

1. Find the z-transform of the following $x[n]$ stating the region of convergence:

- (a) $x[n] = \{\frac{1}{2}, 1, -\frac{1}{3}\}$
- (b) $x[n] = 2\delta[n+2] - 3\delta[n-2]$
- (c) $x[n] = 3(-\frac{1}{2})^2 u[n] - 2(3)^n h[-n-1]$

2. Find the z-transforms of the following $x[n]$:

- (a) $x[n] = (n-3)u[n-3]$
- (b) $x[n] = (n-3)u[n]$
- (c) $x[n] = u[n] - u[n-3]$
- (d) $x[n] = n\{u[n] - u[n-3]\}$

3. Using the relation

$$a^n u[n] \leftrightarrow \frac{z}{z-a}, \quad |z| > |a| \quad (1)$$

find the z-transform of the following $x[n]$:

- (a) $x[n] = na^{n-1}u[n]$
- (b) $x[n] = n(n-1)a^{n-2}u[n]$
- (c) $x[n] = n(n-1)\cdots(n-k+1)a^{n-k}u[n]$

4. Find the inverse z-transform of

$$X(z) = e^{\frac{a}{z}} \quad |z| > 0 \quad (2)$$

Hint : Use the power series expansion of the exponential function e^r

5. Find the inverse z-transform of the following $X(z)$:

- (a) $X(z) = \frac{a}{(z-1)(z-2)}, \quad |z| < 1$
- (b) $X(z) = \frac{a}{(z-1)(z-2)}, \quad 1 < |z| < 2$
- (c) $X(z) = \frac{a}{(z-1)(z-2)}, \quad |z| > 2$

6. Consider a discrete-time LTI system whose system function $H(z)$ is given by

$$H(z) = \frac{z}{z - \frac{1}{2}}, \quad |z| > \frac{1}{2} \quad (3)$$

- (a) Find the step response $s[n]$.
- (b) Find the output $y[n]$ to the input $x[n] = nu[n]$.

7. Consider a casual discrete-time system whose output $y[n]$ and input $x[n]$ are related by

$$y[n] - \frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] = x[n] \quad (4)$$

- (a) Find its system transfer function $H(z)$
- (b) Find its impulse response function $h[n]$