9:00

3 hrs	03 / 11 / 2023	FNB BASEMENT Annex	Exams Office Use Only

University of the Witwatersrand, Johannesburg

Course or topic No(s)	ELEN3013A
Course or topic name(s) Paper Number & title	Signals and Systems IIB
Examination/Test* to be held during month(s) of (*delete as applicable)	November 2023
Year of Study (Art & Sciences leave blank)	Third
Degrees/Diplomas for which this course is prescribed (BSc (Eng) should indicate which branch)	B.Sc (Eng) Elec.
Faculty/ies presenting candidates	Engineering
Internal examiners and telephone number(s)	Dr T. Chingozha
External examiner(s)	Dr A. Alonge
Special materials required (graph/music/drawing paper) maps, diagrams, tables, computer cards, etc)	None
Time allowance	Course Nos: ELEN3013A — Hours: 3
Instructions to candidates (Examiners may wish to use this space to indicate, inter alia, the contribution made by this examination or test towards the year mark, if appropriate)	Answer *ALL* questions and show all working. Type '2' Examination. Total marks: 110 - Full marks: 100

Internal Examiners or Heads of Department are requested to sign the declaration overleaf

Question 1

Consider a discrete time system with the input-output relation:

$$y[n] = T\{x[n]\} = x^2[n].$$

(a) Determine whether the system is linear.

(4 marks)

(b) Determine whether the system is time-invariant.

(4 marks)

(Total 8 marks)

Question 2

Consider the linear time-invariant system

$$y[n] = ax[n-1] + (1-a)x[n]$$

assuming all initial conditions are 0.

(a) Find the system impulse response function $h[n]$.

(4 marks)

(b) Find the impulse response of the system needed to recover $x[n]$ from $y[n]$.

(11 marks)

(Total 15 marks)

Question 3

A linear time-invariant system has response $y[n]$ to input signal $x[n]$ which are defined as follows.

$$\begin{aligned} x[n] &= \left(\frac{1}{16}\right)^n u[n] \\ y[n] &= \left(\frac{1}{4}\right)^n u[n] + \left(\frac{1}{8}\right)^n u[n]. \end{aligned}$$

(a) Determine the system transfer function $H(z)$.

(8 marks)

(b) Determine the system impulse response $h[n]$.

(8 marks)

(c) Determine the difference equation.

(4 marks)

(Total 20 marks)

Question 4

Find the inverse z-transform for

$$X(z) = \frac{z^2 + 2}{(z - 0.3)(z + 0.4)^2}.$$

(Total 15 marks)

Question 5

A discrete time low-pass filter with frequency response $H(\Omega)$ is to be designed to meet the following specifications:

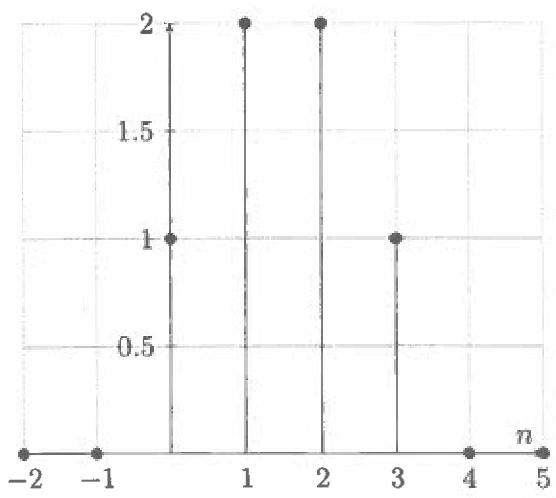
$$\begin{aligned} 0.8 < \quad & |H(\Omega)| < 1.2 \quad 0 \leq |\Omega| \leq 0.2\pi \\ & |H(\Omega)| < 0.2 \quad 0.8\pi \leq |\Omega| \leq \pi \end{aligned}$$

Design by applying the bilinear transformation to an appropriate continuous time Butterworth lowpass filter.

(Total 28 marks)

Question 6

Consider the signal depicted below



(a) Draw the 4-point FFT signal flow diagram.

(12 marks)

(b) Use this to calculate the DFT.

(12 marks)

(Total 24 marks)

Trigonometric relationships

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\tan(A \pm B) &= \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\end{aligned}$$

Integrals

$$\int \frac{1}{x^2 + a^2}\, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

Integration by parts

$$\int u\, dv = uv - \int v\, du$$

Determinants

$$\det(\mathbf{AB}) = \det(\mathbf{A})\det(\mathbf{B})$$

Geometric series

$$\begin{aligned}
\sum_{i=0}^{\infty} ar^i &= \frac{a}{1-r} && -1 < r < 1 \\
\sum_{i=0}^{n-1} ar^i &= \frac{a(1-r^n)}{1-r} && r \neq 1
\end{aligned}$$

Exponential Fourier series

$$\begin{aligned}
x(t) &= \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} \\
c_k &= \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-jk\omega_0 t}\, dt
\end{aligned}$$

Parseval's theorem for energy signals

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2 \, d\omega$$

Normalized average power for periodic signals

$$P = \sum_{k=-\infty}^{\infty} |c_k|^2 = c_0^2 + 2\sum_{k=1}^{\infty} |c_k|^2$$

Laplace, Fourier and z-transforms

Notation: T = sampling period $\omega_s = 2\pi/T$

$E = e^{-\alpha T}$ $C = \cos(\beta T)$ $S = \sin(\beta T)$ $F = \alpha T - 1 + e^{-\alpha T}$ $G = 1 - e^{-\alpha T}(1 + \alpha T)$

$u(t)$ = unit step

$\mathrm{rect}(\frac{t}{\tau})$ = rectangular pulse length τ, height 1

$\mathrm{sinc}(x)$ = $\sin(x)/x$

$\mathrm{zoh}(\dots)$ = zero-order hold acting on samples

$\delta(t)$ = unit impulse

$\mathrm{tri}(\frac{t}{\tau})$ = triangular pulse of length τ, height 1

$\delta_T(t)$ = unit impulse train of period T

ω_o = a particular frequency

$x(t)$	$X(s)$	$X(\omega)$	$X(z)$
$x(t)$	$\int_0^\infty x(t)e^{-st}\,dt$	$\int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$	$\sum_{n=0}^{\infty} z^{-n}x(nT)$
$\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega$		$X(\omega)$	
$\frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st}\,ds$	$X(s)$		
$\delta(t)$	1	1	1
1		$2\pi\delta(\omega)$	
$\delta_T(t)$		$\omega_s\delta_{\omega_s}(\omega)$	
$u(t)$	$\frac{1}{s}$	$\pi\delta(\omega) + \frac{1}{j\omega}$	$\frac{z}{z-1}$
$\mathrm{sgn}(t)$		$\frac{2}{j\omega}$	
$tu(t)$	$\frac{1}{s^2}$		$\frac{Tz}{(z-1)^2}$
$\frac{t^2}{2}u(t)$	$\frac{1}{s^3}$		$\frac{T^2z(z+1)}{2(z-1)^3}$
$t^n u(t)$	$\frac{n!}{s^{n+1}}$		
$\mathrm{rect}(\frac{t}{\tau})$		$\tau\mathrm{sinc}\left(\frac{\omega\tau}{2}\right)$	
$\mathrm{sinc}\,(\omega_o t)$		$\frac{\pi}{\omega_o}\mathrm{rect}\left(\frac{\omega}{2\omega_o}\right)$	
$\mathrm{tri}(\frac{t}{\tau})$		$\frac{\tau}{2}\mathrm{sinc}^2\left(\frac{\omega\tau}{4}\right)$	
$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$	$\frac{1}{\alpha+j\omega}$	$\frac{z}{z-E}$
$te^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^2}$	$\frac{1}{(\alpha+j\omega)^2}$	$\frac{TEz}{(z-E)^2}$
$t^2e^{-\alpha t}u(t)$	$\frac{2}{(s+\alpha)^3}$	$\frac{2}{(\alpha+j\omega)^3}$	$\frac{T^2Ez(z+E)}{(z-E)^3}$
$t^{n-1}e^{-\alpha t}u(t)$	$\frac{(n-1)!}{(s+\alpha)^n}$	$\frac{(n-1)!}{(\alpha+j\omega)^n}$	
$e^{-\alpha\|t\|}$		$\frac{2\alpha}{\alpha^2+\omega^2}$	
$e^{-\alpha t^2}$		$\sqrt{\frac{\pi}{\alpha}}e^{-\frac{\omega^2}{4\alpha}}$	
$\frac{u(t)}{b-a}\left[e^{-at} - e^{-bt}\right]$	$\frac{1}{(s+a)(s+b)}$		
$\frac{u(t)}{b-a}\left[(c-a)e^{-at} - (c-b)e^{-bt}\right]$	$\frac{s+c}{(s+a)(s+b)}$		