ELEN3013: SIGNALS AND SYSTEMS IIB - Tutorial 4

1. A discrete filter with frequency response $H(\Omega)$ is to be designed to meet the following specifications:

$$\begin{aligned} 0.89125 \leq |H(\Omega)| \leq 1, &\qquad 0 \leq |\omega| \leq 0.2\pi \\ |H(\Omega)| \leq 0.17783, &\qquad 0.3\pi \leq |\omega| \leq \pi \end{aligned}$$

   (a) Design by applying the bilinear transformation to an appropriate continuous time Butterworth filter.

   (b) Using the same continuous time Butterworth filter designed above, use the window method to design a 5^{th} order FIR filter to meet the specifications.(use the Hamming window).

2. Consider a continuous time integrator with system function

$$H_c(s) = \frac{1}{s}$$

   Suppose a discrete-time system is obtained by applying the bilinear transformation to $H_c(s)$.

   (a) What is the system response of the discrete system $H_d(z)$. What is the impulse response $h[n]$?

   (b) Write the difference equation of this system.

   (c) Sketch the magnitude and phase of the discrete time system for $0 \leq \Omega \leq \pi$. Under what conditions could the discrete-time "integrator"be considered a good approximation to the continuous-time integrator?

3. Consider a continuous time differentiator with system function

$$G_c(s) = s$$

   Suppose a discrete-time system is obtained by applying the bilinear transformation to $G_c(s)$.

   (a) What is the system response of the discrete system $G_d(z)$. What is the impulse response $h[n]$?

   (b) Write the difference equation of this system.

   (c) Sketch the magnitude and phase of the discrete time system for $0 \leq \Omega \leq \pi$. Under what conditions could the discrete-time "differentiator"be considered a good approximation to the continuous-time differentiator?

   (d) The continuous-time integrator and differentiator are inverses of each other, is this true for the discrete-time counterparts?

4. Design a digital high-pass filter Butterworth filter using the bilinear transformation to satisfy the following specifications:

   - Passband $\omega \geq 150\pi rad/s$.
   - Stopband $\omega \leq 100\pi rad/s$.
   - Passband gain of at least -2dB.
   - Stopband gain of at most -28dB

5. Design a digital band-pass filter Butterworth filter using the bilinear transformation to satisfy the following specifications:

$$\begin{aligned} |H(\omega)| \leq -12dB, &\qquad \omega \leq 45rad/s \\ |H(\omega)| \leq -12dB, &\qquad \omega \geq 450rad/s \\ |H(\omega)| \geq -2dB, &\qquad 120rad/s \leq \omega \leq 300rad/s. \end{aligned}$$

6. Design a digital band-stop filter Butterworth filter using the bilinear transformation to satisfy the following specifications:

$$\begin{aligned} |H(\omega)| &\leq -22dB, && 80rad/s \leq \omega \leq 120rad/s \\ |H(\omega)| &\geq -1dB, && \omega \leq 40rad/s \\ |H(\omega)| &\geq -1dB, && \omega \geq 195rad/s. \end{aligned}$$