→ start with some desired frequency response $H_a(j\omega)$

→ recall the impulse invariance ~~with~~ condition

$$h[n] = T_s\, h_a(nT_s) \quad [\,T_s \rightarrow \text{sampling period}\,]$$

$$h_a(t) = \underbrace{\frac{1}{2\pi}\int H_a(j\omega)\cdot e^{j\omega t}\, d\omega}_{\text{inverse fourier transform.}}$$

$$\Rightarrow \cancel{h_a(t)}\; h[n] = \frac{T_s}{2\pi}\int H_a(j\omega)\cdot e^{j\omega n T_s}\, d\omega$$

all $H_a(j\omega)$ are band limited $\Longrightarrow$ $h[n]$ is not time limited $\Longrightarrow$ $h[n]$ is not causal.

→

→ the solution to this is to truncate $h[n]$ by some function $w[n]$ (window function)

$$\cancel{h_{final}[n] = h_d[n]\, w[n]}$$

$$h_{Trunc}[n] = h[n]\, w[n]$$

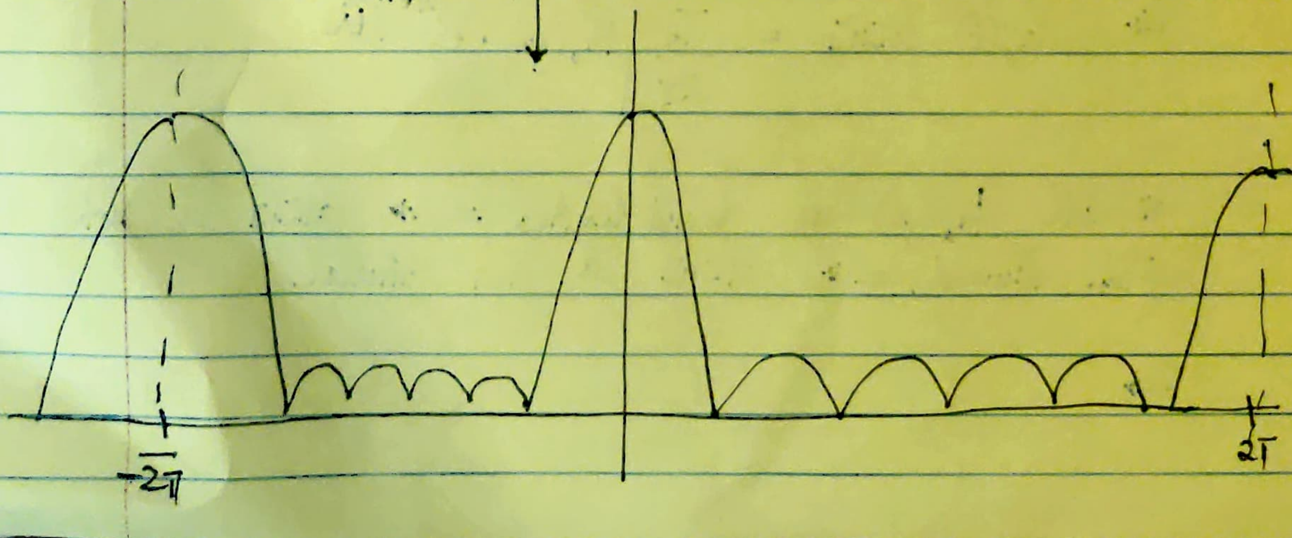
$$H_{Trunc}(j\omega) \rightarrow H_d(j\omega) * W(j\omega)$$

let $w[n] = \begin{cases} 1, & 0 \le n < M \\ 0, & \text{otherwise} \end{cases}$

$$w[n] = \sum_{0}^{M} \delta(t - nT_s) \qquad \text{set } T_s = 1$$

$$= \sum_{0}^{M} e^{-j\omega n} = \frac{1 - e^{j\omega(M+1)}}{1 - e^{-j\omega}}$$

geometric series



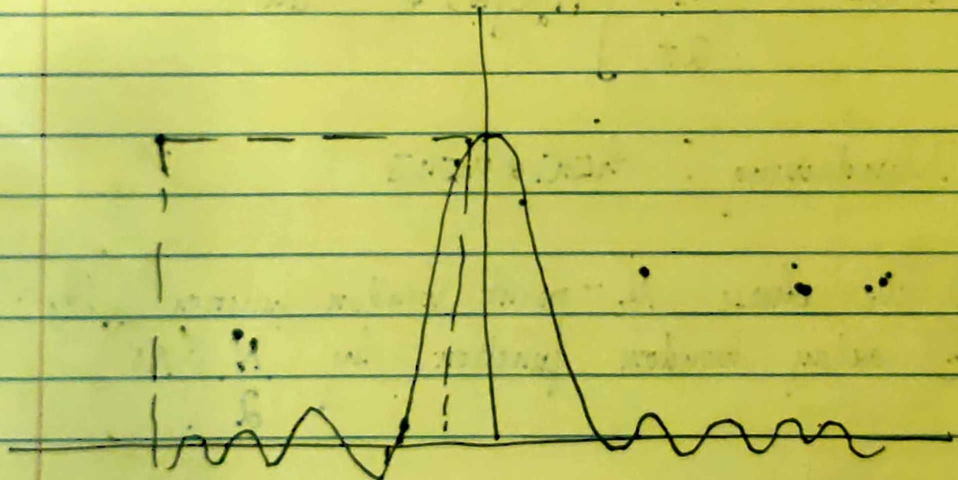
②

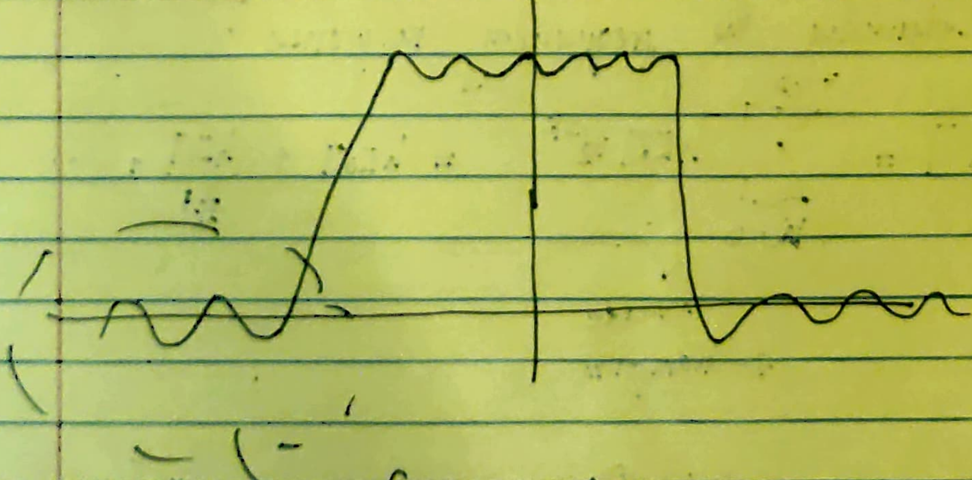
→ if $w[n]$ was 1 from $-\infty$ to ∞ i.e. no truncation we would have $W(j\omega)$ as an impulse train.

we want $w[n]$ to be short (efficient computation)
we want $W(j\omega)$ to be as close to an impulse train

→ conflicting requirements.

→ observe the graphical convolution





→ Gibbs phenomenon.

→ to reduce the effect of the side lobes we ~~want~~ a smooth funcation.

→ so all windowing function are way of implementing this.

PROCEDURE (N - order)

→ ① Determine the discrete filter impulse response h[n]

$$h[n] = \frac{T_s}{2\pi} \int H_a(j\omega)\, e^{j\omega n T}\, d\omega$$

② Windowing : $h[n] * w[n]$

ⓐ we choose N_o - point window function $(N_o = N + 1)$
ⓑ delay window function by $\frac{N}{2}$ (M)

③ Determine the frequency response :

$$H[z] = \underbrace{\sum_{K=0}^{\infty} h[K]\, z^{-K}}_{\text{inverse z-transform}} = h[0] + \frac{h[1]}{z^1} + \cdots \frac{h[N]}{z^N}$$

$$H(z)\Big|_{z = e^{j\omega T}} = H(e^{j\omega T}) = h[0] + h[1]e^{-j\omega T} + \cdots + h[N]e^{-j\omega N T}$$

$$= \sum_{K=0}^{N} h[K]\, e^{-jK\omega T} \longrightarrow$$

3.

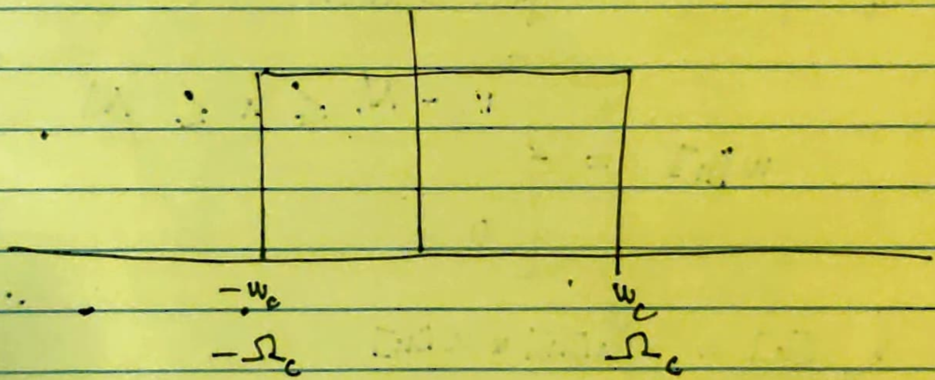
Example

- cut-off frequency ~~100kHz~~ ~~5~~ 20kHz
- filter order $N = 6$

(i) choose T_s (Nyquist) 40kHz

$$\omega_c \to 2\pi(20\,\text{kHz})$$

$$\Omega_c \to \omega_c T$$



$$H_a(\omega) = \text{rec}\left(\frac{\omega}{2\omega_c}\right) \quad \left[\text{rect}\left(\frac{t}{\tau}\right) \text{ pulse length } \tau, \text{ height } 1 \right]$$

$$h_a(t) = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} \text{rect}\left(\frac{\omega}{2\omega_c}\right) e^{j\omega t}\, d\omega$$ Fourier transform

$$\cancel{2w_c \operatorname{sinc}(w\ldots} \quad \frac{\cancel{w_0}}{\cancel{w_c}} \cdot \operatorname{sinc}(w_0 t)$$

$$h_a(t) \quad \cancel{\frac{\pi}{w_c}} \quad \frac{w_c}{\pi} \operatorname{sinc}(w_c \cdot t)$$

$$h[n] = T_s h_a(n T_s)$$

$$= T_s \cdot \frac{w_c}{\pi} \cdot \operatorname{sinc}(w_c \cdot n \cdot T_s)$$

N_0 - point rectangula window $(N_0 = N + 1)$

$$w[n] = \begin{cases} 1 & -M \leq n \leq M \quad \left(M = \frac{N}{2}\right) \\ 0 & \end{cases}$$

$$h_{Trunc}[n] = h[n] * w[n]$$

→ delay $h_{Trunc}[n]$ by $\frac{N_0 - 1}{2}$ units (causal)